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CONTENTS

PAPERS AND DISCUSSIONS

(Alphabetical Indexes Begin on Page 1917)

No. PAGE

2236 RAINBOW ARCH BRIDGE OVER NIAGARA GORGE: A SYMPOSIUM 1

FOREWORD 2

DESIGN.
By Shortridge Hardesty, J. M. Garrelts, and I. G. Hedrick, Jr. 6

MOMENTS IN FLEXIBLE TWO-HINGED ARCHES.
By Elmer K. Timby, Lawrence M. Mead, Jr., and Robert McEldowney, Jr. 35

FABRICATION AND ERECTION CONTROLS.
By John I. Copp, Karl de Vries, William H. Jameson, and Jonathan Jones 50

ERECTION OF STEEL SUPERSTRUCTURE.
By E. L. Durkee 82

Discussion:
By Egidio O. Di Genova 120
Charles Mackintosh 121
C. M. Goodrich 123
T. Kennard Thomson 125
C. M. Spofford 127
L. J. Mensch 128
O. H. Ammann 135
Louis Balog 137
William G. Rapp 139
Leon Beskin 139
Neil Van Eenam 150
C. H. Gronquist 150
M. Hirschthal 159
George Stern 163
Shortridge Hardesty, J. M. Garrelts, and I. G. Hedrick, Jr. 167
Elmer K. Timby, Lawrence M. Mead, Jr., and Robert McEldowney, Jr. 176
John I. Copp, Karl de Vries, William H. Jameson, and Jonathan Jones 178

2237 DEWATERING, INCINERATION, AND USE OF SEWAGE SLUDGE: A SYMPOSIUM 179

FOREWORD 180

DEVELOPMENTS INDICATED BY EXPERIENCE AT BUFFALO, N. Y.
By C. R. Velzy 181

VACUUM FILTERS USED IN PRIMARY SLUDGE DEWATERING.
By Ralph E. Fuhrman 188

POSSIBLE ECONOMIES IN SLUDGE DISPOSAL PRACTICE.
By George J. Schroepfer 194

USE OF SEWAGE SLUDGE AS FERTILIZER.
By LeRoy W. Van Kleeck 208

EXPERIENCE OF CHICAGO, ILL., IN THE PREPARATION OF FERTILIZER.
By William A. Dundas and C. P. McLaughlin 218

BIBLIOGRAPHY.
By the Symposium Authors 241

No. PAGE

Discussion:
By A. L. GENTER 243
WILLEM RUDOLFS 251
LLOYD M. JOHNSON 254
ISADOR W. MENDELSOHN 255
LANGDON PEARSE 261
C. R. VELZY 263
GEORGE J. SCHROEPFER 264

2238 GEOLOGY IN HIGHWAY ENGINEERING.

By Marshall T. Huntting 271

Discussion:
By W. W. CROSBY 298
CARL B. BROWN 299
F. H. KELLOGG 300
BERLEN C. MONEYMAKER 302
E. F. BEAN 305
A. T. BLECK 305
LYMAN W. WOOD 306
PHILIP KEENE 306
JACOB FELD 307
H. E. MARSHALL 309
K. B. WOODS 312
D. J. BELCHER 316
R. WOODWARD MOORE 320
GEORGE E. EKBLAW 322
ALLEN S. CARY AND ADE E. JASKAR 323
KARL TERZAGHI 324
D. P. KRYNINE 328
ROBERT F. LEGGET 332
IRVING B. CROSBY 334
HYDE FORBES 337
A. A. EREMIN 341
MARSHALL T. HUNTTING 342

2239 SURGES IN PANAMA CANAL REPRODUCED IN MODEL.

By F. W. Edwards and Edward Soucek 345

Discussion:
By HAROLD A. WEGGEL 356
HARRY LEYPOLDT 356
C. J. POSEY 358
F. W. EDWARDS AND EDWARD SOUCEK 359

2240 ORIGINAL AND READJUSTED REPAYMENT CONTRACTS, BOULDER CANYON PROJECT.

By Clay C. Elder 363

Discussion:
By W. L. CHADWICK 384
A. R. ARLEDGE 388
CLAY C. ELDER 390

2241 SHEAR EFFECT ON THE STRENGTH OF STRUTS.

By S. Sergev 391

Discussion:
By LEON BESKIN 404
MARSHALL HOLT 408
DAVID B. HALL 410
S. SERGEV 413

No. PAGE

2242 ECONOMICAL CANAL CROSS SECTIONS.

By Victor L. Streeter 421

Discussion:

By V. A. Endersby 431

Julian Hinds 433

D. C. McConaughy 435

C. J. Posey 436

Victor L. Streeter 438

2243 RIGIDITY AND AERODYNAMIC STABILITY OF SUSPENSION BRIDGES.

By D. B. Steinman 439

Discussion:

By Edward Adams Richardson 476

R. K. Bernhard 485

J. M. Robertson 487

C. H. Gronquist 490

F. B. Farquharson 492

H. Reissner 501

Blair Birdsall 509

Hardy Cross 511

Frederick C. Smith, Hermann Friedlaender, and George S. Vincent 512

Louis Balog 522

D. B. Steinman 532

2244 ERECTION OF HYDRAULIC TURBINES AND GENERATORS.

By Andrew M. Komora 581

Discussion:

By Paul E. Gisiger 593

Frank I. Morgan 594

J. F. Roberts 595

D. E. Brainard 596

Andrew M. Komora 598

2245 SHEAR AND BOND STRESSES IN REINFORCED CONCRETE.

By Stanley U. Benscoter and Samuel T. Logan 599

Discussion:

By F. R. Shanley 607

B. J. Aleck 607

Dean Peabody, Jr. 610

William E. Wilbur 612

L. E. Grinter 614

Anders Bull 616

Victor R. Bergman 619

Phil M. Ferguson 622

Stanley U. Benscoter and Samuel T. Logan 627

2246 COEFFICIENTS FOR VELOCITY DISTRIBUTION IN OPEN-CHANNEL FLOW.

By William S. Eisenlohr, Jr. 633

Discussion:

By A. A. Kalinske 645

Edward H. Taylor 646

E. R. Van Driest 648

Hunter Rouse and John S. McNown 651

William S. Eisenlohr, Jr. 657

2247 MILITARY AIRFIELDS: A SYMPOSIUM 669

CONSTRUCTION AND DESIGN PROBLEMS.

By James H. Stratton 670

DESIGN OF DRAINAGE FACILITIES.

By Gail A. Hathaway 697

No. PAGE

BIBLIOGRAPHY.

By the Symposium Authors 730

Discussion:

By James B. Newman, Jr. 734
Thomas E. Stanton 737
W. E. Howland 738
David S. Jenkins 743
William E. Rudolph 751
Raymond L. Irwin 752
G. G. Greulich 758
Hibbert Hill 758
Jacob Feld 763
Robert Horonjeff 766
Clarence S. Jarvis 768
D. D. McGuire 776
E. D. Parmer 776
A. Casagrande 780
Harry E. Cotton 784
D. O. Murray 790
Thomas B. Pringle 792
John R. Haswell 796
W. J. Turnbull and William H. Jervis 797
Ralph R. Proctor 799
D. M. Burmister 809
Ralph Hansen 812
H. V. Pittman 814
G. R. Schneider 818
H. M. Williams 820
Ira B. Mullis 826
C. F. Izzard 827
L. J. Mensch 831
Hibbert Hill 836
Gail A. Hathaway 845

2248 FLOOD FORMULAS BASED ON DRAINAGE BASIN CHARACTERISTICS.

By H. B. Kinnison and B. R. Colby 849

Discussion:

By Clarence S. Jarvis 877
L. R. Beard 882
M. J. Ord 886
Howard M. Turner 888
Byron O. McCoy 891
C. O. Clark 892
H. B. Kinnison and B. R. Colby 900

2249 ANALYSIS OF THE GENERAL TWO-DIMENSIONAL FRAMEWORK.

By Yves Nubar 905

Discussion:

By Leon Beskin 933
Jaroslav Polivka 940
A. A. Eremin 943
Yves Nubar 944

2250 UNUSUAL CUTOFF PROBLEMS—DAMS OF THE TENNESSEE VALLEY AUTHORITY: A SYMPOSIUM 947

DEEP SOLUTION CHANNEL, KENTUCKY DAM, KENTUCKY.

By James B. Hays 948

FLOWING WATER IN UNDERGROUND CHANNELS, HALES BAR DAM, TENNESSEE.

By L. A. Schmidt, Jr. 965

No. PAGE
Discussion:
By BERLEN C. MONEYMAKER 995
PORTLAND P. FOX 997
J. K. BLACK 1003
L. C. GLENN 1004
E. H. BURROUGHS 1005
JAMES B. HAYS 1015
L. A. SCHMIDT, JR. 1016

2251 DESIGN CONSTANTS FOR BEAMS WITH NONSYMMETRICAL STRAIGHT HAUNCHES.
By **August L. Ahlf** 1019
Discussion:
By ROBERT O. JAMESON 1037
JOHN SHERMAN 1041
DALE H. REA 1042
I. B. HOSIG 1044
AUGUST L. AHLF 1045

2252 PRESSURE ON THE LINING OF CIRCULAR TUNNELS IN PLASTIC SOILS.
By **D. P. Krynine** 1048
Discussion:
By M. A. DRUCKER 1066
JOHN M. BIRD 1068
D. P. KRYNINE 1075

2253 STABILITY AND STIFFNESS OF CELLULAR COFFERDAMS.
By **Karl Terzaghi** 1083
Discussion:
By G. G. GREULICH 1120
RAYMOND P. PENNOYER 1124
DEAN P. TSAGARIS 1133
R. T. COLBURN 1136
A. F. HEDMAN 1145
ADOLPH J. ACKERMAN 1155
SAVILE PACKSHAW 1161
MARTIN W. LAUTZ 1165
J. S. MILLER 1170
ROBERT F. LEGGET 1173
D. P. KRYNINE 1175
J. J. POLIVKA 1178
KARL TERZAGHI 1187

2254 TRANSLATORY WAVES IN NATURAL CHANNELS.
By **J. H. Wilkinson** 1203
Discussion:
By HAROLD A. WEGGEL 1226
W. M. LANSFORD 1227
M. A. CHURCHILL 1229
J. H. WILKINSON 1235

2255 CONSTRUCTION OF THE FLOW NET FOR HYDRAULIC DESIGN.
By **H. Alden Foster** 1237
Discussion:
By J. W. HOWE 1253
CLARENCE S. JARVIS 1254
H. ALDEN FOSTER 1255

No. PAGE

2256 CHARACTERISTIC REDUNDANTS USED FOR ANALYZING STATICALLY INDETERMINATE STRUCTURES.

By John B. Wilbur 1257

Discussion:

By Charles A. Ellis 1280
Ralph W. Stewart 1282
Myle J. Holley, Jr. 1283
M. W. Huggins 1284
Leon Beskin 1284
Frederick S. Merritt 1291
John B. Wilbur 1293

2257 HYDRAULIC MODEL INVESTIGATION OF MITER GATE OPERATION.

By Maurice N. Amster 1297

Discussion:

By Edward Soucek 1314
Isaac DeYoung 1316
F. W. Edwards 1319
William A. Tripp 1320
Maurice N. Amster 1322

2258 SIMULTANEOUS CONSOLIDATION OF CONTIGUOUS LAYERS OF UNLIKE COMPRESSIBLE SOILS.

By Hamilton Gray 1327

Discussion:

By Edward S. Barber 1345
Jacob Feld 1349
George W. Glick 1351
Hamilton Gray 1352

2259 LOCK MANIFOLD EXPERIMENTS.

By Edward Soucek and E. W. Zelnick 1357

Discussion:

By J. Pietrkowski 1377
John S. McNown 1378
F. W. Edwards 1385
W. M. Lansford 1388
Robert F. Kreiss 1389
Ralph G. Berk 1393
Ralph W. Powell 1394
Edward Soucek and E. W. Zelnick 1396

2260 AMPLIFIED SLOPE DEFLECTION.

By Ralph W. Stewart 1401

Discussion:

By Leon Beskin 1408
John E. Goldberg 1410
Camillo Weiss 1411
William C. Spiker 1411
Arthur B. McGee 1412
Jaroslav Polivka 1414
A. A. Eremin 1415
Ralph W. Stewart 1417

No. PAGE

2261 STORAGE AND THE UNIT HYDROGRAPH.
By C. O. Clark 1419
Discussion:
By James S. Sweet 1447
Otto H. Meyer 1448
L. K. Sherman 1449
Gordon R. Williams 1451
Ray K. Linsley, Jr. 1452
E. F. Brater 1455
L. C. Crawford 1456
Robert E. Kennedy 1459
Victor H. Cochrane 1461
Franklin F. Snyder 1465
Howard M. Turner 1471
Don Johnstone 1474
C. O. Clark 1477

2262 ANALYSIS OF RIGID FRAMES BY SUPERPOSITION.
By David M. Wilson 1489
Discussion:
By F. S. Merritt 1509
Ralph W. Stewart 1510
John E. Goldberg 1512
A. W. Fischer 1514
Leon Blog 1520
Arthur B. McGee 1522
Jaroslav Polivka 1529
Odd Albert 1533
W. C. Spiker 1539
David B. Hall 1540
A. A. Eremin 1542
Victor R. Bergman 1544
David M. Wilson 1545

2263 CONCENTRATION OF SEWAGE SLUDGE: A SYMPOSIUM 1557
EXPERIENCE IN LOS ANGELES, CALIF.
By R. F. Goudey 1558
BACK RIVER SEWAGE WORKS, BALTIMORE, MD.
By C. E. Keefer 1562
SOME RESULTS IN THE STATE OF NEW JERSEY.
By Willem Rudolfs 1565
CONCENTRATION OF SEWAGE SLUDGES, NEW YORK, N. Y.
By Wellington Donaldson 1572
Discussion:
By C. R. Velzy 1579
R. F. Goudey 1580

2264 WEDGE-SHAPED STRUCTURAL MEMBERS UNDER DIRECT STRESS AND BENDING.
By Robert B. B. Moorman 1581
Discussion:
By Phil M. Ferguson 1591
Stanley U. Benscoter 1593
George Winter 1595
Walter H. Wheeler 1599
Leon Beskin 1599
Robert B. B. Moorman 1602

2265 PRIVATE ENTERPRISE: ANNUAL PRESIDENTIAL ADDRESS.
By J. C. Stevens 1605

MEMOIRS OF DECEASED MEMBERS

Page
John Francis Coleman, Past-President and Hon. M. Am. Soc. C. E. 1608
Alonzo John Hammond, Past-President and Hon. M. Am. Soc. C. E. 1615
Mortimer Elwyn Cooley, Hon. M. Am. Soc. C. E. 1619
John Edwin Greiner, Hon. M. Am. Soc. C. E. 1625
Henry Matson Waite, Hon. M. Am. Soc. C. E. 1631
Edward Everett Wall, Hon. M. Am. Soc. C. E. 1635
Jeremiah Ahern, M. Am. Soc. C. E. 1638
James Pennell Anderson, M. Am. Soc. C. E. 1641
Edgar Alcander Bayley, M. Am. Soc. C. E. 1643
Paul Leonard Bean, M. Am. Soc. C. E. 1644
James Wallace Beardsley, M. Am. Soc. C. E. 1647
Ralph Norman Begien, M. Am. Soc. C. E. 1649
William Ethelbert Belknap, M. Am. Soc. C. E. 1653
Edward Franklin Berry, M. Am. Soc. C. E. 1655
Paul Alexander Blackwell, M. Am. Soc. C. E. 1657
Clarence Moore Blair, M. Am. Soc. C. E. 1658
Eric Edwin Blomgren, M. Am. Soc. C. E. 1660
Sherman Worcester Bowen, M. Am. Soc. C. E. 1661
Sam Wigfall Bradshaw, M. Am. Soc. C. E. 1663
Benjamin Edward Briggs, M. Am. Soc. C. E. 1664
Orrin Lawrence Brodie, M. Am. Soc. C. E. 1665
Albert Marshall Brosius, M. Am. Soc. C. E. 1667
Frederick Charles Carstarphen, M. Am. Soc. C. E. 1668
Charles Robert Foran Coutlee, M. Am. Soc. C. E. 1670
James Cowin, M. Am. Soc. C. E. 1672
Leonard Martin Cox, M. Am. Soc. C. E. 1673
Clare Harmon Currie, M. Am. Soc. C. E. 1677
Edouard Jean Bernard de Mey, M. Am. Soc. C. E. 1679
Herman Francois Doeleman, M. Am. Soc. C. E. 1680
Thomas Linus Keith Donnelly, M. Am. Soc. C. E. 1682
Edward Brown Donohue, M. Am. Soc. C. E. 1683
Max Harry Doyne, M. Am. Soc. C. E. 1686
Samuel Morrison Ellsworth, M. Am. Soc. C. E. 1687
Frank Harvey Eno, M. Am. Soc. C. E. 1690
Frederic Harold Fay, M. Am. Soc. C. E. 1691
Edwin Stanton Fickes, M. Am. Soc. C. E. 1695
Harold Chamberlayne Fiske, M. Am. Soc. C. E. 1699
Charles Broe Galvin, M. Am. Soc. C. E. 1705
James Madison Garrett, M. Am. Soc. C. E. 1706
William Gerig, M. Am. Soc. C. E. 1708
Herbert Thurston Gerrish, M. Am. Soc. C. E. 1711
Leslie Warren Goddard, M. Am. Soc. C. E. 1712
Paul Hansen, M. Am. Soc. C. E. 1713
Clinton Lee Harris, M. Am. Soc. C. E. 1715
Elmo Golightly Harris, M. Am. Soc. C. E. 1717
Berthold Francis Hastings, M. Am. Soc. C. E. 1720
Emile Henry Heilbron, M. Am. Soc. C. E. 1722
George Hewitt, M. Am. Soc. C. E. 1724
George Higgins, M. Am. Soc. C. E. 1725
Fred Keating Hilt, M. Am. Soc. C. E. 1728
John Benjamin Hittell, M. Am. Soc. C. E. 1730
Elmer Thomas Howson, M. Am. Soc. C. E. 1731
Harold Walton Hudson, M. Am. Soc. C. E. 1735
Martin Joachimson, M. Am. Soc. C. E. 1737
Frank Minitree Johnson, M. Am. Soc. C. E. 1738
Frank Rhymal Judd, M. Am. Soc. C. E. 1741
Julius Kahn, M. Am. Soc. C. E. 1742
Winfield Scott King, M. Am. Soc. C. E. 1748
Walter Harold Kirkbride, M. Am. Soc. C. E. 1748
Frederic Arnold Kummer, M. Am. Soc C. E. 1751

Page
Edward Rowland Lewis, M. Am. Soc. C. E. 1755
Frederick Charles Lohmann, M. Am. Soc. C. E. 1757
Moses Jerome Look, M. Am. Soc. C. E. 1758
John Herbert McManus, M. Am. Soc. C. E. 1760
David White McNaugher, M. Am. Soc. C. E. 1762
Charles Sterling Millard, M. Am. Soc. C. E. 1763
Egbert Jessup Moore, M. Am. Soc. C. E. 1764
John Patrick Myron, M. Am. Soc. C. E. 1766
David Leonard Neuman, M. Am. Soc. C. E. 1768
Joseph Pettis Newell, M. Am. Soc. C. E. 1770
Walter Henry Norris, M. Am. Soc. C. E. 1775
Clifford Older, M. Am. Soc. C. E. 1777
David Kirk Orr, M. Am. Soc. C. E. 1778
George Alexander Orrok, M. Am. Soc. C. E. 1779
Arthur Edmund Owen, M. Am. Soc. C. E. 1783
Archibald E. Palen, M. Am. Soc. C. E. 1785
Theodore Bissell Parker, M. Am. Soc. C. E. 1786
George Wellington Pickels, M. Am. Soc. C. E. 1790
John Richards Gray Pill, M. Am. Soc. C. E. 1792
James Charles Pinney, M. Am. Soc. C. E. 1794
Fred Ashley Pneuman, M. Am. Soc. C. E. 1795
Corydon Tyler Purdy, M. Am. Soc. C. E. 1797
Ira Lawrence Pyle, M. Am. Soc. C. E. 1801
Milton Allender Pyle, M. Am. Soc. C. E. 1802
Alfred Merritt Quick, M. Am. Soc. C. E. 1803
Aaron Israel Raisman, M. Am. Soc. C. E. 1806
Frederick Hosea Richardson, M. Am. Soc. C. E. 1807
Allan Townshend Ricketts, M. Am. Soc. C. E. 1810
Franklin Henry Robbins, M. Am. Soc. C. E. 1811
John McClure Salmon, M. Am. Soc. C. E. 1813
George Arthur Seaver, M. Am. Soc. C. E. 1814
Charles Frank Seifried, M. Am. Soc. C. E. 1815
Gilman Walter Smith, M. Am. Soc. C. E. 1818
Howard Everett Smith, M. Am. Soc. C. E. 1820
George Francis Sparhawk, M. Am. Soc. C. E. 1821
Allen Whitmore Stephens, M. Am. Soc. C. E. 1823
Charles Henry Stevens, M. Am. Soc. C. E. 1824
Willard Wilberforce Stone, M. Am. Soc. C. E. 1831
Merton Rogers Sumner, M. Am. Soc. C. E. 1835
John Calvin Sweeney, M. Am. Soc. C. E. 1836
Charles Mason Talbert, M. Am. Soc. C. E. 1840
Peter Gansevoort Ten Eyck, M. Am. Soc. C. E. 1842
Frank Ellsworth Trask, M. Am. Soc. C. E. 1843
Harry Edward Tyrrell, M. Am. Soc. C. E. 1845
Thomas Utegaard, M. Am. Soc. C. E. 1846
Jeptha Alan Wade, M. Am. Soc. C. E. 1850
Joseph Palmer Wadhams, M. Am. Soc. C. E. 1852
Elton David Walker, M. Am. Soc. C. E. 1853
Joseph Weidel, M. Am. Soc. C. E. 1855
William Henry Wetzler, M. Am. Soc. C. E. 1856
Lindsey Wood Whitehead, M. Am. Soc. C. E. 1858
Julius Edgar Willoughby, M. Am. Soc. C. E. 1859
Donald Witten, M. Am. Soc. C. E. 1861
Joseph Henry Young, M. Am. Soc. C. E. 1863
Elisha Brown Baker, Assoc. M. Am. Soc. C. E. 1864
Ernest Daniel Bean, Assoc. M. Am. Soc. C. E. 1865
Frank Joseph Boyle, Assoc. M. Am. Soc. C. E. 1866
Thomas Byrnes Brogan, Assoc. M. Am. Soc. C. E. 1867
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David Wilson Crofoot, Assoc. M. Am. Soc. C. E. 1871
Luis Agustin Deliz, Assoc. M. Am. Soc. C. E. 1872

Page

Carl E. Downing, Assoc. M. Am. Soc. C. E. 1873
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ERRATA

Transactions, Vol. 109 (1944)

Page 424, change the caption of Fig. 25 to read "Nature of the Deflection of a Large Uniformly Loaded Flexible Foundation Mat Resting on Sand: (*a*) As Suggested by the Authors; and (*b*) As Believed To Be More Likely."

PAPERS AND REPORTS OF COMMITTEES PUBLISHED IN PROCEEDINGS, AM. SOC. C. E., JUNE, 1944, TO SEPTEMBER, 1945, INCLUSIVE, NOT PUBLISHED IN TRANSACTIONS, VOL. 110

	Number of *Proceedings*.
TRANSPORTATION OF SUSPENDED SEDIMENT BY WATER. By Vito A. Vanoni	June, 1944
EVAPORATION FROM A FREE WATER SURFACE. By G. H. Hickox	October, 1944
STRESSES IN THE LININGS OF SHIELD-DRIVEN TUNNELS. By Anders Bull	November, 1944
ANALYSIS OF STATICALLY INDETERMINATE STRUCTURES USING REDUCED EQUATIONS. By Lee H. Johnson, Jr.	November, 1944
PRINCIPLES OF DEPRECIATION: FINAL REPORT OF THE SPECIAL COMMITTEE AUTHORIZED BY THE BOARD OF DIRECTION TO ANALYZE AND DISCUSS THE 1943 REPORT OF THE NATIONAL ASSOCIATION OF RAILROAD AND UTILITIES COMMISSIONERS' COMMITTEE ON DEPRECIATION	November, 1944
FLOW IN A CHANNEL OF DEFINITE ROUGHNESS. By Ralph W. Powell	December, 1944
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COLUMN FORMULAS. By William R. Osgood	December, 1944
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Number of *Proceedings.*

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By Isaac DeYoung . . . . February, 1945

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By Harry Hayes . . . . March, 1945

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By Thomas R. Camp . . . . April, 1945

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By R. C. Brumfield . . . . May, 1945

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By John Oran Eichler and Howard W. Eves . . . . May, 1945

WATER SUPPLY ENGINEERING: REPORT OF THE COMMITTEE OF THE SANITARY ENGINEERING DIVISION FOR THE TWO YEARS ENDING DECEMBER 31, 1944 . . . . May, 1945

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By Milan A. Johnston . . . . June, 1945

STRUCTURAL SKEW PLATES.
By F. L. Ehasz . . . . June, 1945

CORRELATING FLOOD CONTROL AND WATER SUPPLY, LOS ANGELES COASTAL PLAIN.
By Finley B. Laverty . . . . June, 1945

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Paper No. 2236

RAINBOW ARCH BRIDGE OVER NIAGARA GORGE

A SYMPOSIUM

Page
Foreword 2

Design.
By Shortridge Hardesty and J. M. Garrelts, Members, Am. Soc. C. E., and I. G. Hedrick, Jr., Jun. Am. Soc. C. E. 6

Moments in Flexible Two-Hinged Arches.
By Elmer K. Timby, Assoc. M. Am. Soc. C. E., Lawrence M. Mead, Jr., Jun. Am. Soc. C. E., and Robert McEldowney, Jr., Esq. 35

Fabrication and Erection Controls.
By John I. Copp, Assoc. M. Am. Soc. C. E., and Karl de Vries, William H. Jameson, and Jonathan Jones, Members, Am. Soc. C. E. 50

Erection of Steel Superstructure.
By E. L. Durkee, M. Am. Soc. C. E. 82

With Discussion by Messrs. Egidio O. Di Genova, Charles Mackintosh, C. M. Goodrich, T. Kennard Thomson, C. M. Spofford, L. J. Mensch, O. H. Ammann, Louis Balog, William G. Rapp, Leon Beskin, Neil Van Eenam, C. H. Gronquist, M. Hirschthal, George Stern, and Shortridge Hardesty, J. M. Garrelts, and I. G. Hedrick, Jr., Elmer K. Timby, Lawrence M. Mead, Jr., and Robert McEldowney, Jr., and John I. Copp, Karl de Vries, William H. Jameson, and Jonathan Jones.

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FOREWORD

It is appropriate that the engineers responsible for the construction of the longest fixed arch span built to date should give the engineering profession a record of the design and erection of that monumental structure.

The selection of the proper bridge to be built at this crossing involved not only the economics of materials and erection but also that elusive item, the economics of appearance. The problem of esthetics was solved in this case by selecting the graceful arch structure for the main span, and approach spans which, in the opinion of the architect, harmonized with the surrounding structures particularly on the Canadian side.

Some of the problems encountered by Mr. Hardesty in his development of the design are discussed in the opening paper. While making his studies relating to the design, he became convinced that additional information was needed to guide engineers in the design of arch ribs. As a result, the problem was presented to Professor Timby, who supervised and directed studies on two-hinged arch models. These studies comprise the subject matter of the second paper in this Symposium.

The third paper of the Symposium gives a record of some of the problems and studies necessary to the construction of such a structure. The mathematical calculations necessary for positive control of such a structure during erection are very extensive. However, one who has seen the close check between the mathematical prediction and the measured forces in such a structure as this will have a greater confidence in future designs of statically indeterminate structures.

The fourth and last paper in the Symposium presents the solution to the many problems in the erection of a monumental bridge and describes the erection procedure.

Notation.—The following letter symbols, used in this Symposium, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials (ASA—Z10a—1932), prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932:

A = area; cross-sectional area of the arch rib:
    A_a = cross-sectional area of rib at point A;
    A_w = web area;
b = a subscript denoting base arch;
C = coefficient: C_1, C_2, C_3, and C_4 are empirical coefficients in Eq. 24;
$\overline{DL}$ = dead load; $\overline{DL}_b$ = dead load on the base arch;
E = Young's modulus of elasticity:
E_b = Young's modulus of elasticity referring to the base arch;
E_s = modulus of elasticity in shear;

- e' = accidental errors; maximum probable ordinate of a departure;
- F = a force;
- H = horizontal force, or component of force:
  - H_b = a horizontal force required, with V_b, to balance a system;
  - H_c = thrust due to camber;
  - H_r = (see V_r);
  - H_t = change in thrust due to temperature change t;
  - H_1 = horizontal force required to balance the cable;
  - H' = differential force;
  - ΔH = a horizontal increment of force resulting from the change in geometry of the rib;
- h = a subscript denoting "horizontal shear";
- I = moment of inertia of the rib cross section:
  - I_b = moment of inertia referred to the base arch;
- $\overline{I}$ = impact load;
- L = span length:
  - L_b = span length of base arch;
  - ΔL = increments of span length;
- $\overline{LL}$ = live load;
- l = length of the horizontal projection of the half arch;
- M = bending moment:
  - M_a = moment at point A;
  - M_b = bending moment in the base arch;
  - M_e = moment by usual elastic analysis;
  - M_m = total bending moment at point m;
  - M' = simple beam bending moment;
  - Δm = differences between bending moments;
- N = factor of safety;
- P = a concentrated force;
- Q = a bending moment used in Table 5;
- R = a ratio, $\frac{\overline{DL}}{\overline{LL}}$;
- r = radius of curvature of the arch rib: r_o = original radius;
- S = section modulus;
- s = unit stress; s_y = elastic limit;
- T = total direct thrust:
  - T_a = total axial thrust at the point A;
  - T_t = thrust due to temperature effect;
  - T_w = thrust due to wind;
- t = change in temperature, in degrees Fahrenheit;
- u = radial component of displacement of the arch rib:
  - u_a = radial component at point A;
  - u_e = radial component of displacement computed by the elastic theory;
  - u_f = radial component of displacement due to force F;
  - u_m = radial displacement at point m;

V = vertical shear:
    V_b = "basic" vertical shear;
    V_r = vertical shear which, with H_r, it is assumed would raise the rib to point A;
    V_1 = vertical force required to balance the cable;
v = a subscript denoting "vertical shear";
W = a total unit load, $\overline{DL} + \overline{LL} + \bar{I}$:
    W_w = working load, ordinary load;
    W_{yp} = load W that produces a yield-point stress in the arch rib;
X_c = centroid ordinate;
Y_c = centroid ordinate;
y = vertical ordinate of arch rib: y_a = ordinate at point A (nominal outline);
α = slope angle of the arch rib at any point;
Δ = horizontal deflection;
δ = deflections due to thrust, bending, shear, camber, or temperature:
    δ_h = horizontal deflection;
    δ_v = vertical deflection;
    δ' = deformation produced by a load of 1 kip;
ϵ = linear coefficient of temperature expansion;
η = vertical displacement;
θ = angle change:
    θ_m = total angle changes;
    $\Delta\theta$ = small increment of angle change;
ρ = horizontal displacement;
τ = change in angles in relation to adjacent members, due to bending:
    τ_m = angle change at point m; and
ϕ = angle change: ϕ_c = angle change at the crown.

Acknowledgments.—The bridge was built under the direction of the Niagara Falls Bridge Commission, a body created by authority of an act of Congress.

The consultants retained by the Commission were: Waddell & Hardesty of New York, N. Y., who prepared the designs and plans; the Edward P. Lupfer Corporation of Buffalo, N. Y., who supervised the construction; and Aymar Embury II, M. Am. Soc. C. E., of New York who was consulting architect.

The deformeter analysis of the superstructure, mentioned in the first paper of the Symposium, was made by Egbert Hardesty, Jun. Am. Soc. C. E., and was reported fully in a thesis entitled "Model Analysis of the Proposed Rainbow Arch Bridge Over the Niagara River," which was presented to Rensselaer Polytechnic Institute in Troy, N. Y., in 1941, in partial fulfilment of the requirements for the degree of Bachelor of Civil Engineering.

The work described in the second paper was done at Princeton University in Princeton, N. J., under the sponsorship of the School of Engineering, of which Kenneth H. Condit is Dean of Engineering and Philip Kissam, M. Am.

Soc. C. E., is chairman of the Department of Civil Engineering. Complete results of these tests, including considerable material not touched on in the Symposium paper, are presented in the form of a thesis entitled "Moments in Flexible Arches," submitted to Princeton University in June, 1941, jointly by Messrs. McEldowney and Mead in partial fulfilment of the requirements for the degree of Civil Engineer. John W. Graham, Jr., Jun. Am. Soc. C. E., made the preliminary design for the loading frame and assisted in its construction. The authors of the first Symposium paper gave many valuable suggestions during the planning of this investigation and in the preparation of the digest of the results.

Award of the general contract for the fabrication and erection of the 950-ft main arch span of the Rainbow Bridge, including the concrete deck, was made to the Bethlehem Steel Company early in May, 1940. The ribs of the Rainbow arch were fabricated in the United States by the Bethlehem Steel Company and the remainder was fabricated in Canada under a contract let to the Canadian Bridge Company. The latter, in turn, sublet the fabrication of the spandrel girders to the Hamilton Bridge Company. In the construction operations, it was required that all workmen employed should be residents of the country where such operations were being done. It was desirable, however, that the field work be under one management, and it was agreed that all construction operations would be by the Bethlehem Steel Company.

The entire erection scheme was worked out under the direction of D. S. Gendell, Jr., M. Am. Soc. C. E., General Manager of Erection for the Bethlehem Steel Company. The author of the last paper of the Symposium, Mr. Durkee, was in charge of the erection of the steelwork as Resident Engineer, assisted by W. W. Oskin, J. T. York, and W. H. Thorn as junior engineers. R. S. Smith was foreman in charge on the New York side of the river and D. S. Shuman on the Canadian side.

The McLain Construction Corporation of Buffalo was contractor for the concrete approaches on the New York side and subcontractor for the concrete deck on the United States half of the steel arch. On the Canadian side, Aiken and MacLachlan, Ltd., of St. Catherines, Ontario, were contractors for the concrete approaches and subcontractors for the concrete deck.

DESIGN

BY SHORTRIDGE HARDESTY[1] AND J. M. GARRELTS,[2] MEMBERS,
AM. SOC. C. E., AND I. G. HEDRICK, JR.,[3] JUN. AM. SOC. C. E.

SYNOPSIS

The first part of this paper treats the design of the fixed arch bridge spanning the gorge between Niagara Falls, N. Y., and Niagara Falls, Ont., Canada. The design loads and allowable unit stresses are given, together with the stresses as calculated by the conventional elastic theory.

In the second part, the effects of rib deflections are discussed. The resulting moments at the quarter point are evaluated for a preliminary, two-hinged arch rib, and for the final fixed arch rib as built. A relatively simple procedure is proposed for calculating approximate values of such moments. These results can be used for preliminary studies, and in many cases are sufficiently accurate for use in final design.

PART I.—DESIGN

INTRODUCTION

The new Niagara arch bridge (Fig. 1) spanning the Niagara gorge is about one-half mile downstream from the Falls and 400 ft north of the location of the former International Falls View arch which it has replaced. The previous structure (wrecked by river ice on January 27, 1938) was a two-hinged arch

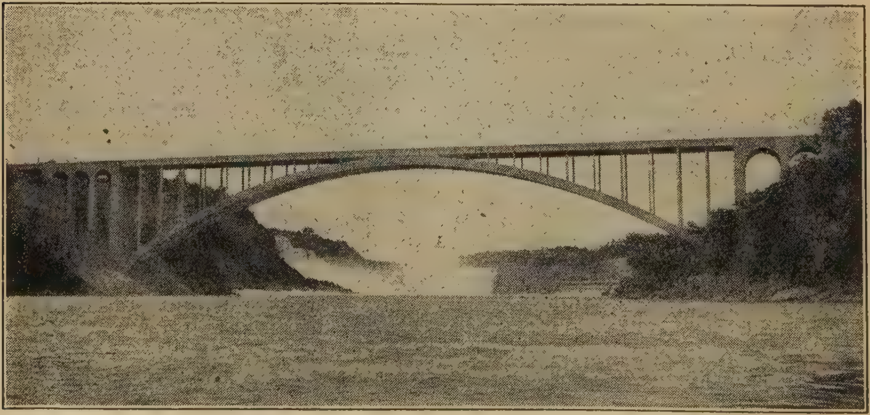

FIG. 1.—VIEW OF ARCH

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[1] Cons. Engr. (Waddell & Hardesty), New York, N. Y.

[2] Associate Prof., Civ. Eng., Columbia Univ.; Designing Engr., Waddell & Hardesty, New York, N. Y.

[3] Designing Engr., Waddell & Hardesty; Associate in Civ. Eng., Columbia Univ., New York, N. Y.

with an 840-ft span. Each rib of this structure consisted of a truss with chords 26 ft center to center, and with a rise of 150 ft. In accordance with present-day standards, the spandrel columns and the truss members were of light design.

At the site of the new bridge, the gorge is 1,250 ft wide at the rim and about 830 ft wide at the surface of the water. The depth from the rim to mean water level is 180 ft, which, with a water depth of 175 ft, makes a total of 355 ft. The walls of the gorge on both sides of the river drop almost vertically for a distance of 80 ft at the top, and then slope down at approximately 45° to the water line. The talus covering on the slopes is from 10 ft to 35 ft thick. The vertical cliffs are of dolomite, and the rock beneath the talus slopes consists of Rochester shale, Irondequoit limestone, Wolcott limestone, Sodus shale, Thorold sandstone, and Albion shale and sandstone. All strata are practically horizontal. A cross-sectional view of the various strata is shown in Fig. 2, a further description being as follows:

Clinton Formation.—

Rochester shale is rather soft on top. It weathers badly, but is a good firm rock requiring blasting where unexposed. It has irregular horizontal and vertical seams.

Irondequoit limestone is hard, firm, durable, and deeply bedded.

Wolcott limestone is a durable rock which does not occur in such a heavy strata as the Irondequoit limestone.

Sodus shale is bluish in color and is firm where not exposed.

Albion Sandstone Formation.—

Thorold sandstone is white, hard, and durable.

The Albion shale and sandstone stratum is a rather soft red sandstone interlaid with reddish shale. The stratum directly beneath is a much harder gray and red sandstone, which on the east shore is mottled, with thinner shale seams than in the overlying shale and sandstone stratum.

Preliminary Studies

Considerable thought was given to the esthetics of the new bridge and to the creation of a structure that would blend with the natural beauty and grandeur of the Niagara gorge. Architectural studies and preliminary designs showed definite advantages in favor of an arch-type structure for this location. In addition to its esthetic superiority, the arch is structurally appropriate; and the inclined rock walls provide a perfect support for the arch abutments. The approach structure favored by the Bridge Commission was one with massive concrete shafts and arches. This contrast in type of construction served to define the main arch span (Figs. 1 and 3).

The plate girder rib type was selected, rather than the trussed rib or the spandrel braced arch, because of its architectural simplicity. Transverse and longitudinal bracing between the spandrel columns was avoided in order to maintain the clean-cut lines of the structure (see Fig. 4). Horizontal forces are transferred from the floor to the plane of the arch through shear in the columns, except at two points near the center where inconspicuous sway bracing was used.

The effects of the participation of the deck with the superstructure, and of expansion joints in the deck floor system, were investigated by a deformeter analysis, using a celluloid model of one rib with spandrel columns and spandrel girders. This analysis demonstrated that the joints for the spandrel structure should be made as flexible as would be consistent with the desired rigidity of the construction and that at least two expansion joints in the spandrel girders and the floor system would be desirable.

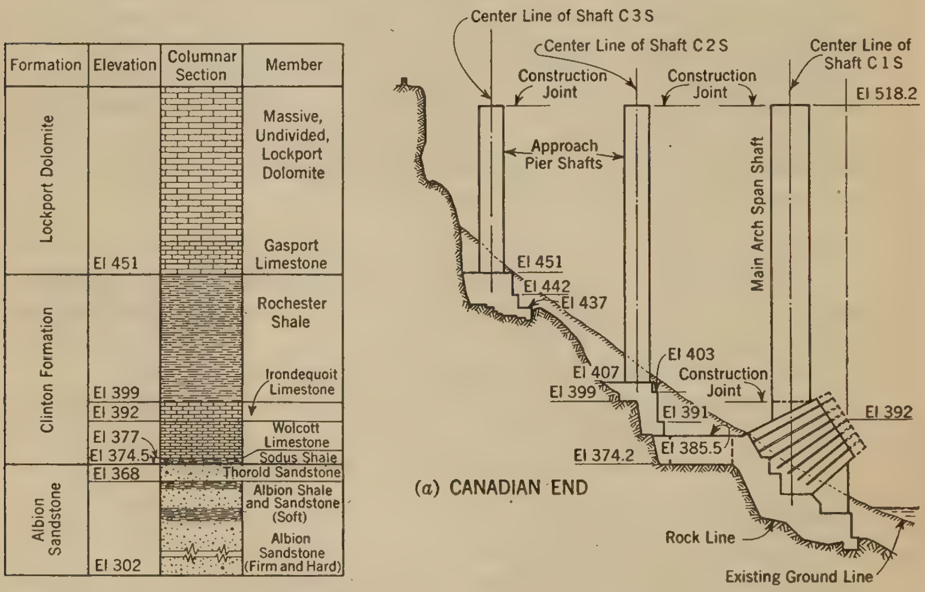

Formation
Elevation
Columnar Section
Member
Lockport Dolomite
Massive, Undivided, Lockport Dolomite
Gasport Limestone
El 451
Clinton Formation
Rochester Shale
Irondequoit Limestone
El 399
El 392
Wolcott Limestone
El 377
El 374.5
Sodus Shale
El 368
Thorold Sandstone
Albion Sandstone
Albion Shale and Sandstone (Soft)
Albion Sandstone (Firm and Hard)
El 302
Center Line of Shaft C 3 S
Center Line of Shaft C 2 S
Center Line of Shaft C 1 S
Construction Joint
Construction Joint
El 518.2
Approach Pier Shafts
Main Arch Span Shaft
El 451
El 442
El 437
El 403
Construction Joint
El 407
El 399
El 391
El 392
El 374.2
El 385.5
(a) CANADIAN END
Rock Line
Existing Ground Line

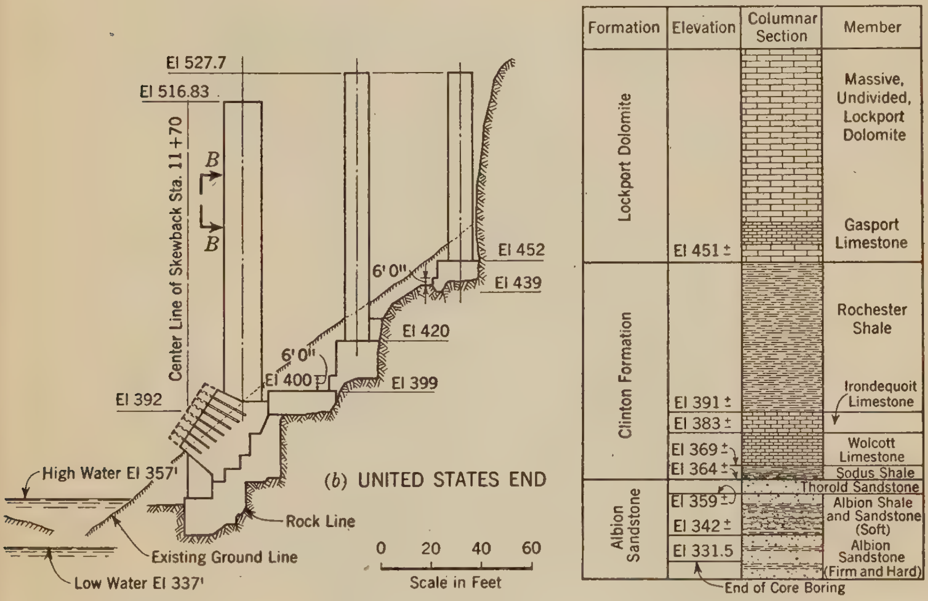
FIG. 2.—ELEVATION OF

The first intention was to build a two-hinged arch structure; but a preliminary design of this type of structure proved to be so flexible, for a span of this length, as to show that a design which would be satisfactory would require ribs of excessive moment of inertia and sectional areas. The hingeless type then was investigated, and it was found that a rib with considerably less area and moment of inertia would be safe, and would be subject to much smaller deflections. Since both economy and rigidity favored the hingeless type, it was selected.

DESIGN LOADS AND ALLOWABLE UNIT STRESSES

The live load used for design was the H-20 loading of the 1935 Standard Specifications for Highway Bridges of the American Association of State Highway Officials. For the design of the arch rib, this is equivalent to a uniform live panel load of 54.4 kips and a concentrated live panel load of 26.7 kips, the panel length being 39.4 ft. The impact effect was calculated in accordance with the forementioned specifications. The intensity of the wind force was assumed to be 30 lb per sq ft on one-and-a-half times the projected area of the structure in a vertical plane, plus 200 lb per lin ft applied 6 ft above the roadway.

The design provided for a temperature variation of ±60° F from an arbitrary normal of 50° F. Allowance also was made for a possible error of ±1 in. in span length. Such an error would produce stresses equivalent to those due to a change in temperature of ±15° F.


El 527.7
El 516.83
Center Line of Skewback Sta. 11+70
B
B
El 392
High Water El 357'
Low Water El 337'
Existing Ground Line
Rock Line
6'0"
El 400
6'0"
El 452
El 439
El 420
El 399
(b) UNITED STATES END
0 20 40 60
Scale in Feet
Formation
Elevation
Columnar Section
Member
Lockport Dolomite
Massive, Undivided, Lockport Dolomite
Gasport Limestone
El 451±
Clinton Formation
Rochester Shale
Irondequoit Limestone
El 391±
El 383±
Wolcott Limestone
El 369±
El 364±
Sodus Shale
Thorold Sandstone
Albion Sandstone
El 359±
Albion Shale and Sandstone (Soft)
El 342±
Albion Sandstone (Firm and Hard)
El 331.5
End of Core Boring

South Piers and Shafts

The allowable unit stresses and the requirements for the details of design are in accordance with the 1938 Specifications for Steel Railway Bridges of the American Railway Engineering Association (A.R.E.A.). The basic unit stresses, in pounds per square inch, were:

	Carbon	Silicon
Tension, net section..............	18,000	24,000
Compression, gross section........	$15{,}000 - \frac{1}{4}\left(\frac{l}{r}\right)^2$	$20{,}000 - 0.46\left(\frac{l}{r}\right)^2$

The allowable unit stresses for the silicon steel arch ribs, for use with stresses computed by the elastic theory, in pounds per square inch, were:

Dead load+live load+impact load+temperature+error..............	20,000
Dead load+live load+impact load+temperature+error+wind load..	25,000

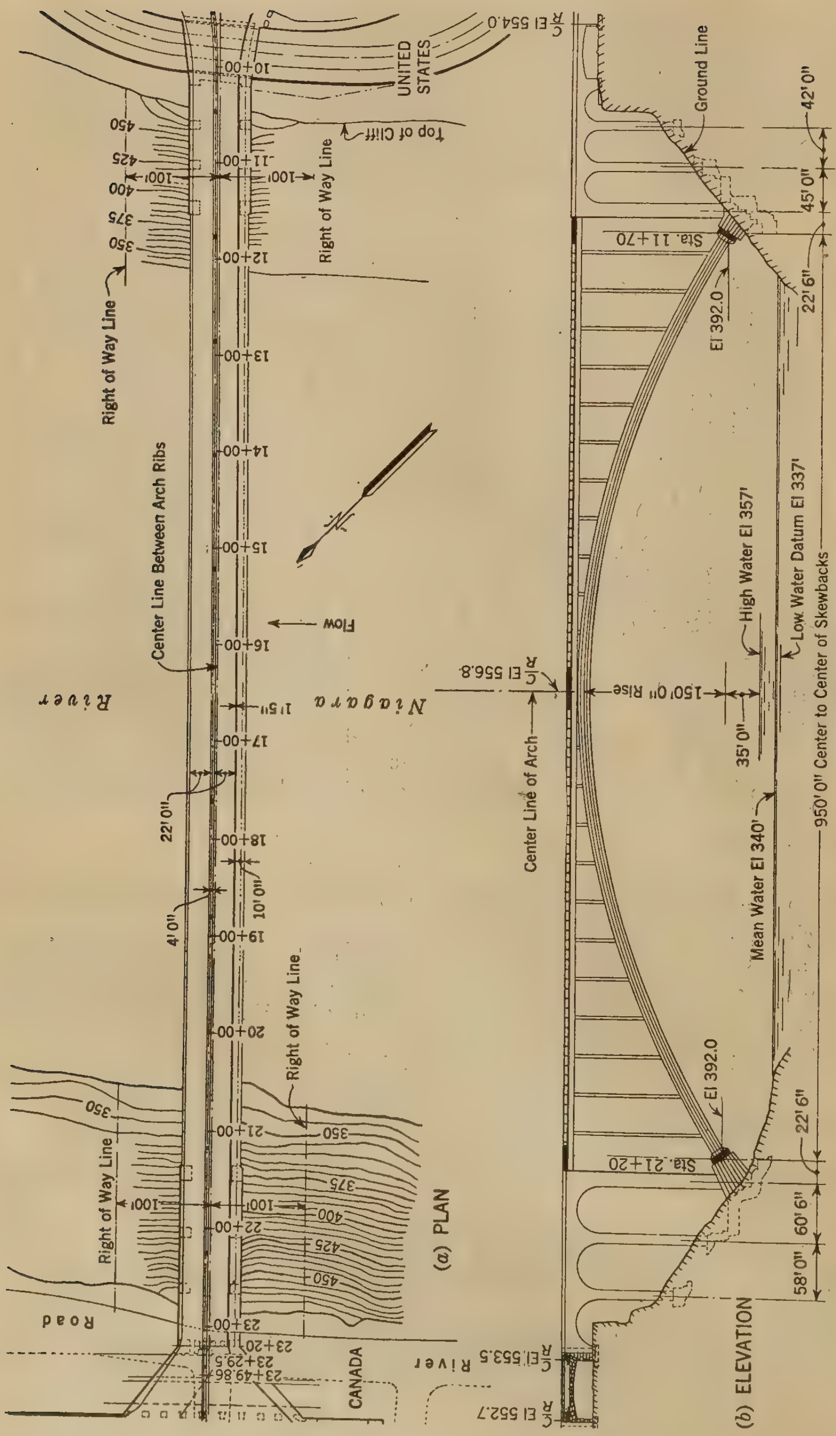

(a) PLAN
UNITED STATES
CANADA
Niagara
River
Road
Flow
Top of Cliff
Right of Way Line
Right of Way Line
Right of Way Line
Right of Way Line
Center Line Between Arch Ribs
100'
100'
100'
100'
11'5"
22'0"
4'0"
10'0"
10+00
11+00
12+00
13+00
14+00
15+00
16+00
17+00
18+00
19+00
20+00
21+00
22+00
23+00
23+20
23+29.5
23+49.86
350
375
400
425
450
(b) ELEVATION
Ground Line
℄ El 554.0
℄ El 556.8
℄ El 553.5
℄ El 552.7
Center Line of Arch
150'0" Rise
Sta. 11+70
Sta. 21+20
El 392.0
El 392.0
High Water El 357'
Low Water Datum El 337'
Mean Water El 340'
35'0"
42'0"
45'0"
22'6"
22'6"
60'6"
58'0"
950'0" Center to Center of Skewbacks

FIG. 3.—PLAN AND ELEVATION OF SUPERSTRUCTURE

DESIGN OF MAIN ARCH SPAN

Floor System.—The bridge carries two 22-ft roadways (see Fig. 5) separated by a 4-ft mall, and a 10-ft walkway on the south side facing the Falls. The roadway floor consists of 7½-in. concrete slab reinforced with welded-bar reinforcing trusses, which is supported on 30-in. wide-flange 108-lb rolled-beam stringers, a maximum of 5 ft 5 in. apart. The walkway is a 4-in. concrete slab. The stringers frame into silicon steel floor beams 6 ft 0¾ in. in depth, 39 ft 4 in. apart. Box-type spandrel girders, 6 ft 0¾ in. in depth, consist of a top cover plate, two top-flange angles, two web plates, and two bottom-flange angles. The spandrel girders are connected to the columns at most points with short, flexible angle connections which allow some relative movement in the longitudinal direction. However, rigid connections were used at panel point 5 (see Fig. 6) in order to provide longitudinal stiffness in the floor system, and at the

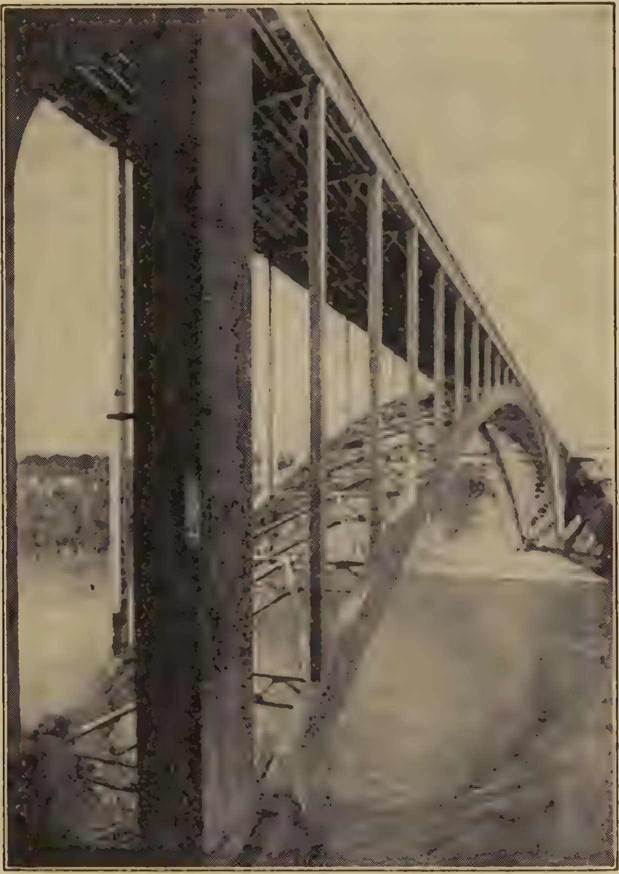


FIG. 4.—VIEW OF SPANDRELS AND WIND BRACING

long columns near the ends which were so flexible that rigid connections were found desirable.

Spandrel Columns.—The spandrel columns are rectangular box sections. They were designed for dead load, live load, wind loads, and stresses due to participation effects. Shears and moments are produced in the columns when the supporting arch ribs rotate or are displaced. Transverse wind loads also produce shears and moments in the columns, due to unequal transverse dis-

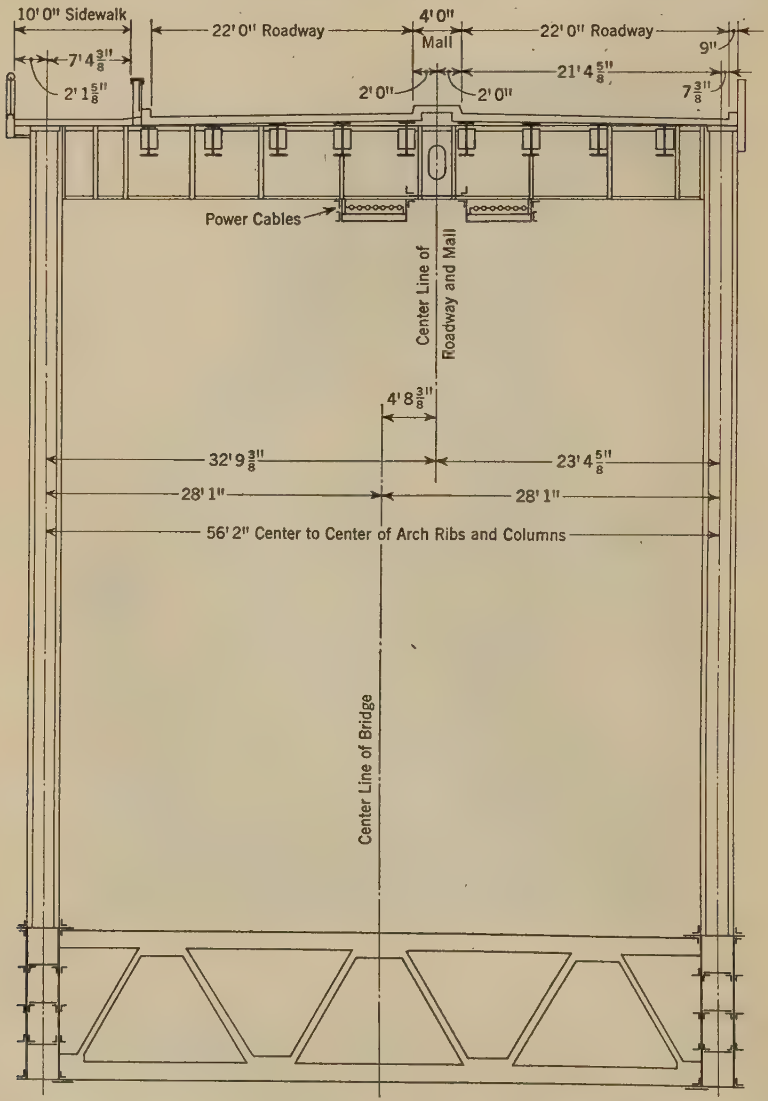

10' 0" Sidewalk
7' 4 3/8"
2' 1 5/8"
22' 0" Roadway
4' 0"
Mall
22' 0" Roadway
9"
21' 4 5/8"
2' 0"
2' 0"
7 3/8"
Power Cables
Center Line of
Roadway and Mall
4' 8 3/8"
32' 9 3/8"
23' 4 5/8"
28' 1"
28' 1"
56' 2" Center to Center of Arch Ribs and Columns
Center Line of Bridge

Fig. 5.—Cross Section of Deck

placements of the floor system and arch ribs. The base of each column was milled to a plane, tangent to the curved top cover plate of the arch rib; and high restraining moments in the connections for the columns were reduced by using flexible base connections, the connection angles having sufficient spring to allow the columns to rock slightly in the longitudinal direction on the arch rib. The tops of the columns at panel point 5 were fixed to the spandrel girders, as has been stated previously.

Wind loads on the deck are carried partly by the roadway lateral system and partly by shear in the spandrel columns. The part of the load carried by each was determined by distributing the loads to the laterals and to the columns in such a manner that the resulting deflections would be consistent. An expansion joint was provided in the deck and the floor lateral system at panel point 3.

Laterals.—The arch rib laterals consist of a plane of K-type bracing at both the top and bottom flanges of the rib, with transverse sway frames at each panel point. These laterals are designed for the wind forces acting directly on the rib and also for the wind forces transferred from the deck to the rib through the spandrel columns.

Arch Ribs.—The arch ribs are silicon steel box girders 12 ft deep. Design loads and stresses are shown in Table 1. In addition to the stresses computed by the elastic analysis, stresses due to the deflection effects were calculated at certain points. The resulting effect of these secondary stresses is discussed subsequently. Each arch rib, except at the springings, has two 144-in. by $\frac{13}{16}$-in. web plates and eight 8-in. by 8-in. flange angles. Top and bottom cover plates 54 in. wide vary from $1\frac{1}{8}$ in. to $2\frac{1}{2}$ in. in thickness. A longitudinal diaphragm at the center of the web plates and longitudinal angles at the quarter points provide adequate stiffness to prevent buckling of the web plates between the normal diaphragms, which are placed at panel points and mid-panel points.

The rib splices were designed to transmit at least 60% of the total stress through the splice material. The abutting ends of the rib sections at the splices were faced accurately to secure an even bearing when assembled in the structure. For access to the rib sections, manholes were cut in the inner web plates, the area around the holes being reinforced to compensate for the area taken out. The web manholes are spaced at intervals of about 120 ft, and openings in the diaphragms permit longitudinal passage through the inside of the rib sections. Typical details for a section of the rib are shown in Figs. 7 and 8.

The make-up of Section 1 is as follows:

Main material	Details
Two web plates, 144 in. by $\frac{13}{16}$ in.	Two angles, 7 in. by 4 in. by $\frac{5}{8}$ in.
Two cover plates, 54 in. by $1\frac{1}{8}$ in.	Two angles, 4 in. by 4 in. by $\frac{1}{2}$ in.
One plate, 34 in. by $\frac{3}{4}$ in.	Four angles, 4 in. by 4 in. by $\frac{7}{16}$ in.
Eight angles, 8 in. by 8 in. by 1 in.	....
Four angles, 7 in. by 4 in. by $\frac{5}{8}$ in.	....

The make-up of Section 2 is identical except that $1\frac{1}{4}$-in. cover plates were used.

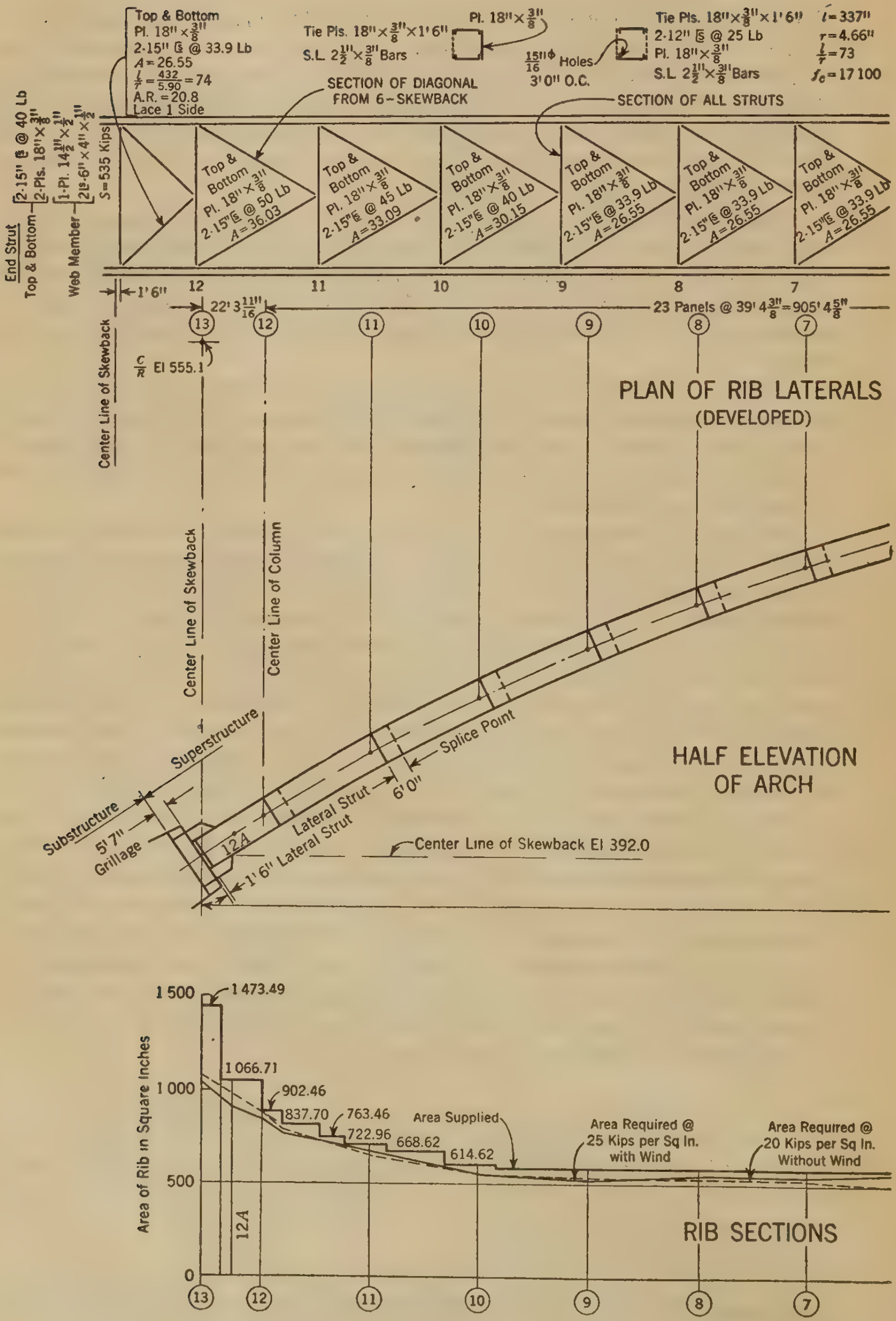
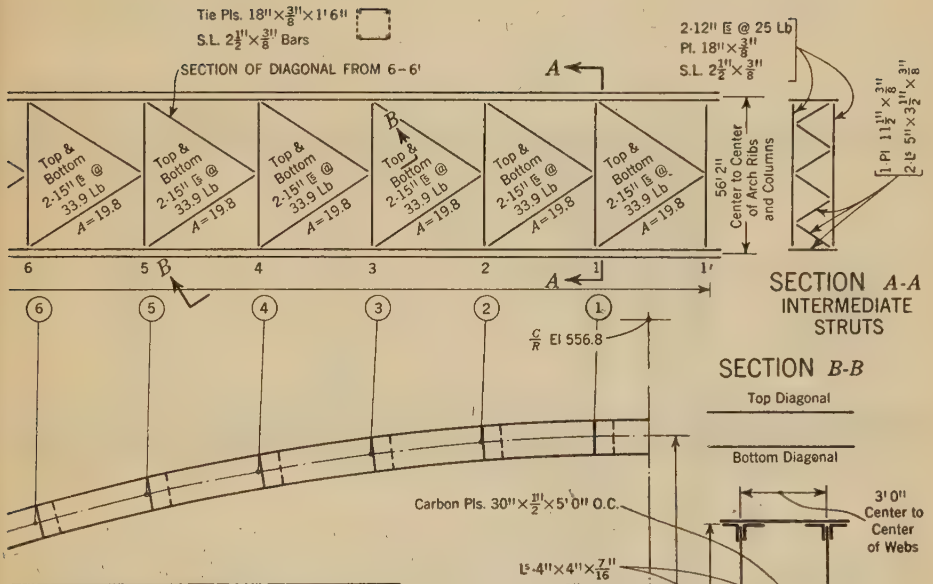
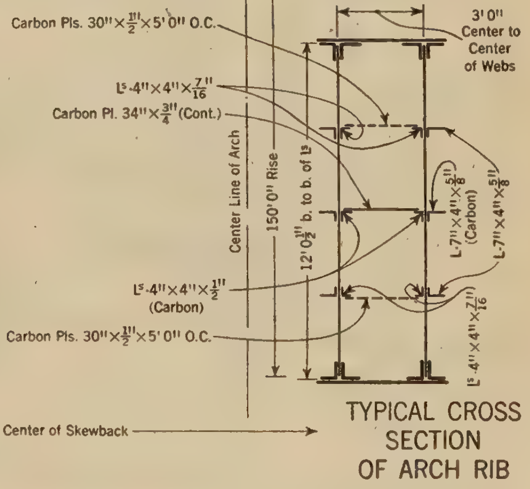

Top & Bottom
Pl. 18"×$\frac{3}{8}$"
2-15" ⊏ @ 33.9 Lb
A=26.55
$\frac{l}{r}=\frac{432}{5.90}=74$
A.R.=20.8
Lace 1 Side
Tie Pls. 18"×$\frac{3}{8}$"×1'6"
S.L. 2$\frac{1}{2}$"×$\frac{3}{8}$" Bars
Pl. 18"×$\frac{3}{8}$"
SECTION OF DIAGONAL FROM 6-SKEWBACK
$\frac{15}{16}$"ϕ Holes
3'0" O.C.
Tie Pls. 18"×$\frac{3}{8}$"×1'6"
2-12" ⊏ @ 25 Lb
Pl. 18"×$\frac{3}{8}$"
S.L. 2$\frac{1}{2}$"×$\frac{3}{8}$" Bars
SECTION OF ALL STRUTS
l=337"
r=4.66"
$\frac{l}{r}$=73
f_c=17 100
End Strut
Top & Bottom
2-15" ⊏ @ 40 Lb
2-Pls. 18"×$\frac{3}{8}$"
Web Member
1-Pl. 14$\frac{1}{2}$"×$\frac{1}{2}$"
2L^s-6"×4"×$\frac{1}{2}$"
S=535 Kips
Top & Bottom Pl. 18"×$\frac{3}{8}$" 2-15"⊏ @ 50 Lb A=36.03
Top & Bottom Pl. 18"×$\frac{3}{8}$" 2-15"⊏ @ 45 Lb A=33.09
Top & Bottom Pl. 18"×$\frac{3}{8}$" 2-15"⊏ @ 40 Lb A=30.15
Top & Bottom Pl. 18"×$\frac{3}{8}$" 2-15"⊏ @ 33.9 Lb A=26.55
Top & Bottom Pl. 18"×$\frac{3}{8}$" 2-15"⊏ @ 33.9 Lb A=26.55
Top & Bottom Pl. 18"×$\frac{3}{8}$" 2-15"⊏ @ 33.9 Lb A=26.55
1'6"
12
11
10
9
8
7
22' 3$\frac{11}{16}$"
23 Panels @ 39' 4$\frac{3}{8}$"=905' 4$\frac{5}{8}$"
⑬
⑫
⑪
⑩
⑨
⑧
⑦
$\frac{C}{R}$ El 555.1
Center Line of Skewback
PLAN OF RIB LATERALS
(DEVELOPED)
Center Line of Skewback
Center Line of Column
Superstructure
Substructure
5'7" Grillage
12A
Lateral Strut
1'6" Lateral Strut
6'0"
Splice Point
Center Line of Skewback El 392.0
HALF ELEVATION OF ARCH
1 500
1 000
500
0
Area of Rib in Square Inches
1 473.49
1 066.71
902.46
837.70
763.46
722.96
668.62
614.62
Area Supplied
Area Required @ 25 Kips per Sq In. with Wind
Area Required @ 20 Kips per Sq In. Without Wind
12A
RIB SECTIONS
⑬
⑫
⑪
⑩
⑨
⑧
⑦

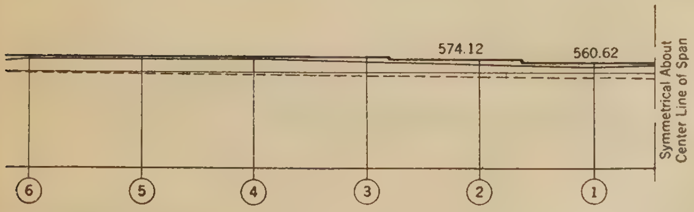
FIG. 6 (WITH TABLE 1).—STRESS


Tie Pls. 18"×3/8"×1'6"
S.L. 2½"×3/8" Bars
SECTION OF DIAGONAL FROM 6-6'
A
2-12" [@ 25 Lb
Pl. 18"×3/8"
S.L. 2½"×3/8"
Top & Bottom 2-15" [@ 33.9 Lb
A=19.8
B
56'2" Center to Center of Arch Ribs and Columns
1-Pl 11½"×3/8"
2-Ls 5"×3½"×3/8"
6
5
4
3
2
1
1'
⑥
⑤
④
③
②
①
SECTION A-A
INTERMEDIATE
STRUTS
C/R El 556.8
SECTION B-B
Top Diagonal
Bottom Diagonal

GEOMETRY OF ARCH RIB			
PANEL POINT	HORIZONTAL DISTANCE FROM CROWN	VERTICAL DISTANCE FROM CROWN.	TANGENT OF AXIS WITH HORIZONTAL
1	19.682'	0.249'	0.0253
2	59.047	2.237	0.0758
3	98.412	6.217	0.1264
4	137.776	12.192	0.1772
5	177.141	20.194	0.2284
6	216.505	30.183	0.2802
7	255.870	42.249	0.3330
8	295.235	56.418	0.3871
9	334.599	72.749	0.4429
10	373.964	91.320	0.5010
11	413.328	112.232	0.5619
12	452.693	135.606	0.6263
13	475.000	150.000	0.6645


Carbon Pls. 30"×½"×5'0" O.C.
3'0" Center to Center of Webs
Ls-4"×4"×7/16"
Carbon Pl. 34"×¾"(Cont.)
Center Line of Arch
150'0" Rise
12'0½" b. to b. of Ls
L-7"×4"×5/8" (Carbon)
L-7"×4"×5/8"
Ls-4"×4"×½" (Carbon)
Carbon Pls. 30"×½"×5'0" O.C.
Ls-4"×4"×7/16"
TYPICAL CROSS SECTION OF ARCH RIB

950'0" Center to Center of Skewback


574.12
560.62
Symmetrical About Center Line of Span
⑥
⑤
④
③
②
①

Sheets for Arch Ribs.

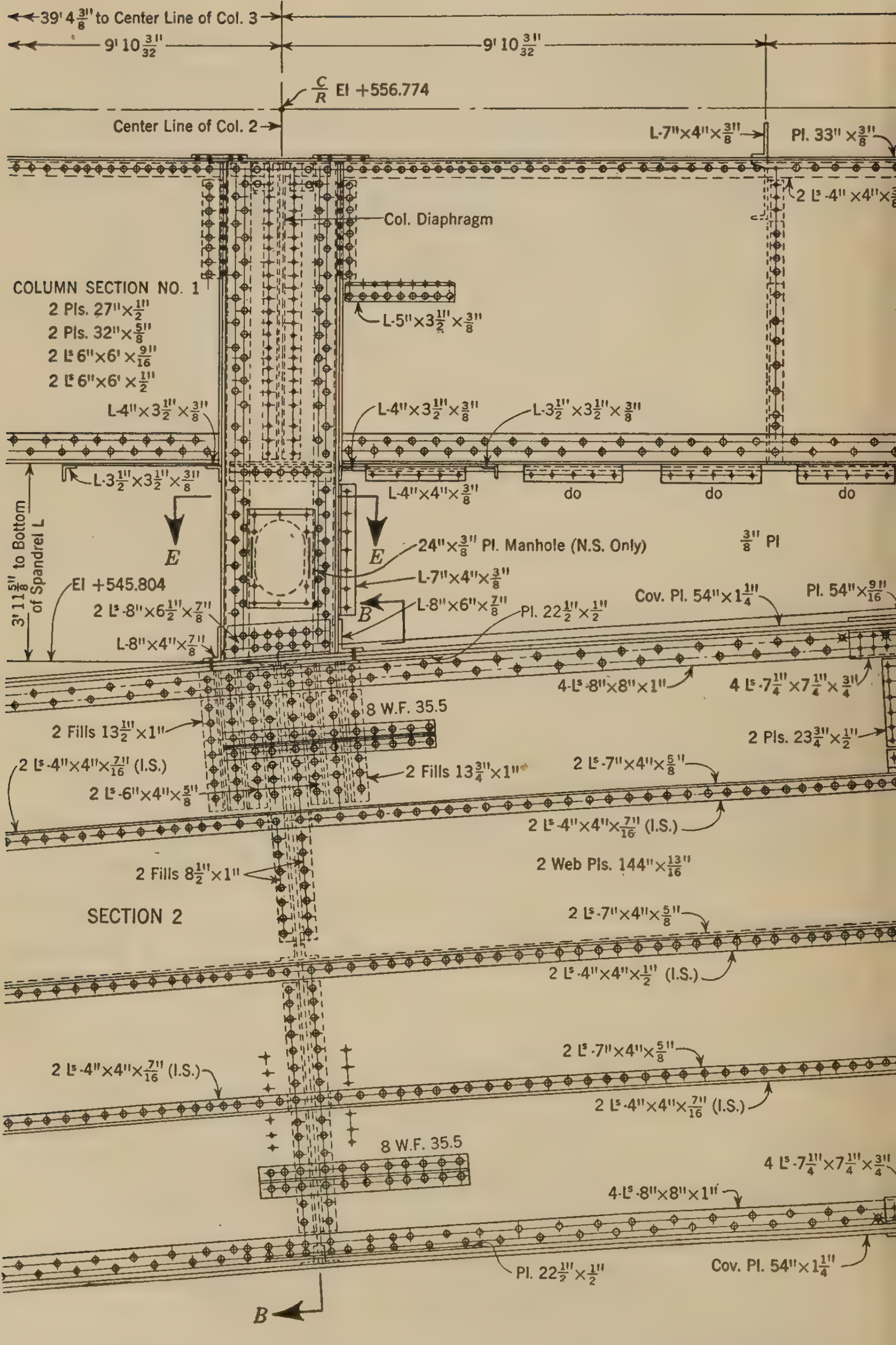
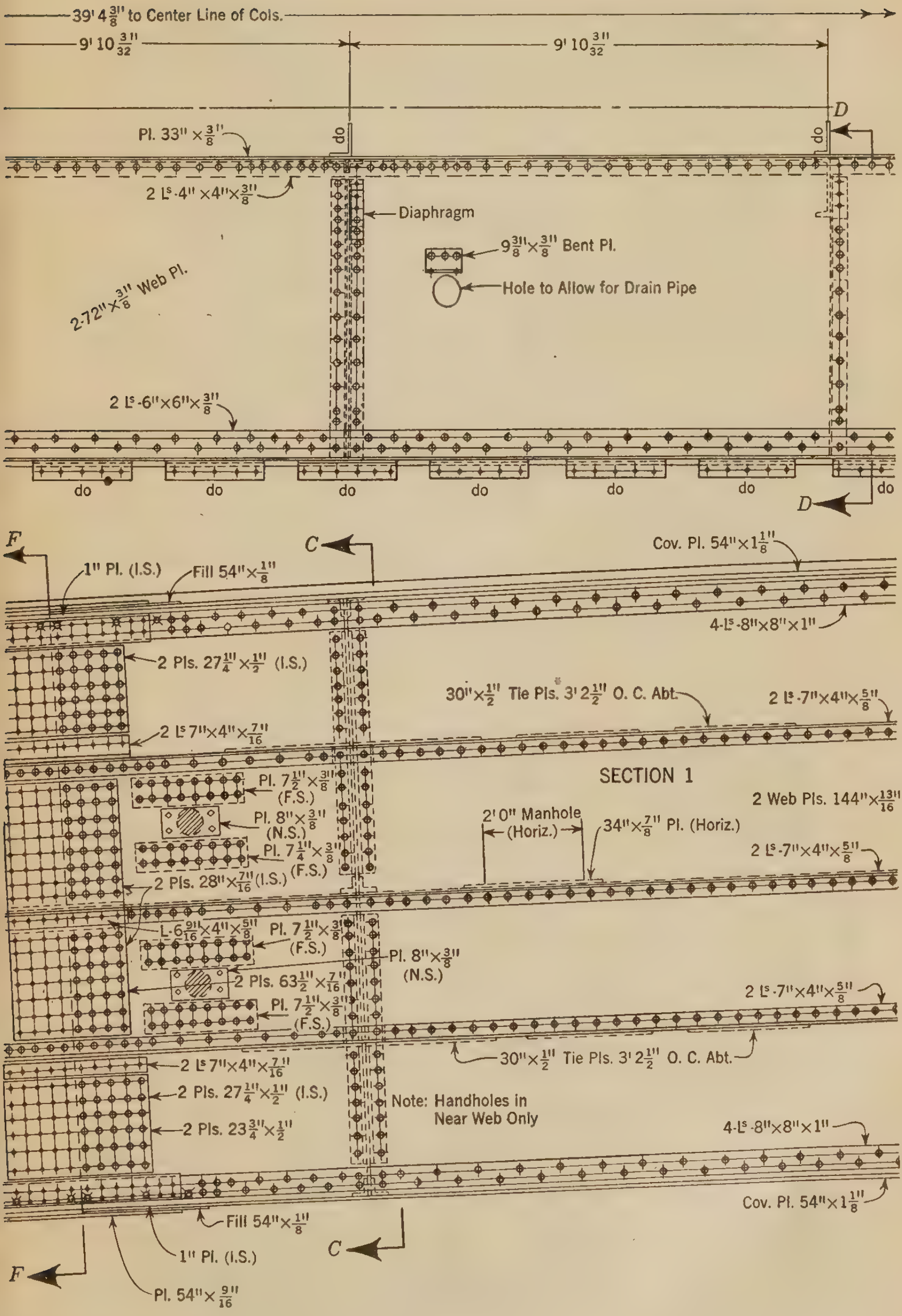

39' 4 3/8" to Center Line of Col. 3
9' 10 3/32"
9' 10 3/32"
C/R El +556.774
Center Line of Col. 2
L-7"×4"×3/8"
Pl. 33" ×3/8"
2 Ls-4" ×4"×
Col. Diaphragm
COLUMN SECTION NO. 1
2 Pls. 27"×1/2"
2 Pls. 32"×5/8"
2 Ls 6"×6'×9/16"
2 Ls 6"×6'×1/2"
L-5"×3 1/2"×3/8"
L-4"×3 1/2"×3/8"
L-4"×3 1/2"×3/8"
L-3 1/2"×3 1/2"×3/8"
L-3 1/2"×3 1/2"×3/8"
L-4"×4"×3/8"
do
do
do
E
E
3' 11 5/8" to Bottom of Spandrel L
El +545.804
24"×3/8" Pl. Manhole (N.S. Only)
3/8" Pl
L-7"×4"×3/8"
2 Ls-8"×6 1/2"×7/8"
L-8"×6"×7/8"
B
Pl. 22 1/2"×1/2"
Cov. Pl. 54"×1 1/4"
Pl. 54"×9/16"
L-8"×4"×7/8"
4-Ls-8"×8"×1"
4 Ls-7 1/4"×7 1/4"×3/4"
8 W.F. 35.5
2 Fills 13 1/2"×1"
2 Pls. 23 3/4"×1/2"
2 Ls-4"×4"×7/16" (I.S.)
2 Fills 13 3/4"×1"
2 Ls-7"×4"×5/8"
2 Ls-6"×4"×5/8"
2 Ls-4"×4"×7/16" (I.S.)
2 Fills 8 1/2"×1"
2 Web Pls. 144"×13/16"
SECTION 2
2 Ls-7"×4"×5/8"
2 Ls-4"×4"×1/2" (I.S.)
2 Ls-7"×4"×5/8"
2 Ls-4"×4"×7/16" (I.S.)
2 Ls-4"×4"×7/16" (I.S.)
8 W.F. 35.5
4 Ls-7 1/4"×7 1/4"×3/4"
4-Ls-8"×8"×1"
Pl. 22 1/2"×1/2"
Cov. Pl. 54"×1 1/4"
B

Fig. 7.—Detail of Ar


39' 4 3/8" to Center Line of Cols.
9' 10 3/32"
9' 10 3/32"
D
do
do
Pl. 33" × 3/8"
2 Ls-4" × 4" × 3/8"
Diaphragm
9 3/8" × 3/8" Bent Pl.
Hole to Allow for Drain Pipe
2-72" × 3/8" Web Pl.
2 Ls-6" × 6" × 3/8"
do
do
do
do
do
do
do
D
F
C
Cov. Pl. 54" × 1 1/8"
1" Pl. (I.S.)
Fill 54" × 1/8"
4-Ls-8" × 8" × 1"
2 Pls. 27 1/4" × 1/2" (I.S.)
30" × 1/2" Tie Pls. 3' 2 1/2" O. C. Abt.
2 Ls-7" × 4" × 5/8"
2 Ls 7" × 4" × 7/16"
SECTION 1
Pl. 7 1/2" × 3/8" (F.S.)
Pl. 8" × 3/8" (N.S.)
Pl. 7 1/4" × 3/8" (F.S.)
2 Web Pls. 144" × 13/16"
2' 0" Manhole (Horiz.)
34" × 7/8" Pl. (Horiz.)
2 Ls-7" × 4" × 5/8"
2 Pls. 28" × 7/16" (I.S.)
L-6 9/16" × 4" × 5/8"
Pl. 7 1/2" × 3/8" (F.S.)
Pl. 8" × 3/8" (N.S.)
2 Pls. 63 1/2" × 7/16"
Pl. 7 1/2" × 3/8" (F.S.)
2 Ls-7" × 4" × 5/8"
2 Ls 7" × 4" × 7/16"
30" × 1/2" Tie Pls. 3' 2 1/2" O. C. Abt.
2 Pls. 27 1/4" × 1/2" (I.S.)
Note: Handholes in Near Web Only
2 Pls. 23 3/4" × 1/2"
4-Ls-8" × 8" × 1"
Cov. Pl. 54" × 1 1/8"
Fill 54" × 1/8"
1" Pl. (I.S.)
C
F
Pl. 54" × 9/16"

Rib Sections 1 and 2

TABLE 1 (WITH FIG. 6).—STRESS SHEETS FOR ARCH RIBS

	POINT	SPRINGING		12 A (MIDWAY BET. 12 & 13)		12		11	
	SECTION	Grillage 12'×22'		2 Webs $144 \times \frac{13}{16}$	: 234.00	2 Webs $144 \times \frac{13}{16}$	: 234.00	2 Webs $144 \times \frac{13}{16}$	: 234.00
		Area : 264□' : 38016□"		2 Webs $128 \times \frac{9}{16}$ 2 Webs $128 \times \frac{5}{8}$	: 144.00 160.00	2 Webs $128 \times \frac{9}{16}$	: 144 00	$8L^s \cdot 8 \times 8 \times 1\frac{1}{8}$	: 133.84
				$8L^s \cdot 8 \times 8 \times 1\frac{1}{8}$	: 133.84	$8L^s \cdot 8 \times 8 \times 1\frac{1}{8}$	: 133.84	4 Cov Pls $54 \times 1\frac{1}{4}$	: 270.00
		32 Anchor Bolts		4 Cov Pls $54 \times 1\frac{1}{4}$	: 270 00	4 Cov. Pls $54 \times 1\frac{1}{4}$	: 270.00	$6L^s \cdot 7 \times 4 \times \frac{5}{8}$	: 38 88
		3"φ Upset		2 Cov. Pls. $18 \times 1\frac{1}{8}$	: 40.50	2 Cov. Pls $18 \times 1\frac{1}{8}$	: 40.50	1-Pl. $34 \times \frac{3}{4}$	: 25.50
		Area : 226.21□"		$6L^s \cdot 7 \times 4 \times \frac{5}{8}$	: 38 88	$6L^s \cdot 7 \times 4 \times \frac{5}{8}$	: 38 88	$2L^s \cdot 4 \times 4 \times \frac{1}{2}$	: 7 50
				1-Pl. $33 \times \frac{3}{4}$	: 24.75	1-Pl $34 \times \frac{3}{4}$	: 25 50	$4L^s \cdot 4 \times 4 \times \frac{7}{16}$	: 13 24
				$2L^s \cdot 4 \times 4 \times \frac{1}{2}$	: 7.50	$2L^s \cdot 4 \times 4 \times \frac{1}{2}$	: 7 50		
				$4L^s \cdot 4 \times 4 \times \frac{7}{16}$	: 13 24	$4L^s \cdot 4 \times 4 \times \frac{7}{16}$	: 13 24		
1	GROSS AREA - SQ. IN.				1,066.71		907 46		722 96
2	MOMENT OF INERTIA (IN^2, FT^2)				22,138		20,621		17,810
3	SECTION MODULUS (IN^2, FT)	139,392			3,554		3,310		2,859
		THRUST	MOMENT	THRUST	MOMENT	THRUST	MOMENT	THRUST	MOMENT
4	D.L. (KIPS)	+ 7,172	- 4,516	+ 7,160	-3,200 -3,484	+ 7,098	- 852 -3484	+ 6,864	+ 489 -1274
5	L+I. (+M) "	+ 846	+ 28,850	+ 865	+ 25,500	+ 883	+ 21,940	+ 808	+ 12,480
6	L+I. (-M) "	+ 638	- 27,690	+ 637	24,500	+ 636	- 21,360	+ 666	- 12,650
7	TEMPERATURE (T) "	± 77	± 9,818	± 78	± 9,180	± 79	± 8,484	± 81	± 6,318
8	ERROR (E) "	± 19	± 2,455	± 20	± 2,300	± 20	± 2,121	± 20	+ 1,580
9	WIND (W) "	± 2,865	± 20,160	± 2,860	± 14,600	± 2,381	± 12,400	± 1,943	± 1,600
10	TOTAL STRESSES IN KIPS								
11	D+L+I+T+E (+M)	+ 8,114	+ 36,608	+ 8,123	+ 33,780	+ 8,080	+ 31,693	+ 7,773	+ 20,867
12	D+L+I+T+E (-M)	+ 7,714	- 44,479	+ 7,699	- 39,464	+ 7,635	- 35,449	+ 7,429	- 21,822
13	D+L+I+T+E+W (+M)	+ 10,979	+ 56,768	+ 10,983	+ 48,380	+ 10,461	+ 44,093	+ 9,716	+ 22,467
14	D+L+I+T+E+W (-M)	+ 4,839	- 64,639	+ 4,839	- 54,064	+ 5.254	- 47,849	+ 5,486	- 23,422
15	UNIT STRESS (K/Sq. IN)	FROM + M	FROM - M	FROM + M	FROM - M	FROM + M	FROM - M	FROM + M	FROM - M
16	D+L+I+T+E								
17	f DIRECT	fc = 480	fc = 530	7.62	7.22	8.90	8 41	10 75	10 28
18	f BENDING	fs = 1,000	fs = 1,000	9.50	11.10	9 57	10 71	7 30	7 63
19	TOTAL			17.12	18 32	18.47	19.12	18 05	17 91
20	D+L+I+T+E+W								
21	f DIRECT	fc = 720	fc = 840	10 30	4.54	11.53	5.79	13 44	7 59
22	f BENDING	fs = 1,000	fs = 16,000	13.61	15.21	13 32	14 46	7 86	8 19
23	TOTAL			23.91	19 75	24 85	20.25	21 30	15 78

TABLE 1.—(*Continued*)

	10		9		8		7		6	
	2 Webs 144 × 13/16	: 234.00	2 Webs 144 × 13/16	: 234.00						
	8 L^s - 8 × 8 × 1	: 120.00	8 L^s - 8 × 8 × 1	: 120.00						
	2 Cov. Pls. 54 × 1 5/8	: 175.50	2 Cov. Pls. 54 × 1 3/8	: 148.50						
	6 L^s - 7 × 4 × 5/8	: 38.88	6 L^s - 7 × 4 × 5/8	: 38.88	←				Same as for Section 9	
	1 - Pl. 34 × 3/4	: 25.50	1 - Pl. 34 × 3/4	: 25.50						
	2 L^s - 4 × 4 × 1/2	: 7.50	2 L^s - 4 × 4 × 1/2	: 7.50						
	4 L^s - 4 × 4 × 7/16	: 13.24	4 L^s - 4 × 4 × 7/16	: 13.24						
1		614.62		587.62		587.62		587.62		587.62
2		13,721		12,701		12,701		12,701		12,701
3		2,229		2,070		2,070		2,070		2,070
	THRUST	MOMENT	THRUST	MOMENT	THRUST	MOMENT	THRUST	MOMENT	THRUST	MOMENT
4	+ 6,675	+ 793 - 540	+ 6,519	+ 972 - 280	+ 6,384	+ 1157 - 38	+ 6,268	+ 1248 + 132	+ 6,178	+ 1181 + 197
5	+ 757	+ 6,380	+ 570	+ 5,590	+ 403	+ 9,170	+ 472	+ 11,760	+ 456	+ 13,400
6	+ 858	- 6,400	+ 1,103	- 5,460	+ 1,048	- 8,400	+ 894	- 11,050	+ 893	- 11,950
7	± 83	± 4,381	± 85	± 2,660	± 87	± 1,147	∓ 88	± 166	∓ 89	± 1,285
8	± 21	± 1,095	± 21	± 665	± 22	± 287	∓ 22	± 41	∓ 22	± 321
9	∓ 1,539	± 4,200	∓ 1,168	± 5,980	∓ 843	± 3,660	∓ 575	± 4,280	∓ 365	± 490
10										
11	+ 7,539	+ 12,649	+ 7,135	+ 9,887	+ 6,896	+ 11,761	+ 6,630	+ 13,215	+ 6,513	+ 16,187
12	+ 7,429	- 12,416	+ 7,456	- 9,065	+ 7,323	- 9,872	+ 7,272	- 11,125	+ 7,172	- 13,359
13	+ 5,997	+ 16,849	+ 5,967	+ 15,867	+ 6,053	+ 15,421	+ 6,055	+ 17,495	+ 6,148	+ 16,697
14	+ 8,968	- 16,616	+ 8,624	- 15,045	+ 8,166	- 13,532	+ 7,847	- 15,405	+ 7,537	- 13,849
15	FROM + M	FROM - M	FROM + M	FROM - M	FROM + M	FROM - M	FROM + M	FROM - M	FROM + M	FROM - M
16										
17	12.27	12.09	12.14	12.69	11.74	12.46	11.28	12.38	11.08	12.21
18	5.67	5.57	4.78	4.38	5.68	4.77	6.38	5.37	7.81	6.45
19	17.94	17.66	16.92	17.07	17.42	17.23	17.66	17.75	18.89	18.66
20										
21	9.76	14.59	10.15	14.68	10.30	13.90	10.30	13.35	10.46	12.83
22	7.56	7.45	7.66	7.27	7.45	6.53	8.45	7.44	8.06	6.69
23	17.32	22.04	17.81	21.95	17.75	20.43	18.75	20.79	18.52	19.52

TABLE 1.—(*Continued*)

	5		4		3		2		1	
							2 Webs 144 × $\frac{13}{16}$	234.00	2 Webs 144 × $\frac{13}{16}$	234.00
							8 Ls 8 × 8 × 1	120.00	8 Ls 8 × 8 × 1	120.00
							2 Cov Pls 54 × $1\frac{1}{4}$	135.00	2 Cov Pls 54 × $1\frac{1}{8}$	121.50
	—Same as for Section 9—→						6 Ls 7 × 4 × $\frac{5}{8}$	38.88	6 Ls 7 × 4 × $\frac{5}{8}$	38.88
							1-Pl 34 × $\frac{3}{4}$	25.50	1-Pl. 34 × $\frac{3}{4}$	25.50
							2 Ls 4 × 4 × $\frac{1}{2}$	7.50	2 Ls 4 × 4 × $\frac{1}{2}$	7.50
							4 Ls 4 × 4 × $\frac{7}{16}$	13.24	4 Ls 4 × 4 × $\frac{7}{16}$	13.24
1	587.62		587.62		587.62		574.12		560.62	
2	12,701		12,701		12,701		12,194		11,688	
3	2,070		2,070		2,070		1,991		1,912	
	THRUST	MOMENT	THRUST	MOMENT	THRUST	MOMENT	THRUST	MOMENT	THRUST	MOMENT
4	+6,085	+917 +28	+6,018	+587 −254	+5,967	+151 −614	+5,931	−284 −1074	+5,911	+434 −1383
5	+535	+13,470	+618	+12,490	+699	+10,970	+740	+9,440	+729	+8,700
6	+803	−12,110	+716	−11,230	+678	−9,550	+677	−7,960	+731	−6,860
7	∓91	±2,212	∓91	±2,952	∓92	±3,505	∓93	±3,874	∓93	±4,058
8	∓23	±553	∓23	±738	∓23	±876	∓23	±968	∓23	±1,015
9	±208	±720	±94	±1,640	±20	±2,440	∓17	±4,150	∓17	±3,500
10										
11	+6,506	+17,152	+6.522	+16,767	+6,551	+15,502	+6,555	+13,998	+6,524	+13,339
12	+7,002	−14,847	+6,848	−15,174	+6,760	−14,545	+6,724	−13,876	+6,758	−13,316
13	+6,714	+17,872	+6,616	+18,407	+6,571	+17,942	+6,538	+18,148	+6,507	+16,839
14	+6,794	−15,567	+6,754	−16,814	+6,740.	−16,985	+6,741	−18,026	+6,775	−16,816
15	FROM +M	FROM −M	FROM +M	FROM −M	FROM +M	FROM −M	FROM +M	FROM −M	FROM +M	FROM −M
16										
17	11.07	11.92	11.10	11.65	11.15	11.50	11.42	11.71	11.64	12.05
18	8.29	7.17	8.10	7.33	7.49	7.03	7.03	6.97	6.98	6.96
19	19.36	19.09	19.20	18.98	18.64	18.53	18.45	18.68	18.62	19.01
20										
21	11.43	11.56	11.26	11.49	11.18	11.47	11.39	11.74	11.61	12.08
22	8.63	7.52	8.89	8.12	8.66	8.21	9.12	9.05	8.81	8.79
23	20.06	19.08	20.15	19.61	19.84	19.68	20.51	20.79	20.42	20.87

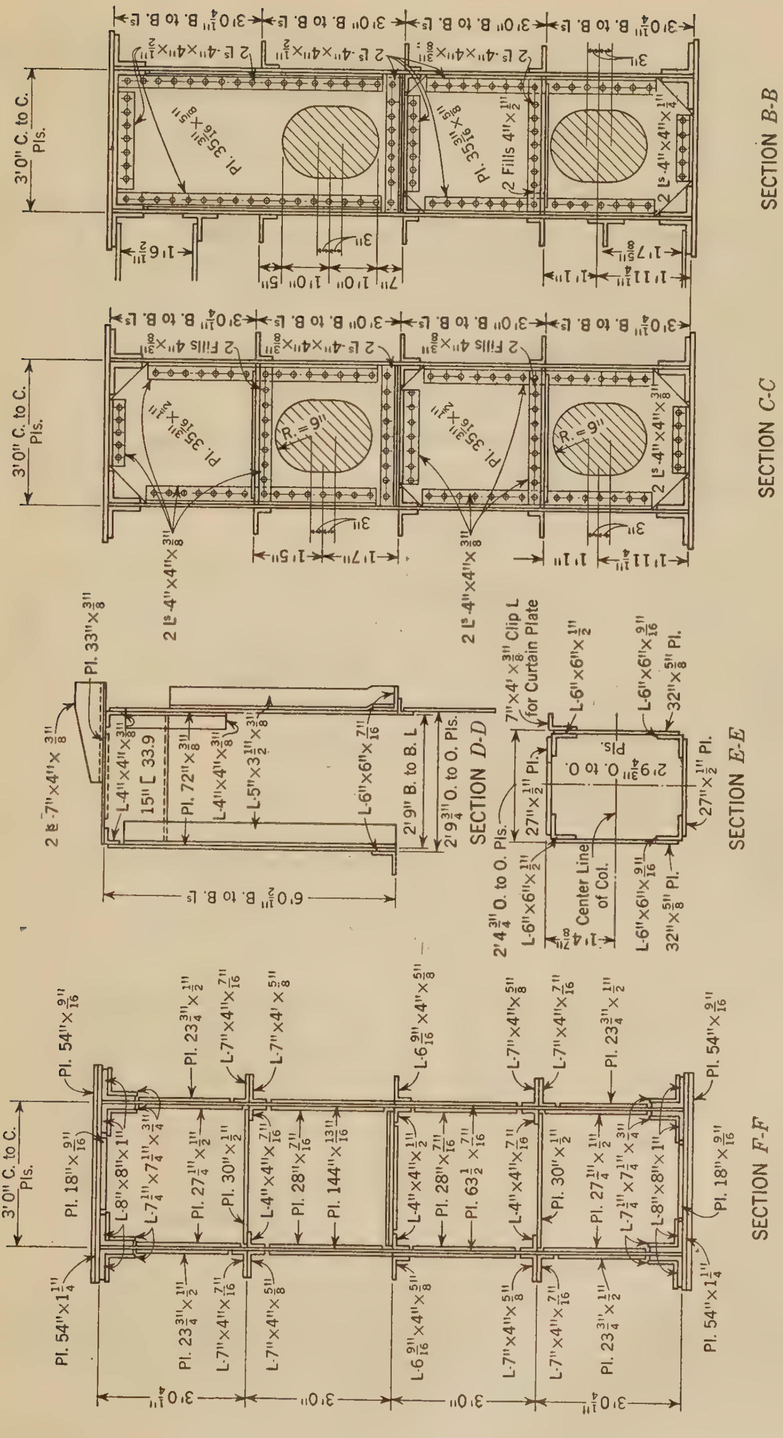

SECTION B-B
3' 0" C. to C. Pls.
3' 0¼" B. to B. Ls
3' 0" B. to B. Ls
2 Ls 4"×4"×½"
2 Ls 4"×4"×⅜"
Pl. 35 3/16"×5/8"
2 Fills 4"×½"
2 Ls 4"×4"×¼"
3"
1' 6½"
5" 1' 0" 1' 0" 7"
1' 1" 1' 11¼"
1' 7 5/8"
SECTION C-C
2 Fills 4"×⅜"
2 Ls 4"×4"×⅜"
Pl. 35 3/16"×½"
R.=9"
2 Ls 4"×4"×⅜"
1' 5" 1' 7"
1' 1" 1' 11¼"
SECTION D-D
2 Ls 7"×4"×⅜"
Pl. 33"×⅜"
L-4"×4"×⅜"
15" ⊏ 33.9
Pl. 72"×⅜"
L-4"×4"×⅜"
L-5"×3½"×⅜"
L-6"×6"×7/16"
2' 9" B. to B. L
2' 9¾" O. to O. Pls.
6' 0½" B. to B. Ls
SECTION E-E
7"×4"×⅜" Clip L for Curtain Plate
L-6"×6"×½"
L-6"×6"×9/16"
32"×⅝" Pl.
2' 9¼" O. to O. Pls.
27"×½" Pl.
2' 4¾" O. to O. Pls.
L-6"×6"×½"
Center Line of Col.
1' 4⅞"
SECTION F-F
Pl. 54"×9/16"
Pl. 54"×1¼"
Pl. 18"×9/16"
L-8"×8"×1"
L-7¼"×7¼"×¾"
Pl. 23¾"×½"
Pl. 27¼"×½"
Pl. 30"×½"
L-7"×4"×7/16"
L-7"×4"×⅝"
L-4"×4"×7/16"
Pl. 28"×7/16"
Pl. 144"×13/16"
L-6 9/16"×4"×⅝"
L-4"×4"×½"
Pl. 63½"×7/16"
3' 0¼"
3' 0"

Fig. 8.—Typical Cross Sections

ARCH ABUTMENTS

The abutments of each rib were constructed as independent units, and all four of the abutments were founded on solid rock which sloped up from the water at an angle of about 45°. All abutments were keyed into the rock by

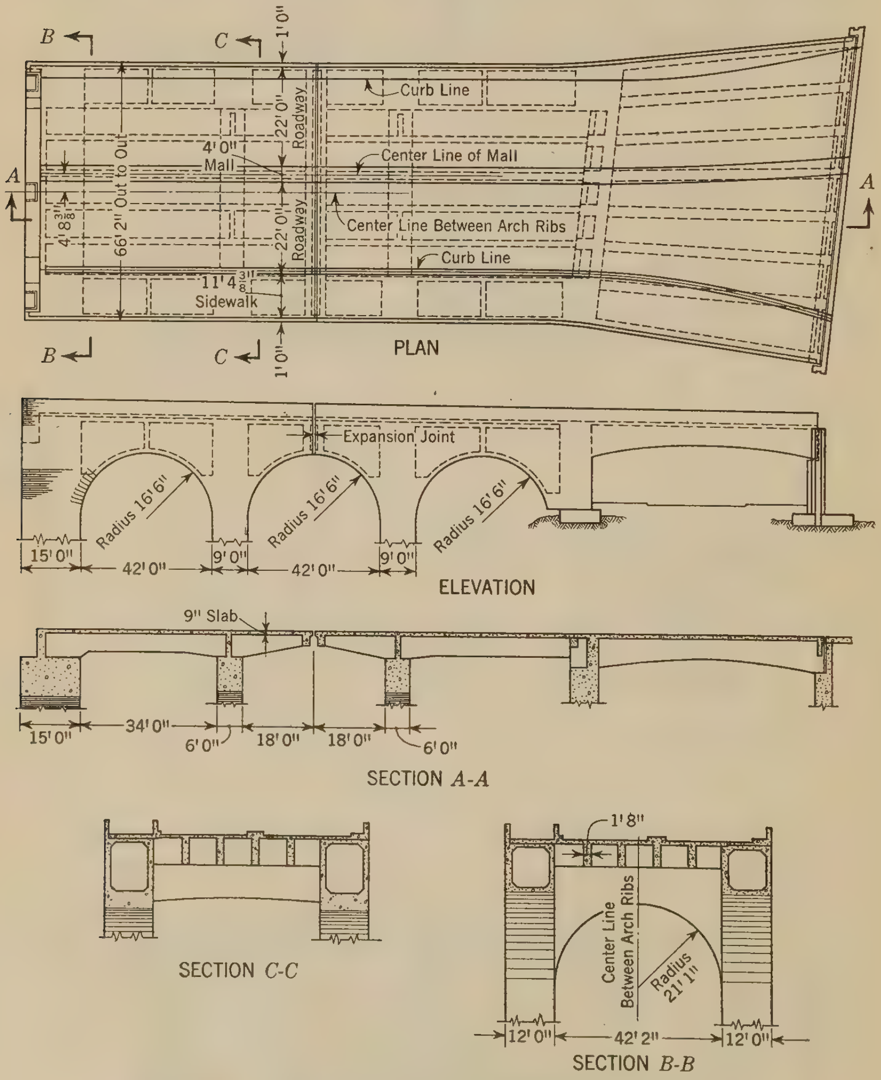

B
C
A
1'0"
Curb Line
4'0"
Mall
22'0" Roadway
Center Line of Mall
4'8 3/8"
66'2" Out to Out
Center Line Between Arch Ribs
Curb Line
11'4 3/8"
Sidewalk
PLAN
Expansion Joint
Radius 16'6"
Radius 16'6"
Radius 16'6"
15'0"
42'0"
9'0"
42'0"
9'0"
ELEVATION
9" Slab
15'0"
34'0"
6'0"
18'0"
18'0"
6'0"
SECTION A-A
SECTION C-C
1'8"
Center Line Between Arch Ribs
Radius 21'1"
12'0"
42'2"
12'0"
SECTION B-B

FIG. 9.—CONCRETE APPROACHES

cutting the surface of the rock into a series of horizontal and vertical steps that provided sufficient bearing area for both components of the rib thrust under any combination of loads. By referring to Fig. 2, it will be noted that the abutments rest on the upper layers of the Albion sandstone, and exert thrust against the Irondequoit and Wolcott limestones and the Thorold sandstone.

APPROACHES

The reinforced concrete approach structures on the two sides of the river are similar. Each approach consists of three arched spans over the sloping walls of the gorge and a beam span over a transverse highway. The concrete box arches are 12 ft wide and frame into columns 12 ft in width. The bridge deck between the arches is carried by reinforced concrete beam and girder construction, as shown in the cross-sectional view in Fig. 9. The longitudinal view at the center line of the bridge indicates the method of framing for this part of the structure. An expansion joint at the crown of the middle arched span completely separates the two halves, so that they act as cantilevers. The design for the concrete structures was made to provide for continuity. The girders for the spread girder span over River Road on the Canadian side are particularly heavy, the north and south girders having span lengths of 85 ft and 91 ft, respectively.

SUMMARY OF QUANTITIES

The principal quantities in the bridge are as follows:

Main Arch Span.—	
Concrete in deck, in cubic yards	1,800
Reinforcing bars in deck, in pounds	210,000
Reinforcing trusses for roadway slab, in pounds	300,000
Carbon steel in floor system, in pounds	2,410,000
Silicon steel in floor system (floor beams), in pounds	480,000
Carbon steel in spandrel columns, in pounds	1,140,000
Carbon steel in rib laterals, in pounds	900,000
Silicon steel in arch ribs and skewbacks, in pounds	6,220,000
Concrete in arch abutments, in cubic yards	7,100
Reinforcing steel in arch abutments, in pounds	300,000
American Approach.—	
Concrete, in cubic yards	7,800
Reinforcing steel, in pounds	840,000
Canadian Approach.—	
Concrete, in cubic yards	10,800
Masonry facing, in cubic feet	5,400
Reinforcing steel, in pounds	1,290,000

PART II.—DEFLECTION EFFECTS AND DESIGN FORMULAS FOR FLEXIBLE ARCHES

The elastic theory has been in use for many years in the design of structures. In general, any change of geometry of the structure due to strain is neglected in this theory. When the elastic displacements are small, the error involved is usually small; but, when the elastic displacements are appreciable, the error introduced may be an important factor in the design.

More exact methods have been developed for some of the problems of structural design in which deflections should be considered. The "deflection theory" for suspension bridges, which takes into account the elastic displacements of the structure, was first used in the United States in the design of the Manhattan Bridge in New York. It was amplified and developed further in the design of later suspension bridges. This theory, which results in lower bending moments in the stiffening truss of a suspension bridge than those calculated by the elastic theory, is considered applicable for practically all spans.

Columns under eccentric load or with initial curvature and columns under the combined action of axial and transverse loads continue to receive considerable attention. The rib of an arch bridge, being a compression member subject to both direct and bending stress, will have deflection characteristics similar to those of a column under the same type of loading.

Several deflection theories are available for arch ribs (see Appendix I). In each of these theories, certain assumptions have been made as to the variation of moment of inertia in the arch rib, and, in some cases, the part of span loaded with live load is specified. However, in one of these theories, referred to as "Stern's" in Appendix I, the arch axis can be divided into any desired number of sections, each having a constant moment of inertia, and live load can be placed on any part of the structure. This latter method was used to compute the quarter-point deflection for a preliminary two-hinged arch design, using the same loading as that required to produce the maximum elastic-theory moments at the quarter point. This computed deflection was 60% greater than the value given by the elastic theory.

When the Niagara design was started, a two-hinged arch rib was analyzed first by the elastic theory, with total stresses not exceeding those set up as allowable working stresses. The resulting rib, which was 14 ft in depth and weighed 3,000 lb per ft, then was checked by an analysis which included the effect of the deflection. This more exact calculation showed that the quarter-point deflection of 12 in., due to (dead + live + impact)-load, as computed by the elastic theory, increased to 21 in. under the exact method, and that the true unit stresses were 28% greater than those indicated by the elastic theory. Perhaps more important, this calculation showed that an increase in dead and live loading of 30% above the design loads would increase the stresses to the yield strength of the steel, so that the safety factor of the structure, instead of being 1.875 as indicated by the elastic-theory unit stresses, was only 1.30. A two-hinged arch rib with a true safety factor of 1.875 under the design loads could have been obtained with a rib of 15-ft depth, weighing 3,800 lb per ft.

In the case of the two-hinged arch rib, the calculation of the quarter-point vertical deflection of 12 in. indicated that the rib had moved radially away from the thrust line approximately 13.6 in. This would produce an additional moment equal to the thrust at that point times the 13.6-in. displacement. In the case of this preliminary design, such a moment amounted to more than 8,600 ft-kips as compared to the elastic-theory moment of 22,950 ft-kips. Furthermore, this additional moment would produce still more deflection, which finally would total 23.4 in. in the radial direction, and a total additional moment of more than 14,500 ft-kips.

In contrast to the flexibility of the aforementioned two-hinged arch rib, the hingeless arch rib which was used in the final design of the bridge is 12 ft deep and weighs 2,600 lb per ft. It has a quarter-point deflection due to (dead + live + impact)-load of 5.5 in. by the elastic analysis. This increases to 6.9 in. by the more exact computation taking deflections into account.

The preliminary calculations for the two-hinged arch rib were sufficient to show that the elastic theory was inadequate for the design of such a structure. Had the two-hinged rib of the first preliminary design been built, the structure undoubtedly would have been unsatisfactory in its performance.

The results of the calculations also suggested the advisability of further study by the use of models, especially for those structures in which large displacements occur. Such a project was undertaken at Princeton University under the supervision of Professor Timby, as reported elsewhere in this Symposium. Three different models were used and a number of different loading conditions were applied to each.

The factor of safety (N), as used in the preceding paragraphs and elsewhere in this paper, is defined by the following relation:

$$N = \frac{W_{yp}}{W_w} \quad \text{(1)}$$

in which W_{yp} is the (dead + live + impact)-load which, together with the effects of temperature and wind, produces a yield point stress in the arch rib, and W_w is the (dead + live + impact)-working load or design load. For this purpose W can be assumed to be the panel load.

The computations for the deflection effects in the preliminary two-hinged arch rib and for the hingless rib as built were based upon finding the equilibrium position of the arch rib by trial. That is, after computing the displacements from the elastic-theory moments and the change in moments resulting from the displaced position of the arch axis, correction moments to displace the arch rib to its equilibrium position were estimated. From these moments, displacements were calculated and second correction moments estimated; and this procedure was repeated until the equilibrium position of the rib had been determined. The values for deflections and moments computed by this procedure will be referred to herein as "exact."

Complete calculations were made for the loading condition that produces maximum bending moment at the quarter point of the span. After exact values were calculated and confirmed by the results of model tests, a simple, approximate procedure was developed which gives an accuracy sufficient for preliminary design and, in the ordinary structure, for final design.

DEFLECTION EFFECTS IN TWO-HINGED ARCH RIB

The relations between the forces acting on the structure and the resulting radial displacements are expressed by the differential equation (see Fig. 10):

$$\frac{d^2u}{ds^2} + \frac{u}{(r_o)^2} = -\frac{M}{E\,I} - \frac{T}{r_o\,E\,A} \quad \text{(2)}$$

in which ds = differential element of length of the arch rib; u = radial component of displacement of the rib; r_o = original radius of curvature of rib; M = final bending moments; T = final direct thrust; I = moment of inertia of rib cross section; A = cross-sectional area of rib; and E = modulus of elasticity. The vertical and horizontal displacements η and ρ are

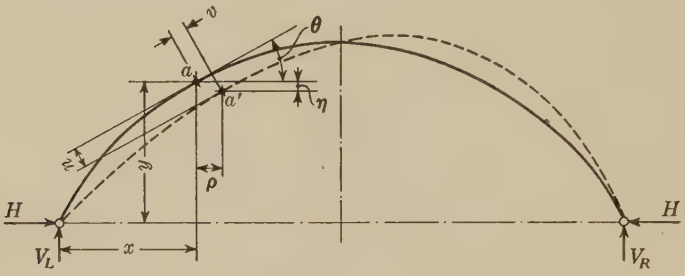


FIG. 10.—DEFLECTION EFFECT IN A TWO-HINGED ARCH RIB

$$\eta = u \cos \theta - v \sin \theta \ldots\ldots\ldots\ldots (3a)$$

and

$$\rho = u \sin \theta + v \cos \theta \ldots\ldots\ldots\ldots (3b)$$

in which v is the tangential displacement.

In the usual elastic analysis for the two-hinged arch, the condition for equilibrium at any point, A, is expressed by the equation,

$$M_{ae} = M'_a - H\, y_a \ldots\ldots\ldots\ldots (4)$$

If the effects of displacements are included, Eq. 4 becomes,

$$M_{af} = M'_a + \Delta M'_a - (H + \Delta H)\,(y_a - \eta_a) \ldots\ldots\ldots\ldots (5)$$

in which M'_a = simple beam moment at point, A; H = horizontal component of end reactions; y_a = vertical ordinate of point, A, nominal outline; η_a = vertical component of deflection of point, A; and $\Delta M'_a$ and ΔH = change in M'_a and H resulting from the change in geometry of the rib.

The correct position of the arch axis is one for which Eqs. 2 and 5 are satisfied. In the analysis, the terms $\frac{u}{(r_o)^2}$ and $\frac{T}{r_o\, E\, A}$ of Eq. 2 were neglected, since these terms are relatively small for ribs of the curvature considered.[3a] Eq. 2 then reduces to:

$$\frac{d^2u}{ds^2} = -\frac{M}{E\,I} \ldots\ldots\ldots\ldots (6)$$

As a first approximation, the moments M_e determined by the elastic analysis were applied to the rib and the resulting displacements determined. If the rib is considered to be in this deformed position, it is not in equilibrium under the action of the computed forces because Eq. 5 is not satisfied.

A second approximation was made by calculating the moments M_1 for the rib in the deformed position, the coordinates being $x + \rho$ and $y - \eta$, instead of x and y.

[3a] See corrections and revised Eqs. 16, 17, and 21, in the closing discussion.

The moments M_1 differed from the moments M_e by $\Delta m_1 = M_1 - M_e$. This difference for the two-hinged arch can be considered to consist of the following terms:

$$\Delta m_1 = \Delta M' - y\,\Delta H + H\,\eta + \eta\,\Delta H \ldots\ldots\ldots\ldots (7)$$

By applying these moments to the rib and finding the resulting deflections, moments M_2 can be computed from the new deformed position. By continuing this procedure, the equilibrium position of the arch when the moments and deflections are consistent can be determined.

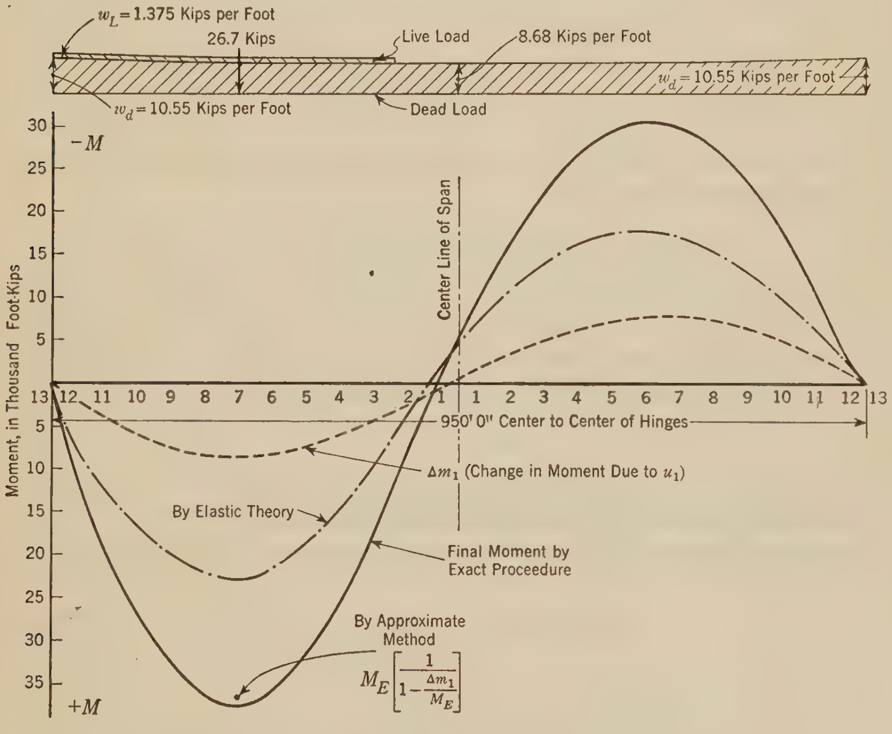


Fig. 11.—Variation in M_e for a Two-Hinged Arch Bridge (L = 950 Ft; Rise = 150 Ft; I = 116.14 Ft^4 at the Crown and 120.24 Ft^4 at the Quarter Point)

For the Niagara two-hinged arch rib, it was noted that the variation in Δm_1, under loadings which produce maximum moment in sections near the quarter points of the span, was similar to the variation in M_e (Fig. 11), and that a close approximation could be obtained by assuming that the deflections at point A in the vicinity of the quarter point, produced by moment Δm_1, are given by:

$$\Delta u_{a1} = u_{ae}\left(\frac{\Delta m_{a1}}{M_{ae}}\right) \ldots\ldots\ldots\ldots (8)$$

Similarly the additional moment Δm_{a2}, resulting from the deflection Δu_{a1}, is given by:

$$\Delta m_{a2} = \Delta m_{a1} \left(\frac{\Delta u_{a1}}{u_{ae}} \right) = \Delta m_{a1} \left(\frac{\Delta m_{a1}}{M_{ae}} \right) = M_{ae} \left(\frac{\Delta m_{a1}}{M_{ae}} \right)^2 \ldots\ldots.(9)$$

Assuming successive corrections to be obtained in a similar manner, the final moment M_{af} at point A can be written:

$$M_{af} = M_{ae} + M_{ae} \left(\frac{\Delta m_{a1}}{M_{ae}} \right) + M_{ae} \left(\frac{\Delta m_{a1}}{M_{ae}} \right)^2 + M_{ae} \left(\frac{\Delta m_{a1}}{M_{ae}} \right)^3 + \cdots.(10)$$

or

$$M_{af} = M_{ae} \left(\frac{1}{1 - \dfrac{\Delta m_{a1}}{M_{ae}}} \right) \ldots\ldots\ldots\ldots(11)$$

An approximate value for the radial displacement can be obtained in a similar manner, giving:

$$u_{af} = u_{ae} + u_{ae} \left(\frac{\Delta m_{a1}}{M_{ae}} \right) + u_{ae} \left(\frac{\Delta m_{a1}}{M_{ae}} \right)^2 + \cdots = \frac{u_{ae}}{1 - \dfrac{\Delta m_{a1}}{M_{ae}}} \ldots.(12)$$

Furthermore, it was found that, for a rib of the Niagara arch properties, the following relation was a very close approximation:

$$\Delta m_{a1} = T_a \, u_{ae} \ldots\ldots\ldots\ldots(13)$$

in which T_a = the total axial thrust at the point A and u_{ae} = the radial displacement of point A as computed by the elastic theory.

Substituting Eq. 13 in Eqs. 9 and 10, the following expressions result:

$$M_{af} = M_{ae} \left[\frac{1}{1 - T_a \left(\dfrac{u_{ae}}{M_{ae}} \right)} \right] \ldots\ldots\ldots\ldots(14a)$$

and

$$u_{af} = u_{ae} \left[\frac{1}{1 - T_a \left(\dfrac{u_{ae}}{M_{ae}} \right)} \right] \ldots\ldots\ldots\ldots(14b)$$

A comparison of the moments computed by Eq. 14*a* and those obtained by model tests at Princeton University (described in the succeeding paper in this Symposium) shows that the agreement is quite good when the value of the term in brackets in Eqs. 14 is less than 2.

For a perfectly fabricated rib, the final (dead load + live load + impact load) moment being given by Eq. 14*a*, the total resulting (dead load + live load + impact load) stress in the extreme fiber at a point A (this point being assumed as between 0.10 L and 0.30 L from either hinge) can be written as:

$$s = \frac{T_a}{A_a} \pm \frac{M_{ae}}{S_a}\left[\frac{1}{1 - T_a\left(\frac{u_{ae}}{M_{ae}}\right)}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

in which A_a = cross-sectional area of rib at the point A; and S_a = section modulus of rib at the point A.

If the useful capacity of the structure is defined as the load that produces a stress at any section equal to the elastic limit s_y of the material and the factor of safety N, as previously defined, is introduced, the following equation should be satisfied by a properly designed rib:[3a]

$$s_y = N\left(\frac{T_a}{A_a}\right) + N\,\frac{M_{LL+I}}{S_a}\left[\frac{1}{1 - N\,T_a\left(\frac{u_{ae}}{M_{LL+I}}\right)}\right] + \frac{N\,M_{DL}}{S_a} + \frac{T_t + T_w}{A_a} + \frac{M_t + M_w}{S_a} \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

in which T_a is the elastic-theory thrust at point A due to (dead + live + impact)-load; T_t and T_w are the thrusts at that point due to temperature effect and wind, respectively; and M_{DL}, M_{LL+I}, M_t, and M_w are the elastic-theory moments due to (dead + live + impact)-load, temperature effect, and wind, respectively.

In the special case of a perfectly formed rib carrying loads that produce no bending moment, the following condition should also be satisfied: $N\,T_{0.25}\left(\frac{u_{0.25E}}{M_{0.25E}}\right) < 1$—in which $u_{0.25E}$ and $M_{0.25E}$ are radial displacement and the moment that would be produced at the quarter point of the span by an assumed uniform live load over half the span, and $T_{0.25}$ is the thrust at the quarter point due to dead load only. Inspection of Eq. 14*b* shows that as the term $T_{0.25}\,\frac{u_{0.25E}}{M_{0.25E}}$ approaches unity, any small displacement of the rib at the one-quarter point becomes very large. The rib, therefore, would buckle in a two-loop form. The value of thrust which makes this term equal to unity agrees closely with the critical buckling thrust obtained for special cases by more nearly mathematically exact theories.[4]

Practically, of course, it is impossible to fabricate and erect an arch rib that will conform exactly to a prescribed outline. The worst form of the variation for accidental errors in rib outline would be the same variation as the live-load displacements, u. With accurate workmanship, the maximum probable ordinate of a departure of this type can be expressed approximately as: $e' = \frac{\sqrt{L}}{30}$, but not less than 0.25 . . . when e' is in inches and L, in feet.

For the Niagara arch span, L = 950 ft, e' = 1.03 in.; and, for a span of 225 ft, the value of e' would be 0.5 in.

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[4] "Theory of Elastic Stability," by S. Timoshenko, 1st Ed., McGraw-Hill Book Co., Inc., New York and London, 1936, p. 225.

An error of this magnitude, although insignificant in an elastic-theory analysis, may produce appreciable secondary moments and should be considered in design. The effect of such error incorporated in Eq. 16 gives finally:[3a]

$$\frac{s_y}{N} = \frac{T_a}{A} + \frac{M_{LL+I} + T_a \frac{e'}{N}}{S_a} \times \frac{1}{1 - N\,T_a \frac{u_{ae}}{M_{LL+I}}}$$

$$+ \frac{M_{DL}}{S_a} + \frac{1}{N}\left(\frac{T_t + T_w}{A_a} + \frac{M_t + M_w}{S_a}\right) \ldots\ldots\ldots\ldots\ldots (17)$$

Eq. 17 is proposed for design use. The following values of s_y and N, which have been taken from the 1941 A.R.E.A. Specifications for Steel Railway Bridges, Appendix A, indicate the numerical values used in current design for (dead + live + impact)-load stresses:

	s_y (lb per sq in.)	N	$\frac{s_y}{N}$
Structural steel	33,000	1.833	18,000
Silicon steel	45,000	1.875	24,000
Nickel steel	55,000	1.83	30,000

When unusual outline or loadings are encountered, or when the importance of the structure warrants, a check of Eq. 16 can be obtained at a few critical points by successive approximations of the elastic theory, previously described and referred to as the "exact method."

As a numerical example, Eq. 15 will be applied to a preliminary two-hinged arch design for the Niagara span: The stresses at the quarter point are shown in Table 2. For this case: The depth of the rib is 14 ft; the section area is 614 sq in.; the section modulus is 29,000 in.^{3}; u_{ae} = 13.6 in. for positive moment and 14.0 in. for negative moment. Using the moments and thrusts computed by the elastic theory, the following stresses are found:

TABLE 2.—STRESSES AT THE QUARTER POINT IN A PRELIMINARY TWO-HINGED ARCH

Loading	Elastic Theory				Deflection Theory	
	Thrust (Kips)		Moment (Ft-Kips)		Thrust (Kips)	Moment (Ft-Kips)
	+M	−M	+M	−M	+M	+M
Dead load	−7,126	−7,126	+353	+353	−7,554	353
Live load	−425	−709	+22,594	−22,076		+37,123
Impact	−37	−62	+1,990	−1,943	−37	+3,270
Σ	−7,588	−7,897	+24,937	−23,666	−7,591	+40,746

For maximum positive moment—

$$s = \frac{7,588}{614} + \frac{24,937 \times 12}{29,000} = 12.36 + 10.31 = 22.67 \text{ kips per sq in.}$$

For maximum negative moment—

$$s = \frac{7{,}897}{614} + \frac{23{,}666 \times 12}{29{,}000} = 12.86 + 9.79 = 22.65 \text{ kips per sq in.}$$

The stress, computed from the moments and thrusts obtained by the deflection analysis, is:
For maximum positive moment—

$$s = \frac{7{,}591}{614} + \frac{40{,}746 \times 12}{29{,}000} = 12.36 + 16.86 = 29.22 \text{ kips per sq in.}$$

Using the approximate Eq. 15, the resulting stresses are:
For maximum positive moment—

$$s = \frac{7{,}588}{614} + \frac{24{,}584 \times 12}{29{,}000} \times \frac{1}{1 - 7{,}588 \times \dfrac{13.6}{12 \times 24{,}584}}$$

$$+ \frac{353 \times 12}{29{,}000} = 12.36 + 10.17 \times \frac{1}{0.6502} + 0.15 = 28.15 \text{ kips per sq in.}$$

For maximum negative moment—

$$s = \frac{7{,}897}{614} + \frac{24{,}019 \times 12}{29{,}000} \times \frac{1}{1 - 7{,}897 \times \dfrac{14.0}{12 \times 24{,}019}}$$

$$- \frac{353 \times 12}{29{,}000} = 12.86 + 9.94 \times \frac{1}{0.6164} - 0.15 = 28.83 \text{ kips per sq in.}$$

DEFLECTION EFFECT IN A HINGELESS ARCH RIB

In the elastic analysis for the fixed arch rib, the condition for equilibrium at any point A is expressed by the equation:

$$M_a = M'_a + M_{oa} - H\, y_a \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

When the effects of displacements are included, Eq. 18 becomes:

$$M_{af} = M'_a + \Delta M'_a + M_{oa} + \Delta M_{oa} - (H + \Delta H)(y_a - \eta) \ldots\ldots (19)$$

in which M_{oa} = moment at point A due to end moments; and $\Delta M'_a$, ΔH, ΔM_{oa} = increments resulting from the change in geometry of the rib. The correct position of the arch axis is that for which Eqs. 2 and 19 are satisfied.

The equilibrium position was determined for the Niagara arch rib for the live-load condition which produces maximum positive moment at the quarter point of the span. This was accomplished in a manner similar to that used for the two-hinged rib. The results of this analysis are shown partly in Fig. 12 in which the first correction moment Δm_1 is approximately proportional to the moments of the elastic theory. Assuming then that this relation exists, the final moment and radial displacement can be estimated by the same procedure used in obtaining Eqs. 10 and 12 for the two-hinged arch.

For the hingeless arch, these equations cannot be simplified by introducing $T\, u_e$ as an approximation for Δm_1, as in the case of the two-hinged arch.

The stress is given by:

$$s = \frac{T_a}{A_a} \pm \frac{M_{ae}}{S_a}\left(\frac{1}{1 - \dfrac{\Delta m_{a1}}{M_{ae}}}\right) \quad \text{(20)}$$

The moment produced by an initial error in rib outline was found to be about 70% of the moment that would be produced in a two-hinged rib by the same

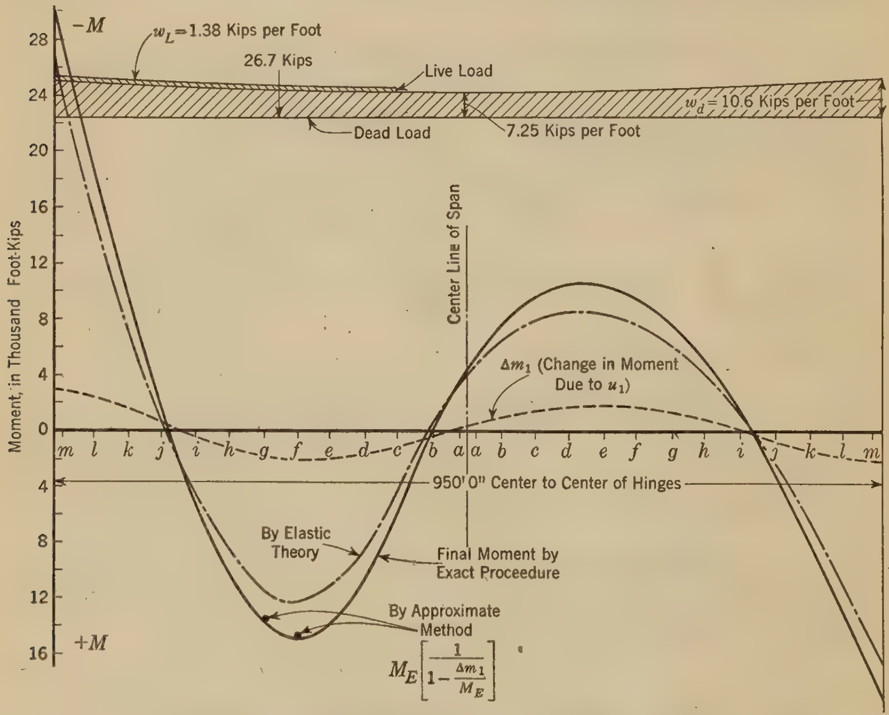

FIG. 12.—VARIATION IN M_e FOR A FIXED ARCH (L = 950 FT; RISE = 150 FT; I = 81.17 FT^4 AT THE CROWN AND 88.20 FT^4 AT THE QUARTER POINT)

error. Therefore a properly designed hingeless arch rib should satisfy the following equation:[3a]

$$\frac{s_y}{N} = \frac{T_a}{A_a} + \frac{M_{LL+I} + 0.7\,T_a\dfrac{e'}{N}}{S_a}\left[\frac{1}{1 - N\left(\dfrac{\Delta m_{a1}}{M_{LL+I}}\right)}\right] + \frac{M_{DL}}{S_a} + \frac{1}{N}\left[\frac{T_t + T_w}{A_a} + \frac{M_t + M_w}{S_a}\right] \quad \text{(21)}$$

The calculations for Δm_{a1} are quite lengthy as the rib outline is generally unsymmetrical after the first deflections are added to the ordinates. However, when a sufficient variety of ribs has been computed exactly, or when model tests, comparable to those made at Princeton University for the two-hinged rib, have been made for the hingeless rib, it may be possible to write an approximate expression for the value Δm_1.

As a numerical example Eq. 20 will be applied to the final Niagara ribs. Similarly to Table 2, the stresses at the quarter point are shown in Table 3. For this case: The depth of rib is 12 ft; the section area, 553.96 sq in.; the section modulus, 24,600 in.^{3} and $\Delta m_{a1} = 2{,}018$ ft-kips, for positive moment, and $\Delta m_{a1} = -1{,}888$ ft-kips for negative moment.

TABLE 3.—STRESSES AT THE QUARTER POINT IN THE HINGELESS NIAGARA ARCH

Loading	Elastic Theory				Deflection Theory	
	Thrust (Kips)		Moment (Ft-Kips)		Thrust (Kips)	Moment (Ft-Kips)
	$+M$	$-M$	$+M$	$-M$	$+M$	$+M$
Dead load	−6,062	−6,105	+2,138	+657	−6,455	+16,628
Live load	−398	−763	+11,948	−10,640		
Impact	−35	−67	+1,051	−937	−35	+1,230
Σ	−6,495	−6,935	+15,137	−10,920	−6,490	+17,858

As before, using the moments and thrusts computed by the elastic theory, the following stresses are found:

For maximum positive moment—

$$s = \frac{6{,}495}{553.96} + \frac{15{,}137 \times 12}{24{,}600} = 11.72 + 7.38 = 19.10 \text{ kips per sq in.}$$

For maximum negative moment—

$$s = \frac{6{,}935}{553.96} + \frac{10{,}920 \times 12}{24{,}600} = 12.52 + 5.33 = 17.85 \text{ kips per sq in.}$$

The stress obtained from the deflection-theory moments and thrusts is as follows:

For maximum positive moment—

$$s = \frac{6{,}490}{553.96} + \frac{17{,}858 \times 12}{24{,}600} = 11.72 + 8.71 = 20.43 \text{ kips per sq in.}$$

Using the approximate Eq. 20, the following stresses are obtained:

For maximum positive moment—

$$s = \frac{6{,}495}{553.96} + \frac{12{,}999 \times 12}{24{,}600} \times \frac{1}{1 - \dfrac{2{,}018}{12{,}999}} + \frac{2{,}138 \times 12}{24{,}600}$$

$$= 11.72 + 6.34 \times \frac{1}{0.8448} + 1.04 = 20.27 \text{ kips per sq in.}$$

For maximum negative moment—

$$s = \frac{6{,}935}{553.96} + \frac{11{,}577 \times 12}{24{,}600} \times \frac{1}{1 - \dfrac{1{,}888}{11{,}577}} - \frac{657 \times 12}{24{,}600}$$

$$= 12.52 + 5.65 \times \frac{1}{0.8369} - 0.32 = 18.95 \text{ kips per sq in.}$$

CONCLUSIONS

Four basic conclusions seem justifiable from this paper:

1. The useful strength of an arch rib is dependent in part upon the magnitude of the product of deflection under live load and total thrust.

2. In flexible arches the usual method of applying the elastic theory will lead to an appreciable overestimate of the strength of the structure. This error is especially serious in the case of long spans with heavy dead load, such as the Niagara arch bridge.

3. An attempt has been made to set up practical design formulas, for both two-hinged and hingeless ribs, that include deflection effects.

4. Additional model tests would aid greatly in confirming and simplifying the basis of design suggested in this article.

APPENDIX I

RÉSUMÉ OF AVAILABLE DEFLECTION THEORIES

J. Melan[5] developed a deflection theory for two-hinged arch ribs with parabolic axis and constant cross section. Horizontal displacement components were neglected and the further approximation was used that $\frac{d^2u}{ds^2} = \frac{d^2\eta}{dx^2}$.

S. Kasarnowsky[6] obtained a solution for the deflection stresses in a constant-section two-hinged arch rib with parabolic arch axis. He assumed that $\eta = u \cos \theta$ and that there was no horizontal displacement. The analysis is restricted to the case of live load extending over one half of the span.

B. Fritz[7] developed equations for three-hinged, two-hinged, and hingeless arch ribs of constant cross section and parabolic arch axes. Horizontal displacements were neglected and the assumption was made that $\eta = u \cos \theta$. Fritz's equations are applicable for a uniform live load over any part of the span length.

A. Freudenthal[8] assumed that the radial displacement is equal to the vertical component of deflection η. He obtained a solution for the stresses in either a two-hinged rib or a hingeless arch rib with parabolic axis and a variation of cross section given by $I = I_c \sec \theta$.

In a deflection theory for two-hinged arch ribs, George Stern,[9] Assoc. M. Am. Soc. C. E., assumed that there were no horizontal displacements of vertical loads and that $\eta = u \cos \theta$. The variation in cross section of rib was taken into account by assuming the arch to be divided into any number of subdivisions with constant section over each element. This theory may be used for any shape of arch axis and any loading condition.

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[5] "Genauere Theorie des Zweigelenkbogens mit Beruecksichtigung der durch die Belastung erzeugten Formaenderung," by J. Melan, *Handbuch der Ingenieur-wissenschaften*, Vol. 2, 1906.

[6] "Beitrag zur Theorie weitgespannter Brueckenbogen mit Kaempfergelenken," by S. Kasarnowsky, *Stahlbau*, Copy 6, 1931.

[7] "Theorie und Berechnung vollwandiger Bogentraeger bei Beruecksichtigung des Einflusses der Systemverformung," by B. Fritz, Berlin, 1934.

[8] "Deflection Theory for Arches," by A. Freudenthal, publications by International Association for Bridge and Structural Engineering, Vol. 3, 1935.

[9] "A Deflection Theory of Two-Hinged Arch Ribs," by G. Stern, thesis No. 516, Dept. of Civ. Eng., Columbia Univ., New York, N. Y. (a résumé of theoretical studies made at the Technical Univ. of Vienna, Austria).

MOMENTS IN FLEXIBLE TWO-HINGED ARCHES

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Lawrence M. Mead, Jr.,[11] Jun. Am. Soc. C. E.,
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Synopsis

The research described in this paper was planned to furnish information to assist in the design of arch ribs sufficiently flexible to sustain appreciable deflections before becoming overstressed. Experience has shown that the classic elastic theory, when applied to such arches, yields calculated stresses which are considerably less than existing stresses unless extended by successive approximations which consider the effect of the rib deflections on the moments. The work consisted of experimental observations of the behavior of loaded circular two-hinged steel arch models of constant section. The results of these observations are presented graphically and have been treated analytically to provide: (*a*) Practical limits of flexibility for the design of two-hinged arch ribs by straight elastic theory; (*b*) material assistance in the preliminary design of arches; and (*c*) actual design methods for certain two-hinged arch ribs which are too flexible for analysis by elastic theory. Within reasonable limits, the results may be applied to noncircular profiles and to varying sections by use of proper relative dimensions.

Test Procedure

Experimental observations were made on three models, having properties described in Fig. 13. Each rib had a section 1.00 in. wide by 0.25 in. deep and was formed cold to fit a template of the desired circular radius to within 0.01 in. Material was cold finished steel with an average value for E of 29,520,000 lb per sq in., maximum variation noted being 1.03%. One of these models is shown mounted in the special loading frame built for these tests in Fig. 14.

Both dead load and live load, accurate to 0.01 lb, were suspended from the model on hanger rods (Fig. 14) at the E-points (Fig. 13). Several different values of dead load, $\overline{DL}$, were used, and each was defined by its intensity in pounds per unit of length at the crown. The intensity of any particular dead load was varied along the arch in a manner calculated to keep the resultant dead-load thrust line on the center line of the rib. Live loads were maintained uniform for the extent of the loading.

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Dead loads and live loads designed and used on the models are listed in Table 4, in which R denotes the ratio of dead load to live load ($\overline{DL}/\overline{LL}$). These various loads were designed to extend beyond practical limits in both directions of magnitude, varying from loads which would be considered very light, in view of the stiffness of the section, to loads which caused the arch to approach a buckling condition. The particular loads that corresponded most closely to the original two-hinged design for the Niagara Rainbow arch mentioned in the preceding paper in the Symposium were: $\overline{DL} = 2.50$ lb per in.; $\overline{LL} = 0.50$ lb per in.; $R = 5.00$.

TABLE 4.—LIVE LOADS, IN POUNDS PER INCH

Dead-load intensity, $\overline{DL}$, in lb per in.	Values of $R \left(= \frac{\overline{DL}}{\overline{LL}}\right)$				
	2.50	5.00	7.50	10.00	15.00
1.25	0.50	0.25	0.167	0.125	0.083
2.50	1.00	0.50	0.333	0.250	0.167
3.75	1.50	0.75	0.500	0.375	0.250
5.00	2.00	1.00	0.667	0.500	0.333

The models were mounted in the apparatus shown in Fig. 14, and observations were made to determine deflection, horizontal component at the reactions, and maximum moment at a point (only the latter being presented in this paper). All observations for maximum moment were made between the condition of dead load on the arch, and the condition of dead load plus live load placed to produce the maximum value of moment at the point being studied—the extent and position of the live load being determined by trial. The hinges were made free to rotate by using clean ball bearings and vibrators at each reaction. Checks indicated that friction was negligible. A constant span was maintained by a horizontal adjustment of one support during each loading cycle to provide a specified constant distance between hinges. This distance was controlled by two 0.0001-in. micrometer dials, shown at the left reaction in Fig. 14.

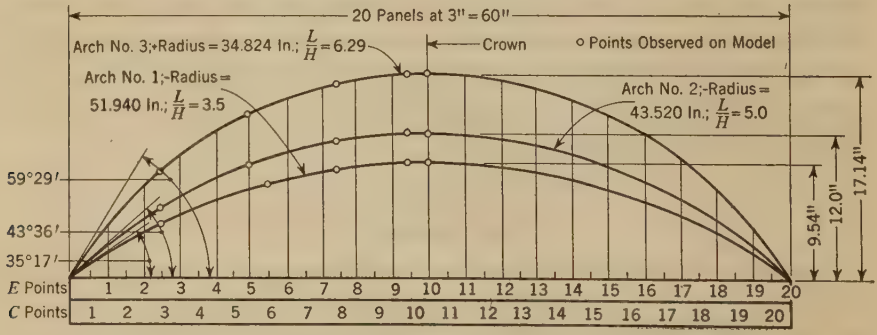
20 Panels at 3" = 60"
Arch No. 3;-Radius = 34.824 In.; $\frac{L}{H}$ = 6.29
Crown
○ Points Observed on Model
Arch No. 1;-Radius = 51.940 In.; $\frac{L}{H}$ = 3.5
Arch No. 2;-Radius = 43.520 In.; $\frac{L}{H}$ = 5.0
59°29′
43°36′
35°17′
9.54"
12.0"
17.14"
E Points 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
C Points 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

FIG. 13.—CONTROLLING DIMENSIONS OF MODEL ARCH RIBS TESTED

Deflection measurements were made in two ways: (1) By direct observation with filar micrometer microscopes; and (2) by observing with a microscope a photographic film on which exposures had been made for both the unloaded and the loaded condition, in respect to live load. The latter method checked the first for accuracy and proved to be much more rapid, in addition to preserving the information for observation and study at a later date. Fig. 15 indicates such a double exposure for one of the more flexible conditions. Although photographs were made for each condition of loading, time has not permitted the study of these data, except in certain instances when they were used in conjunction with the observed horizontal reaction component and the statically determined vertical components, to check the directly observed moments.

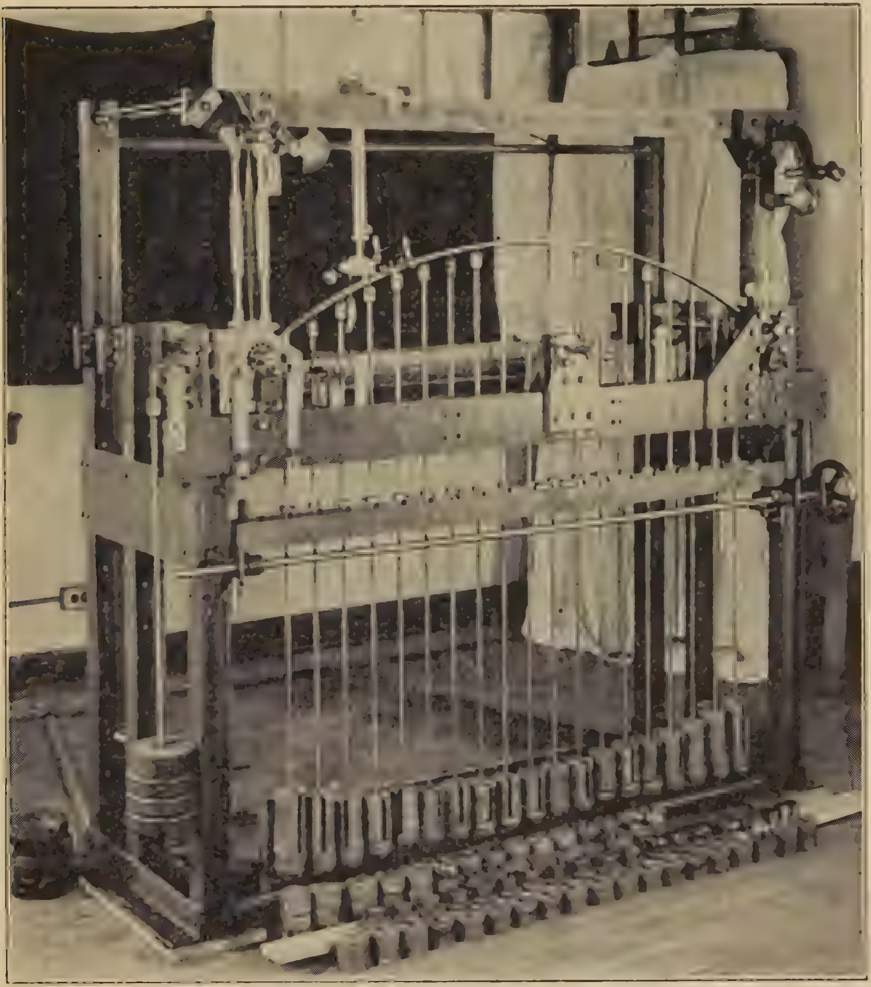

Fig. 14.—View of Model in Special Loading Frame

The horizontal component was measured as follows: One hinge was mounted in a block fixed in the loading frame, whereas the other hinge was mounted elastically in a block relatively free to translate in the direction of the span length. Calibrated piano wires, long enough to allow the use of 20-in. extensometers, with one end attached to the movable block and the other attached to blocks capable of being moved by worm threads and gears, were used to adjust the movable block to correct position for accurate span length. The strain readings on the extensometers indicated the magnitude of the horizontal component.

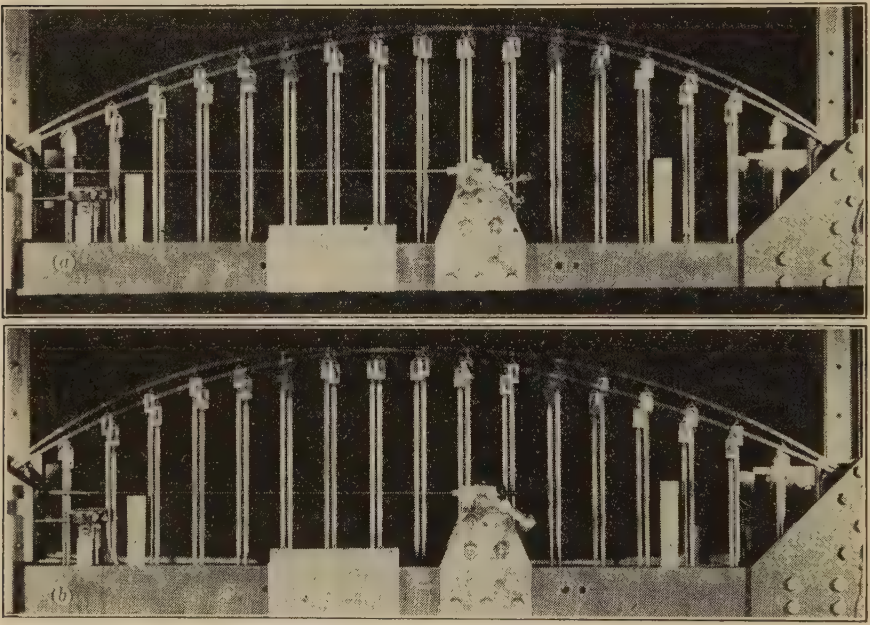

Fig. 15.—Double-Exposure Photographs to Observe Deflections; Model of Arch No. 1 at Point 6 C

Moment in the arch rib was determined by

$$M = E\,I\,\frac{\Delta\theta}{\Delta L} \qquad (22)$$

in which the various terms on the right side of the equality sign were evaluated as follows:

- $E\,I$—by calipered dimensions and repeated stress-strain measurements on a piece of steel cut from that used in making the model.
- $\Delta\theta$—by use of two optically flat mirrors attached to the two ends of the elastic element (see Fig. 13 at point 5 E). In the plane of the model and about 15 ft from its near end, $\frac{1}{20}$-in. scales were supported on an engineer's transit. The images of these scales were observed in the mir-

rors through the transit, and the readings translated into differential angular deflections.

ΔL—by a mechanical spacing bar used in setting mirror mountings in position. The value used was 2.00 in. measured along the rib.

The accuracy of the equipment and the reliability of the experimental technique were such that reported results should be accurate within an error of not more than 3%.

To enable the apparatus to be used for the study of flexible fixed-end arches, levers and weight pans were provided, designed to introduce, at the hinged reactions, moments sufficient to maintain the springing planes in their normal position. This produced fixed-end conditions and a means of measuring the fixed-end moments. Time permitted only a few tests of fixed-end conditions, but these indicated that further study would be profitable.

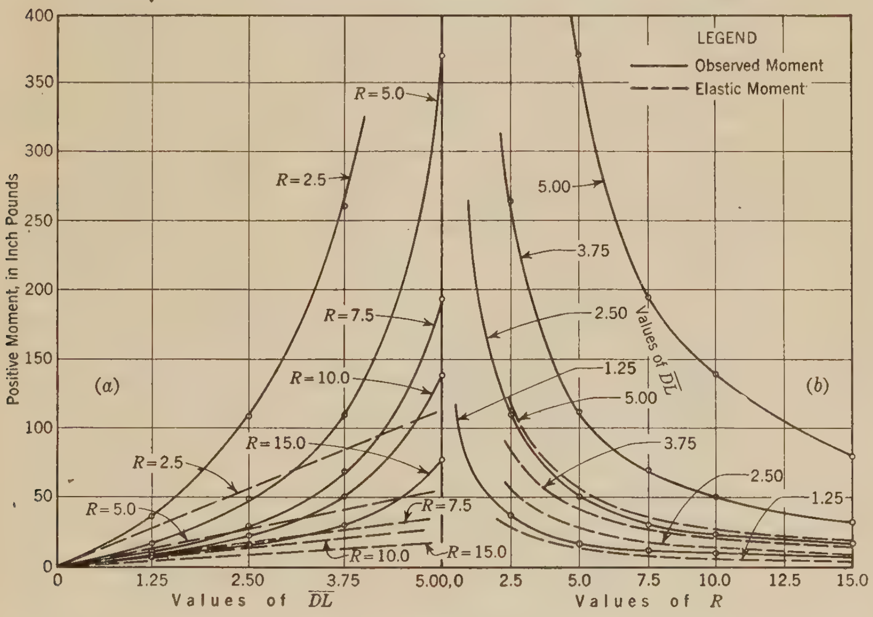

Fig. 16

Experimental Data

Maximum positive and negative moments were observed at five different points in the semispan of each of the three arches tested and for the complete range of loadings in Table 4. Fig. 16 shows two typical linear plots of the same data for maximum positive moment M at point 6 C of arch No. 1, and indicates the consistency of the observed data. Values of the moment as given by the elastic theory also are shown in these figures for comparison. In most cases the differences are rather marked.

Fig. 17 is a log-log plot of the same data as in Fig. 16, using the same coordinates but omitting the elastic-theory data and including the maximum negative moment ($-M$) data. It will be observed that straight lines approximate the trends with remarkable accuracy. This tendency was found to be general for all the points on all arches within certain ranges whose extremes have been adopted as the limits of practicality for the use of the data presented in this paper.

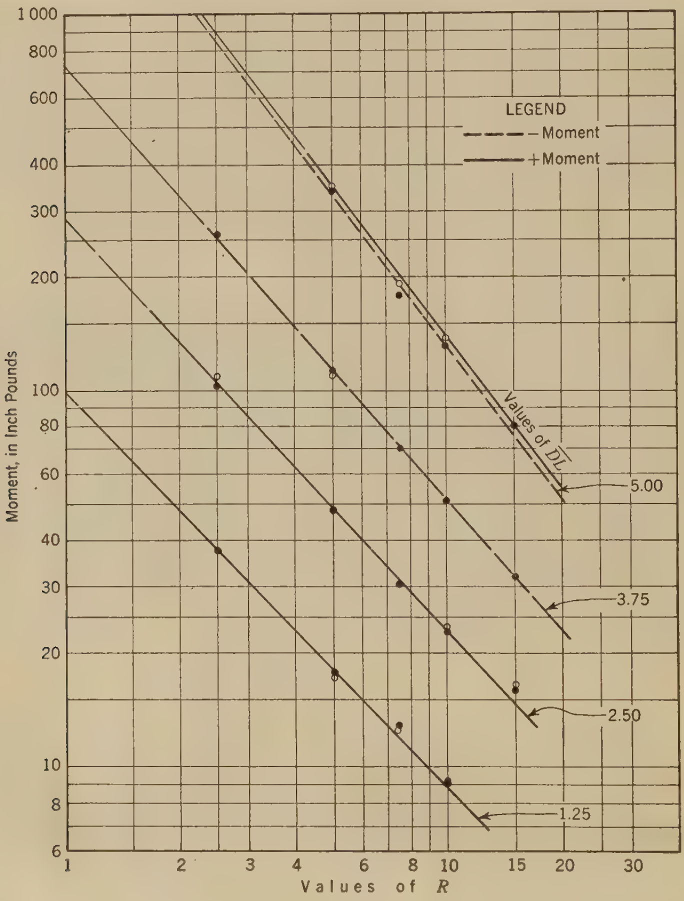

Fig. 17.—Log-Log Plot of Data in Fig. 16(b)

These data are more useful to practical engineers if presented in terms of arches whose dimensions correspond more closely to those of actual structures. For this reason, the complete data for maximum moments were translated to refer to so-called "base arches," of similar geometric shape (in elevation) to that of the models. In these base arches, L = 1,000 ft; I = 100 ft^4;

$E = 29{,}000{,}000$ lb per sq in.; and loads and other values are proportional to those of the models. This translation was accomplished by applying to the model results the laws of similitude, familiar to any engineer using these data, in which

$$\overline{DL}_b = \overline{DL}\,\frac{E_b\,I_b}{E\;I}\left(\frac{L}{L_b}\right)^3 \qquad (23a)$$

and

$$M_b = M\,\frac{E_b\,I_b\,L}{E\;I\;L_b} \qquad (23b)$$

Values without a subscript in Eqs. 23 refer to any arch such as the model, and the values with the subscript, b, refer to the base arch. All subsequent data presented in this paper refer to these base arches.

Space prevents including a complete set of curves for the base arches similar to those of Fig. 17. These curves, complete sets of curves of the type shown subsequently in Figs. 19 and 20, and additional data not presented herein are available in the form of a thesis entitled "Moments in Flexible Arches," by Messrs. McEldowney and Mead, presented to the School of Engineering, Princeton University, in partial fulfilment of the requirements for the degree of Civil Engineer.

Equations for Maximum Moment.—The fact that the observed data plot as straight lines on log-log paper (see Fig. 17) suggested the possibility of writing equations for these lines by the usual intercept-and-slope relation. Calculations for the lines of Fig. 17 show that their slopes, $\frac{\log M}{\log R}$, also vary in a linear manner with the value of log M for $R = 1$ (the intercept of Fig. 17). Furthermore the values of log M for $R = 1$, the intercept, are found to vary approximately linearly with $\overline{DL}$. It was possible therefore to write two equations for the intercepts and slopes of the lines of Fig. 17 in terms of $\overline{DL}$ and log M for $R = 1$, respectively. These two equations then could be substituted into the slope-intercept equation for the lines of Fig. 17, giving a single equation for all the lines for M in terms of $\overline{DL}$, R, and four empirical coefficients, C_1, C_2, C_3, and C_4. This equation is general for all points on each of the three arches and, for the base arches, reduces to

$$\log_{10} M_b = (C_1\,\overline{DL}_b + C_2)\,(1 - C_3 \log_{10} R) - C_4 \log_{10} R \qquad (24)$$

in which C_1, C_2, C_3, and C_4 are selected for any particular point and any arch from the curves of Fig. 18.

By use of Eq. 24 and its accompanying coefficient curves it is possible to obtain values of the maximum bending moment, M_b: For any point on the three arches; for any selected value of $\overline{DL}_b$; and for R within the range of the tests. It should be emphasized that the position of the loading is not needed by the designer, since this was determined experimentally when testing. A designer need know merely $E\,I$, $\overline{DL}$, $\overline{LL}$ and the ratio of rise to span. Having converted these to the base arch, the designer can enter Fig. 18 at the selected span point and pick the coefficients for use in Eq. 24. The calculations are rapid and can

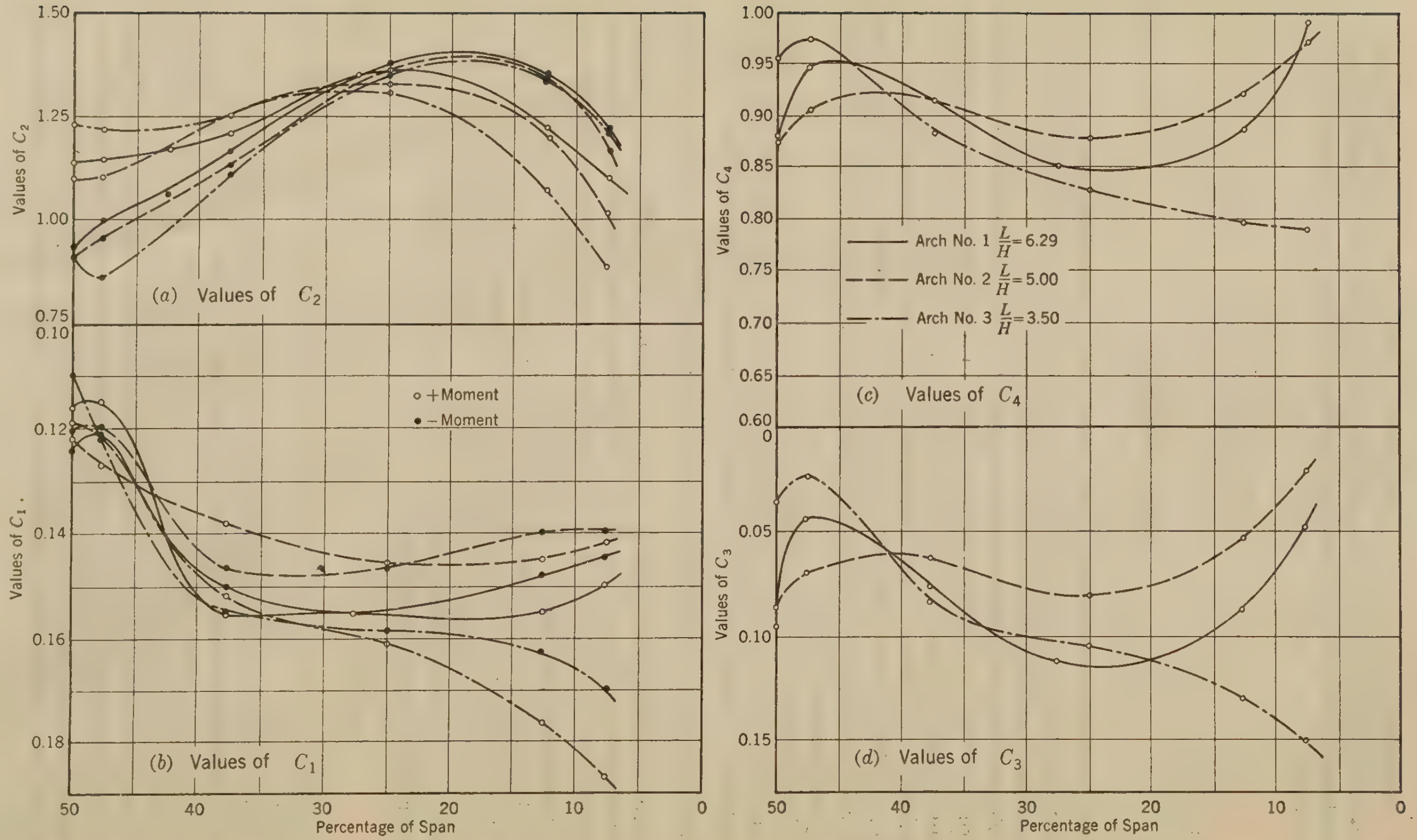

FIG. 18.—CURVES FOR DETERMINING COEFFICIENTS IN EQ. 24

be tabulated for convenience. The value of M_b then may be reconverted to the value for the designer's arch.

It should be noted that the small inconsistencies in the coefficient curves, as plotted from calculations from test results, are not an indication of unreliability in the final equation resulting from their use. The four coefficients are all interdependent, and small changes in one are accompanied by compensating shifts in the others. Eq. 24 and the coefficients have been checked thoroughly by taking values of the coefficients from the various curves for points on the arches between plotted points and by computing the value of M_b for some particular values of $\overline{DL}_b$ and R. This computed value then was plotted with corresponding observed data and found to be entirely consistent.

Accurate results (2%) should be obtained for values of $\overline{DL}_b$ between 2.50 and 7.0 and for all values of R between 1.5 and 15. Use of the equations for other values outside this range will give less accurate results and is not recommended. Interpolating between the coefficient curves for arches with intermediate values of span to rise $\left(\frac{L}{H}\right)$ also is not recommended, as the curves do not vary in a linear manner. The moment M_b for other $\frac{L}{H}$-ratios may be obtained with fair accuracy by computing M_b for all three ratios covered herein, plotting the results against $\frac{L}{H}$, and then selecting the value of M_b for the desired $\frac{L}{H}$ from this curve.

Eq. 24 should offer valuable assistance to a designer in selecting preliminary sections and in checking designs of arches, even if not used for final design.

Dimensionless Plots.—Another useful method of presenting the data is shown in Fig. 19, in which dimensional analysis has been used. It will be noted that the ordinate, $\frac{M}{\overline{DL}\,L^2}$, and abscissa, $\frac{E\,I}{\overline{DL}\,L^3}$, are both dimensionless, since M is in inch-pounds, $\overline{DL}$ in pounds per inch, L in inches, and $E\,I$ in pound-inches^2. Ratio R is also dimensionless, since it is the ratio of two load intensities in pounds per inch. The points in Fig. 19 were obtained from M on the model, by calculating $\frac{M}{\overline{DL}\,L^2}$ and $\frac{E\,I}{\overline{DL}\,L^3}$ for the corresponding model values of $\overline{DL}$, L, E, and I, and by drawing curves of constant R. The ordinates are plotted to logarithmic scale in order to keep the values within reasonable plotting limits and so that the percentage error in plotting would be constant for all values of the ratio R.

The importance of the dimensionless plotting lies in the fact that Fig. 19 applies to any arch that is geometrically similar to the test arch, regardless of size and relation of load to stiffness. For example, suppose a designer has selected certain values of E, I, L, $\overline{DL}$, and $\overline{LL}$ (or R) for his bridge and wishes to find M at a certain point, say near the quarter point, and that his arch has an $\frac{L}{H}$-ratio corresponding to test arch No. 1. Then, he would compute the value of

$\frac{E\,I}{\overline{DL}\,L^3}$; enter Fig. 19 until he reached the appropriate value of R; and, from the corresponding $\frac{M}{\overline{DL}\,L^2}$-value, he would compute the desired M. Care should be exercised to keep all the variables in consistent units (pounds and feet, for example).

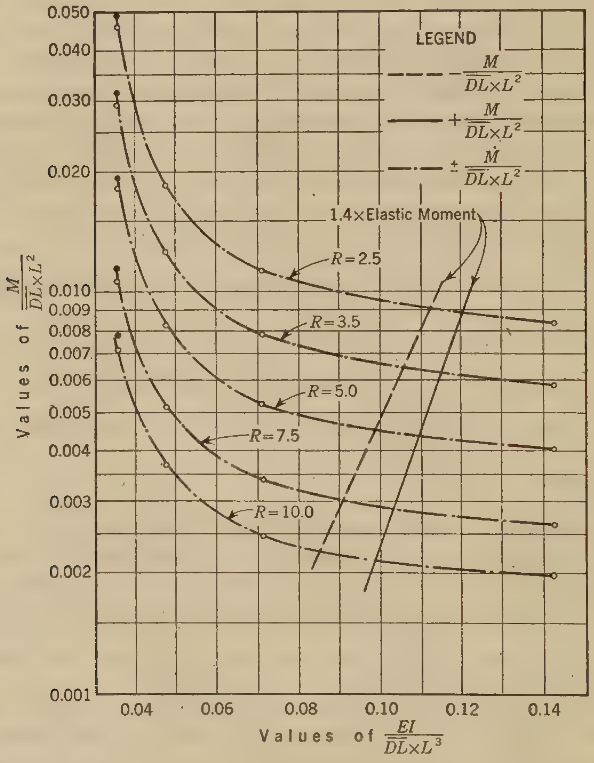

FIG. 19.—DIMENSIONLESS CURVES; ARCH NO. 1, AT POINT 6 C

On the other hand, suppose the designer wishes to investigate the effect on M resulting from a change in $E\,I$. The curves show that an increase in $E\,I$ produces a decreasing effect on the value of M and that $E\,I$ will decrease M effectively only up to a certain point, beyond which thickening the section plays little part in reducing M. In other words, the secondary moments due to deflection become negligible.

In this connection, the elastic-theory moments plot as straight horizontal lines on these dimensionless coordinates, since they are entirely independent of $E\,I$ (as long as it is constant) and $\overline{DL}$. To show how much the plotted curves vary from the elastic-theory values, the points, where the curves are equal to 1.4 times the elastic-theory values, are shown by the two lines in Fig. 19 which intersect the curves of constant R.

Lack of space prevents including the complete set of curves which were calculated for each point on each arch tested.

Curves of Percentage Increase.—The preceding part of the paper has dealt entirely with the absolute values of the moment in the model arches or in arches of geometrically similar shape. This form of presentation is perhaps the most direct, but has certain limitations that can be eliminated by plotting the test results in the form of a percentage increase of observed moment over calculated elastic-theory moment. This is done in a set of curves, an example of which is shown in Fig. 20, giving values for approximately the $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{1}{2}$ span points for each of the three base arches. These curves are similar also to Fig. 16 except that percentages are used instead of absolute values for the ordinate. Values for other arches may be obtained readily by use of Eq. 23.

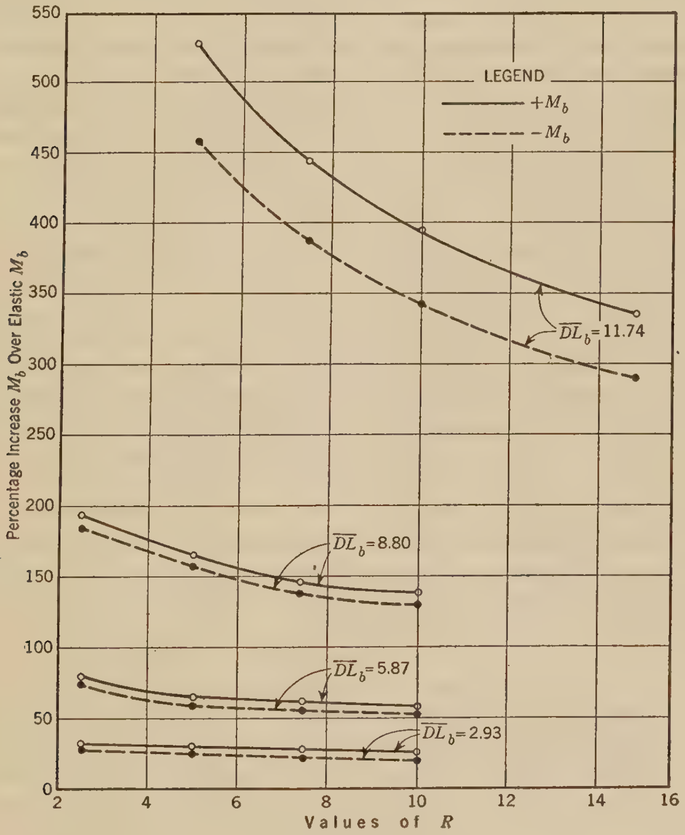

FIG. 20.—PERCENTAGE INCREASE OF ACTUAL M_b OVER ELASTIC M_b, BASE ARCH NO. 1, POINT 6 C

There are three major advantages to such curves:

1. Arches can be classified into three main groups—the first group including those that are stiff enough to permit the use of the elastic theory with no cor-

rections for secondary effects; the second group comprising those arches which can be used satisfactorily, after proper corrections for deviations from the elastic theory have been made; whereas the third group includes those arches which are so flexible as to be entirely unsatisfactory. Conclusions of this nature can be drawn directly from curves of percentage increase, whereas those for actual observed moment must be compared with elastic-theory values to be interpreted.

Most of the tests made in this investigation fall in the second and third categories. For example, assume an arch rib for which $\overline{DL}_b = 5.87$ and $R = 5.00$ and design it in silicon steel for 20,000 lb per sq in. with a direct stress of 10,000 lb per sq in. and a bending stress of 10,000 lb per sq in. If both live load and dead load are increased by 50% so that $\overline{DL}_b = 8.80$ and $R = 5.00$, the direct stress will increase to 15,000 lb per sq in., but, from Fig. 20, the bending moment is now 264.4% of the elastic moment instead of 168.5%. Therefore, the bending stress is now $\frac{2.644}{1.685} \times 1.5 \times 10{,}000 = 23{,}535$ lb per sq in. The total stress would be 38,535 lb per sq in., a stress readily carried by many alloy steels or aluminum alloys. Such an arch would fall in the second category of foregoing arch types.

Had the dead load and live load of the previous example been increased to $\overline{DL}_b = 11.57$ and $R = 5.00$, the direct stress would become 20,000 lb per sq in., but the bending stress would have increased to 74,540 lb per sq in., giving a total of 94,540 lb per sq in. Even if available materials could carry such stresses safely, the deflections coincident with them would rule out such an arch except in very special cases where large deformations could be tolerated.

2. A second use for curves of the type of Fig. 20 is in applying the test results to arches of varying $E\,I$ and to forms other than exactly circular, such as parabolic or three-centered arches. Calculated elastic-theory moments for such ribs are likely to be fairly different from those for circular arches of constant $E\,I$ and of similar general proportions. It is also probable that observed moments of flexible models of such ribs would vary from those on flexible circular arches. However, the percentage increase of actual moment over elastic-theory moment for circular arches of constant $E\,I$ will be almost identical to percentage increases for noncircular arches of variable $E\,I$, provided the variations in stiffness with span and the change in outline are not too pronounced.

Therefore, a designer could use curves of the type of Fig. 20 by: (1) Idealizing his arch into a circular arch of constant $E\,I$ and of very similar proportions; (2) entering the curves to select the proper value of percentage increase; and (3) applying this percentage increase to the calculated elastic moment for the designer's own structure. Thus he could tell readily whether deflection or secondary moments were important in his particular situation.

3. There is a third advantage of the percentage-increase curves. All the test results were for moments caused by a uniform loading of an extent and location such as to produce maximum moment at the observed point. Many highway loading specifications include concentrated loads applied to the arch rib in addition to a uniform load, whereas railroad loadings specify several concentrated loads followed by a uniform load. For such load patterns, the

calculated elastic-theory moment and actual moments are certain to be different from those obtained in these tests. However, it is reasonable to assume that the percentage increase of actual over elastic-theory moment will be approximately the same for any loading pattern. This is particularly true since this increase is largely a function of the relation between $E\,I$ and the dead weight intensity $\overline{DL}$, and not the actual live load intensity (R), as illustrated by the relatively small variation of the curves with R for the majority of the test range.

A small point of interest concerning the curves for percentage increase may be revealed by examination of the curves for the various points across a single arch. Except for the midspan point (10 E) the percentage increase for the corresponding curves at the different span points is found to be substantially the same, particularly for low values of $\overline{DL}_b$.

Application of Data to Niagara Rainbow Arch Bridge

The original two-hinged design of the Rainbow Arch Bridge at Niagara Falls is an excellent practical check on the experimental work, since this arch was a good example of flexibility, and has been studied for deflection moments as described in the preceding paper of this Symposium. By applying the geometrical shape, loadings, and sections of this prototype design to the equations and curves of this paper, the values of moment along the arch can be evaluated and checked with the values as obtained by the designers in their "deflection theory." Furthermore, the calculations which follow more clearly illustrate the use of the experimental data.

The following data were used for the original two-hinged design of the Niagara arch: Span, 950 ft; rise, 150.75 ft; ratio of span to rise $= 6.32 = \frac{L}{H}$; depth of rib, 14 ft (average); I, 119 ft^4; E, 29,400,000 lb per sq in.; dead load $= \overline{DL} = 8.6$ kips per ft at the crown; live load $= \overline{LL} = 1.5$-kips-per-ft intensity; and the ratio $\frac{\overline{DL}}{\overline{LL}} = R = 5.73$.

To illustrate the application of the curves to this arch, the maximum positive moment at the quarter point will be obtained by the use of Eq. 24 and checked with the dimensionless curves and the percentage-increase curves. Since the $\frac{L}{H}$-value for the Niagara arch (6.32) is very close to the experimental value of 6.29 for arch No. 1, the curves and coefficients for this model arch and its base arch will be used.

Eq. 24 is correct in terms of the base arch, so that values of the Niagara arch variables must be converted to the base arch before using the equation. From Eq. 23a:

$$\overline{DL}_b = \frac{8.6 \times 29 \times 10^6 \times 100 \times 950^3}{29.4 \times 10^6 \times 119 \times 1{,}000^3}$$

which, in terms of the base arch $= 6.1$ kips per ft. From Fig. 18, for the quarter point of arch No. 1: $C_1 = 0.156$; $C_2 = 1.37$; $C_3 = 0.846$; and $C_4 = 0.115$. Also, $R = 5.73$ (does not change between model and prototype); and, by

Eq. 24:

$$\log M_b = [(0.156 \times 6.1) + 1.37](1 - 0.115 \log 5.73) - 0.846 \log 5.73$$

or

$$M_b = 30{,}200 \text{ ft-kips (30.2 thousand ft-kips).}$$

Converting this base-arch value of moment back to the prototype, by Eq. 23*b*:

$$M = \frac{30{,}200 \times 29.4 \times 119 \times 1{,}000}{29 \times 100 \times 950} = 38{,}350 \text{ ft-kips.}$$

This value of moment is close to the value arrived at by the designers at working loads, using their "deflection theory," from which the moment was calculated to be 37,130 ft-kips. The elastic-theory value for the same loading condition was 22,600 ft-kips.

The moment can be determined also at approximately the quarter point by using the dimensionless curves, referring to Fig. 19:

$$\frac{E\,I}{\overline{DL}\,L^3} = \frac{29 \times 10^6 \times 144 \times 119}{8.6 \times 1{,}000 \times 950^3} = 0.0685$$

From Fig. 19, interpolating between R-curves for $R = 5.73$: $\dfrac{M}{\overline{DL}\,L^2} = 0.0047$; from which $M = 0.0047 \times 816 \times 1{,}000 \times 950^3 = 36{,}500$ ft-kips.

The moment at this point (one-half panel toward the center from the quarter point) is not appreciably different from the value at the quarter point.

As a final check the moment will be determined from the percentage-increase curves. Referring to Fig. 20, for $\overline{DL}_b = 6.1$ and $R = 5.73$, the percentage increase of actual moment over elastic-theory moment is 70%. Therefore, the actual moment is $M = 1.70 \times 22{,}600 = 38{,}420$ ft-kips, which is almost identical to the foregoing value calculated from Eq. 21.

CONCLUSIONS

1. The experimental results obtained from this investigation, together with the method of presentation, furnish a control and a check on analysis methods for flexible arches which have been developed or may be developed in the future.

2. The data will be of particular assistance in the selection of preliminary or trial sections of two-hinged arch ribs which, in the opinion of the designer, should be sufficiently stiff for practical reasons to make the application of the elastic theory entirely proper.

3. The data supply a basis for the design of relatively flexible, two-hinged arches where such designs are proper—for example, in hanger roofs in which large deflections might not be objectionable if economies can be effected with safety.

4. In flexible arches, the dead load becomes an important consideration when determining moments and any theory which ignores it—such as the elastic theory—is not reliable.

5. Previously published opinions on the effect of reasonable variation of $E\,I$ on the moments in an arch, and of the close agreement between moments in

circular arches and closely similar parabolic arches (in each of which the dead-load resultant follows the center line of the arch) support the contention that the results presented will be of value in the study of arches of noncircular profile and nonuniform section.

6. The relation of load to unit stress in a flexible arch rib is not linear. Accordingly, the safe capacity of such an arch should be estimated by calculating its ultimate (or yield) loads, and then by applying the desired factor of safety to the critical design loads.

7. Conventional methods of calculating temperature stresses are satisfactory for the flexible two-hinged arch rib. (Although not described herein, this matter was checked.)

8. The refined method of analysis developed by the designer of the preliminary two-hinged design for the Rainbow Arch Bridge should yield reliable results.

9. Limited observations were made on the arch models in a fixed-end condition. Although, as might be expected, the secondary moments due to deflections were much smaller than for the two-hinged condition, certain inconsistencies with the elastic theory, of a different sort from those observed on the two-hinged arches, were noted, thus recommending this as a subject for future study.

FABRICATION AND ERECTION CONTROLS

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Synopsis

The hingeless arch ribs were erected by cantilevering symmetrically from each skewback. The first three rib sections were cantilevered from the anchor bolts, a statically determinate condition. Thereafter, the resisting moment at each skewback was augmented by one, two, or three groups of wire-rope strands passing from the forward rib sections to a temporary tower placed behind the skewback, and thence to a concrete anchorage embedded in rock.

A complete description of the erection procedure appears in the fourth paper of this Symposium. It is assumed in this paper that the reader is acquainted with that description. The present paper discusses briefly the shop controls used to insure the correct geometry of the ribs as fabricated and erected.

Throughout the statically indeterminate phases of the erection, it was necessary not only to compute and control the stresses in the permanent steelwork, but to compute the coexisting stresses in the temporary wire-strand tiebacks. This paper describes the procedure using a method of successive approximations for resolving this indeterminacy.

It was also considered desirable to measure, prior to closure, the thrust and moment at the crown of the braced arch-rib sections after they had been made self-supporting by the removal of the wire-strand tiebacks. The thrust and moment thus measured could then be changed, if necessary, by varying the shape of the "keystone" rib section. Thus, the stresses locked in the structure for this loading condition could be controlled, and the effect on the stresses due to accumulated errors in span length, fabricated lengths or angles, skewback angles, etc., could be eliminated. It is believed that this is the first instance of such planned control of crown thrust and moment.

By correcting the shape of the "keystone" rib section properly, agreement of the actual stresses with the calculated stresses for the loads on the arch in this erection stage was assured. However, the fact that no correction was required is an indication of the extreme accuracy of fabrication and pier location.

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INTRODUCTION

As described in the first paper of this Symposium, the Rainbow Bridge is a hingeless arch with a span between skewbacks of 950 ft and a rise of 150 ft. As such, the structure is triply indeterminate and the stresses must be calculated from its elastic properties. Initial errors in span survey, in fabricated length or contour, in skewback angle, or in angles at the field joints as riveted could introduce serious errors in the stresses due to the weights in place at the time of closing and swinging, errors that might be an important percentage of the total stresses at some points. Unusual efforts were made, in closing at the crown, to determine and eliminate the accumulated effect of such errors. The fact that they were found to be negligible and did not require correction testifies to the accuracy attained in survey, fabrication, and erection.

The arch center line was established by the designer to follow very closely the dead-load equilibrium polygon, the ordinates thus determined at all panel points being tabulated on the contract plans. The contractor then undertook, for purposes of drafting and fabrication, to pass a compounded series of circular arcs through these points of control.

The spacing between spandrel columns is approximately 40 ft, and the weight of the rib material necessitated a field splice in each rib in advance of each successive spandrel column. A preliminary study of splices determined a distance (usually about 6 ft) from center of column to center of splice which would keep the splice material clear of the column connections. The rib-center radii were then so selected that changes of radius occurred only on radial lines to alternate splice centers. Thus, two successive pieces of rib as fabricated had a constant radius, and changes of radius never occurred within a fabricated piece. The resulting working curve involved five centers on each side, plus one on the vertical axis, the radii varying from 781.6 ft to 970.6 ft. This curve passed through the twenty-five stipulated points with errors not exceeding $\frac{1}{8}$ in.

The specifications required that the arch ribs conform to the stipulated outline under full dead load at 50° F. This required that each rib section be cambered, or increased in length, by the amount that it would shorten under dead load, and that its length be adjusted for the difference in temperature between 50° and 68°, the temperature at which the fabricator's shop tapes are all standardized. This increase in length of each rib section was assumed as a wedge-shaped piece attained by lengthening the top and bottom arc lengths through the angular increment determined by the increment of length on the center line. This meant that, if the complete rib had been shop assembled under no stress, both the chord distance between skewbacks and the angle between radii through the skewbacks would have been greater than the intended values as erected under dead load.

In connection with the problem of camber, the "effective" areas for computing deformations were of considerable importance. It was decided finally to consider the unspliced longitudinal stiffeners as part of the effective area (see Fig. 6), in addition to all spliced material that is effective in resisting stress.

A set of the contractor's erection drawings and calculation sheets, showing the basic outline, arch-rib radii, splice locations, weights of rib sections, effective areas, and cambering details, has been placed on file at Engineering Societies Library, together with copies of all sheets recording the computations referred to hereafter. A descriptive index of the filed material is given in Appendix II.

Fabrication

In the fabrication of the individual 40-ft rib pieces, the 12-ft web plates were subdrilled to the intended radii of the top and bottom angle gage lines, and the 8-in. by 8-in by 1⅛-in. angles and the 54-in. cover plates were subdrilled straight and pulled in, cold, to the required curvature. It was felt that the local bending stresses thus locked in the angles by the riveting would do no harm in a ductile material, whereas "gagging" to curvature would require stressing beyond the yield point and, in all probability, would produce a series of approximate chords rather than a smooth curve. The smooth curve was desirable not only for appearance, but to accommodate the designer's intention that the straight-line bearing edges of the spandrel columns should "rock" slightly on the curved ribs.

To insure that the rib sections would be of the proper depth and curvature, each web with its flange angles was assembled in a jig. The diaphragm plates between the two parts of the box girder were milled on all four edges, and the sections were then boxed in pairs against squaring brackets to obtain and maintain the rectangular cross section. While held against these squaring brackets, the sections were reamed and riveted. They were then removed to the milling machine and milled accurately to length and bevel. While the sections were in position on the milling machine, center lines and working points were established, and the pads for the column bearings were set.

Assembly

After considerable study of the problem of maintaining accuracy of length and angle, the fabricator decided to shop assemble the rib sections on their sides, starting from the skewback and checking the position of each section accurately as it was placed, until seven sections were assembled. After the splices between these sections had been reamed, the first six sections were pulled out ready for shipment, the seventh section was moved back to the starting point, and the remaining five sections were assembled in sequence to the seventh.

According to the terms of the contract, the rib sections were fabricated in the United States, whereas the rib laterals, spandrel columns, and all floor steel were fabricated in Canada. Each fabricator independently prepared metal templates for the common holes—that is, those at the connections of the spandrel columns and the rib laterals to the rib itself. These templates were set, with great precision, on the assembled rib to center lines which had been located while the rib section was in position on the milling machine. Although there was no physical exchange of templates by the two fabricators, the matching of the holes in the field was practically perfect.

The assembly shop, with four rib assemblies being worked on simultaneously, is shown in Fig. 21. The pieces standing upright at the far end of the shop have been assembled, their splices reamed or drilled, and they are now being painted. Assembling of the piece in the left foreground has also been completed, and the piece removed from the assembly prior to painting. The box sections on the right are the completed grillage and skewback sections, and are loaded on cars ready for shipment. The trapezoidal section in the center foreground is the top section of the cable bent to which the forward ties and backstays attach, standing upside down. Near the center of Fig. 21, a horizontal reamer is shown drilling on a cover-plate splice.

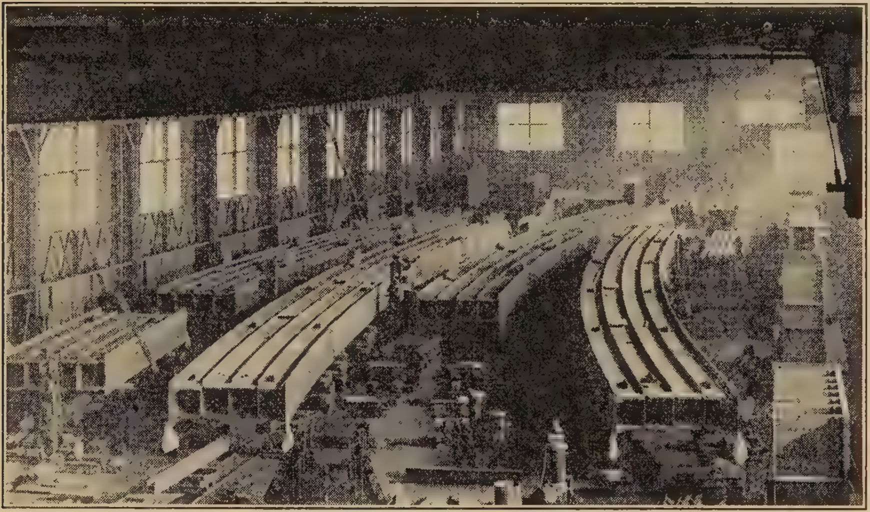


Fig. 21.—View of Four Rib Assemblies in the Assembly Shop

Prior to making the aforementioned two types of assembly—seven-piece and six-piece—the offsets to center line were calculated from as many successive base lines as there would be pieces in the final assembly, each base line running from the left-end center of the first piece laid down to the right-end center of the last piece attached. As successive pieces were attached on the assembly skids, the ordinates from the base thus created were measured and compared with the calculations. Thus, curvature was checked in seven different ways through the seven-piece assemblies and in six different ways through the final, six-piece assemblies. Throughout these successive checks, the measured and calculated ordinates were held to differences not exceeding $\frac{1}{8}$ in. While these checks on curvature were being made, the milled ends of successive sections were kept in close contact and the splice holes thus reamed. The care taken in the laying off, milling, and splice-reaming throughout the fabrication resulted in a gratifying accuracy in the finished structure.

The Erection Problem

To assist in an appreciation of the erection engineering calculations required, Fig. 22 is presented to show the successive cantilever positions, and the following summary of successive operations is given.

First, three rib sections (points 13 to 10, Fig. 6) were erected by cantilevering freely from the anchor bolts, a determinate condition. Next, eight wire tie strands per rib were connected at point 10 and put under a measured, predetermined stress. Henceforth, until closure, the conditions were statically indeterminate. Two more rib sections were freely cantilevered beyond point 10, depending upon the splice bolts and pins for stability; eight tie strands were connected at point 8 and put under predetermined stress. The strands at point 10 were then disconnected from the rib but not from the tieback bent. Two more rib sections were cantilevered beyond point 8, and eight tie strands were connected at point 6; these were the strands originally connected at point 10, with extension "pennants" attached. Four additional strands were attached at point 6, making twelve in all. The tie strands at point 8 remained in place, and the loads were shared between the eight strands at point 8 and the twelve strands at point 6 in predetermined amounts. Three more rib sections were cantilevered beyond point 6, and four tie strands from point 6 were successively connected and stressed at point 3; then the eight strands at point 8 were released from the rib, lengthened with pennants, and reattached and stressed at point 3. The loads were shared between the eight strands at point 6 and the twelve strands at point 3 in predetermined amounts. Two more rib sections were cantilevered beyond point 3, which brought the two half arches, cantilevering from the two shores, within the predetermined 11 in. of meeting at the crown.

At the crown, jacking brackets were bolted to the upper and lower cover plates of each half rib, so that two 500-ton jacks could be inserted above, and two symmetrically below, each rib. After these jacks had been operated to pick up a crown thrust and slack it on to shims, all the tie strands were successively released and removed.

Up to this point, it was necessary to predetermine by calculation, for each successive position, the arch-rib stresses, the stresses in tie strands, tieback bent and anchorages, and, from these latter, the necessary initial lengths of the tie strands. At the successive connections of strands to ribs, there were structural bar-and-pin devices to permit shortening and lengthening the strand length in increments of 1 in. to total adjustments of 12 in. and 18 in., respectively, from the mean, and to permit shortening and lengthening by larger increments to maxima of 24 in. and 30 in., respectively, from the mean. With so limited an adjustment possible in long, flexible ties, it was obviously important to determine the unstressed length with accuracy. Ties too short or too long would have seriously changed the stresses in the arch ribs by straining them too far, or not far enough, against their fixation at the skewbacks.

Scope of Computations

To furnish the erection engineer with the data necessary for the proposed erection procedure, rather elaborate computations were required. The work necessary to obtain this information for each erection stage is outlined herein. The complete tabulation of calculations as prepared by the contractor and checked by the consulting engineers is available at Engineering Societies Library (see Appendix II).

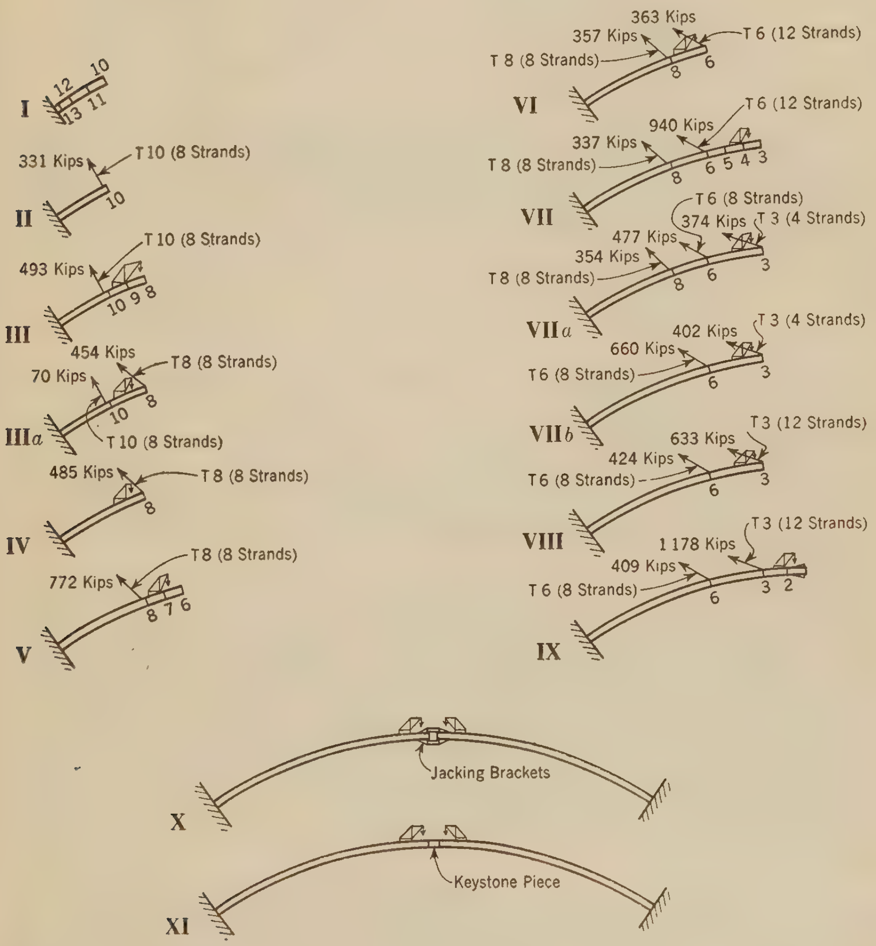

I
10
12
11
13
II
331 Kips
T 10 (8 Strands)
10
III
493 Kips
T 10 (8 Strands)
10 9 8
IIIa
454 Kips
70 Kips
T 8 (8 Strands)
8
10
T 10 (8 Strands)
IV
485 Kips
T 8 (8 Strands)
8
V
772 Kips
T 8 (8 Strands)
8 7 6
VI
363 Kips
357 Kips
T 6 (12 Strands)
T 8 (8 Strands)
8
6
VII
940 Kips
T 6 (12 Strands)
337 Kips
T 8 (8 Strands)
8
6 5 4 3
VIIa
T 6 (8 Strands)
374 Kips
T 3 (4 Strands)
477 Kips
354 Kips
T 8 (8 Strands)
8
6
3
VIIb
402 Kips
T 3 (4 Strands)
660 Kips
T 6 (8 Strands)
6
3
VIII
T 3 (12 Strands)
633 Kips
424 Kips
T 6 (8 Strands)
6
3
IX
T 3 (12 Strands)
1 178 Kips
409 Kips
T 6 (8 Strands)
6
3 2
X
Jacking Brackets
XI
Keystone Piece

FIG. 22.—ERECTION STAGES AND TOTAL TIE STRESSES FOR ARCH RIB, AT 60° F

The calculation work for each erection stage falls into three divisions, as described under 1, 2, and 3 below. In this description, and in Appendix II, references to sheet or drawing numbers prefixed with ED are to the contractor's erection drawings.

1. (Sheets ED10 to ED15). The part of the rib thus far erected was assumed to be cut loose from any ties and to cantilever from the skewback. Assuming that the steel would hold together and remain elastic under such a

condition, the deflected position was calculated for the points importantly in question—namely, the points of tie attachment and the splice points. Thus, for example, with the rib cantilevered three panels to point 10, the vertical deflection of the free end, in feet, was computed to be:

From bending	0.11406
From thrust	0.00028
From shear	0.00521
Total	0.11955

and the stresses in the rib were found to be just under the allowable. Again, for example, with the rib erected to point 1, but before the jacks and jacking diaphragms at the crown were in place, the hypothetical cantilever deflection was computed to be (in feet):

From bending	49.6117
From thrust	0.0032
From shear	0.1200
Total	49.7349

(and horizontally, 11.9038).

2. (Sheets ED4 to ED8). For a unit vertical force, and for a unit horizontal force, applied at a "tie point" (point of attachment of a wire-strand tie), the displacements, due to the elastic deformations of the rib between that tie point and the abutment, of all rib splice points and all tie attachment points were calculated.

3. In the tie or ties to be attached in the erection stage, forces were first assumed and then changed by the method of successive approximations until their effect, as found from step 2, was: (*a*) To raise the tie attachment points on the rib through approximately the deflected distance found in step 1; (*b*) to stretch the tie system until the end of each tie coincided with its attachment point on the rib; and (*c*) to give safe stresses throughout the rib. An average of four steps in the successive approximations was necessary to get perfect agreement on one erection stage, and this work was doubled since two solutions were made, one using the maximum and the other the minimum expected temperature.

From computations made in this manner, the erection engineer was given not only the information shown in Fig. 22, but also the theoretical elevation of each rib splice point at each erection stage; the stresses in the tie strands; the stress in the first strand to be connected and in the last strand to be disconnected for each forward tie; the effect per inch of change of length of one or all of the adjustment links of each forward tie on tie stress and arch elevation; the influence of change in temperature; and the necessary order for the various steps in each new move.

Displacement Calculations for the Rib

All the theoretical deflections required under step 1, and the displacements due to unit tie force components required under step 2, were computed by the following adaptation of the "slope-deflection" method, taking separately into account deformations due to bending, thrust, and shear.

The change in angles (Fig. 23(*a*)) at m in relation to the adjacent members, due to the bending moment, will be:

$$(\tau_m)' = (M_{m+1} + 2\,M_m)\,\frac{L_m}{I_m} \times \frac{1}{6\,E} \ldots\ldots\ldots\ldots (25a)$$

$$(\tau_{m-1})'' = (2\,M_m + M_{m-1})\,\frac{L_{m-1}}{I_{m-1}} \times \frac{1}{6\,E} \ldots\ldots\ldots\ldots (25b)$$

and the total angle change at m will be:

$$\theta_m = (\tau_m)' + (\tau_{m-1})'' \ldots\ldots\ldots\ldots (26a)$$

These angle changes will accumulate from the supported end, and cause an angle change in the member itself equal to:

$$\phi_m = \sum_{m+n}^{m+1} \theta \ldots\ldots\ldots\ldots (26b)$$

They also will cause deflection at the forward end of the member, with respect to the other end, equal to:

$$u_m = \phi_m\,L_m \ldots\ldots\ldots\ldots (27)$$

and hence total deflections due to bending equal to:

$$\delta_h = \sum u \sin\alpha \ldots\ldots\ldots\ldots (28a)$$

and

$$\delta_v = \sum u \cos\alpha \ldots\ldots\ldots\ldots (28b)$$

Thrust and shear deformation (Figs. 23(*b*) and 23(*c*)) will also add up from the support. The total thrust deflections will be:

$$\delta_h = \sum \frac{T\,L}{A\,E} \cos\alpha \ldots\ldots\ldots\ldots (29a)$$

and

$$\delta_v = \sum \frac{T\,L}{A\,E} \sin\alpha \ldots\ldots\ldots\ldots (29b)$$

and the total shear deflections will be:

$$\delta_h = \sum \frac{s\,L}{A_w\,E_s} \sin\alpha \ldots\ldots\ldots\ldots (30a)$$

and

$$\delta_v = \sum \frac{s\,L}{A_w\,E_s} \cos\alpha \ldots\ldots\ldots\ldots (30b)$$

in which A_w = the web area and E_s = the modulus of elasticity in shear.

To apply Eqs. 25 to 30, it was necessary to divide the rib into the thirteen sections defined by the splice points (so as to determine the successive deflected positions of these particular points for field check), and also to subdivide it further at points where there was a change in moment of inertia, and at points of application of concentrated loads (the tie attachment points).

The calculation record sheets (ED10 to ED15) for the rib deflections required by step 1, under the heading "Scope of Computations," may be typified by Table 5, which is a résumé of sheet ED10, applying to erection stages I and II, Fig. 22.

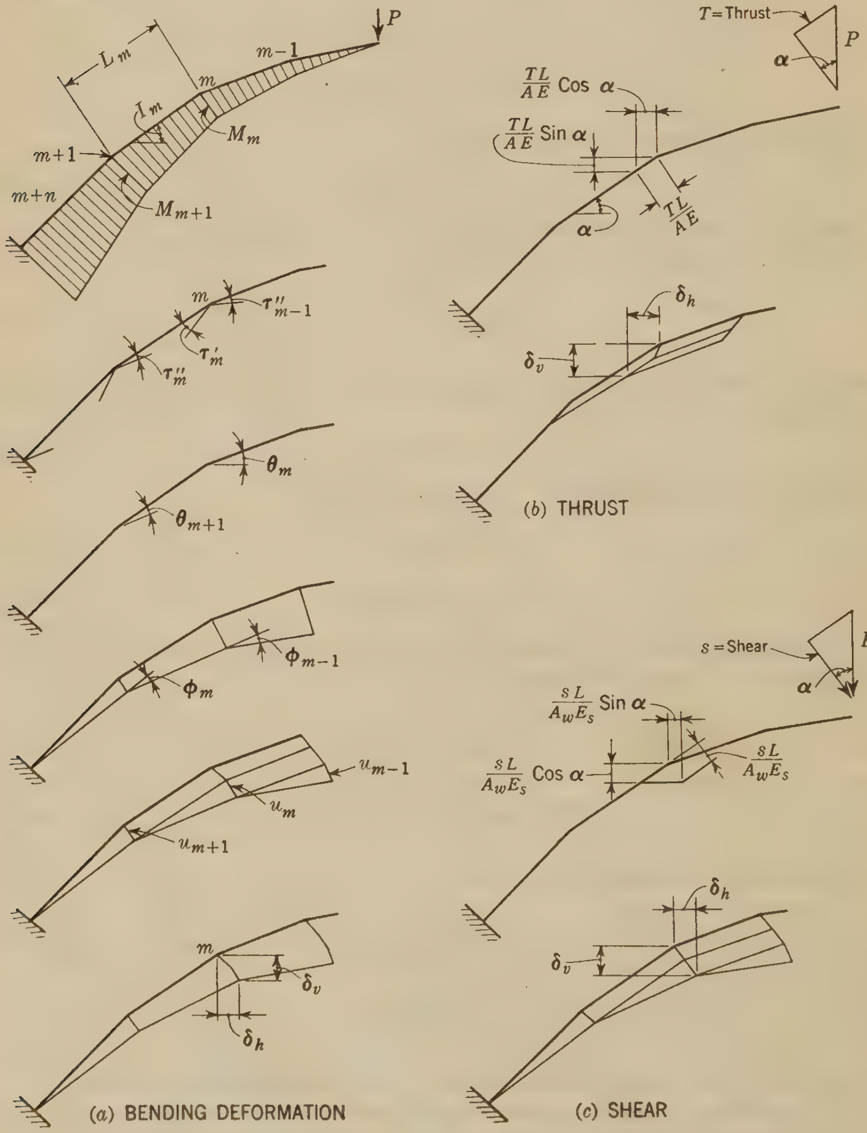

P
L_m
$m-1$
m
I_m
M_m
$m+1$
$m+n$
M_{m+1}
τ''_{m-1}
τ'_m
τ''_m
θ_m
θ_{m+1}
ϕ_{m-1}
ϕ_m
u_{m-1}
u_m
u_{m+1}
δ_v
δ_h
(a) BENDING DEFORMATION
T=Thrust
α
$\frac{TL}{AE}$ Cos α
$\frac{TL}{AE}$ Sin α
$\frac{TL}{AE}$
(b) THRUST
s = Shear
$\frac{sL}{A_wE_s}$ Sin α
$\frac{sL}{A_wE_s}$ Cos α
$\frac{sL}{A_wE_s}$
(c) SHEAR

Fig. 23.—Deformation Diagrams

Sheets ED4 to ED8, covering computations for rib displacements due to unit vertical and unit horizontal force at a tie point, were similar. Because the calculations were made for unit loads instead of for steel-and-traveler dead loads, certain initial column headings differed. For example, in these rib-displacement computations, Figs. 24(*a*), 24(*b*), and 24(*c*), respectively, replaced Cols. 5 to 9, 20, and 28, Table 5. (Note that, in Fig. 24(*a*), 475.00 is the horizontal distance from the midspan to the skewback, SB; and 374.84 is the horizontal distance from the midspan to panel point 10.) Also, the deflections from unit load being very small compared to those for actual loading, the powers of 10 in all columns of sheets ED4 to E8 were changed, and the resulting deflections were obtained in integral numbers of millionths, instead of thousandths, of a foot.

Point	(5)	(6)	(17)	(25)
		$1\,(X - 374.84)$	$1 \times \sin \alpha$	$1 \times \cos \alpha$
	X	M	P	V
	Ft	Kip-ft	Kips	Kips
SB	475.00	100.16		

(*a*) (*b*) (*c*)

FIG. 24.—EXAMPLE OF ALTERNATE COLUMN HEADINGS FROM SHEETS ED4

From the theoretical deflected position of the rib in any erection stage, from the restoring deflections available from unit vertical and unit horizontal tie pulls, and from the approximate angle of the tie after restoring the arch to almost its proper elevation, the tie stresses could now be approximated. It was found that these agreed sufficiently well with much cruder computations made before compiling the bidding estimate. Since the $1\frac{9}{16}$-in. strands had a guaranteed strength of 310,000 lb each, it was hoped to keep working loads within 100,000 lb, but it was considered proper not to change the assumed number of strands if the final calculations should indicate 120,000 lb. The final maximum (see Fig. 22, stage VII-B) was just over 100,000 lb. Actually, the cable stress for the condition in stage V was also slightly greater than 100,000 lb but only for the minimum temperature condition. The modulus of these strands was assumed as 24,000,000 lb per sq in., that value having given satisfactory results when the strands were previously used as footwalk support ropes at the Tacoma Narrows Bridge in Washington.

DISPLACEMENT CALCULATIONS FOR THE TIES

All calculations for balancing the cable stress and tie positions with the corresponding data for the rib itself were made by an exact method which involved a modified form of the "deflection theory" for suspension bridges. This was necessitated by the extreme flexibility of the interlocked system. The methods and formulas used would be grasped quickly by any designer conversant with cable calculation work, but are of such a highly complex and specialized nature that in their stead the method of approach will be given in general terms.

A diagrammatic sketch of the tieback arch with the loads in position is shown in Fig. 25. These loads comprised: The weight of the arch rib, the uniform weight of the backstay and of the forward-tie strands, the weight of the links at the bent and at the rib, the eccentric weight of the bent with its back-leg, and the concentrated load introduced on the forward-tie strands by the pennant sockets when there were any.

In the first instance, three "basic" cases were computed to determine the length to which the several strands should be socketed. These "basic" cases were those of maximum stress in the strands and, in order of attack, were:

(1) Erection stage IX, ties at T6 and T3, arch erected beyond T3 to crown;
(2) Erection stage V, ties at T8, arch erected beyond T8 to T6; and
(3) Erection stage III, ties at T10, arch erected beyond T10 to T8.

TABLE 5.—DISPLACEMENTS UNDER DEAD LOAD, FOR

Point	(a) RIB PROPERTIES				(b) BENDING					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
								(5)×(7)	Σ(8)	
	L	$I \times 10^{-3}$	sin α	cos α	X'	F	ΣF	Q	M	M' and M''
	Ft	In²-ft²			Ft	Kips		Kip-ft		
SB						54.0			24,078	68,453
	9.25	51.26	0.5497	0.8354	7.73		489.1	3,781		
SP13						114.4			20,297	64,672 55,405
	17.30	22.14	0.5331	0.8461	14.64		374.7	5,486		
•									14,811	49,919 40,712
	11.74	20.62	0.5331	0.8461	9.93		374.7	3,721		
SP12						155.5			11,090	36,991 30,416
	15.04	21.11	0.5011	0.8654	13.02		219.2	2,854		
•									8,236	27,562 22,812
	10.00	19.26	0.5011	0.8654	8.65		219.2	1,896		
•									6,340	20,916 15,504
	18.53	17.81	0.5011	0.8654	16.04		219.2	3,516		
SP11						146.4			2,824	11,988 6,983
	23.05	15.78	0.4606	0.8876	20.46		72.8	1,489		
•									1,335	5,494 3,299
	10.93	13.72	0.4606	0.8876	9.70		72.8	706		
•									629	2,593 1,258
	8.01	∞	1.0000	0	0		0	0		
TP10									0	629 1,258
	9.73	13.72	0.4606	0.8876	8.64		72.8	629		
SP10						72.8			0	629 0

These "basic" cases were computed at +60° F for ordering strand lengths. They and all other cases were computed at +60° F and −10° F for determination of arch position, and for a check against any overstress of ribs or strands within this temperature range.

In the first "basic" case investigated (erection stage IX at 60° F), it was assumed that the bent was vertical. This condition was nearly maximum for stress in the backstays (−10° F would have been maximum), and was used for ordering the backstay strands and the full-length forward ties at T6. It also determined the total length of the T10-plus-pennant ties which were used at T6, and of the T6-plus-pennant and T8-plus-pennant ties which were used at T3.

CANTILEVER CONDITION; STAGES I AND II

DEFORMATION									Point
(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	
(1)/(2)	(10) × (11) × 10^{-3}		Σ(13)	(14) × (1) × 10^{-3}	(15) × (3)	(15) × (4)	Σ(16)	Σ(17)	
$\frac{L}{I} \times 10^3$	$6E\tau$	$6E\theta$	$6E\phi$	$6Eu \times 10^{-3}$	$6Eu \times 10^{-3} \times \sin\alpha$	$6Eu \times 10^{-3} \times \cos\alpha$	$6E\delta_h \times 10^{-3}$	$6E\delta_v \times 10^{-3}$	
1/(in²-ft)	Kips per sq in.			Kip-ft per sq in.					
		12.3					0	0	SB
	12.3								
0.180									
	11.6								
		54.9			(etc.)				SP13
	43.3								
0.781					....				
	39.0								
		62.2			....				.
	23.2								
0.569					....				
	21.0								
		42.7			....				SP12
	21.7								
0.713					....				
	19.7								
		31.5			....				.
	11.8								
0.519					....				
	10.9								
		27.0			....				.
	16.1								
1.040					....				
	12.5								
		22.7			....				SP11
	10.2								
1.461					....				
	8.0								
		10.6			....				.
	2.6								
0.797					....				
	2.1								
		2.1			....				.
	0								
0					....				
	0								
		3.0			....				TP10
	0.9								
0.709									
	0.4								
		0.4					11.17	20.12	SP10
	0								

TABLE 5.—

Point	(c) THRUST DEFORMATION								(d) SHEAR			
	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)
	(7) ×(3) $\times 10^{-3}$	(20) ×(1)		(21)/(22) $\times 10^{-3}$	(23) ×(3)	(23) ×(4)	Σ(24)	Σ(25)	(7) ×(4) $\times 10^{-3}$	(28) ×(1)	Web area	(29)/(30) $\times 10^{3}$
	$P \times 10^{-3}$	$PL \times 10^{-3}$	A	$\frac{PL}{A}$	$PL \times \sin\alpha \div A$	$PL \times \cos\alpha \div A$	$E\,\delta_v$	$E\,\delta_h$	$V \times 10^{-3}$	$VL \times 10^{-3}$	A_w	$\frac{VL}{A_w}$
	Kips	Kip-ft	In.²	Kip-ft per sq in.					Kips	Kip-ft	In.²	
SB							0	0				
(etc.)												
....												
....												
....												
SP10							8.22	13.75				

The second "basic" case determined the order length of the strands used at T8, and the third determined that of the strands used at T10. In these stages the backstay stress was much less than in stage IX, and the bent leaned shoreward.

When the backstay strands were first erected, before any forward ties were in place, the bent had to be supported by a back-leg. To keep the initial stress in the backstay strands within the capacity of the pulling tackle which was used in their initial adjustment, the top of the bent was set back 3 ft. When the load from the T10 cables came on, the bent was pulled forward, of course, releasing the back-leg, and, from that time until final dismantling, it was pin-ended at the base.

In the calculations for the three "basic" positions, the problem was to determine a length of tie such that, when the horizontal and vertical components of the force at the point of attachment F (Fig. 26(*a*)) were applied first to the rib and then to the tie, the resulting location of F as determined from the tie system would coincide with that as determined from the rib.

The first step was to determine a position of the rib for each "basic" case, and to compute *H*-forces and *V*-forces (interrelated by an assumed slope of the link at its connection to the rib) which would raise the free-cantilevered rib to this position. The determination of this position, which was always at or near

(Continued)

Deformation				(*e*) Summary of Displacements, in Feet (Thousandths)								Point
(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(40)	(41)	(42)	(43)	
(31) $\times(3)$	(31) $\times(4)$	$\Sigma(32)$	$\Sigma(33)$	(18) $\times 10^3$ $\div 6E$	(27) $\times 10^3$ $\div E$	(34) $\times 2.6$ $\times 10^3$ $\div E$	Σ	(19) $\times 10^3$ $\div 6E$	(26) $\times 10^3$ $\div E$	(35) $\times 2.6$ $\times 10^3$ $\div E$	Σ	
$VL \times \sin\alpha \div A_w$	$VL \times \cos\alpha \div A_w$	$\frac{E}{2.6}\delta_h$	$\frac{E}{2.6}\delta_v$	Horizontal				Vertical				
				Bending	Thrust	Shear	Total	Bending	Thrust	Shear	Total	
Kip-ft per sq in.				→	←	→	→	↓	↓	↓	↓	
				0	0	0	0	0	0	0	0	SB
												(etc.)
												....
												....
												....
		34.58	58.89	63.32[a]	0.47[a]	3.06[a]	65.91[a]	114.06[a]	0.28[a]	5.21[a]	119.55[a]	SP10

[a] These quantities are the final results applicable to SP10, the free end in erection stage I, and will be recognized as including those stated in the text under "Scope of Computations."

"normal" or "geometric" position (namely, that which the point would occupy after completion of the entire structure), involved a consideration of the action of the rib under the then existing loading, and under subsequent loadings, at various temperatures.

In the first "basic" case, the elevations of the tie connection points T3 and T6 were determined by finding their location for erection stage IX (Fig. 22) such that the crown opening would be near "normal" at 60° F and the rib moments would be within safe limits. These tie-point elevations were determined for assumed loads from the cables at assumed slopes, and were essentially unchanged under the exact loads and slopes finally found; it was not essential that the final position in erection stage IX be exactly as stated, since it was important only that the ribs at closure be far enough apart to permit erection of the final members and insertion of the jacks. A final check was made of the crown elevation and opening, of course, after the tie lengths had been accurately determined, and, if it had been found that the elevations of the tie points originally used had led to an impossible closing condition, the tie lengths would have been recomputed for new assumptions of elevations of the tie points.

In a similar manner, the "basic" elevation of T8 was, in the second "basic" case, determined so that, starting from this elevation in erection stage V, the succeeding stages of erection could be completed without requiring that the ties at T8 be readjusted after their initial adjustment had been made. The "basic" elevation of T10 was determined similarly for erection stage III.

It must be noted, however, that the choice of the "basic" elevations thus selected also affected the stresses in the ribs, so that, particularly in the initial erection stages when the rib was very stiff and resistant to change in elevation, it was necessary to investigate and modify the "basic" tie-point elevations to keep the rib stresses within allowable values.

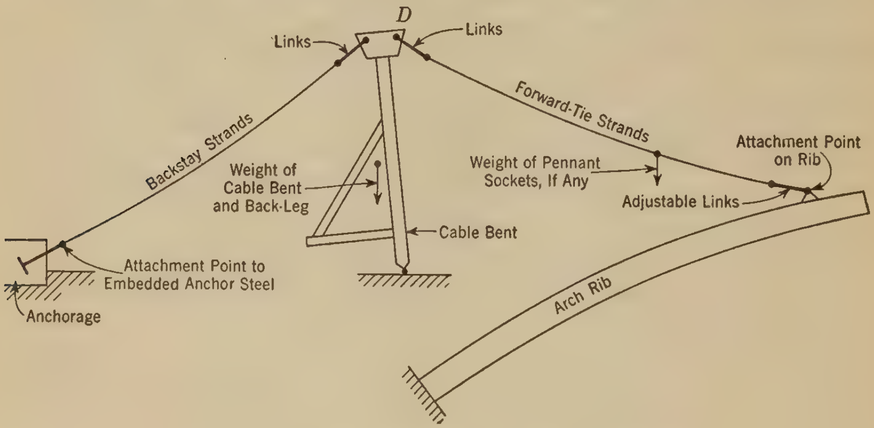
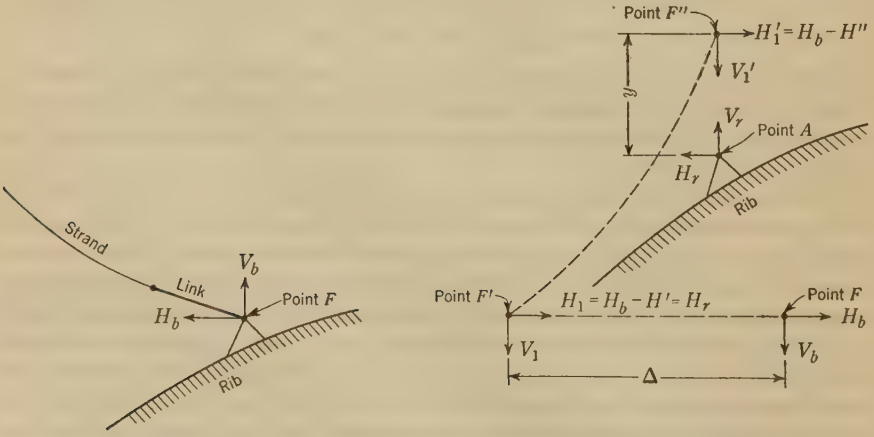

D
Links
Links
Backstay Strands
Forward-Tie Strands
Attachment Point on Rib
Weight of Cable Bent and Back-Leg
Weight of Pennant Sockets, If Any
Adjustable Links
Cable Bent
Attachment Point to Embedded Anchor Steel
Anchorage
Arch Rib

FIG. 25.—LOADS CONSIDERED IN CABLE CALCULATIONS


Point F''
$H_1' = H_b - H''$
V_1'
y
V_r
Point A
H_r
Rib
Strand
Link
V_b
H_b
Point F
Rib
Point F'
$H_1 = H_b - H' = H_r$
Point F
H_b
V_1
V_b
Δ

(*a*) BASIC CONDITION (*b*) BALANCING CONDITION

FIG. 26

Assuming a trial value of H, a length of tie was computed which would bring the tie and rib into coincidence at point F, and, for this length, the value of V for the tie was computed (that is, the slope of the link at the rib was computed, thus automatically determining V). By successive approximations, modifying

H and V on the rib each time but holding the elevation of point F essentially constant, tie lengths were finally determined for forces H_b and V_b which balanced the system. These lengths were the "basic" lengths, and the final position of point F so determined was the "basic" position of the point of attachment of the link to the rib.

By using these "basic" results as a starting point, and with the strand lengths now established, the amount of successive approximation computation for the remaining or "intermediate" cases was considerably reduced. These "intermediate" cases comprised erection stages II, IV, etc., and all stages at which strand ties were added, adjusted, or released. With one minor exception, the strand stresses in each "intermediate" case were less than those in the "basic" case to which it was immediately related (as stage II to stage III, etc.).

The method used for calculating the position and H-forces and V-forces for any intermediate condition is shown graphically in Fig. 26(*b*). With the location of point F, and the "basic" forces H_b and V_b known, values of H_r and V_r were assumed which would raise the rib to point A, approximately its normal position. This H_r-force was then applied to the link and cable system (the "basic" position of the cable was actually used so that only a differential force $H' = H_b - H_r$ was applied to this system), and a horizontal deflection Δ was determined so that point F moved horizontally to point F′; V_1 was then the vertical force required to balance the cable. Since points F′ and A did not coincide, point F′ was moved to point F″ by considering the forward-tie assembly rotated about the point of attachment of the link to the bent, and a new and slightly different $(H_1)'$ and $(V_1)'$ were determined. The vertical distance y was then a measure of the error in the assumption originally made, and new assumptions were set up and the process repeated as many times as was necessary to make points F″ and A coincide. After the cable system and rib were mutually in equilibrium, the moments and shears throughout the erected portion of the rib were investigated to assure that there were no overstresses.

The problem of connecting and disconnecting sets of forward ties and of adjusting them to stress or elevation made it desirable to investigate the effect of changing the length of all strands by 1 in., and of changing the length of one strand by 1 in. When more than one set of ties was connected to the rib at one time, as was the case beginning with erection stage V, calculations for the unit change in length of one set of strands included not only the effect on the stress in these strands and the elevation of the rib at their point of attachment, but also the same effects on all other effective sets of strands. These results were tabulated for field use, so that the erection forces had complete information regarding elevations of the rib at all splice and tie points, stresses in the strands, and effect of changing strand length on both elevation of the rib and strand stress, for all erection stages shown in Fig. 22. They also were given a sequence for releasing, partly, the load on the strands by jacking the ribs apart at the crown, casting off some strands, modifying the jack load, and finally casting off the remaining strands. Thus, it was possible to cast off the strands progressively, without any intermediate partly released stages, and without danger of excessive loads on top or bottom shims at the crown, or of excessive stresses on the last strands left in place.

FIELD CONTROL OF TIE AND RIB STRESSES

To measure the stresses in the tie strands, the contractor provided himself with four patented dynamometers, the principle of which is as follows: Holding the end supports (which are about 4 ft apart) upon the stressed rope, a spring is released which imposes a constant force transversely upon the rope, midway between supports. Since the deflection of the rope produced by this transverse force bears an inverse relationship to the longitudinal stress in the strand, the stress in the rope may be read on a properly calibrated dial. At the top of the tieback bent, platforms were installed on which the contractor's engineers could operate these dynamometers, free from the confusion and danger on the bridge.

If the external loads applied to the ribs by the forward ties were known for any erection stage, the stresses in the rib were no longer indeterminate. The aforementioned dynamometer provided a means for determining the actual loads applied to the rib by the forward ties, and thus made it possible, in the initial erection stages, to adjust the strands to a precalculated load so that the rib would not be overstressed. In the initial erection stages it was not practicable to determine the load applied by the ties by measurements of the rib deflection, since the rib was so stiff that large changes in tie loads produced deflections of the rib too small to measure. However, in later erection stages, the opposite effect occurred; the rib was then so flexible that even very small changes in tie loads produced large deflections. Therefore, in these later stages, elevations on the rib were used as the criterion in determining the forward-tie strand adjusted length, rather than the loads as measured by the dynamometer, and these loads were used only as a check.

This condition was actually a fortunate occurrence, as it happened that this strand-load check proved to be somewhat illusory. In the later stages, with the ribs at correct elevations, the sum of the strand forces was read 10% or more above the computed values. It was felt that the computations could not be in error to such a degree. The possibility to be feared was not that the strands would yield at these stresses, but that the dead loads had been underestimated and therefore the stresses in the finished bridge would be greatly in error. The contractor rejected this hypothesis and confidently maintained that the dynamometer readings were in error. Later, when the crown thrust was measured on the jacks and found to be exactly as computed, there was general satisfaction. In the initial stages, where dynamometer readings were used as the criterion, it is believed that they were in error by no more, and probably less, than 5% since the angle of inclination of the ties with the vertical was small, thus reducing the correction factor for slope.

The apparent error in dynamometer readings may have been in the calibration or in the manipulation, or in both. The dynamometers are understood to have been positively calibrated in a vertical rope-testing machine, and in a horizontal strand-pre-stressing machine. Correction charts were provided for inclined strand positions, such as prevailed on this operation, and it is natural to question their accuracy. Also, it was found that methods of manipulation, and the particular points on the strand to which the bearings were applied, changed the readings considerably. It follows that the technique used in taking the readings, which were recorded, may not have been the technique that should have been selected.

The adjustment of the several strands at one tie point was tedious, usually consuming several working days. The accuracy of the strand-length computations was such that there always was more than enough in the pinned links to put the rib at proper elevation. Sometimes one or more strands in a group would be connected at a 1-in. interval longer or shorter than most of the group, so that the measured stresses could be averaged better. To save time, it was considered permissible to let one or more strands take 15% more load than the average of the group, because that did not exceed the 120,000 lb considered proper. Levels taken along the ribs after each adjustment showed the arch-rib contours to be following the prescribed curve within fractions of an inch at all points.

THE ELASTIC CENTROID—ITS APPLICATION TO THE STUDY OF THE ACTION OF THE CLOSED RIB

To study the stresses in the arch ribs, with the half arches erected to the crown but not connected, and the method of controlling these stresses through jacking thrusts, it was convenient to use the previous type of analysis, with modifications. The previous concept of a rib deflected from the skewback as a cantilever was retained, and the imaginary cantilever deflections from bending, thrust, and shear were found as before. However, instead of introducing forces at the forward-tie points to raise the deflected half rib to its proper position, it was necessary to raise each half rib by applying to it, at the crown, a thrust and a moment coming from the opposite half rib. Since loads were symmetrical about the vertical axis through the crown, there would be no transfer of shear. Each half rib was assumed to carry, in addition to its own weight, the erection travelers and equipment in their final position and the jacks, including their supports and appurtenances, but no tiebacks.

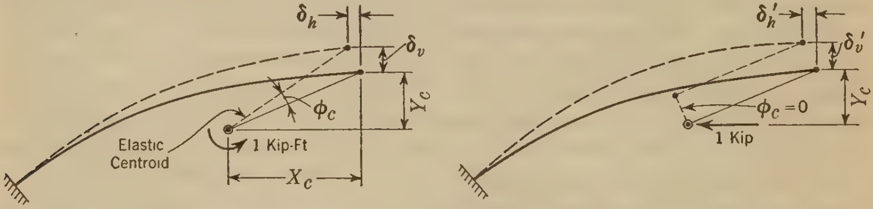
The physical forces available and under control at the crown would be a thrust from two 500-ton jacks above each rib and a thrust from two equal jacks below each rib. These jacks would be equidistant from the rib center line, and the manipulation of one set would be independent of the manipulation of the other. Therefore, both thrust and moment were alterable at will, provided that the effects of opening and closing the upper and lower jack openings, in any combination, could be known in advance in terms of change of thrust and change of moment.

Also, it was necessary to know the effect of jack thrust upon the crown elevation of the arch. Increasing the thrust would spread and raise the half ribs, and such an adjustment might have become desirable if the two arches were somewhat out of level across the bridge. It was not the intention to adjust the crown opening to change the crown elevation of the bridge as a whole, as the ribs were relatively flexible, and any substantial vertical change would require too much change of the crown opening to be practicable.

Another desideratum in the crown adjustment was that the two half ribs should be left in such a relationship, as to distance and angle at the crown, that the prefabricated floor panel 1–1′, several feet above the ribs, should fit without alteration of its over-all length.

To study these effects of jacking thrusts at the crown, the concept of the "elastic centroid" was introduced. The method here used, based on the physical concept of the elastic centroid defined in the paper, is taken from "Die graphische Statik der Baukonstruktionen," by H. Müller-Breslau.[17] The subject of the elastic centroid, also called the elastic center or neutral point, is approached from a somewhat different point of view in texts written in English,[18] but the basic concept is the same.

The elastic centroid of the half arch is a point, located mathematically, where the effect of thrust and moment upon the half arch may be segregated; it is a point in space, but is assumed to be connected to the crown point of the half arch by an infinitely rigid member. A unit moment applied at the centroid produces no translation of the centroid, but does rotate the imaginary connecting arm. A unit thrust (a horizontal force) applied at the centroid produces translation of the elastic centroid (and crown of the arch), but no rotation of the imaginary connecting arm or of the arch itself at the crown. This is shown graphically in Fig. 27.



(*a*) UNIT MOMENT AT CENTROID (*b*) UNIT HORIZONTAL FORCE AT CENTROID

Fig. 27

The problem of determining the thrust, moment, and shear existing at any point in any hingeless arch, of course, is one involving the analysis of an indeterminate structure. The general rule for such analysis is to cut the structure at the point in question, thus dividing it into two determinate structures. By evaluating the forces required to bring the cut ends into coincidence, the internal forces in the uncut structure can be determined.

Closure of the Rainbow arch was made at the crown, and all calculations were based on cutting the structure at that point. Since the structure and erection loads were symmetrical, consideration of shear was eliminated. It was only necessary to determine for one arm the thrust and moment, at the crown, that would return the freely deflected arm to its closed position—erection loads, camber, and temperature being considered.

It is obvious that both the crown moment and crown thrust produce rotation and thrust at the crown. The analysis of these interrelated functions is complex and is greatly simplified by the introduction of the elastic centroid concept in which translation and rotation are segregated. The procedure used in determining the thrust and moment at the centroid, and in translating these functions to the crown, follows:

---

[17] "Die graphische Statik der Baukonstruktionen," by H. Müller-Breslau, Vol. 2, Pt. 2, 1925 Edition, p. 377 *ff.*

[18] "Theory of Modern Steel Structures," by L. E. Grinter, The Macmillan Co., New York, N. Y., Vol. II, 1937, p. 206 *ff.*; also, "Movable and Long-Span Steel Bridges," edited by G. A. Hool and W. S. Kinne, McGraw-Hill Book Co., Inc., New York, N. Y., 1923, p. 440 *ff.*

To establish the location of the centroid, a moment of 1 kip-ft was applied to the arch rib at the crown, and ϕ_c, δ_h, and δ_v were computed for the crown from Eqs. 25 to 28, where (taking into consideration the basic properties of the rib), as defined by Fig. 27(*a*):

$$\phi_c = \frac{222.750}{6\,E\,10^3} = 0.001263 \times 10^{-3} \quad \text{(31a)}$$

$$\delta_h = 0.0566 \times 10^{-3} \quad \text{(31b)}$$

and

$$\delta_v = 0.2854 \times 10^{-3} \quad \text{(31c)}$$

For the purpose of simplifying computations, all calculations involving E or E_s (such as Eq. 31*a*) were made in kip units (E = 29,400 kips per sq in. and $E_s = \frac{E}{2.6} = \frac{29{,}400}{2.6}$ kips per sq in.). After solving Eqs. 31, the centroid ordinates were then (see Fig. 27(*a*)): $X_c = \frac{\delta_v}{\phi_c} = \frac{0.2854}{0.001263} = 226.0$ ft; and $Y_c = \frac{\delta_h}{\phi_c} = \frac{0.0566}{0.001263} = 44.8$ ft.

In skeleton form, Table 6 gives the computations (from sheet ED9) leading to Eqs. 31. A horizontal load of 1 kip at the centroid, using Eqs. 25 to 30, produced bending, thrust, and shear deformations. As defined by Fig. 27(*b*):

$$(\delta_h)' = (2.108 + 0.025 + 0.019)\,0.10^{-3} = 2.152 \times 10^{-3} \text{ ft}$$

$$(\delta_v)' = (6.557 - 0.007 + 0.047)\,0.10^{-3} = 6.597 \times 10^{-3} \text{ ft}$$

Table 7 gives in skeleton form the computations (from sheet ED9) leading to the foregoing numerical results. In the complete table (sheet ED9) from which Table 7 and other similar tables are condensed, the "point" column contains an entry for each splice point, SP13 to SP1, inclusive, and for each intermediate point at which there is a change in the area, and the moment of inertia, of the rib.

If a vertical force P is applied to the half rib at some point A, and if δ_v is the vertical deflection at point A due to a unit moment at the centroid (or by Maxwell's law, the rotation at the centroid due to unit vertical load at point A), then the angular rotation at the centroid due to force P is $P\,\delta_v$. The restoring moment at the centroid to overcome this angular rotation is $\frac{P\,\delta_v}{\phi_c}$ in which, by Fig. 27(*a*), ϕ_c is the rotation at the centroid due to unit moment. These values of δ_v for all critical points on the rib were computed for unit moment at the centroid. The value of ϕ_c is given in Eq. 31*a*.

Similarly, if the same force P is applied at the same point A, and if $(\delta_v)'$ is the vertical deflection at point A due to a unit horizontal force at the centroid (or, by Maxwell's law, the horizontal deflection at the centroid due to unit vertical load at point A), then the horizontal deflection at the centroid due to force P is $P\,(\delta_v)'$. The restoring horizontal force at the centroid to overcome this deflection is $\frac{P\,(\delta_v)'}{(\delta_h)'}$ in which $(\delta_h)'$, by Fig. 10, is the horizontal deflection at the centroid due to a unit horizontal load at the same point. These values of $(\delta_v)'$ were computed for all critical points on the rib for a unit horizontal load at the centroid. The value of $(\delta_h)'$ is 2.152×10^{-3}, as stated.

Thus, for any series of vertical loads acting on the half rib, the foregoing computations offer a rapid and accurate method for determining the moment and thrust, at the centroid, required to bring the crown to a position such that it can be joined to the adjacent half rib. The requirement is, of course, that the crown point have zero horizontal movement and zero rotation. This moment and thrust was:

TABLE 6.—DISPLACEMENTS DUE TO UNIT MOMENT OF 1 KIP-FT (SEE FIG. 27(*a*))

Point	(*a*) Rib Properties					(*b*) Bending			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
								(7) × (3)	
	L	$I \times 10^{-3}$	$\frac{L}{I} \times 10^3$	sin α	cos α	M	M'' and M'	$6E\tau$ $\times 10^3$	$6E\theta$ $\times 10^3$
	Ft	In²-ft²	1/(in²-ft)			Kip-ft		Kips per	
SB						1		....	0.540
							3	0.540	
	9.25	51.26	0.180	0.5497	0.8354				
							3	0.540	
SP13						1			2.883
							3	2.343	
	17.30	22.14	0.781	0.5331	0.8461				
							3	2.343	
.						1			4.050
							3	1.707	
	11.74	20.62	0.569	0.5331	0.8461				
							3	1.707	
SP12						1			3.843
							3	2.136	
	15.04	21.11	0.712	0.5011	0.8654				
							3	2.136	
(etc.)									
....									
....									
....									
							3	11.571	
SP1						1		....	11.571
								$6E\phi_c \times 10^3 =$	

$$M = \frac{\Sigma P\, \delta_v}{\phi_c} \qquad (32a)$$

and

$$H = \frac{\Sigma P\,(\delta_v)'}{(\delta_h)'} \quad \text{(32}b\text{)}$$

The thrust at the elastic centroid is also affected by the shop cambering and by temperature; the moment at the elastic centroid is not affected by changes in

APPLIED AT THE ELASTIC CENTROID (HALF ARCH), IN THOUSANDTHS OF A FOOT

DEFORMATION								
(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	Point
Σ(9)	(10) × (1)	(11) × (4)	(11) × (5)	Σ(12)	Σ(13)	(14) ÷ 6 E	(15) ÷ 6 E	
$6E\phi \times 10^3$	$6Eu \times 10^3$	$6Eu \times 10^3 \times \sin\alpha$	$6Eu \times 10^3 \times \cos\alpha$	$6E\delta_h \times 10^3$ ←	$6E\delta_v \times 10^3$ ↑	$\delta_h \times 10^3$ ←	$\delta_v \times 10^3$ ↑	
sq in.	Kip-ft per sq in.					Ft		
				0	0	0	0	SB
0.540	5	3	4					
				3	4	0.000017	0.000023	SP13
3.423	59	31	50					
								.
7.473	88	47	74					
				81	128	0.00046	0.00073	SP12
11.316	170	85	147					
								(etc.)
								....
								....
								....
				9,983	50,350	0.05659	0.28543	SP1
222.750								

length along the rib. If δL is the camber change in length of one section of the rib, α the angle of this section with the horizontal, and $(\delta_h)' = 2.152 \times 10^{-3}$, then the thrust due to camber is

$$H_c = \frac{\Sigma \delta L \cos\alpha}{(\delta_h)'} \quad (33)$$

Table 8(*a*) gives, in skeleton form, the computations (from sheet ED9) for the calculation of thrust due to increase of length in cambering, and the vertical displacement (upward) resulting from the elimination of spread by the pressing of one crown on the other.

If t is the change in temperature in degrees Fahrenheit, ϵ the linear coefficient of temperature expansion, l the length of the horizontal projection of the half arch, and $(\delta_h)' = 2.152 \times 10^{-3}$, then the change in thrust due to temperature change t is:

$$H_t = \frac{\epsilon t l}{(\delta_h)'} \qquad \text{(34)}$$

Table 8(*b*) gives, in skeleton form, the computations (from sheet ED9) for calculating the crown displacements, horizontal and vertical, for each degree of change of temperature. Note that Cols. 3 and 9, Table 8(*b*), apply also to erection stages I to IX, inclusive.

From Eq. 32*a*, the moment at the centroid due to final erection loads (ties removed and jacks in place at the crown) was found to be 122,760 kip-ft. From

TABLE 7.—DISPLACEMENTS DUE TO A UNIT HORIZONTAL FORCE OF ONE

Point	(*a*) RIB PROPERTIES					(*b*) BENDING				
	(1)	(2)	(3)	(4)	(5)	(18)	(19)	(20)	(21)	(22)
									(20) ×(3)	
	L	$I \times 10^{-3}$	$\frac{L}{I} \times 10^3$	sin α	cos α	Y	$(Y-44.82)$ M	M'' and M'	$6E\tau \times 10^3$	$6E\theta \times 10^3$
	Ft	In²-ft²	1/(in²-ft)			Ft	Kip-ft		Kips per	
SB						150.00	105.18	....		55.9
	9.25	51.26	0.180	0.5497	0.8354			310.46 305.38	55.9 55.0	
SP13						144.92	100.10			282.3
	17.30	22.14	0.781	0.5331	0.8461			291.08 281.86	227.3 220.1	
(etc.)										
....										
....										
....										
SP1						0.00	−44.82	−133.16	−513.6	513.6
										$\phi_c =$

Eqs. 32*b*, 33, and 34, the thrust at the centroid due to these same loads, at 60° (which is 10° above the normal or camber temperature of 50°, previously stated) was found to be 2,719 + 76 + 15 = 2,810 kips. These are the thrust and moment which, if applied at the elastic centroid of the half rib, would make it possible to join the two adjacent half ribs. Therefore, to determine the corresponding moment and thrust at the crown of the closed arch, it was only necessary to transfer them to that point, using the distance $Y_c = \dfrac{\delta_h}{\phi_c} = 44.8$ ft as the lever arm for H. The nominal moment and thrust at the crown of the closed arch as erected at 60° were found to be: $M = 122{,}760 - 2{,}810 \times 44.8 = 3{,}130$ kip-ft; and $H = 2{,}810$ kips.

Since one pair of the jacks at the crown, which were to be used for weighing the thrust and moment, were located above the rib and one pair below, on 13.92-ft centers, the proportion of this load to be taken by each pair of jacks (at 60° F) was:

$$\text{Top jack load, } + \frac{3{,}130}{13.92} + \frac{2{,}810}{2} = 1{,}630 \text{ kips}$$

KIP (SEE FIG. 27(*a*)) APPLIED AT THE ELASTIC CENTROID (HALF ARCH)

DEFORMATION						(*c*) THRUST DEFORMATION					Point
(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)	(33)	
Σ(22)	(23) ×(1)	(24) ×(4)	(24) ×(5)	Σ(25)	Σ(26)		(29) ×(1)		(30) ×10³ ÷(31)	(32) ×(5)	
$6E\phi \times 10^3$	$6Eu \times 10^3$	$6Eu \times 10^3 \times \sin\alpha$	$6Eu \times 10^3 \times \cos\alpha$	$6E\delta_h \times 10^3 \leftarrow$	$6E\delta_v \times 10^3 \uparrow$	P	PL	A	$PL \times 10^3 \div A$	$PL \times 10^3 \div A \times \cos\alpha$	
sq in.	Kip-ft per sq in.					Kips	Kip-ft	In.²	Kip-ft per sq in.		
				0	0						SB
55.9	517	284	432								
				284	432						SP13
338.2	5,851	3,119	4,951								
											(etc.)
											....
											....
											....
											SP1
0											

TABLE 7.—

Point	(c)—(*Continued*)			(d) Shear Deformation						
	(34)	(35)	(36)	(37)	(38)	(39)	(40)	(41)	(42)	(43)
	(32) ×(4)	Σ(33)	Σ(34)		(37) ×(1)	Web area	(38) ×10³ ÷(39)	(40) ×(4)	(40) ×(5)	Σ(41)
	$PL \times 10^3 \div A \times \sin\alpha$	$E\delta_h \times 10^3$ ←	$E\delta_v \times 10^3$ ↓	V	VL	A_w	$VL \times 10^3 \div A_w$	$VL \times 10^3 \div A_w \times \sin\alpha$	$VL \times 10^3 \div A_w \times \cos\alpha$	$E \times \delta_h \times 10^3 \div 2.6$
	Kip-ft per sq in.			Kips	Kip-ft	In.²	Kip-ft per sq.in.			
SB		0	0							0
(etc.)										
....										
....										
....										
SP1		728.48	209.86							215.50

and

$$\text{Bottom jack load, } -\frac{3{,}130}{13.92} + \frac{2{,}810}{2} = 1{,}180 \text{ kips}$$

Closure Computations

Since there was no assurance in advance that it would be possible to survey the abutments and fabricate and erect the arch without any appreciable errors, it had been decided to fabricate the ribs with a normal opening of 11 in. at the crown, and, after weighing in the proper crown thrust and moment, to measure this actual opening and fabricate a piece that would fit it exactly. This closing piece, referred to as the "keystone piece," was prefabricated 13 in. long on its center line, with its two ends not milled. Provision was made that it could be finally as small as 9 in. on the center line, a variation of ±2 in. from the 11-in. normal.

Prior to closure work in the field, the flexibility of the arch rib itself was investigated completely so that it would be possible to determine the proper correctional movements at the crown to produce theoretical thrust and moment after a weighing had been made.

To determine the effect of unit movements at the crown on the thrust and moment, the results of the elastic centroid calculation were used. Thus, the

(*Continued*)

	(e) SUMMARY OF DISPLACEMENTS, IN THOUSANDTHS OF A FOOT								
(44)	(45)	(46)	(47)	(48)	(49)	(50)	(51)	(52)	Point
Σ(42)	$\frac{(27)}{6E}$	$\frac{(35)}{E}$	$\frac{(43)\times 2.6}{E}$	Σ	$\frac{(28)}{6E}$	$\frac{(36)}{E}$	$\frac{(44)\times 2.6}{E}$	Σ	
$E\times\delta_v$ $\times 10^3$ $\div 2.6$	Horizontal = $(\delta_h)' \times 10^3$				Vertical = $(\delta_v)' \times 10^3$				
	Bending ←	Thrust ←	Shear ←	Total ←	Bending ↑	Thrust ↓	Shear ↑	Total ↑	
0	0	0	0	0	0	0	0	0	SB
									(etc.)
									....
									....
									....
537.64	2.1084	0.0248	0.0191	2.1523	6.5566	0.0071	0.0475	6.5970	SP1

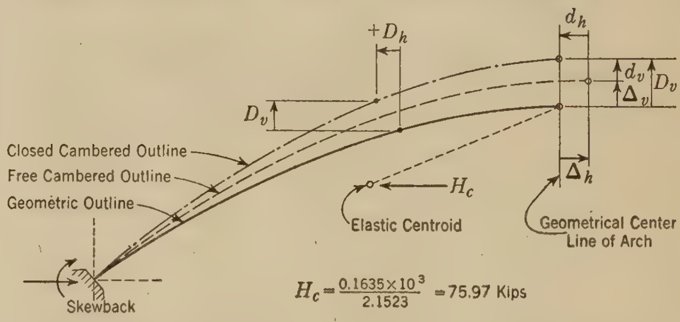
horizontal force required at the centroid to deflect the centroid (and the crown) of the half rib a distance of $\frac{1}{2}$ in. (0.0417 ft) horizontally without rotation was found to be, since $(\delta_h)' = 2.152 \times 10^{-3}$, $H = \frac{0.0417}{2.152 \times 10^{-3}} = 20$ kips. Similarly, the moment on the half rib, which would rotate the crown of the rib through an angle whose tangent was $\frac{1}{2}$ in. in $\frac{12.23\text{ ft}}{2}$, or $0.0417 \times \frac{2}{12.23}$, was (from Eq. 31*a*): $M = 0.0417 \times \frac{2}{12.23} \times \frac{1}{0.000001263} = 5{,}400$ kip-ft. This moment, applied at the centroid, would cause only rotation at the centroid, but would produce deflection at the crown. The horizontal component of this deflection would be (from Eq. 31*b*): $\delta_h = 5{,}400 \times 0.0000566$ ft; and, since for this condition only rotation was wanted at the crown, it was necessary to introduce a restoring force (away from the skewback) at the centroid to eliminate this horizontal deflection at the crown. This restoring force was $H = M \times \frac{\delta_h}{(\delta_h)'} = 5{,}400 \times \frac{0.0000566}{0.002152} = 142$ kips.

There were thus two basic conditions—one to produce a $\frac{1}{2}$-in. translation of the crown without rotation, or a 1-in. opening between the two ribs; and the other to produce a rotation at the crown, without translation, of such magnitude that the opening at the top jack would be increased by 1 in., and that at the bottom jack would be decreased by 1 in. This is shown graphically in Fig. 28.

From these basic values, combinations were made which enabled the fabricators to determine readily the expected movement for a given change in jack load, or the expected change in jack load for a given movement. It was obvious from the values in the first three columns of Table 9 that the crown could be translated easily, whereas those in the last three columns show that it could be rotated only with great difficulty. This indicated that it would be a relatively simple matter to attain the proper relationship between the load on the top and bottom jacks—that is, to locate the thrust line vertically on the rib—but that it would be difficult to change greatly the total thrust on the two sets of jacks. Thus, there was little control over thrust, and better control over moment, at the crown.

TABLE 8.—Displacement (in Feet) Due to

(a) Displacements Due to Camber

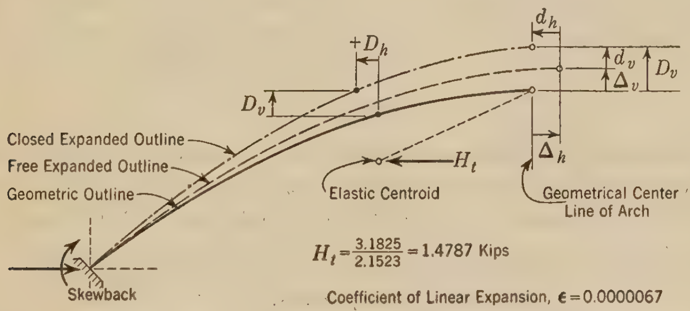


Point	Horizontal Component							Vertical Component					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
	Camber (sheet ED2) ∂l	$\cos \alpha$	$\partial l \times \cos \alpha$	$\Sigma \partial l \times \cos \alpha$	Table 7, Col. 48	$\delta_h \times 75.97$	$d_h - \Delta_h$	$\sin \alpha$	$\partial l \times \sin \alpha$	$\Sigma \partial l \times \sin \alpha$	Table 7, Col. 52	$\delta_v \times 75.97$	$d_v + \Delta_v$
				Δ_h →	$\delta_h \times 10^3$ ←	d_h ←	D_h + ← − →			Δ_v ↑	$\delta_v \times 10^3$ ↑	d_v ↑	D_v ↑
SB				0	0	0	0			0	0	0	0
	0.0015	0.8354	0.0013					0.5497	0.0008				
SP13				0.0013	0.0020	0.0002	−0.0011			0.0008	0.0027	0.0002	0.0010
	0.0076	0.8461	0.0064					0.5331	0.0041				
SP12				0.0077	0.0472	0.0036	−0.0041			0.0049	0.0729	0.0055	0.0104
	0.0131	0.8654	0.0113					0.5011	0.0066				
....													
....													
TP3				0.1249	2.1691	0.1648	0.0399			0.0472	5.8300	0.4429	0.4901
	0.0080	0.9901	0.0079					0.1401	0.0011				
SP3				0.1328	2.1100	0.1603	0.0275			0.0483	6.0845	0.4622	0.5105
	0.0154	0.9963	0.0153					0.0864	0.0013				
SP2				0.1481	2.1456	0.1630	0.0149			0.0496	6.4654	0.4912	0.5408
	0.0154	0.9996	0.0154					0.0289	0.0004				
SP1				0.1635	2.1523	0.1635	0.0000			0.0500	6.5970	0.5012	0.5512

When the foregoing effects of unit jacking had been calculated, it was a matter of simple geometry to construct diagrams tying the movements to the aforementioned criterion—that the upper ends of spandrel columns 1 and 1′ should be at such a distance as to accommodate the prefabricated panel of floor. It appeared that, because of the extreme flexibility of the half ribs in regard to translation, it would not be practicable to change greatly the total crown thrust, and that something quite close to the thrust, as weighed with nominal 11-in. opening, would have to be accepted. Because of the apparently high stresses read on the tieback strands as erection proceeded, there arose some nervousness regarding what the weighed thrust might prove to be. An increase in thrust of 100 kips would increase the computed skewback moment by 15,000 kip-ft. This would be of little import if it arose from a uniformly distributed underestimate of the vertical loads, which would produce counter moments; it could

Camber and Temperature Change

(b) Displacements for Each Degree (F) of Temperature Increase



$H_t = \frac{3.1825}{2.1523} = 1.4787$ Kips

Coefficient of Linear Expansion, $\epsilon = 0.0000067$

Horizontal Component						Vertical Component						Point
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	
X	475 $-X$	ϵx $\times 10^3$	Table 7, Col. 48	δ_h $\times 1.4787$ $\times 10^3$	$(d_h - \Delta_h)$ $\times 10^3$	Y	150 $-Y$	ϵy $\times 10^3$	Table 7, Col. 52	δ_v $\times 1.4787$ $\times 10^3$	$(d_v + \Delta_v)$ $\times 10^3$	
	From SB x	$\Delta_h \times 10^3$ →	$\delta_h \times 10^3$ ←	$d_h \times 10^3$ ←	$D_h \times 10^3$ $\overset{+}{\leftarrow}$ $\overset{-}{\rightarrow}$		From SB y	$\Delta_v \times 10^3$ ↑	$\delta_v \times 10^3$ ↑	$d_v \times 10^3$ ↑	$D_v \times 10^3$ ↑	
475.00	0	0	0	0	0	150.00	0	0	0	0	0	SB
467.27	7.73	0.0518	0.0020	0.0030	−0.049	144.92	5.08	0.0340	0.0027	0.0040	0.038	SP13
442.70	32.30	0.2164	0.0472	0.0698	−0.147	129.44	20.56	0.1378	0.0729	0.1078	0.246	SP12
												....
110.92	364.08	2.4393	2.1691	3.2074	0.768	0.86	149.14	0.9992	5.8300	8.6208	9.620	TP3
90.00	385.00	2.5795	2.1100	3.1201	0.541	5.20	144.80	0.9702	6.0845	8.9972	9.967	SP3
45.08	429.92	2.8805	2.1456	3.1727	0.292	1.30	148.70	0.9963	6.4654	9.5604	10.557	SP2
0.00	475.00	3.1825	2.1523	3.1826	0.000	0.00	150.00	1.0050	6.5970	9 7550	10.760	SP1

be a serious matter if it arose from some other inexplicable cause, and was progressively additive from crown to skewback. The contractor did not fear that this latter hypothesis would be true, and was unwilling to agree that, if the total thrust should read higher than computed, it would be obligatory to reduce it. Fortunately, the computed and the weighed total thrust agreed within 1%; the circumstances are recounted because they brought into full relief the necessity, in closing a similarly indeterminate structure, for maintaining the highest possible accuracy in computing the amount and distribution of all dead loads.

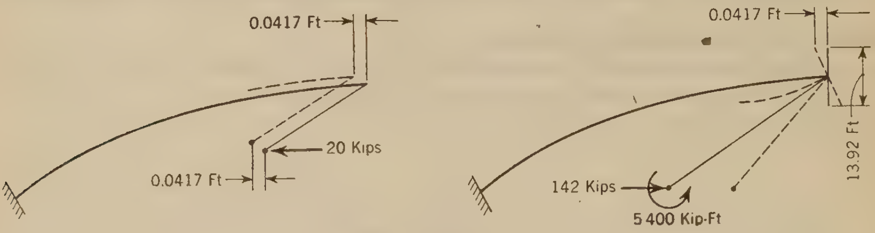


FIG. 28

TABLE 9.—EXPECTED MOVEMENT, IN INCHES, FOR A GIVEN CHANGE IN JACK LOAD (KIPS)

(+ = Increase, and − = Decrease)

Changes in:	Change in Top Jack Load (Kips):					
	+100	0	+50	+314	−366	−52
Bottom jack load (kips)	0	+100	+50	−366	+438	+72
Top jack opening (inches)	+13.2	+11.0	+12.1	+1	0	+1
Bottom jack opening (inches)	+11.2	+9.6	+10.4	0	+1	+1

FIELD CLOSURE

Actual weighing of thrust was done on specially constructed "weighing capsules," one of which was inserted in series with each hydraulic jack. They were sealed, high-pressure hydraulic mechanisms, in which extremely small movements of a plunger under load application were transmitted to a Bourdon tube gage by the enclosed liquid. Each capsule was calibrated with its gage to a load of 400 tons, on the 800,000-lb testing machine at Lehigh University in Bethlehem, Pa. The several gage reading-load curves, as plotted, were straight lines, and were extended to 500 tons, the jack capacity.

Check readings of crown thrusts were also made on 10-in. gages on the hydraulic jack feed lines, and these agreed very closely with the readings on the capsules, although neither the jacks nor the gages had been specially calibrated. The accuracy of hydraulic jacks for reading loads has been questioned many times, but experience on this structure indicates that, if the jacks are in good condition and are calibrated with the gages, and if readings are always taken on ascending loads, they will read the actual loads with only mirror errors.

A fuller discussion, with record of individual readings on all capsule gages and all jack gages, was published in 1941.[19] Crown thrusts were read after midnight; temperature readings at several points indicated quite stable temperatures between 60° and 64° F. Readings were taken under four different top and bottom crown opening combinations, and it was interesting to see how closely the relationships between change of opening and change of thrust agreed with the results of the intricate computations herein explained.

No better crown opening was found, all factors considered, than the originally selected nominal value of 11 in. The total thrusts at that opening, corrected to 60° F, were as follows, in kips:

Rib	Sum of two top jacks	Sum of two bottom jacks	Total
North rib	1,716	1,123	2,839
South rib	1,757	1,081	2,838
Theoretical	1,630	1,180	2,810

The total measured thrust was therefore only 1% in excess of the computed value; the point of application, however, was above that indicated by computations. The dead weights which seem most open to question are those of the erection travelers and appurtenances, all located near the crown, where they would operate to raise the thrust line. It appears highly probable that not only the thrusts, but also the moments, throughout the structure are within negligible fractions of their computed values. If an excess moment does exist at the crown, it is quite small, does not increase at other points, and is a small proportion of the total moment, including that from partial live load, at any rib section.

CONCLUSION

If a long, flexible arch bridge of the hingeless type is built with meticulous accuracy in surveying, in fabrication, and in erection, it can be closed on its properly cambered form without need for final adjustments or fear of considerable error in the locked-up stresses from the loads in place at time of closure. If not laid out and built with such meticulous accuracy, it is not probable that such adjustments as can be made at the time of closure would provide sufficient remedies or create full confidence in the stress distribution.

APPENDIX II

For the benefit of any who may wish to make a further and more detailed study of points raised in this paper, the following descriptive index of the information on file at Engineering Societies Library has been prepared.

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[19] "Weighing Reactions of Rainbow Bridge, Niagara Falls," by Jonathan Jones, *Civil Engineering*, December, 1941, p. 730.

Sheets	Information
ED1	Basic geometry.
ED2	Camber calculations and shop working points.
ED3	Areas, bending properties, and capacities of rib sections and splices.
ED4 to ED8, inclusive	The vertical and horizontal displacements of splice points, tie points, and crown resulting from unit vertical or horizontal forces acting successively at the tie points (see Fig. 24).
ED9	The location of the elastic centroid of the half arch; the vertical and horizontal displacements, for the half arch, of splice points, tie points, and crown, resulting from unit horizontal thrust or unit moment acting at the elastic centroid; these same displacements, for the closed arch, resulting from camber and from change in temperature (see Tables 6, 7, and 8).
ED10 to ED15, inclusive	The vertical and horizontal displacements of splice points, tie points, and crown under erection dead loads, for the various erection stages, considering the erected portion of the arch to be cantilevered from the skewback (see Table 5).
ED16	The resisting moment of the anchor bolts at the skewback.
ED17	The theoretical crown thrust and moment in the closed arch under erection loads; the horizontal and vertical displacements of splice points, tie points TP6 and TP3, and crown under these loads; the moments and thrusts at the splice points for these loads. The erection loads used were based on carefully made preliminary estimates, but were found to be in error by small amounts when final shop bills were completed. The final calculations for theoretical thrust and moment at the crown were made from influence lines constructed from this basic information (see sheets A129, and A137 to A140, inclusive).
ED18	Tabulation of field information for arch elevations, strand stresses, and strand adjustments; index to calculation (A) sheets which are supplementary to these ED sheets.
ED19 and ED20	Wind stresses and displacements during erection.
ED21 to ED23, inclusive	Backstay strand setting and tension charts.

For index to calculation (A) sheets, and their relationship to the foregoing ED sheets, see sheet ED18. The following calculation sheets referred to on sheet ED18 are included with the information on file in the Engineering Societies Library:

A19 to A24, inclusive, with charts; A30 to A135, inclusive.

The following subsequent sheets, not referred to on sheet ED18, are also included:

A137	Final revision in erection dead loads at time of closure at crown.
A138	Effect of these latter revisions in erection dead load on theoretical thrust and moment at crown.
A139	Final calculation of total theoretical thrust and moment at crown.
A140	Theoretical elevations on top of cover plate at splices, at closure.

ERECTION OF STEEL SUPERSTRUCTURE

By E. L. Durkee,[20] M. Am. Soc. C. E.

Synopsis

The erection of the steel superstructure of the main 950-ft span of the Rainbow Bridge is described in this paper. Since it is the longest, single, fixed arch span in the world unusual care was exercised in planning the method of erection as well as carrying out the actual erection operations in the field.

The paper describes the general method of erection and the erection equipment. The most unusual features are the tieback system of cable supports used to cantilever the two half arches from each side out to the center of the span; the method of adjusting these cable supports and of measuring the stresses in them; the method of surveying and controlling the profile of the arch ribs during the various erection stages; the measurement of the crown opening left for the purpose of determining the crown stresses; and the fabrication of a special keystone section to make the actual stresses agree with the computed ones for the dead load of the arch ribs. Also, the erection of the floor steel and the placing of the concrete deck are described.

The general features and the design of the arch are covered in the first paper of the Symposium. The calculation of the erection stresses in the arch and tieback system used for controlling the fabrication and erection of the arch ribs is contained in the third paper.

Introduction

Award of the general contract for the fabrication and erection of the 950-ft main arch span of the Rainbow Bridge, including the concrete deck, was made to the Bethlehem Steel Company early in May, 1940. In the construction operations it was required that all workmen employed should be residents of the country where such operations were being done. It was desirable, however, that the field work be under one management and it was agreed that all construction operations be under the supervision of Bethlehem.

The entire responsibility for the exact position and elevation of the arch was that of the contractor. He was required to check all control lines and grades, including the measurement of the distance across the river between the skewbacks made by the engineers, as well as the exact angle of the concrete skewbacks. The engineers' triangulation of the main span showed a length of 950.016 ft, and the contractor's check measurement gave 950.013 ft based on entirely independent base lines—a difference of only 1 in 317,000.

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[20] Res. Engr., Erection Dept., Bethlehem Steel Co., Alameda, Calif.

Because of the construction program for the concrete approaches, work on the main span had to be started in the middle of December and was carried on under extremely unfavorable working conditions during the next three months. In spite of this, and the difficulty of securing adequate help, work progressed steadily throughout the winter, without any serious delays and without a single fatality or serious accident of any kind. The work was completed within the allotted contract time and the bridge was opened to traffic on November 1, 1941.

Method of Erection

Cantilever Tieback System.—The arch, obviously, could not be erected on falsework. Although a single bent might have been placed on the steep and rocky shore on either side of the river, it would have been too close to the abutments to have been of much use. To construct bents farther out in the river was impossible because of the sharp angle of the rocky bottom which sloped at about 45° to a maximum depth estimated at 175 ft. Since construction was to be undertaken in the winter and spring, there was also the same danger from ice such as destroyed the Honeymoon span (which the Rainbow Bridge replaced).

An overhead tieback system of support was the obvious solution. Since the arch ribs are entirely self-supporting (there being no spandrel truss system), it was decided to cantilever only the arch ribs and their bracing, which would reduce the dead load to be supported during erection considerably and also permit temporary use of the spandrel columns and floor material in the overhead falsework. This method of erection is not new and has been used in its essential principles in several other arch spans, notably the Hell Gate arch in New York, N. Y., the Sydney Harbour arch in Sydney, Australia, and the Croton Lake arch in Westchester County, New York.

It was found that the first three panels of arch rib, from the abutment to point 10, could be cantilevered on the thirty-two 3-in. diameter upset anchor bolts in each abutment, thus locating the first point of support at point 10. Three other points, at panels 8, 6, and 3, were chosen at which to connect the forward cable ties so as to keep the stresses in these ties and the connection details within reasonable limits considering the weight to be supported and the angle of inclination of the ties. Fig. 29 shows a general view of one half the span cantilevered out to the center with the supporting cable ties and other erection equipment used.

Cable Bents.—To support these ties, a steel tower or bent 130 ft high and 56 ft 2 in. between centers of columns was located on the end of the concrete approach. This height was such as to give a favorable angle of inclination to the forward ties at panel point 3 and keep the stress in, and number of, these ties within reasonable limits. Each leg of the tower was composed of two of the permanent spandrel columns, stitch-bolted together along one face, all riveting in this face being omitted in the shop except for enough rivets with countersunk heads to hold the enclosed box sections together. Each individual column was about 3 ft square, giving a bent column 3 ft by 6 ft. A special base and top section was fabricated to provide for a hinged bearing at the base and suitable details at the top to which to connect the forward ties and backstays.

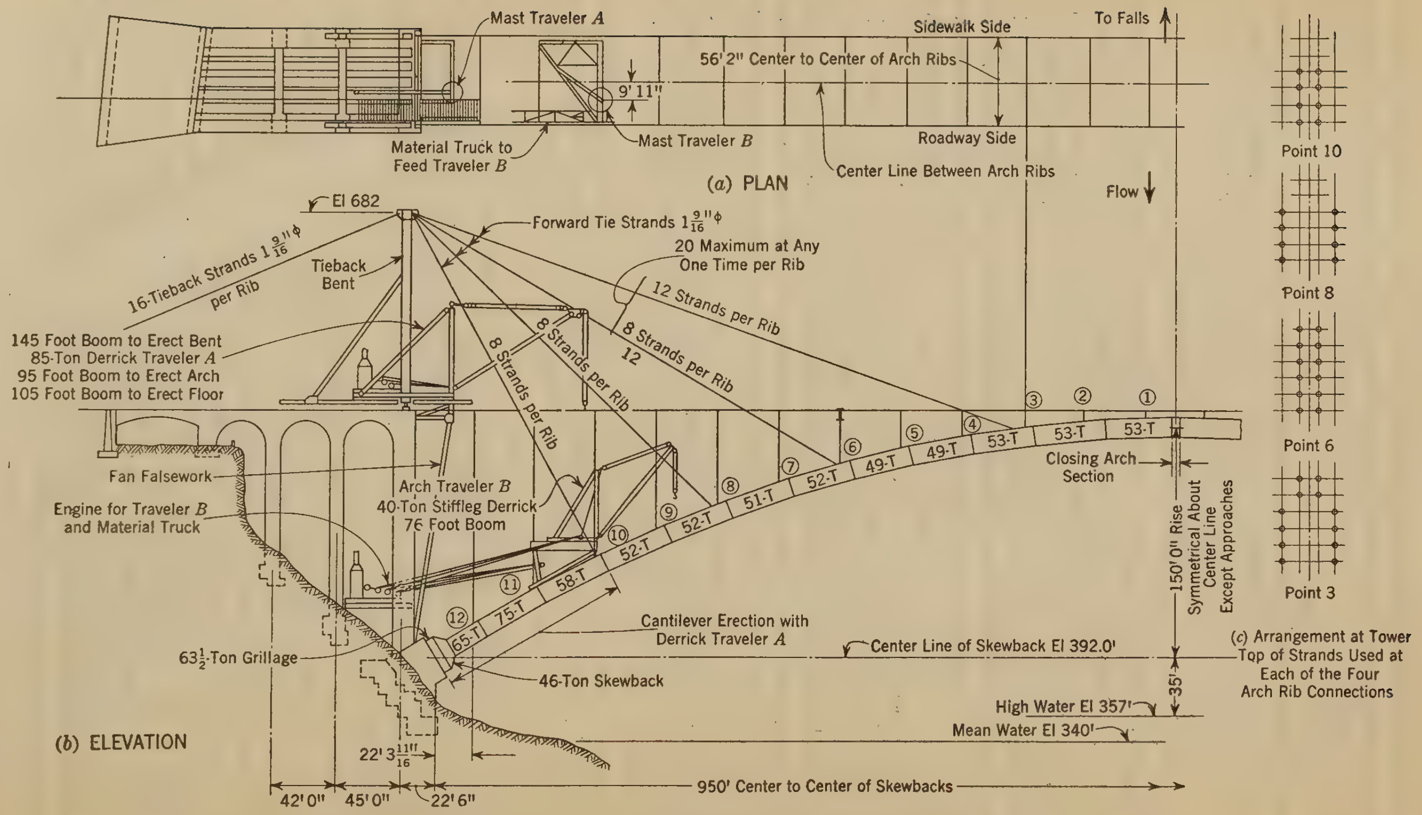
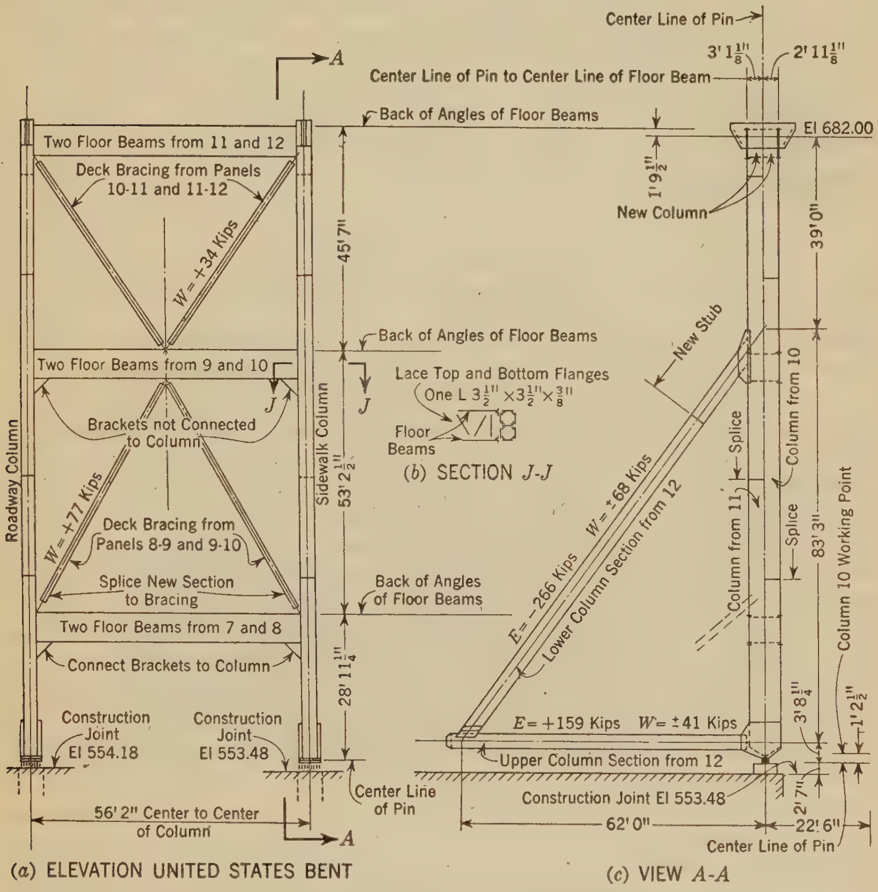

Mast Traveler *A*
Sidewalk Side
To Falls
56' 2" Center to Center of Arch Ribs
9' 11"
Material Truck to Feed Traveler *B*
Mast Traveler *B*
Roadway Side
Center Line Between Arch Ribs
Flow
(*a*) PLAN
El 682
Forward Tie Strands $1\frac{9}{16}"\phi$
20 Maximum at Any One Time per Rib
16-Tieback Strands $1\frac{9}{16}"\phi$ per Rib
Tieback Bent
12 Strands per Rib
8 Strands per Rib
12
8 Strands per Rib
8 Strands per Rib
145 Foot Boom to Erect Bent
85-Ton Derrick Traveler *A*
95 Foot Boom to Erect Arch
105 Foot Boom to Erect Floor
①
②
③
④
⑤
⑥
⑦
⑧
⑨
⑩
⑪
⑫
53-T
53-T
53-T
49-T
49-T
52-T
51-T
52-T
52-T
58-T
75-T
65-T
Closing Arch Section
Fan Falsework
Arch Traveler *B*
40-Ton Stiffleg Derrick
76 Foot Boom
Engine for Traveler *B* and Material Truck
150' 0" Rise
Symmetrical About Center Line Except Approaches
Cantilever Erection with Derrick Traveler *A*
Center Line of Skewback El 392.0'
$63\frac{1}{2}$-Ton Grillage
46-Ton Skewback
35'
High Water El 357'
Mean Water El 340'
(*b*) ELEVATION
$22' 3\frac{11}{16}"$
42' 0"
45' 0"
22' 6"
950' Center to Center of Skewbacks
Point 10
Point 8
Point 6
Point 3
(*c*) Arrangement at Tower Top of Strands Used at Each of the Four Arch Rib Connections

FIG. 29.—VIEW OF CANTILEVER TIEBACK SYSTEM

The top of each tower column consisted of four gusset plates of hammer-head shape, two of 1¾-in. thickness and two of 1⅜-in. thickness, each providing for the connection of four (on the two outside plates) or six (on the two inside plates) forward ties through short, heavy, pin-connected links. Fig. 30 shows the general make-up of this bent, and Table 10 indicates the reactions.


Center Line of Pin
3' 1⅛"
2' 11⅛"
A
Center Line of Pin to Center Line of Floor Beam
Back of Angles of Floor Beams
El 682.00
Two Floor Beams from 11 and 12
Deck Bracing from Panels 10-11 and 11-12
W = ±34 Kips
45' 7"
1' 9½"
New Column
39' 0"
Back of Angles of Floor Beams
New Stub
Two Floor Beams from 9 and 10
J
Lace Top and Bottom Flanges
(One L 3½" × 3½" × ⅜"
Floor Beams
Brackets not Connected to Column
Roadway Column
Sidewalk Column
53' 2½"
W = ±68 Kips
Lower Column Section from 12
Splice
Column from 10
Column from 11
83' 3"
Column 10 Working Point
W = ±77 Kips
Deck Bracing from Panels 8-9 and 9-10
Splice New Section to Bracing
Back of Angles of Floor Beams
E = -266 Kips
Two Floor Beams from 7 and 8
Connect Brackets to Column
28' 11¼"
3' 8¼"
1' 2½"
Construction Joint El 554.18
Construction Joint El 553.48
E = +159 Kips W = ±41 Kips
Upper Column Section from 12
Center Line of Pin
Construction Joint El 553.48
2' 7"
56' 2" Center to Center of Column
62' 0"
22' 6"
Center Line of Pin

(a) ELEVATION UNITED STATES BENT (b) SECTION J-J (c) VIEW A-A

FIG. 30.—MAKE-UP OF CABLE BENTS

The columns were braced in their narrow direction by three struts each made up of a pair of floor beams from the permanent bridge deck, and each pair of which was braced laterally with temporary angle latticing. The two panels of diagonal bracing completing this bent consisted of permanent lateral bracing also from the deck. An open, unbraced panel of 24 ft was left at the base to permit the passage of the arch-rib sections and other material.

At the top and on each side of each column was a double-deck, steel-framed cage providing two working platforms (see Fig. 31) each 3.5 ft wide by 30.5 ft long. These provided complete access to the various groups of cable strands in the backstays and forward ties both for erecting them and later for measuring the stresses in these strands with the special cable dynamometer. On top of each column was a small, steel jinniwink or A-frame derrick with a single sheave at the peak. This rig was detailed with a central, vertical pin about which it could be swung, enabling the rig to face shoreward for the erection of the backstay cables and riverward for erecting the forward ties.

TABLE 10.—REACTIONS IN CABLE BENTS (SEE FIG. 30), IN KIPS

Description	*E*	*W*	Total
Maximum Reactions:			
Under stiffener	+213	±55[a]	+268
Under column	+1,799	±65[b]	+1,864
Minimum Reaction:			
Under column	+97	−55[a]	+42
Maximum Horizontal Shear:			
Longitudinal, at stiffener	+85	±45[a]	+130
Transverse, at column	....	±27[b]	....

[a] Longitudinal. [b] Transverse.

Anchorages.—The top of the cable bent was guyed back on the shore side by means of backstay cables connected to concrete anchorage blocks embedded in the solid rock, there being a separate anchorage for each column. Their location, 360 ft and 375 ft in back of the cable bent, was such as to give the same inclination to the backstays on each side of the river. On the Canadian side of the river, the bare rock surface is exposed, as there is no overburden near the rim of the gorge. Here the anchorage blocks were embedded 6 ft to 8 ft in the solid limestone and extended 17 ft to 20 ft above the rock surface. On the New York side of the river, there is an overburden of 18 ft to 20 ft of clay, gravel, and hardpan which had to be excavated down to the rock surface before blasting out the rock itself. It was also necessary to excavate a trench in front of each anchorage block for the passage of the backstay cables.

In plan, each anchorage consisted essentially of a rectangular block of concrete approximately 16 ft by 19 ft, and 26 ft in height, weighing 550 tons or about 1.5 times the vertical component of the maximum pull of 920 tons in each set of backstay cables (see Fig. 32). The front face of each anchorage in the embedded rock was inclined so as to be normal to the resultant of the cable pull and the weight of the concrete, this resultant thrust being downward at an angle of approximately 12°. In the lower half of each anchorage was embedded a set of structural bars projecting just above the rock surface for connecting the links and bars designed to take the pull of the tiebacks. The embedded anchorage bars spread out fanwise so as to distribute their load throughout the entire area of the anchorage.

In Fig. 33 members AF are sets of three angles and members AB and LK are sets of eight bars.

In excavating the north anchorage pit on the west side of the river, the rock at the surface on the front was found to be of inferior quality and several large uncemented areas between the natural bed planes were encountered during the excavation. Rock excavation for the approach structures in front of this anchorage also was extended somewhat farther than expected and was cut up further with both transverse and longitudinal sewer trenches. The anchorages, therefore, were excavated 2 ft deeper than originally intended in the north pit and 1 ft deeper in the south pit to insure that the reaction from the backstays would be carried safely into the rock.

In order to prove the value of the rock and the safety of the anchorage design in general, a jacking test was devised for the north anchorage pit on the Canadian side. The entire front face of the excavation was timbered, and grout was poured between the timbering and the rock to give a uniform bearing. Three hydraulic jacks of 350-ton and 500-ton capacity, laid horizontally in the pit, were used to apply a maximum horizontal thrust of 900 tons to 1,000 tons to simulate the inclined design reaction of 870 tons. The thrust from these jacks was taken by timbering against the rear face of the pit in which a shelf had to be excavated as shown in Fig. 34. Observation points were established on the rock surface in front of the pit and compared with other points set in the rock well to either side of the excavation so as to detect any possible movement. It was intended to apply the test load for at least 12 hours, but weather conditions made this impossible. For the 5 or 6 hours during which load actually was applied, no movement or distress of any kind in the rock in front of or around the anchorage pit could be observed.

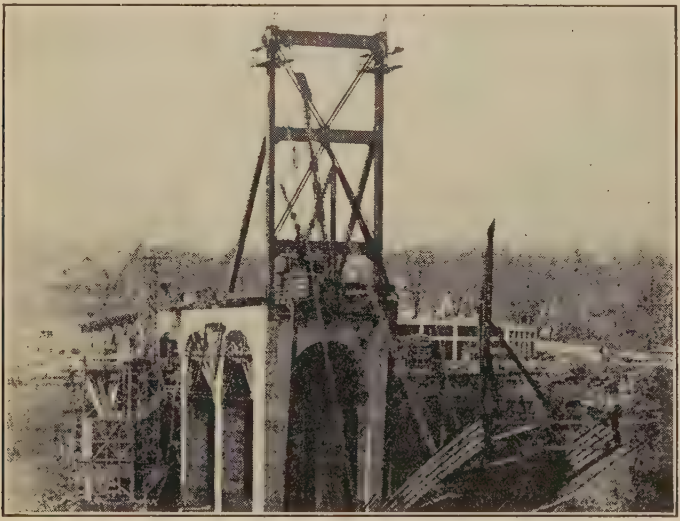
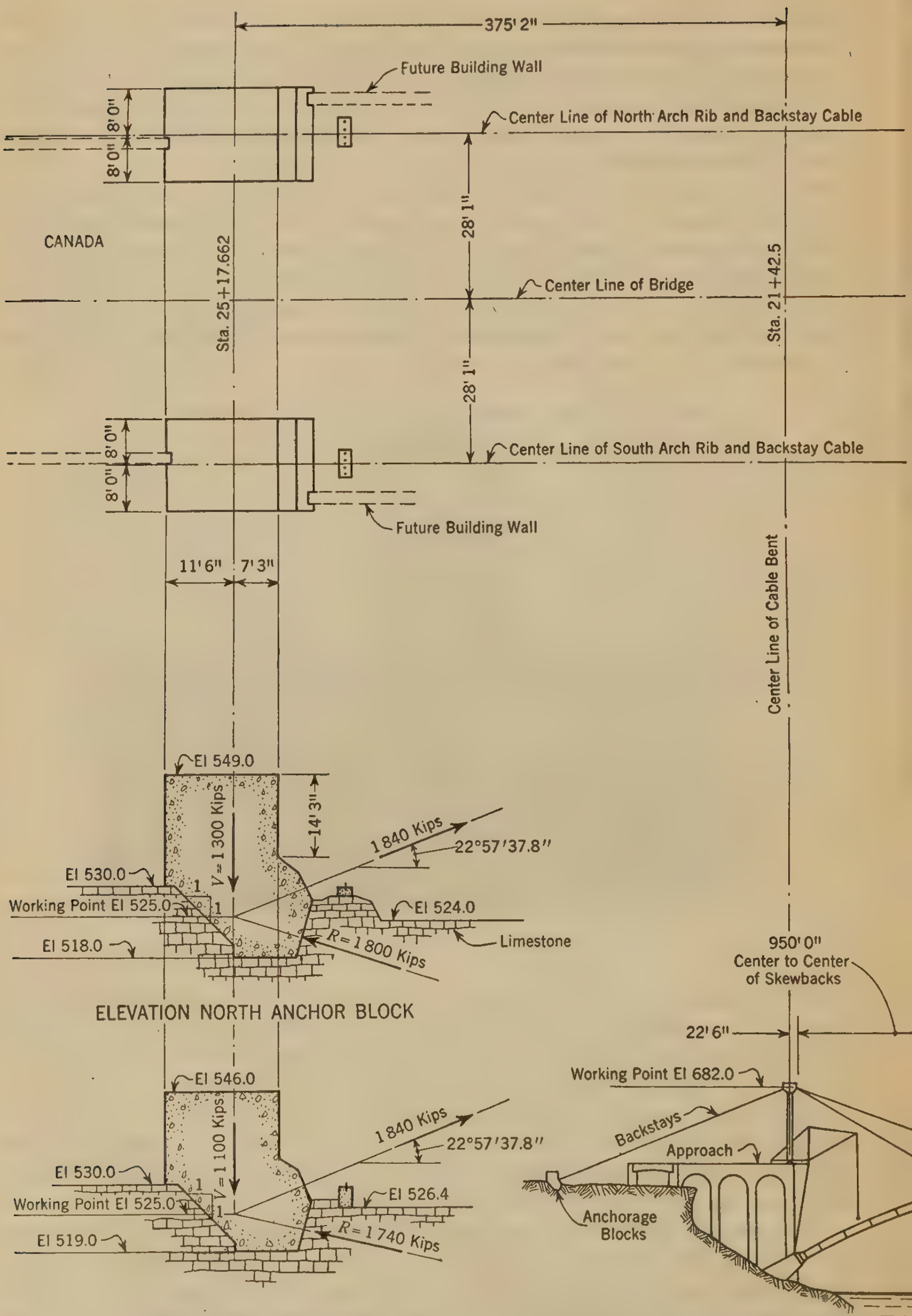


FIG. 31.—VIEW OF CABLE BENT, WITH WORKING PLATFORMS AND JINNIWINKS

As a matter of interest, it may be noted that all anchorage blocks have been left permanently in place. The two on the Canadian side are incorporated in the basement of the approach structure, forming part of the walls. The exterior faces of these anchorages were given the same surface treatment as the other building walls at this level. On the New York side, the anchorage pits

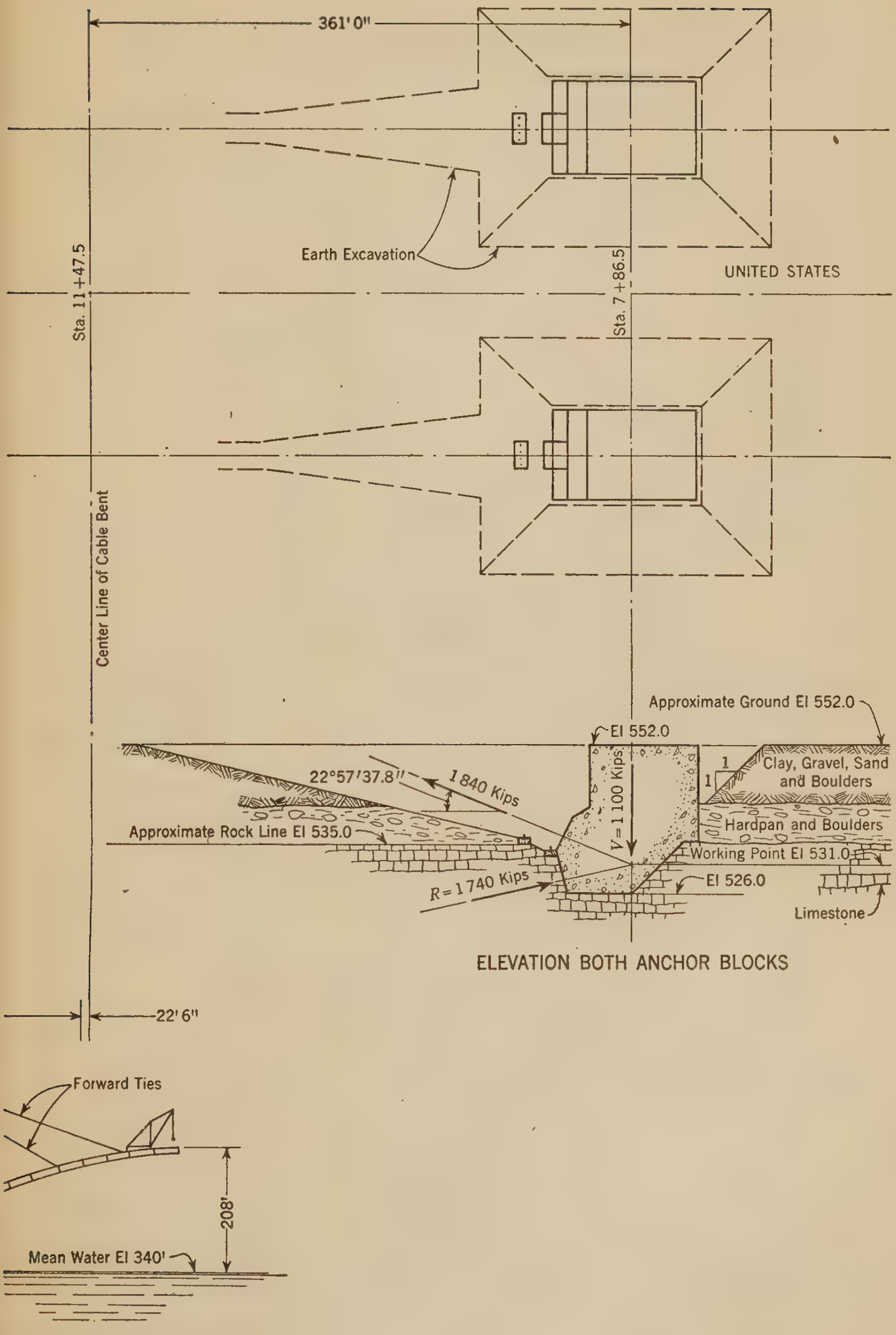

375' 2"
Future Building Wall
Center Line of North Arch Rib and Backstay Cable
8' 0"
8' 0"
CANADA
Sta. 25+17.662
28' 1"
Center Line of Bridge
Sta. 21+42.5
28' 1"
8' 0"
8' 0"
Center Line of South Arch Rib and Backstay Cable
Future Building Wall
11' 6"
7' 3"
Center Line of Cable Bent
El 549.0
14' 3"
V=1 300 Kips
1 840 Kips
22°57'37.8"
El 530.0
1
Working Point El 525.0
1
El 524.0
El 518.0
R=1 800 Kips
Limestone
950' 0"
Center to Center
of Skewbacks
22' 6"
El 546.0
V=1 100 Kips
Working Point El 682.0
1 840 Kips
22°57'37.8"
Backstays
El 530.0
1
Approach
Working Point El 525.0
1
El 526.4
R=1 740 Kips
El 519.0
Anchorage
Blocks

ELEVATION NORTH ANCHOR BLOCK

ELEVATION SOUTH ANCHOR BLOCK

GENERAL ELEVATION CANADIAN SIDE

FIG. 32.—


361' 0"
Earth Excavation
UNITED STATES
Sta. 11+47.5
Sta. 7+86.5
Center Line of Cable Bent
Approximate Ground El 552.0
El 552.0
22°57'37.8"
1 840 Kips
V=1 100 Kips
1
1
Clay, Gravel, Sand and Boulders
Approximate Rock Line El 535.0
Hardpan and Boulders
Working Point El 531.0
R=1 740 Kips
El 526.0
Limestone
ELEVATION BOTH ANCHOR BLOCKS
22' 6"
Forward Ties
208'
Mean Water El 340'

Anchorage Details

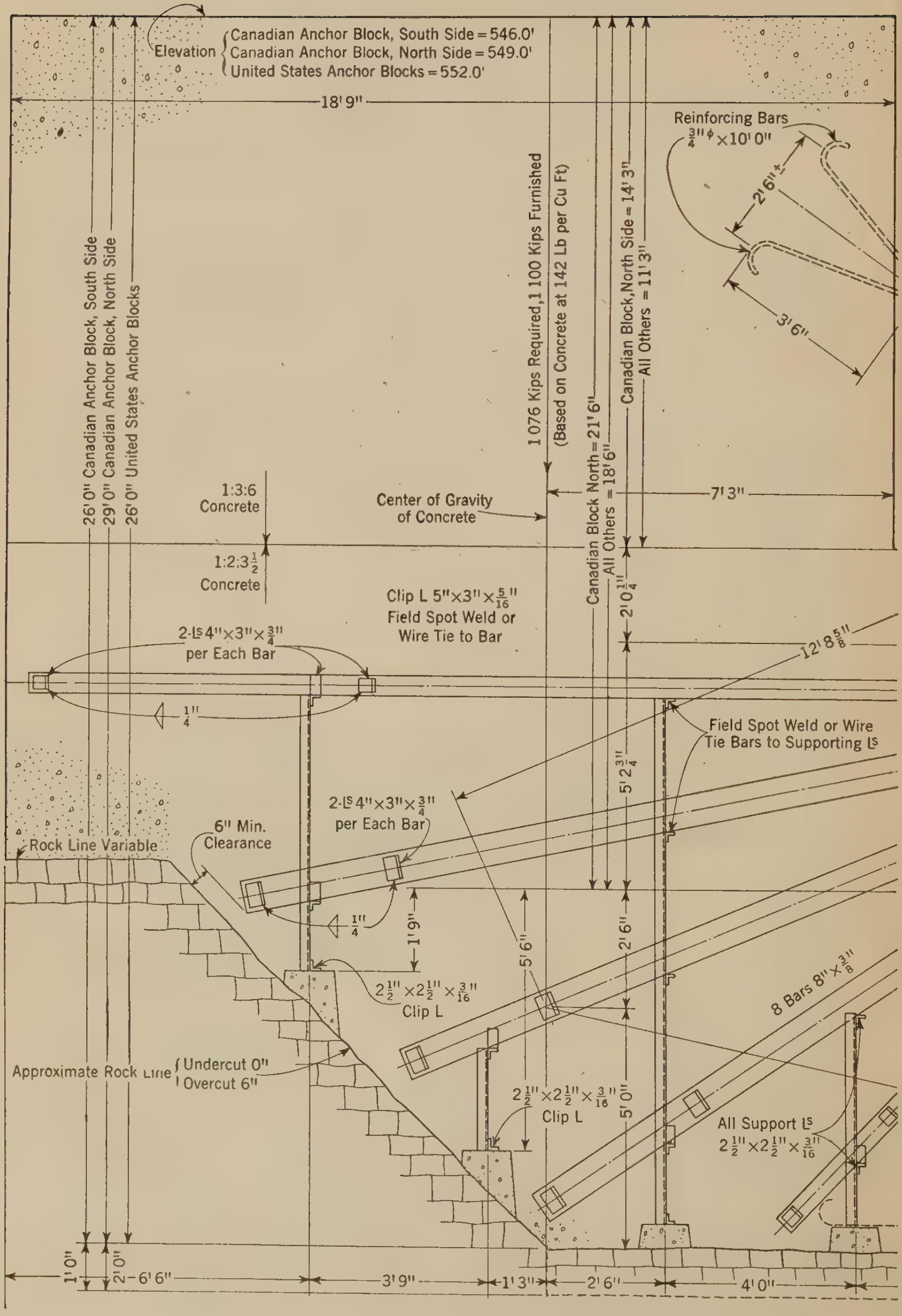
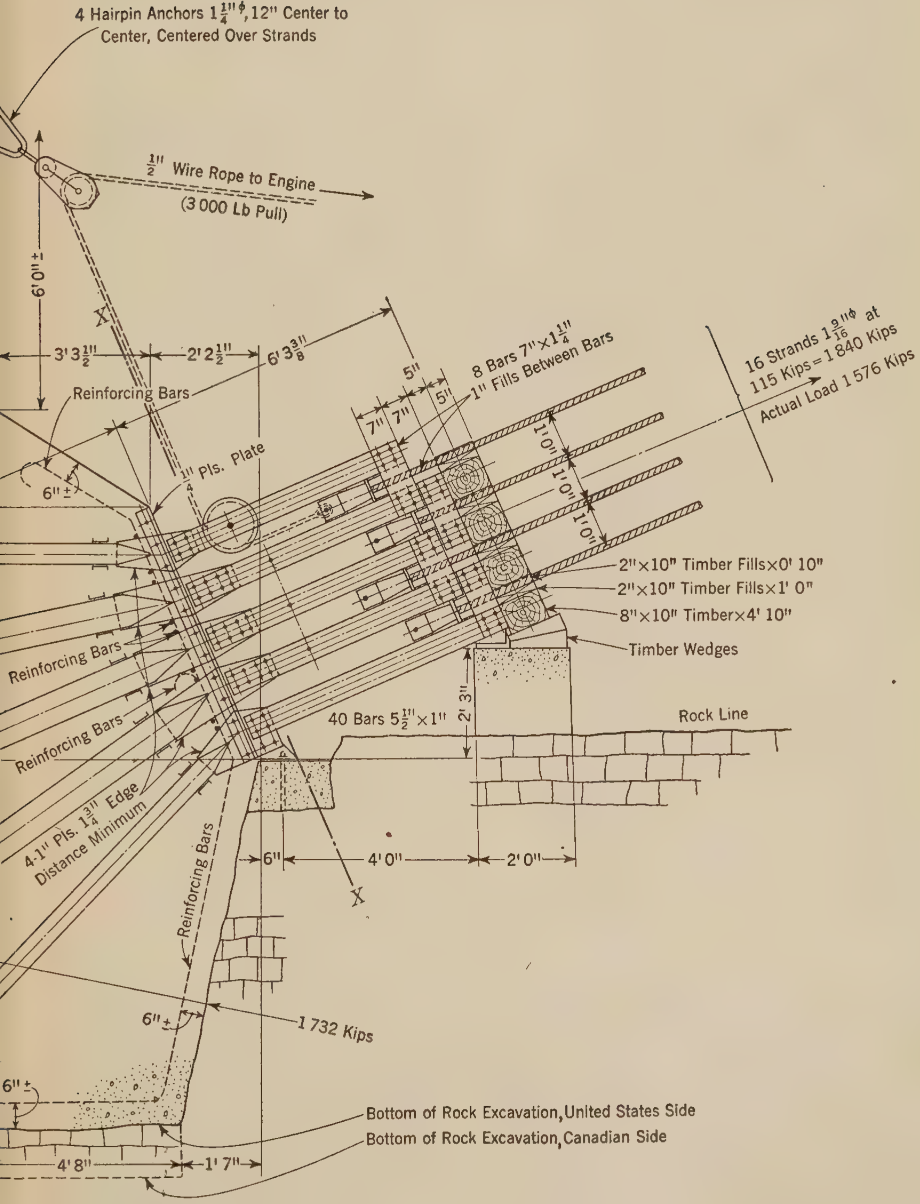

Elevation
Canadian Anchor Block, South Side = 546.0'
Canadian Anchor Block, North Side = 549.0'
United States Anchor Blocks = 552.0'
18' 9"
26' 0" Canadian Anchor Block, South Side
29' 0" Canadian Anchor Block, North Side
26' 0" United States Anchor Blocks
1 076 Kips Required, 1 100 Kips Furnished
(Based on Concrete at 142 Lb per Cu Ft)
Canadian Block North = 21' 6"
All Others = 18' 6"
Canadian Block, North Side = 14' 3"
All Others = 11' 3"
Reinforcing Bars
$\frac{3}{4}$"ϕ × 10' 0"
2' 6" ±
3' 6"
1:3:6 Concrete
Center of Gravity of Concrete
7' 3"
1:2:3$\frac{1}{2}$ Concrete
Clip L 5" × 3" × $\frac{5}{16}$"
Field Spot Weld or Wire Tie to Bar
2' 0$\frac{1}{4}$"
12' 8$\frac{5}{8}$"
2-Ls 4" × 3" × $\frac{3}{4}$" per Each Bar
$\frac{1}{4}$"
Field Spot Weld or Wire Tie Bars to Supporting Ls
5' 2$\frac{3}{4}$"
2-Ls 4" × 3" × $\frac{3}{4}$" per Each Bar
6" Min. Clearance
Rock Line Variable
$\frac{1}{4}$"
1' 9"
5' 6"
2' 6"
2$\frac{1}{2}$" × 2$\frac{1}{2}$" × $\frac{3}{16}$" Clip L
8 Bars 8" × $\frac{3}{8}$"
Approximate Rock Line
Undercut 0"
Overcut 6"
2$\frac{1}{2}$" × 2$\frac{1}{2}$" × $\frac{3}{16}$" Clip L
5' 0"
All Support Ls
2$\frac{1}{2}$" × 2$\frac{1}{2}$" × $\frac{3}{16}$"
1' 0"
2' 0"
2'-6' 6"
3' 9"
1' 3"
2' 6"
4' 0"

FIG. 33.—ELEVATION OF ANCHORAGE ON


4 Hairpin Anchors $1\frac{1}{4}''\phi$, 12" Center to Center, Centered Over Strands
$\frac{1}{2}''$ Wire Rope to Engine (3 000 Lb Pull)
6' 0" ±
X
3' $3\frac{1}{2}''$
2' $2\frac{1}{2}''$
6' $3\frac{3}{8}''$
Reinforcing Bars
6" ±
$\frac{1}{4}''$ Pls. Plate
5"
5"
7"
7"
8 Bars $7''\times1\frac{1}{4}''$
1" Fills Between Bars
1' 0"
1' 0"
1' 0"
16 Strands $1\frac{9}{16}''\phi$ at 115 Kips = 1 840 Kips
Actual Load 1 576 Kips
2"×10" Timber Fills×0' 10"
2"×10" Timber Fills×1' 0"
8"×10" Timber×4' 10"
Timber Wedges
Reinforcing Bars
Reinforcing Bars
2' 3"
40 Bars $5\frac{1}{2}''\times1''$
Rock Line
4-1" Pls. $1\frac{3}{4}''$ Edge Distance Minimum
Reinforcing Bars
6"
4' 0"
2' 0"
X
1 732 Kips
6" ±
6" ±
Bottom of Rock Excavation, United States Side
Bottom of Rock Excavation, Canadian Side
4' 8"
1' 7"

the Center Line of the Arch Rib

were backfilled with earth washed down in place with a stream from a fire hose. The permanent paving in the approach plaza is approximately 2 ft above the top of these two anchorages.

Backstays.—Each of the four sets of tiebacks extending from the top of the cable bent down to the anchorage block consisted of a group of sixteen

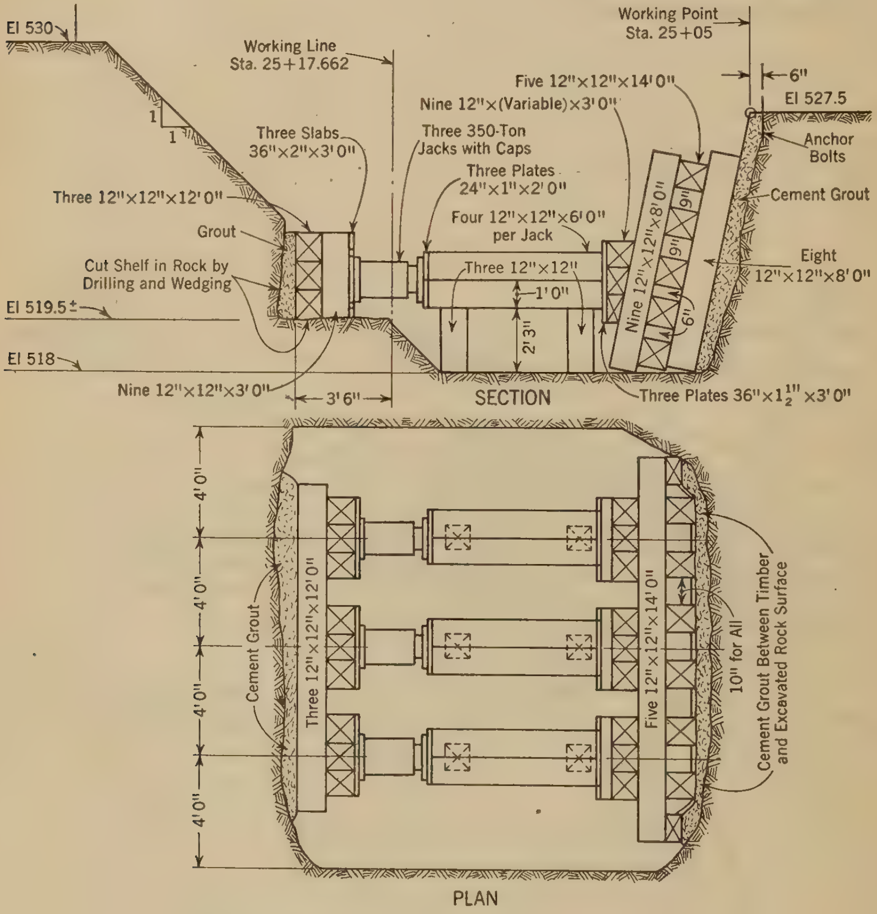

El 530
1
1
Working Line
Sta. 25+17.662
Working Point
Sta. 25+05
6"
El 527.5
Five 12"×12"×14'0"
Nine 12"×(Variable)×3'0"
Three 350-Ton
Jacks with Caps
Three Slabs
36"×2"×3'0"
Three Plates
24"×1"×2'0"
Four 12"×12"×6'0"
per Jack
Three 12"×12"×12'0"
Grout
Cut Shelf in Rock by
Drilling and Wedging
El 519.5±
El 518
Three 12"×12"
1'0"
2'3"
Nine 12"×12"×8'0"
9"
9"
6"
Anchor
Bolts
Cement Grout
Eight
12"×12"×8'0"
Nine 12"×12"×3'0"
3'6"
SECTION
Three Plates 36"×1½"×3'0"
4'0"
4'0"
4'0"
4'0"
Cement Grout
Three 12"×12"×12'0"
Five 12"×12"×14'0"
10" for All
Cement Grout Between Timber
and Excavated Rock Surface
PLAN

Fig. 34.—Jacking Test, North Anchorage, Canadian Side

bridge strands each $1\frac{9}{16}$ in. in diameter with a minimum breaking strength of 155 tons (see Fig. 33). Their normal maximum working stress was 50 tons, increased to 60 tons for wind and ice load. Although they had been prestrained by having been used previously as footwalk cables in the erection of the Golden Gate and Tacoma Narrows suspension bridges, they were prestressed to 150,000 lb for 30 min before being socketed at both ends to carefully calculated lengths under a stress of 100,000 lb. These strands were connected to the tower top through an open socket pin-connected to a short heavy link, which in turn was pin-connected to a pair of links which were pinned off to the hammer-head gusset plates forming the top of the column. The sockets at the lower end of the strands were of the closed bearing type. They were nested between groups of links bolted to the embedded anchorage steel projecting from the face of the concrete, and the head of the socket bore against pairs of transverse plates bolted to these links. The only adjustment was by means of shim plates between the bearing sockets and these bars (see Fig. 33).

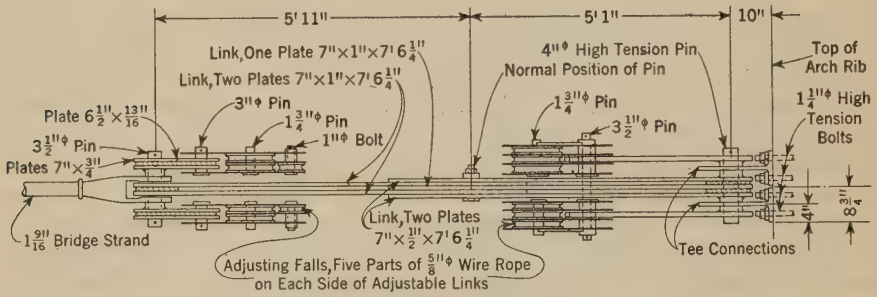
The cable bent was tipped back from its vertical position approximately 3 ft and supported during this time by means of a pair of stifflegs and sills made up of spandrel columns from the permanent structure (see Fig. 30). The rear end of each sill was blocked up from, and tied down to, the concrete approach by means of embedded steel anchors. The actual lean of the tower top was determined by a plumb bob consisting of a steel piano wire and a short section of railroad rail, the latter immersed in a bucket of oil. The lean of 3 ft was designed to give a stress of approximately 5,000 lb in each backstay cable at 60° F. This was well within the pulling capacity of the single runner line used to adjust these strands which first were hung from the top of the tower and the lower end then hauled back into the anchorage connection.

The correct strand stress for the actual tower lean and temperature was insured by surveying the strands for elevation at the quarter point where a wooden tower was located for this purpose. The minimum thickness of the adjusting shim provided at the anchorage was $\frac{1}{4}$ in. which proved to be sufficient to give practically uniform sag to the four cables in each of the four horizontal layers. After this adjustment, no further attempt was made at any time during the erection to survey or to measure the stress in these backstays, entire dependence being placed on the calculations for stress and on measuring and socketing the strands to the calculated lengths.

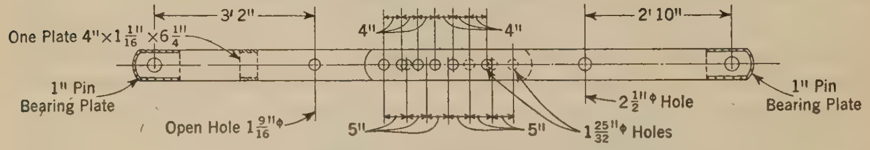
Forward Ties or Tiebacks.—Two sets of forward ties were used to support the cantilevered arch ribs alternately at points 10, 8, 6, and 3. They were the same size ($1\frac{9}{16}$ in. in diameter) as for the backstays. At point 10, eight strands were used for each arch rib and were hung in two vertical rows of four each from the two center gusset plates at the top of the cable bent. After these ties were adjusted and the rib cantilevered to point 8, the second set of ties, also eight in number, was connected and adjusted. This second group was hung in two rows of four each from the two outside gussets of the cable bent so that, when erected, they passed outside of the first group still in place at point 10.

While the ribs were being cantilevered from point 8 to point 6, the ties at point 10 were disconnected from the rib and slacked back to the abutment where they were left hanging vertically from the top of the tower. An adjustable working platform was provided which could be raised or lowered to bring the workmen up to the lower end of these strands. Lengthening "pennant" strands then were attached to this first group of panel 10 ties to make them long enough for use at panel 6. For this point, four additional strands also were hung from the two inside gussets at the top of the bent in addition to the eight lengthened strands from point 10, making a total of twelve strands for point 6.

These twelve strands had to be erected by swinging them out through the group supporting the rib at panel 8. When they were finally connected and adjusted, cantilever erection of the arch rib was continued to panel 3 supported on the strands at both panel 8 and panel 6. At point 3 the final group of twelve strands was connected as follows:


5' 11"
5' 1"
10"
Link, One Plate 7"×1"×7' 6$\frac{1}{4}$"
Link, Two Plates 7"×1"×7' 6$\frac{1}{4}$"
Plate 6$\frac{1}{2}$"×$\frac{13}{16}$"
3"ϕ Pin
1$\frac{3}{4}$"ϕ Pin
1"ϕ Bolt
3$\frac{1}{2}$"ϕ Pin
Plates 7"×$\frac{3}{4}$"
1$\frac{9}{16}$" Bridge Strand
4"ϕ High Tension Pin
Normal Position of Pin
1$\frac{3}{4}$"ϕ Pin
3$\frac{1}{2}$"ϕ Pin
Top of Arch Rib
1$\frac{1}{4}$"ϕ High Tension Bolts
8$\frac{3}{4}$"
4"
Link, Two Plates 7"×$\frac{1}{2}$"×7' 6$\frac{1}{4}$"
Tee Connections
Adjusting Falls, Five Parts of $\frac{5}{8}$"ϕ Wire Rope on Each Side of Adjustable Links

PLAN OF LINKS AND SPLIT SHEAVES


3' 2"
4"
4"
2' 10"
One Plate 4"×1$\frac{1}{16}$"×6$\frac{1}{4}$"
1" Pin Bearing Plate
Open Hole 1$\frac{9}{16}$"ϕ
5"
5"
2$\frac{1}{2}$"ϕ Hole
1$\frac{25}{32}$"ϕ Holes
1" Pin Bearing Plate

ELEVATION OF ADJUSTABLE LINKS

FIG. 35.—ADJUSTABLE LINKS AND SPLIT SHEAVES

The first or uppermost four strands at panel 6 were removed one at a time, slacked back on the arch rib, and lengthened with a pennant. The lengthened strand then was dragged out on the arch rib to panel 3 where it was connected and adjusted. This operation was repeated four times, giving four strands at panel 3, eight at panel 6, and eight still in place at panel 8. These latter then could be disconnected, lengthened, swung out, and connected at panel 3. This gave twelve strands at panel 3 and eight at panel 6, a total of twenty strands per rib, on which cantilever erection was continued to the crown.

The connection of the forward ties to the arch ribs was through a pair of 18-in. heavy T-sections spaced 8 in. on centers and bolted to the top flange of the rib by 1$\frac{1}{4}$-in. diameter high tensile bolts. The arch ribs were reinforced inside by diaphragms which took the load from the T-sections and transferred it to the web plates.

The connection of each forward strand to the T-sections was through an adjustable set of links shown in Fig. 35. The lower end of the strand carried an open socket to which was pin-connected the upper link consisting of two plates 7 in. by 1 in. with pinholes in the lower end spaced 5 in. on centers. Meshing with this upper link was the lower link which consisted of three plates, one 7 in. by 1 in. and the other two 7 in. by $\frac{1}{2}$ in. with pinholes in the upper end spaced 4 in. on centers. The lower end of this link was pin-connected directly to the T-sections. Pinning these two sets of links together through different sets of pinholes gave different over-all lengths of link varying by 1-in. increments from 12 in. shorter to 18 in. longer than the normal length; and by the same or larger increments to maxima of 24 in. shorter to 30 in. longer.

The strand length was adjusted by means of a double set of falls, one on either side of the adjustable links. Each upper, two-sheave block in these falls was supported on the $3\frac{1}{2}$-in. strand socket pin which was extended for this purpose. The lower, three-sheave block was supported on the 4-in. pin connecting the lower link to the T-sections. This pin also was cantilevered out for this purpose. Each set of falls was reeved up with five parts of $\frac{5}{8}$-in. wire rope, the lead lines of which ran directly to the top of the cable bent where they passed over a pair of sheaves and then part way down the bent where they were clamped together to form a bight in which was supported a third or equalizing sheave. This latter sheave was overhauled by a $\frac{7}{8}$-in. wire rope runner line from an engine either on the deck of the approach or on the engine platform. In adjusting a strand, with the adjusting falls in place, a strain first was taken on the equalizing sheave to get the same tension in the two lead lines of the adjusting falls. The two lead lines then were clamped together just above the equalizing sheave so that equal movement was insured in both sets of falls.

The forward ties were erected by a system of horizontal and vertical runner lines, one for each arch rib. The vertical runner line was a $\frac{7}{8}$-in. wire rope run through the jinniwink at the top of the cable bent. The horizontal runner line was also a $\frac{7}{8}$-in. wire rope running from an engine at the abutment out over the arch rib, through a snatch block at the end of the erected steelwork, then back to the abutment where it could be connected to the lower end of the vertical runner line thus forming a "telegraph" system.

The forward ties were shipped on large wooden reels which were unloaded on platforms constructed near the skewbacks. Each strand was unreeled by using the vertical runner line to haul it up to the top of the cable bent where the socket was pinned to the connecting links. To swing a strand out and connect it to the arch rib, the two runner lines were connected through a triangular hitch plate to which the lower end of the strand also was connected. By proper manipulation of the runner lines the strand then was hauled out to the desired point on the rib. Each group of strands was connected at the upper end in two vertical rows whereas the connections at the lower end were in a single row, between the two T-connections. This gave the strands at panel 8 a long, triangular-shaped arrangement with an opening between them of 3 ft at the upper end, tapering to zero near the rib connection.

To erect the strands at panel 6, which had to be swung out through this opening between the strands in place at panel 8, the two runner lines first were hauled up taut so as to bring them above the strands at panel 8. Then they were dropped down through the opening, hooked to the loose strand hanging from the tower top, and pulled back again to a taut position above the panel 8 strands, dragging the loose strand with them. Finally, the runner lines were hauled out to panel 6 where the loose strand was connected and adjusted. The strands at any panel point were removed by a similar use of the runner lines.

Fan Falsework Bent and Platform.—On the river side of the cable bent, and forming an extension to the concrete approach spans, was a steel platform 20 ft long, the outer end of which was supported on two inclined falsework columns themselves supported on a concrete seat just above and in back of the concrete skewbacks for supporting the arch ribs. These two columns, 140 ft long and unsupported for their entire length, each were made from two suitable lengths of the permanent spandrel columns. A special end section (which later was removed and discarded) was detailed, both top and bottom. The upper end of each column was tied back by a steel strut connected to anchor bolts embedded in the concrete approach for the purpose. A floor beam from the permanent structure was framed between the upper ends of the columns. Various other struts and bracing members completed the main platform.

The purpose of this falsework bent and platform was to support a stiffleg derrick for unloading and erecting the steelwork for the arch ribs. This material all was trucked out over the approach and through the cable bent so that the derrick could unload it and lower it into place. A refuge bay 15 ft long was cantilevered out beyond the main fan falsework platform to permit the truck hauling the material to come far enough out so that the center of gravity of the arch ribs would be in front of the cable bent.

ERECTION EQUIPMENT

Yards and Unloading Equipment.—All material for erection in the permanent structure was shipped by railroad to yards on either side of the river. The heaviest pieces to be handled were two 75-ton arch-rib sections (the second rib sections out from the abutment) approximately 53 ft long. Two other arch-rib sections weighed approximately 65 tons and the two steel grillage sections, designed to spread the load of the arch ribs over an area 12 ft by 22 ft on the concrete skewbacks, weighted 63½ tons each, giving a total of six pieces on each side of the river weighing more than 60 tons.

On the Canadian side, an 85-ton stiffleg derrick was set up and operated by a 100-hp, 3-drum steam hoisting engine. Scrap railroad rail was used for counterweight on the end of each sill. This derrick was capable of handling the heaviest, or 75-ton, loads without assistance. It was placed far enough back from the railroad track to permit the trucks and trailers used for hauling to the bridge site to come between it and the track. The material thus could be unloaded directly from cars to the trucking equipment or could be swung around and stored on the ground beside the derrick.

On the New York side it was not possible, in the yard selected (which was long and narrow), to set up a stiffleg derrick of sufficient capacity, or one that could cover the entire yard area. A locomotive crane of 60-ton capacity was provided operating on one track to unload material from the cars which were brought in on a second, parallel track. For the six pieces weighing more than 60 tons it was necessary to bring in a railroad wrecker crane and use both cranes to unload these pieces, one picking each end. Otherwise, the locomotive crane unloaded and loaded all materials. The arch-rib sections could be unloaded and stored alongside the crane track when necessary.

Hauling Equipment and Roadways.—All heavy material was hauled from the railroad yards to the bridge site about a mile distant, by heavy tractors and trailer equipment. All arch-rib sections were loaded on two heavy trailers, one under each end of the rib. A 50-ton, goose-neck type of carry-all trailer was used at the front end and a 100-ton dolly trailer under the rear end. The number and spacing of the wheels in each trailer was such as to spread the load satisfactorily over the pavement. The maximum axle and wheel loads were 24 tons on an eight-wheel axle and 20 tons on a four-wheel axle. Pneumatic tires were used on the carry-all trailer (four wheels) and solid tires on the dolly trailer (eight wheels).

Two separate, heavy-duty tractors were used to haul the trailers, one leading and the other following. The rear tractor was used largely for the purpose of steering, but on the Canadian side of the river it also was necessary to help brake the loads down the steep hill between the yard and the bridge site. The tractors were equipped with air brakes.

On the New York side, the new bridge deck was at the same level as the existing ground surface and the loads could be hauled out over the natural ground directly on to the concrete approach. On the Canadian side, the bridge was approximately 20 ft above the bare rock at the edge of the gorge. A steel ramp 315 ft long with 4% grade was constructed to provide access from the ground surface to the level of the deck. This ramp was made of two-column steel bents spaced from 20 ft to 28 ft on centers with an almost solid floor of 12-in. by 65-lb steel H-beams laid longitudinally to act as stringers. The roadway was formed of 4-in. by 12-in. planks laid transversely across these stringers. A pair of 6-in. by 12-in. timber curbs, spaced 12 ft apart and set 6 in. above the deck, completed the roadway. This ramp was located on the south side of the bridge roadway.

A timber roadway, 12 ft wide between curbs, was provided over the concrete approach on each side of the river. It was located on the south side of the structure, on the Canadian side of the bridge where it formed a continuation of the ramp. On the New York side, the roadway was located on the north side of the approach. This roadway led through the cable bent and out to the fan falsework platform beside the unloading derrick. The 15-ft refuge bay cantilevered beyond the end of the fan falsework columns was designed to support the weight of a tractor only. The remainder of the roadway was designed to carry the heaviest wheel loads. The roadway slab on the concrete approaches had not been poured at this time, as only the main arches, floor beams, and stringers were completed. Hence, the timber roadway was needed over these approach spans.

Unloading Derrick, Traveler A.—For each side of the river, a stiffleg derrick of 85-ton capacity was provided to start the erection and to act as an unloading derrick. It was known as traveler A (see Fig. 29), erected on steel platform beams at the shoreward end of the concrete approach by means of a smaller derrick. In its first position, about 60 ft back from the outer end of the approach, the derrick was provided with a boom 145 ft long by which the first fan falsework columns and platform were erected. Each of these columns weighed 36 tons and was set at a radius of about 65 ft. From this same position, the second operation was to erect the 130-ft high cable bent, the heaviest lift in which was the 27-ton top section. After completing the bent, the traveler backed off and the boom was shortened to 95 ft. Then, the traveler was moved forward to the end of the concrete work, standing in the opening of the cable bent from which the lower panel of bracing was omitted and from where it erected the steel grillage and skewback sections of the arch. Finally, the traveler moved forward to its third position standing directly over the fan falsework columns, 27.5 ft in front of the cable bent. In this position, it was used to erect the first three panels of the arch itself and the arch-rib traveler, after which it acted as an unloading derrick feeding material from the deck down to the arch rib. After the arch ribs were completely erected and swung, the boom was lengthened to 105 ft and this derrick then was used to erect the spandrel columns, girders, and floor starting from the abutment and moving out to panel point 9 where it removed rib traveler B (Fig. 29) and completed the erection of the floor. Traveler A was itself dismantled and removed by a tractor crane brought out on the material roadway for this purpose.

The beams and track stringers forming the deck used to support and move this traveler were all stringers from the floor of the permanent structure. The traveler was operated by a 100-hp, 3-drum, steam hoisting engine.

Rib Traveler B.—To continue erection of the arch ribs beyond the reach of traveler A, a second traveler, moving on the arch rib itself, was provided. This consisted of a stiffleg derrick normally of 40-ton capacity but reinforced to enable it to handle the heaviest remaining arch-rib sections weighing about 53 tons. A 76-ft boom was used. The derrick was supported, in plan, on a triangular-shaped steel underframe (see Fig. 36). This underframe consisted of a front cross girder spanning between the two arch ribs and a single longitudinal girder over one arch rib only, the rear end of which was supported on an adjustable vertical leg which could be pinned off to different lengths and thus could keep the deck of the traveler level for the varying angle of the arch rib as the traveler advanced from the abutment toward the crown. The derrick was powered by a 4-drum steam hoisting engine supported on an engine platform located on the steep, talus slope at the bottom of the gorge, just in back of the skewbacks, and occupying the area under the outermost span of the concrete approach. The framework of this platform consisted of steel columns seated on shelves in the concrete footings, and steel beams and girders. A 4-in. plank deck and a light housing to enclose the engine and protect the operator completed the structure.

In working position on the arch this traveler was pinned to brackets bolted to the top flange of the ribs, which took the entire reaction. To move the traveler, it was jacked up, the bracket pins pulled, and the traveler lowered on to three, two-wheel trucks, one at each corner. A move was made for each panel of the arch erected. At the outer end of each rib was bolted a steel frame or cathead which supported a set of four sheaves. An eight-part set of falls was reeved up between these sheaves and a three-sheave block at each side of the front end of the traveler. The lead lines from these two sets of falls ran back to the hoisting engine at the abutment where lines from two of the drums were removed to accommodate them. A complete move of the traveler, including removal and replacement of the lead lines on the engine, required from 3 to 4 hours.

For the first operation, the rib traveler was used to erect the arch ribs and the bracing between them, advancing from panel point 10 to panel point 2, one panel from the crown. After the erection of the keystone pieces, this traveler moved back down the rib to panel point 8, erecting the spandrel columns, girders, and floor framing behind it from the crown to panel point 7. Standing at point 8, it was dismantled and removed by the overhead derrick, which, in the meantime, had been used to erect the floor and had advanced from the abutment out to panel point 9.

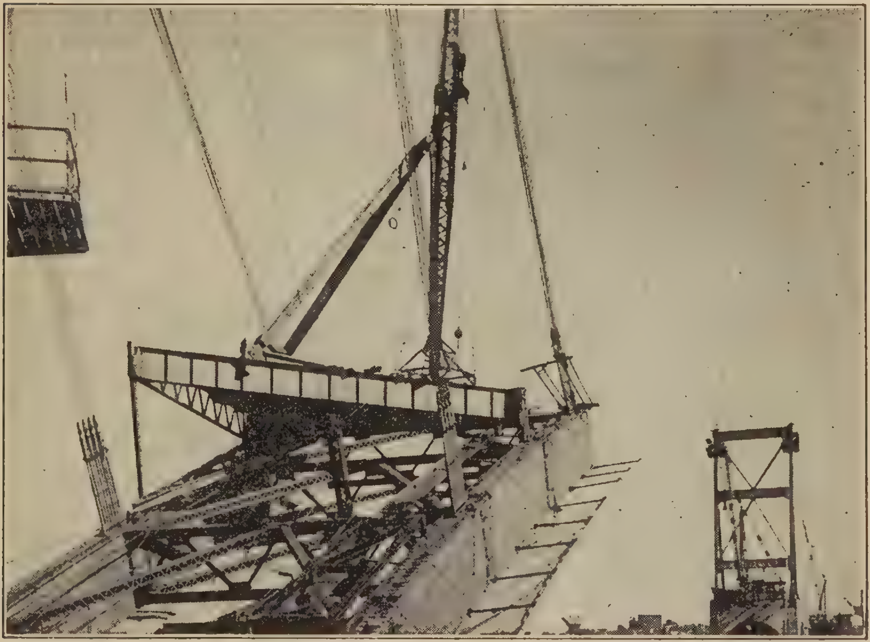


Fig. 36.—View of Rib Traveler B

Material Truck and Track.—All material was brought within reach of the rib traveler by lowering it from the deck with the unloading derrick A and by landing it on the material truck consisting of a steel framework with a single wheeled truck at each corner. The truck was 38 ft long and had an 8-ft gage between wheels. On one side, it ran on a rail supported over the inside web of the arch rib, the two wheels running on this rail being double-flanged. The two inside wheels ran directly on the top flange of a track beam supported on top of the cross struts at each panel point of the arch rib. These wheels had a plain tread with no flange. The truck was moved by a set of six-part pulling falls, the upper three-sheave block of which was fastened to the right-hand front corner of the rib traveler. The triangular shape of the rib traveler underframe enabled the material truck to be brought out immediately behind the front cross girder and within easy reach of the derrick. The arch-rib sections were landed on, and securely belted to, special seats on the truck. They were handled one at a time. All other material, such as struts, bracing, and all floor material, was either bolted or securely lashed to the truck and kept from sliding off endwise by means of special brackets or stops.

Hitches.—Special hitches were provided for the arch ribs and for all other heavy loads as a part of the erection equipment. For the rib sections these consisted of a pair of T-sections bolted to the top flange with high-tensile bolts, and with a hole in the web for a 2¾-in. pin for the connection of the open socket on the wire rope pennants hanging from a triangular hitch plate on the lifting falls.

The center of gravity of each rib section was computed from the shop drawings and the hitch T-sections were so located that the rib would hang free at the same angle as determined by its location in the structure, thus greatly simplifying the entering of each piece into the splice plates at the joint.

Hoisting Engines and Power.—Steam hoisting engines were used on both sides of the river. It had been fully expected that electric power would be used, but it was found disadvantageous for several reasons. On the Canadian side only 25-cycle power was available, whereas the contractor's electrical equipment was all 60-cycle units. Both 25-cycle and 60-cycle power were available on the New York side. The power companies on each side of the river were not anxious to furnish power, however; their lines already were loaded and new lines would have had to be erected, the entire cost to be borne by the contractor. Moreover, the rates for electric power, taking account of the stand-by charges, were found to be high in comparison with the cost of steam power. Therefore, steam hoisting engines were decided upon and used. Four hoists were required on each side of the river, the boilers being operated by fuel-oil burners.

A 3-drum, 100-hp hoist located on the derrick platform was used to operate traveler A. A similar hoist, but with 4 drums and located on the engine platform at the foot of the gorge, was used to operate the rib traveler B. Three lines operated the derrick boom falls, main load, and runner line, and the fourth line operated the falls used to haul the material truck out along the arch rib. A 3-drum, 50-hp hoist also was placed on the engine platform and operated as a skeleton engine, obtaining its steam from the boiler with the hoist used for traveler B. This hoist was used to operate one set of vertical and horizontal runner lines for one of the arch ribs. The runner lines for the second rib were operated by a fourth hoist of either 2 or 3 drums which was placed on the deck of the concrete approach in back of traveler A. This gave a total of 13 drums in use on each side of the river. In addition, 2 of the 4 drums operating the rib traveler served a double purpose in that they were used for moving the traveler as well as operating the main falls and boom falls of this derrick.

Each derrick boom was swung by a bull wheel operated by a steam swinger engine on the traveler platform. The derrick signalman operated the swinger in each case. For traveler A, steam from the derrick engine boiler was used for power. Since the engine for traveler B was not with the derrick, but back at the abutment, compressed air was used to operate its swinger engine.

Signal System and Telphones.—Signals between the signalman on the platform of the rib traveler and the hoisting engineer on the engine platform back at the abutment were relayed by an electric light signal system. A signal box was provided at the derrick with two push-button switches for each drum on the engine. A signal light box faced the hoisting engineer and contained two lights for each drum—one red and one green or amber. The lights were operated by dry cells. The green or amber light was used for "going ahead"—that is for raising a load or the boom and the red light for slacking off or lowering. No light at all stopped all operation. Different speeds of operation could be obtained by flashing the lights on and off, giving very delicate control of the load being handled. Considerable pressure was required to operate the push-button switches, making the system practically foolproof.

On traveler A the signalman was always in sight of the hoisting engineer and no signal system was required. Hand signals, relayed as necessary, also were used for the operation of the runner lines, and adjusting falls were used in the erection and adjustment of the forward ties.

Hand-cranked, dry-cell-operated, field telephones were set up in enclosed boxes for use on the work. One telephone was on the deck, at the fan falsework platform, one at the top of each column of the cable bent, and one on the arch rib at traveler B. These telephones were invaluable for the general interchange of information between the foreman (who was usually on the fan falsework platform where he had full view and command of the entire work) and the pushers or subforemen. This was particularly true in the case of the erection and adjustment of the forward-tie cables, the actual adjustment being made on the arch ribs while the stress was being measured at the top of the cable bent.

Compressors and Pneumatic Tools.—Compressed air for the operation of all pneumatic tools and for operating the swinger engine on the rib traveler was furnished by two, 210-cu-ft capacity, fuel-oil compressors set up on the edge of the gorge near the end of the approach spans. Air was piped down the face of the cliff to the abutments and then out the arch ribs. This line also was used to feed air to the work of erecting the center section of the floor, from the crown back to the quarter point. A second air line was run out over the concrete approaches to the deck of the arch span as this was built out from the abutment until it joined the center section, at which time the line on the arch ribs was removed.

In addition to the usual pneumatic riveting hammers, the following pneumatic equipment was used: All bucking up of field rivets was by means of air dollies of various types. Very few rivets are now bucked up by hand dollies. Fitting-up bolts were tightened entirely by the use of pneumatic impact wrenches which have contributed greatly to decreased costs for this work because of their greater speed over hand wrenches and because they exert a greater force, thus insuring tighter work and consequently fewer loose rivets and cutouts.

Safety Nets.—The specifications required the contractor to use safety nets and barriers to safeguard men against falling into the river, these to be left in place until the completion of the work. A type of net similar to that used in the erection of the floor system of the Golden Gate Bridge was used, and, in fact, five nets still on hand from that work were refabricated for use at the Rainbow Bridge. The remainder of the twenty-five nets were newly fabricated.

The average size of the nets was 45 ft by 85 ft (see Fig. 36). They were constructed of $\frac{3}{8}$-in. manila rope laid to form a 6-in. square mesh, one set of ropes being passed through the other at the intersections and being tied with special twine as well. The sides of the net were finished with $\frac{3}{4}$-in. manila ropes extending 12 ft beyond the ends; and at each end there were additional $\frac{3}{4}$-in. tie ropes 8 ft long and spaced 4.5 ft apart with which to fasten the nets to the supports. The edges of adjacent nets were laced together with separate $\frac{3}{4}$-in. manila lines not a part of the nets themselves.

These nets were supported beneath the arch ribs on 6-in. steel H-beams running outside and parallel to the bottom flange of each rib. The beams were held in position 10 ft outside the rib by angle struts and $\frac{1}{2}$-in. wire rope ties extending from the top flange of the rib out to the end of the struts. A diagonal wire rope guy in the plane of the bottom flange of the rib, one per panel, prevented this assembly from swinging downhill due to the slope of the arch. The struts were hinged at their connection to the bottom flange of the ribs so that the entire framework could be assembled to the rib in the yard and folded up alongside it, where it was fastened until after the rib was erected. Each safety net support projected lengthwise about 8 ft beyond the end of the rib toward the center of the span so that when the net was erected, it extended beyond the end of the steelwork and thus provided protection at the splice where the men were connecting the next rib section.

The nets were folded at the factory and shipped in separate bundles. In erecting them, two lines were passed beneath the arch rib from one net support to the other. The net was unfolded to its full width and one end picked by the rib traveler until the entire net hung vertically; then one end of these two lines was connected to the lower end of the net. As the net was lowered over the side, the two lines hauled the lower end under the two ribs and over to the far support. The tie ropes were first secured to the H-beam support on one side, after which the ties on the opposite side were pulled up, using a line from the tugger hoist to give the proper sag in the net, and then tied off. The adjacent edges of the nets were then laced together with a $\frac{3}{4}$-in. manila rope.

Records were kept of all men who fell, or were knocked off the span, into the nets and it is believed that four lives were saved by their use. One man was actually knocked off the rib by a runner line which sagged and then whipped loose, while the other three men either slipped or fell accidentally. Only one man fell from the deck of the bridge, fortunately near the center of the span, and dropped 40 ft. The man knocked off was standing on the top of the arch rib, and the other two fell from scaffolds at the level of the bottom flange of the arch rib. Fortunately none of these men hit any of the bracing members in their fall and were entirely uninjured. All of them returned to work indicating their confidence in the nets and the job as a whole.

It should be noted that all erection work on the structure, including the concrete deck, was completed without a single fatality or a single serious injury. This is an excellent safety record in view of the locality and the time of year during which the work was undertaken, and the difficult and hazardous nature of the work itself.

Stair Towers, Ladders, Etc.—Access to the engine platforms and skewbacks at the foot of the gorge on each side of the river was by means of a vertical, timber, stair tower approximately 100 ft high down the vertical face of the cliff, followed by a set of steps on the surface of the talus slope. The stair towers were braced from the face of the cliff by struts and guy wires attached to anchors embedded in the rock. These stair towers and steps were constructed by the foundation contractors and then taken over by the superstructure contractor for his work.

Steel ladders were employed to provide access from the deck to the top of the cable bent, running up the inclined stiffleg and then vertically up the column the remaining distance to the platforms at the top of the tower. A steel ladder was provided from the foot part of the way up each fan falsework column to give access to the adjustable working platforms hung at the lower end of the forward ties as they were hanging from the top of the cable bent.

From the skewbacks out to approximately panel point 8, the slope of the arch ribs was so great that a timber ladder arrangement was laid on the top flange of each rib, the cross pieces consisting of 1-in. by 3-in. strips. In addition to this, a 3-ft wide timber walkway with cleats was provided alongside the material track beams. This walkway extended from panel 11 to the rear end of the traveler at panel 2, and enabled the men to keep off the ribs themselves while the forward cable ties were being erected and adjusted.

Short, 12-ft timber ladders were hung over the sides of the arch ribs at various points to give access from the top flange to manholes in the side of the ribs and to the scaffolds hung under the bottom flange at the splices.

At the crown of the arch, opposite the jacking brackets, a timber platform supported on steel beams was constructed, 12 ft wide and 56 ft long, covering the entire distance between the ribs. Large scaffolds were constructed beneath the jacking brackets on the bottom flange of each rib with steel ladders between them and the upper platform. This gave complete and easy access for the erection and adjustment of the jacking equipment and for the measurement of the crown opening during the jacking operations. The upper platform provided a convenient space for the storage of miscellaneous jacking equipment and other tools required at the crown at this time.

ERECTION OF ARCH RIBS

General Method of Erection.—Since the 12-ft deep, box-girder, rib sections formed the sole supporting member for the span, it was logical to erect them only, together with the lateral bracing between them, as the first operation. This reduced by about one half the total weight of steelwork to be supported until the ribs were connected at the crown and made self-supporting. It also permitted the use of the steel not first erected as temporary falsework, traveler track, etc. Approximately 1,000 tons of spandrel columns, girders, and floor beams from the permanent structure were used temporarily for such purposes.

Grillages.—The first piece erected on each concrete skewback was the 63.5-ton steel grillage section made up of plate girders and forming a cellular construction 6 ft deep, 12 ft wide, and 22 ft long. These pieces were designed to distribute the reaction of the arch ribs over the necessary bearing area of concrete. The center third of the top surface of each grillage was milled to a true plane to take the bearing of the steel skewback section, and four corner-bearing surfaces also were finished on the bottom face of each grillage. The original intention was to finish the entire bottom surface to rest directly on the corresponding bearing surface on the concrete skewback itself. This was judged by the contractor to be too difficult of attainment and the following alternative method of securing the proper bearing and alinement was used.

The inclined surface of the concrete was kept about 5 in. below the bottom of the steel grillage except for a 1-ft square pad at each corner. These four pads were to be finished to correct slope and elevation by the foundation contractor. Suitable center lines on each abutment were provided by the engineers and corresponding lines were scribed on the 4-in. slab forming the bottom face of the steel grillage. In order to prove the slope and elevation of these four concrete pads, two triangular-shaped steel frames were built with suitable lateral bracing between them and erected on each abutment, supported on the four concrete pads. They were so designed that, if the pads were finished correctly, the top or horizontal legs of this framework would be in a true horizontal plane, and horizontal and transverse center lines scribed on the steel framework would correspond with the center lines on the concrete work. The original finishing of these concrete pads was neither carefully done nor properly checked and considerable refinishing and adjustment was necessary. As a final check, the milled top bearing surface of each steel grillage also was checked after the grillages had been erected. Instead of a fixed concrete pad, it would have been cheaper and more accurate to provide an embedded, threaded bolt at each corner, with a nut to support and adjust the grillage, and then to survey directly on top of the grillages themselves.

For erection, each grillage was picked up by a special hitch bolted to the top and bottom flange of one of the transverse girder webs. This hitch projected above the grillage and had a pinhole for a pair of short links connecting to the triangular hitch plate on the derrick falls. The hitch was so placed that the grillage hung at approximately its correct inclination in the structure. To guide the grillage in place over the thirty-two 3-in. diameter anchor bolts and to protect the thread on these bolts during this operation, a pilot was provided for each bolt. This consisted of a piece of 4-in. pipe, to one end of which was welded a solid, cone-shaped, steel nose. The pipe simply slipped over the thread on the anchor bolts and did not screw on, so that the pilots could be placed and removed readily.

After the grillages had been set and adjusted and before erection could continue, it was necessary to fill with grout the 5-in. space under the grillage and to fill with concrete the center, vertical row of cells in the grillage, beneath the steel skewback, to give proper bearing for the thrust of the arch ribs. After erection was completed, the remaining cells were also filled, and the entire grillage surrounded with concrete, thus completely enclosing it and making it a part of the concrete skewback.

Steel Skewbacks.—The second steel member to be erected was the steel skewback section 22 ft deep on the lower side and tapering to the 12-ft depth of the arch rib framing into it on the upper side. This member was a box section, 3 ft wide between the web plates the same as the arch rib, designed to spread the load from the rib over the 22-ft depth of the steel grillage. It was approximately 12 ft in height and weighed about 47 tons. The skewback bolted directly to the steel grillage and in addition was secured by double nuts on the end of each of the thirty-two anchor bolts.

Arch Ribs 13 to 10 and Tiebacks at Panel Point 10.—The three arch-rib sections from the skewback out to panel 10, together with the bracing between them, were erected by cantilevering them out under their own weight, still using the unloading derrick or traveler A to handle them. These three sections weighed, respectively, 65 tons, 75 tons, and 58 tons, each. This resulted in a maximum computed stress in the anchor bolts of not quite 20,000 lb per sq in. (erection stage I).

During this operation, the first set of eight forward-tie ropes had been unreeled and hung from the top of the tower. These were swung forward, using the runner line system and connected to the T-sections at point 10 using the outermost pinholes of the adjustable links so that they still hung slack. After all eight were connected, the adjusting falls were applied to the uppermost strand which was pulled to a stress of approximately 50 tons and pinned off. The falls then were applied to the second strand which was adjusted similarly and which, of course, reduced somewhat the stress in the first strand. The strand stress was measured by the use of a dynamometer, applied at the upper end of the strand. The stress in each successive strand adjusted was reduced slightly in an endeavor to secure approximately equal stress in all eight strands when the final one was adjusted. It was necessary for the total stress in the eight strands to equal as closely as possible the calculated stress of 331;500 lb at 60° F (erection stage II), and also for the individual strand stresses to vary by as little as possible. Three adjustments of the strands were necessary to attain this result. The entire operation of erecting and adjusting this group of strands required 2.5 days.

The sole criterion for adjustment of the arch at this erection stage was the total stress in the eight ties at panel 10 which was calculated to pick up the end of the ribs an amount approximately sufficient to reverse the cantilever stress. Subsequent cantilever erection from points 10 to 8, of course, would increase the stress in the ties at point 10 and bring the arch ribs down toward their normal position. No survey for profile of the ribs was made at this stage, the measured tie stresses being regarded as critical. The ribs were so stiff that, if they were not adjusted to a correct profile, an excessive stress would have had to be applied in order to bring them to that profile even though the variation at point 10 was only a small fraction of an inch.

With the strands at panel 10 properly adjusted, the rib traveler was erected at point 10, using the overhead derrick for the purpose. The material track and truck also were erected at this time.

Arch Ribs 10 to 8 and Tiebacks at Point 8.—Standing at point 10, the crew on rib traveler B proceeded to erect the next panel of arch ribs and bracing from points 10 to 9 after which it was moved to panel 9; then the crew erected the next panel of the arch from points 9 to 8. This completed erection stage III in which the theoretical total stress in the tiebacks at point 10 was now 493,500 lb at 60° F. No check was made of the strand stresses at this time or of the elevation of the arch rib. The tiebacks for connection at point 8 had been hung in the meantime from the tower and were now swung forward and connected to the T-sections first loosely and then pulled to proper adjustment with the adjusting falls in accordance with the figures furnished by the engineering department for erection stage IIIA.

The total theoretical strand stress at point 8 was now 454,100 lb which reduced the stress in the tiebacks at point 10 to 69,900 lb. This reduced stress permitted the easy removal of the strands at point 10 by the adjusting falls which were used to disconnect them, one at a time, and swing them back to the abutment, still hanging from the tower.

The removal of the strands at point 10 completed erection stage IV and increased the stress in the strands at point 8 to a total of 485,600 lb. No readings of the strand stress were made at this stage. It was assumed that, if the previous adjustments had been made correctly, the stresses at erection stage IV could not be greatly in error. Neither were the arch ribs surveyed for profile at this stage since they were still so stiff that the total strand stress at point 8 was considered to be the proper criterion.

Arch Ribs 8 to 6 and Tiebacks at Point 6.—Arch-rib traveler B was moved immediately to panel point 8 from which position it was used to erect the ribs and bracing from panels 8 to 7, and then moved to panel 7 and erected the ribs from panels 7 to 6. This completed erection stage V, for which the total theoretical stress in the eight tieback strands at point 8 was 772,100 lb. A set of strand readings made at this time for the Canadian side gave a total stress of 786,600 lb for the north rib and 775,900 lb for the south rib, or a variation of less than 2% from the theoretical. The maximum variation in an individual strand stress was considerably higher, one strand being 19% above the average but all others being within 8% of the average. The average variation in strand stress was about 5%. The one highly overstressed strand was slacked off 1 in. as a precaution, but no other adjustment was deemed necessary.

On the New York side of the river, the total measured strand stress at this stage was 20% high for the south rib and 12% high for the north rib, with a maximum variation for a single strand of −10% from the average. Although the total measured stress was considerably higher than the computed stress, there was some question as to the accuracy of the dynamometers being used to measure these stresses, and, for a number of reasons, the readings seemed high, probably by at least 10%. Therefore, no adjustment was made.

Meantime the eight strands released from panel 10 and left hanging from the tower had been lengthened with pennant strands and made ready to be swung forward and connected at point 6. Four additional strands also were added to this group, making a total of twelve. Because of the arrangement of the strands at the tower top, where they were connected in four vertical rows, these twelve strands for panel 6, which were in the two inside rows, had to be swung out between those at panel 8 which were in the two outside rows. Each strand was swung forward and loosely connected at point 6 until all twelve strands were in place, after which they were adjusted individually to the proper tension. This gave a total of twenty strands now supporting each rib, twelve at point 6 and eight at point 8, and completed erection stage VI.

The first survey for the actual elevation of the arch ribs was made at this stage. The elevations were obtained for each panel point by means of a 200-ft steel tape with a specially constructed target and weight hung from the lower end, the total weight being such as to give the correct pull in the tape. The transit used for leveling was set up near the skewback on the side to be surveyed and the height of instrument was obtained from a near-by bench mark. Instead of depending upon the level bubble to obtain a level line, a specially constructed level board was set up on the far side of the river and the instrument was adjusted so as to read the same elevation on the far side of the river as given by the bench mark on the near side of the river. Two such level boards were used, fastened to the concrete abutments, and only a very slight adjustment of the level bubble was required to give the same reading on the two boards.

Theoretical elevations were computed for each erection stage from points 6 to 10 and for each panel point in the arch rib, the point chosen being on top of the splice plate over the milled joint. The tape was kept wound on a small reel. To obtain an elevation the tape was unreeled, and the target and attached weight were lowered over the outside face of the rib, through the safety net, holding the tape at the proper point on the rib. When the target had been properly lined in vertically, a reading was made on the tape at the top edge of the splice plate. Adding this reading plus the distance from the end of the tape to the center of the target, to the height of instrument, the elevation of the panel point was obtained. It was found that these surveys were made best early in the morning, between 6 a.m. and 9 a.m., at which time there was a nearly constant temperature in the steelwork and usually a complete absence of wind. Even a light breeze was enough to set the target swinging sufficiently to make accurate readings difficult and a moderate or heavier wind made them impossible to obtain. In a light wind, however, the target reading usually could be caught at the bottom of its swing and consistent results obtained.

The first survey of the ribs on the Canadian side of the river for erection stage VI gave results nothing short of remarkable. Of the fourteen panel points surveyed, eight agreed exactly with the theoretical elevations, reading to the nearest 0.01 ft. Of the remaining six points, two were within 0.02 ft, two were within 0.04 ft, and the other two varied by 0.05 and 0.07 ft. The point with the latter variation was at the extreme end of the rib, and the error was never explained satisfactorily. It may have been due to an inaccuracy in the reading, to a minor error in the calculations, or possibly to some local variation in the steelwork itself. No adjustment was made.

On the New York side of the river, the first survey for elevation at erection stage VI showed a maximum variation from theoretical elevation of +0.07 ft at panel 10 and −0.09 ft at panel 6 for the south rib, with corresponding figures of +0.06 ft and −0.05 ft for the north rib. The plotted profiles showed the arch to have slightly too great a curvature due apparently to too great a stress in the strands at point 8 and not enough in those at point 6. Three separate sets of strand adjustments and profile surveys had to be made, after which the measured profiles agreed with the theoretical within 0.04-ft maximum variation. The average variation was only 0.02 ft. It was interesting to note that, at this erection stage, the arch rib was sufficiently limber to respond readily to changes in shape caused by variations in the tieback stresses at panels 8 and 6. The measured strand stresses at this time were higher than those computed by 5%, 6%, 11%, and 16% for the four sets of tiebacks.

Arch Ribs 6 to 3 and Tiebacks at Point 3.—Rib traveler B was moved forward successively to panels 6, 5, and 4, and from these positions the crew erected panels 6–5, 5–4, and 4–3. This completed erection stage VII, with both sets of tiebacks still connected at points 8 and 6. No profile surveys were made for this stage, but complete sets of strand stress measurements were made. These indicated that the tieback stresses at point 6 were somewhat higher, and those at point 8 somewhat lower, than the computed stresses. As the individual strand stress was well within the allowable limit, no adjustment was made.

The next step was the connection of the forward ties at panel 3—one of the most critical and difficult steps in the entire erection procedure. The first four strands at point 3 were obtained from the group at point 6 by: Disconnecting them one at a time; slacking them back on the arch rib; removing the adjustable links; lengthening the strand with a pennant; reconnecting the links; and pulling the lengthened strand forward and connecting it at panel 3. Then, the strand was carefully adjusted to its full stress before the next strand was disconnected at point 6, lengthened, and moved forward. The erection and adjustment of the first four strands at panel 3, with eight strands still in place at panels 6 and 8, was necessary before the strands at panel 8 could be released, and completed erection stage VIIA. A complete survey of the strand stresses was made for this stage to make sure that none of the strands was overstressed, and that the total measured stress was reasonably close to the computed stress.

The remaining eight strands at panel 8 then were disconnected as a group and swung back hanging free from the tower where they were lengthened by pennant strands. This left the arch ribs in erection stage VIIB, supported on four strands at panel 3 and eight strands at panel 6. An entire set of strand stress measurements also was made for this stage. The stress in the strands at point 3 was shown to be somewhat less than the theoretical, and in those at point 6, somewhat greater than the theoretical. As further cantilever erection would decrease the stress in the ties at point 6, and, as their actual stress at this stage was not critical, no adjustment was made.

The eight lengthened strands then were swung forward to panel 3 where they were connected and adjusted, completing erection stage VIII. At this time a complete survey of the arch ribs again was made both for rib profile and strand stresses. Two adjustments were made on each side of the river, before the third and final surveys showed the ribs to be within a maximum variation of $\frac{5}{8}$ in. from the theoretical profile and the measured strand stresses within reasonably close agreement with the computed ones. In general, the average strand stress was found to be from 10% to 20% higher than the computed, with a maximum variation for a single strand of as much as 50% for the strands with low stress, and 25% to 30% for the strands which were highly stressed. In no case, however, was any individual strand stress found to be greater than the specified design limit. The readings are believed to have been high because of errors in the dynamometer and because the readings were taken relatively close to the socketed ends of the strands where they connected to the tower. Experiments showed that the measured stress decreased the farther away from the end of the strand the measurement was made. In addition to this, the angle at which the instrument had to be used also tended to increase the reading as proved by experiment. The effect of curvature in the cables due to the sag from their own weight was eliminated generally by applying the dynamometer from the side—that is, in the vertical plane of the cable—but in some few cases, the instrument had to be rotated in order to clear adjacent strands, thus causing somewhat higher readings. It is believed the measured strand stress was from 10% to 20% higher than the actual stress.

Arch Ribs, Panel 3 to Crown.—With the final adjustment of the strands in erection stage VIII, traveler B was moved forward to panel 3 in which position the crew erected panels 3 to 2. Then it was moved forward to panel 2 and used to complete the erection of the arch ribs and bracing to the crown where the special jacking brackets on the top and bottom flanges were erected. This completed erection stage IX, the last main erection stage before swinging the two half arches one against the other. Because of the careful control of the profile during previous erection stages, the opposite ribs closed in almost perfect alinement and elevation with respect to each other across the 11-in. gap left at the crown. No adjustment whatever in the elevation or alinement was neces-

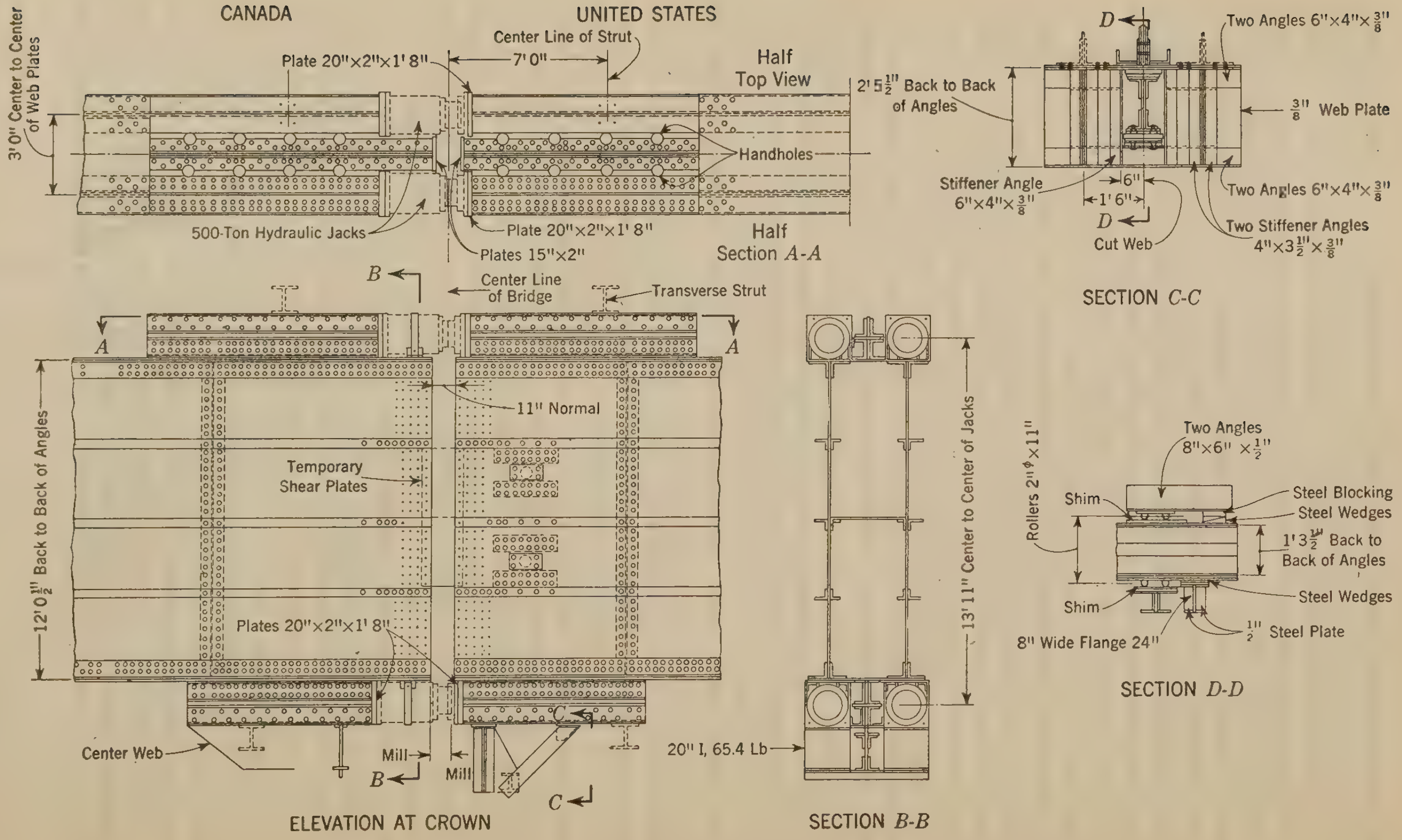

CANADA
UNITED STATES
3' 0" Center to Center of Web Plates
Plate 20"×2"×1' 8"
Center Line of Strut
7' 0"
Half Top View
2' 5½" Back to Back of Angles
Handholes
500-Ton Hydraulic Jacks
Plate 20"×2"×1' 8"
Plates 15"×2"
Half Section A-A
B
Center Line of Bridge
Transverse Strut
A
A
11" Normal
Temporary Shear Plates
12' 0½" Back to Back of Angles
Plates 20"×2"×1' 8"
Center Web
Mill
Mill
B
C
C
ELEVATION AT CROWN
13' 11" Center to Center of Jacks
20" I, 65.4 Lb
SECTION B-B
D
Two Angles 6"×4"×⅜"
⅜" Web Plate
Stiffener Angle 6"×4"×⅜"
6"
1' 6"
D
Cut Web
Two Angles 6"×4"×⅜"
Two Stiffener Angles 4"×3½"×⅜"
SECTION C-C
Two Angles 8"×6"×½"
Rollers 2"ϕ×11"
Shim
Steel Blocking
Steel Wedges
1' 3½" Back to Back of Angles
Steel Wedges
Shim
½" Steel Plate
8" Wide Flange 24"
SECTION D-D

Fig. 37.—Elevation of the Arch Rib at the Crown, Showing Jacking Devices

sary although the ribs were so sensitive at this stage that considerable relative movement was observed from the handling of even light loads by the travelers. Swinging the traveler booms from one side to the other also would produce discernible changes in the elevation. Such relative movements were eliminated by the erection of the special shear devices and temporary lateral bracing erected at the crown for this purpose (see Figs. 37 and 38).

A complete and final survey of the arch ribs was made for erection stage IX, both for rib profile and strand stresses. The actual profile of both ribs, on both sides of the river, was found to be slightly high, by as much as $\frac{5}{8}$ in. to $\frac{3}{4}$ in., between the abutments and panel 7 or 8 where it coincided with the computed profile. From there to the crown it gradually drooped, deflecting to a maximum of 0.16 ft or not quite 2 in. below the computed elevation. The weight of traveler B and the other erection equipment near the crown must have been underestimated. As the cantilevered length was greater than 500 ft and the ribs were only 12 ft deep and supported on only two sets of elastic cables, the structure was extremely flexible in this condition and small loads would cause great changes in profile.

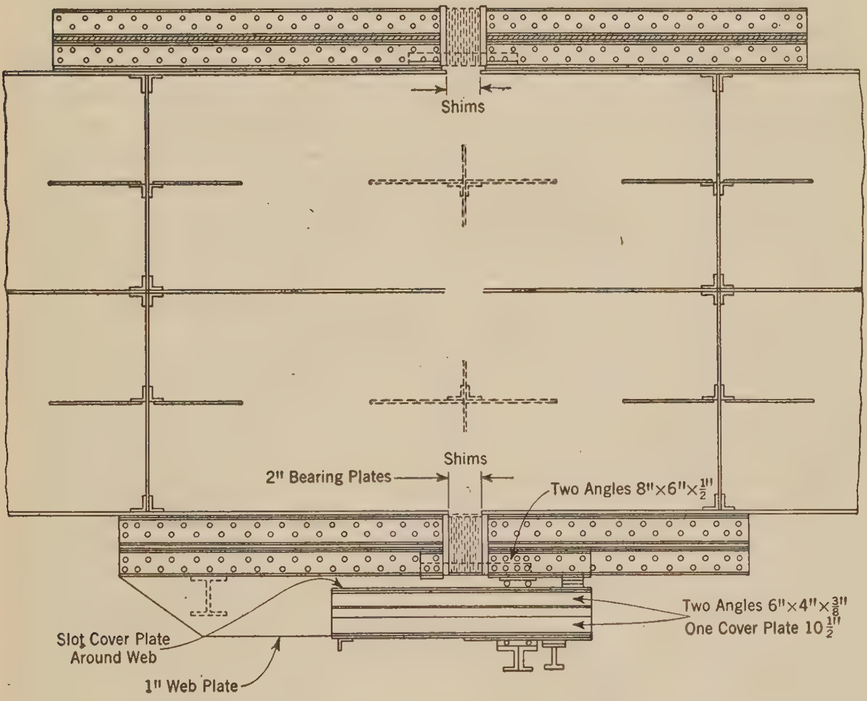


Fig. 38.—Section on the Center Line of the Arch Rib Showing Jacking Devices

The measured strand stresses at panel 3 were roughly from 30% to 50% higher than the computed stresses, and at panel 6 they varied from 6% too high to 20% too low. Even allowing for the dynamometers giving measured stresses from 10% to 20% too high, they still would be greater than the computed stresses, which bears out the assumption that the erection loads near the crown must have been heavier than estimated.

Fitting Up and Riveting.—Each arch-rib section, when it was erected by cantilevering out from the preceding section, was fitted up in the connecting splice with sufficient pins and bolts to carry the weight of all cantilevered steelwork plus the weight of the rib traveler and other erection equipment. All splices were milled, and the splice plates and rivets were designed to carry only about 60% of the total stress. Computations were made for each joint showing the number of pins and bolts required to carry the erection stresses. In general, sufficient pins were employed to carry the entire erection stress without any dependence on the bolts, which were used mainly to pull the various parts into close contact for riveting. The pins for carrying the erection tension stress were all located in the top-flange splice plates and splice angles. Because of the 12-ft depth of the arch ribs, surprisingly few pins were required to carry this stress.

The cantilever erection stress tended to open the milled joints at the top flange by the amount of play of the pins in the holes, but the bottom flange was brought into tight contact and good bearing. The bottom-flange splice plates and angles were riveted in this condition as soon as possible after the rib was erected. Only a small amount of fitting up was performed in the web splice plates at this time.

After erection had reached the point where a set of forward ties was to be attached and after the ties had been connected and adjusted, the cantilever stress in the two or three forward-rib sections was reversed. This reversal of stress actually put the top flange in compression and thus closed the milled joint. As soon as this occurred and the joint was in good bearing, the top-flange splice plates and angles were fitted up and riveted. The holes in the web splice plates were fitted up and riveted as convenient after both the bottom and top flanges had been riveted.

The lateral bracing between the arch ribs was fitted up as soon as erected but was not riveted immediately. Close watch was kept of the lateral alinement of the arch as cantilever erection progressed, and none of the bracing was riveted until it was certain that the alinement was perfect. Both half arches tended to drift slightly to the south, up to a maximum of about $\frac{5}{8}$ in. to $\frac{3}{4}$ in. A stiff south wind was sufficient to bring the ribs back to alinement, however, and no unusual difficulty was experienced in forcing them back to line by means of temporary cable diagonals tightened with turnbuckles. Riveting of this bracing was kept at least three or four panels behind the erection, however, regardless of the alinement, and the 12-ft deep struts were not riveted until it was determined that the two ribs were within $\frac{3}{8}$ in. of the same elevation.

When the arch ribs were closed at the crown and swung and the last panels of lateral bracing erected and fitted up, the entire arch was found to be in perfect lateral alinement within $\pm \frac{1}{16}$ in. All surveys for lateral alinement were made by means of an engineer's transit set up near the top of the cliff on the center line of the bridge. A foresight was obtained on a bench mark on the far side of the river, and the span lined by means of targets painted on the center of the top cross strut at each panel point.

CLOSURE OF ARCH RIBS

Jacking Operations at Crown.—The arrangement of the jacking brackets, the hydraulic jacks, and the shims is shown in Figs. 37 and 38. Two 500-ton hydraulic jacks were provided for both the top and the bottom flange for each rib, giving a total jacking capacity of 2,000 tons per rib. The computed rib thrust was about 1,400 tons, of which 800 tons was in the top flange and 600 tons in the bottom flange, due to the dead-load moment at the crown.

As soon as the jacks were fitted up, those at the top flange were operated to increase the top flange opening to 18 in. The bottom-flange jacks were not operated, but the bottom opening automatically increased to approximately 17.5 in. during this procedure and was shimmed snug. During the entire jacking operation, shims were placed in the space provided between the jacking brackets for this purpose, and the load was carried on these shims at all times except during actual jacking movements.

The purpose of this preliminary jacking apart of the ribs was to release the stress, partly, in the forward ties that were still supporting the ribs at panels 6 and 3, particularly at the latter point where the individual strand stresses were about at their maximum and were beyond the capacity of the adjusting falls. The reason for not putting any stress in the bottom flange of the ribs at the crown at this time was that releasing the strands at point 6 would allow the ribs to deflect and thus automatically would induce stress in the bottom flange at the crown, provided the ribs were just in contact at the beginning.

Eight of the twelve strands at point 3 next were completely disconnected, one strand at a time, using the adjusting falls, and the strands were swung back and left hanging from the tower. This caused the cable bent to lean back somewhat and also increased the stress in the remaining four strands at points 3 and 6. In addition, of course, there was an increase in the crown thrust which was taken through the top and bottom shim packs.

At this time, the remaining four strands at panel 3 were left in place to provide the necessary forward pull at the top of the cable bent while the eight strands at panel 6 were removed. Since removal of those strands would increase the stress in these four remaining strands at point 3, the latter first were lengthened by about 30 in. each in small increments so as to avoid overstressing any one strand during this operation. The bottom-flange shims were reduced at this time to $16\frac{7}{8}$ in., thus keeping the bottom-flange load down so that it would not exceed the flange stress that would exist in erection stage X after all cable ties were removed.

Next, the remaining eight strands at panel 6 were completely disconnected, swung back clear of the arch, and left hanging from the tower. Similarly, three of the four remaining strands at panel 3 then were removed and swung back clear of the arch, leaving only one strand per rib still connected at panel 3. In this condition, the top of the cable bent had leaned back very nearly to its original erection condition, about 3 ft out of plumb, thus reducing the stress in the backstay cables so that they readily could be disconnected with a single runner line. Blocking and lashing was placed at the rear end of the sill and stiffleg to support the tower after the last forward ties were removed. The last strand at point 3 then was disconnected, thus completely freeing and swinging the arch ribs which were now entirely self-supporting. The jacks at the crown again were operated and the crown opening reduced from 18 in. in the top and 16 $\frac{7}{8}$ in. in the bottom to the normal opening of approximately 11 in., top and bottom.

Measurement of Crown Opening.—The 11-in. normal opening at the crown between the arch ribs had been left so that the stresses in the arch ribs might be measured at this point by means of hydraulic jacks and thus compare the actual with the design stresses. The closing keystone piece was prefabricated 13 in. long and provision made so that it could be cut down to a minimum length of 9 in. if circumstances required. By varying the top and bottom lengths of this keystone section, an angular rotation and moment also could be induced at the crown and thus the location of the center of pressure could be changed. A complete description of these calculations is given in the third paper of the Symposium.

In order to compute the crown stresses at this stage of the erection accurately, it was necessary that the dead loads of the erection equipment be known with great accuracy. The weights of the major pieces of erection equipment already had been computed and their effect calculated. All miscellaneous equipment (including drift pins, fitting-up bolts, small tools, timber platforms, and walkways, etc.) was surveyed and its weight computed. Preliminary calculations had shown that the weight on the north rib on the New York side of the river, and on the south rib on the Canadian side, would be somewhat heavier than on the other ribs. It was desirable to have equal weights on the two ribs to avoid two sets of calculations. For this purpose, concrete counterweight blocks weighing 5,000 lb each had been cast and erected at each panel point from points 3 to 10, inclusive, on the south rib on the New York side of the river, and on the north rib on the Canadian side. The traveler booms were swung around to a specified location to produce the desired reaction on the two arch ribs.

In addition to four new 10-in. test gages inserted in the hydraulic pumps, a weighing capsule, which was inserted in line with each jack between the plunger and the jacking bracket, also was provided. These weighing capsules contained an enclosed liquid under initial pressure. They were extremely sensitive to increase of pressure which was measured on accurately calibrated gages. In the actual weighing operations, however, it was found that there was extremely close agreement in the loads measured both by the weighing capsules and by the ordinary hydraulic gages.

The weighing and measuring operations were performed between midnight and 6 a.m. in order to avoid the effects of temperature changes. The crown opening first was set at its theoretical value of 11 in. $+\frac{3}{32}$ in. at the top flange and $-\frac{3}{32}$ in. at the bottom flange, and the jack stresses were recorded. The crown opening was measured between the milled surfaces at each corner of the flanges by means of machinist's scales, reading to the nearest $\frac{1}{64}$ in. Three other sets of readings for different crown openings also were made in order to check better the relation between the actual and computed stresses and also to determine the most favorable dimensions of the keystone piece. Finally, it was decided that the normal 11-in. opening would produce stresses as close to the computed ones as possibly could be desired. Briefly, the total crown thrust was 1% higher than the computed thrust for each arch rib, although the center of pressure was about 4 in. too high for the north rib and 6.5 in. too high for the south rib. The latter variation was probably due to some inaccuracy in the weight and location of the erection equipment. Since the computed thrust due to dead load of steel must be subject to an error of at least 2% to 3% and since the estimated weight of the erection equipment was undoubtedly in error by at least 5%, it can be appreciated readily that the agreement of the measured and computed thrusts within 1% was nothing short of remarkable. On the other hand, reference to the change in crown opening for various changes in the crown thrust indicates that the two half arch ribs are extremely flexible with regard to translation or change in the crown opening. They are much stiffer with regard to an angular change (that is, a change in moment); hence the conclusion that, for a long flexible hingeless arch fabricated and erected with a high degree of accuracy on accurately surveyed skewbacks, it should not be necessary to provide any closing or keystone piece with the idea of varying the dimensions of such a piece to obtain the computed stresses. Only relatively small changes in the thrust result from relatively large changes in the crown opening.

Keystone Pieces.—The two keystone pieces, one for each arch rib, were an ordinary section of arch rib, 11 in. in length on the neutral axis as finally fabricated. They were slightly wedge-shaped, each face being on a radial line of the rib. The splice plates at the crown were continuous between the two ribs and across the keystone section. All holes in these splice plates were reamed to metal templates in the shop, setting them from the milled ends of the ribs and of the keystone pieces as finally finished. There were a total of forty-six splice and filler plates for each keystone piece.

The keystone pieces were erected by swinging them into place sideways into the opening between the arch ribs which was increased to about 11.5 in. to give clearance. A special hitch was provided on the outside of the piece, near the top flange, and a line connected to the inside face at the bottom flange assisted in swinging the piece into place sideways into a vertical position. The splice plates were fitted up to the rib on one side before carefully lowering the jacks to bring the other side into bearing and correct alinement. No particular difficulty was encountered in this operation, and good holes, together with perfect alinement between the opposing ribs, were obtained.

Removal of Erection Steel.—As soon as the crown opening had been measured and while the keystone pieces were being finished in the fabricating shop, all temporary erection steel used in the tieback system was removed. The forward cable ties hanging from the front face of the tower were disconnected and rereeled. The backstay cables were disconnected from the anchorages, lowered from the top of the tower, and also rereeled. The lower strut and panel of bracing in the cable bent was removed, allowing traveler A to back up to a position about 60 ft behind the tower where the boom again was lengthened to 145 ft. Then, the cable bent, together with the stifflegs and struts used to support it, was dismantled, and all material was returned to the yards for refabricating into permanent columns for use in the final structure. The fan falsework platform and columns also were removed and returned to the yards for refabricating. The traveler boom then was shortened to 95 ft, and the traveler advanced to the end of the concrete approach spans ready to start erection of the floor steel. All of this work was completed by the time the keystone pieces were received and erected.

Erection of Floor Steel

Survey of Arch Ribs and Adjustment of Column Shims.—Immediately after the cable tiebacks had been removed and the crown opening measured, the arch ribs were placed in their normal position with an 11-in. crown opening (the same condition as would exist if the keystone pieces actually were in place). A survey then was run over the arch ribs to determine the relative difference in elevation of like-numbered panel points in the two ribs. The absolute elevation was not desired and was not determined at this time, but it was necessary to know whether one rib was higher than the other so that a correction could be made in order to bring the floor beams level. Difference in elevation was determined by setting up a transit on the arch rib near each panel point and reading the elevation of that point on each rib. This survey showed the north rib to be slightly higher than the south rib throughout its entire length, by a maximum of $\frac{7}{8}$ in. on the Canadian side of the river and $\frac{3}{8}$ in. on the New York side. The difference in elevation at the crown was about $\frac{3}{4}$ in. Both ribs had a very uniform curve or profile from end to end.

The columns had been fabricated with $\frac{1}{2}$ in. of normal shims (made up of one $\frac{1}{4}$-in. shim and two $\frac{1}{8}$-in. shims) between the base plate and the cover plate on the rib. By shifting the proper amount of shims from the north to the south rib, the floor-beam connections at the top of the columns could be brought to the same elevation at any one panel point. On the New York side of the river, a $\frac{1}{8}$-in. shim was removed at each panel point from points 9 to 3, inclusive, on the north rib and added to the south rib. For the three points, 2, 1, and 2, at the crown and for points 9, 10, and 11 on the Canadian side of the river $\frac{1}{4}$ in. of shims was shifted from the north to the south rib. For points 3 to 8, inclusive, on the latter side $\frac{3}{8}$ in. of shims was shifted making a relative difference in elevation of $\frac{3}{4}$ in. for the column bases at those points. A $\frac{1}{8}$-in. shim was interchanged at point 12 on the Canadian end of the bridge, and no change was made in points 10, 11, and 12 on the New York end. No explanation is offered for this relatively large difference in elevation between the two ribs which was somewhat surprising after they had followed their theoretical profiles so closely during the various erection stages.

Erection Procedure.—Actual erection of the floor steel was begun as soon as the keystone pieces had been inserted and while they were being fitted up and riveted. At the abutment, traveler A erected the first pair of spandrel columns, which were spliced at the center, for their full height of 142 ft by simply standing them on the arch ribs without any guys or bracing between them. Each column immediately was stayed at the top by erecting the spandrel girder next to the abutment, followed by the floor-beam framing between these columns. These two spandrel girders were supported on sliding seats on the abutment. At this time, they were pulled back several inches to give clearance for erecting the closing floor panel and temporarily anchored in that position to give longitudinal stability to the floor structure being built from the abutment.

At the crown of the arch, travelers B erected the floor steel in front of them, panel by panel, moving backward down the arch rib as each panel was completed. All material for these travelers was fed to them by traveler A and by means of the material truck, as during erection of the arch ribs.

Removal of Travelers; Fitting Up and Riveting.—After traveler A had erected two panels at the abutment, the traveler track was extended and the traveler moved out to panel 11 where it erected two more panels to point 9 and then moved to that position. Traveler B meantime had moved back to panel 7, its final position, from which point its 76-ft boom just could fill in the members in panel 5–6. When this operation was completed, traveler B was dismantled and removed by traveler A which proceeded to fill in the two remaining panels of floor from points 6 to 8.

When the last pair of spandrel girders was erected in panel 6–7, the temporary connection of the two spandrel girders at the abutment was released. These two girders then were jacked forward to close the opening in panel 6–7 and to permit the girders in that panel to be fitted up.

In spite of the fact that there was no sway bracing between the spandrel columns, no difficulty was encountered in keeping them plumb and in holding the entire floor system to correct lateral alinement. This must be attributed very largely to the excellence of the shop work in fabricating the columns and the floor steel.

Fitting up and riveting of the floor steel and bracing followed the erection as closely as possible and was completed within ten days after traveler A had been dismantled and removed by a small tractor crane.

Handrailings.—The two roadway railings and the single sidewalk railing had been erected by the travelers with the other floor steel. With the floor entirely fitted up and known to be in correct alinement, fitting up of the railing-post connections and alinement of the railing itself were begun. The details of the railing-post connections were such that it was practically necessary to fit them up and rivet them before the concrete deck and sidewalk could be poured. In general the railing-post connections were fabricated with small-sized holes and were fitted up with small bolts until proper alinement was attained. The connections then were reamed out to full size and riveted. The subsequent operation of pouring the concrete deck, and the resultant deflections of the arch rib, produced no noticeable change in the alinement of any of the three railings.

CONCRETE DECK

The work of forming and pouring the concrete deck and sidewalk slabs was sublet to other contractors, one for the Canadian and one for the New York side. Wooden forms were used throughout, supported on timber joists which were carried in wire hangers resting on top of the stringers.

The main roadway slab reinforcement consisted of prefabricated, welded, bar joists. All longitudinal bracing consisted of plain bars wired to the trusses at the intersections. Ordinary two-way bar reinforcing was employed in the sidewalk slab.

Each contractor used a mixing plant set up near the shore end of the concrete approach. On the Canadian side of the river the mixed concrete was wheeled out over a runway to the center of the bridge where pouring operations were started. On the New York side all concrete was pumped out over the span through an 8-in. pipe line. Concreting was started at the center of the span in one roadway only and proceeded simultaneously in both directions. The sidewalk slab was poured at the same time as the south roadway slab. In order to prevent excessive deflections in the arch ribs, it was specified that not more than four panels of the roadway be poured on one side of the approach than on the other.

The concrete subcontractors also poured the concrete encasing around the steel grillages at the skewbacks, thus completely filling and enclosing these members and making them a part of the concrete abutments.

PAINTING

Two field coats of aluminum paint were applied to the structure over two coats of proprietary brands of a red lead and iron oxide type of paint.

Application of the first field coat was started before the arch ribs were closed at the crown and continued during the erection of the floor steel and the pouring of the concrete deck. The second field coat was not applied to any exterior surface until the concrete deck immediately above had been poured and the forms removed. All the floor steel received its first coat of paint before the concrete was poured. During concreting operations streams of water were played on the steel to clean it of concrete splash, but, because of an insufficient number of streams and lack of pressure, considerable cleaning had to be done by hand after the concrete had dried before the second field coat could be applied.

Both field coats of paint were applied by brush and no spray painting was done. The aluminum paint used was purchased in the summer of 1940 before its use was prohibited.

Conclusion

Field work was begun on the Canadian side of the river on December 15, 1940, and on the New York side on December 26, 1940. The first permanent steel, consisting of the steel grillage, was erected on the Canadian side on February 14, 1941, and on the New York side on March 6. The last four main-rib sections were erected at the crown on May 19, and the measurement of the crown opening was made on the night of May 26–27. The keystone pieces were received June 11 and erected June 12. Erection of the floor steel was begun on June 13 and completed on July 17. The contract date for starting work on the concrete deck was July 21. The concreting was completed on the Canadian side on September 9 and on the New York side on September 15.

The safety nets were left in place until all concrete work and painting (with the exception of painting the top flanges of the arch ribs) was finished. Removal of the safety nets was completed on October 9, and the final painting was completed on October 24.

The bridge was opened to traffic on November 1, 1941, at which time paving of the approaches and the finishing of the toll booth and quarters for the customs and immigration officials still had not been completed.

DISCUSSION

Egidio O. Di Genova,[21] Assoc. M. Am. Soc. C. E.—An assertion, repeated several times in this Symposium, implies that the elastic theory is not entirely valid for the Rainbow Arch Bridge. The writer's extended studies of the elastic theory lead him to the conviction that the following statement in the first paper (12 lines preceding Eq. 1) at least is out of place:

> "The preliminary calculations for the two-hinged arch rib were sufficient to show that the elastic theory was inadequate for the design of such a structure."

If the writer understands this statement correctly, he believes that it is incorrect. The two-hinged arch is different from the hingeless arch, and it is impossible to draw parallel comparisons between the two. In general, in any elastic system the writer has found that the resultant elastic deformations (and therefore the internal stresses), computed by means of the so-called elastic theory, and the findings obtained by other systems of computations are closely alike, with one exception: When all the possible internal stresses are considered (that is, compression, shear, and flexural moments, generated by an externally applied system of forces), slightly larger results are obtained; but, if the structure is computed by the elastic method without taking in consideration shear and axial stresses, the difference does not exist, or it is of a very small relative order such as can be expected within the common range of approximations.

The elastic theory in itself has certain limitations in relation to the magnitude of the elastic deformations, but those limitations must be considered in relation to the linear variation of strain and stress of the material used, and also in relation to the relative size of the structure.

On an arch of 950-ft span and 150-ft rise, 12 in. (or even 18 in.) of elastic deformation, for a steel structure, is relatively small, and the limits imposed by the theory of elasticity are still satisfied.

The writer has the impression that the geometrical simplicity of this arch, the continuous uniformity of its cross section, the very sturdy condition of fixed restraint, etc., define an ideal case for useful practical application of the theory of elasticity.

Although this theory is not applied to any minor engineering problem, nevertheless the use of it by well-trained engineers in this field and for important works is worth a lot. All engineers know that, in theoretical studies inherent in the elastic behavior of materials, the elastic theory is a very important instrument for developing ideas, but there are also many cases in which its application to practical problems is just as important as the more theoretical applications and the Rainbow Bridge, in the writer's opinion, is one of them.

This writer insists on the idea that the basic principles governing the elastic theory must be understood clearly, and a consistent system of computation must be followed implicitly. Tables and methods of application must be used wisely so as to reflect the theory accurately, or there is likely to be considerable confusion in the mind of the computer; if this advice is not followed, the chance of errors is greater, especially if certain sets of formulas are applied blindly.

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[21] Civ. Engr., Board of Water Supply, New York, N. Y.

The writer admits that it is very difficult to establish a logical system of computations for such an important structure as the Rainbow Bridge. Nevertheless, the various means of checking one's work offered by the elastic theory (as for example: The cut-and-try method; the method of virtual work; the ellipse-of-elasticity method; etc.), which are integral parts of that theory, can be used to best advantage on a project that is as important as the Rainbow Bridge. The results obtained are certainly worth the extra cost, and no competent person can ever charge that they are erroneous, unless some serious error is made.

After a preliminary proportioning of the structural elements, the use of the various analytical and graphic solutions offered by the elastic theory is a blessing. Even if the expense of constructing a model is considered desirable, the theory itself will prove valuable.

The elastic theory, like many other means of analysis, is not magic; the engineer should always remember that in final results he receives from an analysis only the findings inherent in the assumptions made. For example, in the design of the Rainbow Bridge, the effect of vertical loads and the deflections of the arch ribs could have been analyzed by the elastic theory, whereas the laterals, the deck, and the spandrel columns could have been analyzed in some other way. In the preliminary analysis of the bridge, it seems natural that the spandrel columns, for example, should be replaced by their reactions in order to simplify the computation of the arch itself.

The application of the elastic theory requires a little more than just a handbook. It calls for personal ability and engineering art—qualities that cannot be taught easily by any standard handbook.

It is also true that, after all, the origin of any system of computations is essentially based on the elastic theory. At times some standardized system of analysis, made easy for a practical man, is a wonderful thing indeed, for the diffusion of knowledge; but, as a class, professional men of higher standards and reputation must remember that the basic culture of an engineer must be on a plane above the superficial knowledge of a plain mechanic, in the same proportion as the skill of the physician is to that of the registered nurse. To this end, the writer takes this opportunity to express the hope that one more year may be added to the present university engineering course so as to spur the profession to attain a keener technical, cultural, and moral point of view and that engineers of the future will use it to sponsor a more extensive and rational study of the elastic theory.

CHARLES MACKINTOSH,[22] ESQ.—This analysis of the Rainbow Arch Bridge is a valuable work because the writers have reemphasized the fact that any slender arch must be considered as an unstable body since twice the load produces more than twice the stresses. An attempt has been made to reduce the cumbersome procedure required for finding the final moments in the arch. In developing an approximate method of finding the moment, a formula is suggested which is quite satisfactory for the particular arch under consideration with the particular loads to which it was subject. However, these recommended formulas are not applicable to many other loads and shapes of arches.

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[22] Structural Engr. (Mackintosh & Mackintosh), Los Angeles, Calif.

If the arch were of such shape that the moment due to the dead load was high, Eq. 16 would not be satisfactory. According to this formula, for instance, if there were no live loads or impact loads, the arch moments would not be increased by the slenderness of the arch section. Perhaps Eq. 16 would be satisfactory for the general case if all of the moments were grouped together ($M_{DL, LL, I, w, t}$) and these moments, in turn, were multiplied by a factor using the combined moments in the denominator similar to Eq. 16, as previously written.

Thus

$$s_y = N \frac{\Sigma T}{A} + N \frac{\Sigma M}{S} \left[\frac{1}{1 - N \, \Sigma T \, \dfrac{\Sigma u}{\Sigma M}} \right] \text{............} (35a)$$

To include the effect of the inaccuracy in fabricating the arch to the prescribed outline, ΣM should include $\Sigma T \times e'$, in which e' is the tolerance allowed as the authors have indicated in Eq. 17. If no factor of ignorance is present in determining e', the ratio $\frac{e'}{N}$ may be used. Eq. 21 would also be unsatisfactory in any case where M_{DL} is large. This may be treated as suggested herein.

In the course of designing many long-span slender roof arches, the writer has found that a rapid and convenient approximation is given by an adaptation of the usual beam-column formulas[23] as follows:

$$s_y = N \frac{\Sigma T}{A} + N \frac{\Sigma M}{S} \left[\frac{1}{1 - N \dfrac{\Sigma T}{\Sigma T_{cr}}} \right] \text{.............} (35b)$$

in which

$$T_{cr} = \frac{\pi^2 E I}{L^2_i} \left[1 - \left(\frac{\alpha}{\pi} \right)^2 \right] \text{..................} (36)$$

In Eq. 36, L_i = estimated length of arch between points of inflection, and α = central angle between points of inflection. For flat arches the correction for curvature $\left[1 - \left(\frac{\alpha}{\pi} \right)^2 \right]$ is small and may be neglected.

Such methods offer a facile check for stresses in "lamella roofs" and in arched roofs with ribs of wood, concrete, or steel under uniform loads and one-side snow loads and wind loads.

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[23] "Theory of Elastic Stability," by S. Timoshenko, 1st Ed., McGraw-Hill Book Co., Inc., New York and London, 1936, pp. 27–45 and 225–230; "Formulas for Stress and Strain," by Raymond J. Roark, 1st Ed., McGraw-Hill Book Co., Inc., New York and London, 1938, pp. 120–121; "Strength of Materials," by S. Timoshenko, 2d Ed., D. Van Nostrand Co., Inc., New York, 1940, Vol. 2, p. 218; and "Airplane Structures," by A. S. Niles and J. S. Newell, John Wiley & Sons, Inc., New York, 1938, Chapter 14.

C. M. GOODRICH,[24] M. AM. SOC. C. E.—In the discussion of the design of the Rainbow Bridge "elastic theory" is frowned upon, and a "deflection theory" is preferred. Elastic theory deals with deflections, and is based on the law of conservation of energy, a law not lightly to be repealed or suspended. If one made a blunder in a mathematical problem, or failed to find a solution in one particular way, the remedy would be more mathematics. In the case of the arch the remedy is more elastic theory, unless indeed the elastic limit is threatened.

In suspension bridges what is here termed "the elastic theory" is based on the assumption that the stiffening truss rises and falls in a gracefully parabolic manner, which does not accord with elastic theory. It would perhaps be more nearly correct to call it the "inelastic method."

In the very common case of a boom the dead load causes a deflection and a consequent series of moments and deflections due to the axial load; often the biggest effective load on a long boom is due to the swinging and the series of moments and deflections that come from that; the writer proportions also for an inaccuracy of 1/750 the length in each of two directions. A boom 180 ft long was once used to erect the towers for a lift bridge; such a boom deserves respectful consideration. Only three times has the writer gone through the somewhat tedious operation suggested; and the experience of others no doubt will be similar. This case is clearly one of limit design, like the arch.

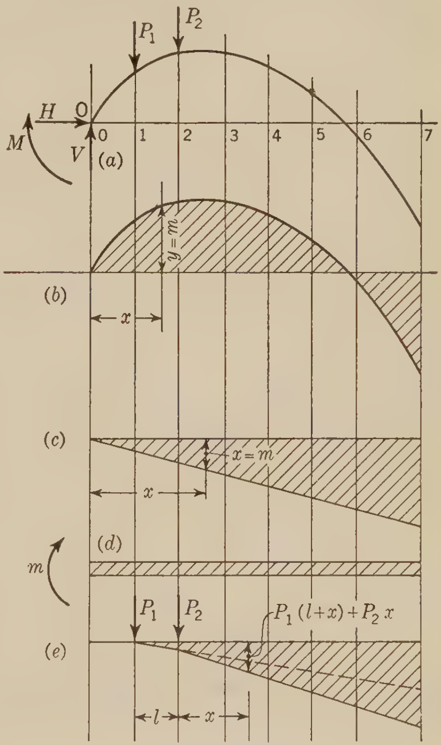

FIG. 39

If "Stern's Theory"[9] contains something more than computing an arch by bits and pieces in the long-familiar way, or the investigation and cure of critical deflection, if any, it would be useful to have the procedure explained.

According to the authors of the third paper (see "The Elastic Centroid—Its Application to the Study of the Action of the Closed Rib"), the method of the elastic centroid, as used by them, was taken from a 1925 edition of Müller-Breslau's book.[17] The writer's first acquaintance with this Müller-Breslau method was shortly after 1910, when the late David A. Molitor, M. Am. Soc.

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[24] Cons. Engr., The Canadian Bridge Co., Ltd., Walkerville, Ont., Canada.

C. E., published[25] a very clear and complete explanation of its use for fixed arches. His work contains also an interesting case of its use in a problem with two redundants. The elastic centroid is also called "center of instantaneous revolution," since it is determined by the moment applied at the base. A still more appropriate name is perhaps "imaginary hinge"; this is the name for like devices used in several problems by August Föppl.[26]

Such fixed arches may be figured by a modest extension of the two area-moment theorems, announced in 1873 by the late Prof. C. E. Greene, M. Am. Soc. C. E. One need only bear in mind that in most cases $\int \frac{m\,M}{E\,I}\,ds$ may be represented conveniently by an area $\int \frac{M}{E\,I}\,ds$, when the expression means the statical moment of a series of areas about either of two axes; or, when m is a unit moment, the sum of these areas.

The arch may be divided into any number of segments n, preferably agreeing with the changes in arch section (see Fig. 39(a)). Taking moments about a horizontal axis through point O:

$$0 = -H\sum_{0}^{6} k\,A\,\bar{y} + H\,k_{6,7}\,A_{6,7}\,\bar{y} + V\sum_{0}^{7} k\,B\,\bar{y}$$

$$+ M\sum_{0}^{7} k\,l\,\bar{y} - \sum_{1}^{7} C\,k\,\bar{y} \ldots\ldots\ldots\ldots\ldots\ldots (37a)$$

in which (in addition to the Symposium notation): $k = \frac{\lambda}{l\,I}$; A and B are the m-areas in section n due to the horizontal and vertical unit dummy loadings, respectively; $\bar{y}$ = vertical distance from a horizontal line through O to that point in the arch intersected by a vertical through the centroid of the moment area with which y is used; l = length of the horizontal projection of section n; λ = axial length of section n corresponding to l; and C = the M-area in section n due to P loads. Taking moments about a vertical through point O:

$$0 = -H\sum_{0}^{6} k\,A\,\bar{x} + H\,k_{6,7}\,A_{6,7}\,\bar{x} + V\sum_{1}^{7} k\,B\,\bar{x}$$

$$+ M\sum_{0}^{7} k\,l\,\bar{x} - \sum_{1}^{7} C\,k\,\bar{x} \ldots\ldots\ldots\ldots\ldots\ldots (37b)$$

in which $\bar{x}$ = the horizontal distance from O to a vertical through the centroid of the moment area with which it is used. Finally, summing areas:

$$0 = -H\sum_{0}^{6} k\,A + H\,k_{6,7}\,A_{6,7} + V\sum_{0}^{7} k\,B + \sum_{0}^{7} M\,k\,l - \sum_{1}^{7} C\,k \ldots (37c)$$

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[25] "Kinetic Theory of Engineering Structures Dealing with Stresses, Deformations and Work for the Use of Students and Practitioners in Civil Engineering," by David A. Molitor, McGraw-Hill Book Co., Inc., New York, N. Y., 1911.

[26] "Vorlesungen über technische Mechanik," by August Föppl, Leipzig, 1902–1912.

In computing the deflection (say at point 2, Fig. 39(a)), consider the tangent at that point to be unchanged, and compute the movements of points 0 and 7 relative to point 2. For a horizontal movement at 0 relative to 2:

$$E\,\rho = -H\sum_{0}^{2} k\,A\,\bar{y} + V\sum_{0}^{2} k\,B\,\bar{y} + M\sum_{0}^{2} k\,l\,\bar{y} - C_{1,2}\,k_{1,2}\,\bar{y}\ldots(38)$$

The equations for the other side would be similar except that the x-values would be measured from the right and the y-values from point 7. For the two-hinged arch, omit the M_0-terms and Eq. 37c. For temperature use $E\,\rho$ in Eq. 37a instead of 0.

Clearly the foregoing method is adapted to finding the first and second shape changes, or to checking the result of an assumption. This ignores, of course, the effects of shear, of possible variations in material and workmanship, of the yielding of steel concrete and rock in the bases, of splices, and of connecting members. Mr. Molitor considered the base as yielding. It would be interesting to know what, if any, measurable displacement occurred in the length of the Rainbow Bridge arch.

The writer would have preferred straight segments in the arch, as being cheaper and without unrelieved stresses; he believes the esthetic effect would have been equally good, due to the fact that the eye sweeps over such a figure in a curve, as shown in psychological laboratories by tracing the movements of the eyeball. A camelback with say three straight segments in the top chord is ugly; with say seven or more it is pleasing, because the eye passes over the top chord in a curve. The mass of the arch is of modest size relative to the mass of the roadway as seen at an angle; the connections between the two are too slight to effect an esthetically satisfying unity.

T. KENNARD THOMSON,[27] M. AM. SOC. C. E.—It is a pleasure to read in the first Symposium paper (heading, "Preliminary Studies") that the engineers realized that such a structure should "blend with the natural beauty and grandeur of the Niagara gorge." The Rainbow Arch Bridge is the fifth to be erected at approximately this site—probably the only case of so many renewals of an iron or steel bridge in America. The first three were undoubtedly iron bridges and the last two steel:

No. 1—A suspension bridge, 10 ft wide, was opened for traffic on January 2, 1869.

No. 2—A suspension bridge replaced No. 1, with a width of 17 ft. This bridge was finished on December 15, 1888, and blew down on January 9–10, 1889.

No. 3—No. 2 was rebuilt also as a suspension bridge, 17 ft wide, in 1889.

No. 4—A steel arch replaced No. 3, and was completed in August, 1897, and destroyed by ice on January 27, 1938.

No. 5—The Rainbow Arch Bridge, as described by the authors, was opened for traffic on November 1, 1941.

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[27] Cons. Engr., New York, N. Y.

The late John A. Stewart, in 1911, acting on his own initiative, invited a group of Americans and Canadians to celebrate the 100th Anniversary of the Treaty of Ghent—100 years of peace between the two countries. He had many prominent men on his Commission, including: Joseph H. Choate, Jacob H. Schiff, Gutzon Borglum, the great sculptor, Sir Edmund Walker, president of the Canadian Bank of Commerce (who was the most outstanding Canadian banker of his time), and many others. He also honored the writer by asking him to serve and to design two bridges for the event. The first World War put an end to the proposed celebration.

One of these bridges, designed by the writer, was to be built between Buffalo, N. Y., and Fort Erie, Ont., Canada, and the other was to replace No. 4, at the site of the Rainbow Arch Bridge. At that time bridge No. 4 was known to be inadequate and unsafe, and the thought was that a bridge built to celebrate 100 years of peace should endure for several hundred years—which no steel bridge, at this site, can ever do, unless a far better paint is developed to resist the spray from the falls, the ice, etc. The writer proposed a heavily reinforced concrete arch, with granite facing (see Fig. 40). The Buffalo Bridge, built by others, also had the same location, number of spans, and similarity of outline as the writer's plan of 1911.

When the writer spoke to Sir Edmund Walker about these proposed bridges, he said that he would not be in favor of bridges; they were too utilitarian; but, when he was shown photos of paintings of the writer's two bridges, he said at once, "Oh, that is different—they are beautiful." When Mr. Stewart showed the two paintings—one 5 ft and one 8 ft long—to Mr. Schiff, he at once told Mr. Stewart: "You cannot get up plans like those without money," and voluntarily gave Mr. Stewart his personal contribution, a check for $5,000.

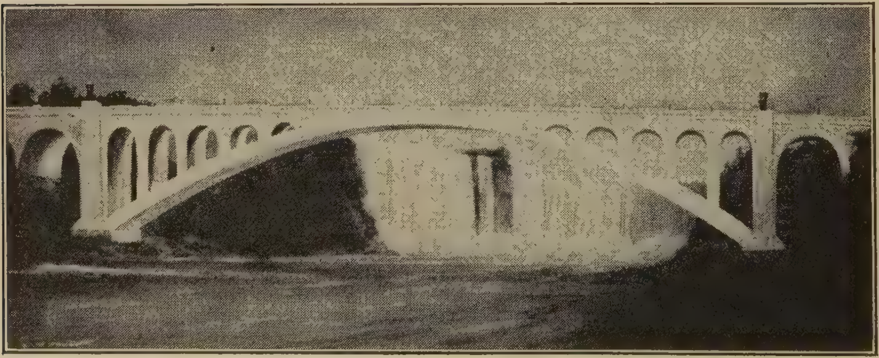

Fig. 40

At a subsequent luncheon, attended by about forty people, one of the most prominent of the New York architects said that suitable architectural plans should be prepared (he would like to have made them). When asked what he thought such plans would cost, he replied, "About $50,000." He was then shown photos of these two paintings, and he exclaimed: "Oh, you have the plans already," and lost interest in the subject. This is a good example (many others could be cited) of how men in other professions are much more adept in knowing what to charge for their services than most engineers are. It also shows that it is well worth while to have simple but beautiful outlines for all engineering structures.

Simplicity of design gives beauty without expense; whereas much intricate filigree work is very expensive and not so beautiful, as exemplified in many cathedrals.

C. M. Spofford,[28] M. Am. Soc. C. E.—The Rainbow Arch Bridge is a beautiful structure, perfectly adapted to its location. Great credit is due to the engineers and architects who evolved the design, to the men who built the bridge so successfully and expeditiously, and to the writers of the papers in this Symposium for their comprehensive description of the design and construction.

Although the laboratory work on the two-hinged segmental arch is valuable in itself, its publication as a part of the Symposium is doubtless due to the fact that it was of assistance to the designers in determining whether or not to use two-hinged arches or perhaps to confirm their opinion that a two-hinged arch would not be satisfactory for this bridge.

The use of the expression "classic arch theory" seems ill-advised, especially as applied to a two-hinged parabolic arch. The writer knows of no classic arch theory for two-hinged arches. Such an arch has only one indeterminate statical force; namely, the horizontal component of either reaction. An expression for this can be readily developed by the conventional method of assuming one of the hinges to be on rollers and equating the expression for its horizontal deflection under any load, with the deflection of the same point due to a horizontal unknown force applied at the hinge. Certain assumptions may be made in a preliminary computation, a simple equation developed, the deflection of the arch computed, and the computation revised, if it seems advisable, by dividing the arch into sections and using summations instead of integrations. The experimental results would have been more useful to arch designers had they been expressed in terms of the horizontal component of a reaction.

In designing two-hinged parabolic arches, both for bridges and buildings, the writer has found no serious technical difficulties, especially when advantage is taken of the properties of the position line and the kernel points.

More experimental work might well have been done to determine the three unknowns of a hingeless arch instead of making the "limited observations upon this type of arch" of which no record appears in the Symposium.

Congratulations should be extended to the engineers of the Bethlehem Steel Company for their unusually thorough investigations and for their great care in the erection procedure. The thanks of the profession are due especially to the authors of the papers on fabrication and erection for their thorough treatment of these phases of the construction. It is worthy of note that safety nets were used and that no fatalities occurred during the construction of the bridge—an unusual record. Another interesting feature is that the erectors used capsules in order to establish the total crown forces and concluded that hydraulic jacks are reasonably accurate when applied forces are measured on the increasing side of the reaction. It is hoped that in the closures a detailed description of these capsules will be given. The writer's experience with hydraulic jacks for determining reactions has not been entirely satisfactory. He notes with interest that the jacks were calibrated in a testing machine which had been previously calibrated by the use of proving rings and assumes that it was not found convenient to use proving rings for this bridge. The writer has used proving rings on various statically indeterminate structures.[29]

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[28] Hayward Prof. of Civ. Eng., Emeritus, Mass. Inst. Tech., Cambridge, Mass.; Cons. Engr. (Fay, Spofford & Thorndike), Boston, Mass.

The flexibility of the arch ribs is not surprising; the same thing was noticed by the writer during the construction of a plate girder roof arch—a horizontal adjustment at one end of which was made without difficulty by several laborers pushing the arch end horizontally.

An unusual feature of this bridge is the use of steel keystone pieces and the method of introducing them into the arch ribs. This is the first time, so far as the writer knows, that such a device has been used, although one important reinforced concrete arch bridge, designed by the writer's firm, was erected as a three-hinged arch and made two-hinged by closing the center hinges after erection.

One of the factors that is always of great interest to engineers is that of cost—either the total cost divided between foundations and superstructure, or, if let at a unit price basis, the pound price of both silicon and carbon steel. It is hoped that the authors, in their closures, will give such information.

The writer assumes that consideration was given by the designers to the advisability of using a light-weight floor and that such a floor—either light-weight concrete with bituminous surfacing or steel grating—was found uneconomical. It would be interesting to have this matter presented in the authors' closures.

L. J. MENSCH,[30] M. AM. SOC. C. E.—The Rainbow Bridge is one of the world's outstanding bridges, and the practical bridge engineer will greatly appreciate receiving information as to the high points of its design, fabrication, and erection. The many illustrations, the history of the design, Table 1, and the summary of quantities are very interesting and one nearly lost feature of the art has been disclosed; namely, the advisability, from the fabricating standpoint, of designing the center line of the arch rib as a series of multicentered circular arcs. The first masonry and the first metal arches were built of single-centered arcs. About 200 years ago French engineers introduced three-centered arcs, and about 50 years ago five-centered arcs made their appearance.

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[29] "Weighing Bridge Reactions with Proving Rings," by C. M. Spofford and C. H. Gibbons, *Engineering News-Record*, March 28, 1935, p. 446.

[30] Civ. Engr. and Contr., Chicago, Ill.

In this bridge there is an eleven-centered arch. It is questionable whether so many centers were really necessary. Although a bridge engineer will note with respect the mathematics and the practical features of the erection problems, he probably would have liked more information on the evidently highly successful job of surveying the bridge site before and during erection and also more details on the concrete deck, the forms used to avoid an accident similar to that which occurred on the Golden Gate Bridge,[31] the kind and spacing of the expansion joints, the kind of railings used, the system of lighting, and other smaller but important features of a successfully constructed bridge. No statement has been made regarding the stiffness of the bridge under traffic—a question of greatest interest for an arch rib with a depth of one eightieth of the span.

According to the Symposium authors, the unusually shallow ribs necessitated the abandonment of the "elastic theory" and the substitution of the "deflection theory." The writer supposes that the "elastic theory" is the mathematical theory of elasticity, except that certain "elastic theory" requirements are not considered:

(1) The effect of unit stresses in excess of the true elastic limit;
(2) Poisson's effect;
(3) Shear deformation;
(4) Vibrations due to loading conditions;
(5) Changes of temperature during critical loading;
(6) Variation of the modulus of elasticity in the various parts of the structure; and
(7) Deformation of the center line of the arch rib for the equation of moments and in the elastic equations.

These phenomena are clearly and completely appraised in the oldest treatise on metallic arched bridges, analyzed by the strict mathematical theory of elasticity.[32] Early writings also emphasize the computation of deflections of the arch due to rib shortening, compression, change of temperature, shrinkage (in wood arches), and faulty joints. J. A. C. Bresse warned his students not to use ribs so shallow as to cause excessive deflection due to a moving load. No real, valid reason has been given in this Symposium why this sound warning has been disregarded. For comparatively large deflections, the problems are treated in textbooks on elasticity under the heading, "Elastica," an expression first used by Leonhard Euler.

The writer holds some reservations as to the validity of Eqs. 18 to 21. For example, Eqs. 18 and 19 contain only two indeterminate quantities—H and M_{oa}. The symbol M'_a refers to a statically determinate moment of the exterior forces in a simple beam and is not concerned with the missing indeterminate value of the reaction at the left springing. To ignore the latter will intensify the conditions in Fig. 12.

To design a bridge, its span, rise, width, dead load, and live load must be known. Often there is a choice of span and rise—greater spans may reduce the cost of the foundation, and greater rise may save in the weight of metal in the arch ribs. Suppose some preliminary designs have prescribed span and rise and number of ribs. The dead load of the roadway does not vary much from 150 lb per sq ft regardless of span. The weight of the ribs may be approximated from handbooks on bridge design. A nearly correct formula for the horizontal component of the thrust is given by:

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[31] *Engineering News-Record*, February 25, 1937, p. 313; March 11, 1937, p. 377; and April 22, 1937, p. 590.

[32] "Recherches analytiques sur la flexion et la résistance des pièces courbes," par J. A. C. Bresse, Paris, 1854.

$$H = \left(w_1 + \frac{\Delta w_1}{6}\right) \frac{L^2}{8 f (1 + k)} \quad \text{(39)}$$

in which w_1 = the dead load per linear foot on the rib at the crown; Δw_1 = increase in dead load per linear foot at the springing, due to the spandrel columns and the greater weight of rib; f = the rise of the arch; and k = a numerical factor between 0.01 and 0.03 that enters the formula to account for the shortening of the arch under load.

If the permissible stress in direct compression is assumed as one half the maximum permissible stress, then Eq. 39 may be used for a close approximation of the area of metal required in the rib at the crown. The depth of the rib is governed by the live load and deflection caused by it. Modifying and simplifying the prior work of M. Bresse,[32] the writer has derived the following fundamental formulas[33] for the three statically unknown values generated by a concentrated load P at any point along the elastic arch:

Vertical component of thrust, V, at the left abutment—

$$V = \frac{P}{L^3} (L - L_a)^2 (L + 2 L_a) \quad \text{(40a)}$$

horizontal component of the thrust, H—

$$H = \frac{15 P L_a^2 (L - L_a)^2}{4 L^3 f (1 + k)} \quad \text{(40b)}$$

and moment at the left springing, in a counterclockwise direction[33]—

$$M_{oa} = \frac{P L_a (L - L_a)^2 \left(L - \frac{2.5}{1 + k} L_a\right)}{L^3} \quad \text{(40c)}$$

In Eqs. 40, in addition to the notation of the Symposium, L_a = the distance from the left end of the arch axis to the load P.

These formulas were derived originally for a very flat circular arch of constant section, and similar ones can be obtained for a parabolic arch of constant ratio $\frac{ds}{I}$. They may be used safely for arches of the Rainbow type.

For a circular flat arch, or for a parabolic arch of constant $\frac{ds}{I}$,

$$k = \frac{45 I}{4 A f^2} \quad \text{(41a)}$$

For the Rainbow arch, $\frac{ds}{I}$ is not constant and Eq. 41a becomes,

$$k = \frac{12.4 I}{A f^2} \quad \text{(41b)}$$

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[33] "The Reinforced Concrete Pocket Book," by L. J. Mensch, Neal Publishing Co., San Francisco, 1909, p. 154 and Fig. 68, p. 155.

The bending at any point x, y of the arch can be computed by Eqs. 40, thus: When the distance x from the left end to point x is less than L_a,

$$M = V\,x - H\,y - M_{oa} \quad \text{(42}a\text{)}$$

and, when x is greater than L_a,

$$M = V\,x - H\,y - M_{oa} - P\,(x - L_a) \quad \text{(42}b\text{)}$$

By solving these equations for every tenth point of the arch, for example, influence lines[34] can be constructed for determining the critical loading for eleven points on the arch. In the case of a uniform load the moment at the left springing can be found either by integrating Eq. 40c between the limits $L_a = 0$ and $L_a = 0.4\,L$, or by computing the area of the influence line between those limits. For this case,

$$M_{oa}\,(\max) = 0.0174\,w\,L^2 \quad \text{(43}a\text{)}$$

Of course, Eq. 43a is strictly correct only for a constant value of $\frac{ds}{I}$. If $\frac{ds}{I}$ decreases to $\frac{1}{5}$ at the springing, the maximum value becomes

$$M_{oa} = 0.0257\,w\,L^2 \quad \text{(43}b\text{)}$$

By interpolation for the Rainbow arch where $\frac{ds}{I}$ decreases to $\frac{1}{1.5}$,

$$M_{oa} = 0.0185\,w\,L^2 \quad \text{(43}c\text{)}$$

Adding 10% for impact stresses, $M_{oa} = 0.0204 \times 1{,}380 \times 950^2 = 25{,}400$ ft-kips for the Rainbow arch, compared with 28,850 ft-kips reported in Table 1, line 5. This difference may have been caused by assuming a concentrated load of 26,700 lb acting about 150 ft from the springing. The moment then is $0.07 \times 26{,}700 \times 950 = 1{,}775$ ft-kips, which, increased by 10%, makes the total moment 27,353 ft-kips. The latter value checks very closely the 28,850 ft-kips given in the Symposium. It is generally believed that the greatest moment and the greatest deflection from a distributed load occur at the quarter point of the span. A concentrated load at the three-tenth point produces a slightly greater moment in the arch, and a distributed load over the half span produces a larger moment at the three-tenth point than at the quarter point. Furthermore, the deflection is greater at the three-tenth point than at the quarter point.

For many years it was standard practice (and it is still a recommendable procedure) to design an arch bridge of the Rainbow type for a movable load on the half span from the springing to the crown, except that the crown section was designed for a uniform load extending over a distance $L/6$ on both sides of the crown. For a uniform load on the half span, the three statically indeterminate quantities can be obtained by integrating Eqs. 40 between the limits $L_a = 0$

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[34] "The Reinforced Concrete Pocket Book," by L. J. Mensch, Neal Publishing Co., San Francisco, 1909, Figs. 69 to 75, pp. 155 and 157.

and $L_a = \frac{L}{2}$. Thus,

$$V = \frac{13}{32} w L \quad \text{(44a)}$$

$$H = \frac{w L^2}{16 f (1 + k)} \quad \text{(44b)}$$

and

$$M_{oa} = w L^2 \frac{1 + \frac{11 k}{3}}{64 (1 + k)} \text{ (tension in the top of the arch)} \quad \text{(44c)}$$

The moment at the right springing is

$$M_s = - \frac{w L^2 \left(1 - \frac{5 k}{3}\right)}{64 (1 + k)} \text{ (with tension at the bottom)} \quad \text{(45a)}$$

and the moment at the crown is

$$M_c = \frac{w L^2 k}{48 (1 + k)} \quad \text{(45b)}$$

The equation for the deflection of the elastic line may be obtained from the fundamental equation of flexure,

$$\frac{d^2 \delta_v}{dx^2} = - \frac{M}{E I} \quad \text{(46)}$$

in which, for a uniform load on the half span,

$$M = V x - H y - M_{oa} - \frac{w x^2}{2} \quad \text{(47)}$$

and, by double integration,

$$E I \delta_v = \frac{w}{192} \left[- 13 L x^3 + \frac{8 L x^3 - 4 x^4}{1 + k} + L^2 \left(\frac{1.5 x^2 + 5.5 k x^2}{1 + k} \right) + 8 x^4 \right] \quad \text{(48)}$$

For the point $x = 0.75 L$,

$$\delta_v = \frac{w L^4 (1 - 3.5 k)}{6{,}144 E I (1 + k)} \quad \text{(49)}$$

and the movement is upward. It is important to know whether there are any moments acting in the arch rib when the bridge is uniformly loaded over the entire span. Integrating Eqs. 40 between the limits $L_a = L$ and $L_a = 0$:

$$V = \frac{w L}{2} \quad \text{(50a)}$$

$$H = \frac{w L^2}{8 f (1 + k)} \quad \text{(50b)}$$

$$M_{oa} = \frac{w L^2 k}{12 (1 + k)} \quad \text{(50c)}$$

and the moment at the crown is

$$M = \frac{w\,L^2\,k}{24\,(1 + k)} \quad \text{.......... (50}d\text{)}$$

Also, at the crown:

$$\delta_v = \frac{w\,L^4\,k}{384\,E\,I\,(1 + k)} \quad \text{.......... (51)}$$

In 1919 discussing a paper on this subject by the late George F. Swain, Past-President and Hon. M. Am. Soc. C. E., the writer presented[35] a new derivation of the Bresse equations, for the deflection and rotation of any element of a curved member, as follows:

$$\theta = \int_0^x \frac{M\,ds}{E\,I} \quad \text{.......... (52}a\text{)}$$

$$\delta_h = -\int^x \frac{T\,dx}{E\,A} - y\int^x \frac{M\,ds}{E\,I} + \int_0^x \frac{M\,y\,ds}{E\,I} \quad \text{.......... (52}b\text{)}$$

and

$$\delta_v = \int_0^x \frac{T\,dy}{E\,I} - x\int_0^x \frac{M\,ds}{E\,I} + \int_0^x \frac{M\,x\,ds}{E\,I} \quad \text{.......... (52}c\text{)}$$

in which *ds* is an increment of length measured along the center line of the arch rib.

Eqs. 52 demonstrate that the deformations are linear functions of the exterior forces and reactions, showing that the law of superposition is valid for the deflections as well as for the stresses. M. Bresse uses them to determine the statically unknown reactions. Subsequent writers have introduced other methods such as the theory of virtual velocities, theory of least work, ellipse of elasticity, Maxwell's equations, Müller-Breslau's equations, slope deflection, etc., all of which lead to equations that must be simplified considerably before they reduce to the simple forms possible by the Bresse equations directly.[32] If the proper value of *M* is introduced, the writer's derivation of the Bresse equations[32] is valid for a simple beam, a continuous beam, a rigid frame, a three-hinged arch, and an arch with two hinges or no hinges.

Contrary to the implied limitations expressed by the Symposium authors, Eqs. 39 to 51 are quite sufficient for the design of an arch of the Rainbow type, and the results agree well with those given in Table 1. To particularize:

(1) From Eq. 39 and Fig. 12 the thrust at the crown is found to be $H = \dfrac{\left(7{,}250 + \dfrac{3{,}350}{6}\right) \times 950^2}{8 \times 150\,(1 + 0.011)} = 5{,}810$ kips for the dead load, as compared with 5,911 kips in Table 1.

(2) The bending moment from the dead load, at the crown, by Eqs. 50, is $M = \dfrac{\left(7{,}250 + \dfrac{3{,}360}{6}\right) \times 950^2 \times 0.011}{24\,(1 + k)} = 320$ ft-kips, as compared with 434 in Table 1.

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[35] *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–1920), Eqs. 17, 18, and 19, p. 660.

(3) The maximum bending moment from live load, at the crown, by Eqs. 44 to 50, is $M = 0.0057 \times 950^2 \times 1{,}380 = 7{,}100$ ft-kips. From Eq. 42*a*, $M = 26{,}700 \times 950 \times 0.047 = 1{,}200$ ft-kips. Adding to the sum (7,100 + 1,200) 10% for impact gives 9,130 ft-kips, as compared with 8,700 in Table 1.

(4) The mechanics for analyzing the effect of a temperature drop in a fixed arch was presented by the writer[36] in considerable detail, in 1929. The analysis offered at that time yields, for the parabolic arch,

$$H_t = \frac{45\, E\, I\, c\, t}{4\, f^2\,(1 + k)} \qquad (53)$$

in which c = coefficient of temperature expansion. The moments at the springing and crown, respectively, are:

$$M_s = H_t \frac{2\, f}{3} \qquad (54a)$$

and

$$M_c = H_t \frac{f}{3} \qquad (54b)$$

For the Rainbow arch the coefficients of H_t in Eqs. 54*a* and 54*b* are 0.7 and 0.3, respectively. The deflection of the crown due to a drop of temperature is obtained easily by means of Eq. 52*c*, and very closely by the relation

$$\delta_v = c\, t\, f + c\, t \frac{L^2}{4\, f} \qquad (55)$$

For the Rainbow arch, when $c\, t = 0.0004$, $\delta_v = 8$ in. Assuming a temperature drop of 60° F and substituting other necessary values,

$$H_t = \frac{13.25 \times E \times 12{,}000 \times 144 \times 0.0004}{150^2 \times 144\,(1 + 0.011)} = 84 \text{ kips},$$

as compared with 93 kips in Table 1; and $M_c = 84 \times 0.3 \times 150 = 3{,}780$ ft-kips, as compared with 4,255 in Table 1.

(5) The effect of an error of 1 in. should be $\frac{1}{4.55}$ of the effect of a drop in temperature of 60° F, as the latter would cause a potential shortening of $0.0004 \times 950 \times 12 = 4.55$ in.

(6) The maximum moment due to live load occurs at the three-tenth point, between points 5 and 6, Fig. 6. By Eqs. 40 to 47, $M = 0.0096 \times 1{,}380 \times 950^2 = 11{,}950$ ft-kips; and, by Eq. 42*a*, $M = 0.0596 \times 26{,}700 \times 950 = 1{,}455$ ft-kips. Adding 10% for impact to the sum, $M = 14{,}745$ ft-kips as compared with 13,470 ft-kips for point 5 in Table 1.

(7) The moment due to live load at the left springing agrees very well with Table 1, as demonstrated previously. It remains only to compare the temperature effects. From Eqs. 54, $M_s = 84 \times 150 \times 0.7 = 8{,}820$ ft-kips, as compared with 8,918 in Table 1.

(8) In view of the very flexible arch rib, the deflections due to dead load, live load, and drop of temperature are of great interest. Deducting the weight

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[36] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1252.

of the rib (because of the method of erection), the dead-load deflection at the crown is computed by Eq. 51 as $\delta_v = 4.2$ in.; and, assuming a length of 270 ft loaded across the crown, the live-load deflection at the crown,[35] $\delta_v = 3.8$ in. For the temperature drop δ_v was found equal to 8 in. and the shortening of the arch rib due to live load was 0.55 in. The total $(4.2 + 3.8 + 8 + 0.55)$ is 16.55 in. By Eqs. 44, 45, and 48, the live-load deflection at the three-tenth point, $\delta_v = \frac{1{,}380 \times 950^4 \times 12^3 \,(1 + 6.8\,k)}{5{,}920\,E\,I\,(1 + k)} = 7$ in. when the half span is loaded and a constant $\frac{ds}{I}$ is assumed. Because of the larger moment of inertia near the springing the actual deflection should be 20% less. With live load in the half span, the deflection δ_v at the three-tenth point caused by the shortening of the arch is equal to $\delta_v = 0.7 \times 0.55 = 0.39$ in., approximately. The corresponding dead-load deflection δ_v caused by the shortening of the arch is equal to $\delta_v = 0.7 \times 4.2 = 2.9$ in., approximately; and the temperature deflection, $\delta_v = 4.19$ in., or a total of about 14.48 in. Although, possibly, these deflections may be reduced 20% on account of the increased stiffness of the rib toward the springing, they are nearly twice as large as those assumed by the authors on the basis of the deflection theory.

A particularly valuable part of this Symposium is the inclusion of Table 1, which enabled the writer to compare the results of the "elastic theory," as propounded by M. Bresse, with the "deflection theory," defended by the Symposium authors.

O. H. AMMANN,[37] M. AM. SOC. C. E.—The designers and constructors of the Rainbow Arch Bridge deserve credit for having produced a most excellent structure, both esthetically and technically. The arch type of bridge, like its predecessor, undoubtedly suggested itself as the natural solution because of the steep rocky sides of the river gorge. The banks are of sufficient height to permit the bridge floor to be entirely above the arch ribs—an arrangement which is esthetically most ideal for any arch structure.

The appearance is enhanced by utmost simplicity of the structural design, more particularly the selection of slender solid-plate arch ribs in place of deep arch trusses which have heretofore prevailed in arches of this magnitude. The adoption of solid-plate arch ribs in this bridge was undoubtedly made possible and economical by the recent advances in theory and structural design as well as in methods of fabrication and erection, and, therefore, evidences a further milestone in the art of bridge construction.

The design of the graceful imposing arch span proper is kept strictly to structural necessities, free from ornamental or architectural embellishments. The short approaches surmounting the rocky slopes are appropriately designed to appear as massive arch viaducts; their contribution to the monumental appearance of the structure undoubtedly justified the additional expense involved when compared to light steel structures exemplified in some similar cases.

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[37] Cons. Engr., New York, N. Y.

Under the given conditions the adoption of the hingeless type of arch rib was unquestionably most appropriate. A two-hinged arch of satisfactory performance could have been designed but, as is indicated by the authors' comparative figures, such an arch with slender solid-plate ribs would have been considerably less economical than the hingeless type.

The arguments might be advanced that the two-hinged arch is statically indeterminate in a lesser degree and that yielding of the foundations, or inaccuracies in workmanship and erection, especially at the rib splices, might cause more serious secondary or supplementary stresses in the hingeless arch. Neither of these arguments could be of practical importance in this case. The determination of stresses, even in complex statically indeterminate structures, has advanced to a stage which, for an important structure, does not admit greater simplicity of analysis as a valid argument. Provided there is no external yielding, the stresses in any well-designed and well-built statically indeterminate structure can be determined with the same degree of reliability as those in a statically determinate system. As regards the effects of yielding, the solid rock foundations, and the high degree of accuracy of fabrication and erection which is now attained in steelwork of this nature by experienced and well-equipped constructors, preclude supplementary stresses of practical significance.

In view of the slenderness of the arch ribs, which was undoubtedly dictated by the practicable make-up of the solid-plate section, the deformations under load could be expected to be appreciable and to warrant investigation of the stresses by a more accurate analysis than the so-called "elastic theory," more particularly since in an arch the differential stresses from the deformation are additive.

The designers have made such an investigation, at least for certain points, and have developed an analysis and approximate formulas for more accurate stresses which, as far as they have been applied, appear to be adequate for this particular structure. It seems to the writer, however, that the permissible limits should be assigned to the accurate stresses and not to the ones determined by the inadequate "elastic method." It is true that in this particular case the more accurate stresses, as far as calculated by the designers, are still within safe limits, but it would seem that, in general, and particularly in comparative designs based on the same specifications, the prescribed permissible stresses should apply uniformly to stresses determined by an adequately accurate method. The differential stresses cannot be classified as "secondary" stresses in the commonly accepted sense.

In this connection, the writer has long held the view that the term "elastic theory," by which the analysis of stresses in an assumed rigid system has become known, is incongruous and misleading. If a special term for it appears desirable it should be the "theory of the rigid system"; and what has become known and referred to in the first paper of the Symposium as the "deflection theory" should be termed, in distinction, "theory of the flexible system." Fundamentally, there is no distinction between the two, except the degree of accuracy, both being based upon the laws or the hypothesis of elasticity. For most common structures in which the changes in the form of structure under loads are negligible, the theory of the rigid system is sufficiently adequate. For more flexible structures there are different "theories of the flexible system" depending upon the degree of accuracy desired or justified. Any desired degree of accuracy may be attained by a method of trial and correction, by more or less elaborate cut and dried formulas, or by model tests. The authors, very wisely, have resorted to all three methods in an effort to check the results, at least at certain critical points.

The research work on two-hinged arch models described in the second paper of the Symposium, and the method of analysis derived therefrom, should prove of aid to the designer, especially in connection with rough preliminary studies. It is the kind of research that deserves to be encouraged in connection with the design of any unusual structure. The results obtained demonstrate again the reliability of stress determinations by means of models and their usefulness in checking the adequacy of theoretical analysis.

It is only regrettable that circumstances probably did not permit the work to be extended to include a complete check of stresses on a model representing the hingeless arch of the Rainbow Bridge as actually built, and it is to be hoped that opportunity for such a study will present itself in the future.

The writer cannot agree with the conclusion expressed by the authors of the second paper that two-hinged arches should be of sufficient stiffness to make the application of the "elastic theory" entirely proper. As long as the accurately determined stresses from the maximum specified loads are within the prescribed limits the design should be considered satisfactory. Accepted practice does not determine the design load by applying a uniform factor of safety to a hypothetical yield load, but rather limits the accurate unit stresses under maximum design load to certain safe stresses based upon the qualities (especially the yield point) of the material.

The third and fourth papers of the Symposium reflect the elaborate and thorough studies and preparations made by the constructors to insure accuracy in workmanship and perfect control of the stress conditions during erection.

Louis Balog,[38] Esq.—The papers on the Rainbow Arch Bridge record considerable progress in the design and construction of long-span arch bridges. The designers of the bridge present, in the first paper, a method of analyzing flexible arches that yields correct results, and they also derive practical design formulas. The results of the experimental investigations given in the second paper check the designers' analysis and are useful for preliminary design of flexible two-hinged arch ribs. In Appendix I of the first paper, methods for the consideration of deflection effects are discussed. In addition to those mentioned a method developed by Fr. Dischinger[39] is of interest. It applies to beam-columns and arch ribs, primarily those having a parabolic axis and a varying cross section expressed by

$$I_c = I_x \cos\theta \text{.........................} (56)$$

The increase of the elastic-theory moments is expressed by a rapidly converging geometric series:

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[38] Cons. Engr., New York, N. Y.

[39] *Der Bauingenieur*, Vol. 18, August 20, 1937, pp. 487–520.

$$M = M_E \left(\frac{\nu + \beta}{\nu - 1} + \frac{\beta_1}{\nu} + \frac{\beta_2}{\nu^2} + \frac{\beta_3}{\nu^3} + \cdots \right) \ldots\ldots\ldots\ldots (57)$$

in which (in addition to the notation of the Symposium): β is a numerical coefficient; and

$$\nu = n \frac{E\,I}{H\,a^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (58)$$

In Eq. 58: a is half the span length; and n is a numerical buckling coefficient.

For ribs that are not too flexible the consideration of the first term in the parentheses at the right side of Eq. 57 is sufficient to estimate the increase of the elastic-theory moments. For a hingeless rib the following approximate formulas are obtained for the half span uniformly loaded:

Moment at springing—

$$M_S \approx M_E \left(\frac{\nu - 0.382}{\nu - 1} \right) \ldots\ldots\ldots\ldots\ldots\ldots (59a)$$

moment at the quarter point—

$$M_{0.25} \approx M_E \left(\frac{\nu - 0.014}{\nu - 1} \right) \ldots\ldots\ldots\ldots\ldots\ldots (59b)$$

and, moment at the three-eighth point—

$$M_{0.375} \approx M_E \left(\frac{\nu + 0.121}{\nu - 1} \right) \ldots\ldots\ldots\ldots\ldots\ldots (59c)$$

For use in Eqs. 59 (see Eq. 58), $n = 20.19$. Introducing the data given in Table 3 for the hingeless rib: $\nu = 20.19 \dfrac{29{,}400 \times 88.2 \times 144}{6{,}460 \times 475^2} = 5.18$; and, by Eqs. 59*a*, 59*b*, and 59*c*, respectively, $M_S = 1.148\ M_E$; $M_{0.25} = 1.236\ M_E$; and $M_{0.375} = 1.268\ M_E$. These approximate values are from 2% to 6% different from the dead-load plus live-load moment values given in Fig. 12 and Table 3, indicating the correctness of the moments used in the design.

For a two-hinged rib the approximate moment at the quarter point, when the half span is loaded, is

$$M_{0.25} \approx M_E \left(\frac{\nu + 0.032}{\nu - 1} \right) \ldots\ldots\ldots\ldots\ldots\ldots (60)$$

in which (see Eq. 58), $n = 9.87$. Introducing the data given for the two-hinged rib in Table 2: $\nu = 9.87 \dfrac{29{,}400 \times 120.24 \times 144}{7{,}551 \times 475^2} = 2.95$; and substituting in Eq. 60: $M_{0.25} = 1.53\ M_E$. This value is 10% less than the correct value of the dead-load plus live-load moment given in Table 2. On account of the flexibility of the two-hinged rib the approximation is not as close as that for the more rigid hingeless rib.

By presenting comparative data for the two-hinged arch and for the hingeless arch, the designers have directed attention to the superiority of the more statically indeterminate structure. The hingeless rib of greater rigidity was designed with considerable saving in metal. The efficient section of this rib resulting from the use of stiffened webs of minimum section should also be noted.

The study of the four papers on the Rainbow Arch Bridge reveals the care and understanding with which this outstanding structure was designed and built.

William G. Rapp,[40] Assoc. M. Am. Soc. C. E.—In its entirety this Symposium is an excellent contribution to the limited existing literature on design, engineering, fabrication, and erection of monumental structures such as the Rainbow Arch Bridge. In particular, a distinct service has been rendered to that part of the engineering profession engaged in the actual prosecution of work in the field—to those engineers on whose shoulders falls the duty of seeing that the structure as designed and fabricated is erected properly, safely, and efficiently. Such practicing engineers must meet unexpected and vital problems with immediate answers, an art that can only be achieved by actual, tedious, and arduous experience, or by the study of papers such as that portion of the third Symposium paper dealing with "Erection Controls" and the entire fourth paper, "Erection of Steel Superstructure."

To the designer, the design of a bridge of this caliber is often an important and seemingly difficult task. Decisions as to type, sizes, etc., are necessary before any further steps can be taken. The engineering work following the basic design decisions is just as important and often just as difficult. Fabrication, similarly, presents important problems, although these can generally be overcome.

The final and most difficult decisions of all, however, are those which must be made by the erector: By the office engineers, who must prepare complete plans for the subsequent erection of the structure, and by the resident engineer, together with all those associated with him at the site. In the preliminary office study: Methods, equipment, stresses and strains in the structure, scheduling, planning and foreseeing all the minor and major details of erection from start to finish; and, for the resident engineer: Unexpected terrain, unforeseen weather, unpredictable working personnel, or difficult field connections—all these problems must be mastered or the design, the engineering, and the fabrication will have been wasted. The structure must be built and too little help has been given, in the past, toward disseminating theoretical and practical information—studies, analyses, and records of actual construction at the site. The more aid that can be given in this phase of engineering, the better the finished project will reflect the original ideas of the designing engineer. Also, the more publicity given to actual field erection problems or difficulties, the better the designing, detailing, and fabricating engineers will be able to solve these problems, by preventing the conditions which too often exist as a result of lack of information on this phase of construction.

Leon Beskin,[41] Assoc. M. Am. Soc. C. E.—The purpose of the following comment is to correct some errors in the first two papers of the Symposium and to show that very simple formulas are available for design and even for definite analysis of flexible arches. The results thus obtained give the same degree of accuracy as the lengthy calculations indicated in the Symposium, and make tests useless.

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Deflection Effects and Design Formulas.—It is regrettable that the expression "elastic theory" has been used in the first Symposium paper to indicate a method of stress analysis in which deflections are supposed to be infinitely small. This expression is certainly a misnomer according to all conventional definitions (see A. E. H. Love[42]). S. Timoshenko studies the "deflection theory," in his "Theory of Elastic Stability."[43] The expression "deflection theory" seems acceptable, but as a subdivision of the elastic theory, and not in opposition to it; the expression "beam-columns" is also widely used.

The statement that several deflection theories are available for arch ribs throws an incorrect light on the problem. Written in differential terms, the problem is elucidated completely, and the theory is unique. Methods of integrating the differential equation may differ, generally because terms are neglected in order to simplify the calculations. The fact that results may differ shows that simplifications made are not justified and lead to inaccurate or erroneous results.

Under the action of axial forces and lateral loads curved beams deflect differently from beams submitted to lateral loads only. Many years ago, the late Milo S. Ketchum,[44] Hon. M. Am. Soc. C. E., presented a formula for the combined effects of bending and axial loads in the elastic range. Some formulas developed subsequently herein will show the simplicity of the problem, even if a fairly close approximation (say, within 1%, generally) is required. They avoid unnecessary loss of time in design and they demonstrate the effect of deflections on stresses as soon as the working stress under axial load, the span, the depth and type of beam, and the end conditions are selected. From those formulas a "factor of amplification," c, can be determined which represents the ratio of actual deflections to deflections calculated for a beam without axial load, and also the ratio of moments in both conditions. That factor is defined as

$$c = \frac{1}{1 + \frac{s_a\,(L_1)^2}{\pi^2\,E\,k^2}} = \frac{1}{1 - \frac{T}{T_c}} = \frac{1}{1 + \frac{T\,(L_1)^2}{\pi^2\,E\,I}} \cdots\cdots\cdots\cdots (61)$$

in which, in addition to the notation of the Symposium: s_a = unit stress due to axial load; k = radius of gyration of the arch; T_c = thrust that causes buckling; and L_1 = effective free length of span. The latter is the total length for a hinged beam under an applied active axial load. It is also the length of the arc between the crown and the skewbacks for a hinged arch under the action of applied transverse loads applied on half of the span and thrust or 0.7 of that length, approximately, if the ends are fixed.

Definition of the Factor of Safety N by Eq. 1.—The persistent use of factors of safety that differ for each type of load is regrettable. Reducing the factor of safety for wind and temperature to 1 is equivalent to reducing those loads in the ratio of the factor of safety used for the other loads. As wind and temperature loads are somewhat arbitrary, the use of a variable factor of safety is a useless complication, especially when various factors of safety are used for different combinations of loads.

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[42] "A Treatise on the Mathematical Theory of Elasticity," by A. E. H. Love, 4th Ed., Cambridge Univ. Press, 1927, p. 92, l. 15.

[43] "Theory of Elastic Stability," by S. Timoshenko, 1st Ed., McGraw-Hill Book Co., Inc., New York, N. Y., 1936.

[44] "Structural Engineer's Handbook," 2d Ed., McGraw-Hill Book Co., Inc., New York, 1918, Section 38, p. 57.

Equations of Equilibrium for Curved Beams.—The factor $\frac{T}{r_o E A}$ in Eq. 2 is obviously erroneous, and must be omitted, since under uniform axial stress, with necessary radial loads to maintain it in equilibrium, the radius of the ring does not change. On the contrary, if there is a strain e_1 along the radius, the latter changes by $e_1 r_o$.

If stresses along the radius are negligible, the quantity e_1 is equal to $-\frac{\mu T}{E A}$ (in which μ = Poisson's ratio), and the corresponding factor is $+\frac{\mu T}{r_o E A}$. If the terms due to shear are taken into account, Eq. 2 must be written:

$$\frac{d^2u}{ds^2} + \frac{u}{(r_o)^2} = -\frac{M}{E I} - \frac{1}{E_s A_w}\left(W_n + \frac{T}{r_o}\right)' + \frac{\mu T}{E A r_o} \quad \text{......(62)}$$

in which W_n is the component of the load normal to the arc, per unit length of arc.

Eqs. 3 are obviously incorrect because the tangential component v of the deflection is omitted. That component is not negligible, since it produces secondary bending in a two-hinged arch when the moment M'_a is proportional to y, and no camber for dead load is provided.

The term $\frac{u}{(r_o)^2}$ cannot be neglected in a refined analysis (see the comparison between the "exact" procedure and the approximate method presented by Mr. Hardesty, et al.) Comparing the value of $\frac{u}{(r_o)^2}$ to that of $\frac{M}{E I}$ in the nondeflection theory, the ratio is

$$\frac{u:(r_o)^2}{M:(E I)} = \frac{12 \times 14:(827)^2}{23{,}000:(29{,}400 \times 120)} = 0.036 \quad \text{............(63)}$$

A direct estimate of that ratio can be made, assuming that the moment M and the deflection u are sinusoidal functions along a circular arch, of the type $\frac{R_{td} \sin \alpha \pi}{\alpha_s}$. Then,

$$\frac{d^2u}{ds^2} + \frac{u}{(r_o)^2} = \frac{1}{(r_o)^2}\left(\frac{d^2u}{d\alpha^2} + u\right) = \frac{R_{td}}{(r_o)^2}\frac{d^2u}{d\alpha^2}\left[1 - \frac{(\alpha_s)^2}{\pi^2}\right] \quad \text{......(64)}$$

in which R_{td} is the ratio of thrust to dead load; and α_s is the slope angle at the skewback. Thus, the relative influence of the second term is $\frac{(\alpha_s)^2}{\pi^2}$, which has the value 0.038 for $\alpha_s = 35°$.

Both methods demonstrate that an error of 4% occurs in the value of the moment, when the term $\frac{u}{(r_o)^2}$ is neglected. When the amplification due to deflection is introduced (as shown subsequently), the error, which is unconservative, is increased to 6%; thus, the "exact method" applied by Mr. Hardesty, et al., is no more accurate than the approximate method.

The secondary moments due to the effect of change in length of the arch under dead load are eliminated by the introduction of a camber. The moments thus eliminated are actually of the same order of magnitude as the moments omitted with the factor $\frac{u}{(r_o)^2}$. The exact value of the horizontal component of the thrust is:

$$H = \frac{\int y\,M'_a\,ds{:}(E\,I) - \int \sin\alpha\,dM'_a{:}(E\,A) + \int \sin\alpha\,dM'_a{:}(E_s\,A_w)}{\int y^2\,ds{:}(E\,I) + \int \cos^2\alpha\,ds{:}(E\,A) + \int \sin^2\alpha\,ds{:}(E_s\,A_w)} \ldots (65)$$

Assuming that the ordinates of the arch are proportional to the curve of moment due to dead load, $M'_a = H_o\,y$, and, with an error of the second order of magnitude, assuming a constant cross section:

$$\frac{H}{H_o} = 1 - \frac{\int dx{:}(A\,E)}{\int y^2\,dx{:}(I\,E)} = 1 - \frac{15\,k^2}{h^2} \ldots\ldots\ldots\ldots (66)$$

The difference between H and H_o produces moments in the arch, which are equal to

$$M = M'_a - H\,y = \frac{M'_a \times 15\,k^2}{h^2} \ldots\ldots\ldots\ldots\ldots (67)$$

For the arch investigated (approximately): $k^2 = 28$; $h = 150$; and, at the crown, $M'_a = \frac{9.5 \times 950^2}{8} = 1{,}050{,}000$. Thus, $M = 2{,}500$. At the quarter point, the value of M is approximately 1,900, which represents 5% of the total moment at that point, and is somewhat smaller than the error of 6% made when neglecting the factor $\frac{u}{(r_o)^2}$. This shows the regrettable fact that, although the authors have been careful to eliminate some secondary bending by special erection methods, moments of equal importance, due to other factors, have been completely omitted from the analysis.

The methods of successive approximation used by the authors to account for the effect of deflections can be simplified by direct integration of the equations of equilibrium, as this is generally done in problems of beam columns. Using Eqs. 5 and 6 (in which the product $\Delta H\,\eta$ is of second order and can be omitted), and substituting the approximations, $\Delta H = 0$, $v \sin\alpha_s = 0$, the equation of equilibrium is:

$$-\frac{E\,I\,d^2u}{ds^2} = M_{ae} + T\,u \ldots\ldots\ldots\ldots\ldots\ldots (68)$$

As a first approximation, the moment M_{ae} due to live load is given by a sinusoidal line. Then the integration is immediate, and with $M_{ae} = \frac{M_o \sin 2\,\pi\,s}{L'}$:

$$E\,I\,u = \frac{M_o \sin 2\,\pi\,s{:}L'}{1 + \frac{T\,(L')^2}{4\,\pi^2\,E\,I}} \ldots\ldots\ldots\ldots\ldots\ldots (69)$$

Thus,

$$M_{af} = \frac{M_{ae}}{1 + \frac{T\,(L')^2}{4\,\pi^2\,E\,I}} = c\,M_{ae} \qquad \text{(70)}$$

Eq. 70 explains why "it was noted that the variation in Δm_1 * * * was similar to the variation in M_e * * *" (see paragraph containing Eq. 8). Furthermore, the value of the quantity $\frac{\Delta m_{a1}}{M_{ae}} = \frac{T_a\,u_{ae}}{M_{ae}}$ is thus determined. Eq. 13 contradicts the fact that M_{ae} and Δm_{ae} are proportional; an average value of T must be used in Eqs. 13 and 14, instead of a variable value.

The coefficient of amplification c (Eq. 61) gives accurate results for a final analysis. For instance with: $T = 7{,}588$; $M = 24{,}950$; $E = 29{,}400 \times 144$; $I = 120$; and $L' = 1{,}020$; Eq. 37 yields $c = 1.64$ and $M = 40{,}900$. This value of M differs by 150 (0.4%) from the value calculated by the authors. If the correction for $\frac{u}{(r_o)^2}$ is introduced, the amplification coefficient is

$$c' = \frac{1}{1 + \frac{(L')^2}{4\,\pi^2} \cdot \left[\frac{T}{E\,I} - \frac{1}{(r_o)^2}\right]} \qquad \text{(71)}$$

Thus the moments are increased by 6% above the values calculated by the authors. This clearly shows that the so-called "approximate methods" are quite sufficient for practical purposes. Otherwise all terms must be taken into account and the computations will be complicated by the need for solving a differential equation of second order. To avoid it, an error of 6% is introduced; in view of that error, all the refinements introduced by Mr. Hardesty, et al., become meaningless.

A method of integration that is always powerful for problems of this type involves the use of developments in trigonometric series. M. Hupner[45] has developed the theory of buckling in hinged and fixed arches and presents general formulas for the amplification of deflections. Values of Δm_a and M_{ae} are proportional when the curve of M_{ae} is a sinusoid. This property is only approximate for other laws of variation of M_{ae}. When $M_{ae} = M_o \left(\frac{\sin 2\,\pi\,s}{L'} + \lambda\,\frac{\sin 4\,\pi\,s}{L'}\right)$ (in which λ is a constant), the amplification factor at the crown is:

$$c_1 = \frac{c}{1 + \lambda}\left[1 + \lambda\,\frac{\pi^2\,E\,I + T\,(L')^2{:}4}{\pi^2\,E\,I + T\,(L')^2{:}16}\right] \qquad \text{(72)}$$

For $\lambda = \pm\,0.1$ and $c = 1.64$, the values of c_1 are, respectively, 1.59 and 1.70. If the moment M_{ae} is constant on each half span, with opposite values on the two halves, the factor of amplification is 1.80, corresponding to $c = 1.64$; for a parabolic distribution of moments, it is 1.658, which shows that a sinusoid gives a very accurate value of the coefficient of amplification for that case.

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[45] "Application des Séries Trigonométriques à l'étude du flambement des arcs," par M. Hupner, *Annales des Ponts et Chaussées*, November, 1935, p. 754.

In reference to Eq. 16, the following can be noted:

(*a*) An average value of T must be used instead of T_a, in the denominator.

(*b*) That average value must include the thrust due to wind and temperature.

(*c*) There is no reason to omit the coefficient N for wind and temperature effects.

(*d*) The separation of moments into those due to dead load and those due to live load for the application of the coefficient of amplification is theoretically incorrect but is certainly acceptable. Theoretically the moments must be developed into components to which various coefficients of amplification must be applied:

$$s = \frac{T_a}{A_a} + \frac{M_{1a}}{S_a} + \frac{M_{2a}}{S_a\left(1 - \frac{T}{T_c}\right)} + \frac{M_{3a}}{S_a\left(1 - \frac{T}{4\,T_c}\right)} + \cdots \ldots\ldots (73)$$

In Eq. 73 the buckling value of the average thrust T_c is approximately equal to $T_c = \frac{4\,\pi^2\,E\,I}{(L')^2}$.

A more exact method of computation is indicated subsequently herein (see Eq. 83). For practical purposes all terms after the third, in Eq. 73, can be omitted, and the total moment M can be divided into a part that is not affected by the coefficient of amplification, and a part that is affected by that coefficient. This shows that an expression of the type:

$$s = \frac{T_a}{A} + \frac{M_{1a}}{S_a} + \frac{M_{2a}}{S_a\,(1 - T{:}T_c)} \ldots\ldots\ldots\ldots\ldots\ldots (74)$$

seems acceptable.

The condition $N\,T < T_c$; or, as written in the Symposium (see paragraph following Eq. 16), $N\,T_{0.25}\left(\frac{u_{0.25\,E}}{M_{0.25\,E}}\right) < 1$ theoretically applies, not only to arches carrying no moment, but also to all arches symmetrically loaded; the condition is actually unnecessary, either because of initial inaccuracies or because of unsymmetrical loading (only canal bridges would not give unsymmetrical loading).

It is possible to apply the foregoing methods of calculation to the case of a hingeless arch. Considering the half arch as a uniformly loaded beam, hinged at the crown and fixed at the skewback, the moment (with the origin at the crown) is:

$$M = -\,w_a\left[\,(1 - \cos s{:}j)\,j^2 - \frac{\sin s{:}j}{\sin p}\,j^2\,J\,\right] \ldots\ldots\ldots\ldots (75)$$

and the maximum occurs for $s = s_o$ such as

$$\tan\frac{s_o}{j} = \frac{J}{\sin p} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (76)$$

in which $j^2 = -\,\frac{E\,I}{T}$; $p = \frac{L_c}{j}$; and L_c is the half length of the arch or a corrected length taking into account the variations of moment of inertia. The load w_a is the antisymmetrical part, equal to one half of a live load distributed on half the span. The quantity J is equal to:

$$J = (1 - \cos p) - p^2 \frac{\tan \frac{p}{2} - \frac{p}{2}}{1 - p \tan p} \dots\dots\dots\dots\dots (77)$$

The maximum moment has the value

$$M = w_a j^2 \left(1 - \frac{1}{\cos s_o : j} \right) \dots\dots\dots\dots\dots (78)$$

An approximate value of the maximum moment is (T_c = buckling thrust of a hinged arch):

$$M = \frac{9}{128} w_a (L_c)^2 \frac{1}{1 - 0.5\, T : T_c} \dots\dots\dots\dots\dots (79)$$

When the arch is reinforced toward the skewbacks, the corrected length L_c is shorter than the actual development of the half arch and can be determined approximately by considering the angular deflection near the skewbacks. The equivalent length giving the same rotation as the actual length is $L_c = \int \frac{I_o}{I}\, ds$ in which I_o is for the basic section. For the Niagara Bridge it is approximately 470 ft. Thus the critical thrust is $T_c = - \frac{88.2 \times 29{,}500 \times 144 \times \pi^2}{470^2}$ $= - 16{,}700$. With $T = - 6{,}460$, the value c is 1.24. The total moment for $\overline{DL} + \overline{LL}$ is then 11,948 × 1.24 + 2,138 = 16,950, which differs by 2% from the value 16,628 calculated in Table 3. The error in the final stress due to the simplified calculation would be less than 1%. Thus, very simple calculations give accurate values of moments. If a higher degree of accuracy is sought, then the expressions developed in the Symposium are no more adequate than the foregoing simplified expressions.

Conclusions from the First Symposium Paper.—Conclusions 1 and 2 are incorrectly formulated, since the factor to be considered is the dimensionless quantity $\frac{T}{T_c} \left(\text{or } \frac{T\,u}{M} \right)$ and not the factor $T\,u$. This factor has been well known for many years. Conclusion 2, as applied to heavy dead loads, is often incorrect. Let s_a = stress due to axial load, s_c = stress due to the axial load that produces a given buckling, and s_b = the stress due to bending, without amplification. It can be shown that, for $\frac{s_a}{s_c} + \frac{s_b}{s_a} < 1$, an increase of dead weight effects a decrease of the influence of the coefficient of amplification on the total stress. Moreover, an increase of dead weight generally means an increase of the depth of the arch; thus, s_c is increased and the coefficient of amplification is reduced, which adds to the effect indicated. Considering the two arches analyzed from that point of view, an increase of dead weight on the hinged arch may cause a small increase of the flexibility effect on the total stress (increase limited to 2% for an increase of 40% of dead weight), whereas the fixed arch is in the zone in which an increase of dead weight causes a decrease of the flexibility effect.

Much simpler design formulas than those indicated are available for many conditions of beam-columns and such formulas can be applied to arches without any difficulty.

Moments in Flexible Two-Hinged Arches.—The methods of analyzing problems of stress distribution by tests are generally more costly and require more time than mathematical approaches to the same problems. In the case of flexible two-hinged arches, the mathematical treatment is so simple and so commonly known that tests do not add any real knowledge and, if not interpreted with the use of the theory, may even be misleading. The interpretation of the tests made on two-hinged arches is an example of such a case.

It is here assumed that the moments due to dead weight have been eliminated by a change of span compensating for the shortening of the arch. Only in that manner can a pressure line coincide with the neutral line of the arch; but it is to be noted that the distribution of dead load is then far from an actual distribution, since the load must be inversely proportional to $\cos^3 \alpha$. For the three arches examined, this signifies that the ratio of dead load at skewbacks and at crown is equal, respectively, to 1.8, 2.6, 7.7. As it is improbable that actual variations of dead load will be greater than 40% between crown and skewbacks, the tests do not seem to represent actual conditions. On the other hand, in order to compare more accurately the results of tests and the theoretical results indicated hereafter, it would be desirable to know the exact law of variation which was used for dead load.

The moment at any point of an arch is equal to the moment of a non-flexible arch multiplied by the coefficient of amplification; using Eq. 14*a*, $M_{af} = \dfrac{M_{ae}}{1 - T_a\, u_{ae}:M_{ae}}$. The ratio, $\dfrac{M_{ae}}{u_{ae}}$, is a constant, equal to the critical thrust T_c, and T_a can be replaced by $R_{td}\,\overline{DL} + 0.5\, R_{tl}\,\overline{LL}$. That expression is exact (changes of length omitted) for a uniform load extending over one half of the span. For other distributions of live load, the factor 0.5 would be changed. It can be shown that (approximately) the ratio of thrust to live load R_{tl} is equal to $R_{td}\left[1 - \dfrac{2\,(\alpha_s)^2}{7}\right]$. Calling $\overline{DL}_o$ the dead load corresponding to the critical thrust T_c, and introducing a coefficient of moment m (ratio of moment to live load) such that $M_{ae} = m\,\overline{LL}$, the value of M_{af} is:

$$M = \frac{m\,\overline{LL}}{1 - \dfrac{\overline{DL} - \overline{LL}\,[0.5 - (\alpha_s)^2:7]}{\overline{DL}_o}} \quad \dots\dots\dots\dots\dots (80)$$

replacing $\overline{LL}$ in terms of $\overline{DL}$ and $R\left(=\dfrac{\overline{DL}}{\overline{LL}}\right)$:

$$M = \frac{m\,\overline{DL}}{R\left(1 - \dfrac{\overline{DL}}{\overline{DL}_o}\right) - \dfrac{\overline{DL}}{\overline{DL}_o}\left[0.5 - \dfrac{(\alpha_s)^2}{7}\right]} \quad \dots\dots\dots\dots (81)$$

Thus, the curve representing M as a function of R is a hyperbola. Eq. 81 contains two constants m and $\overline{DL}_o$ instead of the four constants of Eq. 24, since the factor $\frac{(\alpha_s)^2}{7}$ generally may be neglected. Such constants can be computed easily, and the final expression of the moment becomes

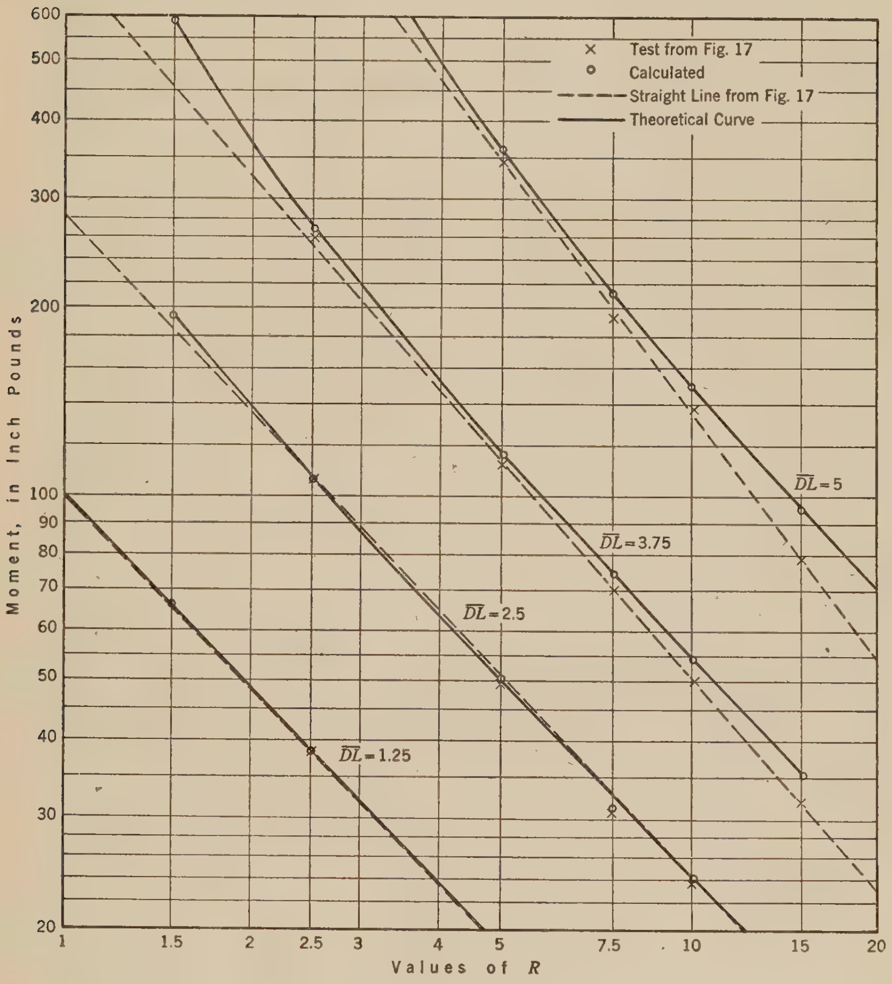

FIG. 41.—LOG-LOG PLOT OF MOMENTS

$$M = \frac{L^2\,\overline{LL}:64}{1 - \frac{(L')^2}{4\pi^2 (r_o)^2} - \frac{(L')^2 r_o}{4\pi^2 E I}\left[1 + \frac{(\alpha_s)^2}{6}\right]\left\{\overline{DL} + \overline{LL}\left[0.5 - \frac{(\alpha_s)^2}{7}\right]\right\}} \quad \ldots(82)$$

The quantity $1 + \frac{(\alpha_s)^2}{6}$ takes into account the average value of the thrust in the arch. Since $\frac{L'}{2\,r_o} = \alpha_s$, Eq. 82 can be written:

$$M = \frac{L^2\,\overline{LL}\,[1 + 0.1\,(\alpha_s)^2]:64}{1 - [1 + 0.6\,(\alpha_s)^2]\left\{\overline{DL} - \overline{LL}\left[0.5 - \frac{(\alpha_s)^2}{7}\right]\right\}L^2\,r_o:(4\,\pi^2\,E\,I)} \;\ldots(83)$$

Fig. 41 affords a comparison between the calculated values and those read from Fig. 17 of the moments. The relative average difference is + 4.5% between the theoretical values and the tests. The average difference would be less if the dead load varied less than indicated by the law $\frac{1}{\cos^3 \alpha}$, and would change its sign for a relative variation equal to half of that given by such law.

As the relation between M and R is represented by a hyperbola, on a log-log diagram, the hyperbola becomes a curve with two asymptotic directions, one parallel to the M-axis and corresponding to buckling, and the other at an angle − 45° with the axes. The straight lines in Fig. 17 are the parts of the transformed hyperbolas which are far from the region of maximum curvature. Thus, they have no significance and do not even represent the asymptotic direction at 45°, since they show the beginning of the amplification effects. On the other hand, moments must become infinite when the loads approach buckling values. For instance, for $\overline{DL} = 5$ and 3.75, buckling occurs when $R = 1.5$ and 0.6, respectively. Therefore, it seems that Eq. 24 has no practical value, since its validity is limited to the range of the tests while Eq. 83 is valid for any values; and moreover it is more complicated and requires more computation than the theoretical expression, Eq. 80. It is to be noted that, in Eq. 24, the loads are in kips per foot and moments are in thousands of foot-kips. The authors have realized the advantage of the coefficient of amplification (called by them "percentage increase") but have made no attempt to introduce it on a rational basis and relate it to the critical thrust. With the use of the theory it is much easier for the designer to examine the influence of changes in shape of the arch and distribution of the load, and tests are unnecessary. Also, the main factors, span L, rigidity $E\,I$, dead load $\overline{DL}$, live load $\overline{LL}$ enter into a theoretical formula in a much more expressive manner than in a formula of the type of Eq. 24, which gives no measure of the relative influence of the various parameters.

The theoretical formula, Eq. 83, reduced to

$$M = \frac{L^2\,\overline{LL}}{64 - (2\,\overline{DL} - \overline{LL})\,L^4:(\pi^2\,h\,E\,I)} \;\ldots\ldots\ldots\ldots\ldots(84)$$

will enable the designer to visualize the influence of the main factors, especially the rise of the arch.

Conclusions, Second Symposium Paper.—The conclusions advanced in this second paper of the Symposium do not throw any new light on well-known results. Tests of the kind made (conclusions 1, 2, and 3) are of little interest, since simple and reliable theories are available, and from such theories definite arches can be selected directly, either for bridges or for hangar roofs. A hingeless arch generally will prove less costly than a two-hinged arch, but there is no special reason to avoid the latter in cases where the coefficients of amplification are small (say, less than 2) when it is difficult to fix the ends. In airplane design, for instance, high coefficients of amplification are accepted.

It is obvious that methods which do not take into consideration beam-column effects are unreliable (conclusion 4), but it is to be hoped that no designer will calculate flexible structures without considering buckling and prebuckling effects—that is, the coefficients of amplication.

There is no doubt that correctly chosen average values of radii and moments of inertia (conclusion 5) will enable the designer to make use of formulas or test data related to circular arches; but no difficulty is encountered in making direct calculations for any type of arch.

It is well known that the relationship between load and stress in beam-columns is not linear (conclusion 6). This fact is taken into account in various specifications for margins of safety—for instance in the Army-Navy-Civil Specifications for Aeronautics (see ANC-5), and also in many recent foreign specifications applied to civil engineering (French Air Department, Air Bases Division, dated about 1934 or earlier).

Since arches buckle in two waves and a change of length introduces a moment corresponding to one wave, the two phenomena are independent, and the nondeflectional theory is valid for temperature stresses (conclusion 7). However, the change of thrust due to temperature will change the coefficient of amplification; and thus there is an influence of temperature on buckling and on live-load moments. That effect, which is small for a bridge, is applied in automatic devices used in temperature control; and it cannot be disregarded entirely.

Nothing fundamentally new has been advanced in the analysis of the Rainbow Arch Bridge (conclusion 8).

It would be very desirable to know precisely what are the "inconsistencies with the elastic theory" (conclusion 9) which occur in the fixed-end type of arch.

Fabrication and Erection Controls.—Unusual care has been taken in the fabrication and erection of the Rainbow Arch Bridge (third Symposium paper), and the engineers who directed that work can consider it as a genuine achievement. It also appears that their computations have been made with all desirable accuracy.

It seems useful to note (see heading, "Application of Data to Niagara Rainbow Arch Bridge") that the elastic centroid concept, which the authors of the third paper took from the 1925 Edition of Müller-Breslau's book,[17] was taught by Maurice Lévy[46] in France as early as the end of the last century in École Centrale des Arts et Manufactures. The correct expression, as used in treatises on mathematics, is "orthogonalisation" and not segregation; and the elastic centroid method is only a special case of the methods of "orthogonalisation" of the unknowns in systems of linear equations.

Moments can be applied at the crown instead of the centroid, since the axis of a moment can be freely displaced without changing the equilibrium.

The elastic center can be determined by a straightforward method (column analogy), which is still valid when deflections due to normal and shearing forces are taken into account.

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[46] "La Statique Graphique et ses Applications aux Constructions, 1886–1888," Gauthier-Villars & Co., Paris.

Neil Van Eenam,[47] M. Am. Soc. C. E.—The problems of research, design, fabrication, and erection involved in the construction of the new Rainbow Arch Bridge are presented in a clear and thorough manner in this Symposium. Until recent years, no solid rib arch would have been structurally and economically feasible, or even possible, for the 950-ft span over the Niagara gorge, because of the excessively thick web plates that would have been necessary. Now, methods are available for stiffening the web plates longitudinally so that plates of reasonable thickness may be employed. On the Rainbow Arch Bridge, longitudinal stiffeners have been used to the best advantage by making them serve an architectural as well as a structural purpose.

The investigation of deflections and of secondary stresses is an outstanding example of the thoroughness of the design. The authors deserve great credit for calling attention to the inadequacy of the elastic theory when deflections are appreciable. This word of caution in regard to the magnitude of the secondary stresses applies with equal force to the design of arch ribs of moderate span, particularly when two-hinged or three-hinged ribs are to be of shallow depth.

The Rainbow Arch Bridge is pleasing and graceful in appearance, and, because of the absence of bracing, it offers only slight obstruction to the view. If the rigidity of the spandrel columns is such that the lateral bracing may be omitted safely, the engineers have succeeded remarkably in building a monumental structure that harmonizes with the landscape.

C. H. Gronquist,[48] Assoc. M. Am. Soc. C. E.—Only at rare intervals does there come to the bridge engineer a problem so satisfying to the engineering mind and heart as the bridging of the Niagara gorge. That the designers and builders of the Rainbow Arch Bridge have produced a solution which is eminently right is evident. The papers in this Symposium present more than adequately the record of this achievement of the authors.

A project which makes similar appeal to the bridge engineer is the Henry Hudson Bridge at Spuyten Duyvil in New York, N. Y. This 800-ft arch was built in 1936 as a four-lane bridge, and prior to the construction of the Rainbow Arch Bridge held the distinction of being the longest steel fixed arch bridge. The general designs of the two bridges are essentially alike, although an upper deck accommodating three lanes was added to the Henry Hudson Bridge in 1938 to transform it into a six-lane bridge, and there are many differences in important features of detail design. Although the design and erection of the Henry Hudson Bridge have been described elsewhere,[49,50] a brief comparison of the two structures should be included in the discussion of this Symposium.

The rib of the Henry Hudson Bridge, like that of the Rainbow Arch Bridge, consists of a silicon steel box girder of constant depth, except for a short section adjacent to each skewback. Unlike that of the Rainbow Arch Bridge, the rib section of the Henry Hudson Bridge (which is 12.6 ft deep and weighs 2,400 lb per ft) is composed of two separate girders 3 ft 8 in. center to center, connected by heavy lacing at both flanges and at mid-height.

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[47] Highway Bridge Engr., U. S. Public Roads Administration, Washington, D. C.

[48] Associate Engr., Robinson & Steinman, New York, N. Y.

[49] "Designing the Longest Fixed Steel Arch," by D. B. Steinman and C. H. Gronquist, *Engineering News-Record*, August 13, 1936, p. 232.

[50] "Special Problems of Hingeless Arch Erection," by W. K. Greene, *ibid.*, November 12, 1936, p. 669.

The skewback section of the rib and the anchorage to the abutment are generally similar for the two bridges, and flexible connections were used at the top and bottom of the spandrel columns for both bridges to reduce temperature and participation stresses in the columns and deck framing. Lateral bracing between the spandrel columns in the Henry Hudson Bridge is laid out in conformance with the bracing in the steel towers surmounting the skewbacks and the bents of the approach spans, which were of steel construction for reasons of economy.

The Henry Hudson Bridge was erected simultaneously from both ends of the span by cantilevering over steel falsework bents supported by cage-braced steel piles driven to rock. The falsework bents were located near the eighth points and quarter points of the arch, and at the quarter-point bents the ribs were reinforced by toggles consisting of eyebar chains and a bent which later became the end spandrel bent of the bridge. The grillages were erected on steel supports set on the concrete of the skewbacks in such manner as to permit jacking the grillages in any direction to secure their accurate location.

Careful check was made of the actual distance between grillages before erecting the skewbacks, and from these data, and the length of the shop rib assemblies, it was determined that the crown closing piece for the Henry Hudson Bridge, nominally 30 in. long, should be fabricated $\frac{1}{2}$ in. short. A practically perfect check of the elevation of the crown of the arch after closure confirmed the efficiency of the closure scheme. This is made further evident by considering that 1-in. error in length of the rib of the Henry Hudson Bridge (1-in. error in length of the span was assumed in the design), corresponding to 15° F change in temperature, would have changed the elevation of the crown 1.5 in., whereas the change in thrust would have been 28 kips, or only slightly more than 1% of the thrust at the time of closure. From the foregoing it is apparent that check of elevation of the axis is at the same time the most sensitive as well as the most simple check on the calculated stress in the rib at closure, and that, if proper precision is used in field layout and shop fabrication, no adjustment need be made at closure. The latter is essentially the conclusion of the authors of the third Symposium paper, to which the writer would take exception only by stating that, if the span between grillages is known to be correct and if it is determined by measurement of length and elevation of rib at closure that the steel is not of correct length, such error should be compensated for by adjustment at closure.

Jacking arrangements at the crown were similar for the two bridges, except that for the Henry Hudson Bridge only one 500-ton jack was used at each flange, the width of opening between the half arches was 30 in., and a small pin was introduced in the bottom flange, where, on release of the jacking force, contact between the two half ribs would be made first. The crown jacks, which were interconnected and operated from a single pump to maintain non-eccentric axial load until initial closure was made, were first brought to capacity load. Because closure was made on a July day when the temperature rose to 90° F, it also was necessary to jack at the quarter-point bents, while maintaining constant load on the crown jacks.

The problem of determining the effect of deflection on the stresses in a slender arch is unquestionably of prime importance. In an effort to circumvent the laborious cut-and-try procedure for obtaining the final moments and deflections termed "exact" by the authors of the first Symposium paper, the writer, some time before the publication of this Symposium, derived a formula for the so-called "deflection-theory" correction of the usual elastic-theory moment and deflection that can be applied as soon as the moment of inertia of the rib at the crown has been estimated.

The usual assumption is made that the arch axis is parabolic and that the moment of inertia of the rib varies with the secant of the slope of the axis. Vertical deflections may then be computed by considering the rib as a horizontal beam of constant moment of inertia equal to that at the crown. Half-span live load on such a two-hinged arch produces deflection in two segments with zero moment and deflection at the crown. Neglecting the effect of distortion and shortening of the rib, the vertical deflection at either quarter point will be:

$$\eta_e = \frac{\dfrac{p\,L^2}{64}}{4 \times 9.6\,\dfrac{E\,I}{L^2}} = \frac{M_e}{P_{e-v}} \quad \text{.....................(85)}$$

The numerator of Eq. 85 is the value of the moment at the quarter point of the arch. The denominator is the buckling load of the rib, for elastic curve deflection, taken as a horizontal strut of length $L/2$ between the end hinge and central point of contraflexure. Since there is zero deflection at the point of contraflexure, the half arch under this loading is analogous to a simple beam with a full-span uniformly distributed load. The center deflection for a fully loaded simple beam is also equal to the moment at the center divided by the buckling load of the beam, for elastic curve deflection, taken as a column of length L.

Written in terms of the quarter-point moment, the horizontal deflection at the loaded quarter point, neglecting the effect of distortion and shortening of the rib, is:

$$\rho_e = \frac{\dfrac{p\,L^2}{64}}{\dfrac{50}{41} \times 9.6\,\dfrac{E\,I}{h\,L}} = \frac{M_e}{P_{e-h}} \quad \text{.....................(86)}$$

in which the denominator is again a buckling load, dependent on the rise h as well as on the span L of the arch.

For loading of this type, producing negligible deflection at the crown, the value of $H_w + H$ is practically unaffected by deflection, and the correction in moment due to vertical deflection is merely $(H_w + H)\,\eta$. To obtain the correction in moment due to horizontal deflection, it is assumed for simplicity that all points on the arch have the same horizontal deflection and that the position of the live load changes by that amount. Then at the quarter point the correction in moment is $\frac{\rho\,(w + p)\,L}{4}$. This assumption errs on the side of maximum deflection effect. The total final moment at the quarter point is:

$$M_f = M_e + (H_w + H)\,\eta + \frac{\rho\,(w + p)\,L}{4} \qquad \text{(87)}$$

If it is assumed that the deflection correction in deflection is in the same proportion as the deflection correction in moment, the expressions for final deflection become $\eta = \frac{M_f}{P_{e-v}}$ and $\rho = \frac{M_f}{P_{e-h}}$; or

$$\eta = \frac{M_e + \frac{\rho\,(w + p)\,L}{4}}{P_{f-v}} \quad \text{and} \quad \rho = \frac{M_e + (H_w + H)\,\eta}{P_{f-h}} \qquad \text{(88)}$$

in which

$$P_{f-v} = 4 \times 9.6\,\frac{E\,I}{L^2} - (H_w + H) \quad \text{and} \quad P_{f-h} = \frac{50}{41} \times 9.6\,\frac{E\,I}{h\,L} - \frac{(w + p)\,L}{4} \qquad \text{(89)}$$

The final buckling loads as corrected for the effect of deflection are reduced from the values computed by the usual elastic theory by the amount of the horizontal thrust and the vertical shear term.

Solving Eqs. 88 simultaneously to obtain the values of the vertical and horizontal deflection at the quarter point:

$$\eta = \frac{M_e}{P_{e-v} \times \frac{P_{f-h}}{P_{e-h}} - (H_w + H)} \quad \text{and} \quad \rho = \frac{M_e}{P_{e-h} \times \frac{P_{f-v}}{P_{e-v}} - (w + p)\,\frac{L}{4}} \qquad \text{(90)}$$

Substituting Eq. 90 in Eq. 87, the ratio of the final moment to that by the elastic theory is:

$$\frac{M_f}{M_e} = 1 + \frac{H_w + H}{P_{e-v} \times \frac{P_{f-h}}{P_{e-h}} - (H_w + H)} + \frac{(w + p)\,\frac{L}{4}}{P_{e-h} \times \frac{P_{f-v}}{P_{e-v}} - (w + p)\,\frac{L}{4}} \qquad \text{(91)}$$

If only the effect of vertical deflection is considered, Eq. 91 reduces to

$$\frac{M_f}{M_e} = 1 + \frac{H_w + H}{P_{f-v}} = 1 + \frac{H_w + H}{4 \times 9.6\,\frac{E\,I}{L^2} - (H_w + H)} \qquad \text{(92)}$$

With Eq. 92 written in terms of total thrust $(T_w + T)$ at the quarter point, and the buckling load computed for the length of the rib from the end hinge to the crown, the ratio of the final positive or negative moment and deflection to that obtained by the elastic theory, on the basis of radial deflections, is obtained. Thus:

$$\frac{M_f}{M_e} = 1 + \frac{T_w + T}{P_f} \quad \text{(93)}$$

in which

$$P_f = \frac{4 \times 9.6\, E\, I}{L^2 \left(1 + \frac{16\, h^2}{3\, L^2}\right)} - (T_w + T) \quad \text{(94)}$$

Radial deflections are computed from the moment at the quarter point:

$$u_e = \frac{M_e}{P_e} \quad \text{and} \quad u = \frac{M_e}{P_f} = \frac{M_f}{P_e} \quad \text{(95)}$$

The buckling load of Eq. 94 is the critical buckling load for the arch, neglecting the minor reduction due to the effect of curvature. This equation, therefore, should be used for stability computations in determining the factor of safety of an arch against buckling under all possible loads, as in Eqs. 16 and 17.

The foregoing relations have been previously developed for the stability of struts subjected to lateral loads,[51, 52] and their application to arches has been indicated.[43] Eqs. 14 will be found to reduce to the form of Eq. 93, since $P_e = \frac{M_{ae}}{u_{ae}}$; and also to the form of Eq. 92, if the horizontal thrust and vertical deflection only are substituted in Eqs. 14.

Applying Eqs. 91 to 95 to the two-hinged design for the Rainbow Arch Bridge for a half-span load of 1.5 kips per foot of rib:

$$\frac{M_f}{M_e} = 1 + \frac{7{,}350}{20{,}850 \times \frac{37{,}850}{40{,}350} - 7{,}350} + \frac{2{,}500}{40{,}350 \times \frac{13{,}500}{20{,}850} - 2{,}500} = 170.6\%;$$

$$\frac{M_f}{M_e} = 1 + \frac{7{,}750}{10{,}650} = 172.8\%; \qquad M_e = \frac{1.5 \times (950)^2}{64} = 21{,}150 \text{ ft-kips};$$

$$M_f = 1.728 \times 21{,}150 = 36{,}500 \text{ ft-kips};$$

$$\eta_e = \frac{21{,}150}{20{,}850} = 1.01 \text{ ft}; \qquad \eta = 1.706 \times 1.01 = 1.73 \text{ ft};$$

$$\rho_e = \frac{21{,}150}{40{,}350} = 0.525 \text{ ft}; \qquad \rho = 1.706 \times 0.525 = 0.90 \text{ ft};$$

$$u_e = \frac{21{,}150}{18{,}400} = 1.15 \text{ ft}; \qquad u = 1.728 \times 1.15 = 1.98 \text{ ft};$$

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[51] "Buckling of Elastic Structures," by H. M. Westergaard, *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 576.

[52] "Airplane Structures," by A. S. Niles and J. S. Newell, 2d Ed., John Wiley & Sons, Inc., New York, N. Y., 1943.

and $$[(\eta_e)^2 + (\rho_e)^2]^{0.5} = 1.14 \text{ ft}; \qquad (\eta^2 + \rho^2)^{0.5} = 1.95 \text{ ft}.$$

A satisfactory check is observed between deflection-theory corrections obtained on the basis of vertical and horizontal deflection components and radial deflection. The results are likewise in good agreement with those of the authors of the first Symposium paper by their approximate method. Considering only vertical deflections: $\frac{M_f}{M_e} = 1 + \frac{7{,}350}{13{,}500} = 154.5\%$. Therefore the effect of including the influence of horizontal deflection is to increase the deflection-theory correction from 54.5% to 70.6% or 72.8%.

Maximum positive moment by the elastic theory is produced at the quarter point for 0.43-span load rather than half-span load, and the horizontal distance to the point of contraflexure is 0.47 L rather than 0.5 L. Using this length in computing the buckling loads:

$$\frac{M_f}{M_e} = 1 + \frac{7{,}250}{23{,}400 \times \frac{40{,}230}{42{,}700} - 7{,}250} + \frac{2{,}470}{42{,}700 \times \frac{16{,}150}{23{,}400} - 2{,}470} = 158.2\%;$$

$$\frac{M_f}{M_e} = 1 + \frac{7{,}700}{12{,}900} = 159.6\%; \qquad M_e = \frac{1.5 \times (950)^2}{60} = 22{,}550 \text{ ft-kips};$$

and $M_f = 1.596 \times 22{,}550 = 36{,}000$ ft-kips. The final moment is little different for the two loadings, since the deflection correction is reduced with reduction in length to the point of contraflexure.

Although not discussed by the authors, maximum moment is produced at the crown by a central 0.3-span loading. Horizontal deflection of the haunches is small, and may be neglected in obtaining the deflection correction approximately. Deflection is in three segments or loops, with an accompanying increase in $H_w + H$ due to crown deflection. Since the distance between points of contraflexure is reduced to about 0.33 L, the arch is stiffer than under half-span load, and the deflection correction is correspondingly smaller. The value of $H_w + H$ is changed by deflection; therefore the deflection-theory increase in moment due to vertical deflection cannot be obtained as for half-span load, but it can be approximated by application of Eq. 92, using the reduced value of 0.33 L in place of $L/2$ for the length of strut in computing the buckling load, and using the value of H computed by the elastic theory. For the Rainbow arch two-hinged design: $H_e = 0.0565 \times 1.5 \times \frac{(950)^2}{150} = 510$ kips; $P_{f-v} = \left(\frac{0.50}{0.33}\right)^2 \times 20{,}850 - (6{,}790 + 510) = 40{,}700$ kips; $\frac{M_f}{M_e} = 1 + \frac{7{,}300}{40{,}700} = 117.9\%$; $M_e = 0.0725 \times 1.5 \times (950)^2 = 9{,}800$ ft-kips; and $M_f = 1.179 \times 9{,}800 = 11{,}550$ ft-kips. This correction would be increased slightly by use of Eq. 92, with the buckling load computed for the length of rib between points of contraflexure to account for the effect of horizontal deflection.

The problem of approximating the deflection-theory correction for the fixed arch near the quarter point is more complex than for the two-hinged structure, since both linear and angular deflection occur at the point of contraflexure near the skewback of the former, whereas only angular deflection can occur at the end hinge of the latter. The value of $H_w + H$ is only slightly affected by deflection for either type, however, vertical crown deflection being negligible. The percentage of deflection-theory correction for the fixed arch, as for the two-hinged arch, is best computed by Eqs. 92 and 93, using the distance between the points of contraflexure for the loading considered. The percentage so computed is practically constant between the respective points of contraflexure, a considerable decrease occurring only near the skewbacks in the fixed arch.

Final deflections for both types of arches may be approximated by increasing elastic-theory deflections by the percentage of deflection-theory correction for moment. The maximum vertical deflection by the elastic theory at the five-sixteenth point of a fixed arch for half-span load may be written:

$$\eta_e = \frac{\dfrac{9\,p\,L^2}{1{,}024}}{\dfrac{192}{35} \times 9.6\,\dfrac{E\,I}{L^2}} = \frac{M_e}{(P_{e-v})'} \qquad \text{(96)}$$

in which the numerator is the moment at the five-sixteenth point of the fixed arch, and the denominator is a buckling load. This buckling load is smaller than the buckling load for the 3/8 L distance between the points of contraflexure for half-span load, however, because the total vertical deflection, not the deflection from a straight line joining the points of contraflexure, is being measured. Similarly, the horizontal deflection at the five-sixteenth point, written in terms of the moment at the five-sixteenth point, is:

$$\rho_e = \frac{\dfrac{9\,p\,L^2}{1{,}024}}{\dfrac{384}{205} \times 9.6\,\dfrac{E\,I}{h\,L}} = \frac{M_e}{(P_{e-h})'} \qquad \text{(97)}$$

For the Rainbow Arch Bridge with a half-span load of 1.5 kips per foot of rib: $P_{f-v} = \frac{64}{9} \times \frac{9.6 \times 29{,}400 \times 11{,}688}{(950)^2} - (5{,}910 + 560) = 19{,}530$ kips;

$$P_f = \frac{26{,}000}{1 + 3\left(\dfrac{150}{950}\right)^2} - 6{,}700 = 17{,}500 \text{ kips}; \qquad \frac{M_f}{M_e} = 1 + \frac{6{,}470}{19{,}530} = 133.2\%,$$

considering vertical deflection; $\frac{M_f}{M_e} = 1 + \frac{6{,}700}{17{,}500} = 138.2\%$, considering radial deflection; and $M_f = 11{,}900 \times 1.382 = 16{,}400$ ft-kips. Comparison of the foregoing results for the deflection-theory correction indicates that in the fixed arch the effect of horizontal deflection is of minor importance and ordinarily may be neglected.

To compute deflections:

$$(P_{e-v})' = \frac{27}{35} \times 26{,}000 = 20{,}000 \text{ kips};$$

$$(P_{e-h})' = \frac{384}{205} \times \frac{9.6 \times 29{,}400 \times 11{,}688}{150 \times 950} = 43{,}400 \text{ kips};$$

$$(P_e)' = \frac{20{,}000}{1 + \frac{35}{9}\left(\frac{150}{950}\right)^2} = 18{,}250 \text{ kips}; \qquad \eta_e = \frac{11{,}900}{20{,}000} = 0.595 \text{ ft};$$

$$\eta = 1.382 \times 0.595 = 0.82 \text{ ft}; \qquad \rho_e = \frac{11{,}900}{43{,}400} = 0.275 \text{ ft};$$

$$\rho = 1.382 \times 0.275 = 0.38 \text{ ft}; \qquad u_e = \frac{11{,}900}{18{,}250} = 0.65 \text{ ft};$$

and

$$u = 1.382 \times 0.65 = 0.90 \text{ ft}.$$

The deflection correction would actually be less than the foregoing due to greater stiffness near the skewback than assumed in the formulas. A closer agreement with the percentage of correction of the authors is obtained for a 0.415-span load with a 0.32 L distance between points of contraflexure as shown in Fig. 12 and Table 3.

Half-span wind load moments as obtained by the usual elastic theory, like live-load moments, are subject to correction for the effect of deflection. Such correction is not considered by the authors of the first Symposium paper in writing Eqs. 16, 17, and 21 for permissible flange stress, including the effect of buckling.

Eqs. 17 and 21 are proposed for design use by the authors of the first Symposium paper. When corrected for the effect of deflection on wind moment, they should prove useful in conjunction with the usual computation of maximum stresses for the design loads. For box-girder rib construction the safety factor of approximately 1.8 currently used for (dead + live + impact)-load stresses appears proper for (dead + live + impact)-load + temperature + error stresses in arches, increased for deflection or buckling effect. For single-web rib construction the working stress should be decreased in proportion to L/b or L/r to allow for the greater tendency toward lateral buckling between bracing points. In conformance with current practice, the safety factor for box-girder ribs would be reduced to 1.5 with the addition of wind stresses, also increased for deflection effect.

It is regrettable that the authors of the second Symposium paper were unable to make a complete determination of the deflection effects on a circular fixed arch, as they have done so excellently for the two-hinged type, or that the findings obtained on the fixed model were not published, unless they were not considered reliable.

The approximate method of the writer for parabolic arches, with moment of inertia varying as the secant of the slope of the rib axis, gives final quarter-point moments for half-span load on two-hinged arches of varying rise ratio which are in reasonably good agreement with those of Eq. 24 for circular arches of constant moment of inertia where the length-rise ratio L/h is 6.3 or 5.0. Although the final positive moment at the quarter point by Eq. 24 for the Rainbow arch two-hinged design is less for $L/h = 5.0$ than for the flatter arch with $L/h = 6.3$, that for the arch of high rise with $L/h = 3.5$ is practically the same as that for the flattest arch ($L/h = 6.3$). This is contrary to the results for the parabolic arch, for which the final moment, including the effect of deflection, decreases with increase in rise for the range of rise ratios considered. Comparative results are shown in Fig. 42(*a*). Although the method of the writer will be less accurate for arches of high rise than for flat arches, it is probable that the difference in moments for the arch of high rise ($L/h = 3.5$) is due principally to the greater curvature of the axis of the circular arch. With increasing rise the greater average moment of inertia of the parabolic arch of variable section also will have a small, increasing effect in reducing the deflection correction over that of the circular arch of constant section. Thus

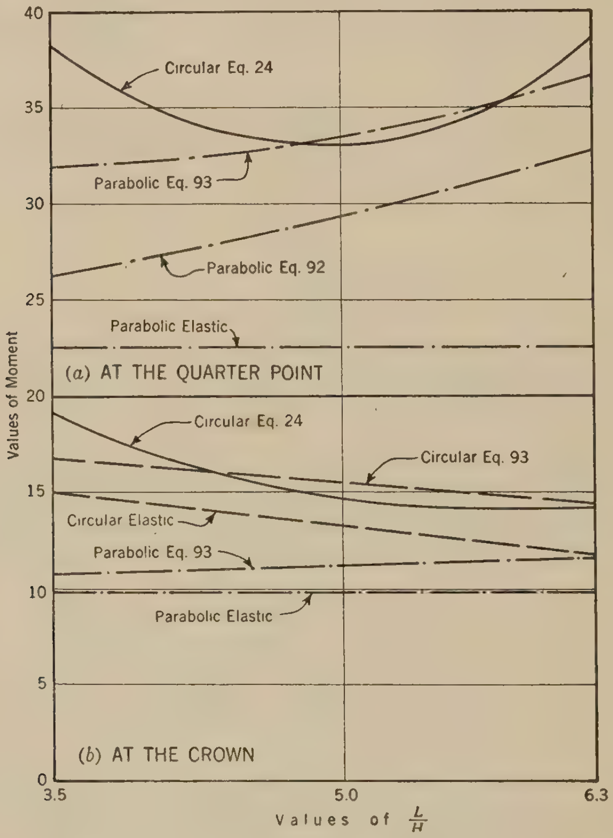

Fig. 42.—Comparison of Moments of Two-Hinged Arch for Varying Rise Ratios ($L = 950$ Ft; $I = 116.14$ Ft^4 at Crown)

the writer concludes, contrary to the authors, that for arches of high rise the deflection correction for parabolic or three-centered arches will be considerably smaller than that for circular arches.

Maximum moments at the crown of the two-hinged design for the Rainbow arch as determined by Eq. 24 and by the writer's approximate method for the circular arch of constant moment of inertia are compared with those for the parabolic arch of variable moment of inertia in Fig. 42(*b*). It is important to note that the crown moment, determined by the usual elastic theory, in the circular arch for one-third span central load, is appreciably greater than that in the parabolic arch for 0.3 span central load, even for the arch of flat rise, and that the difference increases with rise. Quarter-point moment by the elastic theory, for the rise range considered, is slightly less for the circular arch than for the parabolic arch. When the elastic-theory crown moments for the circular arch are increased by the writer's method for deflection-theory effect, and using π^2 in place of 9.6 in computing the buckling loads, good agreement is obtained for all rise ratios, with the possible exception of that for $L/h = 3.5$.

M. HIRSCHTHAL,[53] M. AM. SOC. C. E.—This Symposium on a long span arch bridge is naturally of great interest to engineers, particularly to those concerned with the design of railroad or highway bridges. The profession is grateful to Mr. Hardesty, et al., for the detailed presentation and analysis of the unusual erection and construction problems and their ingenious solution for the Rainbow Arch Bridge over the Niagara gorge from its very inception to its completion. The reason for some of the omissions and the inclusion of some of the material in the Symposium is not clear, however, in view of the thoroughness of the treatment of the entire project.

For instance, there is no information on the various considerations that led to the selection of the particular span length. On the other hand, the model tests seem to have been made on a two-hinged arch to corroborate its rejection rather than on a model of a hingeless arch to verify the stresses and deformations or strains found in the design of the structure selected—the one that was actually built.

Although model tests are very informative, it is dangerous to base all conclusions on them as being applicable quantitatively to prototypes, especially those of extraordinary size. The interest in the Rainbow Arch Bridge is (as has been true in the case of other bridges) due to its unusual span length—the "longest in the world." Had an arch of moderate span length been planned for this site, the writer doubts greatly whether model tests would have been made. Arch action is a normal structural phenomenon that occurs in many forms, although it still has some unexplained features, such as results of temperature effects which model tests (in addition to any mechanical movement of one support) could elucidate. However, even in arch action tests quantitative application of results to prototypes would have to be made with caution.

Since model tests had been made, it would have been exceedingly interesting to have had comparative results of tests on two models, one a fixed-end model and the other the two-hinged model described in the Symposium. Both, of course, should be of identical dimensions, so that resulting deformations for various conditions of loading could be compared. There is no doubt that the very magnitude of the structure produces problems in design that may readily be ignored in smaller structures. For these reasons, results from scale models, particularly when they are of different material from that of the prototype, must not be the sole basis of decision. A 5-ft model is slightly longer than $\frac{1}{200}$th of the span length of the prototype, and, since the deflection varies with the cube of the span for a scaled load, the slightest error will be enormously magnified in the study of deflections.

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[53] Concrete Engr., D. L. & W. R. R., Hoboken, N. J.

Moreover, the writer questions the original selection of a two-hinged (flexible) arch and all the preliminary work attached to it. After complete analysis and detailed estimates were made, this type of design was discarded in favor of the hingeless arch (fixed) because of the economy of the latter. With outcrops of rock on either side of the gorge or waterway available for abutment foundations, a hingeless or fixed arch would suggest itself immediately as the logical and most economical type. This selection would be discarded only if it should prove too expensive.

In the consideration of economy, the writer is inclined to believe that the selection of three (or possibly even two pairs) of ribs would have resulted in some lessening of duplication of sections but would have reduced girder sections for the floor beams and floor system, thus causing a reduction in the dead load and obviating the necessity of using any alloy steel in the superstructure. This arrangement, moreover, would have required the selected arch-rib section for one rib only (if three ribs were used), whereas the side ribs could have had carbon steel sections, while a stiffer cross section of structure would have been provided. The study of the deformation of the hingeless arch should have led to the selection of a section of rib varying from crown to abutment to provide a deeper section at approximately the quarter point. This would have reduced greatly the deformations due to live load in the vicinity of the quarter point even though this varying section would have complicated the detailing somewhat.

Because the only partial live load considered in the arch analysis of the Symposium was that on half the span, it might be interesting to note that the specifications of the American Railway Engineering Association[54] for Masonry Arches for Railroad Loading require the following live-load conditions:

1. For maximum positive moment at the crown, assume the middle quarter of the span loaded.
2. For maximum negative moment at the crown, assume all of the span loaded except the middle quarter.
3. For the maximum positive moment at the springing line, assume five eighths of the span loaded from the opposite end.
4. For maximum negative moment at the springing line, assume the adjacent three eighths of the span loaded.

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[54] *Manual for Railway Engineering*, A.R.E.A., 1943, Committee 8, Masonry, Specification 312, Section 10, pp. 8–61.

A structure as important as the Rainbow Arch Bridge should be designed for at least the maximum live-load specifications of HS20 trucks, as defined by the American Association of State Highway Officials.[55] The Veterans' Gorge Arch Bridge in Rochester, N. Y., was designed (in 1930) for 30-ton truck loads to anticipate future increase in truck loading. Moving artillery over the highways has raised the question of rating highway bridges to permit the passage of guns. The increase in live load would have added comparatively little to the total load and resulting stresses and therefore to the cost of the structure, but would have provided for future loading increases.

As for methods of design, it has hitherto been acceptable practice to design arches by the "elastic theory" (an unfortunate expression, as the "theory" is not elastic). In this Symposium, however, doubt is cast on the "assumptions" underlying this theory and a deflection "theory" or method is presented as being exact.

The cables of a suspension bridge form an inverted arch or catenary and are flexible to such an extent that a stiffening truss is introduced in order to minimize the deformations. The resultant action is a pull or tension exerted by the cables under load, in contrast to the thrust or compression resulting from the loads in the case of the arch. In both cases bending moments are caused by the eccentricity of the applied forces. However, an arch (steel or concrete) has relatively much greater stiffness without considering the stiffening effect of the spandrel system (generally ignored in arch design) to resist deformation or distortion.

All theories applied to structures are based on assumptions, and no assumptions are absolutely exact. The closeness of the results of the proposed to those in the actual structure will depend on the closeness of the assumption to actual conditions. No assumption is made in the "Theory of the Elastic Arch" (so-called elastic theory) which is unsound or questionable, particularly for a steel structure in which the modulus of elasticity changes but very slightly in the various sections of the arch. In fact, the only assumptions other than that of elasticity made in this type of analysis for the fixed arch (aside from that of uniformity of E) are that the supports shall be unyielding, so that the horizontal span length remains constant, and that there will be no vertical displacement of one abutment with respect to the other. For the Rainbow arch, with the two abutments fixed in sound rock, the assumptions are as closely approximated as possible. The resulting thrusts will result in an infinitesimal yielding of the abutments or of the rock foundation, but such yielding would be microscopic.

Hardy Cross, M. Am. Soc. C. E.,[56] showed that differences in the value of E in the various sections of such structures were not serious in affecting the analysis of the arch. He investigated effects of variations as great as one third and one half the values of E. Such large variations within the range of working stresses are unusual. In the light of recent research developments, control over concrete will be more strictly observed in the future, and greater uniformity in the product and therefore in the value of E may be expected.

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[55] Standard Specifications for Highway Bridges, A.A.S.H.O., 2d Ed., 1935, Div. V, Section 2, Paragraph 7a.

[56] "Dependability of the Theory of Concrete Arches," by Hardy Cross, *Bulletin No. 203*, Univ. of Illinois, Urbana, 1930.

In the early stages of concrete construction for railroad structures in the United States, the arch type of design was quite popular. When the New Jersey cutoff of the Delaware, Lackawanna and Western Railroad Company was projected (1907–1911) between Hopatcong, N. J., and Slateford, Pa., of a total of seventy-six bridges and culverts included in the construction of the line, sixty (that is, 80%) were of the concrete arch type. In the construction of the Pennsylvania cutoff of the same railroad (1912–1915) between Clarks Summit, Pa., and Halstead, Pa., there were sixty-six arch structures, inclusive of two viaducts. One of the viaducts, the Tunkhannock Viaduct, is almost half a mile long, and includes the maximum span length (180 ft clear, or 208 ft center to center of piers), but this structure, of course, was designed for railroad engine loading.

Although considerably greater span lengths have been used for highway loading in the United States, the greatest extension of span length of concrete arches has been developed on the European continent, but no structure has approached the proportions of the Rainbow Arch Bridge.

Naturally, the thought would occur to a designer of concrete bridges: What could be done in concrete for a site like the Rainbow gorge? Fig. 43, for example, seems a rather startling extension of span length for a concrete arch and approaches closely the limits visualized by E. Freyssinet.[57] The writer, when designing the Veterans' Gorge Bridge over the Genesee gorge in Rochester, proposed and made preliminary studies of a 600-ft arch for the main opening to span between the outcrops of rock on either side of the gorge. The resulting structure, however, did not meet the esthetic requirements of the architects retained for the work, so that the 300-ft clear semicircular arch was adopted and built as part of the viaduct. Based on 5,000-lb concrete and intermediate grade reinforcing steel, with working stresses of 1,500 lb per sq in. and 20,000 lb per sq in., respectively, the center rib crown section would have been 16 ft deep by 8 ft wide, 23 ft deep at the springing, and the reinforcement would consist of 25, $1\frac{1}{2}$-in. square bars in the intrados and extrados. The floor system would have been a two-way reinforced concrete slab of 24-ft clear spans in both directions. The length of the span of the spandrel arches surmounting the main span would have been 24 ft.

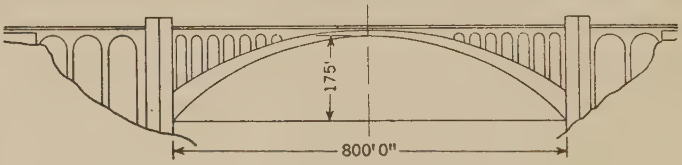

Fig. 43

The foregoing dimensions and sections are based on orthodox design. For economy of loads, the use of lightweight concrete, hollow arch ribs, and a larger percentage of alloy steel reinforcement would have to be investigated in an effort to reduce the dead load. In the concrete arch, dead loads result in stresses amounting to very nearly 50% of the total stress from all loads or forces and from the bending moments due to eccentricity of thrusts, including temperature effects and rib shortening. Incidentally, the field for the use of alloy steel for reinforcement of concrete has not yet been explored and may become of some importance in the future.

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[57] "Memoires de la Société des Ingénieurs Civils de France," by E. Freyssinet, July–August, 1930.

Of course, the construction or erection of such an arch structure would present a complicated set of problems, the solution of which would include the construction of a cableway and the erection of an "umbrella" section over each abutment for the support of steel arch ribs, as falsework for the concrete arch ribs. It would involve the shifting of the arch-rib centers for the construction of the outside ribs and their dismantling—no inconsiderable task (much like the steel arch erection problem). The design of the long columns would be a special problem, as would the order in which the voussoir sections should be placed. All these are difficult but not insuperable problems. At the site of the Rainbow Arch such a proposed bridge would have been an interesting and instructive job.

Incidentally, the late William H. Burr, M. Am. Soc. C. E., proposed a 750-ft concrete arch to span the Harlem River at Spuyten Duyvil in New York, N. Y., some years ago, so that the span length proposed in this discussion is within range of a structure actually designed although not built.

George Stern,[58] Assoc. M. Am. Soc. C. E.—A valuable contribution to the literature on practical design of arches of long span is contained in this Symposium. The design of an arch bridge of 950-ft span represents a structural problem of the first magnitude. The use of high-strength structural steel with high allowable unit stresses leads to structural members of small cross section and to comparatively small rigidity of the structure.

In an arch rib of long span considerable deformation will occur and design by the usual so-called "elastic theory" will involve appreciable errors, as was correctly stated by the authors of the first Symposium paper. Many of these potential errors will be avoided by the use of the approximate method suggested in the Symposium. To obtain correct results, however, the deformation of the arch under the design loading must be considered in setting up the equations of equilibrium—that is, the "deflection theory" must be used.

Although the so-called exact method, outlined in the first Symposium paper obviously can be used to determine the final moments by a trial-and-error procedure, a direct analysis when not too laborious would be preferable. The equilibrium equations required for such a direct analysis may be set up as follows:[9]

The bending moment $M(x)$ at a point x of a two-hinged arch can be expressed as

$$M(x) = m(x) - H\left[y(x) - \eta(x)\right] \ldots\ldots\ldots\ldots\ldots (98)$$

in which H is the true horizontal thrust of the arch and $m(x)$ is the bending moment at the same point x of a simple beam of the same span and cross section as the rib. The vertical displacement $\eta(x)$ of point x is considered positive when downward. The second derivative of Eq. 98 is

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[58] Stress Analyst, Aircraft Design Dept., Otis Elevator Co., New York, N. Y.

$$\frac{d^2M}{dx^2} = -W(x) - H\frac{d^2y}{dx^2} + H\frac{d^2\eta}{dx^2} \tag{99}$$

On the other hand, the following well-known relationships apply for a small element ds of a curved bar subjected to axial force, bending moment, and temperature change:

$$\frac{\Delta\, ds}{ds} = \frac{T}{E\,A} \mp \epsilon\, t \tag{100a}$$

and

$$\frac{\Delta\, d\theta}{dx} = -\frac{M}{E\,I\cos\theta} - \frac{T}{E\,A\,r\cos\theta} \pm \frac{\epsilon\, t}{r\cos\theta} \tag{100b}$$

in which $d\theta$ = the angle formed by the end tangents of the element ds, $\Delta\, ds$ = the change in length at the neutral axis, and $\Delta\, d\theta$ = the angular displacement of the element ds. The remaining notation is that used in the Symposium.

From the geometrical relations,

$$dy = ds \sin\theta \tag{101a}$$

and

$$dx = ds \cos\theta \tag{101b}$$

the following expressions for the vertical and horizontal displacements, $d\eta$ and $d\rho$, are derived:

$$d\eta = \frac{\Delta\, ds}{ds}\, dy + \Delta\,\theta\, dx \tag{102a}$$

and

$$d\rho = \frac{\Delta\, ds}{ds}\, dx - \Delta\,\theta\, dy \tag{102b}$$

By differentiating Eq. 102*a* twice,

$$\frac{d^2\eta}{dx^2} = \frac{\Delta\, ds}{ds}\,\frac{d^2y}{dx^2} + \frac{d(\Delta\,\theta)}{dx} \tag{103}$$

and, combining Eq. 103 with Eqs. 100, the following expression can be obtained:

$$\frac{d^2\eta}{dx^2} = \frac{T}{E\,A}\,\frac{d^2y}{dx^2} \mp \epsilon\, t\,\frac{d^2y}{dx^2} - \frac{M}{E\,I\cos\theta} - \frac{T}{E\,A\,r\cos\theta} \pm \frac{\epsilon\, t}{r\cos\theta} \tag{104}$$

By substituting for $T = H\sec\theta$, $\frac{d^2y}{dx^2} = -\frac{1}{r}$, introducing the notation $I\cos\theta = \bar{I}$ and $\frac{d^2\eta}{dx^2} = \eta''$, and, considering that M, as well as A and I, is a function of the abscissa x, Eq. 104 may be rewritten:

$$E\,\bar{I}(x)\,\eta''(x) = -M(x) - \frac{2\,H}{r}\,\frac{I(x)}{A(x)} \pm \frac{2\,E\,\bar{I}(x)}{r}\,\epsilon\, t \tag{105}$$

In general, the second term of the right side of Eq. 105, representing the effect of the normal stresses due to the axial thrust, has small influence on the results. The second derivative of this equation, substituted into Eq. 99, gives

$$[E\,\bar{I}(x)\,\eta''(x)]'' + H\,\eta''(x) = +W(x) + H\,y''(x) \tag{106}$$

Eq. 106 expresses the vertical displacement of a two-hinged arch rib as a function of the horizontal thrust H. In differentiating Eq. 105, the second term dependent on H was neglected. In the latter part of this discussion, however, the final solutions will also be given for the case in which that term is not disregarded.

Another relationship between horizontal thrust and vertical displacement can be derived from the fact that the relative displacement of the two hinged supports must be zero. Hence, from Eqs. 102, after some transformation,

$$\int_0^L d\rho = \int_0^L \eta' \, y' \, dx - \int_0^L \frac{\Delta \, ds}{ds} [1 + (y')^2] \, dx = 0 \ldots\ldots (107)$$

By substituting Eq. 100*a* and introducing an arbitrary cross-sectional area A_o, Eq. 107 can be transformed mathematically into

$$\int_0^L \eta(x) \, y''(x) \, dx + \frac{H}{E \, A_o} \int_0^L \left\{ \frac{A_o}{A(x) \cos \theta_x} [1 + (y')^2] \right\} dx$$
$$\mp \epsilon \, t \int_0^L [1 + (y')^2] \, dx = 0 \ldots\ldots\ldots\ldots\ldots\ldots (108)$$

Eqs. 106 and 108 represent two formulas with two unknowns, the horizontal thrust H and the vertical displacement η. As soon as the general solution of the differential Eq. 106 is obtained, H and η can be computed.

Eq. 106 is a nonhomogeneous differential equation of the fourth order; the derivation of its general solutions is far too laborious to be given in detail. The procedure followed was first to solve the corresponding homogeneous differential equation. In the case of an arch rib of constant cross section (that is, $\bar{I}(x) = I(x) \cos \theta =$ constant), the characteristic values λ_i and characteristic functions $\phi_i(x)$ of Eq. 106 can easily be computed since they represent simple sine curves. Thus the vertical displacement and the moment can be expressed in the form of Fourier series. On the other hand, if the moment of inertia of the rib varies, the characteristic functions are no longer true sine curves. To obtain these $\phi_i(x)$-values, some additional preparatory computation must be made. The formulas that can be derived to express these sinusoidal curves are not too elaborate, but their complexity increases the more closely the variation of the moment of inertia is considered.

Once the characteristic values λ_i and functions $\phi_i(x)$ of the corresponding homogeneous differential equation are computed, the general solution of the nonhomogeneous differential equation can be expressed by a uniform and convergent infinite series of these previously mentioned functions. This method of solving by series was first used by H. H. Bleich in his deflection theory of suspension bridges.[59] Just as the theory of arches shows a good analogy to that of suspension bridges, so also do the respective deflection theories correspond rather well if similar methods of attack are used. In the case of two hinged arch ribs, the aforementioned series converges so rapidly that, for all practical purposes, it is more than satisfactory to use only a few terms of the series. Thus, the general solution of Eq. 106 may be written

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[59] "Die Berechnung verankerter Hängebrücken," by Hans H. Bleich, Springer, Vienna, 1935.

$$\eta(x) = \sum_{i=1}^{i=\infty} d_i \, \phi_i(x) \quad \text{(109)}$$

in which the factor d_i is given by

$$d_i = \frac{a_i + b_i H}{\lambda_i - H} \quad \text{(110)}$$

The parameters a_i and b_i are defined by the integrals,

$$a_i = \int_0^L w(x) \, \phi_i(x) \, dx \quad \text{(111}a\text{)}$$

and

$$b_i = \int_0^L y''(x) \, \phi_i(x) \, dx \quad \text{(111}b\text{)}$$

As stated previously, Eq. 109 can now be substituted in Eq. 108. Thus, by integrating term by term and by transformation, an expression can finally be derived which contains the horizontal thrust H as the only unknown value:

$$\sum_{i=1}^{i=\infty} \frac{(a_i + b_i H) \, b_i}{\lambda_i - H} + \frac{C}{E \, A_o} H \mp C_t \, \epsilon \, t = 0 \quad \text{(112)}$$

in which

$$C = \int_0^L \frac{A_o}{A(x) \cos \theta_x} [1 + (y')^2] \, dx \quad \text{(113}a\text{)}$$

and

$$C_t = \int_0^L [1 + (y')^2] \, dx \quad \text{(113}b\text{)}$$

In Eqs. 109 and 112, the effect of the normal stresses due to the thrust is disregarded. It was mentioned before that this effect on the vertical displacement is very small. Thus, in most cases, Eqs. 109 and 112 will give satisfactory results.

However, if the effect of the normal stresses is not neglected, the general solution of the differential equation of the two-hinged arch problem may be expressed by

$$\eta(x) = \sum_{i=1}^{i=\infty} \frac{a_i + b_i H}{\lambda_i - H} \phi_i(x) - \frac{2 \, I_A}{r \, A_A} \left[1 - \frac{\cos \alpha \left(x - \frac{L}{2} \right)}{\cos \frac{\alpha \, L}{2}} \right] \pm \frac{2 \, E \, I_A \cos \theta_A}{r \, H} \left[1 - \frac{\cos \alpha \left(x - \frac{L}{2} \right)}{\cos \frac{\alpha \, L}{2}} \right] \epsilon \, t \quad \text{(114)}$$

in which $\alpha = \sqrt{\dfrac{H}{E \, I_A \cos \theta_A}}$. In the terms representing the effect of the normal stresses, average values A_A, I_A, and θ_A are used for area, moment of inertia, and the angle θ. Using Eq. 114, the following final formula can be developed:

$$\sum_{i=1}^{i=\infty} \frac{(a_i + b_i\,H)\,b_i}{\lambda_i - H} + \frac{C}{E\,A_o} H + \frac{2\,I_A}{r^2\,A_A} \left(L - \frac{2 \tan \frac{\alpha\,L}{2}}{\alpha} \right)$$

$$\mp C_t\,\epsilon\,t \mp \frac{2\,E\,I_A \cos\theta_A}{r^2\,H} \left(L - \frac{2 \tan \frac{\alpha\,L}{2}}{\alpha} \right) \epsilon\,t = 0 \ldots\ldots (115)$$

The derivation of Eqs. 109, 112, 114, and 115 shows that they can be applied to any symmetrical two-hinged arch, irrespective of the shape of the arch axis, the variation in cross section of the rib, and the loading condition to be investigated.

The horizontal thrust H can now be calculated from Eq. 112 or 115 by the trial-and-error method. Once its value is known, the bending moment, shear, and unit stress may easily be computed by the conventional methods, and the vertical displacement can immediately be obtained from Eq. 109 or 114. (The writer is informed that Friedrich Bleich, M. Am. Soc. C. E., has developed, but not yet published, a deflection theory of two-hinged and hingeless arches. He uses a method similar to that outlined herein, but, by neglecting the effect of the normal stresses due to the thrust on the vertical displacement, equations are derived from which H can be calculated directly.)

A solution for the final moments in the preliminary two-hinged Niagara arch gave results which agree with those obtained by the authors of the first Symposium paper. Using the method outlined herein, the writer computed a 60% increase in the moment at the quarter point due to the deformation of the arch rib, as compared with the 63% increase obtained by the so-called exact procedure in the first Symposium paper.

Although few engineers will have the opportunity to design an arch bridge of the magnitude of the Rainbow Arch Bridge, the record of its design and construction will prove very valuable to future designers of similar projects.

SHORTRIDGE HARDESTY[60] AND J. M. GARRELTS,[61] MEMBERS, AM. SOC. C. E., AND I. G. HEDRICK, JR.,[62] ASSOC. M. AM. SOC. C. E.[62a]—The criticisms and extensions in the discussions have materially increased the usefulness of this Symposium and are appreciated by the writers.

Several discussers have objected to the use of the term, "elastic theory," to denote a method of structural analysis based on the assumption that the change in the geometry of the arch axis under load does not affect the stresses in the rib, and to the use of the term, "deflection theory," to denote a method which takes into account the effect of such change in geometry and thus gives a more nearly correct solution than does the elastic theory. To the writers the use of two terms seems merely to be carrying into arch analysis the nomenclature long used in the field of suspension bridges. The two usages[63,64] are exactly parallel.

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[60] Cons. Engr. (Waddell & Hardesty), New York, N. Y.

[61] Associate Prof., Civ. Eng., Columbia Univ.; Designing Engr., Waddell & Hardesty, New York, N. Y.

[62] Stress Analyst, Grumman Aircraft Eng. Corp., Bethpage, L. I., N. Y.

[63] "A Practical Treatise on Suspension Bridges," by D. B. Steinman, 2d Ed., John Wiley & Sons, Inc., New York, N. Y., 1929, p. 246.

[64] "Elementary Treatise on Statically Indeterminate Stresses," by John I. Parcel and G. A. Maney, 2d Ed., John Wiley & Sons, Inc., New York, N. Y., 1936, p. 351.

The writers note that some discussers have apparently confused the loading used in the design of the arch rib with the half-span loading considered in Part II of the paper. In analyzing the stresses in the arch ribs, the moments and thrusts at each section were computed for a unit load at each panel point in turn. The live-load stresses at each section were then determined by loading those parts of the span that would result in the maximum stresses. The use of a single arbitrary length of loading for the design of the entire rib would give stresses much less than the maximum stress for most sections; hence such method would be seriously in error.

Mr. Di Genova's discussion, in general, is irrelevant to the subject of the paper. The answer to his claims of exactness for the "elastic theory" can be found by a careful reading of the paper, in connection with discussions on it, and also by reference to any good treatise on the "deflection theory" for suspension bridges. The deflections of the 950-ft two-hinged arch, which Mr. Di Genova says are relatively small, actually have the effect of increasing the bending moment at the quarter point by 72% and of decreasing the safety factor of the rib from 1.875 to 1.30.

Mr. Mackintosh's suggestion of combining M_{DL} with M_{LL} would give design moments which are too large unless the dead loading was unsymmetrical and produced moments of nearly the same spanwise variation as the unsymmetrical live loading of the critical condition. Ordinarily this is not the case, as dead-load moments and deflections are usually symmetrical, whereas the rib has the lowest resistance to buckling with a two-loop bending-moment diagram having positive moment over approximately one half the span and negative moment over the remaining part of the span.

Mr. Mackintosh, and later Mr. Beskin, have suggested an approximate beam-column correction of the form:

$$M_f = M_{ae} \left[\frac{1}{1 - \frac{T}{T_{cr}}} \right] \qquad (116)$$

The writers agree that this correction is sufficient when the rib is of uniform or nearly uniform section and when T is nearly constant throughout the length of the rib; but they were not willing to depend entirely on this form of approximation for the Niagara arch, where considerable variation existed in cross section and curvature of the rib axis. An evaluation of the approximations involved in this form of correction will entail essentially the computations referred to by the writers as their "exact" procedure.

Mr. Goodrich's request for an explanation of "Stern's Theory" has been supplied in a later discussion of the present paper by Mr. Stern.

Professor Spofford has suggested that a cost and quantity estimate would be a worth-while addition to the paper. The writers, therefore, have included the following tabulation of the final costs (1940–1941 exchange values):

Detailed Construction Costs:

Contract 1A. Substructure on the United States Side (McLain Construction Corporation)—

Unclassified excavation (4,735 cu yd at $5)	$ 23,675	
Concrete in footings (2,559 cu yd at $10)	25,590	
Concrete in arch abutments above footings (1,095 cu yd at $11)	12,045	
Concrete in shafts (3,061 cu yd at $8.50)	26,018	
Bar reinforcement for structures (369,039 lb at 4¢)	14,762	
Setting anchor bolts	1,000	
Core borings (60 lin ft at $4)	240	
Total for contract 1A, United States money		$ 103,330

Contract 2A. Substructure on Canadian Side (Aiken and MacLachlan)—

Unclassified excavation (6,117 cu yd at $2.35)	$ 14,374	
Concrete in footings (3,224 cu yd at $12)	38,688	
Concrete in arch abutments above footings (1,780 cu yd at $12.50)	22,250	
Concrete in shafts (3,236 cu yd at $12.75)	41,259	
Bar reinforcement for structures (418,453 lb at $4\frac{1}{2}$¢)	18,830	
Setting anchor bolts	640	
Core borings (105 lin ft at $5)	525	
Total for contract 2A Canadian money	$ 136,566	
Total for contract 2A United States money		$ 120,246

Contract 3A. Superstructure of Arch Span (Bethlehem Steel Company)—

Concrete in arch abutments (390 cu yd at $33.50)	$ 13,065	
Concrete in slabs and curbs (1,405 cu yd at $27)	37,935	
Bar reinforcement for structures (209,264 lb at 4.47¢)	9,354	
Reinforcing trusses for roadway slabs (305,136 lb at 5.75¢)	17,545	
Carbon steel in floor system 2,450 lb × 983 ft (2,407,527 lb at 8.232¢)	198,188	
Silicon steel in floor system 495 lb × 983 ft (485,594 lb at 8.9¢)	43,218	
Carbon steel in spandrel columns 1,195 lb × 950 ft (1,136,301 lb at 8.694¢)	98,790	
Carbon steel in rib laterals 950 lb × 950 ft (903,296 lb at 13.848¢)	125,088	
Silicon steel arch ribs and shoes 5,455 lb × 950 ft + 4 × 260,000 (6,223,496 lb at 14.525¢)	903,963	
Provision for utilities	2,823	
Extra work orders	1,812	
Total for contract 3A, United States money		$1,451,781

Contract 4. Superstructure of American Approach Spans (McLain Construction Corporation)—

Unclassified excavation (1,000 cu yd at $1.50)	$ 1,500	
Concrete in footings (310 cu yd at $11)	3,410	
Concrete in abutments above footing (216 cu yd at $16)	3,456	
Concrete in superstructure (3,368 cu yd at $27)	90,963	
Bar reinforcement for structures (593,303 lb at 4¢)	23,732	
Structural steel (20,741 lb at 15¢)	3,111	
Provision for utilities	500	
Earth excavation—special (5,000 cu yd at 45¢)	2,250	
Rock excavation—special (204 cu yd at $2)	408	
Protection of cliff	3,867	
Total for contract 4, United States money		$ 133,197

Contract 5. Superstructure of Canadian Approach Spans (Aiken and MacLachlan)—

Unclassified excavation (1,299 cu yd at $2.50)	$ 3,247	
Concrete in footings (864 cu yd at $13)	11,232	
Concrete in shafts (1,188 cu yd at $16)	19,008	
Concrete in superstructure of river road span (1,118 cu yd at $16)	17,888	
Concrete in superstructure of arch spans (3,334 cu yd at $27)	90,018	
Bar reinforcement for structures (1,020,300 lb at 6¢)	61,218	
Structural steel (63,018 lb at 30¢)	18,905	
Masonry facing (2,500 cu ft at $6.50)	16,250	
Arch stones (900 cu ft at $8)	7,200	
Special stones (2,600 cu ft at $10)	20,000	
Special stone details	1,550	
Provision for utilities	3,000	
Extra work orders	750	
Sales taxes	2,924	
Total for contract 5 {Canadian money	$ 273,190	
Total for contract 5 {United States money		$ 241,783

Contract AP1. Retaining Walls at American End (Charles H. Wing Company)—

Unclassified excavation for wall footings (3,141 cu yd at $1.25)	$ 3,927	
Unclassified excavation for parkway drive (9,063 cu yd at 54¢)	4,894	
Concrete in wall footings (499 cu yd at $11.15)	5,565	
Concrete in walls above footings (555 cu yd at $21.75)	12,072	
Concrete in railings (75 cu yd at $26.56)	1,992	
Bar reinforcement for structures (84,191 lb at 3.76¢)	3,166	
Extra work orders	2,300	
Total for contract AP1, United States money		$ 33,916

Summary of Construction Costs:

Bridge Between Abutments—		
Contract 1A, substructure on American side	$ 103,330	
Contract 2A, substructure on Canadian side	120,246	
Contract 3A, superstructure of arch span	1,451,781	
Contract 4, superstructure of American approach	133,197	
Contract 5, superstructure of Canadian approach	241,783	
Retaining walls and excavation—American side	33,916	
Lighting system on bridge	12,973	
Total bridge between abutments		$2,097,226
American Terminal—		
Paving and embankment of plaza	$ 63,743	
Paving and embankment of access road	50,519	
Buildings	166,000	
Lighting system	13,000	
Total American terminal		$ 293,262
Canadian terminal, cost to bridge commission		160,000
Name plates		2,000
Toll equipment		10,820
Office equipment and fixtures		16,000
Miscellaneous construction items		11,100
Borings		7,925
Engineering, architectural, and inspection services		193,989
Total construction cost (United States money)		$2,792,322

Mr. Mensch states that "J. A. C. Bresse warned his students not to use ribs so shallow as to cause excessive deflection due to a moving load. No real, valid reason has been given in this Symposium why this sound warning has been disregarded." Deflections were not disregarded in any sense. In fact, Part II of the paper is devoted entirely to the discussion of methods for evaluating the conditions which determine excessive deflections. The deflections of the final design are within permissible limits. Eqs. 18 and 19 are correct, and Eqs. 20 and 21 are clearly stated to be approximations. Mr. Mensch's Eqs. 39 to 51 are perhaps satisfactory for obtaining approximate sections. These equations apply the "elastic theory" to a particular case, the parabolic rib with $I = I_c \sec \alpha$, which yields simple integrals.

Mr. Mensch also appraises the paper erroneously when he states: "Contrary to the implied limitations expressed by the Symposium authors, Eqs. 39 to 51 are quite sufficient for the design of an arch of the Rainbow type, and the results agree well with those given in Table 1." Table 1 contains "elastic theory" results which the writers obtained by a more accurate integration, for this particular arch, than would be given by Mr. Mensch's equations. The comparison, therefore, has no bearing on the sufficiency of the "elastic theory." However, the approximations of Mr. Mensch's equations are hardly negligible even if deflection effects are neglected. His value of H, computed as 5,810 kips instead of the more accurate value of 5,911 kips, would represent a difference in moment at points near the crown of 101 kips × 150 ft, or of approximately 15,000 ft-kips. In comparing deflections, Mr. Mensch has added deflections at the crown point due to dead load, live load, rib shortening, and temperature to obtain a total of 14.48 in., which he has compared to the writers' value of 6.9 in. for (dead + live + impact)-load at the quarter point. This comparison has no meaning since the deflections are for different points of the arch rib and for different loads. The true comparison was given in the paper as 5.5 in. for the elastic theory as compared to 6.9 in. for the correct computations.

Under the heading, "Introduction," in the third paper of the Symposium it is stated that the arch was cambered for full dead load.

Mr. Ammann's contention, that the permissible limits for stresses should be assigned to the accurate stresses and not to those which are approximate, is strictly correct. As a matter of fact the designers applied this test in checking the most critical sections, but did not follow this procedure in preparing the stress sheet of the drawings because of the practical difficulties involved in the computation and preparation of a complete stress sheet showing the deflection stresses at every panel point.

The writers wish to thank Mr. Balog for calling attention to the Dischinger method of calculating deflection effects with its numerical application to the Rainbow arch rib.

Mr. Beskin has objected to the use of the word "theories" by the writers in referring to certain studies (see Appendix I) which have been made in connection with the deflection effects of arches. Since the writers were employing the papers in question as references, they accepted the titles of the original authors, who, correctly or incorrectly, used the term "theory" in describing their work.

Mr. Beskin has rightfully questioned the use of safety factors as they appear in Eq. 16. Although it is generally true that allowance should be made in design for increases in actual dead loads and live loads, in most structures the maximum temperature and wind loads are constant throughout the life of the structure. The factor of safety for these loadings, therefore, need serve only as a factor of ignorance to cover estimation of the present loads and approximations of the analysis. A factor of $N' = 1.33$ used with the 30-lb wind loading of present specifications and the maximum temperature range of the locality would seem to be sufficient.

The effect of axial thrust, although finally neglected, is not correctly expressed in Eq. 2. A more complete equation is:

$$\frac{d^2u}{ds^2} + \frac{u}{(r_o)^2} = -\frac{M}{E\,I} - \frac{T\,\mu}{r_o\,A\,E} + \left(\frac{M}{E\,I} + \frac{T}{r_o\,A\,E} + \frac{T\,\mu}{r_o\,A\,E}\right)\frac{T}{A\,E} + \frac{1}{2\,r_o}\left(\int\frac{T\,ds}{r_o\,A\,E} + \int\frac{M}{E\,I}\,ds + \int\frac{T\,\mu}{r_o\,A\,E}\,ds + C\right)^2 \quad\ldots\ldots(117)$$

in which μ = Poisson's ratio.

To illustrate the effect of axial deformation on radial deflections, consider the semicircular arch in Fig. 44 with radial loads of magnitude T. The bending moment in this case is zero throughout and the axial compression T is constant around the full length of the rib. It will be assumed that $\mu = 0$. If point A is assumed fixed and point B is free to deflect, then the tangential component of deflection, v, of point B is seen to be

$$v = \frac{\pi R T}{A E} \qquad \text{(118)}$$

and the radial component of deflection of point B is

$$u = \frac{v^2}{2 R} = \frac{R}{2} \left(\frac{\pi T}{A E} \right)^2 \qquad \text{(119)}$$

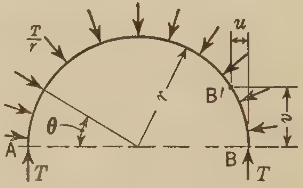

FIG. 44

The same result can be obtained by Eq. 117, which reduces to the following when $M = 0$,

$$\frac{d^2 u}{d s^2} + \frac{u}{R^2} = - \frac{1}{R} \left(\frac{T}{A E} \right)^2 + \frac{1}{2 R^3} \left(\int \frac{T}{A E} \, ds \right)^2 \qquad \text{(120)}$$

The solution of Eq. 120 for constant T is

$$u = \frac{1}{2 R} \left(\int \frac{T}{A E} \, ds \right)^2 \qquad \text{(121)}$$

and, since $ds = R \, d\theta$,

$$u = \frac{R}{2} \left[\left(\frac{T \theta}{A E} \right)^2 \right]_0^\pi = \frac{R}{2} \left(\frac{T \pi}{A E} \right)^2 \qquad \text{(122)}$$

This question of the effect of thrust on radial deflections, however, is academic, since both Mr. Beskin and the writers agree that its influence is negligible for steel or concrete ribs.

The writers inadvertently made the statement in the paragraph containing Eq. 6 that the term $\frac{u}{(r_o)^2}$ of Eq. 2 is neglected in the elastic theory and, therefore, in their "exact" procedure. The basic differential relation[65] of the "elastic" theory is

$$\frac{\Delta \, d\theta}{ds} = - \frac{M}{E I} \qquad \text{(123)}$$

in which $\Delta \, d\theta$ is the change in angle between the end tangents of the element ds. In the paper it was stated incorrectly that $\frac{d\theta}{ds} = - \frac{d^2 u}{d s^2}$—a relation that is exact for straight beams only. It can be proved that for curved beams the following relations[66] exist:

$$\frac{\Delta \, d\theta}{ds} = \left(\frac{1}{r_f} - \frac{1}{r_o} \right) = \frac{d^2 u}{d s^2} + \frac{u}{(r_o)^2} = - \frac{M}{E I} \qquad \text{(124)}$$

in which the element ds is assumed to remain of constant length.

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[65] "The Theory and Practice of Modern Framed Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, 10th Ed., John Wiley & Sons, Inc., New York, N. Y., 1929, Pt. II.

[66] "Strength of Materials," by Stephen Timoshenko, 1st Ed., D. Van Nostrand Co., Inc., New York, N. Y., 1940, Pt. II, p. 457.

The actual working equation used in obtaining deflections by the elastic theory was:

$$\delta_a = \int_A^B \frac{M_{ae}\, m}{E\, I}\, ds \ldots\ldots\ldots\ldots\ldots\ldots\ldots(125)$$

in which m is the moment in the arch due to a unit load at point a in the direction of the deflection δ_a. Since both M_{ae} and m include the effect of $\frac{u}{(r_o)^2}$, the deflections and moments of both the "elastic" analysis and the "exact" procedure have been computed without neglect of the term $\frac{u}{(r_o)^2}$.

With regard to the use of an average value of T in Eq. 16 instead of the thrust at the point A which is being analyzed, the thrust near the quarter point is, in the first place, very nearly the average thrust. Secondly, since the change in H is negligible for deflection effects, the position of the thrust line remains very close to the position obtained in the elastic analysis, which accounts for the relation $\Delta m_{a1} = T_a\, u_{ae}$. Mr. Beskin is correct in stating that the thrust in the denominator should include the thrust due to wind and temperature.

Eq. 16 for the two-hinged rib, when revised in accordance with the foregoing discussion, becomes

$$s_y = N\left(\frac{T_a}{A_a}\right) + \frac{N\, M_{LL+I}}{S_a}\left[\frac{1}{1-(N\, T_a + N'\, T_t + N'\, T_w)\,\dfrac{u_{ae}}{M_{LL+I}}}\right] + \frac{N\, M_{DL}}{S_a} + \frac{N'\,(T_t + T_w)}{A_a} + \frac{N'\,(M_t + M_w)}{S_a} \ldots\text{(16 revised)}$$

and Eq. 17, also for the two-hinged arch, becomes

$$\frac{s_y}{N} = \frac{T_a}{A_a} + \left\{\frac{M_{LL+I} + T_a\,\dfrac{e'}{N}}{S_a}\right\}\left[\frac{1}{1-(N\, T_a + N'\, T_t + N'\, T_w)\,\dfrac{u_{ae}}{M_{LL+I}}}\right] + \frac{M_{DL}}{S_a} + \frac{N'}{N}\left(\frac{T_t + T_w}{A_a} + \frac{M_t + M_w}{S_a}\right) \ldots\ldots\text{(17 revised)}$$

Similarly, for the hingeless rib, Eq. 21 becomes

$$\frac{s_y}{N} = \frac{T_a}{A_a} + \frac{M_{LL+I} + 0.7\, T_a\,\dfrac{e'}{N}}{S_a}\left[\frac{1}{1-\left(\dfrac{N\, T_a + N'\, T_t + N'\, T_w}{T_a}\right)\left(\dfrac{\Delta m_{a1}}{M_{LL+I}}\right)}\right] + \frac{M_{DL}}{S_a} + \frac{N'}{N}\left[\frac{T_t + T_w}{A_a} + \frac{M_t + M_w}{S_a}\right] \ldots\ldots\text{(21 revised)}$$

In these revised equations the new term N' equals the load factor for temperature and wind loads (1.33 recommended). Present specifications account for the improbability of the simultaneous occurrence of maximum dead load, live load, wind load, and temperature load by increasing the allowable total stress. The writers believe that a more rational approach, especially when the stress-load relation is nonlinear, is the use of load factors as in the revised Eqs. 16, 17, and 21.

Mr. Beskin states that conclusions 1 and 2 are incorrect. The arguments he has submitted do not uphold this statement. In stating that the factor $\frac{T\,u}{M}$ should be considered in design, Mr. Beskin agrees with conclusion 1 which states "The useful strength of an arch rib is dependent in part * * *" upon $T\,u$. In regard to conclusion 2, the percentage error in total stress due to neglect of deflection effects is (approximately)

$$\text{Error } (\%) = E = 100 \left[\frac{s_a + \dfrac{s_b}{1 - s_a/s_c}}{s_a + s_b} - 1 \right] \quad \text{(126)}$$

In a given arch rib, if the dead load is increased, s_b and s_c will remain constant and the rate of change of E with respect to s_a will be:

$$dE = 100 \left\{ \frac{\dfrac{s_b}{s_c} \left[\dfrac{s_b + \dfrac{(s_a)^2}{s_c}}{(1 - s_a/s_c)^2} \right]}{(s_a + s_b)^2} \right\} ds_a \quad \text{(127)}$$

Inspection of Eq. 127 reveals that, for this case, an increase in s_a will always cause an increase in the percentage error in total stress. This was the condition the writers had in mind when stating conclusion 2.

However, if it is assumed that $s_a + s_b$ = constant and also that s_c = constant (which means that the rib section increases as the dead load increases), then:

$$dE = \frac{100}{s_a + s_b} \left[\frac{\dfrac{s_b}{s_a} + \dfrac{s_a}{s_c} - 1}{s_a\, s_c\, (1 - s_a/s_c)^2} \right] ds_a \quad \text{(128)}$$

In this case, as pointed out by Mr. Beskin, the percentage error in total stress will decrease for an increase in s_a, if $\frac{s_b}{s_a} + \frac{s_a}{s_c} < 1$.

Mr. Gronquist has suggested an interesting method of estimating deflection effects. The question of whether a correction factor should be applied to wind-load moments depends entirely on whether the critical wind loading produces a moment curve similar to that of the critical live load. The effect of wind loads on the rib will vary with the arrangement of the wind-bracing systems. In the Niagara design a symmetrical wind load was found to be critical for the quarter-point region and, therefore, the moments were not amplified by deflection.

Mr. Hirschthal has inquired about the considerations that determined the span length. The main abutments were located at a minimum safe height above the highest recorded ice level. The abutments so located determined the span length. With regard to his questions concerning the scope of the model tests made at Princeton University, it should be noted that these tests were not conducted as a part of the Niagara project, but as general research. The writers were aware of the work being done and, of course, were quite interested in the results, since the work paralleled closely the design calculations under way at the time. Those in charge of the Princeton University tests expected to continue with corresponding tests on hingeless ribs, but this work became "a war casualty."

The selection of the hingeless type of rib was not as obvious a choice as Mr. Hirschthal infers. Preliminary discussions with experienced fabricating and erecting contractors indicated a strong preference for the two-hinged type. In addition, the preliminary designs by the usual "elastic theory" methods indicated that the total material saving in the hingeless arch would be small. The two-hinged arch could not logically be eliminated from consideration until an investigation of the effects of flexibility proved that a satisfactory design would require an excessive amount of material.

The use of three ribs would have been undesirable for the following reasons: (1) The shallower ribs resulting from a three-rib design would increase the objectionable flexibility effects; and (2) the unit stresses due to wind loading would be 50% to 100% greater in the lighter ribs of a three-rib design.

Esthetically and economically, the rib of constant depth is preferable for steel construction.

Mr. Hirschthal's assumption that the arch was designed for live load over half the span length has already been shown, in the third paragraph of this closure, to be incorrect. The live-load assumptions were more exacting than those that would be required by the specifications of the American Railway Engineering Association for Masonry Arches.

With reference to the design loading, the H20 loading was the standard for heavy traffic structures when the Rainbow arch was designed. The H20–S16 loading did not appear in the bridge specifications of the American Association of State Highway Officials until 1941.

The writers appreciate Mr. Stern's discussion, which presents an interesting method of attacking the problem of flexible arches.

ELMER K. TIMBY,[67] ASSOC. M. AM. SOC. C. E., LAWRENCE M. MEAD, JR.,[68] JUN. AM. SOC. C. E., AND ROBERT MCELDOWNEY, JR.,[69] ESQ.—Many valuable ideas have been presented in the discussions on the model studies of flexible two-hinged arches for which the writers are grateful. They were disappointed because of their inability to include in the investigation the behavior of flexible fixed-end arches, which several of the discussers would have liked. Although the model project was suggested by the design of the Rainbow Arch Bridge and served to confirm the designers' calculations on the two-hinged arch, it was felt that a more general investigation, covering the range of variables of rise to span, load, and stiffness, would be of more value to the profession than the study of only one combination of these variables. In the time available it was impossible to study both two-hinged and fixed-end arches. The fixed-arch program (and perhaps studies of parabolic arches) was planned for investigation in 1942, but World War II intervened, and no one was available for this work. It is hoped that when the war is over the project will be continued.

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[69] 1st Lt., U. S. Marine Corps (formerly Apprentice Engr., Taylor-Wharton Iron & Steel Co., High Bridge, N. J.).

Mr. Ammann's suggestion that the "elastic theory" be called the "theory of the rigid system," and that the "deflection theory" be called the "theory of the flexible system" would clarify much confusion. Mr. Ammann misunderstood the writers' second conclusion, namely—that the test data would enable the designer to select a rib which in his (the designer's) opinion should be stiff enough to permit legitimate use of the "elastic theory."

Mr. Hirschthal questions the accuracy of the model studies and mentions the danger of applying them to so large a prototype. The writers maintain, however, that, if the accuracy of the measurements is increased in proportion to the decrease in the size of the model and the loads on the model, reasonable accuracy will not be sacrificed. For example, in the case of the model test which corresponded most closely to the proportions and loading of the Niagara arch, deflections could be measured to an accuracy of less than one part in fifty (or 2%) at the quarter point for maximum moment at the same point, even though the total deflection was only 0.07 in. on the model. An accuracy of 3% would probably be acceptable.

The writers cannot agree with Mr. Beskin that tests are of little value for analyzing or checking stress distributions of complicated or important structures. Mathematical treatment of many stress analysis problems is either inadequate or so complicated as to make test results far more practicable for actual design. As an aircraft stress analyst, Mr. Beskin must be familiar with the difficulty in treating, accurately, by mathematics, all phases of incomplete diagonal tension beams. For example, the method approved by the U. S. Army Air Corps for designing open section web stiffeners is based on an empirical formula developed to fit the results of a large number of static tests. Although several theoretical solutions of the problem appear reasonable and seem to cover the more important variables, none actually was verified sufficiently by test to be usable without unwarranted conservatisms.

In the case of a structure as important and costly as the Rainbow Arch Bridge, the model tests would seem justified for checking purposes even if they only proved (as they did) that the theoretical treatment included enough variables and was sufficiently accurate to insure a safe design.

In regard to the variation of dead load across the span, the writers felt that in such a general investigation, and with the limited time available, it was better to select a distribution that produced no dead-load moment—a condition sought in practical design—rather than to select some arbitrary percentage variation across the span. Undoubtedly the particular variation chosen will have some effect on the amplified final moments, especially in the case of the large ratios of dead load to stiffness and of rise to span. The actual dead-load distribution used varied as $\frac{r^2}{y^3}$ which reduces to $\frac{1}{r \cos^3}$, as Mr. Beskin deduced.

Mr. Beskin is to be congratulated for developing a theoretical formula which checks the test results so accurately. The writers consider their work well justified if it has stimulated the development of such theoretical solutions and has served as a check for them.

John I. Copp,[70] Assoc. M. Am. Soc. C. E., and Karl de Vries,[71] William H. Jameson,[72] and Jonathan Jones,[73] Members, Am. Soc. C. E.—The writers appreciate that the greater portion of the study given by many members of the Society to the papers of this Symposium was given to the first paper, on design, since that paper is naturally of more direct interest to designing engineers than is one covering fabrication and erection controls.

The method of weighing the crown thrust at closure has also been commented on, and a request has been made for a more complete description of the "weighing capsules." The writers would prefer to refer to the conclusion of the paper, and to emphasize that, in their opinion, weighing of the crown thrust of a properly constructed arch is unnecessary, and on an improperly or carelessly constructed arch it is useless.

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Paper No. 2237

DEWATERING, INCINERATION, AND USE OF SEWAGE SLUDGE

A SYMPOSIUM

PAGE
FOREWORD 180

Developments Indicated by Experience at Buffalo, N. Y.
By C. R. VELZY, M. AM. SOC. C. E. 181

Vacuum Filters Used in Primary Sludge Dewatering.
By RALPH E. FUHRMAN, ASSOC. M. AM. SOC. C. E. 188

Possible Economies in Sludge Disposal Practice.
By GEORGE J. SCHROEPFER, ASSOC. M. AM. SOC. C. E. 194

Use of Sewage Sludge as Fertilizer.
By LEROY W. VAN KLEECK, ASSOC. M. AM. SOC. C. E. 208

Experience of Chicago, Ill., in the Preparation of Fertilizer.
By WILLIAM A. DUNDAS, ESQ., AND C. P. MCLAUGHLIN, ESQ. 218

Bibliography.
By THE SYMPOSIUM AUTHORS 241

WITH DISCUSSION BY MESSRS. A. L. GENTER, WILLEM RUDOLFS, LLOYD M. JOHNSON, ISADOR W. MENDELSOHN, LANGDON PEARSE, C. R. VELZY, AND GEORGE J. SCHROEPFER.

NOTE.—Published in January, 1943, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

FOREWORD

At a technical meeting of the Sanitary Engineering Division on January 22, 1942, with W. Rudolfs, M. Am. Soc. C. E., presiding, the subject of dewatering, drying, incineration, and use of sewage sludge was thoroughly explored. Five of these papers have been revised for publication in the present Symposium. The most important revision is in the isolation of a bibliography on the subject as an independent unit of the Symposium, subject to discussion on a par with any of the papers. Additions to this bibliography that occur in discussion will be appended to the bibliography in the final printing.

DEVELOPMENTS INDICATED BY EXPERIENCE AT BUFFALO, N. Y.

By C. R. Velzy,[1] M. Am. Soc. C. E.

Synopsis

Sludge from the primary treatment plant at Buffalo, N. Y., passes through tanks for a short period of digestion, and is then disposed of by vacuum filtration and incineration. There have been developments in sludge conditioning and in the operation and maintenance of filters. The major problem with incineration has been severe abrasion, particularly in the flash driers, induced draft fans, and cyclones. Resulting costs of maintenance have been reduced by changes in equipment and materials. Operating results and costs are stated briefly.

The sludge disposal facilities at Buffalo are part of a treatment plant designed for a population of 750,000. The plant provides primary treatment with chlorination. The raw sludge is the product, therefore, of primary sedimentation only. The design contemplated a suspended solids content in the raw sewage amounting to 0.30 lb per capita, and a removal of 50%, or 0.15 lb per capita. For the present population of approximately 600,000, this would result in a daily removal of raw solids of approximately 90,000 lb. For the year 1940–1941 the actual amount of dry solids removed from the sewage was 89,300 lb per day, and was somewhat less than that for the previous two years.

Sludge disposal at Buffalo includes short-period or partial digestion, vacuum filtration, and incineration. Sludge is conditioned for filtration with lime and ferric chloride. After filtration the sludge cake passes over a system of conveyer belts, where it is weighed on its way to drying equipment. The dried and powdered sludge is blown into the furnace and burns in suspension. Hot gases from combustion are used to dry the sludge, and all vapors from drying are returned to the furnaces to eliminate odors.

The raw sludge is pumped to four sludge tanks having a total volume of 750,000 cu ft, or a design capacity of 1.0 cu ft per capita. Each tank is 90 ft in diameter, with a side-wall depth of 27.5 ft, and is equipped with a floating cover.

In the sludge disposal building there are three vacuum filters of 500 sq ft of filtering area each and three sludge incinerators having a design capacity of 100 tons of sludge cake per day on the basis of a moisture content of 70%. The furnace units include cyclones for the separation of vapors from powdered sludge and for the removal of fly ash from the flue gases, a pneumatic ash-handling system, preheaters for the recovery of heat, and the fans and blowers for the handling of gases and powdered sludge. The furnaces are equipped with combination oil and gas burners for the use of auxiliary fuel to supply heat to make up deficiencies of heat units in the sludge. During three years of operation this equipment has performed well. Some changes in details have been made, others are under study, and it is expected that more developments will be indicated as the art progresses.

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[1] Asst. Engr., Greeley & Hansen, Norfolk, Va.

The value of sludge tanks has been shown by the writer elsewhere (1).[2] They absorb the irregularities in quantity of raw sludge obtained, permit a six-day operating week in dewatering and incineration, provide gas as fuel for boilers and auxiliary heat for incinerators, and deliver a sludge of reasonably uniform characteristics for filtration and burning.

In the conditioning of sludge, the laboratory has made determinations of the theoretical quantity of chemicals required and has proposed a procedure by which the operator in the plant can make a quick determination of the amount of chemicals required. In connection with the application of chemicals, experiments have been made on details of the mixing tank with particular reference to baffling, mixing-paddle speeds, and detention. A short detention period with no baffles and proper mixing-paddle speeds is indicated, but more work is to be done before definite conclusions are drawn as to the best combination of these factors. In the equipment for the application of chemicals, there has been a tendency away from automatic devices and toward devices that will provide easy manual control and permit the operator to observe rates of application of chemicals and readily make necessary adjustments. For example, ferric chloride solution is now fed through a perforated rubber pipe, arranged so that the number of small streams of the solution is an indicator of the rate of feed. Thus the operator, or the superintendent, can quickly check the application of ferric chloride by counting the streams on this feeder and observing the dial on the weightometer.

In the operation of vacuum filters, studies have included experiments with different types of filter cloth and a number of modifications in the mechanical operation of the filters. Canton flannel has been found to be most satisfactory in operating results and in general economy in regard to cost and life of cloth. In the application of the cloth to the filter drum, it has been found that ordinary sash cord is a satisfactory substitute for the bronze strips which were originally used. The sash cord is inexpensive, is easy to apply, and results in greatly reduced time for applying new cloth.

On one of the filters a screw feeding device has been installed, with a surge bin above the screw. This device receives the sludge cake from the filter and discharges it uniformly to the first conveyer belt, producing a more satisfactory reading on the weightometer chart. With a variable speed drive, it also provides for a closer control of the loading on the incinerators. The device, as now used on one of the filters, is satisfactory, but a definite determination has not been made as to whether or not it is worth the cost.

After about two years of operation, it was found that the wire screen on the surface of the filter drum had become coated with lime scale so that the areas between the wires were materially reduced. This scale was removed successfully by the application of muriatic acid in the filter tub. Acid also has been used successfully in reducing the scale accumulations in filtrate lines and in the supernatant line which receives the discharge from the filtrate pumps.

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[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography at the end of the Symposium.

Fig. 1 shows a flow diagram of the drying and incineration processes. The fundamental principles of these processes have been described by D. C. Reybold (2). This flow diagram represents the present installation at Buffalo and differs in some mechanical details from the original installation. It also differs from other installations previously described. This discussion of problems and improvements is offered, not as a criticism of the original design, but as a description of developments normal to any process as new as sludge incineration.

The developments actually accomplished thus far are largely in the mechanical operation of the drying and burning equipment. The major problem has been due to abrasion of certain parts of the equipment. It appears that this could not be foreseen in the original design, as in other installations abrasion has been much less severe. In fact, it has been stated that in one of the Chicago, Ill., installations certain parts, which suffer from abrasion at Buffalo, give trouble because of the formation of scale. This is probably due to differences in the characteristics of sludge and perhaps in the character of auxiliary fuel.

In this connection, it is interesting to note that, during the first year of operation at Buffalo, when oil was used as an auxiliary fuel, scale formed on the preheater surfaces to such an extent that cleaning was required at frequent intervals. Since gas has been used exclusively as auxiliary fuel, the scale on the preheaters has disappeared, the soot blowers have become unnecessary and have been removed, and abrasion on the heater surfaces has made it necessary to replace some of the parts.

Abrasion has been particularly severe on the flash dryer, the induced draft fan, and at bends in pipe lines and ducts handling gases, powdered sludge, and fly ash. In the pipe lines and ducts this problem has been solved rather easily by the use of concrete wear backs at the bends. The concrete can be used in a thickness which will withstand wear for a considerable time and is easily replaced when it is worn through.

Wear on the flash dryer during the first months of operation was so severe on the bars and rings that the cages required replacement after about two months of continuous operation. At the suggestion of the manufacturer, a lower speed was tried in the cage mill. This has increased the life of the mill to about five months. It is possible that further improvement can be made. Adjustments in the type of assembly have resulted in easy replacement of worn parts and the reuse of portions of these parts.

The wear from abrasion on the induced draft fan has been so severe that originally the fan blades would last only approximately two months. The repair was difficult and costly because it required disassembling of the fan, replacement of blades, and reassembling of the fan. Hard surfacing of blades by arc welding was tried, but the slight additional wear did not warrant the cost. Ribbing of the blades with hard material was as effective as complete surfacing, and much less costly. The real answer was found in this case when

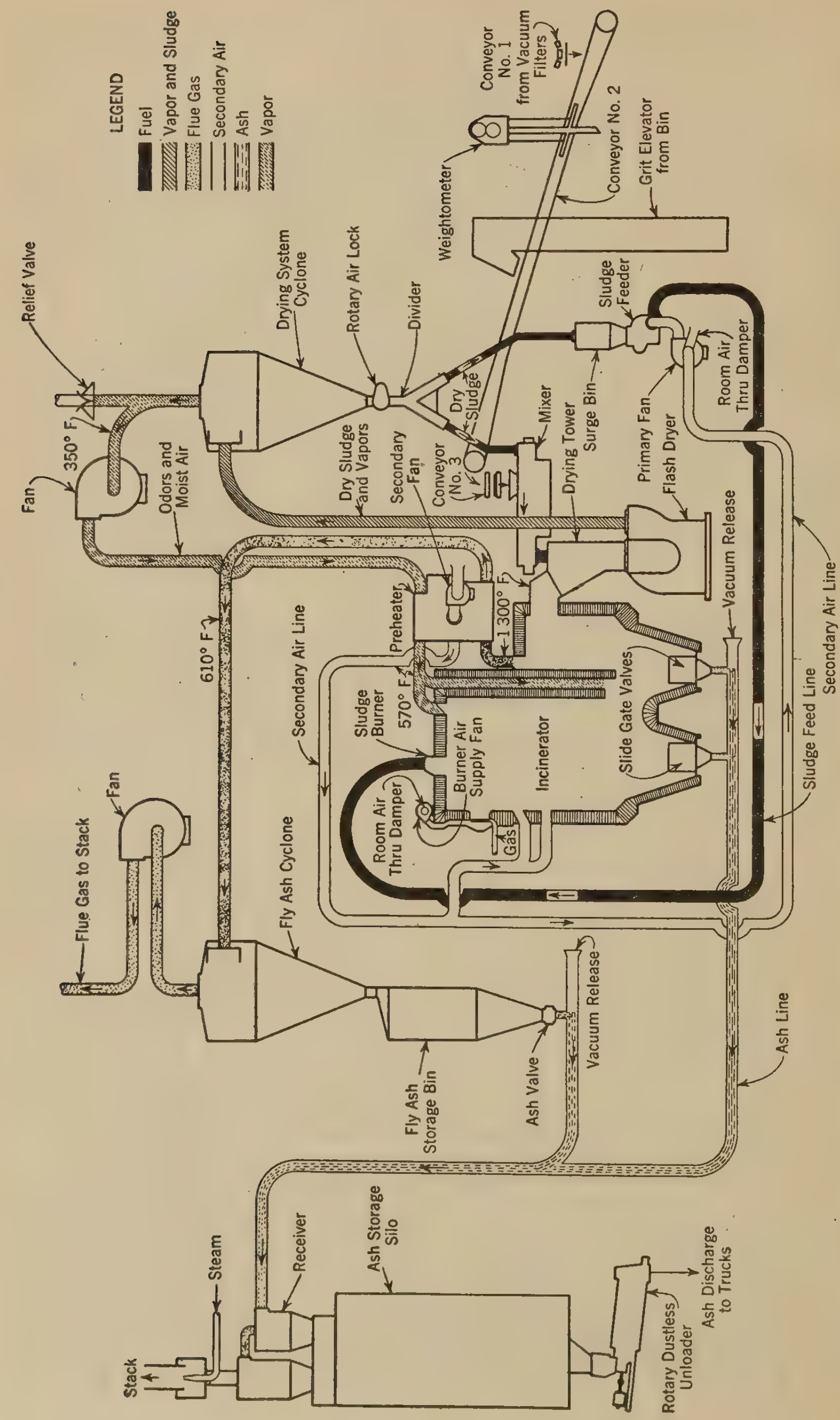
LEGEND
Fuel
Vapor and Sludge
Flue Gas
Secondary Air
Ash
Vapor
Relief Valve
Fan
350° F
Odors and Moist Air
Drying System Cyclone
Rotary Air Lock
Divider
Dry Sludge
Dry Sludge and Vapors
Secondary Fan
Conveyor No. 3
Weightometer
Conveyor No. 1 from Vacuum Filters
Conveyor No. 2
Grit Elevator from Bin
Sludge Feeder
Mixer
Surge Bin
Drying Tower
Primary Fan
Flash Dryer
Room Air Thru Damper
Secondary Air Line
Vacuum Release
1 300° F
Preheater
610° F
570° F
Sludge Burner
Burner Air Supply Fan
Incinerator
Slide Gate Valves
Sludge Feed Line
Secondary Air Line
Room Air Thru Damper
Gas
Fan
Flue Gas to Stack
Fly Ash Cyclone
Vacuum Release
Ash Line
Fly Ash Storage Bin
Ash Valve
Stack
Steam
Receiver
Ash Storage Silo
Rotary Dustless Unloader
Ash Discharge to Trucks

FIG. 1.—FLASH DRYING AND INCINERATION SYSTEM; TREATMENT WORKS OF THE BUFFALO (N. Y.) SEWER AUTHORITY

the fan was moved to the clean air side of the cyclone. One unit in this location has operated for more than a year to date (1943) without showing appreciable wear.

In the fly ash, cyclone abrasion has been less severe; nevertheless, it has constituted a problem calling for careful attention. Gunite has been tried with partial success. Better results probably would have been obtained if trap rock or other suitable aggregate had been available at Buffalo. Experiments have been made with brick linings and some other materials. It is believed that some form of concrete, or brick or ceramic material, will prove more satisfactory and less costly than steel. At the present time (1942), when steel is urgently needed for war, there is particular merit in finding some other satisfactory material. With the adjustments indicated to meet the abrasion problem, the maintenance costs on equipment will be materially reduced and better continuity of operation will be obtained.

Some of the operating results for dewatering and incineration for the three years of operation of the Buffalo treatment works are given in Table 1. Some of the developments in these operating results are worthy of note. The relatively high moisture content in raw sludge during the first year was largely due to inexperienced personnel and changing of operators. As the operators became more skilful in handling sludge and operating sludge pumps, they gained a normal control of the condition of the raw sludge. The increase in the volatile matter of raw sludge and digested sludge was apparently due to a decrease in the amount of grit carried by the sewage. During the first year of operation there was an extensive program of sewer cleaning that probably created a steady flow of grit to the treatment works. There has been some decrease in the solids burned per furnace hour. The heat content of the sludge cake has showed a marked increase, and there has been a corresponding decrease in the amount of gas used for auxiliary fuel. Computations show that the total quantity of heat dissipated by the furnaces has remained practically constant even with the decrease in solids burned.

TABLE 1.—COMPARISON OF OPERATING RESULTS FOR DEWATERING AND INCINERATION

Item	Description	July, 1938, to June, 1939	July, 1939, to June, 1940	July, 1940, to June, 1941
1	Moisture in raw sludge (%)	96.54	93.38	92.77
2	Moisture in digested sludge (%)	87.8	89.87	90.89
3	Volatile matter in raw sludge (%)	57.6	56.4	62.9
4	Volatile matter in digested sludge (%)	41.5	42.9	47.5
	Chemicals Used per 100 Lb of Dry Solids:			
5	Ferric chloride	2.31	3.10	2.93
6	Lime (Ca O)	9.25	11.62	11.21
7	Moisture in sludge cake	61.8	62.2	63.1
8	Volatile matter in sludge cake (%)	38.8	38.1	42.9
9	Filter rate, in lb of dry solids per sq ft per hr	....	7.58	6.77
10	Incineration; total furnace hours	....	9,231	10,278
11	Dry solids burned (1,000 lb)	13,559	23,525	23,226
12	Solids burned per furnace hour	....	2,550	2,260
13	Gas used for incineration, in cu ft per lb of dry solids burned	....	1.30	1.23
14	Average daily gas production (1,000 cu ft)	....	476	516
15	Heat content of sludge cake (Btu per lb)	....	3,790	4,320

Actual expenditures for operating the sewage treatment works for the fiscal year 1940–1941, together with costs per million gallons, are given in Table 2. A summary of costs, according to the processes involved in sewage disposal at Buffalo, is given in Table 3.

TABLE 2.—OPERATING UNIT COSTS; SEWAGE DISPOSAL AT BUFFALO, N. Y., 1940–1941

Item	Description	Total cost	Cost per million gallons
	Pumping:		
1	Outlying pumping stations	$ 22,376.94	$0.44
2	Main pumping station	45,885.70	0.89
3	Administrative expense	36,121.53	0.70
4	Laboratory	25,451.82	0.49
5	General sedimentation	50,959.68	1.00
6	Chlorination	48,633.60	0.95
7	Sludge disposal and heating	95,242.90	1.86
8	Maintenance and repair	61,656.59	1.20
9	Yards and grounds	31,916.46	0.62
10	Miscellaneous	4,648.72	0.09
11	Total	$422,893.94	$8.24

TABLE 3.—SUMMARY OF OPERATING COSTS BY PROCESSES; SEWAGE DISPOSAL AT BUFFALO, N. Y., 1940–1941

Item	Description	Pumping	Primary treatment	Sludge disposal	Total
1	Direct cost[a]	$68,262.64	$50,959.68	$95,242.90	$214,465.22
2	Administration	8,320.00	12,641.53	15,160.00	36,121.53
3	Maintenance and repairs	6,166.00	21,580.59	33,910.00	61,565.59
4	Miscellaneous	7,320.00	14,625.18	14,620.00	36,565.18
5	Laboratory	....	12,725.91	12,725.91	25,451.82
6	Chlorine	....	....	....	48,633.60
7	Total	$90,068.64	$112,532.89	$171,658.81	$422,893.94

[a] Salaries and supplies.

The total flow for the year was 51,262 million gal, and the total quantity of solids removed as raw sludge (dry basis) and grit was 18,300 tons. From these data the following unit costs are obtained:

Total cost of treatment, in dollars per million gallons	8.25
Cost of treatment, not including pumping and chlorination, in dollars per million gallons	5.97
Cost of treatment, in dollars per ton of raw solids (dry basis) disposed of	23.50
Cost of treatment, not including pumping and chlorination, in dollars per ton of raw solids	16.70
Total cost of solids disposal, in dollars per ton of raw solids disposed of	9.38
Cost of solids disposal, including only direct costs and maintenance and repairs, in dollars per ton of raw solids disposed of	7.07

SUMMARY

Operating experience in the disposal of sludge from the Buffalo primary sewage treatment works has led to developments in the selection of materials and in the use and arrangement of equipment. Some of the developments indicated may be briefly summarized as follows:

1. Sludge tanks have proved valuable to take up the irregularities in daily sludge quantities, to provide flexibility in the operation of incinerators, and to supply gas for fuel.
2. In applying chemicals for sludge conditioning, the tendency has been toward the use of manual rather than automatic devices, and arrangements which permit the operator to readily observe rates of chemical application.
3. Canton flannel has proved most satisfactory as a filter cloth.
4. A sludge-feeding device between the filter and the incinerator has been useful, but there is still some question whether it is worth its cost.
5. Muriatic acid has proved satisfactory and economical for cleaning lime scale from filter screens and pipe lines.
6. Although at Buffalo the major problem in sludge drying and burning has been the abrasion of equipment, it appears that this problem may vary widely in severity in different plants. The variation is probably due to differences in sludge characteristics and may also be affected by the nature of auxiliary fuel used.
7. Maintenance costs due to abrasion may be minimized in general by placing fans on the "clean air" side of cyclones, by using minimum speeds on moving parts, and by using concrete or ceramic linings instead of steel on stationary parts subjected to abrasion.
8. Operating results over the first three years show reduction in moisture content of raw sludge and increases in the moisture content, volatile matter and heat content of digested sludge and sludge cake. Incinerators have maintained their rate of heat dissipation.
9. Operating costs have amounted to $8.24 per million gallons treated, including pumping and chlorination, and direct costs of sludge disposal have amounted to $7.07 per ton of raw solids.

VACUUM FILTERS USED IN PRIMARY SLUDGE DEWATERING

By Ralph E. Fuhrman,[3] Assoc. M. Am. Soc. C. E.

Synopsis

The primary dewatering of digested sewage sludge by vacuum filters is one of the many mechanical processes that gained popularity in the decade 1932–1942. In the early 1930's, this service of vacuum filters was still in the experimental or pilot plant stage, whereas a very few years later saw the installation of huge filter areas. Sludge treatment literature of this period abounds with observations on vacuum filtration of digested sewage sludge and it is the purpose of this paper to indicate various experiences in the District of Columbia.

Description of Facilities

Sludge at the sewage treatment plant of the District of Columbia is dewatered by the operation of four vacuum filters, each with a filter area of 500 sq ft. These units, and the related equipment serving them, are sufficient to dewater the primary digested and elutriated sludge produced by the plant within the work week consisting of five 8-hr days. Between operating periods, sludge is held in the digestion tanks to be withdrawn as desired. After elutriation, the sludge is elevated to the filter floor by three closed impeller centrifugal pumps large enough to pass a 3-in. sphere. The sludge delivered to the mixing tank is controlled by a volumetric meter actuated by the level of sludge within the filter pans or troughs. To the required amount of sludge is added ferric chloride as a conditioning agent. Under the usual method of operation the mixing time amounts to 3 or 4 min, after which the sludge passes through rubber-lined pipes to the filter units. Provision for the addition of lime was made in the plant, but so far the use of lime has not been necessary on digested and elutriated primary sludge. As auxiliary equipment, vacuum pumps with an operating displacement of 2 cu ft per min per square foot of filter area are provided, as are filtrate pumps and low-pressure blowers, for freeing the filter cake from the cloth.

Ferric chloride is stored in two 8,000-gal, rubber-lined tanks and is pumped to elevated tanks with float switches by centrifugal pumps. The elevated tanks supply sufficient operating head for the ferric chloride feeders at the mixing tanks. Ferric chloride, which is purchased and fed as a 40% solution, is siphoned into the storage tanks from railroad tank cars. All pipe lines handling ferric chloride are hard rubber, or steel pipe lined with hard rubber, and all lines conveying conditioned sludge or filtrate are rubber-lined steel pipes.

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[3] Supt., Sewage Treatment Plant, District of Columbia, Blue Plains, Washington, D. C.

Elutriation facilities provided at the plant include two air-agitated tanks for the mixing of digested sludge with river water or supernatant from the elutriation settling tanks when countercurrent elutriation is used. The unit is provided with appropriate sludge pumps and wash-water pumps necessary for the elutriation process. Wash water may be drawn directly from the Potomac River or from the plant effluent conduit. Vacuum priming facilities are provided as all of the pumps are centrifugal.

OPERATING EXPERIENCES

In operation of the elutriation mixing tanks, experience indicated that large quantities of fine sand passed the grit chambers and settled out of the sewage in the sedimentation tanks. This material was pumped into the digesters with raw solids and, since it was unaffected by digestion, settled out in the elutriation mixing tanks in spite of air agitation through porous plate diffusers. The mixing tanks accumulated this very fine sand until practically no mixing of the passing liquids was attained. From this observation and others the mixing of sludge and wash water seems to be of minor importance, and as a trial the inlet and outlet of one mixing tank were connected by a 12-in. pipe. In this manner, the sand that formerly settled in the mixing tank is transported gradually with the sludge to the vacuum filters. The mixing of sludge and wash water seemed to be sufficient by the flow of these liquids within the pipe line. This total time of flow is usually about 20 sec in contrast to the mixing-tank period of 15 to 20 min. Plans are being made to try a similar scheme on the other mixing tank and if successful the problem of removing this fine sand will have been eliminated.

Initially the sludge pumps provided to elevate the digested, elutriated sludge from the elutriation tanks to the mixing tanks were sufficient to pass only a 2-in. sphere and in the early service of these pumps incessant clogging was experienced to the point where continuous dewatering was impossible. After considering a number of possible solutions to the problem, including the possibility of pumps equipped with cutters, plain centrifugal pumps similar in design to those originally installed were purchased, the only differences being in the size of the impeller passage, which was 3 in. in the new pumps, and the speed, which was 1,140 rpm as a maximum.

In the first two years after installation, some of the hard-rubber pipe lines showed much sagging between the supports of the pipes. Additional supports were first provided and although this reduced the possibility of a failure in the lines they were still unsightly. However, sufficient sagging was evidenced to put extra strains on the pipes. The remedy chosen was the replacement of these lines by steel pipe with a hard-rubber lining. This pipe has completely overcome the difficulties of the original material.

An operating procedure that has been found to be very important is that of prompt and thorough washing of filters at the end of an operating period. Filters at the plant are washed for 30 to 45 min after the operating period to remove, as far as possible, particulate matter that would "set" and tend to clog or blind the filter cloth. Analyses of discarded filter cloths have revealed 52% iron salts. At the suggestion of A. L. Genter, M. Am. Soc. C. E., reclamation of cloths was attempted with oxalic acid. For this use, a 1% solution of oxalic acid is made up in the filter pan and the filter is allowed to turn in the solution for a 24-hr exposure. For a particular cloth, for example, after 528 hr of service the oxalic acid wash was applied and service resumed until the cloth had served a total of 682 hr, an increase of about 30%. On the basis of a filter cloth cost of $120, the hourly cost of service was reduced from $0.23 to $0.18. Likewise, at a cost of oxalic acid of about $2.50 for the cloth, additional filtering service worth $35 was obtained. Some of these data have been reported previously (3).

During each summer of filter operation, fungus appeared on the filter cloths between the usual week-end shutdown from Friday afternoon to Monday morning. Within this period, the filter cloth disintegrated in spots all over the filter and rendered it practically useless. The responsible organism has not been identified, but it is probably one of the molds or bacteria known (4) to be destructive to wool.

The remedy for this attack is to apply a 100-ppm solution of copper sulfate to the filter cloths whenever the plant is shut down for more than 16 hr; that is, copper sulfate is applied every Saturday and Sunday, as well as any other nonoperating day, by rotating the filter with the solution in the filter pan long enough to saturate the wool cloth.

Before remedial measures could be developed, some cloths gave very short filtering service. The minimum life has been 32 hr of service and the maximum 899 hr. During the calendar years of 1940 and 1941, the minimum life was 227 hr, the maximum 682 hr, and the average life 408 hr. Cloths have cost from $87 to $134, with an average cost of approximately $115 each.

VALUE OF ELUTRIATION

P. D. McNamee has shown (5) that, without elutriation, the sludge produced at the District of Columbia plant would require about three times the ferric chloride now used. So far, it has not been practicable to operate the vacuum filters with unelutriated sludge to obtain plant data to check this value, but under the circumstances of the comparison, there is reason to believe it

TABLE 4.—VALUE OF ELUTRIATION

Item	Description	Dollars
1	Cost of construction, including mechanical equipment	40,500.00
2	Patent rights	12,000.00
3	Total investment (Items 1 and 2)	52,500.00
	Fixed Charges of Elutriation Facilities:	
4	Complete unit, amortized over 20 years at 3% (0.037 × Item 1)	1,498.50
5	Interest on the investment (0.03 × Item 3)	1,575.00
6	Total, per year (Items 4 and 5)	3,073.50
	Operating Cost for the Fiscal Year Ending June 30, 1941:	
7	Labor and supervision	2,496.00
8	Materials and supplies	163.00
9	Electrical energy	252.00
10	Total, for the year (Items 7, 8, and 9)	2,911.00
11	Total annual cost of operation and fixed charges, 1941 (Items 6 and 10)	5,984.50
12	Reduction in ferric chloride required	17,000.00
13	Value of elutriation, in dollars per year (Item 12 minus Item 11)	11,015.50

would be quite close to an actual operating figure. Acceptance of this figure makes possible an evaluation of the sludge washing facilities at this plant. Table 4 indicates the cost of construction and operation of this unit. The operating costs are those for the year ended June 30, 1941, which was a full operating year. During this period, $8,500 was spent on ferric chloride for conditioning the sludge prior to dewatering. Accordingly, without elutriation, ferric chloride, if it had been the sole conditioner, would have cost three times this amount, or $25,500. The value of elutriation may be given, then, as $17,000 less the operating cost of, say, $6,000 per year, or a saving of $11,000 per year. From these data, it is apparent that the elutriation unit will pay for itself every 4 or 5 years. These conclusions are based on the condition that ferric chloride alone would be the conditioner if elutriation were not practiced. Furthermore, it is assumed that costs of filtration, as affected by filter yield and life of cloth, would be the same, although all data indicate that greater yields may be expected from elutriated than from unelutriated sludge. For this reason, additional savings made possible by elutriation arise from reduced filter area and reduced filtering time. Consideration of these advantages would increase further the monetary savings affected by elutriation.

OPERATING RESULTS

During the 1941 fiscal year which ended June 30, 1941, the sludge dewatering facilities along with elutriation enjoyed a year of full operation. In the course of the year, volatile matter in digested sludge varied from 39.0% to 48.4%, manifesting the extra load on the digestion tanks resulting from the large population growth in the area served.

TABLE 5.—AVERAGE RESULTS OF ANALYSES OF ELUTRIATED SLUDGE

Month	Specific gravity	Solids (%)	Volatile matter in solids (%)	Wash water used, in 1,000 gal daily	Ratio of wash water to sludge volume	ALKALINITY, PPM		Effective wash water ratio[a]	Power for pumping wash water (kw-hr per day)
						Digested sludge	Elutriated sludge		
1940									
July	1.04	9.2	39.0	160.9	4.5	3,499	368	8.5	35
August	1.03	8.4	37.6	204.2	4.1	3,583	450	7.0	33
September	1.03	7.9	39.1	229.9	5.0	3,486	395	7.8	37
October	1.02	6.3	45.6	160.0	4.2	3,692	390	8.5	28
November	1.02	6.2	46.6	129.6	4.3	3,428	358	8.6	22
December	1.02	6.4	45.3	81.7	3.3	3,409	399	7.5	19
1941									
January	1.03	6.1	45.9	205.8	4.0	3,708	489	6.6	33
February	1.02	5.8	45.7	226.0	2.9	3,447	564	5.1	35
March	1.02	5.6	46.8	243.9	3.2	3,232	674	3.8	35
April	1.03	6.2	45.3	221.6	3.8	2,759	382	6.2	35
May	1.02	5.2	48.4	197.1	4.5	3,044	488	5.2	29
June	1.02	5.0	47.6	192.0	3.9	3,085	440	6.0	36
Mean	1.03	6.5	44.4	....	3.8	3,364	450	7.5	....

[a] Based on alkalinity removal.

Reference to Table 5 reveals the analytical and other operating data on sludge elutriation during the year. Of particular interest is the fact that 3.8 volumes of Potomac River water to one volume of digested sludge gave a total

TABLE 6.—SLUDGE DEWATERING PLANT OPERATION

Month	Days of dewatering	Filter hours	Cake produced, in tons (wet)	Solids in cake (%)	Dry solids in cake tons	Yield of cake solids[a] (dry)	Suspended solids in filtrate (ppm)
1940							
July	21	467.7	2,370.7	32.0	758.6	6.5	95
August	19	435.7	2,752.8	30.3	834.1	7.7	90
September	20	469.1	2,786.2	29.1	810.8	6.9	41
October	17	298.0	1,513.2	27.0	408.6	5.5	38
November	14	241.7	1,203.3	26.5	318.9	5.3	48
December	15	238.5	1,281.4	27.6	353.7	8.3	39
1941							
January	20	380.9	1,880.8	28.2	530.4	5.6	29
February	18	485.9	2,121.0	28.4	602.4	5.0	13
March	21	592.1	2,549.3	26.9	685.8	4.6	13
April	18	412.0	1,861.7	31.1	579.0	5.6	25
May	18	319.6	1,610.5	28.4	457.4	5.7	37
June	19	363.9	1,839.1	28.4	522.3	5.7	35
Total	220	4,705.1	23,770.0	....	6,862.0	....	....
Mean	18	392.0	1,980.0	28.9	....	5.9	....

[a] In pounds per square foot per hour.

alkalinity reduction equivalent to 7.5 volumes of wash water. The gain represented by these figures may be attributed to counter current elutriation, which was used throughout the year.

TABLE 7.—COSTS OF SLUDGE ELUTRIATION, DEWATERING, AND DISPOSAL

Item	Description	Dollars
1	Ferric chloride (Fe_2Cl_3)................	8,500
2	Other operating costs of elutriation and dewatering.......................	23,313
3	Total (Items 1 and 2)................	31,813
4	Cost of sludge disposal (railroad shipment)...........................	20,929
5	Total operating cost (Items 3 and 4)	52,742

Table 6 shows the detailed operation of the sludge filters for the year. For the entire period, 23,770 tons of sludge cake were produced, with an average solids content of 28.9%. These 6,862 tons of dry solids were dewatered by the vacuum filters at the average rate of 5.9 lb per sq ft per hr. The ferric chloride used during the period was 218.54 tons, or an average dose of 3.18%. Costs of sludge dewatering are as given in Table 7.

TABLE 8.—UNIT COSTS OF SLUDGE TREATMENT AND DISPOSAL FOR 1941

Item	Description	General operating costs	Ferric chloride	Sludge disposal	Total
1	Per ton of sludge cake............................	$0.98	$0.36	$0.88	$2.22
2	Per ton of dry solids in the cake....................	3.40	1.23	3.06	7.69
3	Per million gallons of sewage treated..............	0.59	0.21	0.53	1.33

Table 8 indicates the unit costs of sludge treatment and disposal for the year. It should be emphasized that these costs are for sludge elutriation, vacuum filtration, and cake disposal only and do not include costs of sludge digestion.

SUMMARY

At the District of Columbia Sewage Treatment Plant, elutriation of primary digested sludge prior to vacuum filtration has been found beneficial to the dewatering process and economically justified. Some simplifications in the sludge mixing facilities have been made, and measures for prolonging the life of the woolen filter cloths have been practiced.

POSSIBLE ECONOMIES IN SLUDGE DISPOSAL PRACTICE

By George J. Schroepfer,[4] Assoc. M. Am. Soc. C. E.

Synopsis

Because of the fact that considerable information has been published (6)(7) concerning the processes used and the results of operation of the sludge disposal system at the Minneapolis–St. Paul (Minn.) plant, such data as are presented herein are limited in scope and are condensed and arranged to illustrate certain points which the writer wishes to make.

The sludge disposal processes at the Minneapolis–St. Paul plant were placed in operation in July, 1938. They consist essentially of concentration, dewatering on vacuum filters, and incineration. The vacuum filters, of which there are six, each have an area of 500 sq ft. The three incinerators are of the multiple-hearth type; each has eight hearths and is 22.25 ft in outside diameter.

Operation and Cost Data at the Minneapolis–St. Paul Plant

In Table 9 are shown comparative data on sludge filtration and incineration by years. It is interesting to note that it has been possible to maintain an average solids concentration in the sludge prior to filtration of more than 9% (91% moisture). The moisture content of the sludge cake has consistently averaged approximately 65%. In connection with statements that will be

TABLE 9.—Comparison of Sludge Filtration and Incineration Data by Years; Minneapolis–Saint Paul Sanitary District

Item	Description	1938[a]	1939	1940	1941[b]
1	Sewage treated (mgd)	88.9	102.9	104.2	111.9
2	Raw sewage strength, suspended sewage solids (ppm)	240	285	300	315
3	pH-value of concentrated sludge	6.0	5.8	5.8	5.7
4	pH-value of conditioned sludge	11.7	11.6	10.5	10.0
	Total Solids (%):				
5	Raw sludge	7.15	7.79	8.07	8.11
6	Thickened sludge	9.02	9.13	9.34	9.35
7	Filter cake	35.0	34.7	35.2	35.5
8	Volatile solids, filter cake (%)	52.1	59.7	59.8	58.8
9	Suspended solids, filtrate (ppm)	315	260	165	135
10	Filter cake production (wet tons daily)	172.0	303.3	308.3	305.7
11	Filter cake production (dry tons daily)	58.6	103.3	108.4	108.6
12	Filter rate (lb per sq ft per hr, dry)	5.50	4.73	4.29	3.90
13	Detention period in conditioning tanks (min)	13	12	10	9
	Conditioning Chemicals; Per Cent of Dry Sewage Solids:				
14	Lime as Ca O	10.3	5.68	4.76	3.81
15	Ferric chloride	3.17	2.10	1.92	1.60
16	Incineration, power (kw-hr per ton of dry solids)	18.5	17	17	14
17	Incineration, fuel oil (gal per ton of dry solids)	3.7	2.3	2.5	1.5

[a] July to December, inclusive, 1938. [b] January to October, inclusive, 1941.

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[4] Chf. Engr. and Supt., Minneapolis–St. Paul San. Dist., St. Paul, Minn.

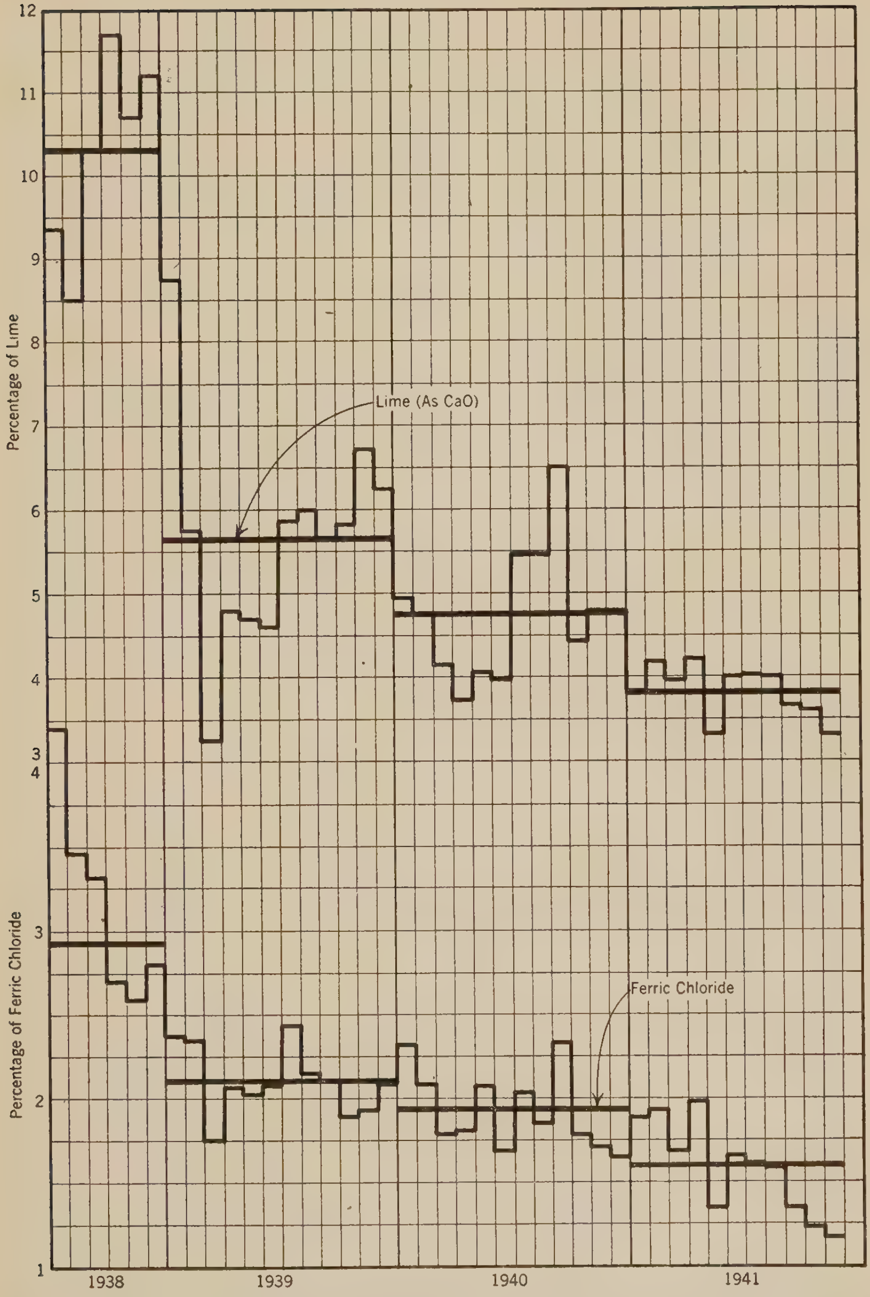

FIG. 2.—CHEMICALS FOR SLUDGE CONDITIONING, MINNEAPOLIS-SAINT PAUL (MINN.) SANITARY DISTRICT (EXPRESSED AS PERCENTAGES OF DRY SEWAGE SOLIDS)

made subsequently concerning this matter, the progressive and planned reduction in the filter rate from 5.50 lb per sq ft per hr in 1938 to 3.90 lb per sq ft per hr in 1941 should be noted. This ties in with the progressive reduction in chemicals shown in Table 9 and illustrated more graphically in Fig. 2. The reduction of chemicals to less than one half of their previous values is of some economic magnitude.

Costs of filtration and incineration for the various years of operation are shown in Table 10. The costs of operation and maintenance of the sludge disposal processes have been decreased from $6.97 per dry ton in 1938 to $3.41 in 1941. It is of interest to point out the relative magnitude of the cost of sludge disposal (which in this project has been between 40% and 45% of the total cost), as compared to the total cost of the collection, treatment, and sludge disposal processes. As shown in Table 10, the cost of chemicals for sludge conditioning has decreased progressively from $2.20 to $0.97 per dry ton. These are based on present chemical costs, which are somewhat lower than early costs. The magnitude of these reductions is shown in the daily costs to filter the present quantities of sludge. From a value of $242 per day in 1938, the costs have been reduced to $106.20 in 1941. This amounts to a total annual reduction of $50,000.

TABLE 10.—COMPARISON OF FILTRATION AND INCINERATION COSTS BY YEARS; MINNEAPOLIS–SAINT PAUL SANITARY DISTRICT

Item	Description	1938[a]	1939	1940	1941[b]
	Filtration Cost, in Dollars:				
1	Operation	47,428.74	92,177.80	76,568.91	60,744.22
2	Maintenance	2,990.74	6,932.35	5,306.02	5,706.86
3	Total	50,419.48	99,110.15	81,874.93	66,451.08
	Incineration Cost, in Dollars:				
4	Operation	19,233.60	32,393.09	35,697.84	27,007.48
5	Maintenance	1,808.60	4,691.93	14,524.63	13,522.59
6	Total	21,042.20	37,085.02	50,222.47	40,530.07
7	Dry sewage solids disposed of, in tons	10,248.0[c]	37,703.0[c]	37,319.0	31,347.0
8	Filtration cost, in dollars per dry ton	4.92	2.64	2.19	2.12
9	Incineration cost, in dollars per dry ton	2.05	0.99	1.34	1.29
10	Total cost of sludge disposal, in dollars per dry ton	6.97	3.63	3.53	3.41
11	Cost of entire project (collection, treatment, and sludge disposal), in dollars per dry ton	15.40	7.98	8.18	8.25
	Conditioning Chemicals, % of Dry Sewage Solids:				
12	Lime as Ca O	10.30	5.68	4.76	3.81
13	Ferric chloride	3.17	2.10	1.92	1.60
	Cost of Chemicals, in Dollars per Dry Ton:				
14	Lime at $9.30 per ton (Ca O)	0.96	0.53	0.44	0.35
15	Ferric chloride at $39	1.24	0.82	0.75	0.62
16	Total	2.20	1.35	1.19	0.97
17	Cost of chemicals to filter 110 tons of dry solids, in dollars per day	242.00	149.00	131.00	106.70

[a] July to December, inclusive, 1938. [b] January to October, inclusive, 1941. Supplementary data for 1941, 1942, and 1943 are given in Table 24. [c] Includes conditioning chemicals.

IMPROVEMENTS IN FILTRATION AND INCINERATION AS APPLIED TO THE MINNEAPOLIS–ST. PAUL PROJECT

In order that the remarks made herein can be evaluated properly in the light of definite conditions, certain improvements and operating goals as they apply to this project are mentioned briefly. This has the further advantage of restricting the observations to certain particular conditions, since the inconsistency of sludge behavior characteristics makes general observations inapplicable or even misleading. For this reason improvement changes effected at this plant should not be considered as "cure-alls," but rather as possibilities for exploration and development as they might apply to a different plant.

Concentration of Sludge.—It is a commonly recognized fact that, within limits, the more concentrated the sludge that can be produced the greater will be the efficiency of vacuum filtration, since, with a concentrated sludge, the gallonage of liquid sludge per ton of dry solids is reduced, resulting in decreased chemical costs and less work for the vacuum filters. The concentration of sludge in the Minneapolis–St. Paul plant is controlled by weighing a known quantity of sludge that is pumped from the sedimentation tanks at frequent intervals, as well as by soundings and determinations of moisture content of sludge in the tanks. The moisture content is further reduced by the control of the concentration tanks in which a further separation and concentration are effected by reason of the withdrawals of supernatant liquor. That such procedures are effective is shown by the fact that the moisture content of the sludge as removed from the settling tanks averaged 8.1% in 1940 and 1941, with an increase in solids, by concentration, to an average of 9.35% (see Table 9).

Conditioning of Sludge.—Considerable improvement in the conditioning of sludge, and resulting reduction in the cost of filtration, can be effected by improving the process of mixing in the conditioning tanks. In the case of air mixing, which is used at the Minneapolis–St. Paul plant, better results were obtained by increasing the number of points through which air is admitted to the sludge for mixing and by decreasing the pressure and violence of the mixing. In addition, the path of the sludge, which formerly passed through the tanks with over and under baffles, was modified to create a sinuous path, both vertically and horizontally, which procedure resulted in improved mixing. Furthermore, the detention period in the conditioning tanks has been reduced from 13 min in 1938 to 7 or 8 min at the close of 1941.

Instead of adding the chemicals to the tanks through one or two pipes, riffle boards for both lime and ferric chloride have been installed. This has resulted in considerably improved dispersion of chemicals.

Suitable chemical feed equipment must be provided with an adequate range and careful checking to keep the machines in accurate adjustment. In this connection, it is desirable to make tests with different chemicals to protect the plant against the inability to secure the chemicals regularly used. Instead of lime and ferric chloride, which are the usual conditioning chemicals, plant scale tests have been conducted using ferric sulfate and lime, and ferric chloride alone. As a safeguard, a quantity of ferric sulfate has been stored for more than a year. Although not as economical as the present combination (approximately 25%

more expensive), the usefulness of substitute chemicals in emergencies has been demonstrated.

Filter Mediums.—The type of cloth used as a filter medium should be considered. After some, but not extensive, experimentation, 8-oz Canton flannel has been adopted at this plant. However, the difficulty of obtaining this material during wartime, and its increased cost, have necessitated additional investigations leading to the possible use of a new cloth type. The life of filter cloths is also an important subject for investigation. For experimental purposes, cloths have been used for as long as 800 hr, but such procedures are distinctly not economical for the type of sludge disposed of at the Minneapolis–St. Paul plant. Newer cloths give somewhat higher yields for the same quantity of chemicals and, conversely, require less chemicals for the same yield. With high chemical dosages, the importance of this factor is not readily apparent, but with dosages approaching 1% of ferric chloride and 3% of lime, such factors show up quite markedly. In spite of the fact that the cloth is structurally in good condition, its use has been restricted to 350 hr. In recent months, because of increased cost of cloths, a longer period of usage has been adopted. When it is realized that cloth and wire constitute only approximately 3% to 10% of the cost of chemicals, the economy of maintaining satisfactory cloth will be readily apparent.

Tests have been made on a plant scale to demonstrate the effect of cloth age on filter yields. As a result of such tests, it has been shown that there is a loss in yield of 2.5% for each 100 hr of additional cloth life. Thus, at 500 hr, the yield of a filter is approximately 10% to 15% less than that of a new cloth, other conditions being constant. These results apply to an interval of 4 hr between washings (which is current practice at this plant). With longer intervals between washings, the yield with low chemical dosages falls off rather rapidly for older cloths. In fact, even after the first hour or two, the yield of older cloths drops off.

The aforementioned tests were conducted in an investigation to determine economical cloth life. The results of this study for present Minneapolis–St. Paul conditions are as summarized in Table 11. This comparison is based on present cloth costs, in 50-yd bolts, of $0.235 per sq yd, and indicates the economical life to be about 400 hr. At different cloth prices, such as the previous price of $0.135 per yd, or with varying chemical unit prices or required additions, the economical balance, of course, would vary.

TABLE 11.—COMPARISON OF DAILY COSTS OF FILTER CLOTH AND CHEMICALS (DOLLARS)

Description	Cloth Life, in Hours							
	100	200	300	400	500	600	700	800
Wire and cloth	36.00	18.00	12.00	9.00	7.20	6.00	5.15	4.50
Chemicals	73.00	75.50	78.00	80.00	82.00	84.00	86.50	89.00
Total	109.00	93.50	90.00	89.00	89.20	90.00	91.65	93.50

Regardless of the age of the cloth, an appreciable improvement can be effected by proper washing. It has been found that the washing can be improved considerably by rearranging the washer sprays to impinge along the filter drums almost at right angles to the drums, instead of tangentially, as in the usual installations. However, regardless of the efficiency of washing, certain material remains in the cloth even after extended periods of washing. Analysis of the blinding material in a number of cloths after approximately 300 hr to 400 hr of service indicates that approximately 60% is calcium carbonate and approximately 20% is grease. At the present price of cloth, it appears to be economical to give each cloth an acid bath, using one to two carboys of hydrochloric acid (in 600 gal of water), together with an inhibitor, after approximately 350 hr of service, thereby reducing the blinding and increasing the economic cloth life to approximately 500 hr to 600 hr. This expedient has the further advantage of providing an inexpensive means of removing calcium carbonate from the screens and splines of the filters, which formerly became so incrusted with material as to interfere with the free passage of water through the filter. This material was previously removed by sand blasting and other mechanical means, but because of savings in time and expense, and for other reasons, the chemical method of removal is more desirable.

It has been found economical, in the case of the Minneapolis–St. Paul plant, to wash filters frequently. During the first several months of operation, washing was confined to intervals of approximately 24 hr or longer. This was changed later to at least one washing of each filter every 8 hr, and in the last several months a washing has been required at least every 4 hr. This has marked effects on the quantity of chemicals required. In fact, the importance of maintaining a clean filter medium has resulted in the development, by the staff of the Sanitary District, of a method of washing filters while they are in service, either continuously, or at frequent intervals, as dictated by the relative economics. Although this method has not been incorporated in this plant to date (1942), because of its recent conception and the effect of priorities on procurement of the necessary material, its simplicity, and therefore its likelihood of operating satisfactorily, justify a description herein. Instead of its usual location, on the far side of the filter from the take-off plate, the wash-water pipe and nozzles would be installed immediately below the top edge of the take-off plate, so that the washing operation would be done in the section of the drum between the take-off and the submergence in the sludge, which section is not utilized in the usual installation. A trough suspended immediately above the sludge level would return the spent wash water to the overflow box on the filter. It is expected that, in addition to the advantages gained through maintaining a clean filter medium, the installation of this method of washing, in effect, will result in the addition of one filter, since it is necessary to have one filter in reserve a major part of the time to take the place of filters being washed.

Economic Speeds of Rotation of Filter Drums.—It is commonly recognized that, within limits, a change in speed of the filter drum increases or decreases the filter yield. It is possible to determine for each size of installation and type of sludge the economic range of speeds at which the filters ordinarily should be operated.

Adequate Filter Area.—The filter rate has a marked effect on the economics of filtration. Where incineration or drying is the practice, it is desirable to have a multiplicity of units so that uniform feed to subsequent units can be maintained at all times. An important loss of filter time results from washing of the filters and, to hold feeds constant, it is advisable to have a spare unit, in addition to the regular operating filters, to use during washing of regular units.

Return of Incinerator Ash.—Other papers by the author mention the advantage of returning incinerator ash to the raw sewage after filtration and incineration. This possibility, as well as a variation of it—adding ash to the sludge immediately prior to conditioning—bears investigation.

Incineration.—The desirability of maintaining a uniform feed of solids to the incinerator has been mentioned. By doing so, it is possible to improve operation and reduce maintenance cost. Sudden and large changes in sludge feed result in rapid changes in furnace temperatures, with consequent long-time damage to refractory materials.

As an experiment, the preheater has been removed on one of the three incinerators, which results in considerable savings in power (which constitutes 25% of incinerator costs) and maintenance costs. Although the original preheater was still serviceable, the result of investigations seemed to justify removal of the preheater from one furnace. This was done in May, 1941. Operation of this unit indicates a marked reduction from 17 kw-hr to 5.5 kw-hr in the quantity of power required. Operation of the unit over a period of six months has justified similar changes in the other two units at an estimated savings in power, replacement, and maintenance costs of $12,000 annually. This will result in a reduction in operation and maintenance costs of incineration of approximately 33¢ per ton.

Attention is called to the reduction in the quantity of fuel oil required by the incinerators (Table 9). Only 1.5 gal of fuel oil are now required per ton of dry solids incinerated, this quantity being required in heating up, cooling down, and holding temperatures in incinerators not in operation. A substantial savings in this item was effected by the installation of duct work so that the heat available in operating furnaces could be utilized to heat up, cool down, or maintain temperatures in any desired unit. In view of the fact that fuel costs are a minor item in sludge disposal expense at this plant (averaging 5¢ to 10¢ per dry ton), these savings are not of major importance, but are cited to indicate the possibilities available.

An interesting possibility relates to the utilization of the heat resulting from incineration. Various methods have been developed which indicate the possibility of generating sufficient steam for all heating purposes and electric power requirements.

In general, it can be stated that the possibility of effecting major economies and that the operating difficulties encountered are somewhat less in connection with sludge incineration as compared to vacuum filtration. The difficulty of making marked savings in incineration costs is evident from the fact that approximately 50% of the cost is labor, of which the major part is the time of one man on each shift to operate the incinerators.

EFFECT OF FILTER RATE ON COST OF FILTRATION

In the past, it was erroneously believed that the measure of the efficiency of sludge filtration was the thickness of the filter cake produced, a heavy mat being considered highly desirable. This mistaken impression has sometimes resulted in the use of higher filter rates in both design and operation than are economical. As an example, a number of plants are operating with filter rates of 5 to 10 lb per sq ft per hr. In some cases the installed area and filter size preclude the use of lower rates by the operator. Because of its importance to filter costs, the writer devotes considerable space to a discussion of this factor.

In the early period of operation of the Minneapolis–St. Paul plant, when chemical additions of about 3% of ferric chloride and 10% of lime were utilized, filter rates of 8 to 10 lb per sq ft per hr were common, the usual cake thicknesses being between 0.5 in. and 0.75 in. In addition to causing other difficulties (which are enumerated in earlier papers by the author), this type of operation was distinctly not economical. As shown in Table 12, filter rates were progressively reduced so that by October, 1941, the rates were only 3.55 lb per sq ft per hr. The attendant reduction in chemical requirements is also shown, but it should be emphasized that not all of this reduction can be attributed to the reduction in rate, as certain improvements and better operation were accomplished simultaneously. The test referred to in Table 12 was made during one day of operation with other influencing factors maintained constant, so that the effect of filter rates on chemical costs could be observed. The influence of filter rates on costs of operation and maintenance is shown in Table 13 and on total annual charges in Table 14. Referring to Fig. 3, it will be observed that chemical costs at high filter rates far outweigh other affected cost items, whereas at low rates the reverse is true. Conditions at the Minneapolis–St. Paul plant indicate that the economical filter rate is approximately 2 lb per sq ft per hr, based on total annual charges (including fixed charges). Considering operation and maintenance costs only, however, the economical rate is approximately 1 lb per sq ft per hr. It will be observed that the operation and maintenance costs at a 2-lb rate are less than half of what they would be at a 7-lb rate. Based on the tonnages produced at the Minneapolis–St. Paul plant, a

TABLE 12.—EFFECT OF FILTER RATE ON REQUIREMENTS OF CHEMICALS FOR SLUDGE CONDITIONING

No.	Description	(a) No. of Filters Used (Test on Plant Scale)[a]			(b) Average Results, by the Periods:			
		6	4	3	1939	1940	1941[b]	Oct., 1941
1	Filter rate (lb per sq ft per hr)	3.00	4.60	6.15	4.73	4.29	3.90	3.55
2	Cake thickness (in $\frac{1}{32}$ in.)	5	8	12	8	7+	7−	6
	Chemical Requirement:[c]							
3	Ferric chloride	0.89	2.33	2.35	2.10	1.92	1.60	1.23
4	Lime as Ca O	2.63	5.03	7.55	5.68	4.76	3.81	3.60

[a] For a production of 110 tons of dry solids per day. [b] January to October, 1941, inclusive.
[c] Percentages by Dry Weight of Sewage Solids.

reduction in the filter rate from 7 lb to 2 lb per sq ft per hr would result in a saving in operation and maintenance costs of approximately $125 per day, or about $45,000 annually. Expressed in another way, the installation of one more filter in this plant would result in savings in operation and maintenance of $14 per day, or approximately $5,000 per yr, which would pay for the additional investment in two to three years. The magnitude of the costs involved compels attention to this important phase of filtration economics.

TABLE 13.—OPERATION AND MAINTENANCE COSTS, MINNEAPOLIS–SAINT PAUL SANITARY DISTRICT

(Based on the Production of 110 Tons of Dry Sewage Solids per Day; Operation Labor Not Included)

		CHEMICALS				COSTS, EXCEPT CHEMICALS[b]					TOTAL COSTS	
		Quantities[a]		Costs								
Filter rate (lb per sq ft per hr)	Filters required (1)	Lime as Ca O (2)	Ferric chloride (3)	Dollars per dry ton (4)	Dollars per day (5)	Power (6)	Maintenance (7)	Changing cloth and wire (8)	Washing (9)	Total, Cols. 6 to 9 (10)	Cols. 5 and 10 (11)	Dollars per dry ton (12)
2	9	2.30	0.85	0.55	60	25.90	7.40	15.60	1.35	50.25	110.25	1.00
3	6	3.40	1.25	0.80	88	25.10	4.95	10.50	0.90	41.45	129.45	1.18
4	4.5	4.50	1.65	1.07	118	24.80	3.70	7.85	0.70	37.05	155.05	1.41
5	3.6	5.70	2.05	1.34	147	24.40	2.95	6.10	0.55	34.00	180.00	1.64
6	3	6.80	2.50	1.62	178	24.20	2.45	5.20	0.45	32.30	210.30	1.91
7	2.5	8.00	2.90	1.87	206	24.00	2.10	4.35	0.40	30.85	236.85	2.15

[a] Percentage of dry weight. [b] Dollars per day.

TABLE 14.—TOTAL ANNUAL CHARGES, IN DOLLARS; MINNEAPOLIS–SAINT PAUL SANITARY DISTRICT

(Based on a Production of 110 Tons of Dry Sewage Solids Daily)

					DOLLARS PER DAY				
Filter rate (lb per sq ft per hr)	Area of filters required (sq ft) (1)	First cost of equipment installed (2)	Total cost of filters and structures (3)	Annual fixed charges (6%) (4)	Fixed charges (5)	Chemicals[a] (6)	All other[b] (7)	Total (8)	Total charges[c] (9)
1	9,000	250,000	370,000	22,200	61.00	30	75.00	166.00	1.51
2	4,500	130,000	240,000	14,400	39.50	60	50.25	149.75	1.36
4	2,250	75,000	145,000	8,700	23.80	118	37.05	178.85	1.63
6	1,500	55,000	105,000	6,300	17.30	178	32.30	227.60	2.07

[a] From Table 13, Col. 5. [b] Other differential operation and maintenance costs from Table 13, Col. 10. [c] Dollars per dry ton.

What has been stated concerning this matter is subject to the following local and practical limitations: It is based on the conditions (type of sludge, chemical costs, etc.) existent at this plant. A comparable analysis can, and should, be made by other operating plants, based on the conditions obtaining there, to determine (*a*) the economics of operation of the present installation, (*b*) the possibilities of effecting economics by the enlargement of present facilities at some future date, and (*c*) economical rates for different types of sludges for the benefit of designers of future plants. Other practical limitations relate to the possible inability of present equipment to remove cake when it becomes exceedingly thin, although that limitation would not be effective at the economical value of 2 lb per sq ft per hr (⅛-in. thickness) determined herein. Furthermore, the cost of operation labor (maintenance labor and labor for changing cloth have been assumed to vary) has been assumed constant in this analysis, since it is quite definite for this plant that the same personnel which now operates six filters could operate eight filters (approximately the 2-lb rate), which number is dictated by the present size of this plant. Of course, the same number of men could not operate eighteen filters (1-lb rate) as would be required to operate three filters (6-lb rate). However, the usual decision in an operating installation will be on the question of whether one filter or, at most, two filters should be added. Therefore, usually no increase in personnel will be involved, since a large part of the operation labor is affected by the tonnage of material handled, which is assumed constant.

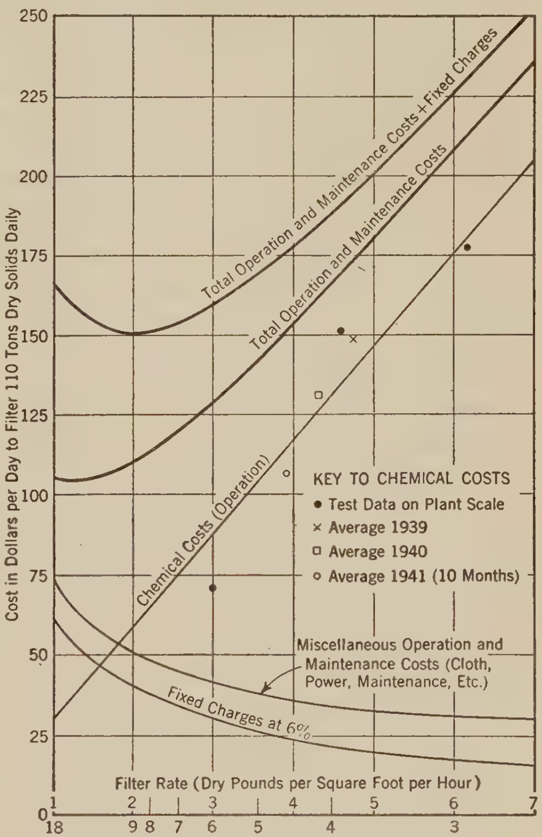

Fig. 3.—Daily Cost of Materials and Supplies, Power, Maintenance, Etc., to Produce 110 Tons of Dry Solids per Day

Costs of Dewatering and Incineration as Compared to Other Processes of Sludge Disposal

The relative complexity of efficient dewatering by vacuum filters presents again the question of the relative economy of this process of sludge disposal as compared to others available, particularly for the comparatively small plants. Sludge digestion and drying on sand beds are still satisfactory processes and should receive consideration in a decision on the most advantageous type of sludge disposal for a particular project. A number of factors, including practical and economic ones, usually will influence the decision in favor of dewatering, followed by drying or incineration, in large projects. In the case of very small plants, the decision is likewise comparatively simple in favor of the older type of processes. Dependent on the type of treatment by which the sludge is produced, and other factors, the decision in the case of plants serving populations in the range from 50,000 to 200,000 should be based on a thorough analysis of the many influencing and controlling factors involved. The quantities of material to be disposed of, as they influence the likelihood of securing efficient and economical operation of a more complicated mechanical and chemical process, must be considered. To expect that even the most conscientious operator of a plant in a community of, say, 25,000 people, in which 1 to 2 tons of dry solids must be disposed of daily, can even approach the efficient and controlled operation of a plant handling 50 or more tons daily is erroneous. Extending this idea further, a plant of the size of the Minneapolis–St. Paul plant handles quantities in a single week that the small plant referred to would encounter in a year or two of operation.

For illustrative purposes the costs of various sludge disposal processes are mentioned briefly. A. J. Fischer, Assoc. M. Am. Soc. C. E. (8), has estimated the costs of different sludge disposal processes as shown in Table 15. These are cited to show that, at the very least, an economic question is involved in medium-sized plants. Using actual costs as reported by the late H. F. Ferguson and W. H. Wisely, Assoc. Members, Am. Soc. C. E. (9), for communities using digestion and drying in a population range from 47,000 to 200,000 (to which range the writer makes special reference), the costs of operation and maintenance vary from $2.45 to $5.10 per dry ton (higher values for the smaller plants) and the total annual charges vary from $7.85 to $12.50 per dry ton. When it is realized that the operation and maintenance costs alone for several considerably larger plants using dewatering on vacuum filters, followed by incineration, range from $5 to $15 per dry ton, the point to which the writer alludes will be evident.

TABLE 15.—COSTS OF SLUDGE DISPOSAL, IN DOLLARS PER DRY TON

Population served	DIGESTION AND DRYING ON BEDS		DIGESTION, FILTRATION, AND INCINERATION	
	Operation and maintenance	Total annual charges[a]	Operation and maintenance	Total annual charges[a]
100,000	3.40	7.00	7.10	11.50
200,000	3.50	6.70	6.80	9.80

[a] Includes fixed charges on the investment at 7.5%.

Personnel Required.—The greater complexity of sludge filtration and incineration processes, both from mechanical and chemical standpoints, as compared with the older processes available, requires a higher quality of personnel for economical operation and maintenance. Of course, it is possible to filter sludge with large dosages of conditioning chemicals, and to incinerate sludge with high maintenance charges, but to perform this work economically requires a relatively high degree of skill. New problems are constantly arising and new possibilities must forever be explored. Whatever success the Minneapolis–St. Paul Sanitary District has had in economical operation of its sludge disposal processes is attributed largely to the advantage of having a relatively large number of professional engineers and chemists in supervisory positions.

It is not enough, however, to have men with technical training in charge. It is necessary that the interest and conscientiousness of the entire personnel, including especially the filter and incinerator operators, be maintained at a high level at all times. For example, as a means of awakening the interest of the filter operator, such possibilities as control of filtration by Buechner tests, frequent calculation of chemical dosages (as often as hourly), and shift competition on chemical requirements should be utilized. Operation should never be considered to have reached what appears to be the ultimate, so that standardization in practice, with consequent stagnation, will take place. New ideas (and there can be many) must be explored and developed continually, so that the entire staff, from the superintendent down, never loses sight of the possibilities of improvement. After all, a routine standardized task is certain to become uninteresting.

Dependability of Operation.—To accomplish the purposes for which a sewage treatment plant is constructed, it is necessary that the sludge disposal processes be able to handle the raw material produced as necessary. Digestion or a sufficient period of storage prior to filtration provides some leeway for the unexpected failures of equipment. Duplicity and multiplicity of units can be secured in the larger plants, but in the smaller plants this is sometimes not considered to be economically feasible. In addition to the economics of selection, dependability is another factor that must be taken into consideration in the decision between digestion and drying on sand beds and the mechanical processes. With from three to as many as several dozen vacuum filters in a larger plant, dependability as a factor is reduced in importance by thoughtful design; but in the smaller plants that are provided with only one vacuum filter and one drying or incineration unit the operator must be eternally vigilant to take advantage of all favorable opportunities to place his equipment in such condition that it will survive the period when its operation will be most in demand. Essentially, the point that the writer wishes to make is that, although operators of plants using digestion and drying may complain of the perversity of the bacteria in the digestion tanks, or the unpredictableness of the weather as it affects drying on beds, their problems dwindle into insignificance in comparison to those of the superintendent of a small plant with complicated and numerous mechanical devices which, even with proper care, display a persistent propensity to break down at the most inopportune times.

The statements concerning the efficiency and dependability of various processes of sludge disposal are made, keeping in mind certain limitations in their applicability. In large plants the decision is quite likely to be in favor of the mechanical processes, and properly so. In the small and medium-sized plants, thorough consideration must be given to all factors, including those peculiarly local factors which influence the decision. In the very small plants the writer feels that consideration should be given to the position of the operator, who is in the embarrassing position of complying with the requirements of public health officials as to dependable and satisfactory operation on the one hand, and meeting the demands of local public officials as to the economical operation on the other. The writer realizes, also, that all equipment improves in dependability and economy with development, and this has been the case with sludge disposal equipment. Furthermore, it is more than likely that, with careful observance of actual operation of equipment, along with more attention to details of design as they pertain to the smaller plants, on the part of designing engineers and equipment manufacturers, and that, with the present tendency to raise the standards of qualifications of small plant operators, mechanical processes, in the future, will be shown to be economical and dependable, even for the smaller plants.

Relative Costs of Filtration and Incineration

A review of the costs of filtration as compared with incineration at several plants will indicate the relatively large proportion of costs devoted to dewatering prior to incineration. The writer feels that with present practice too much of the cost of sludge disposal is devoted to the preliminary processes which serve simply to prepare the material for incineration. In spite of appreciable economies effected in filtration costs, the costs of dewatering at the Minneapolis–St. Paul plant are still approximately twice the costs of incineration. At several other plants this relation is still greater. As previously indicated, the possibility of savings in filtration is greater than in incineration. Developments in sludge disposal practice are likely to be along the line of reducing or eliminating the preliminary expenses. As a matter of fact, one or two small plants have been incinerating liquid sludges for some time. In connection with the studies of type of sludge disposal for the Minneapolis–St. Paul plant, the writer concluded in 1935 that, if the moisture content of the sludge could be reduced to 85% or 90%, dewatering on vacuum filters could be economically dispensed with, assuming that equipment was available for incinerating this type of sludge. The Sanitary District has begun experiments in which a portion of liquid sludge will be added to the filter cake in the incinerators as a means of determining the possibilities of this procedure.

Filter Rate Index

Because of the importance of the filter rate on the cost of filtration, it is desirable to use a more accurate index of filter rates than the commonly used unit of pounds per square foot of installed filter capacity per hour. This does not furnish an accurate measure of the number of square feet of filtering surface utilized in a given period, since it does not take into account varying filter speeds (changed quite frequently to hold uniform tonnage) which, of course, affect the filter area. Filter speeds used in two different plants may easily vary by as much as 50%. For this reason the writer suggests the use of the index, "pounds per square foot," based on actual filter surface used. To make the use of this unit possible, it is necessary only to install inexpensive revolution counters on the filter drums. Using the revolutions in a given period of time, the tonnage produced in that same period, and the area of the filter, it is possible to obtain an accurate filter rate for comparisons within a plant and with other plants. Incidentally, this furnishes a readily applied method of determining when it is desirable to change filter cloths, in terms of actual work done by the filter.

Summary and Conclusions

The writer has made certain observations and indicated certain possibilities of improving operations or effecting economies in sludge disposal processes. The presentation of this paper has afforded an opportunity to state certain general impressions that the writer has had for some time. He hopes that the discussions and considerations by others in this field will serve to clarify the relation between, and the applicability of, the various sludge disposal processes available, and thereby take advantage of the temporary lull in sewage disposal construction which will precede the period of expansion in the postwar era. The impressions of the writer are summarized as follows:

(1) Consideration should be given, in existing and projected plants, to the possibility of effecting economies by the utilization of lower filter rates than have heretofore been the general practice.

(2) A closer study and a better explanation for some of the occurrences in vacuum filtration can be made by the use of a filter-rate index, based on pounds per square foot of filter surface utilized. The index commonly used does not take into account the variation in filter speeds and other factors of importance in a close scrutiny of filtration characteristics.

(3) Since the decision between the mechanical processes and the older methods of sludge disposal is usually quite clear cut in the very large, as well as in the very small, plants, the writer suggests increased consideration in selection, in the case of medium-sized plants. Certain reasons are offered for the present inapplicability of the mechanical processes to the smaller plants.

(4) The writer feels the necessity of a higher degree of directive ability, for economical operation and maintenance of mechanical sludge disposal processes, than has heretofore been considered necessary. Even in the very small plants this should approach that of professional grade. In addition, the interest of all members of the staff must be promoted continually to improve the position of mechanical processes of sludge disposal.

USE OF SEWAGE SLUDGE AS FERTILIZER

By LeRoy W. Van Kleeck,[5] Assoc. M. Am. Soc. C. E.

Synopsis

The disposal of the settleable solids or sludge produced by sewage treatment plants is a major operating problem. Air-drying, incineration, use as fill, and other means of disposal for sludges have been discussed by others. The writer's plea is for more general use of sewage sludge, properly prepared as a soil conditioner, as well as for its fertilizer value. Failure to make use of it is a waste of a natural resource.

Introduction

One bomb dropped on London in 1939 was not released by a German war plane but by Sir Albert Howard, an eminent English agricultural scientist, who, in a paper (10) presented before the British Institute of Sanitary Engineers stated:

> "When the problems of waste disposal as a whole are considered, it is difficult to escape the conclusion that the West has completely lost direction. Why are the methods of the forest, in dealing with its wastes, more effective than those of the various Councils in our large cities? The answer is obvious if we study Nature's round—the wheel of life * * *. Trouble inevitably follows any attempt to short-circuit or to interrupt this wheel. This is always followed by disease in crops, in live stock and in mankind * * *.
>
> "The farmers of forty centuries, the cultivators of the East, the Chinese in particular, have closely followed the methods of the forest when dealing with their wastes. Nothing is ever thrown away. Vegetable and animal residues are mixed and converted into humus before being added to the land, so that the soil is not overworked by having to decay this material and grow a crop at the same time * * *. The consequence is that countries like China and Japan maintain over 2,000 people to the square mile as compared with about 60 in the United States. The Far East has been doing this for centuries without any loss of fertility. After less than one hundred years, the soils of the United States are almost worn out and a great national effort is now in progress to restore the lost fertility. You have all heard about the dustbowls and the floods of the Mississippi Valley—both the inevitable consequences of improper methods of agriculture which have taken no account of the correct utilization of wastes.
>
> "In Great Britain we are rapidly following in the steps of North America * * *."

In the voluminous discussion of this provocative paper, Lt.-Col. F. C. Temple stated (10*a*): "* * * engineers who have to deal with sewage or town refuse are not doing their job properly until they get the products from both back on to the land."

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In a written discussion of Howard's paper, Arthur J. Martin, past-president of the Institute, stated (10*b*):

> "The Author's protest against the neglect of our domestic wastes is very timely. The Sanitarian is notoriously a wasteful animal. He delights in destruction. He flouts the wise economy of nature and disregards the food cycle which she has established. We draw heavily year by year on the fertility of our soil and we squander plant food to the tune of scores of millions of pounds. We seek to redress the balance by means of artificial manures * * *.
>
> "The persistent neglect of sewage sludge is deplorable. It is doubtless attributable in part to the intense conservatism of the average farmer, but it is largely the result of the habit, to which the Author refers, of rating the value of a manure in terms of nitrogen, phosphorus and potassium and overlooking the humus of which our fields are so sadly in need * * *.
>
> "An engineer from Czechoslovakia told me last year that the whole of the sludge from Prague, a city of a million inhabitants, is utilized year by year on an area of eighty square miles. Some of the sludge is carried to a distance of fifty miles, and the supply cannot keep pace with the demand. Such instances go to show that sludge has only to be known to be appreciated * * *."

In view of the rather detailed information available on the use of sewage sludge as fertilizer (see bibliographical references (11) through (17), it would seem that for the purposes of this Symposium this paper can be confined to summarizing the latest information on the subject, together with the writer's personal experience with the use of sludges, particularly primary digested sludge, as fertilizer, or in a more appropriate term, as a soil conditioner.

USE OF LIQUID SLUDGE VERSUS DEWATERED SLUDGE

Liquid sludge normally contains from 85% to 98% water and is infrequently used as a fertilizer either in raw or digested form. The principal objections to its use are:

1. The large quantity of fluid bulk that must be transported unless pipe lines can be laid to near-by agricultural fields;
2. Difficulties of uniform application to the area to be fertilized;
3. The greater likelihood of objectionable odor; and
4. The possibility of pathogenic organisms being present. C. P. Mom and C. O. Schaeffer (18) isolated typhoid bacteria in liquid Imhoff tank sludge drawn to drying beds, but after drying no typhoid bacteria could be found. Although liquid sludge does contain additional fertilizer constituents which are lost in the drained liquor by dewatering processes, past experience encourages dewatering and storage of sludge as additional safeguards against the transmission of infection. No cases of sickness have been traceable to the use of properly conditioned digested or activated sludge.

Sludge is generally utilized, after it has been partly dewatered, by draining it on beds of sand (Fig. 4), by pressing, by centrifuging, or by vacuum filtration (Fig. 5). The partly dried sludge is then referred to as "sludge cake," and ordinarily contains 70% or less of moisture. Such cake is dry enough to be shoveled and spread like manure or moist soil.

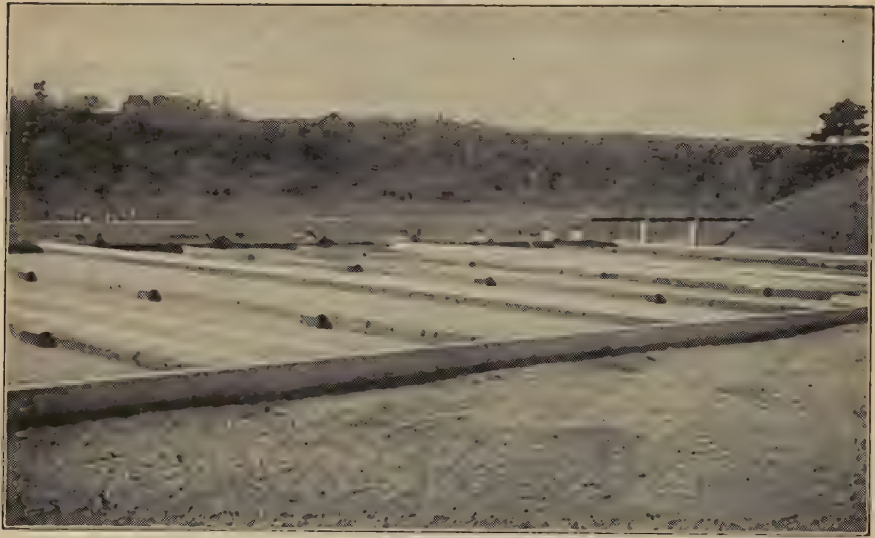

FIG. 4.—OPEN SLUDGE-DRYING BEDS

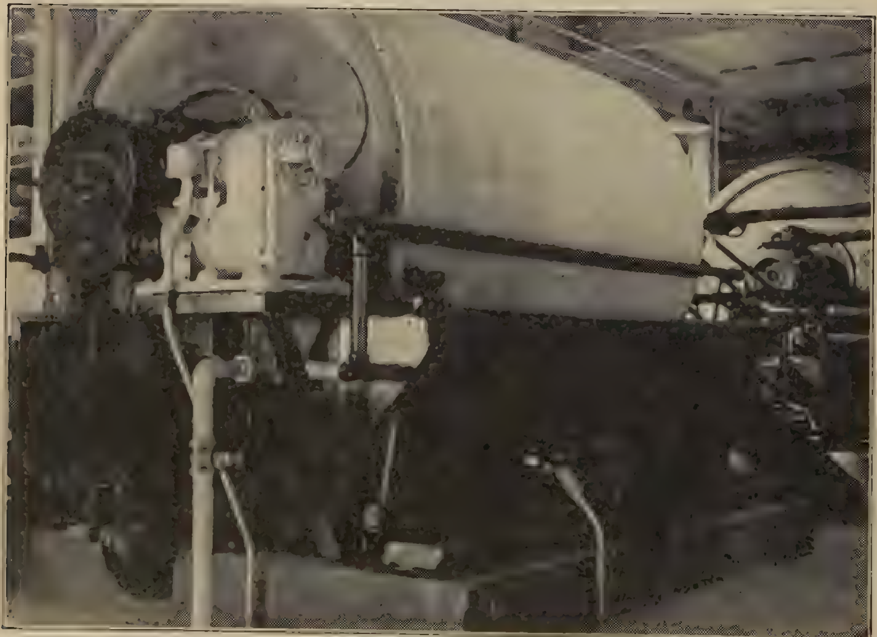

FIG. 5.—MODERN VACUUM FILTER FOR SLUDGE DEWATERING

USE OF RAW SLUDGE AS FERTILIZER

The use of raw primary sludge as fertilizer is not recommended, because:

1. It is odorous;
2. It has a higher grease content than digested sludge and is not readily assimilated by the soil;
3. It tends to bring about early and persistent soil acidity, especially if the sludge has been conditioned with iron salts;
4. Industrial wastes in the sludge may poison the soil;
5. It is a potential carrier of disease unless highly limed in sludge conditioning, when the high pH (usually ranging from 10.3 to 10.6) may destroy large numbers of bacteria. If heat dried, the bacterial content may be reduced; and
6. A sixth objection is that it is not humified for soil use. In this respect, Howard of England states (19):

> "The utilization of vegetable and animal wastes in crop production involves two definite steps, (1) the formation of humus and its incorporation into the soil, and (2) the slow oxidation of this product accompanied by the production of available nitrogen. Both of these stages are brought about by micro-organisms for which suitable environmental conditions are essential. The requirements of the first phase—the preparation of humus and its incorporation into the soil mass—are so intense that if the process takes place in the soil itself, it is certain to interfere with the development of the crop. The soil in fact gets indigestion! This explains to a large extent why crude sewage, crude refuse, and some of the sewage and refuse derivatives have been found wanting, or even harmful, by cultivators. In some places worse has happened; but it is clear that in many the first phase has not been sufficiently prolonged to allow the second to come into operation before the stuff was offered to the crop."

ACTIVATED SLUDGE

Raw or heat-dried activated sludge has a much higher fertilizing value than ordinary primary or secondary sludge for two reasons: One, the higher content of ammonia, and the other, its much higher availability. When digested, much of this fertilizing value is lost. Heat-dried and ground raw activated sludge is an organic fertilizer material especially suited to lawns because of its nitrogenous content. The analysis of Milwaukee sludge is as follows (dry basis):

Constituent	%
Nitrogen	6.2–6.5
Available phosphoric acid	2.5–2.75
Potash	0.5–0.75

The drying process is generally believed to free such sludge of harmful bacteria, although it does not completely sterilize it.

DEWATERED DIGESTED SLUDGES

Much of the digested sludge produced in the United States is so-called primary digested sludge—that is, from sewage solids that have been settled in tanks and subjected to anaerobic digestion. Secondary or humus sludge is frequently mixed with primary sludge and digested with it. Most of this paper is concerned with digested sludge. A comparison based on chemical analysis of digested primary sludge and related materials is afforded in Table 16. Items 2, 3, and 4, Table 16, are as reported by E. E. DeTurk (20) and Thomas P. Maloy (21).

TABLE 16.—COMPARISON OF TOTAL CHEMICAL CONTENTS (PERCENTAGE BY WEIGHT, DRY BASIS)

No.	Substance	Nitrogen	Phosphoric acid	Potash
1	Digested sludge[a]	1.7 (total)	1.5	0.15
2	Manure[b] (20)	1.97 (total)	1.29	1.96
3	Horse manure (21)	0.7 (available)	0.11	0.45
4	Sheep manure (21)	1.8 (available)	1.25	3.0
5	Commercial potato fertilizer	5.0 (total)	8.0	7.0

[a] Average of samples from a number of sewage plants in Connecticut. Some sludge cake produced in Connecticut also contains about 10% calcium oxide (lime). [b] Supposedly a mixture of animal wastes.

The nitrogen in sludge comes into action slowly, thus removing the danger of so-called burning of crops. Alleged burning of grass is usually caused by over-application of sludge cake, thus actually smothering the growing blades of grass, particularly if it is young.

Apparently sludge compares favorably (Table 16) with the ordinary run of manures. Its principal deficiency is in potash. According to DeTurk (20*a*): "The most important differences in the two materials are that the more easily decomposable nitrogen compounds of sludge have been more completely removed by its digestion process than is the case with manure, leaving a nitrogenous residue of lower availability (this is not shown by the table); and that the potash being mostly water-soluble has been largely washed out of the sludge while it is retained by the manure."

The chemical analysis of sludge does not, however, tell the entire story. Primary digested sludge is of considerable value as a soil conditioner, from its content of humus. Approximately 50% of the dry weight of sludge may be humus. Most of the organic matter in digested sludge is humus except for the ether-soluble fats. A sandy soil is particularly benefited by sludge, since its water holding capacity is increased. Unlike commercial fertilizers, sludge is not a balanced food for plants, nor is it an all-purpose plant food. Its application, however, works a mechanical change into soil, and improves its texture. The continued exclusive use of commercial fertilizers represents unsound and dangerous farming practice. Soils repeatedly planted to growing crops require the organic humus such as is found in manures, peat moss, green cover crops, and sludges. The value of sludge should not be judged only by comparison of chemical analyses with artificial fertilizers, but by the results it produces in plant growth.

Howard declares (10*a*), in connection with the use of unhumified materials:

> "We seem to be always thinking in terms of soil nutrients. This is a great mistake. In agriculture it is a question of the utilization of fungous tissue mobilized in the form of humus which really matters and which is one of the secrets of the agriculture of the East. This fungous question

> is far more important than nutrients and in this matter we have gone on completely wrong lines."

Rudolfs (22) believes that the soil improving and growth promoting values of sewage sludge are distinct and additional values to its fertilizer or plant food content by replenishing of organic matter and the minor elements needed by plants.

Digested sludge is comparatively free from weed seeds which are largely destroyed in the digestion process. Most manures on the other hand contain weed seeds which prove very objectionable, particularly when spread on lawns. Tomato plants and a few of the cucurbits may sprout in digested sludge, especially if the digesting temperatures are below 80° F, but they are not ordinarily troublesome.

Application of Digested Sludge and Hygienic Considerations

Sludge may be applied as a top dressing in the case of trees, shrubs, or lawns; in the hill in the case of melons, squash, or other vine crops; or it may be spread and then plowed or forked into the soil. When used as a top dressing for lawns, it should not be applied too thickly or it may choke the growing vegetation. Sludge may be applied when the ground is frozen or covered with snow. In such cases, it is raked and pulverized into the soil in the spring. Grinding sludge cake with machines (Fig. 6), or breaking it into smaller particles by hand, greatly enhances its application as a top dressing. Many sewage plants now wasting their sludge output would find a ready market if grinding were instituted. Except where applied to grass, trees, shrubs, or

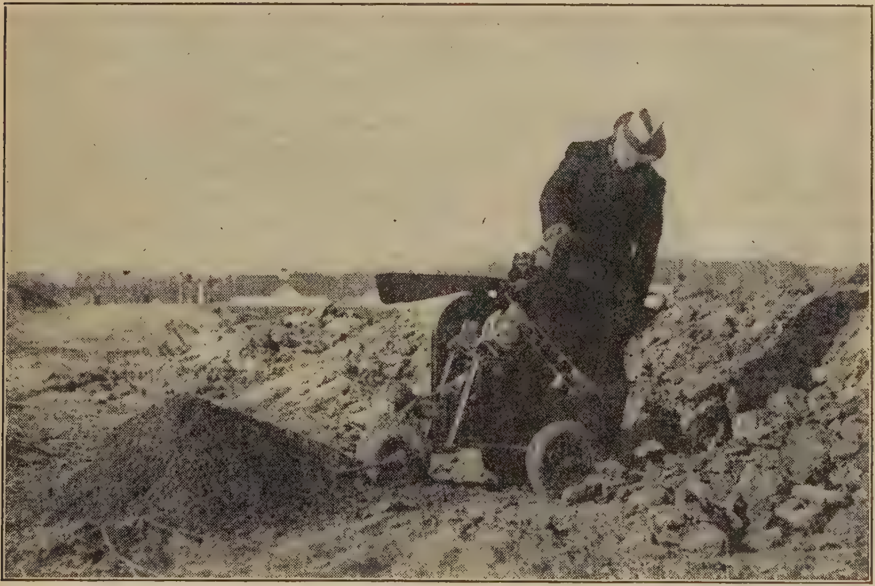

Fig. 6.—Grinding Makes Sludge Cake Suitable for Ready Use as a Fertilizer

perennial beds, sludge should be placed in the hill or plowed under in the fall or early spring before planting the crop.

Digested sludge is safe from the health standpoint if it is mixed with the soil and no more is applied to the growing crop. It is safer not to plant root crops eaten raw, such as carrot or radish, or vegetables grown in close contact with the soil, and eaten raw, such as lettuce, celery, or onions, in ground recently fertilized with digested sludge, unless the sludge cake itself has been subjected to storage for at least a month.

The digestion of sludge, combined with the more common methods of dewatering, such as vacuum filtration (in which case most sludges are conditioned with lime, giving a high enough pH-value to kill great numbers of bacteria) or air-drying, and its storage afford sufficient protection against intestinal diseases. However, reasonable care should be exercised in handling it.

Amount of Digested Sludge to Be Used

Although an excess of sludge will rarely do harm, except in cases of outright mechanical smothering of growing things, the following suggestions have been made for its application:

1. For orchards (15)	10 tons wet sludge cake per acre (or about 11 cu yd per acre)
2. For grass (16)	10 to 20 cu yd per acre (or 0.6 to $1\frac{1}{4}$ cu ft per 100 sq ft of soil)
3. For vegetables, shrubs, and flowers (15)	20 to 60 cu yd per acre (or $1\frac{1}{4}$ to $3\frac{3}{4}$ cu ft per 100 sq ft of soil)

Fortifying Digested Sludge

By mixing dried sludge with chemicals a balanced fertilizer can be obtained with the desired plant food. Fortified sludge should be applied in smaller amounts to the soil depending on the type of crop and the local conditions. The following formula is suggested for general use and is suitable for almost any crop or soil:

Chemical	Pounds
Sludge (dry basis)	360
Ammonium sulfate	25
Acid phosphate (20%)	50
Muriate of potash	15

In the case of dry sludge, 360 lb amounts to about 900 lb of sludge cake as removed from the average sand-drying bed, or 1,200 lb of cake as removed from the average vacuum filter. This is, roughly, two thirds of a cubic yard, or about 20 cu ft. Other formulas may be used for special cases, or for a particular soil.

Use of Sludge as Fertilizer

Sludge is particularly adapted to lawns, golf courses, pastures, and meadows, as well as to the flower and vegetable garden. Maloy (21) reports that trees fertilized with sludge had a healthier foliage, both in amount and color, and retained their leaves for a longer time in the fall than near-by unfertilized trees. A. M. Rawn, M. Am. Soc. C. E. (23), in 1941 stated that sludge has been used as fertilizer for citrus groves for about thirteen years with convincing results.

Sludge deepens the green color of grass and stimulates a luxuriant growth. Its benefits are noted into the second year. For flower beds it provides a much-needed humus for the hot summer months as well as a moderate but long-yielding amount of nitrogen. In vegetable gardens, it is beneficial for corn, potatoes, beans, spinach, asparagus, and vine crops.

For years the writer has grown plots of vegetables, shrubs, flowers, and grass with and without sludge. Its worthiness as a soil conditioner has been demonstrated by excellent plant growth and high yields.

When used continuously on land, sludge tends to acidify the soil. Hence, corrective measures must be taken every three or four years to avoid damage to plants requiring a neutral or near neutral soil, by checking acidity (caused largely by the breakdown of the greases and fats present in sludge) through judicious applications of powdered limestone, or hydrated lime.

Economic Value of Digested Sludge

While digested primary sludge does not have the market value of raw activated sludge, it may have an estimated value (based primarily on its chemical analysis only and without regard to its valuable humus content) of at least 50¢ per cu yd for unground sludge. Ground sludge brings a better price and is generally in demand. Mr. Rawn (23) states that dairy manure in Southern California in 1941 was selling for $1.50 per ton, delivered at the roadside. This is the commercial variety containing about 1.25% nitrogen. Digested sewage sludge (roadside delivery) was selling for $7.00 per ton in the same area. The difference in selling price, according to Mr. Rawn, is not indicative of high-pressure salesmanship; to the contrary, it portrays better than any other factor the difference in value, and is excellent proof of the fertility value of the product.

At most sewage plants, sludge is given away, or used as fill. The potential promoter or user of sludge can reasonably consider the cost of manure in his community as a rough index of the value of local sludge.

Why Digested Sludge Has Not Been More Widely Used as a Soil Conditioner

What are some of the reasons why digested sludge has not been used more extensively as a soil conditioner? Undoubtedly an esthetic dislike to its use, because of its source, and the fear of producing offensive odors are paramount in the minds of many laymen and agriculturists. Since properly digested, dried, and stored primary sludge and heat-dried and ground, raw activated sludge have been established as safe hygienically for application to the soil, there appears little room for argument from the health standpoint. With regard to odor, many of the more common animal manures, including cow, pig, and sheep wastes, have as much or more odor than sludge, especially if applied before thoroughly rotted or composted.

Another reason why sludge is not used is the lack of promotion for its use by many sewage plant superintendents and supervising officials. In some cases this is no fault of the operating personnel but is caused by a lack of time. The primary job of sewage treatment plant personnel is the protection of the public health and the prevention of nuisance from raw sewage, and not the manufacture or sale of fertilizer. Sludge disposal as fertilizer, when it involves salesmanship, storage facilities, grinding equipment, and in some cases bagging facilities, delivery and bookkeeping, is beyond the scope of many of the smaller treatment plants.

An objection to the use of sludge as a fertilizer is the lumpy or chunky physical form of much sludge cake, which makes it unsatisfactory for lawns, shrubs, and flower beds. Heat-drying and grinding, or grinding alone, produces a product much more suitable for home use.

There is a dearth of information among laymen on the proper application of sludge; its limitations; the need of occasional liming, and the types of vegetation for which it is adapted. Many homeowners and market gardeners could make excellent use of local sludge, but do not appreciate its humus value.

Because of the large quantity of sludge to be disposed of, or transportation costs, or other factors, the disposal of sludge cake by means other than as fertilizer has been found more practical or economical in most sewage treatment plants located in industrial areas or on the seaboard. Demands for sludge as fertilizer are seasonal. Storage is expensive. The cost of purchase and operation of equipment for drying and grinding wet sludge cake is appreciable. The cost of bags and bagging machinery is considerable. There is also the question of the market locally available. All these factors must be weighed in considering the problem.

In spite of these acknowledged limitations, however, the 1940 sewage plant operation reports for the cities of Minneapolis and St. Paul (Minn.) and New York, N. Y., carry favorable comments on the possibilities of using sludge at these plants for fertilizer rather than to continue present disposal methods of incineration and barging to sea.

According to Mr. Schroepfer (24), author of a companion paper in this Symposium:

> "When the Twin Cities sewage treatment plant was designed it was realized that an assured practical and economical method must be installed for regular day in and day out disposal of the large quantities of raw sludge filter cake to be produced. Incineration offered and still affords the most practical and economical method of disposal.
>
> "However, the Sanitary District has realized from the start that the sludge cake would have a certain value as a soil conditioner or low-grade fertilizer, somewhat comparable to manure, and that eventual development of its use for such a purpose would be desirable and might be of economic advantage to the District as well as to the farmers and people of the State. Manure is becoming scarce and a real need exists for a cheap, humus-containing fertilizer. The present method of disposal by incineration seems an economic waste of a natural resource, but the Sanitary District has no alternative unless another method can be developed at no added cost to the people of the Twin Cities. It is the District's hope that at least a portion of the sludge cake may be disposed of for use as a soil conditioner, to everyone's mutual benefit."

The report continues with remarks on experiments conducted to date. Results, it is reported, appear favorable, and it is noted that some of this raw sludge has been used in a fresh form. However, it is being recommended to experimental users that the sludge be stock-piled or composted for one month or longer before use. Not only will composting render it more friable and therefore easier to apply but, as has been discussed, it will permit changes to occur in the fresh organic matter so that a humus material for plant growth is available. Precautions from the health standpoint are mentioned, and it would seem that this phase should receive study before it is used extensively.

The New York City Department of Public Works states in its 1940 report (25):

> "The two primary objectives of sewage treatment are first, to reduce the sewage wastes to a liquid sufficiently pure to be unobjectionable, harmless, and relatively pure and secondly, to dispose of the impurities removed as sludge in an economical and unobjectionable manner. In this latter respect, this department has not lost sight of the possible value of the by-products of sewage treatment. In addition to the value of the combustible methane gas produced by the digestion of sewage sludge, the sludge residue after digestion has a distinct value as a soil conditioner."

The larger plants, of practical necessity, must grind and dry the sludge cake to create a sufficient market; otherwise the sludge is not in satisfactory physical form for immediate use and the high moisture content (60% to 70% in most cases) reduces materially the weight of actual dry sewage solids per pound of material. However, there are numerous small sewage treatment plants scattered over the United States from which natural digested sludge cake is obtainable for the cost of hauling. Certainly, such material is worth reasonable hauls of five to ten miles.

CONCLUSION

Admittedly, there are problems to be overcome at many treatment plants before all or (in some cases) even a part of the sludge cake can be disposed of as a soil conditioner. However, in view of the dearth of farm manure, the outlook for such disposal is encouraging with sound development.

EXPERIENCE OF CHICAGO, ILL., IN THE PREPARATION OF FERTILIZER

BY WILLIAM A. DUNDAS,[6] ESQ., AND C. P. MCLAUGHLIN,[7] ESQ.

SYNOPSIS

In the sludge disposal of The Sanitary District of Chicago (Ill.) a method has been worked out by a chain of experiments to dewater and heat dry activated sludge for use as fertilizer material or for incineration. The principles of design are indicated from the heat standpoint together with the control of odors. A typical heat balance is presented, with a discussion of the relative merits of various fuels. At the Southwest Works a combination of power generation with heat drying has been successfully used.

GENERAL

The Sanitary District of Chicago was established in 1890 to divert the sewage from Lake Michigan and later to treat sewage from a present area of approximately 442 sq miles. The present sewage load is from a population of approximately 4,000,000 and an industrial waste load that is equivalent to an additional population of nearly 3,000,000, making a total population equivalent to 7,000,000.

Practically all of the sewage is treated in four main plants (see Fig. 7)—the North Side, Southwest, West Side, and Calumet Sewage Treatment works. All plants, with the exception of the West Side Works, utilize the activated sludge process. Solids tributary to the West Side Works are disposed of by digestion in Imhoff tanks and drying on underdrained sand beds. Waste activated sludge is disposed of by dewatering, heat-drying, and processing for fertilizer uses or incineration at the Southwest and Calumet works. The waste activated sludge from the North Side Works is pumped to the Southwest Works for disposal with its waste sludge.

The sludge disposal facilities of the Calumet Works are designed to dispose of a daily average of 40 tons of dry solids by dewatering to 80% moisture on vacuum filters and heat-drying for fertilizer uses or incineration. The plant went into operation in July, 1936.

The sludge disposal facilities of the Southwest Works were designed to dispose of an initial average of 375 tons of dry sewage solids per day (100 tons of which comes daily from the North Side Works) by dewatering to 80% moisture on vacuum filters, followed by heat-drying for fertilizer uses or incineration. This plant went into operation in June, 1939.

When construction of the North Side Works was begun in 1923, sludge was to be disposed of by digestion in the Imhoff tanks of the West Side Works,

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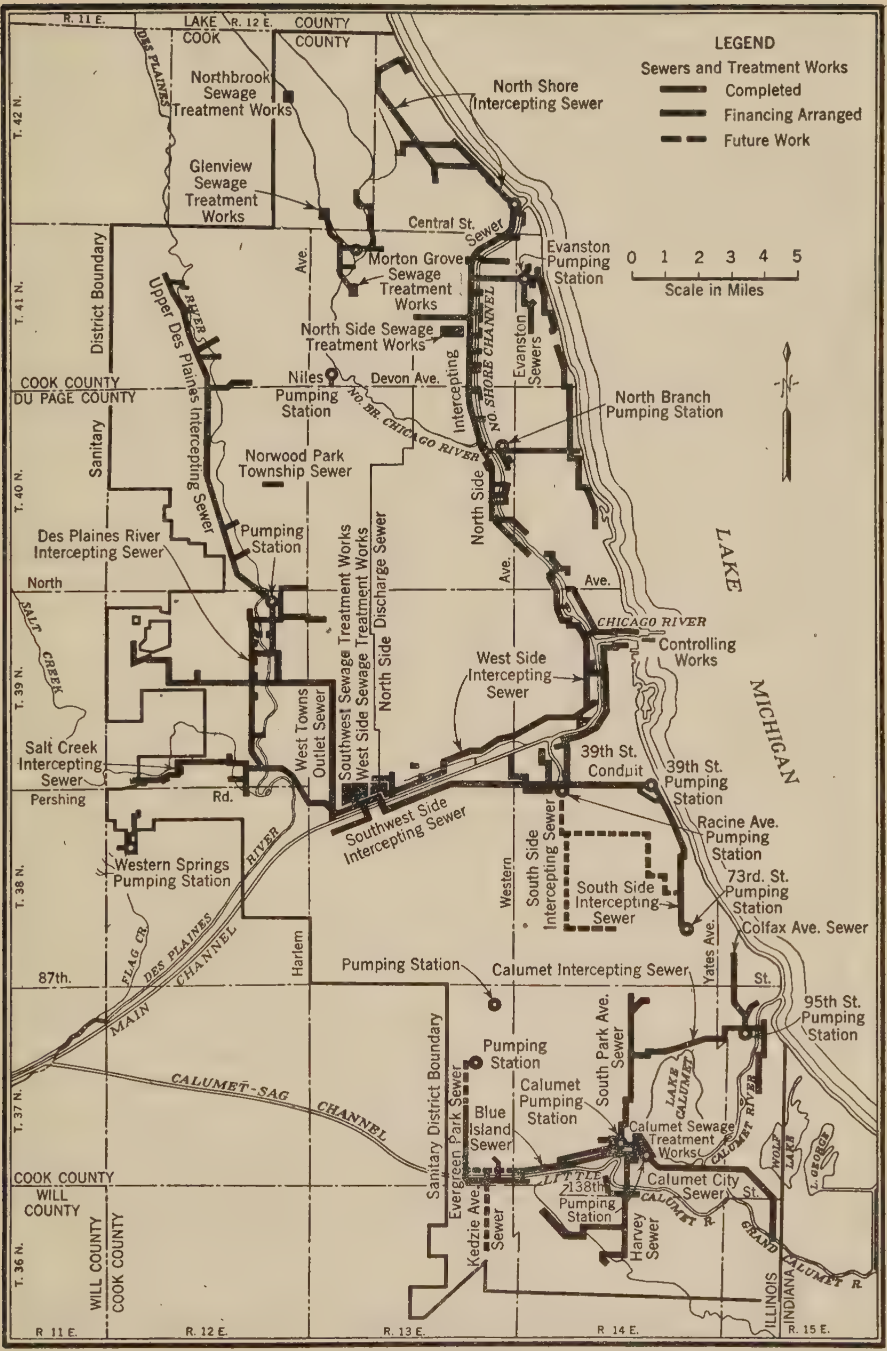

LEGEND
Sewers and Treatment Works
Completed
Financing Arranged
Future Work
0 1 2 3 4 5
Scale in Miles
LAKE MICHIGAN
R. 11 E.
R. 12 E.
R. 13 E.
R. 14 E.
R. 15 E.
T. 42 N.
T. 41 N.
T. 40 N.
T. 39 N.
T. 38 N.
T. 37 N.
T. 36 N.
LAKE COUNTY
COOK COUNTY
COOK COUNTY
DU PAGE COUNTY
COOK COUNTY
WILL COUNTY
WILL COUNTY
COOK COUNTY
ILLINOIS
INDIANA
DES PLAINES
Northbrook Sewage Treatment Works
Glenview Sewage Treatment Works
North Shore Intercepting Sewer
Central St. Sewer
Morton Grove Sewage Treatment Works
Evanston Pumping Station
Sanitary District Boundary
Upper Des Plaines Intercepting Sewer
RIVER
Ave.
North Side Sewage Treatment Works
Niles Pumping Station
Devon Ave.
NO. BR. CHICAGO RIVER
NO. SHORE CHANNEL
North Side Intercepting Sewer
Evanston Sewers
North Branch Pumping Station
Norwood Park Township Sewer
Pumping Station
Des Plaines River Intercepting Sewer
North
Ave.
Ave.
SALT CREEK
West Towns Outlet Sewer
Southwest Sewage Treatment Works
West Side Sewage Treatment Works
North Side Discharge Sewer
CHICAGO RIVER
Controlling Works
West Side Intercepting Sewer
39th St. Conduit
39th St. Pumping Station
Salt Creek Intercepting Sewer
Pershing
Rd.
Southwest Side Intercepting Sewer
Racine Ave. Pumping Station
Western Springs Pumping Station
RIVER
Western
South Side Intercepting Sewer
South Side Intercepting Sewer
73rd. St. Pumping Station
Yates Ave.
Colfax Ave. Sewer
FLAG CR.
DES PLAINES
CHANNEL
MAIN
Harlem
87th.
Pumping Station
Calumet Intercepting Sewer
St.
95th St. Pumping Station
South Park Ave. Sewer
Pumping Station
CALUMET-SAG CHANNEL
Evergreen Park Sewer
Blue Island Sewer
Calumet Pumping Station
LAKE CALUMET
CALUMET RIVER
Calumet Sewage Treatment Works
WOLF LAKE
L. GEORGE
LITTLE CALUMET R.
138th Pumping Station
Calumet City Sewer
St.
Kedzie Ave. Sewer
Harvey Sewer
GRAND CALUMET R.

FIG. 7.—SEWAGE TREATMENT PLANTS, SANITARY DISTRICT OF CHICAGO, ILL.

together with fresh sewage solids. The waste activated sludge was therefore pumped to the West Side Works. Although such mixtures of fresh sewage solids and activated sludge can be digested in this manner, the high moisture content of the digesting mixture indicated that a large tank capacity would be required. Moreover, the drying of such large volumes of digested sludge on underdrained beds would require large additional areas of land, and the use of the beds would be restricted to the warmer months of the year.

An intensive study of mechanical dewatering, heat-drying, and incineration was undertaken in 1931. An experimental plant handling 20 tons of dry solids daily by dewatering, drying, and incineration was constructed at the West Side Works and placed in operation in the summer of 1932. By February, 1934, this plant was remodeled to develop the flash system as applied to the drying of sewage sludge and to incineration by burning in suspension. Each of these plants was operated for a year on various sludges and on sludge mixtures. Throughout these tests a continuous type vacuum filter with an area of 570 sq ft was used for dewatering. Other tests were conducted on dewatering with a bag type of filter press and a paper-making machine.

The discovery by John R. Palmer, in 1925, of the efficacy of ferric chloride as a sludge conditioner was upheld by tests of other reagents. From this exhaustive group of experiments and from experience at the pilot plant at Calumet, certain fundamental facts and principles stood out as requisite for the efficient dewatering, drying, and combustion of sludge.

In the processing of activated sludge for fertilizer or burning, its moisture content must be reduced from approximately 97% to between 5% and 10%. To produce 1 ton of sludge containing 5% moisture, from sludge containing 97% moisture, requires the removal of 64,561 lb of water. Such a quantity of water is best removed by a combination of filtration and heat-drying. Economic and practical operating considerations have established the continuous type vacuum filter as the most satisfactory mechanical dewatering equipment for sewage sludges.

Before conditioning and filtration, sludge should be concentrated to reduce the quantity of water to be removed by the filter and to provide a filter feed of uniform consistency, which is a vital factor in successful filter operation. Sludge as drawn directly from final settling tanks or digesters may vary greatly in consistency. It is recommended, therefore, that some form of concentration be provided in all instances to assure uniformity in filter feeds.

Most sewage sludges do not filter freely and therefore require the addition of a conditioning chemical which coagulates the solids and releases the water. In dewatering sewage sludge by vacuum filtration, some form of conditioning has always been necessary. Ferric chloride alone is probably the most effective and economical reagent available for conditioning activated sludge. With other types of sludges, combinations of ferric chloride and lime are used and in some instances lime alone. Other available but probably less effective reagents are chlorinated copperas and ferric sulfate.

Another paper in this Symposium indicates that elutriation of digested sludge seems to offer a successful means for reducing the amount of chemical required for conditioning and improvement in filter operation. Elutriation or washing of the sludge seems to effect the removal of certain compounds in digested sludge that consume conditioning chemicals without producing conditioning effects.

Ferric chloride is the most expensive item of filtration cost in handling activated sludge. Close control of dosing is required. Underdosed or overdosed sludge will result in high cake moistures and low yields. The amount of ferric chloride required will be dependent upon the characteristics of the sludge involved. In Chicago the amount of ferric chloride varies daily, with a seasonal swing. Actual operating experience only will demonstrate the requirements for a particular case.

In the layout of the Chicago works, devices are installed to proportion the ferric chloride accurately and to mix it thoroughly with the sludge. Most devices that are available for the performance of this important function, although theoretically correct, fail in practical operation due to their complexity and inability to withstand the action of the chemical. Filtration should take place as quickly as possible after dosing and mixing. Continuous conditioning is used in Chicago. Routine filter operation requires checking at frequent intervals to determine the amount of chemical required for best operating results. Ferric chloride requirements of The Sanitary District of Chicago for conditioning activated sludge vary between 4% and 7% by weight of dry solids in the cake. Under these conditions, cake moistures may vary between 78% and 85%. Indications are that the amount of ferric chloride required may be related to the volatile content of the sludge.

On a continuous vacuum filter, as the cake mass forms on the filter cloth, each particle is under the pressure of the surrounding liquid due to the vacuum behind the cloth. As a particle lodges in the face of the cake, its motion decreases and the water flows by. A frictional drag is produced that causes the individual particles massed in the cake to press against each other. The finer particles will be carried on with the liquid into the cake to the point where they are stopped by frictional resistance either in the cake or within the filter medium. Examination of cake removed from a filter will show that it is more dense on the side next to the cloth than on the side away from the cloth. As the density increases, the size of the capillaries decreases and the resistance to flow of water increases, thereby increasing the time rate of filtration. In practical filter operation, when the density increases to such an extent that the passage of air and water ceases, the cake in some cases will crack and open at the weaker points in its structure. When this occurs, the pressure on the cake is relieved and further extraction of water is impossible.

The filtration rate for a compressible substance like activated sludge does not increase in proportion to the pressure because the increased pressure decreases the size of the voids between the particles, thereby restricting the flow of water. Although this phenomenon may not be the complete explanation for the difficulties encountered in the removal of water from activated and similar sludges, it is well substantiated in part by operating results in Chicago.

Filter operating costs at The Sanitary District of Chicago per ton of dry solids on a percentage basis are as follows:

Item	Calumet	Southwest
Ferric chloride	40	55
Labor	21	23
Power	8	8
Filter cloths and wire, etc.	2	3
Maintenance	29	11

At Chicago, ferric chloride is purchased in solution on a semi-annual contract basis. The last contract in 1941 was at the rate of 1.255¢ per lb of anhydrous ferric chloride, with freight on water allowed, making a cost per pound at the Chicago works of approximately 1.54¢.

The life of the filtering medium is largely a function of time rather than tonnage output. Cloths are subject to rotting, and their life is frequently determined by a crust of iron oxide formed on the back of the cloth. At Chicago, with ferric chloride conditioning of activated sludge, a long-nap woolen blanket has proved most satisfactory and economical. The life of a woolen blanket is approximately 3½ months. The cost per square foot, based on recent bids, is 18¢. The life of winding wire is approximately the same as the life of the filter medium. No attempt is made to reuse this wire. The use of woven wire for cloth backing has not been satisfactory.

Wood backing is being tried on a limited number of filters in Chicago, and indications are that it may prove more satisfactory and economical than wire cloth.

HEAT-DRYING

After sludge has been dewatered as far as economically possible by mechanical means, it must be dried thermally before it can be burned or processed for fertilizer. This drying process consists of transferring sufficient heat to the sludge to evaporate the entrained moisture. It is necessary to heat the sludge to the saturation temperature corresponding to the pressure in the system and to supply the latent heat of vaporization to the moisture. Since the evaporated water always leaves the dryer at a temperature above the saturation temperature, it is superheated and this requires additional heat.

In experimental work of The Sanitary District of Chicago, it was found desirable to separate the functions of drying and combustion so that each one would progress independently of the other in a separate unit especially designed for the efficient performance of its particular function.

The fundamental principles and factors found necessary for economical and efficient drying, free from nuisances, to prepare sludge for efficient combustion or for use as a fertilizer are as follows:

(1) Maximum surface exposure;
(2) Thorough agitation of the wet material;
(3) Temperature potential of gas;
(4) Velocity of drying gas;
(5) Responsive and flexible temperature control;
(6) Deodorization of gases vented from the drying operation;
(7) Instantaneous water removal; and
(8) Preservation of fertilizer qualities.

All drying operations utilize the first four of these principles in a more or less efficient manner. All eight of them are efficiently performed in the flash-drying equipment in use at Chicago.

Flash drying is a continuous operation in which the sludge in a finely divided state is dried instantaneously in a hot gas stream. At Chicago a mixture of superheated water vapor and other gases is used as the drying medium. The temperature of the drying medium as it enters the system is approximately 1,100° F, and the velocity is approximately 5,000 ft per min. At this temperature the evaporation of the moisture is almost instantaneous, and the passage through the drying cycle is a matter of seconds. Research by F. W. Mohlman has shown that dried sludge from a flash-drying system shows practically no trace of pathogenic organisms. The sludge never reaches a temperature sufficiently high to affect any of the properties valuable as fertilizer. The average temperature of the dried material as removed from the drying system is approximately 180° F. The temperature of the vapor and gases vented from the drying systems to the furnaces for deodorization is approximately 350° F. The drying systems are operated slightly under atmospheric pressure to prevent the escape of dust and gases. All operations are automatically controlled. Briefly, the sequence is as follows:

Activated sludge cake containing 78% to 85% moisture as discharged from the filters is conveyed to individual mixers where it is mixed with sufficient sludge, previously dried in the system, to reduce the moisture content and prepare a nonsticking friable mass. In this form the sludge is introduced into the drying system, where it is picked up in the high-velocity, high-temperature gas stream. A cage type of disintegrating mill interposed in the drying circuit breaks the sludge into small particles, causing maximum exposure of wet surface and complete dispersion of the particles in the drying medium. The sludge particles carried in the superheated vapor and gaseous drying medium are discharged into a cyclone separator where dried sludge is separated from the vapors and gases in which it was conveyed. Part of the sludge removed from the separator is returned to the sludge mixer, and the surplus is withdrawn as fertilizer material or pneumatically conveyed to a separate furnace where it is burned in suspension. When fertilizer is being produced, the heat required for the drying system is furnished by the combustion of auxiliary fuel. A portion of the vapors and gases vented from the sludge separators is recirculated through a heat exchanger where it is reheated by an exchange of heat from the hot products of combustion vented from the furnace and returned to the drying system. The surplus or waste vapor is vented to the furnace for deodorization.

Heat for sludge drying may be carried directly by furnace gases or it may be transferred from furnace gases to air or water vapor by means of heat exchangers. Air-drying has two distinct disadvantages. It results in lower efficiency since it is an additional medium introduced to the system, which must be raised from room temperature to the stack temperature, and it supplies oxygen which might cause fires.

The rate of evaporation is dependent on the temperature of the drying medium, the size of the sludge particles, and the rate of removal of the water vapor. The greater the temperature differential between the drying medium and the sludge, the greater the evaporation will be. The sludge particles should be kept as small as possible during the drying process, since the smaller they are, the greater will be the area of wetted surface exposed to the heat. In other words, more moisture is in direct contact with the hot gases. Water vapor formed in the center of a particle of sludge must force its way to the surface to be liberated, and thus the smaller the sludge particles, the shorter will be the distance traveled by the vapor. When the particles are too large, the surface may become dried and case-hardened before the moisture in the center has been evaporated. This hard surface offers resistance to the release of the interior moisture and results in the dryer discharging a product that is dried on the surface but has a wet center. The rate of drying is also affected by the relative velocity between the material and the drying medium. When vapor is released from the sludge, it tends to form an insulating medium around the sludge particle, retarding further evaporation unless it is instantly removed. Failure to remove the water vapor results in an increased pressure in the system that requires more heat to evaporate the water in the sludge.

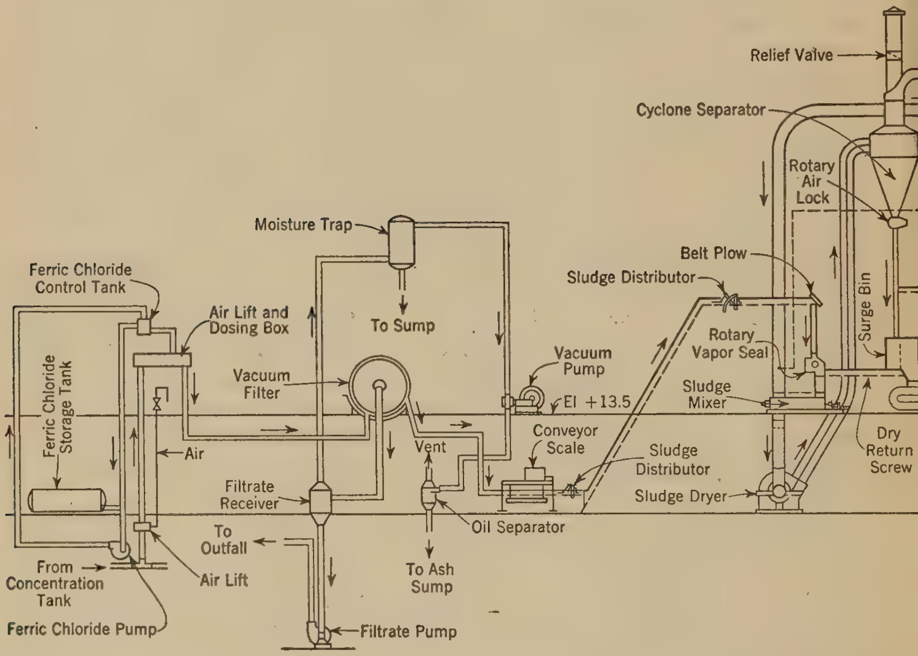
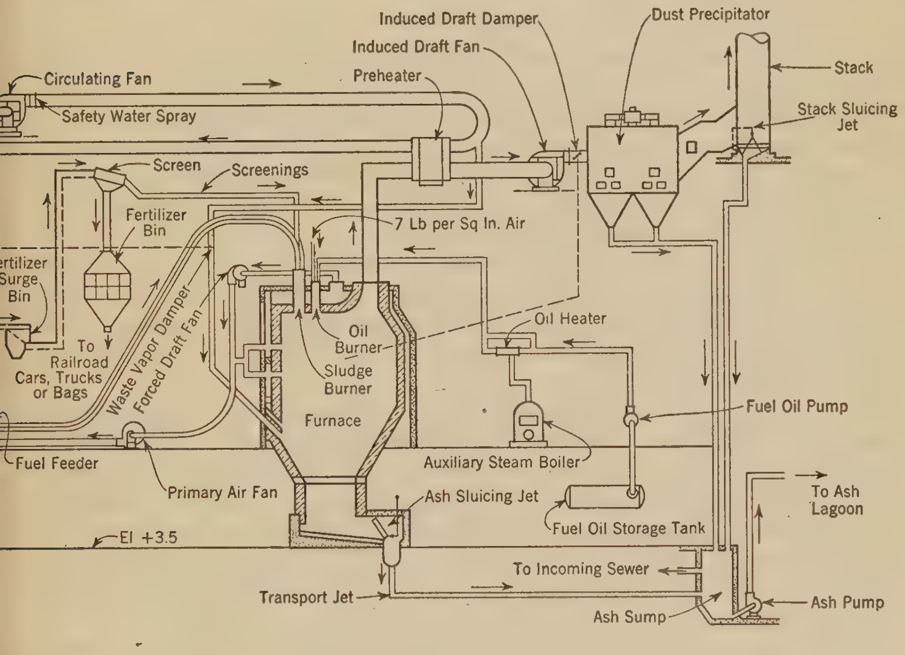

Relief Valve
Cyclone Separator
Rotary Air Lock
Moisture Trap
Belt Plow
Sludge Distributor
Ferric Chloride Control Tank
Air Lift and Dosing Box
To Sump
Surge Bin
Rotary Vapor Seal
Vacuum Pump
Vacuum Filter
Sludge Mixer
Ferric Chloride Storage Tank
El +13.5
Conveyor Scale
Dry Return Screw
Air
Vent
Sludge Distributor
Filtrate Receiver
Sludge Dryer
Oil Separator
To Outfall
To Ash Sump
From Concentration Tank
Air Lift
Ferric Chloride Pump
Filtrate Pump

Fig. 8.—Flow Diagram of Sludge Disposal Plan

Responsive and flexible control is a vital necessity in any drying and combustion operation. Where sludge is dried for incineration, it is desirable that the character of the material and its moisture content as fired be maintained as uniform as possible. Where sludge is dried for fertilizer uses, it is necessary that the size of particles be kept uniform and that the temperature of the drying system be automatically controlled to avoid driving off valuable fertilizer ingredients, and to insure a moisture content in the dried material between 5% and 10%. Where drying and combustion occur independently of each other, automatic temperature control is a relatively simple matter, inasmuch as the demand for heat in the drying system can be met by liberating more or less heat in the furnace. With the cage type of drying mill used by the Sanitary District, the size of particle is dependent upon the speed of the mill, its interior arrangement, and character of the sludge feed.


Induced Draft Damper
Dust Precipitator
Induced Draft Fan
Stack
Circulating Fan
Preheater
Safety Water Spray
Stack Sluicing Jet
Screen
Screenings
Fertilizer Bin
7 Lb per Sq In. Air
ertilizer Surge Bin
Waste Vapor Damper
Forced Draft Fan
Oil Heater
Oil Burner
To Railroad Cars, Trucks or Bags
Sludge Burner
Fuel Oil Pump
Furnace
Auxiliary Steam Boiler
Fuel Feeder
Primary Air Fan
Ash Sluicing Jet
To Ash Lagoon
Fuel Oil Storage Tank
El +3.5
To Incoming Sewer
Transport Jet
Ash Sump
Ash Pump

tension of Calumet Sewage Treatment Works

Sludge, like other similar organic materials, has an inherent tendency to heat and burn spontaneously when stored in large quantities over a period of time. This tendency may be caused by bacterial or chemical action or a combination of both. It is also a function of the moisture content, fineness, and initial temperature of the material stored. Experience at Chicago indicates that the moisture content should be held below 10%, the heat-dried sludge should be as free as possible from dust, and the temperature at the time of loading in cars should be below 100° F. This is accomplished by screening, dedusting, and cooling in water-jacketed conveyers.

All heat-dried activated sludge as produced for fertilizer uses at Chicago is loaded in cars in bulk, after screening and cooling, for immediate shipment to consumers. No material at present is bagged or stored in bulk. Facilities for a limited tonnage of bulk storage are now under construction at the Southwest Works.

Fig. 8 is a schematic diagram of the process at the Calumet plant. Three drying systems of the flash type, with individual furnaces, are installed at this plant. Each is designed to evaporate 6,500 lb of water per hour and to handle 20 tons of dry solids per day.

Fig. 9 is a schematic diagram of the Southwest plant. Eight drying units are installed at this plant. Each unit has an evaporation capacity of 20,000 lb of water per hour and handles approximately 62.5 tons of dry solids per day.

Combustion

In the combustion of sewage sludge as well as commercial fuels many factors are involved, due to the great variation in their characteristics. Specific requirements for the proper combustion of a particular sludge must be considered as a distinct problem requiring individual consideration. However, it is possible to draw certain general conclusions as to the combustion requirements of sludges. These conclusions form the basis of design of all combustion apparatus and thus are worthy of careful study.

The term "combustion," as applied to furnaces, is used in referring to the vigorous reactions of oxygen with different substances, whereby heat and light are liberated. Combustion therefore differs from other slower reactions with oxygen only in the vigor of the reactions and in the forms and quantities in which energy is liberated.

The chemistry of combustion of sewage sludge is elementary and differs but slightly from that of other solid fuels. The elementary substances encountered in the combustion of sewage sludge and commercial fuels are oxygen, nitrogen, hydrogen, carbon, sulfur, and combinations of these elements (see Table 17). Oxygen is the sole supporter of combustion. A combustible substance is one capable of combining with oxygen to produce heat. Combustion is complete when the combustible elements and compounds have united with all the oxygen they can use.

In the combustion of sludge, carbon and hydrogen are the most important of the combustible elements from the standpoint of heat liberation. These elements occur either in a free or combined state as in all solid, liquid, and gaseous fuels. For the most part, the hydrogen, carbon, and sulfur compounds in sludge are unstable in nature. In sludge cake, these compounds are associated with a relatively high percentage of water and ash, as compared with commercial fuels.

Every combustible substance has a definite ignition temperature to which it must be raised before it will combine chemically with oxygen. Below this temperature no combination takes place. Furthermore, this ignition temperature must exist in the presence of sufficient oxygen or there will be no combustion. The ignition temperatures of combustible elements and compounds commonly found in sludges are given in Table 17(*a*), Col. 5.

In the drying and burning of sludge, odors are produced largely by the liberation of hydrocarbon compounds. By observation, this liberation is found to begin at relatively low temperatures and to increase in intensity with

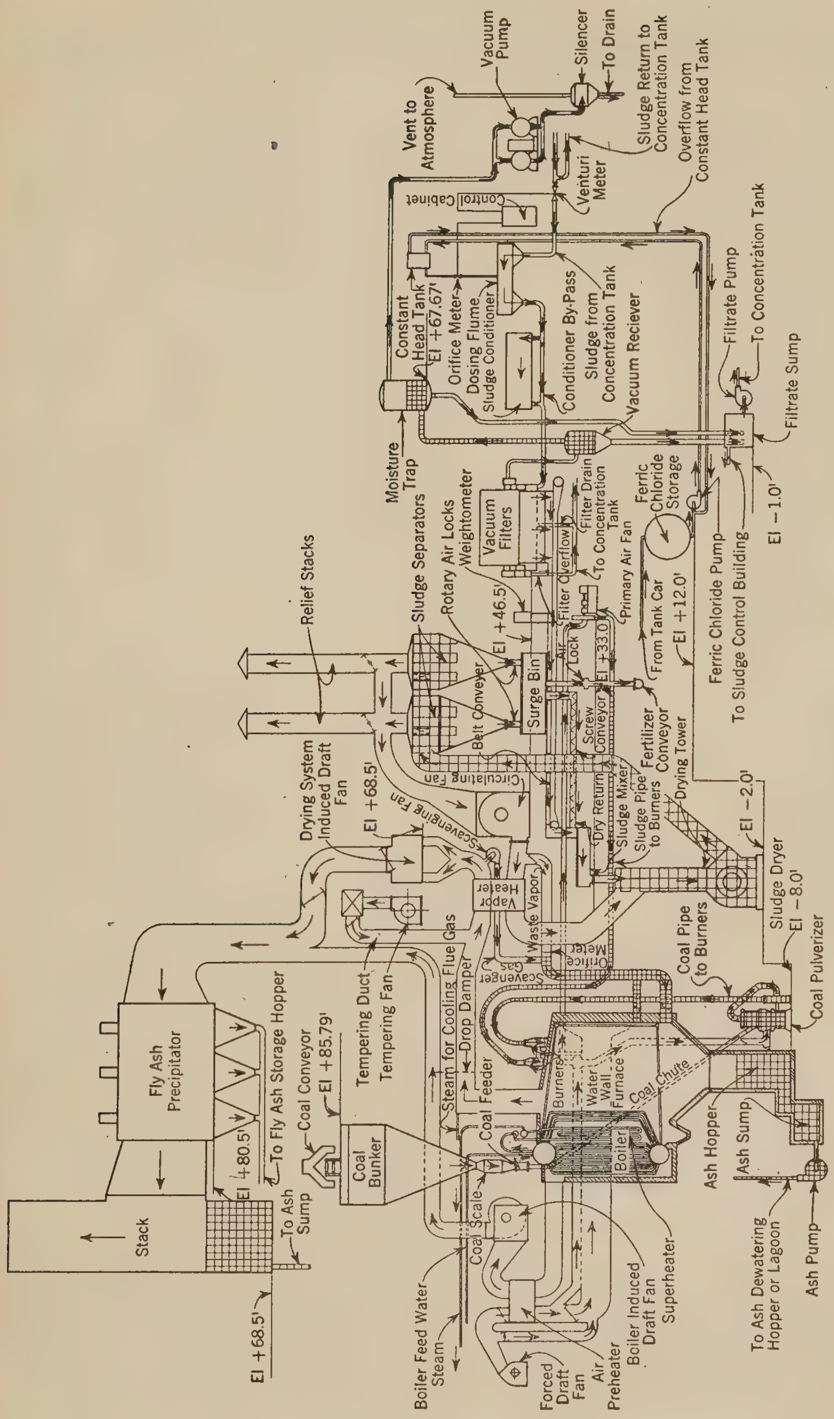

Stack
El +68.5'
To Ash Sump
El +80.5'
Fly Ash Precipitator
To Fly Ash Storage Hopper
Coal Conveyor
El +85.79'
Coal Bunker
Tempering Duct
Tempering Fan
Steam for Cooling Flue Gas
Drop Damper
Coal Feeder
Coal Scale
Boiler Feed Water
Steam
Forced Draft Fan
Air Preheater
Boiler Induced Draft Fan
Superheater
Boiler
Water Wall Furnace
Burners
Coal Chute
Ash Hopper
Ash Sump
To Ash Dewatering Hopper or Lagoon
Ash Pump
Coal Pulverizer
Coal Pipe to Burners
Scavenger Gas
Orifice Meter
Waste Vapor
Vapor Heater
Drying System Induced Draft Fan
El +68.5'
Scavenging Fan
Circulating Fan
Relief Stacks
Sludge Separators
Rotary Air Locks
Weightometer
Belt Conveyor
Surge Bin
El +46.5'
Air Lock
Screw Conveyor
El +33.0'
Dry Return
Sludge Mixer
Sludge Pipe to Burners
Drying Tower
Fertilizer Conveyor
Sludge Dryer
El −8.0'
El −2.0'
Moisture Trap
Vacuum Filters
Filter Overflow
Filter Drain To Concentration Tank
Primary Air Fan
From Tank Car
El +12.0'
Ferric Chloride Storage
Ferric Chloride Pump
To Sludge Control Building
El −1.0'
Vacuum Reciever
Filtrate Sump
Filtrate Pump
To Concentration Tank
Constant Head Tank
El +67.67'
Orifice Meter
Dosing Flume
Sludge Conditioner
Conditioner By-Pass
Sludge from Concentration Tank
Control Cabinet
Vent to Atmosphere
Vacuum Pump
Venturi Meter
Silencer
To Drain
Sludge Return to Concentration Tank
Overflow from Constant Head Tank

FIG. 9.—FLOW DIAGRAM OF SLUDGE DISPOSAL PLANT, SOUTHWEST SEWAGE TREATMENT WORKS

increased temperature. Odors are also released from the solids and gases in solution with the water, which is evaporated into steam. Emulsified organic fats likewise give off offensive odors with a slight application of heat. The ignition temperatures of the CH compounds lie between 1,100° and 1,200° F, approximately. The end products of complete combustion of all CH compounds are water vapor and CO_2, both of which are odorless. Therefore, all gases and water vapor produced in the heat-drying of sludge should be raised to a temperature of not less than 1,300° F in the presence of sufficient oxygen to effect complete combustion of the offensive CH compounds.

TABLE 17.—CHARACTERISTICS OF CHEMICAL ELEMENTS IN SLUDGE

No.	Substance (1)	Symbol (2)	Atomic weight (3)	Molecular weight[a] (4)	Ignition temperature (degrees F) (5)	Proportion by weight (%) (6)
	(a) COMBUSTIBLE ELEMENTS[b]					
1	Carbon	C	12	12	900	35.50
2	Hydrogen	H_2	1	2	1,130	5.13
3	Sulfur	S_2	32	64	470	0.85
4	Carbon monoxide	CO	..	28	1,210	....
5	Methane	CH_4	..	16	1,202	....
6	Hydrogen sulfide	H_2S	..	34	....	....
	(b) OTHER CONSTITUENTS					
7	Nitrogen	N_2	14	28	....	4.61
8	Oxygen	O_2	16	..	....	17.85
9	Chlorides	..	..	..	....	0.57
10	Ash	..	..	..	....	35.49
11	Air	..	..	29	....	....
12	Carbon dioxide	CO_2	..	44	....	....
13	Water vapor	H_2O	..	18	....	....
14	Sulfur dioxide	SO_2	..	64	....	....

[a] Approximate. [b] The potential heat content of sewage sludge is 7,000 Btu (British thermal units) per lb.

Likewise, sludge and similar organic materials must be burned at high temperatures with sufficient oxygen present to complete combustion if odors are to be avoided in the products of combustion discharged to the atmosphere. Sulfur dioxide, one of the end products of complete combustion of sulfur and its compounds, is objectionable if discharged into the atmosphere in large quantities. The percentage of sulfur and its combustible compounds as found in most sludges is relatively low compared with the content of many bituminous coals. Therefore, it is of little consequence. The amino compounds are dissociated at these temperatures and likewise are of little consequence.

The ignition temperature of combustible substances (Table 17(*a*), Col. 5) is well above the boiling point of water (212° F at 14.7 lb per sq in. absolute pressure). Thus, the material is thoroughly dry when ignition and combustion take place. A large ball of sludge with a hard dry surface may become ignited and smolder while the moisture is being driven off from the interior. Such combustion is inefficient and may produce odors unless sufficient auxiliary fuel is burned within the furnace to maintain a temperature sufficiently high to ignite the combustible gases emitted by the smoldering material and dissociate any noncombustible compound of an offensive nature. The experience of The Sanitary District of Chicago has shown that complete deodorization of gases can be assured only after the gases have been raised to a temperature of more than 1,300° F in the presence of sufficient oxygen to complete combustion of the volatile compounds. Efficient combustion of sludge, free from objectionable odors, therefore, is possible only with thoroughly dried material unless the obnoxious gases are deodorized after combustion.

The chemical reaction of combustion results from a rearrangement of the atoms of the constituent elements into new compounds. It is in accordance with fixed laws and may be computed from the molecular weights given in Table 17.

Air is the source of oxygen supply for commercial combustion operations, and is considered as a mechanical mixture of oxygen, nitrogen, carbon dioxide, water vapor, argon, and other inert gases. For combustion purposes, the carbon monoxide and the inert gases are usually included as nitrogen. The generally accepted percentages for atmospheric air are:

For oxygen (Q_2)—
by volume, 20.91
by weight, 23.15
and for nitrogen (N_2)—
by volume, 79.09
by weight, 76.85

The following reactions occur only when the exact amount of oxygen required for combustion is present and the oxygen is wholly utilized:

Combustible	Reaction
Carbon (to CO)	$2\,C + O_2 = 2\,CO$
Carbon (to CO_2)	$2\,C + 2\,O_2 = 2\,CO_2$
Carbon monoxide	$2\,CO + O_2 = 2\,CO_2$
Hydrogen	$2\,H_2 + O_2 = 2\,H_2O$
Sulfur (to SO_2)	$S + O_2 = SO_2$
Sulfur (to SO_3)	$2\,S + 3\,O_2 = 2\,SO_3$
Methane	$CH_4 + 2\,O_2 = CO_2 + 2\,H_2O$
Hydrogen sulfide	$2\,H_2S + 3\,O_2 = 2\,H_2O + 2\,SO_2$

Carbon enters into combustion in two forms, CO and CO_2. The former is incomplete, the latter complete. The combustion of hydrogen is always complete to H_2O. If insufficient air is supplied, a portion of the hydrogen combines with the supply of oxygen present to form water, and the remainder passes to the stack.

From the proportionate parts by weight of oxygen and nitrogen in atmospheric air, 4.32 lb of air are required for each pound of oxygen. This air also supplies 3.32 lb of nitrogen that does not enter into combustion. The ratio 3.32 : 1 is known as the "nitrogen ratio." Nitrogen serves no useful purpose and is a direct loss in combustion work, since it must be raised to the furnace exit temperature. Based upon approximate molecular weights, carbon, hydrogen, and sulfur require the following weights of oxygen for complete combustion:

One pound of:	has the following oxygen requirement (lb):
Carbon	$\frac{O_2}{C} = \frac{32}{12} = 2.667$
Hydrogen	$\frac{O}{H_2} = \frac{16}{2} = 8$
Sulfur	$\frac{O_2}{S} = \frac{32}{32} = 1$

The heat value of any sludge may be determined by actual experiment in a calorimeter. Where such apparatus is not available, approximate heat values may be calculated for certain combustible compounds from the ultimate chemical analysis, using Dulong's formula:

$$q = 14{,}600\ C + 62{,}000 \left(H - \frac{O}{8} \right) + 4{,}050\ S \qquad (1)$$

in which q is the heat of combustion in British thermal units (Btu) per pound, and C, H, O, and S, respectively, denote the proportional parts by weight of carbon, hydrogen, oxygen, and sulfur.

In the complete combustion of 1 lb each of carbon, hydrogen, and sulfur, the resulting products of combustion will be as follows:

Computation	Pounds
$1 \text{ lb } C + 2.667 \text{ lb } O_2 =$	$3.667\ CO_2$
$2.667 \times 3.32 \text{ lb } N_2 =$	$8.85\ N_2$
$1 \text{ lb } H_2 + 8 \text{ lb } O_2 =$	$9\ H_2O$
$8 \times 3.32 \text{ lb } N_2 =$	$26.56\ N_2$
$1 \text{ lb } S + 1 \text{ lb } O_2 =$	$2\ SO_2$
$1 \times 3.32 \text{ lb } N_2 =$	$3.32\ N_2$

Methane, one of the members of the CH series, is computed in a similar manner: $CH_4 = C + 4H$; or, from atomic weights, $16 = 12 + 4$. Thus, 1 lb of methane is composed of 0.75 lb C + 0.25 lb H. To burn 0.75 lb of carbon will require $0.75 \times 2.667 = 2.00$ lb O_2. To burn 0.25 lb of hydrogen will require $0.25 \times 8 = 2.00$ lb O_2. The total oxygen requirement is $2 + 2 = 4$ lb. The total air required is $4 \times 4.32 = 17.28$ lb. The products of combustion will be as follows:

Computation	Pounds
$0.75 \text{ lb } C + 2.00 \text{ lb } O_2 =$	$2.75\ CO_2$
$0.25 \text{ lb } H + 2.00 \text{ lb } O_2 =$	$2.25\ H_2O$
$17.28 \text{ lb air} \times 0.7685 =$	$13.28\ N_2$

These computations, although simple, are important in combustion work, particularly where digester gas and liquid fuels are used as major or auxiliary sources of heat for drying and incineration. The theoretical dry air required for a combustible compound, therefore, is:

$$A = 11.52\,\mathrm{C} + 34.56\left(\mathrm{H} - \frac{\mathrm{O}}{8}\right) + 4.32\,\mathrm{S} \qquad (2)$$

in which A = theoretical weight of dry air required per pound, C, H, O, and S being the proportional parts by weights of the carbon, hydrogen, oxygen, and volatile sulfur in the compound. The term $\left(\mathrm{H} - \frac{\mathrm{O}}{8}\right)$ represents the available hydrogen and corrects for hydrogen combined with oxygen in the fuel. Part is probably in the form of moisture, or is combined with organic nitrates, and part may be present in carbonates of mineral matter found in the sludge or fuel. This assumption probably is sufficiently accurate for ordinary calculations.

A series of calculations for different fuels will show a wide variation in theoretical air requirements per pound of fuel. When expressed in terms of theoretical air requirements per 10,000 Btu, a close agreement will be found among all fuels, including sludge, peat, coal, and oil. The average value of such a comparison will indicate a theoretical air requirement of 7.5 lb per 10,000 Btu, with a maximum departure not exceeding 2%.

Excess Air.—From the chemical standpoint, to supply the proper amount of oxygen for complete combustion appears relatively simple; but this is not the case. The primary problem in combustion is to maintain the supply with its accompanying nitrogen as near to the theoretical amount required as possible. The efficiency of combustion, therefore, may be measured by the relation between the air actually supplied and that theoretically required for combustion. It is vitally important that the air supply be admitted at the proper time and in such a manner that the oxygen comes into intimate contact with the combustible substances in the sludge, which include the combustible solid particles and gases as they are distilled from the sludge.

The air supplied to a furnace for combustion is classified broadly as primary and secondary. Primary air is that required for propagation of flame, whereas secondary air is that introduced to complete combustion after the flame has been propagated.

As stated, nitrogen introduced into the furnace with the oxygen required for combustion performs no useful function in combustion. It passes through the furnace unchanged except in temperature and volume. It absorbs heat and reduces the temperature of the products of combustion. All these factors necessitate designing the furnace space, connecting flues, and ducts sufficiently large to accommodate the excess gases. Any excess oxygen supplied to the furnace results in the same loss as for nitrogen. Such oxygen is accompanied by additional nitrogen, which further accentuates the loss. On the other hand, where insufficient oxygen is supplied for complete combustion, the loss due to the presence of the nitrogen is inappreciable as compared with that due to incomplete combustion. For example, if 1 lb of carbon is burned to carbon monoxide, the heat produced is 4,380 Btu, whereas if the combustion is carried to CO_2 the heat produced is 14,540 Btu. This factor will be discussed further in connection with furnace losses.

Smoke and Odors.—Smoke and odors are a widely discussed topic in connection with the combustion and drying of organic refuse and commercial fuels. Most of the definitions of smoke are unsatisfactory and vague.

The common conception of objection to smoke is based on its visibility. Most smoke ordinances are based on the visibility, density, and color of escaping stack gases, with little or no attention to the invisible or more objectionable compounds which are by far the worst offenders in atmospheric pollution. The amount of solids discharged from a stack is not a complete indication of its nuisance value. A stack may appear smokeless and still discharge a large volume of objectionable gases into the atmosphere. The color of smoke is usually imparted by carbon particles. The presence of a small amount of carbon is sufficient to give color to large volumes of invisible gases, which may or may not represent a nuisance or a combustion loss. A certain amount of color may also be given to gases by particles of fine ash and other mineral matter carried in suspension in the gases, neither one of which represents a combustion loss. When burning pulverized fuel, the amount of this material in escaping gases is considerable unless efficient means are provided for its interception, as the passing of all waste gases through some form of dust collector such as a settling chamber, centrifugal separator, or electrical precipitator.

The effectiveness of such means varies over a wide range and is dependent upon various factors, principally the dust density of the gases. The plain settling or expansion chamber, unless of tremendous volume with resultant low gas velocities, will remove little if any of the fine flocculent dust. A centrifugal separator, showing removal efficiencies as high as 94%, is more effective. This provides maximum removal per dollar invested. The most effective of all separating devices is a commercially patented electrical precipitator. Tests on this equipment installed in the Calumet and Southwest works in Chicago indicate a removal of 97% of suspended solids from the stack gases. The selection of dust-separating equipment should be based on the degree of separation desired. This is determined to a great extent by the remoteness of the plant from habitated areas. Investigations in connection with large power plants, where coal is burned in suspension or on stokers, indicate that a large quantity of the flocculent solids discharged from stacks is deposited at a distance varying from 5 to 15 miles from the source, unless efficient means for interception are provided.

The presence of a large volume of visible water vapor in stack gases is not objectionable from the standpoint of atmospheric pollution, if discharged at a height to permit complete dispersion.

In the early experimental work of the Sanitary District, all gases vented from the sludge dryer and furnace were passed through a vapor condenser and scrubber in an attempt to remove odors. This method of odor elimination proved ineffective and expensive in operation and maintenance. All the metallic parts (condenser, piping, and spray nozzles) were attacked by sulfurous acid and other compounds that formed when the vapors reached the dew point. The interior surfaces of the equipment became coated with a very corrosive, tarry compound with an offensive odor. When heated, this substance gave off an offensive brown gas which ignited at a temperature of approximately 1,200° F and burned freely in the atmosphere. The vapor condenser and scrubber were found to act solely as a wet-type dust collector.

Heat of Combustion.—As previously stated, the term "combustion" is limited to the rapid chemical combination of the combustible constituents of a fuel with oxygen, resulting in the production of heat. The heat of combustion is therefore the heat resulting from the complete oxidation of such combustible elements and compounds. The amount of heat so generated is a constant for any given combination of combustible elements and compounds, irrespective of the manner in which combustion takes place so long as it is complete. The unit measure of quantity of heat is the British thermal unit, or the Btu, which is $\frac{1}{180}$th of the amount of heat required to raise the temperature of 1 lb of water from 32° to 212° F.

The heat of combustion of any fuel can be computed approximately from the ultimate analysis by use of the Dulong formula (Eq. 1). In computing heat values of sludges, great care should be exercised in applying Eq. 1, inasmuch as sufficient check data are not available to determine the precision of results computed by this formula. Accurate calorific values are obtainable only by calorimetric methods, which give the following values for common combustible materials:

Combustible	Heat value (q), in Btu per lb
Carbon to CO	4,380
Carbon to CO_2	14,540
Hydrogen	62,000
Methane	23,850
Sulfur to SO_2	4,050

Temperatures of Combustion.—The actual temperature resulting from the combustion of sludge or any other fuel is obtained most satisfactorily by means of a pyrometer. The theoretical temperature may be calculated from the formula,

$$T = \frac{q}{W\,C} + t \qquad (3)$$

in which T = final temperature of products of combustion, in degrees Fahrenheit; q = British thermal unit per pound of fuel; W = weight of products of combustion, in pounds per pound of fuel; C = mean specific heat of the products of combustion; and t = initial temperature of fuel and air supply, in degrees Fahrenheit.

Assuming the complete combustion of 1 lb of carbon with the theoretical amount of air supplied at an initial temperature of 60° F, then $T = \frac{14{,}540}{12.52 \times 0.27} + 60 = 4{,}360°$ F, approximately.

Such calculated temperature is entirely theoretical and is the temperature that would exist in the furnace if combustion were complete, if no dissociation occurred, and if no heat were lost by radiation. Actually, some heat is radiated to the furnace walls before combustion is complete. At temperatures greater than 3,000° F, dissociation begins to occur, whereby water vapor breaks down into hydrogen and oxygen and carbon dioxide breaks down into carbon monoxide and oxygen. Just as the combining of these elements into water vapor and carbon dioxide liberates heat, so heat is absorbed when the reactions are reversed, resulting in a lower temperature. In computing heat balances, dissociation may be ignored since, after the gases are cooled during the process of heat recovery, the reactions are again reversed and the heat is again liberated. Since it is generally impossible to burn each particle of fuel in the presence of the proper amount of oxygen, incomplete combustion takes place, resulting in a lower temperature. Under actual combustion conditions there is always an excess supply of air present that also reduces the temperature.

In practice, factors may occur with different types of sludges and equipment which, for practical reasons, require that furnace temperatures be kept below the maximum attainable. Under such conditions high percentages of excess air are introduced into the furnace, resulting in reduced combustion efficiency. An important factor is the fusion temperature of the ash. For practical reasons, furnaces containing metallic parts must operate with a high percentage of excess air to maintain a safe operating temperature well below the maximum available.

Specific Heat.—Calculations involving combustion and heat transfer require the use of specific heat values. The specific heat of a substance is the number of British thermal units required to raise the temperature of 1 lb of the substance 1° F. Actually, the specific heat is not a constant for all conditions of pressure and temperature and, if accurate results are desired, a correction must be made for the actual pressure and temperature conditions that exist.

When heat is added to a gas while its volume is kept constant, the external work is zero, but if heat is added while the pressure is held constant, the volume increases and external work is performed. Therefore, the amount of heat necessary to raise 1 lb of a gas 1° F at constant pressure is greater than that required at constant volume because it requires heat to perform external work.

In heat computations dealing with furnace gases and sludge drying gases, the variations in pressure are small. The change in specific heat due to pressure changes thus can be ignored. The specific heat of a gas, however, varies greatly with the temperature. Any calculation in which extreme accuracy is desired should use the mean specific heat of the gas between the temperatures involved. For example, the specific heat of air is usually taken as 0.24, yet it varies from 0.238 at 50° F to 0.275 at 2,000° F.

Heat Balance.—In designing a drying and incineration system, it is necessary to make sufficient heat calculations to determine the quantity of fuel required, the air needed for combustion, the size of furnace required, and also gas quantities for determining fan and duct sizes. A summary of these data, showing the heat input to the system and the distribution of this heat among losses encountered and the useful work done, is known as a heat balance. The ratio of the heat utilized in the performance of useful work to the total heat supplied is the efficiency of the system.

A heat balance may be an actual balance obtained from the results of a test on the unit involved or it may be a theoretical balance. In the first case the fuel is weighed and its heat per pound obtained from calorimeter tests. The work done is accurately measured, as are all losses. In making a theoretical balance, certain assumptions must be made, such as radiation losses, stack temperatures, and the heat value of the fuel.

A heat balance for a flash-drying system as used in The Sanitary District of Chicago, in which the water vapor evaporated from the sludge is returned to the furnace for odor removal, would have for its heat input the heat released from the burning of the sludge and auxiliary fuel or from the burning of auxiliary fuel only if the dried sludge is prepared for fertilizer.

The heat output includes the heat used to evaporate the moisture in the sludge and the following losses:

(*a*) Radiation loss in the drying system. This consists of heat radiated from the surfaces of the dryer, ducts, separators, etc., and amounts to approximately 125,000 Btu per ton of dry solids.

(*b*) Loss due to moisture and hydrogen in the fuel. The hydrogen in the dried sludge forms water with the oxygen in the air. This water, together with the water in the sludge, must be raised to the boiling point, converted into steam, and then superheated to the temperature of the stack gases. The heat required to perform this work is taken from the heat generated from the combustion of the sludge and is therefore a loss.

(*c*) Loss due to incomplete combustion. Even when excess air is present, it is not possible to obtain complete combustion of the sludge. Some particles, for instance, drop unburned into the furnace ash hopper and others pass out as soot with the stack gases. These losses generally amount to approximately 4%. When producing fertilizer, using oil or gas as fuel, this loss is negligible.

(*d*) Stack losses. These losses correspond to the heat that is added to the fuel, combustion air, and waste vapor to raise them from the temperature at which they enter the furnace to the temperature at which they leave the system and enter the stack. These losses are unavoidable but, since they represent the greatest loss of heat in the system, they should be kept at a minimum by maintaining the exit temperature as low as possible, without risking danger of condensation, and by keeping the excess air at a minimum.

(*e*) Radiation and unaccounted losses. These losses allow for radiation from the furnace setting, air leakage into the drying system, and any losses not included in the other items.

The following is a heat balance for one drying unit as installed at the Calumet plant of The Sanitary District of Chicago, based on producing fertilizer and burning oil for fuel:

Assumptions.—

(1) Sludge feed, 20 tons per day at 80% moisture and 55° F.
(2) Average temperature, 50° F.
(3) Air, 30% and vapor, 70%, by volume in hot tower at 1,000° F.

(4) Temperature at outlet of separator, 300° F.
(5) Dried sludge removed from system as fertilizer.
(6) Fuel oil at 19,500 Btu per lb.
(7) Dried sludge, 180° F and 5% moisture.
(8) Drying system radiation losses, 125,000 Btu per ton of dry solids.
(9) Furnace radiation, incomplete combustion, hydrogen, and unaccounted losses, 10%.

Computations.—

Item	Computation	Quantity
(*a*)	Dry solids, in lb per hr $\frac{20 \times 2,000}{24}$	1,667
(*b*)	Moisture, in lb per hr $\frac{1,667 \times 80}{20}$	6,667
(*c*)	Moisture evaporated in drying system, in lb per hr $6,667 - \left(\frac{5}{95} \times 1,667\right) = 6,667 - 88$	6,579
(*d*)	Vapor—Air Mixture (% by Weight):	
	Air (30%) 1 molecular lb = 29 = 8.7 lb	41
	Vapor (70%) 1 molecular lb = 18 = 12.6 lb	59
	Total 1 molecular-lb mixture = 21.3 lb	100
(*e*)	Specific heat of drying mixture $0.41 \times 0.24 + 0.59 \times 0.47$	0.375
(*f*)	Air leakage, in lb per hr $\frac{0.41}{0.59} \times 6,579$	4,570
(*g*)	Radiation losses from drying system, in Btu per hr (at 125,000 Btu per ton of dry solids) $\frac{1,667}{2,000} \times 125,000$	104,000
(*h*)	Waste vapor, in lb per hr $6,579 + 4,570$	11,149
(*i*)	Heat required by drying system, in Btu per hr $6,579\,[212 - 55 + 970 + (300 - 212) \times 0.47]$ $= 1,670\left(0.30 + \frac{0.05}{0.95}\right)(180 - 55)$ $+ 4,570\,(300 - 50) \times 0.24 + 104,000$	8,131,500
(*j*)	Drying mixture required, in lb per hr $\frac{8,131,500}{(1,000 - 300)\,0.375}$	30,900
(*k*)	Mixture to circulating fan, in lb per hr $30,900 + 6,579 + 4,570$	42,049
(*l*)	Heat required from furnace, in Btu $8,131,500 + 11,149 \times 0.375 \times (475 - 300)$	8,863,500

At 20% air and 17.8 lb products of combustion per lb of oil (26):

Item	Computation	Quantity
(*m*) Useful heat, in Btu per lb of oil	$0.9 \times 19{,}500\ [(475 - 50) \times 0.25 \times 17.8]$	15,660
(*n*) Oil required, in lb	$\dfrac{8{,}863{,}500}{15{,}660}$	567
(*o*) Products of combustion	567×17.8	10,080
(*p*) Furnace temperature, in degrees Fahrenheit	$\dfrac{567 \times 0.9 \times 19{,}500 + [11{,}149 \times (300 - 50) \times 0.375]}{(10{,}080 \times 0.25) + (11{,}149 \times 0.375)} + 50$	1,690
(*q*) Recirculated gases to give 1,475° F entering the heater, in lb	$\dfrac{(10{,}080 + 11{,}149)\ (1{,}690 - 1{,}475)}{1{,}475 - 475}$	4,570
(*r*) Total heat supplied	$567 \times 19{,}500$	11,050,000
(*s*) Heat balance, in Btu:		
Furnace radiation, incomplete combustion, hydrogen, and unaccounted losses, 10%		1,105,000
Drier radiation losses		104,000
Dried sludge losses	$1{,}670 \left(0.30 + \dfrac{0.05}{0.95}\right)(180 - 55)$	73,500
Useful drying	$6{,}579 \times 1{,}127$	7,400,000
Stack losses	$6{,}579\ (475 - 212)\ 0.47 + 4{,}570\ (475 - 50)0.24 + 567 \times 17.8\ (475 - 50)\ 0.25$	2,350,000
Total heat balance		11,032,500
(*t*) Evaporated heat, in Btu per lb of H_2O evaporated	$\dfrac{11{,}050{,}000}{6{,}579}$	1,680
(*u*) Efficiency (%)	$\dfrac{1{,}127}{1{,}680}$	67

Fuel.—The heat for drying sludge preparatory to incineration or use for fertilizer may be derived from burning the sludge itself or from some commercial form of fuel. In many incineration operations, the heat content of the sludge may prove sufficient to supply the needs for drying and maintaining a furnace temperature sufficiently high to assure complete deodorization. Under these conditions, auxiliary fuel is needed only during starting periods. Digester gas, if available, is an excellent fuel for this purpose. Where the supply of digester gas is intermittent and variable, it is advisable to supplement it with natural gas or fuel oil. City gas, if available, is also satisfactory, but its cost is usually prohibitive.

The quantity of fuel required for starting depends largely upon the size and type of furnace and the mass of refractory to be heated to the working temperature. Some furnaces can be safely brought to working temperature in less than an hour, whereas some types require 24 hr to 72 hr, depending upon their size. Furnaces of the latter type also require the burning of fuel during the cooling down period, which varies between 24 hr and 48 hr.

For drying and incinerating sludge in an installation of moderate size, where supplementary fuel is needed frequently in regular operation, such fuels in order of their desirability, without reference to cost, are digester gas, natural gas, and fuel oil, or combinations thereof. In the larger installations, powdered coal is very satisfactory and economical. Where combinations of gaseous and liquid fuels are utilized, automatic means for change-over usually should be provided for protection against failure of either source of fuel supply.

Where sludge is processed for fertilizer and no sludge is burned, the entire heat requirement must be derived from some form of commercial fuel, or digester gas, if available. For the small or moderate-sized drying plant, as a practical consideration, it is imperative that gaseous or liquid fuels, or their combinations, be used. The first cost of such an installation is relatively low compared to an installation for burning coal. Usually the higher cost of gaseous and liquid fuels for this purpose over that of coal is offset by higher combustion efficiency, simplicity in operation, and flexible temperature control.

Where sludge is dried for fertilizer, the effect of the type of fuel on the final product is negligible. Therefore, with installations where large amounts of fuel are required, the relative economy of the fuel per unit of heat should determine the type. Coal burned in suspension or on a grate usually will be found more economical than other types of fuels in the larger installations, and when burned on a grate it may be the most economical for small units. Some of the advantages and disadvantages of various fuels are listed in Table 18.

Power Generation at Southwest Works

Whether the generation of power should be combined with drying and incineration of sludge is largely a question of economics. Such a combination ordinarily is not economical except for large installations like the Southwest Works at Chicago where conditions for the combination were very favorable. In the large flash type of drying system, with incineration furnaces, most of the elements of a steam generating plant, both as to equipment and personnel, are present.

At the Southwest Works, the annual costs of steam generation consist principally of fixed charges on the steam generating equipment plus fuel costs. A further advantage is inherent in the treatment plant design due to an abundant supply of plant effluent available for condensing purposes, at ample head to provide gravity flow through the condensers, before release to the outfall. Thus, the necessity for condenser circulating water pumps is eliminated at a considerable saving in the cost of power. The combined effect of these items results in a very low cost for power.

Steam for plant power purposes is generated in four units each of the water-tube, vertical, bent-tube type, combined with a waterwalled furnace in which dried sludge can be burned in suspension together with pulverized coal.

Each unit has a maximum steam generating capacity of 110,000 lb of steam per hour at a pressure of 425 lb per sq in. and a maximum temperature of 725° to 750° F. In addition to supplying heat for generating steam, each furnace is capable of liberating sufficient heat for sludge drying to evaporate as much as 49,000 lb of water per hour at atmospheric pressure and of passing this quantity of water and other gases through the furnace for deodorizing. The total evaporation for each unit under maximum steam and average drying conditions is approximately 150,000 lb of water per hour, of which 110,000 lb is high-pressure steam and 40,000 lb is steam at atmospheric pressure.

TABLE 18.—ADVANTAGES AND DISADVANTAGES OF VARIOUS FUELS

No.	Description	COAL BURNED:		Fuel oil	Natural gas	Digester gas
		In suspension	On grates			
	(*a*) ADVANTAGES					
	Combustion:					
1	Combustion control is simple	√[a]	..	√[b]	√	√
2	High-flame temperatures easily controlled	√	..	..	√	√
3	High combustion efficiency	√	..	√	√	√
4	Heat value is high	..	..	√	√	√
	Fuel:					
5	Can be stored easily	..	..	√	..	..
6	Convenient to handle	..	..	√[c]	..	..
7	Available anywhere	..	..	√	..	..
8	Cost is low	..	√	..	..	..
9	Supply is reliable	..	√	..	..	..
10	Cost is stable	√	√[a]	..	..	..
	Plant and Equipment:					
11	Burning equipment simple	..	..	√[b]	√	√
12	Cost of burning equipment maintenance is low	..	..	√	√	√[d]
13	Furnace can be started easily	..	..	..	√	√
14	No fuel preparation plant is needed	..	..	..	√	√
15	First cost of installation is low	..	√	..	..	..
16	Cleanly	..	..	..	√	√
	(*b*) DISADVANTAGES					
	Fuel Supply:					
17	Not always reliable	..	..	..	√[e]	√
18	Not always available everywhere	..	..	..	√[e]	..
19	Diminishing, with resultant gradual increase in cost	..	..	..	√	..
20	Must be treated	√[f]	..	√[g]	..	..
21	Large storage space required	..	√	..	..	..
22	Excessive labor required for handling	..	√[h]	..	..	..
23	Costs may fluctuate	..	..	√[c]	..	..
24	High in sulfur compounds	..	..	..	..	√
25	Chemical contents variable	..	..	..	..	√
	Operation Problems:					
26	Combustion control is difficult	..	√	√[i]	..	..
27	Fire hazards high	..	..	√	..	..
28	Careful plant maintenance needed	√[j]	..	√[j]	..	..
29	Expert labor required	..	√[k]	..	..	..

[a] Relatively. [b] Compared with coal. [c] Particularly the lighter oils. [d] Exceptionally low. [e] Natural gas is not available in all sections of the United States, and where it is available, the supply is usually unreliable. [f] Coal must be pulverized with costly equipment, and preheated air is necessary. [g] Fuel oil must be strained and heavy fuel oils must be heated. [h] For coal and ash handling. [i] Compared with gas. [j] Burning equipment must receive careful attention and, with coal, fly ash wears the fans and ducts. [k] Requires competent firemen.

Each furnace is equipped with six burners for pulverized coal and four for dried sludge, arranged for vertical firing through the top arch of the furnace.

A water-tube slag screen is provided in each furnace immediately above the hopper bottoms. The furnace volume of each unit, exclusive of back passes of the boiler and hopper bottoms, is approximately 8,000 cu ft. The total heat absorption surface of each boiler is approximately 13,875 sq ft, 3,185 sq ft of which is waterwall surface. The superheater, in addition, contains approximately 4,000 sq ft of surface.

The furnace gases which supply heat for the drying systems are taken off at the top of the furnaces and drawn through the vapor heaters by variable-speed induced draft fans. The gas then passes through dust precipitators for ash removal and thence to the stacks. Gases passing through the boiler and superheater are drawn through an air preheater by a variable-speed induced draft fan. They are exhausted from the preheater to an electrical precipitator where the ash carried in the gas stream is removed before release to the atmosphere by way of the stacks.

Sufficient heat is extracted from the gases in passing through the air preheater to provide primary and secondary combustion air at a temperature of approximately 400° F. Air for combustion is drawn directly from the boiler room by a forced draft fan discharging through the air preheater into a system of ducts where the proportion required for primary and secondary uses is automatically controlled.

The effectiveness of combining sludge drying with steam generation at this plant has been proved by nearly three years of operation. Aside from the production of cheap power, the arrangement reduces furnace maintenance costs due to the protection of the refractories by the waterwalls. Greater flexibility is afforded since furnace operation is not dependent on the drying load alone. More than half the fuel burned is for steam generation. Therefore, sudden changes in drying load do not upset furnace conditions as they would if the furnace were designed for drying only. With some combinations of filter-cake moistures and heat values of sludge, there will be more heat available in the sludge than is necessary for drying. Where drying only is done, this excess heat must be wasted to the stack; but if steam is generated, this heat can be utilized.

Conclusion

The intent of this paper has been to present, in an easily understandable manner, the major fundamentals relative to the drying and incineration of sewage sludge as has been developed to date in The Sanitary District of Chicago. Considerably more can be accomplished along these lines and it may be concluded that the art in its present stage will undergo many changes in developments over a period of time.

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DISCUSSION

A. L. Genter,[8] M. Am. Soc. C. E.—Mr. Schroepfer's contribution to this subject is excellent in that it shows how far the economics of raw sludge dewatering can be pushed with engineering acumen and operating skill. However, as he states, it relates strictly to raw sludge handling in a specific plant; that is (see heading, "Effect of Filter Rate on Cost of Filtration"),

> "It is based on the conditions (type of sludge, chemical costs, etc.) existent at this plant [Minneapolis-St. Paul]. A comparable analysis can, and should, be made by other operating plants, based on the conditions obtaining there, to determine (*a*) the economics of operation of present installation, (*b*) possibilities of effecting economies by the enlargement of present facilities at some future date, and (*c*) economical rates for different types of sludges for the benefit of designers of future plants."

The question of comparison with other plants is not so much how other vacuum filter practice fits into the picture of Minneapolis-St. Paul, but how the practice at this plant fits into the economics of another type of plant where the sludge solids are further mineralized—that is, the bulk of the sludge is reduced by sludge digestion, and the fouled natural sludge moisture is removed by elutriation prior to filtration. This procedure is essentially the reverse of practice at Minneapolis-St. Paul where solids reduction follows filtration. The First Progress Report of the Committee of the Sanitary Engineering Division on Sewerage and Sewage Treatment (27) shows that sludge elutriation is practiced for some 3,000,000 population in various parts of the United States and Canada, namely for a greater total population than at Minneapolis-St. Paul. Instead of referring to Mr. Fischer's six-year-old and rather incomplete paper for illustrative purposes of comparison, as Mr. Schroepfer has done (8), the writer deems it more appropriate to compare the 1941 averages at Minneapolis-St. Paul to some of these widely spread typical plants for the same year. The tables and remarks herewith submitted are intended therefore for this purpose.

In the light of what is now known about the chemical conditioning and vacuum filtration of various types of sewage sludges, it should be emphasized at the outset that the raw sludge at Minneapolis-St. Paul is relatively easy to condition with low chemical doses prior to filtration. This can be attributed to (1) its relatively low volatile and high ash content, (2) consequent high concentration of solids, and (3) low alkalinity of the water associated with the sludge solids. If the volatile percentage at Minneapolis-St. Paul were the same as that of the primary raw sludge at the Richmond-Sunset treatment plant in San Francisco, Calif. (82% instead of 62.1%), the annual chemical conditioning costs would be at least 250% greater than those listed by Mr. Schroepfer. Furthermore, such a sludge would be materially thinner, would require more filter area, filter hours, and incineration hours, with a consequent increase in power expended.

The items of volatile-to-ash ratio in the sludge solids, alkalinity of sludge water, and the amount of sludge solids per capita per year control local vacuum filter operating economies in all plants. They are relatively favorable at

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[8] Cons. Engr., Baltimore, Md.

Minneapolis-St. Paul notwithstanding the facts that the solids per capita are unduly high and the natural tendency in dewatering raw sludge is to increase the vacuum filter equipment. Mr. Schroepfer's record of pushing the economics of sludge dewatering to limits below those of other plants dewatering relatively large amounts of raw sludge is primarily an excellent engineering account of all the economic consequences of the vagaries of raw sludge—(1) "inconsistency of sludge behavior characteristics" (see heading, "Improvements in Filtration as Applied to Minneapolis-St. Paul Project"), (2) frequent washing of filter cloths (every four hours), (3) lime incrustation with the consequent necessity of frequent acid washing of cloths, screens, drums, and filter piping while using analine inhibitor, (4) problem of returning incinerator ash to help mineralize the sludge, thereby increasing the solids to be dewatered, and (5) the use of "a relatively large number of professional engineers and chemists in supervisory positions" (see heading, "Costs of Dewatering and Incineration as Compared to Other Processes of Sludge Disposal: Personnel Required"), together with (6) skilled operators to control all these functions.

With sludge digestion and elutriation, this list of functions and problems cannot help but be in the opposite direction; in other words, there is much less sludge to be handled, greater mineralization of the sludge solids, far less dependence on the vagaries of sludge, far less chemical and power requirements, and greater filter yields. The writer agrees thoroughly with the argument that operating intelligence is a prerequisite to sludge filtration economy in all plants. However, Mr. Schroepfer's following statement (heading, "Costs of Dewatering and Incineration as Compared to Other Processes of Sludge Disposal") needs qualification: "To expect that even the most conscientious operator of a plant in a community of, say, 25,000 people, in which 1 to 2 tons of dry solids must be disposed of daily, can even approach the efficient and controlled operation of a plant handling 50 or more tons daily is erroneous." With equal operating acumen in both cases, this depends principally on the sludge processing methods employed. Undoubtedly, in a larger plant with more equipment and greater operating staff, a greater amount of sludge solids will be disposed of per man-hour of work. However, when it comes to dewatering various types of sludges in vacuum filters, "efficiency" is a relative term. Doing a job efficiently means doing it very well at the least possible cost. Mr. Schroepfer has demonstrated ably that it is difficult and requires "eternal vigilance" to dewater large quantities of raw sludge each day, economically, and that he and his associates are doing this job exceedingly well. However, the application of efficiency to dewatering elutriated digested sludges is far easier from the operator's standpoint and cheaper from the taxpayer's viewpoint. For this reason, the small plant serving about 25,000 population at Annapolis, Md., is included in Table 19.

This plant is under a superintendent devoting his time to a distant sewage pumping station, sewage chlorination, and the entire treatment plant operation without the benefit of chemical control other than that which his time allows and the Maryland State Department of Health requires. At the treatment plant, a helper aids him in operating a small elutriation tank and vacuum filter with ancillary equipment. This work requires only three days weekly and then only 4 or 5 hr daily, without the eternal vigilance and fight against lime incrustation evils common to Minneapolis-St. Paul. The latter plant serves a population about 32 times greater than does the plant at Annapolis, and yet the chemical cost during 1941 at Minneapolis-St. Paul was 187 times greater than at Annapolis (see Table 19(*c*)). Undoubtedly the same operator at this small plant could operate, in the same time schedule and at the same wage, a vacuum filter six times larger (390 sq ft), and handle six times as much elutriated sludge with practically the same ease as at present, thus reducing labor costs per ton of sludge solids to one sixth the present cost. This fact, plus the high power cost (3.5¢ per kw-hr), naturally increases costs per ton of sludge solids dewatered. However, such factors have little to do with efficiency and ease of filter control and operation. The correction of these latter factors primarily starts in the sludge itself; in other words, they depend on the constancy of sludge characteristics or uniformity conditions which produce the chief aid to constancy of operating averages.

Such operating comparisons, as herein offered, ultimately come back to the taxpayer through the annual operating budget, meaning that the essential items controlling sludge dewatering costs should be measured in terms of per capita per year. This is the yardstick adopted in Table 19.

Table 19(*a*) presents the basic data from which most of the information in the remainder of the table is drawn. Sludge dewatering practice in the listed plants is limited to handling primary raw sludge (Minneapolis-St. Paul), digested primary (Buffalo, N. Y.), and elutriated digested primary in the remainder of the plants serving populations of about 800,000 at the District of Columbia (about the same as Minneapolis-St. Paul) down to 25,000 at Annapolis. The flocculation and primary treatment scheme used at the Richmond-Sunset treatment plant is somewhat similar to that used at Minneapolis-St. Paul, although the primary sludge at the former plant is of much higher volatile content and therefore more difficult to settle, digest, and dewater. The greatest sewage flow is at Buffalo. The flows at Minneapolis-St. Paul and the District of Columbia are practically identical. Both Baltimore, Md., and the District of Columbia treatment plants serve about the same population and are the largest elutriation installations. However, Baltimore treats some of its sewage by sludge activation and the digested sludge contains some digested waste activated as well as digested primary, and the digested sludge is given only single-stage elutriation treatment. Consequently the results for the District of Columbia plant are listed herein. Springfield, Mass., is included with Hartford, Conn., as sludge incineration is used at the former plant and not at the latter.

For 1941 the per capita expenditures at the listed plants for sewage treatment (operation and maintenance, exclusive of pumping and chlorination) were: Minneapolis-St. Paul, 38¢; Buffalo, 47.35¢; District of Columbia, 17.26¢; Hartford, Conn., 30¢; Springfield, Mass., 43¢; Winnipeg, Manitoba, Canada, 15.91¢; Richmond-Sunset, 31¢; and Annapolis, 33.6¢. As the various charges included in such accounting are dependent on numerous factors of local significance, such a tabulation is of little value except to compare, roughly, the total treatment costs in those plants employing digestion and elutriation with the Minneapolis-St. Paul plant. To arrive at better comparisons of items essential

TABLE 19.—COMPARISON OF ESSENTIAL ITEMS THAT CONTROL DEWATERING COSTS (1941 DATA)

(a) BASIC DATA

Item No.	Description	Minneapolis-St. Paul, Minn.	Buffalo, N. Y.	District of Columbia	Hartford, Conn.	Springfield, Mass.	Winnipeg, Manitoba, Canada	San Francisco, Calif.	Annapolis, Md.
1	Approximate population	805,000	600,000	800,000	150,000	130,000	255,000	145,000	25,000
2	Total sewage treated (million gal)	39,286.7	51,262	39,568	5,200	5,799	9,572	2,940.6	703.8
3	Suspended solids (ppm)	315	168	180	160	155	329	254	....
4	Percentage removal in plant	74.6	34.8	51	66.5	58.1	54.6	70.5	....
5	Type of sludge dewatered	Raw primary	Digested	Primary digested[a]	Primary digested[a]	Primary digested[a]	Primary digested[a]	Primary digested[a]	Primary digested[a]
	Percentage of Volatile:								
6	Raw	62.1	62.9	66.2	66.3	73.4	69.7	82.0	....
7	Digested	....	47.5	45.1	45.8	45.8	51.1	63.5	51.4
8	Elutriated	....	....	44.4	39.8	48.3	50.5	....	50.0
	Raw Solids:								
9	Tons produced	35,993	18,961	15,000	2,505	3,184	8,769.5	2,748	548
10	Tons dewatered	35,993	....	....	....	....	....	....	....
11	Lb per capita per yr	89.5	63	37.5	33.4	49	69	37.9	43.8
	Digested Solids Dewatered:								
12	Tons	....	10,051	6,870	1,296	946	1,655.2	888	173.2
13	Lb per capita per yr	....	33.5	17.1	17.3	14.6	13	12.3	13.8
	Filter Operation:								
14	Days	351	306	220	156	220	182	197	145
15	Hours	39,919.1	6,320	4,705.1	994	2,277.5	5,407	1,682	710.5
16	Percentage solids in feed	9.35	9.11	6.5	6.8	11.4	10.0	3.15	9.0
	Percentage of Chemical Dose on Solids:								
17	Ferric chloride	1.53	2.93	3.18	2.23	1.63	2.6	4.3	....
18	Lime (Ca O)	3.77	11.21	....	....	....	....	....	....
19	Ferric sulfate	....	....	....	....	....	....	....	2.86
20	Filter yield	3.9	6.77	5.9	7.5	5.0	4.2	5.4	7.5
21	Power cost (cents per kw-hr)	1.163	0.73	0.63	2.32	1.9	0.625	1.1	3.5
22	Fe Cl_3 (dollars per ton)	38.21	31.80	39.00	71.40	72.00	83.60	60	99.6
23	Ferri-floc (dollars per ton)	....	....	....	....	....	....	....	36.10

(b) TOTAL COST OF SLUDGE FILTRATION AND SLUDGE DISPOSAL BY INCINERATION OR HAULING (CENTS PER CAPITA SERVED BY PLANT)

Item No.	Description	Minneapolis-St. Paul, Minn.	Buffalo, N. Y.	District of Columbia	Hartford, Conn.	Springfield, Mass.	Winnipeg, Manitoba, Canada	San Francisco, Calif.	Annapolis, Md.
24	Filtration	9.64	....	3.98[b]	4.25[b]	5.7[b]	5.4[b]	7.0[b]	7.2[b]
25	Incineration or disposal	6.04	....	2.61	1.40	3.22	1.62	1.7	0.0
26	Total	15.68	15.63	6.59	5.65	8.92	7.02	8.7	7.2

(c) CHEMICAL COSTS FOR SLUDGE FILTRATION

Item No.	Description	Minneapolis-St. Paul, Minn.	Buffalo, N. Y.	District of Columbia	Hartford, Conn.	Springfield, Mass.	Winnipeg, Manitoba, Canada	San Francisco, Calif.	Annapolis, Md.
27	Total spent (dollars per yr)	33,410.70	18,382.43	8,500.00	2,060.90	1,108.00	3,803.74	2,300.00	178.70
	Cost, in Cents per Capita:								
28	Actual	4.15	3.06	1.06	1.37	0.852	1.49	1.59	0.714
	With Fe Cl_3 at—								
29	\$38.21 per ton	4.15	3.32	1.04	0.74	0.45	0.68	1.01	0.45[c]
30	\$31.80 per ton	3.71	3.06	0.869	0.61	0.38	0.57	0.84	0.38[c]

(*d*) POWER CONSUMPTION FOR FILTRATION AND FILTER HOURS

	Power Used:								
31	Total kilowatt-hours annually	751,889	....	302,300[d]	48,750[d]	90,000[d]	96,309[d]	63,000[d]	3,695[d]
32	Kw-hr per 1,000 population per year	934	....	378	325	692	378	434	148
33	Filter hours per 1,000 population per 100 sq ft of filter area per year[e]	248	52.7	29.4	23.2	28.9	31.8	23.2	18.5

(*e*) LIFE OF FILTER CLOTH AND CONSUMPTION PER THOUSAND POPULATION SERVED

34	Kind of cloth (CF = canton flannel)	CF	CF	Wool	Wool	Wool	Wool	Wool	CF
35	Acid wash necessary	Yes	Yes	No	No	No	No	No	No
36	Average filter hours per filter[e]	6,653.2	2,106.7	1,176.3	497	1,138.8	2,703.5	1,682	710.5
37	Economic cloth life (hours)	356	211	600	600	800	284	900	142.1
	Dressings:								
38	Total number annually	112	30	8	1.66	3	12	2	5
39	Number per filter	18.7	10	2	0.83	1.42	6	2	5
	Area of Dressings:								
40	Total used annually (sq ft)	56,000	15,000	4,000	581	495	1,800	400	325
41	Square feet per 1,000 population	69.57	25	5	3.9	3.8	7	2.76	13

(*f*) SERVICE CAPACITIES

42	Number of filters	6	3	4	2	2	2	1	1
43	Total area (sq ft)	3,000	1,500	2,000	700	330	300	200	65
44	Filter hours per operating day[e]	113.73	20.65	21.39	6.37	10.35	29.71	8.54	4.9
	Population Per Filter Hour Per Operating Day:								
45	Total	7,078.2	29,050	37,401	23,548	12,560	8,583	17,000	5,100
46	Per square foot of area	14.15	58.1	74.7	67.2	76.1	57.2	85	78.5

(*g*) POPULATION SERVICE WITH ALL INSTALLATIONS OPERATING ON THE MINNEAPOLIS-ST. PAUL TIME TABLE

	Average Hours per Filter Per Day for:								
47	Actual operating day	18.96	6.88	5.35	3.19	5.18	14.86	8.54	4.9
48	351 operating days	18.96	6.00	3.35	1.42	3.24	7.7	4.79	2.02
49	Increased ratio possible at 18.96 hr per filter daily	None	3.16	5.66	13.35	5.84	2.46	3.96	9.38
	Possible Population:								
50	At the increased ratio[f]	805,000	1,896,000	4,528,000	2,000,000	759,000	627,000	574,000	234,000
51	Per square foot per filter hour at this increase (Item No. 50)	14.15	66.6	119.4	150.7	121.3	110.2	151.4	190

[a] Primary digested and elutriated. [b] In all elutriation installations, the cost of sludge elutriation is included in the filtration costs. [c] Annapolis uses Ferri-floc at $36.10 per ton; but during 1940 used a ferric chloride dose of 1.7% on solids, at a cost of $99.60 per ton. [d] In all elutriation installations, the data include power for all elutriation purposes. [e] For total filter days and hours see Items Nos. 14 and 15, Table 19(*a*). [f] Item No. 49 multiplied by Item No. 1, Table 19(*a*).

to obtaining economic filter operation further breaking down of such data is necessary.

The same statement is valid for Table 19(*b*), which is the first step in the process of breaking down sludge disposal costs. It is evident from Table 19(*a*) that the unit cost of chemicals and power varies materially at the different plants. Aside from these facts the sludge lifts differ in the various plants. At Minneapolis-St. Paul all filtration costs include the minor cost of sludge concentration and all costs from and after the point where sludge is pumped from the settling tanks to the point where sludge cake is discharged in the furnaces. The same principle is followed in the remainder of the installations listed in Table 19(*b*). This procedure necessarily includes the entire operating costs for elutriation in the installations using this process.

At Springfield the power is estimated roughly and is taken amply high. The total treatment cost already listed includes an item for depreciation and fixed charges beyond actual operating and maintenance costs. Although Table 19(*b*) has been adjusted to this fact, the higher power estimate has not been changed. This installation has higher sludge lifts than the other plants listed and includes the cost of all changes made during 1941, which was the first full year's operation of this relatively new plant. According to a private communication from J. D. McDonald, engineer in charge of this plant, the 1941 filtration costs included cost of extra ferric chloride and filter cloths purchased beyond 1941 needs and making various minor changes about the filters, conditioning mixer, and incinerator. Aside from these facts, running any vacuum filter to suit an incinerator's sensitive nature necessarily prolongs hours of filter operation and increases power. This was particularly true during the first year's operation at this plant.

At the other elutriation plants the cost of disposing of sludge cake by freight-car shipment or trucking is included. At Annapolis the sale of sludge cake paid for all trucking charges.

At Hartford, the filtration costs were 1¢ per capita higher in 1941 than in 1940 due to the material reduction in population served during 1941—150,000 against 201,400 in 1940. This decrease was brought about by the necessity of diverting a portion of the sewage flow from plant treatment while work on the Park River conduit was in progress.

It is to be regretted that the total cost of sludge filtration and incineration at Buffalo could not be broken down into the items listed in Tables 19(*b*) and 19(*d*), although other tabulated essential information is available.

As chemical and power costs for sludge filtration make up the major portion of total annual filter operating costs and the unit prices for chemicals and power vary materially in the plants listed, Table 19(*b*) should be tabulated with these costs uniform in all cases. However, it is simpler to present these essential items as in Tables 19(*c*) and 19(*d*).

In Table 19(*c*), Item No. 29 shows the per capita cost for chemicals placed on the price paid for ferric chloride at Minneapolis-St. Paul. As the Buffalo plant pays less for ferric chloride than does any other plant listed, the other per capita chemical costs are adjusted to the Buffalo price in Item No. 30. The differences in favor of the elutriation installations are striking. During 1942 the ferric consumption and costs at both Hartford and Springfield improved.

At Hartford the improvement was principally due to securing and using more wash water in elutriation during the latter half of 1941. In fact, the filter yield was materially increased at the same time.

Table 19(*d*) is more or less self-explanatory. The total kilowatt-hours used for sludge filtration during 1941 are shown for all plants except Buffalo. In the elutriation installations the figures include all power used for elutriation. As the expression "population served per filter hour per day or year" has little meaning, due to the fact that different plants use different sizes of filter units, and measure filter hours in terms of each unit, it is rather obvious that filter hours per capita should be reduced to some identical filter area. For this reason, the annual filter hours per thousand population per 100 sq ft of area in the various plants is shown in Table 19(*d*). For example, Minneapolis-St. Paul, Baltimore, Buffalo, and the District of Columbia all use some multiple of 500-sq-ft filter units (see Table 19(*f*)), and the other plants listed use other sizes. Minneapolis-St. Paul recorded 39,919.1 filter hours in 1941 based on one filter unit of 500 sq ft in area serving 805,000 population. Hence, the population served by 100 sq ft of area during this time was $\frac{805{,}000}{5}$, or 161,000; and, for each 1,000 population per 100 sq ft, the filter hours were $\frac{39{,}919.1}{161} = 248$. The other plant data are calculated and listed in the same manner.

Dewatering less sludge solids at higher filter yields in the elutriation installations causes the power and filter hours to be notably lower in such cases. The lowest figure is for the smallest installation at Annapolis. At this plant the flow of digested sludge to elutriation is by gravity, the sludge lift to the vacuum filter is very low, and one or two other mechanized devices for moving and concentrating sludge are absent. Therefore, the power consumption should be relatively low.

Table 19(*e*) is self-explanatory. Again all of the essential factors relating to life of filter dressings, absence of acid washing, etc., are markedly in favor of the elutriation installations. Baltimore is not listed although it shows the lowest filter-cloth consumption of all—1,200 hr of cloth life, or only 2.5 sq ft of cloth dressing per 1,000 population per year. To keep filter yields up and ferric chloride consumption down at Winnipeg, wool cloth dressings are changed more frequently. The wool blankets used at this plant are manufactured locally and cost only $21.00 each. Annapolis uses canton flannel for its small 65-sq-ft filter. Each filter dressing costs just $1.19, or only $6.95 for the entire year of 1941. As a wool blanket lasted practically a full year and cost about $27.00, it is evident why canton flannel is used. Changing cloths on such a small filter unit is an easy and inexpensive task.

Table 19(*f*) relates to population actually served per filter hour per square foot of filter area, per filter operating day, in each plant listed for 1941. Although this essential figure is again materially in favor of the elutriated sludge plants, such a tabulated comparison is not fair to these plants for the simple reason that the vacuum filters in these plants operate only a fraction of the time listed for raw sludge dewatering at Minneapolis-St. Paul. If all the plants were operating on the Minneapolis-St. Paul time basis, without changing the coagulant dose used in 1941, and at the same filter yield, Table 19(*g*) would result. In other words, the average population per square foot per filter hour for all the elutriation installations would be almost ten times greater than at Minneapolis-St. Paul. Of course, it might be argued legitimately that such practice in the installations using only one or two vacuum filters would not allow time for filter repairs and redressing. However, if a spare filter were provided as a standby in such cases, the results would still be at least seven times greater; and if all plants began with approximately the same amount of raw sludge solids per capita, the digestion-elutriation installations would appear about four times better in this particular respect, which the writer considers one of the basic defects in the economy of dewatering large quantities of raw sludge in vacuum filters. As a matter of fact, Mr. Schroepfer's own analytical conclusions show that for Minneapolis-St. Paul the economics of filtration require still more filter area. It should be evident, from the study of Tables 19(*f*) and 19(*g*), that the tendency in the plants employing sludge digestion and elutriation is in the opposite direction—doing the dewatering job with the least amount of vacuum filter equipment and at the least annual operating expense. After all, these items are of material importance to the taxpayer.

Mr. Schroepfer leans to the opinion that filter equipment at Minneapolis-St. Paul may be eliminated by reducing the sludge moisture to 85% or 90% and incinerating the raw sludge directly by increasing the amount of incineration equipment. Sludge digestion is essentially a wet combustion process itself in which the solids are mineralized before further dewatering. In view of the fact that the bacteria in this process do this work for nothing and with much less wear and tear on the relatively long-lived equipment involved, produce much less solids to be dewatered, and plenty of gaseous fuel for plant heating, the writer seriously questions the economics of raw sludge incineration compared to sludge digestion, elutriation, and vacuum filtration, especially on the sludges listed in Table 19(*a*). In fact, by using the latter procedure at Minneapolis-St. Paul, it is doubtful if final sludge incineration would be needed because the digested sludge cake would have a soil value that is out of the question with raw sludge.

According to the writer's estimates, destroying only 60% of the volatile matter in the unthickened raw sludge by digestion at Minneapolis-St. Paul, followed by elutriation of this sludge with river water and filtration, would save over $25,000 more a year in chemicals and power than the 1941 records show. With Mr. Schroepfer's natural genius for pushing filtration economies to the limit in practice, this estimate would undoubtedly be improved. In this estimate no account is taken of the savings due to elimination of lime incrustation, acid washing of filters, extra labor involved in these unnecessary items, frequent cloth changing, and the ultimate use of digestion gas for plant heating.

In the discussion of the relative economies of the vacuum filtration of raw and digested sludges, the argument that initial costs and fixed charges for digestion and elutriation space and equipment may offset the gains due to sludge mineralization has lost is validity in the numerous installations practicing this procedure. Mr. Fuhrman's paper on sludge dewatering at the District of Columbia clearly demonstrates this fact. Extensive filtration and incineration equipment for raw sludge is very short lived compared to digestion and elutriation tanks. Consequently, amortization charges against the former are much higher than against the latter. Notwithstanding these facts the argument has been further invalidated through the recent development of the relatively inexpensive "coilless" digester by A. M. Rawn, M. Am. Soc. C. E., whose contribution (28) to the advance of sludge digestion is outstanding. Expensive construction of tanks with floating covers, hot water coils, and elaborate heat exchanging equipment may be eliminated. As for tank space required for countercurrent or multi-stage elutriation, this is relatively small (0.02 to 0.1 of that installed for digestion). According to the percentage of solids present in the digested sludge, water ratio used for solids washing, and operating hours per operating day, it varies from 0.02 to 0.2 cu ft per capita, the latter value being for thin digested waste activated sludge and eight operating hours per filter operating day.

Acknowledgment.—For aid generously contributed in furnishing data for Table 19, acknowledgment is made to: Messrs. Schroepfer, Velzy, and Fuhrman, Symposium authors; Ellwood Johnson, Robert J. Ross, and Benjamin Benas, Members, Am. Soc. C. E.; George E. Symons, Assoc. M. Am. Soc. C. E.; and Paul D. McNamee, chief chemist, District of Columbia Sewage Treatment Plant; John D. McDonald, engineer in charge of sewage treatment, Springfield; W. M. Scott, Chairman of Commissioners, Greater Winnipeg Sanitary District, Winnipeg; and C. G. Weber, superintendent of the Annapolis Sewage Treatment Works.

WILLEM RUDOLFS,[9] M. AM. SOC. C. E.—Adequate preparation and proper disposal of sewage sludge continue to be among the principal problems of sewage treatment. In general, the processing of sludge for fertilizer or burning requires that the moisture content of primary, digested, or activated sludge must be reduced as much as possible by chemical and mechanical means. Dewatering and air-drying of sludge on sand beds continue to be practiced to a considerable extent, especially where beds can be used throughout most of the year, where natural sand beds are available, for small plants where considerable land but no technical skill is available, and because the method is economical.

As a rule, dewatering of sewage with the aid of chemicals by means of vacuum filtration is not advisable for small plants, except where local conditions such as area available, proximity to dwellings, and final disposal of the sludge require special methods. Nevertheless, there are a number of small plants utilizing vacuum dewatering without any apparent need. The cost is frequently out of proportion and the operation not as good as efficiency and economy demand.

Sludge pressing with the aid of chemicals is used in a few cases; one plant dries its digested sludge successfully by spraying; but preparation for dewatering by cooking; or by direct dewatering by centrifuge, has not been adopted in the United States.

Dewatering.—Most of the dewatering by vacuum filtration is restricted to larger plants. The several Symposium authors are in agreement that for most efficient practice certain requirements are essential:

1. The sludge should be concentrated as much as possible before filtration;

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[9] Chf., Dept. of Water and Sewage Research, New Jersey Agri. Experiment Station, New Brunswick. N. J.

2. Operation of the filters should be as nearly continuous as possible;
3. A conditioning agent must be used;
4. The conditioning agent must be added in accurate proportions and thoroughly mixed with the sludge in the shortest possible time;
5. Filtering must proceed as soon as possible after mixing, to take advantage of the floc formed; and
6. Flexible machinery and close control are essential.

The present method of vacuum filtration is an adaptation of industrial dewatering of slurries. The mechanical structures have not changed greatly, although some minor modifications have been made. The crux of the vacuum filtration method has been, and still is, the proper type and use of chemicals for conditioning. The type of chemical used at present is limited to iron and aluminum salts, with or without lime. Selection of the chemical has been determined by local conditions, availability, character of sludge, etc., but not always on scientific grounds. Under all conditions good coagulation is necessary to avoid troubles. Although great advances have been made in the knowledge of the behavior of sludges and the action of chemicals, much remains obscure. Operating experience at Buffalo, Minneapolis-St. Paul, Chicago, and Washington, D. C., indicate developments. As filter medium, Canton flannel or woolen blankets are used. Difficulties with clogging by iron or lime can now be overcome, but clogging of the filters by thin jelly-like sludge occurs frequently at smaller plants with less efficient operation. Lime scale is removed successfully from filter screens, troughs, and piping with the aid of muriatic acid, at Buffalo; but such removal has failed at other places.

Thorough distribution and rapid mixing of the conditioning chemicals in the shortest possible time are stressed by all authors. Under certain conditions mechanical stirring appears to be favored over air-mixing: Mr. Schroepfer has improved air-mixing facilities by increasing the number of points by which air is admitted, thus reducing the violence of mixing; Mr. Dundas states that most devices available for mixing, although theoretically correct, fail in practical operation on account of their complexity and inability to withstand the action of the chemical; and Mr. Velzy reports that at Buffalo the tendency is to stay away from mechanical appliances for the application of chemicals. Although all agree that rapid mixing and application of chemicals are necessary, there seems to be no definite agreement on the procedure. Practices in a number of small plants vary much more widely than indicated by the Symposium authors. It is questionable whether standardization of practice is desirable or needed.

Some problems requiring further study are indicated. Information is required as to the reaction between coagulants and sludge ingredients; the effect of greases and soaps on the rate of filtration; the effect of thickness of cake on yield; filter washing to prevent binding and reducing the life of cloth; removal of scale and incrustations; type of filter cloth for different sludges; sludge feeding methods to the filters to obtain even loading at all times; improvement of dispersion of chemicals (riffle boards, perforated pipes, ejectors, pumps, sprays, etc.).

The operating cost for conditioning and vacuum filtration as given by the different authors varies greatly. The low figure of $2.19 per ton of dry solids at Minneapolis-St. Paul for primary and chemical sludge is outstanding. Washington, with primary elutriated sludge at $4.64 per ton dry solids, is near the average and somewhat higher than the Chicago range for primary and activated sludge of $3 to $4 per ton of dry solids. Cost data published from a number of small plants, where flows are treated below 10 mgd, vary from $8 to $20 per ton, some being even higher. To these costs must be added the expense of handling and disposal of the cake by dumping, fertilizer production, or incineration.

Drying and Incineration.—Difficulties with drying and burning of the sludge appear to be less than those encountered in dewatering. Uniform feed and continuous operation are stressed to produce maximum surface exposure of the sludge particles in drying and to prevent rapid changes in furnace temperatures. The usual practice of returning previously dried material, which is agitated and mixed with dewatered sludge, appears to have reduced damage, prevented scorching, and alleviated odors. There are still some difficulties. Abrasion of flash drier fans and pipes had been severe at Buffalo, which was reduced by the use of gunite; nevertheless, "it has constituted a problem calling for careful attention." Removal of the preheater in one of the incinerators resulted in considerable savings in power and maintenance cost at Minneapolis-St. Paul. The odor problem of Chicago was solved with electrical precipitators. Constant control, close supervision, a considerable degree of technical and directive ability, skilled labor, and a high degree of interest of the staff are required for economic operation and maintenance.

It is of considerable interest that the cost of dewatering the sludges appears to be from one and one-half to two times the cost of incineration. The total operating cost reported for disposal of the sludge varied from $3.53 to $7.69 per ton of dry solids. Comparison of these costs with those available for barging primary and activated sludges, which varied for the same years from $2.88 to $4.00 per ton dry solids, indicates the present advantage of cities located near the sea. There appears still to be room for improvement in drying and incineration; but since most of the difficulties and the higher cost occur in the conditioning and dewatering of the sludges, studies on simplified dewatering, sludge concentration with, or without aid of chemicals, to a sufficient extent to allow direct incineration, may lead to fewer difficulties and lower cost.

Fertilizer.—For years far-sighted soil scientists have deplored "the wealth that flows to sea." It has been estimated that sewers in the United States carry away enough nitrogen to fertilize 1,000,000 acres of farm land. Although some of the larger cities realize the possibilities, and are considering more extended use of sludge as a soil conditioner, numerous small plants have had material available, which constitutes, in Mr. Van Kleeck's words, "an economic waste of a valuable natural resource."

The use of raw primary sludge as a fertilizer is not recommended for a number of reasons given by Mr. Van Kleeck. The principal objections are odor, grease, acidity, pathogenic organisms, and insufficient humification. Unquestionably, these objections are valid when the material is applied in the raw state to truck-garden crops or used around dwellings. However, the utilization of raw sludge as a soil builder or fertilizer may be reconsidered if the material is to be used by farmers who may store and compost it. Raw sewage sludge mixed with soil, plant residues, or manure and allowed to decompose aerobically for a number of months makes an excellent material of an equally good or better grade than digested sludge. All objections raised disappear because such composted material has an earthy odor and contains a minimum of grease; acidity has been destroyed; pathogenic organisms are killed; and humification has taken place. The rate of decomposition in compost piles is high and naturally varies with the seasons. The heat produced is material even during the colder season.

Although any type of sludge has some fertilizer and fertility value and is capable of improving soils, especially the moisture holding capacity, its value should not be overemphasized. With the present great demand for soil-building material in "Victory Gardens" it has become a question of restraining rather than stimulating the use of sludge. Thousands of people who had no gardens before turn to the "wealth that flows to the sea" and believe that a bag of sewage sludge automatically insures abundant and luscious crops. Disappointment may lead to a wholesale condemnation of the use of sewage sludge. On the other hand, with restrained recommendations, the use of sludge for agricultural purposes may be greatly increased after the emergency is over.

The chemical composition of digested sludge varies materially from place to place. Analysis of many hundreds of samples collected all over the United States shows the following percentage ranges and averages on the basis of bone dry material:

	Range	Average
Volatile matter	42 –64	51
Total ash	58 –36	49
Grease and fats	3 –17	8
Total nitrogen (N)	1 – 4.5	2.2
Phosphorus (P_2O_5)	0.6– 5.7	1.5
Potassium (K_2O)	0.3– 1.6	0.7

It is clear that digested sludges are not complete fertilizers and not balanced plant foods. They contain major and minor elements, growth promoting substances, and aid in replenishing organic matter, and as such are valuable as subgrade fertilizers and for soil improvement. Their value is not constant, but varies materially even at a given place.

Lloyd M. Johnson,[10] Esq.—A wealth of information, covering the development and design of a system for dewatering and drying activated sewage sludge, is presented in this Symposium. Fundamental principles are presented in a logical and understandable manner by Messrs. Dundas and McLaughlin. This paper leads the sanitary engineer through the hitherto foreign fields of combustion, in easy stages. Therefore, it is a timely contribution to the development of a new art in sewage disposal—namely, drying for fertilizer purposes, or combined drying and incineration.

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Three developments discussed by Messrs. Dundas and McLaughlin should be emphasized: Odor control, fertilizer production, and power generation.

Odor Control.—The control of odors is by high temperature deodorization, which consists of raising the temperature of all the gases and vapors in contact with the sludge during the drying period to at least 1,300° F. There is no argument concerning the necessity for high temperature deodorization in locations where odors would create a public nuisance. This has been recognized for decades in garbage incineration. In consequence, garbage incinerators must operate at deodorizing temperatures of 1,300° F and greater. Where odor control is required, high temperature deodorization is the most economical method. No costly chemicals and no special apparatus are required. High temperature deodorization is an integral part of the sludge-drying system.

Fertilizer Production.—The flash-drying system used by The Sanitary District of Chicago is well suited for fertilizer production. Furthermore, the system has great flexibility because the dried sludge can be incinerated whenever the fertilizer market is unfavorable. Before the sludge-drying and incineration system was developed, the end product of a sewage treatment plant was sludge-bed cake. This cake varied in moisture content from 20% to 60%. Thus, after passing through a highly developed and refined treatment process, the city was still confronted with the problem of disposing of huge quantities of sludge-bed cake. With the application of flash drying and incineration, the end product became either a highly desirable and salable fertilizer or a sterile ash well suited for fill. Every sludge, no matter how low in nitrogen content, has some value as a fertilizer and soil conditioner. Experience has shown that a ready market exists, and the sales value will materially help to compensate for the cost of treatment. On the other hand, it is necessary for the city to pay for the trucking and disposal of ash. Thus, as a general rule, provision should be made for fertilizer production in preference to incineration. For very large operations such as those at Chicago, the quantity of ash also becomes a problem. It must be remembered that dried sludge is 50% ash, and it is sometimes difficult to find sufficient low ground for disposal of this ash over a long period of time. Furthermore, ash is highly abrasive and requires considerable maintenance on any ash handling system.

The desirability of fertilizer production is by no means confined to the large cities. As a matter of interest, fertilizer is successfully produced and marketed in communities of 5,000 population.

Power Generation.—The operations at Chicago are unique in that power generation by steam is practiced. It is quite customary for power to be generated by gas engines fired with digester gas. The size of the operations at Chicago make power generation by steam a most economical solution. This process may be applied to much smaller communities if public utility rates are particularly unfavorable or if the location of the plant makes public utility power impractical.

Isador W. Mendelsohn,[11] M. Am. Soc. C. E.—This Symposium of five selected papers affords an opportunity for deserved consideration of one of the major problems of sewage disposal. Presentation in such grouping makes for clearer comparison and more extensive analysis of data evolved in plant operation at Buffalo (B), Washington (W), Minneapolis-St. Paul (M), and Chicago (C) regarding sewage sludge dewatering, incineration, and final use. The Symposium also presents certain limitations which merit attention in rounding out discussion of sewage sludge disposal.

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To the writer, one of the most significant features of the Symposium is the exceedingly large number of tests performed at the plants to develop satisfactory and economical operation in disposal of sludge. By no means do these include all experimental work on sludge disposal at these plants. For ready comparison these researches and results are grouped as follows.

Concentration of Sludge.—

(B) Sludge concentration tanks have proved their value by delivering sludge of reasonably uniform characteristics for filtration and burning, absorption of irregularities in quantity of raw sludge obtained, provision of a 6-day operating week in dewatering and incineration, and provision of gas as fuel for boilers and auxiliary heat for incinerators.

(M) Concentration of sludge by weighing a known quantity pumped from the sedimentation tanks at frequent intervals, by soundings and determinations of moisture content of sludge in tanks, and by withdrawals of supernatant liquor has increased the total solids of the sludge from 8.11% in raw sludge to 9.35% in concentrated sludge.

(C) Concentration of sludge has reduced the quantity of water to be removed by the filter and provided a filter feed of uniform consistency—a vital factor in successful filter operation.

Conditioning of Sludge.—

(B) A mixing tank with no baffles and with proper mixing-paddle speeds and a short detention period gave best results. In the application of chemicals, equipment has been developed to provide easy manual control in preference to automatic devices, such as feeding ferric chloride solution through a perforated rubber-pipe arrangement so that the number of small streams of solution is an indicator of the rate of feed.

(W) In the operation of elutriation mixing tanks, fine sand settling out in the tanks has been removed gradually to the vacuum filters with the sludge by connecting the tank inlet and outlet with a 12-in. pipe. To reduce sludge pump clogging and interference with continuous dewatering, the sludge pumps elevating the digested, elutriated sludge from the elutriation tanks to the mixing tanks were provided with 3-in. instead of 2-in. impeller passages and a maximum speed of 1,140 rpm. To overcome sagging of pipe lines between supports, steel pipe with hard-rubber lining was substituted for hard rubber ferric chloride pipe lines.

(M) Better results were obtained by increasing the number of points through which air is admitted to the sludge for mixing and by decreasing pressure and violence of mixing. Improved mixing resulted from changing the path of the sludge through the tanks with over and under baffles to a sinuous path both vertically and horizontally, and reducing the detention period in conditioning tanks from 13 min in 1938 to 7 or 8 min at the close of 1941. Installation of riffle boards for both lime and ferric chloride instead of adding chemicals to the tanks through one or two pipes provided better dispersion of chemicals. Ferric sulfate and lime and ferric chloride alone can be used satisfactorily as substitutes in emergencies, but they are not as economical by about 25% as ferric chloride and lime.

(C) In conditioning sludge for many years, ferric chloride alone has been found the most effective and most economical reagent available for conditioning activated sludge. With other sludges, combinations of ferric chloride and lime and in some cases lime alone are efficacious; chlorinated copperas and ferric sulfate are probably less effective. Special proportion devices for continuous conditioning and frequent checking of sludge to determine the amount of chemical have established better dosage control of ferric chloride, which has been found to be the most expensive item of filtration cost in handling activated sludge. The amount of ferric chloride required may be related to the volatile content of the activated sludge; ferric chloride requirements for conditioning activated sludge of The Sanitary District of Chicago vary between 4% and 7% by weight of dry cake solids, and resultant cake moistures vary between 78% and 85%.

Operation of Vacuum Filters.—

(B) Canton flannel filter cloth has proved most satisfactory in operation, cost, and life of cloth; and ordinary sash cord has proved a satisfactory substitute for bronze strips in the application of the cloth to the filter drum. Experiments are in progress with a screw feeding device and a surge bin above the screw, to receive the sludge cake from the filter and discharge it uniformly to the first conveyer belt. Formation of lime scale on the wire screen on the surface of the filter drum was removed successfully with the application of muriatic acid in the filter tub; this acid also has been used successfully in reducing scale accumulations in filtrate lines and in the supernatant line receiving the discharge from the filtrate pumps.

(W) Washing filter cloths in a 1% solution of oxalic acid for a 24-hr exposure after certain period of use extended their life 30%; application of a 100-ppm solution of copper sulfate every Saturday and Sunday by rotating the filter cloth in the solution in the filter pan to permit saturation overcame filter-cloth disintegration by fungus growths in the regular filter shutdown from Friday afternoon to Monday morning.

(M) Eight-ounce Canton flannel filter cloth was found to be most satisfactory at this plant. The difficulty of obtaining this cloth during wartime necessitated additional studies leading to the possible use of a new cloth type. The economic life of filter cloths has been restricted to 350 hr; tests made on a plant scale to demonstrate the effect of cloth age on filter yields show that at 500 hr the yield of a filter is approximately 10% to 15% less than that of a new cloth, other conditions being constant, with 4-hr intervals between washings. An appreciable improvement is effected by proper washing, as noted by rearranging the washer sprays to impinge along the filter drums almost at right angles to the drums, instead of tangentially. It appears to be economical to give each cloth an acid bath after 350 hr of service (using 1 to 2 carboys of hydrochloric acid in 600 gal of water), thereby reducing the binding and increasing the economic cloth life to approximately 500 or 600 hr. (This expedient provides an inexpensive means of removing calcium carbonate from the screens and splines of the filters, which formerly became so incrusted with matter as to interfere with the free passage of water through the filter.) Especially is it economical in chemicals to wash filters frequently, the periods between washings having been reduced from 24 hr to 4 hr. Frequent filter washings led to the development of a method of washing filters while in service by installing the wash-water pipe and nozzles (instead of its usual location on the far side of the filter from the take-off plate) immediately below the top edge of the take-off plate, so that the washing operation is done in the section of the drum between the take-off and the submergence in the sludge, which section is not utilized in the usual installation. A trough suspended immediately above the sludge level returns the spent wash water to the overflow box on the filter. Besides maintaining a clean filter medium, this washing procedure in effect adds one filter. For each size of installation and type of sludge, it is possible to determine the economic range of speeds at which filters ordinarily should be operated. The filter rate has a marked effect on the economics of filtration; instead of expressing the filter rate as pounds per square foot of installed filter capacity per hour, the index of pounds per square foot is favored to include varying filter speeds. Uniform feed is desirable where incineration or drying is the practice, and for this a spare filter unit is desirable in addition to regular operating filters for use during washing of regular filters.

(C) A long-nap woolen blanket is most satisfactory and economical for filtering activated sludge conditioned with ferric chloride. The life of a blanket is approximately 3½ months; the life of binding wire is about the same as the filter cloth. Use of woven wire for cloth backing has not proved satisfactory; wood backing may prove more satisfactory and economical than wire cloth.

Operation of Incinerators.—

(B) Mechanical developments at Buffalo include replacement of oil by gas exclusively as an auxiliary fuel, resulting in the disappearance of scale on preheaters caused by oil fuel, removal of soot blowers as unnecessary, and abrasion on heater surfaces making necessary replacement of some parts. Abrasion in pipe lines and ducts has been solved by use of concrete wear backs at bends; lower speeds in the flash-drier cage will have increased its life to about 5 months; adjustments in the type of assembly have resulted in easy replacement of worn parts and reuse of portions of these parts; moving the induced draft fan to the clean air side of the drying system cyclone has reduced the severe abrasion of the fan. Concrete, brick, or ceramic material linings in place of steel linings for stationary parts subject to abrasion are under experimentation.

(M) Sudden and large changes in sludge feed cause rapid changes in furnace temperatures with consequent long-time damage to refractory materials. Removal of the preheater is expected to reduce the operation and maintenance costs of incineration by about $0.33 per ton; fuel oil required for incineration has been reduced, only 1.5 gal now being necessary per ton of dry solids incinerated for heating up, cooling down, and holding temperatures in incinerators not in operation. Substantial fuel-oil savings were effected by installation of duct work so that available heat in operating furnaces could be utilized for heating up, cooling down, or maintaining temperatures in any desired unit. There is a possibility of the utilization of incineration heat for generating sufficient steam for all heating purposes and electric power requirements. Tests are under progress on incineration of filter cake to which has been added a portion of liquid sludge in the belief that, if the moisture content of sludge is reduced to 85% or 90%, dewatering on vacuum filters prior to incineration could be dispensed with economically, provided suitable incinerators are available.

(C) Extensive tests on sludge drying and incineration have led to the use of flash-drying equipment which satisfies the eight fundamental principles and factors for economical and efficient drying free from nuisances to prepare sludge for efficient combustion or for use as a fertilizer—maximum surface exposure, thorough agitation of the wet material, temperature potential of gas, velocity of drying gas, responsive and flexible temperature control, deodorization of gases vented from the drying operation, instantaneous water removal, preservation of fertilizer qualities. Heat-dried sludge should have a moisture content below 10% and be as free as possible from dust. The temperature of sludge at time of loading in cars should be below 100° F to prevent spontaneous combustion when stored in large quantities over a period of time. This is accomplished by screening, dedusting, and cooling in water-jacketed conveyers. It is necessary to raise the temperature of all gases and water vapor produced in the heat-drying of sludge to not less than 1,300° F in the presence of sufficient oxygen to effect complete combustion of offensive hydrocarbon compounds. Furnace spaces connecting flues and ducts of the sludge burner should be designed sufficiently large to accommodate excess gases such as nitrogen and oxygen. Tests with a commercially patented electrical precipitator indicate a removal of 97% of suspended solids from furnace stack gases. The presence of a large volume of visible water vapor in stack gases has not been objectionable in atmospheric pollution, if it is discharged at a height to permit complete dispersion. Where supplementary fuel is needed frequently in regular operation for drying and incinerating sludge in moderate size installations, such fuels in order of desirability, without reference to cost, are digester gas, natural gas, and fuel oil, or combinations thereof, and in large installations, powdered coal is very satisfactory and economical. At the Southwest Works, almost three years of operating plant tests on combining sludge drying with steam generation have proved its feasibility in providing cheap power, reducing furnace maintenance costs due to protection of refractories by water walls, and furnishing greater flexibility since furnace operation is not dependent on the drying load alone. More than half of the fuel is burned for steam generation so that sudden changes in drying load do not upset furnace conditions as they would if the furnace were designed for drying only. With all these improvements in drying and incinerating sewage sludge on record, still more developments are foreseen.

Operating Results.—

(B) For the first three years of plant operation, moisture in raw sludge was reduced from 96.54% to 92.77%; in digested sludge it increased from 87.8% to 90.89%; and in sludge cake it increased from 61.8% to 63.1%. Volatile matter in raw sludge, digested sludge, and sludge cake increased from 57.6% to 62.9%, from 41.5% to 47.5%, and from 38.8% to 42.9%, respectively. Heat content of sludge cake increased from 3,790 to 4,320 Btu per lb; filter rate in pounds of dry solids per square foot per hour decreased from 7.58 to 6.77.

(W) Elutriated sludge contained 44.4% volatile matter in solids; 3.8 volumes of Potomac River water to one volume of digested sludge gave a total alkalinity reduction equivalent to 7.5 volumes of wash water, which was due to countercurrent elutriation. Sludge cake contained 28.9% solids; filter rate averaged 5.9 lb of dry solids per square foot per hour.

(M) Moisture in raw sludge decreased from 92.85% to 91.89%, in the thickened sludge from 90.98% to 90.65%, and averaged 65% in filter cake. Volatile matter in filter cake increased from 52.1% to 58.8%. Filter rate was reduced from 5.50 to 3.90 lb of dry solids per square foot per hour. Filter rates of 5 to 10 lb per sq ft per hr were not found economical, since reductions in filter rates were attended by reductions in chemical requirements. Conditions at Minneapolis-St. Paul indicated 2 lb per sq ft per hr as economical, based on total annual charges, including fixed charges, the basic influencing factors being type of sludge, cost of chemicals, etc.

Operating Costs.—

(B) The total cost of treatment excluding pumping (sedimentation, sludge disposal, and chlorination) was $6.92 per million gallons, or $19.38 per dry ton.

(W) Sludge elutriation, vacuum filtration, and sludge cake disposal costs only, excluding sedimentation and sludge digestion, were $1.33 per million gallons, or $7.69 per ton of dry solids in the cake.

(M) The cost of filtration alone decreased from $4.92 to $2.12 per dry ton and the cost of incineration decreased from $2.05 to $1.29 per dry ton. The total cost of collection, treatment, and sludge disposal decreased from $15.40 to $8.25 per dry ton; and the cost of sludge disposal varied between 40% and 45% of the total cost.

Use of Sewage Sludge as Fertilizer.—

(M) Tests are in progress on the use of sludge cake as a soil conditioner, the sludge being composted for a month or more to render it more friable and a better humus material for plant growth.

General.—Despite the numerous improvements in sewage sludge dewatering, incineration, and final use cited in the Symposium, it is of outstanding importance to note that each paper definitely indicates that still further developments are foreseen.

The Symposium shows that the many improvements were the result of combined efforts of sanitary engineers, mechanical engineers, chemists, and plant unit operators. It consists of papers on operation of sewage plants each serving populations of more than 500,000. According to the 16th Census of the United States in 1940 (29), there were only fourteen cities of 500,000 population or more. The latest comprehensive tabulation of sewage treatment plants in the United States (30) gives a total of 5,580 plants, of which 97% or 5,403 provide sludge digestion, and 3,522 or 63% have some form of sludge dewatering and disposal. Citation of these figures indicates how much more important and apropos the Symposium would have been if some papers had been included on plants serving populations of 20,000 to 200,000, which group would include a large percentage of the sewage treatment plants in the United States. For example, (M) data indicate that for sewage plants serving populations of 50,000 to 200,000 sludge digestion and drying on sand beds should be considered in comparison with dewatering by vacuum filters and incineration, based on type of treatment by which the sludge is produced, quantity of sludge to be disposed of, dependability of operation, and related factors.

The plant operation cost data presented in the Symposium leave much to be desired for better comparison and analysis. Cost of sewage treatment alone, exclusive of cost of sewage collection, should be furnished, on a million gallon basis and a dry ton sewage solids basis. In addition, cost of filtration should be given on a dry ton sewage solids basis, as well as cost of incineration, and other treatment unit operation costs as well, so that a definite dollars and cents value is presented; also a percentage of the total treatment cost should be made available for each unit.

As the (M) paper illustrates both in presentation of data and in stressing the point, studies can be made profitably, at all sewage plants, of economics of operation of present installation, possibilities of effecting economies by enlargement of present facilities at some future date, and economical rates for different types of sludge for the benefit of designers of future plants.

It is well to note that year after year at least one sewage treatment plant is presenting irrefutable data of the value of trained professional sewage treatment plant supervisors and operating personnel. The author of the (M) paper attributes satisfactory operating results to the relatively large number of professional engineers and chemists in supervising positions at his plant, necessitated by greater complexity of sludge filtration and incineration processes; to maintenance at high level at all times of interest and conscientiousness of entire personnel—particularly filter and incinerator operators; by control of filtration by Buechner funnel tests; hourly calculations of chemical dosages; shift competition; and utilization of new ideas. The splendid data presented in the (M) paper make one wonder whether other sewage treatment plants have done as well as they could, and should, in the employment of a number of qualified professional engineers and chemists as supervisors and in the training of plant operators.

Mr. Van Kleeck summarizes data on the conditioning and application of sewage sludge as fertilizer, a most important and promising phase of disposal of sewage sludge. Sufficient attention to this phase has not been given heretofore on an extensive cooperative basis. The progress made, especially since about 1928, indicates a successful solution with sound development.

Langdon Pearse,[12] M. Am. Soc. C. E.—The range of procedure presented in this Symposium illustrates the rapid advance in the practice of handling sewage sludge since the early 1920's, and particularly in the technique of conditioning, dewatering, and heat-drying sludge. The discovery of the value of ferric chloride as a conditioner by Mr. Palmer and the extended study by Mr. Dundas and his associates in heat-drying and incineration are among the most important advances in the art. The first stage of the development related to design; the second, to operation and maintenance.

Mr. Velzy has described operating conditions at the Buffalo installation, in which primary sedimentation produces a sludge that is treated by partial digestion, vacuum filtration, and incineration. Mr. Fuhrman discusses the plant of the District of Columbia, where the primary sludge is digested and elutriated before vacuum filtration, the cake being dumped. Mr. Schroepfer analyzes the Minneapolis-St. Paul plant, in which sludge from primary settling is concentrated, dewatered, and incinerated. The maintenance problems in each are somewhat different: Buffalo uses the flash dryer and Minneapolis-St. Paul, the multiple hearth; Mineapolis-St. Paul does not employ digestion, Buffalo uses partial digestion, and the District of Columbia has complete digestion with elutriation of sludge. The problem of The Sanitary District of Chicago has been the design and operation of apparatus to dewater, heat dry, and incinerate excess activated sludge, as described by Messrs. Dundas and McLaughlin.

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[12] San. Engr., The San. Dist. of Chicago, Chicago, Ill.

If the end product is fertilizer, certain additional costs are incurred from the need of fuel to replace the heat otherwise supplied by the sludge itself when burned. These costs have been estimated at \$2.50 to \$3.00 per ton of dried sludge. In the operation of the equipment, a drier filter cake can be secured if incineration follows. This affects the output of the equipment, measured in dry weight. Furthermore, the rates of filtration on a vacuum filter are considerably lower, measured in pounds of dry solid in cake per hour, for activated sludge than for fresh or digested sludge, and the moisture content of the cake is higher.

For filter cloths Buffalo and Minneapolis-St. Paul use canton flannel. The District of Columbia and Chicago use woolen cloth. At Chicago wool was chosen, despite the higher first cost, because with a life of three months the total cost of the woolen cloth in place was less per ton of product. Because of war conditions, various materials have been tried for winding wire. However, a method of crimping the cloth in place with grooves and wooden splines is proving satisfactory, without any use of wire. The pipe drainage system of the filters is being replaced with plastic or glass pipe. Wooden backing has proved superior to wire mesh.

Heat-dried activated sludge at Chicago is sold only in bulk in carload lots, based on analysis, under a 5-yr contract with an exclusive buyer. The material is distributed chiefly in the southeastern United States. The output has varied. Prior to World War II, the equipment was worked at a higher rate than proved feasible under war conditions. The sales are shown in Table 20.

TABLE 20.—SALES OF HEAT-DRIED ACTIVATED SLUDGE AT CHICAGO

Year	Tons	PERCENTAGE, DRY WEIGHT	
		Ammonia	P_2O_5
1939	11,303	6.00	....
1940	58,464	6.40	2.25
1941	56,597	6.24	2.27
1942	51,729	5.97	2.45

The ammonia content at the Southwest Works at times is as high as 8.1%, but at no time has it fallen below 4.37%. The Calumet sludge is consistently lower in ammonia than the Southwest sludge.

At present there is considerable demand for organic material for mixing purposes. Milwaukee and Chicago are currently selling all they can produce. Milwaukee markets about half its output in bags through jobbers and retailers. This is practicable because of storage facilities of about 25,000-ton capacity. At Chicago, shipment is made as manufactured.

The status of the conversion of sewage sludge to fertilizer is apparently a matter of economics. At present, with the rigid control over chemical nitrogen, organic nitrogen is in demand. From 1910 to 1920, the trend of the organic nitrogen market was upward, reaching a peak in 1920. It then turned downward, largely because of the competition of chemical fertilizers. The trend was reversed about 1932 and rose until 1937, then dropped slightly, and finally stabilized at a somewhat higher level for the war period under ceiling control. Undoubtedly organic matter or humus is required in many soils. Heat-dried activated sludge is the most valuable of the sludges. However, other sludges properly prepared offer a useful source of humus and nitrogen of lower grade. In general, the distribution of sludge in agriculture is limited largely by economic reasons. Air-dried digested sludge is suitable for local use, as a substitute for manure, whereas heat-dried activated sludge may be shipped as far as 1,000 miles.

Because of the uncertainty of prices for organic nitrogen in postwar times, the planner of large activated sludge works should certainly provide another outlet than fertilizer. The plants of the Sanitary District are designed either to burn the heat-dried sludge or to sell it as fertilizer. Because war conditions prevent any present installation of additional equipment, the Sanitary District has just completed a 16-in. sludge pipe line, 4⅔ miles long, from the Southwest Works feeding a system of lagoons, covering 78 acres, divided into 16 units. The working depth averages 16 ft. This will handle excess activated sludge seeded with Imhoff digested sludge.

The use of digestion tanks as a buffer between the settling tanks and the vacuum filter may be necessary in many projects to average the solid load and reduce the equipment for vacuum filters and incinerators. However, situations may arise in which digestion may prove undesirable, particularly in handling industrial wastes from the brass industries, containing copper, which tend to slow up or stop the digestion of sludge. In such cases the designer should consider taking the raw sludge direct from the settling basin to the vacuum filter, with suitable conditioning followed by incineration.

Although the Sanitary District has given considerable thought to the disposal of fly ash, as yet nothing has developed which replaces the pumping of a wet slurry of fly ash to a neighboring lagoon. At the Southwest plant the fly ash is practically all derived from the burning of coal in making steam as well as for drying.

From the standpoint of operation and maintenance, there are still opportunities for improvement in the details of design. Both capacity and life of equipment should be increased, with a minimum of expensive or hard-to-get material. In the conditioning of sludge, elutriation has come forward, particularly for handling digested sludge. In the field of conditioning agents, ferric chloride still is preeminent. Thus, the chemical industry has materially aided sewage treatment development by substantial reductions in price since 1925.

C. R. VELZY,[13] M. AM. SOC. C. E.—The Symposium and the discussions describe the problems of dewatering and incineration, show where improvements and economies can be achieved, and indicate the possible direction of future developments. In general, conclusions are not drawn but rather experiences are stated with the inference that study and development should be continued.

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The disposal of sludge thus far has been a necessary expense of sewage treatment. Disposal processes do not affect the quality of treatment plant effluent, and the value of sludge has never been more than that of a by-product. In some cases, utilization of sludge for fertilizer has yielded an income, and this use will develop as costs of dewatering and handling are reduced and as the demand increases for sludge as a soil conditioner. Efforts toward improvements in sludge disposal in existing plants, therefore, have had, as a final objective, reduction in cost of the processes.

Attention is called to the need for a high degree of ability in the directive personnel, and skill and interest on the part of operators, of dewatering and incineration plants. This need is obvious, not only on account of the complexity of the processes, but also in order to carry on the research and development necessary to continued progress.

The description of the problems of dewatering and incineration, and the comments on utilization of sludge are instructive but do not point to any common procedure for sludge disposal which may be followed in all cases. Designing engineers must still make careful analyses for each new design. For small plants, the use of sand-drying beds is most economical, with utilization of sludge for fertilizer dependent largely on local demand. In some sea coast cities, disposal of sludge at sea is still the least costly procedure. For the large group of cities not in either of these classifications, a thorough analysis of each case is required. Location of the plant, availability of space, and other factors will affect the selection of sludge disposal processes, but the over-all cost of sludge disposal is likely to be the controlling factor.

Experience in existing plants indicates that design should always provide adaptability for future developments. One important item in this connection is the possibility of the use of sludge for fertilizer. The economy of this development may depend upon cost of auxiliary fuel and the market for fertilizer. An incinerating plant which has flexibility in operation as between incineration and drying of sludge for fertilizer has an advantage in being able to follow market conditions. Other possible developments, such as better temperature control in incinerators, heat recovery, and improvements in mechanical equipment, should be considered in design so that observations may be made and adjustments effected, as indicated by the operating experience.

GEORGE J. SCHROEPFER,[14] ASSOC. M. AM. SOC. C. E.—Many of the problems arising in connection with various types of sludge disposal are presented in the papers of this Symposium and in the discussions. Some solutions have been suggested. Years ago an eminent sanitary engineer made a statement to the effect that "Sewage treatment is sludge disposal," indicating that the controlling problem is the disposal of sludge resulting from various processes of sewage treatment. A reading of these papers renews the conviction of the writer that this statement is still apparently true. Frank discussion of some of these problems, as in this Symposium, is leading to their satisfactory solution, and toward even further improvement in the art of sewage treatment.

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[14] Chf. Engr. and Supt., Minneapolis-St. Paul San. Dist., St. Paul, Minn.

The writer enjoyed reviewing the various discussions, and was particularly pleased by Mr. Genter's thorough consideration of the results of the Minneapolis-St. Paul operations. Mr. Genter has accumulated a mass of valuable information on sludge disposal which he presents very effectively, from his point of view, in Table 19. He uses the results of this analysis as the basis of a major part of his discussion. Unfortunately, Mr. Genter bases his comparisons on per capita values which the writer will attempt to show are not scientific and in accordance with the facts. For this reason, the writer's review of Mr. Genter's discussion is limited to showing the fallacy of the method of comparison used.

Every one conversant with the field of sewage treatment realizes that there are variations, affected by a number of factors, in the quantity and quality of sewage from various municipalities. Furthermore, and in addition, there is a large variation in the removals accomplished by different plants. A comparison of costs and efficiencies of sludge disposal on a per capita basis penalizes a plant with a treatment process that is more effective than another or which by chance happens to receive a relatively stronger sewage. Since the per capita quantities of solids removed may vary as much as tenfold between two plants, the per capita basis of comparing sludge disposal is improper and unsound. Although this method of comparison may have a place in commercial literature, or in general comparisons for popular consumption, its usefulness or propriety in scientific analysis is open to serious question.

In Table 21 some of Mr. Genter's data from Table 19 are included, and, in addition, but using the basic data, other comparisons are shown (with items such as Nos. 2a, 26a, 26b, and 28a). Considering the various data (see item No. 2a), the quantity of sewage in gallons per capita per day varies for the plants listed from 77 to 234. The suspended solids content (item No. 3) varies from 155 to 329 ppm. The percentage removal of solids by the plants listed varies from 34.8% to 74.6%. All of these factors combine to cause a large variation in the quantity of solids removed and to require disposal in the various plants, ranging from 33.4 to 89.5 lb per capita per yr. In the same manner the quantity of sludge filtered varies from 12.3 to 89.5 lb per capita per yr. By the writer's method of comparison the latter six plants in Table 21, using elutriation, are shown to be particularly efficacious. The writer insists that to compare, on a per capita basis, six plants employing elutriation, and filtering an average of about 15 lb per capita per yr, with the Buffalo plant (which filters 33.5 lb per capita per yr) and with the Minneapolis-St. Paul plant (which filters 89.5 lb per capita per yr) is unscientific and fallacious.

On a per capita basis (item No. 24, Table 21), the costs at Minneapolis-St. Paul are the highest, $9.64 per capita per yr, compared to an average of $5.60 for the six plants employing elutriation. However, on the more proper basis of tonnage of solids removed or disposed of, the order of economy is reversed. Thus, the filtration cost per dry ton of raw solids is $2.23 at the Minneapolis-St. Paul plant compared to a $2.60 average for the six plants employing elutriation. Furthermore, even the tonnage of raw solids removed by the settling tanks is not a proper basis for comparing the costs of sludge filtration in several

TABLE 21.—COMPARISON OF ESSENTIAL ITEMS THAT CONTROL DEWATERING COSTS (1941 DATA)

Item No.	Description	Minneapolis-St. Paul, Minn.	Buffalo, N. Y.	District of Columbia	Hartford, Conn.	Springfield, Mass.	Winnipeg, Manitoba, Canada	San Francisco, Calif.	Annapolis, Md.
	(a) BASIC DATA								
1	Approximate population	805,000	600,000	800,000	150,000	130,000	255,000	145,000	25,000
2	Total sewage treated (million gal)	39,286.7	51,262	39,568	5,200	5,799	9,572	2,940.6	703.8
2a	Gallons per capita per day	133	234	135	95	122	106	96	77
3	Suspended solids (ppm)	315	168	180	160	155	329	254	....
4	Percentage removal in plant	74.6	34.8	51	66.5	58.1	54.6	70.5	....
5	Type of sludge dewatered	Raw primary	Digested	Primary digested elutriated	Primary digested elutriated	Primary digested elutriated	Primary digested elutriated	Primary digested elutriated	Primary digested elutriated
	Raw Solids:								
11	Lb per capita per yr	89.5	63	37.5	33.4	49	69	37.9	43.8
	Digested Solids Dewatered:								
13	Lb per capita per yr	....	33.5	17.1	17.3	14.6	13	12.3	13.8
	Percentage of Chemical Dose on Solids:								
17	Ferric chloride	1.53	2.93	3.18	2.23	1.63	2.6	4.3	....
18	Lime (Ca O)	3.77	11.21	....	....	....	....	....	....
19	Ferric sulfate	....	....	....	....	....	....	....	2.86
	(b) TOTAL COST OF SLUDGE FILTRATION AND SLUDGE DISPOSAL BY INCINERATION OR HAULING (CENTS PER CAPITA SERVED BY PLANT)								
24	Filtration	9.64	....	3.98	4.25	5.7	5.4	7.0	7.2
25	Incineration or disposal	6.04	....	2.61	1.40	3.22	1.62	1.7	0.0
26	Total	15.68	15.63	6.59	5.65	8.92	7.02	8.7	7.2
26a	Filtration cost per ton of raw solids	2.23	....	2.12	2.63	2.33	1.57	3.70	3.28
26b	Filtration cost per ton of digested solids	2.23	....	4.63	4.92	7.83	8.33	11.45	10.40
	(c) CHEMICAL COSTS FOR SLUDGE FILTRATION								
27	Total spent (dollars per yr)	33,410.70	18,382.43	8,500.00	2,060.90	1,108.00	3,803.74	2,300.00	178.70
28	Actual cost, in cents per capita	4.15	3.06	1.06	1.37	0.852	1.49	1.59	0.714
28a	Actual cost per ton of filtered sludge	0.93	1.83	1.24	1.59	1.17	2.30	2.59	1.03

plants in some of which digestion is employed and in others not utilized, since, in a particular plant, costs of such a major item as chemicals vary directly with the quantity of chemicals actually filtered. For this reason, the cost per dry ton of solids actually filtered (item No. 26b) is a more proper method of comparison of filtration costs. At the Minneapolis-St. Paul plant, the cost per ton of solids filtered is $2.23, compared to an average of $7.78 in the other six plants (lowest $4.63, highest $11.45). Thus, on a more proper basis, the costs at the Minneapolis-St. Paul plant are less than one third of the average costs at the six plants employing elutriation, instead of twice as much as Mr. Genter's basis of comparison implies. The writer concedes that costs at a large plant, such as that at Minneapolis-St. Paul, should be markedly lower than those at a smaller plant filtering as little as one thirtieth of the quantity handled at Minneapolis-St. Paul. The writer also admits that under certain conditions elutriation will result in reduction of chemical requirements for conditioning (possibly at the expense of the treatment process). However, he does not agree that elutriation is the panacea for all sludge disposal "headaches," and in this discussion is using Mr. Genter's own data to show that comparisons on an unsound basis may be very misleading.

The chemicals required for sludge conditioning vary with the tonnage of sludge filtered. In fact, the chemical requirements are based, as the usual "yardstick" (see Table 19, items Nos. 17, 18, and 19), on the percentage of chemicals as related to the dry sewage solids filtered. Per capita quantities of chemicals have no place in plant comparisons nor in the comparison of year-to-year operation of a particular plant. As a result of the war effort, there has been a marked increase in tonnage of solids received at a particular sewage plant, all out of proportion to the increase in population of the community. Mr. Genter's basis of comparison being on a per capita basis would imply that the engineer or superintendent responsible for the operation of a plant receiving, say, twice the load of solids it formerly received, is operating his plant inefficiently because the quantity of chemicals doubled. Chemical requirements must be based on the tonnage of solids disposed of. Referring to Table 21, and basing the comparison on the costs of chemicals in cents per capita, as Mr. Genter does (item No. 28, Table 19), the costs at Minneapolis-St. Paul are shown to be 4.15¢ as compared to the average of the six plants employing elutriation at 1.18¢. Since these six plants filter only about one sixth of the quantity of solids per capita of the Minneapolis-St. Paul plant, the unscientific nature of this method of comparison will be evident. Basing the comparison on the actual cost of chemicals per ton of solids filtered, the picture is exactly reversed. As compared to a cost per ton of $0.93 at Minneapolis-St. Paul, the average costs for the six other plants shown is $1.64. This difference would possibly represent more the variation in size of the plant with attendant higher unit costs of chemicals than it would the effect of elutriation.

All of the foregoing comparisons have been based on Mr. Genter's data which of necessity were restricted to the period up to 1941. In the case of the Minneapolis-St. Paul plant, further economies in the use of conditioning chemicals have been effected. Thus (see Table 22), as compared to 1941 costs of $0.93 per dry ton for chemicals, costs in 1942 were reduced to $0.75, and further lowered for the first ten months of 1943 to $0.72. This is less than one half the costs in the other six plants listed. Similar reductions may have been accomplished in other plants, but these additional data are shown to indicate that, as a basis of comparison, chemical requirements, even in 1942, had not hit "rock bottom" at this plant.

TABLE 22.—ANNUAL COMPARISON OF CHEMICAL REQUIREMENTS AND COSTS OF SLUDGE DISPOSAL

Year	Percentage of Dry Weight		Actual cost of chemicals per dry ton	Total Operation and Maintenance Cost per Dry Ton		
	Ferric chloride	Lime as Ca O		Vacuum filtration	Incineration	Total
1938	3.17	10.30	2.65	4.92	2.05	6.97
1939	2.10	5.68	1.56	2.64	0.99	3.63
1940	1.83	4.51	1.11	2.19	1.34	3.53
1941	1.53	3.77	0.93	2.16	1.23	3.39
1942	1.20	3.44	0.75	2.10	1.17	3.27
1943[a]	1.10	3.12	0.72	2.05	1.19	3.24

[a] January 1, 1943, to October 31, 1943.

It is desirable and important that the sludge disposal process employed should not have a deleterious effect on the treatment process. In other words, it is important that solids once removed by the treatment process not be returned as a result of the failure or inability of the sludge disposal process to handle such solids. Within limits, of course, it is reasonable that finely divided solids be returned to the treatment process in the expectation that they will again be removed. In reviewing the data in Table 19, it was observed that in the plants employing elutriation the number of pounds per capita per year removed by sedimentation averaged 45.1, whereas the number of pounds remaining after digestion and elutriation averaged only 14.7 lb. This raises a question of the reason for the disparity between the quantities of sludge actually filtered and the quantity of sludge normally to be expected following digestion but without elutriation. For example, at the Winnipeg plant a total of 69 lb of raw solids per capita per year were removed by sedimentation and discharged into the digestion tanks. However, a total of only 13 lb were filtered, instead of the 35 to 45 lb which might ordinarily be expected with the normal destruction of solids by digestion. Another possible explanation is that some of the solids were disposed of by other means. At one inspection of the Winnipeg plant, this possible explanation was rejected, since no solids were disposed of by any means except filtration. The only remaining possibility, therefore, is that an extremely large part of the solids entering the digestion tanks was "lost" by heavy supernatant pumping from these tanks, or that in the elutriation process certain fine solids were washed out and returned to the raw sewage from which they failed to settle out again. As shown in Table 23, in the six plants employing elutriation following digestion, the average loss of solids was 58.5% of the solids actually filtered, ranging up to 165% in the case of the Winnipeg plant.

All of the foregoing is intended to show that comparisons of sludge disposal should be based on the work done by such processes, that the tonnage basis accurately measures such work, and that a sludge disposal process which adversely affects the treatment process should even be penalized for such effect in a proper basis of comparison.

In the paper by the writer, the advantage of skilled engineering and technical personnel throughout the various grades of a staff in a large plant was emphasized. This observation was commented upon by several of the discussers. When the paper was being written (1941), the Minneapolis-St. Paul plant had been in operation for slightly more than three years, and operating and cost data were included only to October 31, 1941. Since then more data on sludge disposal operations have become available, and these are presented in Tables 22 and 24. In Table 22 it is shown that sludge filtration costs have

TABLE 23.—COMPARISON OF EXPECTED AND ACTUAL QUANTITIES OF SLUDGE AT VARIOUS PLANTS, IN POUNDS PER CAPITA PER YEAR

Location	Types filtered[a]	Raw solids	Actually filtered	Expected quantities[b]	AMOUNT UNACCOUNTED FOR	
					Pounds	Percentage of actual
Minneapolis-St. Paul	R	89.5	89.5	89.5	0	0
Buffalo	D	63.0	33.5	31.5	+2.0	+6.0
Washington	D–E	37.5	17.1	18.7	−1.6	−9.4
Hartford	D–E	33.4	17.3	16.7	+0.6	+3.5
Springfield	D–E	49.0	14.6	24.5	−9.9	−67.8
Winnipeg	D–E	69.0	13.0	34.5	−21.5	−165.0
San Francisco	D–E	37.9	12.3	18.9	−6.6	−53.7
Annapolis	D–E	43.8	13.8	21.9	−8.1	−58.7

[a] Types of sludge filtered: R = raw; D = digested; and D–E = digested and elutriated. [b] Amount that should have been filtered, at 50% reduction in volatile and fixed solids by digestion.

TABLE 24.—COMPARISON OF FILTRATION AND INCINERATION COSTS BY YEARS; MINNEAPOLIS-SAINT PAUL SANITARY DISTRICT (SUPPLEMENTING TABLE 10)

Item	Description	1941	1942	1943[a]
	Filtration Cost, in Dollars:			
1	Operation	70,726.70	64,944.29	....
2	Maintenance	6,862.94	6,184.74	....
3	Total	77,589.64	71,129.03	57,312.51
	Incineration Cost, in Dollars:			
4	Operation	31,943.94	29,517.16	....
5	Maintenance	12,283.49	10,062.18	....
6	Total	44,227.43	39,579.34	33,018.43
7	Dry sewage solids disposed of, in tons	35,993.0	33,838.0	27,740.0
8	Filtration cost, in dollars per dry ton	2.16	2.10	2.05
9	Incineration cost, in dollars per dry ton	1.23	1.17	1.19
10	Total cost of sludge disposal in dollars per dry ton	3.39	3.27	3.24
11	Cost of entire project (collection, treatment, and sludge disposal), in dollars per dry ton	7.85	8.17	8.25
	Conditioning Chemicals, % of Dry Sewage Solids:			
12	Lime as Ca O	3.77	3.44	3.12
13	Ferric chloride	1.53	1.20	1.10
	Cost of Chemicals, in Dollars per Dry Ton:			
14	Lime at $9.30 per ton (Ca O)	0.35	0.32	0.29
15	Ferric chloride at $39	0.60	0.47	0.43
16	Total	0.95	0.79	0.72
17	Cost of chemicals to filter 110 tons of dry solids, in dollars per day	104.50	86.90	79.20

[a] To October 31, 1943.

been reduced to $2.05 per dry ton and incineration costs to $1.19. As shown in Table 24, the costs of chemicals, on the basis of 110 tons per day of dry solids and fixed unit costs of chemicals, have been reduced from $242.00 per day in 1938 to $79.20 in 1943. This is a result of continued and expected reduction in chemical dosages, which, as shown in Table 22, have been reduced from 3.17% of ferric chloride and 10.30% of lime in 1938 to 1.10% of ferric chloride and 3.12% of lime in 1943. This reduction of approximately $2.00 per dry ton for chemicals, when it is remembered that an average of about 35,000 dry tons are filtered annually, will result in a saving of considerable magnitude.

The statements of Messrs. Van Kleeck and Rudolfs concerning the practicability of using raw sludge as a fertilizer were particularly interesting to the writer in relation to present practice at the Minneapolis-St. Paul plant. Raw sludge has certain objectionable characteristics including a possible health hazard. To obviate some of the difficulties presented in the use of raw sludge, this district has adopted the policy of allowing farmers to haul and use the filter cake only during the fall and early winter months from October to January, inclusive. No raw sludge is allowed to be hauled after February 1. As a matter of convenience the farmers apply the filter cake as it is hauled. The storage of the material, even for several days, creates odor problems and changes the characteristics of the sludge so that it is no longer workable. The use of fresh filter cake is not permitted on lawns and gardens within the city. Instead, the district stores filter cake for a period of two years, and this material (which is very workable and odor free) is made available for interurban use. Several years of experience have shown that at least two years of storage are necessary to place the material in a satisfactory condition for use. Shorter periods of storage result in a sticky foul-smelling mass.

This policy of the district, and the fact that, to the present time, the material is furnished without charge has resulted in a steady and rapid increase in the tonnage of material used as a fertilizer. The following are the quantities of filter cake used for such purposes:

Year	Tons
1939	21
1940	2,756.8
1941	3,678.3
1942	6,867.3
1943	11,580.3

Although use in 1943 represents only about 10% of the annual production of filter cake, there should be an even greater demand in future years.

In a very systematic and orderly manner, Mr. Mendelsohn gives an analysis of the operating experiences and improvements at the various plants. Such information permits some interesting comparisons.

The Symposium with its discussions has presented a rather complete picture of the field of sludge disposal. The tone of the discussions indicates that it has been thought provoking, and, if no other purpose was served, this alone would justify the presentation of the Symposium.

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GEOLOGY IN HIGHWAY ENGINEERING

By Marshall T. Huntting,[1] Esq.

With Discussion by Messrs. W. W. Crosby, Carl B. Brown, F. H. Kellogg, Berlen C. Moneymaker, E. F. Bean, A. T. Bleck, Lyman W. Wood, Philip Keene, Jacob Feld, H. E. Marshall, K. B. Woods, D. J. Belcher, R. Woodward Moore, George E. Ekblaw, Allen S. Cary and Ade E. Jaskar, Karl Terzaghi, D. P. Krynine, Robert F. Legget, Irving B. Crosby, Hyde Forbes, A. A. Eremin, and Marshall T. Huntting.

Synopsis

Within recent years engineers and geologists have worked to bring about a closer bond between their respective professions. Engineering geology studies have been conducted in many parts of the world by goverment agencies, engineering societies, and independent workers.

Geology has entered into highway engineering problems in many ways, and study of geological features has helped to solve many highway construction and maintenance problems. However, many more such problems could be solved profitably by the use of geologists' services. Twelve of these problems will be treated in this paper, and examples will be given to show how some of them are being solved through the cooperative work of geologists and highway engineers. It is the purpose of this paper to help both engineers and geologists to recognize more frequently those situations in which geology does enter into highway engineering.

Introduction

In contrast to the rapid advance of engineering sciences in the fields of statics, dynamics, and hydraulics within the past century, there was a general lag in advancement of those phases of engineering involving earth materials and earth processes, which together constitute the basis of the science of geology. Comparatively recently, however, has come a fuller appreciation of the value of scientific study of the earth materials upon which must rest the foundations of the structures planned and constructed by engineers. As a result many engineers have worked to perfect a better bond between geology and engineering.

Note.—Published in December, 1943, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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Engineering geology studies, many of which are applicable to highway engineering problems, have been initiated in Europe and in America by governmental and municipal agencies, by engineering societies, and by individual workers. Reports of the influence of geology on roads appear in English publications as early as 1888. Since that time, especially within the past few decades, the results of geological investigations into specific highway engineering problems have been published both in Europe and America, but apparently no treatment of the subject as a whole is available.

The work of the geological surveys of several of the states in the United States will be used to illustrate some geological applications to highway engineering.

PROBLEMS

Of the multitude of problems confronting highway engineers many involve geologic factors and thus should command the attention of geologists. No attempt will be made herein to present a complete study of any of the problems; instead, the purpose is to indicate some of those instances in which geologists' services might be profitably engaged. These follow:

(1) Locating suitable road-surfacing material pits and quarries;
(2) Determining the suitability of various earth materials for surfacing, concrete construction, and other highway uses;
(3) Subgrade treatment and classification;
(4) Frost-heave problems;
(5) Predicting the character of material to be excavated;
(6) Preventing and correcting landslides;
(7) Appraising the competency of materials for bridge foundations;
(8) Judging the desirability of bridge sites and road sites with respect to possible changes of stream channels;
(9) Investigating proposed tunnel sites;
(10) Investigating and predicting subsurface water conditions and locating underground water supplies;
(11) Evaluating mineral lands to be bought or condemned for highway location; and
(12) Acting as witness in litigation.

Although highway activities have always claimed some attention from geologists, many, if not most, of the foregoing problems have been insufficiently considered. Economic and other factors will determine the importance of many of them, and some will be relatively more important than others, depending upon several factors, of which the geologic character of the terrain traversed by a given road is probably the most significant. In some instances geological investigations have been made by independent consulting geologists called in for specific assignments, and some have been made as a part of the routine duties of men who were an integral part of the state highway departments, but more frequently they have been made by the state geological surveys in response to highway department requests for consultory advice on special problems.

LOCATION OF SUITABLE ROAD-SURFACING MATERIAL PITS AND QUARRIES

Probably the most obvious and certainly the most widely recognized application of geology to highway engineering is in making materials surveys, which involve locating, and determining the quality, quantity, and the natural features which affect the exploitation of gravel, stone, clay, and other types of material for subgrade construction, surfacing, and concrete construction. Of the

> "* * * total annual expenditure of $500,000,000 [for roads in the United States], probably over half is spent for materials and their transportation to the construction projects, and it is in the cost of these materials that the largest savings can be effected" (1).[2]

The possibility of effecting financial savings and of obtaining better road materials through materials surveys is almost universally recognized, as are also the advantages of having the surveys made by men who have specialized in geology and have made the study of earth materials their profession. Obviously earth materials surveys can best be made by specialists, who, after learning from the engineers specifically which rock types have the physical and chemical specifications best suited to given needs, can then apply their understanding of the geological phenomena controlling the origin, subsequent alteration, and present physiographic expression of those rock types to locate, classify, and delimit commercial deposits for highway use. The geologist, through his knowledge of the glacial, stratigraphic, petrologic, and physiographic history of a given area, should be able to locate, easily, bodies of any type of rock present, without resorting to the too often used method of haphazard test pitting, which is an expensive and tedious process at best.

In some sections of the United States crushed rock is used to a large extent for surfacing and other construction and maintenance work. Selection of sites in rock to be quarried and crushed has been guided in some instances only by the character of plant site available, distance to place the crushed rock, and other such factors, without sufficient regard for the geologic factors involved. The extent, continuity, and uniformity of rock, where not exposed, are features predictable only on a geologic basis. The degree and extent of jointing and of weathering or other alteration are other such features.

Similarly, the nature of a deposit of sand, gravel, or other nonconsolidated material is determined by its geologic origin and so is predictable only when that origin is known. Geologists have been most widely employed in highway work for locating and testing such deposits.

Among the state organizations conducting materials surveys is the Louisiana Geological Survey, which locates and maps deposits of sand, gravel, rock, and shell, and which, in 1941, published a bulletin (2) on sand and gravel deposits of Louisiana. This work was financed partly by the State Highway Department and has since been put to considerable use not only by that department but also by various military groups in the state for building roads in the vicinity of the army cantonments.

In Texas the Bureau of Economic Geology of the University of Texas, in Austin, has cooperated with the State Highway Department in the location of road materials. On most of their projects geophysical prospecting apparatus of the resistivity type has been used to determine the thickness of sand and gravel beds.

The Areal and Engineering Geology Division of the Illinois Geological

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[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix)

Survey has conducted comprehensive studies in cooperation with, and for the benefit of, the Highway Division. One such study was of the limestone resources of the state, the results of which (3) were published in 1925. Another was a study of sources of material, particularly gravel and thin limestone deposits, for use on local roads. Maps of clay deposits in part of southwestern Illinois were made for use in obtaining clay materials for stabilized roads.

Wisconsin is a pioneer and leader in materials survey programs. As early as 1907 its Geological Survey was appropriated $10,000 by the state legislature for investigations and experimental work in road building. This work has been continued, and the survey now assists the highway engineers in every way possible, including the following, quoted from a letter by E. F. Bean:

> (*a*) Road materials surveys. For nearly all construction projects, surveys are made for surfacing materials, sand-gravel fill, and clay binder for gravel.
> (*b*) Classification of excavation. In some cases division engineers are consulted on classification of material to be excavated.
> (*c*) Subgrade treatment. Conferences and consultations in the field have been held.
> (*d*) Litigation. The state geologist has been a witness for the state on cases involving excavation, gravel, and limestone.

The road materials surveys have been especially fruitful, and for this work the Wisconsin Geological Survey is given an appropriation of $15,000 per yr.

Wisconsin Materials Survey Procedure.—The Wisconsin Highway Department has mobile crushing, screening, and washing plants which are moved to sites located by Geological Survey men at the request of the county highway departments. The highway department gives to the state geologist the exact specifications of the rock required, the quantity needed, the road on which it is to be used, and the use to which it is to be put. A geologist immediately goes into the field to investigate road materials in the near vicinity of the work to be done. He locates a number of pit sites in material which will meet the highway specifications. He then examines in detail each site selected to determine tonnage available, amount of overburden, cost of quarrying, and the pertinent geological features. Of each site he makes maps on which his findings are illustrated. Out of a probable eight or ten sites to be selected, the geologist may recommend two or three which will then be more extensively explored by an engineer to determine which deposit shall be developed.

Suitability of Various Earth Materials for Surfacing, Concrete Construction, and Other Highway Uses

Physical tests to determine the suitability of rocks for road surfacing are many and varied, and in the main they are sufficient in themselves. It is in determining the reason why certain rocks are durable and others are not that the geologist can be of service. This is not a question of academic interest only; on the contrary, it is one of considerable practical value, in that through recognition of the critical factors the geologist, by petrographic examination, can indicate quickly whether or not a given rock is likely to be satisfactory for a specific use.

High toughness is usually associated with traprock, and, in general, the fine grained volcanic rocks are harder and tougher than the coarser grained plutonic rocks. The sedimentary rocks vary widely in physical characteristics, but they are generally softer and less tough than the igneous rocks. Limestone and dolomite have greater toughness and are equal in hardness to the average sandstone, as determined by the standard tests. Of the metamorphic rocks, marble is generally neither as hard nor as tough as its unmetamorphosed equivalent, limestone. In most instances the foliated metamorphic rocks are comparatively soft and of low toughness, and as a rule they are rendered unsuitable for most road building purposes by their characteristic tendency to crush into long, thin, or flat fragments.

The hardness and toughness of these rocks are dependent upon the following fundamental properties: (*a*) Mineral composition (relative abundance of various minerals) and chemical composition (hardness, toughness, and cleavage of individual minerals); (*b*) texture (size and shape of grains and porosity); (*c*) freshness of minerals (degree of weathering or hydrothermal alteration of original minerals); and (*d*) jointing and fracturing. These fundamental properties are easily observed by the trained eye alone or with the aid of a good hand lens or petrographic microscope.

Failure of concrete by disintegration presents problems in which the importance of investigating the properties of the aggregate rocks has been minimized far too greatly. Attempts to prevent concrete failures were originally concentrated on varying the water content of the mix, experimenting with different types of cement and various cement-aggregate proportions, and adding special ingredients to the mix. Little attention was paid to the characteristics of the aggregate rocks. Since 85% to 90% of all concrete consists of aggregate, the quality of the concrete undoubtedly is governed to a marked degree by the quality of the aggregate, and the recognition of this fact has stimulated some fruitful research along this line. In 1931 Oliver Bowles reported (4) that failure of some concretes had been traced to physical defects in the rock aggregate, and in this connection he stated,

> "Physical tests of abrasion resistance and crushing strength should be supplemented by microscopic examination to determine soundness and the presence or absence of undesirable minerals. Microscopic examination is particularly valuable in recognizing partial alteration of the original minerals, which is accompanied by the formation of secondary minerals, usually of inferior quality."

More specific are the warnings of J. C. Pearson and G. F. Loughlin in 1923 (5) reporting the failure, in less than a year and a half, of concrete using the plagioclase feldspar labradorite for aggregate. Microscopic examination showed that the feldspar was largely altered to the minerals, laumontite, kaolin, and calcite, in decreasing order of abundance; and it was indicated that these three minerals were responsible in the same order for the disintegration of the concrete. Feldspar aggregate which did not cause failure was found to be unaltered. Strength tests applied to test specimens failed to indicate the unstable nature of the altered feldspar aggregate, thus emphasizing the advisability of having petrographic analyses made of questionable aggregate material previous to its use.

Further recognition of the rôle of inferior aggregates in concrete failures is noted in a highly informative paper (6) by Thomas E. Stanton, M. Am. Soc. C. E., Materials and Research Engineer for the California Division of Highways. Extensive tests conducted by him demonstrated that some concrete failures result from excessive expansion of concrete caused by chemical reaction between high alkali cements and certain mineral constituents in some aggregates, such as some shales, cherts, and impure limestones. Opaline silica, apparently the trouble-causing mineral, was identified in aggregates of the defective concretes. The mineral identifications were made with the microscope by a geologist, Allen H. Nicol, Mineral Technologist for the Aggregate and Soils Department of the California Division of Highways.

In 1941 the Washington State Department of Highways engaged H. A. Coombs as consulting geologist to determine the cause of failure of highway concrete at several places in the state. Late in the year Mr. Coombs reported (7) the results of his investigations into the cause of disintegration of the concrete in several bridges near Ohanapecosh on the White Pass Highway. Test procedures patterned after those developed by Mr. Stanton in California were used to prove the failures to be the result of excessive concrete expansion. The expansion was thought to result from chemical reaction between cement and aggregate. Thin section study with the petrographic microscope combined with fluorescent tests with ultraviolet light proved the reactive aggregate material to be pebbles of andesite. Further work should indicate what percentage of the deleterious andesite can be used in aggregate without causing undue expansion, and all rock deposits exceeding that percentage of andesite should be condemned as sources of aggregate for concrete.

Subgrade Treatment and Classification

Since the weight of traffic is supported not on the pavement but rather on the subgrade, it is obvious that the properties of the subgrade materials will exert a tremendous influence on the success or failure of the surface. The most common subgrade material is "soil," a term which, depending upon its definition, may mean only that part of the mantle which supports plant roots, or it may be used to cover a wide variety of unconsolidated earth materials ranging from gravel through sand, silt, and clay, and including organic deposits. The term will be used in its broadest sense to include all the unconsolidated mantle overlying solid bedrock.

It is well known that soils possessing obviously different physical properties give equally different performances as road building materials. Recognizing this the Public Roads Administration (PRA) and most state highway departments have set up soil testing divisions whose work has been to classify soils, to test their physical properties, to correlate test determined physical properties with performance records, and to make recommendations on the basis of their findings. Soil studies conducted by them and other interested organizations since the 1920's have brought forth a wealth of information which has been successfully applied to the solution of subgrade problems. Most valuable, perhaps, have been their soil classifications based on physical properties and correlated with performance records.

A general soil classification such as that developed by the PRA (8) is satisfactory for most applications of soil mechanics to highway engineering as pointed out by K. B. Woods, Assoc. M. Am. Soc. C. E., and R. R. Litehiser, M. Am. Soc. C. E. (9), but in exceptional cases there is found need for a more specific classification such as one based on mineralogical composition of soil. Some of the effects of mineral composition on the behavior of clays and soils were reported in 1940 by G. E. Ekblaw and R. E. Grim (10). They classify the clay minerals under the three groups—(*a*) kaolinite group, (*b*) montmorillonite group, and (*c*) illite (11) group of sericite-like minerals. Each group has well-defined but vastly different capacities to absorb water, possess exchangeable bases, and break down into extremely small particles which will determine to a large degree the plasticity and permeability of soils containing these clay minerals. Plasticity and permeability in turn are extremely influential in determining compaction, soil slippage, and frost heave—features of considerable importance to the highway engineer.

In other instances where a soil classification based on physical properties is not entirely adequate, a supplementary classification may be useful, based on the soil's origin such as the following:

I. Residual soils.
II. Colluvial soils.
III. Organic soils.
IV. Transported soils—*A*, alluvial; *B*, lacustrine; *C*, marine; *D*, glacial; and *E*, aeolian.

The soils falling under a given heading in this classification will not have all their physical properties alike, but it will be found that some of their properties will be remarkably uniform over wide areas.

Nothing can replace the engineers' physical tests to identify soil types and to determine their suitability in subgrades on the basis of performance, but testing and sampling will be most effective and economical only when something is known of the soil strata's probable uniformity and regularity—features which are dependent in large part upon the origin of the soil, and the closely related mineralogical and chemical composition of the soil, soil particle size and shape, local topography, subsurface water conditions, and climatic conditions. The geologist's understanding of these features will help in interpreting sample test results and will enable him to suggest where samples should be taken. Thus, the direct economy in the use of geologists is in reducing the number of actual tests.

Geologists have played a leading rôle in the soil testing divisions of many state highway departments and have lent substantial assistance in others.

Outstanding is the work of the Illinois State Geological Survey's Areal and Engineering Geology Division, established in 1927 and headed by Mr. Ekblaw. According to R. F. Legget, Assoc. M. Am. Soc. C. E. (12*a*), this organization is unique in being the only state or national geological organization to recognize, to such an extent, the interdepending relations of geology and engineering. In a letter, Mr. Ekblaw states that his division has given advice on scores of problems involving landslides, rockfalls, slump, sod creep, and similar movements of earth above, beside, or beneath highways; heave and collapse of various kinds of pavements; "mud pumpings"; stability of fills, both as regards material in and foundation below the fill; treatment of peat bogs; foundation material for piers and abutments for bridges, overpasses, and underpasses; quality and location of aggregate material for concrete or other types of pavements; all kinds of drainage conditions; and location of highway routes to avoid undesirable conditions. Some of the advice has been preventive, given before construction was undertaken, and some has been curative, given after damage occurred.

Formerly the Michigan Geological Survey Division had direct cooperative relations with the State Highway Department. It conducted surveys for road gravel and road material and made test borings for swamp and marsh fills. The latter work in 1925 was transferred to the newly created Soil Engineering Branch of the State Highway Department's Testing Division. The soil classification used by them is that of the international system, described in the 1938 *Yearbook of Agriculture*, Soils and Men.

In Ohio two geologists are employed in the soil sampling and testing work of the Soil Section of the State Highway Testing Laboratory. The data and recommendations resulting from the soil, foundation, and geologic studies made on all projects involving federal aid funds and on all major state projects are thought to result in savings to the state amounting to several times their cost. Detailed description (13) of the work of their geologists and soil engineers may be found in the *Proceedings*, Purdue Conference on Soil Mechanics and Its Applications, which also includes articles describing the aims and methods of soil investigations in several other states.

H. A. Buehler, State Geologist for Missouri, believes that his state's highway department probably utilizes geology to a greater extent than any other highway department in the nation. The state geologist has been an ex-officio member of the State Highway Commission since its inception. The department's testing division organized a geological investigation branch, which has been the center around which soil stabilization has grown. Using one geologist for every two field divisions, the headquarters office has five geologists, all of whom are field experts in stabilization work.

Frost-Heave Problems

Frost heave might properly be discussed under the previous heading ("Subgrade Treatment and Classification"), but it is thought to be a problem of sufficient weight to merit separate treatment. Frost heave is a phenomenon to be dealt with in highway construction and maintenance over a large part of the United States, and with improved facilities for snow removal annually increasing the mileage of road left uncovered and open to frost attack, the need for frost-heave studies becomes more and more apparent.

Obviously the features of climate, rainfall, and temperature will broadly control the limits of the area in which frost action is of consequence. Recognizing this, J. A. Sourwine, Senior Highway Engineer, PRA, in 1930 concluded from a study (14) of climatological records that the area subject to frost damage to highways includes Maine, New Hampshire, Massachusetts, Rhode Island, Connecticut, New Jersey, New York, Pennsylvania, Ohio, West Virginia, Kentucky, Indiana, Michigan, Wisconsin, Iowa, Illinois, Missouri, northern Oklahoma, northern New Mexico, northern Arizona, Utah, Colorado, Kansas, Nebraska, South Dakota, North Dakota, Wyoming, Montana, Idaho, eastern Oregon, and eastern Washington. Within this area frost heave will be a problem only in those places where drainage conditions and soil characters are so combined as to make excessive differential frost action possible.

Laboratory and field studies in connection with frost heaving have been active for many years and much information on the subject has accumulated. The first explanation of the true nature of frost heaving appeared in 1918 when Stephen Taber (15), State Geologist and Professor of Geology at the University of South Carolina, at Columbia, showed that heave was the result not of expansion of water on freezing but of growth of lenses or layers of ice crystals at the frost line. He found that in order for ice to accumulate in layers a supply of water must be maintained to the zone of freezing either by a water table so high that it is above the frost line, or by capillary rise from a source below. The first source of supply can be removed comparatively easily by drainage, but the second is not so easily disposed of.

It has been shown that in many, if not most, instances of frost-heave disturbances clay is a major constituent of the subgrade material. The clay acts as the medium through which water rises from the water table by capillarity until it reaches the frost line, where it freezes and causes an irregular upheaval of the road surface. The subgrade soil profile ordinarily varies widely within small areal limits; consequently the amount of water available for freezing under the road surface will vary in a short distance, thus causing differential uplift and a rough road.

A comparatively recent suggestion is that mineral content is an important factor influencing the physical properties of clays subject to frost heave. It is known that two clays may have the same ultimate chemical composition and the same grade size distribution and yet possess quite different physical properties if their mineral compositions differ. Obviously then, a knowledge of the effects on the physical properties of a clay attributable to each constituent mineral is necessary in order to understand exactly why a given soil acts as it does. Such knowledge can then be applied to the planning of effective preventive measures. So far as the writer is aware, no research to determine the relation of clay mineral composition to frost heaving has yet been completed, but in this connection Mr. Grim has made some preliminary studies (16).

Suggested preventive methods against frost-heave damage have been many and varied. During the period from 1928 to 1933 the PRA, cooperating with the highway departments of Minnesota, Wisconsin, and Michigan, conducted a survey to correlate the various methods of different engineers. In the resulting report (17) Henry Aaron gives three classes of preventive measures. Additional preventive measures have been suggested in reports by H. F. Winn (18) and A. Scheidig (19). Selection of the best preventive measure in any given case must depend on economic considerations and on the soil profile in the area being considered; thus it must depend directly or indirectly on information furnished by soil surveys.

PREDICTION OF CHARACTER OF MATERIAL TO BE EXCAVATED

Probably more disputes have arisen in civil engineering contracts through inadequate classification of material to be excavated than through any other feature. The cost of digging solid rock may be ten times or more the cost of digging an equal amount of earth (Fig. 1); consequently, a contractor should have the most accurate available estimate as to the volumes of each he will have to move, and the classification should be made in terms of the methods to be used to excavate the material. On large jobs estimates made by pure guesswork or by rule-of-thumb methods are certainly inadequate. Yardage estimates of solid rock and of unconsolidated material will be most reliable where the estimator, knowing the geologic processes operative in the accumulation of those materials, makes his estimates on the basis of that knowledge.

FIG. 1.—THE COST OF DIGGING ROCK MAY BE TEN TIMES THAT OF DIGGING AN EQUAL VOLUME OF EARTH

In excavating intrusive igneous rocks, such as granite, it may be assumed that the rock will remain quite uniform both at great depths and for long distances laterally with respect to properties affecting excavation, except for jointing and weathering. Jointing is generally best developed along the sides and top of the intrusive rock mass, near its contact with the intruded rock. However, a granite mass which has been deeply eroded and covered by sedimentary rocks probably would not be more jointed near its contact with the surrounding rock than elsewhere, thus illustrating the need for knowing the geologic history of a rock to be excavated. The degree and depth of weathering of granite, as well as of every other rock, depend upon the rate of weathering and rate of erosion which carry away the weathered rock. Glaciated areas generally show little weathering in their rocks, owing to the comparatively recent vigorous scouring action of the ice.

Extrusive igneous rocks, such as basalt, may be extremely varied in nature over short distances laterally or vertically, or they may be quite uniform, depending upon their chemical composition, their mode of origin, the type of topography over which they flowed, their rate of cooling, and the amount of deformation, erosion, and weathering they have suffered. In basalt, jointing may be almost entirely lacking or may be very well developed, depending on the foregoing determinants. The degree of jointing very largely controls the cost of excavating such rock, and can be predicted fairly safely.

Unconsolidated earth materials (soils) may vary considerably according to their origin, but they are nearly all easily excavated. The important factor in considering such deposits is their depth to underlying solid rock. Within limits, the depth and shape of such deposits are reliably predictable by competent geologists.

The various sedimentary rocks and unconsolidated sedimentary deposits vary widely with respect to ease of excavation, but each variety has its own set of characteristics developed at the time of its deposition and altered by its post-depositional history. The origin of the deposit—whether alluvial, marine, lacustrine, glacial, or aeolian—very largely controls the nature of the unconsolidated sedimentary deposits as well as the original nature of the sedimentary rocks. The latter, however, may be deformed, metamorphosed, and weathered so as to be more difficult to predict where not exposed. It is the task of the geologist to learn the geologic history of such a rock, and to base his prediction of its properties affecting excavation on that history.

Of the metamorphic rocks, those formed by regional deformation or by load metamorphism at great depth may be expected to be uniform over comparatively great distances laterally, but in some cases they may change abruptly with depth. Such changes at depth are the result of original variations in the rock before it was metamorphosed, and they are predictable to a certain extent. Rocks of contact metamorphic origin are not to be expected to occur over large areas and may be found to grade into their unmetamorphosed equivalents within short distances.

In a study of excavation problems erosional, glacial, sedimentary, igneous, deformational, and weathering histories should all be considered, since the present condition of the materials to be excavated is entirely dependent upon them.

PREVENTION AND CORRECTION OF LANDSLIDES

Treatment of landslide problems may be divided into two categories, preventive and curative. All too often the opportunities to apply comparatively inexpensive preventive measures are passed by during highway construction, and as a result there occur traffic-disrupting, and sometimes catastrophic, slides that could have been prevented had the proper precautions been taken. The necessary curative measures are generally costly and are seldom completely effective.

In laying out the route for a new highway it is important to watch for places where the probability of landslides is great, and such places should be avoided in locating the highway. Where economic or other factors make it necessary to build a road in an area of probable landslide occurrence, careful preliminary surveys should be made in order to plan proper preventive measures. The preliminary survey must first determine which areas are likely to be subject to landslides. It should then discover the nature of the possible slides and show the factors, geologic, climatic, and man-made, which will initiate rock or soil movement. The investigators should be thoroughly familiar with geology and its rôle in landslide occurrence.

The surface of the earth is constantly undergoing changes in shape. Most of these changes involve the transfer of enormous quantities of material from the highlands to the lowlands and to the sea by erosion and deposition. Quantitatively less important but of far greater direct interest to the engineer are those changes in shape which involve landslides. "Mass movement of earth or rock on hill slopes, at an appreciable rate of speed," will be used as the meaning of the term "landslide"—a definition more inclusive than that generally accepted by geologists but well suited to engineering purposes. The rocks and soil of the earth's crust are under the influence of gravity, so that even in only slightly hilly terrain they are in an unstable state of equilibrium. Highway construction through blasting, altering natural drainage, and transferring earth materials from their normal resting places to new positions upsets temporary stability and initiates many landslides.

Old landslides can usually be recognized by their characteristic irregular, hummock-and-hollow topography in the debris situated at the base of slopes on which the scars of the slides usually may be seen. Where no characteristic topography or landslide scar is detectable, old slides are sometimes indicated by tilting trees or trees with tilted bases and vertical tops.

Landslides may involve enormous masses of solid rock broken from steep faces under their own great weight, or they may involve only loose surficial detritus. Rockfalls, not properly classed as landslides, comprise small quantities of solid rock precipitated from steep slopes. The movement may be in the form of mass sliding or slipping, or it may be in the nature of a flow. In any case it will be conditioned by some or all of these features: (*a*) Presence of clay or decomposable shale, (*b*) underground water, (*c*) topography, (*d*) stratigraphy, (*e*) structure, (*f*) joints, (*g*) foundation, and (*h*) shape of rock particles.

The sliding of rock strata along lines of stratification, sliding of surficial detritus over rock foundation, and flow of detrital masses are all facilitated by the presence of clay or shale which softens when wet. Decomposable shale where overlain in a cut or cliff by rocks more resistant to weathering often causes slides by weathering and sloughing, allowing the overhanging rock to break and slide of its own weight.

Abundant underground water decreases the stability of hillside accumulations of soil and facilitates slides, particularly where clay is present.

A topography of great relief, of course, is most subject to slides, but gentle slopes also are the sites of troublesome slides under some conditions.

A stratigraphic relationship frequently causing trouble is that in which pervious rocks overlie impervious strata, especially where the impervious layer is clay. Percolating water, on reaching the impervious layer, is stopped in its downward movement and is forced to flow laterally to an outlet. Where the outlet is a cliff, slides are quite likely to occur along the upper surface of the water-soaked clay layer.

Unjointed igneous rocks are generally capable of standing vertically without sliding, and hard, unjointed, horizontally stratified rocks are relatively stable, but rock strata dipping toward a steep face are subject to slippage along the planes of stratification. Slides in such rocks seldom give any warning previous to the time of their occurrence. Jointing weakens any rock, and jointed rocks already weakened by stratification are especially likely to cause slides. Rock masses often slip along old fault planes where movement previously occurred.

The angle of repose for rounded rocks is less than that for angular rocks which offer more resistance to slippage and flowage; therefore it seems logical to assume that detritus high in rounded rock and sand grains is less stable and more likely to slide than angular detritus. This assumption has been borne out by the observations of G. E. Ladd (20), Associate Economic Geologist, PRA.

Different combinations of geological, climatic, and man-made conditions can produce slides of quite different natures. Preventive measures should be planned on a basis of a slide's causes, and those causes can usually be discovered by geologic investigation and recognition of the type of slide. The following landslide classification is a modified form of one proposed in 1927 by Mr. Ladd:

(*a*) Slope adjustment of surficial material;
(*b*) Rock strata slides;
(*c*) Rockfalls; and
(*d*) Artificial fill slides.

Slope Adjustment of Surficial Material.—This very common type of slide may involve either flowage or slippage or both. Individual slides vary in size from very small to several thousand feet in length and width. Movement is sometimes rapid and complete within a few minutes or hours, but more often it is slow and intermittent and continues for years. Rate of movement is increased by steep slopes, by the lubrication furnished by clay, and by high water content. Also contributory to sliding are rounded rock and sand grains in the detritus. A smooth, lubricated rock foundation below detritus in many cases forms the base on which slippage takes place, but flowage is comparatively independent of foundation conditions.

Rock Strata Slides.—Rock strata slides range in magnitude from small slips in road cuts to huge, sometimes disastrous slides in which millions of tons of rock are dislodged. An example of the latter is the most disastrous slide in North America, which on April 28, 1903, covered more than a mile of highway and killed more than 70 people in the town of Frank, Alberta, Canada. The town was at the base of the steep face of Turtle Mountain, composed of a great thickness of steeply dipping Paleozoic limestone overlying softer cretaceous sandstone and shale. The limestone was highly jointed at right angles to the bedding, and it was along this series of joints that the failure occurred. The joint cracks weakened the rock; coal mining in the cretaceous rocks near the base of the mountain may have weakened its foundation; and severe freezing and thawing of water in joints in the rock probably furnished the force which made the entire side of the mountain tear loose and crash down into and across the valley at its base.

Rock strata slides occur most commonly along highways where construction or stream erosion has left steep faces of highly jointed stratified rocks or where it has cut across the stratification of rocks dipping toward the face. Combinations of jointing and alternating hard and soft beds are especially dangerous.

Rockfalls.—Rockfalls are seldom individually large, but on account of their wide and frequent occurrence they constitute a persistent problem in highway maintenance. They consist of a more or less steady dribble of rock fragments which increases during wet or freezing and thawing seasons. Hard, jointed rocks are especially subject to rockfalls, and any rock face at the base of which a large talus slope has accumulated should be avoided if possible.

Artificial Fill Slides.—Slides in artificial fills are by far the most prevalent, and they are the source in some states of the most serious highway problems to be met. Sliding conditions arise from the use of improper design, unsuitable materials, poor compaction, unfavorable water conditions, and weak foundations. Test procedures for the purpose of recognizing these slide-causing conditions have been developed and are used to a considerable degree in many states.

The State Geological Survey of Illinois has investigated several particularly troublesome landslides near Peoria (21). One slide caused a 5-ft movement out of line of a 200-ft slab of the highway along the west side of Illinois Valley. Sliding was believed to occur in an impervious clay zone kept wet by seepages from an overlying porous sandstone stratum. Drainage was suggested as the only cure for the slides.

In 1935 W. C. Morse reported (22) a saving of perhaps $100,000 to the State of Mississippi and the nation resulting from his condemnation of a site for a new bridge to cross the Yazoo River at Yazoo City. The proposed site was such that the eastern highway approach to the bridge would have had to be built for from a half mile to a mile over landslide terrane which was so old as to mask its true identity and accordingly might not have been recognized as such had not a careful examination been made.

An excellent discussion of landslide preventive measures by Mr. Ladd (23) in 1928 includes statements of costs and effectiveness of various methods. A most important conclusion resulting from his study of slides in Ohio and West Virginia was that, in most cases, retaining structures, seldom entirely effective, should be replaced by surface and underground drainage systems to keep moisture below the critical point at which rock masses are unstable.

Competency of Materials for Bridge Foundations

A bridge of any but the smallest size represents a considerable investment. Consequently its location should be considered as permanent, or at least as permanent as the location of the highway of which it is a part. To be permanent a bridge should be designed to meet changing natural and man-made conditions as well as present conditions. It has been said that a chain is no stronger than its weakest link, and this general thesis is certainly applicable to bridge design. The superstructure of a bridge can be no more sound than its piers and abutments, whose soundness in turn is dependent upon their design and upon the nature of the rock or soil foundations on which they rest (Fig. 2).

Fig. 2.—The Superstructure of a Bridge, Its Piers and Abutments, Can Be No More Sound Than the Rock or Soil Foundations on Which They Rest

The weight of the bridge and its traffic load must ultimately be supported by the earth foundation, making it imperative that bearing capacity of the foundation be known so that proper design and spacing of piers can be determined.

Ordinarily economic factors combined with limitations set by the highway route location are the first considerations in selection of bridge sites; foundation materials usually are not considered as limiting factors, the assumption being that the design of the piers can be made to fit the foundation conditions. For this reason bridge sites selected sometimes present difficult foundation problems which require unusually detailed investigation preliminary to construction. Precise information as to the strength, attitude, and water-bearing characteristics of local materials should be obtained for all bridge sites.

Exploratory methods to determine these characteristics may be classed generally as geological, geophysical, and mechanical sampling. A purely geological exploration of a proposed bridge site is seldom productive of enough detailed and precise data to be sufficient in itself, but a field study of a given site combined with a knowledge of local and general geology should be the basis for rather reliable estimates of probable foundation conditions and should precede choice of more detailed exploration methods. The geological investigation is of particular value in predicting such conditions as depth; structure; and lateral extent of sand, gravel, or soil overlying bedrock.

More precise information of this kind may be obtained through the use of geophysical test methods of several types, the most applicable to engineering work of this kind being the earth-resistivity (electrical) method and the seismic method. A knowledge of geology, especially of stratigraphy and of geologic structures, is indispensable in interpreting the results of any geophysical test.

Taking samples of foundation materials by one of many mechanical means is the most commonly employed exploratory method; the advantage being that in this way subsurface materials can actually be seen and examined, and, when undisturbed samples are taken, tests of crushing and shearing strengths, as well as other physical property tests, can be made. The geologist's sphere of usefulness in mechanical sampling lies in examining samples; in helping to interpret the samples in terms of dimensions, structures, water-bearing characteristics, and load-bearing capacities of subsurface formations; and in combining his interpretation of subsurface conditions with features of general and local geology to form the basis for recommendations for further sampling and testing.

Although in most cases the location of bridge sites is determined primarily by considerations other than foundation conditions, the final selection of sites for the largest bridges frequently is made only after detailed sampling programs have revealed the location of the most satisfactory foundation conditions. In this respect, the San Francisco-Oakland Bridge (San Francisco, Calif.), built at a cost of more than $73,000,000, and one of the largest bridge projects ever undertaken, stands as a monument to the worth of careful preliminary foundation investigations. Before the design or the location of the bridge was selected, the engineers decided on a program of borings and soil tests, the results of which would guide them in selecting (*a*) the most desirable location for the center line of the bridge, (*b*) the best location for individual piers, and (*c*) a logical basis for the design of the piers (24). Samples obtained by several methods were tested mainly in laboratories at the University of California, at Berkeley, and center-line location, pier location, pier design, and construction methods were made to conform to the conditions indicated by the tests.

The too frequent occurrence of bridge failures caused by inadequate foundations indicates that thorough foundation investigations should be conducted for the sites proposed for all bridges, large or small. The following example of a bridge failure should serve to emphasize this point.

In 1912 a $80,000 bridge was built in a west coast city to span a deep cut through which a transverse street passed. The excavation for the cut was entirely in horizontally, thin-bedded, blue clay. Recognizing the unstable nature of such material in the steep banks of the cut, the engineers designed the 95-ft steel cantilever arms and the 171-ft simple steel span so that the arms were strong enough to carry their load without support at their outer ends. The piers supporting the outer ends accordingly were not anchored to the superstructure. The advisability of such a design was proved when in 1915 a slide at the north end of the bridge carried away the piers supporting that cantilever arm without damaging the bridge. Thus, recognition of unstable foundations was the basis for effective superstructure design to meet those conditions; the inadequacy lay in not using the same basis for design of the piers. This was proved on February 26, 1917, when a slide caused the failure of one of the north main piers, badly buckled the lower chords in the south cantilever arm, and caused the collapse of about 150 ft of the timber approach on the south. More soundly anchored piers of proper design very likely would have prevented this bridge failure.

Bridge foundation investigations are conducted regularly by the state geological surveys of several states. Notable in this connection is the work of the Michigan Geological Survey Division in examining the core samples obtained preliminary to plans for a "Straits Bridge" to cross the five-mile-wide Straits of Mackinac between Mackinaw City, Mich., and St. Ignace, Mich.

Desirability of Bridge Sites and Road Sites with Respect to Possible Changes of Stream Channels

Permanency of bridge piers and abutments is dependent not only upon design of the structures and load-bearing capacity of the foundation, but also, to a considerable extent, upon the permanence of the stream channel at the bridge site. The shifting or meandering of a stream channel may dictate long spans and deep piers across the entire width of the flood-stage channel even though the normal channel be only a small fraction of that width. In most instances meandering streams operate to increase maintenance costs rather than first cost of construction. In any case, a careful study should be made for bridge sites on meandering streams, and conditions conducive to the permanence of a channel should be a deciding factor in the final selection of the crossing. The yearly destruction of many miles of highway by shifting stream channels indicates that more emphasis also might well be put on locating highways with more regard to possible channel changes.

Because most changes in the shape of the earth's surface resulting from stream erosion take place so very slowly, people often fail to recognize the tremendous erosive power of most streams. The effects of erosion are emphatically impressed upon the mind of the observer who, standing upon the brink of the thousands of feet deep and miles wide Grand Canyon of the Colorado, sees the muddy river still actively eroding a deeper channel.

The development of the valleys of all rivers, including that of the Colorado, passes through a series of stages (25) recognizable by the consequent topography. During the stage of youth streams have steep gradients and steep sided V-shaped valleys, and they erode rapidly, dominantly downward. During successively later stages the streams' gradients decrease, valleys widen and become less steep sided, and downcutting is largely replaced by lateral cutting. The ultimate development is the peneplain, where erosion has virtually ceased.

Climate and rock types and structures act to modify the effects of erosion. In arid regions much or most of the drainage is interior; that is, streams flow into closed basins and do not connect with the sea. Thus, all material eroded by a stream in such a region is redeposited on land, usually directly in the bed of the stream. Highways in arid regions, where streams flow only during infrequent storms, must be built with due consideration not only to the erosive power of the streams but also to the inevitable and sometimes extensive deposits of sand, gravel, boulders, and mud left in the wake of the flood waters.

Erosion in humid regions is less erratic, and its effects are more easily predicted. In the youth stage erosion is dominantly downward and is most rapid where velocity of current is greatest, at rapids and at constructions of the waterway. Rapids, as a result of accelerated stream bed erosion under them, move upstream, deepening the channel as they go. During later stages lateral cutting widens valleys and allows flood plains to form. Streams near base level on broad flood plains are subject to comparatively rapid lateral shifts of channel, especially when the rivers are at flood stage. Lateral erosion is greatest on the outside bank at a curve in a stream. Consequently, the broad loops of a meandering stream tend to migrate bodily downstream.

Hardness and chemical reactivity of rocks have obvious effects upon erosion. In regions of sedimentary rocks the tendency is for the hard sandstone strata to resist erosion and form ridges while the somewhat softer and much more soluble limestone beds and the soft shale beds, because they are more easily eroded generally, are the rocks in which the valleys form. In arid regions, where there is little opportunity for solution, limestone is resistant to erosion and is commonly a cliff-forming rock.

Rock structures act to control erosion and in so doing they exert considerable control over stream patterns. The structural features most effective in guiding erosion and most essential to recognize in a bridge-site survey are folds, faults, and joints. Faults and joints provide planes of weakness along which erosion is easily accomplished. Folding governs stream pattern by forcing parallel bands of hard and soft rocks into flexures so that upon erosion the tilted hard strata are separated by nearly parallel softer strata.

The effects upon shifting of streams attributable to stage of erosion and to modifications imposed by climatic and rock structural conditions are recognizable in advance through geologic investigation. Such investigation should precede final selection of every bridge site. In the case to be described briefly preliminary geologic investigation very probably would have condemned the bridge site chosen as liable to dangerous changes of channel, a condition proved to be true after the bridge was built.

In September, 1914, a 280-ft through steel span highway bridge was built across a certain river in the State of Washington. It was built by the county and state at a cost of $22,818. The north end of the bridge was anchored to solid rock, but the south abutment rested on river gravel and sand. High water in 1920 shifted the channel of the river and washed out the south approach. The bridge itself was not damaged, so to cross the new channel a second steel span of 240-ft length was built to connect with the first. The second span was completed in November, 1920, at a cost of $39,402. Later high water again shifted the channel farther southward and this time washed out the south approach of the second span. A third steel bridge was connected to the second to cross the newly formed channel. The third span, completed in 1934, was built at the cost of $36,284. Thus, an unfortunate original selection of bridge site was responsible for later construction cost increase of 331% over the cost of the original construction.

Investigation of Proposed Tunnel Sites

Probably in no other phase of highway engineering can the study of geology be more fitly and profitably applied than in locating, designing, and constructing tunnels. Increasingly higher standards for newly constructed highways necessarily result in the construction of more and more tunnels in order to meet the maximum grade requirements. Each tunnel built presents problems unique to that tunnel alone as well as problems common to all such projects. This fact, in addition to the high cost of tunneling, emphasizes the need of obtaining as much information relative to rock conditions as possible before construction is begun.

As in bridge building, the general location of the highway limits the possible locations of the tunnels to be built, and economic factors further govern tunnel design and construction. As in bridge building, however, geologic conditions play a large part in determining those governing economic factors. After a preliminary survey for a new highway has established the general location of a tunnel, a careful geological investigation should be made in conjunction with a drilling campaign. This investigation should reveal the existing geological conditions at the tunnel site, and those conditions should then be the basis for selection of (*a*) the final location of the tunnel, (*b*) the construction methods used, and (*c*) the design of the tunnel. An example to illustrate how a geologic survey can be of value in the final selection of a tunnel site is the case of a tunnel built obliquely across the line of an active fault. A flow of water in a wide zone of fault breccia and gouge caused trouble by lubricating the soft material and allowing it to drop into the tunnel. A geological examination of the site should have revealed the presence of the fault previous to construction and should have shown the advisability of locating the tunnel so as to avoid the fault.

After a tunnel site has been selected, an investigation should be made to discover as many as possible of the geological features that will affect the method of construction and the design of the tunnel. Unconsolidated materials such as sand, gravel, and mud require tunneling techniques entirely different from those of solid rocks, and variations of the main methods used also depend upon details of rock characters, most of which can be determined in advance of construction. These details of rock characteristics have a bearing upon (*a*) overbreak of rock in excavation beyond the limits specified by the design engineer, (*b*) hazards of construction, and (*c*) necessity of lining the tunnel.

Overbreakage.—It is practically impossible to excavate a tunnel to the exact dimensions called for in the specifications, and the material excavated beyond those dimensions according to most contracts must be moved by the contractor without pay. Accordingly, the extent of overbreakage to be expected is an important consideration. Plutonic igneous rocks, because of their general textural and structural uniformities, usually break closer to the theoretical section than do sedimentary and metamorphic rocks. The foliation and jointing of metamorphic rocks and the jointing and bedding of sedimentary rocks act to control the directions of breakage during excavation, the breakage being easiest along the lines of weakness furnished by these structural features.

Hazards.—The chief hazards in tunneling operations are the possibility of encountering large flows of water, the danger of cave-ins in soft rock and unconsolidated material, and the danger of blocks of rock falling from the roof of tunnels in solid rock. An additional inconvenience frequently met in deep tunnels is increase of rock temperatures at depth. Temperature increase with depth is variable and its causes are not entirely understood, and so cannot be predicted with any great degree of accuracy. The 12-mile Simplon Tunnel in Europe, with a maximum rock cover of 7,000 ft, encountered rock temperatures as high as 132.8° F.

The greatest likelihood of encountering large flows of water is found in tunnels in alluvial deposits. Permeable sediments of any kind may carry large flows, as might also permeable extrusive igneous rocks; but large flows of water are not to be expected in intrusive igneous rocks except along faults or joint planes.

Cave-ins and rockfalls are conditioned by the degree of consolidation, the presence of clay or soft shale, vertical and lateral earth pressures, underground water, and textural and structural features of rock.

Necessity of Lining.—The walls and roof of a highway tunnel must be competent to stand without danger of failure for the life of the tunnel. If rock is such as to be hazardous during construction, it probably will require an artificial lining to make a safe tunnel for traffic; but, just because a rock does not give trouble during construction, that is no assurance that it will remain safe and not require lining. Long exposure to air and water causes some rocks to expand and causes others to decompose and lose their strength. Anhydrite and some clays and shales are especially subject to failure in this manner and must be treated accordingly.

Accurate and complete records of the geological data collected prior to, during, and after construction should be kept for every tunnel built. In this way a valuable collection of information may be made for future use.

INVESTIGATION AND PREDICTION OF SUBSURFACE WATER CONDITIONS AND LOCATION OF UNDERGROUND WATER SUPPLIES

Subsurface water, an entire branch of geologic study, can be only barely mentioned. The subject has been treated comprehensively by C. F. Tolman (26). Additional information may be had from the publications of the Water Supply Branch of the United States Geological Survey, two papers by O. E. Meinzer (27)(28) being especially valuable.

Subsurface water may be divided into vadose water and ground water. The line of demarcation between the two is the water table, the upper surface of the zone of saturation in which ground water fills all the interstices in the rock. Vadose water is that subsurface water which lies above the water table, in the zone of aeration. Movements of ground water are predominantly lateral, whereas movements of vadose water in the zone of aeration above the water table are predominantly downward. Upward movements of vadose water are also of some interest to highway engineers, especially in connection with frost-heave control as previously discussed. Subsurface water has a further important bearing on previously discussed highway problems encountered in open excavations and fills, tunnels, landslides, and bridge foundations. Except in frost-heave problems, vadose water has little effect upon highway projects; the trouble-causing water in most instances is that encountered when highway construction penetrates below the water table.

State highway maintenance stations require supplies of potable water, and many highway construction and maintenance operations require large quantities of water. In some instances where surface supplies are not available this involves locating and developing ground-water sources, and locating a sufficient supply as near as possible to the surface may be quite an accomplishment. A man versed in geology should be competent to make a scientific prediction, within limits, of quantity available, rate of flow, and depth to satisfactory supplies of water in any given area. The geologist's prediction is not infallible, of course, for the reason that it is made on at least three extremely variable bases, any or all of which may not be entirely dependable in every instance: (*a*) Past experience of others; (*b*) the individual's education and experience; and (*c*) his interpretation of subsurface rock conditions.

Movements of subsurface water are controlled by geologic conditions, chief of which are character, distribution, and structure of the rocks enclosing the water. Areal and vertical extent of a rock formation; details of its general structural features such as abundance, size, and attitude of stratification, joints, and faults; and textural details governed by size, shape, assortment, and arrangement of individual rock grains—are all factors governing subsurface water conditions and all reflect the complex geologic history through which the rock has passed. In general, well sorted, unconsolidated, clean gravel has the highest water-yielding capacity of the rock formations. Other rocks that yield much water, under favorable conditions, are sand, sandstone, limestone, and basalt. Poor water-bearers are hard, unjointed igneous and metamorphic rocks, and most unproductive of all are the clays, which although porous are nearly impervious to water.

The state geologists of several states have been given the responsibility of locating wells to supply highway uses. The Florida Geological Survey suggests the locations for water wells, identifies formations and water horizons, and makes geologic well logs for the state's road camps. The Geological Survey Division of Michigan assists the Michigan Highway Department in the development of water supplies at its roadside parks, and also locates new wells for home sites where homes are necessarily moved as the result of rerouting or widening of highways.

Evaluating Mineral Lands for Highway Purposes

The right-of-way division of a state highway department has the task of securing by purchase, or where necessary by condemnation, rights of way, pit sites, and stock-pile sites as needed. Increased activity in improving old roads and building new ones in late years has greatly increased the volume and complexity of the work of the legal staffs of those divisions, and the expert testimony of economic geologists has been of assistance in a number of situations where mineral lands have been acquired for highway construction.

Owners of mineral lands are prone to overestimate the value of their own property; likewise, people unfamiliar with mining enterprises tend to underestimate the value of such property, although in some instances they have been duped by high-pressure salesmanship into accepting unwarrantedly high values set by promoters for practically worthless property. The unbiased consultory advice of a geologist could be a real aid to a right-of-way legal staff confronted with such situations.

A second use for geologic advice in evaluating mineral lands arises in connection with the construction of mine-to-market roads. The setup in the State of Washington will serve to illustrate this point.

In 1939 an act was passed creating a Mine-to-Market Road Commission, consisting of three members—the Director of Conservation and Development, chairman; the Director of Highways; and the Executive Officer of the Washington State Planning Council—the purpose of which was to act on all matters pertaining to the establishment, location, construction, and financing of mine-to-market road projects. A project can be initiated under this act by a petition which states the known facts pertaining to the occurrence of mineral deposits in the area to be served, the extent of explorations and development, and approximate length, termini, and route of the proposed road. All petitions are investigated by members of the commission, and the properties to be made accessible by the proposed roads are examined by geologists of the Department of Conservation and Development to check the statements made in the petition and to determine the worth of the proposed roads.

Witness in Litigation

Because highway engineering involves earth materials in most of its phases, it is not surprising that a large number of the cases brought to court by or against highway organizations arise from disputes concerning those earth materials. Likewise, it is not surprising that geologists have been called in on many such cases to give expert testimony on geologic matters involved. In passing it might be said that had the geologist's advice been asked earlier many court actions could have been prevented by eliminating the points of contention.

A court case in which geologists' testimony played a significant part was that in which a railroad sued a state for disruption of traffic and repair of immediate damage, and for destruction of stability along 700 ft of filled grade. The damage, actual and claimed, was caused by an avalanche of rock initiated by a large dynamite blast shot on November 22, 1927, by a contractor building a part of a state highway. The estimated 165,000 cu yd of rock removed by the blast was precipitated from the cliff, down a talus slope at its base, across a muck-filled valley for a distance of about 1,500 ft to and over a section of a long railroad embankment in the valley. A total of 700 ft of track was destroyed. The railroad asked for the $20,000 on the basis of their claim that not only the tracks but also the entire rock fill in the embankment was carried away by the movement. The plaintiff's thesis, substantiated by the testimony of a well-known geologist, was that the movement was in the nature of a slide or flow of a mixture of rock and soil talus and of soil in the valley, the effect of which was to push the rock fill out into the river beyond the embankment, leaving in its place a heteorogeneous mixture composed 35% of rock and 65% of muck and silt decidedly inferior to the original rock as an embankment material.

In opposition to this was the state's contention, as presented by the testimony of Harold E. Culver, State Geologist, that the movement was not in the nature of flowing or sliding of blasted rock, talus, and valley fill, but rolling and tumbling of blasted rock only, the effect of which was to fill a part of the valley and to overflow the embankment taking part of the track with it, but not disturbing the stability of the embankment itself. The evidence presented in Mr. Culver's testimony was influential in the court's decision to disallow the railroad's $20,000 claim.

CONCLUSION

The task of the engineer is to overcome, by his ingenuity, the obstacles presented by nature. His works are becoming increasingly more complex and of greater magnitude and consequently require the use of highly technical information in an increasingly wider range of fields. It has become impossible for any one man to command a knowledge of much more than the bare fundamentals of all of the many highly specialized lines of endeavor involved in the successful completion of many engineering projects. This increased complexity in all fields has given rise to a so-called age of specialization, in which specially trained men in many fields combine their efforts to produce the best possible results. Highway engineering is sufficiently broad to include a great number of varied kinds of work and so has involved specialization in many fields, among them that of geology, dealing with the earth materials and the forces that act upon them—features which in one way or another enter into almost every phase of highway engineering.

Some of the services already rendered to engineering by geologists have been stated in this paper, with the hope that in this way engineers and geologists alike may be helped to recognize more frequently those situations in which the geologist may be of assistance to the highway engineer.

APPENDIX

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DISCUSSION

W. W. Crosby,[3] M. Am. Soc. C. E.—The plea in this paper for active cooperation of geologists in highway work interests the writer, not because he has any awareness of the lack of such cooperation, but because he personally recollects fundamental contributions by geologists to the establishment and the development of highway engineering. Long ago, the foundations were durably laid by these men who are so often forgotten when the structure is completed. Mr. Huntting's earliest reference (15) is dated 1918, and only one other (5) of the twenty eight antedates 1924.

More than two decades before, N. S. Shaler, a noted geologist and professor at Harvard University, at Cambridge, Mass., wrote "American Highways" (29) which resulted in the creation of the Massachusetts Highway Commission in 1893. It is unnecessary to detail here the outstanding leadership of that body, of which Professor Shaler was an active member.

Only a few years later (in 1898), at the suggestion of William Bullock Clark, Professor of Geology at Johns Hopkins University (Baltimore, Md.) and State Geologist, the Maryland Geological and Economic Survey was authorized to investigate "the question of road construction" in that state. The seven reports (30) of this survey (four on the "Highways of Maryland" and three on "State Highway Construction") were published in the years 1899 to 1910, inclusive, and widely distributed. The 1899 Report has been commended "as the most complete statement ever issued by any State." These reports cover in detail most, if not all, of the "problems" listed by the author and much else of at least equal importance.

Of the nine men on the "Scientific Staff" of the Maryland Geological and Economic Survey for the 1899 Report, the following were geologists: William Bullock Clark, Harry Fielding Reid, Edward B. Matthews, Charles S. Proesser, and George B. Shattuck.

Contemporaneously, the late Joseph Hyde Pratt, M. Am. Soc. C. E., as State Geologist, set the standard for modern highway work in North Carolina. Wisconsin followed the Maryland work closely with W. O. Hotchkiss, M. Am. Soc. C. E., State Geologist, supervising the groundwork. The late Logan Waller Page, M. Am. Soc. C. E., at one time Geologist for the Massachusetts Highway Commission, directed the U. S. Office of Public Roads from 1905 to 1919.

The writer has probably mentioned herein only a very few of the geologists who did so much for highway improvement, and he has scarcely hinted at the enduring character and widespread effects of what they accomplished. However, time gives a great appreciation of the fundamental, although often overlooked, importance of the work of geologists in highway engineering.

There are doubtless numerous geologists who have been, or are, contributing most valuably to the advancement of highway engineering. Indeed, a highway department cannot function without such advisers.

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Carl B. Brown,[4] Esq.—The professions of engineering and geology will both benefit by a paper that calls attention to the many aspects of highway engineering in which the training and viewpoint of the geologist can be of service to the engineer. A list as extensive as that of Mr. Huntting could be readily developed for railroad engineering, flood control engineering, military engineering, and, in fact, any engineering field dealing with the surface configuration of the earth, earth materials, or dynamic geology.

Unfortunately, many engineers, seemingly, have not learned to appreciate the aid that can be rendered to them not only by geologists, but by soil scientists, climatologists, agronomists, and specialists in numerous other fields. If some engineers seem noncooperative, this may be due, in part, to the failure of geologists, figuratively, to "come down to earth" or, literally, to evaluate their "stock in trade" on the basis of what is important in terms of the life of engineering works.

Although most of the paper shows an excellent appreciation of engineering needs, under the heading "Desirability of Bridge Sites and Road Sites with Respect to Possible Changes of Stream Channels," the conventional, and not too applicable, geological interpretation of erosion cycles and causes of stream-channel changes is given as follows:

> "Climate and rock types and structures act to modify the effects of erosion. * * * Hardness and chemical reactivity of rocks have obvious effects upon erosion. * * * Rock structures act to control erosion and in so doing they exert considerable control over stream patterns. * * * The effects upon shifting of streams attributable to stage of erosion [youth, maturity, or old age] and to modifications imposed by climatic and rock structural conditions are recognizable in advance through geologic investigation."

These statements all refer to processes and effects that act or result so slowly that they are generally of little importance in considering engineering structures expected to last 100 years or even 500 years.

Throughout most of the United States, especially in the humid sections, accelerated soil erosion is a far more critical factor in highway and bridge construction and maintenance than geologic structure or rock type. Many instances could be cited of stream channels that once ran over rock shoals with high banks sufficient to confine all but the largest flood flows, and that today are filled with sand to within a foot or two of the top of the banks. As a result, even moderate flows are causing severe bank erosion, washing out bridge approaches, and covering highways with sand and silt.

The streams of the Southern Piedmont, in a geological sense, are degrading. The region as a whole is in a stage of geologic maturity of the second or later cycle, being perhaps half finished with the process of downcutting from a former peneplain surface to a new peneplain level. Many of the streams today, however, are not degrading their beds but are rapidly aggrading, because of the overload of coarse sediment produced by widespread sheet erosion and severe gullying of the deep soil mantle—a result of the clearing and cultivation of the land. The rate of slope erosion has been drastically accelerated, and the response has been stream-channel and valley aggradation. The rate of acceleration, itself, has fluctuated widely during the past 150 years with the ups and downs of agriculture. The future channel regime at any point where a bridge is to be built will depend more on future land use and the extent of soil conservation in the watershed than on the relatively immutable geologic factors of rock type, structure, and climate.

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Engineers will find considerable information of value on the effects of erosion on stream-channel changes in two technical bulletins issued in 1939 and 1940 by the U. S. Department of Agriculture (31)(32).

F. H. Kellogg,[5] M. Am. Soc. C. E.—In furnishing a comprehensive view of the diverse factors involving applications of geology to highway engineering, the author has made a valuable contribution. By treating the various headings in the paper from a standpoint of what is actually being done by highway engineering organizations, he approaches the subject from a practical standpoint. Considering the general nature of the paper, one can scarcely take issue with the statements and conclusions. However, a more definite indication of the limitations of geological work in this field would increase cooperation and reduce antagonisms engendered by divergent viewpoints. With no intention of trying to act as a qualified critic of geological methods, the writer suggests some possible limitations.

Obviously, as emphasized by the paper, the more time taken for a geological study, and the more drilling, laboratory, and geophysical information available, the more complete and reliable will be the conclusions. An illustration of the danger of geological study without a sufficient background of detailed exploration is furnished by the experiences in connection with the Moffat Tunnel in Colorado (33)(34). There, a condition not anticipated by preliminary investigation was a fault zone of "rotten granite," which tended to cave into the tunnel bore, costing a number of lives and raising the cost of the project from an estimated 7 million dollars to more than 17 million dollars. The complaint was made that, although the geologists had given an elaborate report, "the voice of prophecy was not upon them." Under more modern conditions, nobody would expect any prophecies, but a vastly greater amount of time and money would be spent on gathering specific data on the rocks in the tunnel site before starting construction on a project of such magnitude.

There are specialists among geologists just as there are among civil engineers and among highway engineers, and so it is questioned whether many of the functions treated by Mr. Huntting could be performed by the average geologist. Particularly in connection with subgrades, slides, and ground waters, a wealth of firsthand experience is a prime requirement for most professional work, and an acquaintance with the elements of elasticity, hydrodynamics, and soil mechanics is often desirable. Doubtless, members of state geological surveys working closely with the highway departments, and some geological engineers, possess these qualifications. Many other geologists do not.

A type of rock slide which has been overlooked by geologists on some construction jobs occurred on the Madden Dam Project in Panama. A shallow key trench was cut in a bluff rising steeply from the Chagres River to a height of nearly 200 ft. The rock showed almost no signs of seams, crevices, or bedding planes. Soon after the excavation, however, it began to crack and masses of rock threatened to fall on the operations at the base of the bluff. The cracked material was removed until, once more, only smooth, solid rock was exposed. In another day or so, more cracks appeared and the danger of rock slides increased. The situation grew steadily worse until a geologist was called in consultation. He immediately diagnosed this condition as a case of overstressed rock. The very fact that the rock was so solid, so free of any strains which would have relieved the stresses, was the cause of the trouble. The geologist showed that the same rock across the river was well jointed, and never acted this way. Thereafter, no more rock was removed from that area without immediately replacing it with concrete, so that the stress balance was undisturbed, and no more "popping rock" was encountered. This case illustrates the value of a geologist with wide experience in practical excavation and construction problems.

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An engineer dealing with rocks and soils must gain some knowledge of geological principles from the very nature of his work. A geologist could advance, substantially, the cause of understanding between the two professions by avoiding conclusions that would be apparent to any one with the slightest knowledge of geology. In general, delineation of alluvial and residual soils, lengthy observations as to the dangers of cuts through rocks with a shaley cleavage dipping toward the road, and detailed descriptions of readily identified rock formations and aquifers—all would fall in this category.

Mr. Huntting has mentioned the possibilities of the geological approach to problems of subgrades and slides in artificial fills. Although the writer does not believe that engineering work in soils must be a highly restricted specialty, or that any particular group has a monopoly on statistical development and mathematical physics, he contends that the value of the geological approach to these problems is limited. For example, in the paper under the heading, "Frost-Heave Problems," the statement is made that soils with a high clay content are likely to show considerable frost heaving because they have small pore spaces, which cause high capillary rise. On the other hand, clays usually have such low permeabilities that only a small quantity of water is involved in this capillary action. Soils with pore spaces sufficiently small to permit a high capillary rise, yet large enough to involve appreciable quantities of water, are the worst offenders as regards frost heaving. These offenders can be identified quickly and economically by simple physical tests. Again, the statement is made in the paper under the heading, "Prevention and Correction of Landslides," that sands with rounded grains are less stable than those with angular grains. This is obvious, but it gives no basis for determining the slope required for a given fill under given conditions of compaction and pore pressure.

The writer once made a study of the effect of mineral composition on the mechanical properties of soils by concentrating individual minerals, grinding them to certain grain-size distributions in a colloid mill and performing various physical tests on the monomineralic soils thus produced. The minerals were selected with the help of C. S. Ross, of the U. S. Geological Survey, who pioneered in the field of clay minerals and their identification. Observing the tremendous effects of small admixtures of certain minerals in the finer fractions of a soil, and those of the intimate mixtures of various minerals in many of the soils studied, the conclusion was reached that a purely physical approach would be more rapid and practical for most control work. The complex petrographic and X-ray techniques involved in identification of the clay minerals do not favor the mineralogic approach to problems in soils engineering except in the fields of research and development. In these fields, this approach has great promise, particularly in connection with studies of chemical and physico-chemical processes (35)(36)(37). In applying the results of this research, however, rapid and simple physical tests based on performance experience appear preferable.

The foregoing comments seem warranted because there are engineers and geologists who would let geological analyses supplant, rather than supplement, purely physical investigations of the problems in the paper. Evidently Mr. Huntting is not of this opinion, and the writer strongly concurs with his basic conclusion that geology is a powerful tool in many highway engineering activities and that there is not a sufficiently general realization of this fact.

Berlen C. Moneymaker,[6] Assoc. M. Am. Soc. C. E.—The twelve problems listed and discussed in this paper include nearly all the major geologic problems involved in the construction and maintenance of highways. Many of the same geologic problems are also concerned with the construction and maintenance of railroads.

Mr. Huntting's treatment is rather general, on the whole, and few professional engineering geologists are likely to challenge, seriously, either his statements or his conclusions. Engineers, however, might be inclined to feel that he tends to "oversell" the importance of geology and to underestimate the engineer's knowledge of geologic problems. As most colleges and universities require their engineering students to complete satisfactorily one or more courses in geology, most engineers embark on their professional careers with some geologic background. Engineers who specialize in field work—constructing and maintaining highways, building dams, driving tunnels, etc., learn much about geologic problems by actual experience. Admittedly, some of this knowledge is gained the "hard way" and occasionally a disastrous failure results from ignorance of geologic conditions. Nevertheless, minor geologic problems, such as the determination of slopes in excavations, etc., may ordinarily be handled satisfactorily by engineers.

Mr. Huntting's paper raises a number of questions, among which are the following:

1. Should All the Work Included in the Twelve Problems Be Performed by Geologists?—These problems involve several distinct earth sciences, some of which are not geologic in the strictest sense, and it is debatable whether geologists should enter the fields of related or allied sciences. The Geologic Division of the Tennessee Valley Authority (TVA), for many years the largest staff of engineering geologists ever assembled, may be cited as an organization in which a high degree of specialization, even in geology, is recognized. The work done by this group is geologic in the very strictest sense, and work in the related or allied sciences is performed by other staffs of specialists. The actual testing of materials is left to materials testing specialists; the testing of soils is left to specialists in soil mechanics; geophysical work is left to geophysicists, and so on. The Geologic Division of the TVA is sufficiently large to include specialists in several branches of geology. Specifically, the activities of this staff include the following: Engineering geology for dam sites and tunnels; the location and geologic study of quarry sites; the collection of samples for testing; petrographic studies of foundation rocks, concrete aggregate, and riprap materials; study of ground-water problems in areas adjacent to reservoirs; examination and appraisal of mineral lands; the presentation of testimony in court at condemnation trials; and miscellaneous geologic work, such as studies of bridge foundations, deep highway and railroad cuts, and drainage projects for malaria control.

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[6] Chf. Geologist, TVA, Knoxville, Tenn.

In the opinion of the writer, geophysics, soil mechanics, and the testing of materials should not be considered in the field of geology. However, many excellent geologists have become specialists in these related sciences.

2. How Should a State Highway Department Provide for Its Geologic Work?—As the author has shown definitely that geologic information may be used advantageously in the construction and maintenance of highways, it might seem that each state highway department should equip itself with a geologist or with a small geologic staff. However, the matter is not as simple as that. If geologic advice is to be of any great value, it must be that of a specialist. In this day of specialization, no geologist is sufficiently versatile to qualify as a specialist in each branch of the science involved in the author's twelve problems. It would be difficult to assemble a small staff of two or three men capable of even moderately reliable work on all these highway engineering problems. A well-rounded staff, qualified to perform specialized work in all branches of the earth sciences involved, would require the personal services of the following specialists, some of whom are not geologists: Engineering geologist, petrologist, ground-water hydrologist, economic geologist, materials testing specialist, soil mechanics specialist, and geophysicist. Obviously, most state highway departments cannot justify the maintenance of such a large staff.

Although Mr. Huntting notes the close cooperation that exists between certain state highway departments and state geological surveys and mentions the engagement of special consultants, he makes no specific recommendation as to how a state highway department may best obtain geologic advice of high quality. Considering the great variety of geologic and related investigations required for the construction and maintenance of highways, the state geological surveys can afford the best and most economical assistance. In states where the geological surveys are small or inactive, the faculties of state universities may supply able geologic advisers.

3. Is the Geologic Examination Sufficient of Itself?—In some instances, a geologic examination may afford all the information necessary, but in many cases a mere examination is not sufficient.

A geologic examination of a tunnel site without a considerable amount of exploratory core drilling may not yield much precise information. The writer examined the Garzas-Peñuelas power tunnel site in Puerto Rico before construction was undertaken and was unable to predict the geologic conditions with desirable precision. However, a very thorough study of the stratigraphy and the orogenic and geomorphic history of the area enabled him to predict accurately the nature of the geologic problems.

The Garzas-Peñuelas tunnel is 11,674 ft long and extends in a north-south direction across the rugged Cordillera Central, which forms the insular divide of Puerto Rico. Rock decay has progressed to such great depths (200 ft) below the surface that bedrock was exposed only at the intake portal and in the channels of two small streams that cross the tunnel location. Moreover, the entire tunnel site was covered by a luxuriant growth of tropical vegetation. By a painstaking study of the residuum along the location, the character of the underlying rock could be predicted with accuracy. A study of the only rock exposures in the area revealed the location of a few faults. Although it was expected that hundreds of faults were to be crossed, there was no way of determining precisely either their location or magnitude or their importance as geologic problems.

Fortunately, little serious trouble was experienced in the actual driving of the tunnel but this was due to the excellence of the engineering supervision rather than to lack of difficulty. As predicted, hundreds of faults were encountered but most of them were small and presented no problems. Many of them were marked by shear zones and occasioned appreciable overbreakage. For support, these structures required timbering during construction and lining afterward. Two major faults, both apparently active, were crossed with some difficulty and after considerable delay. Both yielded copious quantities of water under considerable head. One of them resulted in the caving of the heading and the partial filling of the bore with 1,000 cu yd of fine gouge and 500 cu yd of broken rock. This fault zone was 152 ft wide and yielded 1,300 gal of water per min. The other fault also resulted in the failure of the heading and the partial filling of about 40 ft of the bore with broken rock and gouge. The fault zone was only 21 ft wide and yielded 200 gal per min of water under a head of 160 lb per sq in. Five months were required to tunnel through the 173 ft of ground represented by these two fault zones.

At the time the geologic examination was made, the tunnel site was definitely fixed by engineering and other considerations, and preliminary construction activities were already under way. No geologic exploration of any type was made along the tunnel location. Although the geologic conditions could have been determined in advance only from drill holes on 25-ft centers, from a few feet to 640 ft deep along the tunnel site, the cost of the program would have been prohibitive.

The Garzas-Peñuelas tunnel is an instance in which the engineer in charge handled difficult practical geologic problems remarkably well.

The Moffat Tunnel through the continental divide in Colorado represents another instance in which a geologic examination, without exploration, was highly inadequate. Numerous other instances in which geologic examinations, without the advantage of exploration, proved inadequate or entirely misleading could be mentioned.

One geologic condition not mentioned by the author in his discussion is the silicosis hazard involved in the quarrying or tunneling of siliceous rocks. Rocks containing more than 10% free silica are likely to cause silicosis among workmen unless the dust count per cubic foot is kept below 10,000,000 particles of two microns (0.002 mm) or smaller in size. The free silica content of the rock to be tunneled or quarried may be readily determined by petrologic studies. The silicosis hazard is very largely eliminated by adequate ventilation, the use of wet drilling methods, and the spraying of the broken rock during such operations as loading and crushing. Failure to control siliceous dust invites serious trouble. The contractor on the Hawk's Nest Tunnel in West Virginia was sued by some 600 former employees who claimed to have contracted silicosis while at work on that project.

This paper is an excellent and timely contribution on the relation of geology to highway engineering. Geology is a valuable tool that is used advantageously in several branches of civil engineering. Its limitations, however, should be emphasized as strongly as its advantages. The most reliable geologic work, in any branch of the science, is done by a specialist, and even a specialist, in many instances, must rely on data made available by exploration.

E. F. Bean.[7]—A brief paper cannot be comprehensive in coverage. Thus, the following notes reflect a desire for a longer paper rather than adverse criticism of this paper, which is an excellent review of the work that has been accomplished.

The "Wisconsin Materials Survey Procedure" would be more accurately described if the first sentence under that heading were omitted, and the following statement were added at the close of the paragraph:

> "This information makes it possible to let contracts at lower figures, or for the county to perform the work at lower cost since the uncertain factor, source of road aggregate, has been eliminated."

The State Highway Department does not own and operate any local plants.

The list of investigators of inferior aggregate in concrete (see heading, "Suitability of Various Earth Materials for Surfacing, Concrete Construction, and Other Highway Uses") should be expanded to include, in addition to Mr. Stanton, A. T. Goldbeck, Stanton Walker, and W. J. Emmons, Members, Am. Soc. C. E., and Fred C. Lang and M. O. Withey.

Of course, the Bibliography cannot include all pertinent references but at least five others might well be included (38)(39)(40)(41)(42). Perhaps, also, reference should be made to the *Annotated Bibliography of Economic Geology*, published semiannually, which contains a section devoted to engineering geology.

A. T. Bleck,[8] Esq.—The conclusion of the paper is a very succinct summary. As the author states, highway engineering is sufficiently broad to include many varied kinds of work requiring specialized knowledge, which the engineer should avail himself of as the occasion arises.

In exploration for materials deposits, complicated bridge foundation problems, classification of excavation materials, tunnels, etc., consultation on geological formation appears to be an obvious need. However, considering problems arising in connection with the soil mantle of the earth, highway engineers in general possibly have relied on, or sought for, soil mechanics data to too great an extent and to the exclusion of other factors, such as the geological formation and subsequent development of soils. This opinion is based on not only personal observations but also the apparent controversial tenor of the literature on the general subject of soils in their relation to highway and airport construction.

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[7] State Geologist, Geological and Natural History Survey, The Univ. of Wisconsin, Madison, Wis.

[8] Constr. Engr., State Highway Commission of Wisconsin, Madison, Wis.

If geology, to a reasonable degree of certainty, can be applied to predict the probable exposure to moisture to which the soil in given locations will be subjected, the engineer can then correlate these data with those supplied by the science of soils mechanics and effect an adequate design without sacrificing economy. Perhaps a greater mutual understanding, on the one hand, of the problems encountered, and on the other, of the knowledge and counsel that could be made available, is necessary and Mr. Huntting's paper seems at least to be a step in that direction.

LYMAN W. WOOD,[9] ESQ.—The twelve problems, listed by the author, constitute an adequate statement of a geologist's duties with the Iowa State Highway Commission. In subsurface explorations of bridge sites, the procedure is perhaps more advanced than that used by other highway organizations. Test borings are made for all bridges where foundations are expected to extend below stream bed. The boring records are reviewed by a geologist and become part of the final bridge plan, supplemented in many cases by geological sections of the strata penetrated and by results of physical tests on samples of critical materials. Valuable information on excavation conditions, as well as on probable pile bearing, is thus made available to contractors.

Road material resource surveys, to be of greatest value, must be continuous. The Iowa Highway Commission has followed a plan for periodic reexamination of all material deposits, with provision for revising estimates of quantity or commenting on quality, as advisable. The more important deposits are examined annually; others are examined less frequently. Proper maintenance of a file of material resource information in an up-to-date condition will require a considerable annual outlay of time, but experience indicates that the results are worth while.

PHILIP KEENE,[10] ASSOC. M. AM. SOC. C. E.—In the United States, highway work has attained the category of "big business," and highway engineering is replacing many of its rule-of-thumb practices with scientific solutions. As the author has demonstrated, geology is an important scientific aid to proper design, construction, and maintenance of highways.

Probably geology in Connecticut highway work today is most useful in determining depths of the various soil strata overlying bedrock when making borings for bridges and structures and in soil surveys. As the author indicates, knowledge of the geology of a bridge site, before or after some borings are made, permits economical and judicious selection of subsequent bore hole locations and depths. Soil surveys have been made by the department since 1942 on all cuts and swamps, on the larger projects. The surveys in cuts are intended primarily to locate rock surface for determining probable yardages of rock and earth excavation, inclination of cut slopes, and width of right of way to be purchased; but they also enable the engineer to determine definitely, or tentatively, the need for subbase, underdrains, and slope drains, the suitability of soil for fills, and the treatment of special features which occasionally are encountered.

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[9] Formerly Geologist, Iowa State Highway Commission, Ames, Iowa.

[10] Soil Mech. Engr., State Highway Dept., Portland, Conn.

The bedrock and ledge of Connecticut consist chiefly of ancient complex metamorphic and igneous formations which are very difficult to decipher. However, covering a strip that is from 15 to 25 miles wide and extends throughout the middle of the state in a north-south direction are traprock and sedimentary "brownstone" formations which form part of the Newark series of the Triassic Period. These formations dip from 15° to 30° in an easterly direction, forming definite characteristic ridges in numerous localities. In this Triassic strip the depths to rock surface can be predicted quite accurately for an area, especially on these ridges, before or after a bore hole has been completed. This characteristic dip also permitted definite calculation of the location of the contact plane between the traprock sill and the underlying red sandstone between bore holes for the proposed tunnel at West Rock, New Haven, Conn.

A use for geology, not mentioned by the author and needed only rarely, is in the examination and interpretation of soil or rock cores to determine the plane or zone of rupture of a slide. Such cores might have many small faults and distortions caused by old disturbances, in addition to those caused by the slide being studied. In such a case, geology would be indispensable in distinguishing between the old and the new movements.

JACOB FELD,[11] M. AM. SOC. C. E.—Undoubtedly a knowledge of geology is helpful in solving the twelve problems enumerated in this paper, most of which are specialized problems of soil mechanics. If the author's statements are taken at face value, soil mechanicians (or is it soil "mechanics") as well as foundation engineers are daily encroaching on a restricted field. Since soil control and analysis has always borrowed information from the science of geology, uses and has used knowledge contributed by the sciences of physics and chemistry, and also employs analogies from electricity and hydraulics, the encroachment on the field of geology is not unique.

To be expert enough to handle the twelve problems properly, the geologist should be a well trained highway engineer, a specialist in soil mechanics, an expert in tunnel design and construction, and a divining rod operator, as well as an expert witness. Might it not be simpler to take a highway engineer who has had the necessary training in soil mechanics and insist that he obtain sufficient training in geology than to expect the geologist to understand the engineering problems enumerated?

Suitable gravel pits generally may be located by the application of geology, but the writer has found in practice that the only way to know the characteristics of sand or gravel deposits is to take samples while digging is in progress. Even boring samples do not give the complete answer. On a recent large project where almost one million cubic yards of run-of-bank gravel was needed, approximate locations of available supply within a radius of ten miles had been determined from topographic and geological considerations. When the time came to use this gravel, almost every one of the locations was found to be a disappointingly thin cover of gravel overlying sand. In a number of locations the gravel was covered with glacial till, the cost of the removal of which made the supply of gravel very expensive.

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[11] Cons. Engr., New York, N. Y.

The author's statements concerning the suitability of various earth materials (in which he includes concrete aggregates) fail to take into consideration the fact that standards have been set up by the American Society for Testing Materials, American Concrete Institute, the Bureau of Public Roads, and forty-eight state highway departments, in the form of specifications listing not only the requirements of physical and chemical characteristics but also, in great detail, the methods of testing the compliance with such requirements. Most of these specifications are based on an accumulation of empirical information which may be determined in some instances by geological considerations. However, the wealth of experience indicates that empirical data based on prior performance is a safer guide to the highway engineer.

On U. S. route No. 9, north of Peekskill, N. Y., there is a monument to a contractor who undertook the excavation through a hillside, definitely glacial in origin according to all geological information available, which, when exposed, was found to be a thin skin of glacial till and boulders overlying nice hard solid rock. The unclassified excavation unit price in the contractor's contract applied to the excavation of this cut, more than 100 ft deep in solid rock, put him in the class of an ex-contractor. In this instance, somewhat too great a reliance on "known" geological characteristics of that area were disastrous, to the contractor.

Under the heading, "Prediction of Character of Material to Be Excavated," Mr. Huntting states:

> "The cost of digging solid rock may be ten times or more the cost of digging an equal amount of earth (Fig. 1); * * *."

In more than one instance, the writer has had actual experience, where the cost of digging rock would have been much less than that of the type of earth encountered. The writer also wishes to dissent from the statement (see heading, "Competency of Materials for Bridge Foundations") that:

> "The superstructure of a bridge can be no more sound than its piers and abutments, whose soundness in turn is dependent upon their design and upon the nature of the rock or soil foundations on which they rest (Fig. 2)."

The soundness of a superstructure has no relation to the type of rock or soil encountered in the foundations.

Under the heading, "Prevention and Correction of Landslides," some mention should be made of the slide that occurred in Pittsburgh, Pa., in 1922 and also of the landslides in Los Angeles, Calif., and Fort Peck, Mont.

When discussing bridge foundations, the author seems to indicate that taking and testing soil samples is entirely the work of geologists. In all his soil mechanics studies on that subject, the writer has never heard this claim.

The writer's experience in 1943, when searching for an available supply of gravel, afforded an interesting side light on the evaluation of mineral lands with gravel pockets. He found buying farms, including the houses and livestock, less costly than buying gravel pits. No amount of test pit probing or boring data would give a true indication of the amount of gravel available, except at a most unreasonable cost. Evaluating the land based on expected yield of gravel involved so many factors of both time and exploratory costs that it was much more expedient to buy the land. For sites where agreement as to the amount of available gravel could not be reached, experts were called in to analyze the situation from the geologist's point of view. The result was testimony in the form of reports of one geologist against another; and, as in the litigation examples given in the paper, one geologist gave testimony which was accepted and the other geologist gave testimony which was not accepted. So the score was even.

The writer's objection to this paper is again demonstrated by the "Conclusion." The attempt to delegate all questions "dealing with the earth materials and the forces that act upon them" to geologists, on the plea that highway engineering is sufficiently broad to include this overspecialization work by geologists, seems an attempt at overspecialization.

A quotation from a paper written by Edwin G. Conklin (43) is quite pertinent:

> "The extreme specialization of science has tended to obscure the larger aspects of Nature, as a whole. It has been said that modern science consists in knowing more and more of less and less. Certainly it is concerned largely with analysis and always more analysis.
>
> "Of course, analysis is necessary, but so also is synthesis if we are to see organisms as living beings. We lose sight of these larger aspects of life in the process of analysis, unless we reverse this procedure from time to time and consider organisms and environment synthetically.
>
> "Life has been compared to a beautiful tapestry, woven in intricate design of many threads and colors. By means of physics, chemistry, physiology, anatomy, embryology and genetics we unravel this texture, separate its constituent threads and colors, but lose the pattern as a whole. These analytical sciences have enormously increased our knowledge of life's constituent elements and processes, but the pattern of the tapestry is usually neglected or ignored."

In a similar sense, the writer makes a plea that the subject of highway engineering be considered as a unit and, if the highway engineer needs training in the science of geology to properly solve the problems encountered, he should have that training. If he delegates the soil problems to geologists, why not also delegate such chemical problems as those of cement and its uses to chemists and static problems on the design of pavements and bridges to physicists, and thus leave no place for the highway engineer.

H. E. MARSHALL,[12] ESQ.—The subject of this paper is worthy of the careful attention of both engineers and geologists interested in the efficient operation of a highway system. It presents a comprehensive, up-to-date description of the numerous but often unrecognized problems with which engineers must deal in the design, construction, and maintenance of a highway system in which the application of geological data and methods of study are necessary.

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[12] Geologist, Bureau of Tests, Ohio Dept. of Highways, Columbus, Ohio.

The value of utilizing the services of trained geologists in its highway department has long been realized in Ohio. The department, created in 1904, established the testing laboratory in 1909. The first assignment of the laboratory was to make a geologic study of the sources of aggregate, both developed and undeveloped. The report, prepared by the geologist at that time, has been valuable for the comparison of the wide range of aggregates available in Ohio.

In more recent years studies of the mineral composition of a number of aggregates for service records of portland cement concrete have been made using the petrographic microscope. Although this work has been discontinued at present because of the shortage of trained personnel, it is hoped that it can be continued after the war and that it will indicate a method whereby certain aggregates containing minute quantities of deleterious material, whose effect is not clearly enough recognized by the usual tests, can be identified.

When fourteen dams were constructed for the Muskingum Valley flood control project, approximately 150 miles of highway relocation was necessary. Since much of this relocation involved the construction of fills, a part of which would be inundated, the slopes of these fills had to be protected from erosion by wave action. To find suitable rock for this purpose a considerable amount of geological surveying was done by highway department geologists in the various flood control basins. In addition, the various stones available (principally sandstone) were studied by the use of the petrographic microscope to determine their composition and probable durability when exposed to the various agencies of weathering on the fill slopes. The location of sources of suitable stone close to the projects has resulted in a considerable saving in the cost of highway work in the district. Similar studies were also made in connection with reconstruction of highway fills damaged by the 1937 flood of the Ohio River.

Geologists have been employed continuously in the study of soils and related problems by the testing laboratory since 1937. Much soil testing for highways consists essentially of an engineering analysis of the physical properties of soil, its strength, and the influence of moisture on its performance. Furthermore, the engineer's training in advanced mechanics and mathematics is a necessity for the testing of the stability of soil by shear and consolidation tests and especially for the application of data obtained from these tests to the solution of problems in the supporting strength of soils for structures and embankments. However, the geologist's working knowledge of the historical processes which have led to the development of soil deposits gives him a picture of their character and distribution which is frequently of more immediate value in dealing with routine questions of soil stability than some of the more elaborate methods of theoretical soil mechanics. For a reasonable and well-rounded study of soils for highway work both the geological approach and the engineering approach must be used.

In the Ohio organization geologists have been found particularly valuable in making field surveys of the soil and rock conditions which will be encountered in the construction of proposed new highways. The mapping of soil profiles, particularly in hilly regions where deep cuts extend into the bedrock, can only be done intelligently if the general geology of the region is understood. Frequently, a detailed study of the geology of the area will be necessary in connection with the soil survey for the project.

In areas where landslides are common, detailed information on the character and structure of the bedrock, in addition to data on the physical properties and thickness of the soil overburden, is necessary to a proper evaluation of the conditions which may cause sliding or under which slides have already developed.

To illustrate the influence of local geology on a specific landslide, the following brief report of a slide which occurred in the southern part of the state is given. A section of the Ohio River Road lying just below a long sweeping curve in the river, where lateral erosion of the channel has kept the river against the valley slope, had been settling and sliding periodically for many years. This section is founded on talus near the toe of the slope. Investigation along the line of the old road showed the depth of poorly consolidated detritus composed of clay and boulders to be so great as to preclude the possibility of satisfactorily holding the road on this line by any practical means. A new road was constructed higher on the valley slope so that most of its length was on a prominent sandstone ledge. However, because of the considerable excavation necessary to obtain a roadway of sufficient width on the bench, a part of the roadway was constructed of fill extending beyond the old sandstone cliff and resting on the same talus slope occupied by the old road. Several years after construction a slide occurred in this overlapping fill in which about 400 ft of the roadway settled a distance of 5 to 10 ft within a period of two weeks. Investigation of the conditions leading to the development of this slide showed not only that the fill was resting on a great thickness of clay and rock talus but also that water had been flowing through the sandstone stratum down the dip of the rock from a tributary valley which parallels the river at this point about one-half mile away (see Fig. 3). The probability that water saturating the fill adjacent to the old rock cliff was coming through this porous sandstone eliminated from consideration any permanent corrective procedures which did not either cut off entirely the water from this source or place the full roadway width on top of the solid rock ledge. The potential water bearing nature of this sandstone was evident only when the inclination of the bedrock as determined from a geological study of the area had been made.

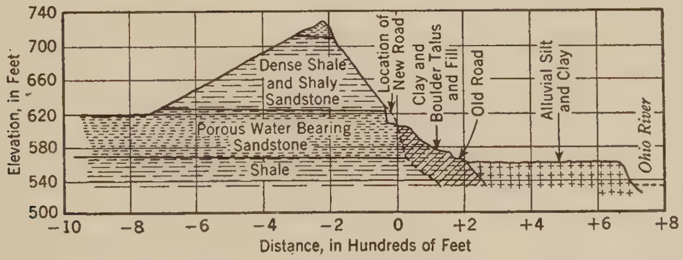

Fig. 3.—Water Seeping through a Dipping Sandstone Bed Contributed to the Saturation of the Talus Slope and Fill on Which a Highway Was Constructed in Southern Ohio

As Mr. Huntting has shown, the geologist by the application of his science can contribute much to the solution of many highway engineering problems, particularly those involving earthwork. However, for engineering purposes, detailed and accurate data must nearly always be obtained by such procedures as test borings and laboratory tests to supplement the more general data obtained from a geological examination. Exact information necessary for the safe construction of an improvement cannot be secured in many cases without detailed investigation. The geologist's services in these problems will be of the greatest value in indicating the need for, and extensiveness of, these studies and in interpreting the results.

K. B. Woods,[13] M. Am. Soc. C. E.—The approach used by the author in discussing the relationship of geology to highway engineering by citing typical cases can be extended indefinitely. The periodic recording of these experiences can constitute a valuable addition to engineering literature. The significance of this procedure is emphasized by the fact that highway engineers generally are not familiar with, and lack an appreciation of, the usefulness of geological literature in solving a wide variety of their problems.

Most states do have, or have had, a geological survey organization which has produced many geological publications containing a vast store of information that can be used to practical advantage by the engineer. Added to these should be other available publications, including in particular the maps and reports produced by the soil surveys of the various states. Pedology and geology go hand in hand and can be useful tools to the highway and airport engineer, particularly in such soil areas as the glacial drift, the great plains, the coastal plains, and in some residual soil areas—including such parent rocks as limestones and shales. In mountainous and semimountainous areas, a knowledge of the rock conditions usually is a primary requirement for satisfactory design and ultimate good performance of the completed engineering structure.

Failure to use available geological information and talents is not entirely the fault of the highway engineer. There is a fundamental difference in the perspectives of geologists and highway engineers. A concrete example is the time element. The geologist thinks in terms of hundreds, thousands, and even millions of years, whereas the highway engineer, in his endeavor to keep pace with the mechanical developments of the automotive industry, is fortunate indeed if he can build and design a highway useful for 50 or even 25 years. The first essential for widespread use of geology in highway engineering is a blending of the two perspectives. The geologist should endeavor to acquaint himself with the broader aspects of engineering and the engineer in turn should decide what phases of geology are applicable to his problems. The engineering and geology schools of the educational institutions in the United States might well be ideally situated to initiate this common understanding.

Referring to the section on "Problems," the following comments will be directed to four of the first six items, namely: (1) Locating suitable road-surfacing material pits and quarries; (2) determining the suitability of various earth materials for surfacing, concrete construction, and other highway uses; (3) subgrade treatment and classification; and (6) preventing and correcting landslides.

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In regard to Problem (1), two factors are of major importance—namely, certain land forms and bedrock geology. The geologist well knows that prospecting for gravels and sands in the glacial drift area can be limited to certain land forms including kames, eskers, river terraces, outwash plains, and sand dunes. The engineer is not generally familiar with the geological history of these land forms and as a result his quest for possible sources of granular materials is all too often hit or miss. The geologist, on the other hand, may not be familiar with the engineering requirements of items such as base courses, for instance, and may locate for the engineer's use only those deposits that may yield commercial aggregates. Many states already have available geological maps which show these land forms. The author has listed several of these. A more recent publication than those in the "Bibliography" is the report by H. W. Leavitt and E. H. Perkins (44) which contains granular materials maps of Maine.

Recent developments in the use of aerial photographs for locating available granular deposits, as well as for interpreting the engineering characteristics of soils, have increased the possibilities of a combined engineering-geological approach to the problem (45). The trained eye can locate granular materials in such photos by noting the drainage patterns, color tone, topography, vegetation, etc., with unfailing accuracy. All deposits of this type are good potential sources of granular materials for highway base courses or even for commercial aggregates in many instances. A knowledge of geology and pedology in this technique is necessary for a correct and accurate interpretation of the evidence included in the photograph.

Another important consideration in the use of information concerning land forms and their effect on highways and airports is that of location. River terraces, for instance, frequently are excellent airport sites because of the granular texture of the soil which provides adequate drainage without the use of many additional man-made appurtenances. Soil survey reports should not be neglected in this respect since detailed maps frequently are available and they can be used to establish grades and to locate highways and airports on the best available materials. Likewise, these same maps, when available, do show the locations of available granular materials.

In regard to Problem (2), the author stresses the fact that research and physical testing of aggregates, used in portland cement concrete construction, have not kept pace with the developments in concrete construction in general. The aggregate has been considered more or less as an inert material, the active ingredient being the cement. As stated by the author, available information disproves this idea. Performance surveys of concrete roads in Kentucky (46) indicate that certain river gravels produced pavements much inferior in quality to those made with other aggregates. Some surveys recently completed in Indiana show that a large mileage of concrete pavements made with one particular aggregate always produced inferior concrete, regardless of the time of year of construction, the underlying subgrade soil, the traffic conditions, the type of cement used, the fine aggregate used, and any other variables that may have occurred in connection with a construction program involving this aggregate during the 12-yr period in which it was used. Other aggregates employed for short sections on these same contracts were invariably more satisfactory. Since the results of physical tests on such aggregates are not particularly alarming, there appears to be much need for the development of better tests for acceptance. As suggested by the author, the study of thin sections by a competent mineralogist might at least be helpful in determining the suitability of such aggregates in concrete construction. In this respect, the approach used by D. G. Runner (47) promises to give considerable assistance in solving such problems.

In regard to subgrade treatment and classification, a "pin-point perspective" seems to have been taken in many instances in correlating pavement performance with soil areas. In any given large geographical area, well-defined geological boundaries can be used to distinguish one soil area from another. Since the soil profile either is the product of disintegration and weathering of underlying bedrock or consists of transported materials primarily derived from rock, it follows that similar soil profiles should be developed from similar materials provided the weathering has been of the same duration and that at least similar conditions of climate, topography, and vegetation have prevailed. From this it can be deduced that standard designs of the highway—particularly the base course, pavement, subgrade treatment, and drainage—can and should be made for each large uniform soil area. The only variable to be considered, then, would be the amount and weight of traffic loads that are to be anticipated. Many performance surveys made on flexible pavements in Indiana (48) lend support to this thesis.

Approximately 2,000 miles of secondary pavements in Indiana were surveyed during the 1943 spring breakup, yielding many interesting results. For instance, about 50 miles of traffic-bound gravel or stone bases with a bituminous top (total pavement thickness of 6 to 8 in.) located on gravel terraces suffered no noticeable ill effect from the combination of a severe winter and traffic. Similar pavements in silty clay soil areas suffered severely even with pavements as thick as 12 to 15 in. Another soil area in north-central Indiana consists of shallow sands on till, the sand being of variable thickness. Much pavement distress was encountered in this soil area, particularly where the shallow sand was 3 ft thick or less. Such an area may escape the engineer's attention since sand is normally an excellent subgrade material. However, a combination of traffic with adverse water and temperature conditions was sufficient during this particular spring to cause havoc with many miles of pavement, including some primary types. Obviously, a standardization of design with particular reference to grade and drainage is essential for proper pavement performance in this particular soil area.

Another pavement problem that can be studied on a geological-soil-area basis is that of pumping of rigid pavements. This action consists of the ejection of muddy water from beneath the pavements through cracks, joints, and along the edge of the pavement. Within a short time a void develops beneath the pavement, leaving the slab unsupported in certain places. Ultimately, cracking and complete disintegration of the pavement result (49). In the Middle West this problem prevails on heavily traveled roads located in lacustrine soil areas, such as in the Lake Chicago (northern Illinois and Indiana) and Lake Maumee (northern Ohio) soil deposits and in deep cuts in moraines. The extreme importance of the type of soil as a factor in pavement pumping cannot be overemphasized. The highly plastic coastal plain clays of Georgia, Alabama, Arkansas, and Texas, for example, produced badly pumping pavements where traffic loads are heavy. The engineer will be able to cope with this problem in the design of his pavement, but, since this new design may be more expensive than the older design, he should know where pumping pavements are likely to occur. The correlation of pavement performance with the geological-soil areas is a feasible approach to this type of problem.

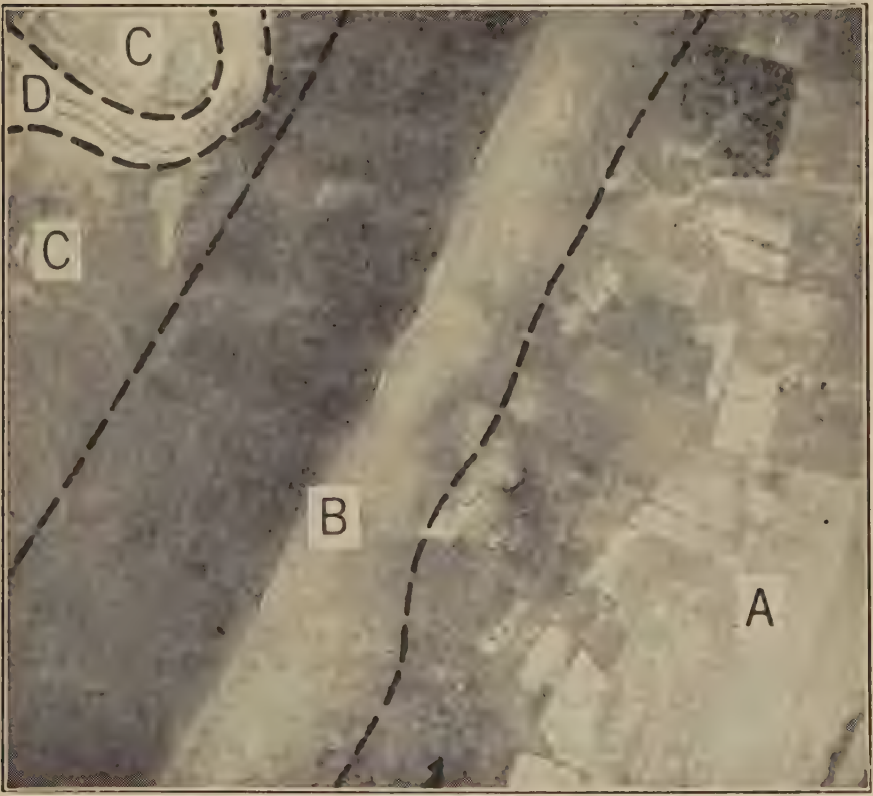

FIG. 4.—AIR PHOTOGRAPH OF A BEDROCK AREA—A IS LIMESTONE, B IS SANDSTONE, C IS SOFT SHALE, AND D IS RECENT RIVER ALLUVIUM

Landslides may be studied frequently on a geological-area basis. The Conemaugh formation of the Pennsylvanian system, for instance, in some areas contains dozens of landslides, particularly in the "red-bed" outcrops. The Ordovician system outcropping in southeastern Indiana, southwestern Ohio, and north-central Kentucky is another landslide-producing geological area. Corrections for landslides in such areas can almost be standardized, and certainly precautionary measures can be employed in the original road layout when it is realized that the geological strata in question have caused trouble repeatedly during and after previous construction operations. However, it should be emphasized that landslides will not necessarily prevail in other outcrops of Ordovician or Conemaugh rocks since this geological age classification does not require a similarity in rock textures.

In correlating pavement performance with geological areas, one of the important weaknesses of geological literature is the stress that is placed on substantiating data showing similarity of age rather than similarity of texture of the particular strata in question. Age is important to the geologist in reconstructing the sequence of geological events. On the other hand, texture and composition of the rocks are of major importance to the engineer, particularly if he is to use geology in applying the results of experience gained in one area of the world to a similar area elsewhere. This difficulty is fast being overcome by the analysis of the patterns of aerial photographs. Experienced technicians can use such photos and predict with remarkable accuracy the type of rock strata. A typical bedrock pattern is shown in Fig. 4. Even the inexperienced can see the contrasting patterns of the limestone, sandstone, and shale, designated as A, B, and C. The continued development and use of these techniques will require even greater coordination between the engineer and the geologist. In turn, active association of the two groups should do much to create a better understanding between them.

D. J. Belcher,[14] Jun. Am. Soc. C. E.—The general thesis that geology is applicable to the solution of many highway engineering problems marks this paper as one to be read and considered by every engineer engaged in highway and airport engineering. It is hoped that some significance can be attached to this apparent trend toward a more diversified treatment of the various branches of endeavor classified as civil engineering.

The writer wishes to approach the discussion of this paper from a different direction—that of the engineer looking at geology. Without question, Mr. Huntting is representative of the progressive group of geologists that are contributing to the improvement of engineering practices in the construction field. That geology can contribute immensely to highway engineering is becoming more apparent each year. In fact, there are few, if any, who have explored the complete significance of geologic principles to these civil engineering problems.

The situation today is one in which the over-all work of the highway engineer is complicated by new and less tolerant standards. The influences and new problems should be met by recourse to available information and skills that may help to solve them. Unfortunately they are often unsolved. For example, many design engineers, field engineers, project engineers, and inspectors, who have opportunities to apply a knowledge of soils engineering, overlook opportunities to make corrections that will save time and money and prolong pavement life. Knowing the principles of soil formation, they can solve simple problems and recognize the potential problems that require the services of the soil engineer or soil scientist.

So it is with geology and the geologist; too few engineers realize the usefulness of geologic principles in everyday problems. Can they then see its application to larger problems?

As a science (and an art) geology is complex and requires a specialist's skill, but engineering is not geology and therefore does not often require the details or terms common in strictly geologic work. Having had the opportunity to examine geology in the light of engineering use in many of the states from east to west the writer has formed the personal conclusion that geology is very important in many areas and relatively unimportant in many others. Where it is important in the bedrock areas, the existing information and methods can be vastly simplified for their application to engineering work. Areas such as those covered by coastal plains sediments, great plains materials, glacial, and aeolian deposits are here considered separate from the general field of geology. Some difference in opinion may occur in the definition or scope of geology. However, the fact remains that soil forming processes in most of these areas create problems not amenable to geologic treatment. The official geologic map of the United States Geological Survey does not always concern itself with surface deposits but presents information on bedrock that is buried at depths of from 5 to 500 ft below the surface in the areas mentioned. Obviously the highway engineer is concerned with the character of the surface or near-surface materials.

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The development of a second point is somewhat contingent upon the attitude taken on the scope of geology. If the field of the geologist is all inclusive, as it may well be by liberal definition, the situation is aggravated when one insists that a competent geologist is always essential. A competent geologist is a highly trained person whose talents may well be wasted on engineering problems in many states. Substitute instead a practical knowledge of geologic processes and principles in a number of the state highway department's engineering personnel and a greater potential improvement is in view.

Specialists can advance their field at a much greater rate by sharing their knowledge, acquainting others with their work, and thereby stimulating an interest in its application. The services of trained geologists will be in greater demand when there are more engineers with an interest in geology working in the field. That potential interest can be developed or retarded by the teachers of applied engineering geology in the colleges and universities.

If the average highway engineer does not use geology it is because he has not been shown the usefulness of the subject. The subject is not often deliberately and wilfully ignored. The fault lies largely with the geologists and the teachers of geology because they are not well acquainted with the problems of engineering. From this situation three conditions have developed:

(*a*) In their natural enthusiasm, the geologists have endeavored to apply the unmodified science to engineering problems;

(*b*) The situation, so common in many specialized fields, arises in which it is felt that only a highly trained geologist can deal with these problems; and

(*c*) The application of geology to highway engineering suffers when the specific engineering problems make the geologic concept seem both vague and general by comparison.

At this point it is well to make it clear that this commentary seeks first to advocate a greater application of geologic principles to the solution of highway problems to which they are pertinent, and then to add to and modify certain phases of Mr. Huntting's paper in the light of personal experience. To that end the following comments are offered:

Problem (1) Location of Pits and Quarries.—There can be no reasonable disagreement with the premise that sands and gravels as well as rock strata are predictable on the basis of a pattern or mode of occurrence that is in part geologic. These facts are seldom if ever taught in the classroom and it is indeed rare to find them presented to engineers in technical literature. The methods are simple and their mastery well within the capabilities of a competent materials engineer. There will be instances in which the special skills of the geologist are necessary, especially in bedrock areas, but the bulk of the work will not require the specialist.

Problem (2) Suitability of Various Earth Materials for Surfacing, Concrete Construction, and Other Highway Uses.—The original presentation under this heading deals with a very significant phase of concrete aggregates that has a much more promising future than it has had a fruitful past. Unfortunately engineers have often chosen highly irrelevant test procedures as a means of testing aggregate quality for specific uses. The problems can be solved only when the origin, mineralogy, and chemistry of the aggregates are studied by competent persons. Obviously these skills can be found among the geologists, mineralogists, and chemists. It is suggested that this phase might be given additional consideration, whereas its inclusion with the less definite "earth materials * * * and other highway uses" to some extent detracts from the importance of the subject.

Problem (3) Subgrade Treatment and Classification and Problem (4) Frost-Heave Problems.—Since these are included in the writer's field of special endeavor, the manifest interest in Mr. Huntting's paper is understandable. These are problems that contain considerations other than geologic. In these two items the geologist may find himself far afield. The broad approach of the geologist has not demonstrated any appreciable suitability in dealing with problems of subgrade treatment or frost action despite the studies of crystal growth conducted by Professor Taber (15) and the resulting contributions to the knowledge of frost action. In the solution of field problems, frost action is a complex problem. It occurs as a product of the whims of ground water as well as of soil texture and is found more often in sands (shallow deposits on till), silty sands, and silts, rather than in clays as indicated.

The writer feels that ultimately a geologic method of classification suitable for engineering usage will form the basis of soil classification, but to date it has not been fully developed. When offered initially, engineers in general may express little sympathy for such a classification because of their classroom impressions of geology.

Returning to the geologist's present method of soil classification—residual, colluvial, organic, transported—it may be cited as an example of unrelated standards of classification. In terms of engineering problems this method lacks realism because:

Residual soils range in texture from the common clays (limestone, granite, basalt, etc.) through the silts, sands, and sandy clays (sandstones, schists, granites, etc.) to residual gravels.

Colluvial soils occupy relatively insignificant areas, vary so widely in texture, lack uniformity, occur on steep slopes, and are so highly susceptible to wide variations in water conditions that little of engineering significance can be said in the general terms necessary to a classification.

Organic soils, all significantly associated with a high water table, are worthy of distinction as a separate class and a further subdivision should probably differentiate between organic-sand mixtures of the glacial outwash areas and the mucks, peat, and muskeg swamps.

The term, "transported soils," like other classifications, is broad and includes all textures and all ground-water conditions. To illustrate:

An alluvial soil area, for example Mississippi River alluvium, varies in texture from medium sand in Minnesota through silts in Illinois to clays from Missouri to the Gulf. At any one point in the flood plains (south of the Wisconsin State border) one may expect sand on the natural levee and silt or plastic clay in the "slack water" areas. Since these alluvial soils vary in texture with the size of the stream and the nature of the soil in the watershed, the method of classification cannot be too general and remain cogent.

Lacustrine soils are uniform by comparison since they are generally of silt or silty-clay texture, whereas marine sediments (of the coastal plains) provide highly plastic clays as well as sands and silts. Likewise glacial soil as a classification embraces the plastic silty clays of the till plains, sands of outwash plains, and gravels of the valley plains, kames, and eskers.

The aeolian group, if subdivided into sand and loess, provides two remarkably uniform soil materials.

It has been acknowledged (see heading, "Subgrade Treatment and Classification") that "The soils falling under a given heading in this classification will not have all their physical properties alike * * *" and that, "Nothing can replace the engineers' physical tests * * *." Physical tests show similarity among the vast areas of lacustrine silt in the glacial lakes Agassiz, North Dakota and Minnesota, Maumee, northern Ohio, Lahonton, Utah, and Bonneville, Washington; in the loess in Mississippi, Tennessee, Illinois, Minnesota, Iowa, Indiana, Kansas, Missouri, Nebraska, Idaho, Washington, and North China—to cite a few; and in the alluvium in Ohio, Indiana, Illinois, and Iowa; and some residual soils derived from fine-grained sandstone. Obviously the engineering problems in these different areas are not all similar, so physical tests alone are often misleading.

This is more a criticism of engineers relying on test data than of the geologic approach, and the geologist will more readily understand it. The engineer relying on test data alone often finds himself standing on one leg (technically and figuratively), and his designs are continually off-balance because other conditions upon which good performance is contingent do not appear in design formulas.

If the geologist's approach is too broad and the contemporary engineer's too narrow, the solution may then be found in a blending that can be obtained by a better mutual understanding. Add geologic principles to highway (soils and materials) engineering. Combine test data and the soil position or land form. Consider the natural position, the soil structure, the profile horizons, the drainage, and the climate. The solution lies largely in the hands of the geologist-engineer-professor—not the geologist, not the engineer, not the professor.

Problem (5) Prediction of Character of Material To Be Excavated.—This is an important phase of highway location and planning. Many engineers familiar with a few fundamentals of rock weathering and geology react unfavorably to the implication that a competent geologist is always required to perform these services.

In the "Prevention and Correction of Landslides," Problem (6), the geologist can render his most valuable service in many afflicted areas. In rapid succession all can approve the listing of his value in tunneling operations, Problem (9); evaluation of mineral lands, Problem (11); and as an expert witness in related court actions, Problem (12). However, the cause suffers when the geologist presumes to render comparable service for highway engineers except in special cases in judging bridge foundations, Problem (7); predicting channel changes, Problem (8); and underground-water supplies, Problem (10). In each of these it is felt that the descriptions offered do not substantiate the positive character of the statement of the problem.

R. Woodward Moore,[15] Esq.—The twelve cases discussed in this paper constitute important problems continually confronting the highway engineer wherever road construction and road maintenance are undertaken. The writer was particularly interested in the references to geophysical methods of test applied to their solution. The possible value of such exploration methods to the geologist and engineer alike should be emphasized.

For some years the Division of Physical Research and Tests of the Public Roads Administration has been engaged in research on the use of geophysical methods of test in highway engineering. Conclusions resulting from these studies and the work of several of the state highway departments, as well as of other agencies in the United States and elsewhere, indicate that the methods of test mentioned by the author—the earth-resistivity and the refraction seismic tests—may have considerable economic value in the solution of many of the problems listed by Mr. Huntting. Specifically, Problem (1) and Problems (5) to (10), inclusive, appear to be susceptible of solution or partial solution by geophysical tests. The following paragraphs are submitted in amplification of the information concerning geophysical tests as presented by the author in the belief that such methods of subsurface exploration may prove to be increasingly useful to the geologist in further demonstrating the usefulness of geology in highway engineering.

Problem (*1*).—The location and delineation of deposits of road-surfacing materials by earth-resistivity tests have been described by various investigators during the ten years since 1934 (50)(51)(52). In a letter to the writer, Henry C. Porter, M. Am. Soc. C. E., reported rather favorably on such tests by the State Highway Department of Texas. Investigations conducted by the Public Roads Administration have further established the usefulness of this test in such work.

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Problem (*5*).—Both the refraction seismic test and the earth-resistivity test have been found to be applicable to the classification of excavation materials (53). The resistivity test has been applied rather successfully in the study of highway grading projects by the Missouri State Highway Department, the test data obtained being of considerable aid in predicting the character of the materials to be encountered (54).

Problem (*6*).—Although little work has been done, as yet, in determining the usefulness of geophysical tests in connection with landslide areas, it would seem highly probable that such tests could be of appreciable value in disclosing potential slide areas. The refraction seismic test has been used in preliminary investigations to determine the presence of impervious shales beneath talus slopes which constitute potential sliding surfaces upon which the overlying porous material may move (55). The earth-resistivity test should be of practical value in locating impervious strata of shale or clay beneath less consolidated and more porous soils. Where such conditions exist, road structures may be benched into the impervious layer instead of being built entirely upon the porous materials subject to slipping along the lubricated surface of the underlying impervious stratum. In the case of large fill structures in highway construction routine resistivity tests made over the fill immediately after construction and at intervals thereafter may prove of value in disclosing variations in moisture content. In this way abnormal increases in moisture may be detected within the fill where increasing hydraulic pressures may cause a slip-out requiring costly reconstruction work and causing considerable inconvenience to an established flow of traffic. No data are at hand to substantiate such a conjecture, however.

Problem (*7*).—Both methods of test may be used in the study of bridge foundation materials, the seismic test being more positive in its identification of solid rock formations in areas where either solid rock or gravel formations may be expected. The low cost involved and the rapidity with which these tests can be executed make them of particular value in preliminary surveys, often eliminating the need for the time-consuming and costly methods ordinarily used in direct explorations of the substrata. Where conditions warrant, the direct methods of exploration may be used on the final location selected.

Problem (*8*).—The earth-resistivity traverse or constant-depth test should be of aid to the geologist in locating buried river channels throughout a given area, particularly where sand and gravel are concentrated within such channels and where rapid erosion might conceivably occur during flood periods. The determination of the presence, depth, and lateral extent of such deposits might well influence decisions as to the desirability of a particular bridge or road location.

Problem (*9*).—Earth-resistivity tests have been used successfully in connection with explorations involving a tunnel site in a hydroelectric development in Canada (56)(57). The use of this test in explorations on highway tunnel sites is likely to be largely influenced by the depth and character of the overburden. However, the presence of faulting and zones of shattered and highly altered or weathered rock where water may prove troublesome in tunnel construction should be rather easily determined by the resistivity test. The refraction seismic test has been used in several instances by the Public Roads Administration in surveys made in connection with the location of proposed highway tunnels. The data obtained from these surveys have not been published but are contained in the files. In such studies the broken and weathered rocks have seismic wave velocities considerably different from the characteristic velocity in the solid rock sought for the portal construction. Predictions as to the need for a lining for a particular tunnel may be made, based upon intermediate seismic wave velocities indicating a fractured, jointed rock mass through which the tunnel is to pass.

Problem (*10*).—There have been numerous instances in which the resistivity test has been used successfully in the search for underground water supplies. References (58)(59)(60) are examples of such studies. At times the water may be located more or less directly by reason of low resistivity resulting from various salts carried in solution, but more often it is located through its association with some stratigraphic or structural formation such as clean sand and gravel or a rock formation conducive to the entrapment and storage of water.

George E. Ekblaw,[16] Esq.—The purpose of this paper (see "Synopsis") is "to help both engineers and geologists to recognize more frequently those situations in which geology does enter into highway engineering." How extended or complete the discussion should be is an open question—it could range from the extreme minimum of a single statement of obvious fact to a maximum of volumes in which all known or possible examples were cited. Whether or not the length and style of this paper is sufficient for the purpose, therefore, is largely a matter of personal opinion and hardly subject to technical discussion.

The author has chosen to subdivide the topic into twelve "problems" ("classes of problems" might be a more appropriate term). By inference these comprise the entire field of cooperative endeavor. Actually there are numerous other classes of problems which are equally or more important—such as (*a*) selecting the most desirable of two or more alternative routes for a highway, either locally or regionally, from the viewpoint of geologic conditions, (*b*) locating or alining a highway to avoid undesirable local conditions, (*c*) peat-bog problems, and (*d*) grade-fill problems—to cite from experience in Illinois alone.

Mr. Huntting has cited incidents to exemplify these "problems." Naturally these examples are those with which he is personally familiar or which have impressed him. Any one working in the dual field of geology and highway engineering could supplement these by other incidents, perhaps even more striking or typical. The number and variety of such incidents would be determined by geographic and temporal range and variety of experience, but to a greater or lesser degree Mr. Huntting's examples could be duplicated anywhere, so they may be considered sufficient to illustrate the classes of engineering problems discussed.

Within the limits of the paper, Mr. Huntting has done probably as well as any one could. Because of the brevity of the paper, many generalized statements have been made which, like all generalities, may not be universally true and to which exceptions of limited application could be made. Excluding such exceptions, experience in Illinois would corroborate all the statements applicable within the state, in a general way, except one: In Illinois it has been observed that silt, not clay, is the major constituent of subgrade material most affected by frost heave, and the writer does not subscribe to the opinion (see heading, "Frost-Heave Problems") that "The clay acts as the medium through which water rises from the water table by capillarity until it reaches the frost line, where it freezes and causes an irregular upheaval of the road surface." The presence of some clay in the silt may add to the effective capillarity and to the ice-layer expansion which is the principal cause of frost heave, but material that has a large percentage of clay seems to favor neither capillarity nor ice-layer expansion. Theoretically, because of the smaller grain size of clay, capillary water should rise higher in clay than in silt, but other factors apparently operate to offset the grain size.

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Allen S. Cary,[17] Assoc. M. Am. Soc. C. E., and Ade E. Jaskar,[18] Jun. Am. Soc. C. E.—Geology contributes to the solution of airport engineering problems as much as to those of highway engineering. The author has presented twelve possible problems in which a geologist's services might be profitable in highway design. Most of these have been of major importance in airport design for several years. The urgency of the military airport program, the heavy wheel loadings, and the tremendous amounts of construction materials involved have forced the designing engineers to obtain a large quantity of accurate data in a minimum length of time. In many, but not enough cases, the services of full-time geologists have been requisitioned.

In the preparation of soil profiles, in procurement of truly representative soil samples, in exploration for fill materials and aggregates, and in providing drainage data, the geologist has been like the man who charged one dollar for fixing the typewriter in a few moments. Objection elicited the statement that he charged only one cent for the fixing, but ninety-nine cents for knowing how. The geologist has been able to minimize extensive "pattern" explorations involving test pits, auger borings, etc., by his professional knowledge of probable extents of uniformity in sediments (both horizontally and vertically) and his understanding of ground-water behavior.

In addition to advising on the design of the runways, the geologist has been of inestimable aid in water supply and building foundation problems.

The author has so ably discussed the various applications of geology to highway engineering that it is not necessary to enumerate further examples. However, the relation between the engineer and the geologist does deserve elaboration. By far the best single publication available on this subject was written by R. F. Legget in 1939 (12).

It is satisfying to note that the constructive service of the geologist is more appreciated now than at any time in the past and bids for further recognition in the future. A review of civil engineering in the past, and its status in the present, demonstrates that the future trend is inevitable. In the earliest years of modern civil engineering, when most projects were of small or moderate scope, the engineer was able to cope with his problems by being his own geologist or by considering a large factor of safety (more correctly, factor of ignorance) in his work. When modern society began to demand larger and more elaborate structures, it became necessary for the civil engineer himself to specialize. Today a civil engineering graduate may have such a subtitle as structural, sanitary, railroad, highway, irrigation, commercial, municipal, hydraulic, or materials engineer. These special fields may even be subdivided further. Many years of general experience are required before a man can qualify for the title of civil engineer.

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Upon resuming responsibility for the design of major projects, the civil engineer is so busy with general aspects that he has neither the time, nor, possibly, the training in details, necessary to obtain the geological data for design. He must either delegate this responsibility to specialists or gamble. The latter course has been taken too frequently as is demonstrated by failures in the past (12). Too often the principal use of consulting geologists is in determining the causes of failure of important engineering structures.

As stated previously, the constructive service of the geologist is becoming more appreciated, and bids for higher recognition in the future. This prophecy is supported not only by the results of mistakes that have been made in the past, but also by the fact that the most favorable sites for engineering projects are gradually being "used up." The ingenuity of the civil engineer is being taxed to the utmost by the design of large concrete and earth-fill dams, long-span bridges, large airports (often built on the poorest foundations), intricate highway systems, elaborate water-supply systems, and many other projects. Thus, the geologist is needed more than ever.

Where to find the experienced engineering geologist, who can fulfil such responsibilities, and devote full-time service to the engineer, is an important consideration, but one beyond the scope of this discussion.

KARL TERZAGHI,[19] M. AM. SOC. C. E.—This review of the relation between geology and highway engineering deserves the attention not only of highway engineers, but of every engineer who deals with rock and soils. The influence of geologic factors on the success or failure of engineering operations is so manifold and decisive that it cannot be overemphasized. Hence, the writer agrees with Mr. Huntting on this subject. However, the manner in which soil mechanics has been treated, or rather ignored, calls for some comments regarding the competency of the geologist to express an opinion on the design of structures and regarding the circumstances which brought soil mechanics into existence.

In his college days the writer enjoyed the benefits of a specialized training in geology and petrology under the guidance of prominent experts in these fields. His first professional papers were devoted to purely geological subjects and throughout his subsequent professional career as a civil engineer he never ceased to draw upon whatever geology and experienced geologists had to contribute toward the solution of his problems which, in his early days, were primarily connected with hydroelectric power developments. However, as his engineering experience increased, the writer became painfully aware of the existence of a large group of engineering problems in the region between the domains of geology and of applied mechanics. These were the problems involving the mechanical effects of loads and seepage pressures on soils. Geology furnished valuable information regarding the origin, sequence, and shape of the individual soil strata, and applied mechanics made it possible to compute the stresses in the soil produced by given systems of forces. However, there was no method of evaluating the corresponding deformations and of defining the stress conditions for failure. The empirical rules used for bridging the gap were sadly inadequate. Since this fact was continuously impressed on the writer by observation, he became more and more obsessed by the desire to remedy the intolerable situation.

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His first impression was that the no man's land between geology and engineering could most successfully be invaded from the geological viewpoint because he thought that definite relationships might exist between the geological character and the petrographic character of the soil strata in the field and their reaction on the forces which act on them during and after construction. To discover the existence of such relations, it is necessary to proceed in a purely empirical manner, by correlating construction experience with whatever can be ascertained by methods of field geology at the sites of construction. After the writer had spent several years collecting data of this type at random, he found the process too slow, and he decided to take advantage of what at that time was the boldest and most promising experiment in engineering geology ever made—namely, the construction of dams, open cuts, and tunnels, under extremely different geological conditions, by the U. S. Reclamation Service. During 1912 he visited practically every one of the larger construction camps of the Reclamation Service. With the active cooperation of the late F. H. Newell, M. Am. Soc. C. E., Director of the Reclamation Service, who was keenly interested in the writer's quest, geological data pertaining to the observed sites were secured. After the voluminous material was assembled, the writer tried to extract from it whatever definite relations may exist between the geological character and strength and bearing capacity of the different strata. The results of his labors were so utterly disappointing that he did not even consider it worth while to publish them. They can be condensed into a single sentence. There are no definite relations, because, for a given geologic origin and petrographic character of a stratum, the strength of the material, rock, or soil depends on conditions that are not, and cannot be, recognized by a purely geological investigation. For almost every geologically well-defined stratum the possible scattering of its physical properties from the statistical average is so important that one cannot even make an acceptable guess based on precedent.

At that stage of his investigation—a stage of bitter disappointment and disillusionment—the writer became interested in what is now known as soil mechanics. During the following 25 years he spent most of his time on observation and experimentation in this particular field. He aimed to develop suitable experimental methods for investigating the physical properties of soils and to explain the behavior of soils under the influence of load and seepage pressure on the basis of the laws of mechanics of solids and fluids. Between

1926 and 1929, the writer spent much time and labor on advocating the purely empirical procedure of correlating soil constants and soil profiles with the observed performance of the road surface in the field. Since the relative merits of the different methods of soil mechanics are already evident by their success or failure in connection with design and construction, their legitimate place among the tools of the practising engineer is no longer a matter of personal opinion.

Soil mechanics, without the assistance of geology and general statistics, can be used only in those rare instances in which a structure must be built in, or above, fairly uniform and homogeneous soil strata. In every other instance reasonably accurate values for the soil constants and information on the vital details of the structure of the subsoil can be obtained only by combining the results of soil tests with the results of investigations on the geological origin of the subsoil. However, the results of these investigations merely represent the prerequisite for solving the engineering problem. The solution itself requires an evaluation of the mechanical effect of the forces acting on the soil during and after construction. This major item in the solution is far beyond the domain of the geologist. As a matter of fact, many geologists know so little about the mental equipment necessary for this decisive part of the design that they consider themselves competent to make positive recommendations. Since this fact is often ignored even by engineers, its practical implications deserve serious consideration.

Although a competent geologist knows more about rocks than any engineer, no engineer would expect him to prepare an adequate design for a stone arch bridge, because the design of such a bridge requires a thorough training in mathematics and mechanics. Nevertheless, it is not uncommon for engineers to consider seriously the opinion of a geologist regarding the safety of a dam foundation, and this is a serious mistake. Few dam foundations can be designed on a purely mathematical basis, because generally the foundation conditions are too complex to permit an accurate forecast of the mechanical effects of load and percolating water. However, precisely because of this complexity, the design of such a foundation requires a far more thorough training in the mechanics of solids and fluids than does the design of a stone arch bridge, because the capacity for evaluating correctly the mechanical implications of various geological details without any computation does not develop in the human mind until a stock of subconscious knowledge has been accumulated as a result of the habitual application of the laws of mechanics to the solution of a great variety of problems. Since the geologist has neither the training nor the opportunity to acquire such a background, his forecast of a mechanical effect can be very misleading. The following example should serve to illustrate this fact.

In 1915, the National Academy of Sciences in Washington, D. C., appointed a committee of distinguished scientists to report on the causes of slides on the Panama Canal. Since one of the members (a prominent geologist) had previously acquired some knowledge of mechanics, he presented his findings in mathematical terms, involving an elaborate theory of the relation between the compressive strength of the rocks and the stability of the slopes. The basis of his theory was "that the tension T, acting along an arc δs of the surface of sliding is equivalent to a normal pressure $T\ \delta\psi$." To justify his statement the geologist referred to a classical textbook on mechanics. Since the writer was not aware of the propriety of applying this principle to the mechanics of solids, he consulted the reference and found that the statement refers not to solids, but to the relation between the surface tension and the normal pressure at the boundary between gases and liquids.

This incident demonstrates that an acquaintance with the literature of mathematics and mechanics, a necessary tool to the engineer, is not sufficient to establish competency as an engineer. Professional training has recognized the need and wisdom of following the acquisition of scientific knowledge by an apprenticeship for the purpose of developing the art of the profession. Competency for the practice of a profession is measured by both education and experience. Neither one alone is sufficient. The experiences necessary to become a competent geologist are not likely to be the same as those necessary to become an engineer. Hence, it would be dangerous for the engineer to accept an opinion of a geologist on a question of engineering. In science, a serious error in computation or judgment merely constitutes a blunder, but in civil engineering it may lead to a catastrophe.

Several geologists have published instructive descriptions of landslide preventive measures. However, if the reader of these discussions is familiar with the subject, he will notice that almost all these measures were developed, practised, and described by civil engineers prior to 1880. The geologists merely discovered that successful preventive means had already been used by civil engineers at a time when the landslides had not yet aroused the interest of the members of their own profession. As a matter of fact, there are exceptions to this rule. About 1935 a German engineering geologist proposed preventing slides on the slopes of a deep cut in stiff, fissured clay in northern Germany by grouting the fissures in the clay with portland cement. This procedure was entirely new. The writer saw the cut after the grouting was done, the cut was made, and the slopes had failed by sliding.

Under the heading, "Investigation of Proposed Tunnel Sites," Mr. Huntting discusses the influence of geological factors on tunneling. Essentially the same information was published in 1874 by F. Ržiha (61) and since that time no new material has been added.

Mr. Huntting implies (under the heading, "Frost-Heave Problems") that the danger of frost heave increases with increasing clay content of the soil. This and several other statements indicate that he does not pay as much attention to the laws of physics as these laws deserve. However, it would be preposterous to blame him, because he is a geologist and not an engineer. His deficiencies in physics are more than compensated by his broad vision in the field of geology, and merely indicate that the field of competency of every professional man, the geologist included, has certain limitations. If the geologist is careful enough not to trespass, his potential usefulness in connection with civil engineering cannot be overestimated.

Because of the obvious value of reliable geological information, large construction organizations, such as the Tennessee Valley Authority or state highway departments, are inconceivable without a staff of competent geologists. However, a vast amount of construction work is still being handled by very small staffs or by individual engineers. The success or failure of some of these jobs may depend to a large extent on a correct interpretation of the geological conditions. Thus, one may ask whether or not the investigation of the geology of a given site requires the services of a professional geologist. The answer to this question depends quite obviously on the nature of the job and a reliable answer can be expected only from an engineer who is thoroughly familiar with the elements of physical geology and with the influence of geological factors on the mechanical effects of loads and seepage pressures. Any other engineer might ignore the existence of geological complications on the job or overestimate the soundness of his own judgment in connection with problems in the domain of the professional geologist. Therefore, an elementary training in geology should be considered an essential prerequisite to the practice of highway, tunnel, or foundation engineering. In recent years most of the universities in the United States and abroad established courses in soil mechanics. This fact alone makes it necessary to teach engineering geology. The reason is obvious. Soil mechanics, as taught and practised at present, is likely to create the dangerous illusion that the processes in natural soil strata are always, or at least usually, as clear and simple as those in the cylindrical specimens subject to investigation in the laboratory. This idea, however, is due not to an inherent defect of soil mechanics, but to the incurable tendency of the human mind toward unwarranted generalization. Soil mechanics is not a substitute for engineering geology but a supplement to it. Sometime in the future when its novelty has worn off and its extravagances have been forgotten, soil mechanics may even turn into a useful tool in connection with purely geological research. Pending this future development, a course in engineering geology is needed as an antidote, and Mr. Huntting deserves the gratitude of his readers for his apt presentation of the manifold practical values of purely geologic information.

D. P. Krynine,[20] M. Am. Soc. C. E.—The predominant industry in the United States is agriculture, the construction business ranking second. Of the two billion dollars spent on public construction in 1938, about 45% was invested in roads and highways. The nation makes a considerable effort, financially, to keep its highways in tune with requirements of its life. The user wants a highway located where he needs it; he wants good riding conditions; and he wants low taxes. Hence, the major problem of the highway engineer is by no means a technical one. First, the highway engineer thinks of the purely human part of his problem, including human relations and economics, and for this purpose good planning is necessary. Again, this planning is not primarily of a technical nature. When finally the time comes for technical planning, the problem is again broken into two parts—what to build and on what to build. The former part of the question, of course, is more important than the latter. When a proper design of the superstructure and a proper choice of pavements (including materials) has been made, according to the needs of traffic, the question of the subgrade and of the foundation arises, and here the advice of the soil mechanician and of the geologist is necessary. Essentially, the highway engineer asks the soil mechanician and the geologist to advise him whether the soil, near the surface or remote from it, will support a given structure safely.

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Soil Mechanics, Soil Science, and Geology.—The engineering study of the properties of earth masses began with the early construction of fortifications in France, perhaps in the sixteenth century. The term "geology" in its present sense was probably first used sometime in the seventeenth century. Thus, the engineering study of earths (later termed "soil mechanics") and geology are approximately of the same age—about three hundred years old. Only recently did these fields begin to approach each other, in scope and content, to the mutual benefit of each. In this connection the purpose of this paper as defined in the "Synopsis" is timely and appropriate.

Near the beginning of the twentieth century, a group of geologists and agriculturalists started a new branch of knowledge which they named "Soil Science," and this particular branch of human knowledge, concerning the agricultural study of topsoils and their properties, has become an independent one. Highway engineers and soil mechanicians can find many interesting ideas in the field of agricultural soil science, such as the capillary moisture in soils, which is exceedingly important to those interested in the structural properties of soils (62). Geologists, particularly hydrogeologists, are more interested in free or gravitational water.

Thus, whereas soil scientists study topsoils mostly for agricultural purposes, geologists are interested in the earth as a whole, including the mantle covering the solid rock—with the exception, perhaps, of topsoils. Highway engineers and soil mechanicians are interested in the upper part of the mantle (about 300 ft thick), including any rock within that range of depth. At greater depths, stresses caused by the heaviest engineering structures are insignificant, and, in a practical sense, there may be no measurable strain from additional loads. Topsoils must be considered when simple earth roads are being built; but neither geologists nor soil scientists can advise the highway engineer effectively about topsoils.

Engineering and Geological Points of View.—There is considerable difference between the engineering and geological approach to the study of that part of the earth mantle which is important to engineers. Three differences are basic: (*a*) Relative scale of outlook, (*b*) relative appreciation of the strength of a material, and (*c*) relative concept of the time element.

The geologist tends to characterize a locality in broad terms whereas the engineer needs data concerning a particular point. In other words, the geologist thinks on a wider scale than does the engineer. Even excellent geological maps cannot be used for engineering purposes directly (except for preliminary studies) without supplementary borings and other soil investigations. An experienced boring contractor or foreman should make the borings rather than an average geologist or an average soil mechanician. Of the latter two, the person best qualified to do so must supervise the borings, interpret the results, and conduct the laboratory and field tests. If there is a geologist who is familiar with the test technique and who in addition understands the function of engineering structures, he may well be entrusted with the tests. In a general case, however, a well-trained soil mechanician is preferable because of his specialized and intimate acquaintance with the strength of soils.

The third difference between a geologist and an engineer is in the relative concepts of time. The geologist's unit for measuring time is large—the geological "age." Ancient and medieval engineers also tried sometimes to build for eternity. The modern engineer, however, has a feeling that in a relatively much shorter time every structure built by man must outlive its usefulness and be replaced. Hence, as a rule he builds for a limited time only, generally for the period of amortization (dams with silting reservoirs, apartment houses, etc.). Especially is this true of highway pavements, for which there is a certain maximum service life (63).

Right Type of Geologist.—The impression gained by a reader of this paper is that a geologist can perform a great number of useful tasks in highway engineering. Granted that this may be so, the question still remains as to the choice of a proper geologist. An experienced Swiss engineering geologist, M. Lugeon (64), has stated (writer's translation):

> "A geologist who works with the engineers must remain a naturalist and must use the methods of natural science only. This is not only to avoid assuming the rôle of an engineer with subsequent conflicts, but mostly because geologists have been trained in schools of naturalists * * *. A geologist who has only studied the history of the earth and of those who have lived on it is of no use to the engineer * * *; but, if this man was also preoccupied in studying the mechanics of the formation of earths and seas, if he tried to know the creative and destructive agencies of the mantle, then practical men will take him from the ranks and use him."

The chief engineer of a highway project is usually a man with wide general interests and technical experience. He will accept no statements which in his opinion are too abstract and cannot be used immediately in practice. Thus, the geologists must know something about the behavior of structures and pavements to speak the highway engineer's language. In addition, considerable previous practice and a certain degree of reorientation will be needed.

Through intimate contact, the writer has come to look upon geologists as exceedingly fine scholars and useful people. In all frankness, however, it should be admitted that in everyday highway practice the services of a geologist, as such, are not essential. Excellent results are attained by engineers with some construction practice and school training in soil mechanics and elements of geology. However, in exceptional cases (for instance, if there are important structures such as tunnels or large dams or if a considerable number of landslides occur), it would be a serious mistake not to seek the advice of an able and competent geologist.

True Value of Soil Investigations.—As a matter of course, whoever is in charge of the preliminary soil investigations must furnish correct results and must report a true picture of both surface and subsurface soil conditions. Nevertheless, the accuracy of soil investigations is not a guaranty of the safety of a given structure. An excellent proof of this statement is furnished by Mr. Huntting himself (see heading, "Competency of Materials for Bridge Foundations") when he mentions an $80,000 bridge that failed due to poor design although presumably soil conditions were fairly well known. Structures stand successfully either because they were adequately designed and built by skilful engineers or purely because they have been lucky enough to escape critical conditions. The writer disagrees with the statement that the San Francisco-Oakland Bridge "stands as a monument to the worth of careful preliminary foundation investigations." If that bridge is to be considered as a monument, it is a monument to the ability of its designers and builders. During the period of most active railroad construction, many structures were built practically without soil investigations. If many of them still stand, can they be considered as monuments to the absence of soil investigations?

Uncertainties.—Superficially, one would conclude that in any investigation of the physical properties of a pavement subgrade an extremely meticulous study of all hard and soft spots should be made. On the contrary, Francis M. Baron, Assoc. M. Am. Soc. C. E., has shown (65) that a hard spot corresponding to a 25% reduction in computed deflections results in the rather small reduction of about 18% in the computed critical stresses.

In some cases, therefore, the study of the pavement subgrade may be simplified and standardized considerably if in a given locality there are only a few types of soils to deal with in the subgrade. In such cases simple auger borings, to a depth of, say, 4 ft, are made, and the thickness of the pavement is determined from practical observations of the behavior of existing pavement slabs on similar soils.

Frost-Heave Problems.—Under the heading, "Frost-Heave Problems," Mr. Huntting states that "The clay acts as the medium through which water rises from the water table by capillarity until it reaches the frost line where it freezes * * *." As a matter of fact, from the point of view of frost heaves, clays are less dangerous than silts since the latter possess both sufficiently great height of capillary rise and fair permeability. Clays, as a rule, lack permeability (66).

Balancing Cuts and Fills.—A field in which geologists could learn something from the engineering profession is in the approach to determining the degree of compaction of the earth mantle. In glacial zones, such as the Southern New England Zone, excavated earth (not rock) when placed in a fill shrinks —perhaps an average of 15%. This means that, in nature, glacial soils are not compacted enough although they have been compressed by huge masses of ice. Furthermore, the degree of compactness is not uniform throughout a given region.

The designing engineer must know how many cubic yards of fill can be made from 100 cu yd of excavation at a given place. Although some information on this subject can be obtained by conducting small excavating and filling experiments, a better approach is needed. In this connection, it is regrettable that (see heading, "Prediction of Character of Material to Be Excavated") Mr. Huntting did not mention anything about the influence on the excavated material, especially on its volume changes, of the following two factors: (*a*) Depth of the water table at the borrow pit; and (*b*) season of excavation. The writer recalls a case in which 100 cu yd of a material containing a certain percentage of fine particles, excavated in the neighborhood of a lake, pond, or river, furnished less than 100 cu yd of fill in summer and more than 100 cu yd of fill in early spring, due, of course, to additional water pumped from the water table during the frosts.

Conclusion.—Various specialists play a useful rôle in highway work—surveyors, designers, construction and maintenance engineers, mechanics, contractors of all kinds, chemical engineers and asphalt technologists, mathematicians and stress analysts, economists, statisticians and accountants, weather men, soil mechanicians, and geologists—and each of them has his place in the entire picture.

ROBERT F. LEGGET,[21] M. AM. SOC. C. E.—There will be little quarrel with most of the general statements formulated by Mr. Huntting. Many of them present, with renewed emphasis, well-known facts which may often be overlooked by engineers in practice. The examples presented in the paper are all interesting and useful to supplement the general propositions.

In this paper Mr. Huntting has covered an extremely wide field. It must be admitted that highway engineering itself covers many phases of engineering activity. At the same time, it must also be observed that the term "highway engineering" is used generally to denote engineering work specifically related to the construction of highways. Mr. Huntting goes far beyond this special field and really covers much of the general practice of civil engineering. In view of this, his statement (see heading, "Introduction") that "* * * no treatment of the subject as a whole is available" might have been qualified in some way. The writer may perhaps be forgiven for mentioning a book, "Geology and Engineering (12)," in which six separate chapters are devoted to six of the main topics discussed by Mr. Huntting, most of the remaining topics being dealt with in other parts of the book, although not to the extent of entire chapters.

Again, Mr. Huntting appears to suggest that the geologist, by himself, can undertake many of the tasks discussed in this paper. Therefore, the following countersuggestions (see heading, "Investigation and Prediction of Subsurface Water Conditions and Location of Underground Water Supplies") are satisfactory: "The geologist's prediction is not infallible, * * *" and (see heading, "Competency of Materials for Bridge Foundations") "A purely geological exploration of a proposed bridge site is seldom productive of enough detailed and precise data to be sufficient in itself, * * *." It is hoped, therefore, that Mr. Huntting will be willing to apply such qualifications to most if not all the problems discussed in his paper. In such work, the cooperation of geologist and engineer is most desirable. The problems, however, are essentially engineering problems in the solution of which the engineer calls to his aid the information which he can obtain through the detailed scientific study of the geologist. Doubtless this is what Mr. Huntting intended to suggest in his paper but certain parts may cause different impressions in the minds of readers not privileged to observe such cooperation as he suggests.

A particular instance of possible overemphasis on geology is the "Bibliography" of the paper which lists a number of publications quite definitely in the field of soil mechanics (essentially an engineering study), for which geology provides only a most necessary background. It is of interest perhaps to recall what Karl Terzaghi, M. Am. Soc. C. E., called "* * * the striking contrast between what we expected when digging into the earth or loading it, and what really happened * * *." Professor Terzaghi then stated (67):

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> "I * * * hoped to discover the philosopher's stone by accumulating and co-ordinating geological information * * *. It took me two years of strenuous work to discover that geological information must be supplemented by numerical data which can only be obtained by physical tests carried out in a laboratory."

These comments are intended only to emphasize Mr. Huntting's arguments as noted, to the effect that geology is but one means of assisting engineering work.

The date 1888 (see heading, "Introduction") presumably refers to the publication of a paper in the *Proceedings* of the Geologists' Association (Great Britain) (68). Quite a number of publications before this date are available which include references to geology and highway engineering, most notably the writings of John Loudon McAdam; but, despite this long history, it is indeed significant that so little has been published with regard to this application of geology.

The author makes a reference (see heading, "Location of Suitable Road-Surfacing Material Pits and Quarries") to test pits, the use of which is described as haphazard. It is suggested that a very important function of geology in relation to highway engineering is guidance in the proper selection of test pits as a more useful means of exploring subsurface conditions. Test pits are sometimes placed in a haphazard manner, but this does not seem to justify the rather critical comment on them in the paper.

With reference to the surveys of road materials by the state highway departments, the Geological Survey of Canada has made investigations of this type for a considerable period, some of these publications dating back to 1916 (69).

The writer would appreciate Mr. Huntting's explanation of the term "traprock" (see heading, "Suitability of Various Earth Materials for Surfacing, Concrete Construction, and Other Highway Uses"), since its use does not appear to be generally accepted in geological circles as a specific name. It is frequently used in engineering circles but seems to be one of those very general terms, the use of which is unfortunate and, therefore, to be avoided in all carefully prepared publications. It is interesting to note (see heading, "Subgrade Treatment and Classification") the suggestion that the fundamental geological classification of soils (with reference to their origin) should be a "supplementary" classification. Surely this classification is of prime importance and should be utilized before other classifications since so many subsidiary features depend on it.

Finally, and with respect, the writer would ask what is meant by the term "within limits" in the suggestion (see heading, "Prediction of Character of Material to be Excavated") that "Within limits, the depth and shape of such deposits are reliably predictable by competent geologists." Determination of the extent of deposits of soil is always a most difficult matter. Here indeed geology can be of assistance in guiding the necessary subsurface exploration work.

These critical comments all refer to matters of detail only. They do not question Mr. Huntting's main thesis with which the writer is in entire agreement. It is indeed encouraging to find such a paper as this in the *Proceedings* of the Society, its publication being in itself evidence of the growing appreciation of geology by the engineering profession. The paper should do much to develop this appreciation.

Irving B. Crosby,[22] Affiliate Am. Soc. C. E.—Geology in highway engineering is one phase of engineering geology, that is, the application of geology to the problems of civil engineering. It appears to have become fashionable to consider engineering geology as something new, but for more than half a century geology has been applied increasingly to a great variety of civil engineering problems on many types of projects. The use of geology in highway engineering was among the first of these applications. As early as 1889 an article was written in the United States about geology and highways (70). By 1918 considerable literature was available on the geology of road materials.

Although the geology of highway engineering had an early start more attention was given to other phases of engineering geology, particularly to the geology of dams. This was probably due to the fact that these problems are generally more critical and the need for geologic investigation more pronounced. Dam projects, however, involved many cases of the application of geology to road construction both in the relocation of highways about reservoirs and in the building of access roads.

Engineering geology has developed into a separate profession which requires competence in many of the specialized fields of geology and also experience in allied sciences and an understanding of engineering. The breadth of the field makes cooperation with other sciences and techniques essential to the successful practise of engineering geology. To meet the many demands of this profession successfully, the practitioner must give his full time and energy to its study. It cannot be practiced successfully by one whose primary interest is in some other field of geology or engineering.

The twelve problems listed by the author do not cover all the possible applications of geology in highway engineering, but they do include a wide range of problems in which geology is useful. Some of them involve other sciences or techniques, as is frequently the case in engineering geology, but all of them involve geology and experienced engineering geologists can contribute to their solution. The pertinency of geological study of frost-heave problems (Problem 5) may be challenged by some, but the basic investigations which gave the first true understanding of the cause of this phenomenon were made by a geologist (15) (38). An engineering geologist can locate areas where frost heaving is likely to occur, and if these areas cannot be avoided in laying out a road provisions can be taken by the engineers to minimize frost heaving.

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The foundations of bridge piers (Problem 7) present problems which may be simple and easily handled by an engineer or which may be complex and may require geologic investigation. An experienced engineer should recognize the cases where geologic cooperation is needed. Buried, former river channels, which were missed by drilling, have been found by geologic study, and the detection of buried channels may be an important problem in connection with the location of bridge piers. The physiographic interpretation of aerial photographs is often helpful in this connection.

Problem 8, the location of bridges and highways with respect to possible changes of stream channels, is most important with aggrading streams. These changes may occur, due to local conditions, in areas which are in the youthful stage of erosion and where the general tendency is downward erosion. Therefore, recognition of the true regimen of the particular part of the stream in question is important. The behavior of meandering streams is not yet thoroughly understood but progress recently has been made on this problem by the Sub-Committee on Landforms of the Committee on the Dynamics of Streams of the American Geophysical Union. This committee, composed of engineers and geologists, has published several reports (71) and one of its members has published the results of field and laboratory investigations on meanders (72). Application, by an engineering geologist trained in physiography, of the known principles of stream meandering to this problem can make possible a selection of locations which will obviate or minimize future trouble from shifting channels.

Ground-water problems are handled by both professions, but the occurrence and movement of ground water depend upon the character and structure of geologic formations, and geologic investigation is desirable except in the simpler problems. A special problem of relatively rare occurrence is to detect places where highway excavation and drainage lowers the ground-water level and injures ground-water supplies.

Engineering geology is a valuable tool which, when used by an experienced, competent man, can aid in the solution of these and many other problems of engineering. It is no panacea, however, and it has its limitations. Some problems can be solved by surface, geological observation alone, many others require subsurface exploration by drilling, test pitting, tunneling, or geophysics, and for some difficult problems it is economically not feasible to obtain an accurate solution. In this latter case, however, the engineering geologist can, and should, show the limits of the conditions that can occur and explain to the engineer the type of difficulties which he must prepare to overcome. When extensive subsurface exploration is necessary some think that a geologist is not needed. On the contrary, however, an engineering geologist, by means of his understanding of the geologic forces that have acted on the formations, can lay out the investigation to reduce greatly the amount of exploration necessary; and then he can obtain the maximum information from it by correct interpretation. This requires understanding on the part of the engineer; he should allow sufficient exploration to enable the geologist to reach an intelligent and useful conclusion. On the other hand, there have been cases in which a geologist has demanded an inordinate amount of exploration to cover up his lack of experience. There is a happy medium which, unfortunately, is not always attained.

Obtaining competent engineering geologists to solve the problems of highway engineering or the other problems of engineering geology constitutes a problem with many difficulties. Engineering geology apparently appeals to many geologists as an attractive, easy, profitable field, and many geologists without experience in engineering have attempted to conduct investigations in it to the great detriment of the profession of engineering geology. At the other extreme are those who believe that, after a few courses in geology, engineers become competent to handle problems in engineering geology. This is just as dangerous as the other practice mentioned. Some understanding of geology by the engineer is essential, however, and the better he understands it, the less the danger of his overlooking some problem in which geologic aid is necessary. The most important requirements for competence in engineering geology are a thorough training in geology and a very broad experience in its application to many types of engineering projects on all types of geologic formations. A man experienced on a few types of problems in a limited area may make disastrous mistakes when he encounters different problems or different conditions. The apparent failures of engineering geology which are sometimes charged against it are often the failures of individuals who were not competent to handle the problems.

The early work in engineering geology was done by consultants who specialized in such work. Later some of the larger governmental organizations developed geological staffs composed largely of young geologists with little or no previous experience in engineering geology who were trained on the job at the expense of the job. A few state geological surveys have engineering geologists who are used in connection with highway work as has been explained by the author.

The use of consultants on special assignments is well adapted to the problems of dams where the work is concentrated and usually of considerable size. The more widely scattered problems of a highway department may not appear to be so well suited for investigation by a consultant, but when the work is well organized and especially when a permanent geologist or staff has collected data and done much routine work the more important problems can be handled, quickly and efficiently, by an experienced consultant.

The wide variety of problems in highway engineering may appear to require a large number of specialists but such is not the case. It is true that there will be problems in engineering geology, structural geology, sedimentation, physiography, ground-water hydrology, economic geology, petrology, materials testing, soil mechanics, and geophysics, but a competent engineering geologist must be trained and experienced in all but the last three and should know something about them. When particular problems in petrology arise, for example, they can usually be handled by a specialist in a university. A state highway department may have sufficient geophysical work to justify the full-time employment of a geophysicist; then, too, it may be more economical and satisfactory to engage outside consultants for that work. A department with an experienced engineering geologist and a specialist in soil mechanics and testing with assistants in proportion to the amount of work and arrangements for geophysical investigations would be competent to meet the wide variety of problems. On problems of special importance an experienced consultant in engineering geology should be used to obtain the advantage of broader experience. Some departments may handle their work most efficiently by depending entirely upon consultants for investigations in engineering geology.

Mr. Huntting has done a service both for engineers and geologists by describing many of the problems in which geology can be useful in highway engineering. He has outlined briefly the way in which geology is related to some of the problems; he has explained its value as a tool for the engineer, and discussed some of its limitations.

HYDE FORBES,[23] M. AM. SOC. C. E.—It is a matter of satisfaction to the writer that the Society has published another paper in the interest of applied geology. The paper is specific in its application to highway engineering, but the treatment is broad enough to apply to many fields in which civil engineers are active. It is of distinct value in further calling the attention of engineers and educators to the application of geology in engineering development and construction.

The late J. C. Branner, head of the Department of Geology at Stanford University, Stanford University, Calif., until his death in 1922, was a scientist of broad vision, and his efforts and accomplishments in the practical application of geology in the fields of petroleum and mining engineering are attested through the accomplishments of his students after graduation. The curriculum he developed was very broad, in that abridged courses in civil, mechanical, chemical, and electrical engineering were required, with opportunities to the individual for specialization in mining, metallurgy, petrography, geochemistry, ore deposits, paleontology, and petroleum.

The writer was particularly fortunate in having Professor Branner select him to direct in work in a new field in which the professor was endeavoring to obtain interest, research, and study; namely, Engineering Geology. After being graduated in 1913, the writer was employed under Professor Branner and was subsequently recommended by him to other similar positions prior to World War I. During these engagements, primarily directed to the investigation of the geology and ground-water conditions of certain areas, it was found that employers had problems involving geology upon which the engineers solicited the writer's services and advice.

The first of these positions, with the Spring Valley Water Company of San Francisco, Calif., afforded opportunities to do work in connection with the watertightness and stability of the abutments for the Calaveras Hydraulic Fill Dam then under construction, and the physical and geochemical characteristics of the materials used in the fill. Similar opportunities were afforded in connection with the foundation investigation for, and the construction of, the base of the Gibralter Dam by the City of Santa Barbara, Calif., in 1915, when the writer was investigating ground-water problems. Professor Branner was consulted and, with the advice of consulting civil and hydraulic engineers contacted during those employments, the writer attempted the preparation of a curriculum aiming at collegiate specialization in engineering geology.

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[23] Cons. Engr. and Geologist, San Francisco, Calif.

After Professor Branner died in March, 1922, the attempt to institute a curriculum at Stanford University emphasizing the practical application of geology in the field of engineering development and construction was not successful. However, the writer has continued to expand the outline for applied engineering geology which he established in association with Professor Branner and in the collection of material as his experience has broadened. His files and library are indexed and cross-indexed under the headings of that outline, in all of which he has made practical and technical application of the science of geology and under which he has found much of value in the engineering literature. The writer presents that outline here in acknowledgment of his debt to Professor Branner and in the hope that it will be of service to engineering educators:

(1) Geology of Rock Formations.
  (*a*) Character and structure—jointing and fractures.
  (*b*) Weathering—effective depth and geochemical change.
  (*c*) Interstitial binder—primary and secondary cementation, vein filling, healing of fractures.
  (*d*) Strength, specific gravity, watertightness, stability, solubility.
(2) Geology of Unconsolidated Formations.
  (*a*) Source of material and method of deposition.
  (*b*) Erosion, transportation, and silting.
  (*c*) Geochemical change and physical change in transportation and deposition, solution, oxidation, carbonization, hydration, abrasion, and wear; further changes to be expected.
  (*d*) Structure, porosity, and permeability.
  (*e*) Water content—drainage and recharge.
(3) Engineering Seismology.
  (*a*) Recognition and location of faults.
  (*b*) Determination of activity of faults and intensities of earthquakes.
  (*c*) Affect of vibrations on engineering structures.
  (*d*) Location of highways, pipe lines, and structures in reference to active faults.
  (*e*) Earthquake resistant construction.
(4) Applied Ground-Water Hydrology.
  (*a*) Occurrence, movement, and head of ground water.
  (*b*) Chemical and mechanical properties of ground-water movement and their effect upon the containing media; affect of ground-water pressure.
  (*c*) Development of water supplies—through physical works such as wells, spring development, tunnels, and galleries.
  (*d*) Safe yield and expected life of ground-water developments.
  (*e*) Spreading of water for ground-water recharge.
  (*f*) Drainage of saturated ground.
(5) Landslides and Earth Movements.
  (*a*) Occurrence, investigation, and correction.
(6) Soil Technology.
  (*a*) Occurrence, and classification of soils.
  (*b*) Development of soils and geochemical changes, from origin in rock fragments or products through stages in development.
  (*c*) Soil mechanics—the mechanical properties of soils as foundation material and for use in engineering structures.

(7) Natural Structural Materials and Methods Adapted in Construction.
    (*a*) Earth-fill construction, compaction, stability, geochemical change.
    (*b*) Hydraulic fill construction, effect of water and transportation.
    (*c*) Rock-fill construction, durability, insolubility, crushing strength.
    (*d*) Subgrade for roads, highways, and air ports.
(8) Foundations.
    (*a*) Geology of dam sites, requirements of structures.
    (*b*) Foundations for bridges.
    (*c*) Foundations for highways.
    (*d*) Foundations for buildings and other structures.
(9) Tunnels and Tunneling.
    (*a*) Investigation, location, design of support required.

If the engineering student is given an opportunity to obtain a basic knowledge in the foregoing subjects, he will enter his professional career well equipped to understand natural conditions and the use of natural materials, and, what is more important, with sufficient knowledge to be aware of his own limitations and to know wherein the services of a specialist are required. This lack is responsible for a large number of failures of structures. The geology student, on the other hand, will have his eyes opened to the possibilities of a specialization that will bring much satisfaction and opportunities for service. In the past, the engineer has made the mistake of assuming that a man who has acquired a college degree in geology "knows all the answers." A geologist cannot see underground any better than the next man, but he can recognize certain conditions from surface indications and, with adequate experience, will learn how to gain subsurface knowledge and apply his basic knowledge correctly.

Under the head of "Problems," the author lists twelve items in which the geologist's services might be profitably engaged. He then discusses these from the standpoint of an experienced engineering geologist. The writer agrees with the first premise, but the testing of material goes beyond the province of the geologist in the opinion of the highway engineer, and it is true that the geologist requires additional specialized training and knowledge in applying his basic training and knowledge to problems (1), (2), and (3). The geologist must have more than a basic training and knowledge of hydrography and ground-water hydrology, too often lacking in the collegiate training, in order to be of value in connection with problems (4), (6), (8), and (10).

Training and experience can be acquired just as well, and in as short a time, on engineering development work and construction as in the field of applied geology, which has been most often associated with geologists by the engineering profession—namely, mining engineering and petroleum engineering. It has always appeared to the writer that the failure to apply geological training and knowledge properly in highway engineering was the cause of considerable economic waste.

Beginning in 1928 and continuing well into the depression years, the oil companies of the West found that the surface geological structures were practically all determined, overproduction was causing difficulties in the industry, and geological staffs were drastically cut. The older geologists were absorbed in administrative and executive positions, but the younger men were dismissed. Many of the latter came to the writer to discuss opportunities for employment and the broadening of their activities. The advice given was that they should take the examination for junior engineer in the State Highway Department and then make a job for themselves as an opportunity arose to show that the application of their special knowledge was of value. Several men took the examination, got on the lists, but were not called after long periods of waiting.

It was felt that some preference in the matter of hiring should be given men having such qualifications, and a letter was written to the Chief Engineer of the State Highway Department setting forth this matter and submitting an outline covering the application of this special knowledge in highway engineering. A conference was arranged at the State Capitol at which the subject was received with enthusiasm by the division engineers in attendance but with a lack of enthusiasm by the executive head. The writer was given to understand that such a setup would require a centralized control, which highway engineers were trying to avoid, or the retention of a consultant for the direction of the field men, which was against their general policy. As far as is now known, no benefits were derived from this effort.

In 1928 the writer was appointed to a committee of the San Francisco (Calif.) Section of the Society to study the foundation and subsidence question over the built-up area of San Francisco. The writer presented the viewpoint of the geologist, and these discussions, resulting in a report published subsequently by the Section, opened up to the writer a field of application of geology and study—namely, foundations for buildings on unconsolidated materials.

The author discusses the problem of acting as a witness in litigation (see problem (12) under the heading "Problems"). Litigation affords the time and money for extensive and exhaustive research such as is never available in engineering development and construction. Court presentation of the engineer's findings is subjected to the keenest, critical analysis in cross-examination. The writer has sometimes observed expert witnesses attempt to solve mathematically the problems of the underground. When purely mathematical solutions are attempted and broken down under cross-examination, or matched by one factually determined, it soon becomes apparent that nature does not act in an empiric manner and that empirical formulas do not apply. One is greatly impressed with the degree of confusion and the ridiculous results achieved when values of reasonable variance from those selected by the proponent, as assumption or small-scale test result, are substituted in a formula. A mathematical computation, involving factors evaluated through assumption, or determined by small-size sample test, is no better than the value of that assumption or test. Its result might have been better stated as the proponents' unsupported opinion in the first place.

A study of geology reveals that geochemical change breaks down grain size with comparative rapidity. Under one set of conditions geochemical change occurs in natural material to produce a certain product and under similar, but slightly varying conditions, the change undergone by the same material yields a different product. Moisture contents of materials, determined as percentages of their oven-dried weights, do not take into consideration any of the water adsorbed or water that remains in fixed chemical combination. There is no constant law as to the quantity of water that may be taken up in chemical combination by the natural materials. For the foregoing reasons it is difficult to accept mathematical calculations based on sample tests.

There is no such thing as an undisturbed sample. The act of sampling implies disturbance from its position of confinement and support, and a new relationship to air—its oxygen, carbon dioxide, and moisture. The degree of disturbance can be ascertained only through knowledge of the geologic history, the physical condition and ground-water condition of the wider area sampled. That does not mean that sampling and testing should be abandoned. Boring and sampling, and more exploration boring, are needed to give as complete knowledge of the subsurface conditions as possible; but the limitations of the results obtained must be realized and used, within those limitations, to denote action, reaction, trends, and ranges, as tools to work with in the exercise of judgment. The writer uses shear tests, compression tests, physical, chemical, mechanical, and moisture analyses in his work for that purpose; but, in bearing-capacity determinations in foundation investigations, he depends upon positive vertical cyclic loading of the undisturbed foundation material; and for settlement he depends upon the experience recorded upon the same or similar foundation conditions determined through complete boring exploration.

To summarize, engineering education has not provided comprehensive curriculums in applied geology but has gone "overboard" on "soil mechanics." Former students and graduates of the Stanford Geology Department have told the writer, during the past several years, that only two fields of activity appear to be open to them—teaching and the United States Geological Survey. More than fifty graduates have joined the Survey, and, after World War II, with the restriction of appropriations such as occurred after World War I, many will be seeking new employment. If they are forced into commercial lines, as geologists were during the early 1930's, much training, knowledge, experience, and ability will be lost to the profession.

The profession as a whole does not recognize and appreciate the importance of applied geology in connection with engineering development and construction and it has overemphasized soil mechanics far more than its importance warrants.

A. A. Eremin,[24] Assoc. M. Am. Soc. C. E.—By illustrative examples, Mr. Huntting has shown the importance of geological investigations in highway construction. In the past few decades considerable interest has been expressed in the value of geology. In various universities the introduction of courses in soil mechanics and geology has increased the curriculum requirements for engineering students.

Economic considerations fully justify geological study for highway structures. Various devices have been developed to insure safety in structures. Streams are regulated and protected by various types of embankments. Bridge seats are designed to follow pier displacement resulting from the movements of various geological formations. In tunnel construction a temporary lining is used to study the characteristic features of the soil penetrated by the tunnel.

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[24] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

Without these engineering devices, geological science is still unable to yield reliable results. Further progress in soil mechanics and geology may be obtained through the cooperative effort of engineers and geologists by interpreting construction failures. A tendency still prevails to overshadow or to hide the real causes of some important structural failures. It is true that learning through failures is an expensive method; but an investigation of the failure of a structure designed according to standard assumptions is still the most reliable and effective method of improving design and construction. Many types of accidents are repeated constantly because no one studies the causes sufficiently. The final design of many large tunnels is the result of careful investigation of partial failures that occurred during construction.

Marshall T. Huntting,[25] Esq.—Every thinker or writer must inevitably be biased by his own particular training and experience. Hence, the writer may have overemphasized the place of geology in highway engineering, or he may have placed relatively too much emphasis on certain subjects. The discussions charging overemphasis or improperly placed emphasis are subject to the same analysis, of course. One man charges that soil mechanics or soil science has been neglected; others object to including the subject of soils in a geological discussion. As was pointed out by Professor Terzaghi, soil mechanics is not a substitute for engineering geology. The opposite is also true. The two should be complementary, with soil tests and geological investigations combined, to obtain best results.

The experience of at least five discussers shows that clay is not a major constituent of the soils where frost heaving is worst. The formation of ice layers causes heaving. Wherever the water table is higher than the frost line, heaving will almost certainly occur, and the more permeable the soil the more water can be furnished to freeze into ice layers. Therefore, even gravels may be heaving soils. In fact, the sand and silt soils where the water table is high will be the worst offenders for that reason, and clay is preferable because of its nearly impervious "insulating" effect. On the other hand, in areas where the water table is well below frost level and heaving is less common the water must rise by capillarity to reach the trouble zone. In this case, the writer contends that clay, mixed with other soil constituents, adds to the capillarity of the soil and makes a given soil a frost-heaving offender, whereas, under identical drainage and water-table conditions, the same soil without its clay content would not cause the water to rise high enough to allow heaving to occur. To prevent frost heaving, the examiner first must make sure the water table is below frost line. Then he must balance two costs against each other:

(1) The cost of further lowering the water table to below the depth from which water may rise by capillarity to the frost level; and

(2) The cost of breaking the capillary rise from the water table to frost level by filling with gravel or by some other means.

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Where sands, silty sands, and silts without clay are soils that heave, the writer suggests that this fact in itself may be sufficient evidence of a high water table and thus may simplify preventive studies by indicating directly the need for better drainage—no other preventive measure being applicable. Such reasoning would not necessarily apply in the case of silts, which have considerable capillarity even when clay is present in very small percentages.

Admittedly, the writer's statement (see heading, "Frost-Heave Problems") —"It has been shown that in many, if not most, instances of frost-heave disturbances clay is a major constituent of the subgrade material"—is in error. It should be amended by the addition of the clause "where the water table is well below the frost line." Also "major constituent" is much too vague a term for use when not accompanied by statistical data. Possibly, a fruitful line of research would be a study of the relation between soil particle size and capillarity, using samples taken from heaving soils. A comparison of the effects of the several individual classes of clay minerals would be important in such a study. A soil with a certain percentage of clay of one mineral composition might be expected to give results different from those given by soils with the same percentage of clays composed of other minerals.

How the soundness of a bridge superstructure can be independent of foundation materials, as stated by Mr. Feld, is difficult to conceive. A number of bridges have failed because piers and superstructure were not properly designed for the existing foundation conditions. In the discussion of bridge foundations, the writer did not wish to "indicate that taking and testing soil samples is entirely the work of geologists." His statement (see heading, "Competency of Materials for Bridge Foundations") was,

> "The geologist's sphere of usefulness in mechanical sampling lies in examining samples; in helping to interpret the samples in terms of dimensions, structures, water-bearing characteristics, and load-bearing capacities of subsurface formations; and in combining his interpretation of subsurface conditions with features of general and local geology to form the basis for recommendations for further sampling and testing."

The actual work—taking samples, testing physical properties, and most of the interpretation of physical tests—should be done by soil mechanicians and other qualified men.

It is interesting to paraphrase Mr. Feld's instance of the contractor who found himself in the class of an ex-contractor because he relied on a geological prediction that a certain hill was composed of glacial till, when excavation found solid rock instead: A patient died as the result of using drugs prescribed by a doctor who made an incorrect diagnosis of the patient's ailment. Perhaps the doctor was not very competent to diagnose that type of case, or perhaps his examination was not complete enough. In any event, such an example is scarcely a basis for condemning all doctors or the medical profession as a whole; nor does it indicate that it would have been preferable for either the patient himself or the druggist to diagnose the case and prescribe the medicine to be used.

Mr. Feld's experience undoubtedly has been shared by others. The cost of excavating solid rock would be less than that of excavating the type of earth encountered, but evidently solid rock was not preferred in the case Mr. Feld himself cited in the preceding paragraph of his discussion. In most instances, excavation of unconsolidated materials would be preferable; but the question is not what is more desirable but what will be found and in what quantities in a given excavation. The geologic history of the area will contain the answer, and the better the investigator knows that history the better equipped he is to find a solution.

In reply to Professor Legget's question as to the use of the term "traprock," admittedly the term should be avoided, and the writer's only excuse is that, in an earlier, unpublished, more detailed version of the paper, a table was included from a governmental publication giving the average results of physical tests on thousands of samples of various rock types. One of the rock types listed was "traprock." In the published version of the paper this table was not included, but inadvertently the term was retained in the text. For the purpose of this paper, "extrusive igneous rocks" should be substituted.

The "within limits" which Professor Legget questions should refer to the limits of the examining geologist's ability and the limits of geology in general, not to the limits of the depth and shape of the deposits.

Mr. Moneymaker partly answers his own question: "How should a state highway department provide for its geologic work?" Mr. Crosby offers further clarification. The answer depends upon the amount of geological investigation needed, the organization of the highway department, and the geological personnel and facilities available and economically practicable. These conditions vary widely from one state to another; each state has its own special problem.

The geologist who is to work with engineers when needed should certainly be an engineering geologist who devotes his full time and interest to that specialty. He cannot be primarily a petrologist or primarily a physiographer, for example, since geology like engineering is a highly diversified subject. Many geologists have specialized to such an extent that they have little knowledge of, and less interest in, the geological fields not included in their own lines of specialization. The engineering geologist must be competent in most of the special fields and should have a wide range of interests and a broad outlook. His willingness to recognize his own limitations is extremely important. He must not overstep the bounds of his competency and should know when to call for the assistance of other specialists.

There are many geologists who think of their science as an end in itself. Obviously, such an attitude is unsuited to the practical field of engineering, where science is but a means to an end.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2239

SURGES IN PANAMA CANAL REPRODUCED IN MODEL

By F. W. Edwards,[1] M. Am. Soc. C. E., and Edward Soucek,[2] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Harold A. Weggel, Harry Leypoldt, C. J. Posey, and F. W. Edwards and Edward Soucek.

Synopsis

A 1 : 60 scale model of Gaillard (formerly Culebra) Cut was built in the Panama Canal Hydraulics Laboratory for the study of surges caused by lock operations. The verification tests described in this paper consist of water-surface observations of model and prototype at corresponding locations and times, during and after corresponding lock operations. The success of these tests was made possible by the cooperation of various divisions of the Panama Canal.

The Problem

Gaillard Cut is an 8¾-mile reach of the summit level of the Panama Canal, extending from the Pedro Miguel Lock to Gatun Lake at Gamboa. Fig. 1 shows cross sections at the observation stations. The operation of filling Pedro Miguel Lock creates surges which increase the difficulty of handling ships in the Cut. Although the condition is not particularly serious at present (1942), it will be aggravated by the operation of a proposed larger lock. The navigation conditions at the junction of the proposed by-pass channel for the proposed third lock and the existing channel are of special importance. No further consideration will be given to the navigation problems involved. This paper is confined to a presentation of the results of verification tests.

Prototype Tests

The prototype observations were made in December, 1939, prior to the construction of the model. All tests were conducted at night after traffic in the Cut had ceased and the water surface had become quiescent.

Note.—Published in January, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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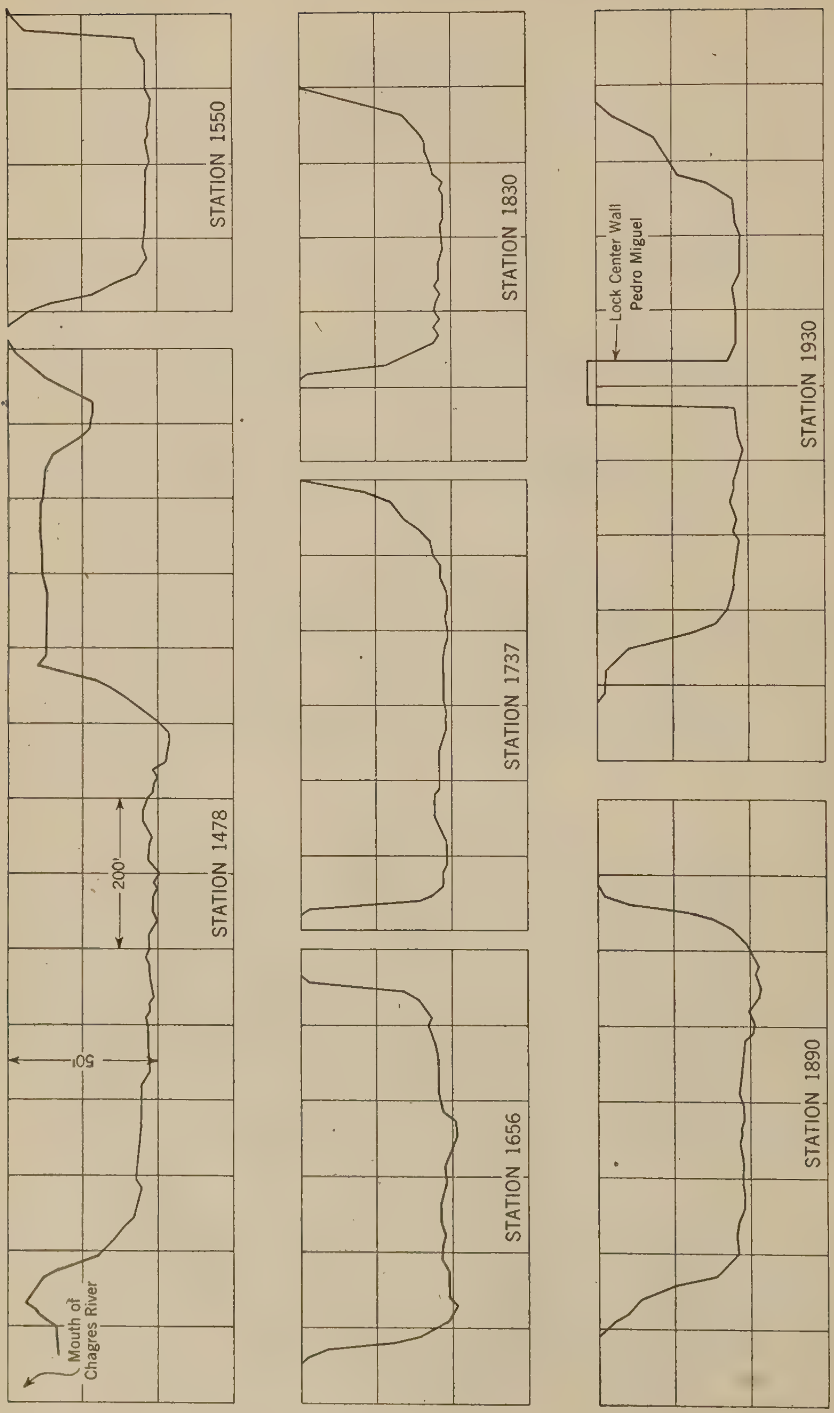


Fig. 1.—Channel Cross Sections, Viewed Toward Pedro Miguel Lock

Pedro Miguel Lock consists of twin chambers separated by a center wall. There are three filling culverts, one in each side wall and one in the center wall. The center-wall culvert is valved in such a manner that it can be opened to either chamber or to both simultaneously. The schedule of lock operations used in the tests was as follows:

Test No.	Description
I	A two-culvert fill of the East Chamber
II	A three-culvert fill of both chambers
III	Two three-culvert fills of both chambers; interval between fills 88 min
IV	Repetition of Test No. III
V	Two three-culvert fills of both chambers; interval between fills 44 min

It was necessary to plan the tests carefully because there was no means of communication with the observers. Observers met at Pedro Miguel Lock at 11:30 p. m. each night, received special instructions regarding the test scheduled for that night, were issued equipment and supplies, and were transported to their stations by launch. Synchronized stop-watches, previously checked for accuracy, were used for timing all readings and operations. All watches were started simultaneously, wound every hour, and stopped simultaneously for checking after the observers had reassembled. No adjustment of observations for errors in timing was found necessary. All equipment, such as watches, data sheets, flashlights, etc., was furnished in duplicate to each party. A party, consisting of two observers, was located at each station and at the lock control house.

Prior to the tests, a series of water-level observations was made at 10-sec intervals at one of the stations. This series was made in the daytime, under normal traffic conditions, to find the interval for test readings which would define the surges adequately. A 1-min interval was found satisfactory and this was adopted for the tests. The duration of observations each night averaged more than 5 hr.

With the exception of permanent gages at Pedro Miguel and Gamboa, temporary staff gages were installed for the verification tests. The gages were graduated in tenths of feet but readings were estimated to hundredths. The gages were leveled, prior to the tests, on a quiescent water surface during early morning hours before traffic started through the Cut. A check upon the accuracy of gage settings was obtained each night by taking readings for several minutes prior to lock operation. Small adjustments were made on the basis of these observations. Various methods of cross-checking the gage elevations were used and all indicated that the adjusted elevations were correct to the nearest one-hundredth of a foot.

THE MODEL

The model size was selected on the basis of economy, sensitivity, and relative freedom from surface tension. A distorted model was considered but rejected. Although friction was relatively unimportant, the concrete surfaces in contact with water were finished so as to give proportional friction losses in model and prototype. Lock hydraulic systems were not simulated, but filling curves—obtained during prototype tests—were approximated in the model by the use of manually operated gates. The feature most difficult to simulate was the junction of the Cut with Gatun Lake. An uncontrolled forebay large enough to simulate the lake was impracticable. A relatively small forebay (684 sq ft), with overflow troughs to maintain a constant level, was adopted. The overflow length and rate of inflow to the forebay required for similitude were determined by trial. Any constant inflow within a considerable range could be used without disturbing the similitude. This characteristic of the model supports the confidence which the writers have in the ability of the model to indicate surge conditions for which no prototype comparison is possible.

Water-surface observations in the model were made on inclined staff gages which were designed so as to minimize local disturbances. The gages were graduated at vertical intervals of one-sixtieth of a foot, representing one foot on the prototype. Readings were estimated to the nearest tenth of a foot, prototype dimension. This method of observation may be considered crude, but has been entirely justified by the results obtained. Other proposed methods had important disadvantages.

Model observations were timed by the use of a single electric timer, located at a control station. A buzzer system was used to signal the observers when readings were desired.

Nature of the Surges

Test No. II will be used to illustrate the surge phenomenon. Fig. 2 shows water-surface profiles at 5-min intervals following the beginning of lock filling. Although these profiles are based on actual prototype observations, the small number of stations permits considerable latitude in interpretation. For this reason, the profiles should be considered pictorial rather than exact.

Considerable variation in section and in average depth exists in the Cut as is apparent from Fig. 1. From consideration of approximately twenty cross sections, the average depth, y, was estimated as 41.0 ft. The corresponding wave velocity computed by the formula

$$V = \sqrt{g\,y} \qquad (1)$$

is 36.4 ft per sec. Velocities of flow are so low that wave velocities may be considered to be absolute. The point of the arrow shown with each profile of Fig. 2 indicates the position which the front of the wave would occupy if it traveled along the Cut at a uniform velocity of 36.4 ft per sec. Fair agreement between calculated and observed positions exists even after 100 min. Friction, variation in channel characteristics, and uncertainty regarding the exact stations from which reflection should be considered to occur, especially at the lake end of the Cut, would be expected to produce some discrepancy.

The surface width near the lock is about 750 ft. The rate of inflow to the locks varies, but the maximum rate in the test under consideration was approximately 22,500 cu ft per sec. From discharge, wave velocity, and surface width, the drawdown calculated by continuity is about 0.8 ft, substantially the depth which occurred. Some sections of the Cut are much narrower, the average top width being only about 480 ft. However, no increase in surge height occurred in the narrower portions of the channel. The surge height is not to be confused with velocity head.

In the 5-min profile (Fig. 2), the water at the initial level is still at rest, while flow occurs toward the lock in the depressed reach. Five minutes later, inflow to the lock has ceased and water is beginning to pile up against the lock.

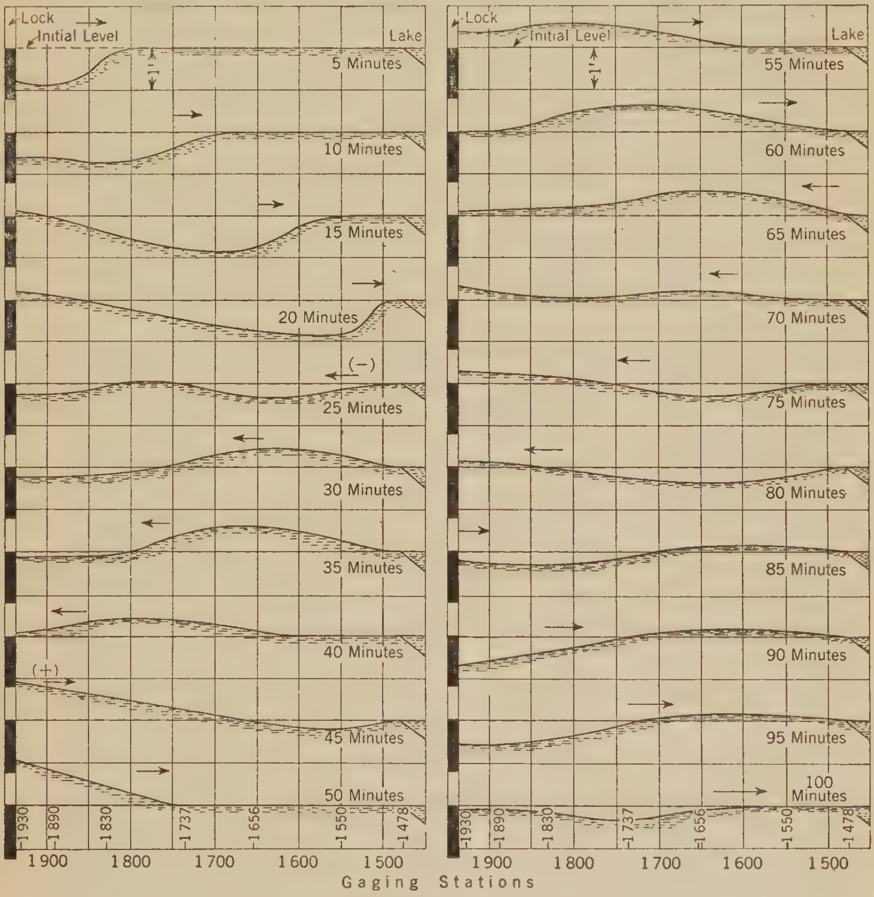
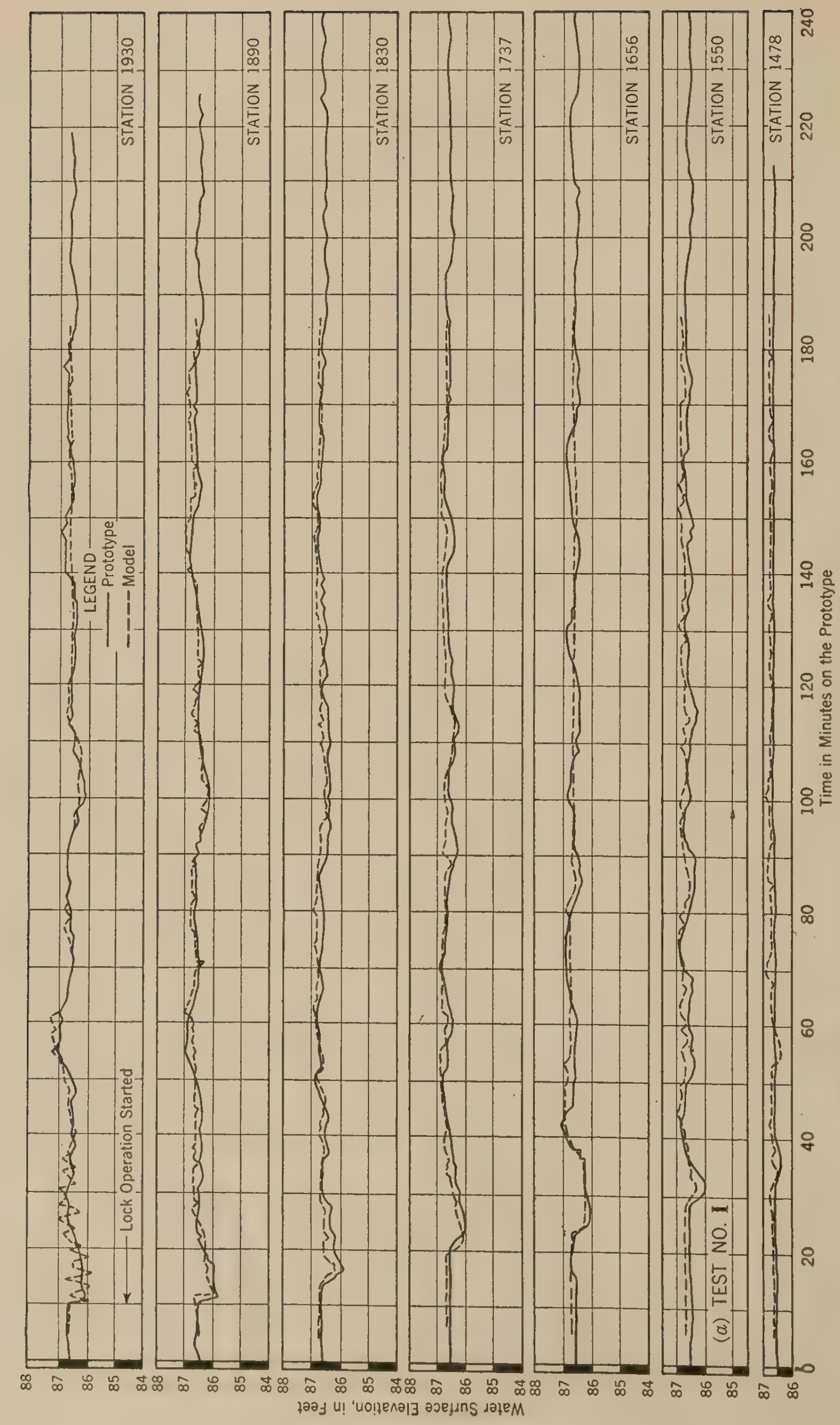


FIG. 2.—WATER-SURFACE PROFILES AT 5-MIN INTERVALS

In the 15-min profile, the surface at the lock has returned to its initial level. The "sag" in the water level at this time is a measure of the volume drawn into the locks—about 8,000,000 cu ft. Water is at rest both ahead of and behind the "sag," but it is in motion toward the lock under the "sag." Water is being transferred past the moving "sag" at a rate approximately equal to the maximum rate of inflow to the locks which occurred nearly 15 min earlier. The difference in slopes at the front and back of the "sag" is indicative of the


STATION 1930
STATION 1890
STATION 1830
STATION 1737
STATION 1656
STATION 1550
STATION 1478
LEGEND
Prototype
Model
Lock Operation Started
(a) TEST NO. I
Time in Minutes on the Prototype
0
20
40
60
80
100
120
140
160
180
200
220
240
Water Surface Elevation, in Feet
88
87
86
85
84

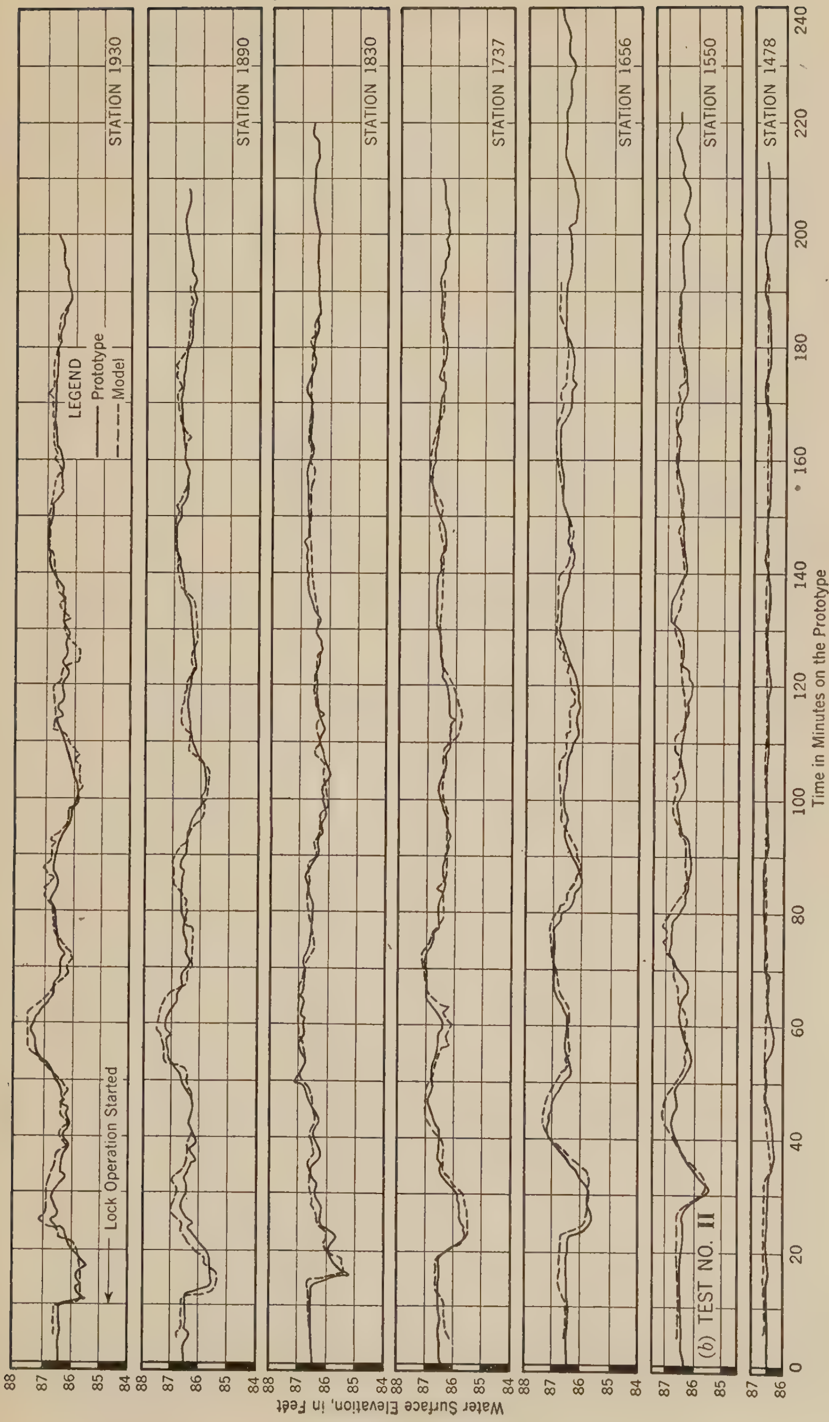
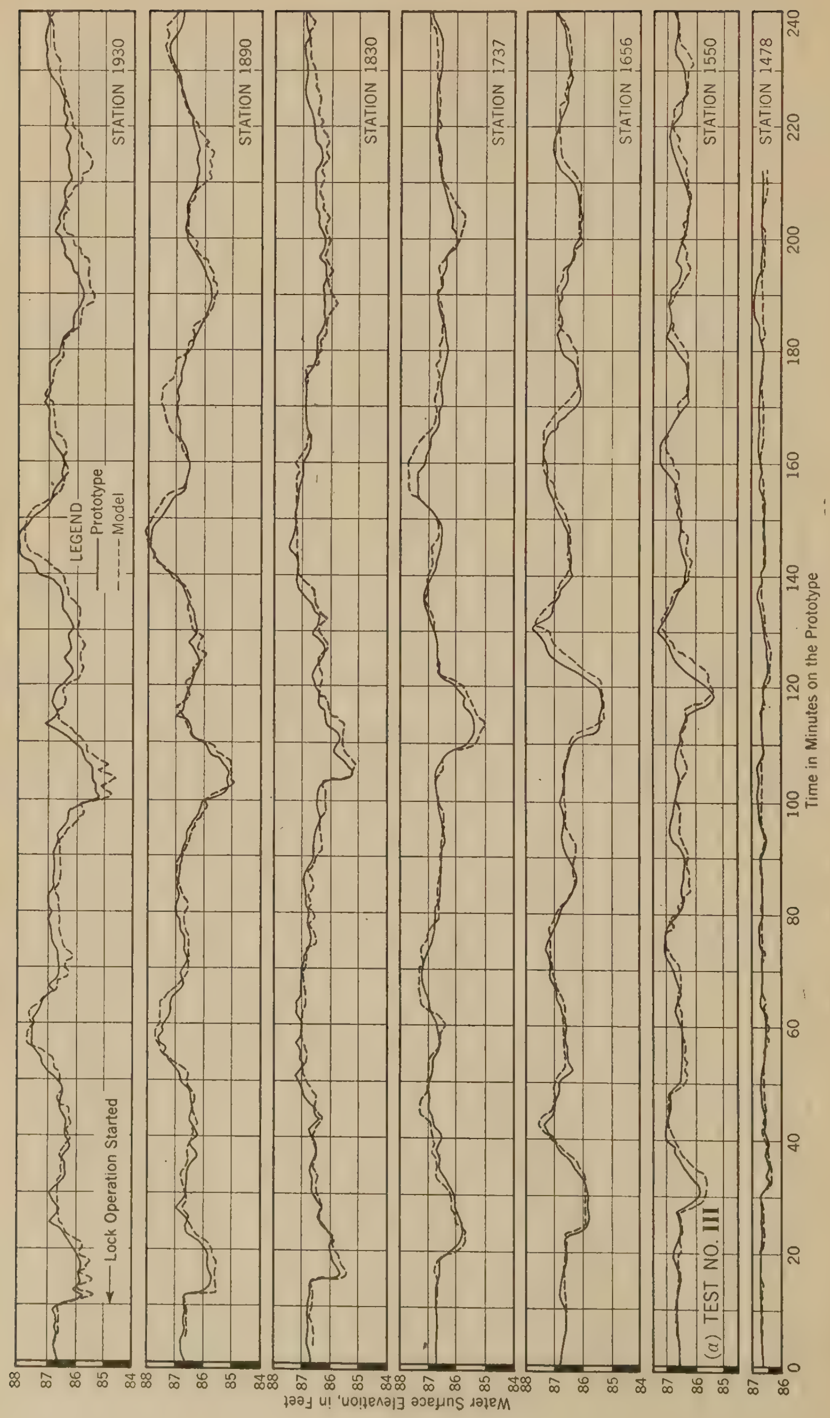


FIG. 3.—COMPARISON OF WATER-SURFACE PROFILES, MODEL AND PROTOTYPE


STATION 1930
STATION 1890
STATION 1830
STATION 1737
STATION 1656
STATION 1550
STATION 1478
LEGEND
Prototype
Model
Lock Operation Started
(a) TEST NO. III
Water Surface Elevation, in Feet
88
87
86
85
84
Time in Minutes on the Prototype
0
20
40
60
80
100
120
140
160
180
200
220
240

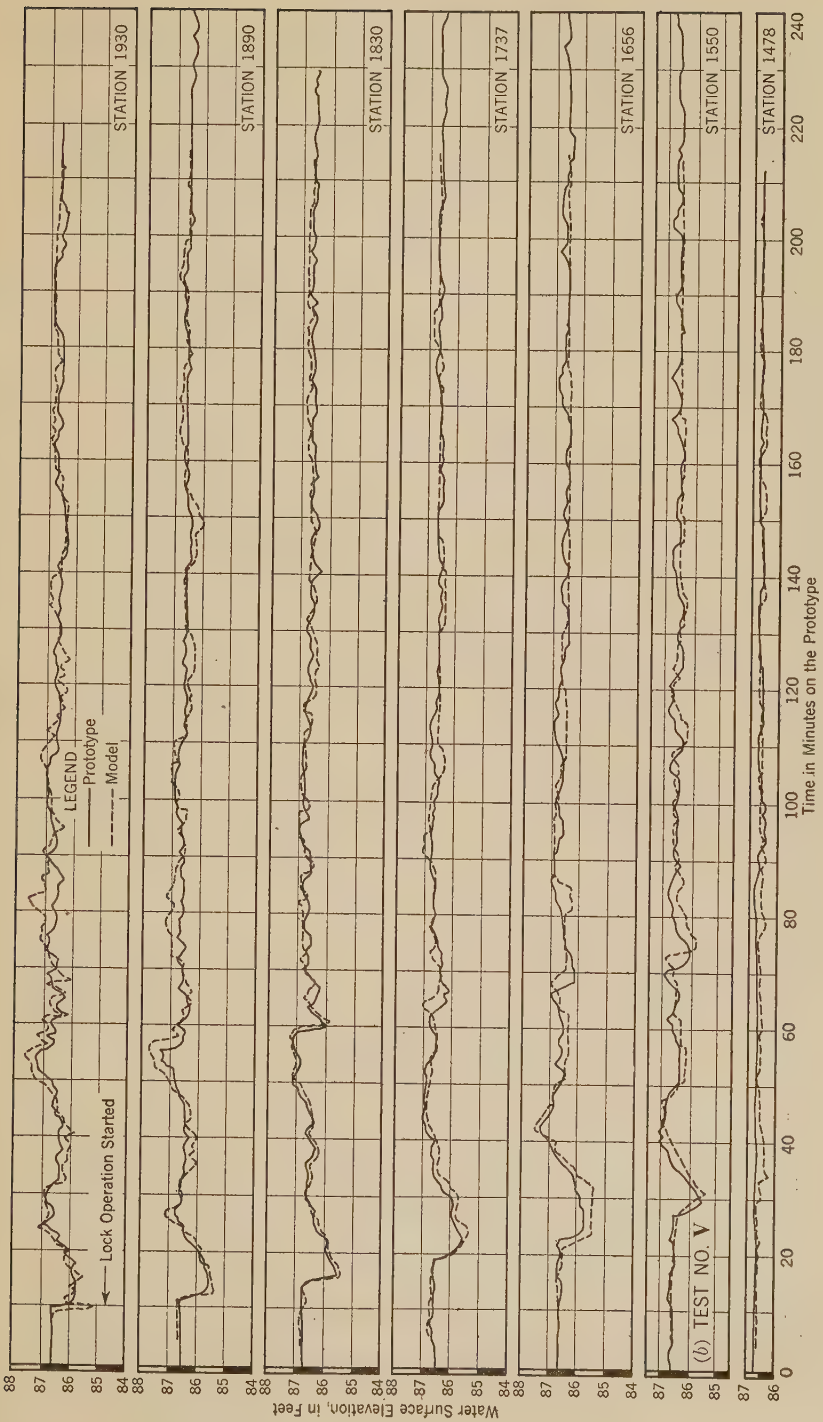


FIG. 4.—COMPARISON OF WATER-SURFACE PROFILES, MODEL AND PROTOTYPE

variation in rate of flow during the lock filling period. The length of the "sag" is a measure of the duration of inflow to the lock. The 20-min profile is a further development of the condition which has been explained.

A rigorous treatment would require that the front and back faces of the "sag" be treated separately. However, no harm to the description, and considerable simplification, result if the disturbance is considered from now on as a single wave. For discussions of the mechanics of wave reflection, reference is made to selected literature by Horace W. King,[3] Earl I. Brown,[4] and H. Rouse,[5] Members, Am. Soc. C. E. For the purposes of this paper, it may be stated simply that waves are reflected without change in sign from closed ends and negatively from junctions with large bodies of water, or those in which a constant level is artificially maintained.

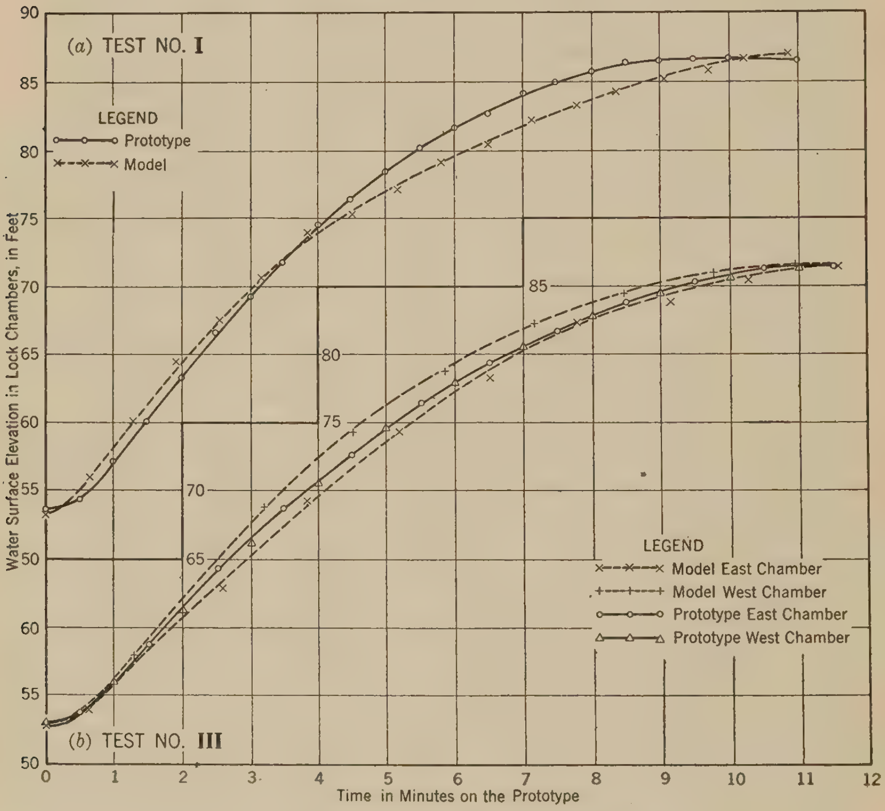


Fig. 5.—Comparison of Lock Filling Curves

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[3] "Translatory Waves in Open Channels," by Horace W. King, *Civil Engineering*, June, 1933, pp. 319–321.

[4] "Flow of Water in Tidal Canals," by Earl I. Brown, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), pp. 749–834.

[5] "Fluid Mechanics for Hydraulic Engineers," by H. Rouse, *Engineering Societies Monograph*, 1938, pp. 353–402.

The 25-min profile (Fig. 2) is a stage of the reflection in the negative sense and the 30-min, 35-min, and 40-min profiles show the return of a somewhat attenuated "hump" from the lake toward the lock. The 45-min and 50-min profiles show the positive reflection at the lock. The 55-min, 60-min, and 65-min profiles show the travel of the reflected "hump" from the lock toward the lake. The 70-min profile is the second occurrence of negative reflection and the 75-min, 80-min, and 85-min profiles show the passage of a now considerably attenuated "sag" toward the lock. The 90-min profile is the second occurrence of positive reflection. The 95-min and 100-min profiles, comparable approximately to the 5-min and 10-min profiles, show the beginning of another cycle.

The effect of successive lock fillings during either of the positive reflection phases, exemplified by Tests Nos. III and V, is predictable, at least qualitatively.

Comparison of Model and Prototype

The conformity of model observations with prototype Tests Nos. I, II, III, and V is shown in Figs. 3 and 4. Test No. IV, which was a repetition of Test No. III, is not reproduced here as the results were identical. The agreement is considered to be very satisfactory. It is evident that conclusions based upon the model observations would be the same as those based upon the prototype tests. Refinements could have been adopted which would have produced better results, but they were not warranted for the studies undertaken. It is believed that more accurate reproduction of the lock filling curves would have eliminated almost entirely the relatively small discrepancies which are indicated. Comparisons of typical filling curves are shown in Fig. 5.

A comparison of Figs. 4(*a*) and 4(*b*) is of special interest because it illustrates the result of synchronization and of anti-synchronization of the effects of consecutive lock operations.

Acknowledgments

The success of these tests was made possible by the cooperation of various divisions of the Panama Canal. Maj.-Gen. Glen E. Edgerton, M. Am. Soc. C. E., is Governor of the Canal Zone. At the time these tests were conducted Maj.-Gen. T. B. Larkin, M. Am. Soc. C. E., was supervising engineer on the Third Locks project and E. S. Randolph, M. Am. Soc. C. E., was designing engineer.

DISCUSSION

Harold A. Weggel,[6] Jun. Am. Soc. C. E.—It is very interesting to note the close conformity of model observations with prototype Tests Nos. I, II, III, and V as shown in Figs. 3 and 4. However, close conformity in five observations does not necessarily mean that all conclusions based on the model would hold true in the prototype. Test No. III and Test No. V are of special interest because they illustrate the results of consecutive lock operations; but deductions are incomplete unless the data are available for the periods of synchronization and anti-synchronization.

Observations made by the U. S. Geological Survey on the Ohio River near Huntington, W. Va., in 1938 between lock 27 and lock 28 indicate the difficulties of predicting the heights of translatory waves. At times of low discharge on the Ohio River, the slightest change of discharge, caused by operation of lock chambers and controls on the dams both upstream and downstream, was enough to create a surge. Occasionally, anti-synchronization of lock operations resulted in negative slope readings, and at other times similar operations would be the cause of superelevated translatory waves. Not always is it the amount of change of discharge that determines the height of wave or depth of trough. The rate of change of discharge may be the prime factor, especially on closing operations in conjunction with reflected waves.

The time of travel of translatory waves for the 4.65 miles between the gages at lock 28 and at 24th Street is between 17 min and 21 min; and the time of travel for the 10.6 miles between lock 27 and lock 28 (and return) is between 88 min and 100 min. Theoretical calculations give the following results for $V = \sqrt{g\,y}$ (Eq. 1): For the upper reach with $y = 10.0$ ft, $V = 17.95$ ft per sec; and, for the lower reach with $y = 13.9$ ft, $V = 21.2$ ft per sec. The time of travel is 19.3 min for the lower reach and 29.1 min for the upper reach or 48.4 min for the entire distance. At times of low flow no difference was noticed between the time of wave travel downstream and upstream.

If the canal surges in the Panama Canal are comparable to those on the Ohio River, it is doubtful whether any model could be accurate enough to reproduce the surges that would be encountered in the operation of the proposed larger lock. The only possible method is the study of the prototype itself with sufficient gages installed to plot the passage of translatory waves and to correlate the effects with the causes. When the causes of the disturbing waves are determined, it should be easy to eliminate the waves through control of the causes.

Harry Leypoldt,[7] Esq.—The data shown in Figs. 3 and 4 lend themselves to an interpretation far different from the one advanced by the authors, because the water-surface profiles show that a seiche with a period of about 29 min is present in the waterway. Let $y = 41.0$ ft; $V = \sqrt{g\,y} = 36.4$ ft per sec; and

$$t = \frac{2\,L}{\sqrt{g\,y}} \qquad \text{(2)}$$

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[6] Associate Structural Engr., Diamond Magnesium Co., Painesville, Ohio.

[7] Los Angeles Harbor Dept., San Pedro, Calif.

in which t is the time in seconds (29 × 60). Computing the uni-nodal length of basin that will accommodate a seiche with a 29-min period, these values in Eq. 1 give $L = 31{,}668$ ft. This brings the end of the basin at about station 1614, which is obviously not a place where wave reflection can occur. From this consideration, at least a bi-nodal seiche is necessary to provide the oscillations shown. Assuming the same depth in the dredged cuts leading to the lower reaches of Gatun Lake, the loop end for a bi-nodal seiche would be at the head of the bend in the channel.

That this is the correct interpretation can be established. The gaging stations between the node at station 1772 and the loop at the locks, station 1930, should show high water when all the other gaging stations show low water. This is the condition shown in the profiles. The location of a gaging station at station 1478, very close to the node at station 1456, should result in water-surface profiles with the amplitude of the seiche nearly zero. This is certainly the case in every profile for station 1478. Also, the profiles at station 1550 and station 1656, lying on either side of the center loop, should be nearly similar, which they are. Furthermore (see the profiles from Test No. III) where a fresh impulse is given at an 88-min interval, the amplitudes should be greatest since 88 is 3 × 29 +; that is, the seiches from the two impulses would augment, whereas those in Test No. V (Fig. 4(*b*)), at the 44-min interval, should be least, since the seiches would interfere to the maximum extent possible. This is likewise the case on the given profiles. No reason was advanced for this choice of intervals.

Without further data, it is conceded that the system may have additional nodes. This can be verified easily on the prototype by gaging stations in the southern part of Gatun Lake. However, treating the problem as a bi-nodal seiche is valid for this discussion. There are minor seiches riding the major one of the prototype, which were probably produced in the canal by reflecting surfaces at places where the width or direction of the canal changes. They do not affect the problem.

The water-surface profiles of Fig. 2 do not show the actual conditions. They are seriously influenced by the findings of Professor King, who states definitely that[3] the water is moving in an open canal when water is added or removed. In the prototype, as in all other seiches, the wave form is nearly sinusoidal, instead of the odd assortment shown in Fig. 2. It is also irrelevant to cite the work of Colonel Brown[4] since his paper treats of tidal canals.

The position of Gatun Lake proper does not influence the part of the canal under consideration in the same manner that lakes do in Colonel Brown's studies. About 6 miles of narrow, shallow waterway, except for the dredged cut, intervenes.

The nomenclature adopted by students of seiche phenomena limits the term "surge" to the "to and fro" motion of the water (current), whereas the rise and fall is called "seiche," and corresponds to tides in tidal waters.

Seiches in nature are produced chiefly by differences in barometric pressure on water surfaces in the same oscillating area, or by tidal effects. In canals, as in the case under discussion, the change in pressure is effected by release of water into the locks.

Since the model was built to scale, and the seiche period is a function only of length and depth of the oscillating system, the model should be a good replica of the prototype. It would be astonishing if it were not; so no conclusions regarding model operations are valid from these experiments. Width of channel does not enter the problem, and therefore added breadth in the model will not change the period, provided depth is held constant. This may be of value in future experiments.

Roughness variations between model and prototype would only cause a difference in the number of oscillations before friction consumed the energy.

The primary object of the investigation was to study the effect on ship operation. In Los Angeles Harbor, numerous continuous seiches occur. Those with relatively long periods have little or no effect on ship maneuvering, whereas the short period seiches are a serious problem, especially in the naval base area where caisson gates for graving docks must be fitted into rather small keyways during the closing operation. The surges accompanying the seiches, with their rapid direction reversal, make the locking operation difficult. In the Panama Canal case, additional locks, with shorter filling times or greater water removal, would induce the same period seiches, but with greater amplitude, and therefore with increased surge. This is shown by the small amplitudes in Test No. 1 where slow removal occurs. However, two sets of locks, with filling operations regulated to time periods that would produce interfering seiches, should answer the problem. Another solution would be to have the filling water removed from the main canal into a paralleling conduit of such length (to be determined by model experiments) and at such places as to produce the least seiches due to interference phenomena, the water having sufficient capacity to fill the locks. The locks would be filled from the auxiliary conduit, the seiches would be reduced through interference, and little or no surge would be experienced when the lock gates were opened.

From the nature of the factors governing the period of the seiches (depth and length of oscillating basin), it is apparent that changing the depth or the length would change the period. The depth enters to the one-half power, requiring greater change to produce the same effect that a change in length would produce. Such change in length of oscillating basin could be attained by introducing a gate in the waterway at a suitable place, the gate to remain closed during lock filling.

However, in view of the fact that the surges (currents) are theoretically zero at the loop ends and greatest at the nodes, the existence of a loop at or near the locks would signify that very little current occurs here and therefore that no cause for anxiety is apparent.

C. J. Posey,[8] Assoc. M. Am. Soc. C. E.—Prototype-model comparisons of this type are all too infrequent in engineering literature. When the underlying theory is generally accepted, there is a tendency to adjust the design to fit the results of the model tests and to omit subsequent tests on the prototype. Only when the model theory is the subject of controversy is the laborious and difficult job of securing prototype data for comparisons likely to be undertaken. Many hydraulic engineers would consider satisfactory model-prototype correspondence, in this particular problem, to be almost a foregone conclusion and prototype tests unnecessary. The writer disagrees with this viewpoint, and believes that such comparisons should be undertaken whenever practicable.

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[8] Associate Prof., Hydraulics and Structural Eng., Research Engr., Iowa Inst. of Hydr. Research, State Univ. of Iowa, Iowa City, Iowa.

Although substantial agreement with respect to wave velocities is evident from inspection of Figs. 3 and 4, a statistical study of comparative water-surface elevations might give a more definite basis for the authors' conclusion (see heading, "Comparison of Model and Prototype") that "The agreement is considered to be very satisfactory." Plotting of the model and prototype test data in the form shown in Fig. 2 should make it easier to judge the degree of correspondence.

The authors' comparison of the results of synchronization and of anti-synchronization, shown in Fig. 4, certainly is of special interest. It seems safe to assume that these are the extreme cases and that surges resulting from operation at intermediate periods will not exceed the values obtained for synchronized operation.

F. W. Edwards,[9] M. Am. Soc. C. E., and Edward Soucek,[10] Assoc. M. Soc. C. E.—It is interesting to note that one discussion expresses doubt regarding the ability of the model to reproduce surges that would be caused by operation of the proposed larger lock, whereas another expresses astonishment that it was considered necessary to conduct verification tests.

Mr. Weggel states, "Test No. III and Test No. V are of special interest because they illustrate the results of consecutive lock operations; but deductions are incomplete unless the data are available for the periods of synchronization and anti-synchronization." The data are available for these periods and were presented in Fig. 4.

In order to explain more fully, reference is made to Fig. 4(*a*). The initial lock operation depressed the water-surface elevation immediately. The second lock operation was performed 88 min later when the water surface was again approaching the low elevation. Withdrawing more water from the canal at this time lowered the water surface and increased the magnitude of the surge. It should be noted that 88 min after the second lock operation the water surface was low again but was not below the elevation caused by the initial withdrawal. The surge as augmented by the second operation, therefore, represents the worst condition likely to be produced by operation of the existing locks.

Reference is made next to Fig. 4(*b*). By performing the second lock operation 44 min after the first, when the water-surface elevation was approaching a maximum, the surface was prevented from rising to the elevation attained in Test No. III and the surge was neutralized. The choice of intervals between lock operations was not accidental, as implied by Mr. Leypoldt, but was based on calculation and on study of the water-surface profiles of Tests Nos. I and II.

Tests Nos. III and V substantiate Mr. Weggel's belief that, "Not always is it the amount of change of discharge that determines the height of wave or depth of trough." The data regarding waves on the Ohio River are interesting but do not support the opinion that a model could not reproduce, accurately, results identical to those observed in the prototype.

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[9] Associate Prof. of Civ. Eng., Pennsylvania State College, State College, Pa.

[10] Hydr. Engr., Office, Chf. of Engrs., Washington, D. C.

The writers agree fully with Professor Posey's viewpoint that comparisons of models and prototypes should be undertaken whenever practicable. Although it was expected that the model would represent the prototype correctly, it was considered good engineering practice to check results by every reasonable method available.

The entire discussion by Mr. Leypoldt is at variance with the beliefs of the writers. He states,

> "The water-surface profiles of Fig. 2 do not show the actual conditions. They are seriously influenced by the findings of Professor King, who states definitely that[3] the water is moving in an open canal when water is added or removed. In the prototype, as in all other seiches, the wave form is nearly sinusoidal, instead of the odd assortment shown in Fig. 2."

In spite of Mr. Leypoldt's belief that the wave form should be sinusoidal, actual data, as represented in Fig. 2, show this to be untrue in the present case.

The writers stated that the profiles should be considered pictorial rather than exact, but this qualification was made because a profile representing nine miles of canal is not precise when based on relatively few observation points. The statement was not made in order to justify a fictitious representation of the data. The wave forms represented in Fig. 2 were influenced only by the observed data; not by any literature, theory, or preconceived notion on the subject. The agreement between the water-surface profiles and the field observations for Test No. II can be seen in Fig. 6. In this figure the observed water-surface elevations from Fig. 3 and the profiles originally shown in Fig. 2 are plotted.

Fig. 6 shows clearly that the writings of Professor King did not influence the drawing of the profiles. However, the theories which Professor King has described are undoubtedly applicable to the generation of the initial wave. The relationships pertaining to the height of surges formed in an open channel by sudden changes in discharge are based only on the conservation of matter (implicit in the law of continuity) and Newton's Laws of Motion. The application of the equations is affected but not invalidated by the initial zero velocity of flow. It is well known that the initial shape of a wave in a channel is a function of the manner in which it is generated; whether the wave approaches regularity in form or tends to break up as it travels along the channel depends upon conditions beyond the scope of the paper. Whether or not the wave form is an "odd assortment" probably depends on the point of view. To the writers, any apparent oddity can be due only to an attempt to classify the wave forms in accordance with a preconceived notion—an error which Mr. Leypoldt attributes to the writers.

The writers disagree with Mr. Leypoldt's contention that the data indicate a seiche of two or more nodes, one at station 1772 and another at station 1456. Since no significant oscillation occurs beyond station 1478, it is clear that none occurs at station 1456, but here the agreement ends. If a node were located at station 1772, the following conditions would exist: (*a*) The water surface at the node would oscillate less than at stations on either side, (*b*) the amplitude would increase progressively from the node to adjacent loops, and (*c*) the concurrent oscillations on the two sides of the node would differ in sign. The water-surface profiles of Fig. 2 show that none of these necessary conditions prevail for a node claimed to exist at station 1772.

Mr. Leypoldt believes the reference to Colonel Brown's work[4] to be irrelevant. He states that "Gatun Lake proper" does not influence the part of the canal under discussion in the same manner that lakes do in Colonel Brown's studies. It is clear that Mr. Leypoldt means the main body of water approximately six miles from station 1478. However, Gatun Lake is considered generally and properly to extend from Gamboa, the point where the Chagres River flows into the canal at station 1478. The second paragraph of the paper and the notation of Fig. 2 testify that this designation was used by the writers. It is Gatun Lake at station 1478 to which the work of Colonel Brown is relevant and which does act in a manner identical to that of the lakes in his studies. It is Gatun Lake at station 1478 which was represented in the model by a forebay in which the water surface was maintained at a constant level. If the waves had not been reflected from station 1478 in the manner described by Colonel Brown, the model could have agreed with the prototype only by chance, the chance being the existence of a node at that station.

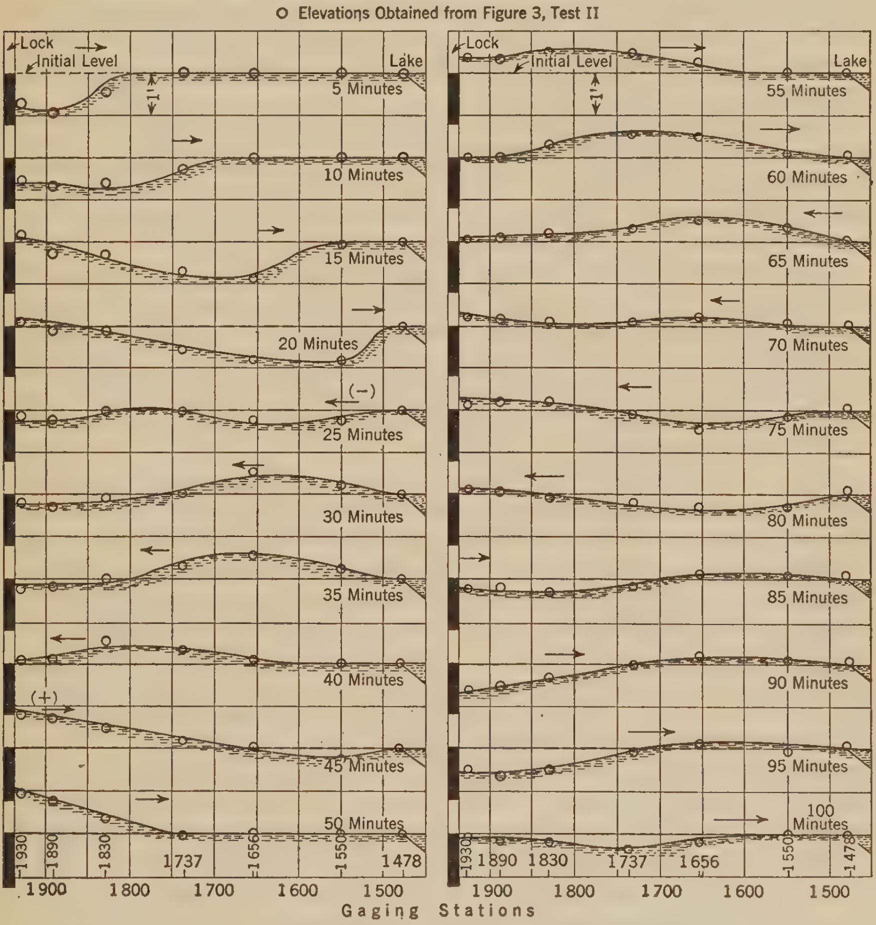

Fig. 6.—Water-Surface Profiles at 5-Min Intervals

Since Mr. Leypoldt's seiche theory does include a node near station 1478, it is necessary to go one step further to show that Colonel Brown's theory of negative reflections does apply. This step is proof that significant oscillations do not occur beyond station 1478. Mr. Leypoldt suggests that the existence of additional nodes could be verified easily on the prototype by additional gages in the southern part of Gatun Lake. Preliminary reconnaissance prior to the tests showed that significant oscillations did not occur beyond station 1478, but to remove all possible doubt, a gage at station 1367+50 was established and operated during the first two tests.

The following quotation from the official report on the prototype tests[11] explains the abandonment of this station: "As had been expected, the surges beyond Gamboa were small so this station was discontinued after the second test." This gage was located near the point where Mr. Leypoldt's analysis indicates a loop.

The "narrow, shallow waterway except for the dredged cuts," and the Chagres River each have a mean width of more than one-half mile. The quiescence of these bodies of water during the surge tests shows conclusively that the bodies are large enough to reflect waves negatively in the manner described by Colonel Brown. The creation of significant oscillations in these waters by means of impulses no greater than those caused by filling the locks is physically impossible.

From Mr. Leypoldt's comments concerning Gatun Lake proper, from his opinion that no conclusions regarding model operations are valid from these experiments, from his statements that the width does not enter the problem and that roughness is unimportant, it is not evident that he understands that the model extended only to station 1478. The model did not encompass the length of the oscillating system which he believes to exist in the prototype. Accordingly the remark that it would be astonishing if the model were not a good replica of the prototype is especially inappropriate. Although it is true that all parts of the model could have been reduced proportionately in width on the basis of gravitational and inertial considerations alone, it is by no means evident that the rate at which the surges dissipate their energy would be unaffected. If roughness affects the number of oscillations, which Mr. Leypoldt concedes, it must also have a progressive effect on their magnitude. Therefore, neither the width nor the roughness may be altered without careful consideration of the results. Since, in a model of the type described, many problems in addition to the period of oscillation are investigated—for example, the flow pattern at the junction of the by-pass for the proposed new lock—Mr. Leypoldt's suggestions regarding future experiments have no value. It does not appear desirable to adopt the restricted meaning of the term "surge," as advocated by Mr. Leypoldt, in a paper written for engineers.

Except to state that the specific suggestions offered by Mr. Leypoldt for solving the surge problems in the Panama Canal are considered impracticable, no comment on this question can be made at this time. The paper was specifically confined to an explanation of the surges and to verification of the model.

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[11] "Surge Tests—Gaillard Cut, Report on Procedure and Results," by Edward Soucek, Third Locks Project; The Panama Canal, January 16, 1940, p. 3 (unpublished).

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2240

ORIGINAL AND READJUSTED REPAYMENT CONTRACTS, BOULDER CANYON PROJECT

By Clay C. Elder,[1] M. Am. Soc. C. E.

With Discussion by Messrs. W. L. Chadwick, A. R. Arledge, and Clay C. Elder.

Synopsis

The Boulder Canyon Project, after a generation of investigation, planning, and political debates, progressed to actual construction in 1930. First commercial power production occurred in October, 1936, with additional generating units still being installed as of 1944. Since June, 1943, the power output has exceeded 1,000,000 kw, the first time in history for a single power plant; so the project's record of breaking records is continued. The ultimate capacity of the plant is announced as 1,332,300 kw.

The great dam, 726 ft high, controls the flow of the Lower Colorado River on the Arizona-Nevada boundary and creates a storage reservoir of 32,000,000 acre-ft capacity, called Lake Mead. It was built and is owned and operated by the United States because of the complexity of its interstate and international problems, which were the more involved due to its being the pioneer in the field of large multiple-purpose hydro-power projects. In addition to improving navigation (for judicial purposes, at least), controlling floods, and regulating the river flow for irrigation, domestic, and municipal uses, the power production is sufficient to amortize the project's construction cost and guarantee payments to Arizona and Nevada in lieu of taxes, plus contributions to a basin development fund. Silt control and regulation of water quality are important incidental results.

The power rates were adjusted in 1941, on an amortization instead of the previous competitive market rate basis. The reasons for this change are explained in this paper, with detailed comparison of the two plans as regards allocation of construction costs.

Note.—Published in December, 1943, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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The Boulder Canyon Project has now gone to war in a big way, of course; but, even before the war boom and the earlier defensive rush period, its success was assured, with power output and cost repayments running far ahead of advance estimates. As of 1944, however, one near-by metallurgical (magnesium) plant requires 200,000 kw and every available kilowatt-hour is devoted to the war industries of Southern California. Stored water is also pumped for these industries and the municipalities around them through the Colorado River Aqueduct.

BOULDER CANYON PROJECT ACT

The Congress of the United States, by the terms of the "Boulder Canyon Project Act," approved on December 21, 1928, authorized the Secretary of the Interior

> "* * * to construct, operate and maintain a dam and incidental works in the main stream of the Colorado River at Black Canyon or Boulder Canyon * * * for the purpose of controlling the floods, improving navigation and regulating the flow of the Colorado River, providing for storage and for the delivery of the stored water thereof for reclamation of public lands and other beneficial uses exclusively within the United States, and for the generation of electrical energy as a means of making the project herein authorized a self-supporting and financially solvent undertaking; * * * also to construct and equip, operate and maintain at or near said dam, or cause to be constructed, a complete plant and incidental structures suitable for the fullest economic development of electrical energy from the water discharged by said reservoir; * * *."

As indicated by this preamble to the Act, the solution of the Colorado River problems, as plans were gradually formulated after investigations over a period of many years, necessarily required a multiple-purpose project. This was perceived long before such a conveniently descriptive term became so widely used and accepted in discussions of engineering economics. Largely to this diversity of objectives, and the serious international as well as interstate complications that were involved, was due the final authorization for the construction by the Bureau of Reclamation, U. S. Department of the Interior, with federal appropriations. Indeed, the Boulder Canyon Project is generally recognized as having been the pioneer in the field of major multiple-purpose hydro-power developments.

In addition to the numerous general purposes as listed, the removal of Colorado River silt (long infamous as often "too thick to flow even though too thin to plow") has proved of vital importance, for, without such silt control, neither power installations nor water diversions could be considered as economically and permanently feasible. (The reservoir has an estimated 300-yr life, as a minimum, which will be indefinitely prolonged by the construction of numerous planned upstream dams.) Likewise, the improvement of the quality of Colorado River water by decrease of its low-flow salinity and hardness, due to storage and mixture of the previously wasted and relatively pure flood flows, was immediately of real value. Such quality improvement has become even more evident as the first filling of Lake Mead (Arizona-Nevada) and two years (1941 and 1942) of large discharges (fortunately chiefly through the gates rather than from the purer surface stratum over the spillways) have continued the leaching of the gypsum-mixed sands included in the desert reservoir's flooded area. This improvement is only temporarily retarded by the 1943 low inflow of but 70% of normal. Even the improvement of navigation, generally dismissed as merely a legal fiction, has permitted the barging of ores, previously inaccessible, on a commercial scale, as well as a recreational patronage for fishing, bathing, sailing, and motor boating, to a most unexpected extent. Visitors to the power plant until it was closed to the public on December 8, 1941, averaged more than 1,000 per day for a total of nearly two million (Fig. 1), besides still larger numbers who merely admired the dam and scenic vicinity.

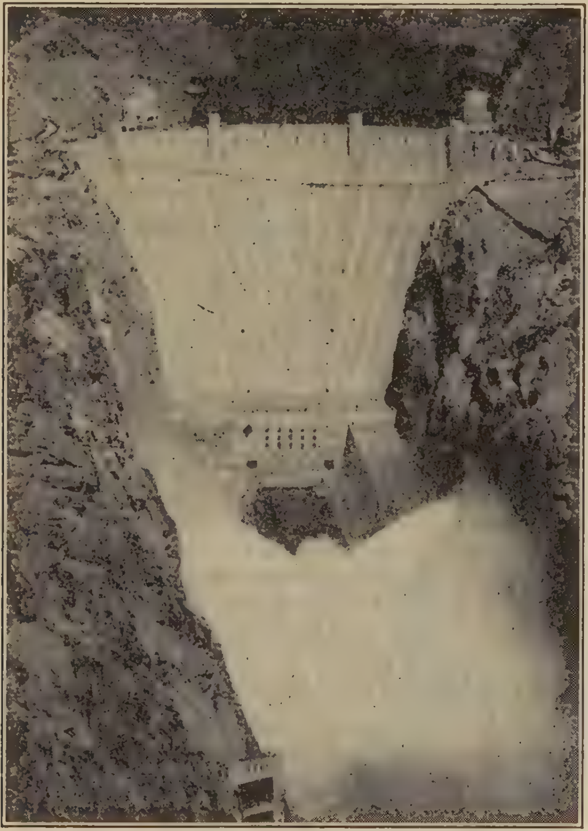

FIG. 1.—VISITORS TO THE POWER PLANT, UNTIL IT WAS CLOSED TO THE PUBLIC ON DECEMBER 8, 1941, AVERAGED MORE THAN 1,000 PER DAY

Evidently the Boulder Canyon Project has really accomplished a multiplicity of purposes.

A draft of the Swing-Johnson bill, as the project Act was generally known, was introduced in Congress as early as 1923, but enactment was delayed during five years of nearly continuous and heated debate over the basic policy of governmental ownership and construction of power dams. The question of a single-purpose versus multiple-purpose project also played a part in the controversy, as some opponents of government control of the high power dam favored instead a separate, relatively low dam at another site to be built by the United States solely for flood control, the immediate need for which seemed unquestioned. This would have left the canyon section of the Colorado River for private power development, probably taking the form of a series of several relatively low dams. The apparent economy of a multiple-purpose dam was a deciding factor in finally securing favorable Congressional action, as advocates of government ownership asserted (correctly, as is now proved) that power and other revenues would be amply sufficient to repay the cost of the project with interest. That the approval of the Boulder Canyon Project Act established a notable and important precedent is now very evident. Although the Bureau of Reclamation had previously constructed about fifteen power plants, incidental to its irrigation projects, their combined cost was less than 1% of the authorized investment for the Boulder Canyon Project. Few of these small installations had encountered serious opposition from public utilities, as they had offered little or no competition and were mostly in remote and undeveloped localities, at the time unprofitable for private power installations. On the other hand, the Lower Colorado River was considered a choice prize, for the control of which the competition was energetic and prolonged. Congressional and public sentiment finally swung definitely in favor of governmental construction of such hydro-power projects, as the decade following the passage of the Boulder Canyon Project Act has confirmed, with its series of great federal dams.

In line with the arguments that the project would be self-liquidating in fact, a definite plan of cost repayment was written into the Act (section 4-b)—

> "Before any money is appropriated for the construction of said dam or power plant, or any construction work done or contracted for, the Secretary of the Interior shall make provision for revenues by contract, in accordance with the provisions of this act, adequate in his judgment to insure payment of all expenses of operation and maintenance of said works incurred by the United States, and the repayment, within fifty years from the date of the completion of said works, of all amounts advanced to the fund under sect. 2(b) for such works, together with interest thereon * * * (at 4%)."

This requirement for bona-fide, enforceable repayment contracts, covering the estimated project cost, was followed strictly by the then Secretary of the Interior, Ray Lyman Wilbur, M.D., before President Herbert Hoover, Hon. M. Am. Soc. C. E., approved the first appropriation for initiating construction work on July 3, 1930.

Such an advance guarantee of cost repayment with interest made the Boulder Canyon Project unique among national developments, a record unshared as yet but only emphasized as to its rarity by the cases of numerous subsequent and comparable power dams constructed under federal auspices. The record is more notable in view of the rigid interpretation and enforcement given the repayment clauses of the Act, quite at variance with the long-established repayment practices on irrigation and other government projects (leniency having been often allowed, generally for good and sufficient reasons). The several contracts negotiated with the Los Angeles (Calif.) City Power Bureau, the Southern California Edison Company, and The Metropolitan Water District of Southern California, which were later supplemented by contracts with various other power distributors, represented probably the largest power transaction to date. They called for then-estimated payments over a fifty-year period of about one-half billion dollars for merely the primary project, not including costs of long transmission lines and complex distribution facilities. The enormity of such repayment promises and the seriousness and importance to all concerned of these contracts are now difficult to realize under subsequent changed conditions of more easily promoted projects as items of progressively greater relief, defense, and war budgets. The support of the Southern California Edison Company, in underwriting a large portion of the cost of this governmental and municipally-favored development, is especially notable, as inaugurating a period of enlightened public relationship and ending, regionally at least, a protracted and "totalitarian" war of uneconomic competition.

COLORADO RIVER DAM FUND

Early drafts of the Swing-Johnson bill provided for financing by a project bond issue but the Act as finally passed relied instead on normal Congressional appropriations. Patterned to some extent after the regular Reclamation fund, a special fund was established, designated as the Colorado River Dam Fund, to be available only for carrying out the provisions of the Act. Greatly simplifying the accounting procedure, and cost or repayment auditing reports, it has somewhat the relation to the U. S. Treasury of a subsidiary to a parent corporation. It appears to have served as a "small-scale" precedent for the subsequent depression era's Reconstruction Finance Corporation (as a means of financing public construction) and other government corporations and funds, which have become such a favorite and accepted method of federal administrative and accounting practice.

All advances to the Colorado River Dam Fund from the U. S. Treasury were required to be repaid with 4% interest, but a definite allocation of $25,000,000 was made to flood control to be repaid out of 62.5% of excess revenues, if and when available. If this repayment were not completed within the general fifty-year amortization period, ending May 31, 1987, repayment was to be continued out of the same proportion of the project's net revenues. Thus deferment was permissible for the repayment of the flood-control allocation, if this became necessary, but provision was made for full repayment finally, with 4% compound interest. The amount of this flood-control allocation appears to have been fixed arbitrarily, so far as is now evident, but is approximately the sum that might have been needed for a program of flood control by means of a low dam for this purpose solely, and a system of raised and strengthened levees. This solution would have given immediate, but evidently only temporary, relief from flood damages, due to relatively rapid filling with silt of a small reservoir. The federal obligation to furnish some degree of flood control on the Lower Colorado River was recognized, however, and this is probably the only case to date in which repayment (even though possibly on a deferred basis) of a government flood-control structure, or its allocated share of the construction cost, has ever been required or promised.

In addition to the flood-control allocation, other excess revenues of the Colorado River Dam Fund were allotted as follows:

> "If during the period of amortization the Secretary of the Interior shall receive revenues in excess of the amount necessary to meet the periodical payments to the United States as provided in the contracts executed under this act, then, immediately after the settlement of such periodical payments, he shall pay to the State of Arizona 18¾% of such excess revenues and to the State of Nevada 18¾% of such excess revenues.
>
> "After the repayments to the United States of all money advanced with interest, charges shall be on such basis and the revenues derived therefrom shall be kept in a separate fund to be expended within the Colorado River Basin as may hereafter be prescribed by the Congress."

Records of Congressional Committee hearings indicate that the intent of Congress was that Arizona and Nevada should receive some payments in lieu of taxes which would have been levied if the project had been privately developed. The amount is stated as having been determined on the basis of a fairly parallel case, the leasing of public oil lands, where the rule had become established that the state involved should receive 37% of the net revenue in lieu of taxes. The disposition of the balance of excess revenues, after completion of repayment of U. S. Treasury advances to the fund, was such a remote problem as not to excite immediate interest, so that it was merely left to future action of Congress, although allotted for Colorado River Basin developments if and when available.

Original Repayment Contracts

As repayment contracts were required by the Act in advance of even the beginning of construction, their terms were necessarily based on estimated project costs. Power production had also to be estimated from stream-flow records and reservoir operation studies, at first being called 4,240,000,000 kw-hr annually of firm power initially, subject to some gradual depletion. This amount was taken as 100% in the allocations of power, although a later decision to add 25 ft to the proposed height of the dam (550 ft) yielded a further block of 90,000,000 kw-hr of firm power annually. Applications for the power originally totaled more than three times the estimated production, with the City of Los Angeles and the Southern California Edison Company each asking for all of the available power. Final power allotments were made only after prolonged hearings and many protests and compromises, with eight contracts resulting from twenty-seven applications, some of which were conditional or indefinite, however. The repayment guarantees involved three main contracts:

City of Los Angeles	37%
Metropolitan Water District	36%
Southern California Edison	27%
Available power	100%

The City of Los Angeles underwrote 6% for other municipalities (which finally amounted to 4.0946%), the Southern California Edison Company similarly underwrote 0.9% for each of two other power companies, and the city and the company each were subject to a drawback of 18% by the states of Arizona and Nevada. As the Metropolitan Water District had not at the time held its aqueduct bond election, legal questions were anticipated as to the strict enforceability of its contract. The guaranteed portion of the project cost was therefore finally covered by the City of Los Angeles and Southern California Edison Company contracts for but 64% of the estimated power production.

These contracts were required to provide revenues sufficient in the judgment of the Secretary of the Interior to amortize the project cost, except for the flood-control allocation; but in addition the Act required that the contracts "shall be made with a view to obtaining reasonable returns" as found to be "justified by competitive conditions at distributing points or competitive centers." The latter condition would necessarily have been a determining factor in fixing the power rate even in the absence of such a statutory direction. In this case, the only significant power market was in Southern California, located over large oil deposits, so that the cost of energy provided by oil and gas necessarily fixed the comparative value of Colorado River hydro power, with proper allowance for transmission costs. Studies made for the Secretary of the Interior on this basis indicated a value at the switchboard for the use of the available falling water of about 1.63 mills per kw-hr. Therefore, power allocations were made on the basis of this rate, with the corresponding secondary power rate fixed rather arbitrarily at 0.5 mill per kw-hr.

ORIGINAL COST ALLOCATIONS

With firm contracts executed on April 26, 1930, for the entire power production, and the power rate fixed and agreed to as described, it was widely assumed that an allocation had been arrived at not only for the project cost but also for a dependable surplus, payable to Arizona, Nevada, and the Colorado River Basin Development Fund. Technically, to be strictly accurate, no such definite allocation of costs really had been made, because the Act prescribed that upon the demand of either party to the power contracts, at the end of fifteen years (in 1945) and every ten years thereafter, the power rate should be readjusted as found by the Secretary of the Interior to be justified by changed competitive conditions. In 1930 this factor seemed to mean little, as the first adjustment date was so far into the future, and elaborate, detailed computations and studies were prepared on the assumption of a uniform rate of 1.63 mills per kw-hr. Misleading conclusions resulted and the expenditure of anticipated surplus revenues was budgeted by those interested as if the fund accumulation were certain and automatic. As the national economic depression became more severe and protracted, affecting seriously both power markets and the all-important price of oil, or equivalent cost of gas, it became apparent that under prevailing competitive conditions the power rate had been set higher than was warranted or than could be maintained, in all probability, after the 1945 readjustment. Various alternate studies of anticipated revenues, with corresponding allocations of cost, were prepared as a basis for discussions of the resulting problems among representatives of the government, the power contractors, and the states interested in excess revenues. Several of these studies that were officially issued by the Bureau of Reclamation and are still of interest, for purposes of comparison, have been tabulated in summary form herein, following a listing of the project costs that serve as a base for the amortization computations.

Cost estimates used in negotiating the original power contracts for the Boulder Canyon Project are derived from a statement[2] by Secretary Wilbur:

Estimated cost (including interest during construction but excluding flood control)	$ 82,674,907
Interest at 4% during 50-yr period, excluding flood control	108,107,007
Interest charges on accumulated deficit	63,973
Estimated operation and maintenance (O & M)	7,262,557
Estimated depreciation	8,875,553
Total requiring "guaranteed" repayment	$206,983,997
Anticipated power revenues at 1.63 miles per kw-hr	
City of Los Angeles	$121,310,549
Southern California Edison Company	88,523,915
Subtotal	$209,834,464
Metropolitan Water District of Southern California	118,031,886
Total estimated falling water revenues	$327,866,350

The distribution of surplus revenue, if and when realized, according to the terms (as then interpreted) of the Boulder Canyon Project Act is:

Repayment of flood-control allocation	$ 25,000,000
Interest at 4% on flood-control allocation	20,981,303
18¾% of surplus to Arizona	22,665,441
18¾% of surplus to Nevada	22,665,441
Balance for Colorado River Basin Development Fund	29,570,168
Total estimated surplus at 1.63-mill rate for fifty years	$120,882,353

In addition to the foregoing totals, power machinery costs were estimated at $17,717,000. It was originally contemplated that this investment would be financed by the power-plant lessees, so that its repayment was not covered in the original power contracts. Therefore, although finally financed with federal funds, along with the remainder of the project, power equipment costs are being repaid over a 50-yr period with interest by means of separate contracts with the generating agents, the City of Los Angeles, and the Southern California Edison Company. Actual project construction costs, as of September, 1940[3]

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[2] House Hearing, 71st Cong., 2d Session, Second Deficiency Appropriation Bill, 1930, p. 950.

[3] *Bulletin No. 1*, Boulder Canyon Project Final Reports, Pt. IV, 1941, pp. 26–28; also Exhibit 33, September 14, 1940.

(presented at the Rate Adjustment hearings at Los Angeles) can be summarized in abbreviated form as follows:

Feature	Costs
Dam and Reservoir	
Right of way and mining claims	$ 782,219
Surveys, tests, and examinations (field)	766,335
Stripping canyon walls	627,499
River diversion, cofferdam, dredging	2,348,984
Diversion works (four tunnels)	26,996,604
Main dam and miscellaneous	23,578,724
Spillways	5,868,792
Outlet works	16,337,587
Permanent cableway and landings	607,913
Subtotal	$ 77,914,657
Power Production	
Power-house substructure	$ 5,094,098
Power-house building, water, oil, and air systems	3,726,170
Penstocks	3,330,950
Electrical equipment	13,624,964
Hydraulic, mechanical, and general station equipment	6,620,512
Switchyards	3,265,609
Subtotal	$ 35,662,303
General Plant	
Roads, railroad, construction and concrete plant	$ 2,491,967
Boulder City, Nev., government buildings, telephone system, etc.	2,182,565
Materials and supplies (field)	305,511
Subtotal	4,980,043
Total investment (field)	$118,557,003
Overhead and Undistributed	
Engineering and inspection (Denver, Colo., office)	$ 2,875,341
Concrete and other research	624,431
Colorado River Board and consultants	83,887
Legal, general, printing, furniture, etc. (Denver)	406,602
General and overhead (Washington, D. C., office)	282,076
Secondary investigations, Colorado River Basin	314,416
Subtotal	4,586,753
Total project cost to September, 1940	$123,143,756

Installations of additional power-plant equipment, continuing into 1944, and other permanent work of a minor nature, raised the project cost to approximately $130,000,000, with the completed cost to be somewhat greater. The most recent available studies, dated February 19, 1941, indicate a final reimbursable cost of $82,878,131 for the new power (falling water) rate determinations, not including generating machinery and the deferred flood-control allocation. This is not exactly comparable with the 1930 estimate of $82,674,907, as the latter includes interest during construction at 4% and also interest during construction at the same rate on the deferred flood-control allocation of $25,000,000. Under the present revised Act and regulations, the latter is considered to be the sum of the first project expenditures instead of being an amount deducted from the construction cost on May 31, 1937, as previously, so that interest during construction on this amount is now omitted, and on the remainder of the cost is computed at a 3% rate. This adjustment for revised interest during construction is estimated as amounting to about $5,000,000 which may be considered the approximate overrun on the cost of the dam and power-plant building, as compared with the 1930 estimates. Most of this increase appears to have resulted from the inclusion of exceptionally thorough and costly research, basin surveys not directly related to project construction, very considerable overhead, and unanticipated expenditures for Boulder City and for guide, scenic, and protection facilities, etc. The actual field construction cost of the dam and power-plant building evidently has exceeded the 1930 estimates but slightly, if at all.

Early rapid progress on the dam was aided by the lowest runoff year (1934) on record, and, with the favorable depression prices of the same period, financial studies made in 1936 (Denver Drawing No. 45-D-10,327 dated September 18, 1936) were allowed to become optimistic to the extent of estimating the reimbursable cost as of May 31, 1937, at $75,927,921. Due apparently to cost accountants being overworked, a considerable lag occurred before the effect of large overhead and indirect costs became fully evident. Annual O & M costs were estimated at but $102,000, compared with $145,000 in 1930 (present studies include O & M costs at $554,000 annually) and, on the other hand, allowances for secondary power and water storage revenues were somewhat more liberal than in either 1930 or at present (1944). The combined effect of these various factors, in addition to the previously mentioned faulty assumption of a 1.63-mill rate for fifty years, was to increase the estimated payments to Arizona and Nevada by 30% and to the Colorado River Basin Development Fund by about 90%, as compared with the quoted 1930 anticipations. Naturally, these larger amounts became fixed in the minds of those interested as being so certainly available within the near future that subsequent smaller but more accurate versions of probable surplus revenues were correspondingly unpopular and dismissed as propaganda.

Beginning in 1937, however, such financial studies were revised to conform to the quoted requirement of the Act (section 5) that payments into the separate Colorado River Basin Development Fund should not be made until completion of all repayments to the United States. Earlier interpretations had assumed that amortization of the dam's cost would cover the permitted 50-yr period, whether such a long time was necessary or not, with surplus funds first amortizing the flood-control allocation and then being paid to the separate fund.

On the corrected basis, however, the federal government share of the surplus was devoted to accelerated project cost amortization, with initial payments into the separate fund being postponed from about 1960 to the very end of the amortization period, 1985 or later, as well as being greatly decreased in amount. The last such study, prepared by the Bureau of Reclamation as of May, 1940, conforming to the original Act but with then-current assumptions for the reimbursable total and estimated O & M costs, and involving varying future power rates, both firm and secondary, after 1945, is available in print.[4] Detailed tabulations are shown for firm power rates of 1.30 and 1.08 mills after 1945, with the secondary rates proportionately decreased. Using identical assumptions and computation methods, similar tabulations have been prepared for 1.63 mills and other future firm rates. These several studies were then brought a step further up to date by corrections for the 1941 estimate of future O & M costs of $554,000 annually (instead of $270,000 annually). They are summarized in Table 1 on a present-value basis, to show the varying allocations of revenues and costs that might have occurred under the terms of the original Act, according as future competitive power conditions more or less severely affected the contracted revenues. Conforming to the May, 1940, O & M estimate, the fixed charges have a present value, computed at 4% as of May 31, 1937, of 100.3 million dollars, including past and deferred investment of 91.2

TABLE 1.—ESTIMATED SURPLUS REVENUES AND VARYING ALLOCATIONS[a] IN PERCENTAGE OF ACTUAL-PLUS-DEFERRED INVESTMENT[b]

Firm power rate after 1945 (mills per kw-hr)	SURPLUS		ARIZONA AND NEVADA		DEVELOPMENT FUND[c]		REPAYMENT[d]	
	Millions of dollars	Percentage of investment	Millions of dollars	Percentage of investment	Millions of dollars	Percentage of investment	Millions of dollars	Percentage of allocation
(a) ANNUAL COST OF OPERATION AND MAINTENANCE AS ESTIMATED IN MAY, 1940 ($270,000)								
1.63	45.0	49	17.3	19.0	2.7	3.0	25.0	100
1.555	40.0	44	15.0	16.4	0.0	0.0	25.0	100
1.40	29.7	33	11.1	12.2	0.0	0.0	18.6	74
1.30	22.9	25	8.6	9.4	0.0	0.0	14.3	57
1.163	14.0	15	5.2	5.7	0.0	0.0	8.8	35
1.08	9.0	10	3.4	3.7	0.0	0.0	5.6	22
(b) REVISED ANNUAL COST OF OPERATION AND MAINTENANCE AS ESTIMATED IN FEBRUARY, 1941 ($554,000)								
1.63	40.0	44	15.0	16.4	0.0	0.0	25.0	100
1.555	35.0	38	13.2	14.5	0.0	0.0	21.8	87
1.40	24.7	27	9.3	10.2	0.0	0.0	15.4	62
1.30	17.9	20	6.7	7.3	0.0	0.0	11.2	45
1.163	9.0	10	3.4	3.7	0.0	0.0	5.6	22
1.08[e]	9.0	10	3.4	3.7	0.0	0.0	5.6	22

[a] Expressed as present values, at 4% interest rate, as of May 31, 1937. [b] Taken as 91.2 million dollars on a present-value basis as of May 1, 1937. [c] Colorado River Basin Development Fund. [d] Flood-control allocation. [e] With a firm power rate of 1.08 mills, $5,000,000 (present value) of project investment would have been unpaid in a 50-yr period.

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[4] Hearings on H. R. Doc. No. 9093, Committee on Irrigation and Reclamation, 76th Cong., 3d Session, p. 177.

million dollars in dam and power-plant building (but not hydraulic machinery), plus operation and maintenance, depreciation, and replacements. All of these costs would have been repaid with any firm power rate, after 1945, of 1.08 mills per kw-hr or greater. Correcting the O & M estimate to the 1941 basis, the present value of all such fixed charges is increased to 105.3 million dollars. A firm power rate of at least 1.163 mills per kw-hr is required to amortize this amount within fifty years at 4% interest. The lower rate of 1.08 mills, if applied after 1945, would have repaid only 95% of the reimbursable project cost, in addition to O & M and depreciation charges. The varying amounts of surplus, and its allocation for the several specified purposes for different firm power rates after 1945, are given in Table 1 for comparison.

This tabulation is an extremely condensed summary of voluminous, detailed cost and revenue studies, referred to previously but not included herein because they are now obsolete and of interest only in reviewing the evolution of the present power contracts and rates. A full explanation of their engineering, legal, and accounting complexities would also be most voluminous, and likewise obsolete. In further explanation of Table 1, however, it may be noted that in all cases included, except the first (1.63 mills power rate for fifty years and preliminary estimate of O & M cost), the present value of the hypothetical payments to Arizona and Nevada is 37.5% of the corresponding present value of the surplus, as is specified in the Boulder Canyon Project Act. With the assumptions of the first case, however, the flood-control allocation of $25,000,000 would have been amortized in about thirty-eight years, as a result of the larger revenues. Then for the next nine years the government's 62.5% share of the surplus would have been applied to accelerating the amortization of the project construction cost. During the last three years of the life of the contracts, Arizona and Nevada would not have shared in the surplus (the Act limited their payments to the amortization period), but instead the entire surplus would have been paid into the Colorado River Basin Development Fund, for subsequent appropriation by Congress. The actual disposition of the surplus revenues would thus have been as follows, on a present-value basis:

	Million dollars
Net surplus (1.63 mills rate for fifty years)	45.0
"Surplus" applied to project cost amortization	+3.8
Surplus of last three years for Colorado River Basin Development Fund	−2.7
Portion of surplus shared by Arizona and Nevada	46.1
37.5% to be divided between Arizona and Nevada	17.3
(or 38.5% of net surplus and 35.5% of full 50-yr surplus)	

As some estimates of the probable firm power rate after 1945, revised on the basis of changed competitive conditions, ranged as low as 1.0 mill per kw-hr, and as this rate would have repaid (within fifty years and at 4% interest rate) only 88% of the investment in the dam and power-plant building, considerable concern arose as to the solvency of the project finances. By the terms of the original Act. the readjustment of the power rate in 1945, and at 10-yr intervals thereafter, was to be the responsibility of the Secretary of the Interior, as might be justified by competitive conditions. The opinion was frequently expressed that a rate lower than required to amortize the reimbursable portion of the project investment within fifty years was not permitted by the Act; but the legality of this idea was in disagreement and was indeterminate. This result was not improbable, however, as the manner of such a rate redetermination was left vaguely undefined by the Act and would be complex and difficult as well as seriously controversial. Even if the rate could be maintained high enough to insure repayments to the U. S. Treasury, however, the payments to the states and the basin fund were obviously in jeopardy. General recognition of this fact was the cause and basis of the negotiations for a revision of the original Act.

Revenues from Water Storage

Project cost allocations and amortization have been analyzed without special reference to anticipated water revenues because estimates of the latter, as included in the various official financial studies of the Boulder Canyon Project, have amounted to only 2.5% to 3% of the total expected revenues. Furthermore, no adjustment was required or has been made with respect to water charges, all recent discussions having involved power questions only. Water storage revenues are incidental and relatively minor in importance, in the case of this multiple-purpose project, partly because conditions were so favorable for power development. The high head and large generating units, rated at 82,500 kva, made possible a low power cost; and the large power demand, even with competitive marketing conditions of the depression period, has kept the project self-liquidating. More direct and fundamental factors control the water revenue that can be secured for the project.

The Boulder Canyon Project Act specifically exempted lands in Imperial and Coachella valleys, California, from any water storage charge. The first area is now, and the latter will soon be, supplied by the All-American Canal, also constructed by the Bureau of Reclamation under the authorization of the same Act. The justification for this exemption is that large appropriative water rights of very early priority had been established by such Lower Colorado River projects as Palos Verdes, Imperial, and Yuma valleys. Upstream developments gradually encroached on these prior water rights, causing more and more frequent shortages of irrigation water. These were climaxed by the most severe drought on record in 1934, just a few months before storage began in Lake Mead on February 1, 1935. Such water shortages were largely limited to the Imperial Valley because its diversion point was then below all others on the river, and litigation against junior upstream diversions had seemed unavoidable until the Colorado River Compact was negotiated. Due to these circumstances, the California ratification of the Compact was conditional on the construction by the federal government of a reservoir of at least 20,000,000 acre-ft capacity, at or below Boulder Canyon on the Colorado River.

A considerable portion of the investment in the Boulder Canyon Project may therefore be conceived as a contribution to prevent threatened litigation, and to stabilize past as well as future water rights and diversions in the Upper as truly as in the Lower Colorado River Basin. Except for the effect of such upstream diversions, now legalized by the Compact and the storage operations of Lake Mead, the natural, unregulated flow of the Colorado River would have been practically sufficient for Imperial Valley irrigation. Present exemption from reservoir storage charges is merely a recognition of this fact.

Other diversions from the Lower Colorado River are not definitely exempt from Boulder Canyon Project storage charges, and the reasonableness of such a charge (fixed at $0.25 per acre-ft) as a contribution toward the repayment of the cost of the project was recognized, for example, in the water contracts for the Colorado River aqueduct of The Metropolitan Water District of Southern California and for the City of San Diego, Calif. Even with the later priority of the water-right appropriation for the aqueduct, however, diversions might have been made during half or more of the time in most years from the river's natural flow, without dependence on storage. The present (1944) charge of $0.25 per acre-ft for all water diverted by the aqueduct, therefore, is in effect, about a $0.50 charge for water really derived from Lake Mead storage. Incidental benefits are obtained as regards desilting and improvement of water quality during non-flood periods, so that the charge on all water diverted was agreed to, it being realized that no accurate or practical separation or determination of natural flow could now be made. A rate of $2.00 per acre-ft was demanded at various times by Arizona representatives, but this was considered to be inequitable and impossibly exorbitant, amounting to a 22% increase in the $8.00 per acre-ft base price of water furnished by the $200,000,000 aqueduct. The rate of $0.25 per acre-ft as determined by the Secretary of the Interior was found justified on a showing that, if the portion of the Colorado River runoff made utilizable for the first time by Lake Mead regulation (about 10,000,000 acre-ft annually) was charged approximately this storage fee, the actual reservoir cost, omitting power production features and the flood-control allocation, could be amortized within fifty years from this revenue source alone, disregarding power revenues. Power revenues are available, of course, amounting (at the average reservoir power head) to $0.73 per acre-ft with the original rate of 1.63 mills per kw-hr, and to $0.52 per acre-ft with the 1941 adjusted rate of 1.163 mills per kw-hr.

On this comparative basis, the water charge does not seem disproportionate to the relative services rendered. It must be emphasized, of course, that the charge is not for the sale of water but rather for storage service only. Prior to 1943, the only contracts for water deliveries other than to the exempt irrigation projects were with The Metropolitan Water District of Southern California and the City of San Diego. Whether a water charge would be made for other projects, proposed or under construction, that will involve diversions from the Lower Colorado River, was not previously evident, as no delivery contracts had been issued for such projects. To be conservative, therefore, all previous financial studies for the Boulder Canyon Project have neglected the possibility of such additional water revenues. In May, 1943, a water contract was announced with the State of Nevada for diversions from Lake Mead, above Boulder Dam, with a charge of $0.50 per acre-ft, including about $0.15 per acre-ft for storage service and $0.35 per acre-ft as compensation for kilowatt-hours lost at Boulder Dam. In present drafts of a proposed water contract with the State of Arizona (not as yet approved by the Secretary of the Interior) the same charge is fixed for diversions from Lake Mead, and a maximum of $0.25 per acre-ft for downstream diversions. An actual storage charge of $0.15 per acre-ft for such agricultural diversions now seems probable. In view of these recent developments, the project's total water revenues by 1987 may considerably exceed the $7,500,000 previously estimated.

BOULDER CANYON PROJECT ADJUSTMENT ACT

The 50-yr amortization period of the project began on June 1, 1937, with the announcement of the Secretary of the Interior that storage and installed generators were sufficient for the production of 1,250,000,000 kw-hr annually for the first power contractor, the City of Los Angeles. Power had already been sold at the secondary rate, however, amounting to $247,000 under interim contracts. During the construction period of seven years after the execution of the contracts, various events had occurred which forced the conclusion that the 1930 power rates were inequitable and excessive, and could not long be sustained. The competitive value of the hydro power had fallen, as a result of decreases in the cost of fuel and in the capital costs of steam plants, as well as improvements in the art of steam power generation. The record indicated that advances to the construction fund by the U. S. Treasury had been secured at an interest rate averaging about 2.75% and that to charge the project a rate in excess of 3% would involve an unintended and undue profit. The United States had constructed or initiated numerous large multiple-purpose projects in other sections of the nation on a rate basis which abandoned the competitive plan previously applied, in favor of a power rate fixed by the amount needed to amortize such part of the investment as was determined to be allocable to power, plus costs of operation, maintenance, replacements, etc. This policy was later generalized by statute as to all reclamation projects. Several early determinations of allocations, particularly for navigation and flood control, appeared to be on a relatively liberal basis, resulting in power rates very unfavorable for the Southwest, as regards regional competition for heavy industry and electro-metallurgical plants. Consequently, the Boulder Canyon Project power contractors found themselves obligated to pay, until 1945, a rate considered excessive whether measured by the requirements of the Act or externally, by comparison with the newer projects.

Accordingly, a request for a review of the rates was filed in 1937 with the Secretary of the Interior, in line with a right granted when the initial allocation of the project energy was made. Preliminary conferences showed that a change from the competitive to the amortization basis, however, involved unavoidable differences of interest among the seven states of the Colorado Basin. Two years of almost continuous discussion and negotiation followed, with frequent formal conferences, before all such interests were brought into agreement. As the surplus revenues of the original Act would be entirely eliminated on an amortization basis, determination by compromise and agreement was necessary as regards payments to be made in lieu of taxes to Arizona and Nevada, and additional payments to the Colorado River Basin Development Fund, for the benefit of all seven of the states involved.

Numerous water problems had required previous conferences, in efforts to avoid litigation among the states, so that the power problems came naturally before the same conference groups under the auspices of the several governors and most frequently under the chairmanship of the late Hon. Henry H. Blood, then governor of Utah. As the governors were generally unable to give the time personally for the more frequent conferences, and the difficulty of changing personnel and lack of continuity of program between sessions became apparent, the now well-known "Committee of 14 and 16" began to function. It consists of two representatives of each of the seven basin states, appointed by each governor or by state boards appointed by him, as its only authority is derived from the various governors, being without legislative authorization. To this Committee of 14 are added, for consideration of power problems, two representatives of the California power contractors (Los Angeles and the Metropolitan Water District) which has the further effect of equalizing the basin representation. The committee has operated under the very able chairmanship of Clifford H. Stone of Colorado, to whom in a large measure is due the credit for reaching an harmonious agreement among the states in 1939 on the question of allocation of Boulder Canyon Project power revenues. The committee recommendation that Arizona and Nevada should each receive $300,000 annually, with $500,000 annually going to the Colorado River Basin Development Fund, was adopted by Congress in the Boulder Canyon Project Adjustment Act, approved July 19, 1940.

In view of the immediate benefits to be derived by the three Lower Basin states, a further agreement was made that expenditures from the Development Fund should be limited to the Upper Basin states until 1955. The first three annual payments were applied to the completion of surveys and studies for a "Comprehensive Plan of Basin Development," long under way by the Bureau of Reclamation. Division of the fund among the Upper Basin states was required to be on an "equitable" basis (whatever that might mean), and not until October, 1941, was agreement finally reached to interpret this as meaning "equally." This is an example of the committee's help in avoidance of litigation over the allocation of the project revenues, although protracted discussions were required at times. Without such a forum to give an opportunity for thorough debate of the controversial problems, and final agreement by compromise before the questions could become political issues at Washington, it seems that passage of the Adjustment Act would have been impossible. Even then, however, difficulties were not ended, as this Act required definite repayment contracts to be entered into by June 1, 1941, covering 90% of the firm power or otherwise the Act would be void. Not until May 29, 1941, were the contracts finally signed, making the Adjustment Act effective.

Hearings on these contracts had been convened at Los Angeles on August 12, 1940, continuing until December 6, before a special board of three representing the Secretary of the Interior. The resulting record includes about 2,000 pages of transcript of oral evidence and argument, besides 91 exhibits, many of these being lengthy reports. Chief questions at issue involved details of the agency (generating) contracts; generating charges; time and method for future readjustments of rates, to insure full repayment to the government; O & M cost estimates; allowances for replacement and depreciation; estimates of firm and secondary energy, available and also marketable; and of greatest interest, the new energy rate. Only the latter point appears to be of importance to the purpose of this paper, as its determination finally achieved the definite cost and revenue allocation that had been lacking under the terms of the original Act. Slight readjustments are still provided for, as of June 1, 1947, and at 5-yr intervals, to correct for excesses or deficiencies in estimated revenues; and annually, to allow for the effect of variations in actual cost of operation, maintenance, and replacements. The latter adjustments increased the firm power base rate (fixed at 1.163 mills per kw-hr for 1937 to 1942) to 1.172 for 1943 and 1.190 for 1944. A proportionate decrease to 0.34 mill per kw-hr for secondary power in the years 1937 to 1942 was changed to 0.346 for 1943 and 0.357 for 1944. The annual secondary rate is to be computed from future adjusted firm rates as the average of (1) a rate which bears the same ratio to the firm rate as 0.3 to 1.1, and (2) a rate which is 0.8 mill less than the firm rate; or otherwise expressed, as the firm rate times 0.6363636, less 0.4; but not less than 0.2 mill per kw-hr.

With similar infinitely detailed and legalistic exactitude, every possible or imaginable contingency of the future as affecting the power rate structure has been provided for at painful length in the several contracts and general regulations, now in force and effect. Revising the original power allocations to allow for the increased power production with the 25-ft higher (575 ft) dam, which change had caused the first allocations to total in excess of 100%, the present power contracts result in the following firm energy allocation:

Allocation	Percentages
Metropolitan Water District of Southern California (for aqueduct pumping only)	35.2517
State of Nevada (for use in Nevada only)	17.6259
State of Arizona (for use in Arizona only; no contract as yet)	17.6259
City of Los Angeles, Calif.	17.5554
Southern California Edison Company	7.0503
City of Glendale, Calif.	1.8475
City of Pasadena, Calif.	1.5847
California Electric Power Company	0.8813
City of Burbank, Calif.	0.5773
Total	100.0000

The power blocks for the two states are equally underwritten by the City of Los Angeles and Southern California Edison Company, with drawback privileges as formerly. Not to exceed 20,000-kw peak demand is reserved for the United States, to be charged equally against the Los Angeles and Edison Company allocations for distribution in Boulder City or for other government camps and uses. This is in addition to power for dam operation, which is to be furnished without charge and included with station losses.

Conforming to the exact terms of the Adjustment Act, the general regulations and the foregoing contracts, a detailed financial operations study now indicates estimated project revenues over the 50-yr period to May 31, 1987, of $256,303,360, which are sufficient to repay the reimbursable investment in the dam and power-plant building (estimated for rate determinations at $82,878,131 as of June 1, 1941, including 3% interest during construction, or about 82.5 million dollars present value as of June 1, 1937, including some allowance for deferred construction), within fifty years amortized at a 3% interest rate. In addition, all estimated operation, maintenance, and replacement costs are covered, and Arizona and Nevada will each receive $15,000,000 during the fifty years, with $25,000,000 going to the Colorado River Basin Development Fund.

These allocations are compared in Table 2, on a present-value basis, as in Table 1, with the results under the original Act for various firm power rates after 1945, and with the various payments capitalized at 3%. Table 2 shows that, under the terms of the Adjustment Act, Arizona and Nevada will each receive 4.2 times as much revenue in lieu of taxes from the Boulder Canyon Project as they were assured of receiving by the original Act, and that only in case the average revised firm power rate after 1945 had exceeded 1.52 mills per kw-hr would the states have received as much revenue as they are now assured of getting. This is equivalent, on a present-value basis, to 18% of the investment in the dam and power-plant building. Likewise, the Colorado River Basin Development Fund will receive annual payments with a present value of 15% of the project investment, instead of the probability (almost certainty) of obtaining nothing under the original Act. A general understanding of these comparisons, at least on an approximate qualitative basis, explains the high degree of agreement that was finally attained in compromising the interests of the various states of the Colorado River Basin.

TABLE 2.—ALLOCATION OF ESTIMATED REVENUES AND COSTS, AS PRESENT VALUES, AT 3%, AS OF MAY 31, 1937

Firm power rate (mills per kw-hr)	DAM AND POWER-PLANT BUILDING[a]		REPAYMENT[b]		ARIZONA AND NEVADA		COLORADO FUND[c]	
	Million dollars	Percentage of investment	Million dollars	Percentage of allocation	Million dollars	Percentage of investment	Million dollars	Percentage of investment
(a) POWER RATE, ETC., OF THE ADJUSTMENT ACT								
1.163	82.5	100	....[d]	....[d]	14.9	18	12.4	15
(b) VARIED POWER RATES AFTER 1945[e] FOR COMPARISON WITH ADJUSTMENT ACT								
1.00	97.0	118	5.7	23	3.5	4	0.0	0
1.08	103.6	126	5.7	23	3.5	4	0.0	0
1.163	110.3	134	5.7	23	3.5	4	0.0	0
1.30	110.3	134	12.8	51	7.7	9	0.0	0
1.40	110.3	134	18.6	74	11.3	14	0.0	0
1.555	110.3	134	26.3	105	15.8	19	0.0	0
1.63	110.3	134	30.2	121	18.1	22	0.0	0

[a] Repaid within fifty years. [b] Flood-control allocation repayment. [c] Colorado River Basin Development Fund. [d] Deferred beyond the 50-yr amortization period. [e] Present values of costs and payments required by the original Act but capitalized at 3% interest rate (as allowed by the Adjustment Act).

By the Adjustment Act, the government obtained certainty as to the amortization of its investment, less the flood-control allocation, within a 50-yr period, at a 3% interest rate, pronounced reasonable under the circumstances. Repayment of the flood-control allocation is definitely deferred beyond the 50-yr period; but the 1945 revision of power rates, under the original Act, would almost certainly have accomplished the same effect, at least as to 77% of the allocation.

On the other hand, assuming for purposes of illustration that the new firm power rate of 1.163 mills per kw-hr is fair and reasonable on a competitive market basis (as seems indicated by its acceptance by all of the power contractors), it may be considered that, on a 3% present-value basis, the government surrendered the possibility of a profit of about $27,800,000 (the difference between 4% and 3% interest charges) besides deferring for fifty years the dependable repayment of $5,700,000 toward the flood-control allocation. The recipients of this relinquished profit vary for different assumptions of future competitive rates, but, on the basis of the present rate as stated, the "split-up" would appear to be as follows:

Arizona (additional revenue)	$ 5,700,000
Nevada (additional revenue)	5,700,000
Colorado River Basin Development Fund (additional revenue)	12,400,000
Power contractors (reduced payments, 1937 to 1945)	9,700,000
Total	$33,500,000

The savings to the power contractors will be divided among power consumers in Arizona, Nevada, Utah (slightly), and Southern California, the chief beneficiary; but this is considerably less than the 100-million-dollar gain which the power contractors were often asserted to be obtaining, during the years of consideration of the Adjustment Act. With the Colorado River Basin Development Fund probably to be divided about equally among the four Upper Basin states until 1955, the indicated advantage of the Adjustment Act to each Upper Basin state is computed at $2,600,000, and to each of the states of Arizona and Nevada, $6,700,000 in addition to $1,750,000 apiece to the latter two under the terms of the original Act (all amounts as of June 1, 1937, discounted at 3%). After 1955 it may benefit Nevada and Arizona projects also. California is most unlikely to share in the fund then, because of the earlier completion of all projects having an available water supply. It is not implied that the 1945 revised firm power rate, by the original Act on the basis of competitive market conditions, would have been exactly 1.163 mills per kw-hr or even approximately this amount; but a similar comparison of individual benefits can be derived from Table 2 for any other revised rate that may be assumed. The illustration given, however, shows at least that the division of the benefits allocated by the Adjustment Act was not merely or solely in proportion to the Congressional voting strength of the various states, for example. Other factors also received consideration, such as anticipated receipts under the terms of the original Act and competitive power market conditions, regional as well as local.

The government, of course, ends by owning the entire project including replacement and depreciation funds for items subject to loss with use, and also retains final discretion as to the expenditure of the Colorado River Basin Development Fund. The foregoing "split-up" of this fund within the Upper and Lower basins, prior to and after 1955, is simply in accordance with the recommendation of local interested officials, which Congress may or may not follow exactly.

In refreshing contrast, from an engineering point of view, to the foregoing allocations, which are so largely dependent on legal and accounting, as well as political, considerations, is a novel rate item in the November, 1941, issue of *Intake* (Los Angeles Water and Power Department publication). New concepts in physics establish a definite relationship between energy and matter, making it possible to assign a weight to electrical energy and compare it with other commodities on the basis of cost per pound. At present Los Angeles retail prices, electricity costs about $20,000,000 per lb or two thirds the price of radium. As the power production from the Boulder Canyon Project in 1943 was about six "pounds" of electrical energy, on this futuristic basis, the adjusted falling water charge is equivalent to $900,000 per lb, or about the market valuation for fine, large cut diamonds. Although this fact certainly was not given much, if any, consideration in the project's cost allocation and power rate determination, to an engineer fully appreciative of the Boulder Canyon Project, it does not seem too far-fetched to value the output of the project as highly as precious jewels.

SUMMARY

Revised power contracts for the Boulder Canyon Project (Arizona-Nevada), dated May 29, 1941, provided for the amortization of the cost, at 3% interest rate, by terms of the Adjustment Act of July 19, 1940. A power (falling water) charge of 1.163 mills per kw-hr was reduced from the 1930 rate of 1.63 mills per kw-hr, as the original Act's competitive basis for power rate determination was abandoned. Depression conditions had affected adversely the competitive rate factors (price of oil and natural gas, cost of steam plants, etc.) so that the rate revision required by the original Act, in 1945, would probably have resulted in approximately the same or perhaps a greater rate reduction.

This loss of revenues would have decreased or even eliminated the project payments to Arizona and Nevada, which are considered to be in lieu of taxes, and also would have ended the chance of payments to the Colorado River Basin Development Fund which were not available in any case until about 1985. Even the repayment of the government's investment in the dam and power-plant building might not have been completed within fifty years as planned. Therefore, by agreement among the United States, the power contractors, and the seven Colorado River Basin states, as represented by their "Committee of 14 and 16," the interest rate was reduced from 4% to 3%, repayment of $25,000,000 flood-control allocation was deferred beyond the 50-yr period, but amortization of balance of investment was made certain within this period, and definite annual payments to states were provided. These include $300,000 apiece to Arizona and Nevada, and $500,000 to the Colorado River Basin Development Fund, limited to use in the Upper Basin only, until 1955.

In this paper, project revenues made available by the present revised contracts and the Adjustment Act, derived from power sales and also water storage service charges, are tabulated by allocations to repayment of investment, payments in lieu of taxes, contributions to the Colorado River Basin Development Fund, and flood-control allocation repayments. For comparison, similar allocations under the terms of the original Act and contracts are presented in Table 3, assuming varied power rates after the 1945 adjustment date, and computing present values at both the adjusted 3% interest rate and at the 4% rate of the original Act. The change in interest rate was made on a showing that the interest cost to the U. S. Treasury for long-term loans during the project construction period did not exceed 2.75%.

TABLE 3.—ALLOCATION OF BOULDER CANYON PROJECT COSTS EXPRESSED AS PERCENTAGES OF INVESTMENT FOR DAM AND POWER-PLANT BUILDING,[a] ON THE BASIS OF PRESENT VALUES OF JUNE 1, 1937, AT INTEREST RATES OF 3% AND 4%

Power charges, in mills per kw-hr	INVESTMENT IN DAM AND POWER PLANT[a]		ARIZONA AND NEVADA, IN LIEU OF TAXES		DEVELOPMENT FUND[b]	FLOOD-CONTROL ALLOCATION OF $25,000,000			
						Normal[c]		Alternate[d]	
	3%	4%	3%	4%	3%	3%	4%	3%	4%
(a) ADJUSTMENT ACT AND REVISED CONTRACTS									
1.163	96%[e]	....	18%	....	15%	....[f]	....[f]	....[f]	....[f]
(b) ORIGINAL ACT AND CONTRACTS									
1.63[g]	130%[e]	97%[h]	22%	16.4%	0	36%	27%	121%	100%
1.163[g]	130%[e]	97%[h]	4%	3.7%	0	7%	6%	23%	22%
1.0[g]	114%[e]	85%[h]	4%	3.7%	0	7%	6%	23%	22%

[a] Less $25,000,000 flood-control allocation. [b] Colorado River Basin Development Fund. [c] In terms of the remainder of reimbursable project cost. [d] Expressed as a percentage of the flood-control allocation of $25,000,000. [e] Water storage charge, 4%. [f] Deferred until after 1987, or the end of the 50-yr amortization period. [g] Power rate, as it might have been revised after 1945 by the terms of the original Act. [h] Water storage charge, 3%.

DISCUSSION

W. L. Chadwick,[5] M. Am. Soc. C. E.—A full and interesting description of the power contracts for the Boulder Canyon Project is presented in this paper, which contains valuable information on the reasons for many of the paragraphs and clauses. As the author states, the problem of allocating the cost of the multiple-purpose Boulder Canyon Project to the several beneficiaries was encountered in only an unimportant way because the Boulder Canyon Project Act required, before money for construction could be appropriated, that provision be made by contract for revenues sufficient to insure payment of all expenses of operation and maintenance and repayment of construction advances by the United States, with interest at 4%. Since the rate for firm energy was determined by the competitive costs of steam generated energy at load centers in Southern California, there was no problem of allocating costs for rate-fixing purposes. Furthermore, the flood control allocation of $25,000,000 was statutory. Although a price was set for water storage service, this rate was not based on capital investment allocated to the purpose. Allocation complexities, however, did enter importantly into the drafting of the numerous contracts under the Boulder Canyon Project Adjustment Act.

Under the original contracts the "power privilege" was leased to the City of Los Angeles and Southern California Edison Company. Under this lease the city undertook to generate the energy requirements of the Metropolitan Water District of Southern California; the municipalities of Pasadena, Burbank, and Glendale; the states; and itself. The company undertook to generate the energy required by the two other companies and itself. The United States agreed to pay the lessees "in the form of credits upon the account of such lessee * * * the cost incurred by it in generating energy for other allottees * * *" and agreed to "require such other allottees to repay such cost to the United States."

The lease contract included the obligations of the lessees to take and pay (pay, in any case) for the energy allotted to each. Separate contracts were then executed by the other allottees requiring, similarly, that each take or pay for the energy allotted.

As the author states, conditions during 1936 and 1937 indicated that the price of 1.63 mills per kw-hr established for falling water would considerably more than liquidate the project costs. The late F. E. Weymouth, M. Am. Soc. C. E., then general manager and chief engineer of the Metropolitan Water District, realizing that the taxpayers and ratepayers of the district would be required to contribute 36% of all revenues collected in excess of actual costs, began negotiations for a downward adjustment of rates. The district was particularly interested, because the economic depression had made large inroads into anticipated water sales and it was apparent that foresight in contracting in advance for a pumping power supply that would be adequate for many years would require payment over a long period for large quantities of energy which could not be used and for which there then seemed little probability of making effective temporary releases for salvage sales.

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[5] Chf. Civ. Engr., Southern California Edison Co., Los Angeles, Calif.

The first efforts were directed toward obtaining a revision of rates under the lease and energy contracts then in effect. Meeting opposition from those interested in excess revenues under the original contracts, Mr. Weymouth proposed revised contracts based on the repayment of all costs only, rather than on the requirement that charges be fixed by competitive conditions. Although long and tedious negotiations were required before this principle was enacted in legislation, it is the basis for rates under the Adjustment Act.

For reasons not known to the writer, during preliminary conferences and negotiations on the adjustment legislation, it was proposed that the lease of power privilege be canceled and that the United States operate the generating plant, as well as the dam, penstock, outlets, town site, utilities, etc. Alternatively, it was proposed that generation be by means of agency agreements between the United States and the lessees. Legal difficulties involving security of tenure and loss of the valuable leasehold rights of both lessees were resolved by a form of agency contract with about the same tenure to the lessees as previously enjoyed under the lease. Under the agency agreements, the agents agree to generate energy under substantially the same bases for the same allottees as formerly.

Under the original agreements, the rate of 1.63 mills per kw-hr covered the use of falling water delivered to the butterfly valve on the upstream side of the turbines. Accordingly, associated with this rate were costs for construction, operation, and maintenance of the dam; penstocks; outlet structures; power plant building; common station facilities, such as cranes, elevators, and offices; and Boulder City, with its utilities, parks, streets, and administration; etc. However, the rate did not include costs for investment in, or operation, maintenance, and replacement of, turbines, generators, switchgear, switch racks, power conductors, and related equipment. The latter were costs repaid by the lessees and collected, in turn, from the allottees by the United States.

Under the Adjustment Act it was required that:

> "The Secretary of the Interior is authorized and directed to, and he shall, promulgate charges, or the basis of computation thereof, for electric energy generated at Boulder Dam during the period beginning June 1, 1937, and ending May 31, 1987, computed to be sufficient, together with other net revenues from the project, to accomplish the following purposes:
>
> "(a) To meet the cost of operation and maintenance and to provide for replacements * * *.
>
> "(b) To repay to the Treasury, with interest at 3 per cent, the advances to the Colorado River Dam Fund, made prior to June 1, 1937, within fifty years from that date.
>
> "(c) To provide $600,000 for each of the years [payable $300,000 each to States of Arizona and Nevada] * * *.
>
> "(d) To provide $500,000 for each of the years * * *." (To the Colorado River Development Fund for additional development in the Colorado River Basin.)

The Adjustment Act required no adherence to the original principle of dividing the rate into a falling water charge and charges for generation. Accordingly, the entire subject of rates and "rate architecture," as referred to by W. F. Durand, was opened during the hearings held in Los Angeles during 1940 with R. V. L. Wright presiding as special representative of the Secretary of the Interior. Mr. Durand and Leland Olds served as special advisers. An early principle agreed to, however, was that the revisions in cost allocations or in rate design should not change the obligations of any allottee at the expense of another.

Allocations of cost for rate purposes were complicated by the size of generating units, the design of the generating plant, the number of allottees, and differences in standards of service on the several power systems involved.

Boulder Canyon power plant (January, 1944) contains six generators on the Nevada side and six on the Arizona side, all of 82,500-kva capacity except for one of 40,000 kva on the Arizona side (unit A-8). The distribution of these machines between generating agents and the allottees for whose benefit the machine is operated is as follows:

Generating units	Allottee for whose benefit the machine is operated
Nevada Side—	
N-1	City of Los Angeles, Burbank, Glendale, Pasadena, Boulder City, California Pacific Utility Company, and Citizens Utility Company (contractors for the Metropolitan Water District), and the State of Nevada for the Southern Nevada Power Company and the Lincoln County Power District No. 1
N-2	
N-3	
N-4	
N-5	Metropolitan Water District and Basic Magnesium, Inc.
N-6	Metropolitan Water District and Basic Magnesium, Inc.
Arizona Side—	
A-1	City of Los Angeles et al. (see N-1 to N-4)
A-2	City of Los Angeles et al. (see N-1 to N-4)
A-5	Southern California Edison Company
A-6	Southern California Edison Company
A-7	Southern California Edison Company
A-8	California Electric Power Company

The Southern California Edison Company is the agent operating units A-5, A-6, A-7, and A-8. All the remainder are operated by the City of Los Angeles.

Including three contractors for energy allotted to but not used by the Metropolitan Water District, there are thirteen agencies receiving energy directly from the Boulder plant. Transmission to City of Los Angeles is at 287,000 v; to Edison Company at 220,000 v; to Metropolitan Water District and Basic Magnesium, Inc., at 220,000 v; to California Electric Power Company and Lincoln County Power District at 138,000 v; to Burbank, Glendale, and Pasadena via City of Los Angeles lines at 287,000 v; to Southern Nevada Power Company at 33,000 and 69,000 v; to Citizens Utilities at 69,000 v; to California Pacific Utilities Company at 69,000 v; and to the United States at 33,000 v. Further complicating the situation are the different frequencies. At the time construction of the plant was undertaken, all Southern California utilities, except California Electric Power Company (then Southern Sierras Power Company) and Los Angeles Gas and Electric Company (later part of the City of Los Angeles system) were operated at 50 cycles. In 1936 and 1937, the City of Los Angeles converted its system to 60 cycles, which, because of joint transmission obligations, required similar conversion by Burbank, Glendale, and Pasadena. Accordingly, the Southern California Edison generators are operated at 50 cycles and all others at 60 cycles. Hence, parallel operation of all units is not possible. Such parallel operation is further undesirable because of the tremendous energy concentrations to be interrupted during short circuits.

Accordingly, after determining the costs to be met (that is, for investment, interest, payments to states, Colorado River Development Fund, operation, maintenance, and replacement) equitable allocation of shares in these costs between allottees and contractors was necessary. At the request of the special board, most of the allottees submitted suggestions for rate design. These varied from complicated bases involving consideration of all differences between allottees to others which would have allocated all on a straight kilowatt-hour allocation. The complications were decidedly undesirable, and the straight kilowatt-hour charge was inequitable because of differences in load factor, cost of generating equipment, differences in operating costs, etc.

An equivalent of the original falling water charge was determined by estimating all costs and charges for the 50-yr contract period, estimating the generation of both firm and secondary energy during the same period, and determining the quotient of the two.

The energy component was finally resolved by dividing the generation and transforming and switching facilities into sections according to uses, determining the investment costs for each, estimating the operation, maintenance, and replacement costs for a year in advance, and billing monthly for the total of such costs per section. Adjustments are made at the end of each operating year.

Costs for facilities used for joint service, such as generators N-1, N-2, N-3, and N-4, were finally allocated on the basis of kilowatt-hour use, although much testimony was taken favoring a kilowatt basis for this purpose.

Essentially, with two minor exceptions, it may be said that

(1) Allocation of all common costs for the dam and appurtenances, general plant, and other facilities necessary to all generation is entirely on a delivered kilowatt-hour basis;
(2) Allocation of costs for generation is on a direct recorded cost basis; and
(3) Allocation for jointly used generating facilities is on a kilowatt-hour basis.

The exceptions are that the cost of common station facilities operated by agents is allocated on the basis of generator months and that operation costs of Southern California Edison Company for operating its own generators and those of the California Electric Power Company are allocated on the basis of generator units operated.

Because billings must be made from month to month, they are based on estimates made a year in advance, adjusted at the end of each year.

Realizing the impossibility of estimating costs and energy output accurately, fifty years in advance, the Adjustment Act contract and regulations provide for adjustments in energy rates at specified times.

The Adjustment Act contracts have now (1944) been in effect for nearly three years. Although, like all negotiated contracts, those under the Adjustment Act involve numerous compromises (based on neither pure economics nor engineering), no fault has yet developed in the allocation bases to indicate their impracticability.

A. R. ARLEDGE,[6] M. AM. SOC. C. E.—A record of the history of the Boulder Canyon development and the not generally known facts regarding the background of the pioneer in multiple-purpose projects are presented in this excellent paper. Although the author has mentioned the varied interests involved and the need for negotiating the contracts, further emphasis should be placed on the long negotiations for the Adjustment Act and the readjusted repayment contracts. The question may be asked: "Are not these contracts reasonably simple legal documents in which the engineering factors are either known or lend themselves readily to fairly close engineering estimates, although these factors involve long periods of time and economic factors of great magnitude?"

From the data submitted, the answer to this question does not seem so simple. The participants in the negotiations at no time considered the task an easy one; but they became more aware of the involved nature of the problems confronting them as time passed, and the pattern of the varied sectional economic and engineering difficulties assumed shape. The engineers, attorneys, and others called upon to solve these problems, of necessity, had to be men, not only informed on the subject matter, but endowed with both prophetic acumen and diplomatic dexterity of first magnitude, in order to cope with the situations as they arose.

In recognition of these difficulties, the Hearings before the Committee on Irrigation and Reclamation, United States Senate,[7] state in part, with reference to the Boulder Canyon Project Act:

> "As is hereinafter more fully pointed out, the text of the act contains many uncertainties, ambiguities, and inconsistencies."

And further[8] in regard to controversies which arose:

> "The chief causes for controversy have been, first, the method of determining charges for electric energy at the dam; second, the extent of, and method of determination of, the rights of the States of Arizona and Nevada in the revenues from the project; and third, the extent of the interest of the several States of the Basin of the Colorado River, in the revenues from the dam."

The original Project Act involved the interests of the seven states of the Colorado River Basin, the United States of Mexico, and the United States of America. Although the Adjustment Act, as its name implies, is primarily an act of Congress amending the Project Act, it required the close cooperation of the seven basin states and the power contractors as represented by the Committee of 16, and the other power allottees and irrigation interests. To complete this complex picture, it is necessary to add the satellite interests which revolve around and affect each of the foregoing major groups.

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[6] Prin. Civ. Engr., Los Angeles Bureau of Power & Light, Los Angeles, Calif.

[7] Boulder Canyon Project Adjustment Act, Hearings Before the Committee on Irrigation and Reclamation on S. 4039, United States Senate, 76 Cong., 3d Session, May 28, 1940, p. 6.

[8] *Ibid.*, p. 7.

In 1937, when the request for a review of the rates was filed, it was thought that the Secretary of the Interior with his broad discretionary powers could correct the difficulties in the energy rate structure as well as those encountered in administering the Project Act, but competent legal authority ruled otherwise. This ruling started, in earnest, the long and tedious negotiations which culminated in the approval of the Adjustment Act on July 19, 1940. This Act carried a provision, in substance as follows: It shall cease to be operative and shall be of no further force or effect, if contracts (the readjusted contracts of the City of Los Angeles, the Metropolitan Water District of Southern California, and the Southern California Edison Company) in accordance with the requirements of Section 10 of the Act shall not have been entered into prior to June 1, 1941. The principal function of the Adjustment Act seemed to be to provide requirements, rules, regulations, and a time limit under which the readjusted repayment contracts could be perfected. The time limit was beaten by a day.

In spite of the seemingly insurmountable difficulties and the duration of the negotiations for both the Adjustment Act, setting up requirements and limitations for the new contracts, and the new contracts, there were leveling elements which provided the incentive for a mutual agreement. First, the contractors for energy at Boulder Dam were paying a rate about 50% greater than actually was required under the conditions as stated in the Project Act. Second, there was grave doubt that the states of Arizona and Nevada would receive payments in lieu of taxes commensurate with the intent of the Project Act, or that the seven states of the Colorado River Basin would receive any payments that might be used as a development fund, because of uncertainties and ambiguities of the Act. Third, as the author has stated, there was some uncertainty, under the original Act, as to whether the federal government would be fully reimbursed in fifty years because of the requirement to readjust rates periodically in accordance with rates prevailing at competitive centers.

The leveling elements, which provided each main group with trading points, under certain conditions may have given the contractors of energy some slight advantage. The "Contract for Lease of Power Privilege," under date of April 26, 1930, in Article 16, has the following provision:

> "At the end of fifteen (15) years from the date of execution of this contract and every ten (10) years thereafter. the above rates of payment for firm and secondary energy shall be readjusted upon demand of any party hereto, either upward or downward as to price, as the Secretary may find to be justified by competitive conditions at distributing points or competitive centers."

Under this provision, a readjustment of rates would have taken place automatically in 1945. Unless the prices of fuel oil and gas had increased materially, the rates for firm and secondary energy, probably, would have been too low to guarantee the repayment of the federal government's advances in fifty years, and also to make payments to the states of Arizona and Nevada, and to the Colorado River Development Fund to be used in the interest of developing a comprehensive plan for the Colorado River Basin as a whole. If the contractors had chosen this alternate, they would have been penalized by the payment of many millions of dollars in excess revenues before 1945. It seems that the best long-term plan has been accepted, measured in terms of benefits to the entire basin.

Engineers may not be in agreement on the merits of one project subsidizing other projects, but, perhaps, all will agree that, to develop a contract extending fifty years into the future, under the present conditions of social and economic flux, it required men of vision and resourcefulness in each major group, working earnestly and diligently for a comprehensive plan of development of the Colorado River Basin.

Clay C. Elder,[9] M. Am. Soc. C. E.—The discussions by Messrs. Chadwick and Arledge are helpful in presenting the viewpoints of the two generating agencies at Boulder Dam. Their clarifying statements are greatly appreciated. To one who recalls the controversies that raged around articles on Boulder Dam a decade or two ago, when the project was being planned, authorized, and constructed, the absence of similar present discussions is at first surprising. This lack of controversy indicates at least that the Adjustment Act of 1941 has been a practical success, in spite of its numerous compromises, and that the compromises have left no serious ill feeling among the seven states and numerous interests involved. The Adjustment Act relates chiefly to rate revisions and revenue allocations, however. On more fundamental questions, the lack of present criticism seems to indicate a successful accomplishment of the project's multiple purposes—flood control, water conservation, power development, silt control, water quality improvement, recreational uses, and even the improvement of navigation.

With the closing of the Boulder Dam magnesium plant, as recently announced, the project is leading the way into the period of reconversion, even as it was previously in the forefront of total war production. The Boulder Dam power output and demand are not decreased by the change but regionally an important saving of essential oil and gas fuel results from the decreased operation of steam standby plants.

The only remaining serious controversies involving the Boulder Canyon Project relate to the allocation of permanent water rights to its conserved flood waters. Recent ratification by Arizona of the Colorado River Compact seemed a major step toward real peace, although current proposals for large irrigation diversions in Arizona above Boulder Dam may finally cancel many of the benefits of the Adjustment Act and may make the project insolvent as to cost repayment to the federal government. Of more immediate importance is the pending treaty with Mexico relating to the utilization of the waters of the Rio Grande and the Colorado River. Hearings on this proposed treaty are scheduled for early in 1945 before the Senate Foreign Relations Committee and, judging from the preliminary rounds of the battle, will be as hotly contested as any similar arguments of the "good old days." So at Boulder Dam, and along the Colorado River, as elsewhere in the world, it appears that "peace in our times" has yet to be achieved.

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SHEAR EFFECT ON THE STRENGTH OF STRUTS

BY S. SERGEV,[1] ESQ.

WITH DISCUSSION BY MESSRS. LEON BESKIN, MARSHALL HOLT, DAVID B. HALL, AND S. SERGEV.

SYNOPSIS

In studying the strength of initially straight struts, pin-supported at one end and laterally restrained at the other, loaded by a concentrated axial load at some intermediate point between the supports, it is desirable to know the effect of shearing forces on the critical load on the struts. This paper treats that problem. A general expression for buckling criteria is derived for struts of different but constant cross sections. A special case and an example is solved in detail comparing the strength of a latticed strut, considering shear effect, with a strut the strength of which is determined by bending considerations only.

INTRODUCTION

General.—Ordinarily, a strut is an initially straight slender member, principally subjected to axial loads. Struts may have different means of support with the loads applied anywhere along the length of the member. Slender struts fail by buckling caused by a critical load or loads on the strut. The usual method of determining critical loads on initially straight struts is to assume, at every normal cross section, the curvature to be proportional to the bending moment only. This is satisfactory for solid struts, but not for latticed struts in which shearing forces may substantially reduce the critical load. Formulas have been derived for initially straight latticed struts subjected to end loads.[2]

In practice, some struts are pin-supported at one end and laterally restrained at the other, and loaded by a single concentrated axial load applied at some intermediate point between the supports. The effect of shearing forces on the critical load has not yet been established for this case. It is desirable, therefore, to derive formulas that yield this information.

NOTE.—Published in November, 1943, *Proceedings.* Positions and titles given are those in effect when the paper or discussion was received for publication.

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[2] "Theory of Elastic Stability," by S. Timoshenko, 1st Ed., McGraw-Hill Book Co., Inc., New York and London, 1936, pp. 139–147.

Notation.—The following letter symbols, used in this paper, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials,[3] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932:

A = cross-sectional area of a strut; also, where specially defined, A, B, C, and D are integration constants:
  A_a = area of top strut section;
  A_b = area of bottom strut section (see Fig. 1);
  A_d = area of two diagonals;
  A_p = area of two batten plates;
a = length of top strut segment (b = length of bottom segment); as a subscript, a refers to the upper strut segment;
B = integration constant (see A);
b = length of bottom strut segment (see a); as a subscript, b refers to the bottom strut segment;
C = integration constant (see A);
c = spacing of batten plates (see Fig. 2);
D = integration constant (see A);
E = Young's modulus of elasticity;
e = deflection of strut at point of load P;
G = shear modulus of elasticity;
h = length of batten plates and spacing of channels (see Fig. 2);
I_a = moment of inertia of the strut cross-sectional area about the z-axis, in the top segment equal to $u\, I_b$;
I_b = moment of inertia of the strut cross-sectional area about the z-axis, in the bottom segment;
I'_b = inertia moment of two batten plates;
I_c = moment of inertia of the cross section of a vertical channel about its own gravity axis parallel to the web;
K = beam constant for determining shear deflections;
k = a span ratio equal to $\frac{b}{L}$;
L = total length of strut equal to $a + b$;
M = bending moment about the z-axis;
m = a substitution constant (Eq. 5*b*); $m^2 = \frac{P}{E\, I_b}$;
n = a substitution constant; $n^2 = \frac{P}{E\, I_a} = \frac{m^2}{u}$;
P = critical load on the strut:
  P' = buckling or critical load on solid struts;
q = a substitution constant (Eq. 10*e*);
r = least radius of gyration with respect to the z-axis (Eq. 34*a*);
t = a parameter dependent on the dimensions of the column and the material (Eq. 30);
u = ratio of inertia moments $= \frac{I_a}{I_b} = \frac{m^2}{n^2}$;

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[3] ASA—Z10a—1932.

V = total transverse shearing force;
x = rectangular coordinate measured along the axis of the strut:
  x_a = values of x measured from the top down;
  x_b = values of x measured from the bottom upward;
y = deflection of the strut at any point x:
  y_m = deflection at x due to bending moment;
  y_v = deflection at x due to shearing force;
z = rectangular coördinate measured along the z-axis;
α = a substitution constant (see Eq. 30);
β = a parameter dependent on the dimensions of the column and the material (see Eq. 32);
γ = a substitution constant for solid struts (Eq. 25);
ϕ = (see Fig. 2) angle between batten plates and diagonals;
ρ = radius of curvature; and
ψ = a substitution constant for latticed struts.

Assumptions.—All equations and formulas derived in this paper are based on the following assumptions:

1. The strut is initially straight;
2. The material composing the strut is homogeneous;
3. The weight of the strut is neglected;
4. All stresses and strains are within the limits of proportionality; that is, they conform to Hooke's law; and
5. The deflection, and hence the curvature, of the strut is small.

Except No. 5, these assumptions need no comment. To avoid repetition, and because the implications in Assumption 5 are significant, this assumption will be discussed at some length.

The deflection of a strut at the incipient buckling state is small, but for all practical purposes the strut is considered to have failed at the first indication of buckling. After buckling begins, with slight increase in the loading, the deflection increases greatly, but its relationship to the effects producing it is no longer of practical interest. Since the deflection is small within the range considered, it can be shown easily that the curvature ρ of the strut axis is approximately equal to

$$\frac{1}{\rho} = \frac{d^2y}{dx^2} \quad \text{.........................(1)}$$

(This is the usual assumption made in all approximate computations.) The exact relationship of curvature to the bending moment M causing it is expressed by

$$\frac{1}{\rho} = \frac{M}{E\,I} \quad \text{.........................(2)}$$

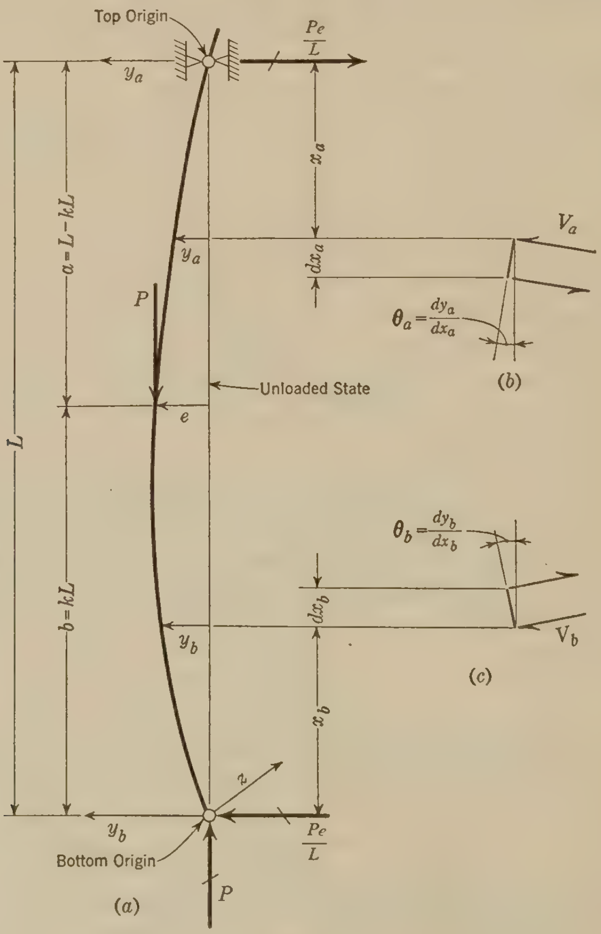
in which E = modulus of elasticity; and I = moment of inertia of the strut cross-sectional area about the z-axis. Eliminating the curvature between these two expressions:

$$\frac{d^2y}{dx^2} = \frac{M}{E\,I} \quad \text{.........................(3)}$$

Eq. 3 defines a valid relationship between the deflection and bending moment when the deflection is small relative to the span. $\Bigl($Another way of stating this is that, for flat curves, the rate of change of the slope at any point equals the curvature of the curve at that point, which in turn equals $\frac{M}{EI}$.$\Bigr)$

CRITICAL LOAD ON SOLID STRUTS

Consider a strut that is pin-supported at one end and restrained laterally at the other, and loaded by a single concentrated load at some intermediate point between the supports (see Fig. 1). The load is assumed to be symmetrically applied, thus simulating concentric load condition. The flexural rigidities of the strut in the two segments a and b are constant but unequal. Assume a solid strut for the present discussion.



Top Origin
$\frac{Pe}{L}$
y_a
x_a
V_a
y_a
dx_a
P
$\theta_a = \frac{dy_a}{dx_a}$
(b)
Unloaded State
e
L
$a = L - kL$
$b = kL$
$\theta_b = \frac{dy_b}{dx_b}$
dx_b
y_b
V_b
(c)
x_b
z
y_b
Bottom Origin
$\frac{Pe}{L}$
P
(a)

FIG. 1

Under a critical load, P, due to initial crookedness or small eccentricity, the strut axis will deflect slightly so that P will be a distance e from the initial or unloaded state of the strut. The indicated reactions are determined from the laws of equilibrium.

First consider the bottom segment b, and express the curvature of the strut axis in terms of the bending moment. The bending moment at any section distance x_b from the bottom origin is

$$M = -P\,y_b + \frac{P\,e\,x_b}{L} \qquad \text{(4)}$$

in which the positive sign denotes compression on the side adjacent to the positive y-coordinate. The general relationship, for flat curves, between the bending moment and the deflection, y_{bm}, that it produces is

$$E\,I_b \frac{d^2 y_{bm}}{dx^2_b} = -P\,y_b + \frac{P\,e\,x_b}{L} \qquad \text{(5a)}$$

Dividing Eq. 5a by $E\,I_b$ and using the notation $\frac{P}{E\,I_b} = m^2$:

$$\frac{d^2 y_{bm}}{dx^2_b} = -m^2\,y_b + \frac{m^2\,e\,x_b}{L} \qquad \text{(5b)}$$

Considering the effect of shearing forces on the deflection of the strut, the total transverse shearing force is:

$$V_b = \frac{dM}{dx_b} = \frac{d}{dx_b}\left(-P\,y_b + \frac{P\,e\,x_b}{L}\right) = -P\left(\frac{dy}{dx}\right)_b + \frac{P\,e}{L} \qquad \text{(6)}$$

from which

$$\left(\frac{dV}{dx}\right)_b = -P\left(\frac{d^2y}{dx^2}\right)_b \qquad \text{(7)}$$

The slope of the strut axis due to shear strain, from fundamental concepts, is equal to

$$\frac{dy_{bv}}{dx_b} = -\frac{K\,V_b}{A_b\,G} \qquad \text{(8)}$$

in which the negative sign follows from Fig. 1(c) where V_b is positive and acting alone would give a negative slope. The factor K is a constant depending on the cross section of the strut. (This factor is 1.2 and 1.11 for rectangular and circular cross sections, respectively.)[4] Taking the first derivative with respect to x_b in Eq. 8, an expression is obtained for the shear curvature as a function of the total transverse shearing force:

$$\frac{d^2 y_{bv}}{dx^2_b} = -\frac{K}{A_b\,G}\left(\frac{dV}{dx}\right)_b \qquad \text{(9)}$$

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[4] "The Theory and Practice of Modern Framed Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, 10th Ed., John Wiley & Sons, 1910, Vol. 2, p. 27; also "Theory of Statically Indeterminate Structures," by W. M. Fife and J. B. Wilbur, McGraw-Hill Book Co., Inc., 1st Ed., p. 27.

In Eq. 9, the expression found in Eq. 7 is substituted for $\left(\frac{dV}{dx}\right)_b$:

$$\frac{d^2 y_{bv}}{dx^2_b} = \frac{K\,P}{A_b\,G}\left(\frac{d^2y}{dx^2}\right)_b \quad\text{(10a)}$$

This relationship gives the curvature of the elastic line of the strut due to shearing force. The total curvature resulting from bending moment and shearing force is the sum of the two curvatures; that is,

$$\left(\frac{d^2y}{dx^2}\right)_b = \frac{d^2y_{bm}}{dx^2_b} + \frac{d^2y_{bv}}{dx^2_b} \quad\text{(10b)}$$

If the expressions for the curvatures found in Eqs. 5*b* and 10*a* are substituted into Eq. 10*b*—

$$\left(\frac{d^2y}{dx^2}\right)_b = -\,m^2\,y_b + \frac{m^2\,e\,x_b}{L} + \frac{K\,P}{A_b\,G}\left(\frac{d^2y}{dx^2}\right)_b \quad\text{(10c)}$$

or rewriting

$$\left(\frac{d^2y}{dx^2}\right)_b\left(1 - \frac{K\,P}{A_b\,G}\right) = -\,m^2\,y_b + \frac{m^2\,e\,x_b}{L} \quad\text{(10d)}$$

Dividing each term in Eq. 10*d* by $1 - \frac{K\,P}{A_b\,G}$ and using the notation $q^2 = \frac{m^2}{1 - \frac{K\,P}{A_b\,G}}$, Eq. 10*d* may be written in the form:

$$\left(\frac{d^2y}{dx^2}\right)_b + q^2\,y_b = q^2\,\frac{e\,x_b}{L} \quad\text{(10e)}$$

The general solution of Eq. 10*e* is

$$y_b = A\sin q\,x_b + B\cos q\,x_b + \frac{e\,x_b}{L} \quad\text{(11a)}$$

The constants A and B may be determined from the boundary conditions $y_{x_b=0} = 0$ and $y_{x_b=b} = e$. The constants are found to be $B = 0$, and $A = \frac{a\,e}{L}\sin q\,b$. Substituting these into Eq. 11*a*,

$$y_b = \frac{a\,e\sin q\,x_b}{L\sin q\,b} + \frac{e\,x_b}{L} \quad\text{(11b)}$$

The slope of the elastic line of the strut, corresponding to the deflection y_b, is found by taking the first derivative with respect to x_b in Eq. 11*b*. The slope is

$$\left(\frac{dy}{dx}\right)_b = \frac{a\,e\,q\cos(q\,x_b)}{L\sin q\,b} + \frac{e}{L} \quad\text{(12)}$$

Consider next the deflection of the top segment. The bending moment of any distance x_a from the top origin is

$$M = -\,\frac{P\,e\,x_a}{L} \quad\text{(13)}$$

The curvature produced by this moment, using the argument discussed before, is

$$E\,I_a \frac{d^2 y_{am}}{dx^2_a} = -\frac{P\,e\,x_a}{L} \quad \text{(14)}$$

Dividing both sides by $E\,I_a$ and using the notation $\frac{P}{E\,I_a} = n^2 = \frac{m^2}{u}$,

$$\frac{d^2 y_{am}}{dx^2_a} = -\frac{m^2\,e\,x_a}{u\,L} \quad \text{(15)}$$

Now, considering the effect of the shearing force,

$$\frac{dM}{dx_a} = V_a = -\frac{P\,e}{L} \quad \text{(16)}$$

From the study of shear strain, as discussed before,

$$\frac{dy_{av}}{dx_a} = -\frac{K\,V_a}{A_a\,G} = \frac{K}{A_a\,G} \times \frac{P\,e}{L} \quad \text{(17)}$$

The curvature of the elastic line due to shearing force is equal to

$$\frac{d^2 y_{av}}{dx^2_a} = \frac{d}{dx_a}\left(\frac{K}{A_a\,G} \times \frac{P\,e}{L}\right) \quad \text{(18)}$$

Since the term in the parentheses will be a constant at the instant buckling begins, it is seen that the curvature due to shearing forces in the top segment is zero. Consequently, the total curvature of the elastic line is that due to bending only. Hence, $\frac{d^2 y_{am}}{dx^2_a} = \frac{d^2 y_a}{dx^2_a}$, and from Eq. 15, integrating once—

$$\left(\frac{dy}{dx}\right)_a = -\frac{m^2\,e\,x^2_a}{2\,u\,L} + C \quad \text{(19)}$$

Integrating again,

$$y_a = -\frac{m^2\,e\,x^3_a}{6\,u\,L} + C\,x_a + D \quad \text{(20)}$$

The constants C and D may be evaluated from the following conditions:[4a]

$$(y_a)_{x_a=0} = 0; \qquad \left(\frac{dy_a}{dx_a}\right)_{x_a=a} = -\left(\frac{dy_b}{dx_b}\right)_{x_b=b} \quad \text{(21)}$$

from which $D = 0$, and

$$C = \frac{m^2\,e\,a^2}{2\,u\,L} - \frac{a\,e\,q}{L}\cot q\,b - \frac{e}{L} \quad \text{(22)}$$

Finally, substituting these values[4b] into Eq. 20:

$$y_a = -\frac{m^2\,e\,x^3_a}{6\,u\,L} - \left(\frac{m^2\,e\,a^2}{2\,u\,L} - \frac{a\,e\,q}{L}\cot q\,b - \frac{e}{L}\right)x_a \quad \text{(23)}$$

[4a] The second condition in Eq. 21 has been found incorrect; see discussions and Eq. 65.

[4b] See Eq. 64.

The deflection e may be eliminated from Eq. 23, using the condition that $y_a = e$ at $x_a = a$. Performing this operation and simplifying:[4c]

$$\tan q\, b = \frac{a^2\, q}{\dfrac{m^2\, a^3}{3\, u} - a - L} \quad \text{. . . . .} (24a)$$

If $b = k\, L$, then $a = L\,(1 - k)$, and Eq. 24a may be rewritten as follows:[4d]

$$\frac{\tan k\, q\, L}{q\, L} = \frac{1}{\dfrac{m^2\, L^2}{3\, u}\,(1 - k) - \dfrac{2 - k}{(1 - k)^2}} \quad \text{. . . . .} (24b)$$

The smallest useful roots satisfying Eq. 24b, using predetermined values of k and u, give the desired solution of the buckling problem under consideration. Before making specific applications of Eq. 24b, which, it will be remembered, was derived for solid struts, it will first be interesting to show that, with minor changes, it will apply to latticed struts.

Latticed Struts.—Eq. 24b applies to latticed and solid struts alike, provided the appropriate constants are used for each case. In straight solid struts loaded as shown in Fig. 1, the slope at any distance x_b from the bottom origin due to shearing force, from Eqs. 6 and 8, is

$$\left(\frac{dy}{dx}\right)_b = -\frac{K\, V_b}{A_b\, G} = -\frac{K}{A_b\, G}\left[-P\left(\frac{dy}{dx}\right)_b + \frac{P\, e}{L}\right]$$

$$= -\gamma\, P\left[\frac{e}{L} - \left(\frac{dy}{dx}\right)_b\right] \quad \text{. . . . .} (25)$$

and the resulting curvature (see Eq. 10a) is

$$\frac{d^2 y_{bv}}{dx^2_b} = \frac{K}{A_b\, G} \times P\left(\frac{d^2 y}{dx^2}\right)_b = \gamma\, P\left(\frac{d^2 y}{dx^2}\right)_b \quad \text{. . . . .} (26a)$$

in which $\gamma = \dfrac{K}{A_b\, G}$. For built-up or latticed struts similarly loaded, the corresponding slope due to shearing force is

$$\frac{dy_{bv}}{dx_b} = -\psi\, V_b = -\psi\, P\left[\frac{e}{L} - \left(\frac{dy}{dx}\right)_b\right] \quad \text{. . . . .} (26b)$$

and, for the curvature,

$$\frac{d^2 y_{bv}}{dx^2_b} = \psi\, P\left(\frac{d^2 y}{dx^2}\right)_b \quad \text{. . . . .} (26c)$$

The constants γ and ψ may be thought of as the slope of the strut axis caused by a shearing force of unity in the solid and latticed struts, respectively. Similar logic may be applied to the top segment.

The curvature due to bending of a built-up strut is independent of the type of lattice used, the lattice bars merely serving the purpose of keeping apart the principal longitudinal component parts of the strut. (It is implied that there is a large number of lattice bars.) Consequently, no change in fundamental approach to curvature caused by bending need be made for latticed struts from that used for solid struts.

[4c] See Eq. 69a.

[4d] See Eq. 69b.

The foregoing demonstrates that the only difference in analysis and design of solid and latticed struts lies in the constants γ and ψ. Thus, Eq. 24*b* is valid for solid and latticed struts with the following differences: For solid struts

$$q = \frac{m}{\sqrt{1 - \frac{KP}{A_b G}}} = \frac{m}{\sqrt{1 - \gamma P}} \qquad (27a)$$

and, for latticed struts,

$$q = \frac{m}{\sqrt{1 - \psi P}} \qquad (27b)$$

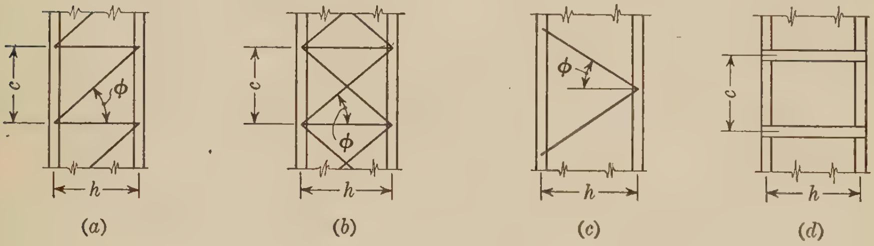

FIG. 2

The expressions for ψ for different built-up struts shown in Fig. 2 and for different types of lattice are, respectively,[5]

$$\psi_a = \frac{1}{E}\left(\frac{1}{A_d \sin\phi \cos^2\phi} + \frac{h}{c\,A_p}\right) \qquad (28a)$$

$$\psi_b = \frac{1}{E}\left(\frac{1}{2\,A_d \sin\phi \cos^2\phi}\right) \qquad (28b)$$

$$\psi_c = \frac{1}{E}\left(\frac{1}{A_d \sin\phi \cos^2\phi}\right) \qquad (28c)$$

and

$$\psi_d = \frac{1}{E}\left(\frac{c\,h}{12\,I_p} + \frac{c^2}{24\,I_c}\right) \qquad (28d)$$

in which c is the spacing of battens; h, the length of the battens between the hinges; A_p, the cross-sectional area of the two battens; A_d, the cross-sectional area of two diagonals; ϕ, the angle between batten plates and the diagonals; I_p, the moment of inertia of two batten plates about axes through the center of gravity and parallel to the z-axis; and I_c, the moment of inertia of the cross section of a vertical channel about its own gravity axis parallel to the web.

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[5] "Theory of Elastic Stability," by S. Timoshenko, 1st Ed., McGraw-Hill Book Co., Inc., New York and London, 1936, pp. 141–145.

SPECIAL APPLICATION

A special application of the formulas derived will be made now for the case when $k = 0.5$, and $\frac{I_a}{I_b} = u = 1.0$. Substituting these factors into Eq. 24*b* and rearranging slightly,

$$\frac{\tan \frac{1}{2} q L}{q L} = \frac{6}{m^2 L^2 - 36} \qquad (29)$$

The relationship between m and q, for solid struts, is

$$q L = \frac{m L}{\left(1 - \frac{K P}{A_b G}\right)^{0.5}} = \frac{m L}{\left(1 - \frac{K E I_b m^2 L^2}{A_b G L^2}\right)^{0.5}} = \frac{t}{(1 - \alpha t^2)^{0.5}} \qquad (30)$$

in which $m^2 = \frac{P}{E I_b}$, $t = m L$; $\alpha = \frac{K E I_b}{A_b G L^2} = \frac{K E}{G \left(\frac{L}{r}\right)^2}$. Eq. 29 is now rewritten:

$$\frac{\tan \left[\frac{\frac{1}{2} t}{(1 - \alpha t^2)^{0.5}}\right]}{\frac{t}{(1 - \alpha t^2)^{0.5}}} = \frac{6}{t^2 - 36} \qquad (31a)$$

The constant α in Eq. 31*a* is small, being in the order of 0.001 for $\frac{L}{r} = 50$ and $\frac{E}{G} = 2.5$. The corresponding smallest useful root satisfying Eq. 31*a* is $t = 4.3$, whereas for $\alpha = 0$, $t = 4.32$ in the writer's paper on this subject.[6] (The root is inadvertently shown as 2.2 instead of the correct root 2.16 for slenderness $\frac{b}{r}$. For slenderness $\frac{L}{r}$, the root is 2 (2.16) = 4.32.) It is thus seen that, for solid stout struts, the shear effect on the strength is small. For more slender struts, the effect of shear on the strength will be even less. Thus, it is permissible to neglect the shear effect when measuring the strength of solid struts.

Latticed Struts.—The situation is somewhat different for built-up or latticed struts, the strength of which may be significantly reduced by shearing deflections. To write the theoretical equation depicting the buckling state, use Eq. 31*a* substituting β for α:

$$\frac{\tan \left[\frac{0.5 t}{(1 - \beta t^2)^{0.5}}\right]}{\frac{t}{(1 - \beta t^2)^{0.5}}} = \frac{6}{t^2 - 36} \qquad (31b)$$

in which

$$\beta = \frac{\psi E I}{L^2} = \frac{\psi E A}{\left(\frac{L}{r}\right)^2} \qquad (32)$$

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[6] *Bulletin No. 101*, Eng. Experiment Station, Univ. of Washington, Seattle, p. 18.

The smallest useful roots of t satisfying Eq. 31*b*, using different values of β, determine the buckling state and the critical load, P. Values of β varying from 0.0 to 0.05 were assumed and the corresponding t roots found. The results are shown in Fig. 3. An approximate relationship between t and β may be expressed by the equation

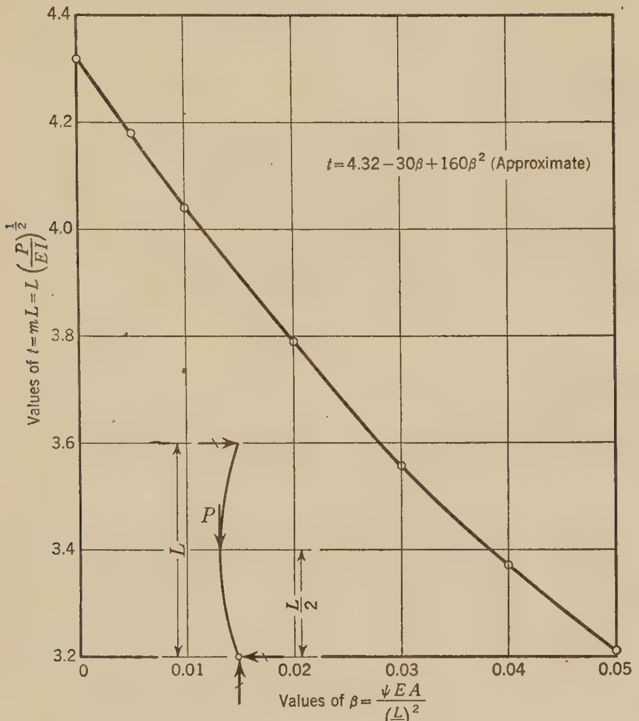

FIG. 3.—t ROOTS FOR DIFFERENT VALUES OF β

$$t = 4.32 - 30\,\beta + 160\,\beta^2 \qquad (33a)$$

Having found the root t for a given value of β, the critical load on the strut may be found from the relation

$$t^2 = m^2\,L^2 = \frac{P\,L^2}{E\,I} \qquad (33b)$$

Solving for P, the buckling or critical load is

$$P = \frac{t^2\,E\,I}{L^2} = \frac{t^2\,E\,A}{\left(\frac{L}{r}\right)^2} \qquad (34a)$$

The strength of a latticed strut may be obtained from the strength of a solid strut having the same flexural rigidity, by using a fictitious length, L_f.

This length may be determined as follows: The critical load for the solid struts is

$$P' = \frac{(4.32)^2 E A}{\left(\frac{L}{r}\right)^2} \qquad (34b)$$

and, for the latticed struts,

$$P = \frac{t^2 E A}{\left(\frac{L}{r}\right)^2} = \frac{(4.32)^2 E A}{\left(\frac{L_f}{r}\right)^2} \qquad (34c)$$

The effect of shear deflections on the critical load is neglected in Eq. 34*b*. From Eq. 34*c* the required fictitious length is found to be

$$L_f = \left(\frac{4.32}{t}\right) L \qquad (35a)$$

or, approximately,

$$L_f = \left(\frac{4.32}{4.32 - 30\,\beta + 160\,\beta^2}\right) L \qquad (35b)$$

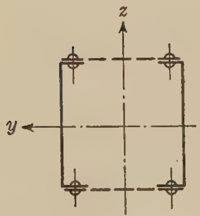

FIG. 4

Numerical Example.—Consider a straight strut (see Fig. 4) composed of two 15-in. channels weighing 35 lb per ft laced together by 2-in. by $\frac{3}{8}$-in. batten plates and lattice bars as in Fig. 2(*a*), so that $c = h = 8.5$ in., $\phi = 45°$, $A_p = A_d = 1.5$ sq in.; $A = 20.46$ sq in.; $I_y = 637$ in.^{4}, $I_z = 627$ in.^{4}, $(r_z)_{\min} = 5.5$ in., approximately, and $E = 30 \times 10^6$ lb per sq in. Then (see Eq. 32),

$$\beta = \frac{\psi E A}{\left(\frac{L}{r}\right)^2} = \frac{\psi E\,(20.46)}{\left(\frac{L}{r}\right)^2} \qquad (36)$$

and, from Eq. 28*a*,

$$\psi_a = \frac{1}{E}\left(\frac{1}{1.5 \times 0.707 \times 0.707^2} + \frac{8.5}{8.5 \times 1.5}\right) = \frac{2.550}{E} \qquad (37a)$$

Finally—

$$\beta = \frac{52.2}{\left(\frac{L}{r}\right)^2} \qquad (37b)$$

By assuming different slenderness ratios, $\frac{L}{r}$, the corresponding t roots can be obtained, and subsequently the critical loads for the latticed struts, with respect to the z-axis. The critical loads on solid struts can be calculated easily. These are listed in Table 1, Cols. 5 and 6, respectively. The ratios of the critical loads on latticed and solid struts of the same slenderness and cross-sectional areas are given[6a] in Col. 7, Table 1. The effect of shearing deflections on the critical load is thus shown to be more pronounced for the stout struts than for the more slender ones. For the latticed strut of usual slenderness, a decrease in strength occurs from 5% to 10% of the strength of a strut with a solid web and equal flexural rigidity; the higher percentage pertaining to steels with a high proportional limit.

TABLE 1.—COMPARISON OF STRENGTH OF STRUTS

$\frac{L}{r}$ (1)	β (2)	t (3)	L_f (4)	P' (5)	P (6)	$\frac{P}{P'}$ (7)	$\frac{P}{A}$ (8)
60	0.0145	3.92	$1.1\,L$	$0.106\,E$	$0.0875\,E$	0.825	128,000[a]
100	0.00522	4.17	$1.036\,L$	$0.0382\,E$	$0.0356\,E$	0.93	52,000[a]
120	0.00362	4.215	$1.025\,L$	$0.0265\,E$	$0.0252\,E$	0.95	37,000[a]
200	0.00131	4.28	$1.01\,L$	$0.00955\,E$	$0.00935\,E$	0.98	13,700[a]
Source	Eq. 37*b*	Fig. 3	Eq. 35*a*	Eq. 34*b*	Eq. 34*a*	....	....

[a] Pounds per square inch.

CONCLUSION

The strength of latticed struts, pin-supported at one end, laterally restrained at the other, and loaded halfway between the supports, may be substantially reduced by the shearing forces, if battens and lacing bars of small cross section are used. For the lattice systems indicated in Fig. 2, using definite batten and lacing bar sizes, the percentage reduction in strength due to shear effect can be calculated for the type of strut herein considered, by the method presented in the paper.

[6a] See Table 2.

DISCUSSION

Leon Beskin,[7] Assoc. M. Am. Soc. C. E.—Extended treatment of conditions involved in Assumption 5 is superfluous, since it is accepted perforce as the conventional basis of any elementary approach to the problem of the instability of struts (see heading, "Introduction: Assumptions").

The list of assumptions is incomplete, since those leading to Eq. 2, expressing the relation of moment to curvature when the state of stress is not pure bending, and the more important assumptions leading to Eq. 9 are omitted. In that respect, it is to be noted that the values given for the constant K are approximate.[8] This constant varies somewhat with the method of applying the load. It is approximately equal to 1 for I-beams, A being the area of the web.

The introduction of Eq. 8 as the fundamental relationship between shear and slope is incorrect. The basic relationship for shear deflection is given by Eq. 9. The former equation is derived by integration of the latter, and is valid for straight beams on two supports only when (*a*) the end moments are zero, or are equal; and when (*b*) there are no applied moments in the span.

Moreover, in this form, Eq. 9 must be extended to include concentrated loads, by adding the equation—

$$\Delta\left(\frac{dy_v}{dx}\right) = \frac{-K\,\Delta V}{A\,G} \qquad \text{(38)}$$

—at each point of application of a concentrated load P, thus giving a variation of shear ΔV. Eq. 38 represents a sharp angle of the neutral line of the strut. It can be derived simply by integrating Eq. 9 along an infinitely small element dx on which a constant load is applied. It purports to express a variation ΔV of the shearing force, although the correct proof of its validity can only be established from considerations of theory of elasticity.[9] By failing to make the foregoing distinction the author has developed Eq. 21, and consequently Eqs. 25 and 26, incorrectly.

For problems in which the principle of superposition does not apply, as is the case for buckling, the distribution of the deflection y into two components, y_m due to bending and y_v due to shear, is improper. It is easily seen that, if the deflection due to bending is neglected, there is no deflection due to shear in the problem discussed. More generally, when side load exists, two conditions can be considered:

(*a*) Without longitudinal load, $y = y_m + y_v$. The deflections y_m and y_v are computed, assuming first that $\frac{1}{A} = 0$, and second that $\frac{1}{I} = 0$. In this case, the two components have definite meanings.

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[7] Senior Stress Analyst, Consolidated Vultee Aircraft Corp., Allentown, Pa.

[8] "Theory of Elasticity," by S. Timoshenko, 1st Ed., McGraw-Hill Book Co., Inc., New York and London, 1934, pp. 37, 41, 74, 104, and 150.

[9] *Ibid.*, p. 103.

(*b*) With a longitudinal compressive load, $y > y_m + y_v$, if the two components are computed as in Condition (*a*).

The only correct relationship is

$$\frac{d^2y}{dx^2} = \frac{1}{\rho} = \frac{1}{\rho_m} + \frac{1}{\rho_v} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

the two component curvatures $\frac{1}{\rho_m}$ and $\frac{1}{\rho_v}$ being defined by the right members of Eqs. 5*b* and 10*a*. Incidentally, the two quantities y_v and y_m that would result from the integration of Eqs. 5*b* and 10*a*, once y is known, have no significance, although their sum is equal to y, provided they are both equal to zero at the supports.

A concentrated transverse load deforms the neutral axis of the strut. Thus, Eq. 21 $\left[\left(\frac{dy_a}{dx_a}\right)_{x_a=a} = -\left(\frac{dy_b}{dx_b}\right)_{x_b=b}\right]$ is not correct. The shearing forces in parts a and b of the strut at the point of application of the load are, respectively,

$$V_a = -\frac{P\,e}{L} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (40a)$$

and

$$V_b = P\left[\frac{e}{L} + (y_b)'\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots (40b)$$

(In Eqs. 40, $(y_a)'$ and $(y_b)'$ are the slopes on both sides of P.) The variation of shearing force at P is

$$\Delta V = -(V_a + V_b) = -(y_b)'\,P \ldots\ldots\ldots\ldots\ldots (41)$$

Then

$$-(y_b)' = (y_a)' + \frac{K\,\Delta V}{A\,G} = (y_a)' - \frac{(y_b)'\,P\,K}{A\,G} \ldots\ldots\ldots (42)$$

Eq. 42 can easily be transformed into

$$(y_b)'\,m^2 = -(y_a)'\,q^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

which should be substituted for Eq. 21. If the cross-section area changed at P, the calculation would be changed since $-\frac{\Delta V}{A}$ would be replaced by $\frac{V_a}{A_a} + \frac{V_b}{A_b}$. Using Eq. 43, Eq. 24*b* is changed to

$$q\,L \cot k\,q\,L = \left(\frac{q^2\,L^2}{3\,u}\right)(1 - k) - \frac{1 - k + \frac{q^2}{m^2}}{(1 - k)^2} \ldots\ldots\ldots (44)$$

It is apparent that the final equation sought by the author is modified considerably by the foregoing correction.

The constants γ and ψ are not directly related to the slope of the axis and are equal to the slope only when definite conditions are fulfilled. They define the angle between the perpendicular to the average direction of the cross section, as distorted by the effect of shear, and the neutral axis.

It is incorrect to state in a general manner that "the curvature due to bending of a built-up strut is independent of the type of lattice used" (see paragraph following Eqs. 26). For instance, the buckled shape of the members of a strut that is latticed only by batten plates is a wavy curve, and, if there are a few battens, only an average, badly defined curve will be identified with the elastic line of a solid strut. Theoretically, it is impossible to separate, definitely, the effect of shear from that of bending, and the values of ψ are valid only when several conditions are satisfied. For instance, the buckling between diagonals or battens must be such as can be disregarded. A thorough investigation of the problem has been presented by Julius Ratzersdorfer.[10]

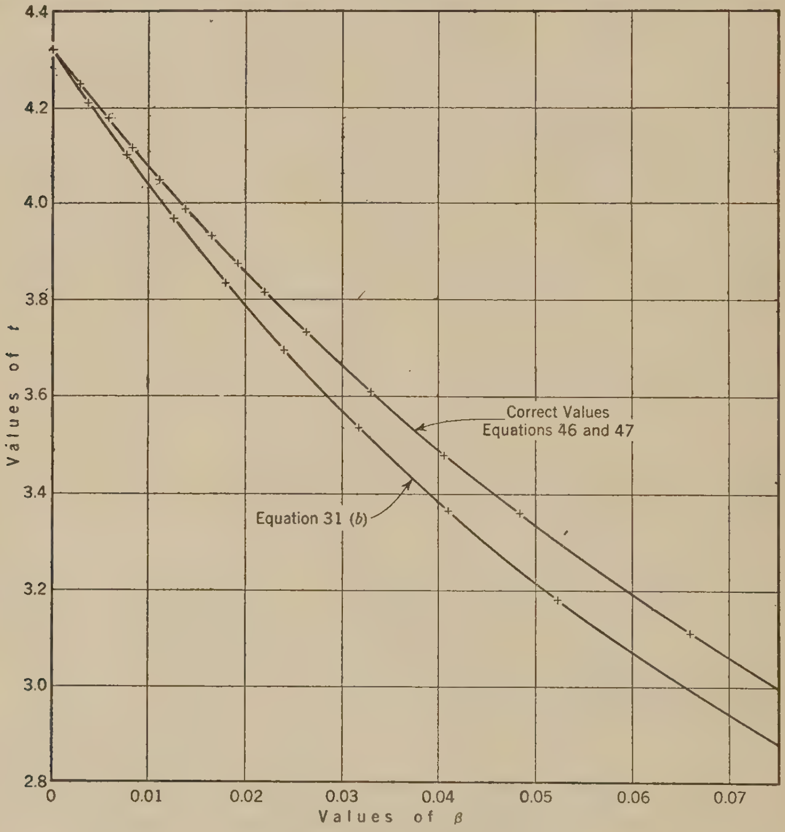

FIG. 5.—RELATIONSHIP BETWEEN t AND β

Application of Eq. 44 will change Eq. 29 as follows:

$$6\,q\,L\cot\tfrac{1}{2}\,q\,L = q^2 L^2 - 12 - \frac{24\,q^2}{m^2} \ldots\ldots\ldots\ldots (45)$$

and Eqs. 31 will be changed to:

$$6\,w\cot\tfrac{1}{2}\,w = w^2 - 12 - \frac{24\,w^2}{t^2} \ldots\ldots\ldots\ldots\ldots (46)$$

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[10] "Die Knickfestigkeit von Stäben und Stabwerken," by Julius Ratzersdorfer, Wien, 1936, pp. 201–243.

in which

$$w = \frac{t}{(1 - \alpha\, t^2)^{0.5}} = \frac{t}{(1 - \beta\, t^2)^{0.5}} \qquad (47)$$

Fig. 3 must be corrected correspondingly. The values of β and t that satisfy Eqs. 46 and 47 are easily computed when w is entered as an independent variable:

$$t^2 = \frac{24\, w^2}{w^2 - 12 - 6\, w \cot \tfrac{1}{2}\, w} \qquad (48a)$$

and

$$\beta = \frac{1}{t^2} - \frac{1}{w^2} \qquad (48b)$$

Using the incorrect Eq. 31*b*, t becomes equal to

$$t^2 = 36 + 6\, w \cot \tfrac{1}{2}\, w \qquad (49)$$

β remaining the same as in Eq. 48*b*. The corrected and erroneous curves are both plotted in Fig. 5. The error can be estimated to vary between 10% and 15% of the variation of t due to the effect of shear—that is, of the difference $t(\beta) - t(0)$, which is always too great in the author's calculation.

The general conclusions stated by the author are well known. The method of calculation presented is an application to a special case of the general method originally derived by S. Timoshenko;[11] and Eq. 24*a* is the application of the general formula established by Professor Timoshenko[12] to the condition in which there is no end load:

$$\left(\frac{k_4}{k_1}\right)^2 (1 - k_1\, L_1 \cot k_1\, L_1) - k_1\, L \cot k_1\, L_1 = \left(\frac{k_2}{k_3}\right)^2 + \left[k_3\, L - \frac{(k_2)^2\, L_2}{k_3}\right] \cot k_3\, L_2 \qquad (50)$$

In Eq. 50, when $k_1 = 0$:

$$(k_2)^2 = (k_3)^2 = \frac{P}{E\, I_b} = m^2;\ (k_4)^2 = \frac{P}{E\, I_a} = n^2;\ L_1 = a;\ \text{and}\ L_2 = b.$$

Noting that, for infinitely small values of k_1,

$$\frac{1 - k_1\, L_1 \cot k_1\, L_1}{(k_1)^2} = \frac{(L_1)^2}{3} \qquad (51)$$

Eq. 50 becomes

$$\frac{n^2\, a^2}{3} - \frac{L}{a} = 1 + m\, a \cot m\, b \qquad (52)$$

As demonstrated by Professor Timoshenko,[13] the influence of the shearing force is equivalent to the replacement of the moment of inertia I by a decreased moment of inertia $I\left(1 - \frac{K\,P}{A\,G}\right)$, which shows that, in Eq. 52, m must be replaced by q, whereas n remains unchanged. By substitution, Eq. 24a is derived directly, obviously containing the error related to the change of slope at P, and making unnecessary all the development presented in the paper. If the correct formula, Eq. 43, is applied to Eq. 52, the latter becomes

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[11] "Theory of Elastic Stability," by S. Timoshenko, 1st Ed., McGraw-Hill Book Co., Inc., New York and London, 1936, Paragraph 22.

[12] *Ibid.*, p. 113, Eq. 79.

[13] *Ibid.*, p. 140.

$$q^2\left(\frac{n^2\,a^2}{3} - \frac{L}{a}\right) = m^2\,(1 + q\,a \cot q\,b) \quad \text{.............} (53)$$

which is identical to Eq. 44 when the necessary changes are made in the notation.

Marshall Holt,[14] Assoc. M. Am. Soc. C. E.—In the design of latticed struts it may be desired to have the effective slenderness ratios about the two principal axes equal so that the computed column strengths for bending in the two directions will be equal. In other words, the question may be raised as to how far apart two channels, as used in the numerical example, need be placed so that the column strengths about the axes y and z will be equal.

Eq. 34a will be useful in this study. Instead of finding a fictitious length of the member, as the author does, one can find the necessary radius of gyration r_z. Thus, instead of Eq. 34c, one can write

$$P_y = \frac{(4.32)^2\,E\,A}{\left(\frac{L}{r_y}\right)^2} = P_z = \frac{t^2\,E\,A}{\left(\frac{L}{r_z}\right)^2} \quad \text{...............} (54)$$

in which r_y = the radius of gyration with respect to the y-axis; and r_z = the required radius of gyration about the z-axis. It then follows that

$$\frac{r_z}{r_y} = \frac{4.32}{t} \quad \text{........................} (55a)$$

or, approximately,

$$\frac{r_z}{r_y} = \frac{4.32}{4.32 - 30\,\beta + 160\,\beta^2} \quad \text{................} (55b)$$

Eqs. 55 are quite like Eqs. 35. A direct solution is impossible because the value of β depends on the size and arrangement of the lacing bars, and these have not yet been determined. A trial solution is possible which consists essentially in checking a trial value of r_z by means of Eq. 55b or the curve of Fig. 6.

Since β is an inverse function of slenderness ratio, Fig. 6 shows that the ratio of r_z to r_y should be relatively larger for small slenderness ratios than for large slenderness ratios. In other words, for a given system of lacing and a given size of lacing bar, the relation between the two radii of gyration should not be specified by a single value.

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In problems of design it might be practicable to plot curves of the type shown in Fig. 6 for the system of lacing and size of bars under consideration. For this purpose, the first scale of abscissas (scale a) is in terms of β, the second (scale b) is in terms of the slenderness ratio about the z-axis, and the third (scale c) is in terms of the slenderness ratio about the y-axis. Scale c was obtained from scale b and the indicated values of the ratio r_z/r_y since the slenderness ratios are in this inverse ratio. For a certain length of column, say 320 in., the slenderness ratio about the y-axis is 57.4, and the required ratio of r_z/r_y is 1.15. The necessary value of r_z is found to be 6.41 in., and the distance back to back of the channels (14.28 in.) follows from simple computations.

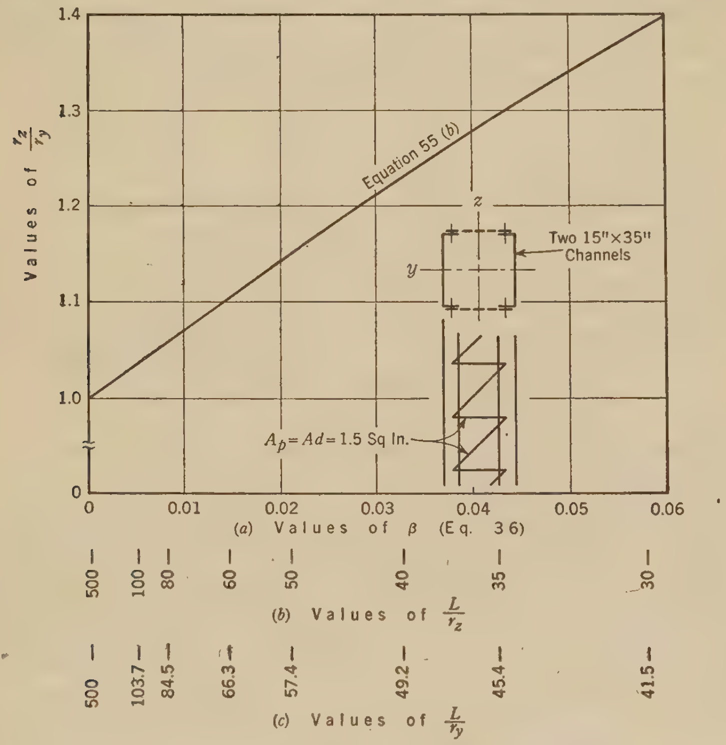


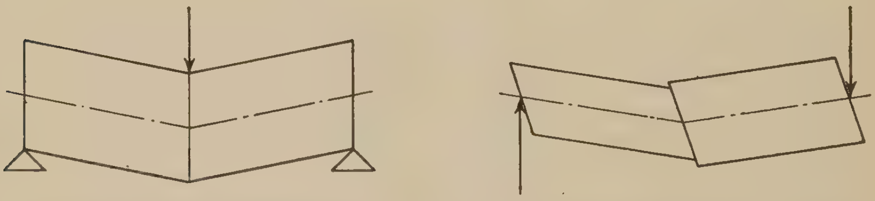
FIG. 6.—RATIO OF r_z TO r_y FOR EQUAL COLUMN STRENGTHS FOR BENDING ABOUT THE z AND y AXES

In some cases it may not be desirable to have the column strength for bending about the two axes equal because of different conditions of restraint or support in the two directions. In this case a factor of proportionality of the column strengths can be introduced into Eq. 54. The procedure is the same as that for equal column strengths.

A procedure similar to that followed by the author was applied to columns loaded on the ends by Ernst Amstutz[15] in 1941.

David B. Hall,[16] Assoc. M. Am. Soc. C. E.—This paper, together with another by the same author,[17] which it supplements, is a useful addition to the literature of column analysis. Hence, with some regret the writer notes an error in the derivation which will necessitate some revisions.

The error is in Eq. 21, which was introduced to establish continuity between the two segments of the strut shown in Fig. 1. The author has assumed that the elastic curves of both segments must have the same slope at the point of application of the concentrated load P, where they join. This assumption is incorrect. The slope of the elastic curve of shearing deflection of a beam is discontinuous at points of concentrated loading (Fig. 7(a)), and at points of abrupt change in section (Fig. 7(b)).



(a) EFFECT OF CONCENTRATED LOAD

(b) EFFECT OF CHANGE OF SECTION

Fig. 7.—Shearing Deflection Curves

The correct method of establishing compatibility at $x_a = a$ is to fit together the two faces of the section cut through that point. The expressions for the slopes of cross sections of the upper and lower segments of the strut are shown in Fig. 8. This figure is diagrammatic. In solid members, for example, plane sections under shearing loads become S-shaped. The sloping plane surfaces shown represent effective values, obtained with the aid of the shape factor K in the case of solid members, or by means of Eqs. 28 in the case of built-up members.

To make the lower face of the upper segment match the upper face of the lower segment one must write, in place of Eq. 21:

$$\left(\frac{dy_a}{dx_a} - \frac{P\,e\,K_a}{L\,A_a\,G}\right)_{x_a=a} = -\left[\frac{dy_b}{dx_b} - P\left(\frac{dy_b}{dx_b} - \frac{e}{L}\right)\frac{K_b}{A_b\,G}\right]_{x_b=b} \quad \ldots(56)$$

The substitution of Eq. 56 for Eq. 21 must have some effect on all further development in the paper. The remainder of this discussion will be confined to ascertaining whether the effect is of any actual consequence, leaving the detailed work of revising the affected equations for the author to perform, if warranted, in his closing discussion.

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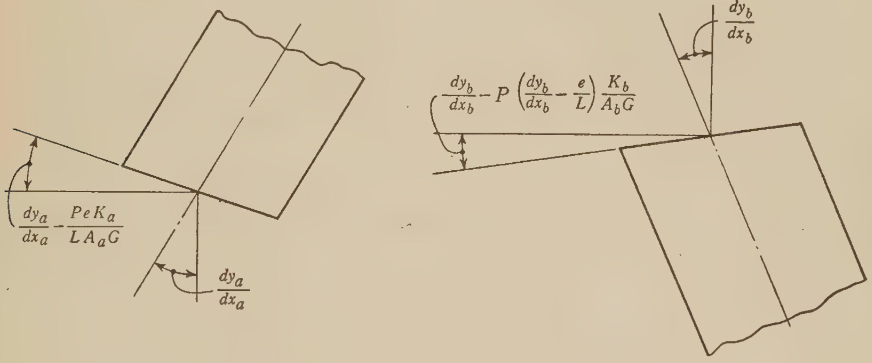
[15] "Die Knicklast gegliederter Stäbe," by Ernst Amstutz, *Schweizerische Bauzeitung*, Vol. 118, August 30, 1941, pp. 97–101.

[16] Stress Analyst, Otis Elevator Co., New York, N. Y.

[17] *Bulletin No. 101*, Eng. Experiment Station, Univ. of Washington, Seattle.


$\frac{dy_b}{dx_b}$
$\frac{dy_b}{dx_b} - P\left(\frac{dy_b}{dx_b} - \frac{e}{L}\right)\frac{K_b}{A_b G}$
$\frac{dy_a}{dx_a} - \frac{PeK_a}{LA_aG}$
$\frac{dy_a}{dx_a}$

(*a*) UPPER SEGMENT (*b*) LOWER SEGMENT

Fig. 8.—Relation of Slope of Cross Section to Slope of Elastic Curve

Attention first will be directed to the simplified problem which results when the strut is of constant section throughout its length. For this condition, Eq. 56 reduces to

$$\left(\frac{dy_a}{dx_a}\right)_{x_a=a} = -\frac{dy_b}{dx_b}\left(1 - \frac{P\,K}{A\,G}\right)_{x_b=b} \qquad (57a)$$

or

$$\left(\frac{dy_a}{dx_a}\right)_{x_a=a} = -\frac{m^2}{q^2}\left(\frac{dy_b}{dx_b}\right)_{x_b=b} \qquad (57b)$$

Also, $u = 1$. The operations which immediately follow are analogous to those of the paper, and, in place of Eq. 24*b*,

$$\frac{\tan k\,q\,L}{q\,L} = \frac{1}{\frac{q^2\,L^2}{3}(1-k) - \frac{\frac{q^2}{m^2} + 1 - k}{(1-k)^2}} \qquad (58)$$

For purposes of comparison it will be convenient to convert both Eq. 24*b* and Eq. 58 into the terms introduced in Eq. 30, with some modification. In the paper t is a measure of the critical column load, and α is a measure of the relative shearing flexibility. Now designate all corresponding terms pertaining to a strut subject to bending deflection only by the subscript M. Let:

$$t = t_M + \delta t \qquad (59)$$

Then

$$m_M\,L = t_M \qquad (60a)$$

$$m\,L = t_M + \delta t \qquad (60b)$$

and

$$q\,L = \frac{t_M + \delta t}{[1 - \alpha\,(t_M + \delta t)^2]^{0.5}} \quad \text{(61}a\text{)}$$

It will be of further advantage to confine the comparison to struts for which α is a small number. As α approaches zero, δt will also approach zero, and a linear relation between them can be found by the same processes used in deriving the rules for differentiation. Eq. 61*a*, for example, may now be rewritten:

$$q\,L = t_M + \delta t + \tfrac{1}{2}\,\alpha\,t^3_M \quad \text{(61}b\text{)}$$

By (1) making the indicated substitutions, (2) carrying out the process of differentiation, and (3) subtracting from each side of the equation the terms that would appear in the equation for a strut subject to bending deflection only—one will finally arrive at the following new equations: From Eq. 24*b*—

$$(\delta t + \tfrac{1}{2}\,\alpha\,t^3_M)\left(\frac{k\sec^2 k\,t_M}{t_M} - \frac{\tan k\,t_M}{t^2_M}\right) = \frac{-\,2\,t_M\,\delta t\,(1-k)}{\left[\dfrac{t^2_M\,(1-k)}{3} - \dfrac{2-k}{(1-k)^2}\right]^2} \quad \text{(62}a\text{)}$$

and, from Eq. 58—

$$(\delta t + \tfrac{1}{2}\,\alpha\,t^3_M)\left(\frac{k\sec^2 k\,t_M}{t_M} - \frac{\tan k\,t_M}{t^2_M}\right)$$
$$= \frac{-\,2\,t_M\,\delta t\,(1-k) - \alpha\,t^2_M\left[\dfrac{t^2_M\,(1-k)}{3} - \dfrac{1}{(1-k)^2}\right]}{\left[\dfrac{t^2_M\,(1-k)}{3} - \dfrac{2-k}{(1-k)^2}\right]^2} \quad \text{(62}b\text{)}$$

Numerical values of t_M, or $k\,t_M$, as a function of k are already known,[6] and may be substituted in Eqs. 62*a* and 62*b*. Thus values of $\delta t/\alpha$ may be calculated as a function of k for each of these equations, Eq. 62*a* representing the development from Eq. 21, and Eq. 62*b* representing the development from Eq. 56. The two functions are plotted together in Fig. 9. The principal relations between these two curves can easily be interpreted: (*a*) The lateral component of P, producing the effect shown in Fig. 7(*a*), acts inward when P is applied in the upper part of the strut, and outward when P is applied in the lower part of the strut, thus increasing the stability in the former case and decreasing the stability in the latter case; and (*b*) the part of the shearing effect considered by the author—namely, the change in curvature in the loaded segment—diminishes in importance as the load approaches the lower support; hence, the relative error is greatest for small values of k.

The problem is more complicated when there is a change of section as in Fig. 7(*b*). Under ordinary conditions of design, the loaded segment of such a strut as that shown in Fig. 1 will be made stronger than the unloaded segment, and in many cases will be more resistant to shearing deflection. Under this, the normal condition, the angle at the junction of the two segments, produced by the shearing force $\frac{P\,e}{L}$, will always be in such a direction as to increase the deflection, and hence decrease the stability of the strut. In a case where the upper segment of the strut is considerably more yielding than the lower segment, and the load is applied at approximately midheight, this phenomenon may prove a substantial, or even the predominant, part of the total shear effect. Where the load is applied near either end, this effect will be less.

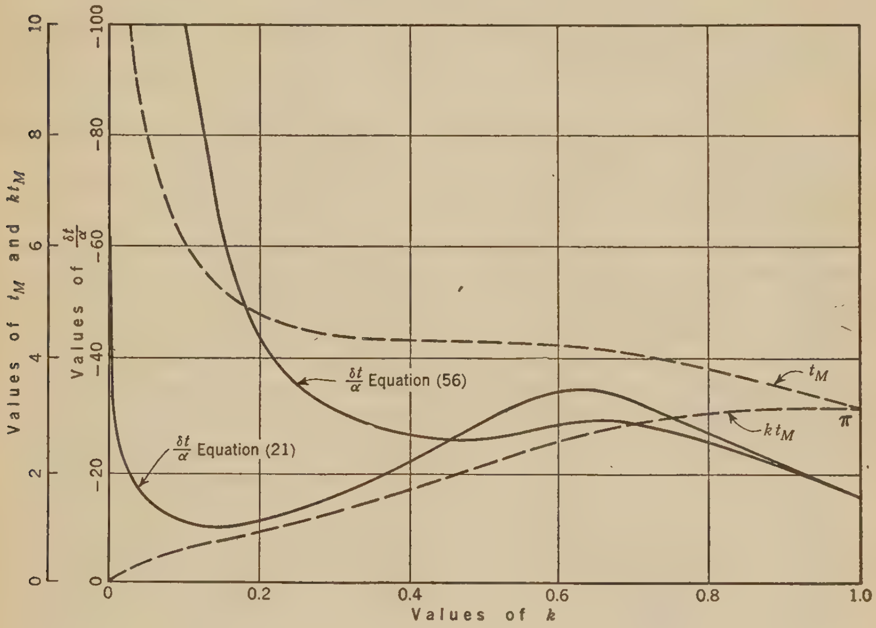


FIG. 9.—VALUES OF $\frac{\delta t}{\alpha}$ COMPUTED FROM EQS. 21 AND 56

Basing his judgment on the foregoing comparisons, the writer believes that the error in the paper is sufficiently large to warrant revision of the equations affected.

S. SERGEV,[18] ESQ.—An error in analysis was made in overlooking the discontinuity in slope of the strut axis caused by shearing forces. (The check method used was not independent as subsequent examination revealed.) Discontinuity of slope due to shearing action in members of homogeneous material may result from discontinuities of shearing forces and cross-sectional areas, separately or in combination. In the strut under discussion, both factors enter into consideration because the shearing forces and cross-sectional areas above and below the point of loading are different. For the special application, when the strut cross section is constant throughout, there is a sudden change in the shearing forces at the point of loading, and consequently a sudden change in the shearing strains is to be expected. Thus, the final slopes due to bending and shearing actions above and below the point of loading are different from each other and a kink in the elastic curve exists.

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[18] Associate Prof., Dept. of Civ. Eng., Univ. of Washington, Seattle, Wash.

To introduce the correct concept of slope relationships at points of discontinuity into the analysis, again consider Eq. 20. The constants C and D may be evaluated from the conditions (compare Eq. 21):

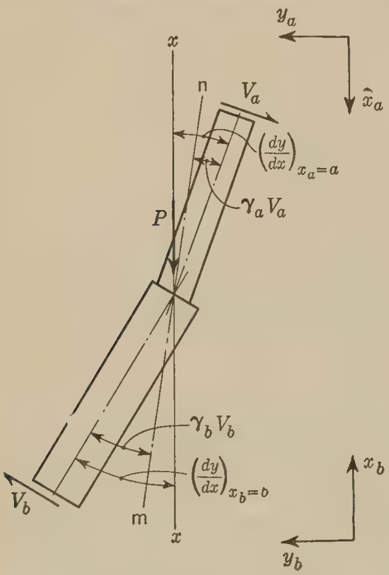
$$(y_a)_{x_a=0} = 0; \quad \text{and} \quad (y_a)_{x_a=a} = e \ldots\ldots\ldots\ldots\ldots (63)$$

from which $D = 0$, and $C = \frac{e}{a} + \frac{P\,e\,a^2}{6\,E\,I_a\,L}$. Substitution of these values into Eq. 20 gives (compare Eq. 23):

$$y_a = -\frac{m^2\,e\,x^3_a}{6\,u\,L} + \left(\frac{e}{a} + \frac{m^2\,e\,a^2}{6\,u\,L}\right) x_a \ldots\ldots\ldots\ldots\ldots (64)$$

This expression for y_a will be simpler to use in the strain energy which follows later.

The interrelationship between the slopes of the upper and lower segments can be established using the following arguments: Consider a small part of the strut in the vicinity of the loading (Fig. 10) under the action of the shearing forces as indicated. The line mn represents the tangent to the elastic curve if the shearing strains are neglected. The angles between the line mn and the vertical are numerically equal but opposite in sign, the latter fact following from the choice of the coordinate axes. This is the requirement of continuity for bending actions only. However, if it is desired to consider shear deformations, then the shearing strains in the upper and in the lower segments must be taken into account, and the following equality, with due regard to signs, should be used:



y_a
x
x_a
n
V_a
$\left(\frac{dy}{dx}\right)_{x_a=a}$
$\gamma_a V_a$
P
$\gamma_b V_b$
$\left(\frac{dy}{dx}\right)_{x_b=b}$
x_b
V_b
m
x
y_b

FIG. 10

$$\left(\frac{dy_b}{dx_b} + \gamma_b\,V_b\right)_{x_b=b} = -\left(\frac{dy_a}{dx_a} - \gamma_a\,V_a\right)_{x_a=a} \ldots (65)$$

In Eq. 65:

$$\gamma_b\,V_b = \frac{K\,V_b}{A_b\,G} = \frac{K}{A_b\,G}\left(-P\,\frac{dy}{dx} + \frac{P\,e}{L}\right)_{x_b=b} \ldots (66a)$$

and

$$\gamma_a\,V_a = \frac{K\,V_a}{A_a\,G} = \frac{K}{A_a\,G}\,\frac{P\,e}{L} \ldots (66b)$$

Substituting these equivalents into Eq. 65, keeping in mind that

$$\left(\frac{dy}{dx}\right)_{x_b=b} = \frac{e\,a\,q\cot q\,b}{L} + \frac{e}{L} \ldots\ldots\ldots\ldots\ldots (67a)$$

and

$$\left(\frac{dy}{dx}\right)_{x_a=a} = -\frac{m^2\,e\,a^2}{2\,u\,L} + \frac{e}{a} + \frac{m^2\,e\,a^2}{6\,u\,L} \ldots\ldots\ldots\ldots (67b)$$

gives

$$\frac{e\,a\,q\cot q\,b}{L} + \frac{e}{L} + \frac{K\,P}{A_b\,G}\left(-\frac{e\,a\,q\cot q\,b}{L} - \frac{e}{L} + \frac{e}{L}\right)$$
$$= -\left(-\frac{m^2\,e\,a^2}{2\,u\,L} + \frac{e}{a} + \frac{m^2\,e\,a^2}{6\,u\,L} - \frac{K\,P\,e}{A_a\,G\,L}\right) \quad \ldots\ldots\ldots\ldots(68)$$

Eliminating e and noting that $\frac{K\,P}{A_a\,G} = \frac{A_b}{A_a}\left(1 - \frac{m^2}{q^2}\right)$, Eq. 68 may be written as follows:

$$a\,q\cot q\,b = \frac{q^2\,a^2}{3\,u} - \frac{q^2}{m^2}\left(1 + \frac{L}{a} - \frac{A_b}{A_a}\right) - \frac{A_b}{A_a} \quad \ldots\ldots\ldots(69a)$$

Let $b = k\,L$, $a = L\,(1 - k)$, and, rearranging slightly, Eq. 69a becomes

$$L\,q\cot k\,q\,L = \frac{q^2\,L^2\,(1-k)}{3\,u}$$
$$- \frac{q^2}{m^2}\left[\frac{2-k}{(1-k)^2} - \frac{A_b}{A\,(1-k)}\right] - \frac{A_b}{A_a\,(1-k)} \quad \ldots\ldots\ldots(69b)$$

which should replace Eq. 24b as first derived. It can now be seen that not only does the ratio of the moments of inertia u enter into consideration but also the ratios of the cross-sectional areas $\frac{A_b}{A_a}$. Thus, it is possible to study a strut with $I_a = I_b$ and with the areas A_b and A_a unequal.

The result given in Eq. 69b completely describes the buckling criteria for struts of uniform but unequal cross sections in each segment loaded as previously described (see Fig. 1).

Check by Strain Energy.—If the equations for the elastic curve of the strut axis are established in both segments, then a solution of the problem may be obtained without resorting to the relationship expressed in Eq. 65, by using the strain energy method. If it is admitted that under a critical load P a slight deflection of the strut is possible, then a small amount of strain energy of flexure and shear will be stored in the strut in the form of potential energy ΔV. This potential energy must be equal numerically to the loss in potential energy of the force P due to loss in elevation.

The potential energy of strain can be calculated in the usual way from the following expression:

$$\Delta V = \int_0^b \left(\frac{M^2_b}{2\,E\,I_b} + \frac{K\,V^2_b}{2\,A_b\,G}\right) dx_b + \int_0^a \left(\frac{M^2_a}{2\,E\,I_a} + \frac{K\,V^2_a}{2\,A_a\,G}\right) dx_a \quad ..(70)$$

in which

$$M_b = -P_y + \frac{P\,e}{L}\,x = -\frac{P\,a\,e\sin q\,x}{L\sin q\,b} \quad \ldots\ldots\ldots\ldots(71a)$$

$$V_b = -P\frac{dy_b}{dx_b} + \frac{P\,e}{L} = -\frac{P\,a\,e\,q\cos q\,x}{L\sin q\,b} \quad \ldots\ldots\ldots\ldots(71b)$$

$$M_a = -\frac{P\,e\,x}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(71c)$$

and

$$V_a = -\frac{P\,e}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(71d)$$

Substituting these equivalents into Eq. 70, and performing the indicated operations, give

$$\Delta V = \frac{m^4\, E\, I_b\, e^2}{L^2}\left(\frac{a^2\, b}{4\sin^2 q\, b} + \frac{a^2 \cot q\, b}{4\, q} + \frac{a^3}{6\, u} + \frac{K\, a\, E\, I_b}{2\, A_a\, G}\right.$$
$$\left. + \frac{a^2\, q^2\, b\, K\, E\, I_b}{4\, A_b\, G \sin^2 q\, b} + \frac{K\, a^2\, q^2\, E\, I_b \cot q\, b}{4\, q\, A_b\, G}\right) \ldots\ldots\ldots\ldots (72)$$

The loss in potential energy of the force P is equal to the product of the force and the distance that it moves downward. This loss in potential energy will be designated by ΔU, and it can be calculated in the following manner:

$$\Delta U = \int_0^b P\,(ds_b - dx_b) = P \int_0^b \left[\sqrt{1 + \left(\frac{dy_b}{dx_b}\right)^2} - 1\right] dx_b \ldots . (73)$$

If higher powers than the first of the small quantities are neglected, the loss in potential energy becomes

$$\Delta U = P \int_0^b \frac{1}{2}\left(\frac{dy_b}{dx_b}\right)^2 = \frac{P}{2}\int_0^b \left(\frac{a\, e\, q \cos q\, x}{L \sin q\, b} + \frac{e}{L}\right)^2 dx_b \ldots\ldots (74a)$$

or

$$\Delta U = \frac{m^2\, E\, I_b\, e^2}{L^2}\left(\frac{a^2\, q^2\, b}{4\sin^2 q\, b} + \frac{a^2\, q^2 \cot q\, b}{4\, q} + a + \frac{b}{2}\right) \ldots\ldots (74b)$$

Setting Eqs. 72 and 74*b* equal to each other and simplifying, an expression identical to Eq. 69*b* is obtained.

This check method is not entirely independent, as it uses the results obtained from the geometrical properties of the elastic curve of the strut. Its merit lies in the fact that it is independent of the discontinuities of slope at the point of loading.

Approximate Solution.—A check of the results previously obtained can be made by using an approximate method based on strain energy and an assumed shape for the elastic curve. (A check of this kind may not reveal errors in analysis if the influence of the errors on the solution is small, say, less than 1%.) As a first approximation, assume that the deflection curve is a sine curve,

$$y = f_1 \sin\frac{\pi\, x}{L} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (75)$$

in which f_1 is a constant. The general expression for the strain energy is that expressed by Eq. 70, in which the bending moments and shearing forces will be now expressed as follows:

$$M_b = -P\, y + \frac{P\, e\, x_b}{L} = -P\, f_1 \sin\frac{\pi\, x}{L} + \frac{P\, e\, x_b}{L} \ldots\ldots . (76a)$$

$$V_b = \frac{P\, e}{L} - P\frac{dy_b}{dx_b} = \frac{P\, e}{L} - \frac{P\, f_1}{L}\cos\frac{\pi\, x_b}{L} \ldots\ldots\ldots\ldots (76b)$$

$$M_a = -\frac{P\, e\, x_a}{L} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (76c)$$

$$V_a = -\frac{P\, e}{L} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (76d)$$

and

$$e = f_1 \sin \frac{\pi b}{L} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (76e)$$

Substituting Eqs. 76 into Eq. 70, and performing the indicated operations, the expression for strain energy becomes

$$\Delta V = \frac{P^2}{2 E I_b} \left[\frac{b f^2_1}{2} - \frac{f^2_1 L}{4} \sin \frac{2 b \pi}{L} + \frac{e^2 b^3}{3 L^2} - \frac{2 f_1 e L}{\pi^2} \sin \frac{\pi b}{L} \right.$$
$$\left. + \frac{2 f_1 e b}{\pi} \cos \frac{\pi b}{L} + \frac{e^2 a^3}{3 u L^2} \right] + \frac{K P^2}{2 A_b G} \left[\frac{b e^2}{L^2} + \frac{f^2_1 \pi^2}{L^2} \left(\frac{b}{2} + \frac{1}{4} \sin \frac{2 b \pi}{L} \right) \right.$$
$$\left. - \frac{2 f_1 e}{L} \sin \frac{b \pi}{L} + \frac{e^2 a A_b}{L^2 A_a} \right] \ldots\ldots\ldots\ldots\ldots\ldots (77)$$

The loss in potential energy ΔU of the load P due to a lowering of its point of application can be calculated as previously described. The expression for ΔU is

$$\Delta U = P \int_0^b \frac{1}{2} \left(\frac{d y_b}{d x_b} \right)^2 d x_b = \frac{P f^2_1 \pi^2}{2 L^2} \int_0^b \cos^2 \frac{\pi x_b}{L} d x_b$$
$$= \frac{P f^2_1}{2 L} \left(\frac{\pi b}{2 L} + \sin \frac{2 \pi b}{L} \right) \ldots\ldots\ldots\ldots\ldots\ldots (78)$$

Substituting $e = f_1 \sin \left(\frac{\pi b}{L} \right)$ into Eq. 77, and equating it to Eq. 78, a general expression for P may be obtained. If in this expression $b = \frac{L}{2}$ is used, it will give, after a slight rearrangement,

$$P = \frac{\pi^2 E I_b}{L^2} \frac{1}{\left(\frac{7}{6} - \frac{8}{\pi^2} + \frac{1}{6 u} \right) + \frac{K E I_b}{A_b G L^2} \left(\pi^2 - 6 + \frac{2 A_b}{A_a} \right)} \ldots (79)$$

in which, for struts with solid webs, $\frac{K E I_b}{A_b G L^2}$ is denoted by α and for latticed struts by β.

Special Application.—As a special application of the formulas previously derived, consider the case in which $I_b = I_a = I$, $A_a = A_b = A$, and $k = \frac{1}{2}$. Substitution of the values into Eq. 69*b* gives

$$6 L q \cot \frac{1}{2} q L = q^2 L^2 - 24 \frac{q^2}{m^2} - 12 \ldots\ldots\ldots\ldots (80)$$

Let $m L = t$ and $q L = w$; then Eq. 80 may be transformed into Eq. 46, in which for the latticed struts $\beta = \frac{1}{t^2} - \frac{1}{w^2}$. The roots of Eq. 46 were determined by Mr. Beskin.

Now consider Eq. 79. Making the substitutions noted herein and simplifying gives the following relationship between P and β,

$$P = \frac{\pi^2 E I}{L^2} \frac{1.913}{1 + 11.23\,\beta} = \frac{1.012\,P'}{1 + 11.23\,\beta} \ldots\ldots\ldots\ldots (81a)$$

As a function of ψ, the critical load,

$$P = \frac{\pi^2 E I}{L^2} \frac{1.913}{1 + P'\,(0.602\,\psi)} = \frac{1.012\,P'}{1 + 0.602\,\psi\,P'} \ldots\ldots\ldots (81b)$$

in which P' is the critical load on the strut similarly loaded but neglecting shear deflections; that is,

$$P' = \frac{(4.32)^2\,E\,A}{\left(\frac{L}{r}\right)^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (82)$$

The degree of accuracy of Eq. 81*a* will be studied now by comparing the critical load P obtained from it with that obtained from Eq. 46. Using the data in the numerical example following Eq. 35*b*, and the $\frac{L}{r}$ ratios in Table 1, the critical loads determined by Eqs. 46 and 81*a* may be ascertained (see Table 2). From these calculations it can be concluded that, for practical applications, Eq. 81*a* gives sufficiently accurate results.

TABLE 2.—NUMERICAL EXAMPLE; SPECIAL APPLICATION

$\frac{L}{r}$ (1)	β (2)	t (3)	P (4)	P_1 (5)	P' (6)	$\frac{P}{P'}$ (7)	$\frac{P_1}{P'}$ (8)
60	0.0145	3.99	0.0905 E	0.0922 E	0.106 E	0.855	0.87
80	0.00815	4.13	0.0546 E	0.055 E	0.0597 E	0.915	0.92
100	0.00522	4.2	0.0361 E	0.0366 E	0.0382 E	0.945	0.96
120	0.00362	4.24	0.0255 E	0.0258 E	0.0265 E	0.962	0.973
160	0.00204	4.28	0.01465 E	0.01475 E	0.0149 E	0.983	0.99
200	0.00131	4.295	0.00942 E	0.00951 E	0.00955 E	0.986	0.995
Source	Eq. 37*b*	Fig. 5	Eq. 34*a*	Eq. 81*a*	Eq. 34*b*	Exact	Approximate

An advantage of the approximate method is the fact that it gives explicit functions such as Eq. 81*a* or 81*b* for evaluating the critical load. The more exact method reduces to an implicit function, Eq. 46, the roots of which must be found first before it can be used practically.

In Col. 7, Table 2, ratios of the buckling load on latticed struts to that on struts with solid webs are given using the more exact analysis. Similar ratios are given in Col. 8, Table 2, where the buckling load on the latticed struts was determined by the approximate method. When the results in Cols. 7 and 8 are compared, the approximate method slightly underestimates the shear effect on the strength of the strut but is sufficiently accurate for most practical purposes. Finally, comparing the final results in Table 1, Col. 7, with the corrected values as shown in Table 2, Col. 7, the conclusion is that the solution, as first given, exaggerated the shear effect on the strength of the struts.

Comment on Discussions.—Mr. Beskin's criticism that the extended treatment of the conditions involved in Assumption 5 is superfluous is partly justified. Eqs. 1 to 3 were intended only to show the approximate relationship existing between curvature and the corresponding bending moment. It would have been logical to have included the shear effect on the final curvature. To make Eqs. 8 and 17 general a constant should be added to the right side. This constant is found from the boundary conditions, is dependent on both the shear and moment effects, and in the general case can be determined only after the final solution is obtained. It is the angle through which the member must be rotated so that the shear deflection will satisfy the end condition. Since the shearing force V is affected by this rotation, Eq. 8, as altered, may be considered fundamental but not always useful as a starting point in the solution of a given problem. In straight simple beams of constant cross section symmetrically loaded, the constant is zero. However, if the loading is symmetrical but the cross section varies, it is necessary to rotate the member about one end so that the shear deflection will vanish at the other. A simple beam of constant cross section unsymmetrically loaded by a concentrated force also requires that the constant be zero as do simple beams of constant cross section loaded by couples, the latter showing distortion in the longitudinal direction.

The separation of the deflection y into components is permissible provided those components are properly evaluated. In problems to which the principle of superposition does not apply, these components cannot be found independently and resort is made to adding together the elastic properties of the member which are free from constants not known in advance. For the problem at hand, the addition of the curvatures due to the corresponding shear and moments is such a sum.

The expression $\frac{K\,V}{A\,G}$ represents the relative sliding angle of two adjacent transverse cross sections of the member and is the average effect of the shearing stresses over the entire cross section. The constant K is found by the energy method equating the work of the shearing force to the total work of the shearing stresses, or by using the more refined methods of the theory of elasticity.[19]

The mode of restraint at the ends of a member plays a significant rôle in the solution of shear deflection problems, and can only be treated adequately by the more exact methods developed in the theory of elasticity. Experience shows, however, that comparatively simple problems in elasticity become extremely difficult or defy solution altogether except by approximate methods. For practical needs, the approximate methods often give the desired accuracy.

Mr. Beskin is correct in calling attention to the misstatement that "the curvature due to bending of a built-up strut is independent of the type of lattice used." If (as implied from the outset) there are many lattice bars in the length of the member, the average curvature may seem continuous and independent of the type of lattice. Actually it is not.

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[19] "Theory of Elasticity," by S. Timoshenko, 1st Ed., McGraw-Hill Book Co., Inc., New York and London, 1934, pp. 42 and 150.

The author laid no claim on originality in handling the problem as the references show. The prime purpose was to ascertain quantitatively the influence of the shearing forces on the strength of the type of struts under consideration.

Mr. Holt calls attention to the important practical problem of estimating the distance between the principal parts of the built-up strut for equal strengths about the principal axes, and develops a relationship between the radii of gyration. Regretfully, because of the error already noted, Fig. 6 may need to be modified.

Mr. Hall early found the error in analysis and much later submitted his discussion with convincing evidence that a change in the affected equations is warranted. His treatment of the comparison of the corrected values with those derived by the writer is interesting and conclusive.

In conclusion, the writer feels that, although the continuity of the paper has been disturbed by the error in analysis, this defect is compensated for by again emphasizing the discontinuity of slopes in elastic bodies due to shearing forces. The writer appreciates the participation of the discussers. At this time it also seems appropriate to thank D. H. Young for his interest. Modesty prevented his sending a discussion because the writer called attention to the error in advance.

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ECONOMICAL CANAL CROSS SECTIONS

By Victor L. Streeter,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. V. A. Endersby, Julian Hinds,
D. C. McConaughy, C. J. Posey, and Victor L. Streeter.

Synopsis

The use of machines for trimming the subgrade and for placing concrete lining in canals has dispensed with the need for simple cross sections composed of straight-line segments. By the use of curved canal cross sections, the hydraulic properties are improved, resulting in a substantial saving in concrete and excavation. This paper shows the method of analyzing several typical curved sections, as well as the most economical trapezoidal section for comparable paving-machine speeds. As the paving machines have cover forms for the concrete which move continuously, their speeds will depend upon the properties of the concrete and upon the steepness of the cross section. An index of steepness is derived in the paper and, assuming the mix to be constant, the speed of the machines is assumed to be constant for cross sections having the same steepness index. This provides a basis for comparing various sections all having the same steepness index.

Introduction

Most of the problems encountered in civil engineering practice require the exercise of judgment and experience. No amount of economic studies will displace this sound judgment and experience, but economic studies of certain factors of an engineering project will provide additional information for the designing engineer which will permit him to design a structure of comparable safety at less total cost.

Large paving machines were used in the construction of the open-channel portions of the Colorado River aqueduct. In 1938, W. L. Chadwick, M. Am. Soc. C. E., and G. E. Archibald, Assoc. M. Am. Soc. C. E.,[2] described this paving machine as follows:

Note.—Published in May, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Associate Prof., Illinois Inst. of Technology, Chicago, Ill.

[2] "Machines and Methods for Canal Construction, Colorado River Aqueduct," by W. L. Chadwick and G. E. Archibald, *Civil Engineering*, February, 1938, p. 104.

> "In this machine, the important element was a form or screed plate, shaped exactly to the cross-section of the finished canal, fastened to the under side of a trussed frame, and provided with feed hoppers and a motorized distributor car. * * * The machine, drawn by power winches, operated like a great trowel. * * * The machine was moved forward continuously, shaping both invert and sides as it passed. It was capable of placing lining as accurately as the grade of the tracks [supporting it along the berm] could be maintained. * * * One record shows a combined trimming and placing accuracy of better than 1 per cent."

As the initial cost of a paving machine is large, its use is limited to the construction of long canals of constant cross section. The two principal items of cost are concrete and excavation. The cost of lining is proportional to the wetted perimeter and the cost of excavation depends upon the area, water-surface width, and height of natural ground surface above the hydraulic grade line. The ideal cross section from the standpoint of minimum excavation and concrete would be a semicircle. In general, however, the excavation would not stand, and cover forms for the concrete would be required, making the construction very expensive. By selecting a section which permits the trimmed subgrade to stand by itself, and which permits the use of a paving machine operated at a reasonable speed, a smaller total cost of canal is obtained.

In this paper the speed of paving machine is taken to be a function of the steepness index which is derived in the next section. Several types of cross sections are computed for three values of the steepness index, and their perimeters and water-surface widths are compared.

Notation

The letter symbols used in this paper conform essentially to American Standard Letter Symbols for Hydraulics, prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942.[3]

The Steepness Index

When concrete is placed on a flat slope (say, 1 on 2) cover forms are not required. The friction developed between the subgrade and the fresh concrete, as well as the adhesion to the reinforcing steel (usually added to control temperature effects), provides stability immediately after placing. There is also some resistance to change of shape in the concrete itself before the initial set. When concrete is placed on a steep slope the effect of friction and internal resistance may not be sufficient to hold the concrete in place. An index to this stability is derived as follows:

Consider a strip of concrete of unit width placed on a sloping surface (see Fig. 1). The force tending to cause motion is the component of the weight tangent to the surface. Let the frictional force resisting motion (including effect of reinforcing bars) be designated by F per unit area of surface; and let γ_c equal the unit weight of concrete. Then for a volume element $t\,dl$ the resultant force tending to cause motion is $t\,dl\,\gamma_c \sin\theta - F\,dl$, provided the first term is greater than the second term. The accumulated force for the slope would be $\int (t\,\gamma_c \sin\theta - F)\,dl$, integrated over the portion of one side of the cross section where the integrand is positive. Taking θ_0 as the angle for which the integrand is zero,

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[3] ASA—Z10.2—1942.

$$F = t\,\gamma_c \sin\theta_0 \qquad (1)$$

and the accumulated force may be expressed as $t\,\gamma_c \int (\sin\theta - \sin\theta_0)\,dl$, carried out from $\theta = \theta_0$ to the water surface. As $\sin\theta = \frac{dy}{dl}$ the force becomes $t\,\gamma_c\,(y_0 - l\sin\theta_0)$, in which y_0 is the vertical distance from the point on the section where $\theta = \theta_0$ to the water surface, and l is the length of section from $\theta = \theta_0$ to the water surface, as shown in Fig. 2. These calculations are for sections which are concave upward, or straight lines, which are the only practical shapes for canal cross sections. Dividing the accumulated force by $t\,\gamma_c$, a steepness index, σ, is obtained having the dimensions of a length, thus

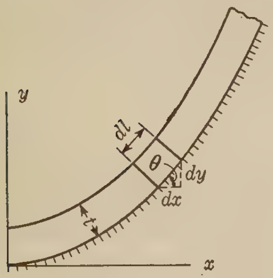

FIG. 1.—SECTION OF CANAL LINING

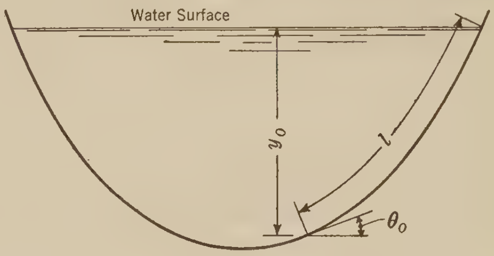

FIG. 2.—A CURVED CANAL CROSS SECTION

$$\sigma = y_0 - l\sin\theta_0 \qquad (2a)$$

If it is considered, for example, that $\tan\theta_0 = 2/3$, then the steepness index becomes

$$\sigma = y_0 - 0.555\,l \qquad (2b)$$

In this paper two canal sections having the same values of σ are presumed to be capable of construction with the same speed of paving machines.

RELATIVE COST OF LINING AND EXCAVATION

The horizontal and vertical alinement of the canal is assumed to have been determined by other studies.[4] The problem of determining the most economical cross section to convey a given discharge at a given slope of the hydraulic gradient for a given or assumed roughness of lining is approached in the following manner.

The Manning formula may be written

$$A\,R^{2/3} = \frac{Q\,n}{1.486\,S^{0.5}} = K \qquad (3)$$

in which: A is the cross-sectional area of canal below water line in square feet; R is the hydraulic radius of the cross section—that is the cross-sectional area divided by the wetted perimeter; Q is the discharge in cubic feet per second; n is the Manning roughness factor; S is the slope of water surface for uniform, steady flow; and K is a known constant for any given study, defined by Eq. 3. Writing $R = \frac{A}{P}$, in which P is the wetted perimeter in feet, Eq. 3 may be written

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[4] "Aqueduct Size and Slope," by Julian Hinds, *Engineering News-Record*, January 28, 1937, pp. 113–120; and "Economic Sizes of Pressure Conduits," by Julian Hinds, *ibid.*, March 25, 1937, pp 443–449.

$$A = P^{0.4}\,K^{0.6} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The cost of concrete lining for comparable cross sections (same steepness index σ and same freeboard) is proportional to the wetted perimeter, P. For minimum cost of lining, a solution of Eq. 4 is required for minimum P. The cost of excavation depends in general upon the yardage, which in turn depends upon the cross-sectional area and the water-surface width. Referring to Fig. 3, the cost of the canal in dollars per foot of length may be expressed as

$$c_l = C + b_w \frac{y_z\,c_e}{27} + \frac{A\,c_e}{27} + \frac{P\,t\,c_c}{27} \quad \ldots\ldots\ldots\ldots\ldots (5)$$

in which the constant C is independent of the cross-sectional shape; and: b_w is the water-surface width; y_z is the average depth of hydraulic gradient below the natural ground surface; t is the thickness of lining in feet; c_e is the cost of excavation per cubic yard; and c_c is the cost of concrete lining in place, per cubic yard.

In Eq. 5, A, P, and b_w are factors dependent upon the cross section. The problem resolves itself into selecting a cross section for a predetermined steepness index which makes Eq. 5 a minimum. Theoretically, the problem could be solved by methods of calculus of variations, but practically it reduces to an examination of the properties of various types of cross sections.

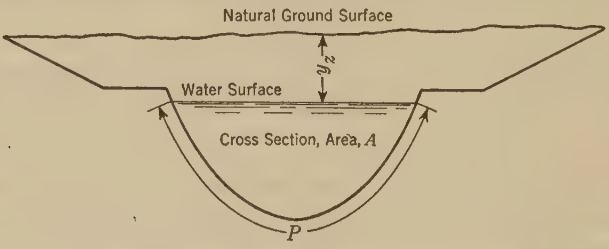

FIG. 3.—CURVED CANAL CROSS SECTION IN CUT

COMPUTATION OF PROPERTIES OF CANAL CROSS SECTIONS

Typical procedures to use in the computation of canal cross-section relationships, such as perimeter, area, water-surface width, and steepness index, are illustrated for the following sections: (1) Trapezoidal cross section, (2) straight sides, circular bottom cross section, (3) circular segment cross section, (4) parabolic cross section, (5) semicubical parabola cross section, and (6) catenary cross section.

The problem reduces to one of finding the dimensions of the cross section such that the given discharge may be conveyed at the specified slope and with a given steepness index. For the first two types of cross sections these conditions may be satisfied in many ways; hence the condition of minimum perimeter of section is also used. For the first two sections the conditions to be satisfied are

$$(a)\quad A\,R^{2/3} = \frac{Q\,n}{1.486\,S^{0.5}} = \text{a known constant} = K \ldots\ldots\ldots (6)$$

$$(b)\quad \sigma = y_0 - l \sin\theta_0 \ldots\ldots\ldots (2a)$$

and

(c) P = a minimum, subject to conditions (a) and (b) (see Eqs. 2a and 6).

For the remaining four sections, only conditions (a) and (b) (Eqs. 2a and 6) are to be satisfied.

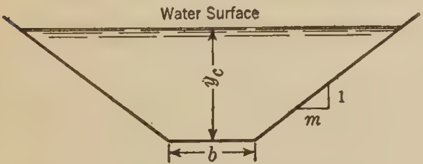

FIG. 4.—TRAPEZOIDAL CROSS SECTION

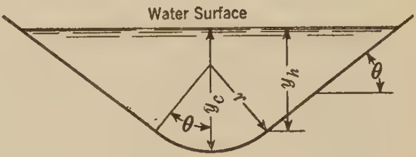

FIG. 5.—STRAIGHT SIDES AND CURVED BOTTOM, CIRCULAR

(1) *Trapezoidal Cross Section.*—Using the notation of Fig. 4,

$$A = b\,y_c + m\,y^2_c \ldots\ldots\ldots (7a)$$

$$P = b + 2\,y_c\sqrt{1+m^2} \ldots\ldots\ldots (7b)$$

$$\sigma = y_c\,(1 - \sqrt{1+m^2}\,\sin\theta_0) \ldots\ldots\ldots (7c)$$

and

$$A - K^{0.6}\,P^{0.4} = 0 \ldots\ldots\ldots (8)$$

Eliminating A, b, and y_c from Eqs. 7 and 8,

$$\frac{P\,\sigma}{1 - \sqrt{1+m^2}\,\sin\theta_0} + \frac{(m - 2\sqrt{1+m^2})\,\sigma^2}{(1 - \sqrt{1+m^2}\,\sin\theta_0)^2} - K^{0.6}\,P^{0.4} = 0 \ldots (9)$$

which expresses the perimeter, P, as a function of m for given values of σ, θ_0, and K. For minimum P, Eq. 9 is differentiated with respect to m, and $\frac{dP}{dm}$ is set equal to zero. The resulting equation is

$$\frac{P}{\sigma} = \frac{2}{\sin\theta_0} - \frac{\sqrt{1+m^2}}{m\sin\theta} + \frac{4\sqrt{1+m^2} - 2\,m}{1 - \sqrt{1+m^2}\,\sin\theta_0} \ldots\ldots\ldots (10)$$

which is independent of K. Eliminating P in Eqs. 9 and 10 leaves an equation with m the only unknown. Its value may be determined by trial for given values of σ, θ_0, and K.

(2) *Straight Sides, Circular Bottom Cross Section.*—This cross section, shown in Fig. 5, is completely specified by values of θ, r, and y_h. In terms of these quantities (notation shown in Fig. 5)

$$A = y^2_h \cot\theta + y_h\,r \times 2\sin\theta + r^2\,(\theta - \sin\theta\cos\theta) \ldots\ldots\ldots (11a)$$

$$P = y_h \times 2\csc\theta + r \times 2\,\theta \ldots\ldots\ldots (11b)$$

and

$$\sigma = y_h \left(1 - \frac{\sin \theta_0}{\sin \theta}\right) + r\,(\cos \theta_0 + \theta_0 \sin \theta_0 - \cos \theta - \theta \sin \theta_0) \ldots (11c)$$

Using the Manning formula, condition (*a*), it may be shown that the condition for minimum P is

$$y_h = r \cos \theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

which states that the center of curvature of the circular portion lies in the water surface, and that the result is independent of θ_0 or K. To obtain Eq. 12, eliminate A between Eqs. 6 and 11*a*. Considering y_h and r as functions of P, σ, and θ, differentiate with respect to σ, holding θ constant, and set $\frac{dP}{d\sigma} = 0$. Differentiating Eq. 11*b* with respect to σ and setting $\frac{dP}{d\sigma} = 0$ the terms $\frac{dy}{d\sigma}$ and $\frac{dr}{d\sigma}$ may be eliminated. Simplifying, it is found that $y_h = r \cos \theta$. Eq. 11*c* is not used. Using Eq. 12:

$$A = r^2\,(\theta + \cot \theta) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13a)$$

$$P = 2\,r\,(\theta + \cot \theta) = \frac{2\,A}{r} \ldots\ldots\ldots\ldots\ldots\ldots (13b)$$

$$\sigma = r\left[\cos \theta \left(1 - \frac{\sin \theta_0}{\sin \theta_0}\right) + \cos \theta_0 + \theta_0 \sin \theta_0 - \cos \theta - \theta \sin \theta_0\right] .. (13c)$$

and

$$r = \frac{\sigma + \frac{P}{2} \sin \theta_0}{\theta_0 \sin \theta_0 + \cos \theta_0} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Eliminating A and r in Eqs. 6, 13*b*, and 14,

$$P^{0.6}\left(\sigma + \frac{P}{2} \sin \theta_0\right) = 2\,(\theta_0 \sin \theta_0 + \cos \theta_0)\,K^{0.6} \ldots\ldots\ldots (15)$$

from which P may easily be obtained by trial for given values of σ, θ_0, and K. With P known, Eqs. 14, 6, 13*a*, and 12 yield values of r, A, θ, and y_h, respectively. The water-surface width is given by

$$b_w = 2\,r \csc \theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

(3) *Circular Segment Cross Section.*—The properties of this cross section are completely specified by r and θ, as shown in Fig. 6. Thus

$$A = r^2\,(\theta - \sin \theta \cos \theta) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17a)$$

$$P = 2\,r\,\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17b)$$

in which θ is expressed in radians, and

$$\sigma = r\,(\cos \theta_0 + \theta_0 \sin \theta_0 - \theta \sin \theta_0 - \cos \theta) \ldots\ldots\ldots\ldots (17c)$$

Using Eq. 6 (condition (*a*)), sufficient equations are available to determine r

and θ for given values of σ, θ_0, and K. This is done by trial from the relations

$$P = \frac{2\,\theta\,\sigma}{\cos\theta_0 + \theta_0 \sin\theta_0 - \theta \sin\theta_0 - \cos\theta} \qquad (18a)$$

and

$$\frac{P^2}{4\,\theta^2}(\theta - \sin\theta\cos\theta) - K^{0.6}\,P^{0.4} = 0 \qquad (18b)$$

which are derived from Eqs. 6 and 17.

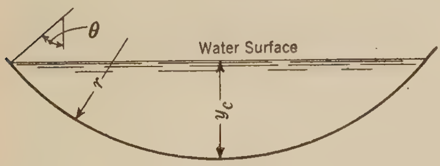

Fig. 6.—Circular Segment Cross Section

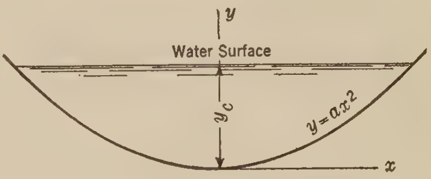

Fig. 7.—Parabolic Cross Section

(*4*) *Parabolic Cross Section.*—Referring to Fig. 7, the properties of the cross section are determined for given values of the constant a in the formula

$$y = a\,x^2 \qquad (19)$$

and the depth y_c at the center. In terms of y_c and a,

$$A = \frac{4\,y^{1.5}_c}{3\,a^{0.5}} \qquad (20a)$$

$$P = \sqrt{\frac{y_c}{a} + 4\,y^2_c} + \frac{1}{2\,a}\left[\log_e 2\,\sqrt{a\,y_c} + \sqrt{1 + 4\,a\,y_c}\right] \qquad (20b)$$

and

$$\sigma = y_c + \sin\theta_0 \left[\frac{1}{4\,a}\log_e(\tan\theta_0 + \sec\theta_0) - \frac{P}{2}\right] \qquad (20c)$$

In conjunction with Eq. 6 (condition (a)) for specific values of a, K, and θ_0, values of y_c are found from Eqs. 20*a*, 20*b*, and 6. The corresponding values of σ may then be determined by Eq. 20*c*. Different values of a are assumed until the required value of σ is obtained.

(*5*) *Semicubical Parabola Cross Section.*—The computation procedure for the semicubical parabola

$$y = \alpha\,x^{3/2} \qquad (21)$$

is similar to that for the parabola. Referring to Fig. 8, the area, perimeter, and steepness index are:

$$A = 1.2\,\frac{y^{5/3}_c}{\alpha^{2/3}} \qquad (22a)$$

$$P = \frac{0.5926}{\alpha^2}\left[\sqrt{(1 + 2.25\,\alpha^{4/3}\,y^{2/3}_c)^3} - 1\right] \qquad (22b)$$

and

$$\sigma = y_c - \frac{8}{27}\,\frac{\tan^3\theta_0}{\alpha^2} + \sin\theta_0\left[\frac{0.2963}{\alpha^2}(\sec^3\theta_0 - 1) - \frac{P}{2}\right] \qquad (22c)$$

With known values of K and θ_0, an α is assumed, and σ is computed, first by determining y_c and P from Eqs. 6, 22a, and 22b by trial, and then substituting in Eq. 22c. This procedure is continued for several values of α, and the curve α versus σ is plotted. From this curve the value of α for an arbitrary σ may be obtained.

(6) *Catenary Cross Section.*—The particular catenary curve

$$y = \frac{1}{\beta}(\cosh \beta x - 1) \qquad (23)$$

as shown in Fig. 9 is selected so that the perimeter may be obtained easily by integration. Values of β and y_c completely determine the cross section. The area, perimeter, and steepness index are expressed more simply in terms of x_0, half the water-surface width b_w. These relations are

$$A = \frac{2\,x_0}{\beta}\cosh \beta\,x_0 - \frac{2}{\beta^2}\sinh \beta\,x_0 \qquad (24a)$$

$$P = \frac{2}{\beta}\sinh \beta\,x_0 \qquad (24b)$$

and

$$\sigma = \frac{1}{\beta}[\cosh \beta\,x_0 - \sin \theta_0 \sinh \beta\,x_0 - \cos \theta_0] \qquad (24c)$$

From Eqs. 24 and 6, σ may be determined by trial for assumed values of β when K and θ_0 are known. Plotting σ against β, the values of β are determined for a predetermined σ.

EVALUATION OF PROPERTIES OF CROSS SECTIONS FOR A SPECIAL CASE

In this section, Eqs. 6 to 24 are applied to a specific problem, in which $K = 4{,}150$; and $\theta_0 = \tan^{-1} 2/3$. If a slope of 1 on 1.5 is the maximum upon which any length of the concrete may be placed without danger of slumping, the steepness index is given by (see Eq. 2a):

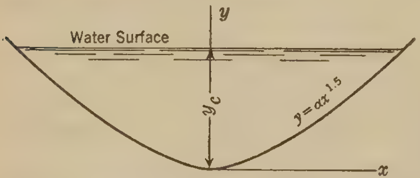

FIG. 8.—SEMICUBICLE PARABOLA CROSS SECTION

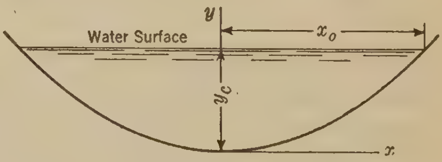

FIG. 9.—CATENARY CROSS SECTION

$$\sigma = y_0 - \frac{l}{\sqrt{3.25}} \qquad (25)$$

in which y_0 is the vertical distance from the water surface to the point where the cross-section slope is 1 on 1.5, and l is the length along the perimeter of the cross section from this point to the water surface. The six cross sections are determined for $Q = 3{,}000$ cu ft per sec, with the canal slope $S = 0.00004$ and a value of canal roughness $n = 0.013$. These values result in $K = \frac{Q\,n}{1.486\,S^{0.5}} = 4{,}150$ (see Eq. 3). Three values of steepness index are used for each section: $\sigma = 0.75$, 1.50, and 2.25. The equations for the curvature of sections 4, 5, and 6 are given in Table 1. The various properties of the cross sections are given in Table 2.

TABLE 1.—MODIFIED EQUATIONS FOR CURVED CROSS SECTIONS IN ILLUSTRATIVE EXAMPLE

$Q = 3{,}000$ cu ft per sec; $S = 0.00004$; $n = 0.013$; $K = 4{,}150$; and $\theta_0 = \tan^{-1} 2/3$

See Table:	σ	Section 4, Eq. 19 and Fig. 7	Section 5, Eq. 21 and Fig. 8	Section 6, Eq. 23 and Fig. 9
2*a*	0.75	$y = 0.01166\,x^2$	$y = 0.09234\,x^{3/2}$	$y = \frac{1}{0.0211}(\cosh 0.0211\,x - 1)$
2*b*	1.50	$y = 0.01456\,x^2$	$y = 0.10584\,x^{3/2}$	$y = \frac{1}{0.026}(\cosh 0.026\,x - 1)$
2*c*	2.25	$y = 0.0175\,x^2$	$y = 0.1187\,x^{3/2}$	$y = \frac{1}{0.03083}(\cosh 0.03083\,x - 1)$

TABLE 2.—PROPERTIES OF CANAL CROSS SECTIONS; ILLUSTRATIVE EXAMPLE

Section No.	Type; see Fig.:	Area A, in sq ft	Depth y_c at center, in ft	Radius r of circular bottom, in ft	Water-surface width b_w, in ft	Wetted perimeter P, in ft	Slope of section at water surface
(*a*) $\sigma = 0.75$							
1	4	875.8	20.40	(13.97)[a]	71.89	84.82[b]	1.42
2	5	872.4	20.77	20.77	71.58	84.00[b]	1.40
3	6	890.2	16.15	57.46	79.95	88.42	1.03
4	7	880.7	17.20	....	76.81	86.10	1.12
5	8	870.5	20.07	....	72.30	83.67	1.20
6	9	883.5	16.86	....	77.75	86.77	1.09
(*b*) $\sigma = 1.50$							
1	4	870.14	20.34	(15.57)[a]	69.97	83.50[b]	1.34
2	5	865.35	21.01	21.01	69.12	82.36[b]	1.31
3	6	870.08	17.12	47.80	73.26	83.47	0.84
4	7	863.39	18.28	....	70.86	81.94	0.97
5	8	860.02	21.04	....	68.12	81.13	1.08
6	9	865.14	17.88	....	71.57	82.35	0.93
(*c*) $\sigma = 2.25$							
1	4	864.7	20.30	(17.14)[a]	68.02	82.24[b]	1.25
2	5	858.2	21.27	21.27	66.56	80.69[b]	1.20
3	6	854.8	17.88	41.23	67.99	79.93	0.69
4	7	851.6	19.25	....	66.34	79.13	0.86
5	8	852.5	21.91	....	64.84	79.39	0.98
6	9	852.3	18.80	....	66.85	79.33	0.82

[a] Bottom width of trapezoid. [b] Minimum wetted perimeter for the types indicated.

As a means of illustrating the method of comparing sections, assume the cost of concrete lining in place to be $c_c = \$15.00$ per cu yd, the cost of excavation to be $c_e = \$0.15$ per cu yd, and the lining thickness, t, to be 0.33 ft. Substituting into Eq. 5 the values of A, b_w, and P obtained for the cross sections for $\sigma = 0.75$ (Table 2(*a*)), it is seen that section 5, the semicubical parabola, costs less than the others for y_z less than 18 ft. Section 2, with straight sides and circular bottom, costs less than the others for y_z greater than 18 ft. Since y_z enters linearly into the cost equation its average over the length of canal in cut may be used.

For $\sigma = 1.50$ (see Table 2(*b*)), substitution of A, b_w, and P into Eq. 5 shows that section 5, the semicubical parabola, is most economical for all values of y_z. For $\sigma = 2.25$ (Table 2(*c*)), section 4, the parabola is most economical for y_z less than 6.4 ft, whereas section 5, the semicubical parabola, is most economical for y_z greater than 6.4 ft.

Although these comparisons take into account the more important factors in selection of a cross section, there are other factors which should be carefully considered by the designing engineer. For strength against uplift the radius of curvature at the vertex would be important. The average velocity for partial capacities would affect the deposition of sand and silt. In some cases the cost of excavation might be more costly for the deeper cross sections. All of these factors cannot be evaluated in terms of dollars, but should be given careful consideration before the final cross section is decided upon.

Conclusions

With the availability of paving machines for the lining of long canals, curved canal cross sections may be placed at no greater expense than the customary trapezoidal sections. In addition, if a canal cross section is selected which has less water-surface width and less perimeter than the trapezoidal section, both excavation and concrete yardages will be reduced. The most economical canal cross section from the standpoint of concrete yardage is a semicircle. This section, however, could not be used generally because the excavation would not retain this shape, and the concrete would require a fixed cover form.

In order to set up a criterion that would allow the comparison of various sections, a "steepness index" was developed. Two cross sections having the same value of steepness index are assumed to require the same speed of paving machine.

Six particular types of cross sections were investigated to demonstrate methods of determining the properties of sections with a given value of steepness index. The general methods employed will apply to different cross sections and to other capacities than those used in the example. If the same six types of sections were analyzed for a smaller channel, it is likely that different types would prove to be most economical.

Acknowledgments

The study of economical canal cross sections was first undertaken by the writer while employed at the headquarters office of the International Boundary Commission, United States and Mexico, United States Section, at El Paso, Tex. The work was under the supervision of Richard Stephens, Assoc. M. Am. Soc. C. E., and under the general direction of J. L. Burkholder, M. Am. Soc. C. E. L. M. Lawson, Hon. M. Am. Soc. C. E., is the United States Commissioner. The writer wishes to express his appreciation for their aid and suggestions in making this study.

DISCUSSION

V. A. ENDERSBY,[5] M. AM. SOC. C. E.—The conclusions, arrived at analytically by Professor Streeter and substantiated by field studies made by the writer, are of particular interest in connection with flexible types of canal linings.

Some Field Observations.—The problem of an optimum canal section arose in connection with the investigation of the possible use of asphaltic lining membranes to fit the needs of laterals and other small canals that cannot carry the capital costs of cement or asphalt concrete linings of the thicker types (2 in. to 4 in.). Since the proposed thin linings can be expected only to serve as impermeabilizing agents and not as slope stabilizers, except in a minor degree, the matter of self-stabilizing canal sections is of primary importance. The thin, flexible linings will prevent erosion, weathering, and raveling of the surface soil, etc., but will give no appreciable resistance to hydrostatic pressures or mass soil movements.

The observations of ditches in use revealed a tendency for sections that were constructed as trapezoids to become modified by nature, in four different ways:

Case 1.—Cohesive soil of an unstable type will bulge into the canal near the bottom, and will slump near the top. This bulge slumps further as it becomes soaked, tending to assume a talus formation.

Case 2.—Granular, noncohesive soil erodes below the water line, thus releasing slides that fill in and overrun the eroded cavity.

Case 3.—Granular, cohesive soil, resistant to erosion, often weathers and ravels above the water line, moves down the slope, and forms a talus.

Case 4.—In combination with case 1, 2, or 3, vegetable growth tends to build out a berm at the water line. Since all cases result in rounded bottom corners, this combination of factors tends to form a semicircular cross section; and in fact sections are often found which are nearly perfect semicircles.

All other factors being equal, a section, originally build to the form most acceptable to nature, will be the most likely to escape natural changes. A rounded section is mechanically stable against gravitational forces because it forms a series of inverted elementary arches. The weakness of sharp angles in any structure, as compared with that of rounded forms, has long been established in relation to rigid structures and in fact to tunnel work. If a section that is fundamentally stable is constructed, the problem of maintaining it is merely that of preventing changes in the mechanical properties of the materials composing it. In this case the sole agent materially active in producing such changes is moisture, either by increase or reduction of content, or by erosion. Thus, the sole need for meeting the problem is one of adequate waterproofing.

Application to Thin or Flexible Linings.—Professor Streeter develops his theory in relation to the tendency of fresh concrete to slump on a slope. The fundamental principles are applicable to granular materials such as sand, to granular-plastic materials such as plastic soils, and to bituminous linings. The latter are granular-plastic in nature rather than rigid, especially while being placed hot, or when exposed to very hot weather.

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It also happens that a great part of such flexible linings as have been placed has been compacted by transverse rolling. To roll transversely, it is necessary to round the corners to some degree; and, although this may not be a consideration when more suitable means of compaction are employed, the fundamental structural considerations remain.

During the recent construction of an asphalt-lined canal for military uses, the lining was placed by machinery in a trapezoidal section, but the problem of the rounded corner was approached by the subsequent addition of fillets, indicating an appreciation, by the authorities concerned, of the underlying structural considerations.

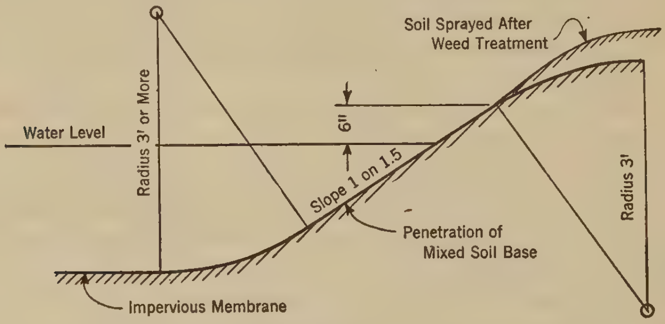

Fig. 10.—Ideal Canal Section for Low-Cost Linings

Professor Streeter expresses the greater stability of the rounded section in Eq. 2*a* and in Fig. 2, in which σ is the steepness index. In the case of a trapezoidal canal, y_0 becomes the full depth of the canal and θ_0 becomes zero except in the case where $\int (t \gamma_0 \sin \theta - F) \, dl$ is zero or minus. In other words, the trapezoidal section gives the maximum slumping force; and, even where the frictional resistance is greater than the accumulated slumping force, the rounded section gives greater resistance to possible ground movements under the lining.

The Ogee Section.—The logical section devised by the writer was determined both by the foregoing considerations as to the economy of the wetted perimeter, and by the special needs of a thin, flexible lining, and is shown in Fig. 10. The total thickness of base and membrane should be not less than 1 in. except when the section is located on firm and cohesive soil. The reversed curve at the berm is intended to prevent edge damage to the lining from stock, etc., and to prevent the infiltration of surface water behind it. The wetted permeter may equally well be any one of the sections discussed by Professor Streeter—circular, parabolic, or catenary.

Summary.—Professor Streeter's conclusions appear to be sound from the points of view of the stability of fresh concrete on the canal slopes, of economy of construction, and of hydraulic efficiency. Furthermore, they conform to the section which nature appears to produce for reasons of inherent stability, and have a special importance where thin, plastic, or flexible linings are used.

Julian Hinds,[6] M. Am. Soc. C. E.—The author has introduced a new member into the family of criteria for "best shapes" of water conduits. Engineers generally are familiar with "best shapes" as determined from minimum area and minimum hydraulic radius alone, and know that only for pipe lines and flumes do these shapes have a great deal of practical significance. For lined excavated conduits (the type considered by the author), the best hydraulic shape usually is poor from a cost-of-construction viewpoint. The engineer is interested in the cheapest and most practical conduit rather than the one with the smallest waterway area and perimeter. For this reason, constructed canal sections usually bear little resemblance to the theoretically "best sections."

If all of the elements affecting costs were included in the theory, a complete formula for the "cheapest section" might be written. Generally, the results are so involved and conditions so variable from place to place that it is preferable to depend on trial-and-error procedure. However, knowledge of the theory of best shape is useful.

Modern development of machines for placing concrete lining has stabilized, or at least limited, some of the variables. The author has taken advantage of this fact to move a step nearer complete determination of the best shape for this particular type of conduit. This is a laudable proposal and the results deserve the careful consideration of engineers interested in problems of this kind.

Although the author's statement of basic principles is clear, the writer wishes to restate them from a slightly different point of view. Consider a differential mass of concrete standing alone on a sloping bank, as at dl, Fig. 1. This mass tends to move down the slope under the influence of gravity. Its movement is resisted by friction along the surface of the bank. If the sliding force exceeds the resisting force, the mass will move unless some other form of support is provided.

If the slab is filled in above and below the mass in question, as indicated in Fig. 1, the differential mass may be made stable by the compressive strength of the concrete below it. The top surface of the mass must in turn support the accumulated excess of sliding force over resistance for the concrete above. Thus the required crushing strength of the differential block will increase as its position moves downward from the top. If the slope is straight, the maximum is reached at the bottom or wherever an outside support occurs. If the slab is curved, the maximum occurs at the position where the individual mass is just stable within itself (against sliding), that is, where $\theta = \theta_0$, using the author's notation.

If the unit weight of the concrete, the resistance to sliding, and the form and dimensions of the bank are known, the required internal resistance can be computed; or, if the permissible internal stress is known, the allowable slope and dimensions may be determined.

The author finds the total force to be resisted in a unit width of slab down to $\theta = \theta_0$ by integration. This he divides by the thickness, t, to get unit stress, and by the weight of a cubic unit of the concrete to get a "steepness factor." The last division is not essential. An "allowable stress" is perhaps more comprehensible to most engineers than a "steepness factor."

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It is noted that Eq. 2*a* gives σ in linear units—that is, in feet, for American usage. If the weight is left in, the results are in pounds per square foot, which might be converted to pounds per square inch. The numerical values of σ from Eq. 2*a* are very nearly equal to the allowable stress in pounds per square inch if the canal dimensions and t are in feet, as a 1-in. by 1-in. by 12-in. prism of concrete weighs approximately 1 lb.

The writer is of the opinion that making the sliding resistance, F, constant, regardless of slope, is incorrect. In other sliding problems, the resistance is assumed to be proportional to the normal force against the supporting surface. If this change is accepted, F becomes $F' \cos \theta$ and Eq. 2*a* becomes

$$\sigma' = y_0 - x_0 \tan \theta_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

in which x_0 is the horizontal projection of the portion of the lining above the point where $\theta = \theta_0$.

If applied to a vertical concrete wall, Eq. 26 gives a stress at the bottom of the wall equivalent to that corresponding to the weight of the wall. For Eq. 2*b*, the stress (or steepness factor) is less than half that corresponding to the weight of the wall; that is, more than half the weight of the lining would have to be held by friction on the vertical bank. The author perhaps does not contemplate a vertical wall but it comes within the theoretical scope of his equations. Also, the effect noted applies in lesser degree to other slopes. Maximum slopes for some of the channels of Table 2 appreciably exceed 1 : 1. The resistance offered by securely anchored, imbedded steel may be more or less independent of slope, which might call for a compromise between Eqs. 2*a* and 26 for heavily reinforced linings. The writer knows of no data on which such a compromise could be based.

Changing from Eq. 2*a* to Eq. 26, or to a compromise, if found desirable, would not vitiate the general idea of the paper but would change some of the detailed computations.

Apparently a placing machine with a tailpiece or apron to smooth and temporarily support the freshly placed lining is contemplated. It is also presumably contemplated that concrete has some resistance to deformation when first placed, and that this resistance increases with passing minutes. Therefore, the slower the machine moves, other things being equal, the stronger will be the concrete as it clears the apron. Thus the speed of machine operation, the height and slope of the bank, and the strength of the soft concrete are interrelated.

Ignoring for the moment the probability that Table 2 should be revised to conform to Eq. 26, it is possible to select the "best" of the listed sections in any of the groups, Table 2(*a*), 2(*b*), or 2(*c*). However, no means for choosing between the three groups is offered. Such a choice requires a knowledge of the details of the machine, the original consistency and rate of hardening of the concrete, and the effect of speed on the cost of lining. The details of the machine are known or determinable. The consistency of the concrete and the rate of stiffening, for the first 10 min or so, are controllable to some extent and are no doubt ascertainable under fixed conditions. The effect of permissible speed of placement on the cost of lining requires careful study. The costs of materials, of trimming banks, of troweling surfaces, and of other items are more or less independent of speed. Nevertheless, other cost items are reduced by rapid placement. By balancing, properly, all the cost and time factors for lining against variations in excavation quantities and other items, if there are any, a "cheapest" section can be devised. Here, as in the more usual case of "best shape," topography and other variables are likely to complicate theoretical formulas beyond the limit of practicability, requiring resort to trial-and-error methods.

As a matter of fact, the average computer will find most of the equations beyond Eq. 8 too complicated for routine use. This is not because the theory is complicated but because of involved mensuration. The use of these equations in a theoretical study, such as that presented in this paper, is entirely permissible, but "practice" is likely to revert to "trial and error."

Another practical point that should be considered is that a good lining machine is costly and durable. Unless the job to be done is large, it will be desirable to salvage the machine for subsequent use or to use a machine salvaged from previous work. This can be accomplished more readily with a trapezoidal machine than with one of circular, parabolic, catenary, or other curved form. Curved fillets of unchanging radius at the bases of the trapezoidal side slopes are permissible and beneficial.

Everything considered, the trapezoidal section—possibly with slight modification—is likely to remain the preferred section for excavated concrete-lined canals. However, it is important that "everything" actually be "considered," in order that the extent of the sacrifice to practicability may be evaluated fully. To this end this paper will be a helpful one.

D. C. McConaughy,[7] Esq.—For the numerical example chosen, the author finds that the parabola and the semicubical parabola are the most economical shapes. However, the slopes at the water surface for these sections, given in Table 2, vary from 0.86 to 1.20. These slopes, in some cases, would not be practicable for the excavation. The slopes at the water surface for the trapezoidal section and for the section with straight sides and curved bottom vary, respectively, from 1.25 to 1.42 and from 1.20 to 1.40, which ranges are more desirable for some soils. It becomes of interest, therefore, to compare the cost of the most economical section with that of the sections more nearly resembling those ordinarily used. If the term C in Eq. 5 is assumed the same for all sections, relative costs in the author's illustrative example are obtained from the remaining terms and are shown in Table 3. The volume of excavation is, roughly, 60 cu yd per ft, so that a small additional cost of excavation for the curved sides would offset any supposed economy due to the shape. The lining thickness is taken as the same for all sections. Whether this would be the case, assuming that the excavated slopes could be maintained at least temporarily, would depend on the judgment of the designer as to whether or not the slope could be regarded as permanent.

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In Eq. 2, the stiffness factor σ seems to be a measure of the total force tending to slide the lining down the side slopes; in other words, an increase in σ seems to mean that there is less friction of the concrete on the sides of the excavation. This might result from a low coefficient of internal friction for the ground. In that case, the author's example shows that the section should be made deeper and the sides steeper—a rather peculiar result.

If the example chosen may be considered representative, the curved bottom with straight sides would seem nearly as cheap as the more elaborate sections, and more practicable.

The author deserves the thanks of the profession for making available a method either of designing a section of minimum cost or of ascertaining how much the cost of any given section that may be considered desirable departs from the absolute minimum.

TABLE 3.—COST, IN DOLLARS PER FOOT, OF CANALS WITH VARIOUS CROSS SECTIONS

Section	Type	VALUES OF y_z:				
		0	10	15	20	25
(a) STEEPNESS INDEX (σ) = 0.75						
1	4	20.57	24.57	26.57	28.56	30.55
2	5	20.40	24.40	26.40	28.39	30.39
5	8	20.33	24.35	26.35	28.36	30.37
(b) STEEPNESS INDEX (σ) = 1.50						
1	4	20.30	24.19	26.13	28.07	30.02
2	5	20.06	23.90	25.82	27.75	29.67
5	8	19.80	23.58	25.47	27.37	29.26
(c) STEEPNESS INDEX (σ) = 2.25						
1	4	20.03	23.81	25.70	27.59	29.48
2	5	19.71	23.41	25.26	27.11	28.96
4	7	19.38	23.07	24.91	26.76	28.60
5	8	19.44	23.04	24.84	26.64	28.44

C. J. POSEY,[8] ASSOC. M. AM. SOC. C. E.—According to the author, concrete can be placed higher on the side slopes of a canal, without using cover forms, than would be permitted by friction alone. This seems quite reasonable, although contrary to the textbook statements that freshly poured concrete has the properties of a fluid. The extra force necessary to hold the concrete up along the slope must be a compressive resistance in the fresh concrete. For a V-shaped cross section, with sides too steep to support the concrete by friction, this force would increase uniformly to a maximum at the bottom. If the concrete were placed too high on the sides, failure would result when the accumulated compressive force in the concrete at the bottom became so great as to cause the section to bulge out of shape and allow the concrete on the sides to slip down. For the V-shaped cross section, the compressive force from one side is balanced entirely by the compressive force from the other, assuming, of course, that the lining was poured to the same height on each side.

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If the section were trapezoidal, with sides steeper than the angle of friction, the failure would follow nearly the same pattern. The compressive force at the base of the slope would have to be resisted by a compressive force in the bottom lining. The intensity of this force would decrease away from the corner, due to friction on the bottom, becoming zero in the central part if the bottom were wide enough.

In the author's more general case, the compressive force in the concrete increases from zero at the top to a maximum at some point part way down the slope. Below this point the frictional forces exceed the component of the weight. At this point the frictional forces acting on a differential length dl are equal to the component of the weight along the slope. The compressive stress in the fresh concrete at this point, a distance y_0 from the water surface, helps support the concrete of the upper lining, which is on too steep a slope to remain there by friction alone.

In the development of the "steepness index," the author assumes that the frictional force resisting motion is independent of the angle of the subgrade. This use of a constant value for the frictional force, as given in Eq. 1, permits a very simple integration for the maximum accumulated force, but is open to question, for it is equivalent to assuming that the frictional force is all cohesion. It is difficult to think of the concrete as being capable of sustaining any appreciable accumulated compressive force if its properties do not include internal friction as well as cohesion. The frictional force between the concrete and the subgrade must vary, at least to some extent, with the normal pressure. The author's assumption of a constant value throughout may or may not be satisfactory. Experiments simulating the action of the paving machines in depositing concrete on different slopes would be helpful in evaluating the adequacy of the steepness index.

In Eq. 5, it would seem reasonable to assume that the cost of excavation varies as the nth moment of the cross section about the natural ground surface, n having a value near unity. For the comparison of canals of generally similar proportions, n might be taken as zero (as the author has done, since the area is the 0th moment), but, in considering deeper cross sections, the increased power requirement for elevating the excavated material should be taken into account.

The author is to be commended for his pioneering effort in a field which is destined to become more active as the labor of canal digging and paving is more completely mechanized. He is undoubtedly correct in suggesting that the old sharp corners, which are inefficient from both hydraulic and structural viewpoints, will be eliminated in favor of curves. It is well that the theoretical investigations keep ahead of the immediate demands of practice. Otherwise, designers might specify old-style cross sections in order to avoid the labor of computing the characteristics of a section better suited to their particular problems. On the other hand, the theoretical investigations should be based on as solid a foundation as possible. It might be important, for this problem, to examine critically the hydraulic friction formulas used, since results may be very sensitive to changes in formulas, and present confidence in the Manning and Kutter formulas may be based on data from too few types of cross sections. The writer has computed tables for the section illustrated in Fig. 5.[9] While making these computations, he wondered whether the Manning formula would give reliable results, when applied to such a cross section. No evidence either way has come to his attention.

Victor L. Streeter,[10] Assoc. M. Am. Soc. C. E.—Much interesting information on the application of the analytical approach for determining the best canal cross section has been presented in the discussions. The writer was pleased that the competent designing engineers who submitted discussions supported his method.

Mr. Hinds' suggestion that the stress in the material be used as a relative index of stability is an important improvement over the steepness index. Messrs. Posey and Hinds both question the use of a friction force proportional to the normal force instead of a constant. Resistance to slumping will depend on the mix, the reinforcing steel, the earth surface, and the time and vibration under the cover forms. The friction developed at the surface of contact would probably be close to Coulomb friction, but the other factors appear to be independent of the normal force. Before studies of this type can be applied quantitatively to a particular canal, full-scale tests should be performed on strips of canal lining to determine the formula more accurately and to ascertain the expected paving machine speed for a given steepness index.

Mr. Endersby has discussed an interesting application of the study to flexible linings. The cross section below the water line mentioned in his discussion conforms closely to the cross sections in the paper which proved economical. Certain safety features, such as resistance to hydrostatic uplift and mild earth pressures, are inherent in the curved section.

Mr. McConaughy's analysis shows that the cost is not greatly different for any of the sections listed. However, all these sections have been carefully selected. As an example, the lining of a trapezoidal section with side slopes of 1 on 1 and a steepness index of 1.50 would cost much more than the sections in the paper. In general, a "most economical" solution involves the selection of a design that is near the minimum cost. In this manner, other factors too complicated for analysis may be computed by the designing engineer without materially raising the cost due to economic factors.

This analytical study can certainly be no more accurate than the Manning formula upon which it is based, as stated by Professor Posey. Additional data regarding the extension of the Manning formula to unusual sections will improve any analytical studies on open-channel economy.

The writer feels that analytical methods should be applied to engineering structures in such a way that all factors may be expressed in mathematical form. Thus, such factors would no longer depend upon engineering judgment, and the designing engineer would be able to concentrate on less explicit factors.

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[9] "Hydraulics of Steady Flow in Open Channels," by S. M. Woodward and C. J. Posey, John Wiley & Sons, Inc., New York, N. Y., pp. 13, 14.

[10] Supervisor of Fluid Mechanics, Armour Research Foundation, Chicago, Ill.

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RIGIDITY AND AERODYNAMIC STABILITY OF SUSPENSION BRIDGES

By D. B. Steinman,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Edward Adams Richardson, R. K. Bernhard, J. M. Robertson, C. H. Gronquist, F. B. Farquharson, H. Reissner, Blair Birdsall, Hardy Cross, Frederick C. Smith and Hermann Friedlaender and George S. Vincent, Louis Balog, and D. B. Steinman.

Synopsis

Coefficients of rigidity, natural oscillation periods, excitation and amplification of vertical and torsional oscillations, aerodynamic forces and moments, lift and torque graphs, velocity relations, energy relations, positive and negative damping effects, critical wind velocities, rates of amplification, limiting amplitudes, effects of phase difference, and methods of predicting, testing, checking, and preventing aerodynamic instability—all of these factors are treated herein, as they affect the design of suspension bridges against aerodynamic effects.

New basic formulas for computing the various modes of oscillation are presented, which are believed to be simpler and more complete, at the same time being of equal or greater accuracy, than those hitherto available to the profession.

The physical relations affecting analysis and design are expressed in working formulas of a form convenient for direct application by bridge engineers. New graphic solutions that broaden scope and clarify conception supplement the analysis. The formulas derived from fundamental theory and those deduced empirically from available data are illustrated and checked by numerical examples applied to known structures.

Introduction

Investigation and analysis have been concentrated in the recent past on the related problems of the rigidity and aerodynamic stability of suspension bridges.

Note.—Published in November, 1943, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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The analysis presented herein has been found useful in explaining past experiences and in predicting new results. By such verification, the mathematical and physical relations developed and coordinated appear to have high consistency and validity—both as a diagnosis and as a guide for correction and prevention. At the same time the writer recognizes the gaps that still remain to be filled by further experimental investigation and the empirical deductions that need to be made more accurate.

Notation.—The letter symbols used in this paper follow customary notation as far as practicable, and are assembled in the Appendix for convenience of reference.

PART 1.—OSCILLATIONS OF SUSPENSION BRIDGES AND MEASURE OF RIGIDITY

The Spring Constant (K).—The "spring constant" K of any span is a measure of the elastic resistance of the structure to deformations or oscillations. It is used in the formula for natural period P or frequency of oscillations N:

$$P = \frac{1}{N} = 2\,\pi \sqrt{\frac{m}{K}} \qquad (1)$$

in which K is defined as the mean effective ratio of the distributed force of restitution or "loading increment" Δw to displacement η, m = mass of the span per unit length, and N = natural frequency of vertical oscillations. The equation of kinetic and potential energy in a simple harmonic cycle yields the general formula

$$K = \frac{\int_0^l \eta\,\Delta w\,dx}{\int_0^l \eta^2\,dx} \qquad (2)$$

in which x = abscissa of any point of the span, measured from one end.

Application to Suspension Bridges.—For a suspension span, by simply substituting the fundamental equilibrium equation

$$\Delta w = -\frac{d^2}{dx^2}(H\,\eta + y\,\Delta H) + \frac{d^2}{dx^2}\left(E\,I\,\frac{d^2\eta}{dx^2}\right) \qquad (3)$$

in Eq. 2, the general formula—

$$K = \frac{-\int_0^l \eta\,\frac{d^2}{dx^2}(H\,\eta + y\,\Delta H)\,dx + \int_0^l \eta\,\frac{d^2}{dx^2}\left(E\,I\,\frac{d^2\eta}{dx^2}\right)dx}{\int_0^l \eta^2\,dx} \qquad (4)$$

—is obtained. In Eqs. 3 and 4: H = horizontal component of tension in one cable; E = modulus of elasticity of the stiffening girders; I = moment of inertia of the section of one stiffening girder; and y = coordinate of any point of the cable, measured from the cable chord. This yields K per cable.

The Sine Wave Curve.—In any span l vibrating in n equal segments, if the displacements follow a sine curve,

$$\eta = a_m \sin\frac{\pi\,n}{l}\,x \times \sin 2\,\pi\,N\,t \qquad (5)$$

in which a_m is the vertical amplitude at the middle of a segment. In this case the elastic increments of loading Δw and their resulting bending moments M are proportional to the displacements η, and follow proportionate sine curves with the same nodes, period, and phase.

For any case in which $\Delta H = 0$, Eq. 3 then yields

$$K = \frac{\Delta w}{\eta} = \frac{\pi^2 n^2}{l^2} H + \frac{\pi^4 n^4}{l^4} E I \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

For an unstiffened suspension bridge, the H-term alone remains. For an unsuspended span, or for a self-anchored suspension span, the EI-term alone remains.

If n is a multiple of 4, also for other even values of n if the midpoint of the cable is free to move horizontally, Eq. 6 is complete, since the condition of inextensibility of the cable ($\Delta l_c = 0$) then is satisfied without involving the adjoining spans and without requiring elastic stretch of the cable.

Other Deflection Curves.—Corresponding expressions for K for other assumed curves of displacement η may be written directly from Eq. 4.

For displacements following, in each segment, the elastic curve of a uniformly loaded beam of constant cross section,

$$K = \frac{306}{31} \frac{n^2}{l^2} H + \frac{3{,}024}{31} \frac{n^4}{l^4} E I \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

which is almost identical numerically with Eq. 6.

For displacements following parabolic curves between nodes,

$$K = 10 \frac{n^2}{l^2} H + 96 \frac{n^4}{l^4} E I \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

in which I is the mean effective value of the variable moment of inertia. This value of K varies less than 1.6% from the value by Eq. 6.

K for Odd Number of Segments.—If $n > 1$ is odd, the governing requirement $\Delta l_c = 0$ may be satisfied, without involving the adjoining spans, by superimposing a single-segment oscillation of relative amplitude $\frac{1}{n}$, yielding

$$\eta = a_m \sin \frac{\pi n x}{l} \mp \frac{a_m}{n} \sin \frac{\pi x}{l} \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Substituting in Eq. 4,

$$K = \frac{\pi^2}{l^2} \left(\frac{n^4 + 1}{n^2 + 1} \right) H + \frac{\pi^4}{l^4} \left(\frac{n^6 + 1}{n^2 + 1} \right) E I \ldots\ldots\ldots (10a)$$

(Compare Eqs. 9 and 10*a* with the solution offered by H. M. Westergaard,[2] M. Am. Soc. C. E.)

Eq. 10*a* may be generalized for any case of superimposed undulations of different values of n (including even values of n). If the same relation is maintained—namely, that the respective amplitudes of the component harmonics are proportional to their values of n—the generalized form reduces to

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[2] "On the Method of Complementary Energy," by H. M. Westergaard, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), pp. 786–793.

$$K = \frac{\pi^2}{l^2}\frac{\Sigma n^4}{\Sigma n^2} H + \frac{\pi^4}{l^4}\frac{\Sigma n^6}{\Sigma n^2} E\,I \qquad (10b)$$

The resulting value of Δl_c or Δl will be the algebraic sum of the component values.

Three Spans Coupled.—The main and side spans may oscillate in unison, with a single segment or any odd number of segments (n and n_1, respectively) in each span. If the bridge is symmetrical, the relative amplitudes are determined by balancing $\Delta l/2$ with Δl_1 at each tower top. If K_0 and K_1 are the values by Eq. 6 for the respective spans, the energy equation yields, for the three spans coupled:

$$K = \frac{K_0 + q\,K_1}{1 + q} \qquad (11a)$$

or

$$K = \frac{\pi^2 \left[\dfrac{n^2}{l^2} + q\dfrac{(n_1)^2}{(l_1)^2} \right] H + \pi^4 \left[\dfrac{n^4}{l^4} + q\dfrac{(n_1)^4}{(l_1)^4} \right] E\,I}{1 + q} \qquad (11b)$$

in which the coupling factor q is given by

$$q = \frac{(n_1)^2\,l}{2\,n^2\,l_1} \sec^4 \gamma_1 \qquad (11c)$$

in which γ_1 is the slope of the cable chord in the side span. Eqs. 11 also may be written by extending the integrations in Eqs. 2 and 4 over all three spans. If $l_1/n_1 = l/n$, the main and side spans are in resonance and $K = K_0 = K_1$.

If $n = 2, 6, 10, \cdots$, $\Sigma\Delta(l/n)$ is zero for the full span but is not zero for the half span. If the midpoint of the cable is not free to take the longitudinal motion $\Delta(l/2)$, as when effectively anchored by center stays, the lengthening and shortening of the respective half spans will tend to be taken up by side-span participation to produce $\Delta l_1 = \Delta(l/2)$ at each tower top. Eqs. 11*a* and 11*b* then will apply, with

$$q = \frac{2\,(n_1)^2\,l}{n^2\,l_1} \qquad (12)$$

Oscillations with Longitudinal Motion of Span.—If $n = 2, 6, 10, \cdots$, without participation of the side spans, the longitudinal motion $\Delta(l/2)$ of the midpoint of the cable (Fig. 1) may be imparted to the suspended span through any physical coupling at midspan, as when center stays are provided. The energy equation then yields

$$K = \frac{K_0}{1 + 2\left(\dfrac{16}{3}\dfrac{f}{n\,l}\right)^2} \qquad (13)$$

in which f = the vertical sag of the cable. If the two side spans oscillate in opposite phase, with the main span free of vertical oscillation, the parallel and synchronous movement Δl_1 of the two tower tops produces longitudinal motion of the main span equal to Δl_1. For this case the energy equation yields

$$K = \frac{K_1}{1 + \frac{l}{l_1}\left(\frac{16}{3}\frac{f_1}{n_1}\frac{}{l_1}\cos^2\gamma_1\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Eqs. 13 and 14 express an apparent reduction in K as the equivalent of an increase in the effective value of the oscillating mass m in Eq. 1.

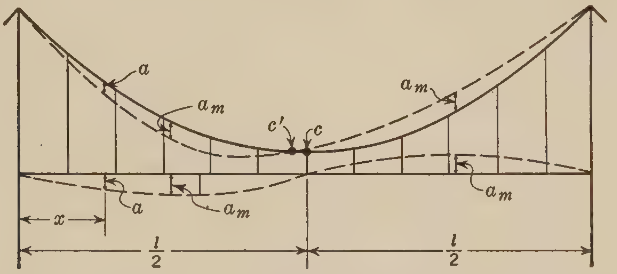

FIG. 1.—SPAN OSCILLATING IN TWO SEGMENTS ($n = 2$) SHOWING CENTER OF CABLE DISPLACED TOWARD THE DEPRESSED SEGMENT

Single Span with Straight Backstays.—For this case, for any odd number of segments including $n = 1$, the necessary increment Δl_c of the cable curve is provided by an increment of cable tension, ΔH, to produce an equal elastic elongation Δl_c of the cable between anchorages. Eq. 4 then yields:

$$K = \frac{\pi^2 n^2}{l^2}\left[H + \frac{32}{\pi^3}\frac{1}{n^4}\frac{E_c A_c}{1 + \frac{3\,l}{16 f^2}(l + 2\,l_1 \sec^2\gamma_1)}\right] + \frac{\pi^4 n^4}{l^4} E\,I \quad . . (15)$$

in which E_c = modulus of elasticity of the cables and A_c = the section area of the cable.

Example 1: Numerical Application of Formulas for K.—In Table 1 the pertinent formulas for K are applied to the recorded modes of vertical oscillation of the bridge across Tacoma Narrows in the State of Washington (1940). Remarkably consistent agreement between the corresponding calculated and observed frequencies is indicated, within the range of variations of observed and assumed data.

(The accuracy of the foregoing formulas for the various modes of oscillation is attested by the exceptionally close agreement noted in Table 1. Compare the results with those from formulas by W. D. Rannie[3] and others. For this comparison, the same assumed value of I = 1,000 has been used.)

For the bridge engineer, it is useful to note that, for $n > 1$, the values of K and N are given with sufficient accuracy for all practical purposes by simply using Eq. 6 without further correction. In fact, for values of n from 2 to 8, inclusive, the values of K given by Eq. 6, without correction, appear to check the observed frequencies of oscillation even more closely than do the values of K calculated more scientifically (except when the effect of center stays in forcing side-span participation needs to be considered).

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[3] "The Failure of the Tacoma Narrows Bridge," a report to the Hon. John M. Carmody, Administrator of the Federal Works Agency, Washington, D. C., March 28, 1941, by a board of engineers consisting of O. H. Ammann, Theodor von Kármán, and Glenn B. Woodruff, Appendix VI, p. 19.

TABLE 1.—APPLICATION OF FORMULAS FOR K AND N TO THE OBSERVED MODES OF OSCILLATION OF THE TACOMA NARROWS BRIDGE

Mode Type	n	n_1	Equation No.	K_0	K_1	K	FREQUENCY N PER MINUTE	
							Calculated	Observed
1	1	1	11*a*	15	100	65	8.2	8
2	2	0	6	62	....	62	8.0	8.7
		1	12	62	100	83[a]	9.3	
4	0	1	14	....	100	94	9.8	....
5	3	1	11*a*	140	100	135	11.8	11.9
6	1, 3	0	10*a*	128	....	128	11.5	....
7	4	....	6	254	....	254	16.2	16.5
8	5	1	11*a*	408	100	392	20.1	20.8
9	6	1, 2	10*b*, 12	605	359	575	24.4	24.6
10	7	1	11*a*	853	100	832	29.3	....
		1, 2	10*b*, 11*a*	853	359	839	29.4	
11	8	....	6	1,159	....	1,159	34.6	34
12	9	3	11*a*	1,530	1,042	1,466	38.3	38

[a] Note effect of center stays, by forcing side-span participation, to raise K from 62 to 83 (if fully effective).

Use of K for Estimating Deflections.—The deflection at the quarter point of a span produced by applied load p covering that half span is given approximately by the formula

$$\eta \approx \frac{p}{K} \qquad (16)$$

using K as given by Eq. 6 for $n = 2$. When the applied load p is large relative to the dead load, K needs to be recomputed for the increased H before use in Eq. 16.

As a rule, the greatest deflections of a suspension bridge occur under half-span loading, and the comparative rigidity of suspension bridges is gaged by such deflections. The value of K as defined, simply calculated and conveniently applied, may therefore, appropriately, be used as the "coefficient of rigidity" of a suspension span.

Example 2: Use of K for Estimating Deflections.—For the Tacoma main span, for $p = 100$ and $K = 62$, Eq. 16 yields a quarter-point deflection $\eta \approx 1.61$ ft, as compared with a computed value of 1.59 ft by the deflection theory. For $p = 750$, K is increased to 69, and Eq. 16 yields $\eta \approx 10.9$ ft (as against 10.9 ft by the deflection theory).

Values of K for Various Suspension Bridges.—Numerical values of the coefficient of rigidity K per cable (or per group of cables), calculated by Eq. 6 for $n = 2$, for various suspension bridges, are recorded in Table 2. For equal applied loads per square foot (live load or aerodynamic pressure), the significant value is K per foot of width (b), or $2\,K/b$. In Table 2, the values of K per foot range from 32.40 to 2.56. The physical meaning of $2\,K/b$ is the loading increment or vertical pressure in pounds per square foot required to produce 1-ft deflection.

It is a significant fact that all of the plate-girder suspension bridges with K per foot less than 15, and all of the open-truss suspension bridges with K per ft less than 6, have been subject to undesirable or dangerous aerodynamic oscillations. (The George Washington Bridge in New York, N. Y., is considered as belonging in neither category.) Both of these indicated critical values happen to coincide with the normal design values for suspension bridges with a vertical slenderness ratio (l/d) of 100 (see Fig. 2). For assured safety at high wind velocities, considerably higher values of K per foot are indicated (see Parts 2 and 3).

TABLE 2.—VALUES OF K FOR VARIOUS SUSPENSION BRIDGES

(K BY EQ. 6, FOR $n = 2$)

Date	Bridge	K per cable	K per foot	PERCENTAGE DUE TO: Cable	PERCENTAGE DUE TO: Girder (R)
1903	Williamsburg	1,082	32.40	22	78
1939	Wabash River[a]	327	26.20	0	100
1909	Manhattan	618	25.76	31	69
1931	Maumee River[a]	648	22.00	51	49
1926	Philadelphia	735	16.52	44	56
1931	Waldo-Hancock	212	15.14	56	44
1929	Grand Mère	188	14.48	34	66
1930	Mid-Hudson	262	12.50	56	44
1929	Mount Hope	201	11.82	54	46
1936	Triborough	547	11.18	66	34
1936	Transbay	324	9.81	62	38
1931	St. Johns	252	9.70	65	35
1938	Thousand Islands[a]	141	9.25	74	26
1938	Thousand Islands[a]	127	8.30	78	22
1929	Detroit	212	7.13	71	29
1939	Deer Isle[a]	69	5.85	80	20
1938	Vancouver	112	5.62	67	33
1931	George Washington	244	4.60	100	0
1939	Bronx-Whitestone[a]	141	3.81	97	3
1940	Tacoma[a]	62	3.16	98.5	1.5
1937	Golden Gate	116	2.56	95	5

[a] Denotes construction with solid-web stiffening girders.

The last two columns in Table 2 record the percentage contributions of the cables and the stiffening girders, respectively, to the coefficient of rigidity K. The proportionate contribution of the stiffening girders or trusses, designated by the ratio R, ranges from 100% in a self-anchored suspension bridge and 78% in the Williamsburg Bridge in New York City to 1.5% in the Tacoma span and practically to zero in the George Washington Bridge. Girder or truss suspension bridges with values of R less than 0.25 are peculiarly sensitive to aerodynamic oscillations.

Approximate Formulas for K and N.—Using the cable term alone, the foregoing formulas yield the convenient approximate relations:

$$K \approx 1.234\, n^2 \frac{w}{f} \qquad (17a)$$

and

$$P = \frac{1}{N} \approx \frac{1}{n} \sqrt{f} \qquad (17b)$$

in which f is simply the sag of the cable. These simplified formulas may be used for estimating the coefficient of rigidity and the natural period or frequency of an unstiffened suspension bridge or of one with negligible stiffening system, for all values of $n > 1$.

Similarly, using the girder term alone:

$$K \approx 1.27\, n^4 \frac{w}{\eta_w} \ldots\ldots\ldots\ldots\ldots\ldots (18a)$$

and

$$P = \frac{1}{N} \approx \frac{1}{n^2} \sqrt{\eta_w} \ldots\ldots\ldots\ldots\ldots\ldots (18b)$$

in which η_w is the central deflection theoretically producible (with E constant) by the dead load w acting on the span as a simple beam. Eqs. 18 may be used for simple spans and for self-anchored suspension spans, for all values of n.

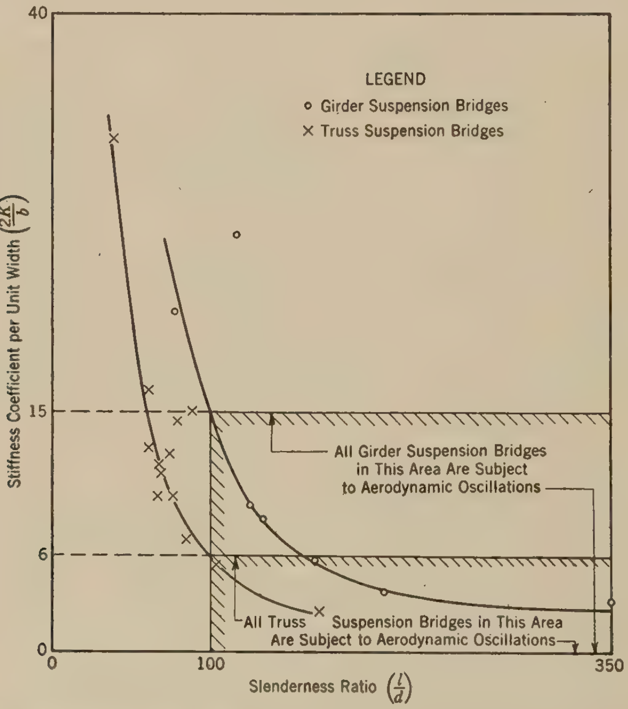

FIG. 2.—CORRELATION OF STIFFNESS COEFFICIENTS (2 K/b) WITH SLENDERNESS RATIOS (l/d) OF VARIOUS BRIDGES, SHOWING RANGE OF BRIDGES SUBJECT TO AERODYNAMIC OSCILLATIONS

For a stiffened suspension bridge (except the self-anchored type), the composite rigidity K is the sum of the respective contributions (Eqs. 17a and 18a) of cable and stiffening girder. Hence

$$\frac{1}{P} = N \approx \sqrt{\frac{n^2}{f} + \frac{n^4}{\eta_w}} \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Example 2a: Calculation of Rigidities and Frequencies by Simple Approximate Formulas.—For the Tacoma main span, for $n = 2$, with $f = 232$ ft and

$\eta_w = 61{,}400$ ft:

By Eq. 17*b*, the natural frequency of cable = 7.9 cycles per min.
By Eq. 18*b*, the natural frequency of girder = 1.0 cycle per min.
By Eq. 19, the natural frequency of combination = 7.96 cycles per min.
Also, by Eqs. 17*a* and 18*a*, $K = 60.6 + 0.98 = 61.58$.

The foregoing results are almost identical with those given by Eqs. 1 and 6.

Fundamental Equations for Torsional Oscillations.—The general equation of torsional oscillations of a span is

$$\theta = \frac{2\,a_m}{b} \sin \frac{\pi\, n}{l}\, x \times \sin 2\,\pi\, N_t\, t \qquad (20)$$

in which θ is the instantaneous angular displacement of any section x at time t, N_t is frequency of torsional oscillations, and a_m is the maximum vertical displacement of cable or girder at the middle of each segment, since torsional oscillation may be treated simply as out-of-phase vertical oscillation of the two cables.

The values of K and N determined for vertical oscillations, multiplied by simple dimensional constants, yield with sufficient accuracy the corresponding values for torsional oscillations. The spring constant of the span in torsion, or the coefficient of torsional rigidity, will be

$$K_t = \frac{b^2}{2} K \qquad (21a)$$

and the natural frequency will be

$$N_t = \frac{b}{2\,r} N \qquad (21b)$$

in which r is the polar radius of gyration of the mass of the cross section of the span. It is important to note that, for torsional resistance of a suspension bridge, the polar moment of inertia ($r^2\, m$) pertains, not to a localized built-up cross section at the panel points, but to the average cross section throughout, including the long ranges between panel points. For the usual proportions, $b/(2\,r) = 1.3$ to 1.5.

Example 3: Calculation of Natural Frequency of Torsional Oscillations.—For the Tacoma Narrows Bridge, for $n = 2$, Table 1 gives two values: $N = 9.3$ (per min) with center stays forcing side-span participation, and $N = 8$ (per min) without center stays. Applying Eq. 21*b* to these frequencies, and using an indicated effective value of $b/(2\,r) = 1.5$, one obtains $N_t = 14$ and 12 (per min), respectively, which are identical with the two successive frequencies recorded at the time of the failure. The higher frequency was observed at the commencement of the torsional oscillations and corresponds to the functioning of the center stays to force side-span participation. Hold-down stays in the side spans prevented their normal participation beyond a limited amplitude. After failure of the center stays, the torsional oscillations, released from their initial coupling restraint, assumed the lower natural frequency of $N_t = 12$ cycles per min.

Use of K_t for Estimating Torsional Deflections.—When the applied torsional loading is relatively small, the angular deflection θ at the quarter point of a span, produced by applied uniform torsional loading T covering that half of the span, is given approximately by the formula

$$\theta \approx \frac{T}{K_t} \qquad (22)$$

in which K_t is the coefficient of torsional rigidity of the span (calculated for $n = 2$).

Example 4: Use of K_t for Estimating Torsional Deflections.—For the Tacoma main span, for $T = 10{,}000$ ft-lb per lin ft and $K_t = 47{,}000$, Eq. 22 yields $\theta \approx 0.213$ radians, as compared with a computed value of $\theta = 0.209$ by the deflection theory.

PART 2.—AERODYNAMIC EXCITATION OF VERTICAL OSCILLATIONS

Initiation of Oscillations.—Small vertical oscillations of a span may be initiated by traffic, by wind gusts, or by the longitudinal component of the wind acting on the cambered span as an aerofoil. The importance of the last cause is confirmed by the fact that the recently built spans that have given trouble all had been provided with steep camber (for reasons of economy), and by the further fact that the oscillations usually have started with a quartering wind. The longitudinal component of the wind initiates the oscillations and the transverse component then rapidly amplifies them in a logarithmic ratio.

Effect of Vertical Oscillations Upon Aerodynamic Lift.—The instantaneous vertical velocity of any point in the span is

$$v = \frac{d\eta}{dt} = 2\,\pi\,N\,a\cos\,(2\,\pi\,N\,t) \qquad (23a)$$

in which a, the local amplitude, is ordinarily defined by

$$a = a_m \sin\frac{\pi\,n}{l}\,x \qquad (23b)$$

Composition of the wind velocity V and its relative vertical velocity, $-v$, yields a resultant change in effective angle of attack (α):

$$\Delta\alpha = -\frac{v}{V} \qquad (23c)$$

Downward velocity of the section will thus produce or augment an upward angle of attack. In an "aerodynamically unstable section," such upward angle of attack tends to produce a downward resultant pressure, in the direction of the motion.

In the following basic analysis (Eqs. 24 to 36, inclusive), high relative wind velocity V is assumed; the effect of phase difference across the section is introduced subsequently.

The change $\Delta\alpha$ in the effective angle of attack will change the instantaneous lift force on a unit length of the span by an amount

$$\Delta L = s'\left(\frac{\rho}{2}V^2 b\right)\Delta\alpha = -s'\frac{\rho}{2}v\,V\,b \ldots\ldots\ldots\ldots\ldots (24)$$

in which ρ is the mass density of air and s' is the corrected slope of the lift graph:

$$s' = s + C_D \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

This correction is necessary because L, the vertical lift force, and D, the horizontal drag force, in the wind tunnel are measured perpendicular and parallel to the wind whereas ΔL in Eq. 24 is measured vertically. The correction represented by Eq. 25 may be made directly upon the lift graph (Fig. 3) by simply augmenting each ordinate (C_L) by the vertical component $C_D\,\alpha$.

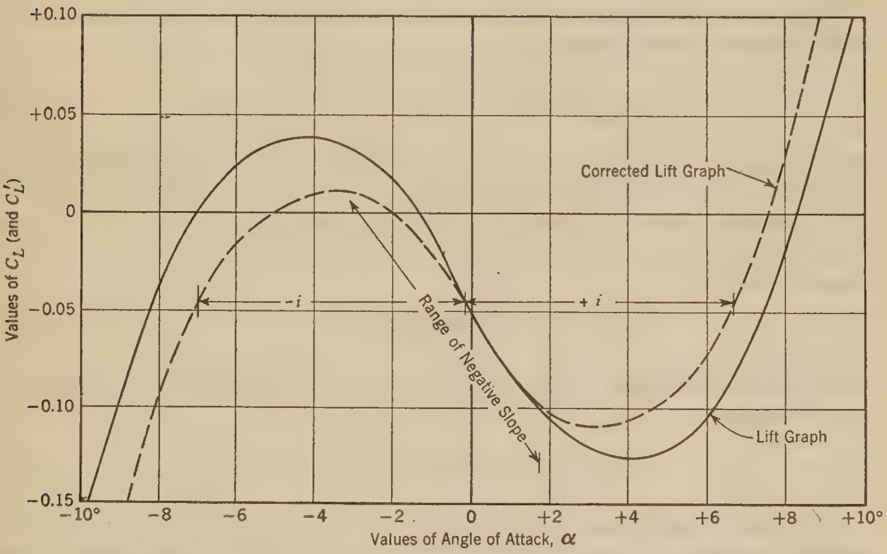

FIG. 3.—LIFT GRAPH (AND CORRECTED LIFT GRAPH)

If the slope s of the lift graph is negative and numerically exceeds the drag coefficient C_D, then s' will be negative and, by Eq. 24, ΔL will be positive when v is positive, and negative when v is negative. This will produce amplification of vertical oscillations. Such a condition constitutes aerodynamic vertical instability.

From the foregoing relations, the maximum value, in a cycle, of the amplifying force ΔL acting on a unit length of the span will be

$$\Delta L(\max) = -\pi s' N \rho b V a \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

Self-Excited Amplification of Vertical Oscillations.—Since the increment of lift ΔL is automatically resonant with the harmonic motion of the vertical oscillations and, moreover, is 90° out of phase with that harmonic motion, it will produce an increment of energy in each cycle. This increment of energy, per unit length, will be

$$\Delta W = \pi \, \Delta L(\max) \, a = -\pi^2 s' \rho a^2 b N V \qquad (27a)$$

and, over the span l,

$$\Delta W(\text{total}) = -\frac{\pi^2}{2} s' \rho (a_m)^2 b N V l \qquad (27b)$$

The maximum kinetic (or potential) energy W in a cycle, per unit length, is

$$W = K a^2 \qquad (28a)$$

Hence, over the entire span,

$$W(\text{total}) = \tfrac{1}{2} K l (a_m)^2 \qquad (28b)$$

The relative increase per cycle in the amplitude, or the logarithmic increment per cycle, is denoted by δ:

$$\delta = \frac{\Delta a_m}{a_m} = \frac{\Delta a}{a} = \frac{\Delta W}{2W} = -\frac{\pi^2}{2} s' \rho \frac{b}{K} N V \qquad (29a)$$

If s' is negative, δ will be positive, representing a self-excited amplification of oscillations. By Eq. 29*a*, the amplification ratio δ increases with the width b, or inversely with the coefficient of rigidity per foot of width, $2K/b$ (see Table 2 and Fig. 2). Substituting numerical values for ρ and g, Eq. 29*a* may be written:

$$\delta = -0.0096 \frac{s' b V}{w N} \qquad (29b)$$

The negative damping coefficient, or the logarithmic increment per second, is denoted by λ:

$$\lambda = (1 + \delta)^N - 1 \approx N \delta = -0.0096 \frac{s' b V}{w} \qquad (30)$$

Limiting Amplitude of Vertical Oscillations.—The amplitudes of self-excited vertical oscillations, under aerodynamic negative damping, are limited. As the amplitudes increase, they will cause the resultant values of α, by Eq. 23*c*, to extend more and more beyond the range of negative slope of the lift graph, until the positive contribution of ΔW from the range of negative slope is balanced by the tapering negative contributions of ΔW from the flanking ranges of positive slope.

To take into account the variation of slope s', Eq. 27*a* is replaced by the more general equation

$$\Delta W = \oint L' \, d\eta \qquad (31a)$$

in which $\oint$ denotes integration over the cycle and in which $L' = L + D \alpha$, corresponding to the corrected lift graph. This expression for energy input may also be written

$$\Delta W = \oint \eta \, dL' \qquad (31b)$$

The condition $\Delta W = 0$ will define a limiting local amplitude a_o; and the condition,

$$\int_0^l \Delta W \, dx = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

will define the limiting segment amplitude a_{om}.

If the lift graph is not readily expressible as a mathematical function, graphic or tabular methods are best adapted for the solution of Eqs. 31 and 32. A graphic solution of Eq. 31*a* is shown in Fig. 4. The variation of ΔW with amplitude a is first found by any of these methods, and is then plotted for the sine-curve amplitudes a over a span segment, using trial values of a_m at mid-segment, until the limiting value a_{om} satisfying Eq. 32 is found.

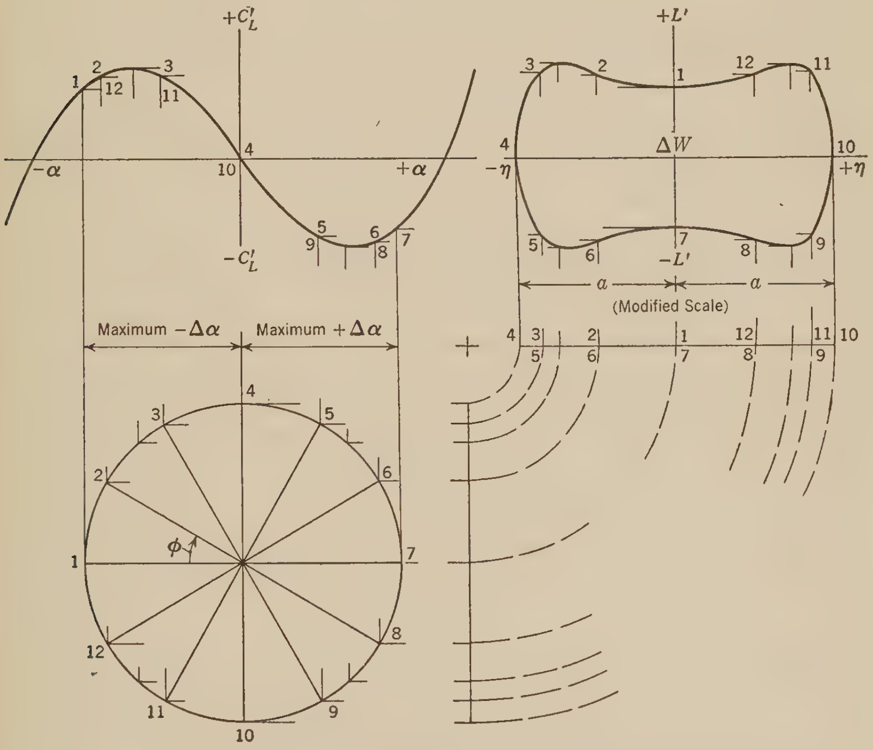

FIG. 4.—GRAPHIC DETERMINATION OF ENERGY INPUT (ΔW) FOR A GIVEN AMPLITUDE (a) OF VERTICAL OSCILLATIONS

If the lift graph is of the typical form shown in Fig. 3, identified with aerodynamic instability, it may be represented by a cubic equation:

$$(C_L)' = s' \alpha \left(1 - \frac{\alpha^2}{i^2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

in which $-i$, 0, and $+i$ are the intercepts on the horizontal axis. (The curve is assumed centered through the origin. This merely changes the inclination datum from which α is measured, and otherwise does not affect the final results.) When used in Eqs. 31 and 32, this yields

$$a_o = \sqrt{\frac{4}{3}\, i} \left(\frac{V}{2\,\pi\, N} \right) \qquad (34a)$$

and

$$a_{om} = \frac{4}{3}\, i \left(\frac{V}{2\,\pi\, N} \right) \qquad (34b)$$

in which a_{om} is the limiting segment amplitude without structural damping.

Structural Damping.—Accurate information on the structural damping of suspension bridges is lacking. For oscillations of flexible suspension spans, the structural damping is represented by a varying logarithmic decrement δ_s of the indicated order of magnitude of 0.002 to 0.130. It is essentially a function of R (the proportion of K due to the stiffening girder), since the principal contribution is from the energy dissipated in deformations of the stiffening system whereas the gravitational resistance to deformation of cable curve (without increment of cable stress) contributes little to damping. Moreover, the energy in the structure (compare Eq. 28*b*) is proportional to $(a_m)^2$, whereas the energy loss due to structural hysteresis is proportional to a fractionally higher power of a_m—$(a_m)^{2.3}$ according to some recorded tests[4]; hence δ_s is a function of a fractional power of a_m (multiplied by $n^2\, d/l^2$ in order to translate a_m into a function of strain). If other losses by dissipation of energy are included, such as those caused by secondary vibrations, roadway deformations, atmospheric damping, joint and bearing frictions, and cable hysteresis, the expression for the structural damping decrement will include terms independent of R and a_m.

In the light of the limited data thus far available, the following semi-empirical expression for the logarithmic decrement δ_s is indicated:

$$\delta_s \doteq (0.001 + 0.06\, R) \left(1 + 1{,}700\, \frac{n}{l} \sqrt{a_m\, d} \right) \qquad (35)$$

An indicated increase with increasing amplitudes is provided by the second factor in the formula. The coefficients and exponents in this formula are written to fit available data and are subject to future revision. A higher value of δ_s is indicated when trusses are used instead of girders.

The critical wind velocity at which aerodynamic amplification of vertical oscillations can commence, defined by the condition $\delta - \delta_s = 0$, will then be, by Eq. 29*b*,

$$V_c = -\, 104\, \frac{w\, N}{b\, s'}\, \delta_s \qquad (36a)$$

in which δ_s is calculated for zero amplitude. At any wind velocity lower than V_c, the aerodynamic negative damping factor is insufficient to overcome the structural damping. Eq. 36*a* may be written in the significant form,

$$V_c = -\, \frac{1 + 60\, R}{15\, s'} \sqrt{\frac{w}{b}} \sqrt{\frac{2\, K}{b}} \qquad (36b)$$

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[4] "Mechanical Vibrations," by J. P. Den Hartog, McGraw-Hill Book Co., Inc., New York, N. Y., 2d Ed., 1940, pp. 253–254.

in which $2\,K/b$ is the coefficient of rigidity per foot of width. Correcting for structural damping, the net energy input per cycle will be, by Eqs. 28*b* and 29*a*,

$$\Delta W = K\,l\,(\delta - \delta_s)\,(a_m)^2 \qquad (37)$$

Limiting Amplitude Corrected for Structural Damping.—At some limiting amplitude below a_{om}, the increasing value of δ_s will become equal to the diminishing value of δ, terminating further self-excited amplification. For the typical lift graph with reversals of slope (Eq. 33 and Fig. 3), the diminished δ will be

$$\delta = \delta_0 \left[1 - \frac{(a_m)^2}{(a_{om})^2} \right] \qquad (38)$$

in which δ_0 is the value at zero amplitude (or maximum effective s'). The limiting amplitude corrected for structural damping then will be given by the relation:

$$a_r = a_{om} \sqrt{1 - \frac{\delta_s}{\delta_0}} \qquad (39)$$

in which a_r is the limiting vertical amplitude reduced by structural damping. This yields for the Tacoma span a maximum limiting amplitude of 1.5 ft (for any mode), which checks the recorded observations.

The Effect of Phase Difference on Vertical Oscillations.—The following relations, whereby aerodynamic instability at low wind velocities may be created by the phase difference across the width of the section, hitherto have been unsuspected and unexplored.

The aerodynamic disturbance traveling across the width of the vertically oscillating section, especially at low wind velocities, encounters a progressive phase difference, with both relative displacements and relative velocities of the different points along the width when referred to the respective instants of successive impingement. The total phase difference across the section, in cycles, is represented approximately by $N\,b/V$. To correct for the effect of this phase difference, a correction factor β needs to be inserted in the formulas based on the lift graph of a stationary model in order to obtain the relations for an oscillating section, similar to the corresponding correction factor subsequently used in the formulas for torsional oscillations in Part 3.

In the case of vertical oscillations, the indicated critical wind velocity due to phase difference is $V_p \approx 2\,N\,b$, corresponding to a total phase difference of one half of a cycle; and a corresponding indicated expression for the correction factor (assuming a single $-\beta$-range below V_p) is of the form,

$$\beta \approx \frac{V^2 - (V_p)^2}{V^2 + (V_p)^2 \left(0.2 + 0.45\,\frac{V_p}{V} \right)} \qquad (40)$$

asymptotic to unity at higher velocities. This yields $\beta = 0$ at $V = V_p$, and β negative in the low-velocity range from $V = 0$ to $V = V_p$. This negative value of β means a reversal of stability or instability in this range. If the section is aerodynamically stable as previously defined (s' positive), the application of the negative value of β in the formulas for ΔL, ΔW, and δ, will reverse the stability and create a condition of aerodynamic instability in the low-velocity range. The wider the bridge, the greater will be the range of wind velocity to which the normally stable section will be aerodynamically sensitive.

Analytical Derivation of β for Vertical Oscillations.—The physical concept underlying the correction factor β may be visualized as an initiated pressure impulse traversing the width of the section in the time dimension (measured in cycles or fractions of a cycle). If $V \geqq 4\,N\,b$, the action extends through one fourth of a cycle or less, so that the traversing pressure impulse does not encounter any reversal of phase; hence β remains positive. If $V = 2\,N\,b$, the action extends through one half of a cycle, and therefore enters a reversal of phase (direction of motion) at the midpoint of this range and if the pressure impulse traverses the section with undiminished intensity, the integrated effect will be zero ($\beta = 0$ at $V_p = 2\,N\,b$). If $V = 4\,N\,b/3$, the action extends through three fourths of a cycle; the pressure impulse traverses the first third of the width in assumed phase with the direction of motion and the remaining two thirds with the motion in reversed or opposed phase, so that the integrated effect will be negative ($\beta = -$). This concept may be expressed mathematically in simple form:

$$\beta = \frac{\int_0^{\phi'} \cos\phi \, d\phi}{\int_0^{\phi'} d\phi} = \frac{\sin\phi'}{\phi'} \qquad (41)$$

in which the numerator represents the integration of the action traversing the oscillating section and the denominator represents the corresponding integration (over the same assumed transit time) across the stationary section. Here $\phi = 2\,\pi\,N\,t$ is the relative phase angle at any instant of the action, and ϕ' is the total effective phase difference across the width of the oscillating section. Assuming unmodified V and undiminished effectiveness of the traveling pressure impulse throughout the range,

$$\phi' = 2\,\pi\,\frac{N\,b}{V} \qquad (42)$$

Eq. 41 yields:

Velocity V	Correction factor β	Velocity V	Correction factor β
∞	$+1$	$\frac{4}{5}N\,b$	$+\frac{2}{5\,\pi}$
$4\,N\,b$	$+\frac{2}{\pi}$	$\frac{4}{6}N\,b$	0
$\frac{4}{2}N\,b$	0	$\frac{4}{7}N\,b$	$-\frac{2}{7\,\pi}$
$\frac{4}{3}N\,b$	$-\frac{2}{3\,\pi}$	$\frac{4}{8}N\,b$	0
$\frac{4}{4}N\,b$	0	...	...

thus making $V_p = 2\,N\,b$. In the low-velocity range below V_p, the resulting values of β (or $V\,\beta$) follow an oscillating curve converging toward the origin. This yields a series of critical ranges for vertical oscillations at low wind velocities. The Tacoma Narrows Bridge likewise showed a series of critical ranges for oscillations at low wind velocities (see Table 3). The actual correspondence (covering all of the observed modes) is too striking to be accidental. Table 3 is the distribution for an aerodynamically stable section—one for which

TABLE 3.—COMPARISON OF THEORETICAL AND OBSERVED RANGES OF VERTICAL OSCILLATIONS

Values of $V/(N\,b)$	First Critical Range	Intermediate Range	Second Critical Range	Intermediate Range	Third Critical Range	Intermediate Range	Fourth Critical Range
	$\beta = -$	$\beta = +$	$\beta = -$	$\beta = +$	$\beta = -$	$\beta = +$	$\beta = -$
By Eq. 41	2.00–1.00	....	0.67–0.50	....	0.40–0.33	....	0.29–0.25
Observed	2.35–0.93	....	0.89–0.53	....	0.43–0.37	....	0.30–0.26
Number of observations	79	0	56	0	4	0	3

s' is positive. The vertical oscillations are limited to the ranges in which β is negative. For an aerodynamically unstable section (s' negative), the distribution would have been reversed.

Eq. 41 may be improved toward perfect correspondence with the observed critical ranges by advancing the integration limits by the amount of the phase lag between any instant of incidence of wind pressure on the outside of the section and the actual traverse of the interior width by the consequent vertical pressure reaction. Assume that the mean path of the deflected air flow is lengthened from b to $d + b$, and that, in partial compensation, the linear velocity of the deflected air flow is speeded up in the proportion of b to $b + 0.4\,d$. For the Tacoma section, the following integration limits are thus obtained:

$$\beta = \frac{\displaystyle\int_{0.19\phi'}^{1.11\phi'} \cos\phi\,d\phi}{\displaystyle\int_{0.19\phi'}^{1.11\phi'} d\phi} = \frac{\sin 1.11\,\phi' - \sin 0.19\,\phi'}{0.92\,\phi'} \quad \text{.........(43)}$$

This yields, for the critical ranges, $V/(N\,b) = 2.60\text{–}0.92$, $0.87\text{–}0.52$, $0.46\text{–}0.37$, $0.31\text{–}0.29$, etc., or a practically perfect correspondence with the ranges of the observed oscillations.

Eq. 43 also yields striking agreement with the preliminary empirical formula, Eq. 40, from $V = \infty$ to V at the middle of the first critical range (see Table 4). Below V_p, Eq. 40 approximately envelopes the oscillating graph representing the critical ranges.

By advancing the integration limits as in Eq. 43, the major critical ranges are widened, and tend to coalesce. Only the widths and ordinates above or below a secant line representing δ_s will be critical. The recorded observations of oscillations in the third and fourth critical ranges (and one in the fifth) show that, in the Tacoma span, the structural damping was almost negligible.

Conclusions Relative to Vertical Aerodynamic Oscillations.—The foregoing results confirm the significance and validity of the β-concept and its suggested analytical evaluation. Moreover, there now appears to be a conclusive and completely consistent diagnosis of the action of the Tacoma span:

TABLE 4.—AGREEMENT BETWEEN VALUES OF β COMPUTED BY EQS. 40 AND 43

Equation No.	VALUES OF $V/(N\,b)$								
	∞	8	6	4	2.6	2	1.85	1.7	0
40....	1	0.86	0.75	0.48	0	−0.31	−0.38	−0.43	0
43....	1	0.86	0.75	0.48	0	−0.30	−0.38	−0.43	0

(*a*) For vertical oscillations, the section was aerodynamically stable. The span was therefore aerodynamically unstable at low wind velocities (below V_p) and stable at high wind velocities (above V_p).

(*b*) For torsional oscillations (as will be developed in Part 3) the entire picture was exactly reversed, since the torque graph, unlike the lift graph, had a range of negative slope.

The validity of the series of lift graphs obtained in the wind-tunnel tests at the California Institute of Technology at Pasadena also is confirmed. These point to a critical ratio of girder depth to section width for vertical oscillations: If $d/b > 1/4$, with solid girders, aerodynamically unstable sections are indicated, and such bridges (if K and R are inadequate) may be expected to show vertical aerodynamic instability in the high-velocity range (above V_p) and possibly, to a limited extent, in the low-velocity range (below V_p). If $d/b < 1/4$ (or a higher ratio with open trusses), the section is aerodynamically stable, and any self-amplified vertical oscillations will be limited to the low-velocity range. The Tacoma span belonged to the latter category.

Critical Velocities Calculated with β.—Eq. 29*b*, with β inserted, may be written

$$\delta = -\,0.025\,\beta\,s'\,\frac{b^2}{w}\,\frac{V}{V_p}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(44a)$$

in which $V_p = 2.6\,N\,b$. All critical velocities, defined by $\delta - \delta_s = 0$, will then be given by

$$\frac{V_c}{V_p} = -\,40\,\frac{w}{b^2}\,\frac{\delta_s}{\beta\,s'}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(44b)$$

From Eq. 40 or 43, a simplified expression for β may be written, sufficiently accurate in the useful range from $V/V_p = 0.8$ to 4:

$$\beta = 1.3\,\frac{V - V_p}{V}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(45a)$$

(Compare Eq. 58*b* introduced subsequently.) Eq. 44*b* then yields

$$\frac{V_c}{V_p} = 1 - 31\,\frac{w}{b^2}\,\frac{\delta_s}{s'}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(45b)$$

At very low wind velocities (up to $V/V_p = 0.2$), Eq. 40 yields, with sufficient accuracy,

$$\beta = -2\frac{V}{V_p} \qquad (46a)$$

Hence, by Eq. 44*b*,

$$\left(\frac{V_c}{V_p}\right)^2 = 20\frac{w}{b^2}\frac{\delta_s}{s'} \qquad (46b)$$

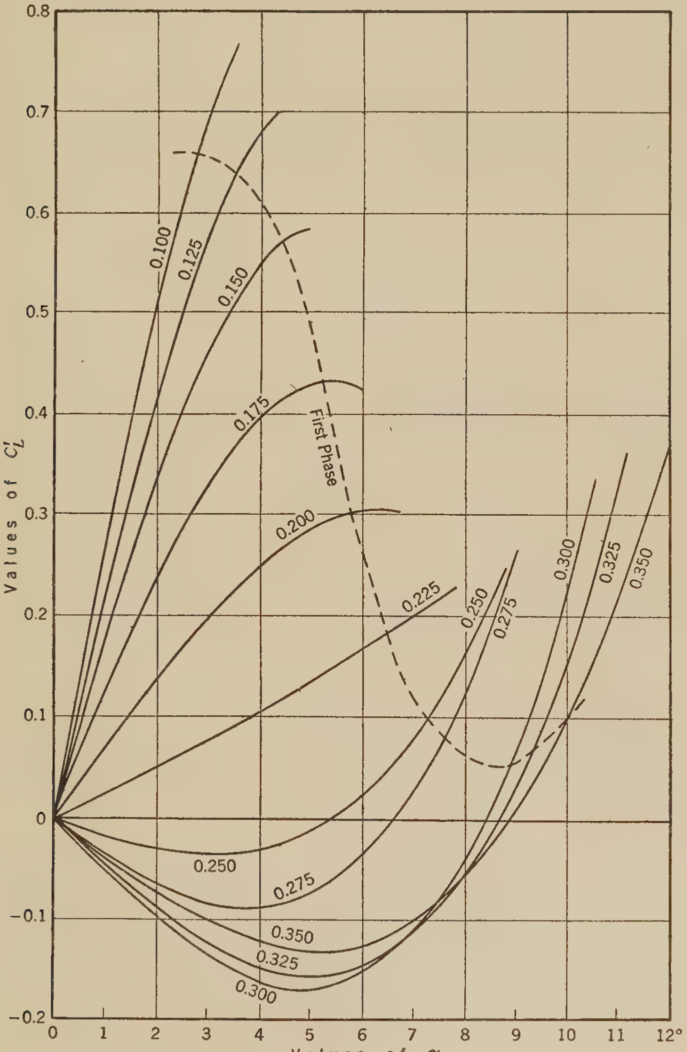

FIG. 5.—LIFT GRAPHS (CORRECTED FOR DRAG) FOR VARIOUS VALUES OF d/b

Example 5: Critical Velocities Calculated with β.—For the Tacoma Narrows Bridge (with $s' = +3.50$, $\delta_s = 0.002$, $b^2/w = 0.53$), Eq. 45*b* yields a maximum critical velocity of $V_c = 2.51\,N\,b$, or 42 miles per hr for $n = 9$, as compared with 40 miles per hr reported, and Eq. 46*b* yields a minimum critical velocity of $V_c = 0.38\,N\,b$, or 1.3 miles per hr for $n = 1$ or 2, as compared with a minimum reported velocity of 1 mile per hr for observed oscillations.

For the Deer Isle Bridge in Maine (without stays), with $s' = -2.52$, $\delta_s = 0.013$, and $b^2/w = 0.46$, Eq. 45*b* yields a minimum critical velocity of $V_c = 3.50\,N\,b$, or 12.2 miles per hr for $n = 1$ or 2, which agrees with observations. For the Toledo Bridge, with $s' = +7.3$, $b^2/w = 0.535$, and $\delta_s = 0.030$, Eqs. 44 yield $\delta < \delta_s$ and $V_c > V$ for all values of V, representing aerodynamic stability at all wind velocities.

Method of Predicting Lift Graphs.—From a study of recorded lift graphs of bridge cross sections, the parameters s' and i (or s'/i^2) of these graphs may be written as functions of the ratio d/b of the H-section. Accordingly, referring to Eq. 33, a single formula is obtained for this entire family of lift graphs:

$$(C_L)' = \frac{28 - 118\dfrac{d}{b} + 0.045\dfrac{b}{d}}{1 + 21{,}500\left(\dfrac{d}{b}\right)^8 + 0.012\dfrac{b}{d}}\,\alpha - \frac{1{,}300 - 5{,}900\dfrac{d}{b}}{1 + 33{,}000\left(\dfrac{d}{b}\right)^8 + 0.005\dfrac{b}{d}}\,\alpha^3 \quad \ldots (47)$$

This is plotted in Fig. 5, giving the lift graphs for values of d/b from 0.100 to 0.350. (The antisymmetrical half for $-\alpha$ is omitted.)

As the angle of attack increases from $\alpha = 0$, the aerodynamic action is in its first phase (bridge floor completely screened by the front girder) up to $\alpha = d/2\,b$, which is the angle of attack at which air-stream lines, if they continued straight, would first impinge upon the horizontal member of the H-section. Beyond this value of α, the lift graph changes in form, usually with a point of inflection. Eq. 47 and Fig. 5 give the lift graphs (corrected for drag) with sufficient accuracy up to $\alpha = 0.6\,d/b$. The range of Eq. 47 may be extended to cover higher values of α by including terms of higher power (α^5 and α^7).

It is significant to note that, as indicated in Fig. 5, all negative lifts and negative slopes are limited to the first phase, $\alpha < d/(2\,b)$. If the horizontal member of the H-section is not at midheight, the lift graphs will not be centered through the origin. More wind-tunnel results are needed to determine such variations.

All known lift graphs for girder suspension bridges fit Eq. 47 and Fig. 5. The known lift graphs for truss suspension bridges also fit Eq. 47 and Fig. 5, provided $0.50\,d$ is used instead of d. This relation is significant and warrants further tests for confirmation or modification of this truss factor.

Following the same general procedure as illustrated by Eq. 47 and Fig. 5, torque graphs for all values of d/b may be similarly formulated and plotted for use in predicting, reducing, or avoiding torsional aerodynamic instability.

PART 3.—AERODYNAMIC EXCITATION OF TORSIONAL OSCILLATIONS

Self-Excited Amplification of Torsional Oscillations.—It is primarily and principally the frontal angle of attack at the leading edge that determines the deflection and disturbance of the air stream, including any eddies produced. The instantaneous vertical velocity v of the leading edge of the cross section (the front girder), combined with the horizontal velocity of the wind V, will change the effective frontal angle of attack by an amount $\Delta\alpha$, given by Eq. 23*c*. Downward velocity of the leading edge thus will produce or augment an upward angle of attack. On an aerodynamically unstable section, such upward angle of attack produces a torque in the direction of the motion. (As the angle of attack increases—due to increasing amplitude—the negative lift increases to a maximum, and then diminishes and reverses; but the negative torque continues to increase through an additional range.)

The change $\Delta\alpha$ in the effective angle of attack will change the instantaneous torque acting on a unit length of the span by an amount

$$\Delta T = -\beta s_t \left(\frac{\rho}{2} V^2 b^2 \right) \frac{v}{V} \qquad \text{(48)}$$

in which s_t is the slope of the wind-tunnel torque graph, and β is a correction factor from the stationary model in the wind tunnel to the oscillating structure.

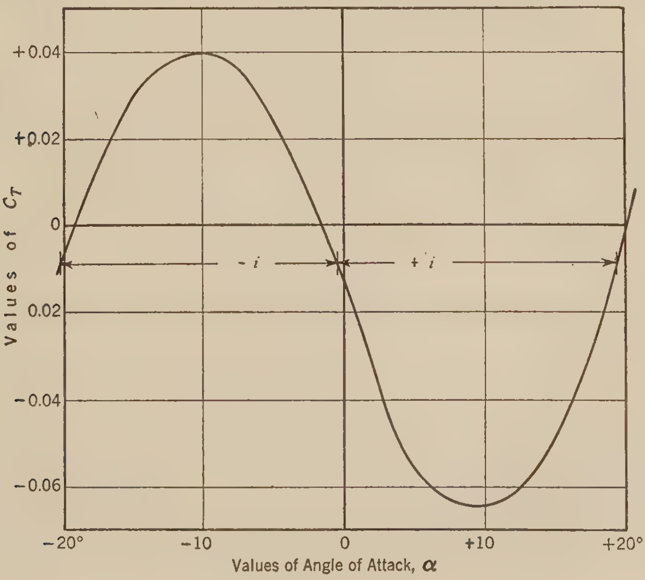

Fig. 6.—Torque Graph

Consideration of the effect of phase difference indicates that the relative vertical motion across the section will modify and actually may augment the aerodynamic torque effect produced by $\Delta\alpha$ at the leading edge, and wind-tunnel tests on oscillating models confirm this conclusion.

If the slope s_t of the torque graph (Fig. 6), when multiplied by β, is negative in the range of oscillation, ΔT by Eq. 48 will be positive (clockwise) when v is positive (upward at the left or leading edge) and negative when v is negative. This will produce self-excited amplification of torsional oscillations. Such condition constitutes aerodynamic torsional instability.

The maximum value, in a cycle, of the amplifying torque ΔT on a unit length of the span will be

$$\Delta T(\max) = -\pi \beta s_t N_t \rho b^2 V a \dots\dots \tag{49}$$

Since the increment of torque ΔT is automatically resonant with the harmonic motion of the torsional oscillations and is, moreover, 90° out of phase with that harmonic motion, it will produce an increment of energy in each cycle:

$$\Delta W = \pi \Delta T(\max) \frac{2a}{b} = -2\pi^2 \beta s_t \rho b a^2 N_t V \dots\dots \tag{50a}$$

and, over the span l,

$$\Delta W(\text{total}) = -\pi^2 \beta s_t \rho b (a_m)^2 N_t V l \dots\dots \tag{50b}$$

The total kinetic plus potential energy in torsional oscillation, per unit length and over the span, will be given by Eqs. 28.

The relative increase, per cycle, in the amplitude of oscillation, or the logarithmic increment per cycle, then will be

$$\delta = \frac{\Delta a}{a} = \frac{\Delta a_m}{a_m} = \frac{\Delta W}{2W} = -\pi^2 \frac{b}{K} \beta s_t \rho N_t V \dots\dots \tag{51a}$$

If βs_t is negative, δ will be positive, representing a self-excited amplification of torsional oscillations. By Eq. 51a, the torsional amplification ratio δ increases with the width b, or inversely with K/b.

Substituting numerical values for ρ and g, Eq. 51a reduces to

$$\delta = -0.0048 \frac{\beta s_t b^3 V}{r^2 w N_t} \dots\dots \tag{51b}$$

and the negative damping coefficient (λ), or the logarithmic increment per second, then will be

$$\lambda \approx -0.0048 \beta \frac{s_t b^3 V}{r^2 w} \dots\dots \tag{52}$$

Limiting Amplitudes of Torsional Oscillations.—The self-excited amplification of oscillations will continue until the positive contribution of ΔW from the range of negative slope of the torque graph is balanced by the tapering negative contribution from the flanking ranges of positive slope (see Fig. 7). The limiting local amplitude (θ_o or a_o) then is defined by the condition $\Delta W = 0$, in which

$$\Delta W = \oint T\, d\theta \dots\dots \tag{53a}$$

which also may be written

$$\Delta W = \oint \theta\, dT \dots\dots \tag{53b}$$

The limiting segment amplitude (θ_{om} or a_{om}) then is defined by Eq. 32. A graphic solution of Eqs. 53 is shown in Fig. 7.

If the torque graph is of the typical form (Fig. 6) represented by a cubic equation similar to Eq. 33, with intercepts $-i$, 0, and $+i$, then

$$\theta_o = \sqrt{\frac{4}{3}}\,i \qquad (54a)$$

and

$$\theta_{om} = \frac{4}{3}\,i \qquad (54b)$$

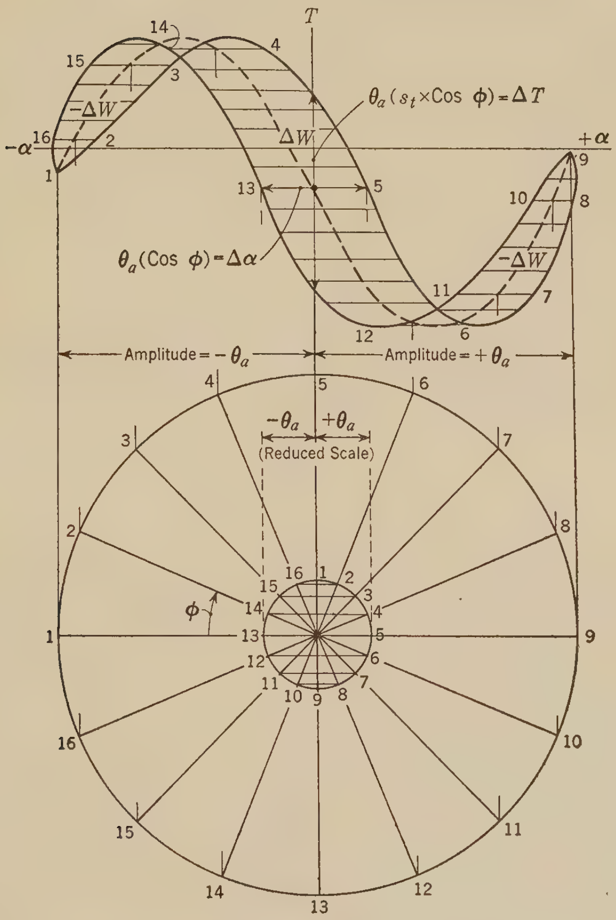
T
14
15
4
3
$-\Delta W$
$-\alpha$
16
2
1
ΔW
$\theta_a(s_t \times \text{Cos}\ \phi) = \Delta T$
$+\alpha$
9
8
10
13
5
$\theta_a(\text{Cos}\ \phi) = \Delta\alpha$
$-\Delta W$
11
7
12
6
Amplitude = $-\theta_a$
Amplitude = $+\theta_a$
4
5
6
$-\theta_a$
$+\theta_a$
(Reduced Scale)
3
7
2
8
16
1
2
15
3
14
4
ϕ
1
13
5
9
12
6
11
7
10
9
8
16
10
15
11
14
13
12

Fig. 7.—Graphic Determination of Energy Input (ΔW) for a Given Torsional Amplitude (θ_o)

The limiting angular amplitudes (θ_o at any point, and θ_{om} at midsegment) thus are independent of wind velocity, span length and rigidity, and modes of oscillation, but depend only on the form of the torque graph. Wind-tunnel tests yield similar torque graphs for all sections having torsional aerodynamic instability, with $i = 21°$ to $36°$, and therefore $\theta_{om} = 28°$ to $48°$. (The value of i appears to be a function of d/b.) The corresponding vertical amplitudes of torsional oscillations then will be

$$a_o = \frac{b}{2} \sin \theta_o \quad \text{(55a)}$$

and

$$a_{om} = \frac{b}{2} \sin \theta_{om} \quad \text{(55b)}$$

in which a_{om} is the limiting amplitude before correction for structural damping.

The torque graph obtained (1943) for the Tacoma cross section by Harold A. Thomas, M. Am. Soc. C. E., at the Carnegie Institute of Technology in Pittsburgh, Pa., has $i = 34°$. Hence, by Eq. 54*b*, the limiting torsional amplitude would be 45°, which checks the reported value for the Tacoma bridge and which also checks the oscillating-model test results obtained by Professor Thomas.

Change of Period by Forced Vibrations.—In any range of constant or mean effective s_t, the aerodynamic torque T varies in phase with the angular displacement θ:

$$T = \left(s_t \frac{\rho}{2} b^2 V^2 \right) \theta + T_{\theta=0} \quad \text{(56a)}$$

This will not affect the energy or amplitude, but will change the frequency N_t from natural frequency to a "forced" frequency:

$$N_t = \frac{1}{2\pi} \sqrt{\frac{K_t - s_t \frac{\rho}{2} b^2 V^2}{2 r^2 m}} \quad \text{(56b)}$$

If s_t is negative, the frequency N_t thus is increased. At high wind velocities, this increase in N_t may amount to 2% or 3%; normally it is negligible.

As a suggested explanation of self-excited amplification, this effect may be eliminated, since the amplitudes are unaffected. The work diagram follows the torque graph through the angular range and then retraces the same path, and is therefore reduced to a line of zero area. It is only the ΔT-effect, represented in Eqs. 48 to 53, that produces a relative displacement of the two tracings of the path along the torque graph (Fig. 7), similar to the displacements in a "hysteresis diagram." In one half of the cycle the torques are $T + \Delta T$, in the other half, $T - \Delta T$. The resulting intercepts through the angular range produce ΔW, the increment of energy that amplifies the oscillations.

Effect of Phase Difference.—Significant torsional instability associated with a negative slope of the torque graph does not occur at low wind velocities. Wind-tunnel tests on oscillating models, with the effect of structural damping eliminated by subtraction, show that there is a critical wind velocity at which aerodynamic amplification (negative damping) appears. This critical velocity V_p is explained by the relative phase difference (represented by

$N_t\,b/V$) of aerodynamic disturbance and torsional oscillation between the two sides of the cross section.[5] The theory of aerodynamic damping further indicates that the "density-mass ratio" $\rho\,b^2/(2\,m)$ of the structure has some influence on V_p, and this is confirmed by the model tests. The recorded values of $V_p/(N_t\,b)$ actually ranged from 3.3 to 5.1 for different values of the density-mass ratio. The limited test results thus far available yield, approximately,

$$V_p = N_t\,b\left(4.8 - 82\,\frac{\rho\,b^2}{2\,m}\right) \qquad (57a)$$

On substitution of numerical values for ρ and g, this reduces to

$$V_p = 1.41\,\frac{b}{r}\,\frac{b}{\sqrt{w}}\left(1.53 - \frac{b^2}{w}\right)\sqrt{K} \qquad (57b)$$

(For the Tacoma span, for $N_t = 12$ cycles per min, Eq. 57*a* or 57*b* gives $V_p = 16.6$ miles per hr. With omission of the correction for density-mass ratio in Eq. 57*a*, or its reduced form b^2/w in Eq. 57*b*, either formula gives $V_p = 25.4$ miles per hr. This significant correction has been overlooked by other investigators.)

The correction factor β in the negative damping formulas for the oscillating structure is also a function of the phase difference across the section; hence it is a function of V_p/V. A qualitative analysis of the aerodynamic action, as confirmed and supplemented by the test results, indicates that β is negative for $V < V_p$; is zero at $V = V_p$; then increases to a maximum greater than unity; and finally diminishes toward unity as a limit at high wind velocities. This variation is represented by the following empirical formula (which fits the test results):

$$\beta = \frac{5.5\left(\dfrac{V}{V_p} - 1\right) + 0.005\left(\dfrac{V}{V_p} - 1\right)^2}{\dfrac{V}{V_p} + \dfrac{V_p}{V} + 0.005\left(\dfrac{V}{V_p} - 1\right)^2} \qquad (58a)$$

or, with close accuracy from $V = 0.8\,V_p$ to $V = 4\,V_p$,

$$\beta = 5\,\frac{V - V_p}{V} \qquad (58b)$$

For various cross sections of the Tacoma type, the wind-tunnel torque graphs are almost identical in their middle parts of negative slope, with $s_t \approx -0.50$, very closely. Substituting this value of s_t, and the value of β from Eq. 58*b*, Eq. 51*b* may be written

$$\delta = 0.012\,\frac{b^3}{r^2\,w\,N_t}\,(V - V_p) \qquad (59)$$

This is identical with the plotted results obtained in wind-tunnel tests on oscillating models.

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[5] "The Failure of the Tacoma Narrows Bridge," a report to the Hon. John M. Carmody, Administrator of the Federal Works Agency, Washington, D. C., March 28, 1941, by a board of engineers consisting of O. H. Ammann, Theodor von Kármán, and Glenn B. Woodruff, Appendix VIII (analysis by Louis G. Dunn).

Analytical Derivation of β for Torsional Oscillations.—Analogous to Eq. 43 for vertical oscillations and representing a similar physical interpretation, a tentative analytical derivation of β may be written for torsional oscillations:

$$\beta = \frac{\int_{0.19\phi'}^{1.11\phi'} (1 - u\,\phi^3)\,(0.65\,\phi' - \phi)\cos\phi\,d\phi}{\int_{0.19\phi'}^{1.11\phi'} (1 - u\,\phi^3)\,(0.65\,\phi' - \phi)\,d\phi} \quad \text{.........} \quad (60a)$$

In this case a factor of the indicated form $(1 - u\,\phi^3)$ is included to represent a variation of the intensity or effectiveness of the vertical pressure impulse as it traverses the section; and the factor $(0.65\,\phi' - \phi)$ must be included to represent the varying lever arm about the center of the section. Values of β as computed by Eq. 60*a* are given in Table 5. In conformity with the test results, values of β exceeding unity are obtained. Varying values of $V_p/(N_t\,b)$ are also obtained, as follows:

TABLE 5.—ANALYTICAL DERIVATION OF β FOR TORSIONAL OSCILLATIONS ($u = 1/3$)

Equation No.	VALUES OF $V/(N_t\,b)$						
	12	8	4	3	2.5	2.0	1.5
60*a*......	3.7	2.8	0.7	−0.1	−0.6	−0.9	−0.6
60*b*......	3.7	2.5	0.8	−0.02	−0.5	−0.8	−0.5

u	$\frac{V_p}{N_t\,b}$
1/7	2.57
1/5	2.80
1/3	3.13
1	3.66
∞	5.34

The value of $V_p/(N_t\,b) = 3.13$ (and therefore $u = 1/3$) corresponds to the Tacoma section.

Each integrand in Eq. 60*a* consists of two modified triangular influence areas, concentrated toward the outer edges of the section. By considering these influence areas as concentrated at their respective centroids, an approximate equivalent formula is obtained as a welcome short cut:

$$\beta = \frac{\cos 0.3\,\phi' - [1 - 0.9\,u\,(\phi')^3]\cos\phi'}{0.9\,u\,(\phi')^3} \quad \text{............} \quad (60b)$$

for which the computed values are given in Table 5. The denominator is the algebraic sum of two resultant pressure forces acting near the outer edges of the section, and the numerator is the algebraic sum of the same two resultants corrected for phase difference by their respective cosine factors.

The several theoretical and empirical formulas for β yield remarkably consistent results. Any further refinement of the values in Table 6 would not be warranted by the present available data.

TABLE 6.—COMPARISON OF THEORETICAL AND EMPIRICAL FORMULAS FOR β

Equation No.	VALUES OF V/V_p							
	∞	5	4	3	2	1	0.5	0
60*a*........	1	3.9	3.8	3.2	2.1	0	−0.6	0
60*b*........	1	4.4	3.8	2.9	1.8	0	−0.5	0
58*a*........	1	4.2	3.8	3.3	2.2	0	−1.1	0
58*b*........	....	4	3.8	3.3	2.5	0	....	....

The coefficient u, introduced in Eq. 60*a* to represent a rate of progressive reduction or modification in pressure reaction by transfer of energy as the air disturbance traverses the section, is related to the density-mass ratio $\rho\, b^2/(2\, m)$, and therefore to the critical dimensional ratio b^2/w. The indicated relation (based on the comparison for the known data of the Tacoma section) is

$$\frac{1}{u} = 355{,}000 \left(\frac{\rho\, b^2}{2\, m}\right)^3 = 20 \left(\frac{b^2}{w}\right)^3 \ldots\ldots\ldots\ldots\ldots (61a)$$

In the useful range, the derived variation of V_p with u, as recorded under Eq. 60*a*, is given very closely by

$$\frac{V_p}{N_t\, b} = 4.8 - 1.16 \sqrt[3]{\frac{1}{u}} \ldots\ldots\ldots\ldots\ldots\ldots (61b)$$

With the correlation recorded in Eq. 61*a*, Eq. 61*b*, independently derived, proves to be identical with the empirical Eqs. 57 previously written from the wind-tunnel tests on oscillating models.

The development of the concepts and relationships introduced herein offers many interesting possibilities and will repay further exploration by both experiment and analysis.

Determination of Required K for any Specified Critical Velocity V_p.—Eqs. 57 enable the required design value of K to be determined for any specified value of V_p (the critical velocity before correction for structural damping). From Eq. 57*b*

$$K = \frac{r^2}{2\, b^2} \frac{w/b^2}{(1.53 - b^2/w)^2} (V_p)^2 \ldots\ldots\ldots\ldots\ldots (62a)$$

As a rule, for the suspension bridges under consideration, w/b^2 ranges from 1.67 to 2.50; and, for this range, the function of w/b^2 and b^2/w in Eq. 62*a* remains practically constant (between 1.88 and 1.93) and may be taken as approximately equal to 1.90. Using this value, and $r^2/b^2 \approx 0.125$, Eq. 62*a* yields

$$K = 0.12\, (V_p)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (62b)$$

Hence, for $V_p = 50$ miles per hr, the required K is 645.

Wider bridges (unless made heavier per square foot) fall outside of the values of w/b^2 assumed herein, and require a higher K. (For the Bronx-Whitestone Bridge in New York City, the required K by Eq. 62*a* is 80% higher than that given by Eq. 62*b*, or $K = 1{,}170$ for $V_p = 50$ miles per hr.)

Critically Dangerous Value of b^2/w.—Eq. 62*a* indicates that there is a critical value of b^2/w for aerodynamically unstable bridges, namely $b^2/w = 1.53$ (equivalent to $\rho\, b^2/(2\, m) = 0.0585$). For bridges having this dimensional ratio, V_p will be zero for all values of K. As b^2/w approaches the critical value of 1.53, sensitiveness to torsional aerodynamic instability increases and becomes more difficult to cure. (The structure that comes nearest to this critical ratio is the Bronx-Whitestone Bridge, with the abnormally high value of $b^2/w = 1$. The prevailing values of b^2/w for other suspension bridges of this type are close to the optimum value of 0.51, for which the required K is a minimum, as given by Eq. 62*b*.) The observed high aerodynamic sensitiveness of certain suspension bridges during erection, with floor forms in place and before pouring the concrete, is also explained by the critical value of b^2/w in that stage.

Structural Damping.—Until more adequate experimental data are available, the tentative indicated expression for δ_s, Eq. 35, previously written for vertical oscillations, also may be used for torsional oscillations of flexible suspension bridges.

The critical velocity due to structural damping, defined by the condition $\delta - \delta_s = 0$, then will be given by

$$V_c - V_p = \frac{r^2\, w\, N_t}{0.012\, b^3}\, \delta_s \tag{63a}$$

where δ_s is calculated for zero amplitude, and the limiting amplitude corrected for structural damping will be given by

$$a_r = a_{om} \sqrt{1 - \frac{V_c - V_p}{V - V_p}\left(1 + 1{,}700\, \frac{n}{l}\, \sqrt{a_r\, d}\right)} \tag{63b}$$

This yields for the Tacoma span, at $V = 40$ miles per hr, with $i = 34°$, a maximum limiting torsional amplitude of 13.1 ft, compared with 14 ft estimated as observed.

Corrected for structural damping, the net energy gain per cycle will be given by Eq. 37.

Determination of Required K for Any Specified Critical Velocity V_c.—From Eqs. 57*b* and 63*a*,

$$\sqrt{K} = \frac{V_c}{1.41\, \frac{b}{r}\, \frac{b}{\sqrt{w}}\left(1.53 - \frac{b^2}{w}\right) + 37.7\, \frac{r}{b}\, \frac{\sqrt{w}}{b}\, (0.001 + 0.06\, R)} \tag{64a}$$

For the prevailing values of $b^2/w = 0.45$ to 0.55, and $r/b \approx 0.354$, Eq. 64*a* reduces to

$$\sqrt{K} = \frac{V_c}{2.93 + 1.13\, R} \tag{64b}$$

Hence, for $R = 0$,

$$K = 0.116\ (V_c)^2 \quad \text{(64c)}$$

and, for $R = 1$,

$$K = 0.061\ (V_c)^2 \quad \text{(64d)}$$

or a reduction of 3% to 49% in the required value of K, as given by Eq. 62*b*, when structural damping is taken into account. To make $V_c = 50$ miles per hr, the required value of K ranges from 626 for $R = 0$, down to 328 for $R = 1$.

With abnormally high values of b^2/w, higher values of K will be required. At the critical value of $b^2/w = 1.53$, to make $V_c = 50$ miles per hr (which then will be due to structural damping alone), the required value of K ranges from 12,500 for $R = 1$, to 46,500,000 for $R = 0$. The obvious measure is to increase R and w or to eliminate the cause of the aerodynamic instability.

PART 4.—METHODS OF SECURING RIGIDITY AND AERODYNAMIC STABILITY

Anticipating Aerodynamic Instability.—The phenomena of aerodynamic instability, both vertical and torsional, are a combined function of flexibility of the span and disposition of the cross section.

The conventional H-type of cross section, with solid floor between vertical stiffening girders, tends to produce the effects of aerodynamic instability. Modification of the section by moving the floor slab to the top or to the bottom of the cross section may reduce the effect, but will not necessarily eliminate it.

Wind-tunnel tests on a scale model of the cross section will yield characteristic curves, useful in analysis, and offer a method of testing or predicting aerodynamic instability of the section. If the graph of aerodynamic lift for varying angles of attack has a range of negative slope, aerodynamic vertical instability of the section may be anticipated. Similarly, if the graph of aerodynamic torque for varying angles of attack has a range of negative slope, aerodynamic torsional instability of the section may be anticipated. In addition, an aerodynamically stable section may develop aerodynamic oscillations (limited to a range of low wind velocities) if the structure has a low value of R. For a complete investigation, wind-tunnel tests on oscillating models are desirable.

The use of open trusses instead of solid-web girders for the stiffening construction reduces, but does not necessarily eliminate, susceptibility to aerodynamic instability.

Influence of Rigidity.—Susceptibility to the effects of aerodynamic instability is obviously reduced by increasing the rigidity of the structure—primarily the rigidity of the main span. Rigidity of the side spans and towers is helpful in the lowest mode ($n = 1$), and may be helpful in some higher modes ($n = 2, 6, \cdots$) if center stays are provided.

For resistance to these aerodynamic effects, the most significant single factor is the coefficient of rigidity, K, or the coefficient of rigidity per foot, $2\,K/b$.

In a bridge with solid-web stiffening girders, the numerical value of K should be kept well above 300, and the numerical value of $2\,K/b$ should be kept well above 15 (lb per ft^3), unless aerodynamic stability is otherwise assured (see Eqs. 36, 62, 63, and 64, and the related text).

In a bridge with open-web stiffening trusses, the numerical value of K should be kept well above 120, and the numerical value of $2\,K/b$ should be kept well above 6 (lb per ft^3), unless aerodynamic stability is otherwise assured (see Table 2 and Fig. 2).

Another significant factor is R, the ratio of the rigidity contributed by the stiffening construction to the total rigidity. Higher values of R, by increasing the structural damping, raise the critical wind velocity, restrict the maximum amplitudes, and prevent development of oscillations at low wind velocities. The value of R should preferably be kept well above 0.25, unless aerodynamic stability is otherwise assured. With low values of R, the required values of K and $2\,K/b$ may be two or more times the foregoing values (see Eqs. 64).

The depth of the stiffening truss or girder, in proportion to the span, is also directly effective in augmenting structural damping and thereby restricting amplitudes (see Eq. 35). Experience to date indicates the hazard of making the depth of the stiffening truss or girder less than 1/100 of the span, unless the cross section is aerodynamically stable or unless other means of resisting oscillations or assuring aerodynamic stability are provided (see Fig. 2).

Lateral flexibility of the span does not appear to be a factor in aerodynamic instability. A suspension span is inherently stable against lateral displacement.

Use of Center Stays.—In amplitudes attainable and magnitude of kinetic energy accumulated, two-segment (single-node) torsional oscillations are generally the most dangerous potential manifestation of aerodynamic instability.

A highly effective method of preventing or resisting such two-segment torsional oscillations is the provision of adequately proportioned center stays or other equivalent means of preventing relative longitudinal motion of cable and suspended structure at midspan.

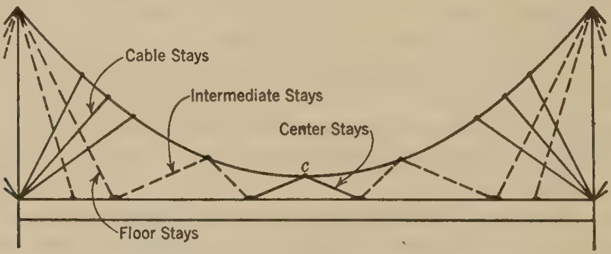

FIG. 8.—VARIOUS SYSTEMS OF LONGITUDINAL STAYS FOR STIFFENING SUSPENSION BRIDGES TO INCREASE THEIR AERODYNAMIC STABILITY

Without such center stays or their equivalent, two-segment ($n = 2$) oscillations of a main span cable involve longitudinal displacement of its midpoint (see Fig. 1). If both cables are in phase, as in vertical oscillations of the span, the longitudinal displacements of their midpoints will be in phase, tending to produce corresponding longitudinal oscillation of the suspended span. If the two cables are out of phase, as in torsional oscillations of the span, the longitudinal displacements of their midpoints will be out of phase, tending to move in opposite directions. Such relative longitudinal movements of the midpoints of the two cables in opposite directions are prevented by anchoring the two midpoints to the suspended span (Fig. 8). When such midspan anchorage of the cables is provided, by center stays or their equivalent, two-segment torsional oscillation is resisted. It cannot then take place without the concurrence of substantial increments of cable tension (such increments being of opposite sign in the two halves of the span), synchronous related torsional distortions of the towers, and synchronous related torsional deformations of the side spans.

The resisting force to be provided by the center stays can be calculated, and they should be proportioned accordingly.

Apparently no substantial two-segment aerodynamic torsional oscillations have developed in any span equipped with center stays as long as such stays and their connections have remained firm and intact.

Use of Longitudinal Diagonal Stays.—An economical method of providing concentrated, localized resistance to vertical or torsional oscillations is the use of longitudinal diagonal stays, inclined in either direction, in the planes of the cables (Fig. 8). Such stays may be located either above or below the roadway and may have their attachment to points of the cable or to points of the stiffening girder, or to both.

If such longitudinal diagonal stays run from the main tower ends of the span upward to the quarter points or other intermediate points of the cable, they are equivalent to stays running in the same direction under the roadway from shore or piers to corresponding points of the stiffening girders. The effect will be a direct resistance to upward displacement or upward swing of the points of attachment and an indirect and lesser resistance to downward displacement or downward swing of the same points of attachment, by virtue of any symmetrically disposed stays in opposing segments or in adjacent spans. Also there is a direct resistance to downward swing, by virtue of any initial tension in the stays, since a member under tension is elastically equivalent to a compression member (until it becomes slack).

If the longitudinal diagonal stays run downward from the tops of the towers to the quarter points or other intermediate points of the stiffening girders, their effect will be a direct resistance to downward deflection or downward swing of the points of attachment, and indirect and direct resistance to upward deflection or upward swing of the same points of attachment, for the same reasons as the foregoing.

In either case, the effect of the longitudinal diagonal stays will be equivalent to a substantial increase in the coefficient of rigidity K during one half of the cycle and to a smaller or equal increase in the value of K during the other half of the cycle. Inadequate values of K may thereby be brought up to safe values. The late John A. Roebling, M. Am. Soc. C. E., had this concept in mind when he designed the Niagara (New York), Cincinnati (Ohio), and Brooklyn (New York) bridges.

With the increase in the value of K, there is a sharp concomitant increase in the value of R, since the cable contribution to K remains unchanged. This increase in the value of R, which will be relatively very substantial when the initial value of R is small (as it is in flexible suspension bridges), will tend to increase structural damping and thereby will raise critical velocities and reduce the amplitudes attainable.

The relative merits of different types of longitudinal diagonal stays are not discussed herein.

For maximum effectiveness, the two types or directions of longitudinal diagonal stays may be combined, so as to provide maximum resistance to both directions of displacement or swing.

If the individual stays, in either a single or double system, are given sufficient initial tension to prevent reversal of their stress in the cycle for any anticipated amplitude of oscillation, then each stay will act with full effectiveness throughout the cycle.

The augmentation of K contributed by each individual stay can be calculated for any given mode of oscillation. A convenient simplified formula is

$$\Delta K \approx \frac{3+n}{5} \frac{E_s A_s}{l_s l} S \left(\frac{a_2 - a_1}{a_m} \right)^2 \sin^2 \psi \qquad (65)$$

in which a_1/a_m and a_2/a_m are the relative amplitudes at the two ends of the stay in terms of any unit amplitude at midsegment, ψ is the angle of the stay with the horizontal, and S is a multiplying factor (ranging from 1 to 2) depending upon the initial tension (and any initial sag) in the stay. The multiplier S has its minimum value of $S = 1$ if the initial tension is zero, and its maximum value of $S = 2$ if the initial tension is high enough to prevent reversal or if the amplitudes of any remaining oscillations of the stiffened structure are too small to cause reversal of the tension in the stay. (Any small initial sag in the stay will produce a slight reduction in S.)

Example 6: Increase in K by Longitudinal Diagonal Stays.—For the data and dimensions of the Tacoma main span, a single system of longitudinal diagonal stays near each end of the span, with a mean effective slope of 45° and made of wire rope strand with an aggregate cross section of $A_s = 16$ sq in. with sufficient initial tension to make $S = 2$, will produce, by Eq. 65, $\Sigma\Delta K = 270$ for $n = 2$. Thus the effective value of K will be increased from the original value of 62 to a mean augmented value of 332, and the corresponding value of R will be increased from 0.015 to 0.816. A double system of longitudinal diagonal stays, of the foregoing proportions running in both directions, will produce $\Sigma\Delta K = 540$ for $n = 2$, so that the effective value of K will be increased from 62 to 602, and the value of R will be increased from 0.015 to 0.900. This would bring K, $2\,K/b$, and R to relatively safe values.

Use of Transverse Diagonal Stays.—Another economical method of providing concentrated, localized resistance to torsional oscillations is the use of transverse diagonal stays (Fig. 9) located between opposite suspenders in the transverse planes near the quarter points and at other intermediate points of the span. The function of such stays is to prevent or resist out-of-phase displacement of the two sides of the span. Transverse horizontal struts between the cables may be used in conjunction with such transverse diagonal stays (see Fig. 9).

Stiffening the cross section of the suspended deck structure at the panel points, as has been suggested by some engineers, resists distortion of that cross section but does not prevent relative vertical displacement of the two sides of the undistorted cross section. In this respect a suspension bridge differs from other types. Resistance to distortion must be distinguished from resistance to torsion. Relative vertical displacement of the two sides of the section does not require distortion of the cross section but does require relative vertical displacement of the two cables, and this is resisted by transverse diagonal stays, with or without transverse struts between the cables.

Such transverse diagonal stays and horizontal struts do not help against vertical oscillations, since they do not affect the value of K against vertical oscillations; but they do help against torsional oscillations, since they directly augment the value of K_t.

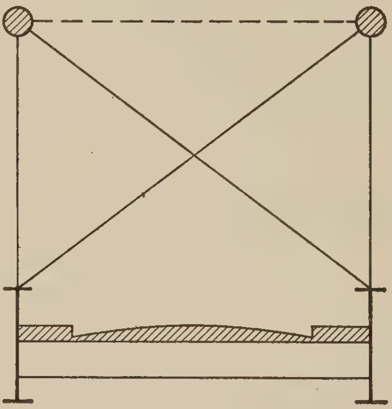

FIG. 9.—TRANSVERSE DIAGONAL STAYS TO RESIST TORSIONAL DEFORMATION

The action of transverse diagonal stays at any vertical section is effectively to resist or prevent distortion of the rectangle between cables, suspenders, and bridge floor (Fig. 9), so that torsional displacement (consisting of relative vertical displacement of the cables) then cannot take place without forcing a lateral displacement of the cables. This evokes a substantial additional force of restitution, which means a commensurate augmentation of K_t. For an adequate system of such transverse stays, with top struts included, a convenient simplified formula is

$$\Delta K_t \approx w \left[\left(\frac{4 + n^2}{5} \right) f + (4 + 2\,n^2)\,c + 5\,n^2 \frac{c^2}{f} \right] \ldots\ldots (66a)$$

in which c is the mean effective height of roadway below horizontal cable tangent. For proportioning the transverse diagonal stays, their maximum tension (per foot of span) producible by any assumed amplitude a_m is given by

$$S_t \approx 4 \frac{w}{b} \left(1 + \frac{n^2}{4} + \frac{c\,n^2}{f} \right) a_m \sec \psi \ldots\ldots\ldots\ldots (66b)$$

Without top struts, the values given by Eqs. 66 are halved.

(For the Tacoma span, Eq. 66*a* yields $\Delta K_t = 2{,}640{,}000$, corresponding to $\Delta K = 3{,}470$, and Eq. 66*b* indicates that a section of less than 1 sq in. per panel point will be adequate if no subsequent torsional amplitudes exceed 1 ft.)

Use of Damping Devices.—To supplement structural damping, artificial damping devices may be installed, for increased absorption of the energy of undulations or oscillations. Such friction damping devices have been used against the longitudinal oscillations of the span. For greater direct effectiveness, these friction damping devices can be used in connection with longitudinal diagonal cable stays or floor stays and also in connection with transverse diagonal stays.

Use of Open-Floor Construction.—The principal aerodynamic vertical pressure forces causing vertical or torsional instability originate in the horizontal floor sections near the inside faces of the stiffening girders. By opening these horizontal sections—either by leaving open spaces or by using open-grating construction—the aerodynamic forces causing amplification of undulations or oscillations can be eliminated or reduced to negligible proportions.

Experiments indicate that the required horizontal width of such open space adjacent to the inside face of each girder is not less than one half the depth of the girder.

By eliminating the solid sidewalk or roadway over this indicated width inside of each girder, differences of pressure above and below the section are largely equalized, and the area on which such differences of pressure would act is eliminated or materially reduced. Without considering the equalization of pressures, but considering only the elimination of the critical areas on which the largest pressure differences would otherwise act, a simple calculation based on the influence areas represented in Eqs. 60 shows that two lateral vents of width $3\,b/16$ to $b/4$ will reduce aerodynamic lift by at least 60% to 75% and aerodynamic torque by at least 75% to 87%; and, in addition, by Eqs. 60, the multiplier β due to phase difference across the net effective width of section will be reduced by 40% to 60%.

The entire roadway width may be made of open-decking construction, but it appears preferable to limit the open construction to the aforementioned lateral strips. The solid middle strip of roadway contributes to aerodynamic stability by atmospheric damping action. Its weight is also desirable to increase the rigidity of the span.

Use of Outside Sidewalks.—Subject to experimental verification, another indicated method of eliminating or reducing aerodynamic instability is the use of wide horizontal fins on the outside faces of the girders. Such horizontal fins conveniently may take the form of outside sidewalks or roadways on cantilever brackets or floor beams.

The purpose of such outside horizontal fins is to reverse the aerodynamic effect otherwise produced. Downward velocity of the windward girder combined with horizontal velocity of the wind then would contribute an upward pressure resultant, opposing and damping the oscillation, instead of a downward pressure resultant, amplifying the harmonic motion.

As with the use of open construction, the negative slopes of lift and torque graphs would be reduced, eliminated, or reversed.

Very conveniently, the use of open construction in the normal sidewalk spaces and the removal of the sidewalks to outside brackets may be combined in the modification of the cross section.

CONCLUSIONS

By increasing weight, depth, rigidity, and bracing, by adding center stays, longitudinal diagonal stays, transverse diagonal stays, or other stays, and by introducing artificial damping devices, resistance to the effects of aerodynamic instability may be built up to any desired amount. These methods resist or check the effects, but do not eliminate the cause.

By modification of the cross section, as by using open spaces in the floor or by adding horizontal fins or other wind-deflecting elements to the vertical faces of the girders, the cause of aerodynamic instability may be eliminated and an aerodynamically stable section may be secured.

The one line of attack places the emphasis on providing increased resistance to a dangerous inherent characteristic; the other line of thought is devoted to avoiding or eliminating the dangerous characteristic—aerodynamic instability of the cross section.

It is more scientific to eliminate the cause than to build up the structure to resist the effect.

In conclusion, the writer asks his readers to share with him his faith and conviction that suspension bridges of all span lengths can be designed economically to any desired degree of rigidity and with assured aerodynamic stability.

APPENDIX

NOTATION

The following letter symbols, adopted for use in the paper, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials,[6] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932; supplemented by customary symbols used in Aerodynamics and in the Theory of Vibrations:

A_c = section of cable (square inches);
A_s = section of diagonal stay (square inches);
a = vertical amplitude at any point of span (feet);
a_m = vertical amplitude at middle of segment (feet);
a_o = limiting local vertical amplitude producing zero ΔW;
a_{om} = limiting midsegment amplitude producing zero ΔW;
a_r = limiting midsegment amplitude reduced by structural damping;
a_1 = local amplitude at first point of attachment of a diagonal stay;
a_2 = local amplitude at second point of attachment of a diagonal stay;
b = width of section, center to center of cables (feet);
C_D = aerodynamic drag coefficient (nondimensional);
C_L = aerodynamic lift coefficient (nondimensional);
$(C_L)'$ = lift coefficient corrected for drag (nondimensional);
c = mean height of roadway below horizontal cable tangent (feet);

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[6] ASA—Z10a—1932.

D = horizontal aerodynamic drag force on section (pounds per foot);
d = vertical depth of stiffening girders (feet);
E = modulus of elasticity of stiffening girders (pounds per square inch);
E_c = modulus of elasticity of cables;
E_s = modulus of elasticity of diagonal stays;
f = vertical sag of cable (feet);
f_1 = vertical sag of cable in side span;
g = gravity acceleration $\approx$ 32.2 ft per sec^2;
H = horizontal component of tension in one cable (pounds);
I = moment of inertia of section of one stiffening girder (inches^2-feet^2);
i = horizontal intercept of lift or torque graph (radians);
K = coefficient of rigidity of the span or structure (per cable) = spring constant for vertical oscillations (pounds per square foot);
K_t = spring constant of the span or structure for torsional oscillations (pounds);
K_0 = K for main span alone;
K_1 = K for side span alone;
L = vertical aerodynamic lift force on section (pounds per foot);
L' = aerodynamic lift force corrected for drag (pounds per foot);
l = length of span (feet);
l_c = length of cable (feet);
l_s = length of a diagonal stay (feet);
l_1 = length of side span (feet);
m = mass of the span per linear foot per cable = w/g;
N = natural frequency of vertical oscillations (cycles per second);
N_t = frequency of torsional oscillations (cycles per second);
n = number of segments in span l;
n_1 = number of segments in span l_1;
P = natural period of vertical oscillations (seconds) = $1/N$;
p = applied load per cable (pounds per foot);
q = coupling factor for coupled oscillations of main and side spans (nondimensional);
R = proportion of K due to stiffening girders (nondimensional);
r = polar radius of gyration of mass of the cross section (feet);
S = factor depending upon the initial tension in a stay (nondimensional);
S_t = stress in transverse diagonal stay for torsional amplitude a_m;
s = slope of lift graph = ΔC_L per radian;
s_t = slope of torque graph;
s' = slope of lift graph corrected for drag = $s + C_D$;
T = torque acting on any section of span (foot-pounds per foot);
t = time from beginning of a cycle (seconds);
u = variation factor of pressure impulse traversing the section (nondimensional);
V = horizontal velocity of wind (feet per second);
V_c = total critical wind velocity including structural damping (feet per second);

V_p = critical wind velocity due to phase difference (feet per second);
v = instantaneous vertical velocity of any point of span (feet per second);
W = energy of oscillations (foot-pounds);
w = dead load of span per cable (pounds per foot);
Δw = increment of load per cable (pounds per foot);
x = abscissa of any point of span, measured from one end (feet);
y = ordinate of any point of cable, measured from cable chord (feet);
α = angle of attack of wind, positive when upward (radians);
β = correction factor in aerodynamic amplification formulas, to correct for phase-difference effects on oscillating sections (nondimensional);
γ_1 = slope of cable chord in side spans;
δ = logarithmic increment per cycle = $\Delta a/a$ (nondimensional);
δ_s = logarithmic decrement per cycle, by structural damping (nondimensional);
δ_0 = value of δ at zero amplitude;
η = instantaneous vertical displacement of any point of span (feet);
η_w = theoretical vertical deflection of a simple beam by dead load w (feet);
θ = instantaneous angular displacement of any point of span (radians);
θ_a = angular amplitude at any point of span (radians);
θ_o = limiting local angular amplitude producing zero ΔW;
θ_{om} = limiting midsegment amplitude of torsional oscillations producing zero ΔW;
λ = negative damping coefficient = logarithmic increment per second;
ρ = air density, in units of mass per cubic foot ≈ 0.00238;
ϕ = phase angle = $2\,\pi\,N\,t$ or $2\,\pi\,N_t\,t$ (radians);
ϕ' = effective phase difference across the width of the section; and
ψ = angle of diagonal stay with horizontal.

DISCUSSION

EDWARD ADAMS RICHARDSON,[7] ESQ.—Although this discussion must be limited primarily to aerodynamical considerations, the calculation of vibrating systems deserves comment. For example, the work of H. Reissner,[8] on modes of vibration, should be cited. Mr. Reissner represents the period of oscillation of the cable alone (P_{cn} in seconds) by the equation

$$P_{cn} = \left(\frac{1}{n}\right) C\, 0.9968\, \sqrt{f} \ldots\ldots\ldots\ldots\ldots\ldots (67a)$$

in which (in addition to the notation of the paper), C is a coefficient, depending on the sag ratio $\left(\frac{f}{l}\right)$, not greatly different from 1.0. With $C = 1.0$, this equation can be derived from that given in the *Encyclopædia Britannica*[9] for taut strings; hence it applies to any flexible suspended cable, string, or chain of uniform weight. Theodor von Kármán, M. Am. Soc. C. E., and M. A. Biot[10] have shown that the period of vibration P_{gn} of a uniform beam, uniformly loaded, supported at the ends, may be represented by the equation

$$P_{gn} = \left(\frac{1}{n}\right)^2 \left(\frac{2}{\pi}\right) l^2 \sqrt{\frac{w/g}{E\,I}} \ldots\ldots\ldots\ldots\ldots\ldots (67b)$$

in which w should be the dead load applied to one girder when I is the moment of inertia of that girder, excluding the corresponding weight of the cable and suspenders.

The combined period P_{Bn} of cable and its corresponding girder (in seconds) may be found from the fundamental cable period P_{c1}, the fundamental girder or truss period P_{g1}, and the ratio $q = \frac{P_{c1}}{P_{g1}}$ through use of the equation

$$P_{Bn} = \frac{P_{c1}}{n + (n\,q)^2} \ldots\ldots\ldots\ldots\ldots\ldots (67c)$$

This simple method of combining cable and either the girder or truss periods, to obtain a combined period equal to that of the bridge for vertical oscillations, yields substantially the same results as Eqs. 1 and 6. It is sufficiently accurate for bridges of interest from the standpoint of aerodynamic instability, as Mr. Steinman suggests. Torsional periods may be derived from Eqs. 67 by multiplying by a coefficient[11] of the form $\frac{2\,r}{b}$ provided it is remembered that the weight of cables and suspenders forms part of the span cross section whose polar radius of gyration is to be used.

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[7] Special Engr., Bethlehem Steel Co.; Partner, Edward, Nellie and George Richardson, Bethlehem, Pa.

[8] "Oscillations of Suspension Bridges," by Hans Reissner, *Journal of Applied Mechanics*, A.S.M.E., March, 1943, p. A-23.

[9] *Encyclopædia Britannica*, 14th Ed., Section on Sound, Vol. 21, pp. 9–18.

[10] "Mathematical Methods in Engineering," by Theodor von Kármán and M. A. Biot, McGraw-Hill Book Co., Inc., New York and London, 1940.

[11] "The Failure of the Tacoma Narrows Bridge," a report to the Hon. John M. Carmody, Administrator, Federal Works Agency (see brief résumé in December, 1943, *Proceedings*, p. 1564).

It should be emphasized that the author's K-limits for flexible bridges, constructed in a similar manner and of similar materials (such that the several cable and girder systems are substantially independent) should not be considered to apply to structures departing materially from such type similarity in any important respect.

Part 3.—Aerodynamic Excitation of Torsional Oscillations.—It is regrettable that this paper contains so few references to the theoretical and experimental work that has been done in the field of aerodynamics.

This paper, furthermore, unnecessarily emphasizes the rôle of lift and moment curves, for bridge cross sections, having negative trends over part of their range. Yet such curves are probably never significant in the behavior of actual bridges, and it can be shown by the work noted in the *Encyclopædia Britannica*[9] on wires made to vibrate by uniform wind streams, by the work of D. Thoma[12] on the lateral vibrations of circular cylinders in uniform streams of water when the spring factor and damping were varied, as well as by papers of other writers, that no such necessity exists. Furthermore, the tests,[11] on which this characteristic was found, were made on a model of low aspect ratio (short compared with the model width) and do not agree with those made at the Guggenheim Aerodynamic Laboratory at Pasadena, Calif., on models of effectively higher aspect ratios (which tests were given in the same report).[11]

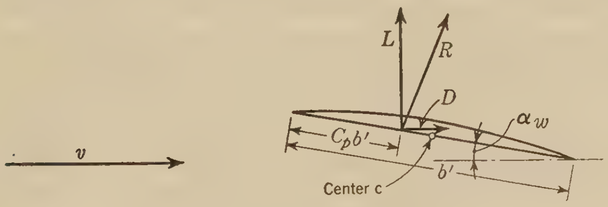

FIG. 10.—AIRFOIL CHARACTERISTICS

Referring to Fig. 10, an airfoil having a chord b' is set at an angle of attack α_w to the direction of the uniform wind stream which has a relative velocity v remote from the airfoil. The area of the airfoil projected on the plane of the chord and span is A_{ls}. The resultant force R crosses the chord at a distance $C_p\, b'$ from the leading edge. The vertical component of R is the lifting force L, and the horizontal component is the drag force D. As used in Fig. 10, q represents the pitot-tube pressure (impact pressure) of the air stream in pounds per square foot. Appropriate formulas for this case are: Impact pressure—

$$q = \tfrac{1}{2}\,\rho\, v^2 \quad \text{(68a)}$$

lift coefficient—

$$C_L = \frac{L}{q\, A_{ls}} \quad \text{(68b)}$$

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[12] "Flatterschwingungen von Zylindern in gleichmässigen Flüssigkeitsstrom," by August Meier-Windhorst, *Mitteilungen des Hydraulischen Instituts der Technischen Hochschule*, Heft 9, München, 1939, p. 1.

drag coefficient—

$$C_D = \frac{D}{q\,A_{l_s}} \qquad \text{(68}c\text{)}$$

and, moment coefficient (any axis)—

$$C_M = \frac{M}{q\,A_{l_s}\,b'} \qquad \text{(68}d\text{)}$$

For bridge oscillations, the moment coefficient C_{Mc} about the center of the chord is desired, or, if found by the formulas—

$$r = C_L \cos \alpha_w + C_D \sin \alpha_w \qquad \text{(69}a\text{)}$$

and

$$C_{Mc} = r\,(0.50 - C_p) \qquad \text{(69}b\text{)}$$

—will approximate the desired value.

When the airfoil shown in Fig. 11 suddenly acquires a velocity v relative to the air at rest (or, when the air "at infinity" has the velocity v relative to the airfoil), a vortex is formed which travels away from the airfoil at a velocity less than v, swirling in the direction shown. Any change in relative velocity will cause the generation of a vortex of appropriate magnitude and sense. In Fig. 11, any sudden stoppage would set up a vortex equal in magnitude to that of the starting vortex, but of opposite rotation. As long as the airfoil maintains its velocity at a constant angle α_w, it will be found that the circulation remains constant around the airfoil.

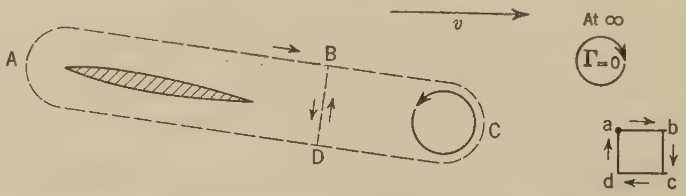

FIG. 11.—"STARTING" VORTEX

Circulation Γ_x may be assumed constant for an element of wing with a length dx along the span, in the plane, at x feet from some spanwise reference point, perpendicular to the span and parallel to the air stream. In vector notation,

$$\Gamma = \int \boldsymbol{v}\,d\boldsymbol{s} \qquad \text{(70)}$$

in which $\boldsymbol{v}$ is the vector point velocity on the path of integration, and $d\boldsymbol{s}$ is the vector element of the path of integration at the same point. Thus, circulation has the dimensions $\frac{\text{ft}^2}{\text{sec}}$.

Remote from the airfoil in a wind stream of uniform velocity v free from turbulence, from Fig. 10 and the square abcd the circulation may be seen to be zero. Since $\boldsymbol{v}$ is normal to $\overline{\text{da}}$ and $\overline{\text{bc}}$, the integral along these sides of the square is zero. Since $\boldsymbol{v}$ is the same for $\overline{\text{ab}}$ and $\overline{\text{cd}}$, whereas $\overline{\text{cd}} = -\,\overline{\text{ab}}$, the sum along these two sides is zero. Hence, for a uniform wind stream the circulation must be zero in any part of the wind stream where the path of circulation does not cut a vortex system.

Referring again to the starting vortex (Fig. 11), the path ABCD encloses the entire system of wing and starting vortices, and is remote from both, so the circulation along it is zero; but along the path BCD the circulation must correspond to the strength of the starting vortex. Hence, along the path ABD, the circulation must equal in magnitude that of the starting vortex, but be of opposite sense so that the sum may be the zero required for the path ABCD.

The theorem of Kutta-Joukowski requires that the lift of an airfoil element of span db'' shall have the value[13]

$$dL = \rho\, v\, \Gamma_x\, db'' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (71)$$

For a finite airfoil with free wing tips, the lift distribution spanwise may be represented by the ordinates of a semiellipse whose major axis is the span b'' and whose semiminor axis is Γ_0 at $x = 0$. The value of L takes the form of $\rho\, v$ times the integral of $\Gamma_x\, db''$ which in turn may be taken as $\Gamma_w\, b''$. Thus, any change in lift may be due either to a change in v, a change in $\Gamma_w \left(= \frac{\pi}{4}\, \Gamma_0 \right)$, or a combination of each.

Fig. 12 shows a rectangular wing with a chord b', and a span b'', in an air stream with a velocity v remote from the airfoil and directed normal to the span.

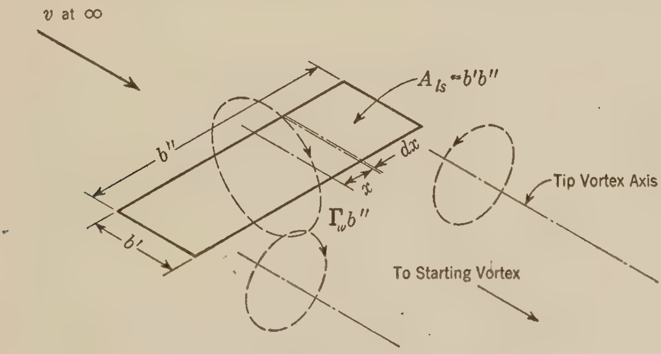

FIG. 12.—AIR FLOW FOR FINITE WING

The wind area A_{ls} equals $b'\, b''$. The total wing circulation producing lift is shown by the curve $\Gamma_w\, b''$, which is defined by:

$$\Gamma_w\, b'' = \frac{L}{\rho\, v} = 2 \int_0^{b''/2} \Gamma_x\, dx \ldots\ldots\ldots\ldots\ldots\ldots (72)$$

A tip vortex is being generated continuously. This is true only for the finite wing, since a wing of infinite span or one with shrouded tips would have no tip vortices. Work is required to generate these vortices. and a part D_i of the drag D furnishes the resistance required, or the induced drag. The remainder of the drag is the profile drag D_p, which is equal to $D - D_i$. The value of D_i may be found from the equation[13]

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[13] "Applied Wing Theory," by Elliot G. Reid, McGraw-Hill Book Co., Inc., New York and London, 1932, Chapters III and IV.

$$D_i = \frac{L^2}{\pi q (b'')^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (73)$$

Eq. 73 is used in converting the data for a wing or wing model of one aspect ratio to those for some other aspect ratio. It would not apply very consistently to the models of the Tacoma Narrows Bridge due to the action of the longitudinal girders. The effect of the tip vortices is to produce a descending velocity component in the wake of the airfoil. For infinite span length, Eq. 73 becomes zero.

Assume a model (Fig. 13), for which the angle of attack is zero in a uniform wind stream of velocity v remote from the airfoil. Assume also that the entire drag D is now the profile drag D_p due to the friction of the air stream on the airfoil. Above the very lowest values of v, the trailing air stream contains vortex elements E and F whirling in one direction, alternating with others, as G and H, whirling in the opposite direction. The velocity u in the vortex system is less than v, relative to the airfoil. For a steady system of this type,[14] $\frac{h}{l} = 0.2803$. If l is known for the vortex period P_K, the drag, and the circulation of each vortex emitted, may be computed very closely. Knowing the circulation for the vortex E, for example (and it could be measured along a path BCD, Fig. 13), the circulation along the path ABD about the airfoil section must be of about the same magnitude as that for E but opposite in direction, since the circulation along ABCD is almost zero. The other vortex elements (F, G, and H) have slight influence. Hence the circulation about the airfoil section must vary with the same frequency as that for vortices E to F from $+\bar{E}$ to $-\bar{E}$. By the Kutta-Joukowski theorem, the lift L normal to the chord must vary correspondingly. Hence a periodic lateral force acts on any "lifting surface" (even a circular cylinder) when placed in a uniform air stream and this force acts without any reference to the airfoil characteristics of lift.

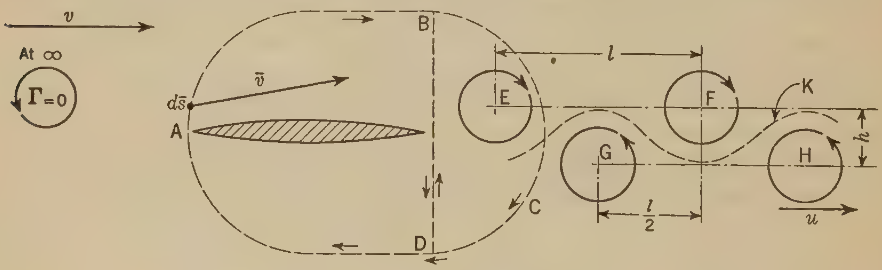

FIG. 13.—THE VON KÁRMÁN TRAIL

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[14] "Applied Hydro- and Aeromechanics," based on lectures of L. Prandtl, by O. Tietjens, McGraw-Hill Book Co., Inc., New York, N. Y., 1934, p. 130 et seq.

It is convenient to use the parameter η introduced by Mr. Thoma to describe the behavior of airfoils, in which

$$\eta = \frac{b' P_K}{v} \text{.........................} (74)$$

Other writers[15] have used the parameter $\left(\frac{1}{k}\right)$ which is the same as $\frac{\eta}{\pi}$. In terms of this parameter, Mr. Thoma[12] found that $\eta = 5.0$ for a stationary circular cylinder. With low damping, oscillation of cylinders occurred at values as low as 4.73, initially, with maximum oscillations at values greater than 6.00; but amplitudes decreased rapidly, toward the end of the critical region, to about 6.8. The *Encyclopædia Britannica*[9] used 5.41 for the vibration of wires in a wind stream. In the case of bridge floors, it is probable that a value of 6.0 would be approximately correct. Tests on wing flutter,[16] and on the oscillation of bridge models,[11] all show the effect.

In other words when P_K, with $\eta = 6.0$, is the same as P_{Bn}, large oscillations may occur, and the corresponding optimum value of v may be found for the particular mode of oscillation. However, since the characteristic dimension, b', of an airfoil is measured parallel to the air flow (and therefore is correct only for wind directions normal to the span), the corresponding dimension for a wind making the angle γ with that normal is $b' \sec \gamma$. Thus the wind velocity must be increased in direct ratio to $\sec \gamma$ to keep the von Kármán trail period equal to that of the bridge. This is equivalent to retaining b' as the characteristic dimension, using with it only the normal velocity component of the wind, $v \cos \gamma$. As the energy of the wind increases with v, it is probable that the greater γ becomes, the more certain the wind will be in starting oscillations. For example, the tests of Louis G. Dunn[11] showed that the Tacoma Narrows Bridge would have negative aerodynamic damping, with torsion at $n = 2$ for values of v greater than 25 miles per hr. With $\eta = 6.00$, a value of v of 27 miles per hr would be needed. At failure, the wind velocity was 42 miles per hr, with $\gamma = 50°$, so that the normal component was 27 miles per hr. The agreement in this case is excellent. The von Kármán trail seems adequate to account for the motion of the bridge. It provides a method of approximating wind velocities at which failure, or serious vibrations, may occur, but no one can be sure that they will actually occur.

It is possible[14] to express the equation for profile drag in the form

$$D_p = 0.959 \left(\frac{l}{b'}\right) - \left(\frac{0.331}{\eta}\right)\left(\frac{l}{b'}\right)^2 - \left(\frac{0.628}{\eta^2}\right)\left(\frac{l}{b'}\right)^3 \text{......} (75)$$

which indicates that, at constant drag, l and η may take various values, with a tendency for the trail period to be unsteady. If, instead of a uniform approaching air stream, the air stream is turbulent, with a circulation changing irregularly in period and intensity, there are two factors tending to combine in producing the lifting surface circulation. It is quite possible that a massive structure (or one that is so stiff that the oscillation per cycle would be small compared with the range of disturbances in the circulation; or, one with large damping effect) would show no signs of oscillation at the critical velocity of air stream. Given a reasonably small starting oscillation, however, it would proceed to increase in amplitude with continued wind action because, when the surface does oscillate under the governing control of spring frequency, periodic "starting" vortices are superposed on the von Kármán trail tendency and the trail unsteadiness tends to disappear, favoring increasing oscillation amplitudes.[17]

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[15] "General Theory of Aerodynamic Instability and the Mechanism of Flutter," by Theodore Theodorsen, *Report No. 496*, National Advisory Committee for Aeronautics, 1935.

[16] "A Study of Wing Flutter," by A. F. Zahm and R. M. Bear, *Report No. 285*, National Advisory Committee for Aeronautics, 1928.

If a bridge is so light or flexible that it tends to undergo forced oscillations of importance, even relatively small degrees of von Kármán trail or of turbulence in the air stream may suffice to set up important amplitudes of oscillation. This was true of the Tacoma Narrows Bridge which revealed many modes of oscillation when the wind velocity exceeded 5 miles per hr. Other bridges have shown trains of waves crossing the span, possibly set up by a short gust at some point of the bridge length (not a gust enveloping the entire structure), as in the case[11] of the Golden Gate Bridge in San Francisco, Calif.

In passing, it is to be noted that the action of wind on towers may be important because the towers generally have b'-values quite different from bridge floors.[18]

The low power input at very small amplitudes, and the rapid rise in power input as the amplitude increases, were demonstrated well by Mr. Dunn.[11] One example is given in Table 7. Since the kinetic energy varies as the square of the amplitude, the gain in energy is about 3.5 times as great in each second as in the preceding second. Many tests showed no certain starting from rest in a uniform wind stream, but did show this form of increase in amplitude when given small initial amplitudes.[17] The tests suggested strongly that turbulence has a marked effect. It should be remembered that in the case of the Tacoma Narrows Bridge there was very little damping in either the models or the prototype.

TABLE 7.—VARIATION OF POWER INPUT WITH AMPLITUDE

Time (sec) (1)	Amplitude (degrees) (2)	Col. 2 squared (3)	Gain in Col. 3 (4)
0	0.67	0.45	1.11
1	1.25	1.56	3.78
2	2.32	5.34	11.88
3	4.15	17.22	46.78
4	8.00	64.00	

Mr. Steinman has given equations for bridge damping that are not general, since material, type of construction (such as welding or riveting), and the introduction of damping devices may change structural behavior greatly. To reduce, or prevent, oscillations, high initial damping or friction is essential. Damping should be sufficiently high to limit oscillations if sudden impulses set up amplitudes which might otherwise lead to wider swings. For example, the dead load of the bridge floor and supports might be transferred to the trusses or girders through friction shoes resting on friction slides attached to these trusses or girders, so that a slight change in relative position, as on changes in deflection, would be opposed by the high starting friction of such sliding devices. Such an arrangement might be sufficient to prevent oscillations, escaping from the irregular turbulence range of forced oscillations, from growing, when the general air-stream speed is synchronized with some particular mode of oscillation.

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[17] "Suspension Bridge Oscillations and Failure," by Edward Adams Richardson, a report to Advisory Board on Suspension Bridges, Bureau of Public Roads, Federal Works Agency, November 13, 1943 (revised).

[18] Reports to the Advisory Committee of the Bureau of Public Roads, Federal Works Agency, on Long Span Suspension Bridges, by Sterling Johnston, January 15 and May 25, 1943 (unpublished letters).

Oscillating airfoils have been studied extensively and, contrary to the author, so have the phase relations presented in the paper. It is unfortunate that Mr. Steinman's studies did not include the von Kármán trail effect, as that concept is essential to a workable theory of the starting of oscillations from rest by a wind stream. Tests under conditions appropriate to these studies and theories, in which the von Kármán effect was small, checked the theories quite well.[12,15,19,20,21]

It is essential that the theory of probability be considered in determining the possibility of oscillations being started by the wind stream. Resonance in general requires that, to produce its maximum effect, the oscillating force shall have a frequency varying, within narrow limits, from that of a given possible bridge frequency. The wind speed at such theoretical synchronism is calculable, as has been noted; but the trail frequency itself tends to be irregular, and for heavy and stiff bridges the force per cycle is capable of producing only a very small oscillation amplitude. Only at appreciable amplitudes will the governing effect of the bridge frequency, as an oscillating airfoil, produce trail frequency regularity. Hence, a light and flexible bridge, which can be given an appreciable amplitude with a short train of impulses that happen to be quite close to the frequency required, is much more likely to begin vibrating than a stiffer and heavier bridge for which the probability of a sufficiently long impulse train with proper frequency is small. The succeeding, out-of-frequency wave trains tend to dampen the slight motion started. If, in addition to the irregularities of the von Kármán trail, turbulence irregularities of the wind stream itself[22] are added (which there is no reason to expect will be of the proper frequency), the chance of producing oscillations greater than the irregular trail and turbulence effects is slight. Hence the Ammann criterion[23] separates the usual types of bridges and bridge construction into two classes (oscillating bridges and non-oscillating bridges) much more sharply than would appear reasonable. However, because a bridge has not oscillated or failed in a wind stream of critical velocity cannot be considered a guarantee that it may not fail, since turbulence varies considerably with conditions; and it may

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[19] "Force and Moment on an Oscillating Airfoil," by H. Glauert, *Reports and Memoranda No. 1242*, British Aeronautical Research Committee. 1929.

[20] "On Some Reciprocal Relations in the Theory of Nonstationary Flows," by I. E. Garrick, *Report No. 629*, National Advisory Committee for Aeronautics 1938.

[21] "Experimental Verification of the Theory of Oscillating Airfoils," by Abe Silverstein and Upshur T. Joyner, *Report No. 673*, National Advisory Committee for Aeronautics, 1939.

[22] "Turbulence Factors of N. A. C. A. Wind Tunnels as Determined by Sphere Tests," by Robert C. Platt, *Report No 558*. National Advisory Committee for Aeronautics, 1936.

[23] Report of March 15, 1943, by O. H. Ammann to the Subcommittee on Model Studies, Advisory Committee on the Investigation of Long Span Suspension Bridges, Bureau of Public Roads, Federal Public Works Agency.

be possible for vibrations to start with reduced turbulence It is of the utmost importance that von Kármán trails be studied further with respect to forces applied and steadiness. At the same time, turbulence at the bridge should be studied, for various wind velocities and directions at different times, so as to evaluate possible ranges and the relation of such phenomena to the magnitude of the von Kármán trail.[17]

In any case where oscillation can start, the limiting amplitude will be that at which the work done by the given wind on the bridge is just dissipated in damping. Of two winds with the same normal component, the one having the higher velocity will produce the larger oscillations, since the forces are larger. The tangential component has been neglected herein, but it cannot be ignored.

In general, vertical oscillations in phase with the wind will not be free from small torsional oscillations of the forced vibration kind, and vice versa.

Part 4.—Methods of Securing Rigidity and Aerodynamic Stability.—The fact that rigidity, of the kind discussed by Mr. Ammann,[23] is desirable needs no discussion. The statement by the author that R, the proportion of rigidity contributed by the stiffening structure, should be large, because of the accompanying increase in structural damping, is true enough and was well demonstrated[17] by the behavior of the Lions' Gate Bridge at Vancouver, B. C., Canada (R being about 0.32).

Although the Ammann type of rigidity is desirable, it is not essential, since the limits set assume substantially independent vibration by the two cables normally used. The transverse diagonal stays recommended by the author are intended primarily to prevent torsional oscillations and thereby the possibility of torsional failure. The action is perhaps somewhat different from that described by Mr. Steinman. If displacement of one cable relative to the other begins, anything that reduces the load on the descending cable and transfers it to the other cable will tend to keep the cables in phase. This effect of the interchange of load on stability was observed by A. F. Zahm and R. M. Bear.[16] Since a diagonal consisting of one half the suspenders will be fully loaded with a deflection of the order of 1/10,000th of the possible amplitude of oscillation, the stiffening effect of such diagonals is extremely great. Oscillations are prevented before they can start, and normal torsional moments are handled easily. No increase in suspender size is needed when half the suspenders are in the diagonals, for the action is similar to that of the lacing bars of a lattice column. Furthermore, since vertical oscillations cannot be totally independent of torsional ones (small and forced), such diagonals will reduce but not prevent vertical oscillations—that is, unless the amount of reduction limits the possible oscillations to the order of those forced by the irregular turbulence.

Obviously, the longitudinal diagonal stays of the intermediate type act somewhat similarly to the transverse diagonals, since two floor points are connected to one cable point. If one floor point tends to move downward, its load is transferred to a cable point farther along the span and tends to cause that point to follow suit. Hence, such a system of stays may very well act to prevent vertical oscillations. The two systems together should make a very flexible bridge act as though it were much more rigid. The additional weight involved is relatively small.

Clearance considerations might limit the use of diagonal stays to the end portions of the bridge. In that case[17] diagonal stays in the center section, from a cable across the adjacent walkway to the edge of the floor, could be added for measurable control. The center walkway is thus forced outside of the girders; but since bridge oscillations depend on the floor width, b', the use of one width in the center and another at the ends would in itself operate to limit oscillations. The two sections (having different von Kármán trail periods) would not be in synchronism at the same time.

The author's description of the outer floor construction and of open roadways is not clear. For instance, the statement (side heading, Part 4, "Use of Open-Floor Construction"), "The solid middle strip of roadway contributes to aerodynamic stability by atmospheric damping action" ignores the fact that no such effect occurs when most needed; that is, when the bridge and trail periods are in synchronism.

As noted, center stays are of value as long as they last; but localized control leads to large forces, whereas distributed control, as by the use of the two types of diagonal stays, permits no such forces to develop as may destroy the center-stay system.

Although differing materially from the author on many important points, the writer tends to agree with him in his final conclusion and might add that even much more slender bridges than Tacoma Narrows could be made to behave as if they were much stiffer, at the same time being both comfortable and safe.

R. K. BERNHARD,[24] M. AM. SOC. C. E.—In the author's unusually interesting paper the derivation of simple working formulas for the practicing engineer to predetermine the aerodynamic stability of suspension bridges represents, as far as the writer is aware, the first direct attempt to solve this difficult problem. The author has treated the entire range of this subject in a most comprehensive way. His primary aim—formulating positive rules to prevent failures due to aerodynamic instability or, as he expresses it, the prevention of this disease rather than its cure—seems of utmost importance.

In connection with the vibration of bridges, the logarithmic decrement for structural damping is of particular interest. The author presents a wide range of damping values for suspension bridges of the order of magnitude from 0.002 to 0.130. These damping decrements may be compared with damping values as cited in other publications.

In the Society's report on the Tacoma Narrows Bridge failure values from 0.0055 to 0.057 are published resulting from tests[25] by F. B. Farquharson, Assoc. M. Am. Soc. C. E., on a suspension bridge model.

Experiments, started as early as 1928 by the writer on actual bridges with the method of induced vibrations, yielded values for structural damping from 0.063 to 0.112.[26,27] The lower values refer to welded beams, the higher values to riveted trusses, and the values in between to riveted through girders. It seems obvious that higher stresses in the connections of riveted trusses cause some relative motion and hence energy losses, thus resulting in larger damping. For similar reasons an increase in damping with rising amplitudes could be experienced coinciding approximately with Eq. 35 which contains a factor $\sqrt{a_m}$ (a_m = amplitude). Furthermore a certain change of the damping capacity with respect to the number of vibrations induced in the structure or structural unit could be observed. During a series of long-time vibration tests at first a decrease and, at impending failure, an increase, in structural damping was noticeable. The same method of induced vibrations yielded somewhat similar results for the internal damping capacity in small mild steel specimens.[28]

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[24] Trenton, N. J.

[25] *Proceedings*, Am. Soc. C. E., December, 1943, p. 1580.

[26] "Purely Dynamic Methods of Investigating Stresses in Structures," by R. Bernhard and W. Späth, *Der Stahlbau*, March 22, 1929, p. 61; also "Endurance Tests on Riveted and Welded Bridges," by R. K. Bernhard, *Verein deutscher Ingenieure Zeitschrift*, Vol. 73, No. 47, 1929, p. 1675.

[27] "Determination of Static and Dynamic Constants by Means of Response Curves," by R. K. Bernhard. *Journal of Applied Physics*, Vol. 12, No. 12, December, 1941, pp. 866–874.

In general, damping values determined by experiments on actual bridges of any type may indicate higher figures, not only when compared with metals but also with bridge models. The previously mentioned damping effect of structural connections, the damping of the foundations, and, finally, the damping of the surrounding soil, etc., will be noticeable, all these effects being rather difficult to imitate in model tests. Referring to the influence of soil characteristics, average damping values as high as 1.25 have been observed in actual field tests.[29]

In the Society's report of the Tacoma Narrows Bridge failure[25] (Appendix IV) an "accepted value of 0.3 for beam type bridges" is cited. According to this figure the logarithmic damping decrement for beam-type bridges is approximately 2.3 to 150 times higher than that for suspension bridges mentioned by the author. The value of 0.30 approaches rather closely the value of 0.31 as published in 1937.[29] It should be kept in mind that this high value, however, represented an average value for all types of structural units such as engine parts, ceilings, walls, towers, ship decks, masts, airplanes, and airship parts.

The author rightly emphasizes the fact that accurate information on the structural damping of suspension bridges is lacking. Furthermore a reduction of the wide range of damping values as published by the various investigators might be valuable. Both seem to indicate that experiments on actual suspension bridges are desirable.

In his conclusions the author states that a modification of the bridge cross section, by introducing open spaces in the floor, can be used to eliminate aerodynamic instability at the source.

This fact has also been observed by the writer on a simple model used for classroom demonstration at Pennsylvania State College, at State College, in 1940 and 1941. The model consisted essentially of a small wooden frame of approximately 10 in. by 10 in. suspended by four weak vertical springs, one in each corner. The conditions for a vibrating system with six degrees of freedom were fulfilled; in other words, oscillations of the sides or of the corners of the frame either in phase or in antiphase were possible. Since the wooden frame did not deform during the vibration, only the almost negligible internal damping of the four springs remained active. The front and rear bar of the wooden frame could be replaced by rods of aerodynamically stable or unstable cross section. A removable piece of cardboard, stretched horizontally between the front and rear bar of the frame, imitated, at least to a certain extent, the deck of one bridge panel. Thus an atmospheric damping factor, as mentioned by the author, was introduced. A constant horizontal wind stream, produced by an ordinary fan, was directed against the model. Different types of linear as well as torsional self-sustained vibrations could be built up. These experiments can be considered as elementary pilot tests only.

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[28] "Some Mechanical Properties of Plastics and Metals Under Sustained Vibrations," by B. J. Lazan, *Transactions*, A.S.M.E., Vol. 65, 1943, p. 87.

[29] "Dynamic Tests by Means of Induced Vibrations," by R. K. Bernhard, *Proceedings*, A.S.T.M., Pt. II, 1937, pp. 634–645.

A rectangular section of the front and rear bar of approximately 2 in. by 1 in., the larger side facing the wind stream, simulated unstable conditions for the individual girder. Whenever the cardboard deck remained in place the amplitudes increased up to a definite dynamically stable equilibrium. Removing this cardboard completely resulted in increased amplitudes, probably caused, in part, by the substantial reduction in atmospheric damping.

Another set of bars with half-moon cross sections of approximately 1-in. radius, the flat sides facing the inside of the frame, simulated dynamically stable conditions for the girder nearest the fan. No substantial reduction of the maximum amplitudes (that is, shielding effect) could be observed since the rear girder remained unstable.

Finally a change in cross section of the complete bridge panel, for example by cutting longitudinal holes in the cardboard deck, corresponding approximately to the width of the sidewalks, reduced rather effectively the excitation of any type of self-sustained vibration regardless of the cross section that was substituted for the individual girders.

J. M. ROBERTSON,[30] JUN. AM. SOC. C. E.—Until the phenomenon of the aerodynamic action of suspension bridges is understood in its fundamental aspects, attempts to prevent the destructive vibrations of such structures are just as apt to make the situation worse as better. This paper represents a considerable advance toward the ultimate solution. Structural engineers must now be familiar with aerodynamics and vibrations—subjects they did not need in the past.

Referring to Eq. 29a, the logarithmic decrement δ is only approximately equal to $\frac{1}{2}\frac{\Delta W}{W}$. On the basis of the usual assumptions as to the nature of W, the correct relation between the two expressions is:

$$\frac{\Delta W}{W} = 2\,\delta\,(1 + \delta + \tfrac{2}{3}\,\delta^2 + \cdots) \qquad (76)$$

Only for small values of δ is the relation used by the author accurate.

In the sentence following Eq. 34 the author states that "Accurate information on the structural damping of suspension bridges is lacking." This observation does not necessarily apply only to suspension bridges, as few data seem to be available on the damping of any except the very simplest of structures. Some data are available on the damping contributed by the material of which the structure is composed. An attempt will be made to summarize some of these data in this discussion.

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[30] Asst. Prof., Dept. of Eng. Mechanics, Pennsylvania State College, State College, Pa.

Structural damping consists of at least two parts: The damping of the material in the structure created by the varying stress in the material; and the damping resulting from the comparative looseness of the joints in the structure. Little is known even as to the relative amount of damping attributable to each of these sources. In some vibrating machine structures, estimates of the damping effect of the material have indicated that in one case it was about 33% and in another 75%. Obviously, if the damping necessary to help keep the stress amplitudes in the structure within safe limits does not come from one source or the other the structure may fail at once or through fatigue. Few data are available on the damping effect of even the simplest of structural joints. O. Föppl[31] presents some test results in which the damping in a simple riveted joint was found to be about twice that in a solid member of similar strength. A welded structure, therefore, would have less damping effect than a riveted structure, at the same vibration amplitude.

The damping capacity of materials has been a subject of investigation since the days of Lord Kelvin. Hundreds of papers have been written on it, or on internal friction, or on "mechanical hysteresis," as it is sometimes called. Nevertheless, much information is still lacking on the damping capacity of engineering materials in the range of stresses of structural significance. Damping capacity is generally assumed independent of the frequency of vibration and has been shown to be a function of the stress or strain amplitude. As noted by the author, J. P. Den Hartog states[32] that the energy loss per cycle due to the damping in the material varies as $(a_m)^{2.3}$ "based on some experiments on soft steel in the dim past." Other writers have been more definite in indicating their supposed source for this relation. W. A. Tuplin,[33] for instance, attributes it to F. E. Rowett.[34] However, in his paper, Mr. Rowett noted that the loss per cycle varied as the third power of the stress amplitude and his data do not seem to indicate the 2.3-power law. The research of S. F. Dorey[35] among others has also verified this third-power variation for various steels. At Pennsylvania State College, in State College, the writer and A. J. Yorgiadis have also verified the third-power variation of damping capacity with stress amplitude for a considerable number of metals and plastics (not published).

Data on the variation of the damping capacity of steels ΔW_{sm}, energy loss per unit volume per cycle, with stress amplitude S_m, are shown in Fig. 14. Mr. Rowett's data were obtained on tubular specimens in torsion through static tests of great sensitivity and dynamic tests (by the decaying vibrations method) on a hard drawn (curve 2, Fig. 14) and an annealed mild (curve 1, Fig. 14) steel. Mr. Dorey's data (curves 3, Fig. 14) were obtained on solid cylindrical specimens in torsion by a static test similar to that by Mr. Rowett.

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[31] "The Practical Importance of the Damping Capacity of Metals, Especially Steels." by O. Föppl, *The Journal of the Iron and Steel Institute* Vol. CXXXIV, No. II, 1936. pp. 393-455.

[32] "Mechanical Vibrations,' by J. P. Den Hartog, McGraw-Hill Book Co., Inc., New York, N. Y., 2d Ed., 1940. p. 254.

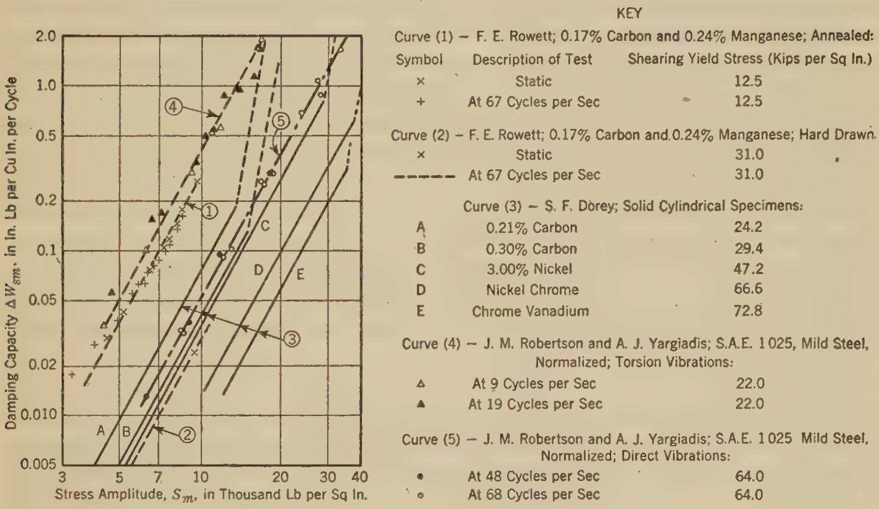
[33] "Torsional Vibration." by W A. Tuplin. John Wiley & Sons, Inc.. New York, N. Y., 1934, p. 205.

[34] "Elastic Hysteresis in Steel," by F. E. Rowett, *Proceedings*, Royal Soc. of London, Vol. 89, Series A, 1914, pp. 528–543.

[35] "Elastic Hysteresis in Crank-Shaft Steels," by S. F. Dorey, *Proceedings*, Inst. Mech. Engrs., Vol. 123, 1932, pp 479–535.

Mr. Dorey studied steels of various compositions as may be noted from the key in Fig. 14. The data observed by the writer and Mr. Yorgiadis on a mild steel, obtained by the method of resonant vibrations using apparatus developed by B. J. Lazan,[36] are also shown. The same material, a normalized mild steel tubing, was tested in torsion (curve 4, Fig. 14) and in direct, tension-compression vibrations (curve 5, Fig. 14). For all the data shown there is no evidence of a frequency effect at least in the range of frequencies covered. Up to a certain stress amplitude (curves 3, Fig. 14) the variation of ΔW_{sm} with S^3_m is apparent.

The damping that occurs in solid specimens vibrating in torsion is much less than that in tubular specimens of the same material for the same maximum stress amplitude, due to the variation of the stress in the solid specimen. As shown by F. M. Lewis in his discussion of the Dorey paper,[35] and by W. Ker Wilson,[37] it is possible to relate the damping in solid and in tubular members under torsion if the energy loss is proportional to some definite power of the stress amplitude. For the third-power law, this indicates that the damping in the solid member should be two fifths that in a tubular member for the same stress amplitude in the outer fibers. If curve 3A, Fig. 14 (for a mild steel), is raised by the inverse of this ratio, there is a rough agreement with the other data given in Fig. 14 for mild steel tubular specimens in torsion. The type of stress as well as its magnitude has considerable effect on the damping capacity as may be seen from the data obtained by the writer and Mr. Yorgiadis on a mild steel (Society of Automotive Engineers (SAE) Specification 1025) under shear stresses and direct, tension-compression stresses. The damping under



KEY

Curve (1) – F. E. Rowett; 0.17% Carbon and 0.24% Manganese; Annealed:

Symbol	Description of Test	Shearing Yield Stress (Kips per Sq In.)
×	Static	12.5
+	At 67 Cycles per Sec	12.5

Curve (2) – F. E. Rowett; 0.17% Carbon and 0.24% Manganese; Hard Drawn.

×	Static	31.0
– – – – –	At 67 Cycles per Sec	31.0

Curve (3) – S. F. Dorey; Solid Cylindrical Specimens:

A	0.21% Carbon	24.2
B	0.30% Carbon	29.4
C	3.00% Nickel	47.2
D	Nickel Chrome	66.6
E	Chrome Vanadium	72.8

Curve (4) – J. M. Robertson and A. J. Yargiadis; S.A.E. 1025, Mild Steel, Normalized; Torsion Vibrations:

△	At 9 Cycles per Sec	22.0
▲	At 19 Cycles per Sec	22.0

Curve (5) – J. M. Robertson and A. J. Yargiadis; S.A.E. 1025 Mild Steel, Normalized; Direct Vibrations:

●	At 48 Cycles per Sec	64.0
○	At 68 Cycles per Sec	64.0

FIG. 14.

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[36] "Some Mechanical Properties of Plastics and Metals Under Sustained Vibrations," by B. J. Lazan, *Transactions*, A. S. M. E., Vol. 65, 1943, pp. 87–104.

[37] "Practical Solution of Torsional Vibration Problems," by W. Ker Wilson, John Wiley & Sons, Inc., New York, N. Y., 2d Ed., 1941, Vol. 2, p. 64.

shear stressing (torsional vibrations) is about seven times that under direct stress. In addition to the variation in damping capacity with the foregoing factors, Fig. 14 indicates the influence of the nature and condition of the material. For the mild steel tested by Mr. Rowett, annealing seems to have increased the damping (at the same stress amplitude) by a factor of about ten. Curves 3, Fig. 14, indicate a variation as great as ten times in the damping with the different compositions of steel. Apparently the higher the strength of the steel, the lower will be its damping capacity.

The damping capacity of steel is also a function of the previous stress history of the material. Continued cyclic stressing may cause an increase or decrease in the damping, depending upon the magnitude of the stress amplitude. This "cold working," as it is sometimes called, has been noted by Messrs. Dorey, Föppl, Lazan, and others, but there is little complete information on it. The most comprehensive study of this phenomenon for any material appears to have been made by A. U. Kutsay and Mr. Yorgiadis,[38] who noted a decrease as great as ten times in the damping capacity of a magnesium alloy as the result of cyclic "cold working." It is rather doubtful that one can expect steel to be affected quite this much, but it is well known to be affected.

Since considerable damping capacity is necessary to prevent excessive vibration amplitudes on a bridge subjected to vibratory forces, from the data given herein a low-strength steel with a high damping capacity may be superior to a high-strength steel with a low damping capacity. In Europe a considerable number of railroad bridge failures was attributed to the substitution of a high-strength nickel alloy steel of low damping capacity for an ordinary mild steel of higher damping capacity.

C. H. Gronquist,[39] Assoc. M. Am. Soc. C. E.—The coefficient of rigidity K in Table 2 is a measure of the resistance of the main span to sine-curve deflection under antisymmetrical sine-curve loading ($n = 2$) that is positive in one half span and negative in the other half span; the quarter-point deflection in either half span under such loading is p/K. It is of interest to compare this deflection with that for antisymmetrical uniform loading ($n = 2$) for which the resulting deflection may be obtained readily by use of a principle demonstrated elsewhere by the writer.[40]

That principle may be stated as follows: The deflection at the midpoint of a uniformly loaded simple beam, or at the midpoint of any uniformly loaded beam segment of constant moment of inertia between points of contraflexure with zero deflection, is equal to the bending moment at the midpoint divided by the buckling load of the beam, for elastic-curve deflection, as a strut of that length. For the loading considered, since the center of the main suspension span is a point of contraflexure with zero deflection and, moreover, since the value of the live load H is zero, each half span may be regarded as a simple beam under the applied uniform load p. The initial, elastic theory bending moment at the quarter point, consequently, is $\frac{p\,l^2}{32}$. The final buckling load for the suspension bridge is equal to the initial buckling load of the girder increased by the amount of the horizontal component of cable stress, in contrast to the buckling load for the arch, which is decreased by the amount of the thrust. The final value of the quarter-point deflection of the suspension span for this antisymmetrical loading (including the effect of deflection) is then:

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[38] "On the Torsional Damping Capacity of Solid Magnesium-Alloy Rods as Affected by Cold-Working," by A. U. Kutsay and A. J. Yorgiadis, *Journal of the Aeronautical Sciences*, Vol. 10, 1943, pp. 303–310.

[39] Associate Engr., Robinson & Steinman, New York, N. Y.

[40] Discussion by C. H. Gronquist of "Rainbow Arch Bridge over Niagara Gorge: A Symposium." *Transactions*, Am. Soc. C. E., Vol. 110 (1945), p. 150.

$$\eta = \frac{M_e}{P_f} = \frac{\frac{p\,l^2}{32}}{4 \times 9.6\,\frac{E\,I}{l^2} + H_w} = \frac{p}{\frac{32}{l^2}\left(H_w + 4 \times 9.6\,\frac{E\,I}{l^2}\right)} = \frac{p}{K_u} \quad \ldots (77)$$

in which the amount of the horizontal component of cable stress remains unchanged, since the live load H remains zero, even considering the effect of deflection.

The stiffness constant for uniform antisymmetrical loading is practically the same as that of the author for sine-curve loading, except that the multiplier is 32 for the former as compared with $4\,\pi^2$ for the latter. This is in conformance with the greater deflection produced by the increase in load for the uniform load condition. A stiffness constant such as the one of Eq. 77, which is a measure of the resistance of the suspension bridge to deflection under antisymmetrical uniform load, was developed by the writer and later proposed for comparing the stiffness of suspension bridges in connection with a discussion on the stiffness of the self-anchored suspension bridge.[41]

The properties of the stiffness constant or final buckling load of the suspension bridge, with a two-hinged stiffening girder, for antisymmetrical loading, yield extremely simple data for computing the ratio of the final bending moment or deflection by the "deflection theory" to the moment or deflection by the usual elastic theory. The final moment is equal to the initial moment decreased by $(H_w + H)\,\eta$, thus:

$$M_f = M_e - H_w\,\eta = M_e\left(1 - \frac{H_w}{P_f}\right) = \frac{P_e}{P_f} \times M_e = R \times M_e \quad \ldots (78)$$

in which P_e is the initial buckling load of the girder, as by the elastic theory, unassisted by the cable pull. Such correction of the initial moment or deflection, by considering the effect of deflection, is analogous to that for a strut subjected to a lateral load or to initial deflection in addition to the direct loading. Formulas similar to Eq. 78 have been derived for the compression member,[42, 43] and are closely applicable to the case of the suspension bridge, when it is considered that the direct load is tensile.

The girder rigidity ratio R in Table 2, or $\frac{P_e}{P_f}$, therefore, is a close approximation of the ratio of the final moments and deflections in the stiffening girder to moments and deflections for antisymmetrical loading computed by the elastic theory. The ratio may be taken as practically constant over the length of the span, and it will change little for different loadings producing maximum live-load moments at various points along the span. The higher value of the ratio for the maximum center moment is accounted for by a reduction in the distance between points of contraflexure. A practically perfect check is obtained between the values of R in Table 2 and the Baker constant[44] for the ratio of the deflection theory to the elastic theory for moments. By including the value of the live load H for any particular loading in computing the final buckling load P_f, a closer approximation of the moment or deflection ratio will result.

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[41] "Simplified Theory of the Self-Anchored Suspension Bridge," by C. H. Gronquist, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 989.

[42] "Buckling of Elastic Structures," by H. M. Westergaard, *ibid.*, Vol. LXXXV (1922), p. 576.

[43] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1936.

Deflection at the quarter points of the main span of a two-hinged girder suspension bridge produced by antisymmetrical concentrations at the quarter points is equal to 0.8 of the elastic theory moment under the load divided by the final buckling load, as for a simple beam with a central concentration. The final moment is:

$$M_f = M_e - H_w \eta = M_e \left(1 - \frac{0.8\,H_w}{P_f}\right) = \left(\frac{P_e + 0.2\,H_w}{P_f}\right) \times M_e \ldots (79)$$

indicating that the deflection effect for concentrated loads is less than that for distributed loading. The ratio derived in Eq. 79 for antisymmetrical concentrations may be applied approximately to any concentrated loading.

The foregoing formulas for deflections, stiffness constants, and deflection theory corrections, as derived for antisymmetrical main span loading, are applicable to the side spans of the suspension bridge, for which antisymmetrical full span loading ($n_1 = \pm 1$) is critical, when l_1 is substituted for $l/2$ in the formulas.

F. B. FARQUHARSON,[45] M. AM. SOC. C. E.—An extremely able mathematical treatment of the problem of aerodynamic stability of suspension bridges is contained in this paper. It appears, however, that some of the data available to the author relating to the behavior of the original bridge were in error and that the author was not informed on some of the uniform conclusions from laboratory tests conducted since the failure of the Tacoma Narrows Bridge. It is unfortunate that the pressure of wartime has made it impossible to release the results of certain very extensive investigations which, although not complete, clarify some important phases of the problem. The following discussion refers to corresponding parts of the paper.

Parts 2 and 3.—One of the most important errors in the information available to the public regarding the behavior of the Tacoma Narrows Bridge lies in the published log of observed modes of motion and associated wind velocities.[3] A most serious error arose in the velocities obtained with an anemometer for which no calibration curve existed. Subsequent calibration of this instrument revealed errors of about 50% in the lower velocity range.

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[44] "Suspension Bridge Analysis by the Exact Method Simplified by Knowledge of Its Relations to the Approximate Method" by Arvid H. Baker, *Engineering and Science Series No. 24*, Rensselaer Polytechnic Inst., Troy, N Y., 1928.

[45] Prof., Civ. Eng., Univ. of Washington, Seattle, Wash.

No uniform procedure was followed in obtaining these readings of velocity. Many of them were taken on the top of the toll house, which was 1,400 ft distant from the nearest tower and where the influence of the terrain was such, in southerly winds, as to alter, completely, the velocity as well as the direction of the wind. Many other readings were taken just beyond the first tower on the main span and again offered no reliable indication regarding the average wind on the bridge as a whole. Furthermore, none of the direction readings was obtained with any accuracy. Each reading was a pure estimate, and no close correlation was maintained between observations of velocity and direction and readings of amplitude. In short, these recordings should be treated as only very sketchy records of the behavior of the wind whose true characteristics must remain forever in doubt.

The net result of the aforementioned discrepancies is that all of the velocity and direction readings in the log are subject to a correction. In general, the velocity error revealed by the anemometer calibration tapers from approximately 50% in the lower range to zero at about 30 miles per hr. It is impossible to estimate the magnitude of the errors due to location of the reading instrument, but it can be stated confidently that many of the readings were very low for that reason.

During the months following the failure of the Tacoma Bridge extensive research was undertaken in the Structural Research Laboratory at the University of Washington in Seattle under the joint sponsorship of the Washington Toll Bridge Authority and the Public Roads Administration. This program involved the building of a wind-tunnel laboratory capable of testing a 1 : 50 scale model of a suspension bridge of the proportions of the one lost at Tacoma Narrows. To date (June, 1944) two complete models have been built, the first a duplicate of the original bridge and the second in connection with the proposed new design. In conjunction with these full model studies, three section models have been constructed as a part of the investigation of the proposed new design for a bridge at Tacoma Narrows.

Paralleling this work at the University of Washington, a program was sponsored by the Washington Toll Bridge Authority at the California Institute of Technology under the direction of Theodor von Kármán, M. Am. Soc. C. E., using section models.

The difference between section and full models should be clarified. The full model is a complete duplication of the prototype, in every significant detail, and includes towers, cables, suspenders, and suspended structures constructed to appropriate model scale. The section model is composed of part of the suspended structure mounted on springs in such a manner that the various degrees of freedom existing on the prototype can be duplicated.

Several significant conditions have been revealed by the tests embraced in the two programs, and, although full correlation between section and full model has not yet been evaluated, on certain basic facts there is full agreement. With respect to the original Tacoma Narrows Bridge the most pertinent of these facts may be outlined as follows:

(*a*) A linear relationship exists between wind velocity V_c, corresponding to the velocity at which motion starts in a given mode, and the frequency of the several vertical modes.

(*b*) Each vertical mode appears twice but at the same frequency. The $V_c/(N\ b)$-value for the second appearance is twice that of the first. From this it follows that for a given configuration the critical velocity for the second appearance of any mode is twice that associated with the first appearance.

(*c*) For any given shape of the suspended structure and angle of attack of the wind, the value of $V_c/(N\ b)$ is constant for all vertical modes in their first appearance and constant at twice that value for all vertical modes in their second appearance.

(*d*) The first appearance of each vertical mode is very weak and difficult to isolate on the full model. These first appearances are readily observed on the section model.

(*e*) Each configuration and angle of attack of the wind has its own constant value of $V_c/(N\ b)$.

(*f*) The torsional modes also appear twice and at the same frequency. The first appearance is weak and damps out with increasing velocity, but the second appearance which occurs almost immediately following the damping of the first becomes catastrophic with increasing wind velocity. In general, the critical velocity for the first appearance of a torsional mode is half that of the second appearance. Some of the tests involving alterations in configuration suggest that this relationship may not always hold.

Oscillation Coefficients.—In aeronautical literature associated with the wind flutter problem the dimensionless ratio $V/(N\ b)$ is sometimes called the "reduced velocity." This ratio is also referred to as a "reduced wave length," the wave occurring in the wake. Some investigators have found it convenient to refer to the reciprocal of this ratio as a "reduced frequency."

Since the ratio is dimensionless and may be viewed as the ratio of two velocities, two distances, or two times, depending on the point of view of the individual or the requirements of the problem, it seems poor practice to limit the meaning of the label by making it too specific. It is proposed that in the developing field of suspension bridge aerodynamics the $V/(N\ b)$-ratio be called the "oscillation coefficient." A parallel practice has already been adopted by Theodore Theodorsen when he refers to the critical "flutter coefficient."[46]

A part of the information in Items (*a*) to (*f*) is in Fig. 15 which is obtained from the results of tests under several radically different conditions. In view of this evidence, Table 3 should not be accepted as valid since it involves a variation of $V_c/(N\ b)$ over a range of 2.00 to 0.25. Laboratory results indicate that this value is constant in the vicinity of 2.20 for the second appearance of the vertical modes and in the vicinity of 1.09 for the first appearance.

The linearity of critical velocity with frequency is completely confirmed by section model tests at the California Institute of Technology (unpublished report), which of course confirm as well the constant value of $V_c/(N\ b)$ for a given configuration.

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[46] "The Mechanism of Flutter, a Theoretical and Experimental Investigation of the Flutter Problem," by Theodore Theodorsen and I. E. Garrick, *Report No. 685*, National Advisory Committee for Aeronautics, 1940.

In Example 5, the author's check for high velocity is good, but the minimum velocity check is in error by a factor of at least five. The writer has not found the time for a critical examination into the possibility of making this mathematical approach fit the known facts more exactly. It is to be hoped that the author may be able to do so.

If the tests reported in Figs. 15 and 16 had been extended to higher wind velocities and if means had been provided to prevent the development of the torsional mode, successive higher vertical modes would have developed. (In Fig. 16, N = frequency in cycles per second.) Following this procedure, motions involving eleven nodes have been noted with no indication that higher modes were not possible. The highest mode involved frequencies in excess of 360 cycles per min (since all field frequencies must be multiplied by the square root of the scale factor this would correspond to approximately 51 cycles per min in the field). These high frequencies were especially severe on the model. Therefore, they were never maintained for any long period of time and thus were never open to careful analysis.

The writer feels that there is no evidence to substantiate statement (*a*) (see heading in Part 2: "Conclusions Relative to Vertical Aerodynamic Oscillations"). There is every indication that vertical modes could have developed well above the eight-noded mode observed in the field at 42 miles per hr. If N is assumed at 0.85 cycles per sec for eleven nodes on the prototype and a value of $V_c/(N\,b) = 2.20$ is assumed from Fig. 15, then $V_c = 2.2 \times 0.85 \times 40 = 75$ ft per sec or 51 miles per hr.

In view of the accumulated evidence, the writer cannot agree with statement (*b*) either, since the evidence from both section and full model tests indicates that torsional tendencies are present on this structure from the time the velocity reaches about 6 miles per hr.

It should also be emphasized that torsional motions can develop on a section having a positive slope on the torque graph. Fig. 17 shows torque graphs for a large model of the original bridge which was tested at both the California Institute of Technology and the University of Washington. These curves show that a full circular fairing applied to the leading and trailing girders reversed the slope of the torque graph in both tests; yet when the full model was tested in this form the amplitude response curves were similar to those of Fig. 16. Recent section model tests at the University of Washington on sections involving trusses have shown a decided torsional motion under certain small angles of attack although the torque graph showed a definite positive slope.

The chief differences between the faired and the unfaired sections seem to be that on the faired section the $V_c/(N\,b)$-value for the vertical modes is considerably lower, and the maximum amplitudes developed in the various modes somewhat lower, than on the unfaired bridge. The degree of torsional instability was somewhat altered, and there is reason to believe that the second appearance of torsion may have damped out again after reaching an angle of about 10° and a wind velocity of approximately 20 miles per hr. These tests were run just before the model was dismantled. They were rather hurried and were never repeated for confirmation. Therefore, they should be taken as indications only. The test was not extended far enough to determine whether torsion appeared again at higher wind velocities.

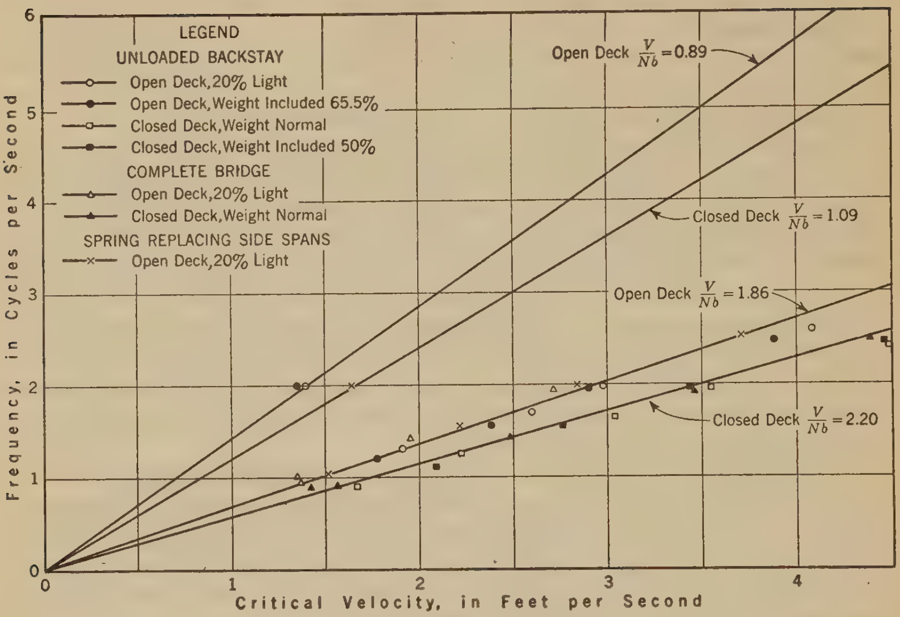

Fig. 15.—Vertical Oscillations

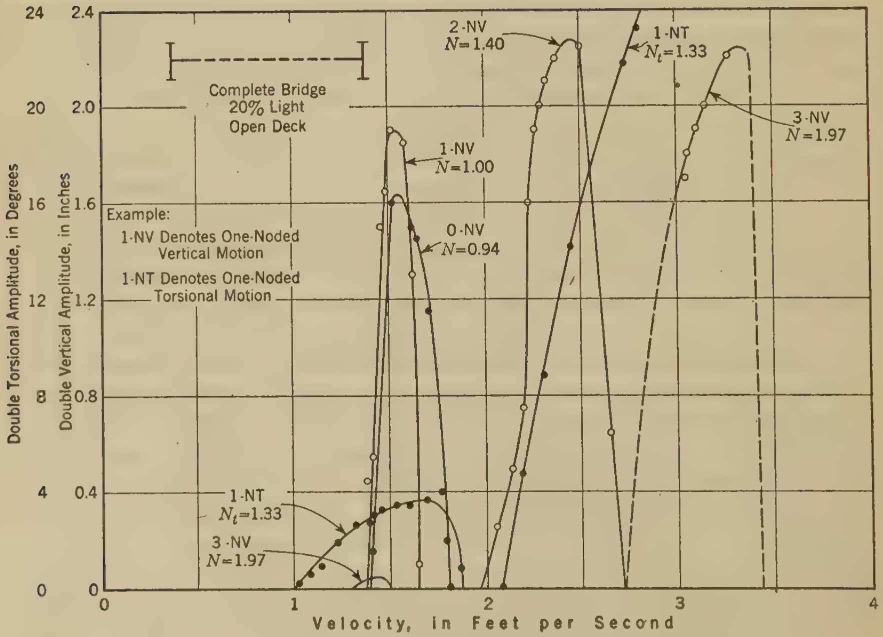

Fig. 16

No attempt has been made recently to explain the vertical motions observed on the basis of the static wind tunnel tests—that is, on the basis of negative aerodynamic damping. Experiment has demonstrated that only very slight differences showed in vertical modes between a section that had a definitely negative slope on the lift line (say, a d/b-ratio of 0.35) and the section of the original bridge (a d/b-ratio of 0.2) that showed a positive slope. Thus other possible sources have been investigated anticipating a more satisfactory explanation.

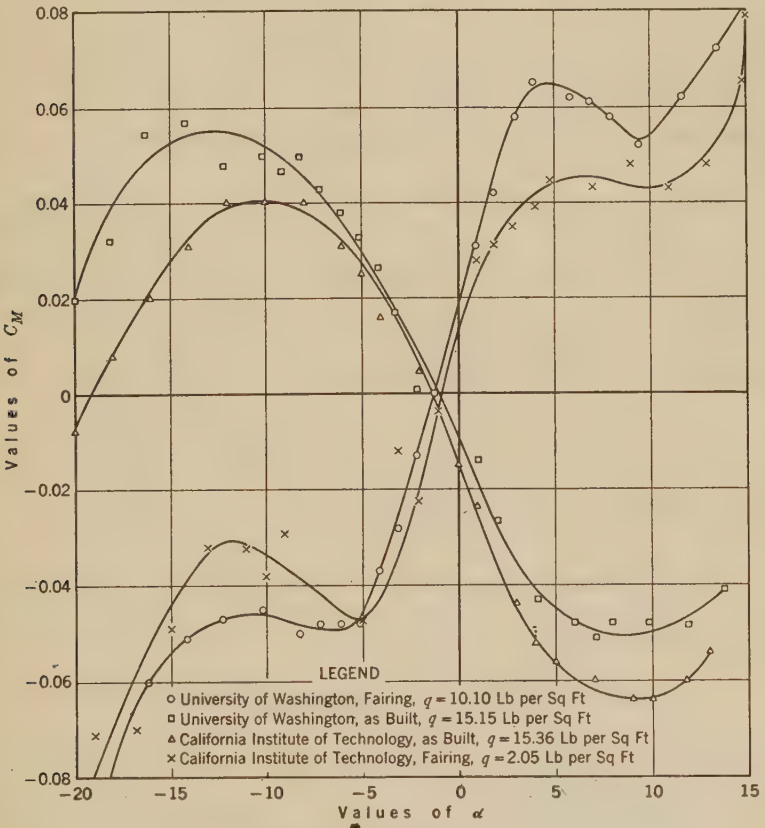

FIG. 17.—MOMENT COEFFICIENT, MODEL D

Early studies had led to the setting aside of the "vortex discharge" theory; but subsequent investigations have indicated that this theory is tenable in connection with the excitation of vertical modes on the basis of "subharmonic resonance"—that is, it has been demonstrated that excitation of a given mode can occur under starting conditions involving a vortex discharge at a rate that shows some multiple of the frequency of the resulting oscillation.

Numerous investigations of the flow were conducted using a hot-wire anemometer[47] of conventional form and recording the resultant velocity fluctuations through the use of a piezoelectric oscillograph. Typical records obtained in this manner are shown in Fig. 18.

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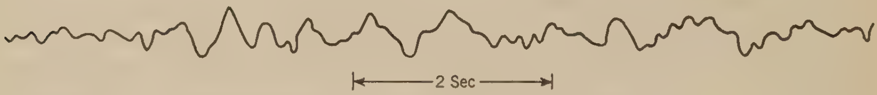
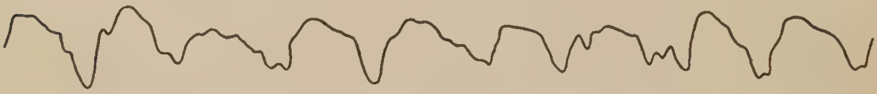
[47] "Applied Hydro- and Aeromechanics," based on lectures of L. Prandtl, by O. Tietjens, McGraw-Hill Book Co., Inc., New York, N. Y., 1934.

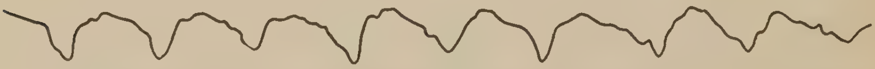
As a result of these the hot-wire tests made simultaneously at the University of Washington and the California Institute of Technology (the latter used a



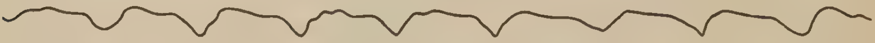
(*a*) MODEL FIXED; SUBHARMONIC EXCITATION, 93%; *V* = 1.52 FT PER SEC



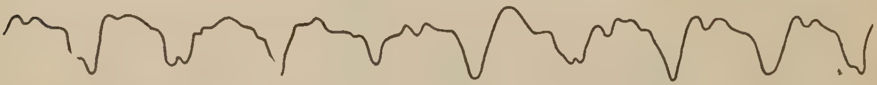
(*b*) SUBHARMONIC EXCITATION STILL PRESENT; DOUBLE AMPLITUDE = 0.6"; *V* = 1.37 FT PER SEC



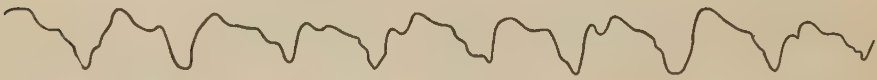
(*c*) SUBHARMONIC EXCITATION STILL PRESENT; DOUBLE AMPLITUDE = 1.0"; *V* = 1.43 FT PER SEC



(*d*) MOTION OF MODEL CONTROLS VORTICITY; DOUBLE AMPLITUDE = 1.4"; *V* = 1.52 FT PER SEC



(*e*) DOUBLE AMPLITUDE = 1.3"; *V* = 1.52 FT PER SEC



(*f*) DOUBLE AMPLITUDE = 1.2"; *V* = 1.52 FT PER SEC

FIG. 18.—TYPICAL VELOCITY FLUCTUATIONS, SINGLE-NODED VERTICAL MODE, WITH $N = 1.0$ CYCLES PER SECOND

different means of recording), it is now felt that a reasonable explanation of the phenomenon is available.

As the wind strikes the section that is at rest, the rate of vortex discharge from the leading girder will be indicated at a multiple of some natural frequency of the structure, and the structure will begin to oscillate in an appropriate mode. If the velocity is maintained at a value close to the critical velocity for that mode, the amplitude will remain small and excitation will continue on a subharmonic basis. As the velocity is increased, however, the section begins to take charge of the flow, and subharmonic excitation yields slowly to harmonic excitation.[48] At some velocity, generally well below that associated with the maximum steady state, the excitation becomes harmonic and the vorticity is completely in phase with the motion. In other words, the section has taken complete charge of the flow. As velocity is increased still further, the section begins to lose its domination over the flow, the amplitude falls off, and the section approaches rest in so far as that particular mode is concerned. In general, there is no exact rest point in the amplitude response picture for any configuration since there is a certain overlapping of the modes.

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[48] "Some Experimental Results on Wing Flutter," by William Bollay and C. D. Brown, *Journal of the Aeronautical Sciences*, June, 1941, pp. 313–318.

The domination of the flow by the section seems somewhat tenuous even at the maximum steady state since it is frequently noted that amplitude will decrease markedly without any change in velocity only to increase as constant velocity is maintained. In the case of the lower vertical modes especially, it has often been difficult to build the maximum amplitude under what appeared to be exactly similar conditions. For example, the response curves for the fundamental (0-N V) and single-noded (1-N V) modes recorded in Fig. 16 show amplitudes which were only noted on two occasions. During all other attempts the maximum steady amplitude indicated a considerably lower value. In general, the higher vertical modes could be built to maximum amplitude with little difficulty.

In Fig. 18(*d*), the motion of the model is in full control of the flow, and the maximum double amplitude has reached 1.4 in. (70 in. on the prototype). In the same run and at the same velocity a few seconds later, the amplitude has fallen off as shown in Figs. 18(*e*) and 18(*f*), and the record indicates that the control of the flow by the section is not complete—that is, some excitation on the subharmonic level is present again.

Only on the basis of the subharmonic theory of excitation can the double appearance of each of these modes be explained. There is some evidence, not yet well substantiated, that the first appearance is harmonically excited, whereas in the second appearance the excitation is subharmonic.

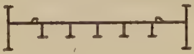
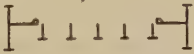
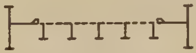
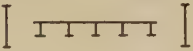
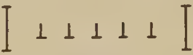
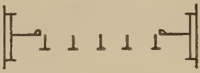
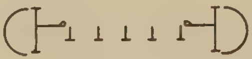
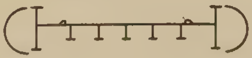
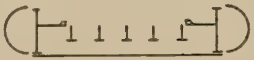
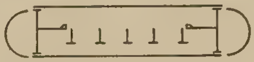
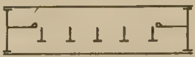
Part 4.—The author's general statements under the heading, "Anticipating Aerodynamic Instability," are not in full accord with experimental evidence derived from the full model which was tested with many variations of the configuration of the suspended structure. These configurations are listed in Table 8.

Lift, drag, and moment tests on fixed sections in the wind tunnel are available for all these configurations with the exception of sections 4, 5, 9, and 11, Table 8. It is significant to note that only in section 6 did a negative slope develop on the lift graph. Likewise, all these sections showed both vertical and torsional motions with the exception of section 10 in which no torsional motion developed in velocities up to a value greater than 110 miles per hr (prototype velocity). With great difficulty and only after very long runs, slight motion was observed in the two-noded and three-noded vertical modes in section 10. The maximum amplitude developed at the end of approximately 30 min (212 min, prototype) was 0.14-in. double amplitude in the two-noded mode.

Section 2 developed the most vigorous motion (see Fig. 16) in both vertical and torsional modes, and it is significant to note that the slope of its lift graph was strongly positive at all angles and that its moment graph was negative in slope over the middle range. All tests with girders of any depth showed negative slopes on the torque graph, and the only section that showed torsional stability (section 10) had a positive slope on the torque graph. In respect to the torque graph for girder bridges, the laboratory tests are in complete agreement with the author's claims.

However, recent tests on section models and full models of a new design using a truss for stiffening show a positive slope on the torque graph between the +12° and the −12° angles of attack; yet the model exhibits strong torsional instability when the angle of attack is approximately +7° or greater and a weaker manifestation of torsional instability when the angle of attack is −7° or greater. This positive slope on the torque graph and resultant instability

TABLE 8.—COMPARISON OF GIRDER SECTIONS

No.	Section	Description
1		The section of the bridge as built
2		Roadway slab completely removed leaving stringers and solid sidewalks with curbs
3		Section 2 with opening covered by screen of proper mesh to simulate a conventional open deck
4		Solid roadway slab with sidewalks and curbs removed
5		Both sidewalks and roadway slab removed, stringers still in place
6		Girder deepened to equivalent of 13 ft with deck open as in section 2
7		Full circular fairing on both girders, open deck as in section 2
8		Full circular fairing on both girders, closed deck as in section 1
9		Section 7 with bottom enclosed
10		Section 7 with top and bottom enclosed
11		Section 10 with circular fairing removed

of the section were likewise demonstrated in fixed model and section model tests at California Institute of Technology (unpublished report).

Use of Center Stays.—Center stays are only effective in inhibiting asymmetric torsional modes, and although they have been installed on a number of bridges it was never with this function in mind. They were first conceived as a means of reducing the longitudinal pendulum-like motion that is observed in connection with asymmetric vertical modes, and there is no evidence that they have ever been designed with regard to the stress that might arise in either the vertical or the torsional modes. There is some evidence to indicate that the center stays on the Tacoma Bridge failed (or yielded) in a strong eight-noded vertical motion which was active for some time before the transition to the first asymmetric mode in torsion. It is possible that this yielding during vertical motion permitted the development of a torsion with its accompanying much higher loading on the stays, which caused further yielding and ultimate failure (possibly accompanied by failure in the lateral system at midspan), leaving the structure quite unprotected against full development of catastrophic torsion.

Open-Floor Construction.—The writer is not so optimistic as the author regarding the beneficial effect of open-floor construction. Tests on the full model have been confined to the sections listed in Table 8, but it is significant that section 4 showed a strong three-noded vertical mode with an amplitude as great as that developed in section 2 (the lower modes were very weak) and with a single-noded torsional motion in excess of 15° of amplitude. A much higher velocity was required to bring in each of these modes but their amplitude was disquieting.

It was confidently expected that the complete removal of sidewalks and roadway slab (section 5, Table 8) would vastly improve the stability condition; but tests involving two complete runs showed three-noded and four-noded vertical modes of large amplitude and also developed a three-noded torsional mode of strong character. The three-noded torsion was especially violent and reached an amplitude in excess of 15°.

Conclusions.—The writer is in complete agreement with the author's concluding statement that the most scientific approach to this problem calls for the elimination of the cause of the motion.

H. Reissner,[49] Esq.—The publication of this paper gives a welcome opportunity for a discussion of the dynamics and particularly the aerodynamics of suspension bridges.

Part 1 is a treatment of free oscillations determined by means of the principle of work developed by Lord Rayleigh[50] for the purpose of finding approximate values of natural frequencies of a beam system of varying cross section to simplify the integration of differential equations with variable coefficients. Probable deflection curves (characteristic functions) could thus be introduced that would satisfy the boundary conditions but not necessarily the differential equations.

When the coefficients of the differential equation of the equilibrium condition are constant, the Rayleigh method becomes superfluous and the only difference (sometimes a convenient difference) is that the frequency equation appears as a quotient of integrals instead of as a determinant.

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[49] Prof., Aerodynamics and Aerostructures, Polytechnic Inst. of Brooklyn, Brooklyn, N. Y.

[50] "The Theory of Sound," by John William Strutt, Baron Rayleigh, Vol. 1, Macmillan & Co., London, 1926, p. 287.

In the application of his method, the author for simplicity omits the cases of continuous or fixed-end girders and of variable stiffness and divides the suspension bridge systems in two classes—those for which the change ΔH of the cable thrust H is zero and those for which it is not zero—without explaining the significance of this division. The fact is that, as long as the horizontal inertia forces and the cable extension are neglected, the values of the frequencies are not influenced by ΔH and the value of ΔH remains indeterminate for symmetric oscillations and must be zero for antisymmetric oscillations.

If this change ΔH is to be determined, the horizontal inertia forces and the local horizontal displacements of the cable must be considered as demonstrated by the writer elsewhere.[51] The resulting differential equations become of the sixth order, because the cable must satisfy two conditions of horizontal displacements on the saddles, and because the girder must satisfy four equations for the vertical displacements and bending moments at the supports. The change ΔH is then no longer constant along the cable.

For the case of symmetrical oscillations in a structure with straight backstays, Mr. Steinman expresses ΔH in terms of the elastic extension. This is only necessary for the case of a one-segment oscillation, which special case has such a high frequency that it practically never occurs.

On the other hand, symmetrical multisegment oscillations in structures with straight backstays are possible without extension of the cable and in these cases the influence of extension (cable stretch) is negligible.

For the case of nonparticipating side spans and the case of the simply-supported girder, the writer has applied the Rayleigh method (Part 1 of the paper) to oscillations,[51] but only because consideration of the horizontal inertia forces, of the varying slope angle γ of the cable and of the local condition of inextensibility, introduces variable coefficients. The effect of inextensibility of a cable element is expressed by

$$\frac{d\zeta}{dx} = -\frac{d\eta}{dx}\frac{dy}{dx} = -\frac{d\eta}{dx}\frac{8f}{l^2}\left(\frac{l}{2} - x\right) \dots\dots\dots\dots\dots (80)$$

in which ξ denotes the horizontal displacement of any point of the cable. In this case the solution of the differential equation by a series development leads to extremely slow convergence and the Rayleigh method is indicated.

The Rayleigh method has been applied to the case of a two-segment oscillation by the writer[51] and elsewhere to the case of a three-segment and a four-segment oscillation by E. Sternberg in cooperation with the writer.[52]

The following expressions, taken from those analyses, show the influence of the aforementioned terms, which are usually neglected:

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[51] "Oscillations of Suspension Bridges," by H. Reissner, *Journal of Applied Mechanics*, Vol. 10, No. 1, March, 1943, pp. A-23 to A-32 (preprinted and presented at the June, 1942, meeting in Cambridge, Mass., of the Applied Mechanics Div. of the A.S.M.E.).

[52] "Special Cases of the Oscillations of Suspension Bridges," by E. Sternberg, Master of Science thesis presented to the Illinois Inst. of Technology in Chicago, in 1942.

For $\eta_2 = a_m \sin \frac{2 \pi x}{l}$:

$$N_2 = \frac{1}{2} \sqrt{\frac{g}{2 f}} \sqrt{\frac{1 + 6.16 \left(\frac{f}{l}\right)^2 + 315 \frac{E I f}{w l^4}}{c_t + c_c 1.12}} \qquad (81a)$$

for $\eta_3 = a_m \left(\sin \frac{\pi x}{l} - 3 \sin \frac{3 \pi x}{l}\right)$:

$$N_3 = \sqrt{\frac{2.05}{4}} \sqrt{\frac{g}{2 f}} \sqrt{\frac{1 + 5.33 \left(\frac{f}{l}\right)^2 + \frac{700 E I f}{w l^4}}{c_t + c_c 1.05}} \qquad (81b)$$

and, for $\eta_4 = a_m \sin \frac{4 \pi x}{l}$:

$$N_4 = \sqrt{\frac{4.05}{4}} \sqrt{\frac{g}{2 f}} \sqrt{\frac{1 + 5.52 \left(\frac{f}{l}\right)^2 + 1{,}250 \frac{E I f}{w l^4}}{c_t + c_c 1.07}} \qquad (81c)$$

in which c_t and c_c are the respective relative contributions of truss and cable weight computed so that $c_t + c_c = 1$.

For the Tacoma Narrows Bridge, the numerator of the last factor in Eqs. 81 has the following relative magnitudes:

n	Numerator, Eqs. 81	N, in cycles per minute
2	1 + 0.038 + 0.01183	7.95
3	1 + 0.0328 + 0.0228	11.6
4	1 + 0.0364 + 0.0468	16.3

The second terms in this numerator give the influence of the horizontal inertia forces, and the third (last) terms, the influence of the girder stiffness. It is noteworthy that the girder stiffness for the two-segment oscillation contributes less than one third of the effect of the horizontal inertia forces of the cable in the case of the relative girder stiffness of the Tacoma Narrows Bridge.

These additional terms are less than 4% and the values without them fit very well into the approximate formula, Eq. 17*b*, although the writer would prefer to write this formula in consistent units—that is, in the form,

$$P \approx \frac{1}{n} \sqrt{\frac{32 f}{g}} \qquad (82)$$

which is true for any system of units.

The foregoing data demonstrate that the free oscillations of suspension bridges which have no redundant constraints, such as stays, can be computed by simple formulas. In fact, they can be computed by formulas which assume that a pre-stressed suspension bridge acts like a mathematical pendulum of length,

$$L = \frac{32}{\pi^2 n^2} f (1 + \epsilon) \qquad (83)$$

in which ϵ is a small quantity depending on the sag-to-span ratio, the cable and girder mass ratio, and the girder stiffness, as may be seen from Eqs. 81.

Under the heading (see Part 1), "Oscillations with Longitudinal Motion of Span," the girder is assumed to execute a free horizontal oscillation regardless of its fixing or friction at the pier supports. The reason for this assumption is not clear. The case in which there is horizontal interference between cable and girder should be treated like a system with redundant members. There is a possibility (worthy of investigation) that the short waves observed running along the girder of the Tacoma Narrows Bridge were started from a combined compression and bending effect caused by such interference.

Example 3, on torsional oscillations, needs clarification. In the cables the masses are concentrated, and only in the girder-roadway system are they distributed transversely. Hence, the transition in frequency from bending oscillation to torsional oscillation should be given by

$$N_t = N \sqrt{\frac{1}{c_t\,(2\,r/b)^2 + c_c}} \ldots\ldots\ldots\ldots\ldots\ldots (84)$$

For instance, for a weight of 2,113 lb per ft of girder roadway and of 747 lb per ft of cable, $c_t = 0.74$ and $c_c = 0.26$. Using a radius of gyration ratio of $\frac{r}{0.5\,b} = \frac{2}{3}$ the numerical result becomes (in cycles per minute): $N_t = 1.305\,N = 7.95 \times 1.305 = 10.4$, whereas Eq. 21*b* would give $1.5\,N = 11.9$.

The fact that the observed frequency ($N = 12$) was somewhat higher than the frequency (10.4) determined by the more exact formula is probably explained by a smaller value of the radius of gyration r, by the torsional elastic stiffness of the stringers and the concrete of the roadway, and by the transverse force components of the hangers.

The author's analysis of partly stable vertical oscillations due to the negative range of the lift characteristic of the cross section and his recommendation that such faulty cross sections be investigated and avoided (Part 2) deserve the full attention of bridge designers. On the other hand, his ideas and explanations concerning the generation of destabilizing phase differences by means of a phase factor β and by a disturbance or an impulse traveling across the width of the roadway are not clear.

In principle the problem of the origin of bridge oscillations, which accumulate energy out of a uniform wind flow, is very hard to handle owing to the rugged and diversified shape of the cross sections. The rational derivation of a flow pattern around these aerodynamically irregular sections, when producing a phase difference of negative damping, appears extremely difficult. The writer may mention that even the so-called flutter theory of the potential flow around perfectly streamlined airfoils governing the rhythm of the circulation by their sharp trailing edges is considered as one of the most intricate problems of aerodynamics. This makes it very doubtful that a valid analysis can be deduced from the mere geometric concept of a phase factor.

The University of Washington in Seattle and the California Institute of Technology have made pertinent experiments[53, 54] on models supported on

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[53] "Failure of the Tacoma Narrows Bridge: Report of the Special Committee of the Board of Direction," *Proceedings*, Am. Soc. C. E., December, 1943, p. 1578.

[54] "The Failure of the Tacoma Narrows Bridge," a report to the Hon. John M. Carmody, Administrator of the Federal Works Agency, Washington, D. C., March 28, 1941, by a board of engineers consisting of O. H. Ammann, Theodor von Kármán, and Glenn B. Woodruff, Chapter IV, p. 96.

springs in such a way that the effect of aerodynamic damping could be observed separately from that of structural damping. These experiments were made for such a wide variety of cases and with such careful attention to the theory of similarity that one may be justified in using them as the only available and reliable basis for a semi-empirical, semi-analytical formula. An expression for the logarithmic decrement, δ_a, as presented in the Carmody Report,[3,53,54] can be written in any arbitrary consistent units in the form,

$$\delta_a \frac{w}{\rho\, g\, b^2} \left(\frac{r}{b}\right)^2 = -\,2.05 \left(\frac{V}{b\,\omega} - 0.958\right) \ldots\ldots\ldots\ldots\ldots (85)$$

in which $\omega = N \times 2\,\pi$. From Eqs. 84 and 85, it follows that the critical velocity V_a (without structural damping) is given by

$$V_a = 0.958\, b\, \omega = 0.958\, \pi\, b \sqrt{\frac{g}{2\,f \left[c_t \left(\frac{2\,r}{b}\right)^2 + c_c\right]}} \ldots\ldots (86)$$

For critical velocities greater or less than that expressed by Eq. 86, the logarithmic decrement becomes

$$\delta_a = 2.05\, \frac{\rho\, g\, b}{w\, \omega} \left(\frac{b}{r}\right)^2 \times (V_a - V) \ldots\ldots\ldots\ldots (87)$$

Eq. 87 is a simple, dimensionally accurate expression which has been confirmed experimentally. Until the development of a completely rational analysis leading to the derivation of the logarithmic decrement, designers must be content with it.

From Eq. 86, it follows that, for the case of aerodynamic damping alone, the only way to increase the critical velocity is to increase the product of the width of the bridge floor and the natural frequency. As the practical frequency variation is very small, the critical velocity can be raised only by increasing the width of the floor, b. The relation between this width and the aerodynamic critical velocity is demonstrated by Table 9. The low value of b and of the critical velocity for the Tacoma Narrows Bridge is particularly significant. For the conditions assumed in Eq. 84, c_t was found equal to 0.74. The writer has assumed the value of 0.75 in Table 9 except in the case of the George Washington Bridge, which has four cables and no stiffening girders.

TABLE 9.—CRITICAL WIND VELOCITY (NO STRUCTURAL DAMPING, SEE EQ. 86)

Bridge	Width b (ft)	Height h (ft)	Distribution of mass	CRITICAL VELOCITY	
				Ft per sec	Miles per hr
Golden Gate	90	475	0.75	60.1	41.0
George Washington	106	325	0.64	98.5	67.1
Tacoma Narrows	39	232	0.75	37.2	25.3
Bronx-Whitestone	74	200	0.75	76.2	52.0

Mr. Steinman seems to distinguish between two kinds of self-excited torsional oscillations, one as treated by the Carmody Report (defined by Eq. 87 and Table 9) and the other as defined by his torque coefficient, $C_t\,\alpha$, and the phase factor β. On the other hand, the Carmody Report[3,55,56] states that curves like those in Fig. 6 represent a stable torque diagram such as might be derived by observations on an ordinary wind vane. A positive torque for a positive angle occurs only when the cross section is streamlined. Such a cross section may be called statically unstable. No self-excited oscillations appear with it but only an inclined equilibrium position which, with insufficient elastic resistance, may lead to overturning.

The Carmody Report[3] was the first to reveal the fact that in suspension bridges it is the structural damping rather than the structural elastic stiffness that contributes to dynamic stability. This is a contribution to a necessary new treatment. To maintain its economic and esthetic advantages, the suspension bridge must necessarily be much more flexible than other forms of truss frame bridges—that is, it must have a lower natural frequency. Therefore, its margin of safety over that furnished by aerodynamic stability must be provided by its internal frictional resistance. The utilization of this important knowledge, however, is impeded by the wide variation in frictional properties of materials and structures.

Experiments tend to show that the logarithmic decrement does not change with frequency but with amplitude. Eq. 35 incorporates this fact although Mr. Steinman does not discuss how this logarithmic decrement conforms with the equation of motion; nor does he cite the experiments on which it is based. Information about damping coefficients or logarithmic decrements of bridge structures—in terms of stiffness $E\,I$, depth d, span l, or stress σ—is certainly scarce if not nonexistent. For reasons of dimensional consistency, the author introduces the square root of the amplitude, thus giving an infinite slope of increase of the decrement toward zero amplitude and a steep slope for small amplitude. The writer doubts the correctness of this assumption. All published material available to the writer indicates that the decrement increases slowly with corresponding increases in stress up to the elastic limit. In fact, such an expression can be derived by starting from the concept of a quadratic velocity damping for which R. Grammel[57] in a specially simple case could show that the logarithmic decrement for small oscillations becomes independent of the frequency and proportional to the amplitude.

To linearize this procedure which is necessary to overcome the mathematical difficulties and justified for small oscillations the structural damping factor d_s may be assumed to be proportional to the average velocity in the form,

$$d_s = \frac{k}{g}\,a\,\omega \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (88)$$

in which a is the maximal amplitude of the girder, g the acceleration of gravity, and k is a dimensionless constant which expresses the functional characteristics of the members and joints of the structure, and the changes of amplitude along the length of the span.

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[55] "The Failure of the Tacoma Narrows Bridge," a report to the Hon. John M. Carmody, Administrator of the Federal Works Agency, Washington, D. C., March 28, 1941, by a board of engineers consisting of O. H. Ammann, Theodor von Kármán, and Glenn B. Woodruff, p. 129.

[56] *Ibid.*, Appendix VIII, p. 23.

[57] "Neuere Versuche über elastische Hysterese," by R. Grammel, *Zeitschrift des Vereines Deutscher Ingenieure*, Vol. 58, 1914, p. 1600.

The combination of aerodynamic and structural damping as formulated in Eq. 89 following will lead to the interpretation of a as the amplitude which is not increasing any more. Such an effect of structural damping has also been stated by Louis G. Dunn.[54]

In 1942[51] the writer evaluated the available observations on aerodynamically critical velocities V_a and the total wind velocity V_c (condensed in Eq. 85 and in the actual observations of the critical velocity of the Tacoma Bridge) by means of the equations of motion. For this purpose constant viscous damping coefficients were used—that is, it was assumed that both decrements were inversely proportional to the frequency, which is correct only for aerodynamic damping. This assumption leads to approximate values of V_c only when applied to bridges with natural frequencies not too different from the frequency of the Tacoma Bridge. This same objection can be raised against the representation, sometimes used, of friction by an additional elastic force with an imaginary factor accounting for the phase difference in an energy-consuming resistance. Furthermore, such a method does not seem rational because it cannot be transformed into expressions involving functions of real quantities.

There seems to be no reason why the viscous damping factor, although constant for given dimensions of a system, cannot be made dependent on the frequency and the amplitude in such a way as to satisfy experimental determinations of logarithmic decrements.[58]

Neglecting the horizontal inertia forces and the cable slope, the equation of motion in its usual form is as follows:

$$\frac{w}{g}\psi\frac{\partial^2\eta}{\partial t^2} - H\frac{\partial^2\eta}{\partial x^2} + E\,I\,n_t\frac{\partial^4\eta}{\partial x^4} + \frac{\partial}{\partial t}\left(d_a\cdot V\,\eta + d_s\,E\,I\,n_t\frac{\partial^4\eta}{\partial x^4}\right) = 0 \,..(89)$$

In Eq. 89, in addition to the notation of the paper, n_t is a factor indicating an eventual torsional stiffness of the wind bracing (for a tubular cross section) and of the bridge floor; d_a is a coefficient of aerodynamic damping; d_s is a coefficient of structural damping (see Eqs. 85 and 88); and

$$\frac{w}{g}\psi = m_1 = m_{\text{girder}}\left(\frac{2\,r}{b}\right)^2 + m_{\text{cable}} \ldots\ldots\ldots\ldots\ldots\ldots(90)$$

In the last term of Eq. 89 the structural damping moment is assumed to be proportional to the time rate of change of the elastic counter forces,

$$\frac{\partial}{\partial t}\left(E\,I\,n_t\frac{\partial^4\eta}{\partial x^4}\right)b \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(91)$$

Solving Eq. 89 in the usual manner and observing Eq. 87, the logarithmic decrement becomes

$$\delta_a + \delta_s = 2.05\,\frac{\rho\,g\,b}{w\,\omega}\left(\frac{b}{r}\right)^2(V_a - V) + k\,a\,\frac{E\,I\,n_t}{w\,\psi\,l^4} \ldots\ldots\ldots(92)$$

The condition for zero decrement now becomes

$$V_c = V_a + k\,\frac{a}{b}\,\frac{E\,I\,n_t}{\psi\,l^4}\,\frac{\omega}{\rho\,g\left(\frac{b}{r}\right)^2} \ldots\ldots\ldots\ldots\ldots\ldots(93)$$

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[58] "Damping Formulas and Experimental Values of Damping in Flutter Models," by R. P. Coleman, *Technical Notes No. 751*, National Advisory Committee for Aeronautics, February, 1941.

The only observations of critical wind velocities available with all necessary dimensions and weight data are those in connection with the Tacoma Narrows Bridge.[59] Using these observations one can discuss the significance of Eq. 93 by inserting the observed simultaneous values of $V_e = 62$ ft per sec, of the circular frequency $\omega = \frac{2\,\pi\,n}{60} = 1.25$, and the observed model value of $V_a = 37.2$ ft per sec from Table 9 into Eq. 93. Eq. 93 contains the product of the friction coefficient k and the ratio $\frac{a}{b}$ which latter is a measure of about half the maximum angle of twist, at which the bridge by the structural damping increasing with amplitude reaches a state of steady oscillation—that is, provided that the stresses in this state are not at the breaking limit as in the case of the Tacoma Bridge.

Solving for this dimensionless quantity $\frac{k\,a}{b}$ one obtains (compare Eq. 93):

$$\frac{k\,a}{b} = (V_e - V_a)\frac{l^4}{E\,I\,n_t\,\omega}\rho\,g\,\psi\left(\frac{b}{r}\right)^2$$

$$= 24.8\,\frac{2{,}800^4 \times 0.00238 \times 32.2 \times 0.59 \times 9}{3.46 \times 10^{10} \times 1.25} = 1.435 \times 10^4 \ldots\ldots(94)$$

With this value the following examples may be computed:

(1) For the Tacoma Bridge what increase of road width b and girder stiffness $E\,I$ would be necessary to raise the critical wind speed from 62 ft per sec to 150 ft per sec (for the same angle of twist)?

Choosing, as suggested by the Tacoma committee, $b = 52$ ft (instead of 39) and doubling the girder depth with the same effective flange area one obtains from Eq. 94: $V_e = 37.2 \times 1.3 + 24.8 \times 4 = 147.5$ ft per sec. If it is required that the tangent of the twisting angle for the steady state is only one fourth of the value observed at the Tacoma Bridge the allowable critical velocity would be $V_e = 48.3 + 24.8 = 73.1$ ft per sec.

(2) What is the critical velocity for the Golden Gate Bridge with the value of critical twisting angle of the Tacoma Bridge? From Table 9 and from the report of the Board of Direction,[60] the following values are used:

$$V_a = 60.1 \text{ ft per sec};\quad E\,I = 118.9 \times 10^{10} \text{ lb ft}^2;\quad n_t = 1,$$

$$\omega = 0.773 \text{ sec}^{-1};\quad \psi = 0.59;\quad l = 4{,}200 \text{ ft};\quad b = 90 \text{ ft};\quad \frac{b}{r} = 3.$$

Eq. 94 gives $V_e = 60.1 + 109.9 = 170$ ft per sec $= 116.5$ miles per hr. If only a quarter of the original value of $\frac{a}{b}$ is allowed, then $V_e = 60.1 + 27.5 = 87.6$ ft per sec $= 60$ miles per hr. It may be mentioned that the highest wind velocity of 110 ft per sec, acting on the Golden Gate Bridge was observed by R. G. Cone, M. Am. Soc. C. E., on February 9, 1938,[61] and that the bridge attained steady deflections of which the magnitude could not be measured.

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[59] "The Failure of the Tacoma Narrows Bridge," a report to the Hon. John M. Carmody, Administrator of the Federal Works Agency, Washington, D. C., March 28, 1941, by a board of engineers consisting of O. H. Ammann, Theodor von Kármán, and Glenn B. Woodruff, pp. 21–27.

[60] "Failure of the Tacoma Narrows Bridge: Report of the Special Committee of the Board of Direction," *Proceedings*, Am. Soc. C. E., December, 1943, p. 1566, Table 1.

(3) Another opportunity for checking Eq. 94 is afforded by the Bronx-Whitestone Bridge. Here the data in feet and pounds are:

$$E\,I\,n_t = 8.5 \times 10^{10}; \quad \omega = 1.19; \quad \psi = 0.59;$$

$$l = 2{,}300; \quad b = 74; \quad \text{and} \quad \frac{b}{r} = 3.$$

With the value of $V_a = 76.2$ from Table 9, Eq. 94 yields $V_c = 76.2 + 100.0 = 176.2$ ft per sec $= 121$ miles per hr and with one quarter of the twisting angle $\frac{a}{b}$ $V_c = 76.2 + 25 = 101.2$ ft per sec.

Concerning the aim to increase the structural stiffness and friction (n_t in Eq. 89 and k in Eq. 88), Mr. Steinman's suggestion in Fig. 9 seems very appropriate. Geometrically rigid cross frames are used, particularly at the quarter points of the spans, running the entire way from and between the cables to the lower truss chords and the transverse roadway girders. In fact, such transverse bracings would force the horizontal wind bracing to resist the twisting of the bridge by means of the bending stiffness and the bending friction in its horizontal plane.

A combination of this device with diagonal stays running from the top points of the cross frames and in the planes of the cables and hangers to the piers at the bases of the pylons, and equipped with vibration absorbers, should be very efficient against all kinds of oscillations with a minimum of redundant members. An indication of a high damping effect of such stays has been found by the experiments of F. B. Farquharson,[53] M. Am. Soc. C. E.

Blair Birdsall,[62] Assoc. M. Am. Soc. C. E.—The profession is indebted to the author for his presentation of the results of an exhaustive study of the problem arising from the failure of the Tacoma Narrows Bridge and the more or less unstable behavior of several other structures. Many interesting and helpful new tools are offered for the use of the designer. Although the validity of these formulas is strikingly confirmed by comparison with observed oscillations, it is undoubtedly true, as stated by the author, that many gaps remain to be filled and that the empirical deductions require more thorough consideration.

For instance, a minimum limit of 1:100 for the ratio of depth of truss or girder to span may be more conservative than necessary. This is evident in the case of the Golden Gate Bridge. A decrease in cable sag from 11.1% to 10% of span would have increased K per cable well above the minimum value of 120 selected by the author and in this case the ratio of truss depth to span is 1:168.

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[61] "The Failure of the Tacoma Narrows Bridge," a report to the Hon John M. Carmody, Administrator of the Federal Works Agency, Washington, D. C., March 28, 1941, by a board of engineers consisting of O. H. Ammann, Theodor von Kármán, and Glenn B. Woodruff, Appendix IX.

[62] Asst. Chf. Engr., Bridge Div., John A. Roebling's Sons Co., Trenton, N. J.

Thus, it is to be hoped that this paper will be the beginning of a series of discussions and additional papers through which the Society will receive the benefit of the vast amount of research and investigation in progress on this subject.

The writer wishes to offer a few remarks on the "spring constant" or "coefficient of rigidity," designated by the author as K. From the derivation of K, it appears perfectly proper to consider this constant as a measure of the resistance of the span to deformations caused by loads applied uniformly along segments of the span: K per cable is a measure of resistance to loads that have a uniform intensity per foot of horizontal cable span; and K per foot is a measure of the resistance to loads that have a uniform intensity per square foot. However, since the present problem involves the aerodynamic behavior of suspension bridges, an attempt should be made to find a constant which will be a measure of the resistance of a particular structure to loads imposed by commonly occurring wind velocities. Obviously, wind of a given velocity will not have the same effect on two bridges that are identical in all respects except for the stiffening member, which is a truss in the one case and a girder in the other. Furthermore, the writer believes that vertical forces resulting from wind action are not uniform per square foot of floor area and that the total vertical wind force is not proportional to width of roadway on two structures that are identical except for width.

TABLE 10.—SUGGESTED REARRANGEMENT OF TABLE 2, WITH MODIFIED VALUES OF K

Year	Bridge	K per cable
1903	Williamsburg.....	1,082
1926	Philadelphia......	735
1909	Manhattan.......	618
1936	Triborough.......	547
1936	Transbay.........	324
1930	Mid-Hudson......	262
1931	Maumee River[a]...	259
1931	St. Johns..........	252
1931	George Washington	244
1929	Detroit...........	212
1931	Waldo-Hancock...	212
1929	Mount Hope......	201
1929	Grand Mère......	188
1939	Wabash River[a]....	131
1937	Golden Gate......	116
1938	Vancouver........	112
1939	Bronx-Whitestone[a]	56.5
1938	Thousand Islands[a].	56.5
1938	Thousand Islands[a].	51
1939	Deer Isle[a]........	27.6
1940	Tacoma[a].........	24.8

[a] Denotes construction with solid-web stiffening girders.

The writer suggests that an attempt be made to modify K so as to include some measure of the effect of wind action. Perhaps such a modification can be obtained from a study of the lift graph for the bridge cross section. In this way it may be possible to classify K-values for existing bridges in the order of their observed behavior. The writer has taken a step in this direction by means of the following simple device.

In Part 4, the author states that the limiting value of K per cable should be 300 for bridges with stiffening girders and 120 for bridges with stiffening trusses. By dividing all the author's values of K per cable for girder-stiffened bridges by 300/120 or 2.5, the writer has obtained a rearrangement of the order of stiffness of the bridges in Table 2 (see Table 10).

Undoubtedly this is not the correct order; nor are the values truly representative, but this arrangement does appear to conform more nearly to the observed behavior of these bridges and indicates that very satisfactory results might be obtained from a more scientific modification of K per cable.

HARDY CROSS,[63] M. AM. SOC. C. E.—The author of this paper deserves thanks for effective condensation and clear presentation. The evidence presented is chiefly analytical; this is useful, but it is hoped that, in the closure, it will be supplemented by evidence from the author's extended practice.

Structural design is based on evidence from several sources; few problems illustrate this better than that of the stability of suspension bridges. Analysis furnishes evidence, but analysis is of necessity based on assumptions. It is important to note that model experiments are merely short cuts in analysis and are also based on assumptions. Experiments seriously attempt to avoid assumptions, but they rarely do so because of differences of useful scale. Full-scale experiments are scarcely practicable for suspension bridges, although it may be possible to make them with airplanes. Experience presents results of full-scale experiment; but the experiment is not planned and the results are fragmentary and often cannot be dependably correlated. Common sense—intuitive qualitative perception of relations—also furnishes evidence; but this evidence alone is not always dependable.

Evidence from no one of these sources is inherently more dependable than that from another; all may mislead—and often do so—and all sources supply valuable evidence. Because engineers are not too wise in the ways of nature, they are not often able to reconcile what seem to be conflicting data from the various sources; the current problem illustrates the point clearly.

Suspension bridges may begin to move in a wind and these movements can build up. "Autoresonance" from wind action is indicated and is generally accepted as an explanation. If autoresonance occurs, the strains in the structure will increase to failure unless the driving force dies out or reverses, or unless damping, either frictional or internal, absorbs the energy as fast as it is supplied, or unless the vibration characteristics of the structure change, as would occur with slack ties.

The first of these phenomena may occur because the wind dies out or because it is gusty. The author thinks that the lift relations change as the vibration builds up; this may be the explanation, or an explanation. The hypothesis is an interesting contribution to speculation and it deserves investigation even though this may be difficult.

Much information is not available on friction damping. Perhaps engineers try too hard to eliminate it in structures; but to absorb by friction the energy of a vibrating suspension bridge is itself a major problem. Even less is known of internal damping, but the writer has not yet seen data to indicate that it is an important factor in this problem.

Changeable vibration characteristics seem to be nature's solution of the resonance problem and perhaps the late John Roebling, M. Am. Soc. C. E., saw their value intuitively. The writer does not know how far this field of thought has been explored but he has reason to think that the author of the paper has studied it and hopes that he will comment on it.

The author feels sure that it is better to eliminate the cause of instability than to provide a cure. Whether this is true depends on the certainty of the diagnosis of cause and the dependability of the recommended cure; good engineering depends on evaluation of insurance risks against probable occurrence and possible consequence. If the engineer can find out when—for what proportions and spans—these bridges are unstable, he may not care so much about why; certainly this is the case in many engineering problems. Usually the designer makes some observations on "when"; then he speculates as to "why"; and finally he revises and successively revises to approach dependability. The author has here contributed to the "why"; it is to be hoped that in his closure he will contribute data from his broad experience as to the "when."

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[63] Prof., Civ. Eng., Yale Univ., New Haven, Conn.

Frederick C. Smith[64] and Hermann Friedlaender,[65] Assoc. Members, Am. Soc. C. E., and George S. Vincent,[66] M. Am. Soc. C. E.—In association with F. B. Farquharson, M. Am. Soc. C. E., the writers have been working on the development of equations which will express adequately the behavior of suspension bridges under vibration. Model studies of the Tacoma Narrows Bridge, one of the structures analyzed by the author, have been undertaken by the University of Washington, at Seattle, and the Washington (State) Toll Bridge Authority with the cooperation of the Public Roads Administration. These studies also involve certain supplemental tests recommended by the Advisory Committee on the Investigation of Long-Span Suspension Bridges.[67] As a part of the supplemental tests the original model was modified to represent a bridge with unloaded backstays.

The author's equations and those developed by the writers, in common with most engineering formulas, are not precise. The engineer starts with certain assumptions, makes a series of approximations of varying degree, and attempts to develop equations that will predict with fair precision the performance of the designed structure. The test of the equations is their reliability for such prediction. For this purpose, complex formulas are not justified except for limited comparative study if simpler formulas give precision equivalent to that of the observations. In the case of suspension bridge vibration due to wind, perhaps the best data for comparison with the results of equations are the observations on the Tacoma Narrows Bridge and its fifty-scale full model in the wind tunnel. The model duplicated with rather startling accuracy the behavior of the bridge in all movements covered by field observations, and provided, in addition, the opportunity to observe movements which had not been adequately observed in the field. The supplemental tests provided observations, not elsewhere available, for checking certain of the equations. It is proposed to discuss the basic assumptions made and the resulting equations, to compare the author's equations with some of the writers' equations, and to test both against observations made on the model.

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[67] *Civil Engineering*, October, 1943, p. 503.

Assumptions.—The basic assumptions by Messrs. Steinman and Rannie[3] agree in that:

(1) The suspenders are inextensible under inertial loads;
(2) The cable stresses due to inertial load are small compared to dead-load stresses;
(3) The tower stiffness against horizontal forces applied at the top is negligible;
(4) The moment of inertia of the truss, hinged at the tower, is uniform throughout the length of the bridge; and
(5) For bridges with suspended side spans, the cable is inextensible under inertial loads.

They differ in that:

(*a*) Mr. Steinman assumes that the deflection curves due to oscillation are sine curves, whereas Mr. Rannie develops the equations for the deflection curves;
(*b*) Mr. Steinman takes truss stiffness into account in all cases, whereas Mr. Rannie neglects truss stiffness in the case of the Tacoma Bridge, although he does develop more general equations which do include the effect of truss stiffness; and
(*c*) Furthermore, Mr. Steinman develops an equation for bridges with unloaded backstays in which he takes into account the extensibility of the cable under inertial loads, a bridge condition that Mr. Rannie did not consider.

In their studies, the writers also found it necessary to take cable extensibility into account in developing equations for the condition with unloaded backstays. Furthermore, the fact was recognized that the backstays carry their own weight and, therefore, are not straight. They are actually loaded equilibrium polygons subject to elastic distortion and, accordingly, were treated as loaded side spans but with unit loading different from that on the main span and correspondingly modified sag (the cable weight on the Tacoma Bridge was about 25% of the total weight).

Equations.—The author simplifies the derivation of many expressions by the use of approximations which, for the most part, appear to be permissible. However there are apparently certain errors in Eqs. 11*c* and 14. Consider first Eq. 11*c*. If, as is suggested by the author, the integration of Eqs. 2 and 4 is extended over the three spans the following equation is obtained:

$$K = \frac{\left[\frac{\pi^2 n^2}{l^2} + \frac{2\,(a_1)^2\,l_1}{a^2\,l}\frac{\pi^2\,(n_1)^2}{(l_1)^2}\right] H + \left[\frac{\pi^4 n^4}{l^4} + \frac{2\,(a_1)^2\,l_1}{a^2\,l}\frac{\pi^4\,(n_1)^4}{(l_1)^4}\right] E\,I}{1 + \frac{2\,(a_1)^2\,l_1}{a^2\,l}} \quad \text{..(95)}$$

If the term, $\frac{2\,(a_1)^2\,l_1}{a^2\,l}$ were equal to q of Eq. 11*c*, Eq. 95 would be identical with Eq. 11*b*, but $\frac{2\,(a_1)^2\,l_1}{a^2\,l}$ is approximately equal to $\frac{(n_1)^2}{2\,n^2}\frac{l}{l_1}\,(\sec^4 \gamma_1)^2$ and not $\frac{(n_1)^2}{2\,n^2}\frac{l}{l_1}\sec^4 \gamma_1$, which the author gives as the value for q. It seems to the writers that $(\sec^4 \gamma_1)^2$ is more nearly correct for the secant term than is $\sec^4 \gamma_1$ for the following reasons:

After developing Eq. 9 (in the development of which the author assumes no tower-top movement), Mr. Steinman states: "The resulting value of Δl_c or Δl will be the algebraic sum of the component values." The writers understand this to mean that, assuming that the tower tops do move, Δl caused by the multiple segment wave represented by the equation,

$$\eta = a \sin \pi n \frac{x}{l} (\sin 2 \pi N t) \qquad (96a)$$

will be the same as Δl caused by a single segment wave represented by the equation,

$$\eta = \frac{a}{n} \sin \pi \frac{x}{l} (\sin 2 \pi N t) \qquad (96b)$$

An approximate formula[68] for the change in length of the span, Δl, when there is a small change in the sag, Δf, is given by the formula,

$$\Delta l = \left\{ \frac{\frac{16 f}{l} \left[5 - 24 \left(\frac{f}{l} \right)^2 \right]}{15 - 40 \left(\frac{f}{l} \right)^2 + 288 \left(\frac{f}{l} \right)^4} \right\} \Delta f \qquad (97)$$

An approximate formula[69] for the length of the side span cable commonly used is

$$L_1 = l_1 \left(\sec \gamma_1 + \frac{8}{3} \frac{(f_1)^2}{(l_1)^2 \sec^3 \gamma_1} \right) \qquad (98)$$

where l_1 is length of side span and γ_1 is the angle the chord of the side span makes with the horizontal.

If this expression is differentiated first in respect to f_1 and then in respect to l_1 and the resulting expressions equaled, the following expression is derived

$$\Delta l_1 = \frac{\frac{16}{3} \frac{f_1}{l_1}}{\sec^4 \gamma_1 - \frac{8}{3} \frac{(f_1)^2}{(l_1)^2}} \Delta f_1 \qquad (99)$$

Neglecting all terms but the first in both the numerator and denominator and substituting $\frac{a}{n}$ for Δf and $\frac{a_1}{n_1}$ for Δf_1 the following approximate formulas are obtained:

$$\Delta l \approx \frac{16}{3} \frac{f}{l} \frac{a}{n} \qquad (100a)$$

and

$$\Delta l_1 \approx \frac{16}{3} \frac{f_1}{l_1} \frac{a_1}{n_1} \frac{1}{\sec^4 \gamma_1} \qquad (100b)$$

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[68] "The Theory and Practice of Modern Framed Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, John Wiley & Sons, Inc., New York, N. Y., 1917, p. 199.

[69] "Suspension Bridges," by D. B. Steinman in "Movable and Long-Span Steel Bridges," by George A. Hool and W. S. Kinne, McGraw-Hill Book Co., Inc., New York, N. Y., 1923, p. 292.

(For the Tacoma Bridge this approximation introduces an error of less than 2% in the value of Δl and less than 1% in Δl_1.) In Eqs. 100, Δl must be equal to 2 Δl_1; therefore,

$$a_1 = \frac{1}{2}\frac{f}{f_1}\frac{l_1}{l}\frac{n_1}{n}\, a \sec^4 \gamma_1 \ldots\ldots\ldots\ldots\ldots\ldots (101)$$

Substituting this value of a_1 in the expression, $\frac{2\,(a_1)^2\, l_1}{a^2\, l}$ of Eq. 95 and remembering that $\frac{f_1}{(l_1)^2} = \frac{f}{l^2}$ gives:

$$q = \frac{(n_1)^2\, l}{2\, n^2\, l_1}(\sec^4 \gamma_1)^2 \ldots\ldots\ldots\ldots\ldots\ldots (102)$$

A somewhat more precise expression for L_1 is[70]

$$L_1 = l_1 \left[\, 1 + \frac{8}{3}\frac{(f_1)^2}{(l_1)^2} + \frac{1}{2}\tan^2 \gamma_1 \right] \ldots\ldots\ldots\ldots\ldots (103)$$

By a similar derivation this gives

$$\Delta l_1 = \frac{\frac{16}{3}\frac{f_1}{l_1}}{\frac{1}{2}(1 + \sec^2 \gamma_1)}\, \Delta f_1 \ldots\ldots\ldots\ldots\ldots\ldots (104)$$

and

$$q = \frac{(n_1)^2\, l}{8\, n^2\, l_1}(1 + \sec^2 \gamma_1)^2 \ldots\ldots\ldots\ldots\ldots\ldots (105)$$

These changes make very little difference in the value of K. In fact, for practical purposes, the secant term could be neglected entirely as its effect on the value of K is small.

If this correction is accepted, not only should Eq. 11c be modified as described, but Eq. 14 which is derived from the same assumptions should also be modified.

The fundamental equilibrium equation, as it pertains to suspension bridges, may be stated as:

$$w + \Delta w = -\,(H + \Delta H)\left(\frac{d^2y}{dx^2} + \frac{d^2\eta}{dx^2}\right) + E\, I\left(\frac{d^4\eta}{dx^4}\right) \ldots\ldots (106)$$

The quantity Δw, being the inertial load per foot per cable, may be written:

$$\Delta w = \frac{w\, a}{g} = -\,\frac{w}{g}\,\omega^2\, \eta_x \sin \omega\, t \ldots\ldots\ldots\ldots\ldots\ldots (107)$$

Eq. 106 recognizes that the curve of the deflected cable is proportional to the moment diagram of the loading carried by the cable, and therefore the load on the cable is proportional to the second derivative of the cable curve. It recognizes also that the load carried by the deflected girder is proportional to the fourth derivative of the deflection curve of the girder. Eq. 107 recognizes that the inertial load in vibration is proportional to the second derivative of the deflection with respect to time. The quantity ΔH, being the result of the inertial loads, also contains the factor, sin ωt, so that the fundamental equation can be written in the form:[71]

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[70] "The Theory and Practice of Modern Planned Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, John Wiley & Sons, Inc., New York, N. Y., 1917, p. 195.

$$-\frac{E\,I}{H}\frac{d^4\eta}{dx^4}+\frac{d^2\eta}{dx^2}+\frac{w}{H\,g}\,\omega^2\,\eta=\frac{\Delta H\,w}{H^2}\ldots\ldots\ldots\ldots\ldots(108)$$

If, instead of the unknown frequency, ω, the dimensionless quantity

$$\mu=\sqrt{\frac{w}{H\,g}}\,\frac{l\,\omega}{2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(109)$$

is used, the following equation results:

$$-\frac{E\,I}{H}\frac{d^4\eta}{dx^4}+\frac{d^2\eta}{dx^2}+\left(\frac{2\,\mu}{l}\right)^2\eta=\frac{8\,f}{l^2}\frac{\Delta H}{H}\ldots\ldots\ldots\ldots(110)$$

Letting $\frac{H}{E\,I}=k^2\left(\frac{2}{l}\right)^2$:

$$\frac{d^4\eta}{dx^4}-\left(\frac{2\,k}{l}\right)^2\frac{d^2\eta}{dx^2}-\left(\frac{4\,\mu\,k}{l^2}\right)^2\eta=-\frac{8\,f}{l^2}\frac{\Delta H}{H}\left(\frac{2\,k}{l}\right)^2\ldots\ldots(111)$$

Eq. 111 is a standard differential equation, the special solution of which is[72]

$$\eta=C_1e^{2k_2\frac{x}{l}}+C_2e^{-2k_2\frac{x}{l}}+C_3e^{2i\mu_2\frac{x}{l}}+C_4e^{-2i\mu_2\frac{x}{l}}+\frac{2f\Delta H}{H\mu_2}\ldots\ldots(112)$$

in which $i=\sqrt{-1}$

$$(k_2)^2=k^2\left[\sqrt{\left(\frac{\mu}{k}\right)^2+\frac{1}{4}}+\frac{1}{2}\right]\ldots\ldots\ldots\ldots\ldots(113a)$$

and

$$(\mu_2)^2=k^2\left[\sqrt{\left(\frac{\mu}{k}\right)^2+\frac{1}{4}}-\frac{1}{2}\right]\ldots\ldots\ldots\ldots\ldots(113b)$$

For the symmetric modes, η has the same value for $+x$ as it has for the corresponding $-x$. When this fact is used the first two quantities on the right side of Eq. 112 combine as a hyperbolic function and a single constant. The terms involving i may be written as trigonometric functions and there results the equation

$$\eta=Y\cosh\frac{2\,k_2\,x}{l}+A\cos\frac{2\,\mu_2\,x}{l}+\frac{2\,f\,\Delta H}{H\,\mu^2}\ldots\ldots\ldots\ldots(114)$$

The general relation between deflection and moment in the stiffening truss is

$$\frac{d^2\eta}{dx^2}=-\frac{M}{E\,I}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(115)$$

Making use of this relationship, the following equation is obtained from Eq. 112:

$$\eta=\frac{2\,f\,\Delta H}{H\,\mu^2}\left[1-\frac{(k_2)^2}{(\mu_2)^2+(k_2)^2}\,\frac{\cos\frac{2\,\mu_2\,x}{l}}{\cos\mu_2}-\frac{(\mu_2)^2}{(\mu_2)^2+(k_2)^2}\,\frac{\cosh\frac{2\,k_2\,x}{l}}{\cosh k_2}\right].(116a)$$

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[71] "The Failure of the Tacoma Narrows Bridge," a report to the Hon. John M. Carmody, Administrator of the Federal Works Agency, Washington, D. C., March 28, 1941, by a board of engineers consisting of O. H. Ammann, Theodor von Kármán, and Glenn B. Woodruff, Appendix VI, p. 15, Eq. 10.

[72] "Elements of the Differential and Integral Calculus," by W. A. Granville, P. F. Smith and W. R. Longley, Revised Ed., Ginn & Co., 1934, p. 403.

Eq. 116*a* holds between the limits of $x = \pm \frac{l}{2}$. For the side spans in which x varies between $\pm \left(\frac{l}{2} + l_1\right)$, the following equation may be derived:

$$\eta = \frac{2 f \Delta H}{H \mu^2} \left[1 - \frac{(k_2)^2}{(\mu_2)^2 + (k_2)^2} \frac{\cos \mu_2 \left(1 + \alpha - \frac{2x}{l}\right)}{\cos \alpha \mu_2} - \frac{(\mu_2)^2}{(\mu_2)^2 + (k_2)^2} \frac{\cosh k_2 \left(1 + \alpha - \frac{2x}{l}\right)}{\cosh \alpha k_2} \right] \ldots\ldots\ldots\ldots (116b)$$

in which $\alpha = \frac{l_1}{l}$.

To this point the foregoing development follows that of Mr. Rannie quite closely, differing only in that a more exact term for truss stiffness is used by the writers. The writers' development differs further from that by Mr. Rannie in that cable extensibility is taken fully into account.

The extension of the entire cable from anchorage to anchorage, $L - L_0$, can be expressed as $\frac{\Delta H}{A\ E} \int_{-0.5\ l - l_1}^{0.5\ l + l_1} \frac{ds^2}{dx}$. The term, $\int \frac{ds^2}{dx}$, is often referred to simply as L_T, and was evaluated by Mr. Steinman in a previous paper.[73] The foregoing relationship may be expressed as

$$L - L_0 = \frac{\Delta H}{A\ E} L_T \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (117a)$$

With a fair degree of approximation the value of $L - L_0$ can be obtained from the geometry of the cable and stated as follows:

$$L - L_0 \approx \frac{8 f}{l^2} \int_{-0.5\ l - l_1}^{0.5\ l + l_1} \eta\, dx \ldots\ldots\ldots\ldots\ldots\ldots (117b)$$

Substituting in Eq. 117*b* the expressions for η in Eqs. 116, integrating over the entire length of the bridge, and then equating Eqs. 117 results in the expression:

$$\tan \mu_2 + 2 \tan \alpha \mu_2 = (1 + 2 \alpha) \mu_2 - \left(\frac{\mu_2}{k_2}\right)^3 (\tanh k_2 + 2 \tanh \alpha k_2) + (1 + 2 \alpha) \frac{(\mu_2)^3}{(k_2)^2} - C (\mu_2)^3 \left[\frac{(\mu_2)^2 + (k_2)^2}{k^2}\right] \ldots\ldots\ldots (118)$$

in which $C = \frac{L_T H_w l}{16 f^2 A E}$ and μ, μ_2, k, and k_2 have the values indicated in connection with Eqs. 112 and 113.

In the development of Eq. 117*b* the approximation is made that $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1.5} = 1$. If this approximation is not used the more rigorous solution gives the same equations as the foregoing except that $L_T = \int \frac{ds^2}{dx}$ is replaced by $\int \frac{ds^3}{dx^2}$, sometimes referred to as L_S. This solution satisfies the requirement that the sum of the kinetic energy, elastic energy, and energy of position must be constant for a given frequency and amplitude. The difference between L_S and L_T, however, is not significant.

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[73] "A Generalized Deflection Theory for Suspension Bridges," by D. B. Steinman, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1145.

If the dead load of the side span varies from that of the main span in the ratio of $w_1/w = \beta$, Eq. 116*b* becomes

$$\eta = 2f \frac{\Delta H}{H \mu^2} \left[1 - \frac{(k_s)^2}{(\mu_s)^2 + (k_s)^2} \frac{\cos \mu_s \left(1 + \alpha - \frac{2x}{l}\right)}{\cos \alpha \mu_s} - \frac{(\mu_s)^2}{(\mu_s)^2 + (k_s)^2} \frac{\cosh k_s \left(1 + \alpha - \frac{2x}{l}\right)}{\cosh \alpha k_s} \right] \qquad (119)$$

in which

$$(k_s)^2 = k^2 \left[\sqrt{\beta \left(\frac{\mu}{k}\right)^2 + \frac{1}{4}} + \frac{1}{2} \right] \qquad (120a)$$

and

$$(\mu_s)^2 = k^2 \left[\sqrt{\beta \left(\frac{\mu}{k}\right)^2 + \frac{1}{4}} - \frac{1}{2} \right] \qquad (120b)$$

Similarly, Eq· 118 becomes

$$\tan \mu_2 + 2\phi\beta \left(\frac{\mu_2}{\mu_s}\right)^3 \tan \alpha \mu_s = (1 + 2\alpha)\mu_2 - \left(\frac{\mu_2}{k_2}\right)^3 \tanh k_2 - \frac{2\phi}{\sqrt{\beta}} \left(\frac{\mu_s}{k_2}\right)^3 \tanh \alpha \sqrt{\beta} \frac{\mu_2}{\mu_s} k_2 + (1 + 2\alpha) \frac{(\mu_2)^3}{(k_2)^2} - C (\mu_2)^3 \left[\frac{(\mu_2)^2 + (k_2)^2}{k^2} \right] \qquad (121)$$

in which $\phi = \dfrac{2 (\mu_2)^2 + k^2}{2 (\mu_s)^2 + k^2}$.

In Eq. 118, μ_2 must be determined by trial. This may be facilitated by plotting each side of the equation against μ_2 and noting the intersections of the curves that give the successive values of μ_2 which satisfy the equation.

Fig. 19 shows the general forms of these curves. Curves (1) show the left side of the equation, curve (2) shows the right side, and curve (3) shows the right side if the last term (which includes the effect of cable extension) is omitted. It will be noted that cable extensibility may have a quite marked effect on the frequency of a lower mode but has relatively little effect on that of the higher modes. With or without this factor the intersections for higher values of μ_2 approach odd multiples of $\frac{\pi}{2}$ or of $\frac{\pi}{2\alpha}$. It should be noted, furthermore, that C is the only term in Eq. 118 that is affected by the dead load. The expression for C contains H_w and, therefore, is directly proportional to w.

Comparison with Observations.—Tests made on the model of the original Tacoma Bridge indicated that the frequency of oscillation of both the asymmetric and symmetric modes can be predicted with about the same degree of precision by either Mr. Steinman's, Mr. Rannie's, or the writers' equations. The author's equations have the advantage of simplicity, at least for the symmetric modes (n is odd).

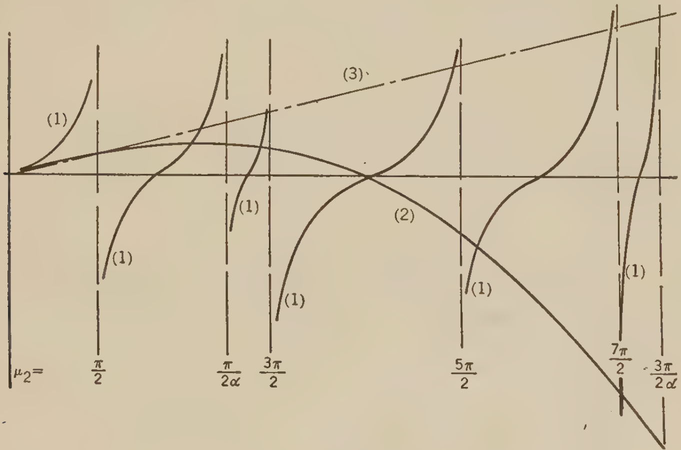

FIG. 19.—SOLUTION OF EQ. 118

When in the supplemental tests unloaded backstays were substituted for the loaded side spans, it was found that, for the asymmetric modes (but not for the symmetric ones), both the author's and the writers' equations had equal validity.

The symmetric modes with unloaded backstays (curves IV, V, and VI, Fig. 20), however, are greatly distorted as compared with the waves on the original bridge (curves I, II, and III, Fig. 20). No fundamental mode could be produced on the model, either by wind action or manually, but two distinct two-node (three-wave) movements developed readily under wind action. One had a very long middle wave and the other a very short one. A four-node (five-wave) movement of nearly equal wave segments also developed readily. Eq. 15 gives values for frequency which usually miss the observed values by plus or minus 10% or more. Although the results of the writers' equations do not check observed values as closely as in the case of the original bridge, they are much more consistent with observations than are the results of Mr. Steinman's equations. It appears probable that the weight and vertical movement of the backstays, neglected in the derivation of the author's equation, should be taken into account.

Table 11 shows prototype conditions with unloaded backstays tested on the model, with a comparison of observed and computed frequencies. The span lengths and main-span sag are the same as for the original Tacoma Bridge. The weight per foot of the main span is varied. The weight of the cable is constant throughout. Blanks in Table 11 indicate modes for which the equation or observation involved gives no value—that is, the equation does not indicate the mode or (in the "observed" column) the mode did not occur.

The wave forms (as distinct from the frequencies) of the asymmetric modes obtained from the different formulas are substantially similar, both for the original bridge and for the bridge with unloaded backstays.

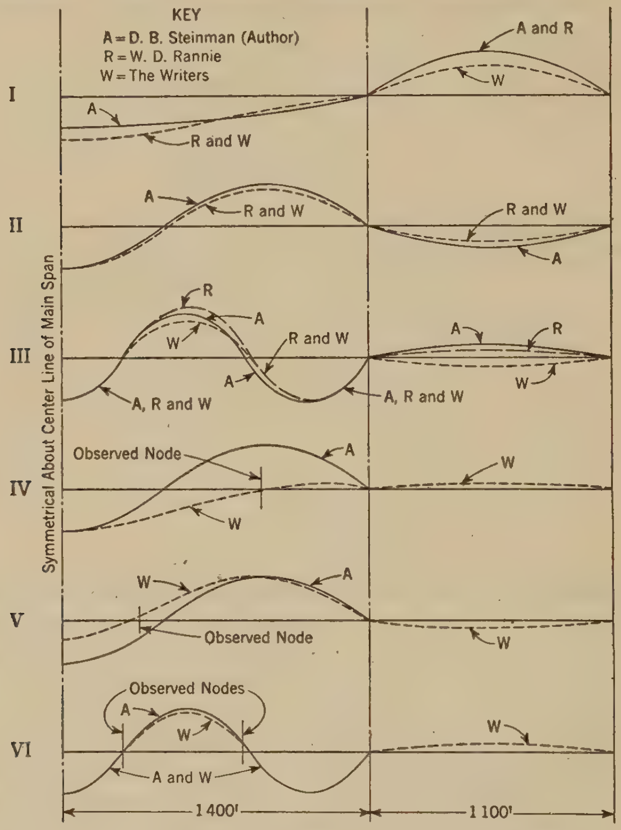
KEY
A = D. B. Steinman (Author)
R = W. D. Rannie
W = The Writers
I
II
III
IV
V
VI
A
R and W
A and R
W
A
R and W
R and W
A
R
A
W
R and W
A
A, R and W
A, R and W
A
R
W
Observed Node
A
W
W
W
A
Observed Node
W
Observed Nodes
A
W
A and W
W
Symmetrical About Center Line of Main Span
1 400′
1 100′

FIG. 20.—SYMMETRIC WAVE FORMS, TACOMA NARROWS BRIDGE

TABLE 11.—FREQUENCIES (IN CYCLES PER MINUTE) WITH UNLOADED BACKSTAYS

Nodes	n	(a) 2,280 LB PER FT ONE TRUSS			(b) 2,850 LB PER FT ONE TRUSS			(c) 3,770 LB PER FT ONE TRUSS		
		Eq. 15	Eq. 121	Observed	Eq. 15	Eq. 121	Observed	Eq. 15	Eq. 121	Observed
0	1	13.5	....	....	12.3	....	....	10.8	....	....
2	3	12.7	10.2	10.9	12.8	10.0	10.5	12.4	9.5	10.0
2	3	....	13.4	14.3	....	13.2	13.8	....	12.8	13.2
4	5	20.8	20.4	21.9	20.9	20.7	20.5	20.4	19.8	21.0

However, for the symmetric modes (n is an odd number), the various formulas give wave forms that differ significantly, as can be seen by referring to the typical charts in Fig. 20. Curves I, II, and III are typical of the original bridge; and curves IV, V, and VI of the original bridge with unloaded backstays with the dead load per foot on the main span increased 32%.

It will be seen that all the equations are reasonably precise for the conditions in which the observed wave segments are about equal. The differences in amplitude, shown best in condition I, Fig. 20, arise principally from two sources:

(1) The curves in Fig. 20, calculated from Mr. Steinman's equations for the main span and side span, are assumed to be sine waves correlated by the required ratio of their amplitudes, $\frac{a}{a_1} = \frac{2\, l_1}{l\, n_1} \frac{n}{\sec^4 \gamma_1}$, whereas those by Mr. Rannie are not true sine curves but are represented by a modified trigonometric equation similar to Eq. 116*a* but with the hyperbolic term omitted. Because of Mr. Rannie's basic assumptions the negative and positive areas under his curves must balance. This difference is most evident in the form of the main-span wave (see Fig. 20, curve I).

(2) The curves in Fig. 20 calculated from the writer's equations differ from both of the others because the extension of the cable recognized in the writers' formulas permits a given downward deflection of the main span to take place without being fully balanced by the upward deflection of the side span. The effect of this factor is shown also in condition III, Fig. 20, in which the writers' side-span deflection is actually reversed from the direction required to balance the waves. A more striking example of this effect is found on the model of the proposed new design for the Tacoma Bridge. No vertical oscillations have been produced by wind action on this model but several modes have been produced manually. Among these is one of the form shown in Fig. 21. This is reproduced faithfully by the writers' Eqs. 116. Of course, this form of wave can occur only if the length of the cable changes during oscillation.

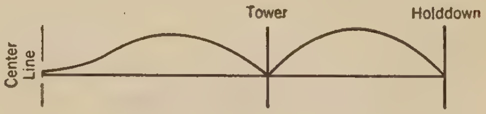

FIG. 21

In cases where the observed lengths are quite unequal, as in conditions IV and V (unloaded backstays), Fig. 20, the writers' equations give very satisfactory results. It is obvious that Eq. 15, based upon wave segments of equal length, cannot represent these conditions adequately.

Eqs. 21*a* and 21*b* properly relate torsional rigidity to vertical rigidity when the torsional stiffness of the suspended structure is neglected. Investigations to date seem to indicate that the torsional stiffness of the suspended structure as usually designed may be neglected without appreciable error. However, the writers' studies indicate that the effect of torsional stiffness does become significant if top and bottom lateral systems are provided.

The author presents, in simple form, equations which adequately predict a large proportion of the movements of a suspension bridge. However, the writers believe that the equations they have derived following Mr. Rannie's basic method have some advantages over those of the paper because, as has been shown by tests on the model, not only can these equations be used to predict performance with results equal in validity to those obtained from the author's equations in those cases in which the author's equations apply, but can also be used to make reliable predictions in those cases in which the author's equations would lead to erroneous conclusions. However, it seems reasonable to anticipate that the author will be able to modify his equations to give them more general and rigorous application.

Louis Balog,[74] Esq.—The basic formula of this paper is Eq. 6, representing the stiffness of suspension bridges by the summation of expressions that characterize the stiffness of the component parts—cable and girder—of the center span. The cable is assumed to be a string under tension, rigidly held at its ends; the girder, to be a simple beam of uniform section. The accuracy of this approximation when determining the frequencies of a practically unstiffened bridge is well indicated by the agreement between the calculated and observed values for the Tacoma Narrows Bridge. However, the equations given for the condition of three spans coupled do not define the values of n_1 corresponding to values of n in Table 1. For bridges with stiffening girders of small rigidity, the exceedingly simple Eq. 17, which was presented by Joseph Melan in 1906,[75] gives useful results. The frequencies of the Tacoma Narrows Bridge differed by only 6% up to the observed $n = 9$, from those obtained by Eq. 17.

The K-values of Eq. 6 are characteristic of deformation when the algebraic sum of the areas of the two segments below and above the initial condition of the cable equals zero, or when the deformation occurs without change in H, without stretch of cable, without movements of the towers, and without involving the side spans. For these reasons, Eqs. 16 and 22 cannot reliably be

TABLE 12.—Application of Eqs. 16 and 22 to Various Bridges

Bridge	(a) Eq. 16 Compared with Deflection Theory					(b) Eq. 22 Compared with Deflection Theory[a]			
	K (Eq. 6)	p (lb per ft)	$\eta_{0.25}$ by: Eq. 16 (ft)	$\eta_{0.25}$ by: Deflection (ft)	Percentage increase	K_t (Eq. 21a)	θ by: Eq. 22 (radians)	θ by: Deflection (radians)	Percentage increase
Golden Gate	116 126	100 2,000	0.86 15.87	0.68 12.41	+27 +28	470,000	0.0213	0.0168	+27
George Washington	244 259	100 2,000	0.41 7.42	0.31 5.88	+33 +31	1,370,000	0.0073	0.0055	+33
Bronx-Whitestone	141 156	100 1,500	0.71 9.62	0.61 8.41	+17 +15	388,000	0.0258	0.0223	+16
Transbay	324 350	100 3,000	0.31 8.56	0.40 10.70	−22 −20	708,000	0.0141	0.0184	−23

[a] In Eq. 22, $T = 10{,}000$ lb-ft per ft.

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[74] Cons. Engr., New York, N. Y.

[75] "Handbuch der Ingenieurwissenschaften," by Joseph Melan, Vol. 5, 3d Ed., Leipzig, 1906, pp. 208–214.

used for estimating live-load deflections and rotations which, besides being influenced by the enumerated changes, are also affected by the ratio of live load to dead load. The close agreement in Example 2 and Example 4 between the results obtained from these equations and those obtained from the deflection theory is accidental as shown in Table 12.

The application of Eqs. 16 and 22 to a number of bridges in addition to those in Table 12 resulted in deviations, up to a maximum of 48%, from the values obtained by the deflection theory. These deviations also prove that the author's physical interpretation of the $\frac{2K}{b}$-values (as the loading from tower to midspan in pounds per square foot which produces a 1-ft quarter-point deflection of stiffened suspension bridges) is incorrect. The percentages given are indicative of the accuracy of this interpretation.

The basic cable term of Eq. 6, $\frac{l^2}{H} = \frac{8f}{w}$, however, can be used for the calculation of quarter-point deflections of the unstiffened cable, due to the loading mentioned, if it is considered to be inextensible and fixed at the towers. For this condition, $\eta_{0.25} = f\left(4 + \frac{8w}{p}\right)^{-1}$. For $p = 100$, considering the prevailing dead loads, $\frac{w}{p}$ is between 12 and 150 and, even in the extreme case, the average value, $\eta_{0.25} = 1.525\,\frac{l^2}{H}$, is a close approximation ($\pm$ 2%) of the correct values.

To connect the stiffness and aerodynamic behavior of suspension bridges, the author evaluates Eq. 6 for twenty bridges, divides these values by the width, and lists the bridges in order of the magnitude of these values (Table 2). From experience the author states that stability requires these K per foot values to be not less than 15 for bridges stiffened by plate girders and not less than 6 for truss-stiffened bridges. In this manner, an empirical connection is established between the $\frac{2K}{b}$-values and the aerodynamic oscillations of suspension bridges.

The use of the basic terms of Eq. 6 to express the stiffness of a suspension bridge cable and its stiffening girder has been practicable in suspension bridge design. The concept of extending these terms by the corresponding first harmonic coefficients and dividing their sum by the width of the bridge is new; as is the significance ascribed to the resulting values by the author.

The practical meaning of the author's requirements for the minimum values of $\frac{2K}{b}$, for the $\frac{l}{d}$-ratio of 100, and for the minimum values for K and R can best be discussed by the application of these rules to several suspension bridges representing the span range of structures actually built.

Table 13 demonstrates that, for spans longer than 1,000 ft, the requirements result in plate girders of impractical dimensions. For the Golden Gate Bridge, trusses would be required which would nearly double the cost of the structure. This bridge has been exposed for seven years to the most unfavorable wind conditions experienced by any of the large suspension bridges and has exhibited dynamic stiffness sufficient for its use and safety. Observed oscillations for winds of from 60 miles per hr to 78 miles per hr were quickly damped out when the wind velocity decreased. Therefore, the Golden Gate Bridge may be assumed to represent the limit of satisfactory dynamic behavior, contradicting the general validity of the author's dynamic stability criteria.

TABLE 13.—APPLICATION OF THE AUTHOR'S MINIMUM REQUIREMENTS FOR $\frac{2\,K}{b}$-VALUES AND $\frac{l}{d}$-VALUES TO VARIOUS BRIDGES

Bridge	Type	Center span (ft)	(a) GIRDER AS BUILT			(b) MINIMUM REQUIREMENTS				
			Depth (ft)	I (in.^{2}-ft^2)	K (Eq. 6)	Depth (ft)	I (in.^{2}-ft^2)	K (Eq. 6)	$\frac{d}{b}$	Increase in cable area (%)
Golden Gate.....	Truss	4,200	25.0	44,000	116	42.0	520,000	270	0.47	75
Tacoma Narrows.	Plate girder	2.800	8.0	1,000	62	28.0	165,000	300	0.72	157
	Truss	2,800	....	....	..		50,000	120		33
Bronx-Whitestone	Plate girder	2,300	11.0	2,615	140	23.0	156.000	550	0.32	117
	Truss	2,300	....	....	..		38,000	222		18
Deer Isle.........	Plate girder	1,080	6.5	424	69	10.8	6,000	300	0.46	75
	Truss	1,080	....	....	..		1,860	120		4
Thousand Islands.	Plate girder	800	6.0	252	127	8.0	1,600	300	0.26	21
	Truss	800	....	....	..		330	145		0

The requirement that plate-girder stiffened bridges be 150% stiffer than truss-stiffened bridges eliminates their use. The requirement that $\frac{l}{d} = 100$ has the same effect because it is destructive to advantageous $\frac{d}{b}$-ratios. Nor could plate girders be used for spans longer than 1,000 ft if $\frac{l}{d} = 100$ and an H-section had to be maintained. Nevertheless, the author concentrates on the investigation of this section which, in view of experience and his recommendations, has lost its practical importance.

The author's analysis of the aerodynamic excitation of vertical and torsional oscillations is based on the characteristic drag, lift, and torque curves obtained on section models of the Tacoma Narrows Bridge. By idealizing the laboratory results, the author obtains a family of lift graphs corrected for drag (Fig. 5). The introduction of the K-values into the frequency, as well as into the energy, relations of vertical and torsional oscillations has the effect of establishing the connection between s' (the slope of the corrected lift graph) or s_t (the slope of the torque graph)—both for $\alpha = 0$—K, V, β, and the aerodynamic amplification ratio δ. The arbitrary function, β, of the phase difference across the section is also introduced for vertical oscillations to explain the behavior of the Tacoma Narrows Bridge in the low-velocity range. The values of δ_s, derived from the moment of inertia of the girders equated with δ, result in Eqs. 44 to 46 for the critical velocities of vertical oscillations and in Eqs. 62 to 64 for the critical velocities of torsional oscillations.

The author proves the validity of these equations by examples which give values corresponding to the observations on the Tacoma Narrows Bridge. Applying the equations to other bridges, however, the results are not convincing. For the Thousand Islands (across the St. Lawrence River), Bronx-Whitestone, and Golden Gate bridges $\left(\frac{0.5\,d}{b} = 0.139\right)$, Eq. 45*b* with $n = 9$ yields 48, 85, and 70 miles per hr, and Eq. 46*b* with $n = 2$ yields 5, 1.7, and 2 miles per hr, respectively. The Tacoma Narrows Bridge subjected to a wind velocity of 42 miles per hr changed from a vertical oscillation with $n = 9$ to a violent torsional oscillation with $n = 2$. The maximum critical velocities determined by the author's equations were experienced by the enumerated bridges without any such effects. For this reason, the practical meaning of Eqs. 45 and 46 is not apparent.

Eqs. 62, 63, and 64, which show increasing safety from torsional oscillations, with decreasing width, also do not give results consistent with the behavior of actual structures. For more than five years the Bronx-Whitestone Bridge was exposed to wind velocities as great as 95 miles per hr, but at no wind velocity did torsional oscillations occur. Using Eq. 62*a*, the critical velocity of torsional oscillations of this bridge is 18.2 miles per hr; and, using Eq. 64*a*, 18.5 miles per hr. Assuming that everything is unchanged except that the bridge is narrowed to half of its width, then, for $\frac{b^2}{w} = 0.25$, Eq. 62*a* results in a critical velocity of 20.8 miles per hr, and Eq. 64*a* yields a critical velocity of 21.6 miles per hr. Such a drastic variation in the $\frac{b^2}{w}$-value did not change the critical velocities appreciably.

The evaluation of observations on the Tacoma Narrows Bridge and on its section models failed to give generally valid relations even for the solid H-section. The author's analysis is more interesting from the point of view of procedure than of practical applicability to design. This fact is apparent from "Part 4.—Methods of Securing Rigidity and Aerodynamic Stability," in which the proposed measures do not follow from the application of the results of the presented analysis. The condition $\delta - \delta_s = 0$ is a basic relation in the critical velocity equations (see heading, "Part 2.—Aerodynamic Excitation of Vertical Oscillations: Structural Damping") where δ_s is essentially a function of R, the proportion of K due to I (the moment of inertia of the girder sections). In building up the resistance of the structure to aerodynamic action with stay ropes, the author assigns a new meaning to δ_s by explaining (see heading, "Part 4.—Methods of Securing Rigidity and Aerodynamic Stability: Use of Longitudinal Stays") the mechanical interference of stays with oscillations; and then writes (but without derivation) Eq. 65 for the amount that K is augmented by the stays. Eq. 65 is similar to the cable term, and not to the girder term, of Eq. 6, S being a function of a unit stress and the initial stay rope sag. Hence, the addition of this quantity to the girder term in the equation for K, to show a desired degree of increase in R is not warranted by the author's explanations or by the fact that pre-stressed stay ropes stiffen a suspension bridge to a certain extent. With the recommended minimum $K = 300$, the cost of the Tacoma Narrows Bridge, stiffened by plate girders, would be more than doubled (see Table 13); whereas by application of 16 sq in. of wire rope strand, in the form of longitudinal diagonal stays, the K-value of the bridge that failed would be raised to 332 and R would be increased to 0.816. In other words, a structural damping well above that of the Williamsburg Bridge would be secured at insignificant expense. This result, which is of great economic importance, demands a more thorough proof than the explanations in the paper. Experience with stays and reference to the Brooklyn Bridge, which was stiffened by six trusses, without stays, having values of $K = 190$ and $R = 0.22$, do not prove the author's claims.

The author's reference to the interference of transverse diagonal stays and top struts, with the out-of-phase movement of the cables, does not prove that Eq. 66*a* represents quantitatively the dynamical effectiveness of this arrangement (Fig. 9). The makeshift appearance of this detail (which has been used on ore-transport suspension bridges) does not befit the permanent and monumental character of a vehicular bridge structure.

The author's statement that the aerodynamic forces causing the vertical and torsional oscillations of the H-section can be eliminated or reduced to negligible proportions by leaving open spaces near the girders or by open-floor construction (see heading, "Part 4.—Methods of Securing Rigidity and Aerodynamic Stability: Use of Open-Floor Construction") is contradicted by experience and wind-tunnel tests. The Bronx-Whitestone Bridge was constructed in the following stages:[76]

(1) All steel, including reinforcing grid with floor plates attached, was erected and sidewalk areas were open, leaving two lateral vents with a total width of 13 ft $\sim \left(\frac{3\,b}{16}\right)$. At this stage, $\frac{b^2}{w} = 1.41$.

(2) Roadway concrete was poured and sidewalk areas were still open. At this stage, $\frac{b^2}{w} = 1.10$.

(3) Sidewalk concrete was poured and the vents in stage (1) were closed. At this stage, $\frac{b^2}{w} = 1.0$.

In none of these stages, nor during the following five years, did the bridge have any torsional oscillations. Its largest vertical oscillations occurred, however, during stage (1) when, according to Eq. 60, the aerodynamic lift was reduced 60%, and, in addition, the multiplier β was reduced 40%.

Homer M. Hadley, Assoc. M. Am. Soc. C. E., describing experiments with the 100-ft scale models of the Tacoma Narrows Bridge, states:[77]

> "The steelwork of the original deck was reproduced to scale—8 ft. marginal stiffening girders, floor beams, stringers and wind trusses—but the floor slab and sidewalks were omitted. This model lacked 20 percent of the stabilizing weight of the prototype $\left[\frac{b^2}{w} = 0.67\right]$ * * *. It was tested with and without midspan diagonal stays. Without stays * * * it quite closely reproduced the vertical undulations of the prototype and torsional movements as well."

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[76] *Construction Methods*, Vol. 21, June, 1939, p. 46.

[77] *Engineering News-Record*, November 4, 1943, p. 80.

Neither closing nor opening the horizontal member of the solid H-section altered the aerodynamic behavior of the section as Eqs. 43 and 60 indicate; and the dimensional ratio $\frac{b^2}{w}$ cannot be offered as a measure of sensitiveness to torsional aerodynamic instability.

The structural arrangements, recommended by the author as methods for securing aerodynamic stability, have been used on actual bridges and in proposed designs for many years. It is not proper to call the H-section "conventional." Many existing suspension bridges have shallow, deck plate-girder, suspended structures, and it is purely accidental that long-span bridges using this form have not been built. Such suspended structures were proposed for the 3,000-ft Wilmington Bridge, in Delaware, and also for the 4,700-ft Battery-Brooklyn Bridge in New York, N. Y., in 1939, because this design satisfied the requirements of those bridges better than the H-section. All stay systems (center, longitudinal, and transverse, Figs. 8 and 9) discussed by the author were suggested by others, or were tested in practice long ago, and then as now they have proved themselves mostly inefficient. In 1926, open sidewalk spaces were used in the design of the proposed bridge over the First Narrows, Vancouver, B. C., Canada, by Henry C. Tammen, M. Am. Soc. C. E. Also, the 1,050-ft Otto Beit Bridge, built in 1938, over the Zambezi River in Rhodesia, Africa, is provided with lateral vents of width $\frac{3\,b}{16} \sim 6$ ft. Outside sidewalks have been used on bridges too numerous to list. Static or structural considerations suggested all these arrangements. Their effectiveness for securing aerodynamic stability is not supported by experience or experimental evidence.

Simple relations derived from the rigidity characteristics of a basic structure actually constructed seem to gage the sufficiency of stiffening trusses for aerodynamic stability. To establish such empirical relations, the dimensional properties of the bridges will be expressed by the cable and girder stiffness parameters: $\pi_1 = \frac{l^2}{H} = \frac{8\,f}{w}$ and $\pi_2 = \frac{l^4}{E\,I}$, and, by their ratios, the dimensionless parameters: $\pi_3 = \frac{\pi_2}{\pi_1} = \frac{H\,l^2}{E\,I}$ and $\pi_4 = \frac{\pi_2}{\pi_{1B}}$. The parameter π_{1B} refers to the basic structure. The writer utilized these parameters by equating the π_3-values of the Golden Gate and Bronx-Whitestone bridges, thus deriving the value of I required to stiffen the latter structure in 1941. At a conference on November 3, 1941, Shortridge Hardesty, M. Am. Soc. C. E., who was asked to pass on the sufficiency of the design, outlined a dependable deflection method on which he based his recommendations. For two basic structures, Golden Gate and Lions' Gate, the two methods gave identical results. General utilization of these parameters will be effected by means of a representation in which the π_3 and π_4 values of the bridges are referred to parabolic curves obtained by assuming the span length to be variable in the following expression of the π_3-value of the basic structure:

$$\pi_{3x} = \pi_{4x} = \left(\frac{H}{E\,I}\right)_B (l_x)^2 \ldots\ldots\ldots\ldots\ldots\ldots (122)$$

Eq. 122 contains the aerodynamic and structural damping characteristics of the basic structure and indicates, for another structure of similar aerodynamic characteristics, the corresponding amount of structural damping that will assure stability, in the form of a requirement for the moment of inertia of the girders. For the reasons stated in connection with Table 13, the Golden Gate Bridge was selected as the basic structure. Introducing its $\frac{H}{E\,I}$-value into Eq. 122, a criterion of satisfactory dynamic behavior is obtained as follows:

$$\pi_{3x} \gtreqless \pi_{4x} \gtreqless 0.3872 \times 10^{-4}\,(l_x)^2 \ldots\ldots\ldots\ldots\ldots\ldots \quad (123)$$

and the required moment of inertia of the stiffening trusses becomes:

By considering π_{3x} and solving Eq. 123 for I_x—

$$I_x = \left(\frac{I}{H}\right)_B H_x = 0.00089\,H_x\ (\text{in.}^2\text{-ft}^2) \sim 0.0009\,H_x\ (\text{in.}^2\text{-ft}^2) \,..\,(124a)$$

by considering π_{4x} and solving Eq. 123 for I_x—

$$I_x = \left(\frac{I}{l^2}\right)_B (l_x)^2 = 0.00499\,(l_x)^2\ (\text{in.}^2\text{-ft}^2) \sim 0.005\,(l_x)^2\ (\text{in.}^2\text{-ft}^2) \,..\,(124b)$$

TABLE 14.—VALUES OF PARAMETERS π_1, π_2, π_3, AND π_4

No.	Bridge[a]	Center span l (ft)	$\pi_1\left(\frac{\text{ft}^2}{\text{lb}}\right)$	$\pi_2\left(\frac{\text{ft}^2}{\text{lb}}\right)$	π_3 (non-dimensional)	π_4 (non-dimensional)
1	Davenport	740	0.1349	2.817	20.9	15.8
2	Maumee River[a]	785	0.0598	2.454	41.1	13.7
3	Waldo-Hancock	800	0.1662	8.357	50.0	46.8
4	Thousand Islands[a]	820	0.2000	28.024	140.1	157.0
5	Lempa River[b]	949	0.2018	4.771	23.6	26.7
6	Grand Mère	919	0.3088	6.279	20.3	35.2
7	Isle d'Orleans	1.059	0.2703	5.163	19.1	28.9
8	Deer Isle[a]	1,080	0.3600	56.659	157.4	317.4
9	Mount Hope	1,200	0.1811	8.392	46.3	47.0
10	St. Johns	1.207	0.1207	7.773	64.4	43.5
11	Triborough	1,380	0.0576	4.169	72.3	23.4
12	Manhattan	1,470	0.0515	0.917	18.0	5.1
13	Mid-Hudson	1,500	0.1364	5.819	42.7	32.6
14	Lions' Gate	1,550	0.2609	21.174	81.2	118.6
15	Williamsburg	1,600	0.0829	0.924	11.1	5.2
16	Philadelphia	1,750	0.0615	1.916	31.1	10.7
17	Ambassador	1,850	0.1347	12.622	93.7	70.7
	Bronx-Whitestone:					
18	Present[a]	2,300	0.1455	181.795	1,249.5	1,018.4
19	Proposed[a]	2,300	0.1344	24.324	181.1	136.3
20	Transbay	2,310	0.0975	6.298	64.6	35.3
	Tacoma Narrows:					
21	1940[a]	2,800	0.3262	1,044.090	3,200.8	5,848.6
22	Proposed[c]	2,800	0.1600	19.500	121.9	109.2
23	George Washington	3.500	0.0872	....	....	....
24	Golden Gate	4,200	0.1786	121.932	683.0	683.0

[a] All bridges are truss type except those numbered 2, 4, 8, 18, and 21 which are plate-girder types. Item 19 combines both types. [b] Estimated from insufficient data. [c] Data supplied by Glenn B. Woodruff, M. Am. Soc. C. E.

Table 14 contains the numerical values of parameters π_1, π_2, π_3, and π_4 for twenty-four suspension bridges.

The π_1-values are plotted in Fig. 22. A circle is placed around the plots of the bridges that moved. Fig. 22 indicates that oscillations occurred in the full range of cable stiffness variation of bridges actually built. It shows that cable stiffness does not govern dynamic stability and demonstrates that weight is not only the most expensive but also is ineffective in controlling aerodynamic oscillations.

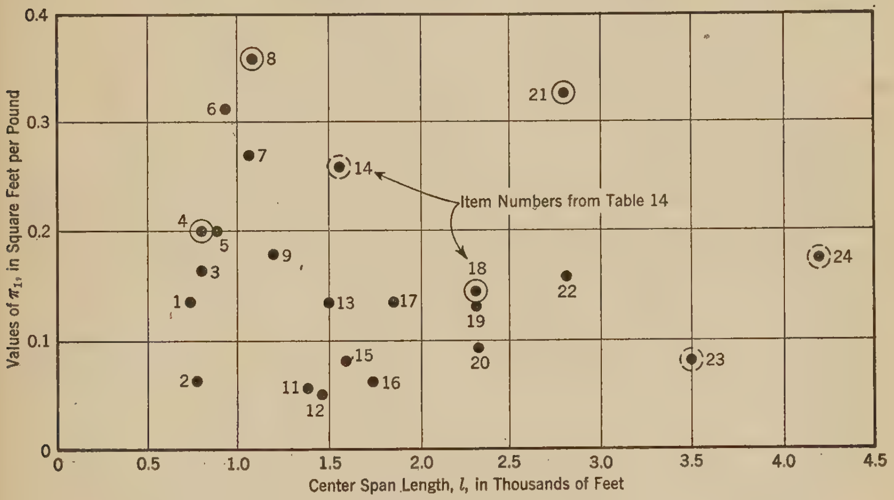

FIG. 22.—CABLE STIFFNESS OF SUSPENSION BRIDGES, AS MEASURED BY PARAMETER π_1 (Full Circle Denotes Objectionable, Dotted Circle Denotes Not Objectionable Movements)

Eq. 123 is represented by a curve passing through the ($\pi_3 = \pi_4$)-value of the Golden Gate Bridge in Fig. 23. The plotted points in Fig. 23(*a*) correspond to the first interpretation of the criterional parabola (Eq. 124*a*), representing the π_3-values. In Fig. 23(*b*) they correspond to the second interpretation (Eq. 124*b*), representing the π_4-values. The plots of those bridges with satisfactory resistance to aerodynamic action are near or below, and the plots of those with objectionable movement are considerably above, the curve, the divergence being nearly proportional to the character of the oscillations. When the distances of the plots are measured with the corresponding ordinates of the parabola, the deficiency or excess stiffness is indicated as compared with that required by Eqs. 124.

Table 15 lists the bridges in the sequence of their dynamic stiffness, as determined by the foregoing simple relations and the ratios of the required moment of inertia of the stiffening girders to the moments of inertia in the actual structures. Examination of Table 15 indicates some evident inconsistencies in both Eqs. 124*a* and 124*b*. However, the behavior of the actual structure is the integrated result of action and resistance since neither the acting forces nor the damping effects are shown quantitatively by the behavior. For this reason a consistent arrangement of the bridges in the order of their dynamic stiffness is unknown. Nevertheless, Fig. 23 disproves visibly "the high consistency and validity" of the stability criteria presented in the paper. For the Bronx-Whitestone Bridge, the author's minimum requirements (Table 13) result in trusses of 140% larger moment of inertia than is sufficient for stability; contrariwise for the Deer Isle and Thousand Islands bridges they result in

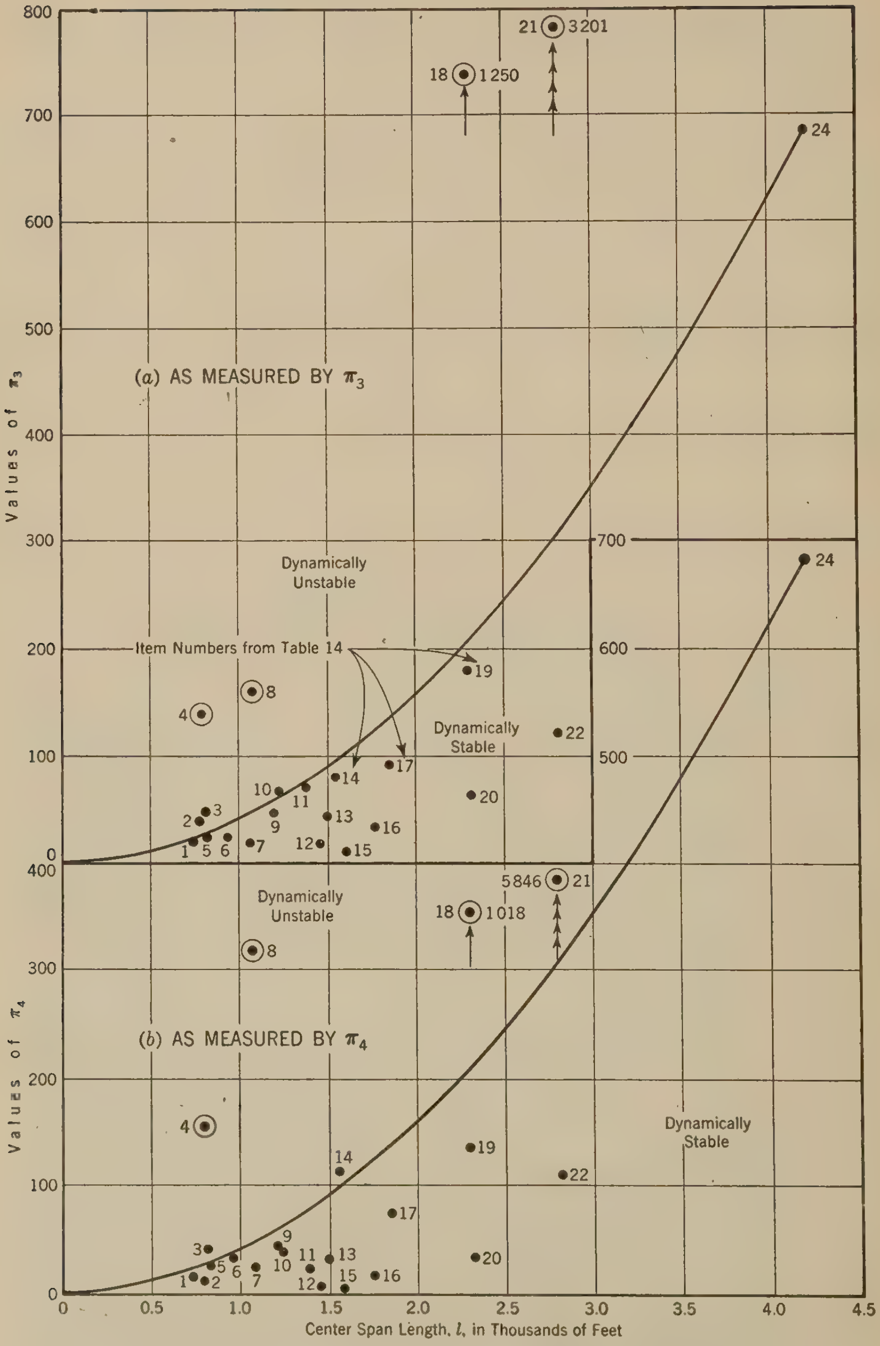

FIG. 23.—DYNAMIC STIFFNESS OF SUSPENSION BRIDGES, AS MEASURED BY DIMENSIONLESS PARAMETERS π_3 AND π_4

trusses, the moments of inertia of which are deficient 60% and 380%, respectively, as indicated by Fig. 23. (Bridges that are stable without satisfying this criterion have been discussed by the writer elsewhere.[78]) Any attempt, however, to derive aerodynamic stability from purely empirical considerations should be regarded as a temporary expedient for estimating purposes and should not take the place of specific experimental research for each structure to be designed.

TABLE 15.—Ratios of Required Moment of Inertia and Moment of Inertia in Actual Structure

(Excess = +; Deficiency = −)

Bridge	Eqs. 123 and 124a	Bridge	Eqs. 123 and 124b
Williamsburg	+ 8.93	Williamsburg	+ 19.15
Manhattan	+ 4.65	Manhattan	+ 16.29
Philadelphia	+ 3.81	Philadelphia	+ 11.05
Transbay	+ 3.20	Transbay	+ 5.85
Tacoma Narrows (proposed)	+ 2.49	Triborough	+ 3.16
Isle d'Orleans	+ 2.27	Tacoma Narrows (proposed)	+ 2.77
Mid-Hudson	+ 2.04	Mid-Hudson	+ 2.67
Grand Mère	+ 1.73	Ambassador	+ 1.87
Ambassador	+ 1.41	Maumee River	+ 1.77
Mount Hope	+ 1.20	Isle d'Orleans	+ 1.50
Lions' Gate	+ 1.14	Bronx-Whitestone (proposed)	+ 1.50
Bronx-Whitestone (proposed)	+ 1.13	Davenport	+ 1.34
Lempa River	+ 1.10	St. Johns	+ 1.30
Triborough	+ 1.01	Mount Hope	+ 1.19
Davenport	+ 1.01	Lempa River	+ 1.03
		Grand Mère	+ 1.01
Golden Gate	1.00	Golden Gate	1.00
St. Johns	− 1.14		
Maumee River	− 1.40	Lions' Gate	− 1.28
Waldo-Hancock	− 2.02	Waldo-Hancock	− 1.88
Deer Isle	− 3.49	Bronx-Whitestone (present)	− 4.97
Thousand Islands	− 5.65	Thousand Islands	− 6.33
Bronx-Whitestone (present)	− 6.10	Deer Isle	− 7.03
Tacoma Narrows (1940)	− 10.54	Tacoma Narrows (1940)	− 19.27

Because the weight of the various characteristics affecting economical design for aerodynamic stability is not apparent from the results reported in the paper, the importance of the variation of the characteristics of the suspension bridge system as they influence aerodynamic stability will be examined. This examination reveals the following:

(1) Cable sag ($\sqrt{f}$) determines the fundamental frequency; hence the actual frequency variation is very small and its effect on stability irrelevant.

(2) Cable stiffness, predominately, determines the structural elastic stiffness. Consequently the requirements of static stiffness can often be met fully without trusses. Fig. 22 shows that within the limits of its practical variation, cable stiffness cannot assure aerodynamic stability.

(3) Trusses, within economically practical limits, contribute little to the structural elastic stiffness but they can be the principal contributors to structural damping. They represent, however, neither a progressive, nor always an economical, solution in the design for aerodynamic stability.

(4) Stay systems may be developed into efficient contributors to structural damping. Dynamic models overestimate the damping efficiency of stay

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[78] *Engineering News-Record*, December 16, 1943, pp. 74–75.

systems grossly. The Bronx-Whitestone model indicated high damping effect for stays which, installed on the prototype, have been found insignificant. Wind-tunnel models equipped with stays should supply basic data for the development of efficient stay systems. Stay systems will then gain importance in the economical design for aerodynamic stability.

(5) Suspended structures which are aerodynamically stable, with or without efficiently provided additional structural damping, constitute the economical solution of the design problem of aerodynamic stability. In other words, the structure whose satisfactory static rigidity has been achieved the most economically will also be aerodynamically stable, or can be made so the most economically.

The author's equations are useless for the solution of the practical design problem, because neither the variation of aerodynamic damping with arbitrary modification of the shape of the suspended structure, nor the variation of structural damping resulting from any practical arrangement, can be calculated by them. In fact, they give inconsistent results even for shapes similar to the one from which they were derived, as proved by the behavior of existing bridges.

The author's conclusions recommending that stability be secured by increasing weight and depth must be rejected. The fundamental design principle of structural efficiency, expressed by lowering the ratio of the weight of metal in the structure to the live load the structure is able to carry, should never be abandoned, especially because weight, within any reasonable limit, does not assure dynamic stability. Analysis of the cost items of long-span suspension bridges clearly indicates the great economic significance of the reduction of weight. Deep girders also affect the cost unfavorably. The determination of stability, therefore, is primarily the task of aerodynamic research which should be connected directly with the design of suspension bridges until the experiments permit a clear theoretical interpretation.

D. B. Steinman,[79] M. Am. Soc. C. E.—To those engineers who have contributed discussions, challenging, confirming, or supplementing the material presented in this paper, the profession is indebted. A careful study of the data in the discussions reveals nothing that contradicts the writer's predictions and much that confirms his recorded conclusions. The writer welcomes the opportunity afforded by this closing discussion to answer the questions that have been raised, so that this study of a critical problem, enriched by the discussions, may be brought closer to complete solution and practical application.

Definitions.—For those who have been puzzled by the seemingly paradoxical conclusions of this paper, certain clarifying definitions may be helpful. Misconception has been caused by failure to perceive the distinction drawn by the writer between basic aerodynamic stability of the section and aerodynamic stability of the structure.

An "aerodynamically stable section" is one having a positive lift (or torque) graph. (Examples: A circular section, a half-round section with convex face toward the wind, a T-section with stem toward the wind, a conventional airfoil, a horizontal flat plate, and an H-section with $d/b < 0.24$.) The defining condition is s' (or s_t) > 0.

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[79] Cons. Engr. (Robinson & Steinman), New York, N. Y.

An "aerodynamically unstable section" is one having a negative lift (or torque) graph. (Examples: A half-round section with flat face toward the wind, a T-section with head toward the wind, a vertical flat plate, and an H-section with $d/b > 0.24$.) The defining condition is s' (or s_t) < 0.

An "aerodynamically stable structure" is one in which no aerodynamic amplification of oscillations can occur. The defining condition is $\delta_s > \delta$.

An "aerodynamically unstable structure" is one in which aerodynamic amplification of oscillations can occur. The defining condition is $\delta_s < \delta$. (There are at least eight such bridges.)

On account of the phase-difference effect across the width of a section, aerodynamic instability of a structure may be produced with either stable or unstable sections. The basic aerodynamic stability or instability of the section influences the degree and the range of potential instability of the structure. If the section is aerodynamically stable, any instability of the structure is more limited in wind-velocity range and in amplitudes, and is more easily prevented or cured.

The Problem and Its Logical Solution.—The primary purpose of the paper was not to explain past events but rather to provide a basis for future practical application. Any reference to past or present structures is merely incidental, as a test of validity.

The practical problem of the bridge engineer must be kept in mind. The bridge engineer cannot, as a rule, afford elaborate wind-tunnel tests, scale models, and tests on oscillating section models for every bridge he has to analyze or design or for each successive preliminary design for a project. Rarely will time or financial limitations permit extensive investigations of such character. The governing relations must be reduced to scientific, mathematical form and then to guiding specifications to permit direct application in design.

For these reasons, the writer has endeavored to reduce the problem to four separate steps:

1. The simple determination of the rigidity characteristics of the structure, represented by the coefficient of rigidity K, enabling the frequency N to be computed accurately for any mode of oscillation;
2. The determination of the aerodynamic amplification characteristics of the structure, represented by the logarithmic increment δ, as a function of the cross section;
3. The determination of the potential instability, or amplification characteristics, at different wind velocities, represented by the variation of δ with $V/(N\,b)$ (supplied by the writer in the form of the β-function, which has a quantitative rational basis in the concept of the phase-difference effect); and
4. The evaluation of the positive damping characteristics of the structure, represented by the logarithmic decrement δ_s (tentatively covered by the writer's semi-empirical formulas for δ_s, in the form indicated by rational considerations and with coefficients and exponents written to fit available data and known behavior of actual structures).

These four steps, when combined, offer a complete solution for any structure. Their separation, for research and analysis, permits a logical and scientific attack. Without such clarification, much experimentation may become mere groping and mystification.

Rigidity of Suspension Bridges.—Table 2 was not presented as a proposed grading of the relative aerodynamic susceptibility of different bridges. It was merely an attempt, by the statistical method, to find the dividing line below which aerodynamic instability may be anticipated unless prevented.

The coefficient of rigidity K, or its quotient $2\ K/b$, constitutes a simple and convenient measure of rigidity. It epitomizes all that needs to be known about the structure from the viewpoint of elastic resistance to oscillations; it represents the combination of the theory of suspension bridges with the theory of mechanical vibrations—the remainder of the problem is concerned with aerodynamics and structural or mechanical damping. If K can be computed, there is little or no need for building costly scale models of the structure. All the aerodynamic questions can be answered better by small section models.

In emphasizing the significance of the quotient $2\ K/b$, the writer was endeavoring to correct the insistence by others that K, the rigidity per cable, is the sole criterion. Both $2\ K/b$ and K need to be considered, in conjunction with other factors. That K alone does not tell the entire story is indicated by the fact that the George Washington Bridge, with $K = 244$, despite its low value of d/b, is admitted to be subject to aerodynamic oscillations, whereas the Ambassador Bridge (between Detroit, Mich., and Windsor, Canada), the Grand Mère Bridge (in Quebec Province, Canada), the Mount Hope Bridge (in Rhode Island), and the Waldo-Hancock Bridge (in Maine), all with lower values of K, have been immune. The Canadian suspension span of the Thousand Islands Bridge and the Bronx-Whitestone Bridge have identical values of $K = 141$; yet the former, with a few light cable stays added, has had no reported oscillations, whereas the latter has had quite a different history. The Lions' Gate Bridge, with $K = 112$ and $2\ K/b = 5.62$, has not had the large oscillations that have been observed on the Golden Gate Bridge, with $K = 116$ but with $2\ K/b = 2.56$. The Tacoma Bridge, with $K = 62$, was destroyed within four months by the first 42-mile wind, whereas the narrower Deer Isle Bridge, with $K = 63$, plus a few light stays added, in a more exposed coastal location, has for five years repeatedly withstood storms of from 70 to 80 miles per hr. In these comparisons, if $2\ K/b$ is used instead of K and if the girder and truss bridges are segregated, the differences in behavior are understandable.

Besides the rigidity coefficients K and $2\ K/b$, other factors affecting the aerodynamic stability are truss or girder construction, the camber, the location and exposure, the cross section of the bridge, side-span and tower rigidity (in some modes), torsional bracing, the ratio d/b determining the aerodynamic constants, the structural damping δ_s (including any artificial damping devices), the ratio R helping to determine the structural damping, and the width factor b^2/w. Some of these governing numerical ratios, directly or indirectly, are represented as a composite function in the slenderness ratio, d/l. Practically all known suspension bridges with d/l less than 1/100 have shown aerodynamic instability. An exception is the Ronosfoss Bridge in Norway; its reported immunity (with $K = 62.5$ and $2\ K/b = 6.36$) is explained by an extremely low value of d/b, almost a flat plate, yielding very low aerodynamic lift and only positive (stabilizing) torque. The George Washington Bridge would have similar immunity, except for a low value of $2\,K/b$ ($= 4.60$) and the floor opening in the wrong place. The opening reduces passive atmospheric damping although it affects little reduction in the laterally concentrated lift pressures. The Grand Mère Bridge, with K only 188, is aerodynamically stable because the high value of $2\,K/b$ for a truss bridge represents high rigidity and the exceptionally high value of $R = 0.66$ creates very high structural damping. The Bronx-Whitestone Bridge, because of its width, has an abnormally high value of b^2/w (in fact, twice the normal value), which practically doubles the instability by doubling the value of δ to which the bridge is subjected.

Generalized K-Formula for Any Harmonic Modes.—The simple K-formulas originally presented (Eqs. 2 to 15) suffice to cover all modes of span oscillation previously recorded. To cover other possible combinations of spans, modes, and superimposed harmonics, a generalized formula for K may be written:

$$K = \frac{\pi^2 \Sigma\left(\frac{n^2\,a^2}{l}\right) H + \pi^4\,\Sigma\left(\frac{n^4\,a^2}{l^3}\right) E\,I + \frac{512}{\pi^2}\,\frac{E_c\,A_c}{L_s}\left[\Sigma'\left(\frac{a\,f}{n\,l}\right)\right]^2}{\frac{1}{w}\,\Sigma(w\,a^2\,l) + \frac{2}{w}\,\Sigma(w\,e^2\,l)} \quad ..(125a)$$

in which a is the vertical amplitude of any component mode in any span, e is the amplitude of longitudinal oscillation of any span; and L_s is a cable-length constant[73,80,81] given by

$$L_s = E_c\,A_c\,\Sigma \int_0^l \frac{1}{E_c\,A_c}\,\frac{ds^3}{dx^2} \approx E_c\,A_c\,\Sigma \frac{1}{E_c\,A_c}\left(l + 8\,\frac{f^2}{l}\right) \sec^3\gamma \quad ..(125b)$$

In Eqs. 125 and those that follow, Σ denotes summations covering all similar expressions in all spans, and Σ' denotes a summation covering only the odd values of n in the several spans.

In Eq. 125*a* and the following equations, and in all similar formulas involving summations of functions of the amplitudes, it is not necessary to know the absolute magnitudes of the several concurrent amplitudes (a and e) but merely the relative magnitudes.

Eq. 125*a* is completely general for any number of spans of any respective lengths, simple or continuous, with any respective cable sags, including straight or unloaded backstays, for any combinations of values of n and a in simple or superimposed harmonic oscillations in the different spans, with any concomitant longitudinal oscillations in any spans, with different values of w, H, $E\,I$, and $E_c\,A_c$ in different spans, and with any value of ΔH producing cable stretch.

The general formula for K, Eq. 125*a*, may be rewritten in a single compact expression of equal generality:

$$K = \frac{\Sigma(K\,a^2\,l)}{\frac{1}{w}\,\Sigma(w\,a^2\,l) + \frac{2}{w}\,\Sigma(w\,e^2\,l)} \quad \ldots\ldots\ldots\ldots\ldots\ldots(126a)$$

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[80] "A Practical Treatise on Suspension Bridges, Their Design, Construction and Erection," by D. B. Steinman, 2d Revised Ed., John Wiley & Sons, Inc., New York, N. Y., 1929, p. 37 and pp. 250–251.

[81] "On the Deflection Theory of Suspension Bridges," by S. O. Asplund, Upsala (Stockholm), Sweden, 1943, p. 27.

in which K for each span or for each component mode has been computed separately by any of the pertinent formulas in this paper, including Eq. 125a or any reduced form of Eq. 125a.

With w equal in all spans, and with $e = 0$, Eq. 126a takes the simpler form:

$$K = \frac{\Sigma(K\, a^2\, l)}{\Sigma(a^2\, l)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(126}b\text{)}$$

which may also be written directly from the energy relation of Eq. 28b.

The last term in the numerator of Eq. 125a represents the effect of ΔH. It is ordinarily either zero (in antisymmetric modes) or of relatively minor magnitude. There will be no cable stretch between anchorages ($\Delta H = 0$) if

$$\Sigma'\left(\frac{a\, f}{n\, l}\right) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(127}a\text{)}$$

over all spans, or if

$$\Sigma'(a/n) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(127}b\text{)}$$

in each span—also, of course, for antisymmetric modes. (If the cable is anchored at midspan, as by center stays, and longitudinal oscillation of the span is prevented, Eq. 127a is applied over the half span and side span, or from the point of fixation to either anchorage, using the obviously modified values of n, f, and l.)

A simple short cut for estimating the value of K is to assume that $\Delta H = 0$. Consistent relative amplitudes are then given by Eq. 127a, and their substitution in Eq. 125a or 126b yields K with sufficient accuracy for all practical purposes (see Table 1).

Determination of K and Relative Amplitudes for Composite Modes.—A simple or primary mode is one represented by a single sinusoidal curve—hence, single values of n and of a, as in Eq. 5. The term "composite mode" is used herein to denote oscillations comprising a plurality (superimposed or coupled, or both) of simple modes, as in Eqs. 9 to 12.

For each symmetric component mode, the general formula for K is

$$K = \left(\frac{\pi^2\, n^2}{l^2} H + \frac{\pi^4\, n^4}{l^4} E\, I\right) + \left(\frac{512}{\pi^2}\, \frac{f}{l^2}\, \frac{E_c\, A_c}{L_s}\right) \frac{1}{n\, a}\, \Sigma'\left(\frac{a\, f}{n\, l}\right) \ldots \text{(128)}$$

This is a reduced and simplified form of the composite formula, Eq. 125a. It may be derived independently (compare Eq. 6), or it may be written by applying the mathematical condition for a minimum (or a maximum) to Eq. 125a.

In Eq. 128, the first two terms are identical with the basic Eq. 6, and the third term represents the correction for ΔH. Denoting the first two terms by K_n (defined by Eq. 6), and the constant dimensional factor in the third term by C, Eq. 128 may be written in convenient, compact form:

$$K = K_n + C\, \frac{1}{n\, a}\, \Sigma'\left(\frac{a\, f}{n\, l}\right) \ldots\ldots\ldots\ldots\ldots\ldots \text{(129)}$$

This is a basic equation in the theory that follows. For antisymmetric modes, K is simply equal to K_n.

For symmetric modes (necessarily composite), Eq. 129 is valid for each component mode; and, with the consistent (normal) relative amplitudes substituted, it must yield the same value of K (and, hence, the same frequency) for each component mode. To satisfy this condition, Eq. 129 yields two simple governing relations between K and the respective values of n, a, and K_n of the component modes:

$$\Sigma'\left(\frac{1}{n^2}\,\frac{C\,f/l}{K - K_n}\right) = 1 \qquad (130)$$

and

$$n\,a \propto \frac{1}{K - K_n} \qquad (131)$$

Eq. 130 yields the desired value of K for the normal (balanced) composite mode, and Eq. 131 gives the relative amplitudes of the component modes.

In practical application, the largest number of component modes necessary to be considered and substituted in Eq. 130 is three, and that is the case of the lowest symmetric mode ($n = 1$, $n = 3$, and $n_1 = 1$). In the higher symmetric modes, the superimposed harmonics prove negligible, and only two primary K_n-values need to be substituted, one for each span (example: $n = 3$ and $n_1 = 1$). Accordingly, Eq. 130 is in reality a quadratic, or cubic, equation for K. Each real root of this equation will represent a normal composite mode with a mathematically and physically consistent set of relative amplitudes.

In practical application, Eq. 130 lends itself to a simplified numerical solution by an expedited method of trial substitution. It is the sum of two or three fractions equated to unity, with the sole unknown, K, in each denominator. If the mode represented by the first fraction is dominant, substitute an approximate trial value of K in the other term or terms, and solve the first fraction for K. The first or second trial will generally suffice. Similarly, the one or two other roots are obtained by treating the other fraction or fractions, respectively, as dominant.

Example 7: Computation of Symmetric Composite Modes for a Three-Span Bridge.—For the dimensions of the Tacoma Bridge, $K_0 = 15$, $K_1 = 100$, and $K_3 = 140$ (see Table 1); and $C = 1{,}420$. Substituting in Eqs. 130 and 131,

$$\frac{a_1}{a} = \frac{K - 15}{K - 100} \qquad (132a)$$

$$\frac{3\,a_3}{a} = \frac{K - 15}{K - 140} \qquad (132b)$$

and

$$\frac{117.7}{K - 15} + \frac{13.1}{K - 140} + \frac{92.6}{K - 100} = 1 \qquad (132c)$$

The three solutions of this equation (for $n = 1$, $n = 3$, and $n_1 = 1$, as the dominant modes, respectively) are $K = 53$, $K = 135$, and $K = 290$ ($N = 7.5$, 11.8, and 17.3 per min), each corresponding to a normal mode repeatedly observed in the actual structure. With each value of K, the corresponding values of a_1/a and a_3/a are determined.

For the mode $n = 5$ and $n_1 = 1$, Eq. 130 yields $K = 415$ for a_5 dominant, or $N = 20.7$, as compared with $N = 20.8$ observed in the actual structure (see Table 1).

Example 8: Calculation of Modes with Straight Backstays.—For the lowest symmetric modes $n = 1$ and $n = 3$, the respective values of K_n (see Table 1) are 15 and 140. With $Cf/l = 118$, the composite value of K is given very simply by

$$\frac{118}{K - 15} + \frac{118/9}{K - 140} = 1 \quad\ldots\ldots\ldots\ldots\ldots\ldots (133)$$

Eq. 133 yields $K = 102$ and $K = 184$; hence $N = 10.2$ and 13.8 per min (versus 10.5 and 13.8 observed; see Table 11). The amplitude relation is $\frac{a_3}{a_1} = 1/3\, \frac{K - 15}{K - 140} = -0.76$ and $+1.28$. The node points are defined by

$$\frac{\sin x/l}{\sin 3\, x/l} = -\frac{a_3}{a_1} \quad\ldots\ldots\ldots\ldots\ldots\ldots (134)$$

yielding $x/l = 0.22$ and 0.42 (versus 0.22 and 0.41 observed). In the first case the end segments are shortened; in the second case they are lengthened. In the same mode type, the two cases represent, respectively, a minimum and a maximum value for K; yet both are normal (balanced) modes. Because two nodes appeared in the model tests, this mode type was wrongly classified as a higher mode with two unexplained variations, and the investigators vainly sought the missing lower mode (see Table 11).

Instead of designating mode types by the number of nodes, it is more informative and scientifically more consistent to designate them by the number of segments—that is, by the n-value or values.

The fact, revealed by the writer's analysis, that a normal mode is produced by a maximum value of K (or N), as well as by a minimum value, appears to be a new discovery, previously unnoted in vibration analysis.

Application to Unloaded Backstays with Cable Sag Considered.—In Eqs. 128 to 131, the usual relation of equal unit dead loads in main and side spans ($w_1 = w$) was assumed. If w_1 is not equal to w, the sole change is to replace K_{n1} for any side-span component by $\frac{w}{w_1} K_{n1}$, in Eqs. 129, 130, and 131.

Example 9: Calculation of Modes with Unloaded (Sagging) Backstays.—Modifying Example 8 by considering sag in the unloaded backstays, with $w_1/w = 1/4$ (hence, $f_1 = 9$) and using $C = 1{,}425$, as before, the key equations for the lowest composite mode ($n = 1$, $n = 3$, and $n_1 = 1$) are

$$\frac{3\, a_3}{a} = \frac{K - 15}{K - 140} \quad\ldots\ldots\ldots\ldots\ldots\ldots (135a)$$

$$\frac{a_1}{a} = \frac{K - 15}{K - 4\,(98)} \quad\ldots\ldots\ldots\ldots\ldots\ldots (135b)$$

and

$$\frac{118}{K - 15} + \frac{13.1}{K - 140} + \frac{23.2}{K - 4\,(98)} = 1 \quad\ldots\ldots\ldots\ldots (135c)$$

The three solutions of Eq. 135c (with a, a_3, and a_1 dominant, respectively) are $K = 99$, $K = 175$, and $K = 427$. With the maximum value of K, all amplitudes are in the same direction, producing maximum cusps at the towers. This checks and explains Fig. 21.

Effectiveness of Inclined Stays.—A confirmation of the writer's theory for calculating the effectiveness of inclined stays is afforded by observations on the Deer Isle Bridge. For the initial stays placed on the structure in 1939, the calculated increase in rigidity by Eq. 65 is $\Delta K = 98$. The actual increase, computed from the observed values of N, is from $K = 63$ to $K = 165$, or $\Delta K = 102$, which is almost identical with the value given by the formula. With the concomitant increase in the value of R, yielding a fourfold increase in structural damping, the critical velocity was thereby materially raised.

Effect of Center of Suspension.—The effective center of suspension of a suspension bridge cross section in torsional oscillation is the midpoint of the line joining the two points of connection of the suspenders to the outside girders. The torsional stability of the section is increased by raising this center of suspension relative to the center of gravity of the suspended section. This is confirmed by simple model tests. The spring constant K_t is augmented by a torque of gravity restitution; a gravity pendulum effect is added to the torsional spring constant.

If h is the vertical height of the center of suspension above the center of gravity of the suspended section, the value of K_t given by Eq. 21a is augmented to

$$K_t = \frac{b^2}{2} K + 2\,h\,(w - w_c) \qquad (136)$$

in which w_c is the cable dead load. The total energy W required for any angular amplitude is increased in the same proportion, and consequently the aerodynamic instability, measured by the rate of amplification δ, is correspondingly reduced.

Instead of increasing h by raising the points of suspender connection, as by deeper girders or by brackets, the same improvement may be accomplished by lowering the center of gravity of the suspended section. Accordingly, torsional rigidity and stability may be improved by changing from deck or half-through construction to through construction. The writer has confirmed this conclusion by model tests. A deck truss section mentioned by Professor Farquharson as unstable becomes aerodynamically stable when simply inverted.

Calculation of R and Energy Distribution in Composite Modes.—Any formula for K may be written

$$K = K_H + K_I + K_{\Delta H} \qquad (137)$$

which is the sum of three terms representing, respectively, the contributions of cable displacement, girder bending, and cable stretch. The same three terms also represent the respective relative distribution of the total oscillation energy stored in the structure at any time (see Eq. 28b). Since the K_H-portion represents merely gravitational potential energy (a raising of the center of gravity of the total suspended dead load), it does not contribute directly to hysteresis damping. The remaining fraction of the total energy is elastic energy, subject to hysteresis damping, and is represented by

$$R = \frac{K_I + K_{\Delta H}}{K} \qquad (138)$$

In antisymmetric modes, and in most other cases, ΔH is zero or negligible, so that R is simply K_I/K (see Table 2). This may be written (for each span),

$$R = \frac{\pi^2 n^2 S^2}{1 + \pi^2 n^2 S^2} \qquad (139a)$$

in which S is the familiar stiffness factor of suspension bridges,[82]

$$S = \frac{1}{l}\sqrt{\frac{E I}{H}} \qquad (139b)$$

(See also Mr. Gronquist's discussion.)

In any composite mode (including coupled modes and superimposed harmonics), the basic relation of the general Eq. 126b for K also applies to the three parts, K_H, K_I, and $K_{\Delta H}$. Consequently,

$$R = \frac{\Sigma(R\,K\,a^2\,l)}{\Sigma(K\,a^2\,l)} = \frac{\Sigma(R\,K\,a^2\,l)}{K\,\Sigma(a^2\,l)} \qquad (140)$$

Eq. 140 gives R for the composite mode as a weighted mean of the individual values of R, defined by Eq. 138, for each component mode.

If the determined rates of hysteresis loss in cable and girder are different R may be separated into R_I and $R_{\Delta H}$, defined by K_I/K and $K_{\Delta H}/K$, respectively. One may also write, for the nonelastic contribution, $R_H = K_H/K$. The fundamental relation of Eq. 140 may then be applied to R_H, R_I, and $R_{\Delta H}$, separately, to obtain their values for the composite mode.

By Eq. 28b, the total energy stored in cable displacement, girder bending, and cable stretch, respectively, is

$$W_H = \tfrac{1}{2}\,R_H\,K\,\Sigma(a^2\,l) \qquad (141a)$$

$$W_I = \tfrac{1}{2}\,R_I\,K\,\Sigma(a^2\,l) \qquad (141b)$$

and

$$W_{\Delta H} = \tfrac{1}{2}\,R_{\Delta H}\,K\,\Sigma(a^2\,l) \qquad (141c)$$

If the logarithmic decrements $\Delta W/(2\,W)$, corresponding to hysteresis (and friction) loss in girder bending and cable stretch, are denoted by δ_I and $\delta_{\Delta H}$, respectively, the combined logarithmic decrement representing these two contributions will be

$$\delta_s = R_I\,\delta_I + R_{\Delta H}\,\delta_{\Delta H} \qquad (142a)$$

or, in general, for all conditions,

$$\delta_s = \Sigma(R\,\delta_s) \qquad (142b)$$

in which R represents the fraction of the total energy for each element in the structure. If all elements are included, $\Sigma R = 1$.

Logarithmic Increment.—In Eqs. 29a and 30, the writer introduced δ as the rate of amplification $\left(\frac{\Delta a}{a}\right)$ per cycle. Normally δ may be taken as equal to the logarithmic increment. If δ (or δ_s) is defined as the logarithmic increment or decrement, the exact relation is

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[82] "A Practical Treatise on Suspension Bridges, Their Design, Construction and Erection," by D. B. Steinman, 2d Revised Ed., John Wiley & Sons, Inc., New York, N. Y., 1929, p. 277.

$$\delta = \tfrac{1}{2} \log \left(1 + \frac{\Delta W}{W} \right) = \log \left(1 + \frac{\Delta a}{a} \right) \qquad (143a)$$

Utilizing the writer's empirical short cut[83] for logarithmic relations,

$$\delta \approx \frac{\Delta W/(2\,W)}{1 + 0.45\,\Delta W/W} \approx \frac{\Delta a/a}{1 + 0.45\,\Delta a/a} \qquad (143b)$$

Eqs. 143 will be found more convenient to apply than Eq. 76 given by Professor Robertson. Eqs. 143 indicate plainly that for small values of δ the amplification ratio is closely equal to the logarithmic increment.

Use of K for Estimating Deflections.—Eqs. 16 and 22 for estimating live-load deflections directly from K and K_t were not a part of the aerodynamic analysis but were introduced as items of interest merely to emphasize the picture of the physical meaning of K as the "coefficient of rigidity." By using the words "estimating" and "approximate," and the sign for "approximately equal," the writer endeavored to make it clear that those two short-cut formulas were presented as approximate. Nevertheless, he believes that, as a rule, those formulas will yield better estimates of the indicated live-load deflections than any other formulas of comparable simplicity.

Mr. Balog's criticism of Eqs. 16 and 22 is therefore inappropriate. It is easy to show that any approximate formula is only approximate, especially if the numerical values used as standards of comparison are themselves erratic. The formulas that Mr. Balog offers for calculating quarter-point deflections are derived from inconsistent and inadmissible assumptions and would yield greater discrepancies. For the Tacoma Bridge the calculated deflections by his formula would be 38% too low; for the Maumee River Bridge (in Toledo, Ohio) the calculated deflections by his formula would be 48% too high; and the error would be still higher for more highly stiffened bridges.

In supplying and then criticizing a definition which he improperly ascribes to the writer, Mr. Balog has misinterpreted the physical meaning of $2\,K/b$. Actually, $2\,K/b$ is the loading increment Δw, in pounds per square foot, required to produce a 1-ft deflection, in which Δw is not a uniform load limited to the half span but is a sine-curve loading proportional to the displacements η, changing sign at midspan or at any node point. The basic relation (see definition of K and Eq. 6) is $K = \Delta w/\eta$. It is only by virtue of the convenient "accidental" relation of Eqs. 16 and 22 (and subject to the same degree of approximation) that a uniform half-span load may be substituted for the nonuniform reversing full-span load in the foregoing definition.

In criteria for aerodynamic stability, the use of calculated quarter-point deflections under uniform half-span load is a crude substitute; it cannot be as scientifically accurate and consistent as the use of K, given simply and directly by Eq. 6. In aerodynamic oscillations, the loading is not a uniform half-span load; the correct loading is represented by K; the correct frequencies are given by K; and all relations involve K. The evasive replacement of K by an arbitrary, and sometimes erratic, substitute serves no useful purpose and is patently unscientific.

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[83] "Mathematical Short-Cuts for Engineers," by D. B. Steinman, *Engineering News-Record*, May 20, 1943, pp. 79–81.

The Vortex Theory.—A number of considerations prompted the writer's decision to omit the study based on the vortex theory. The theory predicated on the von Kármán vortex trail, in which aerodynamic frequency is normally a fixed function of the wind velocity and of the sectional dimensions instead of being automatically resonant with the frequency of the structure, appears to be inconsistent with many known facts, such as (1) instability over wide ranges of wind velocities (extending in some cases to infinity); (2) instability over a plurality of ranges that are not related as simple harmonics; (3) ranges of vertical instability alternating, instead of coinciding, with ranges of torsional instability; and (4) a contrary or reversed relationship in some bridges. The burden of explaining these inconsistencies rests with the proponents of the theory.

The most pressing reason for the omission was the fact that the vortex theory is far from being in a form to yield the quantitative constants and relations necessary to compute the excitation and amplification of vertical and torsional oscillations, aerodynamic forces and moments, lift and torque response graphs for the oscillating section, energy relations, positive and negative damping effects, critical wind velocities, critical angles of attack, rates of amplification, limiting amplitudes, effect of change of section, and criteria for predicting stability and instability. If the vortex theory is not directly applicable to these problems, it is not helpful to the bridge engineer.

The Theory of Self-Excited Oscillations.—The writer employed and developed the theory of self-excited oscillations because this theory offered the most useful quantitative results. The theory utilizes the familiar static lift and torque graphs to furnish the basic quantitative magnitudes of L and T. The fundamental concept, as applied to vertical oscillations of ice-coated transmission lines, was originated by J. P. Den Hartog. The writer developed the Den Hartog concept as a working theory for bridge spans and extended it to torsional oscillations. Furthermore, by introducing the β-concept, the writer showed how instability may be identified with, and calculated from, lift and torque graphs of positive slope.

The theory of self-excited oscillations is valid for either positive or negative slopes of the stationary lift or torque graph. It is not dependent upon any reversal of slope in the lift or torque graph. The writer's calculation of the limiting amplitudes of vertical oscillations on the Tacoma span (with results identical with the recorded values of the limiting amplitudes) was based on the actual lift graph of positive slope. The limitation of amplitudes is produced by the structural damping increasing with amplitude in accordance with Eq. 35. This maximum limiting amplitude, occurring at a maximum ordinate δ in a critical range of $V/(N\,b)$, is not increased by an increase in the wind velocity V. A substantial increase in wind velocity V takes $V/(N\,b)$ out of the critical range for the existing mode and ends the oscillation.

In the case of torsional oscillations in bridges of the Tacoma type, with the upper critical range of $V/(N\,b)$ extending to infinity, there is no maximum ordinate in the δ-graph—that is, there is no limit to the value of δ given by Eq. 51. Moreover, the value of δ may be several times higher than the value of δ for vertical oscillations. If δ_s is small as compared with δ, the effective limit for amplitudes is that provided by the reversal in the slope of the torque graph (Eqs. 54 and 55 and Fig. 7). Amplitudes close to this limit are rapidly attained, lowering the value of δ to the value of δ_s, and any subsequent increase in wind velocity merely overcomes δ_s to squeeze the already high amplitude closer to the limit.

It is important to note that a lift graph with negative slope may produce the same intensified amplification characteristics for vertical oscillations as described herein for torsional oscillations. Some lift graphs (those for sections with d/b greater than 0.24) have a negative slope. This would tend to make vertical oscillations catastrophic. The Deer Isle section belongs to this category; and the writer is convinced that the few light cable stays installed for restraint should be credited with saving the bridge from catastrophe in the severe storms and hurricanes to which it has been subjected since its completion in 1939.

The theory of self-excited oscillations, as extended and developed, appears adequate to explain the known behavior of actual structures and all experimental results known to the writer. Moreover, it provides a complete and consistent analysis yielding a quantitative solution and a prediction of all the data a bridge engineer needs, including aerodynamic forces and moments, critical wind velocities, critical angles of attack, rates of amplification, limiting amplitudes, criteria for stability, evaluation of methods of prevention, specifications for design limitations, and guides for design to assure stability.

Velocity Relations.—Stability or instability of a section, in sign and in intensity, is a function of wind velocity V. All formulas for δ, whether positive or negative, contain the basic factor $V/(N\,b)$ as well as the modifying factor β which is also a function of $V/(N\,b)$.

The governing velocity ratio $V/(N\,b)$ is nondimensional. It is known in aerodynamic analysis as the "reduced velocity." The writer prefers to call it the "velocity ratio." It is the ratio of the wind velocity V to the related constants $N\,b$ of the oscillating section. It is the ratio of the spacing or "wave length" (V/N) of the aerodynamic disturbance to the corresponding dimension (b) of the section. Thus, $V/(N\,b) = 3$ is the velocity ratio of a wind traveling three widths per cycle, or traversing the section in one third of a cycle. The reciprocal of $V/(N\,b)$ is, accordingly, the total phase difference (in fractions of a cycle) encountered across the width of the oscillating section.

The elementary expression for energy input by the aerodynamic action in an element of time on an element of surface requires three-way integration: (1) Over the cycle, (2) over the width of the section, and (3) over the length of the span. The integration over the width of the section, taking into account the varying phase differences encountered in the traverse of the section, yields the important modifying factor β as a function of $V/(N\,b)$. Because β changes sign as well as magnitude with variation in $V/(N\,b)$, potential stability and instability undergo reversal as well as variation in intensity with variation in $V/(N\,b)$.

For differentiation of aerodynamic response, the total range of wind velociy from zero to infinity is divided into a low-velocity range and a high-velocity range. The division point is V_p, the critical velocity due to phase difference. This critical velocity V_p is not a constant; but, for a given section and a given type of oscillation (vertical or torsional), the critical velocity ratio $V_p/(N\ b)$ is a constant for all modes and all rigidities. Hence, higher modes are represented by higher critical velocities V_p. (Compare this statement with Professor Farquharson's discussion, based on his misinterpretation of this point.)

Sections with negative slope of lift or torque graphs, herein termed "unstable sections," are potentially unstable (against the respective type of oscillations) in the high-velocity range. Instability of a structure in the high-velocity range is dangerous and may prove catastrophic. Such instability is not limited, for any mode, to a narrow range of wind velocity but increases indefinitely with V. The Tacoma Bridge, with a negative torque graph, was destroyed by torsional oscillations in the high-velocity range. The Deer Isle Bridge, with a negative lift graph, has inherent vertical instability in the high-velocity range. This bridge was critically tested by vertical oscillations in a wind of 72 miles per hr on the night of December 2, 1942. (The mode was $n = 1$, and the observed vertical amplitude was 12.5 ft, compared with a vertical amplitude of 12.4 ft calculated for this wind velocity by the writer's formulas. The light cable stays initially installed prevented the development of the more natural mode, $n = 2$. The additional stays subsequently installed have apparently proved effective in making the structure safe against all modes of oscillation, despite the severe storms in the exposed coastal location.)

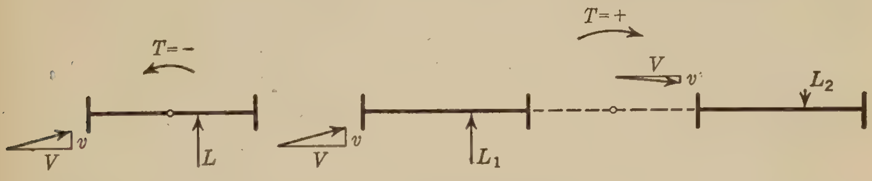
Conversely, sections with positive slope of lift or torque graphs are stable in the high-velocity range. Their potential instability is limited to the low-velocity range. The Tacoma Bridge, with a positive lift graph, had its vertical oscillations (in all modes) limited to the low-velocity range—that is, below the respective values of V_p.

"Unstable" sections (negative slope) also have potential instability in the low-velocity range. The high-velocity range is a single critical range extending from V_p to infinity. The low-velocity range is composed of a series of alternating critical ranges, represented by an oscillating graph converging from V_p down to a zero value of V. If this instability response graph is plotted on $V/(N\ b)$ instead of on V, the same graph will be valid for all modes. The alternate critical ranges belong to "stable" and "unstable" sections, respectively. These relations apply to torsional oscillations, as well as to vertical oscillations. Instability in the low-velocity range is limited, and is more easily prevented or cured.

The foregoing conclusions, derived by analysis, may be subject to minor qualifications; but, in their essential features, they have received multiple confirmation by both experiment on models and experience on actual structures.

Stable Sections.—That aerodynamic stability or instability of a section is a function of the form and proportions of the section and may be controlled by the provision of suitably located and proportioned openings in the horizontal width of the section is a demonstrable fact. Bridge cross sections can be devised or modified to produce assured aerodynamic stability. Tests made by

Prof. J. Ackeret at Zürich, Switzerland, in 1941 proved absolute torsional stability, at the highest wind velocities ($V/(N\,b)$ = from 43 to 175), for a section consisting of two H-sections with the roadway between them removed. With this gap closed, the section was violently unstable, self-starting under wind, and attained angular amplitudes of more than 90° in a few seconds. With the gap open, no applied wind could make the section oscillate, and any applied amplitude was almost instantaneously damped by the wind At the highest wind velocities, an applied amplitude of 70° was brought to rest in two seconds. These results have been confirmed independently in similar model tests by the writer. The aerodynamic stability of this section is easily explained by the writer's theory (see Fig. 24).



(a) UNSTABLE SECTION (T=−)

(b) STABLE SECTION (T=+)

FIG. 24.—AERODYNAMIC STABILITY OF A SECTION SUBJECT TO TORSION

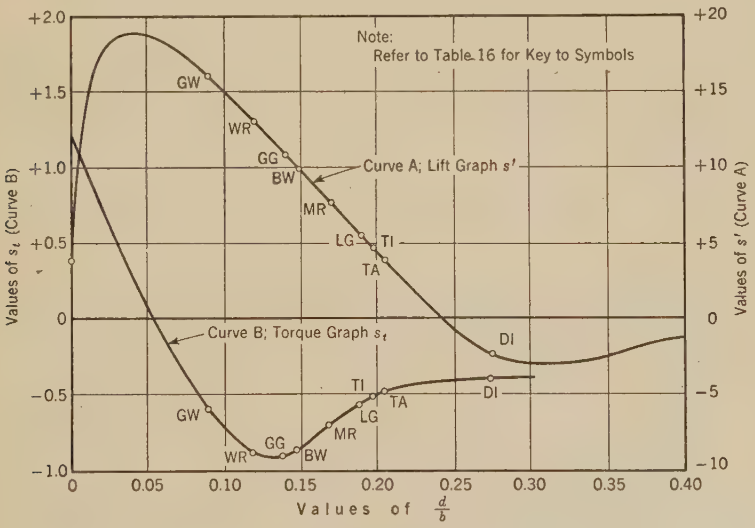


FIG. 25.—VARIATIONS OF SLOPES OF LIFT GRAPHS s' AND TORQUE GRAPHS s_t FOR DIFFERENT SECTION RATIOS d/b

Slopes of Static Lift and Torque Graphs.—By plotting the coefficient of α from Eq. 47, curve A in Fig. 25 is obtained, giving the governing slopes (s') of static lift graphs for different section ratios. Identified points correspond to specific known suspension bridges, using $d/2$ instead of d in the case of truss bridges. Nearly all the bridges marked on the graph fall in the range of positive s', representing vertically "stable sections." Those having the highest positive s' will have the highest potential vertical instability in the low-velocity range. The Deer Isle section is identified in the range of negative slope, representing vertical instability in the high-velocity range. At $d/b \approx 0.24$, the graph crosses the axis, representing zero s' and therefore minimum vertical instability. At $d/b = 0$, the graph comes down to the value for a flat plate.

Curve B in Fig. 25 is the corresponding graph for the governing slopes s_t of static torque graphs. With more direct experimental determination, these curves can be confirmed or perfected.

Modification of the section in the direction of reducing drag, by providing or enlarging openings in the vertical projection or by streamlining, will reduce the effective d/b. The consequent shift of the ordinate toward the left in Fig. 25 may or may not improve aerodynamic stability, depending upon the resulting augmentation or reduction of the ordinate and any change in sign. In some cases, as indicated by the graphs, aerodynamic stability may be improved by limited increases in the vertical projection of the section so as to change the effective d/b to a more stable ratio.

Confirmation by Model Tests.—The model tests (1942–1944) at Seattle, Wash., and at the California Institute of Technology offer confirmation of the theory and analysis developed and presented by the writer. In particular, the experiments confirm the validity and usefulness of the following conclusions which had previously been missed or unsuspected:

1. For vertical aerodynamic oscillations of bridges of the Tacoma type (positive lift graph), the critical wind velocity V_p for any mode is an upper limit, and no material vertical instability in that mode can occur at any higher wind velocity ratio. (This is exactly the reverse of the relation that had been generally assumed.)

2. Data on vertical aerodynamic oscillations of different modes at different wind velocities will fall into a consistent and orderly arrangement if reduced to a $V/(N\,b)$ basis.

3. The possible vertical aerodynamic oscillations of bridges of the Tacoma type, for all modes, fall into a mathematically related series of ranges of $V/(N\,b)$, all below $V_p/(N\,b)$. The maximum potential amplitudes occur approximately at the middle of each range. The second critical range occurs approximately at 0.45 of the mean value of $V/(N\,b)$ for the highest critical range. For all values of $V/(N\,b)$ in the intervals between the critical ranges, sections of the Tacoma type are aerodynamically stable.

4. Only the crests of those critical ranges overtopping a horizontal secant representing δ_s will be effective in producing aerodynamic instability. If δ_s is reduced, the effective critical ranges are widened and lower critical ranges may be brought into play.

5. In bridges of the Tacoma type (with negative torque graph), there are potential critical ranges in torsion below V_p as well as in the known catastrophic range above V_p. (This fact, missed in the tests for the official report on the Tacoma failure, is predicted by Eq. 60*b*.)

6. The aerodynamic behavior of a section in vertical and torsional oscillations can be predicted from the characteristics of the static lift and torque graphs, respectively.

Open Sidewalks.—Questioning the efficacy of the writer's proposal of open sidewalks, Professor Farquharson gives some fragmentary information on unreported model tests he made on such construction as applied to the Tacoma section. Correspondence has elicited the information that the tests were made hurriedly and under difficult conditions, and the further significant fact that, for the sole observed vertical mode ($n = 4$), the critical velocity (prototype scale) was increased from 17 miles per hr for the closed deck and 13 miles per hr for the section with open roadway to 38 miles per hr for the section with open sidewalks. It would have required a large amount of additional stiffening by stays or trusses to produce this 120% augmentation of critical velocity. For the sole observed torsional mode ($n = 2$), the critical velocity (prototype scale) was increased from 15 miles per hr for the closed deck and 10 miles per hr for the section with open roadway to 18 miles per hr for the section with open sidewalks. The results indicate that the aerodynamic instability of the section, measured by the net aerodynamic amplification ($\delta - \delta_a$), was reduced at least 32% in torsion and more than 84% in vertical instability by the mere opening of the narrow sidewalk areas. In the section with open roadway but with closed sidewalks, the reduction in atmospheric damping δ_a outweighed the reduction in negative damping δ; but, in the reverse condition (sidewalks open), the reduction in δ was so great that it far outweighed the reduction in atmospheric damping.

The maximum amplitudes attainable are not a measure of the instability, but depend upon other conditions, including curvature (or lack of curvature) of the lift and torque graphs. A section of lower instability may attain the same amplitudes, but the rate of amplification will be slower, and hours of sustained wind velocity (represented by minutes in the model) may be required to reach the given amplitudes. Obviously, such lower instability can be cured more easily.

Unfortunately, Professor Farquharson omitted any measurements of the rate of amplification, and no static lift or torque graphs of the section with open sidewalks were secured. The opening of the sidewalks increased the critical velocity ratio $V/(N\,b)$ for vertical oscillations from 2.30 to the abnormally high value of 6.60. If the lift graph was thereby reversed in slope (bringing the critical $V/(N\,b)$-ratio into the high-velocity range), some intermediate degree of opening in the sidewalks might give a lift graph of zero slope (s'), yielding zero (or negligible) amplification. As an alternative, a somewhat greater width of "sidewalk" opening might yield zero amplification in lift or in torsion, or both. All these points need further exploration.

Critical Ranges in Vertical Oscillations.—In the large-scale model of the Tacoma Bridge at the University of Washington in Seattle, the structural damping was about thirteen times as high as it appears to have been in the original structure. This explains why only one "subharmonic" range could be evoked for vertical oscillations in the model, as compared with at least five critical ranges recorded in the actual structure. Obviously, the inability of the model to show the lower critical ranges does not disprove the recorded fact of the occurrence and observation of lower critical ranges in the original bridge. The lowest velocity at which vertical oscillations could be produced in the model was (translated to the scale of the prototype) 6.7 miles per hr. In the actual bridge, vertical oscillations were measured and recorded at 1 mile per hr or, adding the 50% calibration correction, at 1.5 miles per hr. By the writer's formulas, the minimum critical velocity (see Example 5) was 1.3 miles per hr. These comparisons answer Professor Farquharson's criticism. The model is in error—not the bridge nor the writer's formulas.

Critical Ranges in Torsion.—A plurality of critical ranges of torsional instability is indicated by the trigonometric formulas for the β-function in torsion. By Eq. 60*b*, the critical ranges of $V/(N\ b)$ are: From ∞ to 3.13; from 1.35 to 0.80; from 0.57 to 0.44; and from 0.36 to 0.31; etc. The graph of $\beta\ V/(N\ b)$, plotted from Eq. 60*b*, showing these critical ranges, is recorded in Fig. 26. The results thus predicted by the writer's analysis have been confirmed by experiment (see Professor Farquharson's discussion). The writer's assumption—that a persistent correlation exists between the stationary lift and torque graphs and the aerodynamic behavior of the oscillating section—thus derives added corroboration.

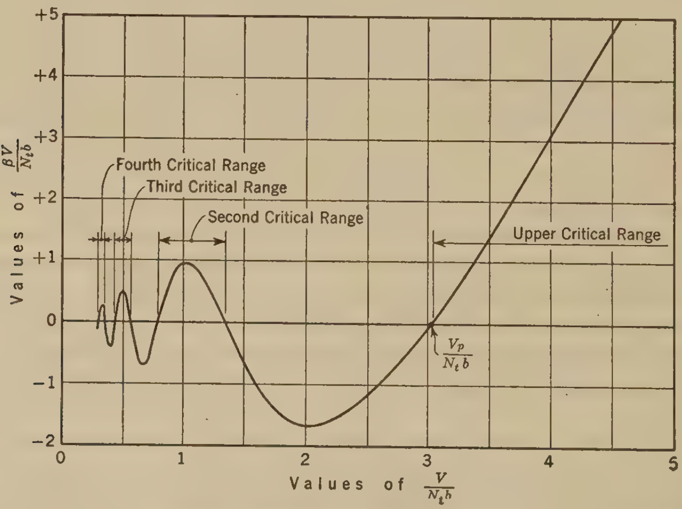


FIG. 26.—GRAPH OF CRITICAL RANGES FOR TORSIONAL INSTABILITY (BY EQ. 60*b*)

Structural Damping.—In writing his tentative formula for structural damping, Eq. 35, based on limited data, the writer assumed that the logarithmic decrement δ_s increases with amplitude and that it is a function of a fractional power of the amplitude. His assumptions have been confirmed by additional data. The experiments of O. Föppl,[31,84,85] J. Schmidt,[86] A. Appenrodt,[87] H. Sonnemann,[88] M. Hempel,[89] B. J. Lazan,[36] and A. J. Yorgiadis and A. U. Kutzay[90]—all confirm the increase of hysteresis damping (expressed as a logarithmic decrement) with stress, strain, or amplitude. From the data obtained by various procedures and presented in different forms, the writer has calculated the corresponding values of δ_s and the law of the variation of δ_s with strain. Professor Föppl's data on mild steel in torsional shear yield values of δ_s increasing from 0.02 to 0.10 with increase in amplitude or strain; and the law of variation is found to be represented by an exponent having a mean value of 1.5. For duralumin in torsion, Mr. Lazan's data yield values of δ_s increasing from 0.02 to 0.08, with the exponent of the amplitude averaging 1.5 for higher stresses. The two metals, mild steel and duralumin, show parallel hysteresis characteristics (in the range of δ_s and the exponential relation of δ_s) when both are tested in torsion. For duralumin in repeated reversals of longitudinal stress, Mr. Lazan's test results yield values of δ_s increasing with amplitude or strain, with 0.55 as a mean value of the exponent. From the foregoing comparisons, steel in reversing longitudinal stress may be expected to yield closely parallel results. Hence, the writer's assumption that δ_s is a function of $a^{0.50}$ is given increased validity. The experimental results also confirm the writer's conclusion that the function expressing δ_s is not zero at zero amplitude.

Professor Sonnemann's tests on solid and riveted flat bars in bending, using steel with a tensile strength of 50,000 lb per sq in., seem to show that riveted joints increase δ_s by a constant amount of 0.015.

In the welcome discussion by Professor Robertson, convincing and consistent data are presented (Fig. 14) confirming the increase of hysteresis damping with strain and yielding a uniform exponent of unity for the law of variation. Of particular interest is the fact that tests on steel in direct stress (as distinguished from shear or torsion) are included. Messrs. Robertson and Yorgiadis have verified this conclusion by tests on two steels, in addition to tests on a variety of other materials.

For application to actual structures, the aggregate of positive damping includes not only hysteresis loss in the solid metal but also energy absorption in pin connections and riveted joints, bearings, foundations, etc., in addition to atmospheric damping. The elements of friction damping yield a logarithmic decrement varying inversely with the amplitude. Atmospheric damping theoretically yields an exponent of unity, but experiments show a fractional exponent. All these additional elements tend to reduce the exponent of unity

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[84] "Damping Capacity of Materials," by G. S. von Heydekampf, *Proceedings*, A.S.T.M., Vol. 31, Pt. II, 1931, pp. 157–175.

[85] "Mechanische Schwingungen," by O. Föppl, Vol. 2, Springer, Berlin, 1936.

[86] "Die Dämpfungsfähigkeit von Eisen- u. Nichteisenmetallen," by J. Schmidt, *Mitteilungen des Wöhler-Instituts Braunschweig*, No. 9, 1931.

[87] *Ibid*, No. 24, 1935.

[88] "Die Schwingungsfestigkeit und Dämpfungsfähigkeit von handelsüblichen Stählen u. Kupfer und ihre Beeinflussung durch Kaltnietung," by H. Sonnemann, *ibid*, No. 28, 1936.

[89] *The Journal of the Iron and Steel Institute*, Vol. 134, No. II, 1936, pp. 427–437.

[90] *Ibid*, pp. 100–101.

yielded by pure hysteresis damping, so that the total resultant positive damping will be represented by a fractional exponent of the amplitude. Professor Robertson states (in correspondence) that he believes the fractional exponent would be nearer zero than unity, and that such fractional exponents were obtained when extraneous losses at the supports were not eliminated from his experiments (and from prior experiments by others).

The writer's prior conclusion that structural damping increases with the amplitude and that it may be taken as varying with the one-half power of the amplitude thus appears to be justified. Moreover, Eq. 35 for structural damping of riveted girder suspension bridges checks the known behavior of actual structures.

Atmospheric Damping.—Contrary to opinions expressed in the discussions, atmospheric damping as a factor in the aerodynamic problem is real and may be significant. As an illustration of the range of information obtainable by simple means, the following conclusions from some tests using a small-scale model of the Tacoma section and taking but a few hours of office time are recorded:

(1) With the wooden section model replaced by a small steel plummet of equal weight hung from the same spring suspension, so as to make atmospheric damping negligible, the number of oscillations required for 50% decay of amplitude was increased from 16 to 231. The respective calculated logarithmic decrements were 0.045 and 0.003, indicating that in this section model the atmospheric damping ($\delta_a = 0.042$) amounted to fourteen times the net structural (hysteresis) damping ($\delta_s = 0.003$).

(2) With the floor fully open (roadway and sidewalks removed), the mean δ_a (after subtracting the constant mean $\delta_s = 0.003$) was reduced from 0.042 to 0.010, a reduction of 76%.

(3) With the roadway removed and the sidewalks in place, the mean δ_a was reduced from 0.042 to 0.020, a reduction of 52%.

(4) With the sidewalks open and the roadway closed, the mean δ_a was reduced from 0.042 to 0.040, a reduction of only 5%.

(5) Per square foot, the roadway area was five times as effective as the sidewalk area in contributing atmospheric damping in vertical oscillations.

(6) In all cases, the positive damping (δ_s or δ_a) clearly increased with the amplitude. The indicated formulas are: For case 0 (streamlined plummet)—

$$\delta_s = 0.005\, a^{0.21} \qquad (144a)$$

for case 1 (model, floor fully closed)—

$$\delta_a = 0.070\, \frac{b'}{w}\, a^{0.42} \qquad (144b)$$

for case 2 (sidewalk open)—

$$\delta_a = 0.160\, \frac{b'}{w}\, a^{0.67} \qquad (144c)$$

for case 3 (roadway open)—

$$\delta_a = 0.126\, \frac{b'}{w}\, a^{0.71} \qquad (144d)$$

and, for case 4 (floor fully open)—

$$\delta_a = 0.034 \frac{b'}{w} a^{1.23} \qquad (144e)$$

in which b' is the net solid width of the section, after deducting the width of all floor openings. Case 2 (sidewalk open) gave the highest atmospheric damping per square foot of net horizontal area. (In all tests, the total suspended weight was kept constant by adding dead load to compensate for the weight of flooring removed.)

(7) In Eqs. 144, the augmenting exponents of the amplitude as the floor area is progressively removed indicate that the component of δ_a due to air friction and disturbance along the vertical faces of the girders increases with an exponent of unity or higher power of the amplitude, whereas the component of δ_a due to reaction of the horizontal area of the section increases with the one-half power or lower exponent of the amplitude.

(8) With the sidewalks open, the mean δ_a per square foot of horizontal area was actually increased from 0.022 to 0.027. This increase may be attributed to the increased air friction on the inner vertical faces of the girders.

(9) Adopting an exponent of 0.50 for the power of the amplitude, the foregoing tests yielded, for the normal cross section with the floor fully closed,

$$\delta_a = 0.086 \frac{b}{w} a^{0.50} \qquad (145a)$$

or, with an exponent of unity,

$$\delta_a = 0.30 \frac{b}{w} a \qquad (145b)$$

The mean of the results for all the sections, open and closed, yielded

$$\delta_a = 0.097 \frac{b'}{w} a^{0.76} \qquad (145c)$$

These empirical results may be compared with the theoretical formula for atmospheric damping based on uniform velocity:

$$\delta_a = 0.087 \frac{b}{w} a \qquad (145d)$$

The correction for acceleration appears to be an increase in the coefficient or a reduction in the exponent, or both.

(10) Using the indicated differentiation between the contributions of girder friction and normal floor resistance, the foregoing tests would yield for an **H**-section with closed floor:

$$\delta_a = 0.075 \frac{b}{w} a^{0.50} + 0.030 \frac{d}{w} a \qquad (146a)$$

and, for an **H**-section with open sidewalk areas:

$$\delta_a = 0.075 \frac{b'}{w} a^{0.50} + 0.060 \frac{d}{w} a \qquad (146b)$$

(11) With weight and springs constant, the frequency of oscillation increased slightly, from 78 and 79 (per min) in the closed model to 81 and 82 in the open models, and to 83 when the streamlined plummet was substituted, apparently because the mass of air to be moved by the oscillating model was reduced.

(12) Under transverse wind, at a velocity contributing aerodynamic stability, an active aerodynamic damping (δ) was consistently added to the passive atmospheric damping in all cases and at all amplitudes.

(13) Under transverse wind, opening the sidewalks (21% of the total area) reduced the superimposed aerodynamic δ by 81%; opening the roadway (45% of the total area) reduced the aerodynamic δ by 82%. Hence, the sidewalk area was more than twice as effective per square foot in producing aerodynamic δ.

(14) Under transverse wind, the aerodynamic δ consistently diminished when the amplitude was increased, as predicted from the curvature of the lift graph. Doubling the amplitude (at 1 in. in the model) reduced δ by 20%.

Atmospheric Damping in Torsional Oscillations.—With the same section model in angular oscillations, a continuation of the foregoing office experiments yielded the following results:

1. In torsional oscillations, the values for atmospheric damping averaged 69% of the corresponding values in vertical oscillations.
2. The mean values of the atmospheric damping in torsion were as follows: Full section, $\delta_a = 0.0288$; roadway removed, $\delta_a = 0.0185$; sidewalk removed, $\delta_a = 0.0158$; and both roadway and sidewalk removed, $\delta_a = 0.0158$.
3. In torsional oscillations, in all cases, δ_a increased with increasing amplitude. The mean rate of variation for the complete section corresponded to $a^{0.84}$. It was somewhat lower ($a^{0.78}$) when the roadway was removed and higher ($a^{1.05}$) when the roadway was restored and the sidewalk removed, indicating (as in the case of vertical oscillations) that air friction on the vertical faces of the girders contributed the higher exponent. Removal of the sidewalks concentrated a relative vertical air flow against the inner faces of the girders.
4. With an exponent of unity, the formula yielded by the test results for the normal section (with floor fully closed) was

$$\delta_a = 0.20 \frac{b}{w} a \qquad (147a)$$

or two thirds of the corresponding value in vertical oscillations, as given by Eq. 145*b*. The mean of the results for all the sections, open and closed, yielded

$$\delta_a = 0.22 \frac{b'}{w} a^{0.89} \qquad (147b)$$

5. Using the indicated differentiation between the contributions of girder friction and normal floor resistance, the foregoing tests in torsional oscillations yielded, for an H-section with closed floor:

$$\delta_a = 0.050 \frac{b}{w} a^{0.50} + 0.030 \frac{d}{w} a \qquad (148a)$$

and, for an H-section with open sidewalk areas:

$$\delta_a = 0.050 \frac{b'}{w} a^{0.50} + 0.060 \frac{d}{w} a \ldots\ldots\ldots\ldots\ldots (148b)$$

(Compare Eqs. 148 with Eqs. 146.)

6. With a light transverse wind blowing across the oscillating section, positive aerodynamic damping was added at small amplitudes (below 1 in. in the model) which changed to negative aerodynamic damping at higher amplitudes. By opening the sidewalks, the positive aerodynamic damping was increased (from 0.0024 to 0.0060), and the negative aerodynamic damping (at the higher amplitudes) was reduced (from 0.0077 to 0.0028). With the roadway removed, or with the entire floor open, the aerodynamic damping δ was negative (tending to produce instability) even at zero amplitude. The most stable combination (against torsional oscillations) was the section with the sidewalks open.

7. The results with transverse wind on the torsionally oscillating section model were consistent with experimental findings by others that in some cases a certain minimum amplitude must be reached before aerodynamic damping becomes negative (δ becomes positive). This critical amplitude in the model was 0.87 in. for the solid section and 1.19 in. for the section with open sidewalks (3.2 ft and 4.4 ft in the prototype), or 37% higher for the section with open sidewalks. Instability was not produced, because the atmospheric damping δ_a far outweighed δ and was increasing far more rapidly.

Structural and Atmospheric Damping.—It appears desirable to separate the atmospheric damping from the structural damping in the formula for the combined positive damping. Such modification of Eq. 35 would be

$$\delta_s = (0.001 + 0.06\,R)\left(1 + 50\,\frac{n}{l}\,\sqrt{a_m\,d}\right) + \delta_a \ldots\ldots\ldots (149)$$

in which the δ_a-term at the end of the expression may be separately formulated and evaluated.

If the amplifying ratio δ is determined from oscillating model experiments as a net logarithmic increment, without separation of active aerodynamic and passive atmospheric effects, it will automatically include a reduction by δ_a; in such case, the δ_a-term in Eq. 149 would be redundant.

Expressions for the atmospheric damping δ_a may be formulated from theoretical considerations and then modified in numerical parameters by experimental determination.

Starting with the elementary formula for air resistance to steady motion of a flat plate, the resulting expression for energy loss ΔW in a cycle of oscillation yields the following formula for atmospheric damping on a flat bridge floor of a span with vertical oscillations:

$$\delta_a = 0.087 \frac{b}{w} a_m \ldots\ldots\ldots\ldots\ldots\ldots\ldots (150a)$$

Accordingly, this contribution to δ_s would vary directly with the amplitude.

The relation given for the simple theoretical case by Eq. 150*a* is modified by the accelerated motion of the oscillating section, and is augmented by the H-form of the cross section and by the side friction of the relative vertical air flow on the vertical faces of the girders. The writer's experiments (see Eqs. 145 to 148) indicate a substantial increase in the numerical coefficient of Eq. 150*a*, together with a reduction in the exponent of the amplitude for the component due to direct pressure on horizontal areas, but with an exponent of unity or higher for the component due to air friction on vertical areas. Provisionally, as a simple and convenient formula to cover the mean effect of all contributions to atmospheric damping, an indicated expression is

$$\delta_a = 0.36 \frac{b}{w} a_m \quad \text{(150}b\text{)}$$

For greater refinement, or for special cases such as partly open floors, Eq. 150*b* may be modified by dividing it into two components as suggested by Eqs. 146.

For torsional oscillations, the same elementary derivation as represented by Eq. 150*a* yields

$$\delta_a = 0.0054 \frac{b}{w} \left(\frac{b}{r}\right)^2 a_m \quad \text{(151}a\text{)}$$

or, with $b^2/r^2 \approx 8$, one half of the value for vertical oscillations. A probable concentration of the air pressure toward the corners of the oscillating section would increase this ratio; and the cable and side friction contributions would tend to make the ratio equal to unity. With the augmented coefficient suggested by the writer's experiments, and pending further test results, the indicated composite formula for atmospheric damping in torsional oscillations is

$$\delta_a = 0.03 \left(\frac{b}{r}\right)^2 \frac{b}{w} a_m \approx 0.24 \frac{b}{w} a_m \quad \text{(151}b\text{)}$$

or two thirds of the value in vertical oscillations.

Failure to separate the passive decrement δ_a, or δ_s (including δ_a), from the active aerodynamic factor δ in tests on oscillating models impairs the consistency, clarity, and generality of the results.

Structural Damping by Diagonal Stays.—For the augmentation of structural damping contributed by diagonal stays, the fundamental energy relations yield the simple formula

$$\Delta\delta_s = \frac{\Delta K}{K} \delta_{st} \quad \text{(152)}$$

in which δ_{st} is the hysteresis damping factor of the material of the stays, and ΔK is the augmentation of K contributed by the stays (see Eqs. 65 and 66). For wire rope ("bridge cable") use $\delta_{st} = 0.025$, and for wire rope strand ("bridge strand") use $\delta_{st} = 0.009$.

For the Tacoma Bridge, by Eq. 152, the single system of inclined stays assumed in Example 6 would have increased δ_s from 0.002 to 0.041, and the double system would have increased δ_s to 0.080, a fortyfold increase.

For the Deer Isle Bridge, by Eq. 152, the initial installation of a few inclined cable stays of wire rope, adding $\Delta K = 102$ to the rigidity, increased δ_s from 0.013 to 0.053, a fourfold increase.

The relationship expressed in Eq. 152 is basic. It may be written in general form:

$$\delta_s \text{ (total)} = \frac{\Sigma(K\,\delta_s)}{\Sigma K} + \delta_a \qquad (153)$$

in which the summations cover all component contributions to K (cables, girders, stays, etc.) and the respective damping factors.

Evaluation of Friction Damping.—If a rope stay, instead of being rigidly secured at both ends, is passed through or around a friction damping device at one end and is kept taut by counterweighting or by connection to another stay of equal and opposite change of length, the resulting contribution of friction damping, expressed as a logarithmic decrement, is

$$\delta_f = 2\,F\left(\frac{a_2 - a_1}{a_m}\right)\sin\psi\,\frac{1}{K\,l\,a_m} \qquad (154)$$

in which F is the constant friction force developed by the friction device. Note, by Eq. 154, that the effectiveness of any friction damping device diminishes with increase of K, l, and a_m. The last is significant; apparently friction damping is the only type of damping that diminishes as the amplitude increases. It is infinite at zero amplitude. It prevents amplification until a certain minimum amplitude is initiated.

For the Tacoma span, with $n = 2$, by Eq. 154, a single 45° floor stay or cable stay between the tower and the quarter point of the span, with a friction clamp developing a friction force of 2,000 lb in the stay, would contribute a positive damping of $\delta_f = 0.016/a_m$. This value would be multiplied by the number of such stays.

The effectiveness of friction brakes installed at the ends of the main span may be similarly determined. For $n = 2, 6, 10, \cdots$ (vertical oscillations, with center stays and no side-span participation) the positive damping contributed by such brakes at the two ends of the span would be

$$\delta_f = 4\,F\left(\frac{16}{3\,n}\,\frac{f}{l}\right)\frac{1}{K\,l\,a_m} \qquad (155)$$

The effectiveness is greatly reduced in the higher modes, since δ_f varies inversely with K and n. The effectiveness also diminishes as the amplitude increases. The value of δ_f in Eq. 155 is diminished by the percentage of side-span participation.

For $n = 1, 3, 5, \cdots$, Eq. 155 may be used upon substituting the span camber f_c for the cable sag f. In these modes, end friction brakes are ineffective if the camber is small.

For the installation on the Bronx-Whitestone Bridge, for $n = 1$ and $n_1 = 1$, Eq. 155 yields $\delta_f = 0.069/a_m$ (neglecting the increase in K by floor stays, which would reduce δ_f).

For the modes $n = 4, 8, 12, \cdots$, the end friction brakes are of zero effectiveness.

With friction brakes installed at the two ends of the span, the longitudinal reaction to be resisted by center stays is augmented by $2F$.

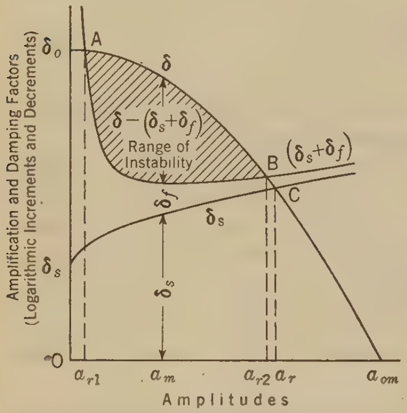

FIG. 27.—GRAPHIC DETERMINATION OF LIMITING AMPLITUDES

Graphic Determination of Limiting Amplitudes.—The relations between the logarithmic amplification and damping factors and the resulting limiting amplitudes are represented graphically in Fig. 27.

The graph of δ, the logarithmic increment, starts at the initial value of δ_0 at zero amplitude. By Eq. 38, it is a parabola, intersecting the amplitude axis at the theoretical limiting amplitude, a_{om}. The reduction of δ with amplitude is due to the curvature of the lift (or torque) graph. By the same assumed correlation, a straight lift (or torque) graph would yield a constant δ, and a lift (or torque) graph curving away from the α-axis would yield a varying δ increasing with amplitude.

The graph of δ_s, representing structural plus atmospheric damping, records the increase with amplitude, as given by Eq. 35 or Eq. 149. The intersection of the ascending δ_s-graph with the descending δ-graph at point C determines the limiting amplitude a_r, terminating amplification.

If friction damping is provided, the ordinates δ_f, varying inversely with amplitude, are added to the ordinates δ_s to give the graph of total positive damping, $\delta_s + \delta_f$. This new graph cuts the δ-graph at two points, A and B, yielding the minimum and maximum limiting amplitudes (for amplification), a_{r1} and a_{r2}. Although δ_f is large at small amplitudes (infinite at zero amplitude), the resulting reduction in the range of instability (from a_r to a_{r2} and from zero to a_{r1}) may be relatively small.

By increasing $\delta_s + \delta_f$ to exceed δ, the range of instability is eliminated.

Oscillating Lift Graphs in the High-Velocity Range.—Vertical amplitude response curves obtained for the writer by Prof. Harold A. Thomas, M. Am. Soc. C. E., are presented in Fig. 28. A wind stream of variable but controlled and measured velocity was applied to the section models which were free to oscillate vertically. The frequency of oscillation was controlled by varying the elastic restraint. The amplitudes were measured on an automatically recorded graph of the oscillations, and are plotted (as corresponding amplitudes in the prototypes) for various velocity ratios, $V/(N\,b)$. For comparison, two different sections were used—the Tacoma section with $d/b = 0.205$, and the Deer Isle section with $d/b = 0.276$. The dimensional difference was so slight that, to the unaided eye, the two models appeared identical. However, the marked difference in aerodynamic behavior of the two sections was "astonishing"; the writer's theory and predictions thus received striking confirmation.

As predicted by the writer's analysis, the Deer Isle section, with $d/b > 0.24$, is vertically unstable in the high-velocity range. The maximum amplitudes increase rapidly with the wind velocity, without any top limit.

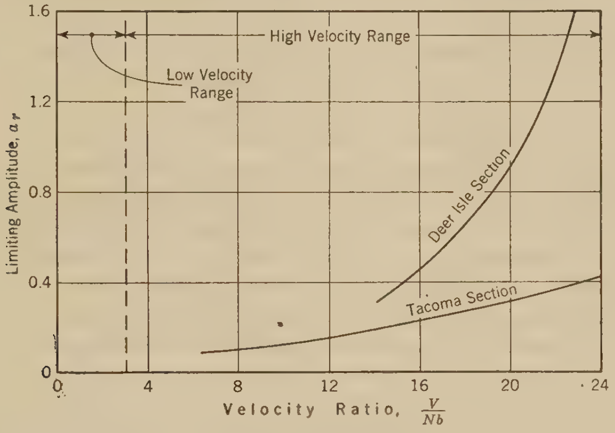

Fig. 28.—Oscillating Lift Graphs (Showing Vertical Instability in the High-Velocity Range)

In the case of the Tacoma section model, with $d/b < 0.24$, the vertical oscillations detected in the high-velocity range represent a secondary effect, little more than a "tremor." The maximum amplitudes in the model, even at velocities corresponding to hurricanes, amounted to only 1/16 in.

Oscillating Torque Graph with Friction Damping.—Fig. 29 records a torsional amplitude response curve, obtained for the writer by Professor Thomas, using a small oscillating model of the Tacoma section. The experimenters found, for each value of $V/(N_t b)$ in the unstable range, two limiting amplitudes, a minimum and a maximum. The lower amplitudes represent an unstable steady state, with stability below the line and amplification above. The upper amplitudes represent a stable steady state, terminating amplification. The region within the bounding curve represents aerodynamic instability.

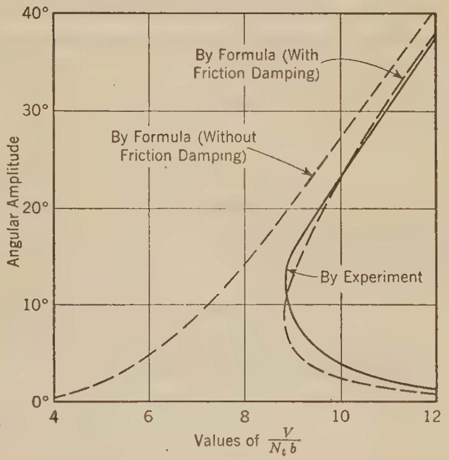

Fig. 29.—Oscillating Torque Graph (Tacoma Section)

This novel response curve is explained by the relatively high friction damping δ_f unintentionally introduced by the use of a center pivot at each end of the oscillating section model. The dotted line shows the theoretical response curve calculated by the writer's formulas without considering friction damping, and the dashed line shows the theoretical response curve calculated by the writer's formulas with friction damping included, assuming constant friction and using estimated values of the damping constants. The agreement between the calculated and observed curves, allowing for the unknown variable elements, is further confirmation of the writer's theory and formulas.

Fig. 29 clearly illustrates the substantial effect of friction damping at low amplitudes and the relatively negligible effect of friction damping at high amplitudes.

Aerodynamic Damping Due to Relative Velocities.—A frictionless pendulum swinging in still air is brought to rest by simple atmospheric damping (δ_a), produced by the velocity (v) of the pendulum. A steady wind (V) in the line of swing will bring the pendulum more quickly to rest; this additional damping is an effect of relative velocities. The pressure opposing the swing into the wind is augmented in proportion to $(V + v)^2$, and the pressure aiding the return swing is reduced in proportion to $(V - v)^2$. Whenever there is energy withdrawal, the addition of velocities augments the resisting pressure; whenever there is energy input, the subtraction of velocities reduces the effective pressure. Hence, this contribution of aerodynamic damping is always positive. It may greatly exceed the simple atmospheric damping δ_a. Because this contribution of aerodynamic damping is, in effect, an addition to δ_a but depends upon V, it is denoted subsequently by δ_{aV}.

In the pendulum analogy, the net energy input in a cycle, per unit of normal surface, is

$$\Delta W = \oint C \frac{\rho}{2} (V^2 + v^2 \pm 2\, V\, v)\, v\, dt \ldots\ldots\ldots\ldots (156a)$$

The three terms in this expression, divided by 2 W, yield, respectively, δ, δ_a, and δ_{aV}. (In this case, a pendulum under steady longitudinal wind, $\delta = 0$.)

Similarly, in the case of a vertically oscillating bridge section, the net energy input per cycle is

$$\Delta W = \int_0^l \int_0^b \oint C \frac{\rho}{2} [(V_L)^2 + v^2 \pm 2\, V_L\, v]\, v\, dt\, db\, dx \ldots\ldots (156b)$$

in which V_L, the mean effective lift component of V at any relative instant, is given by

$$V_L = V \sqrt{(C_L)'/C} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (157)$$

The integration of the first term of Eq. 156*b*, divided by 2 W, yields the negative aerodynamic damping:

$$\delta = -\frac{1}{4} \beta\, s' \left(\frac{\rho\, b^2}{2\, m}\right)\left(\frac{V}{N\, b}\right) \ldots\ldots\ldots\ldots\ldots (158a)$$

which is identical with the several expressions previously presented.

The second term, divided by 2 W, yields the simple atmospheric damping:

$$\delta_a = \frac{32}{9\,\pi} C \left(\frac{\rho\, b^2}{2\, m}\right) \frac{a_m}{b} \ldots\ldots\ldots\ldots\ldots\ldots (158b)$$

in which the aerodynamic pressure constant $C \approx 2$. (This reduces to Eq. 150*a*.) Note that δ_a is independent of N, K, and V and, hence, may be easily determined by testing a section model in still air. The increase of this damping with amplitude helps to restrict maximum amplitudes.

The third term, divided by 2 W, yields the aerodynamic damping due to relative velocities:

$$\delta_{aV} = \frac{1}{2}\left(\frac{\rho\, b^2}{2\, m}\right)\left(\frac{V}{N\, b}\right)\sqrt{\pm\, C\,(C_L)'} \tag{159a}$$

in which $(C_L)'$ denotes the mean effective value of the lift graph ordinate (disregarding sign) as it varies over the range $\alpha \pm \Delta\alpha$. If this range of the lift graph traversed during a cycle is approximately straight and lies entirely above or below the axis, $(C_L)'$ may be taken simply as the ordinate at the specified angle of attack α. However, if the involved range of the lift graph crosses the horizontal axis, so that $(C_L)'$ changes sign during the cycle, the value to be used in Eq. 159*a* is given, with sufficient accuracy, by the mean effective value—

$$(C_L)' = \frac{8}{\pi}\frac{N}{V}\, a_m\, s' \tag{159b}$$

This yields

$$\delta_{aV} = 0.8\left(\frac{\rho\, b^2}{2\, m}\right)\sqrt{\pm\, C\, s'\left(\frac{a_m}{b}\right)\left(\frac{V}{N\, b}\right)} \tag{159c}$$

As a rule, use either Eq. 159*a* or Eq. 159*c*, whichever yields the higher value of δ_{aV}. At angles of attack corresponding to large ordinates of the lift graph, Eq. 159*a* governs; at angles of attack corresponding to small intercepts $(C_L)'$, the effect of $\pm\, \Delta\alpha$ becomes important and Eq. 159*c* governs.

Since δ_{aV} is always positive, it is most conveniently grouped with the positive damping contributions (δ_s and δ_a), instead of being incorporated in the formulas as a reduction of δ.

For the Tacoma section, in the critical range for vertical oscillations, δ_{aV} by Eq. 159*c* is seven or more times as large as δ_a by Eq. 158*b*.

Note that δ_{aV} increases with increasing V and, in the more usual case (Eq. 159*c*), with increasing a, but diminishes with increasing N. This helps to explain limiting amplitudes, as well as the cutting off of a prevailing mode with increasing wind velocity. Most important, however, δ_{aV} supplies a most welcome additional safety factor—a contribution of positive damping that is most effective when it is most needed, namely, at high wind velocities, at high amplitudes, and in the lower modes. This is the only positive damping (other than $-\delta$) that increases with wind velocity.

The same reasoning applied to torsional oscillations yields a corresponding series of expressions:

$$\delta = -\frac{1}{8}\,\beta\, s_t\left(\frac{b}{r}\right)^2\left(\frac{\rho\, b^2}{2\, m}\right)\left(\frac{V}{N_t\, b}\right) \tag{160a}$$

$$\delta_a = \frac{1}{9\,\pi}\, C\left(\frac{b}{r}\right)^2\left(\frac{\rho\, b^2}{2\, m}\right)\theta_m \tag{160b}$$

and

$$\delta_{aV} = \frac{1}{14}\left(\frac{b}{r}\right)^2\left(\frac{\rho\, b^2}{2\, m}\right)\left(\frac{V}{N_t\, b}\right)\sqrt{\pm \frac{3}{2} C \left(C_T, \text{ or } \frac{4}{\pi^2}\, \theta_m\, s_t \right)} \quad \text{..(160}c\text{)}$$

in which θ_m is the angular amplitude at midsegment. In Eq. 160*c*, either the ordinate C_T or the slope term is used, whichever is larger. As a rule, the involved range of the torque graph crosses the horizontal axis; hence, in most applications, the slope term governs.

For vertical oscillations, convenient expressions for δ_{aV} may be written from Eqs. 158*a* and 159:

$$\frac{\delta_{aV}}{\delta} = \frac{\sqrt{\pm 8\, (C_L)'}}{-\beta\, s'} \quad \text{.....................(161}a\text{)}$$

or

$$\frac{\delta_{aV}}{\delta} = \frac{4.5\, \sqrt{a_m}}{-\beta\, \sqrt{\pm\, s'\, V/N}} \quad \text{...................(161}b\text{)}$$

If either of these fractions equals or exceeds unity, vertical stability is assured. Eq. 161*a* determines critical and stable angles of attack, by the ordinates $(C_L)'$ of the lift graph. Eq. 161*b* determines a limitation of amplitudes at any velocity ratio V/N.

Similarly, for torsional oscillations, from Eqs. 160*a* and 160*c*:

$$\frac{\delta_{aV}}{\delta} = \frac{\sqrt{\pm\, C_T}}{-\beta\, s_t} \quad \text{.......................(162}a\text{)}$$

or

$$\frac{\delta_{aV}}{\delta} = \frac{\sqrt{0.4\, \theta_m}}{-\beta\, \sqrt{\pm\, s_t}} \quad \text{....................(162}b\text{)}$$

If either of these fractions equals or exceeds unity, torsional stability is assured. Eq. 162*a* determines critical and stable angles of attack, by the ordinates C_T of the torque graph. Eq. 162*b* determines limitations of amplitude θ_m. These limitations of torsional amplitude are virtually independent of wind velocity and of mode in the high-velocity range. (Structural and atmospheric damping and curvature of the lift and torque graphs will contribute further restriction of critical ranges and limiting amplitudes.)

By Eq. 161*a*, the Tacoma section (using the California Institute of Technology lift graph) had maximum vertical instability at $\alpha = 0°$ and $-10°$, and was vertically stable at all angles of attack exceeding $\alpha = +9°$ and $-12°$. Eq. 161*b* yields, for the Tacoma section, limiting vertical amplitudes of 1.6 ft for all modes, without considering other damping effects.

Correction of Torque Graph for Center of Rotation.—In the static torque graph for a section, the torques represented by C_T are obtained or plotted for an assumed center of rotation. For any specified raising of the center of rotation by an amount z, the ordinates of the modified torque graph are given by

$$(C_T)' = C_T - \frac{z}{b} C_D \quad \text{.....................(163)}$$

Raising the center of rotation thus depresses the torque graph relative to the horizontal axis.

In any application in which the torque-graph ordinates are used to determine the magnitude and distribution of the vertical pressures (or vertical velocity components), the torque due to the horizontal drag forces (represented by C_D) should first be eliminated by subtraction. For this purpose, the assumed center of rotation should be lowered to the level of the center of pressure of the drag forces. With this shift of centers denoted by $-z$, the ordinates of the torque graph "corrected for drag" are given by Eq. 163.

Significant Characteristics of Static Lift Graphs and Torque Graphs.—There are three significant characteristics of static lift graphs and torque graphs that affect the stability or instability of a section—(1) slope, (2) curvature, and (3) ordinates.

(1) A positive slope yields a "stable section"; that is, one that will be essentially stable in the high-velocity range. A negative slope yields an "unstable section"; that is, one that is potentially unstable in the high-velocity range. Both types have (alternate) ranges of potential instability in the low-velocity range. Reducing the slope reduces the potential instability. Changing the angle of attack to a point of reduced slope reduces the potential instability. When the amplitudes correspond to traversal of the lift graph or the torque graph into regions of diminished or reversed slope, the intensity of amplification (δ) diminishes with increasing amplitude and a limiting amplitude is thereby determined.

(2) The curvature of the lift graph or torque graph (represented by s'/i^2 or s_t/i^2) affects or determines the limiting amplitudes by determining the reduction of mean effective slope with increasing amplitude. Curvature toward the horizontal axis has a tempering or limiting effect. Curvature away from the horizontal axis increases instability (δ) with increasing amplitude. A graph of zero initial slope curving away from the horizontal axis represents stability until certain initial amplitudes are imposed; the smaller the curvature is, the greater are the critical initial amplitudes required to start amplification.

(3) The ordinate, $(C_L)'$ or $(C_T)'$, of the lift or torque graph at any angle of attack α determines the significant positive damping contribution δ_{aV}. If the ordinate is sufficiently large, the resulting value of δ_{aV} will exceed δ (or $\delta - \delta_s$), producing stability.

It should be noted that the three parameters—slope, curvature, and ordinates—are to be measured at the abscissa α, representing the angle of attack, and that, for any material amplitude, the corresponding range of the graph is to be considered, and the mean effective value of each parameter over that range is to be used.

Confirmation by Critical Angles of Attack.—Professor Farquharson, in his discussion, mentions a section that was aerodynamically stable in horizontal wind but, unlike all sections previously tested, revealed pronounced torsional instability at certain critical positive and negative angles of attack. Those results are explained and duplicated in Fig. 30. The slope, curvature, and ordinates of the static wind-tunnel graph supply the keys for Fig. 30.

Because the section was vertically unsymmetrical, the torque graph was depressed vertically and displaced horizontally by substantial amounts. Even after correction for the moments of the drag forces about the elevated center of rotation used in the wind tunnel, the reduced negative ordinates are sufficiently large to produce stability, by Eq. 162a, at flat angles of attack.

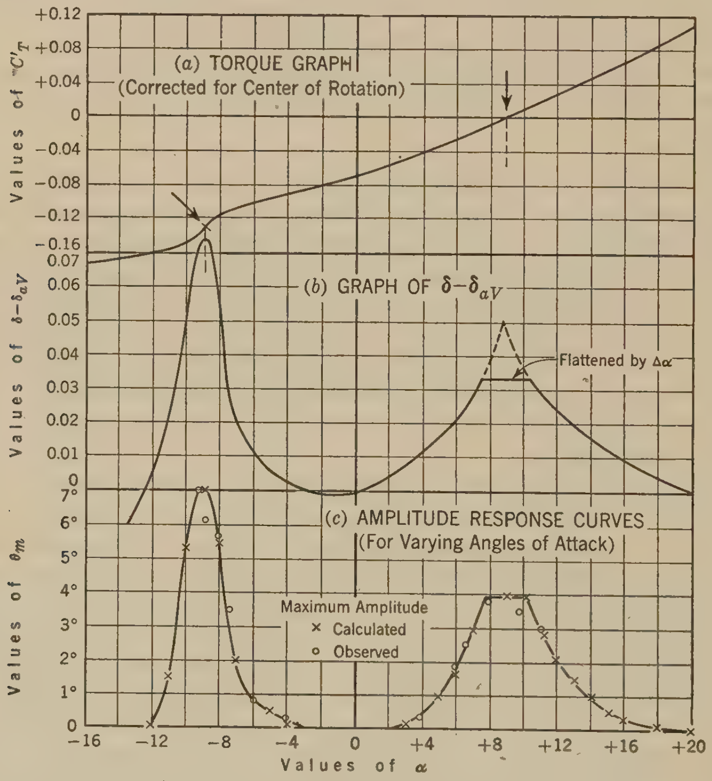

FIG. 30.—CONFIRMATION OF THEORY BY AMPLITUDE RESPONSE CURVES AT CRITICAL ANGLES OF ATTACK

The torque graph, corrected for the drag moments by Eq. 163, is plotted in Fig. 30(a). The ordinates $(C_T)'$ of this modified graph are used in calculating δ_{aV} (by Eq. 160c), and the slopes s_t of the original torque graph are used in calculating δ (by Eq. 160a). The resulting values of $(\delta - \delta_{aV})$ are plotted in Fig. 30(b). These ordinates, representing instability, are zero or negligible in the middle range (α = from $-3°$ to $+1°$), on account of the depressed torque graph yielding high values of δ_{aV}. The localized steep slope of the graph at $\alpha = -9°$ produces maximum instability at that angle of attack; and the zero intercept of the (corrected) torque graph produces a second (lower) peak of instability at $\alpha = +9°$. This latter peak is truncated, however, by the δ_{aV} produced by the amplitude (see Eq. 160*c*).

The maximum amplitudes, calculated by the writer's formulas from the values of $(\delta - \delta_{aV})$, are plotted in Fig. 30(*c*), and in the same curves are plotted the actual limiting amplitudes obtained and recorded in the Seattle model tests. The agreement, remarkable as it is, may be brought even closer to identity by minor adjustments of the assumed constants.

The extreme right-hand branches of the graphs in Figs. 30(*b*) and 30(*c*) are found, in the computations, to be highly sensitive to minor variations in the slope of the corresponding portion of the torque graph, and may become horizontal, ascending, or wavering with such slope variations. This explains why that portion of the amplitude response curve obtained in the model tests varied in range and in form with various modifications of the section, as with the application of streamlining, "spoilers," special railings, open curbs, etc.

Method of Predicting Critical Velocities.—An essential constant in predicting the aerodynamic behavior of any given section at any given wind velocity is the critical velocity ratio $V_p/(N\,b)$, constituting the line of demarcation between stability and potential instability. It has different values for vertical and for torsional oscillations, and it varies with the section ratio d/b. It is defined by the condition $\beta = 0$.

A general expression for β may be written in simple form. In vertical oscillations,

$$\beta = \frac{\int_0^b p_x \cos\phi\, dx}{\int_0^b p_x\, dx} \qquad \text{(164}a\text{)}$$

and, in torsional oscillations,

$$\beta = \frac{\int_0^b p_x \left(\frac{b}{2} - x\right) \cos\phi\, dx}{\int_0^b p_x \left(\frac{b}{2} - x\right) dx} \qquad \text{(164}b\text{)}$$

in which x is measured across the width b of the section, and p_x is the corresponding intensity of aerodynamic pressure. If the law of variation of p_x were known, all unknowns of the aerodynamic behavior of the section would be given by simple integration. The denominator of Eq. 164*a* is the static lift force L', and that of Eq. 164*b* is the static torque T. The numerator determines aerodynamic stability or instability, in both sign and intensity. Any value of ϕ' (total phase difference) that makes the numerator zero gives a critical velocity ratio $(V/(N\,b) = 2\,\pi/\phi')$.

The law of variation of p_x across an H-section, at different angles of attack, may be determined experimentally. (See G. Eiffel's classic experiments[91] on flat plates.)

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[91] "La Resistance de l'Air," by G. Eiffel, Dunod and Pinat, Paris, 1910; see also "The Resistance of the Air and Aviation Experiments Conducted at Champ-de-Mars Laboratory," by G. Eiffel, 2d Ed., translated by Jerome C. Hunsaker, Houghton, Mifflin Co., Boston and New York, 1913.

In the case of an **H**-section, there is a region of suction or negative pressure for some distance (approximately 2 d) behind the windward girder.[91] An upward angle of attack produces a positive (upward) pressure beyond this shielded range. The fraction j of the width, representing the effective division point, may be expressed as a function of the depth ratio d/b. The resultant pressures herein considered also include the contributions (positive or negative) on the reverse face of the horizontal member.

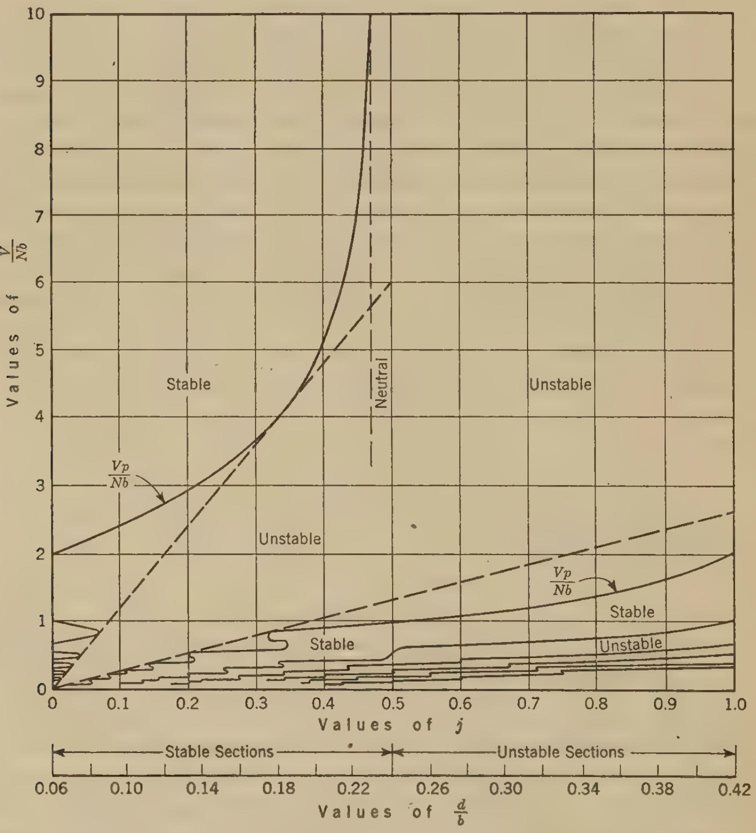

FIG. 31.—CRITICAL VELOCITY RATIOS FOR VERTICAL OSCILLATIONS

For preliminary exploration of the ranges of stability and instability, an assumed law of variation of p_x may be applied. One of the simplest assumptions, yielding reasonably consistent results, is a uniform pressure p_x, positive beyond the shielded fraction of the width and negative (downward) in the shielded range.

Substitution of this simple p_x-relation in the denominators of Eqs. 164 yields values of L' and T that follow the general variation of the known slopes of the static lift and torque graphs for different section ratios. By this matching, an indicated expression for j is

$$j = 2.8\left(\frac{d}{b} - 0.06\right) \ldots\ldots\ldots\ldots (165a)$$

An alternative expression, checking known test results even more closely, is

$$j = 10\left(\frac{d}{b}\right)^2 \ldots\ldots\ldots\ldots (165b)$$

The dividing lines between stable and unstable sections are $j = 0.50$ in lift and $j = 0$ in torsion, corresponding to $d/b = 0.238$ and $d/b = 0.06$, respectively, by Eq. 165*a*.

Substitution of the same simple relations in the numerators of Eqs. 164 yields the critical values of $V/(N\,b)$. These values, for vertical oscillations, are plotted in Fig. 31, thereby charting the critical ranges of stability and instability for varying section ratios d/b. This chart informatively presents the contrasting response of "stable" and "unstable" sections. It clearly shows that V_p is an upper limit of instability in stable sections and a lower limit of instability in unstable sections. Between the two limits is a narrow neutral band of minimum vertical instability at all wind velocities. A corresponding chart for torsional instability is similarly obtained.

Although predicated on a simplifying asumption, this preliminary illustration of a rational method of predicting critical velocities yields results that closely check recorded experimental observations. Fig. 32 shows how the graph of $V_p/(N_t\,b)$ in torsion, given by this simplified theory, agrees almost identically with experimental results obtained by Theodor von Kármán, M. Am. Soc. C. E. Mr. von Kármán's results, plotted on b/d instead of d/b, would fall almost exactly on a straight line through the origin:

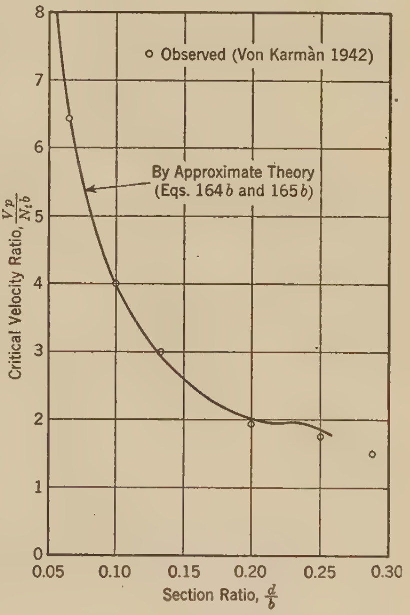

Fig. 32.—Critical Velocity Ratios for Torsional Instability for Different Section Ratios; Comparison of Theory With Recorded Observations

$$\frac{V_p}{N_t\,b} = 0.4\,\frac{b}{d} \ldots\ldots\ldots\ldots (166)$$

In other words, the critical velocities in torsion vary almost exactly inversely with d. This surprising relation, obtained in section model tests, is directly the opposite of all predictions by the vortex theory; the writer's theory supplies the explanation and the predictive foundation.

Method of Predicting Instability (δ) *at Various Wind Velocities.*—By applying Eqs. 164 for β, with the same simplified law of variation of p_x as was assumed for Fig. 31, graphs are obtained showing the variation of δ with $V/(N\,b)$ for different section ratios. The graph for vertical instability is plotted in Fig. 33. It clearly shows the governing difference between vertically "stable" and "unstable" sections. The unstable sections (with $d/b > 0.24$) have an unlimited critical range yielding unlimited increase of δ with wind velocity. The optimum section ratio ($d/b \approx 0.24$) has the lowest value of maximum δ. As the section ratio is made progressively smaller, the critical range of $V/(N\,b)$ shrinks in width but becomes steeper, reaching a maximum δ at $d/b \approx 0.17$. At still lower values of d/b, the height of the instability graph progressively diminishes, approaching a second optimum ratio at $d/b = 0$ (a flat plate).

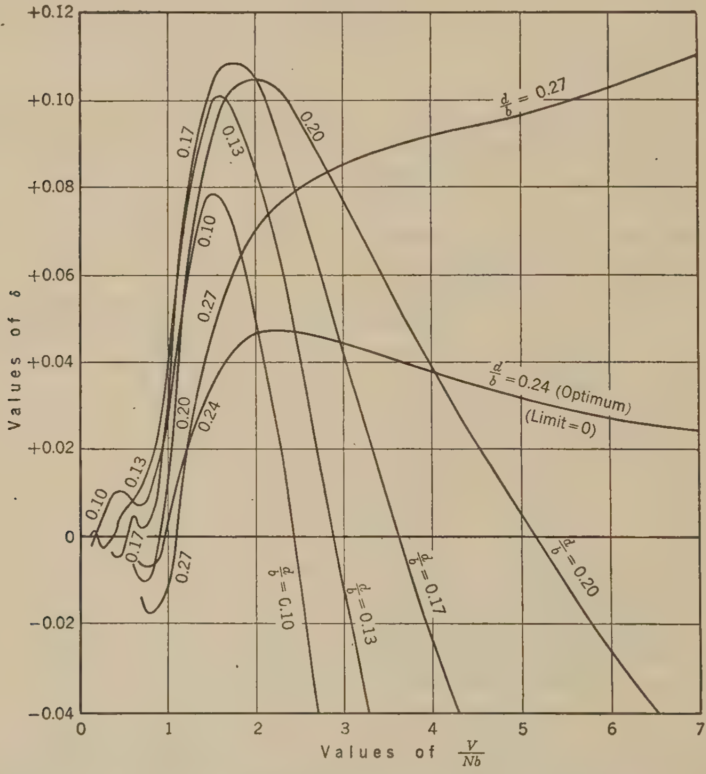

Fig. 33.—Graphs for Predicting Vertical Instability δ at Various Wind Velocities $V/(N\,b)$, for Various Section Ratios d/b (plotted for $b^2/w = 0.50$. For Other Values, Increase δ in Proportion)

Critical Velocities for Dominance of Torsional Instability.—For any given section ratio, if the δ-graphs for vertical and torsional instability are superimposed with the necessary adjustment of horizontal scale, the point of intersection of the two graphs gives the critical velocity ratio at which torsional instability becomes dominant. This is illustrated in Fig. 34 for two section ratios (using $N_t \approx 1.5\,N$). For the Tacoma section ($d/b \approx 0.20$), torsional instability becomes dominant at $V/(N_t\,b) \approx 2.9$, corresponding to $V \approx 15$ miles per hr. For the Deer Isle section ($d/b \approx 0.27$), torsional instability does not become dominant until $V/(N_t\,b) \approx 9.7$, corresponding to $V \approx 31$ miles per hr for the bridge without stays and to $V \approx 63$ miles per hr for the bridge with the present system of longitudinal stays.

The intersecting lift and torque instability graphs plotted for the Tacoma section by the theoretical formula in Fig. 34 are almost identical with the corresponding intersecting graphs of amplitude response obtained in large-scale oscillating model tests.

The Tacoma Bridge had potential torsional instability at 15 miles per hr, but torsional oscillations were prevented until the overstressed midspan stays failed. This happened to occur during a wind of 42 miles per hr. The higher wind velocity and the vertical mode $n = 9$ prevailing at the time had only an accidental relation to the appearance of torsional instability.

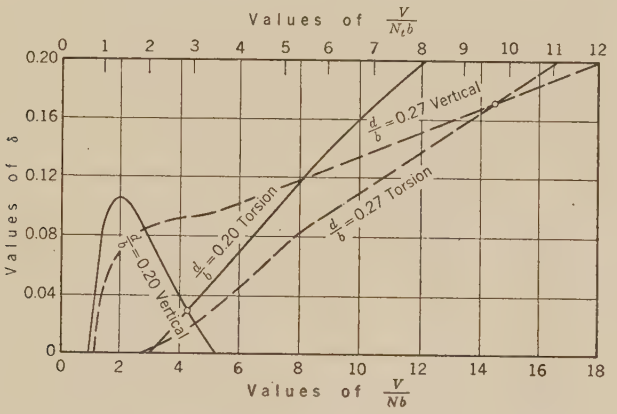

FIG. 34.—CRITICAL VELOCITIES FOR THE DOMINANCE OF TORSIONAL INSTABILITY (BY EQS. 164)

Charts for Predicting Instability at Various Wind Velocities.—Fig. 35, plotted from Eq. 164*b*, charting the values of δ in torsion in the high-velocity range, gives, and explains, results that are almost identical with results obtained experimentally in oscillating model tests. The δ-contours in Fig. 35 for torsionally "unstable" sections show steady slopes of unlimited increase of δ with wind velocity.

All these graphs (Figs. 31 to 35) are presented to illustrate governing relations, but are subject to quantitative modification by perfected analysis and more adequate test data. The graphs introduce and exemplify a method of analysis and prediction, but require further check and adjustment before they can be used for design.

Critical Velocities as Affected by Width of Section.—From the elementary relation of $V/(N\,b)$, some engineers have concluded that critical velocities are proportional to the width b of the section. The true relation is not so simple; in fact, at critical high velocities, the relation is reversed.

The general formula for potential instability in the high-velocity range may be written (see Eqs. 44, 45, 59 and Figs. 26 and 33):

$$\delta = k\,B\left(\frac{V - V_p}{N\,b}\right) \tag{167}$$

in which $B = b^2/w$ (hereinafter termed the "width factor"), and k is a coefficient times s' or s_t. Note that

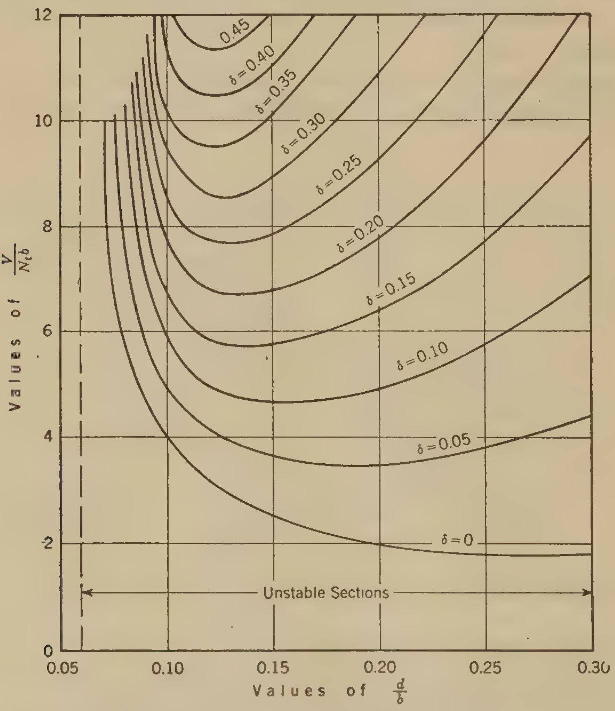

FIG. 35.—TORSIONAL INSTABILITY CURVES OF δ FOR VARIOUS SECTION RATIOS AT VARIOUS VELOCITY RATIOS

$$N\,b = 0.904\sqrt{K\,B} \tag{168}$$

From these relations, the critical velocity is given by

$$V_c = 0.904\sqrt{K}\left(\frac{V_p}{N\,b}\sqrt{B} + \frac{\delta_s}{k\sqrt{B}}\right) \tag{169}$$

in which $V_p/(N\,b)$ is a constant (independent of width) for a given section ratio. For $\delta_s = 0$, Eq. 169 yields $V_c = V_p$. The first term in the parenthesis contains $\sqrt{B}$, whereas the second term contains the reciprocal. For bridges of equal b/w, the width factor B is proportional to the width b. Hence, by Eq. 169, for a given value of K, critical velocity increases directly not with the width but with the square root of the width at zero positive damping ($V_c = V_p$), and inversely with the square root of the width at high positive damping. In other words, V_p is proportional to $\sqrt{B}$, but $(V_c - V_p)$ is proportional to $1/\sqrt{B}$. At values of V_c greater than 2 V_p, the second relation dominates. Design conditions to secure stability will usually require such a high value of $(V_c - V_p)$. Therefore, in the critical high-velocity range governing design, other factors being equal, a wider bridge will have lower critical velocities.

For aerodynamic similitude in model tests, the model must have the same value of the width factor B as the prototype. Erroneous conclusions as to critical velocities in torsion were reached in the tests for the official investigation of the Tacoma failure because all the models had lower values of B than the prototype, and consequently yielded higher critical velocities. As a result, the investigators mistakenly concluded that the critical velocity was 25 miles per hr, whereas correction for B makes the critical velocity 15 miles per hr.

Eq. 169 checks and explains the relation recorded empirically in Eqs. 57. Actually, $V_p/(N_t\, b)$ should be a constant for a given section, independent of B. A true value of $V_p/(N_t\, b)$ of approximately 2.9 for the Tacoma section, with a residual positive damping (in the models) of $\delta_s \approx 0.018$, would yield the values of $V_c/(N_t\, b)$ ranging from 3.3 to 5.1 actually observed for the models with different values of B. Hence, Eqs. 57 do not define the true V_p at zero damping but a higher V_c at a residual positive damping that was overlooked by the investigators. More recent tests on models of the Tacoma section yield values of from 2.6 to 2.8 for $V_p/(N_t\, b)$ in torsion. Other cross sections yield different values of $V_p/(N_t\, b)$, ranging from 0.75 to infinity. These results are shown both by the writer's theory and by experiment.

The conclusion that $V_p/(N_t\, b)$ is a constant for a given section but varies for different sections (see Fig. 32) supersedes Eqs. 57 and 61, and Eq. 169 supersedes Eqs. 62 and 64 and the related discussion.

Eq. 169, valid for both vertical and torsional oscillations, retains an important point—namely, that the steepness of the δ-graph is proportional to the width factor B. This affects critical velocities, amplitudes, rates of amplification, and required values of K and of structural damping. By making comparisons at small values of δ_s and, hence, at small values of $V_c - V_p$ (as Mr. Balog has done in his discussion), the effect of the relative steepness of the δ-graphs is largely lost. At high values of δ_s (or of $V - V_p$) the effect of the width factor B becomes more marked, especially in respect to amplitudes and rates of amplification.

Measures of Instability Related to Structural Proportions.—Three different measures of aerodynamic instability are: (1) Rate of amplification, (2) critical wind velocity, and (3) maximum amplitude. Simplified formulas reveal the variation of these measures of instability with governing structural factors.

The width factor $B = b^2/w$, representing in more practical form the nondimensional density-mass ratio $\rho\, b^2/(2\, m)$, appears as a factor in all formulas for aerodynamic damping, both positive and negative.

By Eqs. 1, 21, 29, and 51, a simplified formula for the rate of amplification is

$$\delta = k\, V \sqrt{B/K} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (170a)$$

in which, for vertical oscillations,

$$k = -0.0106\,\beta\,s' \qquad \text{(170}b\text{)}$$

and, for torsional oscillations,

$$k = -0.0106\,\beta\,s_t\left(\frac{b}{r}\right) \qquad \text{(170}c\text{)}$$

Similar sections at high wind velocities have like values of k. Hence, for bridges with similar cross sections or equivalent section ratios, at equal high wind velocities:

(1) Rate of Amplification is proportional to $\sqrt{B/K}$.

Critical velocity V_c is defined by applying the stable-state condition ($\delta = \delta_s$) at zero amplitude. Hence, by Eq. 170a,

$$V_c = \frac{1}{k}\,\delta_s\sqrt{\frac{K}{B}} \qquad \text{(171)}$$

in which δ_s is the positive damping at zero amplitude. Accordingly, in the high-velocity range, for bridges with similar cross sections or equivalent section ratios, with equal structural damping at zero amplitude:

(2) Critical velocities are proportional to $\sqrt{K/B}$.

Let $F(a)$ be the positive damping function of amplitude; that is,

$$F(a) = \delta_s = k_1 + k_2\,a + k_3\sqrt{a} + \frac{k_4}{a} \qquad \text{(172}a\text{)}$$

In this general expression, the successive terms represent the zero amplitude damping constant, the atmospheric damping, the hysteresis damping, and the friction damping. The hysteresis damping may also be covered by the second term. If all the terms with the exception of the second term are zero or relatively negligible, $F(a)$ is proportional to a. In general, $F(a)$ must increase with amplitude; otherwise (except for curvature of the static lift and torque graphs), there would be no limitation of amplitudes. By Eq. 170a, the maximum amplitude, defined by the steady-state condition, $\delta = \delta_s$, is given in terms of $F(a)$ by

$$F(a) = k\,V\sqrt{\frac{B}{K}} \qquad \text{(172}b\text{)}$$

Hence, for bridges of similar cross sections or equivalent section ratios, at equal high wind velocities:

(3) The maximum amplitudes, or their functions $F(a)$, are proportional to $\sqrt{B/K}$.

Each of the foregoing governing relations of proportionality (conclusions 1 to 3) contains the stiffness ratio K/B (or its reciprocal). The three measures of instability may be consolidated to read:

(1), (2), (3) Safety against aerodynamic instability is
proportional to $\sqrt{K/B}$.

Comparisons of different bridges based on K alone yield roughly satisfactory results because B is fairly constant at approximately 0.4 to 0.6 for most bridges. For wide bridges of normal or reduced weight per square foot, however, the value of B may be doubled—thus adding 41% to the instability as computed from K alone.

For bridges of equal weight per square foot, B is proportional to b. Under that limitation, K/b represents K/B and becomes an equivalent measure of stability (compare with Table 2).

Although actually the square root of the "stability factor" K/B appears in the proportionality relations, the desired objective is accomplished by specifying a minimum value of K/B. On the statistical basis of the known behavior of various structures, the indicated minimum specification is

$$\frac{K}{B} > 300 \qquad (173)$$

The formula value of K for $n = 2$ is to be used, since the mode $n = 1$ will rarely give a value materially lower. The values of K and w are for half widths.

Table 16 lists suspension bridges in the order of their stability factors K/B, and Fig. 36 presents the corresponding information graphically. With the exception of the George Washington Bridge with $K/B = 344$, no bridges with K/B greater than 300 have ever, as far as known, shown any traces of aerodynamic instability. All bridges with K/B less than 300 have evidenced aerodynamic instability, in varying degree roughly corresponding to the inverse order of the (K/B)-values.

TABLE 16.—BRIDGES LISTED IN DESCENDING ORDER OF STABILITY FACTOR, K/B

Key	Bridge	K	B	K/B	$\sqrt{K/B}$
W	Williamsburg	1,082	0.27	4,007	64
M	Manhattan	1,236	0.79	1,565	40
MR	Maumee River	648	0.53	1,218	35
PC	Philadelphia	735	0.61	1,205	35
TB	Transbay	324	0.46	697	26
WR	Wabash River	327	0.47	695	26
MH	Mid-Hudson	262	0.40	655	26
Tri	Triborough	547	0.96	570	24
WH	Waldo-Hancock	212	0.41	515	23
MtH	Mount Hope	201	0.43	463	22
D	Detroit	212	0.56	375	19
SJ	St. Johns	252	0.67	373	19
GW	George Washington	244	0.71	344	18.5
GM	Grand Mère	188	0.55	339	18.5
TI	Thousand Islands	141	0.58	243	15.5
TI	Thousand Islands	127	0.58	219	15
LG	Vancouver	112	0.69	161	13
GG	Golden Gate	116	0.77	151	12
DI	Deer Isle	69	0.46	150	12
BW	Bronx-Whitestone	141	0.99	142	12
TA	Tacoma	62	0.53	117	11

Individual variations, as between the George Washington and Grand Mère bridges (the latter with higher δ_s) or between the Bronx-Whitestone and Deer Isle bridges (the latter with negative s'), are explained by differences in structural damping and in the aerodynamic characteristics of the cross sections. Eq. 173 cannot cover such differences; nor can any other generalized rule.

A more scientific specification, taking into account variations in s', s_t, and δ_s would be, from Eq. 171,

$$\delta_s \sqrt{K/B} > k\,V \qquad (174)$$

to assure a critical velocity (zero amplitude) above any specified V. It is helpful to note that

$$\sqrt{\frac{K}{B}} = 1.1\, N \left(\frac{w}{b}\right) \qquad (175)$$

Criteria for Aerodynamic Stability.—To aid in quickly checking the aerodynamic stability of a proposed bridge design, three alternate criteria, of progressively increasing scientific accuracy, are developed and presented, as follows:

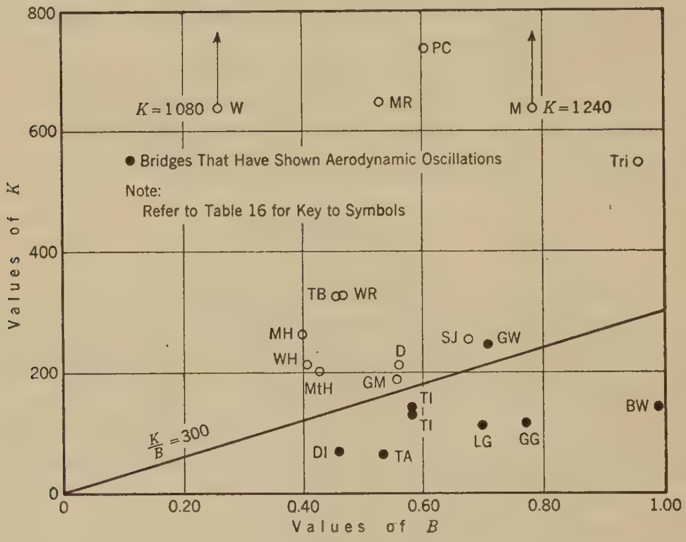

FIG. 36.—CHART OF K/B FOR VARIOUS SUSPENSION BRIDGES

1. The first criterion, recorded in Eq. 173 and illustrated in Table 16 and Fig. 36, is of simplified rational form on a statistical base. It may be simply stated:

The stiffness ratio K/B shall exceed 300.

2. The second criterion is derived from the more complete governing relation recorded in Eq. 174. Since the structural damping is essentially proportional to R, Eq. 174 yields the conclusion that the structural constant $R\sqrt{K/B}$, subsequently termed the "aerodynamic stability constant," determines the highest wind velocity of assured stability. Values of this constant for different suspension bridges are presented in Table 17. The values of $R\sqrt{K/B}$ give an order of listing that agrees with the known relative aerodynamic stability of the different bridges more consistently than any of the preceding criteria. The bridges at the bottom of the list have been the most unstable. All bridges with a value of $R\sqrt{K/B}$ less than 5 have been subject to aerodynamic oscilla-

tions. All bridges with this constant greater than 5 are in the aerodynamically stable classification. This relation yields the second criterion.

The aerodynamic stability constant $R\sqrt{K/B}$ shall exceed 5.

3. For a more completely rational criterion, the variation of potential instability with the form and proportions of the cross section must be considered. For different section ratios d/b, curves of potential torsional and vertical instability, at an assumed wind velocity of 100 miles per hr, are plotted in Fig. 37, using the computed maximum values of $\delta\sqrt{K/B}$ as ordinates. For assured stability, the structural damping δ_s must equal or exceed the maximum δ. Consequently these curves also give the required values of $\delta_s\sqrt{K/B}$. By comparison with the known behavior of these structures, the indicated appropriate values of δ_s are 0.25 R for girder bridges and 0.50 R for truss bridges. Using these coefficients, the estimated values of $\delta_s\sqrt{K/B}$ for the different bridges are given in Table 17 alongside the required values. The ratio, recorded in the final column, is the "factor of safety against aerodynamic instability." The Williamsburg Bridge has the highest ratio, 5.3, and the Tacoma Bridge has the lowest ratio, 0.02. (Multiplying by 100 miles per hr or some lower figure gives a rough idea of the wind velocity at which marked instability may appear.) All bridges for which this ratio is less than unity have been subject to aerodynamic oscillations; and, as far as known, all bridges for which this ratio is greater than unity have been aerodynamically stable. The same test is applied graphically in Fig. 37, with differentiated horizontal and vertical scales for girder and truss bridges, also using the higher ratio of $\delta_s : R$ for truss bridges. All bridges plotted below the stability graph are in the unstable category, and those above the graph are in the stable classification.

TABLE 17.—APPLICATION OF CRITERIA FOR AERODYNAMIC STABILITY

Bridge	R	$R\sqrt{K/B}$	Effective $\frac{d}{b}$	$\delta_s\sqrt{K/B}$		
				Required	Estimated	Ratio, $\frac{\text{Col. 6}}{\text{Col. 5}}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Williamsburg	0.78	50	0.30	4.7	25	5.3
Manhattan	0.69	28	0.13	3.6	14	3.9
Wabash River	1.00	26	0.12	3.5	6.5	1.9
Philadelphia	0.56	20	0.16	3.1	10	3.2
Maumee River	0.49	17	0.17	2.8	4.2	1.5
Grand Mère	0.66	12	0.23	1.8	6.0	3.3
Mid-Hudson	0.44	11	0.24	1.7	5.5	3.2
Transbay	0.38	10	0.23	1.8	5	2.8
Mount Hope	0.46	10	0.27	2.8	5	1.8
Waldo-Hancock	0.44	10	0.16	3.0	5	1.7
Triborough	0.34	8	0.10	3.0	4	1.3
St. Johns	0.35	7	0.18	2.5	3.5	1.4
Detroit	0.29	5.5	0.19	2.3	2.8	1.2
Vancouver	0.33	4.3	0.19	2.3	2.2	0.95
Thousand Islands	0.26	4.0	0.20	2.1	1.0	0.48
Thousand Islands	0.22	3.3	0.20	2.1	0.8	0.38
Deer Isle	0.20	2.4	0.28	3.6	0.6	0.17
Golden Gate	0.05	0.6	0.14	3.6	0.2	0.06
Bronx-Whitestone	0.03	0.4	0.15	3.4	0.1	0.03
Tacoma	0.02	0.2	0.20	2.0	0.04	0.02

Degree of stability is represented by the ordinate of the plotted point divided by the corresponding ordinate of the graph. Table 17 and Fig. 37 represent the third criterion for aerodynamic stability:

> The aerodynamic stability constant $R\sqrt{K/B}$ (or the value of $\delta_s\sqrt{K/B}$ which it represents) shall exceed the required value for the section, as tabulated or as plotted in a graph for different section ratios.

Fig. 37 also shows how a horizontal line (dashed) might be used instead of the curved graph. This horizontal line would yield an apparently correct dividing line for the structures listed, but would give misleading results for any bridge designs that might fall between the horizontal line and the curved graph. The curved graph is composite, representing the criterion for torsional stability in the principal range of section ratios and the superimposed higher criterion for vertical stability for sections with effective d/b exceeding 0.25. This steeper segment of the graph explains the high instability recorded for the Deer Isle Bridge (before the present stays were installed). If the section ratio had been 0.25 instead of 0.276, the instability would have been greatly reduced, as shown in Fig. 37, and the required increase in $R\sqrt{K/B}$ would have been only 4.8 instead of 12.0.

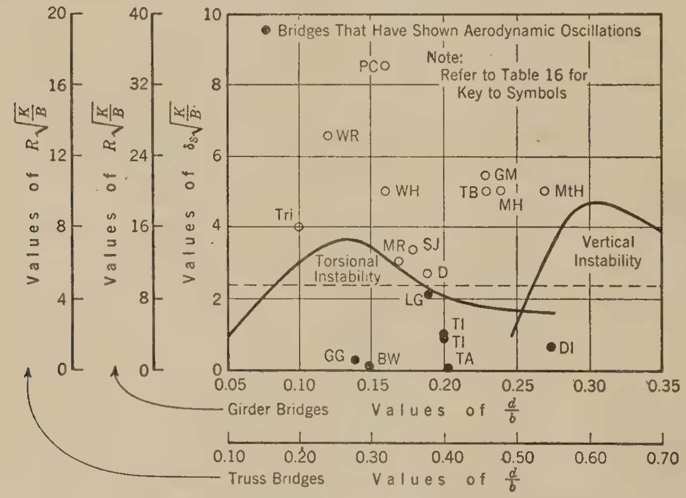

FIG. 37.—CRITERION FOR AERODYNAMIC STABILITY (REQUIRED $R\sqrt{K/B}$ FOR A CRITICAL VELOCITY OF 100 MILES PER HR)

For bridges of very low R, the comparison can be improved by taking into account the value of δ_s at $R = 0$, using a formula of the form

$$\delta_s = 0.005 + 0.25\,R \qquad (176a)$$

for girder bridges, and

$$\delta_s = 0.010 + 0.50\,R \qquad (176b)$$

for truss bridges. For bridges of high R, the first term of Eqs. 176 is relatively negligible. These values of δ_s represent the positive damping at zero amplitude, or very small amplitude, and are high because they include the damping due to friction at joints, connections, and bearings.

The scientific validity of this criterion is not dependent upon the empirically assumed values of the ratio, $\delta_s : R$. If the indicated ratio is too high, the criterion represents a critical velocity proportionately lower than 100 miles per hr.

The bridge engineer thus has his choice of three criteria, the first being the simplest and the last being the most scientific and the most accurate.

Criteria proposed by others have been based on incorrect or incomplete relations. Mr. Balog uses the ratio between the two terms of Eq. 6, represented by his parameter π_3 and by the writer's stiffness factor S (see Eqs. 139), the relation between the two being $\pi_3 = 1/S^2$. Stripped of the obscuring mathematics, all that Mr. Balog states in his criterion, Eq. 122, is that S for any bridge shall not be less than S for the Golden Gate Bridge; this is equivalent to stating that R shall not be less than 0.055. Such a criterion is inadequate, both in magnitude and in scope. From this criterion Mr. Balog derives two criteria, Eqs. 124*a* and 124*b*, the three being mutually inconsistent. Eq. 124*a* states that, for aerodynamic stability, the value of I/H for any bridge shall equal the value of I/H for the Golden Gate Bridge; and Eq. 124*b* states that I/l^2 of any bridge shall equal I/l^2 of the Golden Gate Bridge. Both these criteria, as shown by Table 15 and Fig. 23, and as stated by Mr. Balog, yield some obviously misleading results. The reason becomes clear when the two criteria are further translated: Eq. 124*a* specifies the value of $R\, l^2$ and Eq. 124*b* specifies the value of $K\, R\, l^2$, using the Golden Gate Bridge as the standard in each case. The fact that l^2 is thus arbitrarily introduced into the measure of aerodynamic stability is hidden in Mr. Balog's mathematical presentation. Thus, spurious advantage is given to long spans, and perfectly stable bridges of shorter span are made to appear unstable by the same criterion.

Required Stiffening of Suspension Bridges.—Substituting the reduced expressions for K and R as given by Eq. 6, using $n = 2$, the second criterion ($R \sqrt{K/B} > 5$) for aerodynamic stability reduces very closely to

$$E\, I > \frac{b\, l^4}{120 \sqrt{f}} \qquad \text{(177)}$$

in which I is per half width. The dead load w has disappeared from the relation by cancelation. Eq. 177 shows that augmentation of rigidity by reducing cable sag f (as was done for economy in the Tacoma Bridge) actually reduces safety against aerodynamic instability. Eq. 177 will give the same relative results as the second criterion and defines the identical dividing line between aerodynamically stable and unstable suspension bridges.

Here at last is an answer to the question that has challenged bridge engineers for a century: How much stiffening does a suspension bridge require? Any suspension bridge with a stiffening system satisfying Eq. 177 will be aerodynamically stable in the normally required range of wind velocities. The only suspension bridges that have manifested aerodynamic instability are those that have been proportioned below this requirement.

Any suspension bridge design that does not satisfy this criterion by the product $E\,I$ of the stiffening girders or trusses requires either equivalent stiffening (K and R) by other means, or special design of cross section to minimize, eliminate, or reverse any aerodynamic instability. Long-span suspension bridges will require considerably deeper stiffening girders or trusses than those of the recent past, unless the designs receive special study for aerodynamic stability.

The foregoing may be summarized in a simple practical specification:

> (a) The stiffening system of a suspension bridge shall be proportioned to yield a value $E\,I$ of not less than $b\,l^4/(120\,\sqrt{f})$; otherwise, the design shall receive special investigation for aerodynamic stability.

Required Depth of Stiffening Girders.—In normal designs by the deflection theory, for highway loading, the foregoing criterion (Eq. 177) is very closely satisfied by making

$$d > \frac{l}{120} + \left(\frac{l}{1{,}000}\right)^2 \qquad (178)$$

In other words, the required depth ratio d/l ranges from 1/120 in very short spans to 1/100 for spans between 1,000 and 2,000 ft and to 1/80 for spans longer than 4,000 ft. Matched against the second and third criteria and Eq. 177, Eq. 178 yields the same dividing line between stable and unstable bridges and an excellent correlation of ratios and orders of listing (compare Fig. 2). Thus, an even simpler, practical form of the desired specification is available:

> (b) In highway suspension bridges, the depth of stiffening girders or trusses shall be not less than $l/120 + (l/1{,}000)^2$; otherwise, the design shall receive special investigation for aerodynamic stability.

In self-anchored suspension bridges ($R = 1.00$), and in other suspension bridges proportioned by the elastic theory, a shallower depth ratio will yield the required moment of inertia I.

Methods of Securing Aerodynamic Stability.—The writer has repeatedly demonstrated by model tests that, under appropriate conditions, aerodynamic stability may be secured or materially improved by opening lateral areas. He has explained and confirmed this conclusion by supplementary tests and by analysis. He has never suggested that complete stability will be guaranteed in all cases and under all conditions. The correct answer lies in scientific investigation, not in rejection on the basis of prejudiced conclusions or fragmentary observations.

The proposal of open sidewalks must not be confused with the proposal of an open roadway. For the open roadway, the writer's model tests yielded unfavorable results.

The fact (cited by Mr. Balog) that the Bronx-Whitestone Bridge during erection, before concreting but with the floor forms in place, showed high vertical instability even with the sidewalk spaces open confirms the writer's observations and conclusions. With the low K and the high B due to small dead load, and with the low structural and friction damping due to the absence of the concrete floor, a bridge at that stage may have abnormally high aerodynamic instability, or abnormally high amplitude response, which even the open sidewalks can scarcely be expected to overcome.

Whether or not outside sidewalk brackets will contribute a stabilizing effect depends upon the proportions of the section and the category of instability. Theoretical considerations indicate a beneficial effect under some conditions. Without properly planned experiments, no categorical conclusion is justified.

The effectiveness of longitudinal and of transverse stays should be obvious, as some of the discussers have noted. The late John A. Roebling, M. Am. Soc. C. E., used floor stays and cable stays a century ago, and that was one reason why his bridges stood up while bridges built by his contemporaries were blown down by the wind.

In referring to conventional suspension bridge cross sections as H-sections, the writer uses the term in its broadest sense, so as to include flat plates, deck sections, and through sections, with either solid web or open-truss stiffening girders. After the basic relations are established for the symmetrical H-section, secondary corrections are easily determined (by experiment or analysis) for the effects of shifting the horizontal member to different heights, including the top and bottom positions, as well as for the effects of floor openings and of outside brackets. Thus, all these variations are brought into an organized relationship, instead of being left a disconnected haphazard mystery. The less common double-deck sections, and new forms like the suggested tandem H-sections, will require separate investigation.

Summary and Conclusions.—The studies represented by this paper lead to twenty-five conclusions, as follows:

1. The potential aerodynamic stability or instability of sections of various proportions for all modes of oscillation throughout the range of wind velocities can be quantitatively charted and predicted.

2. The coefficient of rigidity K, measuring the resistance to oscillation and determining the frequency of oscillation in any mode, can be easily and accurately calculated for any existing or proposed structure of known dimensions.

3. Rigidity and structural damping to resist potential aerodynamic instability, both vertical and torsional, may be augmented in precalculated amounts, by using: (*a*) Deeper stiffening girders or trusses; (*b*) cable stays, tower stays, center stays, and intermediate stays; (*c*) continuous construction; and (*d*) unloaded backstays.

4. Rigidity and structural damping to resist potential torsional instability may be augmented in precalculated amounts by: (*a*) Using transverse diagonal stays; (*b*) installing a double system of lateral bracing; (*c*) increasing the torsional stiffness of the towers; (*d*) raising the points of suspender connection relative to the center of gravity of the section; and (*e*) lowering the center of gravity of the section (for example, by the use of through construction).

5. Positive damping, opposing or nullifying potential aerodynamic instability, includes hysteresis damping, friction damping (structural or artificial), passive atmospheric damping, and positive aerodynamic damping due to relative velocities. Subject to determination or confirmation of the basic coefficients, all these factors can be formulated and predictively computed.

6. If the total positive damping is known, the critical wind velocities at which aerodynamic instability may appear can be precalculated—as can the potential amplitude response at any specified higher velocity.

7. The basic information for predetermining potential aerodynamic instability, both in sign and in intensity, is supplied by the static lift graphs and torque graphs. These graphs are known or can be predicted for conventional sections and, for unusual sections, can be easily secured by simple wind-tunnel tests on a stationary section model.

8. The wind-tunnel characteristics of an H-section can be predicted from the section ratio d/b in the case of solid girders and from the equivalent section ratio in the case of open trusses.

9. For zero or minimum vertical instability, the optimum section ratios are from $d/b = 0.22$ to $d/b = 0.24$. Deeper sections are potentially unstable against vertical oscillations in the high-velocity (catastrophic) range. Shallower sections are potentially unstable in the low-velocity range. Section ratios of from 0.12 to 0.17 are the most unstable at low wind velocities. A section ratio of $d/b = 0$ is a second optimum.

10. For zero or minimum torsional instability, the optimum section ratios are from $d/b = 0$ to $d/b = 0.06$. All deeper sections are "unstable" sections; that is, they are potentially unstable in the high-velocity (catastrophic) range. Section ratios of from 0.10 to 0.15 are the most unstable (in torsion) in the high-velocity range. The critical velocity initiating the high-velocity range increases toward infinity as the section ratio is reduced.

11. For any given section ratio (more easily for "stable" sections), vertical and torsional stability up to any specified wind velocity can be assured by providing required values of rigidity, or positive damping, or both. These required values are easily precalculated.

12. Potential vertical instability may be minimized or eliminated by using: (*a*) Section ratios, or equivalent section ratios, equal to 0.24 or less (avoiding section ratios of from 0.12 to 0.17); (*b*) the optimum section ratios of from 0.22 to 0.24 (or, as a second choice, ratios close to zero); (*c*) sections yielding static lift graphs of very flat or zero slope; (*d*) sections yielding lift graphs of small slope and large ordinates throughout a specified range of angles of attack; (*e*) open sidewalks; (*f*) outside brackets; and (*g*) streamlining (but only when the resulting equivalent section ratio is consistent with the foregoing recommendations).

13. Potential torsional instability may be minimized or eliminated by using: (*a*) Section ratios equal to or less than 0.10; (*b*) section ratios as close to zero as possible; (*c*) sections yielding static torque graphs of very flat or zero slope; (*d*) sections yielding static torque graphs of small slope and large ordinates (after correction for moment of drag forces) throughout a specified range of angles of attack; (*e*) open sidewalks; (*f*) outside brackets; (*g*) streamlining when the resulting equivalent section ratio or the resulting torque graph is consistent with the foregoing recommendations; and (*h*) multiple girder sections with central open deck.

14. A completely open deck, or open roadway with closed sidewalks, may

lose more in atmospheric damping than is gained in reduced aerodynamic instability.

15. Friction damping varies inversely with amplitude. It is helpful in resisting the initiation of oscillations, but has little effect after any substantial amplitude is attained.

16. Aerodynamic instability is not explained as the result of narrow width. For equal section ratios and equal weight per square foot, a wider bridge has higher rates of amplification and higher amplitudes, both vertical and angular, and requires a higher coefficient of rigidity K or a higher structural damping, or both, to counteract potential instability.

17. For bridges of equal or equivalent section ratios, potential aerodynamic instability is inversely proportional to $\sqrt{K/B}$, in which B is the width factor b^2/w.

18. At any given wind velocity in a range that is unstable for one or more modes of oscillation, the mode (vertical or torsional) yielding the highest rate of amplification (net negative damping) at that wind velocity will tend to prevail. The potential succession of modes with changing wind velocity may thus be precalculated if desired, but is unimportant. For purposes of design, the highest precalculated value of negative damping up to a specified wind velocity (see charts) should be matched against the potential mode of lowest K or of lowest positive damping.

19. For assured aerodynamic stability (zero amplification) of a "stable" section at all wind velocities, a structural damping δ_s should be provided equal to the maximum negative damping precalculated or charted for the section in the low-velocity range.

20. For assured aerodynamic stability (zero amplification) of an "unstable" section up to any specified wind velocity, a minimum required value of the product $\delta_s \sqrt{K/B}$ should be provided as precalculated or charted for the section at the specified wind velocity.

21. Increase in dead load, without reducing the depth and section of the girders or trusses, will produce a small or negligible improvement in aerodynamic stability, which may be outweighed by the resulting reduction in atmospheric damping. If advantage is taken of the increased dead load to reduce the girder section (by the deflection theory), the coefficient of rigidity K will remain unchanged and the aerodynamic stability will be materially reduced. The use of a lighter floor construction, with the necessary corresponding increase in the girder section, will actually improve aerodynamic stability by a substantial and easily calculable amount.

22. To aid in checking the aerodynamic stability of a proposed bridge design quickly, three alternate criteria of progressively increasing scientific accuracy are developed: (*a*) The stiffness ratio K/B shall exceed 300; (*b*) the aerodynamic stability constant $R\ \sqrt{K/B}$ shall exceed 5; and (*c*) the aerodynamic stability constant $R\ \sqrt{K/B}$ shall exceed the required value for the section, as tabulated or as plotted in a graph for different section ratios. Bridge designs with exceptionally favorable or unfavorable sections or relative magnitudes of structural and atmospheric damping require or warrant individual analyses of potential instability.

23. The required stiffening of a suspension bridge to assure satisfactory aerodynamic behavior is given in a simple, practical specification:

> The stiffening system shall be proportioned to yield $E\,I$ not less than $b\,l^4/(120\,\sqrt{f})$; otherwise, the design shall receive special investigation for aerodynamic stability.

24. The criteria in conclusions 22 and 23 are finally reduced to an even simpler, practical rule:

> The depth of stiffening girders or trusses shall be not less than $l/120 + (l/1{,}000)^2$; otherwise, the design shall receive special investigation for aerodynamic stability.

25. Specific gaps and inadequacies in available experimental data have been indicated as guides for test programs to confirm, amplify, or modify the present indicated conclusions and to bring the practical solution of the problem closer to finality.

Acknowledgments.—In undertaking and completing the task represented by this study, the writer has been as much spurred by challenging questions and criticisms as he has been heartened by the rare instances of encouragement and cooperation. To all who have contributed, either knowingly or unknowingly, including the many anonymous reviewers who recorded their comments, the writer takes this opportunity of acknowledging his indebtedness.

The writer owes a special debt of gratitude to Carl H. Gronquist, M. Am. Soc. C. E., who, during many arduous months of generous collaboration, contributed helpful suggestions, checked derivations of formulas and numerical applications, reviewed manuscript, and prepared illustrations.

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ERECTION OF HYDRAULIC TURBINES AND GENERATORS

By Andrew M. Komora,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Paul E. Gisiger, Frank I. Morgan, J. F. Roberts, D. E. Brainard, and Andrew M. Komora.

Synopsis

Most engineers who have installed hydroelectric generators have discovered that certain units operate perfectly whereas others perform in a manner leaving much to be desired. Construction methods and procedures differ widely, and the writer believes that, by adhering to certain fundamentals in erection procedure, most units can be assembled so as to operate perfectly. The purpose of this paper is to offer suggestions for the perfection of such methods and procedures.

The construction procedure for assembling and erecting turbines and generators recommended in this paper is based principally on the methods used by the Tennessee Valley Authority (TVA). The installations vary from the low-head, large-diameter Kaplan-type turbines at Pickwick Landing, Tenn., to the relatively high-head Francis-type units at Hiwassee, N. C.[2] The fundamental rules apply to all types of installations and give comparable results. The steps in the erection are presented under six headings, or "stages," in the order in which they generally occur.

Stage 1. Placing and Erecting Turbine Embedded Parts

Fig. 1 illustrates cross sections through two typical turbine types, indicating the nomenclature generally used by engineers and construction personnel. Although the paper refers expressly to the typical "Kaplan" installation, the general procedure is applicable to all types.

It is assumed that, just prior to placing the embedded part of the first turbine, the concrete for the power-house substructure has been poured as high

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Note.—Published in March, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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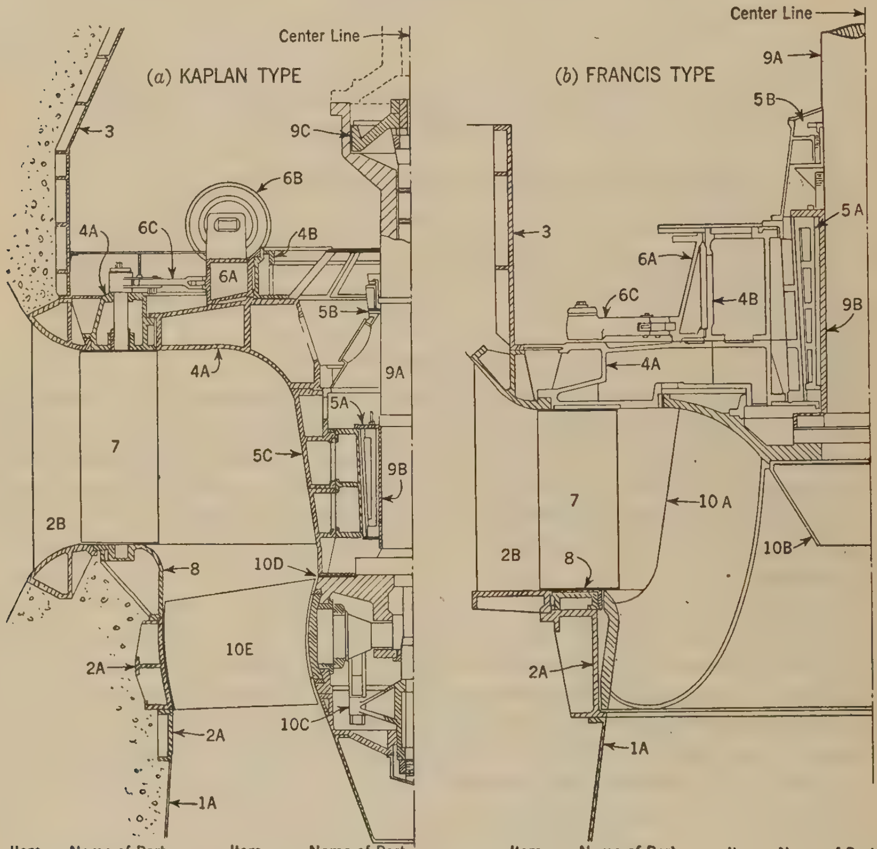
[2] "Allocation of the Tennessee Valley Authority Projects," by Theodore Parker, *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 175, Fig. 1.

as possible. In the case of a concrete scroll case, the largest part of the scroll-case floor will have been poured, including the scroll-case walls. A recess large enough for working room is left to receive the draft-tube liner. Concrete piers are erected high enough to receive the speed ring but with sufficient clearance for placing screw jacks and steel shims under it.



Item	Name of Part
1A	Draft Tube Liner
2A	Discharge Ring
2B	Speed Ring
3	Pit Liner
4A	Cover Plate
4B	Cover Plate Extension
5A	Guide Bearing
5B	Stuffing Box
5C	Head Cover Barrel
6A	Shifting Ring
6B	Operating Cylinder and Rods
6C	Levers and Links
7	Guide Vanes
8	Guide Vane Ring
9A	Main Shaft
9B	Shaft Sleeves
9C	Reservomotor Piston
10A	Runner
10B	Runner Tip
10C	Runner Blade Mechanism
10D	Runner Hub and Cap
10E	Runner Blades

Fig. 1.—Sections of Typical Turbine Types

The draft-tube liner is then set in place, centered, and alined approximately to grade. It is anchored securely to U-bolts placed in the concrete. The speed ring is next assembled, either in place or on an assembly deck, depending on handling facilities and space requirements. Great care is necessary to level the ring as closely as possible. Strong supporting blocks, preferably cut from H-sections, are used in addition to screw jacks. It is not necessary to spend much time centering the ring on the theoretical power-house center lines or to bring it exactly to the grade called for on the plans (see Stage 6). The entire procedure for the unit assembly is dependent on the final elevation and center lines of the speed ring. The lower and upper discharge rings are assembled in place and centered as closely as possible to the speed-ring center. The ultimate runner clearances will depend on how accurately this phase of the work is done. At this point all bolted contact joints have been coated with red lead and linseed oil.

After assembling the rings, the draft-tube liner is fastened to the lower discharge ring. The method of fastening varies with different manufacturers and installations, but most modern draft-tube liners are welded to the discharge rings. Careful "stitch" welding in light layers and peening avoid the danger of distorting the parent metal.

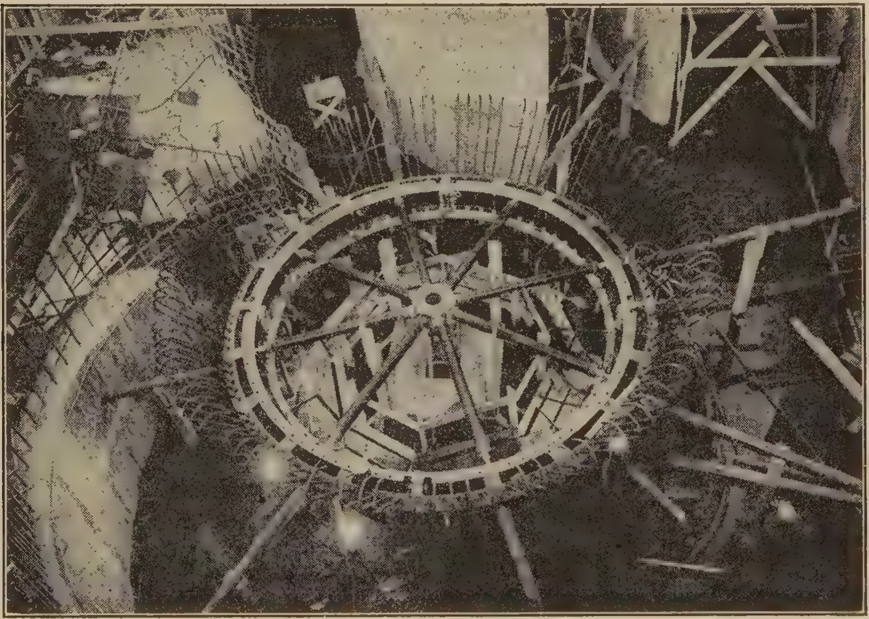


FIG. 2.—SPEED-RING ASSEMBLY SHOWING METHOD OF BRACING

Double extra strong 6-in. pipe, or its equivalent, is best to brace the speed ring rigidly, inside and outside (see Fig. 2). It is false economy to use light struts since good substantial center struts can be made by most job forces. One "spider" will suffice for many installations by merely splicing it to the arms, or by cutting, to fit various speed rings of different diameters. The jacking bolts at the ends of the struts should be about 2 in. in diameter and should be tack-welded to the ring. The external struts should be placed opposite those on the inside of the speed ring to avoid distortion. They should be supported by shores at close centers to preclude the possibility of sag. The ring should be "trued up" for roundness by jacking with the struts, and then it should be leveled.

In leveling, it is necessary to erect a separate tower for the instrument. Whether a level or a transit is used depends on the engineer's preference. The sighting target can be made up of cross hairs mounted on a micrometer head which in turn is mounted on a heavy steel base. The speed ring, the discharge rings, and the draft-tube liner are now ready for concrete and grout (see Stage 2).

The lower and upper pit liners are assembled on to the speed ring after the ring has been completely grouted. Reasonable care is needed in alining and centering the pit liner, but this precaution should not be carried to extremes. The principal thing is to center the operating cylinder bases, as closely as possible, on the center line of the unit. Of primary importance to designers and manufacturers is the fact that the plane of the operating cylinder bases cannot be held perpendicular to the true center lines of the cylinder assembly. The bases may be alined perfectly, but it will be found invariably that they are out of line after the scroll-case concrete has set. To remedy this condition, some manufacturers provide a "dutchman" or filler piece to be inserted between the operating cylinder and the base on the pit liner. Measurements are taken between the operating cylinders and their bases after final alinement and the filler pieces are machined so as to take up any discrepancy. It is recommended that the bases be made thick enough to allow for facing in the field. This operation can be performed with a job-constructed facing rig made of a slow-speed air motor to which a heavy slotted plate with a cutting tool is mounted.

A poor practice among some designers and manufacturers is to require that the pit liner be welded to the speed ring for watertightness. There is always a good chance of distorting the speed ring during the welding operation. A watertight joint can be obtained by providing a V cut in the speed-ring flange or pit-liner base (possibly both), and by inserting a rubber filler slightly larger than the V cut. Care must also be exercised to avoid distorting the speed ring in welding the various membering joints in the pit liner. Welds should be made in thin layers, and peened.

STAGE 2. GROUTING OF SPEED RING

The speed ring on a hydraulic unit is generally designed to support a large part of the superimposed scroll-case roof, the entire generator, including rotor, the water-wheel runner, and the hydraulic thrust. It is highly essential, therefore, that a good grouting job be obtained under the speed ring, to avoid the possibility of an open joint or seam.

The concrete around the lower and upper discharge rings should be poured in relatively short lifts to within about 6 in. of the speed ring, the actual distance depending on the design of the speed ring and on job conditions. Then the concrete is allowed to stand for seven or eight days, being cooled with a water spray to shrink it as much as possible.

Procedures and proportions for mixing grout in turbine installations probably vary more with different construction organizations than any of the other operations. In an effort to devise a consistent standard, a series of grouting tests was conducted at Wilson Dam during the fall and winter of 1941. As a result of these studies the writer recommends that grout headers, about 4 or 5 ft high and spaced at 3-ft or 4-ft centers, be inserted in the speed ring. He prefers them to be 1½ in., or larger, in diameter, with funnels to facilitate pouring grout. Small vent holes, 0.5 in. to 0.75 in., should be provided in the speed ring, spaced in a staggered manner, one being placed in each ribbed corner. Pouring should begin simultaneously at several points on the speed ring and the grout should be worked uniformly away from each header. For one to two hours, or until all signs of free water and laitance have disappeared, the grout should be rodded thoroughly. When there is evidence that pure grout is escaping from any vent hole, this hole is plugged. This procedure is followed on all vent holes around the speed ring, and all vents are reopened at about 10-min intervals and checked for free water.

The following specifications are recommended in grouting turbine speed rings:

1. Water-cement ratios are to be as low as possible, but not lower than 0.50 in confined areas;
2. Coarse aggregates are to be added to the grout as working conditions will permit; and
3. The grout is to be rodded continually for at least an hour, or until all of the free water and laitance is expelled.

Stage 3. Pouring Concrete

Designers will lay out the pours around a unit in equal segments, say 4 to 6 sections, depending on the physical size of the turbine. Alternate segments will then be poured so as to equalize the radial thrust on the speed ring and pit liner. As a general rule, a seven-day interval (or more) is necessary between horizontal lifts and a three-day interval between vertical lifts. This will allow for more rapid cooling of the individual pours, thus minimizing the shrinkage effect on the unit. Alternate lifts are poured simultaneously and at slow rates not to exceed 1 ft an hour. In no case should mechanical vibrators be allowed closer than 10 ft from the unit as there is always danger of jarring loose a strut or stay, thus causing possible misalinement or distortion of the speed ring.

Stage 4. Erection of Turbine Internal Parts Including Stationary and Rotating Elements

The starting point of all assembly of the internal parts of the turbine is the guide-vane ring. In some installations, this is fixed at the time the speed ring and upper discharge ring are installed and is an integral part of these members (see point 8, Fig. 1(*a*)). In other installations, the guide-vane ring is a separate member; it is necessary that the ring be centered as closely as possible to the discharge ring and fixed in its final position by dowels.

The outer cover plate is assembled in place and bolted loosely to the speed ring. A steel mandrel, having the same diameter as the guide-vane trunnions, is inserted through the upper guide-vane bushing in the outer cover plate and into the lower guide-vane bushing in the guide-vane ring. This check is made in each set of bushings. It may be necessary to shift the outer cover plate during this operation in order to aline the upper and lower bushings properly. In some instances, it may be necessary to scrape the bushings to prevent binding of the guide-vane stems. When the bushings are properly alined, the outer cover plate is bolted rigidly to the speed ring and a sufficient number of dowel holes are drilled to fix the position of the outer cover plate with respect to the speed ring.

The outer cover plate is then raised as a unit and blocked up on timber cribbing; the guide vanes are next installed in accordance with factory match marks; the outer cover plate is lowered in place, bolted and doweled to the speed ring; and each of the guide vanes is centered vertically so that the clearance is the same between the guide vane and the guide-vane ring and outer cover plate.

The inner head cover (the lower point 4A, Fig. 1(*a*)) is bolted to the head cover barrel (point 5C, Fig. 1(*a*)). (Dowel holes joining these pieces usually are drilled in the factory.) These two pieces should be installed as a unit. The head cover barrel is centered with the discharge ring and the inner cover plate is then doweled to the outer cover, using undersize dowels. The cover-plate extension (point 4B, Fig. 1(*a*)) and shifting ring (point 6A, Fig. 1) are installed in the order named. During this operation, the runner is generally being assembled on an erection pedestal in another part of the building. The operating cylinders (point 6B, Fig. 1(*a*)) are assembled to their bases on the pit liner. Great care must be exercised in this operation to aline the cylinders concentrically with the shifting ring. The rods are generally alined with the shifting ring at the mid-stroke position in order to equalize the radial throw-out between the fully open and closed position. As mentioned, the pit liner will be distorted somewhat due to the shrinkage of the concrete. It would be a fortunate coincidence if the operating cylinder base stayed in alinement. In most cases it is necessary to insert a machined filler plate between the operating cylinder and the pit-liner support. A better method is to have the manufacturer weld a base plate of sufficient thickness to the pit liner to allow for machining on the job. Accurate measurements, for alinement requirements, can be taken during the assembly of the operating cylinders and the base plate can be milled with a job-facing tool. The operating cylinders and shifting ring are to be removed.

The runner and shaft assembly presents no serious difficulties. The clearances between the runner blades (point 10E, Fig. 1(*a*)) must be equalized, if necessary by inserting shims in the operating links (point 10C, Fig. 1(*a*)). The packing around the runner hub is inserted carefully and tightened with the gland ring. The hub is tested with air pressure to insure the tightness of the packing. The inner cover plate (lower point 4A, Fig. 1(*a*)) is removed from the turbine pit and mounted on the runner assembly and bolted to the head cover barrel (point 5C, Fig. 1(*a*)). The stuffing-box housing (point 5B, Fig. 1(*a*)) is now mounted around the shaft. This provides a temporary support for the runner and shaft assembly. The entire assembly is next placed in the turbine pit.

The inner head cover is bolted and doweled rigidly to the outer head cover. The runner and shaft are centered as closely as possible in the upper discharge ring (point 2A, Fig. 1(*a*)) and the head cover barrel, respectively. The shaft is then plumbed as accurately as possible. It is advisable to place jacking struts between the shaft and some convenient point on the inner head cover to hold the shaft in vertical alinement during the generator assembly. Fixed tram points are placed in the inner head cover just above the lower guide bearing. It will be necessary to use these points after the turbine shaft is coupled to the generator shaft. The guide-bearing housing and shoes should be inserted and centered about the center of the shaft throw-out. Generally, running clearances of from 0.004 in. to 0.006 in. are used in the final adjustment. It is advisable to use long copper shims between the turbine guide-bearing shoes and the shaft. The shoes are jacked firmly against these shims and left in this position until the generator shaft is alined to the turbine. In this connection, "wedge-type" shoes are most practicable—that is, shoes with tapered back seats—because, as wear occurs, they can be readjusted easily to maintain the proper running clearances.

STAGE 5. ERECTION OF GENERATOR-MECHANICAL FEATURES

Only the mechanical features of generator erection which tie in to the turbine are discussed in this paper. Generator erection in detail has been covered capably elsewhere.[3, 4]

Great care should be exercised in establishing the correct grades for the main bracket and stator sole plates (6F). The turbine shaft generally will be resting on a temporary seat from one fourth to three fourths of an inch below its final grade before being coupled to the generator. This clearance, plus an additional allowance of from 0.030 to 0.050 in. for main bearing-bracket deflection (depending on the bracket span and loading) should be taken into account when computing the final elevation of the sole plates.

It is highly desirable to center and aline the generator shaft and bearing bracket to the turbine shaft before final centering and alinement of the stator. The generator shaft is centered on the thrust bearing before being placed in the pit. The thrust-bearing shoes are all set to initial adjustment before the rotor is placed. At this point both the main bracket and stator sole plates are grouted. It has been found that grout with a water-cement ratio of 0.40 is best suited for this purpose. This grout is dry enough so that it can be placed under the bearing plates without the use of forms and does not shrink on setting up.

Some generator manufacturers use a radial type bracket, whereas others use through girders with side arm stiffeners. In the latter case, it is recommended that the side arms be not fastened and doweled to the sole plates until the unit has been loaded to about three quarters of its rated capacity. The added deflection caused by the hydraulic thrust will tend to close-in the guide-bearing shoes (point 6E, Fig. 3) and thus cause excessive heating if the side arms are fastened down initially.

After the bearing-bracket sole plates are grouted, the remainder of the generator erection can be completed. The generator shaft is centered accurately and plumbed to the turbine shaft. Four heavy plumb bobs suspended on piano wires at the quarter points can best be used for this purpose. Using temporary coupling bolts, the turbine shaft and runner are brought up slowly

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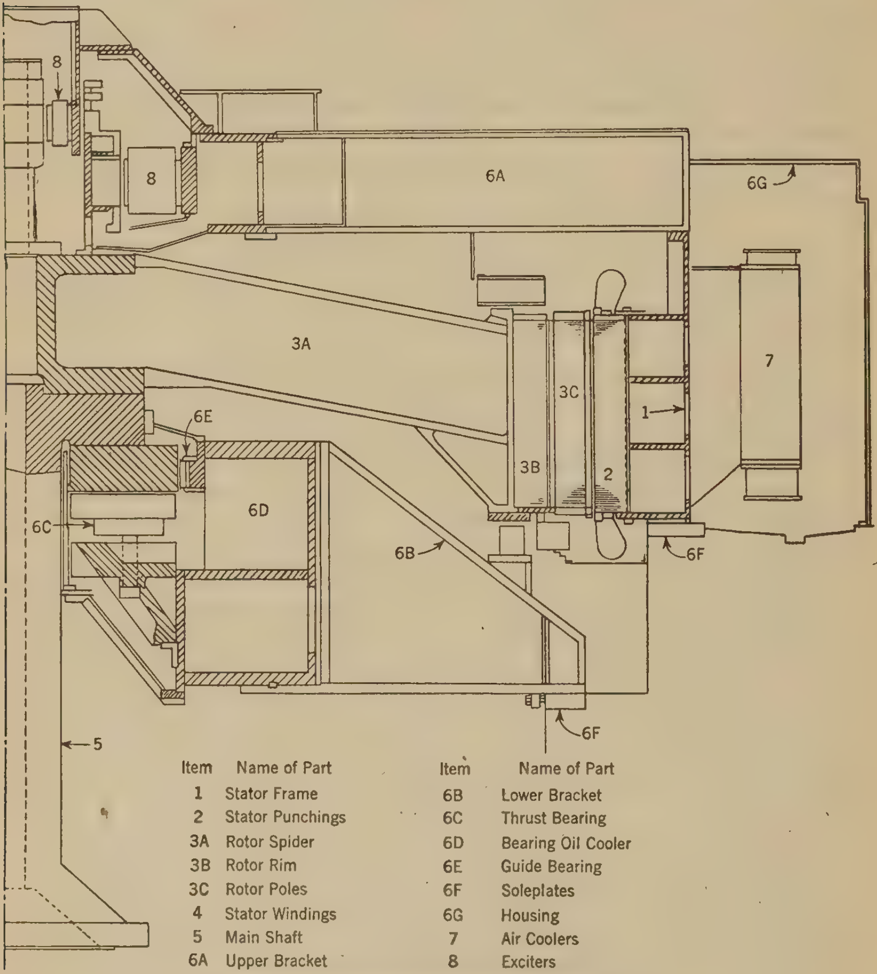
[3] "Order Out of Chaos," by H. A. Wallace, *Allis-Chalmers Electrical Review*, September, 1941.

[4] "Slugged Arc," by H. A. Wallace, *ibid.*, June, 1942.

to the generator shaft. It is customary practice to make the permanent coupling bolts 1 to 2 mils oversize, which necessitates freezing the bolts in dry ice or forcing them in place with hydraulic jacks. Either method is satisfactory and the choice depends on job conditions. The bolts are pre-stressed uniformly



Item	Name of Part	Item	Name of Part
1	Stator Frame	6B	Lower Bracket
2	Stator Punchings	6C	Thrust Bearing
3A	Rotor Spider	6D	Bearing Oil Cooler
3B	Rotor Rim	6E	Guide Bearing
3C	Rotor Poles	6F	Soleplates
4	Stator Windings	6G	Housing
5	Main Shaft	7	Air Coolers
6A	Upper Bracket	8	Exciters

FIG. 3.—SECTION THROUGH TYPICAL GENERATOR

and to such an extent that the total tension developed will be in excess of the combined weight of the turbine, turbine shaft, and the maximum hydraulic thrust, and, in the case of the Kaplan types, the maximum internal pressures. This will assure that the turbine and generator shaft coupling faces are in positive contact under all conditions. In the old days, the general order used to be to "slug her up till she rings and then take another turn." In some instances, this resulted in a permanent deformation in the coupling flanges with resultant misalinement of shafts. The customary stretch in bolts is from 0.006 to 0.012 mill, depending on the number, size, and length of bolts, and on the total suspended load.

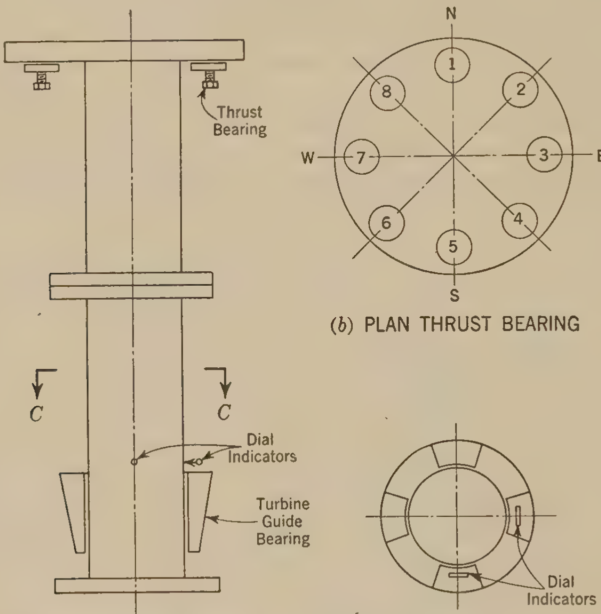
The customary procedure is to have the turbine shaft shipped to the generator manufacturer for final fitting up to, and alining with, the generator shaft —a needless expenditure of time and money. With present precision machining methods, the chance of misalining the coupling flanges is negligible. The holes for the coupling bolts can be subdrilled in the shop $\frac{1}{8}$ in. undersize and line reamed in the field after the two shafts are drawn together. One generator manufacturer uses this method in fitting his rotor to the shaft in the field. For a cutting unit, he uses an automobile cylinder honing machine fitted up with a cutting tool.

After the turbine shaft is coupled permanently to the generator shaft, the entire unit is free to swing on the thrust bearing. The thrust-bearing shoes are set up as uniformly as possible. They are generally adjusted by either the "feel" method or the "slugged arc"[4] method. The first requires a highly skilled mechanic and the second is much more accurate. By "feel" each shoe is adjusted progressively around the thrust bearing. The amount of tension on each adjusting screw is determined by the mechanic by observation and "feel" of the wrench. At best, this is a guess on the operator's part. Basically, the "slugged arc" method consists of measuring an arc at a known radius of a hand-calibrating wrench. Each of the adjusting screws under the thrust shoes is loosened completely, progressively and one at a time. The screws are tightened by a hand-calibrating wrench, care being exercised to obtain the same tension on each screw. The arc is measured by a graduated scale and the values are plotted. The loose shoes are tightened and the tight shoes are loosened by amounts depending on the plotted values. The "slugged arc" measurements are repeated and the values again plotted. Theoretically, they should all fall on the same horizontal line. An experienced operator generally can make a uniform adjustment on the second trial.

The following method of adjusting thrust-bearing shoes was evolved on the job by G. L. Teague and has been used successfully on two large machines (see Fig. 4). The combined assembly consisting of rotor, generator and turbine shafts, and runner is plumbed as accurately as possible. The initial thrust of each individual thrust shoe is immaterial. Two indicators are placed at the lower guide bearing at the quarter points (see Fig. 4(*c*)).

Compression is applied on the thrust shoes progressively, beginning with shoe No. 1 and continuing to shoe No. 8 (Fig. 4(*b*)). If either indicator shows the least movement during the adjustment on any individual shoe, the setting is stopped and the process is repeated on the next shoe in numerical order. Obviously the setting up of shoe No. 1 will not produce any movement in the west-east direction, but will produce motion in the north-south direction. The adjustment of shoe No. 2 will produce a movement which will show up on both indicators in Fig. 4(*c*). The procedure is repeated until the slightest adjustment on any individual shoe produces a movement on either indicator. This simple method will work with any number of thrust shoes.

It is absolutely essential that, whatever method is used, each thrust shoe be adjusted to as uniform a loading as possible. The successful running of the unit will depend to a very high degree on this adjustment.



(*b*) PLAN THRUST BEARING

(*a*) ELEVATION OF SHAFT (*c*) VIEW *C-C*

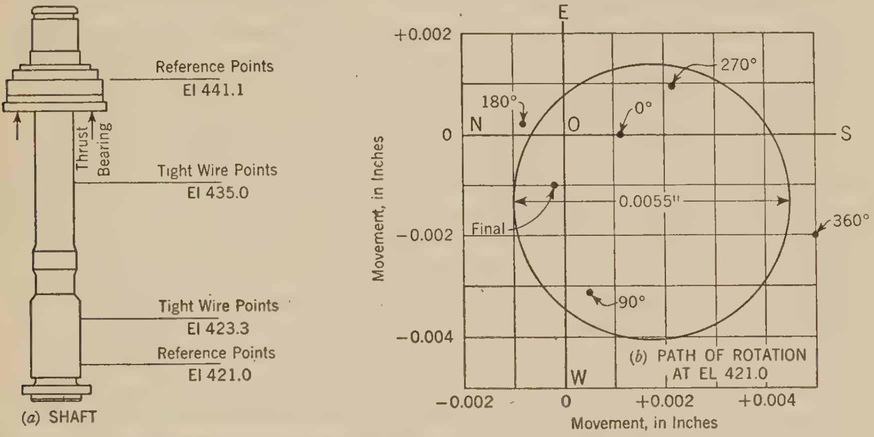
FIG. 4.—THRUST-BEARING ADJUSTMENT

In recent years, it has been customary to make a "rotation" check on the combined assembled unit. This check consists in rotating the unit mechanically (by jacking or attaching cables from the power-house crane to the rotor spider) through four 90° turns and taking plumb readings at several points. The bearing shoes are coated with a heavy graphite and oil lubricant prior to the mechanical rotation to preclude the possibility of scarring the bearing surfaces. It has also been found advisable to disregard the readings on the initial 90° turn because the heavy lubricant is "ironed out" between the shoes and the runner place in this turn.

A typical rotation check is shown in Table 1, in which Fig. (*b*) testifies that the axis of the shaft described a circle of rotation at the lower guide bearing of 0.0055 in. The center of this circle is 2 mils south and $1\frac{1}{4}$ mils west from the theoretical center line. The initial or zero point was $1\frac{1}{4}$ mils south and zero mils east and west. The shaft was replumbed so that the final position was 1 mil west and zero mils north and south.

In a great many installations, the thrust collar is not an integral part of the shaft but is shrunk or keyed to the shaft in the field. In such cases, a rotation check would indicate whether or not the face of the thrust collar was set normal to the shaft. It is conceivable that a thrust collar could be set a few mils from a normal position and the setting of the thrust shoes equalized in this position. Plumb readings would indicate that the shaft was vertical and centralized in the turbine guide bearing. However, in actual operation the shaft would describe a cone; that is, it would wobble within the turbine guide bearing with an amplitude depending on the ratio of the bearing diameter to the combined length of the turbine and generator shafts and the amount the thrust collar was set out of normal to the shaft. The rotation check could be omitted in the case of an integral thrust collar and generator shaft. Present-day machining methods are very precise so that there is little danger of the thrust collar being out of true with the shaft. The shaft is rotated in a lathe after machining and polishing, and careful throw-out measurements are made with dial indicators.

TABLE 1.—OBSERVATIONS FOR A TYPICAL ROTATION CHECK


Reference Points
El 441.1
Thrust Bearing
Tight Wire Points
El 435.0
Tight Wire Points
El 423.3
Reference Points
El 421.0
(a) SHAFT
E
+0.002
270°
180°
0°
N
O
0
S
0.0055"
Final
360°
−0.002
90°
−0.004
W
(b) PATH OF ROTATION AT EL 421.0
−0.002
0
+0.002
+0.004
Movement, in Inches
Movement, in Inches

Direction	Reference		Micrometer Readings for Reference Points						Centering[a]
	Tight wires	Reference points	0°	90°	180°	270°	360°	Final	
(a) Elevation 439.9 (Tight Wire Elevation 435.0)									
N	0.495	0.5315	0.529	0.530	0.5315	0.533	0.533	0.542	0.243
S	0.504	0.5365	0.541	0.540	0.538	0.537	0.537	0.5275	0.244
E	0.492	0.6045	0.6305	0.631	0.6295	0.6305	0.6315	0.5945	0.254
W	0.499	0.576	0.5535	0.5515	0.5535	0.553	0.551	0.588	0.255
(b) Elevation 421.0 (Tight Wire Elevation 423.3)									
N	0.4765	0.7615	0.759	0.759	0.7595	0.764	0.7675	0.7705	....
S	0.4855	0.5985	0.6005	0.600	0.5995	0.5955	0.5935	0.588	....
E	0.4725	0.691	0.7145	0.719	0.7135	0.7135	0.7185	0.6795	....
W	0.480	0.769	0.744	0.740	0.745	0.745	0.7405	0.778	....

[a] These are micrometer readings for the final centering of the turbine and guide bearing (not necessarily at El. 439.9).

The slight added cost of an integral thrust collar and shaft is warranted over that of keyed or shrunk hub. The integral thrust hub and shaft is easier to install and the results are more certain. In some installations having shrunk hubs, several weeks were spent in alining the units.

STAGE 6. ALLOWABLE TOLERANCES

In some installations, much unnecessary time and effort have been spent in alining or centering a unit to theoretical elevations and center lines. Deciding the necessary degree of accuracy requires a certain amount of "horse sense." Obviously, in erecting a high-head unit with a steel scroll case, which in turn connects to a steel penstock, closer tolerances are adhered to than in a low-head unit with a concrete scroll case. In steel scroll cases, especially in the riveted type, a certain amount of creepage will occur as the plates are assembled to the speed ring, making it necessary to field drill the connecting joint to the penstock. In such cases, greater tolerances can be allowed than, say, in cast steel machined scroll cases where it is necessary to meet fixed points.

Speed rings are set and held as level as is humanly possible. A tolerance of 0.001 in. per ft of speed-ring diameter should be the maximum allowable deviation from a true horizontal plane. There are cases in which, in spite of all precautions, the rings get out of level more than the amount specified (0.001 in. per ft) during the concreting operations. The cover plates and the guide-bearing housing can be adjusted, in such cases, to compensate for the added "outage" and still hold the turbine shaft in vertical alinement.

The remainder of the turbine parts, such as guide vanes, head covers, guide-vane linkages, operating cylinders, distributor rings, etc., are assembled as closely to specified dimensions as machined fits will permit. With the foregoing suggestions in mind, it is possible to summarize the principal erection tolerances as shown in Table 2.

TABLE 2.—SUMMARY OF PRINCIPAL ERECTION TOLERANCES, IN INCHES

For:	Speed Rings		Operating cylinders	Generator		Shaft alinement
	Concrete scroll cases	Spiral, steel scroll cases		Stator	Shaft	
Elevation	±0.060	±0.030	±0.010	±0.030[b]	....	....
Centering	±0.090	±0.060	±0.060[a]	....	....	....
Concentricity	....	....	....	±0.030	....	....
Turbine shaft	....	....	....	....	±0.010[c]	....
Plumb	....	....	....	....	....	0.001[d]

[a] For centering horizontally. [b] For elevation, with respect to rotor. [c] Base coupling with respect to turbine shaft. [d] 0.001 in. per 5-ft height of shaft, from the true vertical.

ACKNOWLEDGMENTS

Engineering work for the TVA was under the general supervision of Theodore B. Parker, M. Am. Soc. C. E., chief engineer; George R. Rich, M. Am. Soc. C. E., chief design engineer; A. L. Pauls, M. Am. Soc. C. E., chief construction engineer; and H. J. Petersen, head of the Mechanical Design Section.

The writer wishes to express his sincere thanks and appreciation to the many turbine mechanics and erectors employed by the Authority, and especially to J. S. Kuzawa, formerly master mechanic, for their many helpful suggestions and instructions in the art of assembling turbines; and to Sam McCluer, associate materials engineer, for conducting the grouting tests.

DISCUSSION

Paul E. Gisiger,[5] M. Am. Soc. C. E.—A useful summary is presented of the main points to be observed in the erection of Kaplan and Francis wheel hydroelectric generating units. The writer would like to add one recommendation to this competently presented paper—namely, that those who are in actual charge of erection make and keep a permanent record of all elevations, clearances, tolerances, etc., established at the time of erection. This record should be checked and if necessary supplemented immediately before the unit goes into regular operation.

The writer has several times regretted the lack of such a record. It is fairly obvious that the levels and tolerances established during erection of a hydroelectric unit are not permanent. Even the best of concrete is subjected to slight changes of volume and shape due to the factors of shrinkage and creep, apart from temperature variations and elastic deformations. The dislocations caused by these factors are generally of equal, if not higher, order of magnitude than the tolerances within which a unit can be erected. The resulting changes in alinement are in most cases inconsequential and may be unnoticed for years, but occasionally they are large enough to require correction and then the question arises as to how much change has taken place since the time when the unit was erected. The horizontal alinement of a generator stator may have changed, the runner blades of a Kaplan turbine may have left a trace on some sector of the circumference of the discharge ring, a bearing may show signs of pressure concentrated on one side, or a shaft may be found to be slightly out of plumb. In each case several causes may be responsible for the change. Without an accurate record for reference it is difficult, if not impossible, to judge the nature and causes of the trouble and to apply suitable remedies.

There are normally at least three parties concerned with the erection of a hydroelectric unit—the purchaser, the turbine manufacturer, and the generator manufacturer. If something does get out of plumb or alinement, it is natural for the equipment manufacturers to question the structure on which this equipment is supported. All the interested parties then need a complete record of the condition of the unit at the beginning of its useful life.

From this viewpoint it cannot be recommended too strongly that a complete record be kept of each unit, giving all key measurements and elevations such as, for instance, diameters of the discharge ring in at least four directions, elevations of marked points of the speed ring, elevations of the points of support of the main bearing brackets, and elevations of the stator base plates. In addition to these points on parts of the generating unit itself, it will be helpful to maintain a record of observation points that permit following any slight change in the structural supports. Such points may be located, for instance, on the two main diameters at the lowest accessible points of the pit liner and at corresponding locations higher up on the concrete foundation of the generator stators. The cost of installing such observation points or bench marks at the time of erection is insignificant when compared to the value of the information they may reveal during the life of the unit.

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[5] Structural Engr., Pennsylvania Water and Power Co. and Safe Harbor Water Power Corp., Baltimore, Md.

FRANK I. MORGAN,[6] ESQ.—A longfelt need has been met by the publication of this interesting paper. The writer wishes to add his comments in the hope that Mr. Komora will clarify a few points. For example, under "Stage 1. Placing and Erecting Turbine Embedded Parts," the author states: "Whether a level or transit is used [in setting equipment] depends on the engineer's preference." Is it not true that there is no choice since the transit is not a sufficiently accurate instrument for this purpose? The use of a high-grade level is quite essential to insure accurate results.

The author mentions difficulties experienced with the I-beam type of bearing bracket because of improper shimming, etc., of the side arms. This has been a serious problem and much trouble has been experienced with guide bearings because distortion of the bearing retainer resulted from increased deflection under load. This defect has been corrected in later designs by hinging the side arms. It would be interesting to learn how the bearing clearance is determined, with the machine under load as recommended by the author.

Mr. Komora mentions the use of four plumb lines when plumbing the shaft. Why are four lines necessary? It should be possible to obtain all the data with two plumb lines 90° apart—reducing the amount of work considerably.

The author states that it is customary to make the coupling bolts 1 to 2 mils oversize. What are the particular benefits of this procedure? This practice requires considerable extra time and labor during subsequent dismantling work. Furthermore, such tight fitting bolts tend to distort the faces of the flanges and to make difficult the accurate determination of the stress, in the bolts, during the process of tightening. A light tapping fit for coupling bolts is considered adequate. If the bolts are properly stressed when tightened, the friction between the coupling faces should be sufficient to transmit the torque. In one particular case the bolt holes in one flange of a shaft coupling were inadvertently reamed oversize. The unit, a large one, was assembled with the coupling in this condition, and an inspection quite some time after the unit had been in service, under load, showed that no movement had taken place in the coupling.

Mr. Komora is quite emphatic in his condemnation of the practice of shipping the turbine shaft to the generator manufacturer, there to be assembled with the generator shaft and the entire assembly checked for truth in a lathe. Such procedure is advisable until the erection of large units, on which this course has not been followed, has proved satisfactory. In at least one case, such a check revealed considerable inaccuracy which could be corrected only by remachining. Had the shafts reached the field, serious delay would have resulted. Shop assembly also makes it unnecessary to ream coupling bolt holes in the field.

In Table 1 the actual throw of a particular shaft is given as 0.0055 in. The author states (see heading, "Stage 5. Erection of Generator-Mechanical Features"), "The center of this circle [the shaft throw] is 2 mils south and $1\frac{1}{4}$ mils west from the theoretical center line. * * * The shaft was replumbed so that the final position was 1 mil west and zero mils north and south." Five mils is extremely small for the throw of a shaft, and since the center of the throw is but 2 mils away from the true center the position of the shaft is almost ideal—making any changes in its location somewhat inconsistent with the author's statement that much unnecessary time and effort has been spent in alining units. In this particular case the clearance of the guide bearing in which this shaft operates is probably 12 or 16 mils; therefore, the throw of the shaft plus the error in the position of the center would still be small enough to be within the bearing clearance. Units having shaft throws of 50 mils are operating successfully although this is, by no means, a desirable condition.

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[6] Maintenance Engr., Pennsylvania Water and Power Co. and Safe Harbor Water Power Corp., Baltimore, Md.

Has Mr. Komora had experience in plumbing the shafts and checking the throw of units equipped with spring type thrust bearings? With this type of thrust bearing the shaft may be made absolutely plumb and centered in the turbine guide bearing. As soon as the rotor is turned, however, the shaft will no longer be plumb because of unequal yielding of the springs in the thrust bearing. Thus, the final setting must be a compromise.

The author states that a period of several weeks has been spent in alining units having shrunk hubs. If the surface of the hub (more properly, the thrust block) is not normal to the shaft axis, how can the unit be alined correctly since no amount of adjustment of the thrust bearing will eliminate this error?

J. F. Roberts,[7] M. Am. Soc. C. E.—This description of the field erection practice of the TVA on large hydraulic turbines of both the Francis and Kaplan types is interesting and will be of great assistance to engineers with similar problems. During wartime when the privately owned power companies are not able to do any appreciable amount of extension work or to develop new power facilities, it is gratifying that government owned corporations and commissions are allowed to publish data on their projects. This opinion has been expressed by several engineers connected with private power companies.

Mr. Komora's brief comments regarding the grouting of embedded parts could have been amplified considerably. The writer was intimately connected with the experiments conducted at Wilson Dam by the TVA in an effort to study the results of various grouting procedures. Some special compounds were tested, especially those which tend to cause expansion on setting. Although in some cases these gave excellent results, it has been found difficult in practice to obtain uniformly good results because construction forces object to the careful measurements and accurate proportioning of ingredients that seem necessary. In certain cases a slight excess of water entirely ruined the effects of the agents which were supposed to cause expansion on setting. In the final analysis it was found that more uniform results could be obtained with a straight cement mixture, provided that such a mixture was rodded for a sufficient length of time to work all the free water out of the mixture.

Mr. Komora mentions that mechanical vibrators should not be placed closer than 10 ft to the unit to avoid possibility of distortion of the parts of the machinery. Originally, the writer was in accord with this opinion, but further experience has indicated the advisability and extreme usefulness of the small sized vibrators especially in grouting work where rodding and tamping are difficult. These small vibrators, which will pass through a $2\frac{1}{4}$-in. hole, have been of material assistance in compacting the grout, and, if the embedded parts are properly anchored down and the height of the pour or lift of concrete is kept to reasonable limits, as outlined by Mr. Komora, no difficulties should be encountered in moving, shifting, or distorting the embedded castings.

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[7] Mgr., Hydr. Dept., Allis Chalmers Mfg. Co., Milwaukee, Wis.

Closer cooperation between the designers of the machinery itself and the designers of the concrete structures would be helpful. Wherever possible the undersurface of castings that are to be grouted in should be constructed with a slope such as to avoid dead pockets, as it is very difficult to produce a satisfactory grouting job under a large horizontal surface.

Mr. Komora mentions the use of temporary coupling bolts for lifting the turbine runner and shaft into position. Although this idea works satisfactorily, using a quicker and easier method the turbine shaft and runner are brought up to the generator coupling by means of four or more hydraulic jacks equally spaced around the coupling. These can be supported on the head cover or in some cases satisfactory results have been obtained by mounting the jacks on horizontal plates suspended with two long bolts through the generator coupling bolt holes. Turbine runners and shafts weighing as much as 200 tons have been lifted into final position in less than one hour with this arrangement. By checking the gap between the couplings at frequent intervals, or placing dial indicators to observe the movement, the couplings can be brought up absolutely parallel with no danger of binding.

Many civil engineers fail to appreciate the extreme flexibility of large castings when subjected to hydrostatic pressures such as those which may be exerted by large masses of liquid concrete. Turbines with runner openings from 20 to 24 ft in diameter, having speed rings with over-all dimensions of from 30 to 37 ft in diameter, are easily distorted and have to be securely held in order to maintain a true circle. Runner clearances on large Kaplan and propeller type units vary from 0.120 in. to 0.180 in. on a radius. Few large installations can be maintained truly round, the average discrepancy being 0.020 in. to 0.030 in. on the radius. Where an insufficient number of radial struts are used to hold these castings round, flat spots may be found between the braces. It is advisable to take twenty to thirty equally spaced readings around the circumference to secure an accurate picture of the condition of such rings. Temperature also is an important factor. Daily readings on embedded parts show a gradual distortion as the maximum temperature in the setting concrete is obtained and then a gradual return to the original conditions as this heat is dissipated. Continuous sprays of cooling water on embedded metal parts help to reduce this distortion.

This paper and similar ones materially assist manufacturers in giving field construction engineers a better understanding of the problems to be encountered and of the accuracy of the desired results.

D. E. Brainard,[8] Esq.—Proper installation is an essential prerequisite for successful operation of hydroelectric machinery. Designers may profit greatly from the experience of those who erect their equipment. Therefore, the author's orderly recording of salient points is welcome. The following supplementary comments are offered from the point of view of the generator manufacturer.

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[8] Motor and Generator Eng. Div., General Electric Co., Schenectady, N. Y.

The author has outlined a procedure for minimizing the tendency of the side arms of a girder-type thrust bearing bracket to pinch the guide bearing. In a number of cases, the side arms are attached to the main beams with a hinged joint so that the function of transverse stiffening may be performed without any twisting of the main beams.

Since 1933, at the insistence of certain operating groups, the use of a jacking fit on coupling bolts has flourished. Most manufacturers saw no benefit from the tight fit and the resulting tendency to score the bolts, and preferred to make the bolts and the holes "line and line" so that the bolts could be driven in by a rawhide mallet. With few exceptions, customers now accept the "light driving fit."

Coupling bolts should be stretched in excess of the load imposed on the bolts by the weight of the water-wheel parts, the hydraulic thrust, and the cylinder oil pressure, if any. For Francis-type wheels, however, this stretch may be as small as 0.002 in. The bolts should not be stretched more than necessary in a given case, as excessive tightening may lead to galling the threads.

The author questions the value of factory fitting of coupling bolts and alinement of water-wheel and generator shafts. Certainly, there is a size of unit below which the additional expense of this operation is not justifiable. For large units there appears to be an advantage in employing factory facilities to handle the heavy and sometimes cumbersome parts involved in fitting the bolts. The assurance that the parts will run true when reassembled appears to be a good investment.

The intricate procedure outlined for obtaining proper distribution of load among the shoes of a thrust bearing is required by only one type of bearing. One of the generator manufacturers originated and produces a type of thrust bearing that provides inherent distribution of the load over the bearing surfaces and also automatically compensates for changes in the line-up of the unit which would disturb the loading of a rigidly supported bearing.

The rotation check described in the paper as "customary" has been required by only one customer. It is possible to determine whether a shaft is straight and plumb by checking from tight wires without the laborious process of turning the shaft and rotor. The almost universal practice of making a factory check with the thrust collar on the shaft has been found in practice to give adequate protection against inaccuracies.

A thrust collar forged integral with the shaft does have certain definite advantages, but this feature can be used only when the thrust bearing is below the rotor. This "overhung" design with no upper guide bearing and often no exciter was developed on moderate-sized, low-speed units where the stator and rotor could be made accessible for inspection or repair by disassembling a minimum of related parts without disturbance to the shaft alinement and bearing assembly. This size of unit allowed space for reasonably convenient disassembly of the thrust bearing. As the size and resulting importance of units increased and as the runaway speeds were raised by water-wheel developments, upper guide bearings were added, largely eliminating the element of simplicity. When the output and speed of a generator become such as to require a relatively small diameter, space does not permit reasonably convenient disassembly of the thrust bearing if it is located below the rotor. So, except for a very limited range of size and speed, it is desirable to use the "conventional" mechanical arrangement with the thrust bearing above the rotor. Top location of the thrust bearing reduces its size and permits handling it with the crane.

Andrew M. Komora,[9] M. Am. Soc. C. E.—As Mr. Roberts states, the grouting of embedded parts of a turbine is highly important. It is regretted that, because of space limitations imposed by the restrictions of World War II, it was impossible to include the results of the experiments conducted at Wilson Dam with various grouting procedures. Perhaps these results can be published later as a special phase of turbine erection.

Construction men, especially concrete-placing crews, have a mania for using vibrators close to the turbine-embedded parts. For proper engineering use small vibrators can be permitted under turbine parts if they are securely anchored.

The suggestion made by Mr. Roberts that the turbine and shaft can be raised into position by means of hydraulic jacks is a good one. In certain cases, however, it is more feasible to use temporary coupling bolts as the design of the unit does not lend itself to the use of hydraulic jacks. As stated by Mr. Roberts, it cannot be overemphasized that large-diameter speed rings and pit liners must be secured and braced at close intervals to preclude the possibility of distortion due to concreting.

As emphasized by Messrs. Morgan and Brainard, the use of hinged side arms of a girder-type thrust bearing should eliminate the tendency to pinch the guide bearing when the unit is operating under load. The writer agrees with Mr. Brainard that the coupling bolts should be stretched only slightly in excess of the load imposed on the bolts by weight of the water-wheel parts, the hydraulic thrust, and the cylinder oil pressure, if any. In many installations, the stretch required to take such loads will produce a stress in the bolts close to the allowable working limit.

The only possibility that the turbine and generator shafts made by different manufacturers will not match is the one chance that the coupling faces might not be at right angles to the axis of the shaft. The writer has yet to see an instance in which they failed to be at right angles. It may be accepted that the cost of reaming the coupling holes in the field is far less than that of shipping the entire shaft from one plant to another.

The writer agrees with Mr. Brainard that the rotation check has little, if any, significance in the case of integral thrust collars. It is still of great value in installations where the thrust collar is an independent unit and is either keyed or shrunk on to the shaft.

Thrust collars, forged integral with the shaft, have been used on units up to 60,000 kw with very good results. In general, the designs that are used on integral thrust collars present fewer erection problems to the field forces.

The writer wishes to acknowledge, gratefully, the discussions of Messrs. Gisiger, Morgan, Roberts, and Brainard.

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SHEAR AND BOND STRESSES IN REINFORCED CONCRETE

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Juniors, Am. Soc. C. E.

With Discussion by Messrs. F. R. Shanley, B. J. Aleck, Dean Peabody, Jr., William E. Wilbur, L. E. Grinter, Anders Bull, Victor R. Bergman, Phil M. Ferguson, and Stanley U. Benscoter and Samuel T. Logan.

Introduction

Formulas in common use for computing shear and bond stresses in reinforced concrete beams are based on the assumption that the member is subjected to symmetrical bending without thrust. The object of this paper is to demonstrate the error involved in applying them to members that carry thrust, and to develop general formulas which are applicable to prismatic members with or without thrust. The necessity for such formulas becomes apparent upon realization that practically all members of rigid frames are subjected to thrust as well as bending. The paper provides a new viewpoint for the problem of shear and bond stresses in reinforced concrete and is offered for the purpose of promoting discussion on the problem.

Notation

The following letter symbols, adopted for use in the paper, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials,[3] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932:

A = area of cross section of steel;
b = width of a member;
c = distance from the centroid of a section to the fiber with maximum unit stress;

Note.—Published in March, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Associate Engr., Bureau of Aeronautics, Navy Building, Washington, D. C.

[2] 1st Lt., Corps of Engrs., U. S. Army, Camp Joseph T. Robinson, Ark.

[3] ASA—Z10a—1932.

d = effective depth of a beam:

$k\,d$ = part of the depth that defines the distance from the compressive fiber to the neutral axis;

$\alpha\,d$ = part of the depth that defines the distance from the compressive fiber to the compressive steel;

I = moment of inertia;

$j\,d$ = internal moment arm = $\left(1 - \frac{k}{3}\right) d$;

k = a proportion of the effective depth (see d);

n = the modular ratio;

$p = \frac{A}{b\,d}$ = ratio of steel (compressive or tensile) to the sectional area of the beam;

Q = an integral defined by Eq. 4;

S = section modulus;

T = force acting on tensile reinforcement, dT being a small increment of such force;

u = unit bond stress;

V = total shear;

v = unit shearing stress;

x = a lateral distance, dx (see Eq. 1 and Fig. 1) denoting an element of width b;

y = a vertical distance measured from the centroidal axis: y_0 = the vertical distance between the centroidal axis and a point lying in the tensile region (see Figs. 1 and 2);

α = a proportion of the effective depth (see d);

Σ_o = perimeter of tensile steel.

Subscripts c and t used with the foregoing nomenclature denote "compression" and "tension," respectively. Subscript m denotes "maximum."

BEAM WITHOUT THRUST

A differential slice of a beam, subjected to bending and shear, is shown in Fig. 1 with the forces that act upon the slice. The centroidal axis and the neutral axis coincide. The distances c_c and c_t are the distances from the centroidal axis to the extreme fibers. The forces acting on the tensile reinforcement are T and $T + dT$. Equilibrium of the differential length of the tension bar (or bars) requires that the bond stress u be given by:

$$u = \frac{1}{\Sigma_o} \frac{dT}{dx} \quad \text{.......................... (1)}$$

On any horizontal plane between the neutral axis and the tension steel, such as is indicated by the distance y_0, the shearing stress v is given by:

$$v = \frac{1}{b} \frac{dT}{dx} \quad \text{.......................... (2)}$$

From structural mechanics the well-known formula for shearing stress in a beam may be taken as

$$v = \frac{V\,Q}{I\,b} \quad \text{(3)}$$

in which Q is defined by the integral,

$$Q = \int_{y_0}^{c_t} y\,dA \quad \text{(4)}$$

in which y is measured from the centroidal axis. For the case illustrated in Fig. 1, the quantity Q is merely the moment (about the centroidal axis) of the transformed tensile steel area and, hence, is given by

$$Q = n\,A_t\,c_t \quad \text{(5)}$$

in which A_t is the tensile steel area. Substitution of Eq. 5 into Eq. 3 gives

$$v = \frac{V\,n\,A_t\,c_t}{I\,b} \quad \text{(6)}$$

Two section moduli S_c and S_t may be defined for the beam as

$$S_c = \frac{I}{c_c}\text{; and } S_t = \frac{I}{c_t} \quad \text{(7)}$$

The first modulus S_c may be divided into the bending moment to obtain the maximum compressive stress, and the second modulus may be used similarly to compute the stress in the tensile steel. Introducing S_t into Eq. 6 gives the following formula for the shearing stress,

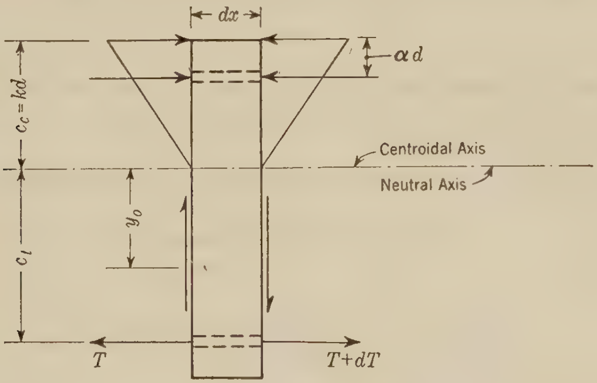

FIG. 1.—MEMBER WITHOUT THRUST

$$v = \frac{V\,n\,A_t}{b\,S_t} \quad \text{(8a)}$$

From Eqs. 1, 2, and 8*a* it may be seen that the corresponding formula for bond stress is

$$u = \frac{V\,n\,A_t}{\Sigma_o\,S_t} \quad \text{(8b)}$$

since

$$u = \left(\frac{b}{\Sigma_o}\right) v \quad \text{(9)}$$

The usefulness of Eqs. 8 depends upon having a knowledge of the section modulus S_t. The following formulas may be derived for S_c and S_t,

$$S_c = \frac{b\,d^2}{6}\left[2\,k^2 + 6\,p_c\,(n-1)\,\frac{(k-\alpha)^2}{k} + 6\,n\,p_t\,\frac{(1-k)^2}{k}\right] \ldots. (10a)$$

and

$$S_t = \frac{b\,d^2}{6}\left[\frac{2\,k^3}{1-k} + 6\,p_c\,(n-1)\,\frac{(k-\alpha)^2}{1-k} + 6\,n\,p_t\,(1-k)\right] \ldots (10b)$$

Since the neutral axis is also the centroidal axis, an equation of moments of areas about this axis can be written as follows:

$$(k-\alpha)(n-1)\,A_c\,d + \frac{b}{2}\,(k\,d)^2 = n\,A_t\,d\,(1-k) \ldots\ldots\ldots\ldots (11)$$

Since Eq. 11 gives a linear relationship between A_c and A_t, it is a simple matter to eliminate either p_c or p_t from Eqs. 10. If p_t is eliminated the formulas for S_c and S_t become:

$$S_c = \frac{b\,d^2}{6}\left[k\,(3-k) + \frac{6\,p_c\,(n-1)(k-\alpha)(1-\alpha)}{k}\right] \ldots\ldots (12a)$$

and

$$S_t = \frac{b\,d^2}{6}\left[\frac{k^2\,(3-k)}{1-k} + \frac{6\,p_c\,(n-1)(k-\alpha)(1-\alpha)}{1-k}\right] \ldots\ldots (12b)$$

It is possible that reinforced concrete beams may be designed more conveniently by using the section modulus concept than by the methods now in common usage. This may be particularly true for designers who work alternatively with steel, timber, and concrete.

If a beam does not contain compressive steel, the formula for S_t becomes:

$$S_t = \frac{k^2\,(3-k)}{1-k}\left(\frac{b\,d^2}{6}\right) (\text{when } p_c = 0) \ldots\ldots\ldots\ldots\ldots (13)$$

From Eq. 11,

$$A_t = \frac{k^2}{n\,(1-k)}\left(\frac{b\,d}{2}\right) (\text{ when } p_c = 0) \ldots\ldots\ldots\ldots\ldots (14)$$

Substituting Eqs. 13 and 14 in Eqs. 8:

$$v = \frac{3\,V}{(3-k)\,b\,d} = \frac{V}{b\,j\,d} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15a)$$

and

$$u = \frac{3\,V}{(3-k)\,\Sigma_o\,d} = \frac{V}{\Sigma_o\,j\,d} \ldots\ldots\ldots\ldots\ldots\ldots (15b)$$

in which $j = 1 - \frac{k}{3}$. These formulas, which are in common use, are seen to be in exact agreement with Eqs. 8 for beams without compressive reinforcement. When a beam contains compressive steel, Eqs. 8 give the correct formulas for v and u, whereas Eqs. 15 are somewhat in error.

MEMBER WITH THRUST

In the case of a member carrying thrust and bending, the neutral axis and the centroidal axis do not coincide. The centroidal axis always lies within the compressive region, as indicated in Fig. 2. The formulas for v and u, given by Eqs. 8, are also applicable to this case. The shear stress v will be the shear stress occurring at any point between the neutral axis and the tensile steel. This is the shear stress which is of interest in reinforced concrete design since it is equal in magnitude to the diagonal tensile stress.

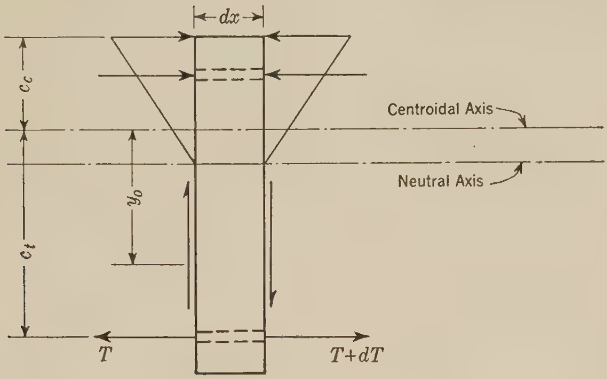
dx
c_c
Centroidal Axis
Neutral Axis
y_0
c_t
T
$T+dT$

FIG. 2.—MEMBER WITH THRUST

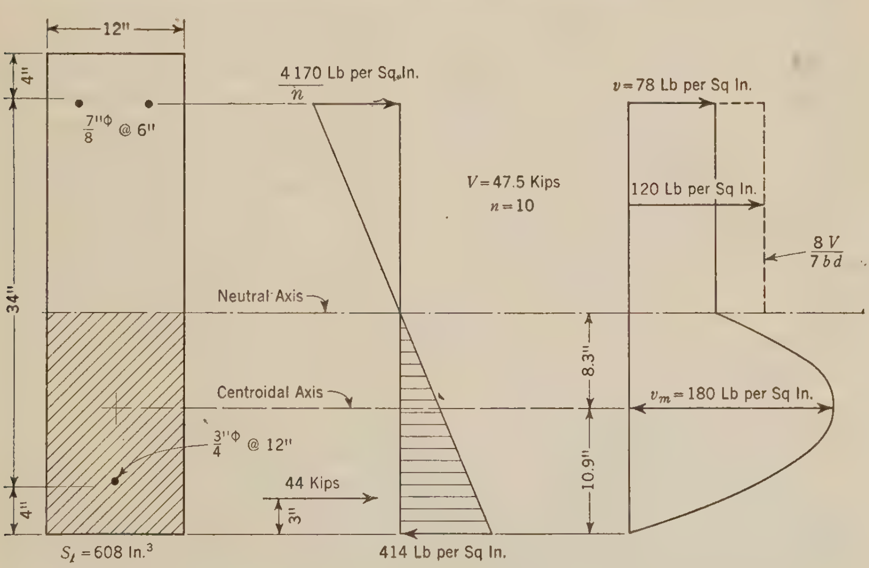
12"
4"
$\frac{7}{8}$"Φ @ 6"
34"
4"
$\frac{3}{4}$"Φ @ 12"
S_t = 608 In.^{3}
4 170 Lb per Sq.In.
n
V=47.5 Kips
n=10
v=78 Lb per Sq In.
120 Lb per Sq In.
$\frac{8V}{7bd}$
Neutral Axis
Centroidal Axis
8.3"
10.9"
v_m=180 Lb per Sq In.
44 Kips
3"
414 Lb per Sq In.

FIG. 3.—INTERNAL STRESS DISTRIBUTIONS

In order that the use of Eqs. 8 may be clear it may be well to emphasize certain characteristics of the section properties of a member with thrust. The distance y in Eq. 4 and the distance c_t in Eq. 5 are measured from the centroidal axis (not the neutral axis). The integral Q, for determining the shearing stresses in any elastic beam, is always computed about the centroidal axis. For given values of thrust and moment the neutral axis can be located. This determines the compressive region. The centroidal axis lies at the center of gravity of the transformed area which includes the total transformed steel area as well as the area of concrete in compression. Any change in the value of the thrust or moment changes the location of the neutral axis. This changes the transformed area and hence changes the location of the centroidal axis. More specifically, all of the flexural properties of the section (the transformed area, the neutral axis, the centroidal axis, the section moduli, and the moment of inertia) change with a change in the eccentricity of the thrust. When the eccentricity is known, all of the flexural properties can be computed. The case that occurs most commonly is that of a beam with constant thrust from end to end. The moments vary along the span and hence the eccentricity of the thrust varies along the span. This means that the flexural properties vary along the span. If the loading on a beam is changed, the flexural properties at a given section will change, assuming that the change in loading produces a change in the moment or thrust at the given section.

Due to the complicated algebraic relationships which arise in a stress analysis of a reinforced concrete member with thrust and bending, it is generally found convenient to use trial-and-error methods. The neutral axis must be located and the flexural properties determined in order to establish the allowable compressive stress in accord with the 1940 Joint Committee Code.[4] A convenient procedure for making this stress analysis was presented in 1942.[5] Such an analysis yields the section modulus S_t which is required in Eqs. 8 for computing v and u. An example will illustrate the significance of the shearing stress given by Eq. 8a.

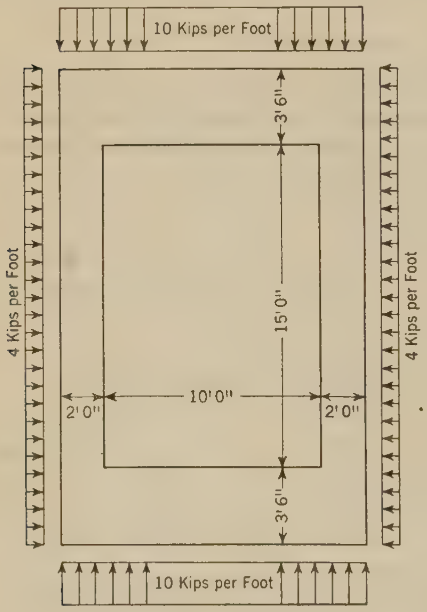

Fig. 4.—Rectangular Conduit with Loads

Fig. 3 shows a section of a member subjected to an eccentric thrust of 44 kips and a shear of 47.5 kips. Since the process of locating the neutral axis and computing the normal stresses is well known these values are shown without discussion. The moment of inertia of the transformed section may be computed readily by straightforward procedures, and the section modulus S_t

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[4] "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," *Proceedings*, Am. Soc. C. E., Pt. 2, June, 1940, Eq. 17, p. 73.

[5] "Thrust and Symmetrical Bending in Reinforced Concrete," by Stanley U. Benscoter, *Civil Engineering*, April, 1942, p. 214.

may be computed to be as shown. The shearing stress diagram is the feature of interest in this example. From the general formula (Eq. 3) it is possible to compute the shear stress at various points within the compressive region and thus to plot the shear-stress diagram. This diagram is of parabolic shape with a slight irregularity at the depth of the compressive steel. The maximum value v_m occurs at the centroidal axis. The stress v, which occurs between the neutral axis and the tensile steel, is only 43% of the maximum v_m. It is this stress v which is equal to the diagonal tensile stress and which governs the beam design. The value of v_m is generally of no interest. A dotted line is shown in Fig. 3 to indicate the shear stress that would be obtained from the formula in the 1940 Joint Committee Code[6] (or Eq. 15a using $j = \frac{7}{8}$). The stress v is 65% of the value given by the code.

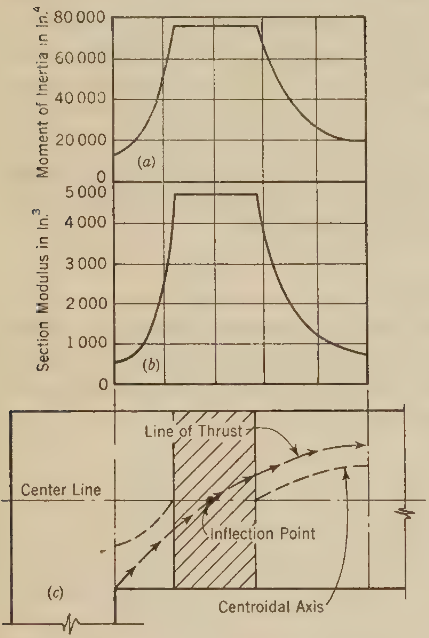

FIG. 5.—SECTION MODULUS AND MOMENT OF INERTIA

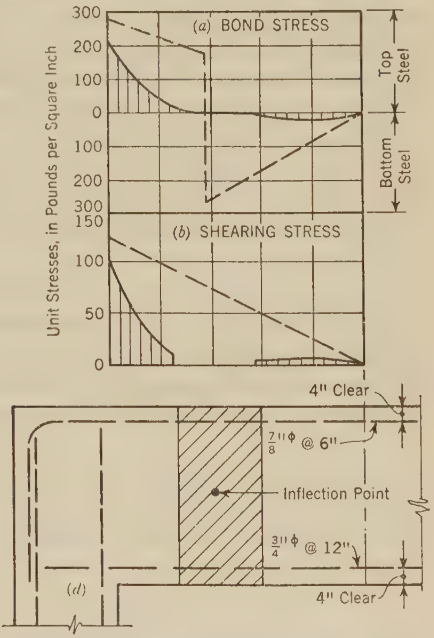

FIG. 6.—BOND AND SHEARING STRESSES

It is of interest to construct bond and shear-stress diagrams along a member. Fig. 4 shows a slice, 1 ft thick, of a rectangular conduit with loads roughly equivalent to the effects of an 80-ft fill. The weight of the conduit has been neglected for convenience. Moments in the frame were computed by moment distribution assuming each member to have infinite rigidity over its supports. Only the clear span of the top slab will be used for discussion. The moment in the top slab at the face of the support is found to be 78.1 ft-kips, the shear is 50 kips, and the thrust is 44 kips. Dividing the moment by the thrust gives an eccentricity of the thrust equal to 21.3 in. From this information the neutral axis may be located and the section modulus S_t found to be 535 in.^{3} The shear and bond stresses are found from Eqs. 8 to be (using $n = 10$): $v = 94$ lb per sq in. and $u = 204$ lb per sq in., respectively.

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[6] "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," *Proceedings*, Am. Soc. C. E., Pt. 2, June, 1940, Eq. 1, p. 52.

Similar analyses were made at various sections of the member which correspond to every 1 in. difference of thrust eccentricity. The section of Fig. 3 was taken at 3 in. from the face of the support in the top slab of the conduit. The locations of the centroidal axis and the line of thrust are shown in Fig. 5(*c*). The variations of the moment of inertia and the section modulus S_t are shown in Figs. 5(*a*) and 5(*b*), respectively. These diagrams illustrate the divergence of a reinforced concrete beam from the ideal straight homogeneous beam which is assumed in the development of flexural theory.

Diagrams of bond and shearing stresses are shown in Fig. 6. The solid lines indicate the stresses as computed by Eqs. 8, and the dashed lines indicate the stresses as computed from the formulas of the 1940 Joint Committee Code. No shearing stress is shown for the shaded region of the beam, since the thrust falls within the kern in this region. There is no diagonal tension in this region.

Stress analysis of reinforced concrete is still in serious need of both theoretical and experimental research to eliminate the unnecessary safety factors which are now inherent in standard design procedures. The experimental research work is needed on actual rigid-frame structures.

Conclusion

Formulas for bond and shearing stresses have been developed which are applicable to members with or without thrust. These formulas are correct in so far as ordinary flexural theory is applicable to reinforced concrete and yield the commonly used formulas as a special case.

DISCUSSION

F. R. Shanley,[7] Esq.—Airplane structural designers frequently abandon the classical formula for transverse shear stress and use a more general method[8] in which the shear flow is determined directly from the differences in axial force at adjacent cross sections along the beam. In effect, this amounts to a direct use of Eq. 2. If the axial stresses have been determined at relatively close stations, the additional work involved to obtain the shear and bond stresses is negligible. The use of the method for bond stresses has been demonstrated by Fred B. Seely.[9]

The main reason for adopting this method in airplane analysis was that the conventional shear stress formula does not account for variations in the cross section along the beam. For instance, a taper in depth may relieve the shear stresses greatly, but this cannot be shown by a formula that deals with only one section at a time. The general method would seem particularly useful for beams in which some of the steel bars are placed at an angle. Another advantage is that the effects of all variables, including compression loading, are included in computing the bending stresses and need not be considered again in determining shear or bond stresses.

The "unit" method, as it is called, is quite sensitive to abrupt changes in cross section. If the sections used for analysis are fairly close together, the resulting shear stresses will appear abnormally high. Some of this effect is relieved by local distortion of the cross section, or by yielding of a ductile material, but the designer regards abnormally high shear stresses as danger signals, indicating poor structural continuity. This might be rather important in dealing with a brittle material like concrete.

B. J. Aleck,[10] Jun. Am. Soc. C. E.—As the authors indicate, common formulas sometimes yield conservative shear and bond stresses, when a member is subjected to axial load. The writer finds the authors' results somewhat incomplete, however, and not always conservative. For example, using the authors' variables:

$$T = n\,A_t\left(\frac{M}{S_t} - \frac{P}{A}\right) \qquad \text{(16)}$$

in which A is the area of the transformed cross section; and the unit shear stress (with P constant) is:

$$v = \frac{1}{b}\frac{dT}{dx} = \frac{n\,A_t}{b}\left(\frac{\frac{dM}{dx}}{S_t} - \frac{M}{S^2{}_t}\frac{dS_t}{dx} + \frac{P}{A^2}\frac{dA}{dx}\right) \qquad \text{(17)}$$

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[7] Div. Engr., Structural Research, Lockheed Aircraft Corp., Burbank, Calif.

[8] "Unit Method of Beam Analysis," by F. R. Shanley and F. P. Cozzone, *Journal of the Aeronautical Sciences*, Vol. 8, No. 6, April, 1941, p. 246.

[9] "Resistance of Materials," by Fred B. Seely, John Wiley & Sons, Inc., New York, N. Y., 2d Ed., 1935, p. 386.

[10] Research Engr. (Materials and Structures), Curtiss-Wright, Buffalo, N. Y.

Considering an element as shown in Fig. 7:

$$\frac{dM}{dx} = V - P\,\frac{dc_t}{dx} \quad\text{......................}(18)$$

Hence

$$v = \frac{n\,A_t}{b}\left(\frac{V}{S_t} - \frac{P}{S_t}\frac{dc_t}{dx} - \frac{M}{S^2{}_t}\frac{dS_t}{dx} + \frac{P}{A^2}\frac{dA}{dx}\right) \quad\text{...........}(19)$$

The authors' solution evidently corresponds to the result if the last three terms in Eq. 19 were to vanish (see Eq. 8*a*). As the prospect of evaluating the derivatives in Eq. 19 is discouraging, the same formula will be derived in terms of simpler variables; thus, if M_s is the moment of all external forces about the tension steel and $j\,d$ is the distance between the resultant compressive force in the concrete and compression steel and the force in the tension steel:

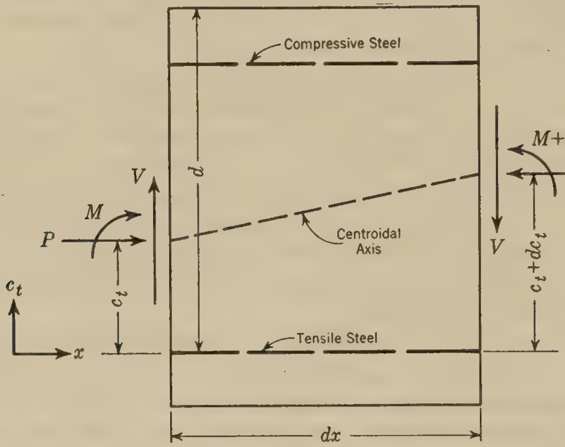

FIG. 7.

$$T = \frac{M_s}{j\,d} - P \quad\text{.........................}(20)$$

and

$$v = \frac{1}{b}\frac{dT}{dx} = \frac{\dfrac{dM_s}{dx}}{b\,j\,d} - \frac{M_s}{j^2\,b\,d}\frac{dj}{dx} = \frac{V}{b\,j\,d} - \frac{M_s}{j^2\,b\,d}\frac{dj}{dx} \quad\text{.........}(21)$$

The resultant resisting compressive force, C, is equal to

$$C = b\,d\,f_c\left[\frac{k}{2} + \left(1 - \frac{\alpha}{k}\right)(n-1)\,p_c\right] \quad\text{............}(22a)$$

Also,

$$P\,e = M_s = b\,d^2\,f_c\left[\frac{k}{2}\left(1 - \frac{k}{3}\right) + (1-\alpha)\left(1 - \frac{\alpha}{k}\right)(n-1)\,p_c\right] \quad\text{..}(22b)$$

and

$$P = C - T = b\,d\,f_c$$

$$\times\left[\frac{k}{2} + \left(1 - \frac{\alpha}{k}\right)(n-1)\,p_c - \left(\frac{1}{k} - 1\right)n\,p\right] = \text{constant}\ldots(22c)$$

Eqs. 22 are well known. Moving from one section to the next, k and f_c change with x. Differentiating Eq. 22c with respect to k:

$$\frac{df_c}{dk} = -\frac{f^2_c\, b\, d}{P}\left[\frac{1}{2} + \frac{\alpha}{k^2}(n-1)\,p_c + \frac{1}{k^2}\,n\,p\right] \ldots\ldots\ldots\ldots (23)$$

Differentiating Eq. 22b with respect to x, using Eq. 23:

$$\frac{dk}{dx} = \frac{V}{f_c\, b\, d^2}$$

$$\times \frac{1}{\dfrac{1}{2} - \dfrac{k}{3} + \dfrac{\alpha}{k^2}(1-\alpha)(n-1)\,p_c - \dfrac{e}{d}\left[\dfrac{1}{2} + \dfrac{\alpha}{k^2}(n-1)\,p_c + \dfrac{1}{k^2}\,n\,p\right]} \ldots (24)$$

$$j = \frac{M_s}{C\,d} = \frac{\dfrac{k}{2}\left(1 - \dfrac{k}{3}\right) + (1-\alpha)\left(1 - \dfrac{\alpha}{k}\right)(n-1)\,p_c}{\dfrac{k}{2} + \left(1 - \dfrac{\alpha}{k}\right)(n-1)\,p_c} \ldots\ldots (25)$$

$$\frac{dj}{dk} = \frac{b\,d\,f_c}{C}\left\{\frac{1}{2} - \frac{k}{3} + \frac{\alpha}{k^2}(1-\alpha)(n-1)\,p_c - j\left[\frac{1}{2} + \frac{\alpha}{k^2}(n-1)\,p_c\right]\right\} \ldots (26)$$

and

$$\frac{dj}{dx} = \frac{dj}{dk}\frac{dk}{dx} = \frac{V}{C\,d}\,B \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

In Eq. 27,

$$B = \frac{\dfrac{1}{2} - \dfrac{k}{3} + \dfrac{\alpha}{k^2}(1-\alpha)(n-1)\,p_c - j\left[\dfrac{1}{2} + \dfrac{\alpha}{k^2}(n-1)\,p_c\right]}{\dfrac{1}{2} - \dfrac{k}{3} + \dfrac{\alpha}{k^2}(1-\alpha)(n-1)\,p_c - \dfrac{e}{d}\left[\dfrac{1}{2} + \dfrac{\alpha}{k^2}(n-1)\,p_c + \dfrac{n\,p}{k^2}\right]} \ldots (28)$$

Therefore,

$$v = \frac{V}{b\,j\,d} - \frac{M_s}{b\,j^2\,d}\frac{dj}{dx} = \frac{V}{b\,j\,d}(1 - B) \ldots\ldots\ldots\ldots\ldots (29)$$

Eq. 29 represents the complete answer. To facilitate simple comparisons it is convenient to compare results when $p_c = 0$. In this case,

$$v = \frac{V}{b\,j\,d}\left[\frac{2\,p\,n\,(1+k)}{k\,(k + 4\,p\,n)}\right]_{p_c=0} \ldots\ldots\ldots\ldots\ldots\ldots (30)$$

whereas the authors' formula leads to

$$v = \frac{V}{b\,j\,d}\left[\frac{\left(1 - \dfrac{k}{2}\right)\left(1 - \dfrac{k}{3}\right)}{1 - k + \dfrac{k^2}{3} + \dfrac{k^3}{12\,p\,n}}\right]_{p_c=0} \ldots\ldots\ldots\ldots\ldots (31)$$

Eq. 30 is readily plotted on a graph as $\dfrac{v\,b\,d}{V}$ versus $p\,n$, for k held constant at 1.0, 0.9, 0.8, etc. Some idea of possible divergence of answers is afforded when

$k = 0.6$ and $p\,n = 0.05$. Then, Eq. 30 yields 52.4% of Eq. 31 and 33.3% of the code value.

More important is the lack of conservatism of both the code and authors' formulas when the axial load is tensile. Thus if $k = 0.20$ and $p\,n = 0.12$, the shearing stress according to Eq. 30 is 212% of that given by the code and 206% of that given by Eq. 31.

After this theoretical discussion, the inadequacy of these results should be emphasized. Some questions that deserve consideration are:

1. How good a criterion is shear for diagonal tension?[11] One authority states: "The vertical shearing stress is not the numerical equivalent of the diagonal tension stress; nor is there any definite relation between them."

2. How accurate is the assumed stress distribution?

   (a) Is the concrete carrying tension (in flexure)? How much?
   (b) Is the stress diagram linear over the transformed section?
   (c) How much load does the compressive steel carry?[12]

3. Is the allowable shear stress given by code to be used with the new formula? Tests should be conducted, especially when axial load is tension.

4. What is the correct value of p to use in the region where bars are lapped? What allowance should be made for variation of effective p-value in this region?

DEAN PEABODY, JR.,[13] M. AM. SOC. C. E.—An alternate method of computing the bond stresses u in tensile reinforcing steel, and the shear stresses v in the concrete between the tensile steel and the neutral axis, is developed in this paper. Also, the variation of shear stress v between the neutral axis and the extreme fiber in compression is discussed. For these computations the transformed area procedure is used. The authors consider only the case of sections that have cracked close enough to the neutral axis for the tensile stresses in the concrete to be neglected.

Beam Without Thrust.—Eqs. 8 use the section modulus S_t, which can be computed by Eq. 12*b* or 13. To compute S_t the neutral axis ratio k must be determined by the usual reinforced concrete equations, by tables or plots, or by the use of the transformed area. A designer working "alternatively with steel, timber, and concrete" could determine j at once, after reviewing the theory sufficiently to compute k. Only for the beam with tension steel does $j = 1 - \frac{k}{3}$, as stated in the "Notation" and, also, immediately following Eqs. 15. However, if compression steel is used, the moment arm $j\,d$ is the distance between the resultant tensile force and the resultant of the compressive forces, and the moment arm ratio j no longer equals $1 - \frac{k}{3}$. If j is determined correctly for beams with compression steel, Eqs. 15 are not "somewhat in error" but give the same result as Eqs. 8. The use of Eqs. 15 or 8 would seem a matter of personal opinion.

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[11] "Design of Concrete Structures," by L. C. Urquhart and C. E. O'Rourke, 4th Ed., McGraw-Hill Book Co., Inc., New York, N. Y., 1940, p. 93.

[12] "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," *Proceedings*, Am. Soc. C. E., June, 1940, p. 45, Section 804(*c*).

[13] Associate Prof., Bldg. Constr., Mass. Inst. of Tech., Cambridge, Mass.

Member with Thrust.—This material is valuable, as the designer has not often investigated the shear stresses in columns subjected to bending, nor included the direct thrust in the design of beams. The authors discuss only the case of compressive stresses on part of the section with the concrete cracked on the tension side. This is a common case for a beam with a direct thrust, but many columns have sections with compressive stresses over the entire area. In such a case the easiest procedure would employ the transformed area, determine the centroidal axis, and use Eq. 3. In fact, the writer would rather use Eq. 3 for all cases of thrust: First, because the maximum shear stress occurs at the centroidal axis (as in Fig. 3) and can be determined by this method; and, second, because Eqs. 8 cannot be used for the shear stress near the tension steel until the transformed area computations furnish the centroidal axis, neutral axis, and moment of inertia. Having proceeded thus far with the transformed area, Eq. 3 can be used as expeditiously as Eq. 8*a*.

In the sixth paragraph from the end of the paper, the authors note that Eq. 15*a* does not give the correct shear stress v in the concrete between the tension steel and the neutral axis (Fig. 3). Their estimate of an error of 65% is not correct, because it is assumed that $j = 1 - \frac{k}{3} = \frac{7}{8}$. The distance between the resultant compressive force and the resultant tension force, if denoted $j\,d$, is less than this and the error is greater (the writer computes $j = 0.83$). However, Eq. 15*a* should never be used for members with thrust. It was derived for the case in which the compressive force C equals the tensile force T; and this is never true for members with thrust. In other words, Eq. 15*a* holds when the centroidal and neutral axes coincide.

Diagonal Tension.—Under the heading, "Members with Thrust," the authors state that the shear stress v is equal in magnitude to the diagonal tensile stress. This is not accurate, because the principal tensile stress s_t on any particle is equal to:

$$s_t = \frac{f}{2} \pm \frac{1}{2}\sqrt{f^2 + 4\,v^2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(32)$$

in which f = fiber stress. To avoid the use of Eq. 32, the shear stress v has been employed as a measure of diagonal tension.[14] The tensile stress s_t may be 10% to 20% greater than the shear stress v at sections where diagonal tension is important. It has been the custom to specify allowable shear stresses sufficiently low so that the allowable tensile stress s_t will not be exceeded. The tables of allowable shear stresses in the past and present Joint Committee Codes and the Regulations of the American Concrete Institute have never clearly stated this fact.

For a beam of single span supported at the ends, the principal tensile stress s_t will make the greatest angle (45°) with the longitudinal axis of the beam at or near the supports. The particles in these sections have small fiber stresses f and comparatively large shear stresses v. One may assume that the concrete on the tension side has not cracked all the way to the neutral axis, and a tensile stress may be assigned to the concrete whose shear equivalent is $0.02\,f'_c$ or $0.03\,f'_c$, even for particles just above the tension steel. It would seem logical, then, in the authors' method, to include some of the tension area of the concrete in the transformed area and in the computation of the centroidal axis, the moment of inertia, and the shear stress v.

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[14] Regulations, A.C.I., 1941, Section 1205*a*.

For continuous beams, the maximum negative moment and the maximum shear force both occur at the support. The assumption of a section cracked to the neutral axis is logical, but designers still assume that the concrete takes a tensile stress whose shear equivalent v equals $0.02 f'_c$ or $0.03 f'_c$ when computing diagonal tension. In continuous beams, the greatest inclination of the principal stress s_t, near the bottom of the beam, will occur close to the point of inflection of the moment diagram. At this point, about one fifth to one fourth the span length from the support, the fiber stresses f are small and the entire concrete area down to the tensile steel might well be used to compute the transformed area and the shear stress. Engineers have never chosen to take time enough to attain this accuracy. However, for members with thrust, cases will arise in which the tension in the concrete is within its allowable stress, and the authors may well consider sections where a part, or the whole, of the concrete tension area is included in the transformed area.

The authors do write briefly about sections whose particles are wholly in compression in the third paragraph from the end of the paper. The statement, "There is no diagonal tension in this region," for such particles is not completely accurate. The principal tensile stress s_t equals:

$$s_t = \frac{f_c}{2} - \frac{1}{2}\sqrt{f^2_c + 4\,v^2} \dots\dots\dots\dots\dots\dots (33)$$

and will have a small, probably negligible, value.

The writer has found this paper stimulating but confesses that he leaves it with a greater respect for the transformed area solution for all cases except the beam without thrust.

William E. Wilbur,[15] M. Am. Soc. C. E.—The subject of combined flexure and direct stress in concrete beams has always been a troublesome one for designers to analyze. In a measure, this is due to two assumptions made in concrete design—namely, a straight-line variation of fiber stress, and no tension in the concrete. It is no part of this discussion to question these assumptions, since they lead to simple and sufficiently accurate formulas. They will be assumed as correct.

In the first part of their paper, the authors derive expressions for unit shear and bond stress in beams subject to bending only. In order not to confuse the nomenclature, let $z\,d$ be the depth from the centroid of the tensile fiber stresses to the centroid of the compressive fiber stresses. Then, for any beam, of any shape, the laws of mechanics give the unit shear at the neutral axis as

$$v = \frac{V}{b\,z\,d} \dots\dots\dots\dots\dots\dots (34a)$$

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[15] Structural Engr., The Fluor Corp., Kansas City, Mo.

When b is constant, this is the maximum unit shear. For a rectangular section, such as a timber beam, z has the value of $\frac{2}{3}$, and the expression becomes

$$v = \frac{3\,V}{2\,b\,d} \qquad \text{(34}b\text{)}$$

For a concrete beam without compressive steel, $z = 1 - k/3$, or "j," and Eq. 34a becomes Eq. 15a. For an I-beam or plate girder with web thickness b, z is nearly unity, and substitution in Eq. 34a gives the common approximation

$$v = \frac{V}{b\,d} \qquad \text{(34}c\text{)}$$

When there is compressive steel in a concrete beam, the value of z will be exactly the same as j only when the compressive steel is at the centroid of the compressive concrete fiber stresses. Substitution of $k/3$ for α in Eqs. 11 and 12 will be found to result in the same values of Eqs. 15 as those given by the authors.

In actual design, the area of compressive steel is usually relatively small, and the value of α is not far from $k/3$. It follows, then, that the value of v will not vary greatly from the value $\frac{V}{b\,j\,d}$ (Eq. 15a) and that expression will nearly always be sufficiently exact for practical purposes.

In the latter part of their paper, the authors discuss the case of flexure and direct stress. They are correct in their statement (see heading, "Member with Thrust") that "Any change in the value of the thrust or moment changes the location of the neutral axis," and hence all the properties of the section. In discussion of a previous paper[16] the writer demonstrated a simple method for determining the fiber stresses in this case, without recourse to special diagrams or complicated formulas.

The authors are incorrect in their statement that "The integral Q * * * is always computed about the centroidal axis," and this error leads to the irrational results shown in their Fig. 3.

Consider a beam subject to flexure, or flexure and direct stress, with maximum fiber stresses at two adjacent sections of f and $(f + \Delta f)$, respectively (see Fig. 8). The hatched area represents the difference between the fiber stresses at the two sections, and is a measure of the horizontal shear. The horizontal shear at S, for example, is measured by the part of the hatched area between S and the extreme fiber. It is obvious that the horizontal shear is maximum at the neutral axis, and cannot be otherwise, on the assumption of no tension in the concrete.

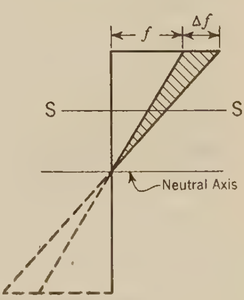

FIG. 8.

If Eq. 34a is applied to the example in Fig. 3, and z is taken as approximately equal to $38 - (\frac{1}{3} \times 19.2)$, the maximum unit shear, between the neutral axis and the tensile steel, is found to be approximately 125 lb per sq in.

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[16] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 385.

The effect of the thrust, then, by decreasing the value of z, is to increase somewhat the maximum unit shear over that in a member with flexure only; and the authors have done well to call this to the attention of designers.

When the thrust is so great as to produce compression over the entire section, the centroidal axis and the neutral axis will coincide, and z will approach the value of $\frac{2}{3}$. The writer cannot agree with the authors' statement (third from the last paragraph) that there is no diagonal tension when the thrust falls within the kern. Diagonal tension will always accompany shear, but in this case the unit diagonal tension will no longer be numerically equal to the unit shear.

L. E. GRINTER,[17] M. AM. SOC. C. E.—The authors approach their problem with the logic of straightforward analysis. They accept the usual assumptions of the analysis of reinforced concrete beams and derive the standard formulas introducing such generalizations as will permit them to study the influence, on the cross section, of a thrust as well as a bending moment. Their thesis is that the thrust must be considered for the design of the cross section to resist shear and bond stresses properly.

Perhaps without intention they have chosen an example that is about the strongest reasonable case for supporting that thesis. It is a beam having a prismatic ratio (length over depth) of less than three as contrasted to a common ratio in building design of from ten to twenty. Moreover, this deep beam must resist a great thrust produced by uniform lateral pressure of the earth over the 22-ft height of the structure. Figs. 5 and 6 establish certain major discrepancies between the theoretical bond and diagonal tension stresses (not shear stress as labeled in Fig. 6) for the structure studied. Less serious discrepancies, probably of little consequence, can be anticipated for building frames, but this is no criticism of the paper, which gives only casual attention to building frames and appears intended as a study of heavy foundation structures. It is an excellent brief presentation of an interesting thesis.

The writer has ardently supported mathematical analysis as about the only device available to the designer for the production of a balanced structure; but the designer must stop and analyze the real meaning of such calculated stresses as $\frac{V}{b\,j\,d}$ to reach a logical decision as to where the refinement proposed by the authors is valuable and where it is merely theoretical "excess baggage."

A few assumptions used by the authors may be reviewed profitably because similar assumptions are always necessary for the analysis of any structure. The analysis of moments and thrusts follows from assumptions as to the loads shown as vertical uniform pressures on the top and bottom of the box (Fig. 4). Such pressure lines are never uniform but are actually curved. Likewise, the side pressures vary; they increase toward the bottom and are also likely to be unsymmetrical with respect to the vertical axis. These factors seldom can be brought into a design because they are not determined by theory but by local conditions.

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[17] Vice-Pres. and Dean, Graduate School, Illinois Inst. of Technology, Chicago, Ill.

Based upon conventional load assumptions, a designer computes a set of moments using stiffness factors, in all likelihood computed from I-values obtained as $\frac{b\,d^3}{12}$. If he tries the refinement of using I-values in agreement with the transformed section (see Eqs. 10), what transformed section shall he use? Evidently, since each member of a frame usually has one or more points of contraflexure, the transformed section could be either the usual one or an upside down section. In reality, over a large part of the length of the beam the effective section is certainly a section of uncracked concrete, since a crack covers only a fraction of an inch. Actually, the beam has a constantly varying moment of inertia, the degree of variation being unknown. In desperation, engineers can be excused for abandoning the idea of using the transformed section and of adopting $\frac{b\,d^3}{12}$ for the moment of inertia in determining stiffness factors; but in doing so they must not forget that this crude assumption has had its inevitable influence on computations for bending moments.

Then there is the problem of the corner action which cannot be solved by common methods of analysis. Perhaps the designer takes the theoretical span of the beam as the clear span; perhaps he chooses the span center-to-center of columns; or perhaps with corner fillets he uses less than the clear spacing between columns as the span. Whichever he chooses, the result expressed as a bending moment diagram is only an approximation—a result of judgment as well as of mathematical analysis.

Such crudities of analysis are even more obvious when the member contains both a bending moment and a thrust. The fit of a structure within confining material may shift a line of thrust by a small distance that would be of no consequence if it represented the vertical reaction under a horizontal beam; but, if it represented the location of a longitudinal thrust acting on a vertical member, it may have an enormous influence upon the stress in the member because a shift of even 1 in. in the eccentricity of a longitudinal thrust could be disastrous.

Finally, Figs. 5 and 6 reveal the theoretical fact that section moduli, fiber stresses, bending moments, bond and shearing stresses all change sharply, whereas common sense dictates that such changes are actually gradual because stress builds up slowly and not suddenly. This inaccuracy is the result of the designer's inability to treat the structure mathematically as a series of small particles. He must adopt gross procedures such as the treatment of an entire beam cross section as a single element.

It is necessary to analyze the suggestions offered in this paper with these facts in mind in order to decide just where the proposed equations should be used. The specification formula, $v = \frac{V}{b\,j\,d} < 120$ or 240, is not a statement about any stress that actually exists in the structure. It is just a rule for controlling a size of cross section, more justified by experiment and observation of its satisfaction in practical cases than by its theoretical derivation. In other words, whether the calculated shearing stress $\frac{V}{b\,j\,d}$ is high or low is immaterial if the specification writer takes care of that fact in his choice of an allowable working stress in shear.

The example offered by the authors seems to be an exaggerated case, and the effect of thrust on bond and shear in beams or columns of common length-depth ratio may not usually be of sufficient importance to consider. If most of the inaccuracies that have been mentioned in this discussion exist in the analysis of a given reinforced concrete structure, it is unlikely that the designer can hope to know the stress at a given point any closer than ± 25%. That being so, he can obtain little benefit from the theory presented in the paper unless the influence of the thrust on bond or diagonal tension proved to be of about this magnitude.

It is worth noting in Fig. 3 that the theoretical shear reaches 180 lb per sq in. on a cross section where the shear formula gives 120 lb per sq in. and the authors' theory shows the critical shear causing diagonal tension to be only 78 lb per sq in. Presumably, shear in reinforced concrete is not very serious if it cannot produce diagonal tension. However, it is well to recognize that the formulas in the paper, which show very low shear stresses, refer only to shears in the diagonal tension areas of the beam; elsewhere, larger shears than those given by $v = \frac{V}{b\,j\,d}$ may exist. Even though such increased shears may not be serious, the authors have called attention to a change in stress pattern that designers have probably been overlooking. It is a serious fault to overlook possible stress conditions. They have a habit of suddenly becoming critical. For example, the authors have not advised designers to look for the possibility that a negative thrust (a tension) can upset their computations by producing a more serious diagonal tension and bond condition than those represented by the formula $v = \frac{V}{b\,j\,d}$. Nevertheless, the theory presented by the authors can be applied to the case of negative thrust.

Anders Bull,[18] M. Am. Soc. C. E.—It is suggested in the paper that reinforced concrete beams may be designed more conveniently by the section modulus concept than by the methods in common use. This may be true for beams in plain bending, where the section moduli S_c and S_t are referred to the centroidal axis which, in that case, coincides with the neutral axis. When thrust is also acting, however, the fact that the two axes no longer coincide leads to complications.

Formulas published by the writer[19] in 1921, although similar to Eqs. 12, are applicable to both types of problems. A bending moment M and a thrust P', applied at a distance a from the center line of tensile steel, may be replaced, according to a well-known principle, by an eccentric force $P = P'$, acting at a distance $e = a + \frac{M}{P}$ from the tensile steel; that is:

$$P\,e = P\,a + M \quad \text{(35)}$$

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[18] Civ. Engr., Board of Transportation, New York, N. Y.

[19] "Computing Stresses in Reinforced Concrete Arches and Other Structures," by Anders Bull, *Concrete* (Detroit, Mich.), December, 1921, p. 242.

When no thrust is acting, the term $P\ a$ drops out $(0 \times \infty)$, in which case $P\ e = M$.

Treating $P\ e$ as the moment, the equilibrium equations for rotation about the respective centers of flange strain yield, for a linear stress distribution, the following formulas in which f denotes the critical stresses:

$$\frac{d}{e} = \frac{6\, p_c\, (n-1)\, (k-\alpha) - 6\, p_t\, (1-k) + 3\, k^2}{6\, p_c\, (n-1)\, (k-\alpha)\, (1-\alpha) + 3\, k^2 - k^3} \ldots\ldots\ldots (36)$$

$$P\ e = b\, d^2 \frac{6\, p_c\, (n-1)\, (k-\alpha)\, (1-\alpha) + 3\, k^2 - k^3}{6\, k} f_c \ldots\ldots (37a)$$

$$P\ e = b\, d^2 \frac{6\, p_c (n-1)\, (k-\alpha)\, (1-\alpha) + 3\, k^2 - k^3}{6\, n\, (1-k)} f_t \ldots\ldots (37b)$$

Designating the fractions in Eqs. 37 by C_c and C_t, respectively:

$$f_c = \frac{P\ e}{b\, d^2\, C_c} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38a)$$

and

$$f_t = \frac{P\ e}{b\, d^2\, C_t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38b)$$

Only for plain bending, when $P\ e = M$, do the denominators in Eqs. 38 represent the section moduli of the cross section about the neutral axis.

The average shearing and bond stresses are found to be:

$$v_{\text{avg}} = \frac{V}{b\, d} \left(\frac{d}{e} + \frac{p_t}{C_t} \right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (39a)$$

and

$$u_{\text{avg}} = \frac{V}{b\, \Sigma_0} \left(\frac{d}{e} + \frac{p_t}{C_t} \right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (39b)$$

There are numerical tables[19] from which C_c and C_t can be determined conveniently, using p_t and $\frac{d}{e}$ for arguments. When the tables are not available, the problem is solved by first determining the value of k which is the only unknown in Eqs. 37. This is easily done by means of Eq. 36, which is resolved into the following third-degree equation:

$$\frac{d}{e} k^3 + 3 \left(1 - \frac{d}{e} \right) k^2 + 6 \left[p_c\, (n-1) \left(1 - \frac{d}{e} + \alpha \frac{d}{e} \right) + p_t\, n \right] k$$

$$= 6 \left[p_c\, \alpha\, (n-1) \left(1 - \frac{d}{e} + \alpha \frac{d}{e} \right) + p_t\, n \right] \ldots\ldots\ldots\ldots (40)$$

For plain bending $\frac{d}{e} = 0$, and Eq. 40 is reduced to:

$$k^2 + 2\, [p_c\, (n-1) + p_t\, n]\, k = 2\, [p_c\, \alpha\, (n-1) + p_t\, n] \ldots\ldots (41)$$

Evidently the hypothetical example chosen by the authors has not been correctly treated, since the external and internal moments do not balance. For a cross section 3 in. from the face of the support (Fig. 3), the moment due to roof load is 791 in.-kips, and the moment due to thrust is $44 \times 16.2 = 713$ in.-kips—a total external moment of 1,504 in.-kips. Since the center of steel is 4.4 in. from the surface, the stresses indicated in the diagram could develop only the following internal moments:

$$
\begin{array}{lr}
\text{Concrete}\ldots\ldots\ldots\ldots 12 \times 19.2 \times \dfrac{414}{2} \times \dfrac{2}{3} \times 19.2 & = 610{,}470 \\
\text{Compression steel}\ldots 0.442 \times 9 \times 414 \times \dfrac{14.8}{19.2} \times 14.8 & = 18{,}800 \\
\text{Tension steel}\ldots\ldots\ldots 1.203 \times 10 \times 417 \times 18.4 & = 92{,}310 \\
\hline
\text{Total internal moment (inch-pounds)}\ldots\ldots\ldots\ldots & = 721{,}580
\end{array}
$$

—an amount which suffices for balancing the moment due to thrust only.

Furthermore, with a thrust of 44 kips applied 3 in. above the bottom of the slab, there could be no point of inflection as in Fig. 5, the roof load moment at the center of span being $\left(78.1 - \dfrac{10 \times 5^2}{2}\right) 12 = -563$ in.-kips on which is superimposed a thrust moment of about $44 \left(\dfrac{42}{2} - 3\right) = +792$ in.-kips, making a total of $+229$ in.-kips. The "line of thrust" shown in Fig. 5 has no physical meaning, as the thrust remains at the same height throughout the span.

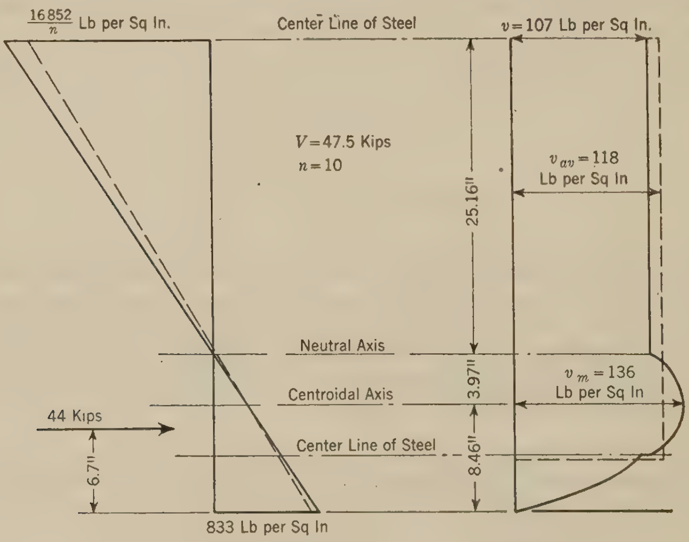
$\frac{16852}{n}$ Lb per Sq In.
Center Line of Steel
$v = 107$ Lb per Sq In.
$V = 47.5$ Kips
$n = 10$
$v_{av} = 118$ Lb per Sq In
25.16"
Neutral Axis
3.97"
$v_m = 136$ Lb per Sq In
Centroidal Axis
44 Kips
Center Line of Steel
8.46"
6.7"
833 Lb per Sq In

FIG. 9

For comparison, Fig. 9 shows the stresses obtained from Eqs. 36 to 40. The thrust is applied 6.7 in. from the bottom of the slab, being the resultant of

$3.5 \times 4 = 14$ kips acting at the center of the cross section and $7.5 \times 4 = 30$ kips acting at the bottom.

A novel feature of the paper is the distribution of the shearing stresses across a transverse section. The findings are interesting from a theoretical point of view, but their practical value may be questioned, as the average shearing stress is so much more easily determined. The assumptions on which reinforced concrete design is based are far from consistent in regard to tensile stresses in concrete. Such stresses are considered when diagonal but are disregarded when longitudinal, although the latter stresses obviously contribute to the bending strength. The aim is to simplify the computations. For the same reason any highly refined treatment would seem somewhat futile. After all, the allowable stresses governing the design are determined by the cut-and-try method. The possibility of future excavation alongside a subway structure like the one in Fig. 4 must also be considered as it may cause the thrust to be removed temporarily, eliminating its relieving effect on the shearing stresses.

The significance of the bulging shape of the shear distribution curve is easily seen by investigating another cross section, located a little closer to the center of span. The new stress distribution line (inclined dashed line in Fig. 9) is then found to intersect the old line at the centroidal axis, showing that the compressive stresses below the axis are reduced whereas those above are increased. These changes conform to the shape of the curve, whose first derivative $\frac{dx}{dy}$ is proportional to the change in compressive stress at any given height y.

Victor R. Bergman,[20] Assoc. M. Am. Soc. C. E.—In calling attention to the errors that can result from a careless application of the commonly used formulas for shear and bond stresses to beams subjected to thrust as well as bending, the authors have performed a useful service. However, their paper contains a few questionable points which the writer wishes to discuss. For example, $j\,d$ is not in general exactly equal to $\left(1 - \frac{k}{3}\right) d$, even for beams without thrust. The relationship does not hold quite true for T-beams or even for prismatic beams with compressive steel. For the case of beams carrying thrust, the expressed relationship is very definitely in error. The writer would prefer to substitute for the authors' definition of $j\,d$ that given in the Joint Committee Code[21]—"$j\,d$ = the moment arm of the internal force couple."

Fig. 2 is drawn incorrectly, because the compressive stress triangles are depicted as if the $k\,d$-distances were equal for the two sections. Such is not the case, because, as the authors state under the heading, "Member with Thrust," "Any change in the value of the thrust or moment changes the location of the neutral axis." This fact is illustrated in Fig. 10, which shows a definite shift in the position of the neutral axis in the distance dx.

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[20] Structural Designer, The Kellex Corp., New York, N. Y.

[21] "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," *Proceedings*, Am. Soc. C. E., Pt. 2, June, 1940, p. 53.

From Fig. 10, using the notation shown, and taking moments about any point at the level of the force $C + \Delta C$, it is possible to write (approximately):

$$\Delta T\left(d - \frac{k\,d}{3} + \frac{h}{3}\right) = V\,dx - C\,\frac{h}{3} \quad \ldots\ldots\ldots\ldots\ldots (42)$$

and, since $v\,b\,dx = \Delta T$:

$$v = \frac{V\,dx - C\,(h/3)}{\left(d - \frac{k\,d}{3} + \frac{h}{3}\right)(b)\,(dx)} \quad \ldots\ldots\ldots\ldots\ldots (43)$$

Eq. 43 is slightly approximate, because it assumes the location of each resultant compressive force at one third of its respective $k\,d$-distance from the top of the member, thus ignoring the modifying effect of the compressive steel. An accurate determination of the positions of the resultant compressive forces can be made easily by well-known methods, if desired.

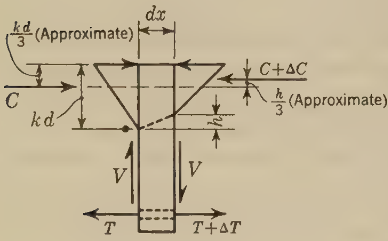
$\frac{kd}{3}$ (Approximate)
dx
C
$C+\Delta C$
kd
$\frac{h}{3}$ (Approximate)
h
V
V
T
$T+\Delta T$

Fig. 10

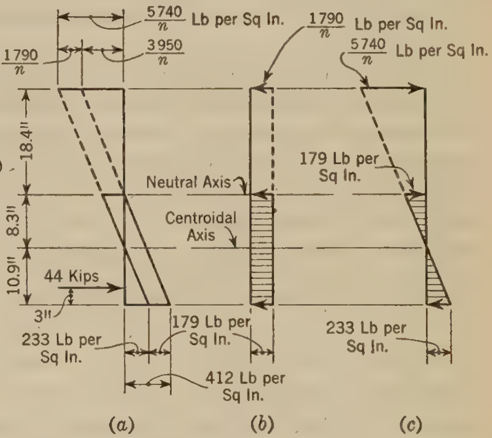
$\frac{5740}{n}$ Lb per Sq In.
$\frac{3950}{n}$
$\frac{1790}{n}$
$\frac{1790}{n}$ Lb per Sq In.
$\frac{5740}{n}$ Lb per Sq In.
18.4"
8.3"
10.9"
Neutral Axis
Centroidal Axis
179 Lb per Sq In.
44 Kips
3"
233 Lb per Sq In.
179 Lb per Sq In.
412 Lb per Sq In.
233 Lb per Sq In.
(a)
(b)
(c)

Fig. 11.—Member with Thrust—Internal Stress Distribution (V = 47.5 Kips; and n = 10)

The writer has applied Eq. 43 to the case shown in Fig. 3, by first locating the neutral axis of a section parallel to that shown in Fig. 3, but 1 in. closer to the left support (Fig. 4). The neutral axis for this new section is approximately 18.15 in. from the bottom of the beam, thus making h = 1.05 in. and $C = 44{,}000 + 1.2 \times 3{,}950 = 48{,}740$ lb. Then, approximately,

$$v = \frac{(47{,}500)\,(1) - (48{,}740)\left(\frac{1.05}{3}\right)}{\left(37.6 - 6.4 + \frac{1.05}{3}\right)(12)\,(1)} = 80 \text{ lb per sq in.},$$

which agrees fairly well with the authors' determination. An even better agreement could be achieved by: (1) Computing the actual position of the resultant compressive forces; and (2) using a dx-distance smaller than 1 in.

It should be noted carefully that the writer has not used the length $j\,d$ in this determination of v.

Another and probably more instructive and useful approach to this problem is to consider the section (Fig. 3) as resisting: (1) A direct compression of 44,000 lb applied at the center of gravity of the transformed area; and (2) a bending moment whose value is 44,000 (10.9 − 3.0) = 348,000 in–lb.

The stress diagram of Fig. 3 (slightly modified by corrections) is reproduced in Fig. 11(*a*) where the combined stress diagram is resolved into two component parts—that due to direct compression and that due to bending, as shown separately in Fig. 11(*b*) and Fig. 11(*c*), respectively. The direct compressive stress of Fig. 11(*b*) is obtained simply enough by dividing the 44,000-lb thrust by the transformed area A_{TR}. The section shown in Fig. 3 has a transformed area, $A_{TR} = (12)\,(19.2) + (10 - 1)\,(0.44) + (10)\,(1.2) = 230.4 + 4.0 + 12.0 = 246.4$ sq in., and a direct compressive stress (in pounds per square inch) of $\frac{44{,}000}{246.4} = 179\,-$. This stress can be obtained also from Fig. 11(*a*) by a simple proportion; thus (in pounds per square inch): $\frac{8.3}{19.2}\,(412) = 178\,+$.

With this stress value and the relationships shown in Fig. 11(*a*), the stresses of Fig. 11(*c*) are found easily. The stresses acting on the section in Fig. 11(*c*) result solely from flexure and therefore should, and do, produce equal compressive and tensile forces, C and T, which can be computed as follows: $C = \frac{233}{2}\,(12)\,(10.9) + (233)\,\frac{6.5}{10.9}\,(4.0) = 15{,}800$ lb; and $T = \frac{179}{2}\,(12)\,(8.3) + 5{,}740\,(1.2) = 8{,}920 + 6{,}880 = 15{,}800$ lb. These two forces form a resisting couple whose lever arm, calculated by standard procedure, is 22.0 in. This couple, whose numerical value is 15,800 × 22.0 = 348,000 in-lb, exactly balances the bending moment already referred to, caused by the thrust of 44,000 lb acting at a distance of 7.9 in. from the center of gravity of the transformed section.

The authors state that "A dotted line is shown in Fig. 3 to indicate the shear stress that would be obtained from the formula in the 1940 Joint Committee Code[6] (or Eq. 15*a* using $j = \frac{7}{8}$)." Although it is true that the use of $\frac{7}{8}$ for $j\,d$ in Eq. 15*a* will give the erroneous result indicated by the authors, it should be understood clearly that the error is caused by a mistake in the evaluation of $j\,d$ rather than in the formula, which is correct.

The Joint Committee Code defines $j\,d$ as "the moment arm of the internal force couple." Bearing this definition in mind, a glance at Fig. 3 should make apparent the fallacy involved in taking $j\,d$ as $\frac{7}{8}\,d$ (= 32.9 in.), or even as $\left(1 - \frac{k}{3}\right) d$ (= 31.2 in.). The stresses shown as acting on the section do not produce the equal, opposite forces required for a couple. The writer's value of 22.0 in. for the moment arm of the resisting couple (Fig. 11(*c*)), is the proper value of $j\,d$ for use in Eq. 15*a*.

Substituting numerical values, $v_m = \frac{47{,}500}{12 \times 22.0} = 180$ lb per sq in., which checks exactly the authors' finding for v_m.

To verify the value of $v = 78$ lb per sq in. given in Fig. 3, it is necessary first to determine what proportion of the total resisting moment is furnished by the tensile steel (see Fig. 11(*c*)). The position of the resultant compressive force is found readily to lie at a distance of 3.7 in. from the bottom of the beam ($10.9/3 = 3.6$ in. could be used with but trifling error). The proportion of the total resisting moment, furnished by the tensile steel, is now found to be: $\frac{(1.2)\,(5{,}740)\,(37.6 - 3.7)}{348{,}000} = 0.67$.

If the tensile steel carried all of the tension involved in the resisting couple, the shearing stress, v, would equal: $\frac{47{,}500}{12\,(37.6 - 3.7)} = 117$ lb per sq in. Actually, $v = 0.67 \times 117 = 78$ lb per sq in., which agrees with the authors' value.

The writer has presented two additional methods of calculating shearing stresses in beams subject to thrust. Whether or not the individual designer will consider either method preferable to that furnished by the authors is a matter of personal choice.

Phil M. Ferguson,[22] M. Am. Soc. C. E.—The limitations that attend the proper use of the ordinary formulas for bond and shearing stress in reinforced concrete beams are discussed in this paper. In view of many discussions in recent years relative to the divergence between calculated and actual steel stresses, especially where direct stress is involved, any one would be cautious about insisting on any value of bond stress as a true value. Shrinkage and flow of concrete alter these stresses significantly, even though designers do not usually include their effect in calculations. This is not to disparage the calculations given in this paper and those presented herewith, but rather to emphasize the artificial base upon which both practice and theory now rest. So long as specifications and practice maintain this artificial base, designers, in order to be consistent, must use calculations of this type.

Since bond stress is measured by the change in tensile steel stress, it is entirely logical that direct compressive stress, as it reduces the steel stress, should also reduce the bond stress. However, differences as great as those shown in Fig. 6 are somewhat startling and might seriously modify a design.

The question immediately arises as to whether there are many situations where the ordinary formulas—

$$u = \frac{V}{\Sigma_o \frac{7}{8} d} \qquad \text{(44a)}$$

and

$$v = \frac{V}{b \frac{7}{8} d} \qquad \text{(44b)}$$

—are inadequate. These formulas are simple and commonly known. They fit beams not subject to direct stress. Does the small direct stress which may be present in many beams in a continuous frame generally limit their usefulness? If not, designers are not apt to adopt the authors' formulas for general use. The proposed formulas are simple enough in appearance, but contain a very troublesome term S_n. It will be recalled that S_n is based on a centroid which shifts with the ratio between M and P. Although not a complex calculation, the determination of S_n is lengthy enough to cause one to avoid it wherever possible. No simpler equation seems to be available for those cases where direct stress really modifies bond or shear significantly. Investigation reveals, however, that small axial compressive loads acting on ordinary beams do not modify bond and shear stresses seriously enough to be a matter of concern. This might be more accurately stated by saying that no compressive force will modify shear or bond stresses as much as 10% unless it is large enough, relative to the moment, to produce an e/h-ratio (Fig. 12) of less than unity.

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[22] Chairman, Civ. Eng., Univ. of Texas, Austin, Tex.

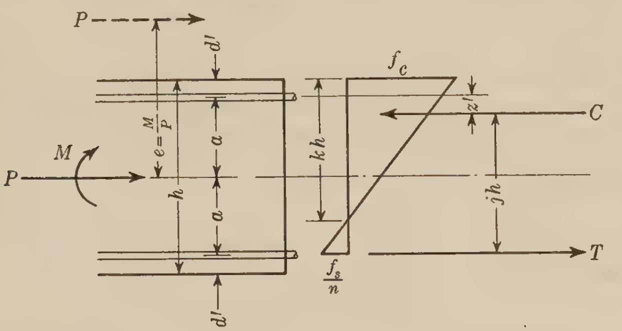

FIG. 12

Although some of this might have been ascertained by applying the authors' method to a few sample cases, it seemed desirable to investigate the theoretical relationships more thoroughly in order to establish the general area of significant difference. The following proofs include the familiar terms k and j as fractional parts of the over-all depth h, the total depth, instead of the effective depth d. This is in accordance with the notation commonly used in combined direct stress and bending.

The applied M and P (at mid-depth) are held in equilibrium by a resultant compression C and a tension T (Fig. 12). Summation of moments about the tension steel gives $C\,j\,h = M + P\,(0.5\,h - d')$, and

$$C = \frac{M + P\,(0.5\,h - d')}{j\,h} \quad \text{.................... (45)}$$

Since $C = T + P$, $\frac{dC}{dx} = \frac{dT}{dx}$. Also $dT = u\,\Sigma_o\,dx$. Hence, $u\,\Sigma_o = \frac{dT}{dx} = \frac{dC}{dx}$

$$= \frac{1}{j\,h}\,\frac{dM}{dx} - \frac{M + P\,(0.5\,h - d')}{(j\,h)^2}\,h\,\frac{dj}{dx} = \frac{V}{j\,h} - \frac{P\,e + P\,(0.5\,h - d')}{j^2\,h}\,\frac{dj}{dx}\text{; and}$$

$$u = \frac{V}{\Sigma_o\,j\,h}\left[1 - \frac{P}{V}\,\frac{(e + 0.5\,h - d')}{j}\,\frac{dj}{dx}\right] \quad \text{.............. (46)}$$

Although Eq. 46 might seem to indicate that the correction term in the bracket depends upon V and possibly upon P, it will be shown that a proper evaluation of the term $\frac{dj}{dx}$ results in the elimination of all the load terms from this expression, with the exception of e, the equivalent eccentricity of the load. An expression for $\frac{dj}{dx}$ will now be found.

The resultant compression C can be located by taking moments of the compressive forces about the center line of the compression steel:

$$\left(\frac{f_c}{2} b\,k\,h\right)\left(\frac{k\,h}{3} - d'\right) = C\,z' = \left[\frac{f_c}{2} b\,k\,h + p'\,b\,h\,n\,f_c\left(\frac{k\,h - d'}{k\,h}\right)\right] z'$$

$$z' = \frac{k^2\left(\frac{k}{3} - \frac{d'}{h}\right)}{2\,p'\,n\left(k - \frac{d'}{h}\right) + k^2}\,h \ldots\ldots\ldots\ldots(47)$$

It will be noted that $p'\,n$ is used instead of the value $p'\,(n-1)$, sometimes preferred. The value of n is not usually accurate enough to justify any distinction. Since $z' + j\,h$ is the distance between tension and compression steel, which is fixed, $\frac{dz'}{dx} + h\,\frac{dj}{dx} = 0$, and

$$\frac{dj}{dx} = -\frac{1}{h}\frac{dz'}{dx} \ldots\ldots\ldots\ldots(48)$$

Differentiating Eq. 47:

$$\frac{dz'}{dx} = h\left[\frac{k^2 - 2\frac{d'}{h}k - 2\left(\frac{z'}{h}\right)(p'\,n + k)}{2\,p'\,n\left(k - \frac{d'}{h}\right) + k^2}\right]\frac{dk}{dx} \ldots\ldots\ldots(49)$$

Hence,

$$\frac{dj}{dx} = -\frac{1}{h}\frac{dz'}{dx} = -\left[\frac{k^2 - 2\frac{d'}{h}k - 2\frac{z'}{h}(p'\,n + k)}{2\,p'\,n\left(k - \frac{d'}{h}\right) + k^2}\right]\frac{dk}{dx} = -R\,\frac{dk}{dx} \ldots(50)$$

In Eq. 50,

$$R = \frac{k^2 - 2\frac{d'}{h}k - 2\frac{z'}{h}(p'\,n + k)}{2\,p'\,n\left(k - \frac{d'}{h}\right) + k^2} \ldots\ldots\ldots\ldots(51)$$

Formulas relating k to $\frac{e}{h}$ are available in several textbooks, but can be derived by writing the equilibrium equation for the moment of T and C about the eccentric position of the load shown in Fig. 12. It is convenient to subdivide C into its components—C_1 on the compressive steel and C_2 on the concrete:

$$\Sigma M_P = C_1\,(e - a) + C_2\left(e - \frac{h}{2} + \frac{k\,h}{3}\right) - T\,(e + a)$$

$$= p'\,b\,h\,n\,f_c\left(\frac{k\,h - d'}{k\,h}\right)(e - a) + \frac{1}{2}f_c\,b\,k\,h\left(e - \frac{h}{2} + \frac{k\,h}{3}\right)$$

$$-\,p\,b\,h\,n\,f_c\left(\frac{h - d' - k\,h}{k\,h}\right)(e + a) = 0 \ldots\ldots\ldots(52a)$$

or

$$p'\,n\left(k-\frac{d'}{h}\right)(e-a)+\frac{k^2}{2}\left(e-\frac{h}{2}\right)+\frac{k^3\,h}{6}$$
$$-\,p\,n\left(1-\frac{d'}{h}-k\right)(e+a)=0\ldots\ldots\ldots\ldots\ldots(52b)$$

Differentiating Eq. 52*b* with respect to x and collecting terms gives the equation:

$$\frac{dk}{dx}\left[p'\,n\,(e-a)+k\left(e-\frac{h}{2}\right)+\frac{1}{2}k^2\,h+p\,n\,(e+a)\right]$$
$$=-\frac{de}{dx}\left[p'\,n\left(k-\frac{d'}{h}\right)-p\,n\left(1-\frac{d'}{h}-k\right)+\frac{k^2}{2}\right]\ldots\ldots(53a)$$

Since $de=\dfrac{dM}{P}$, or $\dfrac{de}{dx}=\dfrac{dM}{dx}\dfrac{1}{P}=\dfrac{V}{P}$,

$$\frac{dk}{dx}=-\frac{V}{P\,h}\left[\frac{p'\,n\left(k-\dfrac{d'}{h}\right)-p\,n\left(1-\dfrac{d'}{h}-k\right)+\dfrac{k^2}{2}}{p'\,n\left(\dfrac{e}{h}-\dfrac{a}{h}\right)+k\left(\dfrac{e}{h}-\dfrac{1}{2}+\dfrac{k}{2}\right)+p\,n\left(\dfrac{e}{h}+\dfrac{a}{h}\right)}\right]$$
$$=-\frac{V}{P\,h}B\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(53b)$$

In Eq. 53*b*, B is the quantity in the bracket. The value of B simplifies either for symmetrical steel or for no compressive steel. If $p'\,n=p\,n$,

$$B=\frac{2\,p\,n\,k-p\,n+k^2/2}{2\,p\,n\dfrac{e}{h}+k\left(\dfrac{e}{h}-\dfrac{1}{2}+\dfrac{k}{2}\right)}\ldots\ldots\ldots\ldots\ldots\ldots(54)$$

The value of $\dfrac{dk}{dx}$ from Eq. 53*b* may be substituted into Eq. 50:

$$\frac{dj}{dx}=-R\,\frac{dk}{dx}=+\frac{V}{P\,h}B\,R\ldots\ldots\ldots\ldots\ldots\ldots(55)$$

Eq. 46 now becomes:

$$u=\frac{V}{\Sigma_o\,j\,h}\left[1-\frac{P}{V}\,\frac{(e+0.5\,h-d')}{j}\,\frac{V}{P\,h}\,B\,R\right]$$
$$=\frac{V}{\Sigma_o\,j\,h}\left[1-\left(\frac{e}{h}+\frac{1}{2}-\frac{d'}{h}\right)\frac{B\,R}{j}\right]\ldots\ldots\ldots\ldots\ldots(56)$$

The bracket might be considered the modification term except that common practice substitutes $\frac{7}{8}\,d$ for $j\,h$ in ordinary beam design and this is not a good substitution here. Including the necessary correction, the equation becomes:

$$u=\frac{V}{\Sigma_o\,\frac{7}{8}\,d}\left\{\frac{0.875}{j}\,\frac{d}{h}\left[1-\left(\frac{e}{h}+\frac{1}{2}-\frac{d'}{h}\right)\frac{B\,R}{j}\right]\right\}=\frac{V}{\Sigma_o\times\frac{7}{8}\,d}$$

times a modification term$\ldots\ldots\ldots\ldots\ldots\ldots(57)$

The modification term is lengthy, but is not otherwise difficult to evaluate. In practice, the authors' equation $u = \frac{V\,A_s\,n}{\Sigma_o\,S_n}$ usually would be preferable if a correction is needed; but it does not indicate what controls the modification and gives no idea of whether this modification is important or otherwise. Eqs. 57, 53*b*, and 50 show that the modification term depends on $p\,n$, $p'\,n$, $\frac{d'}{h}$, and $\frac{e}{h}$; or that, for a given beam, it will vary solely with the $\frac{e}{h}$-ratio, since $\frac{e}{h}$ controls k. Eq. 52*b* can be used to find either $\frac{e}{h}$ or k if the other is known; or almost any textbook contains k-curves for the case of symmetrical reinforcement.

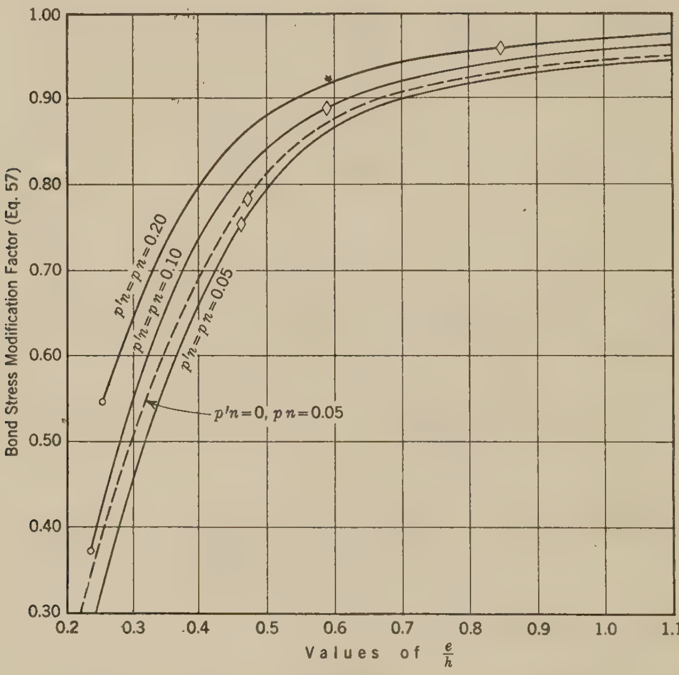

FIG. 13

The modification term is too lengthy to visualize. Fig. 13 shows the modification term $\left(\text{for } \frac{d'}{h} = 0.10\right)$ plotted against $\frac{e}{h}$ for cases of symmetrical reinforcement varying from $p\,n = 0.05$ to 0.20 and for one case of no compressive steel. The value $p = \frac{A_s}{b\,h}$ is based on steel in one face. These curves are terminated on the left at the $\frac{e}{h}$-value which gives zero stress on the tension steel. Any smaller $\frac{e}{h}$-ratio puts this steel in compression. The value of $\frac{e}{h}$ on each curve that corresponds to a neutral axis at the mid-depth of the member has been marked by a small diamond.

These curves indicate that for this $\frac{d'}{h}$-ratio, unless $\frac{e}{h}$ is less than 0.7, the modification term will be 0.9 or more, since the steel usually will not be less than that indicated by $p\,n = 0.05$. Few designers will care to make any correction for larger $\frac{e}{h}$-values than this, since it is well known that the usual bond formula is very approximate at best and a 10% change on the safe side is not very important.

If designers retain the use of the usual formula wherever $\frac{e}{h}$-values exceed 0.7, the proposed formula will not be necessary in most cases. In ordinary beams, values of $\frac{e}{h}$ as low as 0.7 will occur only where thrusts are relatively large or where moments are relatively small. The case of relatively large thrusts at sections critical for bond is rather infrequent; and the bond is rarely of interest where the moment is relatively small. Columns will frequently be loaded to give $\frac{e}{h}$-values as small as 0.7; but bond in columns is seldom a serious matter.

Since horizontal shear on the tension side of the neutral axis is directly related to the bond stress, these same conclusions should apply to shear calculations in this part of the beam. However, it seems improper to say that this shear is a satisfactory measure of the most serious diagonal tension in the beam. It must be noted that the greater shear near the centroid will tend to cause an increased diagonal tension there, and that the small axial compression might only cause the diagonal tension to increase less rapidly than v.

Fig. 3 gives data to illustrate this point. In the compressive area, it seems reasonable to apply the usual formula for maximum diagonal tension:

$$t = \frac{f}{2} + \sqrt{\left(\frac{f}{2}\right)^2 + v^2} \quad \text{......................} (58)$$

At a point 5 in. from the neutral axis (in the direction of the centroid), $f = 108$ lb per sq in., $v = 164$ lb per sq in., and $t = -54 + \sqrt{(54)^2 + (164)^2} = -54 + 173 = 119$ lb per sq in. This is remarkably close to the 120 lb per sq in. given by Eq. 44*b*. This matter needs further study before any modification of present methods is proposed.

Stanley U. Benscoter,[23] and Samuel T. Logan,[24] Juniors, Am. Soc. C. E.—The discussions provide a number of interesting and useful additions to the solution of the problem presented in the paper. It has been properly noted in several discussions that the definition of $j\,d$, as given by the writers, is not strictly correct. However, as a matter of convenience, the definition is commonly used in design practice. Professor Peabody has correctly criticized the writers' misleading statement regarding the validity of the standard shearing stress formula in members without thrust having compressive steel.

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Several discussers have correctly noted that the neutral axis is not parallel to the reinforcement as shown in Fig. 2. Messrs. Aleck, Ferguson, and Bergman have developed formulas based on the slope of the neutral axis with respect to the reinforcement. It should be noted carefully that the formulas proposed by the writers do not include any effect of the rate of change of flexural properties along the member. Consequently, the factors that enter into the formulas, and the development of the formulas, can be completely illustrated by a figure which neglects the slope of the neutral axis.

The application of the classical shearing stress formula to reinforced concrete, as given by Eq. 3, can be developed in undergraduate classrooms in a manner such as that used by the writers. In the graduate classroom a more comprehensive development can be given as illustrated by Eq. 19. This formula considers the rate of change of the flexural properties along the member. An additional term could be obtained by regarding the thrust P as a variable in order to obtain a formula applicable to arches.

Some discussion of Eq. 19 may be worth while. If all terms except the first are dropped, the writers' formula is obtained. It appears that the additional terms beyond the first term are more or less compensating. This is suggested by the fact that calculations presented by Mr. Bergman, based on a consideration of the rate of change of k along the member, agree well with the values computed from the writers' formulas. Thus, a dilemma which the writers are unable to resolve is presented. Why does a method of analysis which considers the rate of change of flexural properties along the member give values which agree with those computed from a formula which does not consider the rate of change of flexural properties?

For further illustration of this dilemma, consider Eqs. 28 and 29. For the member without compressive steel the formula for B becomes

$$B = \frac{\frac{1}{2} - \frac{k}{3} - \frac{j}{2}}{\frac{1}{2} - \frac{k}{3} - \frac{e}{d}\left(\frac{1}{2} + \frac{n\,p}{k^2}\right)} \qquad \text{(59)}$$

The quantity $(1 - B)$ becomes

$$1 - B = \frac{3\,dk^2 - dk^3 - 3\,e\,k^2 - 6\,e\,n\,p}{3\,dk^2 - 2\,dk^3 - 3\,e\,k^2 - 6\,e\,n\,p} \qquad \text{(60)}$$

When $P_c = 0$, Eqs. 22 become

$$P\,e = b\,d^2 f_c \frac{k}{2}\left(1 - \frac{k}{3}\right) \qquad \text{(61a)}$$

and

$$P = b\,d\,f_c\left[\frac{k}{2} - \left(\frac{1}{k} - 1\right) n\,p\right] \qquad \text{(61b)}$$

Dividing Eq. 61*a* by Eq. 61*b* gives

$$e = \frac{dk^2\,(3 - k)}{3\,k^2 - 6\,(1 - k)\,n\,p} \qquad \text{(62)}$$

Substituting Eq. 62 into Eq. 60 gives

$$1 - B = \frac{2\,(k-2)\,(k-3)\,n\,p}{k^3 + 4\,(k^2 - 3\,k + 3)\,n\,p} \qquad (63)$$

If Eq. 63 is substituted into Eq. 29, the resulting formula for v will be the same as Eq. 31. Thus, Eq. 30 is in error, and the computations, based on Eq. 30, lead to incorrect conclusions. It is especially worthy of note that a formula can be derived, based on the rate of change of k and j along the member, which is exactly the same as the writers' formula which does not consider the rate of change of flexural properties along the member. This dilemma must be resolved by future research.

A few comments on formulas involving k and j rather than the section modulus may be in order. Just as in the case of tapered beams without thrust,[25] the shearing stress may be calculated from the formula,

$$v = \frac{V_1}{b\,j\,d} \qquad (64)$$

in which V_1 is a reduced shear. The beam column of reinforced concrete is prismatic externally but nonprismatic internally. Although the surface elements are parallel, thus giving the member a prismatic shape externally, the resultant internal tensile and compressive forces are not parallel.

The reduced shear V_1 is calculated from the total shear V by deducting a reduction factor. This reduction factor is proportional to the tangent of the angle δ_1 made by the internal compressive force with a normal to the cross section. Thus, V_1 may be written as

$$V_1 = V - C \tan \delta_1 \qquad (65)$$

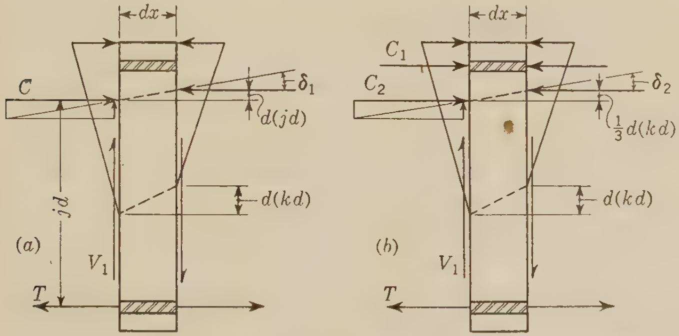

FIG. 14.—SHEAR REDUCTION FACTOR

The validity of Eqs. 64 and 65 may be demonstrated readily by using Eq. 2 and referring to Fig. 14(*a*). Eqs. 64 and 65 correspond to Eq. 21, Eq. 43, or Eq. 46. The compressive force C can be expressed in terms of k. The quantity

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[25] "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," *Proceedings*, Am. Soc. C. E., Pt. 2, June, 1940, Eq. 5*a*, p. 56.

$\tan \delta_1$ can be expressed in terms of derivatives of k and j in various ways such as,

$$\tan \delta_1 = d \frac{dj}{dx} \qquad (66a)$$

$$\tan \delta_1 = d \frac{dj}{dk} \frac{dk}{dx} \qquad (66b)$$

$$\tan \delta_1 = d \frac{dj}{de} \frac{de}{dx} \qquad (66c)$$

$$\tan \delta_1 = d \frac{dj}{dk} \frac{dk}{de} \frac{de}{dx} \qquad (66d)$$

or

$$\tan \delta_1 = \frac{V\,d}{P} \frac{dj}{dk} \frac{dk}{de} \qquad (66e)$$

Eq. 66e has been used by Professor Ferguson. The factor $-\frac{dj}{dk}$ is given as R in Eq. 51. The product $d\frac{dk}{de}$ is given as B in Eq. 54. The first factor is a function of k only, whereas the second factor is a function of both k and e. These derivatives are properties of the cross section and are rather complicated. By a simple expedient the first factor may be eliminated.

Consider the forces acting on the differential slice of the member as shown in Fig. 14(b). The total compression has been divided into two parts—the force C_1 consisting of the stress in the compressive steel and the force C_2 consisting of the concrete stress. The entire reduction factor used in computing the reduced shear V_1 is affected by the slope of C_2 with respect to the tensile steel since the force C_1 is parallel to the tensile steel. Thus, the formula for V_1 may be written

$$V_1 = V - C_2 \tan \delta_2 \qquad (67)$$

The formula for $\tan \delta_2$ may be written

$$\tan \delta_2 = -\frac{d}{3} \frac{dk}{dx} \qquad (68a)$$

$$\tan \delta_2 = -\frac{d}{3} \frac{dk}{de} \frac{de}{dx} \qquad (68b)$$

or

$$\tan \delta_2 = -\frac{V}{3} \frac{d}{P} \frac{dk}{de} \qquad (68c)$$

Thus, Eqs. 67 and 68c appear to provide the most convenient method for calculating shear and bond stresses in terms of k rather than in terms of the section modulus. Actually, the use of the writers' formula in terms of the section modulus will require less time.

Mr. Shanley has suggested the unit shear method which has been used rather extensively in aeronautical engineering in recent years. Unfortunately, the civil engineer seldom has sufficient time in design practice to use such a method. However, the method is commonly used as a research tool to check the validity of formulas upon which the civil engineer must depend. The first of the two numerical calculations presented by Mr. Bergman illustrates the application of the unit shear method to the problem treated by the writers.

Several discussers mention the possibility of some tensile stress existing near the neutral axis. However, it is not safe to consider tensile stresses in designing horizontal members. In such members it must be assumed that a shrinkage crack exists entirely through the member at the section being analyzed. The irregular surfaces in contact at the crack can transmit shearing stress but cannot transmit tensile stress. On the other hand, it is highly improbable that vertical walls or columns have horizontal shrinkage cracks. Consequently, vertical members with large thrust eccentricity, such as cantilever retaining wall stems, are designed in a manner that is unnecessarily conservative.

Mr. Wilbur maintains that the formula $\frac{V}{b\,j\,d}$ is equally satisfactory for members with or without thrust. He presents a stress diagram in Fig. 8 which is correct for members without thrust and mistakenly assumes it to be correct for members with thrust. Hence, an incorrect value for the maximum shearing stress is computed.

Dean Grinter correctly notes that, for straight members having a length-to-depth ratio of ten to one or greater, the formula $\frac{V}{b\,j\,d}$ should give satisfactory accuracy. Actually, the eccentricity of the thrust is the criterion that determines this accuracy. It is well that Dean Grinter has called attention to the fact that, with tensile thrust, the formula $\frac{V}{b\,j\,d}$ underestimates the shear and bond stresses.

Mr. Bull offers Eq. 39*a* for computing shearing stress. This formula may be reduced to $\frac{V}{b\,j\,d}$. Hence, Mr. Bull also maintains that $\frac{V}{b\,j\,d}$ may be used for all thrust eccentricities, which is incorrect, and his stress calculations are likewise incorrect. The "line of thrust" in the writers' paper is the same as the "pressure line" in the paper to which Mr. Bull refers.[19]

Mr. Bergman has presented two instructive methods of stress calculation. These methods provide a clarification of internal stress relationships. In the second calculation Mr. Bergman introduces an unusual definition of $j\,d$ as the distance between the resultant tensile and compressive forces which remain after the thrust has been subtracted from the internal stress diagram. This definition is in strict literal agreement with the 1940 Joint Committee Code. When this definition is employed, the stress computed from $\frac{V}{b\,j\,d}$ is the maximum stress v_m at the centroid rather than the value between the neutral axis and the tensile steel. This maximum stress is not the value that the designer should compare with the allowable stress, and it is highly improbable that the writers of the code intended to recommend such a procedure.

Professor Ferguson correctly notes the artificial base upon which all reinforced concrete stress analysis rests. He presents a development which is of considerable value to the designer in showing the range of thrust eccentricity where the formula $\frac{V}{b\,j\,d}$ gives stresses that are too high with compressive thrust. Fig. 13 illustrates the matter clearly. The writers' formulas should be used in the range of small eccentricities. Structures with heavy loading, such as the structure used by the writers as an illustration, fall in this class. Most arches are in this class. Since the formula $\frac{V}{b\,j\,d}$ requires longer to evaluate correctly than the formula recommended by the writers, there is little reason to use the formula $\frac{V}{b\,j\,d}$ in any member with thrust.

Professor Ferguson has developed formulas in terms of the over-all depth h rather than in terms of an effective depth d. Beam-column stress analysis should be divided into two classifications. One class should consist of members having the thrust within the kern and the other class should consist of members having the thrust outside the kern. In the first class the total depth h should be used in order that the thickness of the tensile steel covering will enter into the stress calculations. In the second class some conveniently defined effective depth should be used specifically to assure that the thickness of the tensile steel covering does not enter into the stress calculations.

Professor Ferguson calls attention to the fact that higher diagonal tensile stresses may exist in the compressive region, near the neutral axis, than in the region between the neutral axis and the tensile steel. In the example given by the writers this condition is very marked, because the compressive stresses in the example are very low whereas the shearing stresses are high. The high diagonal tensile stresses will exist only over a short distance along the cross section. However, they should be given some attention by future research.

Both theoretical and experimental research is needed in reinforced concrete stress analysis. Strains must be measured on actual rigid frame structures in the field. The civil engineer must learn to attach electrical strain gages to reinforcing rods as well as to insert plugs in the compressive face of members to obtain strain readings. Such field experimentation is sorely needed for the improvement of reinforced concrete design methods.

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COEFFICIENTS FOR VELOCITY DISTRIBUTION IN OPEN-CHANNEL FLOW

BY WILLIAM S. EISENLOHR, JR.,[1] ASSOC. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. A. A. KALINSKE, EDWARD H. TAYLOR, E. R. VAN DRIEST, HUNTER ROUSE AND JOHN S. MCNOWN, AND WILLIAM S. EISENLOHR, JR.

SYNOPSIS

Where it is necessary to take into consideration the nonuniform distribution of velocity of water flowing in open channels, it is customary to use the velocity head based on the mean velocity and a coefficient. This coefficient is fixed by the equation selected: It is the ratio of either the mean of the squares of the local velocities to the square of the mean, or the mean of the cubes of the local velocities to the cube of the mean velocity. The Chézy and Manning equations are frequently considered to be energy equations but this paper shows that, from the nature of their coefficients, they are truly momentum equations. They will not yield practical solutions when considered as energy equations except for uniform velocity distribution. The proper coefficient to be used in momentum equations is then shown to be C_m, the ratio of the mean of the squares of the local velocities to the square of the mean velocity.

INTRODUCTION

In the practical solution of most problems involving a stream of water flowing in a pipe or open channel it is not necessary to consider the variation of velocity throughout the cross section; use of the average velocity will give sufficiently accurate results. Textbooks on hydraulics, therefore, merely develop the equations of flow through an elemental stream tube and the resulting equations are then used for total flows. There are problems in nonuniform flow, however, in which a substantial part of the total change in momentum and kinetic energy between two cross sections may be due to a change in the distribution of velocity as distinguished from a change in the mean velocity. In such problems the nonuniform distribution of velocity must be taken into account by integrating, over the cross section, the equation of flow through an elemental stream tube.

NOTE.—Published in January, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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The momentum and kinetic energy of a stream of water are functions of the square and cube of the velocity, respectively. Therefore, the total momentum and kinetic energy possessed by a stream at any cross section are the sums of the momentums and energies of the flow in the different parts of the cross section. Practically, the same result can be obtained by using the average velocity and a coefficient which has been evaluated by the integration process for the particular velocity distribution being studied. These coefficients are always greater than unity. They appear to be so related[2] that for ordinary use the coefficient of momentum can be taken as

$$C_m = 1 + \frac{\epsilon^2}{3} \qquad (1a)$$

and the coefficient of energy as

$$C_e = 1 + \epsilon^2 \qquad (1b)$$

in which $\epsilon = \frac{u_{\max}}{V} - 1$. Some confusion seems to exist as to which coefficient should be used, as it is difficult to appraise correctly the factors involved in such a choice if one is accustomed to the use of formulas based on the mean velocity alone.[2a] The purpose of this paper is to explain the various factors involved and show their proper use. The usual desire is to apply some "correction coefficient" to the ordinary formula and let that suffice. Although the use of a proper coefficient is the practical solution, it should be realized that such a coefficient is the result of integrating the fundamental hydraulic equation involved, and is in no sense a "correction." To understand properly the various coefficients it will be necessary to consider the fundamental principles involved, and to develop the equations for integration. This can be done best by an illustrative example for which purpose the "Bernoulli" equation has been chosen. The numerical values of such coefficients are usually obtained from observations of the velocity distribution in the flow problem being studied or from some characteristic of flow which will yield a reasonable approximation of the velocity distribution.

NOTATION

The following letter symbols, used in this paper, conform essentially to American Standard Letter Symbols for Hydraulics,[3] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942. Subscripts denoting particular values are given at the end.

A = area, total cross sectional;
a = acceleration, linear;
B = bulk or volume (see Eq. 14a);
C = coefficient of flow (Chézy) in $V = C\sqrt{R\,S}$;
C_A, C_B = coefficients of force distribution in the energy equation;

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[2] "Die Bestimmung der Lage der Energielinie bei fliessenden Gewässern mit Hilfe des Geschwindigkeitshöhen-Ausgleichwertes," by Th. Rehbock, *Der Bauingenieur*, No. 15, Vol. 3, August 15, 1922, pp. 453–455.

[2a] See author's redesignation in the closing discussion.—Ed.

[3] ASA—Z10.2—1942.

C_e = coefficient of energy distribution = $\frac{\overline{u^3}}{V^3}$;
C_m = coefficient of momentum distribution = $\frac{\overline{u^2}}{V^2}$;
F = force;
f = Darcy coefficient of friction = 4λ;
g = gravitational acceleration;
h_f = head, lost by friction;
k = a constant ratio of pressure intensities (see Eqs. 4);
L = length; length of stream tube or channel between sections;
m = mass;
n = coefficient of roughness in the Manning equation;
P = wetted perimeter;
p = pressure per unit area;
Q = discharge (rate of flow);
R = hydraulic radius;
S = friction slope; slope of momentum grade line;
t = time;
u = velocity component in the direction of flow; velocity of flow through a stream tube;
V = mean velocity, $\frac{Q}{A}$;
Z = elevation, above datum;
γ = specific weight;
ϵ = velocity distribution constant = $\frac{u_{\max}}{V} - 1$;
θ = angle between axis of a stream tube and the horizontal, $\sin^{-1}\left(\frac{Z_A - Z_B}{L}\right)$;
λ = resistance coefficient = $\frac{f}{4}$; the ratio of boundary shear to kinetic energy based on the mean velocity;
ρ = density, mass per unit volume = $\frac{\gamma}{g}$;
τ_0 = unit shear at the channel boundary.

Subscripts and Special Characters.—Letter subscripts refer a symbol to various conditions as follows:

A = "section A" (Fig. 1);
B = "section B" (Fig. 1);
A-B = "between section A and section B";
e = "energy equation";
m = "momentum equation";
n = "normal to the sides of a stream tube" (see Eqs. 4);
p = "pressure";
r = "resisting";
u = "uniform velocity distribution."

A bar over a symbol signifies the average value of the quantity indicated. Subscripts 1 and 2 refer a symbol to stream tubes 1 and 2 (Figs. 1, 2), respectively.

MOMENTUM VS. ENERGY

Standard textbooks on hydraulics usually demonstrate the proof of the Bernoulli equation either by the momentum theorem or by the energy principle, although they are not always so labeled. In either case it is usual to assume that the stream is composed of a bundle of "stream tubes" of differential area in cross section. A stream tube may be defined as a small tube bounded by a surface which at every point is tangent to the velocity vector at that point. There is no flow across the bounding surface of a stream tube. (A more correct definition for turbulent flow is that the temporal average of flow across a stream tube boundary is zero.) Flow through the stream tube is then similar to flow through a small pipe of varying cross section. The flow through a single stream tube may be analyzed and the resulting differential equation integrated over the cross section. The basic equation by the momentum theorem or energy principle is that of Newton's second law of motion, $F = m\,a$, wherein the substitution $a = \frac{u^2{}_B - u^2{}_A}{2\,L}$ has been made. The difference between the two methods lies in whichever side of the equation the distance L is placed. If it is placed on the right, the momentum equation becomes:

$$F = m\frac{u^2{}_B - u^2{}_A}{2\,L} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in the form of momentum change per unit time—that is, $\frac{F\,t}{t} = \frac{m\,u}{t}$; and $F = \frac{m\,u}{t}\left(\times\frac{L}{L}\right) = \frac{m\,u^2}{L}$. The change of the momentum with time is equal to the resultant force acting on the mass.

If the distance L is placed on the left the energy relation is

$$F\,L = \tfrac{1}{2}\,m\,(u^2{}_B - u^2{}_A) \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3a)$$

which states that the change in kinetic energy is the result of work done by the impressed forces. If the change in unit time[3a] is considered, as was done for momentum, and Eq. 3a is divided by $t = 1$:

$$F\,u = \tfrac{1}{2}\,m\,[u^2{}_B - u^2{}_A]_{t=1} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3b)$$

To apply these equations it is necessary, of course, to have steady flow, or flow which on the average may be regarded as steady. Only under such a condition would the system be in equilibrium.

It is most important to realize the significance of the method of considering the distance L as used in the preceding paragraph. In the momentum equation, L is the length of the stream tube in which the change in momentum takes place. It may have any value desired but it is the same for every stream tube within the stream. In the momentum equation the forces are considered to be acting independent of any motion. On the other hand, in the energy equation the forces must act through a specified distance (u, if Eq. 3b is used). Since the velocity distribution is nonuniform, u is different for each stream tube. This makes an extremely complicated differential equation, the integration of which does not yield a solution suitable for practical use. To simplify the problem, most textbook writers assume a uniform distribution of velocity when deriving the Bernoulli equation by means of the energy principle. Using this assumption, u is the same for each stream tube. The distances through which the forces act are thus the same for all stream tubes. The integration then can be performed in the same way as for the momentum equation and the result is an equation identical with that derived from the momentum theorem. An equation is thus derived by the energy principle, based on assumed uniform velocity distribution, giving the same answer as that derived by the momentum theorem. The result is an understandable confusion as to whether to use the coefficient of momentum distribution or the coefficient of energy distribution.

[3a] See redesignation in closing discussion, expressed by Eq. 41c.—Ed.

The Momentum Theorem

Many ideas frequently thought of in terms of the energy relation are really momentum concepts. The following paragraph is taken almost verbatim from the explanation given by L. Prandtl.[4]

The value of the momentum theorem is that it contains statements about conditions along the boundaries of a region only. The internal forces cancel by the principle of action and reaction, when summing over all the masses of the system, leaving only the external forces to be considered. Now the change in momentum of a mass of fluid which is bounded in any way, and is in steady motion, is due entirely to the fact that the boundaries of the fluid are displaced by the motion; every particle of the fluid in the interior is replaced by another which has taken up its velocity. What happens at the boundaries may be shown in the case of a stream tube. It is an essential feature of the momentum theorem that every individual mass belonging to the system shall remain in the system and that no new masses shall enter it. Thus the boundary surfaces drawn for the purpose of applying the theorem are not displaced with the flow; they are "fluid surfaces." Hence in the case of the stream tube there is a mass dm which leaves the downstream end of the stream tube in unit time. During the same time, by the law of continuity, an equal mass enters the upstream end of the stream tube. Thus, in unit time the stream tube contributes to the total change in momentum the positive momentum $dm\ u_B$ at the downstream end and at the same time the negative momentum $dm\ u_A$ is contributed at the upstream end. These changes in momentum per unit time may immediately be expressed as forces exerted from outside on the mass of fluid in question across the cross sections at the ends of the stream tube.

Development of the Momentum Equation

Proceeding now to develop the equation of flow according to the momentum theorem, consider the channel shown in Fig. 1. The stream can be considered to be divided into stream tubes of differential area dA. The change in momentum between sections A and B is the result of the external forces. The pressure force on the upstream end is $p_A\, dA_A$, and on the downstream end is $p_B\, dA_B$. Another basic assumption that must be made is that the flow is gradually varied. With this assumption the development can be simplified by letting $dA_A = dA_B = dA$ without restricting the variation in velocity.

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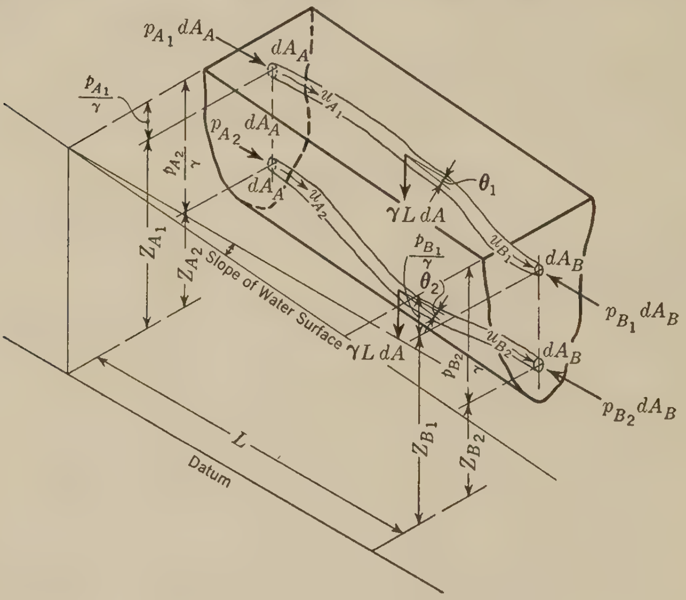
[4] "The Physics of Solids and Fluids with Recent Developments," Pt. 2, by L. Prandtl, 1936, p. 223.

(The validity of this assumption is easily shown. Let $p_A = p + \delta p$; $p_B = p$; $dA_A = dA$, and $dA_B = dA + \delta A$. In addition to the pressure forces on the ends of the stream tube, there is also a component in the direction of motion of the normal pressure p_n on the sides of the stream tube, which is equivalent



$p_{A_1} dA_A$
dA_A
$\frac{p_{A_1}}{\gamma}$
u_{A_1}
$p_{A_2} dA_A$
$\frac{p_{A_2}}{\gamma}$
dA_A
u_{A_2}
θ_1
$\gamma L\, dA$
Z_{A_1}
Z_{A_2}
Slope of Water Surface
$\frac{p_{B_1}}{\gamma}$
θ_2
u_{B_1}
dA_B
$p_{B_1} dA_B$
$\gamma L\, dA$
u_{B_2}
dA_B
$\frac{p_{B_2}}{\gamma}$
$p_{B_2} dA_B$
L
Datum
Z_{B_1}
Z_{B_2}

FIG. 1.—MOMENTUM DIAGRAM; ALSO ENERGY DIAGRAM AT $t = 0$

to $p_n\, \delta A$. Since p_n lies between p_A and p_B it can be expressed as $p + k\, \delta p$. The pressure forces are then:

$$F_p = p_A\, dA_A + p_n\, \delta A - p_B\, dA_B \tag{4a}$$

$$F_p = (p + \delta p)\, dA + (p + k\, \delta p)\, \delta A - p\, (dA + \delta A) \tag{4b}$$

and

$$F_p = \delta p\, dA + k\, \delta p\, \delta A \tag{4c}$$

The first term of Eq. 4c is the equivalent of the resultant pressure force as obtained by assuming equal areas. The only error introduced in this assumption is the neglect of the term $k\, \delta p\, \delta A$ which, as the product of two differentials, is negligible.)

The resultant pressure force is then $(p_A - p_B)\, dA$. The gravity force is the component along the axis of the stream tube of the weight of the tube, $\gamma\, L\, dA$, which is $\gamma\, L\, dA \sin \theta$. In addition to the pressure and gravity forces, there is the resisting force along the boundary of the stream tube. For the present this will be designated simply $dF_{r,m}$.

Inserting the foregoing quantities in Eq. 2:

$$p_A\,dA - p_B\,dA + \gamma\,L\,dA\,\sin\theta - dF_{r,m} = \rho\,L\,dA\,\frac{u^2{}_B - u^2{}_A}{2\,L} \quad\ldots\ldots(5)$$

substituting for $\sin\theta$ and rearranging[4a]:

$$dF_{r,m} = \left[\left(\frac{p_A}{\gamma} + Z_A\right)dA - \left(\frac{p_B}{\gamma} + Z_B\right)dA + \frac{u^2{}_A}{2\,g}\,dA - \frac{u^2{}_B}{2\,g}\,dA\right]\gamma \quad\ldots(6)$$

The total resisting force $F_{r,m}$ can now be obtained by integrating the right side of the equation with respect to dA. This is the important step. Since $\left(\frac{p}{\gamma} + Z\right)$ is the height of the water surface above the datum plane, it is the same for all stream tubes. The integration of the first two terms is easily performed. The last two terms cannot be integrated in their present forms.

The coefficient for nonuniform distribution of velocity is now introduced. The term C_m is defined as the coefficient of momentum distribution due to nonuniform distribution of velocity. It is expressed as:

$$C_m = \frac{\overline{u^2}}{V^2} \qquad \text{or} \qquad C_m = \frac{\int u^2\,dA}{V^2\,A}.$$

Integrating Eq. 6:

$$F_{r,m} = \left[\left(\frac{p_A}{\gamma} + Z_A\right) - \left(\frac{p_B}{\gamma} + Z_B\right) + C_{m,A}\,\frac{V^2{}_A}{2\,g} - C_{m,B}\,\frac{V^2{}_B}{2\,g}\right]\gamma\,A \quad\ldots(7)$$

It was shown previously that integrating the momentum equation canceled all the internal forces and left only the external forces. Therefore, $F_{r,m}$ must represent the total shearing stress along the channel boundary between A and B. If this force is replaced by an equivalent negative force on B equal to $\gamma\,h_{f,A-B}\,A$ (in which h_f is an expression in terms of "loss of head due to friction" of the shear at the boundary) the usual form of the Bernoulli equation is derived with the velocity distribution coefficients and the friction term included:

$$\gamma\,h_{f,A-B}\,A = \left[\left(\frac{p_A}{\gamma}+Z_A\right) - \left(\frac{p_B}{\gamma}+Z_B\right) + C_{m,A}\,\frac{V^2{}_A}{2\,g} - C_{m,B}\,\frac{V^2{}_B}{2\,g}\right]\gamma\,A \quad\ldots(8a)$$

and

$$\frac{p_A}{\gamma} + Z_A + C_{m,A}\,\frac{V^2{}_A}{2\,g} = \frac{p_B}{\gamma} + Z_B + C_{m,B}\,\frac{V^2{}_B}{2\,g} + h_{f,A-B} \quad\ldots\ldots(8b)$$

Instead of replacing $F_{r,m}$ by an equivalent "loss of head force," it can be stated directly in terms of unit shear at the boundary, τ_0. This unit shear is usually expressed in terms of the kinetic energy of the water[5] based on the mean velocity, $\rho\,\frac{V^2}{2}$, by means of a resistance coefficient, λ. The coefficient λ is the same as $\frac{f}{4}$, in which f is the familiar Darcy coefficient of friction.

[4a] See redevelopment of Eqs. 6 and 7, as expressed by Eq. 43 in the closing discussion.—Ed.

[5] "The Physics of Solids and Fluids with Recent Developments," Pt. 2, by L. Prandtl, 1936, p. 275.

The total resisting force may then be put equal to the total shear at the boundary:

$$F_{r,m} = \tau_0 \, P \, L \qquad (9a)$$

and since

$$\tau_0 = \frac{\lambda \rho V^2}{2} = \frac{f \rho V^2}{8} \qquad (9b)$$

substituting values from Eqs. 7 and 9*b* in Eq. 9*a* and transposing,

$$\frac{f \rho V^2}{8} P L = \left[\left(\frac{p_A}{\gamma} + Z_A \right) - \left(\frac{p_B}{\gamma} + Z_B \right) + C_{m,A} \frac{V^2{}_A}{2g} - C_{m,B} \frac{V^2{}_B}{2g} \right] \gamma A \qquad (10a)$$

$$\frac{f}{8} \frac{\rho}{\gamma} \frac{P}{A} V^2 = \frac{\left(\frac{p_A}{\gamma} + Z_A \right) - \left(\frac{p_B}{\gamma} + Z_B \right) + C_{m,A} \frac{V^2{}_A}{2g} - C_{m,B} \frac{V^2{}_B}{2g}}{L} \qquad (10b)$$

and

$$\frac{f}{8g} \frac{1}{R} V^2 = S \qquad (10c)$$

By making the proper substitutions for f in Eq. 10*c*, the Chézy and Manning equations are obtained. Substituting $f = \frac{8g}{C^2}$ gives the familiar Chézy equation—

$$V = C \sqrt{R S} \qquad (11)$$

or substituting $f = \frac{n^2}{0.00858 \, R^{1/3}}$ gives the Manning equation—

$$V = \frac{0.262 \sqrt{g}}{n} R^{1/6} \sqrt{R S} = \frac{1.486}{n} R^{2/3} S^{1/2} \qquad (12)$$

Although the substitution of $f = \frac{n^2}{0.00858 \, R^{1/3}}$ is usually regarded as empirical, it does not affect the rationality of the development of the slope term, which is the concern of this paper. Likewise it is to be emphasized that f is a coefficient of boundary resistance alone and therefore both C and n are coefficients of boundary resistance alone, and are usually considered as such.

The momentum theorem is thus seen to provide a rational method of deriving the Bernoulli, Chézy, and Manning equations in a form which is usable and the one in which they are usually considered. The proper velocity-distribution coefficient to use in such equations is, therefore, $C_m = \frac{\overline{u^2}}{V^2}$

This can also be shown in another and perhaps more direct manner. It was noted previously that the change in momentum per second can be expressed as a force. This force can be measured directly with a pitot tube, and is equal to $\gamma \frac{u^2}{2g} dA$. Therefore, the total force is

$$F_{r,m} = \frac{\gamma}{2g} \int_0^A u^2 \, dA \qquad (13a)$$

For this value can be substituted

$$F_{r,m} = \gamma\, C_m \frac{V^2}{2\,g} A \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13b)$$

in which C_m is the coefficient previously defined. Thus, if one were to integrate over a cross section with a pitot tube, the ratio of the average height of water in the tube, above the free water surface, to the height of water in the tube corresponding to the mean velocity would give the coefficient C_m.

DEVELOPMENT OF THE ENERGY EQUATION

In contrast to the momentum theorem, the energy principle requires that the work done by the forces be evaluated. Work requires motion. The proper method of applying the energy principle to flowing water is to take a mass of water and determine what takes place as work is done upon it. The work done in moving the mass a given distance could be computed if the mass moved as a solid body. Since the present problem concerns nonuniform velocity, such an assumption cannot be made. The work done during a given time interval can be considered, however. Taking the same mass of water as was considered before (see Fig. 1) and letting the forces act for one second will produce the results shown in Fig. 2. Not only has the mass moved with respect to the channel, but each stream tube has also moved with respect to every other stream tube having a different velocity. Thus, the internal forces acting along the boundaries of the stream tubes do not cancel by action and reaction as in the momentum equation, since there must be residual unbalanced forces that

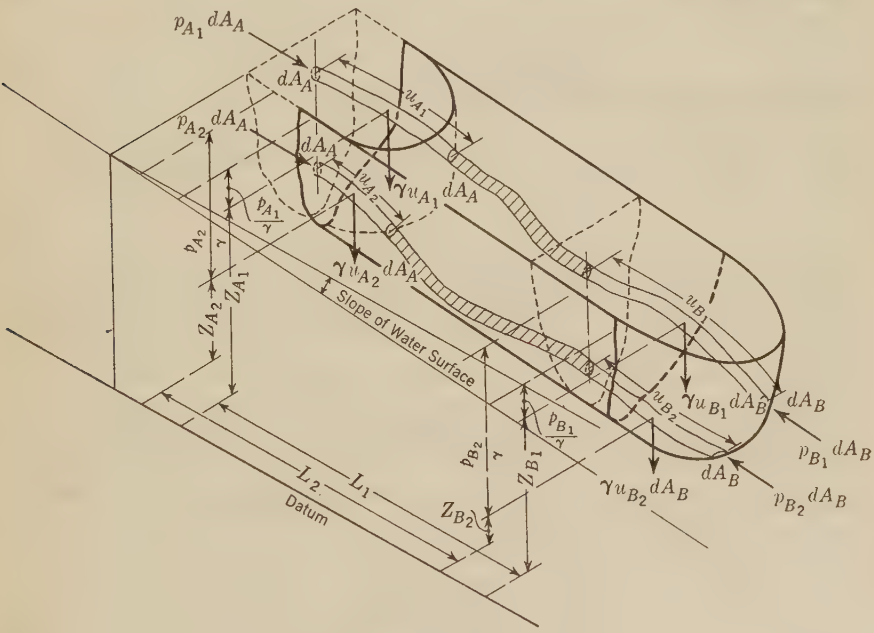

$p_{A_1} dA_A$
dA_A
u_{A_1}
$p_{A_2} dA_A$
dA_A
u_{A_2}
$\gamma u_{A_1} dA_A$
$\frac{p_{A_1}}{\gamma}$
$\frac{p_{A_2}}{\gamma}$
Z_{A_1}
Z_{A_2}
$\gamma u_{A_2} dA_A$
Slope of Water Surface
u_{B_1}
u_{B_2}
$\gamma u_{B_1} dA_B$
dA_B
$p_{B_1} dA_B$
$\frac{p_{B_1}}{\gamma}$
$\frac{p_{B_2}}{\gamma}$
Z_{B_1}
$\gamma u_{B_2} dA_B$
dA_B
$p_{B_2} dA_B$
Z_{B_2}
L_2
L_1
Datum

FIG. 2.—ENERGY DIAGRAM AT $t = 1$

produce the relative motion between the stream tubes. The integration of the differential equation will sum up these unbalanced internal forces as well as the external forces and the final equation must contain some expression including them.

Proceeding now to evaluate the terms in Eq. 3*b*, the work done by the pressure forces is $p_A\, dA_A\, u_A - p_B\, dA_B\, u_B$. In this case it is not necessary to specify the $dA_A = dA_B$ but the assumption of gradually varied flow must still be applied. Since the volume $dA_A\, u_A$ must equal $dA_B\, u_B$ and may for convenience be expressed as B (bulk or volume) it follows that the volume in between (shaded volume in Fig. 2), although containing different particles of water at $t = 1$ than it did at $t = 0$, has the same energy content throughout the period considered. Thus the work of the gravity forces can be considered as that required to move B from the position at $dA_A\, u_A$ to the position $dA_B\, u_B$, which, as far as the gravity forces are concerned, is $\gamma\, B$ times the vertical component of the distance between them $Z_A - Z_B$. The resisting force will be expressed as $dF_{r,e}$ and the distance through which it moves is $\bar{u}_{A-B}$, the average velocity in the stream tube. Similarly as with the gravity forces, the change in kinetic energy can be expressed as the change in energy of a mass $\rho\, B$ being moved from a position at $dA_A\, u_A$ to a position at $dA_B\, u_B$. Inserting these quantities in Eq. 3*b*:

$$p_A\, dA_A\, u_A - p_B\, dA_B\, u_B + \gamma\, B\, (Z_A - Z_B) - dF_{r,e}\, \bar{u}_{A-B} = \tfrac{1}{2}\, \rho\, B\, (u^2_B - u^2_A) \qquad (14a)$$

rearranging, and remembering that $B = dA_A\, u_A = dA_B\, u_B$:

$$dF_{r,e}\, \bar{u}_{A-B} = \left[\left(\frac{p_A}{\gamma} + Z_A\right) dA_A\, u_A - \left(\frac{p_B}{\gamma} + Z_B\right) dA_B\, u_B + \frac{u^2_A}{2\,g}\, dA_A\, u_A - \frac{u^2_B}{2\,g}\, dA_B\, u_B\right] \gamma \qquad (14b)$$

To integrate over the cross section to obtain the total resisting force $F_{r,e}$, it is necessary to make substitutions. The term C_e is defined as the coefficient of energy distribution due to nonuniform distribution of velocity evaluated as $C_e = \dfrac{\overline{u^3}}{V^3}$ or $C_e = \dfrac{\int u^3\, dA}{V^3\, A}$. Still other substitutions must be made in the case of the energy equation, however. During the time period from $t = 0$ to $t = 1$, the original plane cross sections at the ends of the reach have been transformed into warped surfaces, and $\left(\dfrac{p}{\gamma} + Z\right)$ is no longer a constant. As u is likewise a variable, the following substitution is made:

$$C_A = \frac{\int \left(\frac{p_A}{\gamma} + Z_A\right) u_A\, dA_A}{\left(\frac{p_A}{\gamma} + Z_A\right)_{\text{avg}} V_A\, A_A} \qquad (15a)$$

$$C_B = \frac{\int \left(\frac{p_B}{\gamma} + Z_B\right) u_B\, dA_B}{\left(\frac{p_B}{\gamma} + Z_B\right)_{\text{avg}} V_B\, A_B} \qquad (15b)$$

By integration, the total resisting force is then:

$$F_{r,e}\,V = \left[C_A \left(\frac{p_A}{\gamma} + Z_A\right)_{\text{avg}} V_A\,A_A - C_B \left(\frac{p_B}{\gamma} + Z_B\right)_{\text{avg}} V_B\,A_B + C_{e,A}\frac{V^2_A}{2\,g} A_A\,V_A - C_{e,B}\frac{V^2_B}{2\,g} A_B\,V_B \right]\gamma \qquad (16a)$$

Since $A\,V = A_A\,V_A = A_B\,V_B$,

$$F_{r,e} = \left[C_A \left(\frac{p_A}{\gamma} + Z_A\right)_{\text{avg}} - C_B \left(\frac{p_B}{\gamma} + Z_B\right)_{\text{avg}} + C_{e,A}\frac{V^2_A}{2\,g} - C_{e,B}\frac{V^2_B}{2\,g} \right]\gamma\,A \qquad (16b)$$

This can be arranged into the form of the Bernoulli equation:

$$C_A \left(\frac{p_A}{\gamma} + Z_A\right)_{\text{avg}} + C_{e,A}\frac{V^2_A}{2\,g} = C_B \left(\frac{p_B}{\gamma} + Z_B\right)_{\text{avg}} + C_{e,B}\frac{V^2_B}{2\,g} + \frac{F_{r,e}}{\gamma\,A} \qquad (16c)$$

It is easily seen that a formula such as Eq. 16*c* is not easily adapted to practical solutions of open-channel problems since $F_{r,e}$ involves internal as well as boundary resistance. It could be argued that the Chézy and Manning equations are purely empirical and therefore the slope could be evaluated by means of Eq. 16*b*, substituted in them, and the proper coefficients computed empirically. Such coefficients would necessarily be coefficients of internal resistance as well as boundary resistance and presumably could vary as compared with coefficients of boundary resistance alone. It is doubtful if such a method would yield practical results.

COMPARISON OF COEFFICIENTS

Eq. 16*b* is identical in form with the momentum equation (Eq. 7). It differs in that the coefficients C_e are quite different from the coefficients C_m, and the additional coefficients C_A and C_B are also present. It must also be realized that, although $F_{r,m}$ in the momentum equation is the resistance at the boundary only, $F_{r,e}$ in the energy equation includes that force as well as the resisting forces exerted by one stream tube upon another due to their relative motion. It is concluded, therefore, that Eq. 16*b* is not suited to practical application.

Most textbooks on hydraulics which use the energy principle to demonstrate the Bernoulli equation assume a uniform distribution of velocity. With this assumption, in rearranging Eq. 14*a* one could use $B = dA_A\,V_A = dA_B\,V_B$; and the distance through which the resisting force moves is V. Likewise, $V^2_A = u^2_A$ and $V^2_B = u^2_B$ can be substituted. The quantity $\left(\frac{p}{\gamma} + Z\right)$ is now constant over the cross section also, and Eq. 14*a* transforms to

$$dF_{r,u}\,V = \left[\left(\frac{p_A}{\gamma} + Z_A\right) V_A\,dA_A - \left(\frac{p_B}{\gamma} + Z_B\right) V_B\,dA_B + \frac{V^2_A}{2\,g} V_A\,dA_A - \frac{V^2_B}{2\,g} V_B\,dA_B \right]\gamma \qquad (17a)$$

which can be integrated without further substitution:

$$F_{r,u} = \left[\left(\frac{p_A}{\gamma} + Z_A\right) - \left(\frac{p_B}{\gamma} + Z_B\right) + \frac{V^2_A}{2\,g} - \frac{V^2_B}{2\,g}\right]\gamma\,A \ldots . (17b)$$

Eq. 17*b* is similar to Eq. 7 and would be identical if Eq. 7 were based on uniform velocity distribution. In such a case C_m would reduce to 1.00. Since uniform distribution of velocity is assumed, the water in all stream tubes moves forward together, there is no relative internal motion, and therefore $F_{r,u}$ would be the force at the boundary alone, as in the case of the momentum equation.

SUMMARY

Having two identical equations for the flow of water with uniform velocity distribution, one based on the momentum theorem and the other based on the energy principle, it is easy to understand how differences can arise in working backward to nonuniform velocity distribution. Apparently many engineers think of the Bernoulli equation as it is normally written as an energy equation exclusively because it can be obtained using energy principles and assuming uniform velocity distribution. This leads to the desire to correct for nonuniform distribution by introducing the coefficient C_e. However, the Bernoulli equation is also a momentum equation[5a] and when so used the proper coefficient for nonuniform velocity distribution is C_m. If it is desired to use the coefficient C_e it must be realized that the other coefficients C_A and C_B must be added and that the nature of the resistance F_r changes also.

The historical background concerning the confusion of coefficients has been presented by B. A. Bakhmeteff,[6] M. Am. Soc. C. E., and the strict hydrodynamical approach was given by G. H. Keulegan[7] at the suggestion of the writer. The present paper is an attempt to interpret the two references cited in terms of demonstrations found in ordinary textbooks.

[5a] See correction of error in closing discussion pertaining to Eqs. 46.—Ed.

[6] "Coriolis and the Energy Principle in Hydraulics," by B. A. Bakhmeteff, *Theodore von Karman Anniversary Volume*, Contributions to Applied Mechanics and Related Subjects, 1941, p. 59.

[7] "Equation of Motion for the Steady Mean Flow of Water in Open Channels," by Garbis H. Keulegan, Research Paper RP1488, *Journal of Research*, National Bureau of Standards, Vol. 29, July, 1942, pp. 97–111.

DISCUSSION

A. A. Kalinske,[8] Assoc. M. Am. Soc. C. E.—Hydraulic engineers are reminded, by this paper, of an item which, although of minor significance in most practical varied-flow problems, is nevertheless of considerable interest, because it reemphasizes the difference between the two fundamental methods of attacking hydraulic problems—namely, the energy and momentum principles. Nothing new is really presented since, as the author mentions at the end of his paper, this particular problem has been very completely discussed in papers by B. A. Bakhmeteff,[6] M. Am. Soc. C. E., and G. H. Keulegan.[7] Nevertheless, it is well that the author has brought it more definitely to the attention of hydraulic engineers.

The writer would like to reinterpret and restate certain ideas mentioned or implied in the paper. First, the fundamental reason that the wrong application of Bernoulli's equation is made to stream cross sections having nonuniform velocity distribution is a faulty understanding of the derivation of Bernoulli's equation. Basically, the expression known as Bernoulli's equation is the integral along a streamline of the Euler differential equation for steady frictionless fluid motion.[9] Euler's equation is obtained from a simple application of Newton's law ($f = m\,a$) to a fluid particle. Of course, since basically the energy and momentum laws stem from Newton's law, these can be used to derive Bernoulli's equation. Because, fundamentally, Bernoulli's equation holds only along a streamline, its use cannot be extended directly to a finite stream cross section except for the special case in which the velocity of all the fluid particles in the cross section of a stream tube is the same.

If an equation of the Bernoulli type is to be used for stream cross sections having nonuniform velocity distribution, it is necessary to derive such an expression from basic dynamic considerations. In any such derivation, of course, the analyst must consider all frictional forces since these are always present for real fluids. This was done by Mr. Keulegan,[7] using both the energy and momentum principles. The author has derived similar expressions in a somewhat less clear, and less rigorous, manner.

The energy principle, as used by Mr. Keulegan and the author, probably could be called the "power" principle since the terms calculated are really the "flow" of energy per unit time and the work done per unit time. The important item to recognize in the use of this principle is that all external and internal energy losses and work done must be taken into account. In using the momentum law only external forces need be considered. Hydraulic engineers will recognize this basic difference in the use of these two principles in the analysis of the hydraulic jump.

In a truly rigorous sense, Eq. 8*a* or Eq. 16*c* should not really be called a Bernoulli equation. Such an equation is basically true only along a streamline, and was originally derived for ideal fluids. Eq. 8*a* is actually a momentum equation and Eq. 16*b* a power equation.

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[8] Associate Prof., Univ. of Iowa, Iowa Inst. for Hydr. Research, Iowa City, Iowa.

[9] "Fundamentals of Hydro- and Aero-Mechanics," by L. Prandtl and O. Tietjens, McGraw-Hill Book Co., Inc., New York, N. Y., 1934, p. 112.

The author's analyses, in reality, are concerned with deriving varied-flow equations for open channels. However, neither Eq. 8*b* nor Eq. 16*c* could be used for solving varied-flow problems (for example, obtaining backwater curves), except for very short increments of the distance, L. The reason for this is that the equations have not been integrated with respect to L; instead average values have been used for such variables as A and wall shear. Distance L should have been designated as dL.

The author's statements regarding Eq. 2 and Eq. 3 are basically inaccurate. He states that $a = \frac{u^2_B - u^2_A}{2\,L}$, which, of course, comes from the relation $a = \frac{u\,du}{dL}$ when integrated between points A and B, assuming a as constant. Obviously, there is no reason to assume a as constant in any length of channel. It would have been better if Eq. 5 had been written as:

$$(dp)\,(dA) + \gamma\,(dL)\,(dA)\sin\theta - df_{r,m} = \rho\,(dA)\,u\,(du) \ldots\ldots\ldots (18)$$

Any attempt to integrate Eq. 18 with respect to L (which the author really does) would indicate that an assumption should be made regarding the variation of A with L. The author assumes, of course, that such variation is negligible; this is inconsistent since he does not at the same time assume variation in u as negligible. Similar weaknesses are present in the derivation of Eq. 16*c*, involving the energy principle.

Reference to Mr. Keulegan's handling of this same problem indicates that his final equations are in differential form as far as the distance L is concerned. However, the author's results (which show that the coefficient C_m should be used when only external forces are being calculated, and that C_e should be used when the total energy loss is concerned) agree with Mr. Keulegan's final conclusions. This is the all-important part of the paper and is correct, even though the derivations leading up to these facts are quite inelegant mathematically, and not overly convincing.

It is hoped that this paper will clarify the question of which "velocity-head" correction coefficient should be used, and why, when applying the Bernoulli equation to various practical fluid flow problems. Many engineers have not been too clear in their minds regarding this matter and have accepted textbook statements without a careful analysis of the dynamics involved. A more thorough consideration of classical hydrodynamics and the exercise of greater care in transferring relations developed therein into the field of real fluids would have prevented the various misunderstandings such as have occurred in the practical use of the Bernoulli equation.

EDWARD H. TAYLOR,[10] JUN. AM. SOC. C. E.—Certain assertions in this very interesting paper are somewhat novel, and their acceptance would require a change in the thinking methods of the writers of "standard textbooks on hydraulics." The following statement leading to Eqs. 2 and 3 was of particular interest to the writer:

> "The basic equation by the momentum theorem or energy principle is that of Newton's second law of motion, $F = m\,a$, wherein the substitution

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> $a = \dfrac{u^2{}_B - u^2{}_A}{2\,L}$ has been made. The difference between the two methods lies in whichever side of the equation the distance L is placed."

Since momentum and energy are basically different physically—that is, momentum is a vector quantity and energy a scalar quantity—it appears incorrect to state that one can be obtained from the other by a mere transposition of one factor.

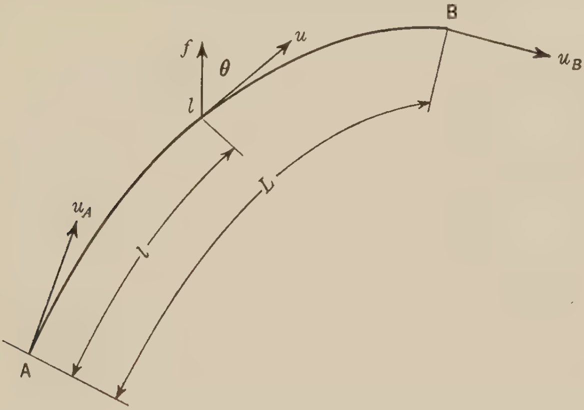

Fig. 3

In other words, the appearance of the length L on one side of the equation or the other should make no difference in the fundamental nature of the equation itself. The F in Eq. 2 and the F in Eqs. 3 are apparently identical. It might be of value to recall that, in a given mechanical system, the force that causes a change in momentum is not, in general, the same force that causes a change in kinetic energy. Forces normal to the path of motion have no effect on the kinetic energy, but definitely influence the momentum. As a simple example, consider a mass particle fastened to a point by a string and describing plane uniform motion about the point.

The key to this situation lies in the substitution, $a = \dfrac{u^2{}_B - u^2{}_A}{2\,L}$. Consider a particle (fluid or otherwise) which follows the path shown in Fig. 3. Here f is the resultant of all forces acting on the particle when it occupies the position l. The equation of motion is:

$$f \cos\theta = m\,\frac{du}{dt} \tag{19}$$

Multiplying both sides of Eq. 19 by dl,

$$f \cos\theta\, dl = m\,\frac{du}{dt}\,dl \tag{20a}$$

or

$$f \cos\theta\, dl = m\,u\,du \tag{20b}$$

Integrating

$$\int_0^L f \cos \theta \, dl = m \left(\frac{u^2_B}{2} - \frac{u^2_A}{2} \right) \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

An average force F may be defined by

$$F = \frac{1}{L} \int_0^L f \cos \theta \, dl \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

provided that its nature be carefully kept in mind. Then,

$$F\,L = m \left(\frac{u^2_B - u^2_A}{2} \right) \text{; or (see Eq. 2) } F = m \left(\frac{u^2_B - u^2_A}{2\,L} \right).$$

Thus, the F of Eq. 2 does not seem to be the resultant force acting on the mass, but rather an average component taken along the path. As far as the writer is aware, the use of the substitution $a = \frac{u^2_B - u^2_A}{2\,L}$ permits no other interpretation. Furthermore, the right-hand member is not the rate of change of momentum (although it is its dimensional equivalent, as shown), but it is the average change in kinetic energy per unit length on the path L. Rate of change of momentum is a vector quantity, by Newton's second law, and Eq. 2 contains no element of direction.

Thus, the writer is led to the conclusion that Eq. 2 is truly an energy equation. This would eventually lead to the further statement that energy coefficients should be used in Eq. 7. The foregoing remarks, although contradictory to the author's general thesis, are in no way intended to be derogatory. They are merely an expression of the writer's understanding of the meaning of momentum and energy.

E. R. VAN DRIEST,[11] ASSOC. M. AM. SOC. C. E.—The theorems of momentum and energy have proved to be very important tools of the hydraulic engineer and therefore their proper application should be understood completely by those who would use them. To be sure, some problems lend themselves more readily to solution by the momentum theorem, whereas others are solved more easily through the use of the energy theorem. Although this paper does not elaborate on the application of these principles, it does attempt to clarify some of the basic facts underlying them.

It is true that the standard textbooks derive the Bernoulli equation by either the application of the Newtonian law to a particle of fluid or the conservation of energy law to a mass; however, in the use of the Newtonian law the acceleration is written as the derivative $u \frac{du}{dL}$ rather than in the restricted form entered in Eq. 2. In most texts the acceleration equation is integrated directly, whereas in certain treatises[9] the line integral is formed along the streamline thus indicating that the work done on the fluid particle is equal to the change in kinetic energy. In this interpretation the left-hand member of Eq. 3a should read $\int_A^B F \, dL$.

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The author's development of Eq. 7 for the case of gradually varied flow is not a strict application of the momentum theorem as such. The theorem states that in steady flow the sum of all external forces acting on a fluid mass is equal to the flow of momentum through the fixed surface bounding that mass at the instant in question. This theorem is expressed by the mathematical relation obtained from the Newtonian law for the L-direction,

$$\sum F_L = \int^{A} \rho\, u^2\, dA \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

If this theorem is applied to the mass of Fig. 1:

$$\int^{A_A} p_A\, dA - \int^{A_B} p_B\, dA + (\text{weight})_L - F_{r,m}$$
$$= \frac{\gamma}{g}\int^{A_B} u^2{}_B\, dA - \frac{\gamma}{g}\int^{A_A} u^2{}_A\, dA \ldots\ldots\ldots\ldots\ldots (24)$$

Rewriting Eq. 24 in terms of the mean velocities across sections A and B, and abbreviating the left-hand member,

$$\sum F_L = \frac{\gamma}{g}\,(C_{m,B}\, V^2{}_B\, A_B - C_{m,A}\, V^2{}_A\, A_A) \ldots\ldots\ldots\ldots (25)$$

It can readily be shown that Eq. 25 can be approximated by Eq. 7 when V_B approaches V_A and the difference between C_B and C_A approaches zero more rapidly than the difference between V_B and V_A. In other words, since the values of C depend only upon the shapes of the velocity distribution curves, as long as the shapes of the curves become similar faster than the velocities (and this is undoubtedly a good assumption for gradually varied flow), the two equations become sensibly the same.

Whereas the momentum theorem has been seen to be applied to the case of gradually varied open-channel flow, the energy theorem is most often used because the energy loss can be evaluated by means of a known formula. Reverting to the general energy theorem—namely, that the work done on a system by external forces is equal to the change in energy of that system—the author has not very carefully derived an energy relation for flow between two sections of an open channel. Reconsidering the case, if an elemental tube of the fluid mass is allowed to move over an interval of time Δt and the work done and energy change over this interval are obtained, then division by Δt will yield the time rate of change of work and energy in the element and summation throughout the entire mass will give the total rate of change of work and energy in the mass at the position considered. The quantity $\frac{p}{\gamma} + Z$ still remains constant over each section so that, upon division by the weight discharge $\gamma\, Q$, the following equation results

$$\left(\frac{p_A}{\gamma} + Z_A\right) - \left(\frac{p_B}{\gamma} + Z_B\right) = \frac{I'}{\gamma\, Q} + \alpha'_B\,\frac{V^2{}_B}{2\,g} - \alpha'_A\,\frac{V^2{}_A}{2\,g} \ldots\ldots (26)$$

in which α' is the "mean-cube" coefficient defined by $\frac{\int u^3\, dA}{V^3\, A}$, and I' represents the rate of internal work done by friction. Although Eq. 26 concerns the energy transformation per second of a mass of fluid per pound of fluid flowing per second, it can be interpreted to state that the work done by the external forces (pressure and gravity), per pound of fluid occupying the space of one pound of discharge as it moves from section A to section B, such that each element has a common displacement irrespective of the time required for each element to travel from A to B, is equal to the change in energy of that pound between sections A and B. Thus, if the internal energy transformation in the fluid volume for the given displacement could be computed or measured, Eq. 26 could be useful, especially for large distortions of the velocity profile.

With reference to the illuminating paper by B. A. Bakhmeteff,[6] M. Am. Soc. C. E., there is attainable an energy relation involving the "mean-square" factor instead of the "mean-cube" factor. The "mean-square" equation is derived readily by considering the work done on an element of fluid as it moves between two sections; thus:

$$\frac{dp}{dL}\,\beta\,dA\,dL + \gamma\,\beta\,dA\,\frac{dZ}{dL}\,dL - dF_r\,dL = \frac{\gamma}{g}\,\beta\,dA\,\frac{d\left(\frac{u^2}{2}\right)}{dL}\,dL \dots\dots (27)$$

in which β is the length of the element, and the differential lengths are inserted to indicate the idea of work. Hence, upon division by $\gamma\,\beta\,dA$ and integrating along the path of motion, there results per pound of fluid on each streamline:

$$\left(\frac{p_A}{\gamma} + Z_A\right) - \left(\frac{p_B}{\gamma} + Z_B\right) = i + \frac{u^2{}_B}{2\,g} - \frac{u^2{}_A}{2\,g} \dots\dots\dots\dots (28)$$

in which i represents the internal work due to friction. Since the integration is between two cross sections of flow, the work and energy equation for the movement of a cross-sectional volume of length dL is given by

$$\int\left(\frac{p_A}{\gamma} + Z_A\right) dA_A\,dL\,\gamma - \int\left(\frac{p_B}{\gamma} + Z_B\right) dA_B\,dL\,\gamma$$
$$= \int i\,dA\,dL\,\gamma + \frac{\gamma}{2\,g}\int u^2{}_B\,dA_B\,dL - \frac{\gamma}{2\,g}\int u^2{}_A\,dA_A\,dL \dots\dots (29)$$

Whence,

$$\left(\frac{p_A}{\gamma} + Z_A\right) - \left(\frac{p_B}{\gamma} + Z_B\right) = \frac{I}{A\,dL\,\gamma} + \alpha_B\,\frac{V^2{}_B}{2\,g} - \alpha_A\,\frac{V^2{}_A}{2\,g} \dots\dots (30)$$

in which α is the "mean-square" coefficient defined by $\frac{\int u^2\,dA}{V^2\,A}$, and I is the internal energy change for the block. It is to be noted that the work done and the energy changed is per pound of fluid which traverses a distance such that each element of it has a common displacement and occupies the cylindrical region that one pound would occupy at any section of the channel. It thus follows that the two equations, Eqs. 26 and 30, compare the energy transformations in fluid volumes for a given displacement, one containing the "mean-cube" coefficients and the other the "mean-square" coefficients; it would seem that either equation is applicable provided the amount of internal energy transformation is known.

Reverting to the purpose of this paper, that of demonstrating the proper use of the correction coefficients for velocity distribution, there seems to be no question but that the "mean-square" factor must be used in the case of the momentum theorem application, whereas in the case of energy, either coefficient may be used depending upon which of the foregoing equations is considered. However, in gradually varied open-channel flow it is apparent that neither factor should be favored in view of the fact that the energy losses are ordinarily computed from empirical equations in which V_B is equal to V_A and in which the coefficients do not enter into the problem at all. In such a method of solution it seems advisable to ignore the correction coefficient entirely; certainly this appears justified since α and α' are close to unity in ordinary cases and experimental evidence[12] is available.

In closing, the writer wishes to emphasize the necessity for mathematical precision in coping with problems of the type investigated in this paper. It is hoped that none of these remarks are taken as discouraging in any manner.

HUNTER ROUSE,[13] M. AM. SOC. C. E., AND JOHN S. MCNOWN,[14] JUN. AM. SOC. C. E.—Attention is again focused by this paper upon the century-old controversy concerning the correct interpretation of the energy and momentum principles in hydraulics. Some aspects of the problem apparently have never yet been presented with adequate substantiation, as witnessed by the disagreement in even the most recent literature on this subject. Perhaps the thought and discussion provoked by the author's analysis will serve to settle at least the principal points of argument which remain. The author's oversimplified approach, unfortunately, has resulted in several conclusions which seem to have fundamental significance but which are in no way justified by a more careful analysis. The writers therefore submit herewith a criticism of the author's presentation together with an original analysis which they believe will clarify the general problem in a simple yet rigorous manner.

The type of motion considered by the author is gradually varied steady flow with a velocity distribution which changes from section to section. The only sound approach to even this particular phase of the general problem is the application of the basic equation of mechanics,

$$F = m \frac{dv}{dt} \dots\dots\dots\dots\dots\dots\dots\dots (31)$$

to a differential volume of fluid, and the subsequent integration of this expression throughout a finite volume according to established principles of the calculus. The author, however, has sought to proceed from particular first integrals of this basic equation—assertedly the fundamental equations of energy and momentum—without fully acknowledging the restrictions which these integrals imposed upon the subsequent analysis. He has, in fact, tacitly attributed to the integrals certain pseudo-differential properties, thereby eliminating from his derivation the very characteristics which distinguish the momentum and energy principles. A typical instance is found under the heading, "Development of the Momentum Equation," which in itself would serve to invalidate the final conclusions of the paper. If the change in velocity along a stream filament is appreciable, it follows that the accompanying change in cross-sectional area must also be appreciable. The fallacy in the author's proof to the contrary lies in treating as a differential expression what is actually the integral of a differential expression over a finite length of stream tube; under such circumstances it is obviously no longer permissible to ignore small quantities as though they were still infinitesimals of higher order than the other terms.

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[12] Discussion by J. C. Stevens of "Back-Water and Drop-Down Curves for Uniform Channels," by Nagaho Mononobe, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 990.

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The general energy equation of hydraulics is fundamentally a scalar relationship, in which the boundary forces exerted upon the fluid between two cross sections do not appear because they do no work; it must include, however, a term embodying the dissipation of energy due to fluid deformation. The general momentum equation, on the contrary, is a vector relationship; therefore, all external forces become significant, whereas the internal stresses which produce the energy dissipation yield a force resultant of zero. Although the basic equation of motion may be written as a differential equation of acceleration, impulse-momentum, or work-energy through proper organization of terms, the essential differences between the integral equations of momentum and energy make it impossible to convert either one into the other by a comparable reorganization of terms—unless the type of flow under consideration is so simplified that the distinguishing characteristics of the integral equations disappear. The latter is the case only if the flow is very nearly uniform and the velocity is essentially constant over every section; under these circumstances, the familiar equation of gradually varied flow may be derived upon either a momentum or an energy basis. Once the acceleration becomes appreciable, or the velocity distribution variable, the two methods of approach necessarily lead to quite different mathematical expressions which cannot be interchanged.

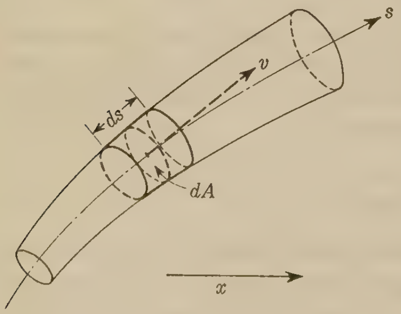

Fig. 4

As has already been pointed out, the basic equations which are sought can be obtained only from consideration of the dynamics of an infinitesimal fluid volume. For simplicity in analysis, the volume selected (see Fig. 4) is bounded by the streamlines forming the wall of a stream tube and by two normal sections a distance ds apart. The external forces acting upon this elementary free body are the attraction of the earth and the pressure and shear exerted by the surrounding fluid, which may be resolved in any direction x and equated to the product of the mass of the element and the corresponding component of its acceleration:

$$\Sigma (dF)_x = dm\, a_x \ldots\ldots\ldots\ldots (32a)$$

The acceleration may be transformed as follows: $a_x = \frac{dv_x}{dt} = \frac{\partial v_x}{\partial s}\frac{ds}{dt} = v\,\frac{\partial v_x}{\partial s}$. Furthermore, dm may be written as $\rho\, ds\, dA$, so that

$$\Sigma (dF)_x = \rho\, v\, \frac{\partial v_x}{\partial s}\, ds\, dA \ldots\ldots\ldots\ldots (32b)$$

Eq. 32*b* expresses, in effect, the equality between the impulse per unit time and the accompanying rate at which the momentum of the fluid element is changed. It should therefore replace Eq. 2, which is neither a basic differential equation nor a correct integral thereof.

A very essential point—possibly the crux of the controversy—must now be noted. Before evaluating any double integral, the two principal variables must first be made independent. Since the quantity dA varies with s, the foregoing equation cannot be integrated correctly along the axis of the stream tube unless means are taken to express dA in terms of a variable which is independent of s. Fortunately, such means are immediately at hand in the fact that $v\, dA = dQ$, the elementary rate of flow through the stream tube, which is necessarily the same at all cross sections; thus,

$$\Sigma (dF)_x = \rho\, \frac{\partial v_x}{\partial s}\, ds\, dQ \ldots\ldots\ldots\ldots (32c)$$

Integration in the s-direction then yields the momentum equation for a finite length of stream tube:

$$\int_{s1}^{s2} \Sigma (dF)_x = \rho\, (v_{x2} - v_{x1})\, dQ \ldots\ldots\ldots\ldots (32d)$$

Since the pressure forces between successive elements are equal and opposite, in the process of summation they cancel, leaving in the term at the left only the external forces of pressure, shear, and gravitational attraction acting upon the filament of fluid. Were it possible to express these forces simply, the foregoing first integral written for flow in the x-direction would yield the correct form of Eq. 5; notably absent is the factor 2 which appears in the denominators of the velocity terms of the author's expression due to his assumption of Eq. 2 as the basic differential equation.

The general equation of momentum for accelerated steady flow is obtained by completing the foregoing integral throughout the volume:

$$\int_{\text{vol}} \Sigma (dF)_x = \int_{Q} \rho\, (v_{x2} - v_{x1})\, dQ = \int_{A2} \rho\, v_{x2}\, v_2\, dA - \int_{A1} \rho\, v_{x1}\, v_1\, dA \ldots (32e)$$

Since internal shears as well as pressures cancel in the process of summation, the left side now represents the x-component of the resultant of all external forces (that is, the end pressures, the boundary pressures, the boundary shear, and the fluid weight), which can be evaluated only through measurement or arbitrary assumption as to type of variation. The right side, which represents the difference in flux of the x-component of momentum past the two end sections, requires equally explicit knowledge as to the corresponding velocity distributions. As a matter of fact, only in case the flow at the two end sections is in essentially the same direction can the equation readily be applied to conditions in which the velocity varies across the flow, under which circumstances it reduces to the alternative forms,

$$\Sigma F = \rho\, Q\, (C_{m2}\, V_2 - C_{m1}\, V_1) = C_{m2}\, \rho\, (V_2)^2 A_2 - C_{m1}\, \rho\, (V_1)^2 A_1 \,.\,.\,(33)$$

in which, as the author has stated,

$$C_m = \frac{1}{A} \int_A \left(\frac{v}{V}\right)^2 dA \ldots\ldots\ldots\ldots\ldots\ldots\ldots (34)$$

It will be seen that Eq. 33 differs considerably from Eq. 8, for the several reasons already discussed.

In an essentially parallel manner, the general energy equation may be derived by equating to the rate of change of energy the rate at which work is done upon the elementary free body of Fig. 4 by the same external forces just considered. It is now necessary, however, to write the component of these forces in the direction of displacement s. Furthermore, it must be noted that the energy of the particle will change as the result of both acceleration and dissipation; the former, of course, is directly expressible in terms of the increase in kinetic energy, but a simple expression for the latter can be written only in terms of the decrease in the total head $H = \frac{v^2}{2\,g} + \frac{p}{\gamma} + Z$ of the element. Thus,

$$\Sigma(dF)_s\, v = \rho\, ds\, dA\, \frac{d\left(\frac{v^2}{2}\right)}{dt} - \gamma\, ds\, dA\, \frac{dH}{dt} \ldots\ldots\ldots\ldots (35a)$$

which is the differential equation of energy suggested to replace Eq. 3. Rearranging the terms on the right side,

$$\Sigma(dF)_s\, v = \rho\, ds\, dA\, \frac{\partial\left(\frac{v^2}{2}\right)}{\partial s}\, \frac{ds}{dt} - \gamma\, ds\, dA\, \frac{\partial H}{\partial s}\, \frac{ds}{dt}$$

$$= \rho\, \frac{\partial\left(\frac{v^2}{2}\right)}{\partial s}\, ds\, dQ - \gamma\, \frac{\partial H}{\partial s}\, ds\, dQ \ldots\ldots\ldots\ldots\ldots (35b)$$

Since dQ, unlike dA, is a constant along the stream tube, Eq. 35b may be integrated at once with respect to s:

$$\int_{s1}^{s2} \Sigma(dF)_s\, v = \frac{\rho}{2}\, [(v_2)^2 - (v_1)^2]\, dQ - \gamma\, (H_2 - H_1)\, dQ \ldots\ldots (35c)$$

Were it practicable to express each individual member of the summation at the left, division by γ and substitution of $v\,dA$ for dQ would yield the correct form of Eq. 14. As in the case of the momentum derivation, however, it should be noted that internal pressures along the stream tube have equal and opposite reactions at points of the same velocity, and hence the net work done by pressures upon the fluid filament will consist only of that at the two end sections. The work done by shear along the filament cannot be evaluated easily, of course; but, in the process of summation over the entire volume, the net internal work done by both pressure and shear is evidently equal to zero. Moreover, since the boundary velocity is zero at all points, the work done by boundary shear must also be zero, leaving at the left of the final equation only the terms for pressure and elevation at the two end sections. The general volume integral, written in terms of the several heads, thus takes the rather obvious form,

$$\int_{A1}\left(\frac{p_1}{\gamma}+Z_1\right)v_1\,dA-\int_{A2}\left(\frac{p_2}{\gamma}+Z_2\right)v_2\,dA$$

$$=\int_{A2}\left[\frac{(v_2)^2}{2\,g}-H_2\right]v_2\,dA-\int_{A1}\left[\frac{(v_1)^2}{2\,g}-H_1\right]v_1\,dA\ldots\ldots\ldots(36)$$

Like the general momentum equation, the foregoing general energy equation may be applied to a given state of flow only if the distribution of velocity and pressure (and hence of total head) is known at both end sections. Unlike the momentum equation, on the other hand, the energy equation involves only the magnitudes of the velocities; however, the velocity distribution here affects both sides of the equation, with the result that the energy principle may usually be applied only if both sections under consideration are located in essentially uniform zones; the pressure distribution is then known (that is, hydrostatic), and the energy equation reduces to the more familiar form of the Bernoulli theorem,

$$C_{e1}\frac{(V_1)^2}{2\,g}+\frac{p_1}{\gamma}+Z_1=C_{e2}\frac{(V_2)^2}{2\,g}+\frac{p_2}{\gamma}+Z_2+H_l\ldots\ldots\ldots(37)$$

in which

$$C_e=\frac{1}{A}\int_{A}\left(\frac{v}{V}\right)^3dA\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(38)$$

and

$$H_l=\frac{1}{A_1}\int_{A1}H_1\frac{v_1}{V_1}\,dA-\frac{1}{A_2}\int_{A2}H_2\frac{v_2}{V_2}dA\ldots\ldots\ldots\ldots\ldots(39)$$

Since Eq. 37 lacks certain unnecessary factors of Eq. 16, yet further clarifies the term for energy dissipation, it is considered preferable by the writers.

From a comparison of the general momentum and energy relationships it will be evident that, although both may describe correctly the same state of flow, the mathematical terminology of the description must be quite different in the two cases. In other words, the inherent distinction between energy as a scalar and momentum as a vector quantity, as well as the presence of a term for lost head in the one equation and terms for boundary pressure and shear in the other, makes it completely impossible to interchange the two equations by any conceivable transformation process. Therefore, despite the author's statements to the contrary, the Bernoulli theorem can in no way be regarded as a form of the momentum equation. As a matter of fact, it is this very difference between these two integral forms of the basic equation of motion which makes them so effective when used in combination. Hence, it is scarcely logical to attempt to show their similarity, but rather to emphasize the distinctions which make them such powerful complementary tools.

In the basic application of these equations to rapidly accelerated flow, pressure changes are generally so great that boundary-shear, lost-head, and velocity-distribution terms may be neglected without appreciable error; the equations of momentum, energy, and continuity then permit direct solution for both the pressure change and the resultant boundary force accompanying any rate of flow. They have also proved to be a useful combination in another extreme type of motion—the gradual establishment of uniform flow beyond the entrance of a pipe or conduit, in which acceleration and boundary pressure become the terms which are ignored. If these various effects are of comparable magnitude, of course, then none can be neglected safely. Typical of such conditions is the problem of rapidly decelerated flow, which still awaits a satisfactory solution by any but empirical methods. In a word, it is still impossible to obtain a quantitative analysis of the general problem of fluid motion without recourse to measurement, because other means are not yet at hand for obtaining the distribution curves for velocity and pressure which are required in the general momentum and energy equations.

Although the type of motion considered by the author is far simpler than the general case, in that acceleration is negligible and the pressure distribution is therefore hydrostatic, the distribution of velocity still remains unknown. Without actual field measurement, in other words, the coefficients C_m and C_e can be estimated only roughly, for scarcely enough is known about boundary resistance and energy dissipation in nonuniform flow to permit even an approximate evaluation by analytical means. To be sure, such estimates will yield average values which are perhaps closer to the truth than the magnitude of unity customarily assumed, but this still provides no trustworthy clue as to the change in either coefficient from section to section. Fortunately, however, the error involved in assuming these coefficients to equal unity is probably no greater than that arising from the present lack of knowledge regarding boundary shear and energy dissipation under nonuniform conditions. Indeed, the common failure to distinguish between the evaluation of boundary shear and energy dissipation in gradually varied flow is actually tantamount to assuming that $C_m = 1 = C_e$, for under such circumstances (but only then) the energy and momentum equations become identical.

Much is made by the author of the asserted fact that the familiar resistance equations stem from momentum (boundary-shear) rather than energy (dissipation) considerations. Actually, the equations in question have been derived solely for the case of uniform flow; under these conditions, of course, boundary shear and energy dissipation are directly proportional, which robs the author's point of all significance. The customary loss coefficients of nonuniform flow, on the other hand, are simply means of evaluating the quantity H_l of Eqs. 37 and 39 in terms of the boundary geometry and the Reynolds number, and hence have no source whatever in the momentum principle. Which type of coefficient to apply in the analysis of gradually varied flow must depend upon whether the momentum or the energy principle is utilized when the true functional relationships for the coefficients are eventually determined. Either principle, of course, would be satisfactory for this purpose; however it is to be noted that the momentum principle is not necessarily the simpler of the two, because of the difficulty in evaluating the longitudinal component of the boundary pressure.

From the foregoing discussion it would appear that such further refinements in applying the momentum and energy equations to even the relatively simple case of gradually varied flow must await considerable progress in analyzing the phenomena of velocity distribution and either boundary shear or energy dissipation. Fortunately, the solution of any one of these three phases of the resistance problem will almost automatically clarify the other two, for all three are closely interrelated—not only through the mechanics of fluid turbulence, but through the correct integral forms of the momentum and energy relationships.

William S. Eisenlohr, Jr.,[15] Assoc. M. Am. Soc. C. E.—The thesis of this paper, as given in the "Synopsis" and "Introduction," is that there are problems in nonuniform flow in which an appreciable part of the total change in momentum and kinetic energy between two cross sections may be due to a change in the velocity distribution as distinguished from a change in the mean velocity. The Chézy and Manning equations are momentum equations. Thus, the coefficient of velocity distribution should be the ratio of the mean of the squares of the local velocities to the square of the mean velocity. That the several discussions of the paper, although critical, have not successfully disproved this thesis will be shown. To do so, however, it will be necessary to present revised demonstrations. At the same time, an effort will be made to clarify some of the original statements.

First, it is necessary to include the concept embodied in Eq. 27 given by Professor Van Driest and derived from the paper by B. A. Bakhmeteff, M. Am. Soc. C. E. Somehow or other, in preparing the paper, the writer lost sight of the concept that there is an energy equation involving the square relation as well as one involving the cube relation of the velocity. Since the difference between momentum and energy no longer is sufficient to account for the use of different coefficients, it becomes appropriate to redesignate the coefficients as follows: C_s (formerly C_m) $= \frac{\overline{u^2}}{V^2}$ = coefficient of velocity distribution based on the second power of the velocity for use in space equations; and C_t (formerly C_e) $= \frac{\overline{u^3}}{V^3}$ = coefficient of velocity distribution based on the third power of the velocity for use in time equations. (The writer prefers the use of the letter C with subscripts, to that of the Greek letter α, since the latter cannot be made on an ordinary typewriter and is difficult to make attractively by hand.)

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The writer stated that the condition of steady, gradually varied flow would be assumed. Perhaps this point should have been given more emphasis. By steady flow is meant that there is no variation in the conditions of flow with respect to time. Gradually varied flow is a condition of flow in which the convergence, divergence, or curvature of the stream tubes occurs so gradually that consideration of the stream tubes as parallel will introduce no appreciable error. In other words, the assumption is made that the vector velocity v can be replaced by u without appreciable error. (A similar assumption is always made in computing the slope S, the quantity $\tan \theta$ being used for $\sin \theta$.) Having made this assumption, the careful distinction between the two made by Messrs. Rouse and McNown is without significance and somewhat confusing. This assumption also gives the element of direction which Professor Taylor finds lacking in Eq. 2.

With the foregoing assumptions always in mind, the equations for gradually varied flow in open channels will be developed. The basic equation is the momentum equation expressing Newton's second law of motion—the rate of change of momentum is proportional to the impressed force, and takes place in the direction of the straight line in which the force acts. Therefore:

$$dF_R = dm \frac{d\mathrm{v}}{dt} \qquad \text{(40)}$$

in which F_R is the resultant force and v is the vector velocity. By the assumption of gradually varied flow, the velocity component parallel to the x-axis, u, can be substituted for v without appreciable error. Thus, the resultant force must be parallel to the x-axis and must be equal to the sum of the components in the x-direction of all forces acting, ΣdF_x. Then, $\Sigma dF_x = dm \frac{du}{dt}$ will describe the change in momentum completely. Substituting $\frac{du}{dt} = \frac{du}{dL}\frac{dL}{dt} = u \frac{du}{dL} = \frac{1}{2}\frac{du^2}{dL}$, the momentum equation is

$$\Sigma dF_x = dm \left(\frac{1}{2}\frac{du^2}{dL}\right) \qquad \text{(41a)}$$

Eq. 41a is a momentum equation but the rate of change of momentum is expressed in terms of distance L rather than in terms of time t. As stated by Messrs. Rouse and McNown, u is a function of L, and Eq. 41a cannot be integrated rigorously without taking that relation into consideration. However, it will be shown subsequently that, if the acceleration is assumed constant (or the equivalent of using the average force $\frac{1}{L}\int_0^L \Sigma dF_x\, dL$ as stated by Professor Taylor), reasonably close results for gradually varied flow will be obtained. Eq. 41a is readily transformed into the energy equation—

$$\Sigma dF_x\, dL = \tfrac{1}{2}\, dm\, du^2 \qquad \text{(41b)}$$

—the energy equation in which the work done by the force dF_x equals the change in kinetic energy of the mass dm.

Whether Eq. 41*a* is transformed into Eq. 41*b* by transposing dL or by multiplying both sides by dL, the result is the same—a vector force is multiplied by the distance through which it acts to give the scalar quantity of work. Likewise, the right side of Eq. 41*a* has the dimensions of the rate of change of momentum, $\frac{M\,L^2}{T^2\,L} = \frac{M\,L}{T^2}$, which is a vector quantity. Whether the quantity is multiplied by L, or the divisor L is removed, the result is $\frac{M\,L^2}{T^2}$, the dimension of energy, which is a scalar quantity. There is nothing novel in such a procedure. In fact, it is the standard derivation of the work-energy equation. This transformation is not rigorous except in the differential form.

If both sides of Eq. 41*b* are divided by dt: $\Sigma dF_x \frac{dL}{dt} = \frac{1}{2}\,dm\,\frac{du^2}{dt}$ or $\Sigma dF_x\, u = \frac{1}{2}\,dm\,[du^2]_{t=1}$, the latter being the differential form of Eq. 3*b*. Without imposing the restriction of $t = 1$, and making the substitution $\frac{du}{dt} = u\,\frac{du}{dL}$, the power equation is obtained:

$$\Sigma dF_x\, u = dm\left(\frac{1}{3}\frac{du^3}{dL}\right) \dots\dots\dots\dots\dots\dots \text{(41}c\text{)}$$

Eq. 41*c* is frequently called an energy equation (as was Eq. 3*b* in the paper); but, since it actually represents changes in work and energy per unit time, it will be designated the power equation—especially to distinguish it from Eq. 41*b*.

From the three basic equations of momentum, energy, and power, the three equations of nonuniform flow in an open channel will be derived. Considering Eqs. 41*a*, 41*b*, and 41*c*, the coefficient of velocity distribution seems to be based on the square relation in the momentum and energy equations and on the cube relation in the power equation. This theory will be shown to be correct.

The simplest way to obtain the equations for the entire channel would be to let $dm = \rho\, dA\, dL$ and then to perform the double integration with respect to A and L. To avoid calculus, where possible, and at the same time to give emphasis to the physical significance of the equations, the mass will be taken as the entire mass of the stream tube $\rho\, L\, \overline{dA}$, in which L is the length and $\overline{dA}$ is the average value of the cross section. Thus, $L\, \overline{dA}$ equals the volume of the stream tube. Such a stream tube is shown in Fig. 5, the boundaries of which are fixed in space. In the paper it was found that selecting a volume of water and following the change in energy in that volume with respect to time did not yield practical results; therefore, such a method will not be given further consideration.

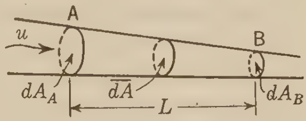

FIG. 5.—STREAM TUBE (VOLUME = $L\,\overline{dA}$)

Although the stream tube is fixed in space, the water particles at any point within the tube will change from instant to instant. In steady flow, each particle arriving at a given point will have exactly the same momentum or energy possessed by the particle which just left that point. As a result, the total momentum or energy contained within the stream tube will be the same at every instant of time. During any interval of time a volume of water dB will flow out of the stream tube, and by the continuity of flow an equal volume dB will flow into the stream tube. Thus, the total change in momentum or energy of the water flowing through the stream tube during a given interval will be represented by the difference between the momentum or energy of a volume dB leaving the tube and the momentum or energy of a similar volume entering the tube. The three equations of flow can be developed as follows.

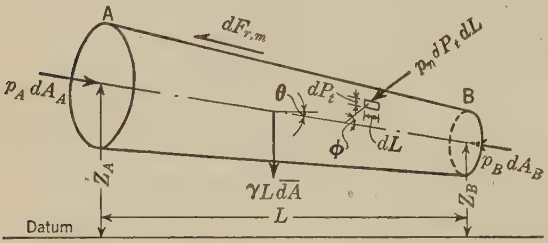

Fig. 6.—Forces Acting on Stream Tube

Revised Development of the Momentum Equation.—In this revised demonstration the effect of the variation of dA with L will be fully considered. The rate of change of momentum is equal to the resultant of all the external forces that are acting. For the stream tube in Fig. 6, the pressure forces are, as before:

$$\int_A^B dF_p = p_A\, dA_A - p_B\, dA_B - \bar{p}_n\, (dA_A - dA_B) \ldots\ldots\ldots (42a)$$

in which the first two terms on the right side represent the pressure forces on the end areas and the third term represents the component in the direction of motion of the normal pressure p_n on the sloping sides of the tube. Since this force is important and usually neglected, it will be considered in detail.

The pressure on any element of area along the side of the stream tube is $p_n\, dP_t\, dL$ in which P_t is the perimeter of the stream tube. For uniform flow, p_n is normal to the direction of flow and the term disappears. In gradually varied flow, p_n is still the normal pressure on the side of the stream tube; but, by reason of the change in area, the side must make a slight angle and p_n an angle ϕ with the direction of flow. The pressure force on the side of the stream tube now has a component in the direction of flow equal to $p_n\, dP_t\, dL \cos \phi$. The total force is then $\int\int p_n\, dP_t\, dL \cos \phi$; but $\int\int dP_t\, dL \cos \phi$ equals the difference in end areas of the stream tube. The value of $\bar{p}_n$ is defined by the relation: $\bar{p}_n\, (dA_A - dA_B) = \int\int p_n\, dP_t\, dL \cos \phi$.

As it is impracticable to attempt to evaluate $\bar{p}_n$, Eq. 42*a* will be written in a more usable form by expressing the forces in terms of the average area $\overline{dA}$. The quantity $\overline{dA}$ has such a value that the total volume of the stream tube between sections A and B is $L\, \overline{dA}$. Let k_1 be a certain portion of the difference in end areas, having a value such that $dA_A = \overline{dA} + k_1(dA_A - dA_B)$ and $dA_B = \overline{dA} - (1 - k_1)(dA_A - dA_B)$. Likewise, let $\bar{p}_n = p_A - k_2(p_A - p_B)$. Substituting these values in Eq. 42*a* gives, for the pressure forces:

$$\int_A^B dF_p = (p_A - p_B)\,\overline{dA} + (k_1 + k_2 - 1)(p_A - p_B)(dA_A - dA_B) \quad \text{..(42}b\text{)}$$

The gravity force $\int_A^B dF_g$ is $\gamma\, L\, \overline{dA} \sin\theta$, and the resisting force is $dF_{r,m}$. The change in momentum during any interval, as explained previously, can be represented by the difference between the momentum of the volume of water dB as it leaves the stream tube at section B and the momentum of an equal volume of water dB as it enters the stream tube at section A. This change can be expressed as $\rho\, dB\, u_B - \rho\, dB\, u_A$. Since the resultant force is equal to the rate of change of momentum, the volume dB in this case must be the volume change per unit time, dQ. Therefore,

$$\int_A^B \Sigma dF_x = \rho\, dQ\,(u_B - u_A) = \rho\, u_B^2\, dA_B - \rho\, u_A^2\, dA_A \quad \text{.....(42}c\text{)}$$

Eq. 42*c* is identical with the expression derived by Messrs. Rouse and McNown through their integration process. In Eq. 42*c*, there was no need for integration because the total change in momentum for the entire length of stream tube L was used. Since the pressure forces are not amenable to practical application when expressed in terms of dA_A and dA_B, Eq. 42*c* will be written in terms of the average area $\overline{dA}$. Let $dQ = u'\,\overline{dA}$ and $u' = u_A + k_3(u_B - u_A)$ (in which k_3 is some portion of the difference in velocity between the ends of the stream tube) and substitute in Eq. 42*c*:

$$\int_A^B \Sigma dF_x = \rho\,\overline{dA}\,[k_3\, u_B^2 - (1 - k_3)\, u_A^2 + (1 - 2\,k_3)\, u_A\, u_B] \quad \text{...(42}d\text{)}$$

Combining the various parts, the general momentum equation is obtained for gradually varied flow in a stream tube between sections A and B in terms of the average area of the tube $\overline{dA}$:

$$(p_A - p_B)\,\overline{dA} + (k_1 + k_2 - 1)(p_A - p_B)(dA_A - dA_B) + \gamma\, L\,\overline{dA}\sin\theta$$
$$- dF_{r,m} = \rho\,\overline{dA}\,[k_3\, u_B^2 - (1 - k_3)\, u_A^2 + (1 - 2\,k_3)\, u_A\, u_B] \quad \text{...(42}e\text{)}$$

Eq. 42*e* is complicated by the proportionality factors k_1, k_2, and k_3. These factors represent the portion of the difference between the values at sections A and B by which the average value of the area, pressure, and velocity is greater or less than the value at section A. It seems reasonable to suppose that for gradually varied flow the values of k_1, k_2, and k_3 will be so close to 0.5 that they can be assumed to have that value. Making this assumption, Eq. 42*e* becomes, after substituting $\sin\theta = \frac{Z_A - Z_B}{L}$ (actually $\tan\theta$) and rearranging:

$$dF_{r,m} = \gamma \left[\left(\frac{p_A}{\gamma} + Z_A \right) \overline{dA} - \left(\frac{p_B}{\gamma} + Z_B \right) \overline{dA} + \frac{u^2{}_A}{2\,g} \overline{dA} - \frac{u^2{}_B}{2\,g} \overline{dA} \right] \ldots (43)$$

which is identical with Eq. 6 except that the area of the stream tube is definitely labeled the average area. The only assumption is that the average area, pressure, and velocity are the averages of the values at the end sections. The term $\left(\frac{u^2{}_A}{2\,g} - \frac{u^2{}_B}{2\,g} \right) \overline{dA}$ is the practical equivalent of $\frac{u^2{}_A}{g}\,dA_A - \frac{u^2{}_B}{g}\,dA_B$. The same answer would be obtained if d'Alembert's principle of reversed effective force were used, and the momentum changes were represented by pressure forces equal to the velocity heads. These pressures on the end areas, of course, are $\left(\frac{u^2{}_A}{2\,g} \right) dA_A$ and $\left(\frac{u^2{}_B}{2\,g} \right) dA_B$; but the average pressure on the difference in end area must also be considered and, when it is included, the result is the same as Eq. 43.

The integration of Eq. 43 with respect to A yields Eq. 7 as before, and Eqs. 9 to 12 remain unchanged. The symbol $F_{r,m}$ denotes the total boundary shear and is usually assumed to be the product of the average unit shear and the area of the boundary. Professor Kalinske has stated that such an assumption limits the length of channel over which the expression is valid. On the other hand, the length of channel must be long enough that the effect of boundary shear can be measured with reasonable accuracy. Although these conditions limit the application of Eq. 9*a*, the principle involved is the momentum theory of external forces that are acting, and the substitutions made in arriving at the Chézy and Manning equations are empirical expressions for boundary shear. As Professor Rouse has stated[16] for the Manning equation, "The factor *n* is evidently a measure of boundary roughness, since it is invariably tabulated solely as a function of boundary composition." Furthermore, Robert Manning's studies leading to the development of the "Manning" equation included results of experiments on the Hayne (Belgium), Parana (South America), and Mississippi rivers; and modern tables of values of *n* almost always include values for natural streams. The statement by Messrs. Rouse and McNown that "Actually, the [familiar resistance] equations in question have been derived solely for the case of uniform flow * * *" thus appears to be unduly restrictive.

Revised Development of the Energy Equation (*See Heading, "Development of the Power Equation" for Revised Development of the "Energy Equation" of the Paper*).—The change in kinetic energy represented by the difference between the energy of volume *dB*, as it leaves the tube at section B, and the energy of an equal volume *dB*, as it enters the stream tube at section A, is the result of the work done by all forces acting on the tube during the same interval of time. Since work is a scalar quantity, the total work done is the algebraic sum of the product of force times distance moved in the direction of the force. The work done by the pressure forces is the sum of the pressure force on each bounding surface of the tube times the distance of movement across that surface in the interval of time considered. Since there is no movement across the sides of the tube, the only work done by the pressure forces is on the ends of the tube. To visualize this work, assume the stream tube to be separated in space, as in Fig. 7. The water will approach the tube as a filament of uniform cross section, dA_A, and will leave with a uniform cross section, dA_B. If the volume passing through the tube in a given time interval is defined as $dB = s_A\,dA_A = s_B\,dA_B$ (in which s is a small distance in the direction of flow, the dL of Eq. 41b), the total work done by the pressure forces is $p_A\,dA_A\,s_A - p_B\,dA_B\,s_B$.

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[16] "Evaluation of Boundary Roughness," by Hunter Rouse, *Proceedings*, 2d Hydr. Conference, June 1–4, 1942, State Univ. of Iowa, Iowa City, *Bulletin No. 27*, Studies in Eng., 1943, p. 108.

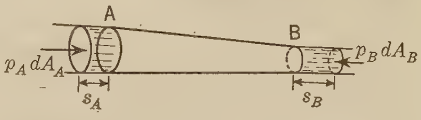

FIG. 7.—STREAM TUBE AS A FREE BODY SHOWING WORK DONE BY PRESSURE FORCES ($s_A\,dA_A = s_B\,dA_B$)

The total work done by the gravity force is equivalent to the weight of volume dB times the distance volume dB moves in the direction of the gravity force while being moved from section A to section B. This distance is the difference in elevation, $Z_A - Z_B$. The work done by gravity is therefore $\gamma\,dB\,(Z_A - Z_B)$. The change in kinetic energy, of course, is $\rho\,dB\,\frac{u^2{}_B}{2} - \rho\,dB\,\frac{u^2{}_A}{2}$.

During the interval required for volume dB to enter or leave the stream tube, work is done by the water in the tube against the shearing force at the sides. There is also a dissipation of energy as heat from the turbulence set up by the resistance of the sides of the tube. The net result of these effects is a loss of energy within the tube that, for convenience, will be expressed as a loss of potential energy $\gamma\,dB\,h_L$, in which h_L is the energy lost per unit weight of water passing through the stream tube, usually referred to as the loss of head. The work-energy equation is, then:

$$p_A\,dA_A\,s_A - p_B\,dA_B\,s_B + \gamma\,dB\,(Z_A - Z_B) - \gamma\,dB\,h_L = \rho\,dB\,\frac{u^2{}_B}{2} - \rho\,dB\,\frac{u^2{}_A}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (44a)$$

Rearranging, and substituting $dB = s_A\,dA_A = s_B\,dA_B$, gives

$$(p_A + \gamma\,Z_A)\,s_A\,dA_A - (p_B + \gamma\,Z_B)\,s_B\,dA_B = \rho\,\frac{s_B}{2}\,u^2{}_B\,dA_B - \rho\,\frac{s_A}{2}\,u^2{}_A\,dA_A + \gamma\,h_L\,dB \quad \ldots\ldots\ldots\ldots (44b)$$

It must be remembered that in Eqs. 44 the change in energy of the volume dB during the displacement from A to B was used as the equivalent of the change in energy of the stream tube during the displacement s. In integrating over the cross section, therefore, the volume B obtained is that resulting from the displacement s of the mass of the entire stream between A and B (see Eq. 41b and subsequent discussion). Hence s_A and s_B are both constant.

Proceeding with the integration, making use of the coefficient C_s previously defined, and letting $h_s = \frac{1}{B}\int_A h_L\, dB$ (in which h_s is the average for the entire stream of the energy lost per unit weight because the stream passed over the length of channel L), the following expression results, after dividing by the weight of the volume B:

$$\frac{p_A}{\gamma} + Z_A + C_{s,A}\frac{V^2{}_A}{2\,g} = \frac{p_B}{\gamma} + Z_B + C_{s,B}\frac{V^2{}_B}{2\,g} + h_s \ldots\ldots\ldots (44c)$$

Development of the Power Equation.—Power is the time rate of doing work. To obtain the power equation it is only necessary to divide the work-energy formula, Eq. 44*b*, by the time interval Δt during which the volume dB entered or left the stream tube. Since s is a small distance, $\frac{s}{\Delta t} = u$. Performing the division:

$$(p_A + \gamma\, Z_A)\, u_A\, dA_A - (p_B + \gamma\, Z_B)\, u_B\, dA_B$$
$$= \frac{\rho}{2} u^3{}_B\, dA_B - \frac{\rho}{2} u^3{}_A\, dA_A + \gamma\, h_L\, u\, dA \ldots\ldots\ldots\ldots (45a)$$

Integrating Eq. 45*a* with respect to A by making use of the coefficient C_t previously defined, letting $h_t = \frac{1}{V\,A}\int h_L\, u\, dA$ (in which h_t is the average for the entire stream of the energy lost per unit weight per unit time because the stream passed over the length of channel L), and then dividing by the weight per unit time, $\gamma\, Q$:

$$\frac{p_A}{\gamma} + Z_A + C_{t,A}\frac{V^2{}_A}{2\,g} = \frac{p_B}{\gamma} + Z_B + C_{t,B}\frac{V^2{}_B}{2\,g} + h_t \ldots\ldots\ldots (45b)$$

Comparison of Equations.—Three equations have been developed for gradually varied flow in open channels as follows:

Momentum—

$$\left[\frac{p}{\gamma} + Z + C_s\frac{V^2}{2\,g}\right]_A \approx \left[\frac{p}{\gamma} + Z + C_s\frac{V^2}{2\,g}\right]_B + \frac{\bar{\tau}_0\,\bar{P}\,L}{\gamma\,\bar{A}} \ldots\ldots (46a)$$

energy—

$$\left[\frac{p}{\gamma} + Z + C_s\frac{V^2}{2\,g}\right]_A = \left[\frac{p}{\gamma} + Z + C_s\frac{V^2}{2\,g}\right]_B + h_s \ldots\ldots\ldots (46b)$$

and power—

$$\left[\frac{p}{\gamma} + Z + C_t\frac{V^2}{2\,g}\right]_A = \left[\frac{p}{\gamma} + Z + C_t\frac{V^2}{2\,g}\right]_B + h_t \ldots\ldots\ldots (46c)$$

The approximation symbol is used with the momentum formula (Eq. 46*a*) to emphasize the fact that the equality exists only under certain specified conditions and to aid in the comparison with the energy formula, Eq. 46*b*. In the following discussion it will be assumed that the conditions of gradually varied flow are such that the approximation becomes an equality.

Eq. 46*a* indicates that the momentum equation can be put in the form of the Bernoulli equation only by making restrictive assumptions. Therefore, the statement made in the "Summary" of the paper that "* * * the Bernoulli equation is also a momentum equation * * *" appears to be erroneous if the Bernoulli equation of ordinary hydraulics is considered, despite the fact that at least seven modern textbooks on hydraulics and fluid mechanics give the derivation based on the rate of change of momentum as one proof of the Bernoulli equation. Examination of these textbooks discloses that in only one is any consideration given to a variation in cross-sectional area and that one[17] disposes of the variation in a manner somewhat similar to the writer's original development. In uniform flow there is no variation in area and consequently no change in velocity; thus, the velocity terms cancel out of the equation. Obviously, the Bernoulli equation without the velocity terms is not the Bernoulli equation. Furthermore, in applying the momentum theorem to local phenomena (the hydraulic jump, for example), approximations cannot be made that are valid for gradually varied flow, and the similarity to the Bernoulli equation becomes more remote. Therefore, it is not proper to derive the Bernoulli equation of ordinary hydraulics from the momentum theorem.

The similarity of equations derived by the momentum and energy principles has been emphasized to show how readily the two can be confused. Thus, by rearranging Eqs. 46*a* and 46*b*, the following expressions are obtained:

$$\frac{\bar{\tau}_0 \bar{P} L}{\gamma \bar{A}} \approx \left[\frac{p}{\gamma} + Z + C_s \frac{V^2}{2 g}\right]_A - \left[\frac{p}{\gamma} + Z + C_s \frac{V^2}{2 g}\right]_B \ldots\ldots (47a)$$

and

$$h_s = \left[\frac{p}{\gamma} + Z + C_s \frac{V^2}{2 g}\right]_A - \left[\frac{p}{\gamma} + Z + C_s \frac{V^2}{2 g}\right]_B \ldots\ldots\ldots (47b)$$

Each term in Eqs. 47 has the dimension of length. Therefore, as lengths, the right sides of the two equations are identical. However, these lengths represent different physical quantities. The lengths in Eq. 47*b* represent the usual energy per unit weight because they were obtained by dividing the energy equation by γB, the weight of water passing through the reach L. On the other hand, the lengths in Eq. 47*a* represent force per unit area because they were obtained by dividing the momentum equation by γA. The significance of the equality of the right sides of the two equations is that, for conditions satisfied by Eq. 47*a*, the loss of energy within the stream is the result of boundary shear alone.

The principal distinction between Eqs. 47*a* and 47*b* is that the latter is rigorous because it is the definition of the quantity h_s and is not necessarily limited to gradually varied flow, whereas the former defines the resistance to flow as the boundary shear for the specified conditions of flow. Since the length given by the right side of Eq. 47*a* is defined as h_s in Eq. 47*b*, it is permissible to write $\frac{\bar{\tau}_0 \bar{P} L}{\gamma \bar{A}} \approx h_s$ as long as h_s is considered as a length only. If

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[17] "Hydraulics and Its Applications," by A. H. Gibson, 4th Ed., Constable & Co., Ltd., London, 1934, 82–84.

h_s is considered as loss of energy per unit weight, the foregoing substitution is not permissible because it implies that h_s is to be multiplied by γB and this would make the expression nonhomogeneous, since there is also the implication that the left side is to be multiplied by γA. In other words, making approximations that involve little error and writing the momentum equation in the form of Eq. 47*a* (the right side of which is identical with the energy formula, Eq. 47*b*) do not make Eq. 47*a* an energy equation. Such a procedure, however, is apparently what Messrs. Rouse and McNown visualize as a basis for the statement,

> "The customary loss coefficients of nonuniform flow * * * are simply means of evaluating the quantity H_l of Eqs. 37 and 39 in terms of the boundary geometry and the Reynolds number, and hence have no source whatever in the momentum principle."

The coefficient of velocity distribution is C_s, the ratio of the mean of the squares of the local velocities to the square of the mean velocity. Further examination shows that both the resisting force and the energy lost are related to the length of channel L without consideration of the time factor. The coefficient C_s, therefore, is the proper coefficient for use in space equations.

Turning to the relation between the energy and power equations, it will perhaps be simpler to show the difference by using an illustration. Consider a stream flowing out of a large lake. At cross section A near the lake, the velocity distribution will be very uniform and no appreciable error will be involved by assuming $C_s = C_t = 1.00$. This particular condition was chosen to simplify the demonstration. Suppose that at a second cross section B some distance downstream, away from entrance conditions, the stream channel is complex and the value of C_s is 1.2 and that of C_t is 1.6 (which is considered reasonable for a complex natural channel). Conditions will then be as shown in Fig. 8.

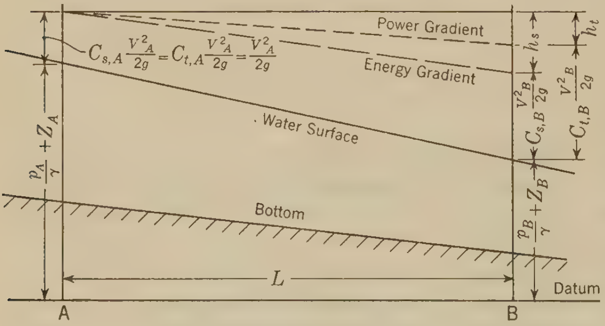
Power Gradient
$C_{s,A}\frac{V^2_A}{2g}=C_{t,A}\frac{V^2_A}{2g}=\frac{V^2_A}{2g}$
Energy Gradient
Water Surface
Bottom
$\frac{p_A}{\gamma}+Z_A$
h_s
h_t
$C_{s,B}\frac{V^2_B}{2g}$
$C_{t,B}\frac{V^2_B}{2g}$
$\frac{p_B}{\gamma}+Z_B$
L
Datum
A
B

FIG. 8.—PROFILE DIAGRAM

In the development of the energy equation, h_s was defined as the average energy lost per unit weight in the distance L between sections A and B. Therefore, from Eq. 47*b* it appears that the quantity $\frac{p}{\gamma} + Z + C_s \frac{V^2}{2\,g}$ must be the energy possessed by a unit weight of water passing any section. Since $\frac{p}{\gamma} + Z$ represents the elevation of the water surface, the point plotted a distance $C_s \frac{V^2}{2\,g}$ above the water surface in a profile diagram such as Fig. 8 must represent the energy content per unit weight passing that section. A line connecting all such points would then be the "energy content line." Obviously, for steady flow the energy content line will not change with time and its slope becomes the "energy gradient" when this term is defined as the rate of loss of energy per unit weight per unit length of channel. The distinction between "energy content line" and "energy gradient" is important only in the case of unsteady flow.

In the same way the power equation can be written in the form (compare Eq. 47*b*):

$$h_t = \left[\frac{p}{\gamma} + Z + C_t\frac{V^2}{2g}\right]_A - \left[\frac{p}{\gamma} + Z + C_t\frac{V^2}{2g}\right]_B \ldots\ldots\ldots (48)$$

and the quantity $\frac{p}{\gamma} + Z + C_t\frac{V^2}{2g}$ is the average energy per unit weight passing any section per unit time, or the power at any section. Likewise, in a profile diagram such as Fig. 8, a line plotted a distance $C_t\frac{V^2}{2g}$ above the water surface will be the "power content line" which for steady flow has a slope that represents the average loss of energy per unit weight per unit time per unit length of channel and is the "power gradient."

Thus, h_s is the average energy lost by the stream for every pound of water passing through the reach from section A to section B, but h_t is the average energy lost per second per pound of water passing through the reach. Although h_s and h_t have different numerical values, they represent the same physical quantity—energy lost by the stream in distance L. Therefore, it should be possible to obtain h_s by multiplying h_t by $\bar{t}$, the average time needed for a pound of water to pass any section. To do this, since both h_L and t vary for each stream tube, the multiplication must be made before integration as in the development of the base equations.

The coefficient of velocity distribution for use in the power equation (where the loss of energy per unit time is considered) is C_t, the ratio of the mean of the cubes of the local velocities to the cube of the mean velocity.

The use of the energy or power equation is largely a matter of choice. In either case, coefficients for loss of energy will be determined experimentally by using the equation selected. There are certain considerations that affect the choice of equation. Since the total loss of energy depends on the volume passing any cross section, the most convenient volume to use is Q, the volume per unit time. This leads to the selection of the power equation and (since power is energy per unit time) the frequent designation of the power equation as the energy equation.

There may be two objections to such a procedure. In the first place, even though the energy equation or the power equation is the logical choice for the solution of a given problem, the effect of friction will usually be obtained by the use of the Manning equation or the Chézy equation. Since these are momentum equations and only the energy equation uses the same velocity distribution coefficient, C_s, the energy equation may be more suitable for general use than the power equation. Secondly, the velocity distribution coefficients are usually ignored. Disregarding them in the power equation will introduce about three times the error caused by disregarding them in the energy equation. To avoid confusion it is suggested that the name "Bernoulli equation" be used only in connection with the flow through a single stream tube or through a channel with uniform velocity distribution.

Several discussers have suggested that since the velocity distribution coefficients were small, especially in relation to other unknowns, one should neglect them rather than become concerned over their proper use. For those accustomed to thinking of open channels in terms of canals or similar channels of regular cross section, such a procedure may seem appropriate. The writer believes, however, that the velocity-distribution coefficient will become an important tool in the analysis of flow in natural channels of complex section. There are indications that values of C_s and C_t for such channels can easily be as great as 1.2 and 1.6, respectively, and can vary quite rapidly from section to section. This paper has been presented for the benefit of those engineers who find themselves confronted with situations in which velocity-distribution coefficients must be considered. In closing, the writer desires to thank all those who offered discussions. The critical comments led the writer to make such extensive revision of the paper. He hopes the result will be found sufficiently rigorous and suitable for explaining some of the confusion on the subject.

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MILITARY AIRFIELDS
A SYMPOSIUM

Page

Construction and Design Problems.
By James H. Stratton, M. Am. Soc. C. E. 670

Design of Drainage Facilities.
By Gail A. Hathaway, M. Am. Soc. C. E. 697

Bibliography.
By the Symposium Authors 730

With Discussion by Messrs. James B. Newman, Jr., Thomas E. Stanton, W. E. Howland, David S. Jenkins, William E. Rudolph, Raymond L. Irwin, G. G. Greulich, Hibbert Hill, Jacob Feld, Robert Horonjeff, Clarence S. Jarvis, D. D. McGuire, E. D. Parmer, A. Casagrande, Harry E. Cotton, D. O. Murray, Thomas B. Pringle, John R. Haswell, W. J. Turnbull and William H. Jervis, Ralph R. Proctor, D. M. Burmister, Ralph Hansen, H. V. Pittman, G. R. Schneider, H. M. Williams, Ira B. Mullis, C. F. Izzard, L. J. Mensch, Hibbert Hill, and Gail A. Hathaway.

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CONSTRUCTION AND DESIGN PROBLEMS

By James H. Stratton,[1] M. Am. Soc. C. E.

Synopsis

The procedures in use by the Corps of Engineers, U. S. Army, in the design and construction of airfield pavements, are described in this paper. The design procedures for heavy airplane wheel loads having been in use only since the beginning of the present war construction program, it is too early to forecast the ultimate service behavior of airfields of recent construction. However, to the extent permitted by the pressure of the war construction program and the demands of air traffic, the design and construction procedures have been verified by accelerated traffic tests utilizing heavy, rubber-tired earth-moving equipment. The service performance, under severe operating conditions, of pavements designed and constructed in accordance with these procedures, on the whole, has been extremely gratifying.

Criteria for airfield site selection and for grading and clearance of runways, taxiways, and aprons; investigation and test procedures; and airfield drainage are also discussed.

Loading Criteria for Military Airfields

Military airplane wheel loads adopted by the Corps of Engineers for pavement design are shown in Table 1. Class IV military airfields are exceptional; a wheel loading of 15,000 lb may be considered the minimum military design standard. Loadings greater than 60,000 lb may be anticipated in the near future, but no design provision is being made yet for such loadings.

TABLE 1.—Military Airplane Wheel Loads for Pavement Design

Class	Wheel load (lb)	Contact pressures (lb per sq in.)	Contact area (sq in.)
I	60,000	75	800
II	37,000	65	570
III	15,000	55	273
IV	5,000	45	111

Impact.—That the buoyancy or wing lift of moving planes markedly reduces rolling impact stresses at runway joints and cracks is indicated by the excellent service performance of runways as compared to apron and taxiways of the same design. The lesser effect of wing buoyancy in reducing such stresses on apron and taxiway concrete pavements results from the slower rate of plane travel on these pavements.

In the design of aprons and taxiways, special allowance is made for vibration stresses caused by the warming of airplane engines, and this allowance is considered as providing for rolling impact on these pavements.

Investigations by the Army Air Forces and the Corps of Engineers disclose that landing impact for good two-wheel landings is not severe, whereas one-wheel landings result in high impact stresses. The generally excellent service performance of the parts of runways on which most landings occur indicates that landing impact does not contribute to pavement failures. Hence, stresses due to landing impact are ignored.

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Frequency of Loading.—In general, the loading frequency of airfield runway pavements is considerably less than on highways, due both to inherent traffic limitations of airfields (air saturation) and to the natural traffic distribution that results from the relatively greater widths of runways (150 ft to 300 ft).

Stresses Due to Warming of Airplane Engines.—Airplane static loadings and the loadings of airplanes when engines are being warmed induce higher stresses than those produced by rapidly moving planes. This has been verified by noting the relatively large increases in deflections of flexible pavements imposed by standing airplanes and by standing planes, and slowly moving planes, when their motors are being warmed at high speed (dynamic loading). The dynamic loading is taken into account in designing turnarounds, aprons, hardstandings, and taxiways that are subject to the stresses thus caused.

The dynamic loading is adjusted by increasing the loadings, in Table 1, 25% for the design of turnarounds at the ends of runways, aprons, hardstandings, and taxiways subject to normal frequency of use. This load increase assumes a 25% increase in the wheel load distributed over a tire print area (see Table 1) which is also increased by 25%. Warming engines on runways is exceptional, and dynamic loading is not considered in runway design.

Design of Certain Facilities for Limited Use.—In the interest of economy, a reduction in design requirements is feasible for certain airfield pavements, such as stub-end and peripheral taxiways and the pavements of auxiliary and all-over fields which are used infrequently ("limited operation"). For flexible pavements the combined thickness of base course and the pavement required for the specified load assumption is decreased 20%. For rigid pavements a higher working stress is used. Pavements are not designed for both dynamic loading (warming of airplane engines) and limited operation.

Criteria Governing Selection of Pavement Types.—Service aprons and hardstandings are subject to the high shearing stresses that occur when a plane is turned with locked wheels, and to the action of oil and gasoline dripping and spilling. Since bituminous pavements are unsatisfactory for such facilities, as well as for turnarounds at the ends of runways (which are subject to similar high shearing stresses) concrete generally is used. Concrete is not warranted for hardstandings at temporary airfields. Since reasonable maintenance is practicable and economical, for the most part they are constructed of surface-treated base courses designed for the specified loadings. Ordinarily, stub-end and peripheral taxiways are constructed with surface-treated base courses.

Turfing of Airfields.—Where climatic and soil conditions permit, runway and taxiway shoulders are turfed to control dust and erosion. Such turfed runway and taxiway shoulders will support reasonably high loadings under reasonably favorable ground conditions, and thus will reduce the likelihood of accidents when the shoulders are used inadvertently by planes. Where turfing is impracticable or uneconomical, the shoulders are treated with a dust palliative of a texture and color which distinguishes them from the taxiways and runways, and thus prevents their being mistaken for landing and traffic areas.

CONCRETE PAVING

The design of concrete pavements generally is based on the methods of H. M. Westergaard, M. Am. Soc. C. E. (1)(2)(4)[2] which contemplate loads applied at the corners, edges, and the interior of pavement slabs. The Westergaard analysis omits consideration of temperature variations and other influences that tend to change the slab volume, effect of edge thickening, effect of local hard or soft spots in the subgrade, horizontal components of the reactions of the subgrade, and inertia of pavements and of subgrades. However, these factors have a direct bearing on pavement performance, and to the degree of their importance are considered in the design and construction of airfield rigid pavements.

The design of airfield concrete pavements is an empirical application of the Westergaard formulas for interior slab loadings for large tire imprint. A design factor is selected to provide a pavement sufficiently thick to withstand the design stresses at interior corners and under the conditions of load repetition normal at military airfields. Typical design curves for the specified allowable working stresses and wheel loads are shown in Fig. 1. After the modulus of soil reaction k has been determined, the required thickness of slab, except at free edges and along longitudinal expansion joints, is taken directly from the design curves. The results of accelerated traffic tests using rubber-tired earth-moving equipment with wheel loads and tire imprint areas comparable to those specified for the design of Class I, II, and III airfields, and of load tests on

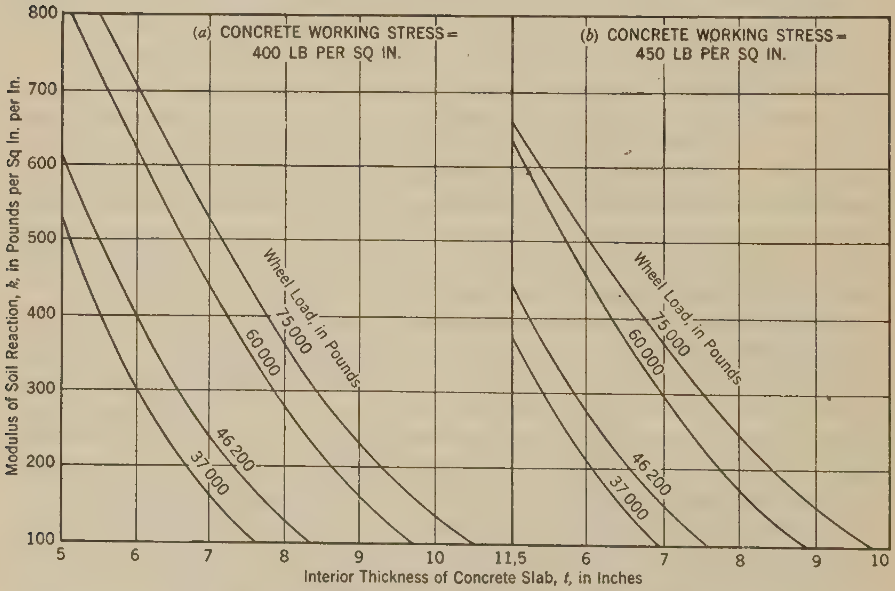

FIG. 1.—DESIGN CURVES FOR CONCRETE PAVEMENTS

concrete slabs of various thicknesses placed on subgrades of various types, were used in selecting the design factors and in checking the design method. Airfield pavement service performance has verified the design method, which is being checked further by additional traffic and plate bearing investigations and by observation of the service behavior of rigid airfield pavements.

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[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography at the end of the Symposium.

Temperature Stresses.—Temperature differences between the upper and lower surfaces of concrete slabs cause warping; the effect of warping varies with the magnitude of the temperature differential. Midday temperature differentials of 24° F in a 6-in. slab, and of 31° F in a 9-in. slab, have been recorded in the vicinity of Washington, D. C. (5). It is probable that greater differentials are experienced elsewhere, as, for example, in the southwestern United States. Temperature differentials and their effects are discussed by Dean Westergaard (6), L. W. Teller, Sr., M. Am. Soc. C. E., and E. C. Sutherland (5), E. F. Kelley (7), and R. D. Bradbury, Assoc. M. Am. Soc. C. E. (8). Mr. Bradbury concludes that, for the purpose of design, a maximum temperature differential of 3° F per inch of slab thickness may be assumed when the top of the slab is warmed by the sun and the bottom is cooled by the subgrade. He suggests a differential of 1° F per inch of slab thickness when, at night, a reverse temperature differential may prevail. The design differentials proposed by Mr. Bradbury are less than those recorded by the Public Roads Administration (5), and probably are considerably less than those that may be expected at some localities.

Actual warping stresses noted by the Public Roads Administration (5), in summer in the vicinity of Washington, D. C., were 320 lb per sq in. for a 6-in. uniform slab, and 416 lb per sq in. for a slab with thickened edge (9 in.-6 in.-9 in.). The investigations of the Public Roads Administration and the analyses by Dean Westergaard indicate that temperature warping stresses may be reduced materially by shortening the slabs. For example, for a 10-ft slab the interior longitudinal stresses are 66%, and the free edge stresses 89%, less than the corresponding stresses in a 20-ft slab.

Excessive warping stresses are avoided by imposing design limitations on slab lengths and by permitting relatively thin slabs for the design loadings when base courses are used to improve subgrade support. This is better practice than to place thick slabs directly on subgrades of low bearing capacity. The relatively low volume of traffic at airfields results in relatively infrequent occurrence of the critical combined stress condition at midday when warping stresses are at a maximum.

Factors Affecting Subgrade Support.—Variations of subgrade reaction as they affect stresses in a slab are regarded by Dean Westergaard (1) and others as not particularly significant. This conclusion is based on values of the subgrade moduli ranging between $k = 50$ and $k = 200$ lb per sq in. per in. The bearing value of a subgrade is influenced by a number of factors, including variations in the soil moisture resulting from fluctuating ground water, seepage through joints, and capillarity—which induces condensation at the underside of the paving slab—and also subgrade deformation due to loading and to temperature warping. Warping of a concrete pavement tends to deform the subgrade, which in turn affects the magnitude and distribution of the stresses in the pavement. Compaction of subgrades and the use of compacted base courses to improve and to develop uniformity of the subgrade support mitigate these adverse influences.

Highway Procedures for Determining the Subgrade Modulus.—In the design of rigid highway pavements it is customary to assume a value for the subgrade modulus generally ranging from $k = 50$ to 200, with $k = 100$ most commonly assumed as representing fair average subgrade conditions. No method of evaluating the subgrade support for concrete pavement design has been devised and generally adopted.

In corner-load testing of concrete slabs on a synthetic subgrade of moist clay, M. G. Spangler, M. Am. Soc. C. E. (9), computed an apparent initial subgrade modulus of $k = 275$ corresponding to a deflection of 0.0232 in. maximum, and an apparent final value of $k = 40$ corresponding to a deflection of 0.0620 in. maximum at 400 to 500 load repetitions. This decrease was the result of deformation of the subgrade under the repetition of load. Although this reduction resulted in a computed increase in slab stress of about 20% to 25%, Mr. Spangler, concurring with Dean Westergaard, concluded that variations of subgrade reaction of this magnitude have relatively little effect upon the stresses in the slab. However, he suggests the need for further investigation. In a discussion of Mr. Spangler's paper, the late Clifford Older, M. Am. Soc. C. E., observes that warping of slabs under field conditions will result in permanent subgrade depression, and that this indicates the fallacy of using stress formulas that are based on the assumption that the subgrade reaction is proportional to slab deflection. Mr. Older concludes that test slabs should be traffic tested to confirm the design assumptions and relationships of load, subgrade reaction, and slab stress.

The stress increases accompanying the decrease of subgrade reaction as noted in the laboratory by Mr. Spangler probably are less than those often observed under field conditions. The result of repeated deflections of a slab, particularly at joints where water has ready access, may be a remolding of a plastic subgrade soil, or a pumping action with a consequent marked loss of support at the locations where slab stresses are most critical. Base courses contribute to the alleviation of conditions destructive to the supporting capacity of a subgrade.

Determination of Subgrade Reaction.—Designs of airfield concrete pavements are based on the modulus of soil reaction k determined by field plate bearing tests. Tests to determine k are conducted prior to design, whenever feasible. The pressure of the war construction program has precluded field bearing observations and other required investigations in some instances and, to estimate the k values, it has been necessary to use, interpretatively, the results of field plate bearing tests at other sites. To represent the subgrade in cut, bearing test areas are stripped of vegetation, scarified, and compacted at optimum moisture to a depth of 6 in. To represent graded fill, test embankments at least 30 in. high are constructed with the same compaction control that is used in the airfield construction. Where low subgrade support is apparent, base courses are placed at selected thicknesses on the compacted test area (both cut and fill) and then evaluated by plate bearing tests. These tests of both subgrade and base course are repeated sufficiently often to give representative values. The results of the plate bearing tests are plotted as stress-deformation curves, and the subgrade modulus k_u (and base-course modulus) at optimum moisture is selected as the slope of a chord passing through the origin and the curve at 0.05-in. deflection. This value of deflection and the size of the bearing plates (30 in. in diameter) were selected after a series of bearing and traffic load tests. The selected deformation tolerance of 0.05 in. for the determination of this modulus may not be a proper one for all types of subgrades nor for base courses placed on subgrades to improve the pavement support.

The optimum moisture condition at which the subgrade and base courses are placed does not represent the moisture condition after the passage of time. To evaluate the subgrade support that might obtain as a result of moisture accumulation, principally in the subgrade, field plate bearing tests were made on subgrades that were saturated after initial testing at optimum moisture. The results of these investigations were not satisfactory, and an alternate procedure has been adopted tentatively to adjust for possible decrease in subgrade support due to moisture accumulation. This consists of utilizing the results of laboratory consolidation tests on subgrade samples compacted at optimum moisture, and on duplicate samples similarly compacted which are saturated by immersion prior to testing to represent probable future moisture accumulation and swelling of the subgrade. The duplicate sample is saturated under a restraining load representing the weight of the overlying pavement. The adjusted value of subgrade modulus is determined by the formula:

$$k = \frac{p_s}{p} k_u \qquad (1)$$

in which p is the load intensity on the 30-in. plate that produces a settlement of 0.05 in.; p_s is the load in pounds per square inch required to consolidate the saturated specimen to the same extent as the load of intensity p when applied to an unsaturated laboratory specimen. The adjustment for possible subgrade saturation takes into account neither the attendant remolding of the subgrade, the effects of cyclical swelling and shrinkage of the subgrade, permanent subgrade deformation resulting from slab warping or load application, nor the effects of frost heaving or thawing.

The interpretation of plate bearing test results requires the exercise of judgment based on experience with accelerated traffic tests and upon observation of pavement service performance for extended periods. Such interpretation requires full information on the subgrades, base courses, and concrete which implies extensive laboratory tests of the subgrade, base-course materials, and the concrete; on such factors as the effects of subgrade restraint and of changes in the subgrade induced by change in moisture content; the amount and effect of temperature warping and of frost action and thawing; and the effect of slab dimensions and of stress transfer devices. Field plate bearing tests are so expensive and time consuming, and require the mobilization of such large amounts of equipment for test embankment construction, compaction, load jacking, and base-course construction that such testing often is precluded.

Base Courses for Concrete Pavements.—When used in pavement designs for Class I and II airfields, wheel loading values of the subgrade moduli commonly assumed in highway practice result in excessively thick pavements (10 in. to 14 in.). Such thick slabs obviously are undesirable because of the high-temperature stresses that would result. Furthermore, experience has shown that concrete slabs more than 8 in. thick are inferior in the lower portion, due to difficulties in vibrating the concrete.

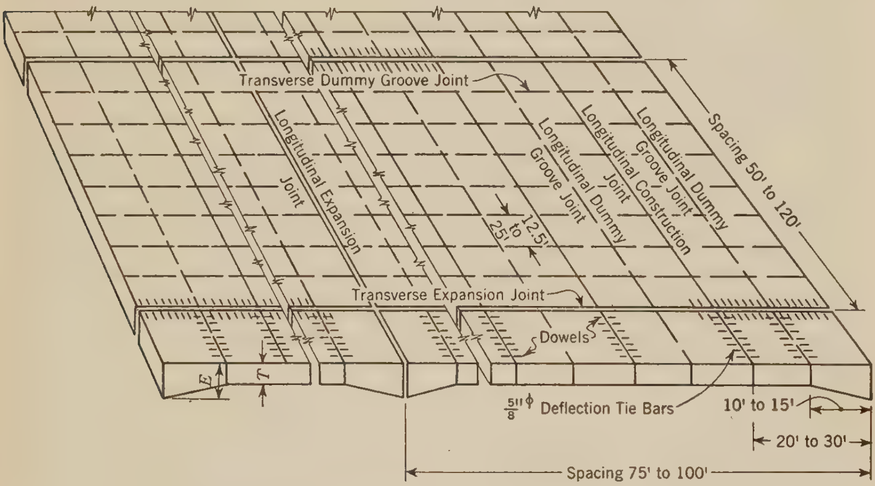
The excessively thick pavements for Class I and II airfields required for subgrades of low bearing value are avoided by the use of base courses to improve the support for the pavements. High-quality base materials are less subject to the adverse influences which readily deteriorate the susceptible low-quality subgrades. They also serve to insulate and protect the underlying subgrades, and thus to preserve, to a large degree, whatever value and uniformity of support the subgrades offer. Recent plate bearing investigations have disclosed that improvement of the pavement support by use of base courses is not always evidenced in the values of k as determined by the plate bearing tests. Accelerated traffic tests and pavement service performance, however, demonstrate the value of base courses under concrete pavements.

Allowable Working Stresses for Concrete Pavements.—For normal operation, the allowable working stress of the concrete is determined by dividing the 28-day flexural strength by a design factor of 1.75. This value was chosen after giving due consideration to warping stresses, effect of loading at joints, concrete fatigue, and increased strength of concrete with age. For limited operation a design factor of 1.40 is used. The results of traffic tests and observations of rigid pavement service performance were considered in selecting these design factors.

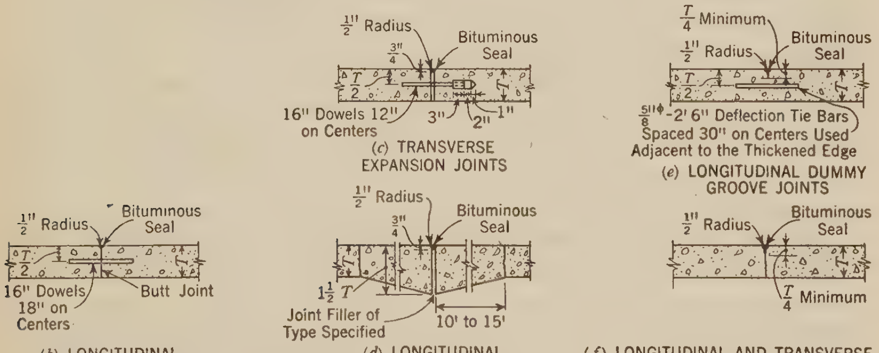
Airfield Concrete Pavement Sections Used by Corps of Engineers.—Standards of the Corps of Engineers for concrete pavement sections are shown in Fig. 2. Edges are thickened only at free sides and at longitudinal expansion joints. Where slabs have thickened edges (joints), the shaping (troughing) for the thickened edges results both in material disturbance to the subgrade or base course and in reduction in base-course thickness, with resulting reduction in the value of the slab-supporting medium. Since pavement slab stresses are critical at edges and corners, it is imperative that the integrity of the slab support at these locations be especially protected. The shaping of base courses is particularly difficult, and it is probable that the benefits to be expected from the use of base courses are not realized when the base material is disturbed to thicken the edges. For this reason, edges are not thickened at interior joints (except at free longitudinal expansion joints), and adequate stress transfer devices are provided.

Stress Transfer Across Joints.—Observations of paving construction have disclosed the difficulties in providing high quality concrete at construction keys. Honeycombing due to lack of consolidation by spading and vibration of the concrete vitiates the functioning of the keys as devices for stress transfer. Messrs. Teller and Sutherland (5) have shown the need for supplemental transfer devices at keyed joints. Their conclusions and the observed poor workmanship at keys have resulted in the decreased use of keys in military airfield pavements.

The widths of pavement strips range generally from 20 ft to 30 ft, the present trend being toward wider lanes. Where strips exceed 18 ft in width, continuous longitudinal dummy joints are run down the center. Tie bars are placed at longitudinal dummy joints in the paving strips adjacent to longitudinal expansion joints and outside pavement edges. The provision of stress transfer devices at other joints than those shown in Fig. 2 is left for local consideration.


Transverse Dummy Groove Joint
Longitudinal Expansion Joint
Longitudinal Groove Joint
Longitudinal Dummy Joint
Longitudinal Construction Joint
Longitudinal Dummy Groove Joint
Spacing 50' to 120'
12.5' to 25'
Transverse Expansion Joint
Dowels
T
$\frac{5}{8}$"ϕ Deflection Tie Bars
10' to 15'
20' to 30'
Spacing 75' to 100'

(*a*) THICKENED OUTSIDE EDGE WITH DOWELED LONGITUDINAL CONSTRUCTION JOINTS


$\frac{1}{2}$" Radius
$\frac{3}{4}$"
Bituminous Seal
T
$\frac{T}{2}$
16" Dowels 12" on Centers
3"
2"
1"
$\frac{T}{4}$ Minimum
$\frac{1}{2}$" Radius
Bituminous Seal
$\frac{5}{8}$"ϕ-2' 6" Deflection Tie Bars Spaced 30" on Centers Used Adjacent to the Thickened Edge
$\frac{1}{2}$" Radius
Bituminous Seal
16" Dowels 18" on Centers
Butt Joint
$\frac{1}{2}$" Radius
$\frac{3}{4}$"
Bituminous Seal
$1\frac{1}{2}$ T
Joint Filler of Type Specified
10' to 15'
$\frac{1}{2}$" Radius
Bituminous Seal
$\frac{T}{4}$ Minimum

(*c*) TRANSVERSE EXPANSION JOINTS

(*e*) LONGITUDINAL DUMMY GROOVE JOINTS

(*b*) LONGITUDINAL AND TRANSVERSE CONSTRUCTION JOINTS

(*d*) LONGITUDINAL EXPANSION JOINTS

(*f*) LONGITUDINAL AND TRANSVERSE DUMMY GROOVE JOINTS (Without Tie Bars)

Fig. 2.—Concrete Pavement Section

Joint Spacing.—The spacing of expansion and contraction joints is largely a matter for local determination. The character of the subgrade and base course (including their restraining influences), the climate (particularly daily temperature range), and the season of construction, as bearing on the question of initial shrinkage, are factors that affect the spacing of expansion joints.

Longitudinal expansion joints are laid out generally at intervals of 75 ft to 100 ft, and for widths of 100 ft or less no longitudinal expansion joints are required. Transverse expansion joints are at intervals of 50 ft to 120 ft depending on the judgment of the district engineer. Transverse expansion joints are equipped with dowels that have been painted, greased, and capped at one end.

Transverse contraction (dummy groove) joints are at intervals of 10 to 25 ft, most pavements having spacings of less than 20 ft. The likelihood of high warping stresses due to temperature differentials has an important bearing on the spacing of transverse joints, and the practice is to use short slabs where the differentials may be high.

Construction of Concrete Pavements.—A well-prepared rigid pavement design may be vitiated by carelessness or improper construction, and emphasis is placed on the quality of subgrade and base-course construction; protection of the prepared subgrade, base courses, and the concrete during the construction; the selection of proper equipment for construction; construction sequences; and adequate curing of the concrete. For thick slabs particularly, there is need for improvement in placement methods. Alternate lane construction of multistrip airfield pavements is conducive to serious disturbance to the subgrade by the operation of construction equipment and by ponding of rain water on the intervening subgrade between completed strips. On the other hand, in successive lane construction serious cracking has resulted from the operation of paving machines mounted at one end on previously placed concrete which has not yet gained sufficient strength to support the paving-machine load. The latter procedure is preferred, however, and the construction schedule is established, where feasible, to permit proper aging of the concrete before imposing the paving-machine loading.

Concrete curing is highly important and construction instructions emphasize the need for protecting the concrete to avoid the adverse effects of temperature warping before strength gain is sufficient to resist warping stresses.

Because of the war shortage, steel reinforcement has been omitted from the rigid pavements of airfields.

Summary of Factors Affecting the Design of Concrete Pavements.—Load, temperature warping, and subgrade reaction are the principal factors affecting the stress condition of concrete pavement slabs. By improving the slab support through the use of base courses it is possible to reduce the theoretical stresses from airplane wheel loadings to values which are commonly accepted in highway practice, and to keep slab thicknesses to dimensions that obviate excessively high-temperature warping stresses.

Subgrades deteriorate and lose their supporting value as a result of moisture accumulation and remolding of the subgrade soil or because of deformation due to loading and to temperature warping. This degradation of the subgrade results in cracking of the overlying pavement. Base courses which improve the support for rigid pavements insulate the underlying subgrades against the adverse influences to which many are highly susceptible. Properly designed base courses are highly resistant to these same influences.

Flexible Pavements

The wide variety of aggregate types suitable for bituminous pavements often permits economies through the use of local materials. The so-called "low type of flexible pavements" is generally unsuitable for military airfields, except for traffic facilities used infrequently. Central hot-plant bituminous concrete construction is preferred for pavements subject to normal use.

Flexible pavement construction for airfields (10) generally requires base courses to distribute stresses to the subgrade. The various formulas in vogue for determining the combined thickness of base course and pavement required for a specified loading generally depend on a rating of the subgrade bearing capacity. The subgrade is usually rated by plate bearing tests.

Bearing plate investigations and accelerated traffic tests using heavy, rubber-tired earth-moving vehicles indicate that flexible pavements on airfields, designed on the basis of subgrades rated at deflections commonly considered acceptable, are not satisfactory for present-day airplane loadings. It has been found infeasible to fix a maximum value for the deflection tolerance that would be applicable in rating subgrades of various soil types for flexible pavement design. Moreover, the effects of moisture change, rate of load application, stress repetition, and frost heaving and thawing cannot be evaluated directly by the plate bearing method of rating subgrades. The effects can be evaluated, if at all, only by empirical correlation of the results of a large number of plate bearing tests with the service performance of pavements on a wide variety of subgrades. These facts led to the consideration of other design methods.

California State Highway Department Method of Design.—The standard method of design adopted by the California State Highway Department (13) has been accepted for the design of flexible pavements at airfields. To extend the experience gained in California to the loadings required for airfield pavements, the design curves for wheel loadings in excess of 12,000 lb, shown in Fig. 3, were prepared by the engineering staff of the Chief of Engineers based upon its design, construction, and operating experience and with the advice and assistance of O. J. Porter and Arthur Casagrande, Members, Am. Soc. C. E. A series of accelerated traffic tests on pavements with varying thicknesses o base courses, founded on several distinctive types of subgrades to represent the range of subgrade soil types which might be encountered, resulted in substantial confirmation of the design curves tentatively established for wheel loadings in excess of 12,000 lb.

As the result of more recent accelerated traffic tests to determine the relationship of the values of the California bearing ratio (CBR) to required combined thicknesses of pavements and base courses, the curves have been revised in the range covering cohesionless subgrades to reduce the required combined thickness of pavements and base courses.

Evaluation of Subgrade.—The CBR of a subgrade soil is its relative stability as determined by an arbitrary penetration test and is expressed as a percentage of the stability value of a selected crushed stone. The penetration values for the crushed stone have been established. The CBR of a subgrade is tested by forcing a circular piston, 3 sq in. in area, into the subgrade soil in a cylindrical test mold at the density and moisture conditions to be expected after the passage of time as a result of moisture accumulation, and noting its load deformation characteristics. The procedure has been fully described by Mr. Porter (11). It is applicable to highway construction methods used in the State of California and to conditions and soils in that state.

Inconsistent test results on certain of the many types of soils encountered in the construction throughout the continental United States, and elsewhere, have made it necessary to modify the California test procedure to insure representative results consistent with the character of the construction. As more tests are performed, it may be necessary to alter further the CBR test procedure.

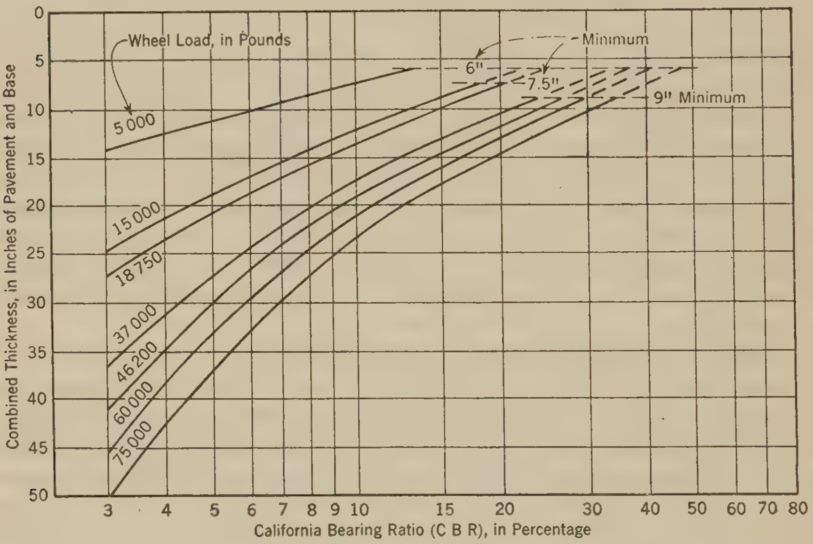


FIG. 3.—CURVES FOR THE TENTATIVE DESIGN OF FLEXIBLE PAVEMENTS (CALIFORNIA METHOD)

Instead of compacting soils in the test mold under a load of 2,000 lb per sq in. (termed static load), as in the California method, it has been found advisable to tamp the soil in the test mold as is done in laboratory testing for density and moisture control for field compaction. The control test for most soils is a modification of the well-known Proctor test. The densities of some soils when compacted by the static load vary as much as 10% from the densities obtained by tamping the soil in the mold. Static loading compaction of some soils results in nonuniform densities or a crust-like surface condition in the upper zone. This is the part of the specimen which is penetrated by the test piston, and the effects influence the CBR test results.

Adoption of a tamper similar to the Proctor tamper, to compact the soil in the CBR test mold, has led to other modifications in the testing procedure and also in the testing equipment. Since the CBR test results are greatly affected by the surface condition of the specimen penetrated, the test mold has been redesigned to facilitate inverting the mold and penetrating the bottom end of the sample, which is relatively smooth, free of crusts, and of relatively uniform density.

For some soils the California method of soaking the CBR test specimen to represent the density and moisture conditions that may obtain in the subgrade after the passage of time results in a very soft, mushy testing surface. This condition is prejudicial to the results of the CBR test and is not representative of the actual subgrade moisture and density conditions. Soils of this type now are soaked by permitting the water to rise from the bottom until expansion ceases, or until free water appears on the surface.

Tests of granular materials in accordance with the original procedure give CBR values which result in excessive base thickness requirements. The weight of the annular ring that encircles the piston when it penetrates the soil sample, and which is used to represent the confining effect of the overlying base and pavement, has been increased to represent the restraining effects of the overlying base and pavement material.

The value of the CBR method of design lies in the correlation with pavement performance by the California Highway Department over a period of thirteen years for the range of loadings common to highway practice, and with the results of accelerated traffic tests conducted by the Corps of Engineers with loadings comparable to those of present-day airplanes (see Table 1).

Incomplete laboratory tests in process suggest the following tentative conclusions:

(*a*) The moisture content of low to medium plastic soils at the time of compaction in the test mold affects the CBR. The CBR decreases with increase of moisture content.

(*b*) A change in density affects the CBR value for most cohesionless and low to medium plastic soils.

(*c*) The magnitude of the penetration surcharge (annular ring) has a marked effect upon the CBR of cohesionless soils, and some effect on low plastic soils.

(*d*) The CBR is increased for some soils if the length of the sample is less than 4 in.

(*e*) Erratic CBR values result from testing soils containing a high percentage of particles greater than $\frac{1}{4}$ in. in size. It is considered necessary to make numerous tests of such soils.

Design for Capacity and Limited Operation.—The required combined thickness of pavement and base course, based on the CBR test of the subgrade soil, is taken from Fig. 3 unless an additional thickness of base course is needed to insulate the subgrade against frost action. The CBR method of design is based on the performance of flexible highway pavements under a wide variety of conditions over a period of years. The combined thickness of base course and pavement requirement, as specified by the design curves for a given loading, provides for the depreciation in the subgrade bearing value by subgrade saturation, subgrade moisture change, stress repetition, and other influences (except frost action) which tend to decrease the stability of the subgrade material. The total combined thickness given by Fig. 3 is used for capacity operation, but for pavements subject to limited operation it is reduced by 20%.

Where field compaction of the subgrade is impracticable or inadvisable, or where unusually weak strata underlie the compacted subgrade layer, the CBR tests are conducted on undisturbed subgrade samples. The effects of saturation on the uncompacted subgrade are determined by duplicate tests on saturated, undisturbed specimens of the subgrade.

Design of Base-Course Layers.—Suitable base courses can be constructed using soils of relatively low CBR in the deeper layers. This practice is often economical if materials with high CBR are costly to process or must be transported from more distant sources. The required combined thickness of base material and pavement above any given base-course layer depends upon the CBR of the base-course layer and may be determined from Fig. 3. If the base course is designed for several layers of material, the economic thickness of each layer is determined. If a material of high bearing value is used for the full depth of base a lesser thickness is required than if lower CBR material is used in the lower portions of the base. If this opinion is verified by further tests, this fact will permit a change in the CBR design curves.

The base course directly under the pavement should be sufficiently stable to withstand the high stresses produced in the zone directly under the wheel of a plane. The required stability depends upon the type and thickness of pavement, the action and effect of a moving or skidding wheel, the type of plane, etc., and cannot be determined by Fig. 3. For highway pavements used by heavy trucks, the base-course material immediately under a bituminous pavement should have a CBR of at least 80%. Preliminary tests disclose that materials of lower CBR directly under airfield pavements would probably give satisfactory performance for airfield traffic conditions. When material with a CBR of 80% is not available locally, and transportation facilities are inadequate for importing suitable material, the requirements for the top 6 in. of base course have been tentatively established as follows:

Gross weight of plane, in pounds	Minimum allowable CBR for upper base material
10,000 to 30,000	50%
30,000 to 74,000	65%
74,000 to 120,000	80%

Recent investigations refute the results of the preliminary tests and indicate that material of less than 80% CBR should not be used directly under pavements when the subgrade soil is plastic.

Pavement Thickness.—The cross section of a bituminous pavement may comprise several courses, depending on the total thickness required, economic use of materials, practical construction limitations, the possibility of stage development of the airfield, etc. Pavement thicknesses are designed with a surface course, or surface and binder courses, to resist displacement and shear under the design wheel loads, giving consideration to the type of bituminous material used. Pavement thicknesses are not less than 1.5 in., for wheel loads up to 15,000 lb, and not less than 3 in., for wheel loads up to 37,000 lb. For wheel loads exceeding 37,000 lb the thicknesses are designed in accordance with the requirements in each specific case, but in general do not exceed 6 in.

Pavement Quality.—A high density in surface courses is required for resistance to water absorption and raveling under traffic, and is achieved by the proper gradation of aggregates, thorough mixing with the bitumen, and thorough compaction to produce a low void ratio. Bituminous surfaces are compacted from 90% to 95% of the theoretical maximum density, depending on the type. Grades of bitumen are selected which will remain soft and pliable and will promote healing of incipient cracks induced by weathering and wear, without benefit of a substantial amount of traffic kneading.

The non-skid requirements for runway surfaces are less important than for highway surfaces, because of the primary difference in type of traffic, alinement, and grades. Pavements with rough surface textures provided by the coarse graded mixtures, which have been widely favored for highways because of their high non-skid properties, are not needed for runways, and are undesirable from the standpoint of excessive tire wear. The reasonably fine-grained, gritty texture produced by coarse sand or small stone chips, gravel, or crushed slag, used as cover aggregate for seal coats, is desirable for runways.

Subgrade and Borrow Investigations

Explorations are made to determine the location of ledge rock, elevation and slope of the water table, and soil profiles, and to provide representative soil samples for testing. The subgrade soil survey is extended to a depth equal to at least twice the equivalent diameter of the tire contact area, but in no case less than the maximum depth of frost penetration or less than 4 ft below the final elevation of grading at each point where grading is required.

Laboratory tests on subgrade soil samples include mechanical analysis and soils classification, Atterberg limits, moisture content, natural density, CBR, expansion, permeability, and compaction control. Shear strength and consolidation tests are made if the pavement is to be supported on embankment fill. Soils are classified in accordance with the Casagrande Soil Classification (12). Local classification often is used to supplement this general classification, where it is necessary to describe the soil adequately.

Compaction Tests and Control

Compaction of soils is controlled, wherever applicable, by the AASHO Method (American Association of State Highway Officials) modified as follows to give densities comparable to those obtained by specified field compaction equipment: The weight of the rammer or metal tamper is 10 lb instead of 5.5 lb; the tamper is dropped from a height of 18 in. above the sample instead of 12 in.; and the samples are compacted in five equal (approximately 1-in.) layers instead of three layers. The densities of some cohesionless sands and gravels as obtained by the Modified AASHO Method often are not representative of obtainable field densities, and the control specimens of such soils usually are compacted by tamping, static pressure, or vibration alone or in combination.

Fill is constructed in 9-in. layers, loose measurement, and compacted to at least 90% of the maximum unit weight at optimum moisture, except that directly beneath the base course or pavement where a base course is not provided, the subgrade is compacted to at least 95% of the maximum unit weight at optimum moisture, for a depth of not less than 6 in. In cut areas most subgrade soils are scarified and recompacted so the unit weight of the top 6 in. is at least 95% of the maximum unit weight at optimum moisture. Scarifying, reworking, and compacting certain types of clay soils often result in lowering the bearing value below that of the natural clay in place. For this type of material, tests are made to determine the relative bearing values of the recompacted soil and of the undisturbed soil; and, if the undisturbed soil has the higher bearing value, construction operations are conducted to produce the least possible disturbance of the subgrade.

Treatment of Soils with Expansive Characteristics

Natural subgrades with expansive characteristics are compacted at a moisture content, not necessarily the optimum moisture, to a unit weight which will minimize expansion. The selected moisture content and unit weight are approximately equal to those of an expanded sample after the soil has been compacted at optimum moisture and saturated.

Pavement Design for Frost Action

The stability of some soils is greatly reduced by frost action, the loss of stability and extent of heave depending upon the type of soil and the drainage conditions. Several investigators have established limiting grain-size curves defining the boundary between frost-action and nonfrost-action soils for uniform grading. The most generally accepted method of insuring pavement or base-course stability over a potential frost-action soil is to provide sufficient thickness of insulating material. Where the required thickness of base-course material as determined by CBR is not sufficient to preclude frost penetration in the subgrade, additional thickness of insulating material often is required. In general, to insure stability of flexible pavements and surface-treated bases, the combined thickness of pavement and base material not subject to frost action is made equal to the average depth of frost penetration as determined from local records.

Base Courses

Wide varieties of gravels, base-course materials, sands, and gravelly and sandy soils as well as crushed rock, shell, caliche, etc., are used alone or blended to provide base courses. Processing by crushing and screening often is required to produce the required grading.

Where crushed gravel or stone is used for base courses directly under concrete pavements, a reasonably smooth top surface of base course is provided to reduce restraint to the pavement. Base-course materials sometimes are stabilized with commercial admixtures such as portland cement, cut-back asphalts, emulsified asphalts, and tars. Such admixtures are most advantageous with relatively sandy or friable soils.

All base courses are compacted to the maximum density obtainable with heavy field compaction equipment. If the Modified AASHO Method is applicable, the base course is compacted to at least 95% of the maximum density at optimum moisture content.

Special Investigation of Airfield Pavements

Various tests and investigations have been made by the Corps of Engineers for the purpose of improving design and construction methods. Procedures for the investigations and tests were devised to determine conclusions which might be applied to the current construction program. The investigations fall into three categories: Laboratory tests, field bearing tests, and accelerated traffic tests using rubber-tired earth-hauling equipment with wheel weights comparable to the airplane wheel loadings given in Table 1.

Laboratory Investigations.—The special laboratory investigations comprised all the routine tests ordinarily made for site selection and pavement design. The extensive use of the CBR test soon disclosed that the results are influenced by sample density; variation in density of the sample; moisture condition of sample; method of saturating sample to represent moisture conditions that may result in the subgrade after the passage of time; weight of the annular surcharge used to represent the confining effects of the overlying pavement and base course; and rate of penetration. Although considerable laboratory investigational work has been done to adjust the CBR testing procedure to permit proper evaluation of these variables, the results have not been fully correlated with the service performance of flexible pavements.

Field Bearing Test.—A large number of plate bearing tests were made to develop the design method for flexible pavements and to develop a satisfactory procedure for determining the modulus of subgrade reaction which could be used in the design of concrete pavements.

The investigation of design methods for flexible pavements included plate bearing tests on subgrades, pavements, special test sections, and on actual highways in use for several years. It was concluded that the supporting power of subgrades could not be evaluated by plate bearing tests which, for their interpretation, predicate a universal value of allowable deformation for all types of soil. Various values of allowable deformation, ranging from 0.1 in. to 0.5 in., have been proposed by different authorities for rating the unit bearing capacities of subgrades by plate bearing tests.

A study was made to determine if subgrades could be rated by determining in each test the point at which the rate of deformation is accelerated under load application. This would correspond to the yield point on the stress-deformation curve. Fig. 4 is a typical stress-deformation curve obtained by a plate bearing test. The yield point, as can be seen by the curve, is not definite, but can be assumed to be at the point indicated. For this particular test, a bearing value between 15.5 lb per sq in. and 40 lb per sq in. will result if the usual assumptions as to allowable deformation are made.

These studies indicate that the bearing value probably could be selected at some value for the deflection (deformation) less than that at the yield point. However, various factors, including moisture condition and density of the subgrade soil at the time of testing, and the effects of moisture change, stress repetition, and other influences on the behavior of each subgrade under in-

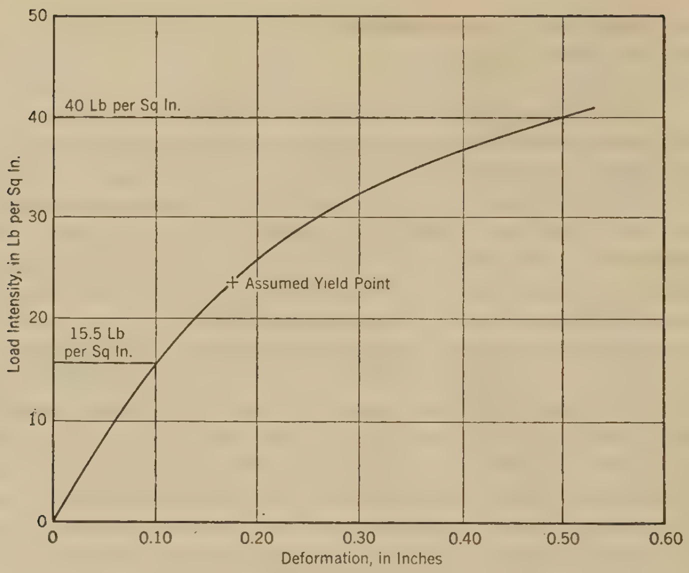


FIG. 4.—LOAD-DEFORMATION CURVE OF A PLATE 30 IN. IN DIAMETER PLACED ON THE SUBGRADE (LOADS APPLIED IN INCREMENTS OF 5 LB PER SQ IN. AND ALLOWED TO ACT UNTIL DEFORMATION CEASED)

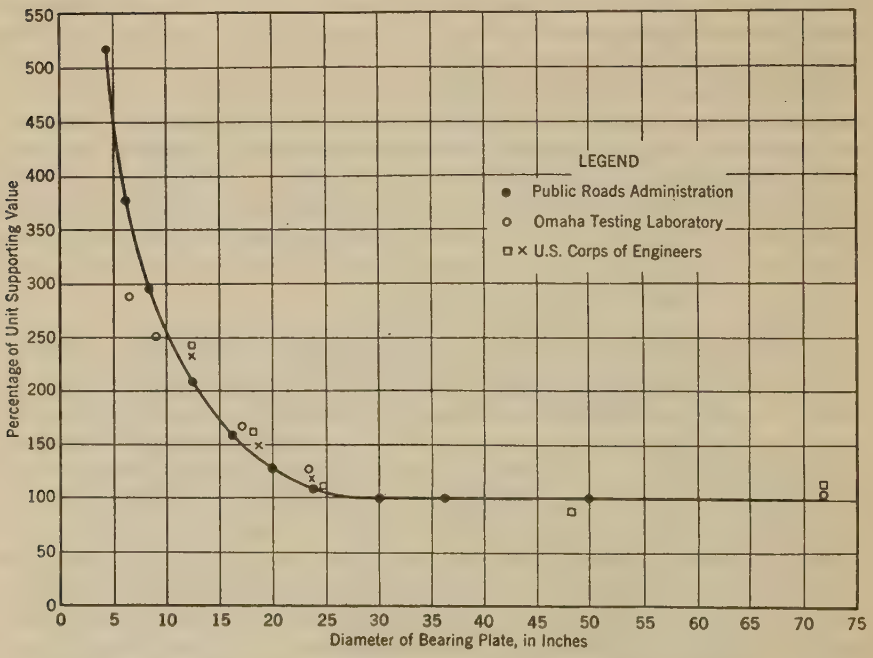


FIG. 5.—RELATION BETWEEN BEARING PLATE DIAMETER AND SUPPORTING VALUE OF SOIL

vestigation, present complications in the selection of a value for the allowable deformation which, in each instance, would represent an evaluation of these influences on the supporting capacity of the subgrade. Small promise of a satisfactory solution was offered by a general investigation of the plate bearing method of rating subgrades with the view of establishing a satisfactory design procedure. Moreover, the pressure of the war construction program made such an investigation infeasible.

Plate bearing tests were made to determine the relation of the area of the plate to the bearing value of the subgrade (see Fig. 5). The results agree closely with those obtained by other investigators and show that, for all practical purposes, the bearing value is the same for all plates with a diameter greater than 30 in. To eliminate the effect of area, a circular plate 30 in. in diameter was selected to determine the modulus of soil reaction for the design of concrete pavement.

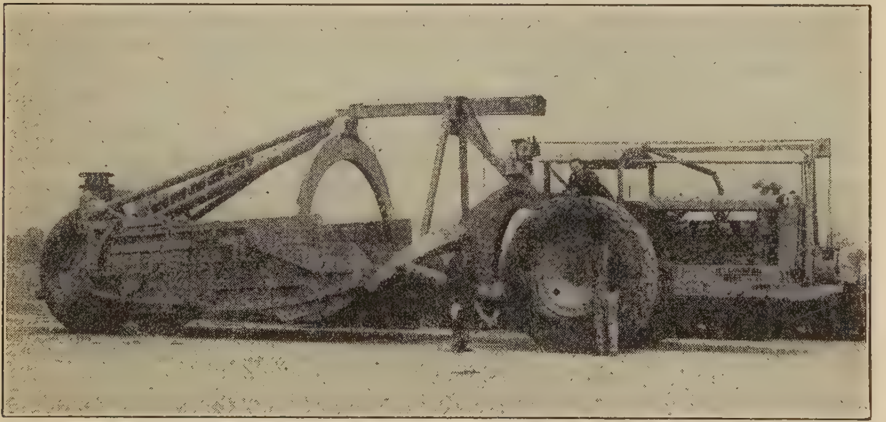


Fig. 6.—Equipment for Accelerated Traffic Tests with Wheel Loads of 37,000 Lb to 60,000 Lb

Accelerated Traffic Tests.—Fig. 6 is a picture of the rubber-tired earth-hauling equipment which is being used to produce traffic wheel loads from 37,000 lb to 60,000 lb. Smaller equipment similar to this is used to produce loads of 15,000 lb to 30,000 lb. Fortunately, the tire imprint of such equipment is practically equal to that of airplane tires carrying similar loads. This type of equipment is used in making accelerated traffic tests of specially constructed test sections of both flexible and concrete pavements and also of actual airport pavements.

The severity of the accelerated traffic test with respect to fatigue stresses in the concrete pavements, and the rapidity of the repetitions of stress in the tests on both rigid and flexible pavements, as compared to the frequency of their occurrence under service conditions, must be weighed in evaluating the test results.

Fig. 7 is a cross section of an embankment constructed for testing CBR design procedures for flexible pavements. This type of construction permits economies and insures a uniform subgrade condition. On such a test section deflection measurements were made with planes standing, with planes moving at slow rates with engines turning at high rates, and with planes standing with engines revolving at high rates to represent the condition of engine warming. For the results of testing one such flexible pavement embankment, reference is made to the Stockton Test Report (3).

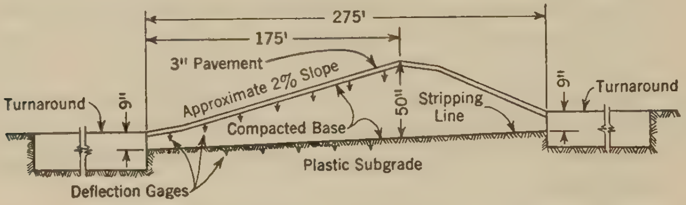


Fig. 7.—Longitudinal Section of Embarkment to Test Flexible Pavement

In the accelerated traffic test the selection of the number of wheel coverages to represent the action of airplane traffic for the full life of a pavement must be based on judgment. A minimum of 3,000 to 5,000 coverages with the design wheel loading over every part of a test strip 5 ft to 8 ft wide on flexible pavements and a minimum of 10 ft wide on concrete pavements have been selected tentatively as representing the use which the airfield pavement may be expected to receive in the course of its normal useful life.

Reinforcement of Existing Airfield Pavements

The increasing number of heavier planes has created a demand for airfields of greater load capacity. In some instances this demand has required reinforcement of existing pavements to take heavier loads than were contemplated in the original designs. Accelerated traffic tests have been an invaluable aid in evaluating the capabilities of such pavements.

Existing pavements are reinforced and pavements that have failed are rebuilt usually by overlay construction, which involves the use of bituminous pavements, or concrete on concrete or bituminous pavements. Where concrete is laid on concrete, an intermediate slippage course of $\frac{3}{4}$ in. to 2 in. of bituminous stabilized sand is used. The joints and joint spacing of the overlay concrete pavement are made to correspond, as far as practicable, with the joints of the concrete pavement being reinforced.

Drainage of Airfields

There has been a gradual development of airfield drainage from the era of the early turfed field relying entirely on the existing natural drainage features, through the period of combination storm and subsurface drains in stone-filled trenches along runway pavements, to the present use of separate storm and subsurface drainage systems with the surface runoff modulated by controlled ponding. These changes in design have been necessitated largely by the exacting requirements of military operations as described by Mr. Hathaway in the companion paper of this Symposium.

Surface Drainage Systems.—The storm drainage system must be adequate in design and areal extent to remove a selected design storm without damage to airfield facilities, undue saturation of the subsoil, or significant interruption of normal traffic. Furthermore, the drainage must be so designed that storms greater than the design storm will not cause undue damage or serious interruption of traffic.

A drainage system plan for a typical airfield is shown in Fig. 8(*a*). For economy in drainage installation there should be a proper relationship between the grading plans and the drainage design to obtain the maximum use of temporary storage of storm runoff.

Depending on the functional requirements of the airfield, storm drainage systems usually are designed to remove runoff from rainfall having a certain average frequency of occurrence. The infiltration capacity estimated to be characteristic of a given soil following a rainfall of one hour serves as the most convenient index to the probable volume of loss through infiltration during the design storm.

Subsurface Drainage Systems.—Subsurface drainage frequently can be eliminated by providing an adequate and properly designed surface drainage system. However, where the ground water is above the frost line or causes soft, boggy conditions in areas where soil stability is required, subsurface drainage is necessary. Two principal types of subsurface drain installations are encountered, the more frequent being a single line of drains adjacent to either side of the runway pavement and so constructed as to drain the pavement base courses and subgrade (see Fig. 8(*b*)). The second type of installation consists of a grid system of drains, not unlike a farm tile drainage system.

The design of a subsurface drainage system is a very approximate procedure based largely on the judgment of the engineer.

Failures in the operation of combined storm and subsurface drainage systems have led to the general requirement that the two systems be so designed that surface runoff cannot backflow into the subsurface drains and result in saturation of pavement base courses and subgrades.

Collectors on pavements are used only to handle the melt of heavy accumulations of snow stacked at the edges of runways. The design of such a collector system is shown in Fig. 8(*c*), the connection being made directly to the storm drains along the edge of the landing strip, either at a manhole or by a special tee in the line. It is imperative that this type of collection system shall not contribute to the moisture accumulation in the subgrade and base course.

SELECTION AND SITE PLANNING OF AIRFIELDS

Airfield sites are selected by the Engineer Department in close coordination with representatives of the Army Air Forces. The requirements of the Air Forces which influence selection and planning of the airfield embrace: (1) Type of aircraft, including wheel load and other characteristics affecting the design of runways, the size of parking areas, hangars, etc.; and (2) type of unit to be accommodated (bomber, fighter, or other) whose different operating procedures demand different facilities. Upon these items are based the number, direction, and dimensions of runways, taxiways, parking aprons, hardstandings, and other

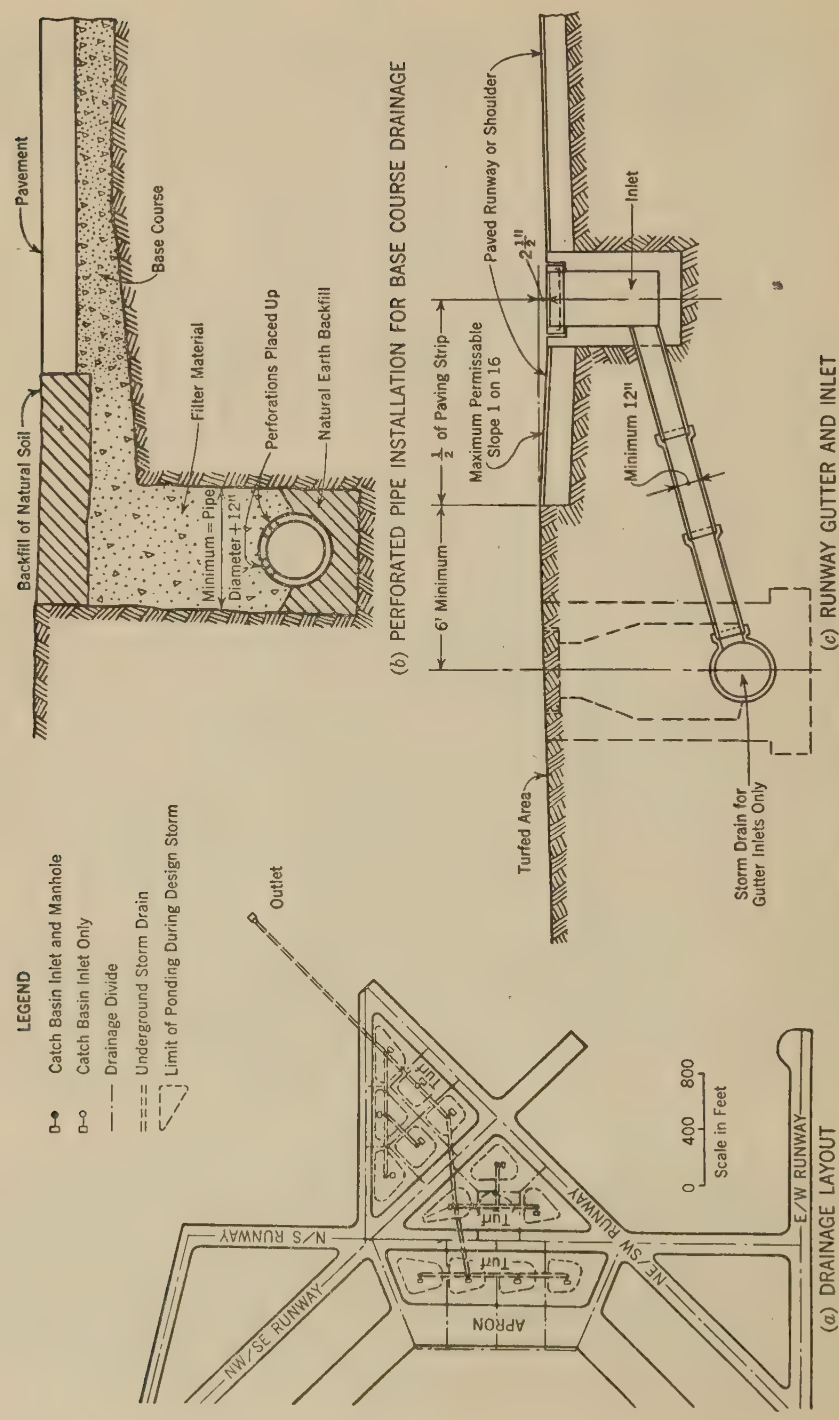

Pavement
Base Course
Backfill of Natural Soil
Filter Material
Minimum = Pipe Diameter + 12"
Perforations Placed Up
Natural Earth Backfill
(b) PERFORATED PIPE INSTALLATION FOR BASE COURSE DRAINAGE
Paved Runway or Shoulder
2½"
Inlet
6' Minimum
½ of Paving Strip
Maximum Permissable Slope 1 on 16
Minimum 12"
Turfed Area
Storm Drain for Gutter Inlets Only
(c) RUNWAY GUTTER AND INLET
LEGEND
Catch Basin Inlet and Manhole
Catch Basin Inlet Only
Drainage Divide
Underground Storm Drain
Limit of Ponding During Design Storm
Outlet
Turf
Turf
Turf
N/S RUNWAY
NW/SE RUNWAY
NE/SW RUNWAY
E/W RUNWAY
APRON
0 400 800
Scale in Feet
(a) DRAINAGE LAYOUT

Fig. 8.—Primary Storm Drain System for an Airfield

facilities. These, in turn, determine the area of the airfield, which normally varies between 1,000 and 1,500 acres for non-dispersed layouts, and proportionally more for dispersed fields. To accommodate constantly increasing demands of new equipment, all new stations are planned with a view to expansion of housing and airplane landing and parking facilities.

Two other main factors influencing site selection and design of airfield layouts are the physical condition of the site and provisions for safety. Crosswind velocities as great as ten miles per hour do not seriously affect the landing or taking off of military aircraft, but winds of greater velocity present serious problems unless the aircraft can be landed or taken off within approximately 22½° of the wind. For this reason, when laying out runways it is necessary to obtain accurate wind information and to interpret it correctly. Although flat terrain is ideal for aircraft operations, it often presents drainage problems that can be solved only by a compromise between the flatness required for aircraft operation and the slopes required for proper drainage. This must be accomplished with a minimum of grading to conserve time, cost, and use of critical equipment. In the choice of a site and in the design of the airfield, the topography of surrounding areas is considered with a view to permitting clear approaches to all runways and future expansion of initial facilities without excessive additional cost of grading or land acquisition.

Safety Provisions.—Safety is provided in: (1) Approach zones, (2) clear zones, (3) lateral clearances from runways, taxiways, aprons, and hardstandings, and (4) building restrictions (see Fig. 9). Approach zones for runways and other prepared landing strips are trapezoidal areas two miles long, 1,500 ft wide at the inner end, 4,000 ft wide at the other end, and symmetrical about the extended axis of the runway or landing strip. Within these limits all obstacles are cleared which would project above a plane sloping upward at the rate of 40 horizontal to 1 vertical, except at certain types of schools and auxiliary fields where a 1 on 30 plane applies. Wherever a clear zone is required, the approach zone begins at its outer limit; otherwise, it begins at the end of the runway or landing strip.

Fundamental regulations for the design of roadways, clear zones, and approach zones are shown diagrammatically in Fig. 10. Except at auxiliary fields and at certain special-purpose airfields, a clear zone usually is required at either end of the runway or landing strip, and extends 1,000 ft in the direction of the center line of the runway or landing strip prolonged.

Lateral clearances from runway center lines, taxiways, aprons, and hardstandings to movable objects (aircraft, vehicles, railway cars, etc.) and to fixed obstacles (buildings, trees, rocks, and terrain irregularities) are prescribed (see Fig. 10) to assure the elimination of all objects that may constitute hazards to the safety of aircraft. Buildings are not permitted between the ends of runways and the boundary of the reservation in the area extending 750 ft each side of the runway center line prolonged, even though such buildings would not project above the approach plane of 1 on 40.

Layout, Dimensions, and Grades of Runways.—Where practicable, runways are laid out to make either 90° or 45° angles with one another, permitting landings and take-offs within 22½° of any prevailing wind. Runways usually are oriented with the eight principal directions, but this may be varied to decrease the amount of grading required, to permit clear approach, or to aline runways in directions best suited to future extension.

Current prescribed dimensions for runways call for 7,000-ft length at sea level, of which only 5,000 ft is initially graded and paved, the remaining 1,000 ft at either end constituting the clear zone. The normal width is 500 ft of graded area, of which the center 150-ft strip is paved. Sometimes, 75-ft shoulders on either side of the paved strip are authorized. For every 1,000 ft above sea level, runway length is increased 500 ft.

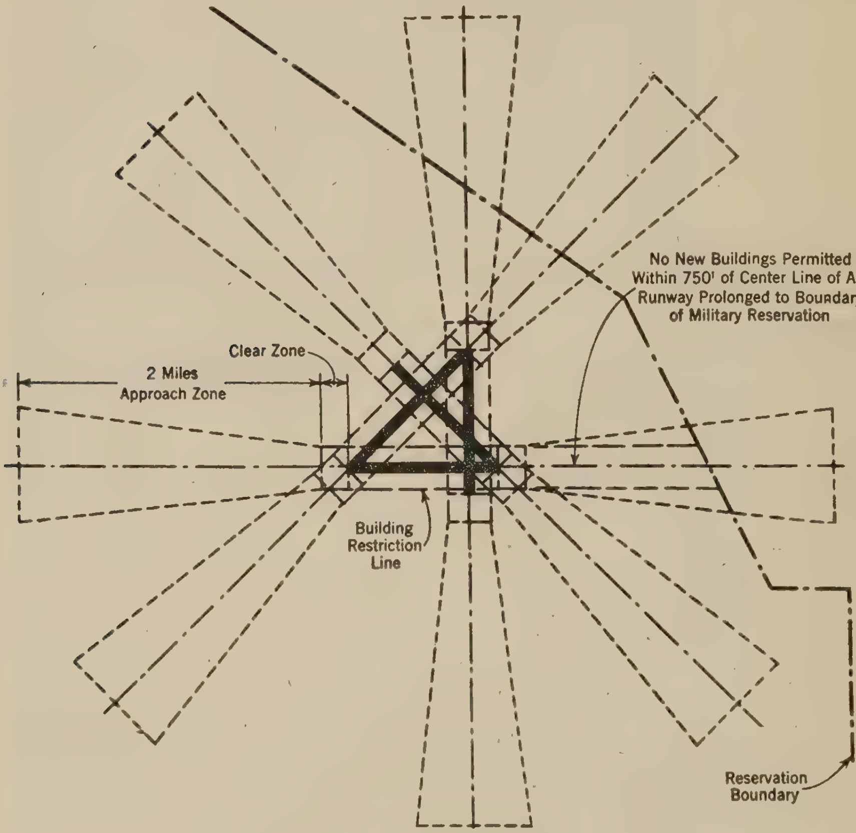
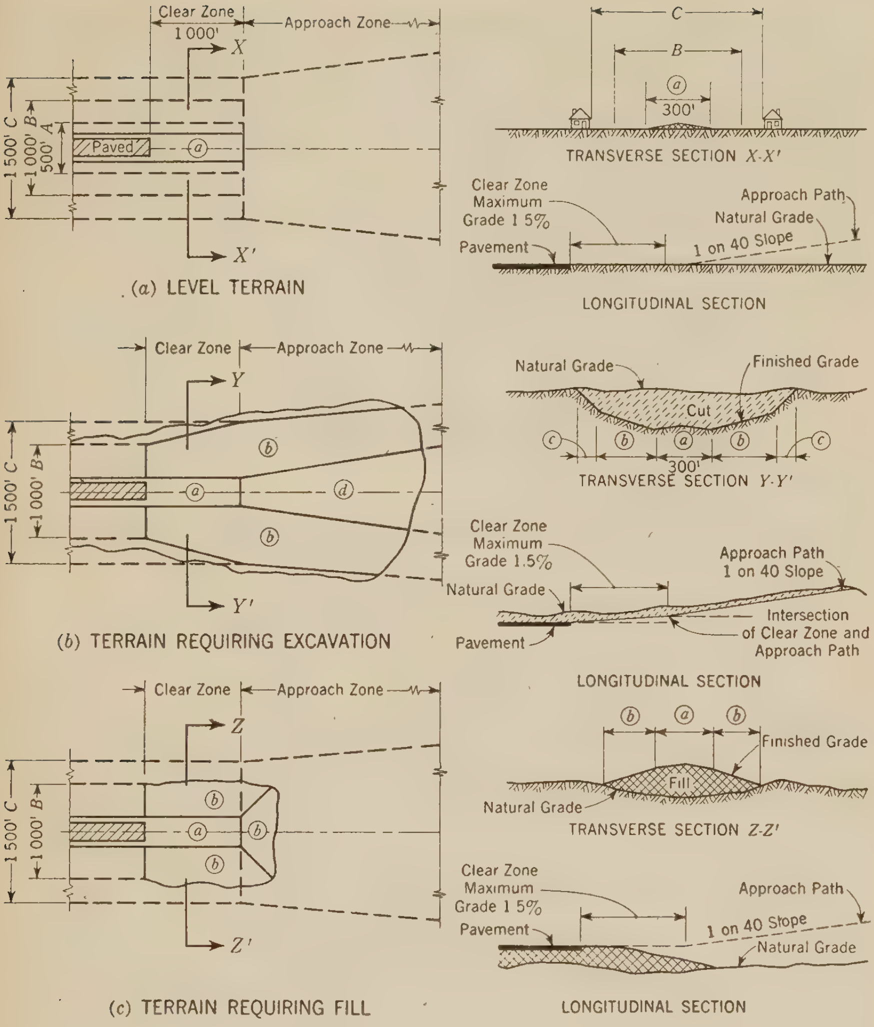

No New Buildings Permitted Within 750' of Center Line of Any Runway Prolonged to Boundary of Military Reservation
Clear Zone
2 Miles Approach Zone
Building Restriction Line
Reservation Boundary

Fig. 9.—Safety Provisions in Airfield Design

Maximum allowable grades are as follows: For runways, 1.5% longitudinal and transverse; for turfed all-over fields, 3% in any direction (except where the field is included within a runway field, when 1.5% in any direction applies); and, for surfaced all-over fields, 1.5%. When runways are designated for 120,000-lb gross load, the maximum allowable longitudinal grade is 1%, although 0.5% is preferred.



Clear Zone
1 000'
Approach Zone
X
X'
1 500' C
1 000' B
500' A
Paved
ⓐ
C
B
300'
TRANSVERSE SECTION X-X'
Clear Zone Maximum Grade 1 5%
Pavement
Approach Path
Natural Grade
1 on 40 Slope
LONGITUDINAL SECTION
Y
Y'
ⓑ
ⓓ
Natural Grade
Finished Grade
Cut
ⓒ
TRANSVERSE SECTION Y-Y'
Clear Zone Maximum Grade 1.5%
Approach Path 1 on 40 Slope
Intersection of Clear Zone and Approach Path
Z
Z'
Fill
TRANSVERSE SECTION Z-Z'
Clear Zone Maximum Grade 1 5%

(a) LEVEL TERRAIN

(b) TERRAIN REQUIRING EXCAVATION

(c) TERRAIN REQUIRING FILL

SAFETY PROVISIONS

A 500' Strip – Graded Area

B 1 000' Strip, (Ground Loop Area) No Obstacles of Any Kind.

C 1 500' Strip, (Safety Clearance Zone) No Fixed Obstacles (Buildings, Trees, Rocks or Terrain Irregularities) Which Constitute Hazards to Safe Operation of Aircraft.

No Obstacles, Natural or Artificial, High Enough to Protrude Into the Approach Path, Permitted in the Approach Zone

GRADING REQUIREMENTS

ⓐ Smooth Graded – Compacted and Turfed. Maximum Longitudinal Grade 1.5% Maximum Transverse Grade 1 5%

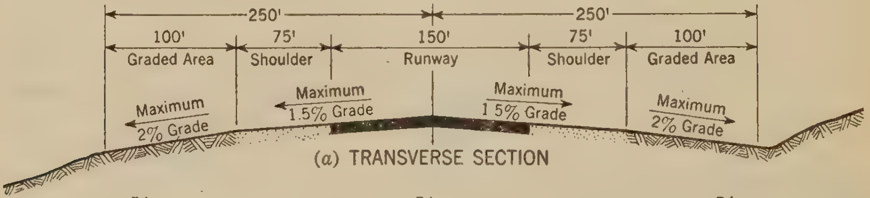
ⓑ Rough Graded – Maximum Longitudinal Grade 1 5% Maximum Transverse Grade 10%

ⓒ Rough Graded – (Where Necessary) to Provide Slope of Natural Repose

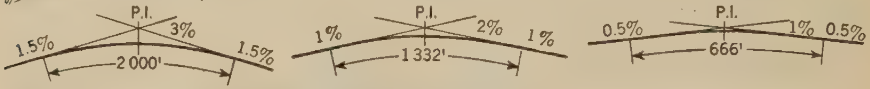
ⓓ Rough Graded – To Slope of Approach Path. Rough Grade All Excavated Areas in the Approach Zone

FIG. 10.—REGULATIONS FOR THE DESIGN OF RUNWAYS, CLEAR ZONES, AND APPROACH ZONES

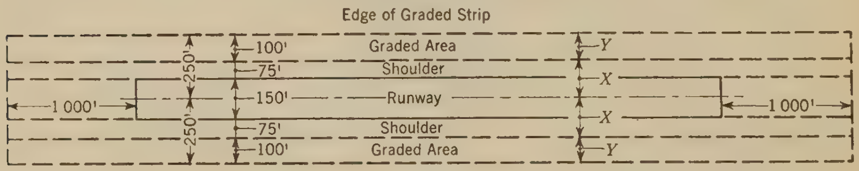
Grade change tolerances are illustrated in Fig. 11. The minimum distance between points of intersection (P.I.) at grade changes is 1,000 ft. Parts of a runway having different longitudinal grades are joined by a vertical curve at least 666 ft long for each per cent of the algebraic difference between the two grades. This relationship is maintained for a distance of at least 150 ft from the center line of the runway. Runways are graded for unobstructed visibility between any two points 10 ft above the surface of the runway. If the runway is longer than 7,000 ft, clear visibility is maintained between any two points 10 ft above the surface and 7,000 ft apart.


250'
250'
100'
Graded Area
75'
Shoulder
150'
Runway
75'
Shoulder
100'
Graded Area
Maximum
2% Grade
Maximum
1.5% Grade
Maximum
1.5% Grade
Maximum
2% Grade

(*a*) TRANSVERSE SECTION


P.I.
1.5%
3%
1.5%
2 000'
P.I.
1%
2%
1%
1 332'
P.I.
0.5%
1%
0.5%
666'

(*b*) LONGITUDINAL SECTIONS
SHOWING THREE EXAMPLES OF PERMISSIBLE GRADE CHANGES


Edge of Graded Strip
250'
250'
100'
75'
150'
75'
100'
Graded Area
Shoulder
Runway
Shoulder
Graded Area
1 000'
Y
X
X
Y
1 000'

(*c*)

Maximum Longitudinal Grade in Area X or $Y = 1.5\%$ (1% for 120 000 Lb Wheel Load)
Maximum Transverse Grade in Area $Y = 2\%$; in Area $X = 1.5\%$

Longitudinal Grade Changes in Area X { Minimum Distance Between P.I's. = 1 000'
Minimum Vertical Curve = 666' per Each 1% Algebraic Difference

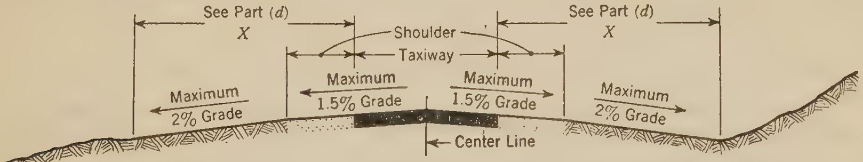
Longitudinal Grade Changes in Area Y at Edge of Graded Strip { Maximum Algebraic Difference = 2%
Minimum Curve = 300' (Minimum Distance Between P.I's. = 300'

FIG. 11.—PERMISSIBLE GRADE CHANGES FOR RUNWAYS

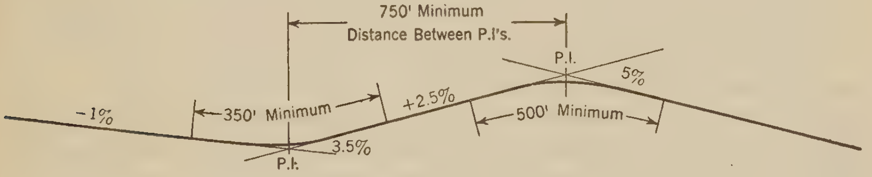
Airplane Parking Areas.—Although frequently it is feasible to park aircraft on turf, the wear resulting from traffic, propeller blasts, and spillage of oil and gasoline, often renders the turf unusable, and paved parking spaces are necessary. Parking spaces usually are large aprons in the vicinity of maintenance and repair facilities; however, individual hardstandings sometimes are provided in lieu of a center parking apron.

Where conditions permit, the parking apron is placed parallel to the runway upon which the greatest volume of traffic is anticipated, thus shortening the amount of taxiing required by aircraft and permitting full observation of traffic from the control tower. The maximum allowable grade on aprons is 1.5% in any direction. At hardstandings, a small servicing apron is constructed adjacent to the maintenance and repair buildings. Size of parking apron is based upon the number and type of aircraft to be parked on it.

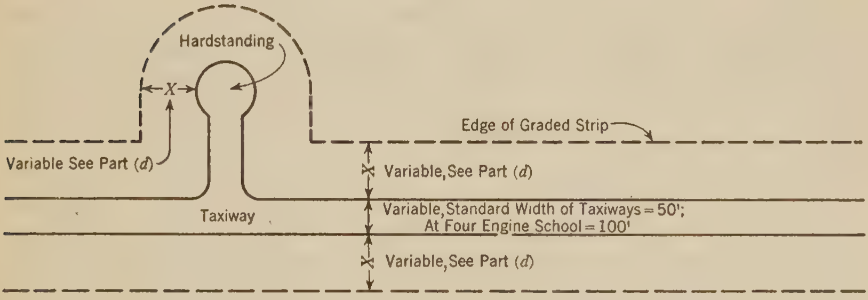
Taxiways.—Ordinarily, taxiways are laid out in straight lines, with angle turns not sharper than 90°, and avoid crossing runways or other taxiways. The standard width for taxiways is 50 ft, with a 12.5-ft shoulder on each side.


See Part (d)
X
Shoulder
Taxiway
See Part (d)
X
Maximum
2% Grade
Maximum
1.5% Grade
Maximum
1.5% Grade
Maximum
2% Grade
Center Line

(*a*) TRANSVERSE SECTION


750' Minimum
Distance Between P.I's.
P.I.
5%
-1%
350' Minimum
+2.5%
500' Minimum
3.5%
P.I.

(*b*) LONGITUDINAL SECTION
SHOWING TWO EXAMPLES OF PERMISSIBLE GRADE CHANGES


Hardstanding
X
Variable See Part (d)
Edge of Graded Strip
Variable, See Part (d)
Taxiway
Variable, Standard Width of Taxiways = 50'; At Four Engine School = 100'
Variable, See Part (d)

(*c*)

Length of Vertical Curve		Maximum Grades		Gross weight of plane (lb)	X (ft)
%[a]	Length[b]	Description	%		
		Algebraic Difference—			
1	300 ft	Planes, 120,000 lb	3.5		
2	300 ft	Planes, 74,000 lb[c]	5.0	30,000	40
3	300 ft	Grades for Taxiways—		74,000	60
4	400 ft	Longitudinal	2.5	120,000	80
5	500 ft	Transverse	1.5		

[a] Algebraic differences between percentage grades. [b] Length of vertical curve required; minimum length, 300 ft. [c] 74,000 lb or less.

(*d*) Standard Dimensions

Fig. 12.—Permissible Grade Changes for Taxiways

Maximum allowable grades are 2.5% longitudinal and 1.5% transverse. Portions of taxiways having different grades are connected by vertical curves having a minimum length of 100 ft for every per cent of algebraic difference between the grades. Vertical curves of less than 300 ft are not permitted under any circumstances, and the minimum distance between points of grade intersections is not less than 750 ft. Examples of permissible grade changes are shown in Fig. 12.

SUMMARY

The trend toward heavier wheel airplane loadings has led to a reexamination of the methods used in the design of airfield pavements. As a result of this reexamination it has become apparent that testing procedures for the evaluation of subgrades and for the economical use of base courses and pavements must be conducted under carefully controlled conditions if efficient and economical designs are to be attained.

Good designs may be vitiated by poor construction and high quality construction control (inspection) is essential.

Accelerated field traffic tests and service performance of airfields have verified the methods of design and construction described. Continuing observations of pavement performance may lead to further revisions and refinements in the design and construction procedures.

Proper design and construction of drainage facilities are essential both to effective pavement performance and the safe use of airfields.

Site layouts must be based on uniform standards with respect to grades and clearances. These standards have been progressively raised as size and landing speeds of airplanes have increased.

DESIGN OF DRAINAGE FACILITIES

By Gail A. Hathaway,[3] M. Am. Soc. C. E.

Synopsis

The primary purpose of this paper is to suggest procedures for the design of drainage facilities for airfields utilizing surface pondage for temporary storage of runoff from high intensity storms of short duration. Diagrams have been developed by means of which relationships can be determined between drain-inlet capacities, maximum storage volumes, and the duration of rainfall, for the frequency curve selected, that will produce the maximum rate of runoff for the specific area involved. Standard rainfall intensity-duration curves applicable to the continental area of the United States were derived to represent average rates of rainfall for various durations that have the same average frequency of occurrence. The relationship of overland flow and peak runoff rates to various standard rainfall intensity-duration curves is demonstrated and a typical layout and sample computations for the drainage system of an airfield have been included.

General

A drainage system for an airfield normally serves three functions: Interception and diversion of surface and ground-water flow originating from lands adjacent to the airfield area; removal of surface runoff from the airfield; and removal of ground-water flow (gravitational water) from the airfield area. The respective facilities may be combined or separate depending upon local conditions.

To assure stable soil conditions, subsurface drainage of an airfield is essential in many cases. The need for, feasibility of construction of, and probable efficacy of subsurface drainage should be determined by a study of soil and ground-water conditions at the proposed airfield, but, in general, the efficient removal of surface runoff by a carefully designed storm drainage system will minimize the need of extensive subsurface drainage. Inasmuch as the scope of this paper pertains primarily to surface drainage, discussion of subsurface drainage is limited to the foregoing comments.

The design capacity of the storm drainage systems should be adequate to accomplish the following objectives within the range of economic feasibility, with proper consideration being given to the purpose and importance of the particular airfield:

(*a*) Removal of surface runoff that would result from the selected "design storm," without damage to field facilities, undue saturation of subsoil, or significant interruption of normal traffic;

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[3] Head Engr., War Dept., Office, Chf. of Engrs., Washington, D. C.

(*b*) Removal of surface runoff from storms greater than the "design storm" with the minimum damage to field facilities and with shortest interruption of normal traffic that is practicable;

(*c*) Maximum reliability of operation practicable under all climatic conditions;

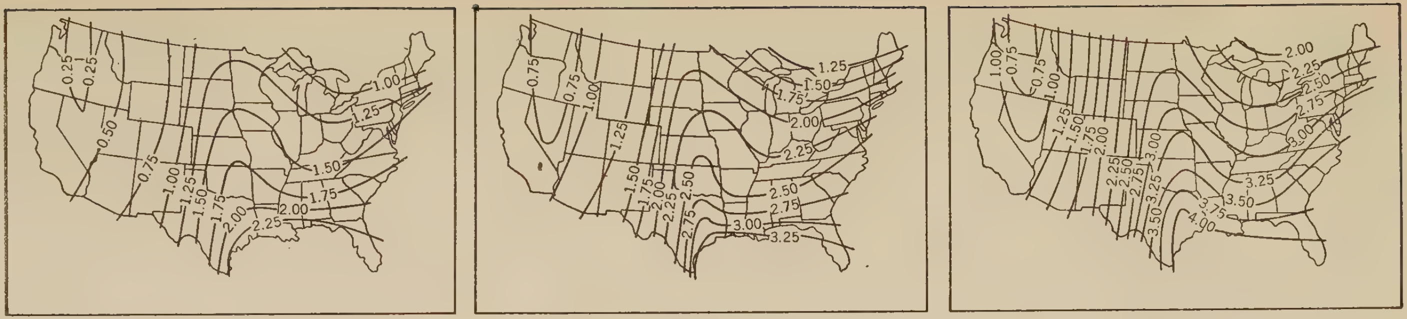
(*d*) Minimum maintenance and operation difficulties and expense; and

(*e*) Adaptability to future expansion of airfield drainage facilities with the minimum of expense and interruption of normal traffic.

The importance of careful planning in the preparation of grading plans for an airfield scarcely can be overemphasized. In many instances, the need for expensive underground drains in sections of a field may be avoided by grading the surface to facilitate natural drainage. In other cases required drain capacities may be reduced greatly by shallow ponding basins in intermediate areas between the runways where runoff from intense short-duration rainfall may be stored for brief periods of time without interfering with the use of runways and other essential areas. On the other hand, improper planning of grading details may cause flooding of an important area in one section of the airfield while the drainage facilities in other sections of the field are taxed to only part of their capacity.

The major limitations on grading are as follows: Maximum longitudinal and transverse grade for runways and aprons is 1.5%, except in the case of fields used by four-motored planes, where the maximum longitudinal grade for runways is 1%. A clear graded shoulder should be provided along each side of each runway to a minimum width of 250 ft from the runway center line. The selected transverse grade for the runway pavement is continued to the edge of the stabilized shoulder, 75 ft from the edge of the runway pavement. Beyond this point the shoulder may be sloped longitudinally to drain into inlets located at least 250 ft from the runway center line. In this area the maximum longitudinal and transverse grade of 1.5% is necessary to develop the ponding areas at drain inlets. Along the line of inlets, generally located about 250 ft from the center line of the runway, a maximum allowable grade change not to exceed a 2% algebraic difference is permitted. Grade changes along this line should be connected by vertical curves of not less than 300 ft. Turfed all-over fields which are used only by light-weight aircraft with traffic dispersed over the entire area, have a permissible maximum grade in any direction of 3%. The maximum change in grade which may be made within a distance of 100 ft is 1%.

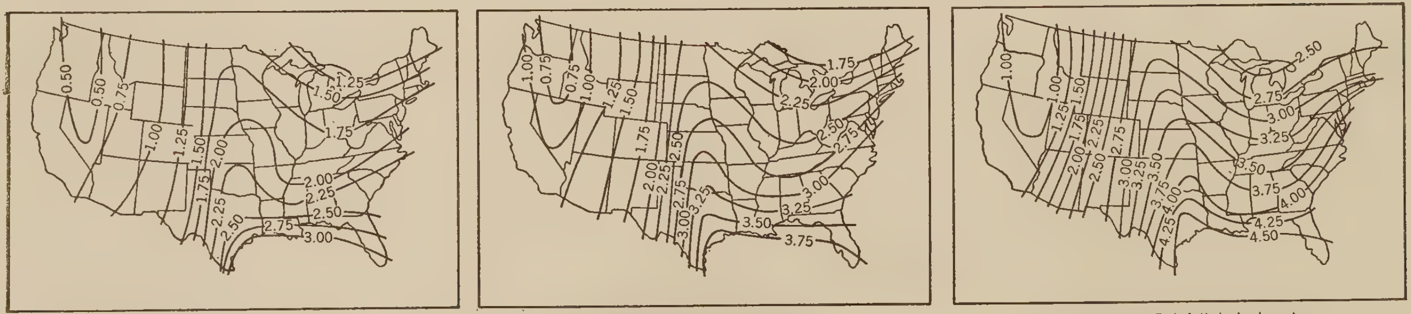
It is common practice to design storm drains to provide for disposal of runoff from rainfall having certain average recurrence intervals or frequency of occurrence. The rainfall intensity corresponding to a particular frequency of occurrence varies appreciably with the duration of the rainfall, the average rate for a period of 5 min usually being several times as high as the average for a period of 1 hr. A study of rainfall intensity-frequency data for a large number of precipitation stations indicates that there are fairly consistent relationships between the average intensity of rainfall for a period of 1 hr and the average rates of comparable frequency for shorter intervals, regardless of the geo-


0.25
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.25
0.75
1.00
1.25
1.50
1.75
2.00
2.25
2.50
2.75
3.00
3.25
1.00
0.75
1.25
1.50
1.75
2.00
2.25
2.50
2.75
3.00
3.25
3.50
3.75
4.00

One-Hour Rainfall, in Inches, to Be Expected Once in 2 Years

One-Hour Rainfall, in Inches, to Be Expected Once in 10 Years

One-Hour Rainfall, in Inches, to Be Expected Once in 50 Years


0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.25
2.50
2.75
3.00
1.00
0.75
1.25
1.50
1.75
2.00
2.25
2.50
2.75
3.00
3.25
3.50
3.75
1.00
1.25
1.50
1.75
2.00
2.25
2.50
2.75
3.00
3.25
3.50
3.75
4.00
4.25
4.50

One-Hour Rainfall, in Inches, to Be Expected Once in 5 Years

One-Hour Rainfall, in Inches, to Be Expected Once in 25 Years

One-Hour Rainfall, in Inches, to Be Expected Once in 100 Years

FIG. 13.—ONE HOUR RAINFALL, IN INCHES, TO BE EXPECTED ONCE IN A GIVEN NUMBER OF YEARS

graphical location of the stations or frequency of 1-hr rainfall. Consequently standard curves may be developed to express the rainfall "intensity-duration" relationships with an accuracy satisfactory for airfield drainage problems.

"Standard rainfall intensity-duration curves" were derived for use in airfield drainage problems in the following manner:

(*a*) The area of the United States was divided into 2° quadrangles;

(*b*) Using data from charts compiled by the late David L. Yarnell, M. Am. Soc. C. E. (15), as shown in Fig. 13, and other available information, 1-hr representative rainfall intensities for each 2° quadrangle were plotted against average rates of comparable frequency for intervals of 5, 10, 15, 30, 120, and 240 min, respectively, using data for average frequency of occurrence of once in 2, 5, 10, 25, 50, and 100 years, respectively.

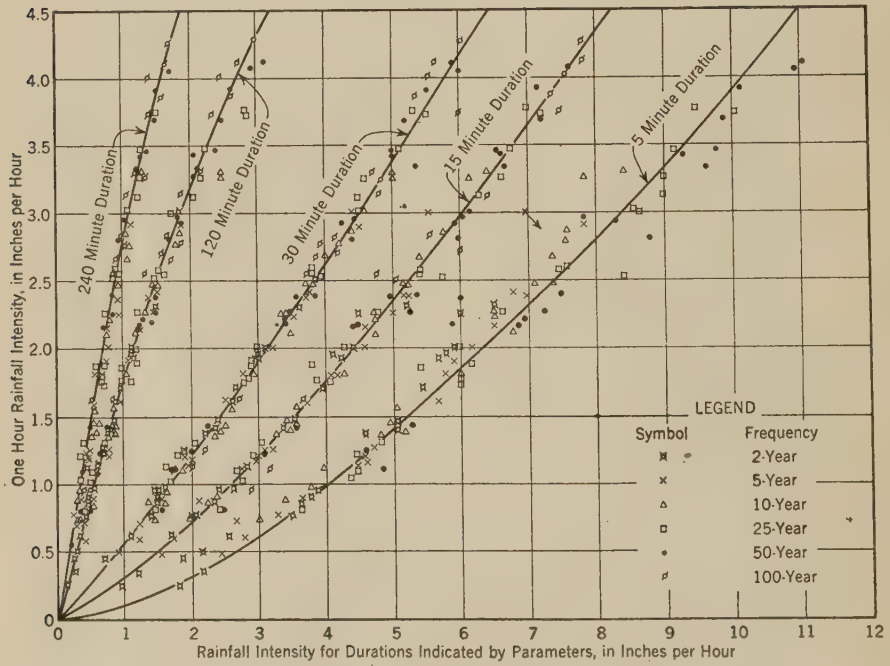


FIG. 14.—RELATION OF 1-HR RAINFALL INTENSITIES TO INTENSITIES FOR DURATIONS OF 5, 15, 30, 120, AND 240 MIN

(*c*) Curves, representing approximately the average of the points plotted in step (*b*), were drawn (see Fig. 14), and data scaled from these curves were replotted in the form shown in Fig. 15. Results of the study indicate that the curves in Fig. 15 may be adopted as general standards in airfield drainage problems with a satisfactory degree of accuracy. For convenience in reference these curves are numbered to correspond with the 1-hr rainfall intensity represented.

Design Storm Criteria

The term "design storm criteria" refers to the rainfall intensity-duration curve adopted as a basis of design for determining the capacity of storm drain facilities for a particular area. The rainfall intensity-duration curves are derived to represent average rates of rainfall for various durations that have the same average frequency of occurrence. Such intensity-duration curves are computed from data for a number of storms; consequently, all of the intensities represented by a particular curve may not be expected in the same storm—at least not with the frequency indicated. For example, if curve No. 2.0 in Fig. 15 represents a 10-yr rainfall frequency at a particular station, it is implied that on the average of once in 10 years a rainfall will occur averaging 3.1 in. per hr for a period of 30 min. This is also shown in Fig. 14 as the point where the 2.0-in. 1-hr rainfall intensity intersects the 30-min duration curve. Although the precipitation continues for a 30-min period at the rate of 3.1 in. per hr, the rain that falls in 30 min totals only 1.55 in. Similarly it is to be expected that on the average of once in 10 years rainfall will average 4.4 in. for a period of 15 min. The precipitation may end at 15 min or continue at some rate equal to or lower than curve No. 2.0, and the intensity for periods less than 15 min may be equal to or less than curve No. 2.0.

The rainfall intensity-duration value required to produce the maximum drain-inlet discharge corresponding to a given design storm criterion, taking into consideration all drainage area characteristics and ponding effects, is referred to herein as the "design storm rainfall." To simplify the preparation of generalized diagrams for estimating drain-inlet capacities required under various conditions for design storm runoff, the following approximations pertaining to the rainfall intensity-duration relations have been accepted:

(*a*) Each standard rainfall intensity-duration curve shown in Fig. 15 is assumed to represent average rates of rainfall for separate storms of various duration that have the same average frequency of occurrence; and

(*b*) Rainfall is assumed to be of uniform intensity during each individual storm, and to end at the duration corresponding to the particular rate on the given intensity-duration curve.

The design-storm frequency serves as an index to the average frequency with which some portions or all of the storm-drain system will be taxed to capacity. The selection of a design-storm frequency for a particular project must be based primarily upon judgment, with consideration of the purpose and importance of the given airfield, and such economic and engineering limitations as may exist. Design-storm frequencies ranging from 2 to 10 years are generally adopted for airfields depending, of course, upon the importance of the field as determined by the class and volume of traffic to be accommodated, the necessity for uninterrupted service and similar factors. It should be observed that, after having selected the design-storm frequency for the particular airfield involved, computations must be made to determine the duration of rainfall for the frequency curve selected that will produce the maximum rate of runoff for the specific area involved. The duration of rainfall required to produce the maximum rate of runoff will depend primarily upon the length of overland flow, taking into account surface detention, roughness factor, and other surface runoff characteristics.

The design-storm frequency alone is not a reliable criterion of the adequacy of storm drain facilities. In many instances, storms appreciably more severe than the design storm may cause very little damage or inconvenience, whereas in other cases flooding of important areas may result. It is generally advisable to investigate the probable consequences of storms more severe and less frequent than the design storm, before making a final decision regarding the adequacy of drain-inlet capacities.

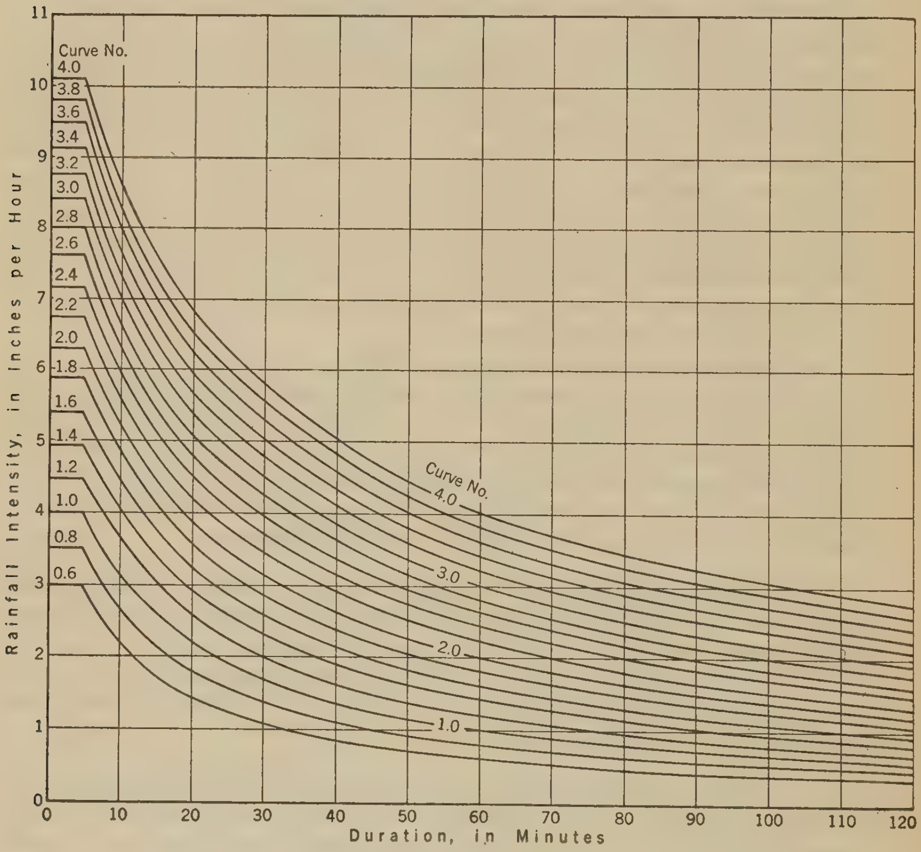


Fig. 15.—Standard Rainfall Intensity-

Notation

The following letter symbols, used in this paper, conform essentially to American Standard Letter Symbols for Hydraulics[4] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942:

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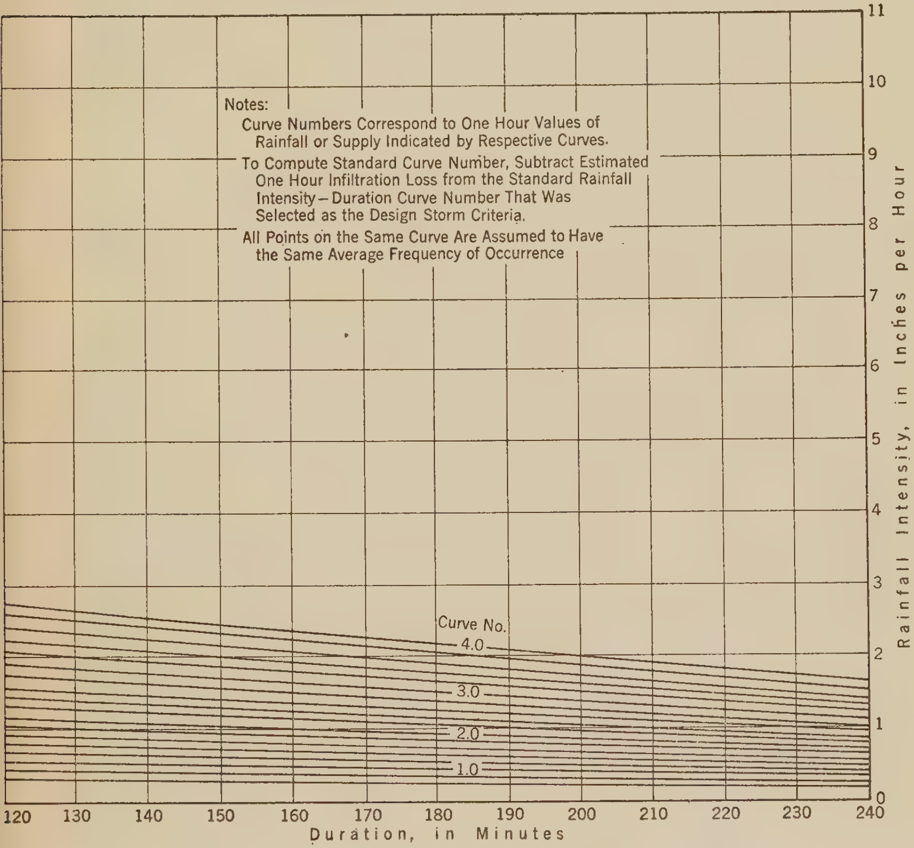
[4] ASA—Z10.2—1942.

I = factor of turbulence;

K = a constant dependent upon characteristics of the drainage area involved;

L = length of an elemental strip of turfed, bare, or paved surface, in a direction parallel to the maximum slope;


Notes:
Curve Numbers Correspond to One Hour Values of Rainfall or Supply Indicated by Respective Curves.
To Compute Standard Curve Number, Subtract Estimated One Hour Infiltration Loss from the Standard Rainfall Intensity—Duration Curve Number That Was Selected as the Design Storm Criteria.
All Points on the Same Curve Are Assumed to Have the Same Average Frequency of Occurrence
Curve No.
4.0
3.0
2.0
1.0
120
130
140
150
160
170
180
190
200
210
220
230
240
Duration, in Minutes
0
1
2
3
4
5
6
7
8
9
10
11
Rainfall Intensity, in Inches per Hour

DURATION CURVES; OR, STANDARD SUPPLY CURVES

M = an exponent dependent upon the type of overland flow;

n = coefficient of roughness;

Q = drain capacity, in cubic feet per second, at a designated point;

Q_d = drain-inlet capacity, in cubic feet per second;

q = rate of overland flow at the lower end of an elemental strip of turfed, bare, or paved surface, in inches per hour or in cubic feet per second per acre of drainage area;

q_d = drain-inlet capacity, or maximum rate of outflow from a ponding basin, in cubic feet per second per acre of tributary drainage area;

q_p = peak runoff rate, in inches per hour or cubic feet per second per acre of drainage area;
S = slope of surface, or the hydraulic gradient;
t = time, or duration, in minutes; time from the beginning of supply:
t_c = critical duration of supply; that is, the duration of rainfall excess (rate of supply) corresponding to a given "standard supply curve" that would produce the maximum rate of outflow from a given drainage area, taking into account surface detention and the surface runoff characteristics;
t_d = time required for water to travel from a specified inlet to a given point in the drainage system;
t_l = critical duration of supply for an elemental strip of effective length L;
t_r = duration of supply, in minutes;
t_σ = duration corresponding to the assumed supply rate σ;
X = ratio of $\frac{q_p}{q_d}$; defined by Figs. 22 and 25;
y = depth, in inches; and
σ = rate of supply, or rainfall in excess of the rate of infiltration.

INFILTRATION CAPACITY

In view of the several variations that affect the infiltration capacity of a given soil, it is impracticable to determine accurately the infiltration capacity that should be assumed to apply during storms of various durations and frequencies. The rate of infiltration is a variable that changes not only during the course of a storm but also during a season. Moreover, the infiltration rate changes with the type of soil structure, soil cover, temperature (air, soil, and water), moisture content of soil, turbidity of the water, and the amount of organic matter in the soil. The use of cover on the land, such as grass turf, serves as a protection from the impact of rain, retards the rate of runoff and thereby reduces the velocity of overland flow and turbidity, thus permitting a greater infiltration of water into the soil. Rates of infiltration on bare soil can be expected to be considerably less than for turfed areas. For use in connection with the design of storm drains for a particular airfield, the infiltration capacity that is estimated to be characteristic of the given soil, following a rainfall of 1 hr, serves as the most convenient index to the probable volume of loss through infiltration during the design storm.

The term "rate of supply" (σ) refers to the difference between the rainfall intensity and the infiltration capacity at the same instant of a particular storm. To simplify computation procedures, the rainfall intensity and the infiltration capacity are assumed to be constant during any specific storm. On this premise, the rate of supply during a particular storm would also be uniform. The average rates of supply corresponding to storms of different lengths and the same average frequency of occurrence may be computed by subtracting estimated infiltration capacities from rainfall intensities represented by the proper curve in Fig. 15. For convenience and since no appreciable error results, "standard supply curves" are assumed to be the same shape as the curves in

Fig. 15. The "standard supply curve" number corresponding to a given "standard rainfall curve" number is computed by subtracting the estimated 1-hr infiltration value from the 1-hr rainfall quantity.

OVERLAND FLOW EQUATIONS

The term "overland flow," as used herein, relates to surface runoff resembling sheet flow before it has reached a defined channel or ponding basin. Robert E. Horton, M. Am. Soc. C. E., has developed the following general equation that indicates the rate of overland flow to be expected from a uniform rate of rainfall-excess, or "rate of supply," assuming flow characteristics ranging from fully turbulent to laminar (18):

$$q = \sigma \tanh^M \frac{M+1}{M} (\sigma\, K)^{1/M} \frac{t}{60} \ldots\ldots\ldots\ldots\ldots (2)$$

In Eq. 2, q is the rate of overland flow at the lower end of an elemental strip, in cubic feet per second per acre, or inches per hour; σ is the rate of rainfall in excess of infiltration, in inches per hour; M is an exponent dependent upon the type of overland flow; K is a constant dependent upon characteristics of the drainage area involved; and t is the time from the beginning of supply, in minutes. According to Mr. Horton, this equation is strictly rational for 75% turbulent flow ($M = 2.00$) and quasi-rational for other degrees of turbulence from 33% to 100%. The constant K involves slope, roughness and length of overland flow, and percentage of turbulence, and may be expressed by the equation:

$$K = \frac{1{,}020 \sqrt{S}}{I\, n\, L} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

In Eq. 3, S is the surface slope; I is the factor of turbulence; n is the coefficient of roughness; and L is the length of an elemental strip of turfed or paved surface in a direction parallel to the maximum slope. The exponent M is equal to 3.00 for laminar flow and $\frac{5}{3}$ for fully turbulent flow; therefore, the factor of turbulence I is given by the equation:

$$I = \frac{3}{4} (3.0 - M) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

For average airfield surfaces, 75% turbulence, corresponding to $M = 2.00$ in overland flow, may be assumed although the percentage of turbulence would be lower for a "dense" grass cover and somewhat higher for very smooth pavement. Substitution of the value $M = 2.00$ in Eq. 2 gives the following formula for the rate of overland flow corresponding to 75% turbulence:

$$q = \sigma \tanh^2 \left[0.922\, t \left(\frac{\sigma}{n\, L} \right)^{0.50} S^{0.25} \right] \ldots\ldots\ldots\ldots\ldots (5)$$

The factor n in the preceding formulas has the same meaning for overland flow as it has for channel flow but is made up differently. Mr. Horton points out that, in the case of a stream channel, n shows the combined resistance effect of true rugosity, bends, noneffective cross section in eddies, bays, and deep holes, and expansion and contraction of cross section. All of these factors except true boundary rugosity may be of small importance in a stream channel, but they are often the major factors in case of overland flow. Available data indicate that the following values of the coefficient of roughness n are approximately representative of surfaces encountered in airfield drainage problems:

Surface	Value of n
Smooth pavements	0.02
Bare, packed soil, free of stone	0.10
Poor grass cover, or moderately rough bare surface	0.20
Average grass cover	0.40
Dense grass cover	0.80

In certain approximate computations of overland flow rates, average roughness values for inlet drainage area have been used. The average roughness factor applicable to a drainage area consisting of a combination of paved and unpaved surfaces (or any combination of surfaces having dissimilar roughness characteristics) may be estimated by selecting values of the coefficient n applicable to the respective surfaces and weighting the values in proportion to the surface areas.

The series of overland flow curves for paved and turfed areas shown in Fig. 16 were computed by the use of Eq. 5, assuming S equal to 1% and n equal to 0.02 for pavements and 0.40 for turf. However, these curves may be applied to surfaces having other roughness factors or surface slopes by using, instead of the actual length L of the surface involved, a hypothetical length that is greater or less than the actual length by a sufficient margin to compensate for the difference between the correct values of n and S and those used in preparing the curves shown in Fig. 16. The values of L may be converted easily by the use of Fig. 17, as follows:

(*a*) *Fig. 17(a)*.—The rate of overland flow indicated by Eq. 5 is a function of the product of $n\,L$; hence, any combination of n and L values that give the same product will result in the same rate of discharge. Thus, a surface having a roughness factor n of 0.40 and a length L of 100 ft would produce the same rate of overland flow as one having an n-value of 0.20 and a length of 200 ft, other factors being the same. The curves in Fig. 17(*a*) represent similar relationships for various values of n.

(*b*) *Fig. 17(b)*.—The discharge rate represented by Eq. 5 is also a function of the quotient $\frac{S^{0.25}}{L^{0.50}}$. Therefore, the overland flow curves in Fig. 16, computed for a slope of 1%, may be applied to a surface having any other slope, if compensating corrections in the value of L are made in accordance with Fig. 17(*b*).

INFLOW HYDROGRAPHS

The rates of runoff from turfed areas of various lengths that would result from continuous and uniform rates of rainfall-excess (rates of supply) are represented by the overland flow rates in Fig. 16. For example, with a continuous and uniform rate of supply (σ) of 2 in. per hr and a length of 100 ft, the

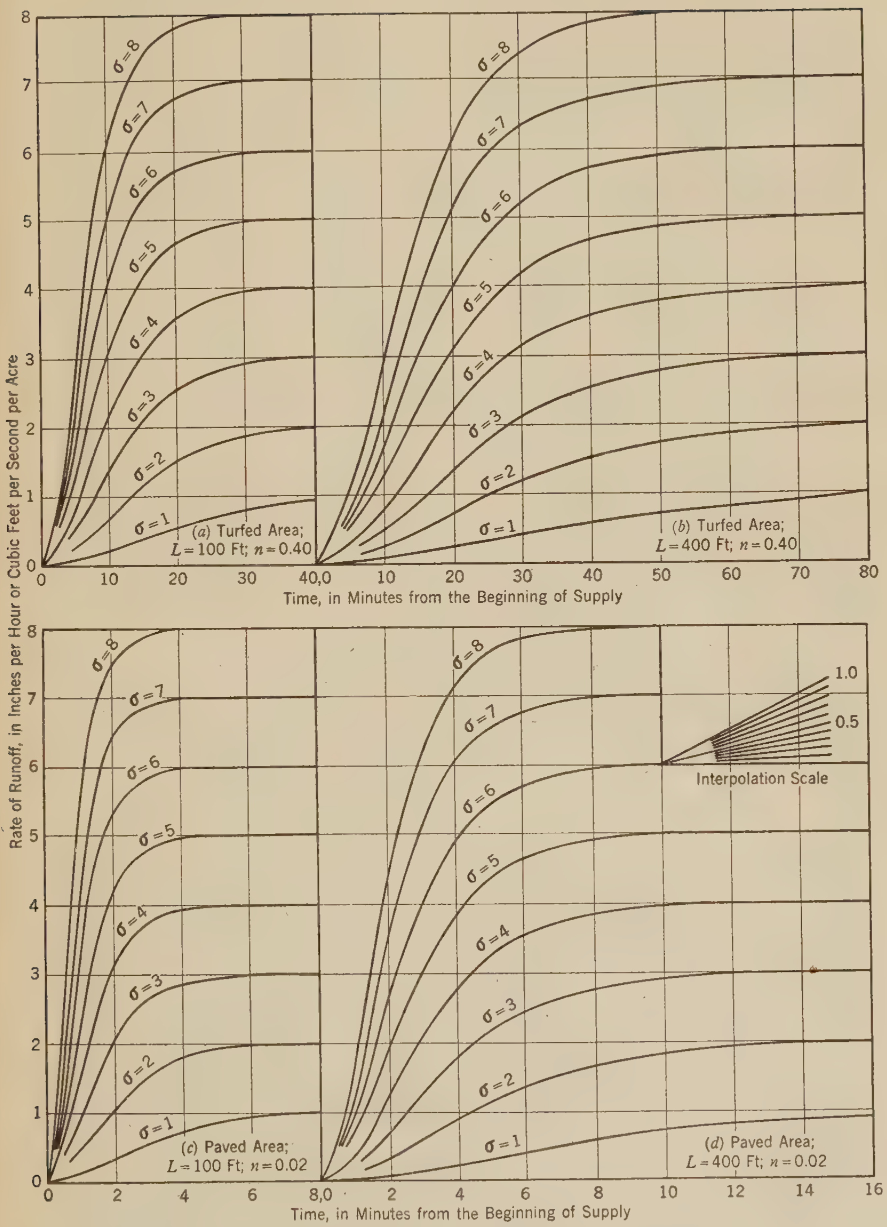


FIG. 16.—OVERLAND FLOW RATES VERSUS DURATION OF SUPPLY FOR PAVED AND TURFED AREAS WITH MAXIMUM SURFACE SLOPE, S, OF 1%

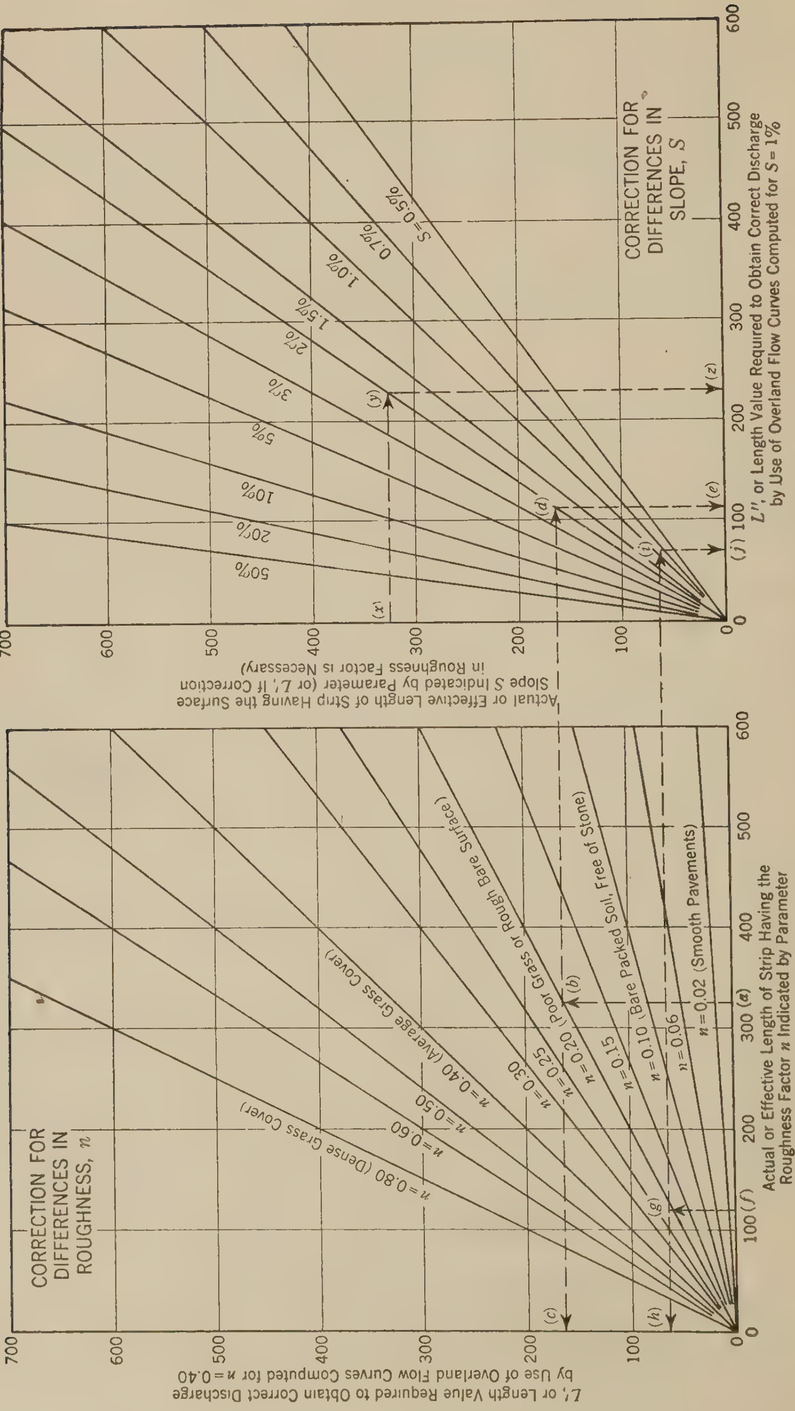


FIG. 17.—MODIFICATION IN L REQUIRED TO COMPENSATE FOR DIFFERENCES IN n AND S

rate of overland flow 25 min after the beginning of supply would be 1.7 in. per hr. Similarly, the rate of overland flow 40 min after the beginning of supply would be 2.0 in. per hr. Hydrographs of runoff corresponding to any selected duration and rate of rainfall excess may be computed from the overland flow rates indicated in Fig. 16 by the procedure shown in Fig. 18. The overland flow curve No. 1, of Fig. 18, represents the runoff hydrograph from an elemental strip of turf 400 ft in length assuming a continuous and uniform rate of supply (σ) of 4 in. per hr. The ordinates of the overland flow curve can be considered to be the sum of the corresponding ordinates of a series of identical runoff hydrographs (2, 2A, 2B, 2C, etc., not as yet determined) resulting from a continuous series of short rainfall periods of specified duration—for example, 20 min. Overland flow curve No. 1A is the same curve as No. 1, but is translated horizontally a time distance equal to the selected duration of 20 min, and represents the runoff for a continuous and uniform rate of supply of 4 in. per hr beginning 20 min after that of curve No. 1. Hydrograph No. 2 is obtained by subtracting the ordinates of No. 1A from the ordinates of No. 1 and approximately represents the runoff hydrograph to be expected from a supply rate of 4 in. per hr for a duration of 20 min. This is verified readily by a consideration of the composition of overland flow curves No. 1 and No. 1A at any particular time, say, for example, at 60 min. Hydrograph No. 2 (identical to 2A, 2B, etc.) must be such that the sum of the first 20-min value, *a*, of No. 2B, plus the second 20-min value, *b*, of No. 2A, plus the third 20-min value, *c*, of No. 2, equals the

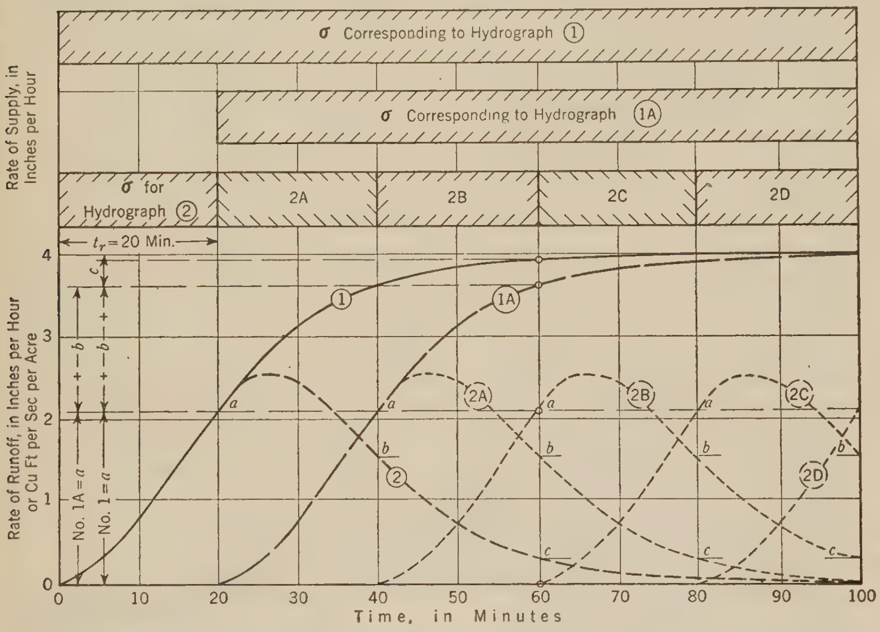


FIG. 18.—COMPUTATION OF HYDROGRAPH TO REPRESENT RUNOFF FROM A SUPPLY OF SPECIFIED DURATION

ordinate of No. 1 at 60 min. However, No. 1A at 60 min equals No. 1 at 40 min by construction, and No. 1 at 40 min is equal to ordinate a of No. 2A plus b of No. 2. Since hydrographs 2, 2A, 2B, etc., are identical, all a ordinates are equal to each other as are the b, c, etc., ordinates. Thus at the time of 60 min No. $1 = a + b + c$ and No. $1A = a + b$. It follows that the ordinates of No. 1 minus the ordinates of No. 1A equal c, which is the 60-min value of hydrograph No. 2.

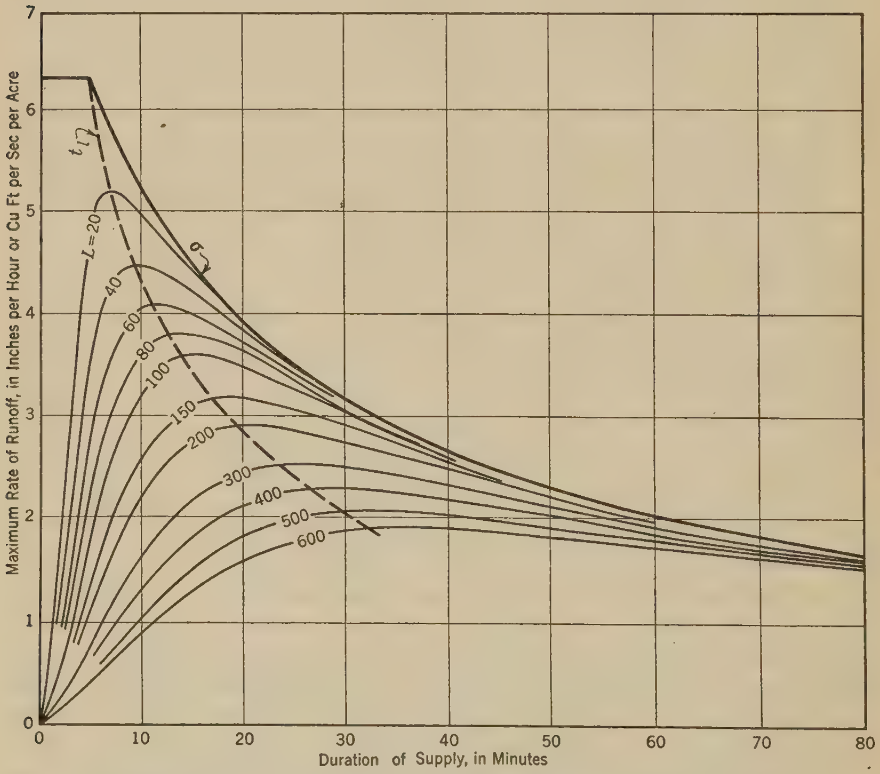


Fig. 19.—Supply Curve No. 2.0; Rates of Overland Flow Corresponding to Standard Supply Curves; $n = 0.40$, and $S = 1\%$

The curves shown in Fig. 19 have been included for the purpose of demonstrating the relationship between the maximum rate and critical time of runoff (t_l) for various lengths L of overland flow and a specific supply curve. The curves were obtained by computing the rates of discharge at successive time intervals that would result from the various rates of supply σ, corresponding to standard supply curve No. 2.0 by application of the overland flow curves shown in Fig. 16. The procedure is illustrated by the sample computations in Table 2. To illustrate the procedure, durations of supply of 12, 20, and 35 min were selected and have been marked by dotted lines in Fig. 20. The rates of supply (scaled from No. 2.0 curve) for durations of 12, 20, and 35 min are 4.83, 3.85, and 2.84 in. per hr, respectively (see Col. 2, Table 2, and points on curve A, Fig. 20). The overland flow rate was then scaled by interpolation between $\sigma = 4$ and $\sigma = 5$ (from curves similar to those shown in Fig. 16) for a length of 200 ft, a duration of 12 min, and a supply rate of 4.83 in. per hr, and determined to be 2.41 in. per hr (see Col. 6, Table 2, and the point on curve B, Fig. 20). The foregoing runoff values are accepted as the maximum under the assumption that surface storage is negligible. Actually the maximum runoff would occur a short time after the rainfall-excess ceases. For all practical purposes, however, the maximum rate of overland flow can be assumed to occur approximately at a time coincident with the ending of the rainfall-excess.

TABLE 2.—RATES OF OVERLAND FLOW CORRESPONDING TO INTENSITIES AND DURATION OF SUPPLY REPRESENTED BY STANDARD SUPPLY CURVE NO. 2.0 IN FIG. 15

($n = 0.40$, $S = 1\%$)

Duration of supply, in min	Rate of supply, in in. per hr[a]	Rate of Overland Flow[b] Corresponding to Durations Shown in Col. 1 and Rates of Supply Given in Col. 2						
		$L = 20$ ft	$L = 60$ ft	$L = 100$ ft	$L = 200$ ft	$L = 300$ ft	$L = 400$ ft	$L = 600$ ft
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
3	6.30	2.68	1.12	0.75	0.39	0.25	0.22	0.13
5	6.30	4.74	2.59	1.76	0.96	0.64	0.52	0.33
7	5.81	5.16	3.41	2.55	1.54	1.12	0.83	0.58
9	5.35	5.06	3.84	3.02	1.94	1.42	1.10	0.76
12	4.83	4.75	4.07	3.43	2.41	1.80	1.49	1.02
15	4.41	4.39	4.02	3.59	2.70	2.12	1.76	1.26
20	3.85	3.85	3.70	3.46	2.86	2.39	2.05	1.55
25	3.44	....	3.38	3.27	2.85	2.49	2.20	1.73
30	3.12	....	3.12	3.02	2.77	2.49	2.25	1.85
35	2.84	....	....	2.81	2.60	2.39	2.20	1.86
40	2.62	....	....	2.62	2.48	2.32	2.15	1.86
45	2.43	....	....	....	2.32	2.21	2.09	1.86
50	2.27	....	....	....	2.20	2.11	2.00	1.82
60	2.00	....	....	....	1.96	1.92	1.86	1.72
80	1.62	....	....	....	1.60	1.59	1.56	1.50
100	1.38	....	....	....	1.38	1.35	1.33	1.28
120	1.16	....	....	....	....	1.16	1.16	1.12
t_c[c]	....	7	11	14	20	24	28	32

[a] Scaled from curve No. 2.0, Fig. 15. [b] As scaled from diagrams similar to those shown in Fig. 16. [c] Critical duration of supply interpolated from Fig. 19.

Attention is directed to the fact that the curves in Fig. 19 are not hydrographs for any specific design storm, but represent the peak rates of runoff from individual storm events of various durations, all of which have the same average frequency of occurrence. The duration of supply corresponding to the maximum discharge for a particular standard supply curve and value of L in Fig. 19 is indicated by the broken line designated as t_l, the "critical duration of supply," defined as "the duration of rainfall-excess (rate of supply) corresponding to a given standard supply curve that would produce the maximum rate of outflow from a given drainage area, taking into account surface detention and surface runoff characteristics."

The curves in Fig. 16 represent the rate of overland flow that would result from various continuous rates of supply of uniform intensity. The maximum rate of runoff for the respective uniform supply rates occurs at a time ordinarily referred to as the "time of concentration," when the rate of runoff becomes equal to the rate of supply. Curves A, B, and C of Fig. 20 are defined as follows:

A.—Standard supply curve No. 2 (Fig. 15 and Col. 2, Table 2);
B.—Maximum rates of overland flow corresponding to specific intensities and durations of supply represented by the standard supply curve No. 2 (Fig. 19 and Col. 6, Table 2); and
C.—Rate of overland flow for turfed areas corresponding to a uniform continuous supply rate of 2 in. per hr.

It will be observed that the rate of runoff represented by curve C, Fig. 20, approaches quite closely the rate of supply of 2 in. per hr at a time 60 min after the beginning of supply. Curve B, Fig. 20, represents the maximum rates of overland flow corresponding to specific intensities and durations of supply selected from curve A. Curve B is approximately equal to the rate of supply

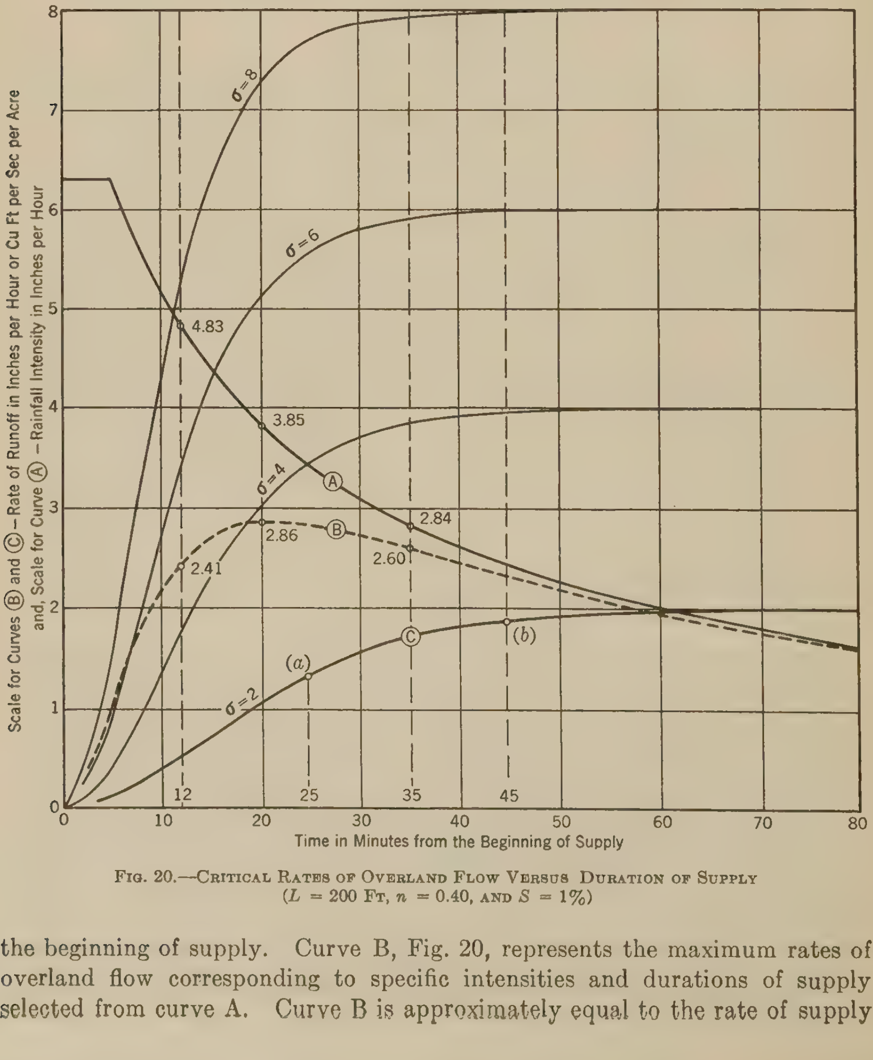


Fig. 20.—Critical Rates of Overland Flow Versus Duration of Supply ($L = 200$ Ft, $n = 0.40$, and $S = 1\%$)

of 2 in. per hr as represented by curve A at a duration of 60 min and is equal to curve C at 60 min. However, further inspection of curve B shows that a discharge greater than that represented by curve C at 60 min will result from durations of supply having the same average frequency (curve A) but higher intensities and shorter durations. In other words, under the assumed conditions, the maximum rate of overland flow for intensities and durations of supply represented by curve A and a length of 200 ft occurs with a supply rate of 3.85 in. per hr and a duration of supply of 20 min (see curves A and B, Fig. 20).

DRAIN-INLET CAPACITY VERSUS SURFACE-STORAGE RELATIONSHIPS

If the rate of outflow from a drainage area is limited by the capacity of a drain serving the area, runoff occurring at rates exceeding the drain capacity must be stored temporarily. As soon as the rate of inflow into the ponding basin becomes less than the drain capacity, the accumulated storage may be drawn off at a rate equal to the difference between the drain capacity and the rate of inflow into the basin. The general relation between inflow, storage, and outflow is expressed by the equation:

$$\text{Outflow} = \text{inflow} \pm \text{storage} \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

The rate of outflow from a ponding basin is affected somewhat by the elevation of the water surface at the inlet to the drain serving the area, the rate of outflow increasing as the head on the inlet increases. However, because of the flat slopes of airfield areas, the surfaces of the storage ponds surrounding the drain inlets are usually very large in comparison with the depth of water at the inlets. Therefore, it is reasonable to assume that the rate of outflow through a particular drain inlet would be approximately constant as long as the rate of runoff and accumulated storage are sufficient to maintain the full discharge capacity of the drain inlet serving the particular area. It is also assumed that the rate of outflow would equal the rate of inflow into the pond until the full discharge capacity of the drain inlet is attained. To illustrate these assumptions reference is made to the curves shown in Fig. 21 for a length $L = 400$ ft, $S = 1\%$, $n = 0.40$, and a uniform "rate of supply" (σ) of 4 in. per hr for a duration of 20 min. Length L was chosen to represent the weighted mean distance from the limits of the drainage area to the average limits of the pond. Hydrographs 1, 2, and 3, Fig. 21, are:

1. Runoff that would have occurred if the given rate of supply (4 in. per hr) had continued indefinitely;
2. Runoff from a supply of 4 in. per hr for a duration of 20 min; and
3. Rate of outflow, assuming the inlet capacity as designated, and a constant head-discharge relation.

Hydrograph 3, Fig. 21, would represent the constant rate of outflow corresponding to the inflow hydrograph 2 if the drain-inlet capacity were assumed to be 1.25 cu ft per sec per acre of drainage area. The volume of storage above the outflow hydrograph 3 and below the inflow hydrograph 2 that would be accumulated at successive intervals of time under these conditions is indicated by curve 4, Fig. 21. The maximum volume of storage that would accumulate under these particular conditions is 1,350 cu ft per acre of drainage area and the end of the accumulation period occurs approximately 43 min after the beginning of runoff.

DISCHARGE-STORAGE DIAGRAM

Hydrograph 2, Fig. 21, represents the rate of runoff from one acre of turf with a length of 400 ft, a coefficient n of 0.40, and a slope of 1%, assuming a uniform value of σ equal to 4 in. per hr for a duration of 20 min. A series of hydrographs similar to hydrograph 2 was computed for a "rate of supply" of 4 in. per hr and for durations of supply of 10, 20, 30, 40, 50, and 60 min, successively. In each case the drainage area was taken as 1 acre and the "effective length" (L) as 400 ft. The volumes of runoff under the respective hydrographs and above abscissas representing discharge rates (drain-inlet capacity), similar to that of hydrograph 3 in Fig. 21, equal to 25%, 50%, and 75% of the peak ordinates thereof, were measured by planimeter. The volumes thus determined represent approximately the maximum quantities of water that would be stored temporarily in surface ponds if drain-inlet capacities equal to 25%, 50%, and 75% of the peak ordinates of the inflow hydrograph were adopted. The data

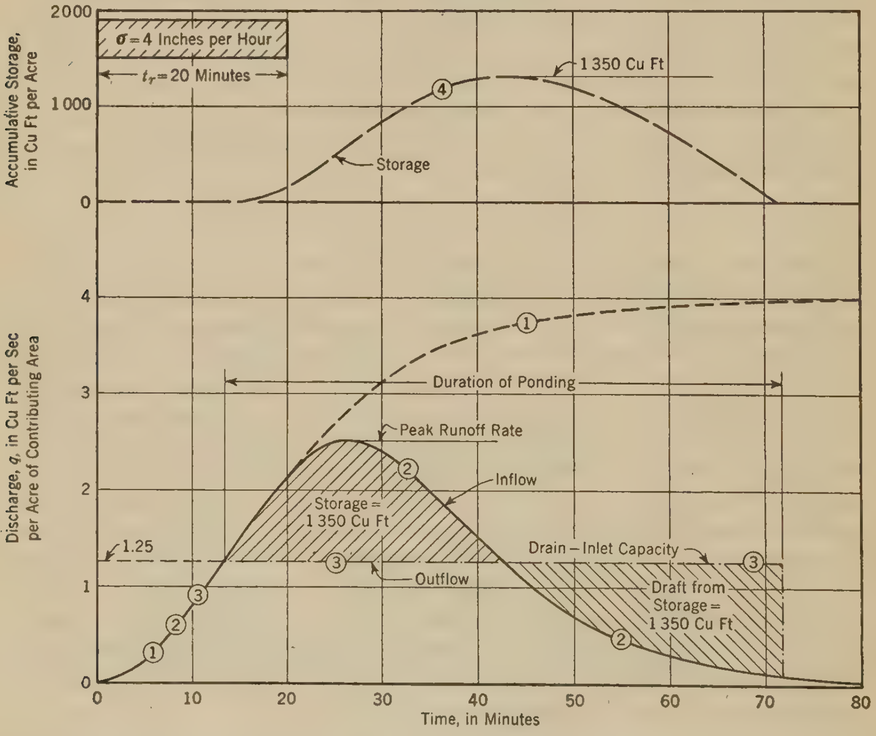


FIG. 21.—COMPUTATIONS FOR STORAGE REQUIRED WITH SELECTED DRAIN-INLET CAPACITY TO PROVIDE FOR RUNOFF FROM 1 ACRE OF TURF; L = 400 FT, n = 0.40, AND S = 1%

were plotted to obtain the series of curves in Fig. 22 for σ equal to 4 in. per hr. The same procedure was followed in developing a series of curves similar to that shown in Fig. 22 for "rates of supply" of 1, 2, 3, 4, 5, 6, 7, and 8 in. per hr assuming effective lengths (L) of 40, 100, 200, 300, 400, and 600 ft, successively. The foregoing procedure is one step in the preparation of "discharge versus storage diagrams" and Fig. 22 has been included to illustrate the step.

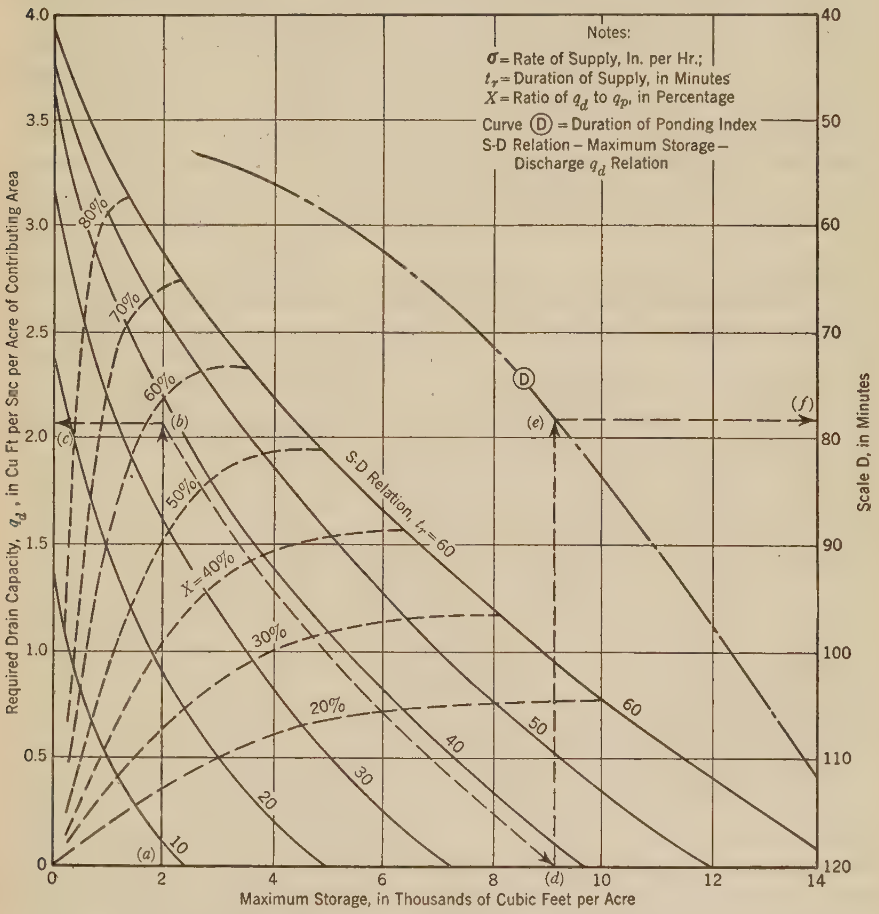


Fig. 22.—Storage-Discharge Relations for Turfed Areas; L = 400 Ft and σ = 4

Inasmuch as the volume of storage accumulated in a ponding area is a function of both the rate and duration of supply, a "trial" computation procedure is necessary to determine the drain-inlet capacity required to limit storage to a specific volume, assuming a design storm criterion represented by one of the "standard rainfall intensity-duration curves" in Fig. 15. The computations involved in this procedure are illustrated in Table 3. The series of diagrams similar to Fig. 22 was used in computing the drain-inlet capacities in Cols. 3 to 9, Table 3. "Rates of supply" (σ), corresponding to values used in preparing the diagrams in Fig. 22, were selected to avoid the need for interpolation (see Col. 1, Table 3). The durations of the supply rate corresponding to the "rates of supply" listed in Col. 1 were scaled from the "standard supply curves" in Fig. 15, and the values were placed in Col. 2, Table 3. The procedure followed in computing drain-inlet capacities required to limit surface storage to volumes indicated in the headings of Cols. 3 to 9, Table 3, assuming the various combinations of rate and duration of supply tabulated in Cols. 1 and 2, Table 3, may be illustrated by the following sample computation. Assume a turfed area for which $L = 400$ ft; $\sigma = 4$ in. per hr; $t_r = 38$ min, and a maximum permissible storage volume equal to 2,000 cu ft per acre of drainage area. The drain-inlet capacity required for the assumed condition would be determined by entering the diagram in Fig. 22 for $\sigma = 4$, at point a (storage 2,000), following a vertical line to point b ($t_r = 38$), and reading the drain-inlet capacity of 2.1 cu ft per sec per acre of drainage area at point c.

TABLE 3.—DRAIN-INLET CAPACITIES FOR VARIOUS SURFACE STORAGE VOLUMES—TURFED AREA; $L = 400$ FT, $n = 0.40$, AND $S = 1\%$

Assumed rate of supply, σ, in in. per hr (Fig. 15)	Duration corresponding to assumed supply rate, in min (Fig. 15)	ASSUMED MAXIMUM SURFACE STORAGE, IN CUBIC FEET PER ACRE OF DRAINAGE AREA						
		500	1,000	2,000	3,000	4,000	5,000	6,000
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(*a*) STANDARD SUPPLY CURVE NO. 1; SEE FIG. 15								
Drain-Inlet Capacity, q_d, in Cubic Feet Per Second Per Acre[a]:								
1	60	0.6	0.4	....	....	....	....	....
2	23	0.7	0.4	....	....	....	....	....
3	11	0.5	0.3	....	....	....	....	....
4	5	....	....	....	....	....	....	....
Critical,[b] q_d	....	0.7	0.4	....	....	....	....	....
Ratio $\frac{q_p}{q_d}$ (X) (%)	....	61	41	....	....	....	....	....
Duration, in Minutes:								
D-scale values		80	80	....	....	....	....	....
Maximum, ponding[b]		59	100	....	....	....	....	....
Critical supply[b]		33	38	....	....	....	....	....
(*b*) STANDARD SUPPLY CURVE NO. 2; SEE FIG. 15								
Drain-Inlet Capacity, q_d, in Cubic Feet Per Second Per Acre[a]:								
1	164	0.9	0.8	0.7	0.6	....	....	....
2	60	1.5	1.3	1.0	0.7	....	....	....
3	32	1.8	1.5	1.0	0.6	....	....	....
4	19	1.7	1.4	0.8	0.4	....	....	....
5	11	1.4	1.0	0.5	....	....	....	....
6	6	1.0	0.6	0.1	....	....	....	....
Critical,[b] q_d	....	1.8	1.5	1.0	0.7	....	....	....
Ratio $\frac{q_p}{q_d}$ (X) (%)	....	80	64	57	40	....	....	....
Duration, in Minutes:								
D-scale values		75	75	118	118	....	....	....
Maximum, ponding[b]		35	53	99	150	....	....	....
Critical supply[b]		32	36	46	56	....	....	....

TABLE 3.—(*Continued*)

Assumed rate of supply, σ, in in. per hr (Fig. 15)	Duration corresponding to assumed supply rate, in min (Fig. 15)	Assumed Maximum Surface Storage, in Cubic Feet Per Acre of Drainage Area						
		500	1,000	2,000	3,000	4,000	5,000	6,000
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		(*c*) Standard Supply Curve No. 3; See Fig. 15						
Drain-Inlet Capacity, q_d, in Cubic Feet Per Second Per Acre[a]:								
2	112	1.8	1.7	1.5	1.4	1.2	1.1	....
3	60	2.6	2.3	1.9	1.6	1.3	1.1	....
4	38	2.9	2.6	2.1	1.7	1.3	1.0	....
5	24	3.0	2.6	1.9	1.4	1.0	0.7	....
6	15	2.7	2.1	1.4	0.9	0.5	0.1	....
7	10	2.3	1.8	1.0	0.5	0.1	....	....
8	6	1.6	1.3	0.6	0.2	....	....	....
Critical,[b] q_d	....	3.0	2.6	2.1	1.7	1.3	1.1	....
Ratio $\frac{q_p}{q_d}$ (X) (%)	....	76	74	59	47	46	....	....
Duration, in Minutes:								
D-scale values		58	78	78	78	115	....	....
Maximum, ponding[b]		32	42	62	89	126	....	....
Critical supply[b]		30	34	42	49	58	....	....
		(*d*) Standard Supply Curve No. 4; See Fig. 15						
Drain-Inlet Capacity, q_d, in Cubic Feet Per Second Per Acre[a]:								
2	198	2.0	1.9	1.8	1.7	1.6	1.5	1.4
3	103	2.9	2.8	2.5	2.3	2.1	1.9	1.7
4	60	3.6	3.3	2.9	2.5	2.2	1.9	1.7
5	41	4.2	3.7	3.1	2.7	2.3	1.9	1.6
6	27	4.2	3.6	2.9	2.3	1.8	1.5	1.2
7	18	3.9	3.3	2.6	1.8	1.4	0.9	0.5
8	13	3.4	2.9	2.0	1.4	1.0	0.5	0.1
Critical,[b] q_d	....	4.2	3.7	3.1	2.7	2.3	1.9	1.7
Ratio $\frac{q_p}{q_d}$ (X) (%)	....	90	78	67	56	47	51	43
Duration, in Minutes:								
D-scale values		80	80	80	80	80	117	117
Maximum, ponding[b]		30	40	54	70	90	112	142
Critical supply[b]		....	33	38	46	53	60	67

[a] Drain-inlet capacity q_d in cubic feet per second per acre of drainage area required to limit storage to the assumed volumes. These values were computed with the assistance of curves similar to Fig. 22. [b] The critical value of q_d, the maximum duration of ponding, and the critical duration of supply were used in preparing diagrams A, B, and C, respectively, in Fig. 23.

As all rates of supply represented by a particular "standard supply curve" (Fig. 15) are assumed to occur with the same average frequency, the drain-inlet capacity required for a particular area should be adequate to limit the volume of storage to the maximum permissible quantity during all storms producing supply rates corresponding to the "standard supply curve" selected as the design criteria. Accordingly, only the "critical" drain-inlet capacities indicated in Table 3 are of prime concern in the determination of design capacities for storm drains. The relationships between "critical" drain-inlet capacities, maximum storage volumes, and "standard-supply-curve" numbers corresponding to those shown in Fig. 15 have been resolved into the convenient curves, an example of which is shown in Fig. 23, diagram A.

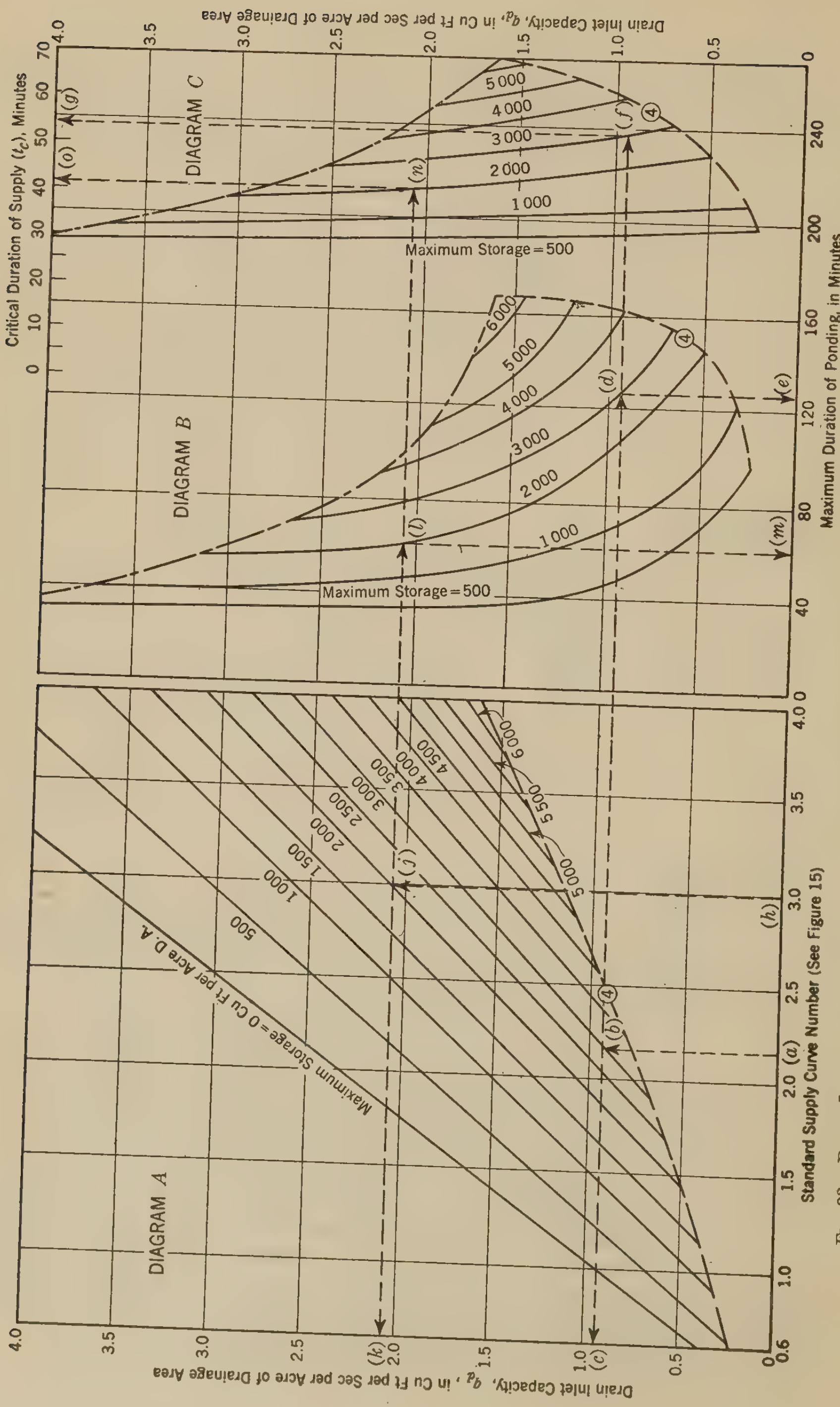


Fig. 23.—Drain-Inlet Capacity Versus Maximum Surface Storage for Turfed Areas; L = 400 Ft, S = 1%, and n = 0.40

The expression "duration of ponding" refers to a period beginning at the time the rate of storm runoff first exceeds the drain-inlet capacity and ending at the instant all the accumulated storage has been discharged (see Fig. 21). The duration of ponding corresponding to various combinations of drain-inlet capacity, permissible volume of surface storage, rate and duration of supply, and rate of runoff may be conveniently computed by use of charts similar to the one shown in Fig. 22 and the charts shown in Figs. 24 and 25.

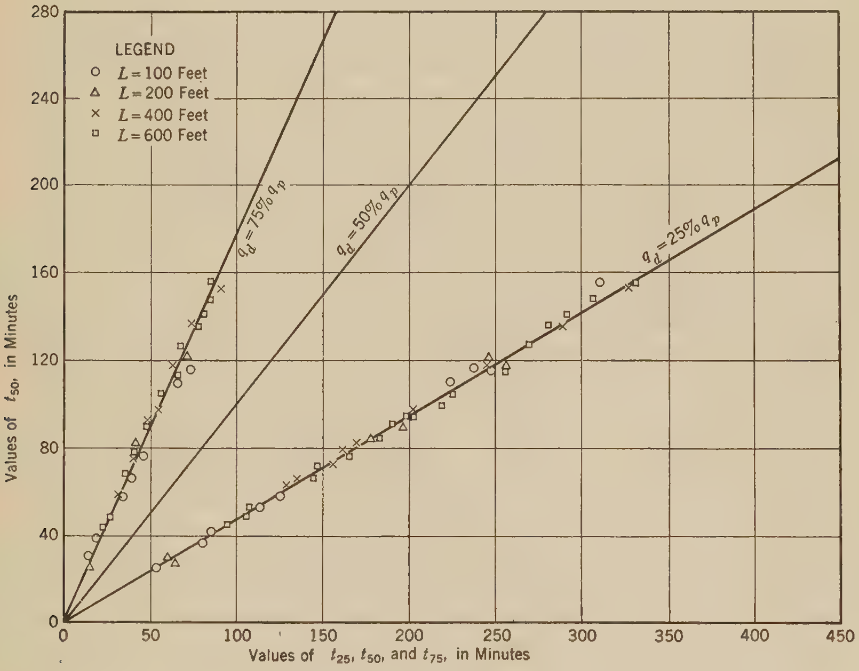


FIG. 24.—BASIC CORRELATION OF D_{50} TO D_{25} AND D_{75}

Figs. 22, 24, and 25 were included only to illustrate the manner in which the curves of diagrams A and B of Fig. 23 were derived; for example:

(*a*) The drain-inlet capacities equal to 25%, 50%, and 75% of the peak ordinates were assumed and the corresponding durations of ponding (t_{25}, t_{50}, and t_{75}) were computed by determining the length of time the drain-inlet capacity would have to remain at, say, t_{50}, in order for the accumulated volume of storage to be drawn off. This storage volume is illustrated clearly in Fig. 21.

(*b*) Values of t_{25} and t_{75}, computed in step (*a*), were plotted against corresponding values of t_{50} to determine average relationships for drain-inlet capacities equal to 25% and 75% of the peak rates of runoff for various sets of conditions (see Fig. 24). (Plotted points in Fig. 24 were computed from hydrographs similar to curve No. 2, Fig. 21, for various rates, durations of supply, and lengths *L*.)

(*c*) To facilitate interpolations, durations of ponding in minutes, scaled from Fig. 24, were plotted against drain-inlet capacity expressed as percentages of peak discharge (q_p) with values of t_{50} as parameters (see Fig. 25(*a*)).

(*d*) A working diagram (Fig. 25(*b*)) was prepared using data scaled from Fig. 25(*a*), with values of t_{50} ("scale *D*," Fig. 22) plotted against duration of ponding in minutes, with ratio (*X* of Fig. 22) of drain-inlet capacity to peak rate of runoff as parameters.

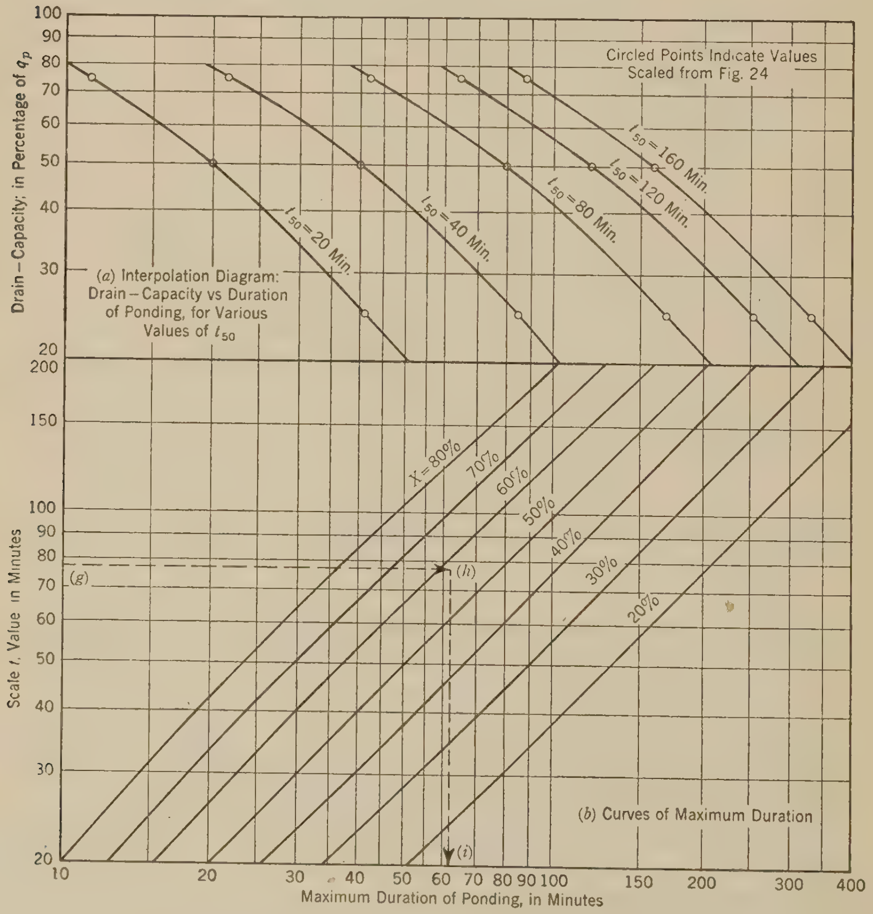


FIG. 25.—METHOD OF DETERMINING MAXIMUM DURATION OF PONDING

(*e*) Values of t_{50} were plotted in Fig. 22 against maximum storage, assuming zero drain-inlet capacity and various durations of supply to obtain the curves designated thereon by the letter *D*.

(*f*) The use of the curves shown in Figs. 22, 24, and 25 may be illustrated by continuing the example referred to previously with a length *L* of 400 ft, σ of 4 in. per hr, a duration of supply (t_r) of 38 min, and a maximum permissible storage volume of 2,000 cu ft per acre of drainage area. Using these values a drain-inlet capacity of 2.1 cu ft per sec per acre of drainage area was determined from Fig. 22. By interpolation determine the value of the *X*-ratio (59%) corresponding to point b, and record the *X*-ratio for future use. Follow the "*S-D* relation" curve to a t_r of 38 min from point b to point d; then proceed vertically to point e and read a "scale *D*" value of 78 min at point f. Enter Fig. 25(*b*) at point g with a "scale *D*" value of 78 min; follow a horizontal line to a point corresponding to an *X*-ratio of 59%; and read a value for the maximum duration of ponding at point i of 62 min on the bottom scale.

(*g*) Results obtained by the procedure outlined in step (*f*), for the critical values shown in Table 3, are tabulated under the subdivisions in Table 3 for the various "supply curve numbers" (see footnote (^{b}), Table 3).

Therefore, to determine the maximum drain-inlet capacity for various durations of supply (Col. 2, Table 3) and for various storages (Cols. 3 to 9, Table 3), the drain-inlet capacity (Cols. 3 to 9) was plotted against the duration of the supply rate in minutes (Col. 2) corresponding to the assumed "rate of supply" in inches per hour (Col. 1). The procedure may be illustrated by specific reference to Table 3: The drain-inlet capacities assuming a "standard supply curve No. 3," for example, for a storage volume of 2,000 cu ft per acre of drainage area are shown in Col. 5. These values were plotted against the duration of the supply rate in minutes indicated in Col. 2, Table 3. A critical (or maximum) drain-inlet capacity of 2.1 cu ft per sec per acre of drainage area was scaled from this curve at an approximate duration of supply of 42 min. Similarly, the critical drain-inlet capacity of 1.7 cu ft per sec per acre for a storage volume of 3,000 cu ft per acre was scaled from the curve plotted for this condition, at a duration of supply of 49 min. The critical drain-inlet capacities obtained by this procedure are tabulated under the subdivisions for the various "supply curve numbers" (see footnote (^{b}), Table 3) and for each of the storage values indicated in the headings of Cols. 3 to 9, Table 3. The results thus presented in Table 3 were plotted in diagram C, Fig. 23 (length of 400 ft), against the corresponding storages and critical drain-inlet capacities. Similar computations were made for lengths of 40, 100, 200, 300, and 600 ft, respectively, the results of which are omitted from the paper.

The use of diagrams A, B, and C, Fig. 23, is illustrated by the following example:

Given.—(1) Length *L* of turfed area = 400 ft; (2) design storm criteria as represented by "standard supply curve No. 3.0" in Fig. 15; and (3) maximum permissible storage = 2,000 cu ft per acre of drainage area.

Solution.—Enter diagram A of Fig. 23 at point h (supply rate of 3.0) and follow a vertical line to point j (storage of 2,000); then follow a horizontal line left to the margin of diagram A to point k and read the required drain-inlet capacity of 2.1 cu ft per sec per acre of drainage area (see Col. 5, Table 3). The maximum duration of ponding is obtained by following a horizontal line from point j to the right to point l (storage 2,000) and reading a value of 62 min on the bottom scale of diagram B at point m (see Col. 5, Table 3). The critical duration of supply (t_c) is obtained by continuing the horizontal line from point l to point n (storage 2,000) to read a value of 42 min at point o on the scale at the top of diagram C (see Col. 5, Table 3).

Effective Length of Drainage Area

As used in the Horton overland flow formulas, L refers to the over-all length of an elemental strip of turfed or paved surface in a direction parallel to the maximum slope. This definition of L must be liberalized somewhat when applied to an area in which a shallow large-area pond will be formed by storage of runoff during the design storm. In estimating the "effective length" for a drainage basin in which an appreciable quantity of temporary storage is permissible, the value of L adopted should be such that the rate of runoff indicated by the corresponding overland flow hydrograph would approximate the actual hydrograph of inflow into the ponding area. In such cases, the selected value of L may be considered as an index to be used in selecting the most applicable set of computation charts rather than as a definite dimension of the drainage area. With storage capacities ordinarily attainable and permissible in intermediate areas, the use of an approximate value of L will not introduce serious errors in the determination of the required drain capacities.

The value of L strictly applicable to a particular area varies as the size of the storage pond fluctuates during the course of storm runoff. As water accumulates in the relatively flat storage area during the process of storm runoff, the size of the pond increases rapidly, thus progressively reducing the distance from the edge of the pond to the outer limits of the drainage area. To simplify computation of the required drain-inlet capacity, it is desirable to select a value of L for the drainage area tributary to each inlet that most closely approximates the average during the period of critical runoff. In most cases it is satisfactory to estimate the "effective length" as the distance from the outer limits of the turfed drainage area to the mean boundary of the ponding area during the period of critical storm runoff. If the drain inlet is not located near the centroid of the turfed drainage area, the "effective length" should be estimated as the weighted average distance from the boundary of the turfed area to the mean limits of the ponding area corresponding to a depth equal to two thirds the maximum during the design storm.

As a general rule, ponding areas are located in turfed intermediate areas that are bordered by paved runways, taxiways, or paved aprons (see Fig. 26). Runoff from these paved areas must pass over turfed slopes to reach the ponding areas and drain inlets, and in so doing is subjected to somewhat the same retardance as the runoff that results from precipitation falling directly on the turfed area. Inasmuch as the time required for water to flow over paved areas is normally short (5 to 10 min), the length of the paved area can be disregarded or given very little weight in estimating the "effective length" (L) for a composite area. In exceptional cases where large paved landing mats or bare areas are involved it will be necessary to determine the modified effective length by means of the diagrams in Fig. 17.

Storm Drains

A sample layout of a storm drain system for a portion of an airfield is shown in Fig. 26. The inlets to the underground drains are located in such a manner as to permit the temporary storage of substantial volumes of runoff during intense storms without interfering with normal use of runways, taxiways, and parking areas. To prevent the concentration of an excessive quantity of storage in the lower parts of the field, the intermediate areas were assumed to be graded in such a manner as to effect drainage toward the several drain inlets and to form shallow, large-area storage basins around each inlet. By providing storage ponds around several inlets a relatively large volume of water can be stored without causing excessive depths of ponding in any area during the design storm.

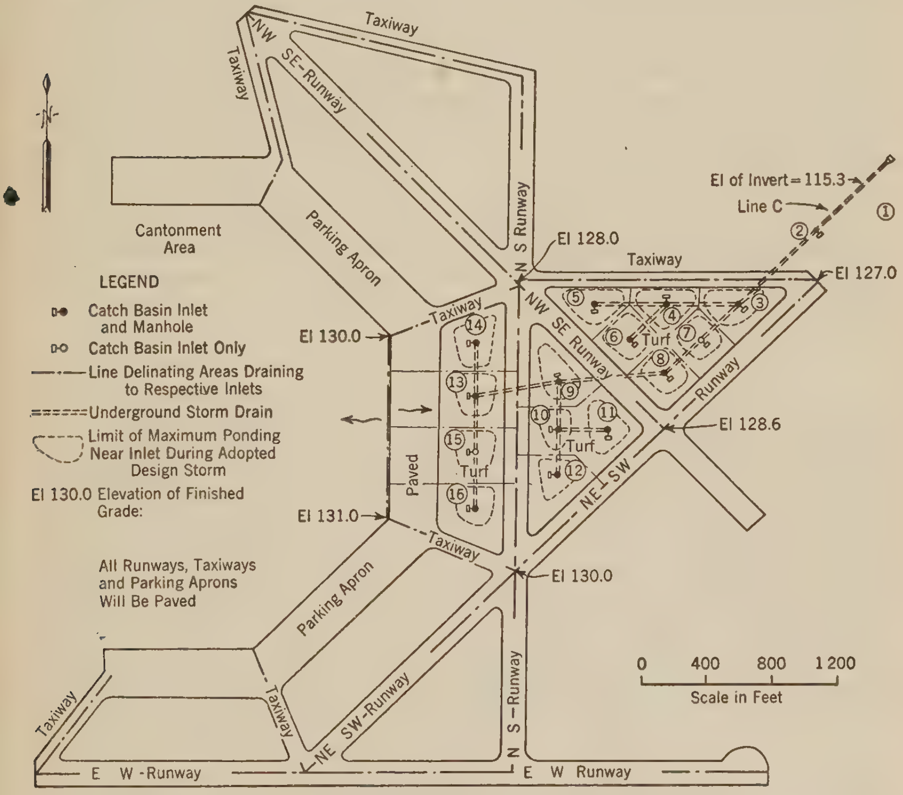

Taxiway
NW SE-Runway
Taxiway
N
Cantonment Area
Parking Apron
N S Runway
El of Invert = 115.3
Line C
①
②
El 128.0
Taxiway
El 127.0
③
④
⑤
⑥
⑦
⑧
Turf
NW SE Runway
Runway
LEGEND
Catch Basin Inlet and Manhole
Catch Basin Inlet Only
Line Delinating Areas Draining to Respective Inlets
Underground Storm Drain
Limit of Maximum Ponding Near Inlet During Adopted Design Storm
El 130.0 Elevation of Finished Grade:
Taxiway
El 130.0
⑭
⑬
⑨
⑩
⑪
El 128.6
⑮
Turf
Turf
⑫
Paved
⑯
El 131.0
NE SW
Taxiway
El 130.0
All Runways, Taxiways and Parking Aprons Will Be Paved
Parking Apron
0 400 800 1200
Scale in Feet
Taxiway
Taxiway
NE SW-Runway
N S-Runway
E W-Runway
E W Runway

FIG. 26.—LAYOUT OF PRIMARY STORM DRAIN SYSTEM

The location of drain inlets must be carefully related to the grading plan for the intermediate area if ponding areas having sufficient storage capacity to effect important reductions in required drain-inlet capacities are to be provided without creating objectionable flooding conditions or undesirable rates of change in surface grades. The elevation of the various inlets will be determined by their location with respect to runways, aprons, etc., and the limiting surface grades applicable to the area. The distance between inlets will be determined by the elevation of the respective inlets with relation to controlling elevations of runways, aprons, etc., the general slope of the field, and the limiting grades prescribed. For major airfields a maximum effective length of overland flow in excess of 400 ft (corrected to a slope of 1% and a roughness factor of 0.40) should not be permitted in areas adjacent to runways and aprons, and 600 ft in all-over fields.

The storage capacity that can be provided in intermediate areas without exceeding limiting grades in any direction is determined partly by the general slope of the field and partly by the location of the drain inlets with respect to runways, taxiways, and aprons. To estimate the maximum volume of storage permissible in a particular area, the maximum elevations of ponding allowable therein should be estimated by inspection of cross-sectional profiles, and the limits of the ponding area should be sketched on a map showing the drain-inlet layout. The surface areas of the various ponding areas may be measured by planimeter. Ordinarily, the storage capacity, in cubic feet below the elevation of maximum ponding, may be computed as one third of the maximum depth of ponding at the inlet, in feet, multiplied by the surface area of the pond, in square feet.

In general, the maximum depth of ponding in intermediate areas during the design storm should not exceed approximately one foot at the inlets if the intermediate areas are to be used for landing purposes and the upper limits of the storage ponds should not approach within 75 ft of runway pavements.

Minimum Drain Capacities

In some instances it would be feasible to provide surface ponds of sufficient capacity to store practically the entire runoff from storms less than a few hours in duration, without exceeding the maximum limits of the ponding areas. However, if the maximum permissible volume of storage were utilized in such cases, the drain-inlet capacities provided would be very small and the corresponding period required to remove the stored waters after each storm would be excessive. Furthermore, flooding of the lower parts of the ponding area would be relatively frequent; it may be practically continuous for several days as the result of a series of comparatively moderate storms. Although the area subjected to frequent and prolonged flooding may constitute a small proportion of the maximum permissible ponding area, the creation of unstable soil conditions in such areas would be objectionable in many cases.

Curve No. 4 in diagram A of Fig. 23 represents the minimum drain-inlet capacities that are considered desirable, regardless of the volume of storage capacity that may be permitted without the depth or area of flooding exceeding allowable limits. The drain-inlet capacities represented by curve No. 4, diagram A, Fig. 23, in cubic feet per second per acre of drainage area are equal to the rates of supply corresponding to a duration of 4 hr on the respective "standard supply curves" given in Fig. 15. If a drain-inlet capacity indicated by curve No. 4 is adopted in a particular case, it may be expected that some storage in the ponding basin will result during all storms less than 4 hr in duration that produce supply rates corresponding to the given "standard supply curve." The proper criteria to be followed in estimating minimum drain-inlet capacities depend largely upon the characteristics of the soil involved and the use that is to be made of the intermediate areas.

COMPUTATIONS OF STORM DRAIN CAPACITIES, ASSUMING SUPPLEMENTAL PONDING

The principal procedures involved in the determination of drain-inlet capacities required to limit storage in individual ponding areas to permissible volumes are shown by sample computations presented in Tables 4(*a*) and 5(*a*) for line C in the sample layout of Fig. 26. Assuming the project to be in north central Alabama, the following summarizes briefly the successive steps to be followed:

(*a*) Select a design-storm frequency (say of 2.2 in. per hr) approximately equivalent to a 5-yr frequency in Fig. 13(*d*).

(*b*) Estimate infiltration capacity and determine "standard supply curves" applicable to paved and unpaved areas. The infiltration capacity for a 1-hr rainfall for turfed areas in this case is assumed to be 0.6 in. per hr. The infiltration capacity for paved areas is zero; therefore the "standard supply curves" for paved and unpaved areas are Nos. 2.2 and 1.6, respectively (see Fig. 15).

(*c*) Compute "weighted standard supply curves" for composite drainage areas tributary to the respective drain inlets (see Cols. 15 to 19, Table 4(*a*)).

(*d*) By use of curves such as Fig. 23, compute the drain-inlet capacities required to limit storage above the respective inlets to permissible volumes (see sample computation in Table 4(*a*)).

In general, the methods used in current practice to design city storm sewers are applicable to the determination of pipe sizes for airfield storm drains. It is desirable to maintain the hydraulic grade line as near as practicable to the top of the drain pipe during the design storm. A minimum velocity of 2.5 ft per sec should be maintained in so far as practicable to minimize the deposition of sediment in the drains. Where ponding is contemplated the design of the storm drains should be reviewed, assuming the hydraulic gradient at the top of each inlet, to investigate the possibility of excessive flooding at any of the low inlets. If such conditions exist, the discharge from the inlets contributing to the excessive flooding should be regulated either by an orifice control in the inlet or by designing the inlet connection pipe for the design discharge reduced by ponding.

COMPUTATION OF STORM DRAIN CAPACITY, ASSUMING NO SUPPLEMENTARY PONDING

In certain instances it may be advisable to provide storm drains having sufficient capacities to remove runoff from the design storm without temporary ponding in the areas surrounding the drain inlets. To accomplish this purpose, drain-inlet capacities equal to the maximum rates of design storm runoff from areas tributary to respective inlets are required. An accurate determination of drain-inlet capacities necessary to remove design storm runoff from several inlet drainage areas without ponding entails a rather tedious procedure. Similar computations and a general consideration of the objectives involved indicate that the simple approximate method illustrated in Tables 4(*b*) and 5(*b*)

TABLE 4.—DRAIN-INLET CAPACITIES; DRAINAGE LINE C, FIG. 26

Inlet No.	DRAINAGE AREA, IN ACRES				PERMISSIBLE PONDING				AVERAGE		LENGTH L, IN	
	Paved; $n = 0.02$	Unpaved		Total	Depth at inlet; ft	Pond area; 1,000 sq ft	Volume, 1,000 cu ft	Volumes, cu ft per acre	Roughness factor n	Slope S; %	Actual, or effective	Equivalent L, for $n = 0.4$ and $S = 1\%$
		Bare; $n =$	Turf; $n = 0.4$									
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)[b]	(10)	(11)	(12)	(13)
(a) REQUIREMENTS FOR LIMITING THE												
16	3.44	....	3.44	6.88	0.9	44	13	1,900	0.21	1.0	125	70
15	3.11	....	3.20	6.31	0.9	55	16	2,500	0.21	0.8	120	70
14	3.20	....	3.53	6.73	1.2	62	25	3,700	0.22	0.9	120	70
13	3.11	....	3.20	6.31	0.9	55	16	2,500	0.21	0.8	120	70
12	2.27	....	3.20	5.47	0.9	44	13	2,400	0.24	0.7	120	85
11	1.76	....	3.28	5.04	1.0	62	21	4,200	0.27	1.0	100	70
10	0.53	....	2.02	2.55	0.9	37	11	4,300	0.32	0.8	80	70
9	2.44	....	3.28	5.72	1.0	55	18	3,200	0.24	0.8	120	75
8	1.26	....	2.27	3.53	0.9	55	16	4,500	0.26	0.8	100	75
7	0.55	....	2.35	2.90	0.9	48	14	4,800	0.33	0.8	75	70
6	0.52	....	2.44	2.96	1.0	55	18	6,100	0.33	0.9	75	70
5	1.60	....	2.44	4.04	1.0	62	21	5,200	0.25	0.6	100	80
4	0.50	....	2.18	2.68	1.0	59	20	7,500	0.33	0.6	50	55
3	1.68	....	2.69	4.37	1.0	66	22	5,000	0.25	0.8	75	55
(b) REQUIREMENTS FOR REMOVING THE DESIGN-STORM												
16	3.44	....	3.44	6.88	....	....	....	0	0.21	1.0	220	115
15	3.11	....	3.20	6.31	....	....	....	0	0.21	0.8	200	120
14	3.20	....	3.53	6.73	....	....	....	0	0.22	0.9	200	115
13	3.11	....	3.20	6.31	....	....	....	0	0.21	0.8	200	120
12	2.27	....	3.20	5.47	....	....	....	0	0.24	0.7	200	140
11	1.76	....	3.28	5.04	....	....	....	0	0.27	1.0	180	120
10	0.53	....	2.02	2.55	....	....	....	0	0.32	0.8	160	145
9	2.44	....	3.28	5.72	....	....	....	0	0.24	0.8	200	135
8	1.26	....	2.27	3.53	....	....	....	0	0.28	0.8	170	135
7	0.55	....	2.35	2.90	....	....	....	0	0.33	0.8	160	145
6	0.52	....	2.44	2.96	....	....	....	0	0.33	0.9	160	135
5	1.60	....	2.44	4.04	....	....	....	0	0.25	0.6	180	145
4	0.50	....	2.18	2.68	....	....	....	0	0.33	0.6	120	130
3	1.68	....	2.69	4.37	....	....	....	0	0.25	0.8	160	115

[a] Value of L used for selecting the proper diagram. [b] This column is not required when appreciable ponding minimum capacity criteria by curves of the type of No. 4, Fig. 23; therefore, actual volumes and depths of storage Cols. 4 to 10 for computations of t_e in Table 4(b).

USING CURVES NOS. 2.2 FOR PAVED AREAS AND 1.6 FOR TURFED AREAS)

EET	SUPPLY CURVE NO. TIMES DRAINAGE AREA					DRAIN-INLET CAPACITY			CRITICAL CONTRIBUTION TO SYSTEM			
	Paved areas, Col. 2	Unpaved		To-tal, Col. 5	Weighted curve, Col. 18 ÷ Col. 5							In-let No.
Dia-gram alue[a]		Bare, Col. 3	Turf, Col. 4			t_c (min)[b]	q_d (cu ft per sec)[c]	Q_d, Col. 21 × Col. 5	t_c (min)[e]	q_d (cu ft per sec)[b]	Q_d, Col. 24 × Col. 5	
(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(1)
ONDING TO PERMISSIBLE VOLUMES												
100	7.6	....	5.5	13.1	1.9	....	1.4	9.6	....	1.4	....	16
100	6.8	....	5.1	11.9	1.9	....	1.1	6.9	....	1.1	....	15
100	7.0	....	5.7	12.7	1.9	....	0.7[d]	4.7	....	0.7[d]	....	14
100	6.8	....	5.1	11.9	1.9	....	1.1	6.9	....	1.1	....	13
100	5.0	....	5.1	10.1	1.8	....	1.0	5.5	....	1.0	....	12
100	3.9	....	5.3	9.2	1.8	....	0.6[d]	3.0	....	0.6[d]	....	11
100	1.2	....	3.2	4.4	1.7	....	0.6[d]	1.5	....	0.6[d]	....	10
100	5.4	....	5.3	10.7	1.9	....	0.8	4.6	....	0.8	....	9
100	2.8	....	3.6	6.4	1.8	....	0.6[d]	2.1	....	0.6[d]	....	8
100	1.2	....	3.8	5.0	1.7	....	0.6[d]	1.7	....	0.6[d]	....	7
100	1.1	....	3.9	5.0	1.7	....	0.6[d]	1.8	....	0.6[d]	....	6
100	3.5	....	3.9	7.4	1.8	....	0.6[d]	2.4	....	0.6[d]	....	5
40	1.1	....	3.5	4.6	1.7	....	0.6[d]	1.6	....	0.6[d]	....	4
40	3.7	....	4.3	8.0	1.8	....	0.6[d]	2.6	....	0.6[d]	....	3
UNOFF WITHOUT TEMPORARY PONDING (ALTERNATE DESIGN)												
115	7.6	....	5.5	13.1	1.9	15	3.2	22.0	20 25 30	3.2 3.0 2.9	22.0 20.7 20.0	16
120	6.8	....	5.1	11.9	1.9	15	3.2	20.2	20 25 30	3.1 3.0 2.9	19.6 18.9 18.3	15
115	7.0	....	5.7	12.7	1.9	15	3.2	21.5	20 25 30	3.2 3.0 2.9	21.5 20.2 19.5	14
120	6.8	....	5.1	11.9	1.9	15	3.2	20.2	20 25 30	3.1 3.0 2.9	19.6 18.9 18.3	13
140	5.0	....	5.1	10.1	1.8	16	2.8	15.3	20 25 30	2.9 2.8 2.6	15.9 15.3 14.2	12
120	3.9	....	5.3	9.2	1.8	15	3.1	15.6	20 25 30	3.0 2.9 2.7	15.1 14.6 13.6	11
145	1.2	....	3.2	4.4	1.7	17	2.6	6.6	20 25 30	2.7 2.6 2.5	6.9 6.6 6.4	10
135	5.4	....	5.3	10.7	1.9	16	3.1	17.7	20 25 30	3.1 3.0 2.8	17.7 17.2 16.0	9
135	2.8	....	3.6	6.4	1.8	16	2.9	10.2	25 30	2.8 2.7	9.9 9.5	8
145	1.2	....	3.8	5.0	1.7	17	2.6	7.5	25 30	2.6 2.5	7.5 7.2	7
135	1.1	....	3.9	5.0	1.7	16	2.7	8.0	20 30	2.8 2.5	8.3 7.4	6
145	3.5	....	3.9	7.4	1.8	17	2.8	11.3	20 30	2.8 2.6	11.3 10.5	5
130	1.1	....	3.5	4.6	1.7	16	2.8	7.5	20 30	2.8 2.5	7.5 6.7	4
115	3.7	....	4.3	8.0	1.8	15	3.0	13.1	30	2.7	11.8	3

Col. 9) is permissible. [c] Cubic feet per second per acre of drainage area. [d] These values were determined by
uring the design storm would be somewhat less than the values indicated in Cols. 6 and 9. [e] See Table 5(*b*),

afford a satisfactory degree of accuracy. In order that the design capacities with and without temporary ponding of runoff may be compared for the same conditions, the drain-inlet layout presented in Fig. 26 and the design storm criteria used in the sample computation given in Table 4(*a*) were used in Tables 4(*b*) and 5(*b*). A comparison of Tables 5(*a*) and 5(*b*) shows that the maximum flow is about 189 cu ft per sec without supplemental ponding as compared to 57 cu ft per sec with supplemental ponding.

TABLE 5.—DESIGN CAPACITY OF UNDERGROUND DRAINAGE; LINE C, FIG. 26

POINT OF DESIGN			CRITICAL RUNOFF TIME,[a] IN MINUTES						
Inlet, or junction No.	Distance, in ft, from:		Critical inlet No.	Critical inlet time, t_i	Assumed velocity in pipe, ft per sec	Drain Time		Approximate t_c, Col. 5 + Col. 8	Adopted t_c
	Main outlet	Preceding inlet				From preceding inlet	Accumulated total		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(*a*) ASSUMING PONDING									
16	3,610	....	....	....	....	....	....	....	....
15	3,260	350	....	....	....	....	....	....	....
13	2,910	350	....	....	....	....	....	....	....
9	2,390	520	....	....	....	....	....	....	....
8	1,740	650	....	....	....	....	....	....	....
7	1,420	320	....	....	....	....	....	....	....
3	1,100	320	....	....	....	....	....	....	....
2	500	600	....	....	....	....	....	....	....
1	0	500	....	....	....	....	....	....	....
Branch Lines:									
14	3,260	0	....	....	....	....	....	....	....
12	2 990	0	....	....	....	....	....	....	....
11	2,990	0	....	....	....	....	....	....	....
10	2,690	300	....	....	....	....	....	....	....
6	1,860	0	....	....	....	....	....	....	....
5	1,980	0	....	....	....	....	....	....	....
4	1,540	440	....	....	....	....	....	....	....
(*b*) ASSUMING NO									
16	3,610	....	16	15	....	0	0	15	t_i
15	3,260	350	16	15	3.5	2	2	17	t_i
13	2,910	350	16	15	3.0	2	4	19	20
9	2,390	520	16	15	3.0	3	7	22	20
8	1,740	650	16	15	4.0	3	10	25	25
7	1,420	320	16	15	3.0	2	12	27	25
3	1,100	320	16	15	3.0	2	14	29	30
2	500	600	16	15	4.0	2	16	31	30
1	0	500	16	15	....	....	....	....	....
Branch Lines:									
14	3,260	0	14	15	....	0	0	15	t_i
12	2,990	0	12	16	....	0	0	16	t_i
11	2,990	0	11	15	....	0	0	15	t_i
10	2,690	300	12	17	3.0	2	2	19	20
6	1,860	0	6	16	....	0	0	16	t_i
5	1,980	0	5	17	....	0	0	17	t_i
4	1,540	440	5	16	2.5	3	3	19	20

[a] Critical time required to produce the maximum flow in the underground drain. [b] Table 4(*a*) contains discharge

A comparison of the pipe sizes required for drainage systems utilizing both instantaneous runoff and temporary ponding discloses that, in general, the system contemplating ponding utilizes approximately one third the discharge capacity necessary for the instantaneous runoff system. This apparent saving is somewhat tempered because of the slightly increased grading costs. However, it can be shown that a considerable saving will result where temporary surface ponding is used.

SUMMARY

The facilities required for reasonably adequate drainage of an airfield vary greatly and cannot be defined precisely. Within the range of economic feasibility, the design capacity of the drainage systems should be adequate to remove surface runoff from the selected design storm without damage to field facilities, undue saturation of subsoil, or significant interruption of normal

USING CURVES NO. 2.2 FOR PAVED AREAS AND NO. 1.6 FOR TURFED AREAS)

RATE OF INFLOW INTO UNDERGROUND DRAINS,[b] IN CUBIC FEET PER SECOND, CORRESPONDING TO THE VALUE OF t_c ADOPTED (COL. 10) FOR INLET NUMBERS:																Inlet, or junction No.
16 (11)	15 (12)	14 (13)	13 (14)	12 (15)	11 (16)	10 (17)	9 (18)	8 (19)	7 (20)	6 (21)	5 (22)	4 (23)	3 (24)	2 (25)	Total (26)	(1)
F RUNOFF																
9.6	...	...	...	...	...	...	...	...	...	...	...	...	...	...	9 6	16
9.6	6.9	...	...	...	...	...	...	...	...	...	...	...	...	...	16.5	15
9.6	6 9	4.7	6.9	...	...	...	...	...	...	...	...	...	...	...	28.1	13
9.6	6.9	4.7	6.9	5.5	3.0	1.5	4.6	...	...	...	...	...	...	...	42.7	9
9.6	6.9	4.7	6.9	5.5	3.0	1.5	4.6	2 1	...	...	...	...	...	...	44.8	8
9.6	6.9	4.7	6.9	5.5	3.0	1.5	4.6	2.1	1.7	...	...	...	...	...	46.5	7
9.6	6.9	4.7	6.9	5.5	3.0	1.5	4.6	2.1	1.7	1.8	2.4	1.6	2.6	...	54.9	3
9.6	6.9	4.7	6.9	5.5	3.0	1.5	4.6	2.1	1.7	1.8	2.4	1.6	2.6	2.1	57.0	2
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1
...	...	4.7	...	...	...	...	...	...	...	...	...	...	...	...	4.7	14
...	...	...	...	5.5	...	...	...	...	...	...	...	...	...	...	5.5	12
...	...	...	...	...	3.0	...	...	...	...	...	...	...	...	...	3.0	11
...	...	...	...	5.5	3.0	1.5	...	...	...	...	...	...	...	...	10.0	10
...	...	...	...	...	...	...	...	...	...	1.8	...	...	...	...	1.8	6
...	...	...	...	...	...	...	...	...	...	...	2.4	...	...	...	2.4	5
...	...	...	...	...	...	...	...	...	...	1.8	2.4	1.6	...	...	5.8	4
PONDING OF RUNOFF																
22.0	...	...	...	...	...	...	...	...	...	...	...	...	...	...	22.0	16
22.0	20.2	...	...	...	...	...	...	...	...	...	...	...	...	...	42.2	15
22.0	19.6	21.5	19.6	...	...	...	...	...	...	...	...	...	...	...	82.7	13
22.0	19.6	21.5	19.6	15.9	15.1	6.9	17.7	...	...	...	...	...	...	...	138.3	9
20.7	18.9	20.2	18.9	15.3	14.6	6.6	17.2	9.9	...	...	...	...	...	...	142.3	8
20.7	18 9	20.2	18 9	15.3	14.6	6.6	17.2	9 9	7.5	...	...	...	...	...	149.8	7
20.0	18.3	19.5	18.3	14.2	13.6	6.4	16.0	9.5	7.2	7.4	10.5	6.7	11.8	...	179.4	3
20.0	18.3	19.5	18.3	14.2	13.6	6.4	16.0	9.5	7.2	7.4	10.5	6.7	11.8	10.0	189.4	2
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1
...	...	21.5	...	...	...	...	...	...	...	...	...	...	...	...	21.5	14
...	...	...	...	15.3	...	...	...	...	...	...	...	...	...	...	15.3	12
...	...	...	...	...	15.6	...	...	...	...	...	...	...	...	...	15.6	11
...	...	...	...	15.9	15.1	6.9	...	...	...	...	...	...	...	...	37.9	10
...	...	...	...	...	...	...	...	...	...	8.0	...	...	...	...	8.0	6
...	...	...	...	...	...	...	...	...	...	...	11.3	...	...	...	11.3	5
...	...	...	...	...	...	...	...	...	...	8.3	11.3	7.5	...	...	27.1	4

values used in Cols. 11 to 25, Table 5(*a*); and Table 4(*b*), the discharge values used in Cols. 11 to 25. Table 5(*b*).

traffic. Careful consideration should be given to drainage in the preparation of grading plans for an airfield. Results of studies indicate that the standard rainfall intensity-duration curves adopted as general standards provide a satisfactory degree of accuracy for airfield drainage problems.

The overland flow equations developed by Mr. Horton provide a useful tool for the determination of the rates of runoff from paved or turfed areas that would result from continuous and uniform rates of rainfall-excess. The procedures presented herein for the design of drainage facilities for airfields utilizing surface pondage for temporary storage of runoff from high intensity storms of short duration will provide a balanced and economical design consistent with the selected design storm rainfall. The experience to be gained in the operation of the drainage systems of airfields now completed should furnish data for the improvement of present procedures.

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DISCUSSION

JAMES B. NEWMAN, JR.,[5] ESQ.—The development of aircraft, accelerated by war, requires modification of design and of construction of airports at every step. Only a short time ago the experienced airport engineer found no particular problem in the design of a runway pavement. His specialized knowledge was supported by the experience of hundreds of able highway engineers, and by years of accumulation of data resulting from traffic tests and scientific research. Today the problem is vastly different. Loads applied to pavements on military airfields have no precedent in either airport or highway engineering. The necessity for wide runways with comparatively flat crowns and almost negligible grades, coupled with a requirement of broad graded areas on either side and at either end of the pavement, renders drainage of airfields and of runways a much more complex problem than drainage of streets and highways, or even prewar airfields; and yet drainage, always of prime importance in design, has become increasingly critical.

In the face of the requirement of constant change of design, the engineer has been confronted with other unusual difficulties. The expansion of the Army Air Forces has exceeded the imagination of most men. This large striking power could be of no use unless large numbers of planes, manned by properly trained crews, are hurled against the enemy. The constant cry has been, therefore, for extensive work rapidly performed. There has been no time for exhaustive research, for the development of ideal methods, for the conduct of extensive surveys, and for examinations. The Corps of Engineers has been repeatedly required to complete super-airports in less than six months after the need for the developments first has been announced. There has been no opportunity for long deliberation on choice of materials. With the priorities system, the engineer has been forced to accept the materials, equipment, and man power available, and, somehow, to produce the required results.

That the Corps of Engineers has met the challenge of this gigantic problem is proved by the success of the construction program. The few unsatisfactory jobs have an explanation. In almost every case the result was caused by the demand for speed. Completion has been required by definite dates because units have been activated and the initiation of their training has been scheduled for those dates. Labor shortages, nonavailability of materials or equipment, rain, and freezing weather have been obstacles, but completion dates, nevertheless, have governed. In spite of these difficulties, almost without exception, the facilities have met the exacting requirements of the Army Air Forces.

It is difficult to add to the very complete outline of design and construction practices presented by Colonel Stratton. His observations are in accord not only with the practices of the Corps of Engineers, but also with Air Force policy, and are supported by Air Force experience.

Colonel Stratton has indicated the reasons for the selection of a modified California test method for the evaluation of subgrade in the design of flexible pavements. This method is subject to erratic results in tests on some of the many types of soils encountered in the United States, and further modifications of the procedure may be required in the future. The personnel of the Army Air Forces charged with the development of airport criteria and the planning of facilities is inclined to be skeptical of the California method, but has had no logical alternate to offer. More positive results might have been obtained through the use of extensive field bearing tests, but, as Colonel Stratton indicates, the time available for design and construction mitigated against this practice.

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Nevertheless, plate bearing tests have a definite place in the determination of strengths of completed pavements. The training program of the Army Air Forces is far from static. Changes in the use of facilities develop suddenly and often. Installations are now numerous enough to permit some degree of flexibility in the assignment of airfields for specific purposes. Positive knowledge of actual developed strengths of pavements would insure, to a marked degree, against unnecessarily high maintenance costs and possible occasional complete failures. These tests can be made while the airfield is in operation, and, therefore, the element of time no longer takes precedence over more exact procedure.

The drainage of airfields is a subject that cannot be too strongly stressed. There is no surer way to failure than inadequate and improper drainage for wide expanses of pavement designed for heavy loads. Ponding on runways, even to a relatively minor extent, creates an operational hazard, and may result in excessive interruptions of activity. The airport engineer must master this subject, or confine his efforts to desert areas.

Engineers who have not been engaged in the design of military airports recently may have some difficulty in visualizing the loads for which pavements are designed. The lightest pavement—that for a Class IV military airfield—compares favorably with many high-grade federal highways. Several states limit truck loads to 26,000 lb gross on four wheels and 32,000 lb gross on six wheels. Yet the Class IV airfield will accommodate nothing bigger than the single-engine trainer.

The Class III airfield, intended primarily for twin-engine trainers and transports, and for most fighter types, is designed to carry two-and-one-half times the normal highway truck "wheel load."

The Class II airfield, designed for a gross load of 74,000 lb, wheel load of 37,000 lb, will accommodate the larger four-engine cargo ships fully loaded, and bombers of the Flying Fortress class. These airfields have about the same load bearing capacity as main line railroads.

Quite obviously there is no counterpart for the Class I airfield, designed to carry the heaviest planes now in production. The designed gross load of 120,000 lb and wheel load of 60,000 lb exceed any known pavement requirements; and heavier loads are to be expected in the near future.

In order to judge just how much a load 60,000 lb per wheel is, visualize the weights involved in a test made by Colonel Stratton's organization. A large size "turnpull" with large single tires was used. The rear tires had a footprint of 750 sq in. each. The front tires on the tractor weighed 6,000 lb each and had a footprint of 1,000 sq in. The "turnpull" was loaded to the top with scrap steel; then 30-ft steel rails were piled across the top until the load was exactly balanced with 50,000 lb on each of the four wheels. The tractor could pull this load at about 2½ miles per hr over the paving to be tested, with only an occasional failure of the coupling device. If this load had been amplified by four 2,000-hp engines turning propellers at high rates of speed with the resulting vibration, the wheel loads and general effect would have been more nearly equal to the loads imposed upon runways, taxiways, and aprons on Class I airfields.

Warming-up engines, locked wheel turns, and taxying at low speeds before the wings begin to lift weight and provide buoyancy are a few of the factors never before considered by paving engineers. The use of dual tires helps to solve the tiremakers' problems, and also assists in designing adequate braking surfaces; but dual tires do not lessen the load on a pavement when weights are as great as these. These examples emphasize the need for new and advanced designs, and illustrate the reason for building taxiways, warm-up blocks, and parking aprons 25% stronger than the runways constructed to the full wheel loading. No allowances are made for landing impact.

Early pilots recited a formula for locating airports which had more than an element of truth mixed with comedy: "You find a high tension line, follow it to the intersection of a railroad and highway; then look for a river or a marsh, there is your airport." The runways were to be located so that the glide paths and approach angles passed through the oldest city landmarks, the fine old trees in the central park, or the tallest smokestacks in town. When most of these early airfields were located, flying was done in small planes by "barnstormers." The locations were cheap and they grew up like "Topsy." When new requirements arose, it was always too late to remove existing facilities to proper locations.

For modern airplanes, runways must be at least 5,000 ft long at sea level. This length should be increased with increased altitude by adding 100 ft to each runway for each 200 ft of altitude measured from sea level. Some military airplanes require a basic length of 6,000 ft at sea level. Shelters have been built for bad weather and night flying with runways up to 10,000 ft long. Different areas require different runway arrangements or patterns. The number of runways needed is controlled by wind conditions. In a few airfields, one runway is ample since the wind blows either up or down the valley. Many airports need only two runways since wind records for that area, compiled in the form of wind roses, indicate that winds blow only from four directions, each fully provided for by two runways. Most areas are amply served by three runways bearing a 60° relation to each other. A few localities definitely require four runways to cover all wind directions adequately.

Before selecting a site, the engineer should know the maximum wheel loadings and the anticipated air traffic. He should have at hand all the local wind information available. He should examine the possibility for future developments that might affect his plan. In addition, he should provide needed utilities, facilities, transportation, and all other requirements likely to influence his selection of a site. If a piece of farm land tempts him, the engineer should remember that generations of farmers have spent much time and money making the soil as unsuitable as possible for compaction and load-carrying stability. If it is level, he should remember that when Nature levels a piece of land she usually uses methods and materials which make it unsuitable for either base stability or compaction. If a parcel is level and in a cold country, the frost action may be bad, and the engineer should check all means of draining areas before selecting them. In most airfield construction the engineer has more to fear from ultimate cost of excessive fills than from large and costly cuts.

To make a proper selection he should know how many runways are needed. One will be required in each primary wind direction, or rather within 22½° of each high wind direction. Very small airplanes that would be bothered by winds blowing 10 miles per hr or less are best handled on all-over fields. Planes heavy enough to need runways can adjust themselves to fairly brisk winds crossing a runway at an angle as great as 30° off the course of such a runway. In the case of high winds, 30 miles or more, an adjustment of 22½° may be all that can be expected of either pilot or airplane, in which case a fourth runway would be required if 10% of the stiffer winds were such that three runways were not enough.

When considering runway lengths, 5,000 ft should be the minimum with 1,000-ft clear zones on each end. Some runways, plus clear zones, are 12,000 ft long. With this information, the engineer can eliminate all possible sites and start compromising, but he must not forget that the airplane should have a clear glide path not steeper than 1 on 40 from the extreme end of each clear zone and that this glide path should extend at least 2 miles beyond the extreme end of the runway clear zone. When a few possible sites have been determined, he must check the soil, the cost, and source of base-course material, in place, and, finally, the hazard removal cost and other items necessary for a decision.

THOMAS E. STANTON,[6] M. AM. SOC. C. E.—No one who has been close to the procedure for designing and constructing military airfields can fail to be impressed with the thoroughness with which the Engineering Division of the Corps of Engineers has explored the subject. The magnitude of the problem is more readily appreciated when it is realized that the Corps has been forced to anticipate the development of heavier and heavier planes without always having definite advice as to whether or not a given field, originally designed for light traffic, would be converted into a heavy bomber field as soon as completed. The cost is exorbitant and the work seriously delayed if all fields are to be constructed for heavy duty. However, a more serious condition arises when a field designed to handle light traffic is inadequate because military needs subsequently require a service too heavy for the constructed capacity.

As stated by Colonel Stratton, some of the factors that are of considerable importance in pavement design for primary highways are of negligible importance in runway pavements. For impact and loading frequency, the width of runway is insurance against too frequent or rapid repetition of loads.

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Furthermore, the relatively large tires on airplane landing gear provide a wider spread and usually a somewhat lower unit loading than vehicle truck tires, thereby permitting greater deflection of the pavement than can be tolerated in highway pavement construction.

On the other hand, the much greater total loading from the large bombers and transport planes can be charged on the adverse side of the ledger and, in the case of relatively weak subgrade soils, requires a much heavier base design than necessary for the heaviest loads permitted on any highway pavement.

Another important factor is the deflection induced by the warming of airplane engines. The assumed designed loadings are increased 25% for all areas subject to warming-up vibrational stresses. This is not a matter of serious concern to the highway engineer.

The heaviest loading permissible on state highways is 9,000 lb per wheel, whereas the designed loading for all but one of the classes of airfields is considerably greater than this amount. The only airfield loading comparable to highway loading is the Class IV field which is based on a design wheel load of 5,000 lb as against 15,000 lb for a Class III field (Table 1). Furthermore, single tires are used on airplanes of this loading, whereas most trucks carrying loads in excess of 6,000 lb to 7,000 lb per wheel have dual tires.

For the same total load, single-tired trucks have proved much more destructive than dual-tired trucks. Part of the reason for this is in all probability the more severe bending of the pavement surface for equivalent total deflections. With heavily loaded single tires, failure results under a relatively small number of load repetitions for deflections of as little as 0.03 in. to 0.04 in. Therefore, with no restriction for the loading on a single tire, except the permissive total load per wheel, designs should be based on permissible deflections of not more than 0.02 in. to 0.04 in., depending upon the flexibility of the base and pavement. However, because of wartime tire conservation measures, trucks have been forced to limit the load per inch of tire width to a much lower value than was prewar practice. Subsequent highway tests with dual tires, and tests with the larger airplane tires, have proved that substantially greater deflections (say, 0.1 in. for the wide airplane tires currently used on the larger airplanes) can be tolerated when the total load is spread over such a broad band. The permissible deflection is also dependent upon the number of maximum load repetitions during the normal life of the pavement.

W. E. Howland,[7] Assoc. M. Am. Soc. C. E.—The main purposes of this discussion are (1) to present a mathematical method for the determination of a runoff hydrograph for a rain of limited duration, and (2) to compare the hydrograph for a particular rain and area so obtained with that presented by Mr. Hathaway. The same differential equation will be used as was developed by Mr. Horton for Eq. 2, which furnishes the basic curve used by Mr. Hathaway (18). Hydrographs have been plotted from the derived formulas and are shown in Fig. 27 for comparison with Fig. 18. Curves 3, 4, 5, and 6, Fig. 27, have been obtained mathematically by the writer's methods, the basic differential equation (18) being:

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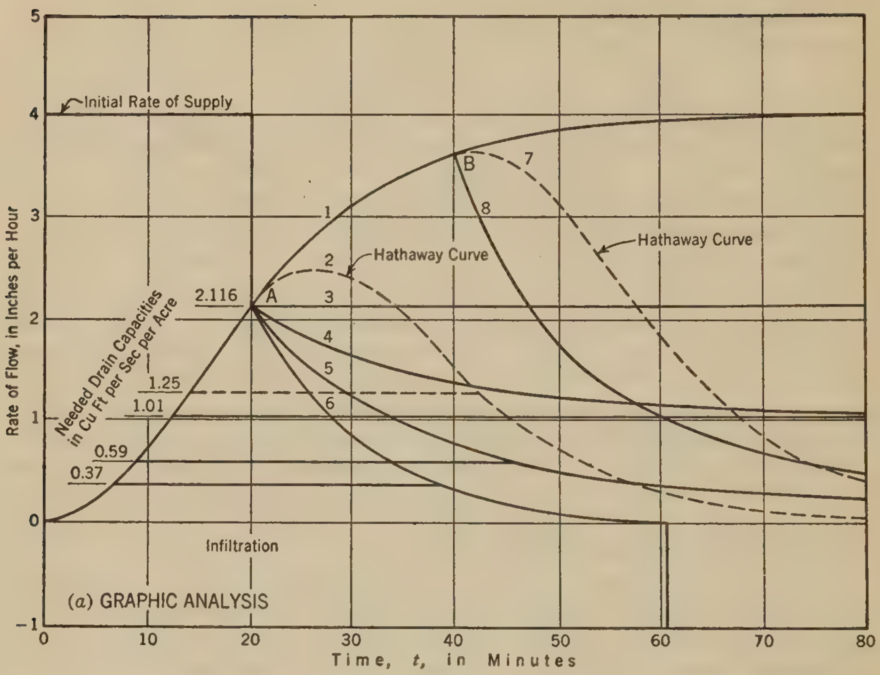
$$\sigma\, dT = K\, y^2\, dT + \tfrac{2}{3}\, dy \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Each of these is intended to be the runoff hydrograph that follows a 20-min-long rate of supply, of 4 in. per hr intensity; but the assumption as to the magnitude of the rate of supply of the rain which follows the initial 4 in. per hr rate is different for each curve. In obtaining curve 3, Fig. 27, the subsequent rate of supply which lasts indefinitely was assumed to be 2.116 in. per hr. (This is a horizontal line.) For curve 4, the subsequent rate of supply of 1 in. per hr is assumed to last indefinitely; for curve 5, the subsequent rate of supply is assumed to be zero. (It might be supposed from reading Mr. Hathaway's explanation that curve 2, Fig. 18, was also based on this assumption of a subsequent zero rate of supply.) In obtaining curve 6, Fig. 27, it was assumed that the ground would absorb water at a rate of 1 in. per hr as long as there was water standing upon it. This assumption might be stated as a negative rate of supply of 1 in. per hr for the subsequent period of ground submergence. This situation might conceivably result from a rain of 5 in. per hr lasting for the preliminary period of 20 min during which time, and also subsequently, the rate of infiltration was 1 in. per hr.

Curve 8, Fig. 27, was computed by the writer's methods for assumptions corresponding to those of curve 5, but with the runoff following a 40-min-long storm. Curve 7 has been constructed for comparison for this case using Mr. Hathaway's methods. The decrease in rate of runoff following the storm is even more abrupt in this case than for that of the 20-min storm.

The sharp break in the writer's hydrographs at the end of the period of high rainfall is the most conspicuous difference between Fig. 27 and Fig. 18. This characteristic has also been noted for comparable areas by others and is recorded in the literature (24)(25).

The effect of the different hydrographs on the estimate of needed drain capacity when storage is provided is also shown in Fig. 27. The same storage was assumed as in the example illustrated by Fig. 21 of the Symposium; and similar methods of analysis were employed. Using hydrograph 2, Fig. 27, the necessary drain capacity is 1.25 cu ft per sec per acre; but, with hydrograph 5, the needed capacity is only 0.59 cu ft per sec per acre; with hydrograph 4, the capacity is 1.01 cu ft per sec per acre; and, with hydrograph 6, the capacity is 0.37 cu ft per sec per acre. On the other hand, with hydrograph 3, the capacity becomes 2.116 cu ft per sec per acre. (The areas were measured in this analysis by a planimeter.) Thus, in this example the drain capacity requirement as determined from Fig. 18 seems about equivalent to that which might be needed for a rate of supply of 4 in. per hr lasting for 20 min, followed by a rain of somewhat more than 1 in. per hr with a rate of supply lasting indefinitely, or for a rain whose rate of supply is 4 in. per hr and lasts for 20 min, followed by another of about half that rate of supply which lasts for another 20 min and then diminishes to zero. Whether or not this example makes possible a true appraisal of the meaning of the results of Mr. Hathaway's methods would have to be determined by more extensive studies.



(b) Mathematical Analysis

Curve No.	General equation[a]	Conditions[b]		Formula for given conditions
		σ	y_a(c)	
1[d]	$q = \sigma \tanh^2 1.5\, T \sqrt{\sigma K}$	4	0	$q = 4 \tanh^2 0.0461\, t$
2	By Hathaway method	....	....	By Hathaway method
3	....	2.116	....	$q = q_a = \sigma = 2.116$
4	$q = \sigma \coth^2 \left(1.5\, T \sqrt{\sigma K} + \coth^{-1} y_a \sqrt{\frac{K}{\sigma}}\right)$	1(e)	1.578	$q = \coth^2 (1.383\, T + 0.842)$
5	$q = \frac{K}{(1.5\, K\, T + 1/y_a)^2}$	0	1.578	$q = \frac{0.85}{(1.275\, T + 0.634)^2}$
6	$q = -\sigma \tan^2$ $\times \left(\tan^{-1} y_a \sqrt{-\frac{K}{\sigma}} - 1.5\, T \sqrt{-K\sigma}\right)$	−1	1.578	$q = \tan^2 (0.969 - 1.38\, T)$
7	By Hathaway method	....	....	By Hathaway method
8[d]	$q = \frac{K}{(1.5\, K\, T + 1/y_a)^2}$	0	2.062	$q = \frac{0.85}{(1.275\, T + 0.485)^2}$

[a] In addition to the notation of the Symposium: T = time, in hours; $t = 60\, T$ = time, in minutes; q and σ are in inches per hour; q_a = flow at the end of 20 min; y is in inches; y_a = depth at the end of 20 min; and K is in (hours-inches)$^{-1}$. [b] The following constant conditions apply to each curve in this example: $K = 0.85$ per hr-in.; $q_a = 2.116$ in. per hr. [c] Initial value. [d] For all curves except 1 and 8, T and t are measured from point A ($t = 20$ min). For curve 8, t and T are measured from point B ($t = 40$ min). [e] $\sigma < K\, y^2_a$.

Fig. 27.—Comparison of Runoff Rates and Drainage Requirements as Derived by Mathematical and Graphical Methods

These curves and examples raise the question as to whether or not the curves used by Mr. Hathaway properly allow for the infiltration which certainly must continue after the storm ceases as long as the ground is submerged. The effect of subsequent infiltration is shown clearly by curve 6, Fig. 27.

Why does not the mathematical method yield precisely the same results as does the Hathaway method for a given intensity and duration of rain? To be specific—why do curves 2 and 5, Fig. 27, differ so markedly? The lack of agreement seems to rest on the failure of the unit hydrograph method employed by Mr. Hathaway to apply to the case. His method assumes that the rate of flow out of a given area which comes from a particular portion of a rainstorm is precisely the same at a given time after that rain occurred, regardless of the amount or distribution of rain that might have followed. Thus, according to this view, the flow rate out of an area from a 40-min-long rain of 4 in. per hr rate of supply would be the sum of two flow rates: (1) The flow rate that did occur during a like storm from this area 20 min after the beginning of that storm; and (2) the flow rate that would occur at a certain time from that area as the result of a storm which lasted just 20 min but which ended 20 min before that time.

This assumption would be a valid one if at the end of the first 20 min an artificial roof, with hydraulic characteristics the same as those of the ground, were suddenly placed over the area. Under these conditions the two flows could not influence one another, and the Hathaway method of adding the two known independent flow rates would be correct.

The actual situation is different. The rain from the second 20 min of the storm finds itself, so to speak, flowing on top of the rain from the first 20 min of the storm. Each layer increases the velocity of the other. (Since the depths are greater at corresponding points on the area, and at corresponding times on this area, than in the hypothetical "double-deck" area, the velocities are greater.) One might first suppose that this increased velocity would not necessarily increase the rate of runoff, since the depths might decrease correspondingly, but, when the depths decrease, the amount of storage also decreases. As a consequence, a greater amount of runoff must have occurred in a given period of time.

From these considerations it appears that the mingling of the two flows, the one from the first 20 min and the other from the second 20 min of the rain, produces rates of runoff which, for a time, are greater than the sum of what each would be if the other were not present. Thus the principle of superposition should not be used with this type of flow.

Of course, the total quantities of runoff, plus storage and infiltration, must check the total volume of supply. The writer has made sure that the areas under each curve conform with this basic requirement. The function $\int q \, dT$ is readily determined mathematically for each of the curves in Fig. 27.

Curve 9, Fig. 28, is the rate-of-supply hydrograph that would yield the Hathaway runoff hydrograph, curve 2, Fig. 18, which is also shown in Figs. 27 and 28. Curve 10, Fig. 28, is the rate-of-supply hydrograph that would yield the Hathaway runoff hydrograph, curve 7, Figs. 27 and 28. These rate-of-supply hydrographs have been obtained by the differential equation:

$$\sigma = K\,y^2 + \frac{2\,dy}{3\,dT} \qquad \text{(8)}$$

Values of $K\,y^2$ are the ordinates to curve 2 which, of course, was plotted by the Hathaway method—that is, by subtracting appropriate ordinates of curves like curve 1 from one another. Since values of $K\,y^2$ are known, corresponding values of y, and also values of $\frac{2}{3}\,y$, may be computed. These have been plotted as curve 11, Fig. 28. (Curve 12, Fig. 28, was plotted similarly, employing as a basis the ordinates of curve 7, Fig. 28.) Then the slope of curve 11 was determined graphically. These slopes constitute the values of the third term in the differential formula, Eq. 8. The last two terms in this equation were then added—with due regard to sign—and plotted as σ. The values of σ are the ordinates of curve 9, Fig. 28. (The ordinates of curve 10 were similarly obtained, using the slopes of curve 12.)

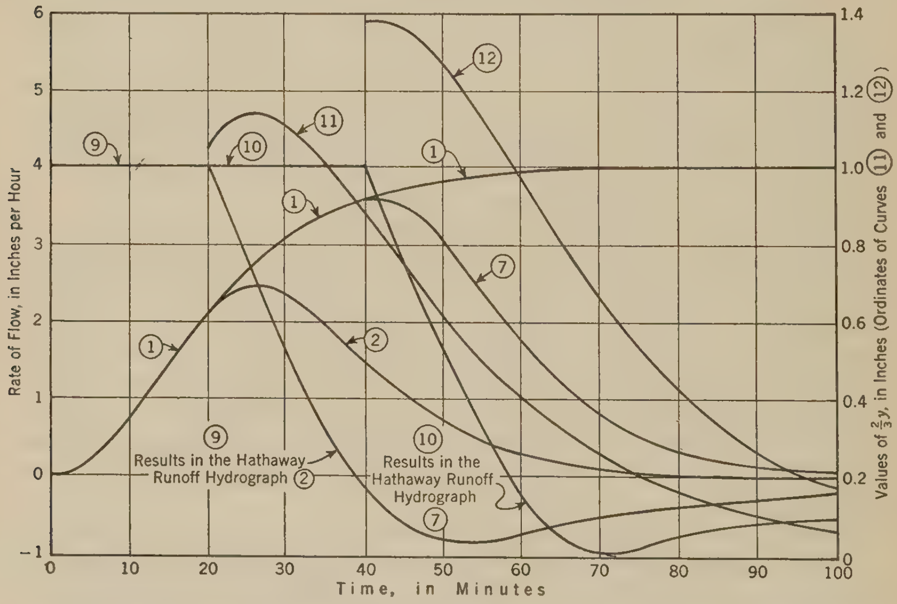


FIG. 28.—RATE-OF-SUPPLY HYDROGRAPHS (CURVES 9 AND 10) WHICH WOULD RESULT IN THE HATHAWAY RUNOFF HYDROGRAPHS (CURVES 2 AND 7)

Such hydrographs of rate of supply—namely, curves 9 and 10—might just as likely occur as perfectly rectangular hydrographs, one of which is represented in Fig. 27. Rains do not stop suddenly, but diminish slowly in intensity after their period of high rate, and sometimes are followed by a period of negative supply (or predominant percolation) somewhat as these curves might indicate. Thus the runoff hydrographs in Mr. Hathaway's paper appear correct for certain plausible rate-of-supply hydrographs which might occur, although not for the simple rate-of-supply hydrographs which Mr. Hathaway seems to have assumed.

To obtain the runoff hydrograph for any arbitrary rate-of-supply hydrograph on the basis of the Horton differential formula, Eq. 8, two methods for analysis of the behavior of flood retention reservoirs are available: (1) The mass curve method presented by H. G. Payrow, M. Am. Soc. C. E. (26); and (2) the method (27) developed by the Miami Conservancy District, Dayton, Ohio.

David S. Jenkins,[8] Assoc. M. Am. Soc. C. E.—That part of the Symposium on airfield drainage, to which this discussion is largely devoted, is a major contribution from the science of hydrology to the advancement of both civil and military aviation. Mr. Hathaway provides the airport engineer with two important new tools for the design of storm drainage systems. First, by the use of Mr. Horton's (18) equations of overland flow as applied by W. W. Horner, M. Am. Soc. C. E. (27), for related purposes, an ingenious consolidation of Mr. Yarnell's (15) rainfall data into one very useful set of curves, and some bold assumptions concerning infiltration, a greatly improved method for calculating storm runoff from airfields is devised. Second, a simple and urgently needed procedure is presented by which the actual quantities of temporary pondage in areas between runways are computed from the established hydrograph. The correct application of these two methods requires certain highly specialized knowledge of hydrology. As the airport engineer engaged in a variety of general civil engineering duties cannot reasonably be expected to acquire this specialized information on short notice, delay in the adoption of these improvements should be expected. If the Hathaway paper were to have no other immediate value than the stimulation of widespread interest in the application of hydrology to airport drainage, much will have been accomplished. Furthermore, the occasional application of the tools will be of great and immediate value in the design of the airports to which they are applied.

Differences in requirements of drainage systems for civil airports and military airfields might be expected because of the different basic purposes of the two establishments. For obvious security reasons, the military airfield must be made serviceable on short notice and must offer minimum delay from flooding, but the commercial airport may be rendered unserviceable by flooding for longer periods with only economic loss. Counterbalancing the need for greater serviceability on the part of the military airfields is the demand for greater safety in civil airports arising largely from the factor of financial liability for passengers and cargo with which the military engineer is not confronted. Because of these compensating requirements, the writer believes that the five general criteria for design of the drainage system stated by Mr. Hathaway may be substantially adopted for civil airports. Postwar economic studies of aircraft movements, cargo values, and fuel reserves are needed to provide a more adequate basis for the design, not only of drainage structures but also of pavements and other airport facilities as well.

The method developed by Mr. Hathaway for calculating storm runoff from airfields is a definite improvement over the rational method, the Burkli-Ziegler equation, and other similar formulas which have little scientific foundation.

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That part of the rainfall which does not appear as runoff is quantitative and specific, and is not a ratio of the rainfall, as represented by a coefficient. As this truth has been well established, the subject is no longer controversial. However, the new method, like the old, requires a personal estimate of critical factors based on judgment and experience. Although separate evaluations of two or more unknowns will doubtless result in greater reliability than a collective estimate of the several, as embodied in the runoff coefficient, it is questionable whether the airport engineer is sufficiently familiar with the phenomenon of infiltration and with the roughness factors applicable to overland flow to make a correct estimate of them without additional guides. Mr. Horner (24*a*) states that possibly as many as 100,000 controlled experiments are available in the records of the Department of Agriculture from which infiltration capacities and overland flow characteristics are derivable. The practical application of these data to essential uses such as the design of military and civil airfields opens a new field to those familiar with these experiments. A contribution of practical value would be made, for example, if a table were prepared from these data showing the range in infiltration capacities for different soils and cover. The data could also be used in preparing a set of photographs showing grass covers representative of the ranges in the roughness coefficient n, similar to visual guides for channel roughness proposed by C. E. Ramser, M. Am. Soc. C. E. (28). If these suggested aids are not adopted, the data should be prepared from infiltrometer tests made on a representative group of airports.

In regions of high rainfall intensity where infiltration may constitute only a small fraction of the rainfall, large errors in estimating its value may result in only minor errors in design; but in regions of medium and low intensities this is not true. In the example given in the paper, Mr. Hathaway assumes an infiltration capacity of 0.6 in. per hr. This value is within the range of infiltration capacities—from 0.5 to 1.0 in. per hr—recommended in the *Engineering Manual* (a more detailed version of the Hathaway paper (23*a*)). From the writer's personal field observation of several thousand watershed rainstorms and the measurement of these and the runoff therefrom, together with the study of similar rainfall-runoff records and infiltrometer determinations, this range appears to be too narrow and its lower limits may be too high. Much greater emphasis could be placed on the influence of grass on infiltration.

TABLE 6.—TYPICAL TERMINAL INFILTRATION CAPACITIES, IN INCHES PER HOUR

Grass cover	MERITA CLAY LOAM		ABILENE SILTY CLAY LOAM		REAGAN SILTY CLAY LOAM	
	Range	Mean	Range	Mean	Range	Mean
Poor.......	0.07 to 0.35	0.15	0.05 to 0.21	0.12	0.12 to 0.20	0.16
Good.......	0.20 to 0.70	0.38	0.23 to 1.64	0.73	0.22 to 2.65	1.26

Unpublished infiltrometer records show wide variation in terminal infiltration capacities, but define a trend indicating that the infiltration capacity of a given soil having a good turf cover is roughly twice that of bare ground and that the over-all average of all terminal infiltration capacities on bare soil is roughly 0.2, with a range from 0.02 to 1.00 in. per hr or higher. Also unpublished are the results of numerous infiltrometer tests on high-plain prairie soils of very low slopes, comparable to slopes of airports. Three of these are given in Table 6 to illustrate the broad range of infiltration capacities on some soils which may be expected by the airport engineer. The average time required to reach constant capacity in each case was as follows:

Soil	Time, in minutes
Merita clay loam	47
Abilene silty clay loam	76
Reagan silty clay loam	63

Since many airport engineers consider that the method proposed by Mr. Hathaway is not usable unless a guide for estimating infiltration capacities is provided, the tentative values in Table 7, for use in airport drainage design only, are suggested. The writer is fully aware that these values are subject to revision and that infiltration rates are influenced by many variables, but the use of Table 7 in conjunction with the Hathaway method should result in fewer and smaller errors than if no guide at all is followed. The values in Table 7 for dry antecedent conditions should be considered in regions where high rainfall intensities occur only as summer thundershowers and where wet antecedent conditions will probably not prevail at the time of the design rainfall.

TABLE 7.—ONE-HOUR INFILTRATION CAPACITIES, IN INCHES PER HOUR
(For Use Only in Design of Drainage Facilities for Airfields)

Character of entire soil[a] profile	DRY ANTECEDENT CONDITIONS		MOIST ANTECEDENT CONDITIONS	
	Dense grass	Poor grass[b] or bare	Dense grass	Poor grass[b] or bare
Tight plastic clays and silty clays	0.4 to 0.6	0.2 to 0.4	0.05 to 0.2	0.02 to 0.1
Porous loams and fine sandy soils	0.8 to 1.2	0.5 to 0.7	0.4 to 0.8	0.2 to 0.4

[a] Gravels and coarse sands not included. For pavement use zero. [b] To a small degree these values for bare soil will vary inversely with intensities. This is not true for dense turf.

In calculating airport runoff to be used in designing the drainage system, an important source of error arises from the necessity of assuming in advance that a certain grass cover will ultimately be established on the area. No opportunity is afforded therefore to examine the grass cover on which the infiltration and roughness values—or the runoff coefficient—must be based. Airport turf is generally not well established until about the end of the first year, and in many cases not for two years following the completion of grading. During these two years which represent one design rainfall cycle, the land is relatively bare, runoff is high, the erosion of pavement shoulders advances rapidly, and the silting of drain lines may be serious. For this reason, the conservative results obtained by the Hathaway method seem further justified for the protection of the installation during this initial period, which may represent as much as 20% of the life of the pavements.

The practice of subtracting the infiltration capacity from the rainfall curve without a downward adjustment of the frequency is questionable, as it implies that the resulting supply curve retains the frequency of the original rainfall. Ground moisture conditions productive of the 1-hr infiltration capacity will not prevail at equal frequency and simultaneously with the design rainfall, although that is the interpretation generally placed upon the supply curve. A factor of safety is thereby introduced, the magnitude of which varies geographically. A similar interpretation is usually applied to other runoff formulas and probably accounts for the fact that some small and apparently inadequate airport drainage systems based on the 2-yr rainfall have not been overtaxed in eight or ten years of service. If the engineer desires to base airport drainage design on the frequency of flooding (as must eventually become standard practice) and, if this design flooding frequency is two years, the selected design rainfall frequency will need to be increased, perhaps to one or even two storms per year, for all but the most important airports. Meanwhile, studies of the frequency of occurrence of selected infiltration rates and probabilities of groups and combinations of individual rainfall events should also be made from the wealth of available data previously mentioned. In any event, if an adjustment of the safety factor is necessary, it should be accomplished by revision of the selected frequency, and preferably not through modifications of other factors that might be misleading.

It is regrettable that the runoff methods proposed cannot be used for the many large intermediate airport areas having effective lengths of overland flow exceeding 600 ft, and there is some question whether the Horton equation would hold true on graded airport slopes 500 ft or 600 ft long. For applicability of that equation the surface (and adjacent surfaces) must be a true plane of such shape as to avoid channeling or accumulations of flow. Furthermore, as the relationship between length and critical time is very nonsensitive for the longer slopes, great refinement of results should not be expected in this range. For the larger areas a correct application of the rational method may be necessary, if indeed it is possible to apply that method correctly. Recent trial studies indicate that the Snyder Synthetic Unit Graph (29) may be adaptable to small watersheds of this type. Considerable success has also been achieved by numerous investigators in applying to small watersheds the "Modified Rational Method" as developed by Merrill Bernard, M. Am. Soc. C. E. (30), and others.

The method of determining the maximum volume of available pondage is not entirely clear although it is stated that this quantity depends upon land slope and permissible grades. General recommendations (31) for commercial airports of the three higher classes require that longitudinal and transverse grades should be limited to 1.5% maximum slope, and that the maximum change in grade at any point shall not exceed 2%. On the basis of these recommendations, in general, negligible pondage can be created along one drain line parallel to a runway when the longitudinal grade is greater than about 0.8% if the transverse grade is 1.5%. Fig. 26 affords favorable opportunity for the use of pondage as the longitudinal grades of the three runways are all less than 0.15%. For use as a guide in determining the maximum pondage requirements, the total runoff for each inlet can be obtained directly from Cols. 23 and 25 of Table 4(*b*).

Care should be taken to avoid the creation of ponding areas where the geologic structure is of such dip and character as to convey water from the pond to the pavement subgrade. A slight dip toward the runway with porous material outcropping in the pond and underlain by less pervious material presents a perfect opportunity for ponded water to reach the pavement base and subgrade. Borings made with this in mind will reveal such a condition if it exists.

Under "Computations of Storm Drain Capacities, Assuming Supplemental Ponding," Mr. Hathaway states:

> "It is desirable to maintain the hydraulic grade line as near as practicable to the top of the drain pipe during the design storm."

This important statement is not further emphasized, and, although this requirement has apparently been satisfied in the example, the procedure by which it is accomplished in the case of ponds has not been clarified. If the hydraulic gradient is permitted to rise to the elevation of one of these ponds, discharge from the pond will cease. Backwater will occur from the drain line into the pond if the gradient is raised further. Fig. 26 shows rectangles beside each inlet which are not discussed, but other than these the only limitation placed on the discharge rate of each pond appears to be the flow capacity of the inlet grating.

To compute the runoff for storm drainage design (32), an adaptation of a rational method (33) is used by the Civil Aeronautics Administration which has successfully pioneered in the field of airport design:

$$Q = \frac{A\,I\,R}{T + t} \quad \text{.........................(9)}$$

in which Q = rate of runoff in cubic feet per second; A = area to be drained, in acres; I = percentage of imperviousness of the area; R = maximum average rate of rainfall over the entire drainage area, in inches per hour, which may occur during the time of concentration; T = duration of rainfall in hours (assumed to be 1 hour in airport design); and t = time allowed for removal of rainfall after the end of the storm (assumed as 2 hours in airport design). The value of R is the 1-hr, 2-yr rainfall intensity for all inlets.

As this method has been used in the design of many large airports, a brief comparison of the results obtained from it and from the Hathaway method is of interest. For this comparison two of the drain lines, C and D, of the midwestern airport in Fig. 29 have been investigated. These lines were selected because they are isolated, or branch, lines carrying neither water from other inlets nor "foreign" water from outside the airport. All computations for line D, including those of pondage, exclude any provision for future grading of the unfinished area to the west. The pondage along line D used in the application of the Hathaway method is the maximum that could be provided with the given inlet elevations. Because of the high longitudinal slope, no pondage can be created along line C. The high infiltration capacity of 0.9 in. per hr was selected to correspond as closely as possible for this rainfall region to the value 0.20 for "imperviousness" used in the method followed by the Civil Aeronautics Administration (33). The roughness factor represents average grass cover. The comparison shows the runoff computed by Mr. Hathaway's method to be appreciably greater, even with ponding, than that computed by Eq. 9. Since the soil surveys indicate that the area is quite sandy, the smaller quantities shown in Table 8 are apparently adequate. If the transverse grade along line D were to be increased to about 1%, pondage sufficient to contain the total design runoff for each inlet could be provided.

Neighboring property interests must be considered in connection with drainage. These interests, which may be seriously affected by the drainage system, may in turn affect the design. The possibility that the artificial conveyance of the water from the airport and vicinity may alter the natural regimen of the outlet streams by either increasing or decreasing the quantities and rates of flow, may cause inundation of rural or urban areas, or otherwise may damage private or public property should be investigated thoroughly before work is started. In his introductory remarks Mr. Hathaway indicates

TABLE 8.—COMPARISON OF RUNOFF COMPUTATIONS FOR DRAIN LINES D AND C, FIG. 29

Inlet No.	USING[a] EQ. 9						USING HATHAWAY METHOD[b]				
	Area (Acres)		Discharge Q, in Cubic Feet Per Second				Discharge Q, in Cubic Feet Per Second				Volume of pondage (1,000 cu ft)
							Without Ponding		With Ponding		
	Paved	Sod	Paved	Sod	Total	Accumulated	Individual area	Accumulated	Individual area	Accumulated	
						(a) LINE D					
35	2.97	8.57	1.34	0.86	2.20	2.20	11.3	11.3	4.6	4.6	13
36	0.76	1.90	0.35	0.19	0.54	2.74	1.9	12.9	1.1	5.7	4
37	0.92	5.68	0.41	0.57	0.98	3.72	3.0	15.9	2.0	7.7	7
38	0.71	5.64	0.32	0.56	0.88	4.60	2.7	17.9	2.0	9.7	6
39	0.77	9.59	0.35	0.96	1.31	5.91	4.2	22.1	2.1	11.8	10
40	0.65	7.03	0.29	0.70	0.99	6.90	3.1	25.2	1.5	13.3	6
						(b) LINE C					
26	0.63	1.94	0.28	0.19	0.47	0.47	1.9	1.9	....	....	....
25	0.63	3.02	0.28	0.30	0.58	1.05	2.0	3.8	....	....	....
24	1.16	7.22	0.52	0.72	1.24	2.29	4.4	8.0	....	....	....
23	1.72	7.61	0.77	0.76	1.53	3.82	6.1	13.7	....	....	....

[a] I equals 0.90 for pavements and 0.20 for turf; $R = 1.50$; $T = 1$; and $t = 2$. [b] The 1-hr infiltration capacity equals, for pavement, 0, and, for turf, 0.90; and the value of the roughness coefficient n equals, for pavement, 0.02, and, for turf, 0.40.

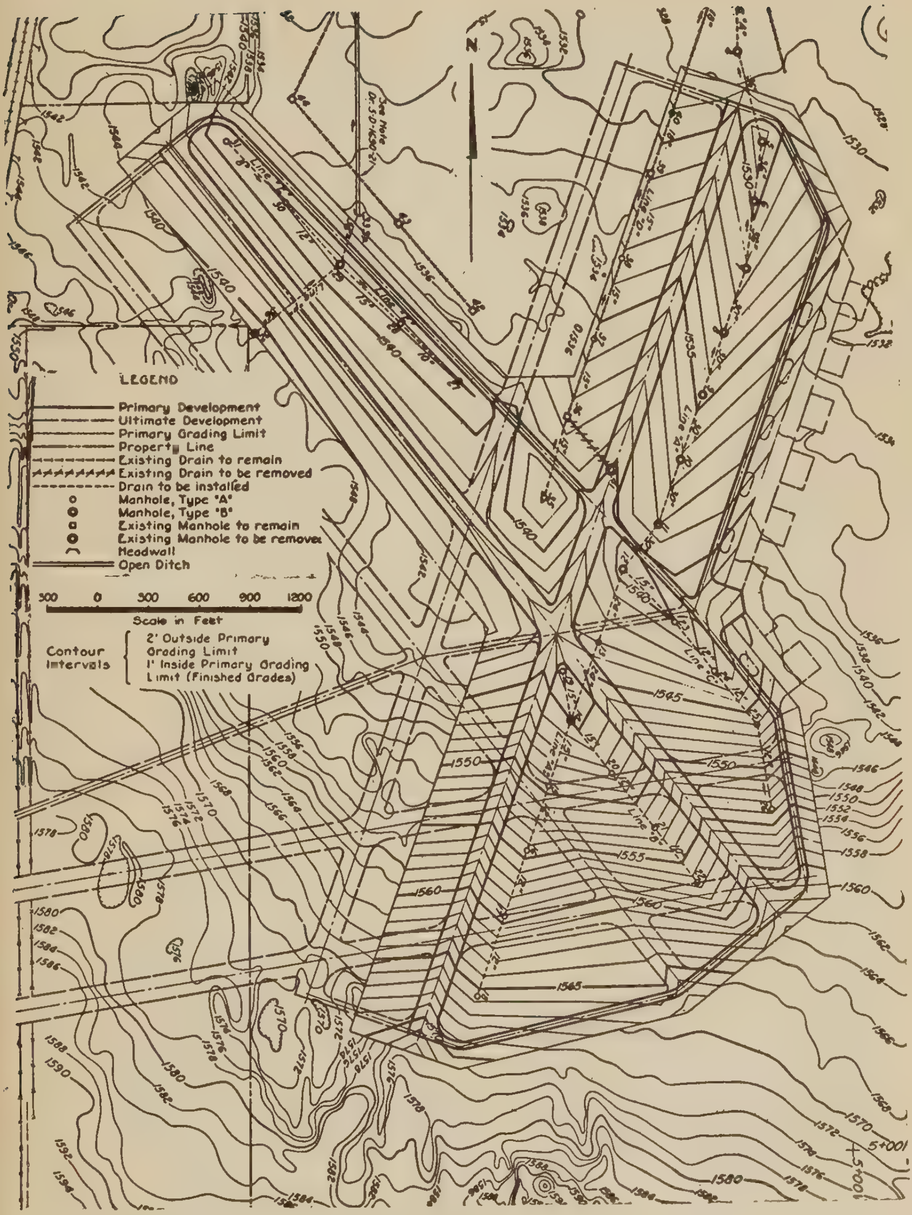
N
LEGEND
Primary Development
Ultimate Development
Primary Grading Limit
Property Line
Existing Drain to remain
Existing Drain to be removed
Drain to be installed
Manhole, Type "A"
Manhole, Type "B"
Existing Manhole to remain
Existing Manhole to be removed
Headwall
Open Ditch
300 0 300 600 900 1200
Scale in Feet
Contour Intervals
2' Outside Primary Grading Limit
1' Inside Primary Grading Limit (Finished Grades)
Line "A"
Line "B"
Line "C"
Line "D"
1530
1540
1545
1550
1555
1560
1565
1570
1580
1590
5+00

FIG. 29

that the problem of diverting water from adjacent areas will be discussed. This water from neighboring lands can usually be disposed of most economically by interception in open channels located outside the area of operations. Then, the quantity of water reaching the final outlet channel under natural conditions should be calculated. As the results may ultimately be used for various purposes, including litigation, the runoff for the 5-yr, 25-yr, and 100-yr rainfall should also be computed and a permanent record made of all assumptions and estimates. The elevations of flood water in the final outlet channel under the natural condition should be computed by using the estimated quantities together with a simple application of the backwater method (34) introduced by the late H. R. Leach. Of interest in this connection is a recent case of an airport in the arid West where, because of neighboring property interests, it has been proposed to construct on the airport property a shallow reservoir of large area with sufficient capacity to collect and evaporate the total annual runoff.

Where snow is cleared into a mound along the edge of the runway, the normal path of overland flow is blocked. The trapped water collects on the runway creating a dangerous operational hazard. Among the numerous means for disposing of this small discharge, a pre-cast concrete pipe of rectangular shape with removable lid and numerous surface inlets has proved satisfactory.

The California bearing ratio (CBR) is being used extensively in its original or modified form in the design of airport pavements. Nevertheless, there is a rather widespread misunderstanding of its purpose. As a result airport drainage is considered by some to be of little or no importance because, it is argued, flexible pavements will be designed on the basis of a saturated subgrade by the use of the CBR, and, therefore, flooding will cause no damage. This Symposium with its discussion of CBR, coupled with its emphasis upon airfield drainage, should aid in correcting this misunderstanding. For further clarification: First, the soaking procedure of the CBR does not necessarily produce saturation. Second, there is a vast difference between saturation and the surcharging of the subgrade through flooding. The construction of a stable pavement on a fluid subgrade would be virtually impossible regardless of the base course thickness, and the CBR does not anticipate this condition. Third, the principal cause of subgrade and pavement instability is the intermittent change in moisture conditions with the accompanying destructive alterations in volume and density. No better method than adequate drainage has yet been devised to minimize that change. Engineers may continue to build thicker and costlier pavements, but, regardless of thickness, strength, and cost, these pavements cannot be successfully floated on fluid subgrades or on those found where airport drainage is inadequate, which are swelling, shrinking, and frost active.

The drainage and pavement designs discussed in this Symposium as well as the plan shown in Fig. 29 are for the better classes of military and municipal airports. There is an urgent and rising demand for economy in airport construction, not only for the several thousand smaller cities but for the numerous smaller airports which will environ larger cities in the postwar era. To satisfy this demand the engineer will be compelled to seek less expensive runways and drainage facilities. The open channel will undoubtedly be used extensively for both surface and subsurface drainage, but an understanding of its limitations as well as of its usefulness is essential. The open channel may be widely adaptable if supplemented where necessary with small closed drains to remove those long depression flows that create dangerous, and frequently obscure, soft areas. Through the developments discussed in this Symposium and the current work of the Civil Aeronautics Administration and other agencies, engineers have or are obtaining much information needed for the design of pavements and drainage facilities to accommodate heavy craft at the large airports. They now must meet requirements for economy—on the smaller fields for the private flier—and to this end much investigational and development work remains to be done.

William E. Rudolph,[9] M. Am. Soc. C. E.—The engineer who has followed the development in airplane landing fields must be impressed by the thorough manner in which the Corps of Engineers has solved the two important problems of paving and drainage, as described in this Symposium.

Colonel Stratton outlines the wheel loadings (as high as 60,000 lb) on which designs of pavements for military airfields are being based. Under the heading, "Loading Criteria for Military Airfields," he states: "Loadings greater than 60,000 lb may be anticipated in the near future, but no design provision is being made yet for such loadings." The writer would raise the question: How will the pavements designed for present loadings be converted to future loadings, when and if such increased loadings are adopted? Recent news items indicate that Idlewild Airport in New York, N. Y., may have runways designed for 300,000-lb plane loads—wheel loads more than double those for Class I airfield pavements. Thus, another and even more important question applying to lengths of runways as well as loadings should be asked: When and where is the limit to be established on increases in design requirements for airfields?

Somewhere the writer has read that development in airplanes generally has been ahead of airport and airport facilities. This statement is doubtless true, but for a reason. After all, the civil engineer has had the continuous task of keeping up with the airplane designer, providing airports and facilities for the landing and the taking off of larger and heavier types of aircraft at ever-increasing landing speeds. Because early aircraft had no brakes, the airfield builder had to provide a large number of runways for landings into prevailing winds. As wing loads and operating speeds increased, he had to lengthen these runways to provide for longer take-offs and landing runs. The resulting field became more and more cumbersome. As wheel loads reached the stage at which turf fields were no longer appropriate, the engineer had to give more attention to soil stability, paved strips, and expensive drainage systems. It is small wonder that throughout the history of aviation, after several years of service, landing fields became obsolescent—too small for the kind as well as amount of air traffic using them.

Will the airfield continue to be laid out and built to fit the airplane, or will the airplane designer build his craft to fit the airfield? Economic laws will dictate the answer to this question. Already engineers are beginning to stress limitations on the size of an airport rather than increases in runway lengths and loadings. The development of catapults for aid in take-offs and of tricycle landing gear for cross-wind landings, as well as of better air and ground brakes, promises to keep the lengths of runways within the limits mentioned by Colonel Stratton; however, wheel loads might be destined to increase further as the cargo plane replaces the bomber in the postwar world.

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Colonel Stratton mentions the current rule by which runway lengths are increased 500 ft for every 1,000 ft above sea level. This necessity seldom involves important increases in airfield dimensions within the United States, but it does create problems in South America, where the mining regions and their population centers are generally located at altitudes above 7,000 ft (many above 12,000 ft). South America is far more dependent on air services for cargo, not to mention passenger and mail, transport than is the United States. Its railways and highways have not been developed to any extent in most sections, and the obstacles of rugged topography and rainy season stream flows make construction very expensive. The European interests which earlier played an important part in developing South American aviation have now been removed, and the engineers of the United States face the task of furnishing the "know how" in completing this development, particularly as applied to postwar cargo transport. The cost of landing fields with the extremely long runways required at high altitudes, located necessarily in regions of rugged topography, presents a problem which must be solved by the civil engineer and the airplane designer working together. The writer recalls an experience with airfield location in Bolivia about 1936, when serving on a committee entrusted with the task of recommending an airfield site near the City of Potosi. Two sites appeared appropriate, one at an elevation slightly below 13,000 ft, the other at one somewhat above 11,000 ft. The higher site had greater advantages—or, more exactly, fewer natural disadvantages—than did the lower, but the army officer making the final decision was interested only in the lower site. Here the additional runway length required at the higher elevation, although relatively small, was the governing factor against this location, available space within the narrow valley being scant for take-off and landing requirements in either case.

A consideration not mentioned by Colonel Stratton in connection with the choice of airfield site is concealment. The writer has read with interest a recent book (35) on this subject by Merrill E. De Longe, in which it is stated that hard surface runways are invariably the most conspicuous features of an air base. Undoubtedly the Corps of Engineers has given serious consideration to reducing such runway visibility.

RAYMOND L. IRWIN,[10] Assoc. M. Am. Soc. C. E.—The procedures recommended in this Symposium by Mr. Hathaway for the design of airfield drainage facilities are impressive. The applications of the more advanced theories on drainage, particularly those concerning infiltration and surface runoff, are, in themselves, valuable contributions toward future drainage studies and computations.

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There is an apparent need for further research to determine the basic data, which play so important a part in the development of the computations and curves. In this connection, attention is invited to the following items which are ordinarily considered essentials in drainage studies and designs: (*a*) Rainfall intensity and frequency data; (*b*) procedures for the determination of rainfall infiltration rates; (*c*) effects of surface detention storage; and (*d*) roughness coefficients applicable to anticipated grass cover.

Available records show that the runoff resulting from an excessive storm is not usually in proportion to experienced rainfall intensities. An intense rainfall which normally occurs during the summer season ordinarily provides less runoff from a given area than a storm of corresponding intensity which occurs during the winter season. There is also a perceptible difference in the intensities of rainfalls with seasonal changes. A comparison of rainfall frequency-intensity data taken for the entire year, as against frequencies and intensities experienced for the six months, November to April, was made by the writer on the basis of data obtained from rainfall records for Pittsburgh, Pa., Cincinnati, Ohio, and Cairo, Ill. For each of these stations, at least seventeen summer rainfall occurrences of 1-hr duration have exceeded the highest winter rainfall intensity for one hour, during the 35-yr period from 1905 to 1939, inclusive. It is an accepted fact also that infiltration rates for summer rainfall occurrences should ordinarily be considered greater than those for storms occurring during the winter seasons. Since infiltration rates and rainfall intensities vary with the seasons of the year, recognition of these factors might be contained in the construction of runoff frequency curves.

The values of runoff used for construction of the curves might be obtained by subtracting the infiltration rate to be applied during a given period, say, a month, from the intensity of the rainfall to be expected during that month. The data could then be combined on a yearly basis if so desired. In this manner it would be possible to refer to the frequency of rainfall contributing to runoff, which is the factor more indicative of the capacity for which protection and design are desired.

The methods for determining infiltration rates have not advanced sufficiently to furnish reliable values for use in design. After establishing infiltration rates based upon careful tests and studies, it has been found by experience that soils which appear porous sometimes are quite susceptible to clogging from the fines and vegetable matter contained in top soil and transported by the surface runoff. This action has been noted to occur particularly where the ground surface is disturbed by grading operations, prior to the establishment of a protective turf covering. The results of such experiences indicate the advisability, even for soils with high infiltration rates, of providing pipe or ditch drainage systems capable, eventually, of disposing of apparently unforeseen surface ponding.

The problem of grading to specified take-off and landing grades for all-over type airfields, although apparently more desirable for plane operations, often has the disadvantage of creating areas that result in surface detention pockets. This is usually caused by uneven settlement of ground in areas where slight slopes are constructed. In most cases, these areas are not harmful in themselves, although basic formulas utilized in drainage computations are disturbed. It seems unreasonable to demand that a contractor maintain surface grades in agreement with theoretical grades within as much as 0.1 ft or 0.2 ft. For fields where grading was considered quite satisfactory, topographic surveys have indicated the existence of these irregularities, which usually have no effect upon plane operations. On steeper slopes care should be exercised in maintaining the proper grades. Slight differences in slope may tend to collect the flow along a given line. This condition might result in channeling, which will tend toward higher velocities and erosion. As a consequence, shorter concentration periods will be applicable. If large ponding areas are involved in the process of overland flow, the concentration time of so-called overland flow apparently would be reduced because of the nearly negligible friction coefficient involved in traversing the ponded areas. Therefore, a greater peak outflow would result.

Various types of soils are encountered in airport construction. Some of these are more responsive to the growth of turf than others. To avoid unnecessary expense in treating the soil, types of grass are grown on some fields which are not practical for other fields. It has been suggested that, if the ground will not grow grass, weeds should be grown to obtain a protective covering. The writer is not familiar with any specific project in which the planting of weeds was found necessary.

For the various conditions of grass cover, which differ widely in structure and texture, difficulty will occur in the evaluation of the roughness coefficients. The Manning formula, used for determining flow, indicates that, for a given depth, the discharge varies inversely with the roughness factor. The roughness factors recommended by Mr. Hathaway range from 0.2 to 0.8. The corresponding discharge values for a given depth, other factors remaining constant, would therefore have the ratio of 4 to 1. Roughness coefficients are frequently selected under the guidance of judgment and experience. Therefore, the values selected might be considered questionable until sufficient supporting data and a dependable procedure are made available for practical application.

Experiments by W. O. Ree (36) on shallow flows over grassed slopes indicate that, after depths of flow of 2.5 in. were exceeded with certain grass cover, the roughness coefficient tended to decrease with increase in depth. At a depth of approximately 5 in., the roughness was less than 80% of that at 2.5 in. (Since runoff velocities for grass are very low, the aforementioned depths are feasible for long slopes.) The change in roughness may be caused by arrangement of the grass blades or reduction of the wetted perimeter by the force and weight of the water. A finer type of grass apparently will be quite sensitive and will cause appreciable changes in discharge at shallower depths. Here again experimental information seems necessary before adequate data can be secured to permit accurate reproduction of profiles for overland flow.

The rates of overland flow corresponding to standard supply curves, Fig. 19, do not represent the absolute maximum rate of runoff for a storm of a given frequency if the rainfall sequence is disturbed. Mr. Hathaway states that the curves of Fig. 19 are not hydrographs, but represent the peak rates of runoff from individual storm events of various durations, all of which have the same average frequency of occurrence. At the beginning of the design storm, which is of uniform rate, the ground surface is free from flow; therefore, storage for overland flow does not exist. This does not represent the usual condition experienced preceding high rainfall intensities. Runoff occurring from the lighter rainfall immediately before the design intensities will often displace a part of the surface detention storage.

In the interest of determining the maximum rate of runoff that could be obtained for a hypothetical storm containing runoff intensity values corresponding to a given supply curve, the writer made the following analysis: Rainfall intensity values corrected for infiltration were obtained from the standard supply curve 1.6 for 5-min periods, and their values were arranged in ascending order. These values were routed graphically through the storage occupied in overland flow, by the method suggested by W. W. Horner, M. Am.

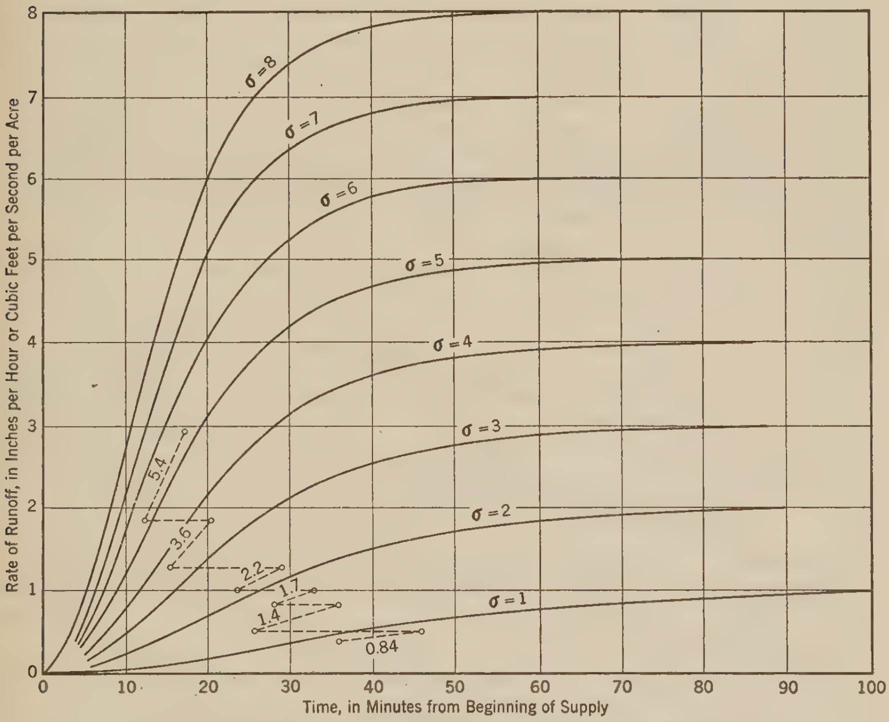

FIG. 30.—ROUTING WITH INCREASING EFFECTIVE RAINFALL RATES

Soc. C. E., and S. W. Jens, Assoc. M. Am. Soc. C. E. (24), using the relation $q = K y^2$. In brief, on the rising side of the hydrograph, the volumes of surface detention or storage for two different but uniform net rainfalls are considered equal, when the runoff rates are equal. Therefore, when changing from a lower to a higher rainfall rate, the discharge rate obtained at the end of the time for the first selected rainfall intensity is proportional to the storage and can be transferred to apply, at an equal discharge rate, to the second rainfall supply curve. This curve is then paralleled for the desired interval of time to obtain the runoff at the end of time for period two. Fig. 30 indicates the steps taken for routing with L = 400 ft and the intensity values obtained from the 1.6 supply curve. The maximum runoff rate obtained by this method was 2.9 in. per hr as compared to a maximum value of 1.75 in. per hr, furnished by Mr. Hathaway's procedure. Fig. 19 indicates a maximum runoff rate of 2.3 in. per hr for supply curve No. 2.0 and L = 400 ft. Although the former value of 2.9 in. per hr represents an extreme condition, but not necessarily the result of a true frequency, it was presented to demonstrate the wide deviation in results possible from the same indicated frequency storm with a different arrangement of the rainfall. Since storms of lower intensity occur more frequently, they should produce a runoff of, say, 1.75 in. per hr where the true frequency would apparently be less than indicated by Mr. Hathaway. There is a greater possibility that the runoff intensities are higher for the shorter lengths of overland flow than for conditions involving travel over the longer lengths. The extreme for minimum flow could be reached by reversing the rainfall distribution, under which condition a value of approximately 1.2 in. per hr would obtain. A careful study of critical rainfall distribution deserves investigation to determine critical conditions for the area under consideration.

A graphical solution used by the writer to determine storage and drain-inlet capacity may be of interest. Where the storage is very small, the rainfall distribution is arranged for maximum conditions as stated in the foregoing paragraph, using descending values of intensities. The method proposed is approximate only and consists essentially of the following:

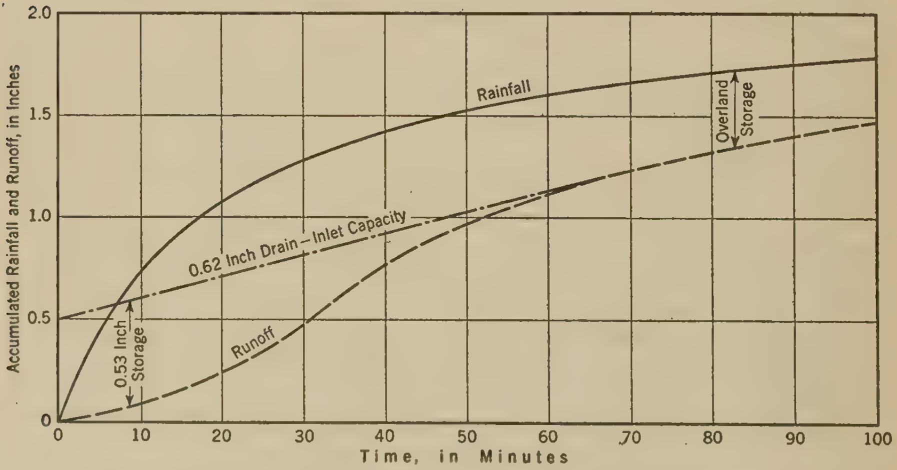

FIG. 31.—GRAPHICAL SOLUTION FOR DETERMINATION OF DRAIN-INLET CAPACITY, FOR STANDARD SUPPLY = 1.6; L = 400 FT

(*a*) Construct a mass curve of rainfall versus time corrected for infiltration rates (see Fig. 31). The rainfall intensities for short intervals of time are taken from the supply curve of Fig. 15 and are arranged in descending order to obtain minimum overland flow storage at the time of maximum ponding.

(*b*) An outflow hydrograph is constructed in accordance with the method previously outlined for the rising side of the hydrograph. When the outflow reaches a peak by this procedure, it is assumed that the outflow or net runoff for the recession side of the hydrograph is equal to the rainfall rate minus the infiltration rate.

(*c*) The storage for overland flow is then obtained from the relation $q = K^2$, in which the symbols are the same as in the Symposium.

(*d*) The instantaneous storage produced by overland flow is subtracted from the mass rainfall. (The outflow hydrograph was not constructed to balance the inflow graph.) The procedure gives a mass runoff curve which is somewhat conservative.

(*e*) By the method of rate lines similar to those used for the solution of water-supply storage problems, the slope of the tangent to the mass curve indicates the drain-inlet capacity. The required storage value is given by the greatest vertical distance from the rate line to the runoff curve.

If accepted, the overland flow curves developed by Mr. Hathaway may be used advantageously in combination with the well-known rational method to determine the flow not affected by ponding. A differential of 10 min or even 20 min in the application of the overland flow curves for the longer lengths of travel is indicated by this method to differ only slightly from values obtained by the procedure developed by Mr. Hathaway. This is true provided the error is on the long side of the time or somewhat past the t_l-value. For the 1.6 supply curve and lengths ranging from 150 to 600 ft, an error of 20 min will not affect the results of computations by more than 16%. In view of this fact, the flow at junction chambers might be obtained accurately enough by direct addition of the critical branch-line values, with adjustments for runoff from paved areas, provided that the entire drainage system is relatively short. In terms of distance a 10-min interval considered for time of flow through a closed conduit, with an average velocity assumed as 3.3 ft per sec, would constitute a travel distance of 2,000 ft. Following this procedure with consideration for junctions of long lines the design capacities can be computed or checked quite rapidly.

At some of the more recently constructed airfields, recording raingraph and recording hydrograph instruments have been installed to furnish accurate records and to present a sound basis for comparing design values with those actually obtained in nature. It will be quite some time before sufficient data are collected, and before satisfactory explanations concerning runoff can be attempted. However, strides are being made in what appears to be the proper direction toward better design for airfield drainage.

G. G. GREULICH,[11] M. AM. SOC. C. E.—As repeatedly indicated in this Symposium, the effects of moisture changes have a great influence on the subgrade soil and base courses. Moisture accumulation, in particular, affects subgrade deformation, remolding, pumping at joints, and density or volume changes due to frost, heaving, and thawing. Deterioration from these influences makes the determination of subgrade ratings by plate bearing or California bearing ratio (CBR) tests a matter of guesswork where year-round performance predictions are necessary.

These factors seem to indicate the need for more attention to subsurface drainage systems. Lowering the ground-water level by adequate drainage may give greatly improved performance at much less cost than can be obtained by thickening pavements and base courses and by compacting the subgrade. Every one is familiar with the crushing strength of dry, as compared to saturated, soils.

Accumulation or ponding of water during construction should be prevented as much as possible. Water seeping through pavement joints should be able to flow readily to subgrade drains. This will prevent excessive moisture accumulations in the soil at points where the surface slabs are weakest and where the greatest relative vertical movement of adjacent slab edges and attendant reworking or churning may result.

In mountainous or foothill country, and even in gently sloping plain country, there may be subsurface flow or seepage of ground water which normally does not approach the surface near enough to affect stability or carrying capacity of the upper soil layers. However, during wet seasons, this flow may occur very near to, or even emerge at, the surface. Under such conditions, during or immediately following the wet season, subsurface drainage will take care of most of such flow and prevent serious changes in the moisture content of the subgrade soil.

Under the preceding conditions, the most excellently designed and functioning surface drainage systems cannot prevent saturation from subsurface flow originating at points that may be remote from the airfield. Infiltration of water in surrounding areas may also accelerate flow under the surface of the airfield.

Whereas inadequate surface drainage may hamper operations only for a limited time, inadequate subsurface drainage may reduce the usefulness of a field for a long period and may even render it completely unserviceable, particularly as more planes with Class I wheel loading come into service.

An extension of the subject of subgrade drainage in the closing discussions would provide a most welcome contribution to general engineering knowledge on the design of military airfields.

HIBBERT HILL,[12] ASSOC. M. AM. SOC. C. E.—The influence of load repetition on the life of pavements is emphasized by Colonel Stratton. This effect is very evident in the accelerated traffic tests described in his paper. In these tests a pavement characteristically will show no distress whatever during the first passages of the load, and yet may fail completely (and progressively) after a relatively small number of passages. The same phenomenon is a matter of common experience to paving engineers, although not usually so readily observed in practice, because of the length of time involved, as in tests.

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The accelerated traffic tests progress from the condition of newly completed construction to total failure in a matter of days or weeks. The condition of the pavement base and subgrade is thus substantially "as built" during the course of the test. On the other hand, perhaps a runway will not be subjected to an equal number of stress repetitions for years. During this time, and in addition to other time effects, moisture conditions in the base and subgrade may have changed, frost may have acted on the base and subgrade, and the paving itself may have deteriorated because of factors other than load stresses.

Thus, in translating accelerated test data into terms of runway pavement life, two questions at once arise: (1) To what actual number of stress repetitions will the runway pavement be in fact subjected? (2) What is the relation between the effects of rapid repetitions of load in an accelerated test and the effects of the much more widely spaced (in time) repetitions to which a runway will be subjected in operation?

Existing data and calculations clarify the first question considerably. The second question can be disposed of rapidly. The answer is not known at the present time. It can only be assumed, on the basis of general experience, that repetitions in one case are approximately equal in effect to repetitions in the second case—that is, if under accelerated conditions a pavement fails at the end of one thousand load repetitions taking place in say ten days, it will also fail at the end of one thousand load repetitions spaced over a much longer period.

In accelerated traffic tests, a load is made to pass repeatedly over a lane of given width in such a manner that the wheel tracks uniformly cover the surface of the lane; for example, assuming a 5-ft lane and a tire imprint width of 1 ft, the first passage is in the 1-ft strip along one edge of the lane, the second passage is in the strip between 1 ft and 2 ft from the edge of the lane, and so on, until, in five passages of the wheel, the entire lane has been covered. If the tire imprint is narrower or wider than this example, a greater or lesser number of passages is necessary to cover the lane. Such a series of passages has been regarded, in the tests, as one coverage, or one maximum load repetition. Load repetitions are reported in terms of number of coverages in lanes of a given width.

Thus, if, in the case of an operating runway, a longitudinal element of the 10-ft-wide runway is considered and it is known that this 10-ft element has been traversed one hundred times by a wheel with a 6-in.-wide footprint, then it can be assumed that the 10-ft strip has experienced five coverages, or five maximum load repetitions. Such an assumption is only valid if the 10-ft strip can be considered as a small element of the entire width. The same assumption could not be properly made in the case of a taxiway, where the narrow pavement would force traffic into narrower lanes.

The usual military runway is 150 ft wide. On such a runway it may be assumed that the pilot will plan to land the center line of the plane on the runway center line and that deviations from this plan will be accidental in a statistical sense. Thus, the distribution of landings over the runway width will approximately conform to the normal law of error, and the maximum number of coverages will be on the central element of the runway.

A study of the possibilities leads to the conclusion that from 16% to 24% of the landings (center line of the plane) will be on the center 12.5-ft strip of a 150-ft-wide runway if there are no constant factors tending to channelize the traffic. Fig. 32 shows the actual distribution of landings on a particular runway used exclusively by fighter planes. These data were obtained by counting tire marks on a concrete runway after about six months of service. Since it is presumed that there are two tire marks per landing, the percentages of landings and of the tire marks are approximately equal. Actually, the tire marks may exceed twice the number of landings, since the plane may bounce in a poor landing, leaving two or more sets of marks.

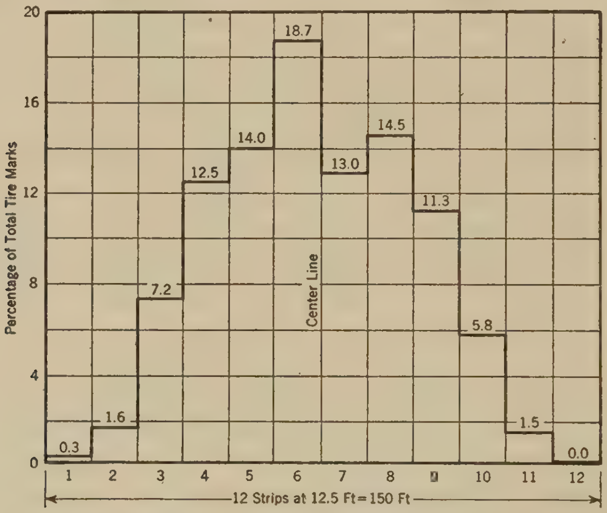

FIG. 32.—PERCENTAGE OF TOTAL TIRE MARKS IN EACH STRIP WITHOUT RESPECT TO LONGITUDINAL DISTRIBUTION

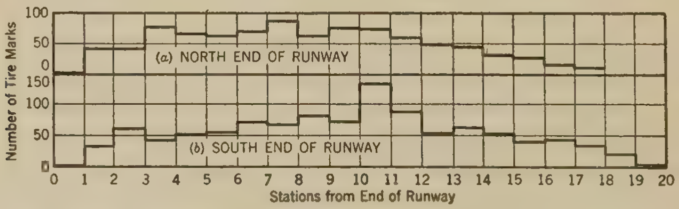

FIG. 33.—NUMBER OF TIRE MARKS IN EACH TRANSVERSE SECTION BETWEEN STATIONS

The total number of tire marks counted to obtain Fig. 32 was 2,010. Of these, 1,132 marks were counted at the south end of the runway and 878 marks at the north end. Longitudinally (Fig. 33) the marks are rather well distributed over about 1,500 ft of each end of the runway, but with a maximum point about 1,000 ft from the end.

Each landing loads two points on the pavement, one point under each wheel (the nose or tail wheel load is small and may be neglected). The total number of load repetitions is therefore twice the number of landings. Fig. 34 represents one hundred landings distributed over a 150-ft-wide runway in accordance with the normal law of error,

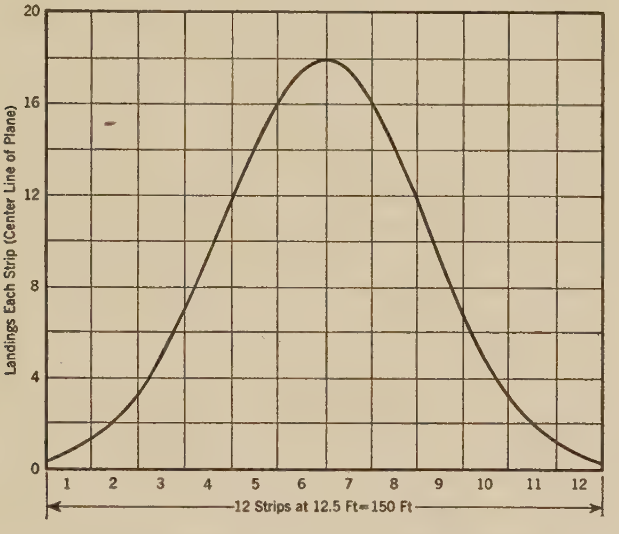

FIG. 34.—DISTRIBUTION OF ONE HUNDRED LANDINGS

$$y = K e^{-h^2x^2} \qquad (10)$$

in which $h = 0.1$. The curve represents the locations of the center line of the plane during the various landings. The wheels of the plane are displaced from its center line by an amount in general dependent upon the size of the plane. In Fig. 35, this displacement is assumed to be 12.5 ft (25-ft spread between wheels), representing a large plane. The number of wheel passages in a given strip then is the sum of the ordinates of the two curves in Fig. 35, representing the individual wheel positions. The maximum ordinate of the resultant curve is less than twice the maximum ordinate of the base curves, varying with the magnitude of the wheel spread. This difference increases as the wheel spread increases and is in general significant only for the largest planes. In the example shown (Fig. 35) the maximum ordinate of the resultant curve is 10% less than twice the maximum ordinate of the base curve.

The most frequently loaded strip will be at the center of the runway, and, for practical purposes, the number of passages over a given width of center strip will be twice the number of landings on that strip. The number of load repetitions (coverages in the sense of this discussion) will be the number of passages over the strip multiplied by the tire imprint width and divided by the width of the strip considered. The latter factor is of great importance. The width of the imprint of a fighter plane is about 8 in., and that of a bomber is about 16 in. The bomber, therefore, occasions about twice the number of load repetitions that the fighter causes during the same number of landings.

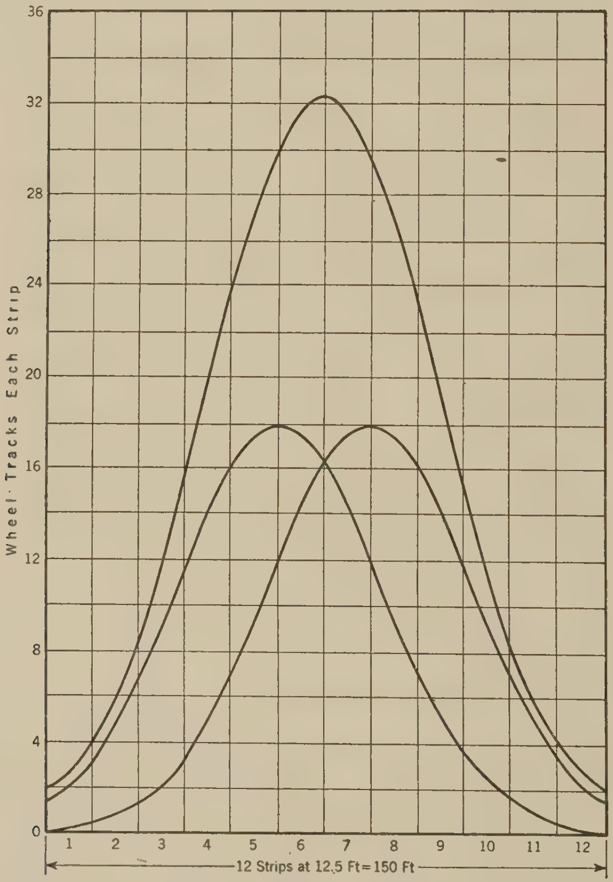

FIG. 35.—DISTRIBUTION OF WHEEL TRACKS; ONE HUNDRED LANDINGS

This discussion relates entirely to landings. Similar considerations will apply to take-offs. Probably take-offs, being more closely under the control of the pilot as to location on the runway, will tend to channelize to a greater extent than do landings. Other factors being equal, the take-off will be approximately along the center of the runway. However, the latter statement would not apply to wide runways; nor does it apply to small planes, which will tend to take off along the side of the runway that is first reached from the taxiway. Thus, local conditions, such as the location of the taxiway exits, will tend to channelize traffic on take-off. Fighters taking off in formation will deliberately use the sides of the runways. At night the pilot will follow the side (left side for large planes) of the runway where he can guide himself by the runway marker lights.

A plane on landing ordinarily first touches the pavement within the first thousand feet of runway. At this point, and for perhaps 1,000 ft beyond, the plane is substantially air-borne, and therefore does not cause maximum load on the runway. The maximum load on the pavement will not be felt until the speed has reduced to that of taxying, and this load will be imposed on a length of runway dependent on the location of the exit taxiway. Similarly, a plane taking off will be substantially air-borne after about 1,000 ft of run. Therefore, the take-offs cause maximum stress to the runway only at the end of the runway from which they occur and the landings only at the opposite end. Each end thus receives 50% of the stress repetitions due to the total operations, regardless of the wind rose. This can be seen readily in the extreme where all operations are in one direction. In such a case one end of the runway will be stressed to the maximum by every landing, but by no take-off, whereas the opposite end will be stressed by every take-off, but by no landing. Dependent, again, on the field layout and on the conduct of local operations, a central part of the runway will receive only accidental and occasionally maximum stresses.

The foregoing considerations have not much influenced airfield pavement design to the present time. As Colonel Stratton states, however, designs of the Corps of Engineers have been influenced to the extent that heavier pavement is provided on taxiways and on aprons (where taxying or standing traffic is channelized) than on the runways proper. The Corps of Engineers considers the effect of repetitions on the life of the pavement in designing advanced airfields for temporary operation. Since such considerations may have considerable economic importance in connection with the heavy construction required for present and future large airplanes, they should receive increasing attention.

JACOB FELD,[13] M. AM. SOC. C. E.—Solutions for practically every design and construction problem which arises in the performance not only of military but of commercial airfields are given by Colonel Stratton in this Symposium. A few comments from the construction point of view are made not as a criticism of the material in the paper but as a possible explanation for the manner in which the design problems have been solved within the limitation of construction feasibility.

In standard sections of concrete pavement, most of which (because of the priority regulations affecting the use of steel reinforcement) are practically devoid of any steel, the thickening of edges has introduced difficulties. Construction of an airfield is always a large-scale operation. The preparation of the subgrade and the application and the compaction of the sub-base—both must be done in large areas if uniform results are expected and if the work is to be done economically. The requirement that the subgrade of the concrete be shaped to provide for thickened edges, especially if the thickened edges occur adjacent to the forms, introduces a new operation which disturbs the compacted subgrade. If the compacted material is removed by the blade of a power unit or even if it is removed by hand, the area must be extended sufficiently to provide space for setting up forms. It is impossible to cut away a wedge-sectioned strip of a compacted sub-base without some disturbance. It is almost impossible to reroll or recompact the irregular shaped subgrade to a density equal to that of the undisturbed adjacent material. In addition, when the concrete is being placed, the vibration required adjacent to the forms to eliminate honeycomb further disturbs the density of the subgrade. Such disturbance may be the cause of honeycomb often found in concrete in the vicinity of the forms, since the reduced density of the subgrade tends to absorb and remove the mortar from the concrete placed on it. Except, of course, for the necessity for hand work in shaping the subgrade, the thickened edges at joints transverse to the direction of the forms are less objectionable from the point of view of the aforementioned difficulties.

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In scheduling the concrete pavement, there is always a difference in opinion as to sequence of lane pouring. Alternate lane construction has the advantage that it releases the forms faster, but as Colonel Stratton states it has some serious disadvantages because of the disturbed subgrade and the possibility of ponding of rain water between completed strips. These objections are overcome by pouring around completed strips, and, if the length of the strip is properly correlated with daily progress, a satisfactory schedule can be maintained. By permitting a time gap after the first strip, used to fill in other concrete operations, continuous operation in placing additional strips can be scheduled by adding strips to the first one poured in a continuous and circular sequence. If the length of the strip is made equal to a 2-day capacity of the unit used for placing concrete, no time loss will occur. After the second strip is started (the concrete being properly set up to support the finishing machine), the operation is away from the completed concrete at all times, and subgrade preparation is never fenced in by completed work.

Under the heading, "Pavement Design for Frost Action," Colonel Stratton states: "* * * the combined thickness of pavement and base material not subject to frost action is made equal to the average depth of frost penetration as determined from local records." This statement should be modified by considering the different heat transmission coefficients of materials like coarse sand or gravel which are ordinarily used for base course, as against the much finer and more easily saturated local materials displaced by the base course.

The writer has noticed that the compaction of added base-course material to specified densities, greater than the natural densities of the immediately underlying material, presents a temporary condition of unbalance which proves a weakness under the impact of traffic. Sudden change in density within the soil body seems contrary to natural conditions and should not be specified. The tendency to readjust density with time is aggravated by traffic vibration when adjacent horizontal layers are of different densities. There is a very similar, and to some extent more serious, adjustment when trenches dug in the natural soil are backfilled to a specified density greater than the density of the undisturbed soil. Such backfilled drainage ditches form a network of relatively hard intrusions in the natural soil condition and almost always cause trouble in the finished pavement.

Fig. 36 is plotted from data taken from Fig. 5, and shows a linear correlation between percentage of unit supporting value and the ratio "perimeter: area" of the test bearing plates smaller than 30 in. in diameter.

In the part of Colonel Stratton's paper on drainage, the entire discussion of subsurface drainage is based on the assumption that the pavement is perfectly impervious. If any rain water seeps through the joints or through cracks into the sub-base or if the underlying base course is saturated and at times possibly below ground-water level, a positive subsurface drainage should be provided. In installations where a porous sub-base is installed on an impervious soil, ponding-of-water accumulation in the sub-base would cause failure of the pavement by its effect on the subgrade material.

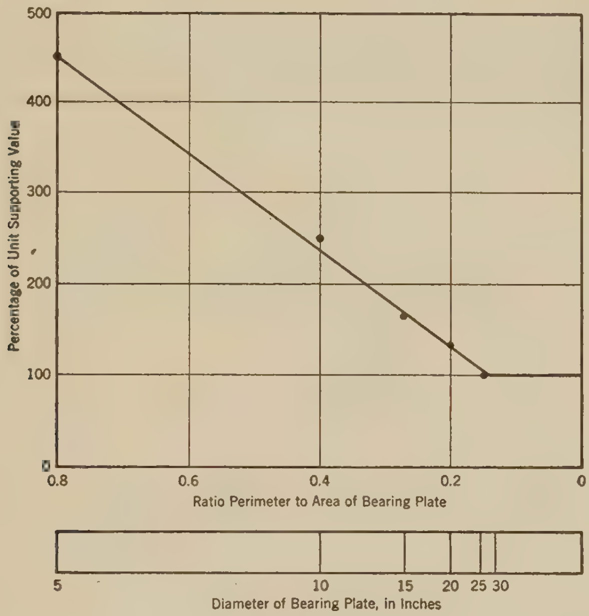

Fig. 36

The system shown in Fig. 8(*b*) for the base-course drainage is a specified finished product which, under the usual construction schedules, is impossible to build. Analyzing the various stages of the work, such an installation would require:

Either—	or—
1. General excavation to the top of pavement level.	1. General excavation to the level of the base-course subgrade.
2. Trench excavation from the top-of-pavement level to the necessary depth for placing the pipe.	2. Trench excavation by hand since the use of the "backhoe" or similar tool is not feasible with the step-in level at the trench, partly at original grade and partly at general excavation level.

Either—	or—
3. Pipe placed and trench backfilled with filter material to the top-of-pavement level so that the excavation for the necessary thickness of pavement and the base course will not seal up the filter material.	3. Pipe placed and the remainder of the work completed as shown in Fig. 8(*b*).

In the first procedure, trench excavation volume increases with consequent reduction in general excavation and loss of filter material which is first placed and then removed. The second procedure specifies much slower operation of trench excavation by hand, which in normal progress schedules would be a bottleneck. A solution of all these difficulties is shown in Fig. 37. This method permits excavation by machine and at the same time introduces an extension of the filter material to help eliminate the sealing up of the filter material by the adjacent soil during heavy rains or from subsurface drainage flow.

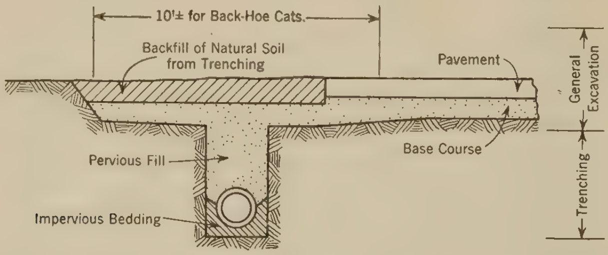

FIG. 37.—BASE-COURSE DRAINAGE DETAIL TO PERMIT MACHINE EXCAVATION

The foregoing comments agree with Colonel Stratton's statement in the "Summary" that "Good designs may be vitiated by poor construction * * *," but good designs must also take account of construction methods, tools, and labor available for the work.

ROBERT HORONJEFF,[14] JUN. AM. SOC. C. E.—As stated by the author, the design of concrete pavements is generally based on the formulas developed by Dean Westergaard. The modulus of soil reaction "*k*" is measured in the field. With a known *k*-value and an assumed working stress for concrete, the required thickness of a slab for any desired wheel load can be computed by substitution in the Westergaard formula. The question in the writer's mind is whether the Westergaard analysis is adequate for determining thicknesses of concrete pavements laid on marshy ground. The load deformation curve (Fig. 4) of a plate 30 in. in diameter placed on a subgrade underlain by mud varying in depth from 40 to 60 ft is considerably different from the curve shown in Fig. 4. When the subgrade consists of mud covered by select material not exceeding 2 ft in depth the load-deformation curve is very flat, indicating that very little load is required to deflect the slab a large amount. In fact, in many cases it is not possible to load the plate with a total of 60,000 lb (Class I airports) without pushing the slab into the mud. With average subgrade conditions the general practice has been to limit the size of individual slabs (by means of dummy joints) to an average dimension of 12.5 ft by 15 ft. The argument presented is that by limiting the size of the individual slab the stresses produced by shrinkage and warping are reduced to a minimum, thus reducing the formation of cracks.

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It is the writer's opinion that this argument cannot be applied to construction of pavements on marshy ground. The individual slab sizes should be increased so that a greater bearing area is provided. It would take a larger load to push a 25-ft by 25-ft square slab into the mud than it would to push a 12.5-ft by 15-ft slab. The next questions that arise are how one can be assured that the larger slab will distribute the concentrated wheel load and how the tensile stresses produced by warping will be taken care of. These conditions can be met by proper distribution of reinforcing steel through the slab. At the top the reinforcing steel should be concentrated at the corners and at the edges and at the bottom toward the center of the slab.

The method of computing the size of reinforcing steel is still an unknown factor. It is true that reinforcing steel may not prevent initial cracking of the slab but it will prevent the enlargement of these cracks and it will certainly prevent the complete severance of a portion of the slab from the parent slab, such as is the case with a plain concrete slab. Reinforcing steel has a definite place in the design of concrete pavements on marshy ground.

The transfer of wheel loads of from 60,000 lb to 75,000 lb by means of ordinary steel dowels is somewhat questionable. There is not sufficient surface area in the dowels to transmit the load to the concrete without crushing the concrete. There is a due need for a satisfactory load transfer device that is practicable to install in the field.

Another problem that is constantly arising in airport work is the strengthening of existing concrete pavements by "second-storying." "Second-storying" means the placing of a new slab over an existing slab and separating the two by means of cushion course. The thickness of cushion course will vary depending upon the particular conditions at the airfield involved. Thus, if the strengthening is required around existing hangars, the finished grade of the new slab has to be maintained as close as possible to the grade of the existing hangar floors, which results in the use of a minimum cushion course of from 1 in. to 2 in. between the two slabs. On the other hand, on a runway the grade of the new slab is not limited by existing structures and the cushion course can be as thick as 2 ft.

A study to determine the behavior of slabs placed on existing slabs with various thickness of cushion course and subjected to moving wheel loads up to 75,000 lb has been undertaken by the San Francisco District in conjunction with the staff of the Engineering Materials Laboratory of the University of California, at Berkeley. The aim is to determine the stresses in the slab at such critical points as those on the edges and corners by measuring internal strains in the concrete. The subgrade reactions and surface deflections are also to be measured. From this study it is hoped that some information will be obtained as to the effect of the thickness of cushion course on the structural behavior of the slab. Various thicknesses of slab both reinforced and unreinforced will be studied in the test section.

Another point that usually is given little consideration in the design of airfields is the radius of the fillets applied to runways and taxiways. Present standards are inadequate to a certain extent. These standards set up by the Civil Aeronautics Authority are as follows:

Angle of intersection	Radius of fillet (ft)
0°–85°	25
85°–115°	50
>115°	100

At a number of airfields designed by the writer the using service has arbitrarily increased the radii of the fillets because most of the planes were "cutting the corners." A number of people attribute this condition to "pure carelessness on the part of the pilots," but it is a known fact that, at a number of airfields used both by large transports and single-seat pursuit aircraft, a sincere effort has been made by the pilots to "stay on the pavement." For large transports the difficulty seems to arise from the fact that the ratio of the wheel spread to the paved width of the taxiway is quite large. Therefore, when a pilot is confronted with the need for making a turn on the taxiway the plane veers off from the center line of the taxiway just enough to throw one of the wheels off the pavement. Pursuit aircraft seem always to taxi at fairly rapid speeds and, therefore, in order to maneuver sharp turns the pilots run off the pavement. The writer recommends that the minimum radius of fillets for airfields to be used by fairly large planes be established at 75 ft.

Clarence S. Jarvis,[15] M. Am. Soc. C. E.—Of untold value is the record in this Symposium on the logical development of both construction and design problems, together with the solutions permitted under the strict limitations imposed by the expanding, ever-changing, and accelerating activities of wartime. Colonel Stratton's interesting notes relating to subgrade support and the various methods of stabilization by compaction, or by the addition of base courses suitably rolled, recalled some of the writer's early construction experience. In addition to the usual provisions for both surface and subdrainage, it was sometimes necessary to insure against both lateral and upward flow of compressible and plastic materials. One of the most satisfactory methods was the placement of crushed slag or stone in trenches for compaction under pile-hammer impact. The virtual interlocking of the angular fragments accomplished in this manner seemed to make such footings react very similarly to plain concrete slabs. Unquestionably the inertia of such heavy masses among the footings accounts in part for their remarkable stability under varying applications of live load and required dead load.

The writer vividly recalls the announcement in the division of design of a federal bureau concerning proposed thickened edges of pavement strips to replace either the prevailing uniform thickness, or even the heavily crowned road pavement, with edges of concrete thinned by the amount of rise in the crown. No less startling was the introduction of dowels to extend across all construction and expansion joints, thus preventing the uneven settlement that had proved so destructive to concrete under heavy metal tires.

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Colonel Stratton's frequent references to traffic distribution, repeated stresses, and fatigue of pavement materials indicate his familiarity with, and his practical application of, the basic principles disclosed by the experiments of the late J. L. Van Ornum, M. Am. Soc. C. E. (37), and others. How else could any one justify the departures from previous practices described under the heading, "Concrete Paving: Allowable Working Stresses for Concrete Pavements"? The use of a safety factor of 1.75, corresponding to 57% of the 28-day flexural strength of the pavement, or even the use, for limited operations, of a safety factor of only 1.40, representing 71% of the stress ordinarily required to cause failure with a single application of loading to destruction, is entirely logical in view of the increasing strength after 28 days and the wide dispersion of traffic, as well as the wing lift accompanying the landing of planes. Who could have foreseen that the vibratory strains of the taxiways and the warming-up aprons, rather than the stresses due to landing impact, would govern both current and future design?

Summarizing the conclusions that may be drawn from the Van Ornum series of tests, application of 50% or less of the loading that would normally cause failure may be repeated indefinitely without appreciably decreasing the resistance of reinforced concrete columns, beams, or slabs; but, for a 75% loading or greater, only a few such applications should be tolerated because of the definite tendency toward fatigue of materials, accompanied by a measurable reduction of ultimate strength.

The observed variation in behavior among individual test specimens employed by the writer in a research conducted in 1905 and 1906 (38), and also among the more numerous samples subjected to the more elaborate tests of the Van Ornum series, would occasionally becloud the results. For example, a loading of 85% would cause failure with a single application, and the next specimen would withstand a similar loading 100 or more times.

In the light of all available tests to date, it is difficult to improve on the range indicated in Colonel Stratton's treatment of allowable working stresses—57% of the ordinary 28-day strength of resistance, or for limited operations, 71%.

Although there are valid objections to the use of concrete pavements as thick as 10 to 14 in., because of the attendant temperature stresses and also because of the difficulty in securing the required compaction and density, there appears to be no practicable limit on the base-course thickness of stone or slag, duly compacted. In the writer's experience, it has been demonstrated repeatedly that a base course of rubble or of quarry stone, with interstices mostly filled with fragments, would support a lightly reinforced pavement of 3-in. thickness plus whatever penetration was attained by keying into the stone base, with better results and at less cost than would plain concrete of double or treble the pavement thickness, and without base course.

Mr. Hathaway deals with noteworthy accomplishments in a broad field that seems to have awaited a well-organized, vigorous, clear-sighted, and analytic approach. Some other attempts have fallen far short of the goal because of the lack of required basic data. However, additional material has been supplied in recent years through the cooperative efforts of various federal departmental representatives including, notably, Mr. Hathaway and Merrill Bernard and N. C. Grover, Members, Am. Soc. C. E., their administrative supervisors and technical staffs, legislative committees, and, finally, the great mass of enlightened citizenry who favored liberal support of hydrometeorological and related research. In addition to this financial and scientific support, the development of fundamental procedures as exemplified by the Symposium certainly required the wholehearted assistance of specially qualified members of the technical staffs in various field offices under the Chief of Engineers, as well as of those men immediately under Mr. Hathaway's direction, as tacitly implied or expressed under "Acknowledgments."

If attempts were made to include in the "Bibliography" such technical papers and other publications as have contributed vitally to the background and to the fundamental knowledge of airfield engineering (including meteorological, infiltration, runoff, and related phenomena), the list would be too long and cumbersome for practical purposes.

The writer realizes that any constructive suggestions for either amplification or clarification of concepts may be mistaken for criticism, no matter how helpfully intended. However, passengers on maritime vessels or on other transportation units, who fail to communicate vital information that may be beyond the range of regular lookouts (an open port too near the water line, signs of shallows or impending collision—anything endangering the common interest) thereby partly forfeit their claims to safe passage. The writer is deeply concerned about the field of hydrologic and related scientific research, particularly regarding,

(*a*) Thorough utilization of existing basic data;
(*b*) Expansion of procurement programs to broaden their scope;
(*c*) Recognition of patterns among natural phenomena, capable of affording guidance in supplying reasonably close approximations of missing basic data; and, finally,
(*d*) Simplification of procedures whereby the divergent factors may be made to yield an integrated, logical, and conclusive result, capable of withstanding repeated practical tests consistently and satisfactorily.

The writer's first glance at Fig. 14 disclosed what appeared to be a break in the continuity of the series between the curves for 30 and 120 min. Obviously, a 60-min curve (intermediate yet not quite midway between the adjacent graphs) should have been included in Fig. 14. This would appear as a straight line connecting the origin of coordinates with the abscissa value of 4.5 at the top margin. By the very nature of this relation, numerical values of ordinates and abscissas at all intersections on the grid for the 60-min curve will be equal. The fact that the hourly rainfall intensities are displayed along the vertical axis was not overlooked.

An additional curve that might well be drawn in Fig. 14 concerns the 24-hr period. Assuming that the only rainfall during such a period occurs within the 240-min (4-hr) interval which the curve represents, the average hourly intensity would be $\frac{4 \times 4.5}{24} = 0.75$ in. Therefore, the new curve or straight line would connect the origin of coordinates with the point on the top margin corresponding to abscissa 0.75, representing the extreme limit toward the vertical axis. Allowing for the gradual increase of precipitation prior to the beginning of the 4-hr period of high intensity and for a corresponding decrease later, as well as for the associated showers which combine to make up the 24-hr maximum for each designated frequency within the series, the final position of the graph would connect the origin with the value of approximately 1.0 along the top margin. This result is logical because 24-in. rainfall depths have been attained rarely, if ever, within the scope of officially accredited records in the United States. The most outstanding example to be found in the writer's investigation (first reported in 1930), covering a period of some five years but dealing with several times as many stations as were included in the final tabulation (39), was the record at Taylor, Tex., of 23.1 in., associated with a maximum hourly rate of 4.25 and a monthly maximum of 25.9 in. Similar maxima recorded for Austin, Tex., in another tabulation of more recent date are 19.0, 4.38, and 20.8 in., respectively (40).

If maximum observed values are desired for some of the world's outstandingly high rainfall stations, one must include 24-hr depths of 21.9 and 28.7 in. for remote stations in Hawaii and Jamaica, British West Indies, respectively, and 25.0 in. for Madras, India, and 40.0 in. for Cherra Punji, India, associated with hourly maxima of somewhat less than 4.0 in. and with daily and monthly maxima of 42.8 and 360.1 in., respectively (39)(40).

Similarly, for the Baguio Station, high among the mountains of Luzon, Philippine Islands, the maximum hourly rate was nearly 4.0 in., whereas the maxima for daily and monthly periods were recorded and published as 46.0 and 133 in., respectively. Obviously, the curve for century-frequency maximum intensities that includes all known values should connect a point on the top margin with abscissa nearly equal to 2.0 in. per hr with the origin; it would therefore lie slightly to the right of the curve of Fig. 14, representing the maximum 4-hr intensities within the United States. The last-described curve, dealing with world maxima, should preferably be excluded, since it would disturb the orderly march of the series developed for the United States.

As far as the writer was able to determine, much of the concept relating to rainfall patterns was new when it was published in 1930 (39). Two illustrations (39*a*)(39*b*), a table (39*c*), and textual references pertaining to all three incline toward the establishment of a nearly straight-line relationship for the series. For many of the stations listed (39*d*), comprising 820 of the principal station-records throughout the world in summarized form, the writer reported that the mean annual rainfall divided by the average number of rainy days per year would represent the maximum rainfall depth to be expected within a 3-min period (39*e*). Obviously, this curve would lie somewhat to the right of the one in Fig. 14 and might well serve as an envelope of the plotted values below the graph for a 5-min duration. Inasmuch as the majority of the stations listed by the writer (39*d*) for world-wide distribution had no data for periods shorter than one day, the intermediate values between the 3-min period and the 24-hr period had to be determined by interpolation. For this purpose, the writer determined by trial that curves for the 15-min and 60-min durations practically trisected the space between the 3-min and 24-hr maxima. For that reason, principally, the 30-min rainfall intensities were purposely omitted, as being less significant members of the series than were those retained. Other members of the plotted series of rainfall data (39*a*) were maxima for both monthly and yearly rainfall depths. Differences in lengths of records for the respective periods, from a day or less to months and years, were inclined to disturb the true relationships. It was not uncommon to find only a decade or so of rainfall intensity records for a day or less, whereas the monthly and yearly values as published covered half a century or more, thus calling for some adjustment to reduce such data to a basis of comparable frequencies.

Under the heading, "General." Mr. Hathaway states:

> "A study of rainfall intensity-frequency data for a large number of precipitation stations indicates that there are fairly consistent relationships between the average intensity of rainfall for a period of 1 hr and the average rates of comparable frequency for shorter intervals, regardless of the geographical location of the stations or frequency of 1-hr rainfall."

This terse statement is of far-reaching importance and somewhat startling on first reading, as it links hourly and shorter intensities regardless of geographical location and frequency of each series. It proves to be almost axiomatic in the light of further study. In fact, it is a part of the concept of rainfall patterns which the writer sought to express in 1930 (39) and earlier. In an effort to define the essential similarities inherent in precipitation intensities or total depths for varying periods of time from 5 min to a year, the writer expressed all these quantities as percentages of the mean annual depth of rainfall. Thus, the maxima for a 5-min period ranged from 0.17 in. at Juneau, Alaska, to 0.91 in. at Denver, Colo., or to 1.05 in. at Tampa, Fla., or to 1.15 in. at Terre Haute, Ind., representing the maximum officially reported and observed depth in the United States. Reduced to percentages of annual mean rainfall depths for the respective stations, the maxima become 0.2%, 6.4%, 2.1%, and 3.0%. Some other stations have established claims for somewhat higher records but have not fully qualified them as official data. Undoubtedly still higher values have occurred for tropical storms, although official observations are lacking.

Isohyetal Maps.—For several years after the publication of technical reports, including the volume entitled "Storm Rainfall of Eastern United States" (41), by the Miami Conservancy District, many hydrologists considered that the Great Plains and Western States were not susceptible of the same treatment. The reason for this opinion was related to the erratic and unpredictable rainfall occurrence, amount, intensity, and areal extent, as well as to the dominant influence of altitude over latitude. In the face of such professional opinion, the writer undertook to extend a series of isohyetal lines representing not only daily but also monthly maxima throughout the United States (39*f*) (39*g*), and then completed the maximum monthly isohyetals for such parts of the various continents as were covered by officially published records. Mr. Yarnell (15) adopted scales that precluded the portrayal of local effects, such as are associated with topographic forms, rain shadows, normal paths of air-mass movements, and the orographic barriers to be surmounted. To overcome this difficulty in part, Mr. Yarnell introduced special maps on a more generous scale, covering the Pacific Coastal States, with regard to 24-hr rainfall depths for various frequencies, from 5 to 100 years. Lack of data had been noted on the Pacific Coastal strip for some of the shorter periods—namely, 16, 8, and 4 hr—and by implication for still shorter periods.

The writer's experience in the construction and drainage of arsenal and airfield facilities lead him to offer the following observations:

(*A*) No matter how liberally one may interpret the series of isohyetals on the small maps of Fig. 13, there is urgent need for either supplemental maps of the Pacific Coastal region on scales permitting the portrayal of the pronounced local variations, or a special note of caution concerning the practical certainty that the indicated limit of 1.25 in. per hr has been and will be greatly exceeded within 25-yr periods of record.

TABLE 9.—SUMMARIZED RAINFALL RECORDS OF THE PACIFIC COASTAL REGION

Item No.	Station	Altitude (ft)	Maximum[a,b]			Annual Rainfall[b]		
			Hour	Day	Month	Minimum	Mean	Maximum
	Washington:							
1	Tatoosh Island..	86	1.44	5.91	25.8	49.6	83.5	114.3
2	Forks[c]..........	480	(2.3)	(6.6)	31.3	108.4	120.3	153.8
3	Spruce[c].........	300	(2.0)	(6.0)	24.6	79.6	124.4	141.0
4	Quinault[c].......	300	(2.4)	(7.0)	42.3	79.2	129.5	157.1
5	Seattle..........	246	0.80	3.52	15.3	20.0	33.8	56.4
6	Tacoma.........	109	0.79	3.79	18.9	22.0	39.9	57.3
7	North Head....	211	0.86	4.29	17.8	39.9	55.5	78.6
	Oregon:							
8	Tillamook[c]......	23	(2.2)	(6.5)	37.2	63.6	95.9	132.1
9	Nehalem[c].......	20	(2.5)	(7.0)	29.3	82.5	109.3	134.4
10	Glenora[c]........	575	(2.7)	(8.0)	50.2	97.3	130.5	167.3
11	Portland.........	57	1.31	7.70	20.1	30.8	42.8	67.2
12	Roseburg........	510	1.14	4.10	12.8	21.1	33.3	44.3
	California:							
13	Monumental....	2,700	(3.0)	(9.0)	43.8	69.3	109.1	153.5
14	Crescent City...	125	(2.6)	(8.0)	31.9	47.5	74.8	107.6
15	Eureka..........	64	1.30	6.87	19.5	22.4	41.8	64.5
16	Red Bluff.......	307	1.72	6.10	20.7	12.9	24.6	49.0
17	Point Reyes....	490	1.40	3.10	11.4	7.3	20.6	32.4
18	San Francisco...	206	1.07	4.70	24.4	9.0	22.5	38.8
19	Sacramento.....	71	0.94	7.20	16.0	7.8	18.3	34.9
20	Fresno..........	295	1.36	2.90	9.5	3.9	9.7	21.5
21	Tamarack......	8,060	(2.8)	(8.5)	39.8	27.8	50.6	94.3
22	Los Angeles.....	361	1.51	7.36	15.8	4.8	15.6	40.3
23	San Diego......	87	1.16	3.20	9.3	4.3	9.8	27.6
24	Campo..........	2,543	(4.6)	(8.6)	20.4	11.3	19.8	35.3
25	Squirrel Inn....	5,280	(3.3)	16.7	30.3	19.6	40.8	82.2
26	Scales..........	....	(2.0)	(6.0)	29.0	49.8	87.6	125.5
27	Kings Canyon..	....	(3.0)	(5.0)	8.6	....	38.4	....

[a] Parentheses enclose unofficial or estimated quantities. [b] All units are in inches. [c] Northwest.

(*B*) A study of the selected station-records in Table 9 (which were chosen intentionally to represent the upper bracket of observed intensities, inasmuch as most of the actual maxima of high mountainous regions are unrecorded) indicates that the influence of latitude is subordinated to that of altitude combined with windward exposure.

(*C*) The seaward slopes of the coastal ranges extending from the Olympic Peninsula in Washington to San Diego, Calif. (and probably even farther) and from the western slopes of higher ranges farther inland (such as the Cascade Range and the Sierra Nevada) are capable of developing a fair approach to the series of intensities plotted around the Gulf Coast. These intensities approach about 4 in. per hr at outstanding stations, although they are ordinarily limited to roughly half of that depth for available periods of hourly record.

(*D*) In Table 9, the rainfall patterns disclosed by neighboring stations are used for practical purposes to estimate appropriate values of the missing items of basic data. All these are enclosed in parentheses to indicate their unofficial character. These estimated values may be subject to uncertainties of approximately 20%.

(*E*) Both limiting and periodic intensities for selected rainfall stations seem to vary with the square root of the number of minutes in the respective time intervals considered, as has been stated in official publications (42)(43). Thus, the envelope of conceivable intensities—perhaps far beyond the attainable limit for most of the outstanding stations of high rainfall—would have depths for respective periods as follows:

Minutes	Inches
1	1.00
5	2.24
10	3.16
15	3.88
30	5.48
60	7.75
240 (4 hr)	15.50
1,440 (24 hr)	37.90
43,200 (month of 30 days)	207.4
525,600 (year of 365 days)	725

Some of the foregoing depths have been attained or even exceeded—for example, the daily, monthly, and yearly depths at Cherra Punji and the daily and monthly depths at Baguio. Some have been either attained or closely approached at several stations in India and the tropical islands of the Pacific Ocean. During seven of the fourteen years of available record for Cherra Punji (1905 to 1919), the maximum daily rainfall depths were in excess of 20 in.; and three of these exceeded 30 in., as follows: 33.00, 31.05, and 32.35 in. during 1910, 1914, and 1915, respectively. Desert regions may attain a much greater proportion of the short-period rainfall intensities, say, for less than an hour, than should be expected for days or months. Otherwise, they would cease to be classified as deserts, inclining more toward the humid in character. Patterns for most stations will follow more or less closely some proportion of the foregoing envelope depths.

Runoff Formulas.—In the interest of simplified procedure and computations, Mr. Hathaway has wisely made a number of assumptions, some of which are admittedly only approximations to cover variable conditions and behavior. (Either compensating or cumulative errors may result for a given problem.)

Among these are the assumed constancy of rainfall, infiltration, and resulting rate of supply for overland flow during a specific storm. Inasmuch as the assumed rectangular form of pluviograph is seldom if ever attained and an alternate form of isosceles triangle on the same time base but with its altitude double the height of the rectangle would enclose an equal area and would measurably alter the proportions for infiltration and supply volumes, this alternate form of pluviograph may bear investigation. Although it would still be an approximation, it would occupy a mean instead of an extreme position.

Mr. Horton's equations for overland flow, as presented by Mr. Hathaway, as well as a single page of textual material in Mr. Yarnell's notable bulletin (15), employ the so-called "rational equation" for runoff, $Q = C\,i\,A$, which expresses runoff Q in cubic feet per second, representing some percentage or decimal fraction (C), the ratio of runoff to rainfall for the critical period, times the product of rainfall intensity i in inches per hour and the number of acres A in the drainage area. The infiltration capacity is likewise expressed in inches per hour; as, for example, 0.6 in. per hr adopted by Mr. Hathaway for turfed areas. Similarly, the allowable pond storage of 2,000 cu ft per acre in the example used by Mr. Hathaway may be expressed as a depth of 0.55 in., since an acre-inch is $\frac{43{,}560}{12} = 3{,}630$ cu ft.

For the example solved by Mr. Hathaway according to the rigorous procedure recently developed with discharge to the inlets of the drainage system given as 2.1 in., from a rainfall intensity of 3.6 in. per hr, C is equal to $\frac{2.1}{3.6} = 0.583$, or 58.3%. Evidently, the surface storage outside of ponds was 0.35 in., which added to $0.60 + 0.55 + 2.10 = 3.60$. Only that portion of the surface storage (10% of the total rainfall) is subject to estimation; it could not be seriously in error; if neglected entirely, the error would be only 10%.

Perhaps it is just plain coincidence, but it is interesting to note that the X-ratio value derived graphically by Mr. Hathaway is 59% (compare with 58.3%), representing the ratio of drain-inlet capacity to peak rate of runoff, the difference being absorbed in pond storage.

The impressive array of well-chosen tables of basic data and related computations, together with the charts and diagrams on which the illustrative solutions of typical problems are readily traceable, and the clear-cut, logical treatment of the entire procedure in the accompanying text by Mr. Hathaway leave but little to be supplied in discussion. It is gratifying to know that the design procedures have already been incorporated in the Engineering Manual of the Construction Division, Office of the Chief of Engineers (23).

The susceptibility of the entire analysis and rigorously derived results to comparison and expression in terms of the so-called "rational formula," as demonstrated herein, should insure their incorporation in professional practice much more widely than would have been possible without such a popularly established device. This formula results in closing a survey circuit well within the tolerance limits, when results derived, or capable of derivation by both methods, are found to be in substantial agreement. Mr. Hathaway's paper represents a noteworthy accomplishment and progress in the field of hydrometeorology.

D. D. McGuire,[16] Esq.—A splendid method for rational design of drainage capacity is contained in Mr. Hathaway's paper. The engineer is given a measuring stick by which he may correlate the effect of the storm deemed critical for a particular installation with the airport topography and the drain capacities necessary for the required use of the airfield. Such a collection of concise data and workable information is appreciated by the drainage engineer.

Proper use of subgrade drainage may often simplify surface drainage facilities. Where extensive flat areas are involved, a better over-all solution of the problem may sometimes be obtained by a combination of several methods. The following analysis and comments give some reasons for serious consideration of subgrade drainage facilities.

The 1-hr infiltration value subtracted from the 1-hr rainfall quantity identifies the standard supply curve and forms the basis for succeeding computations. In the example quoted in Mr. Hathaway's computation of storm drain capacities assuming supplemental ponding, the design storm chosen presumes 2.2 in. of rainfall per hour. The 1-hr infiltration capacity for turfed areas is assumed to be 0.6 in. per hr and represents 36.7% of the rainfall quantity. Surely this is important enough to warrant further investigation and more accurate data on infiltration capacities. The pertinent factors are many and their interrelationship apparently vague. Problems presenting greater difficulties have been solved previously through research.

If 36.7% of the rainfall quantity may be disposed of through infiltration, why not either dispose of the water quickly through subdrainage facilities and thereby reduce the amount of ponding, or, by lowering the adjacent water table, create a ponding area below the surface and away from traffic.

The possibilities of subdrainage pipe seem wide and valuable. Quick disposal of filtered water will reduce overland flow and minimize surface erosion. The subdrainage system creates a storage area below the plane of traffic, thereby reducing the number of required inlets with their attendant safety hazards.

Lowering the water table reduces the damaging action of frost on subgrades under runways and aprons and, through better stabilization, can reduce the cost of surface treatment. A proper subdrainage system will make available for landing outside of runways areas which might never attain a usable condition if the entire disposal depended on inlets and storm sewers.

In view of the many facets of the subdrainage problem, it is hoped that the same clear, usable type of analysis described by Mr. Hathaway for storm sewer disposal systems may be evolved for the design and efficient use of subdrainage pipe.

E. D. Parmer,[17] Esq.—The engineer who is in any way connected with the design or construction of airfields, whether for military use or civilian use, should find this important and timely Symposium indispensable. It contains many sound principles of utmost value to the designer of airfields, presented in a simple, straightforward, and stimulating manner.

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[17] Capt., Corps of Engrs., Army of United States, and Commandant of the Army Soils Control School, Graduate School of Eng., Harvard Univ., Cambridge, Mass. (formerly Junior Resident Engr., Texas State Highway Dept., Beaumont, Tex.).

In connection with airfield design and construction problems, the Corps of Engineers has proceeded quietly and systematically in designing, constructing, and testing, to meet adequately the demands of the airfield war construction program. Although greater and greater wheel loads have been predicted, comparatively little information has been published on methods proposed to meet such an airfield problem.

Suitability of Base-Course Materials.—In discussing base courses for concrete pavements, Colonel Stratton uses the term "high-quality base materials" without sufficient definition. Because of a common misunderstanding as to what constitutes a "high-quality base material" and because of the belief of many engineers that clay should be added to granular materials for use as a base course (simply because the satisfactory performance has been observed of well-graded sand or gravel mixtures containing clay when used as surface courses exposed directly to light wheel loads without the use of substantial pavements), it should be stated that, in general, clean, well-graded granular base materials with a minimum of "fines," say, not over 4% or 5% passing No. 200 mesh, make the most desirable high-quality bases for both rigid and flexible pavements.

Only 3% by weight finer than 0.02 mm in a well-graded granular material used as a base course in northern areas may cause troublesome frost action—breaking of concrete due to irregular heaving in winter and insufficient support due to softening in the spring.

In southern regions, natural clayey gravel or sand-clay mixtures may give satisfactory results if placed in a properly compacted, homogeneous condition. However, the deliberate addition of clays to clean, well-graded granular materials for use as base courses should be discouraged, not only because of the difficulty, expense, and length of mixing process, but also because of the inherent increased sensitivity to moisture control for proper compaction, as well as because of the increased potential susceptibility to subsequent moisture changes.

Design of Concrete Pavements.—War conditions have restricted the use of steel which in turn has caused the revision of concrete pavement sections. More than one good engineer pleaded that the reinforcing steel be left in pavements, and especially those which had to be placed directly on poor subgrades without the benefit of intermediate base courses.

There is no question about the necessity for thickening the edges of concrete pavement as is done in the revised sections if the pavement is placed directly on a poor subgrade. However, it would be very desirable to make additional investigations as to the necessity for thickening any edges when the pavement is placed on a high-quality, well-compacted base course, considering the minimization of differential settlement, together with the loading frequency and the natural traffic distribution on airfields. Is it not probable that the effect of disturbing the base course in shaping for thickened edges counteracts many benefits obtained from the thickened edges?

Control of Frost Action.—In discussing pavement design for frost action, Colonel Stratton states that the most generally accepted method of insuring pavement or base-course stability over a potential frost-action soil is the provision of sufficient thickness of insulating material.

The writer would question the mere use of "insulating material" over a potential frost-heaving soil to insure stability unless the term "insulating material" were defined as a granular material containing no more than a small percentage of material finer than 0.02 mm. Without such a restriction, one can well imagine a granular material with, say, 10% finer than 0.02 mm which, if it were possible to keep dry, would act as an insulating material but which, because of its high percentage of "fines" and high potential capillarity, might cause serious damage from frost heave if used to cover another potential frost-action soil in contact with a water supply. Therefore, in using soils as insulating materials, it is necessary to place a limit, not necessarily on the grain-size curve itself, but on the allowable amount of soil finer than a certain size, unless the soil can be entirely separated from a water supply either by using an intermediate impervious course or by so selecting the soil that the limit of capillary rise will not fall within the zone of frost penetration.

Subsurface Drainage.—In the past much time and money have been wasted in installing subsurface drainage systems which, as Colonel Stratton states, could frequently have been eliminated by providing an adequate and properly designed surface drainage system.

A subsurface drainage system is referred to herein as a system installed primarily for the purpose of removing water from pavement base courses or subgrades—some form of improvised "French drain," a system of open-jointed pipe, or an assemblage of well points.

When studying subsurface drainage, it is expedient to classify soils in one of the three following groups: (*a*) Well-draining soils such as clean sands and gravels; (*b*) Poorly-draining soils like the slightly plastic organic and inorganic silts; and (*c*) Impervious soils—for example, homogeneous clays. (These remarks on drainage are based on lectures by Arthur Casagrande, M. Am. Soc. C. E., on the "Control of Soils in Airport Construction," delivered at the Graduate School of Engineering, Harvard University, Cambridge, Mass.)

Colonel Stratton states that "* * * where the ground water is above the frost line or causes soft, boggy conditions in areas where soil stability is required, subsurface drainage is necessary." It should be noted, however, that (*a*) lowering the water below the frost line may or may not prevent frost heave, depending on the height of capillary rise, and (*b*) it may be much easier and more practicable in some cases to lower the water table by using circumferential ditches or by providing drainage through an underlying impervious stratum when such a favorable condition exists.

For clean well-draining granular materials, subsurface drainage can often be eliminated altogether, or at least reduced to a minimum by providing circumferential ditches to intercept ground water from surrounding areas and by sealing off the surfaces to reduce the infiltration of water into underlying courses or subgrades. In other words, the water should be kept from entering the base courses, but, if some does penetrate, the perviousness of these courses should be advantageous in removing the water. If the well-draining materials contain an appreciable amount of fines, then, to prevent possible serious damage from frost heave, it will be necessary to keep the water table not only below the frost line but at a distance below the frost line greater than the height of capillary rise for the particular soil.

In a strict sense, the statement that poorly-draining soils are "drained" with a subsurface drainage system is incorrect. The height of capillary rise in such soils is usually great, and as long as the water table lies within the height of capillary rise the soil will remain saturated by capillary water. If the frost zone comes within this zone of capillary water, there may be danger of serious frost heave. Therefore, there is no advantage in attempting to use subsurface drains to eliminate danger of frost heave in poorly-draining soils, unless the drains can be placed at such depths that the piezometric water table will be lowered to a distance below the frost line greater than the height of capillary saturation. For some soils, this is impracticable or impossible. However, benefits can be derived from using subsurface drains in poorly-draining soils. By their use soft soils can be consolidated and thereby their bearing value can be increased.

In using subsurface drains for consolidation purposes the following should be emphasized:

(*a*) Since any benefits of consolidation are obtained from capillary forces (that is, as long as the soil remains completely saturated), the deeper the drains can be placed within the zone of capillary saturation, the greater will be the force producing consolidation.

(*b*) The closer the drains can be placed for a given depth, the less time is required to consolidate the soil between the drains.

(*c*) To function properly the system must be so designed that water can be removed as fast as it collects. It may be possible to accomplish this with a gravity system in certain favorable locations, but, more often, it will be necessary to use pumps in sumps. Pumps are also needed when well points are used.

(*d*) Drain pipes for such a system may be very small since the actual amount of water moving through them may be no more than a "trickle." (The overlying soil remains saturated by capillary water so that the greater part of water from that source will be only that which is squeezed out by consolidation.) Since the soil by nature drains poorly, only a small quantity of water can be expected from infiltration and ground-water movement.

(*e*) The ideal situation would be one in which the poorly-draining soil is underlain by a pervious layer, in which case water could be removed from the pervious layer by means of well points or circumferential ditches. The action of a subsurface drainage system in homogeneous clay is so extremely slow that it is of little value for improving the condition of the clay.

Construction Methods.—Good designs may be vitiated by poor construction, and it should be stressed that high-quality construction control and inspection are essential to many types of construction.

The value of "engineered" subgrades and bases is more and more apparent. An excellent pavement may deteriorate on an inferior base and subgrade, whereas a second-grade pavement on a properly designed and constructed foundation may give excellent service. An engineered foundation requires proper control from design through completion. It requires careful selection and placing of materials, proper moisture control, and adequate compaction—the real value of which cannot be overstressed. Many of the defects of settlement, swelling, and shrinking, so often blamed on the pavements, can be traced to the foundations and could have been minimized by proper compaction control.

Remarks on Training of Soils Engineers.—In teaching Army courses on "The Control of Soils in Military Construction," the writer stresses particularly the value of identifying soils correctly because thereby the possibility of making a major mistake may be eliminated. After a soil is correctly identified, the properly trained soils engineer and his inspectors should know how to improve that type of soil, how to judge its drainage characteristics, how to decide the degree of moisture control necessary for efficient compaction, and how to determine what method should be used for compaction.

In prewar days, too little attention was given to the proper training of field men and inspectors for all types of highway and airfield construction. Frequently proper control and inspection was sacrificed for impressive looking plans. If the postwar period is to bring extensive highway and airfield construction with ample available man power, as is anticipated, then surely something will have to be done to assure not only the proper utilization of experienced personnel returning to civilian work, but also the provision of ways and means for the training of additional men. The time to consider these problems has already arrived.

A. CASAGRANDE,[18] M. Am. Soc. C. E.—The construction of transportation facilities—roads, bridges, railways, and waterways—has been an age-old source of problems to the civil engineer, and now the construction of airports, with a scope undreamed of only a few years ago, may well be making an important new chapter in civil engineering. When such rapid developments take place, reliable published information often lags far behind. This Symposium is a fortunate exception.

The following discussion deals primarily with the first paper of the Symposium. Colonel Stratton has succeeded remarkably well in summarizing all pertinent information and essential considerations on this subject, with particular emphasis on the many important contributions which the Army Engineers have made to the design and construction of airfields for heavy plane loads. Although restricted in scope to military airfields, it should be emphasized that most of the Symposium also applies to the commercial airfields of the future. In fact, almost all of the extensive research on the design and construction of airfields, conducted under the direction of Colonel Stratton, will be of inestimable value for the postwar development of commercial air facilities to serve the air liners of the immediate future. Therefore, it is hoped that this paper will be followed by others describing the results of the large-scale traffic tests which have been undertaken by the U. S. Engineer Department, the results of which will be indispensable to every engineer designing pavements for heavy planes.

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Design of Flexible Pavements.—The design procedure adopted by the Office of the Chief of Engineers is based on the empirical method developed by the California State Highway Department. As stated by Colonel Stratton, the design curves of Fig. 3 have been confirmed by large-scale traffic tests.

It may be well to explain why it has not yet been possible to develop a successful theoretical approach to the design of flexible pavements, based on the theory of elasticity. Perhaps the most important reason is that the coarse-grained materials, of which pavement and base courses are generally composed, do not follow Hooke's law. Therefore, the distribution of stresses in the subgrade will show considerably greater concentrations than that computed by the theory of elasticity. In addition, most flexible pavements do not have enough tensile strength to resist the considerable tensile stresses which develop, according to the theory of elasticity, for large ratios in the modulus of elasticity of pavement-plus-base over that of the subgrade. Even if no tension cracks develop, the computed stresses in the subgrade would be on the unsafe side because the deformation of the pavement in tension is much greater than that in compression; in other words, for practical purposes the pavement cannot take tension. These remarks are not intended to discourage further attempts at theoretical studies of the design of flexible pavements, but merely to emphasize difficulties which are often overlooked and which can lead to conclusions very much on the unsafe side.

Stabilized Versus Compacted Base.—Much confusion still prevails on the value of admixture of "fines" or "binder" to a clean coarse-grained material in the construction of base courses. To cite two extremes: (1) Excellent concrete pavements have been built in Long Island on hydraulically placed, clean, beach sand, without even an attempt at compacting the sand; whereas (2) in a different place, well-graded, fairly clean, glacial gravel (the best base material next to crushed stone) was "stabilized" by adding 15% material passing No. 200 mesh, thereby changing it into a frost-heaving material. Even where there is no frost action, the addition of a plastic binder to gravel has often resulted in a type of base which, in a condition of capillary saturation (evaporation from its surface prevented by the overlying pavement), was much less resistant to deformation than the coarse material without the binder. Not only is it costly to mix a binder with coarse material, but it is difficult to make the mixture uniform. Even a small excess of clay binder may cause local destruction of a pavement, which would not happen if the admixture of such binder were omitted.

Instead of paying for the admixture of fines, time and money should be directed toward very thorough compaction of the base-course materials. Although it has been emphasized, far too little attention is paid to proper compaction. In fact, some engineers really believe that a uniform sand cannot be compacted; that it always remains loose. (Perhaps this belief is due to the ambiguity of the term "loose," which is also used to designate grains with no cohesion.) The writer foresees the development of compaction equipment which by means of vibration will produce degrees of compaction of coarse-grained soils unattainable with any methods now used, and at little or no additional cost.

The importance of thorough compaction is best illustrated by the fact that a well-compacted sand has about three times more resistance to deformation and bearing capacity than the same sand in its ordinary loose condition (not bulked). For a well-graded gravel this ratio may be as high as fifteen. The large increase in resistance to deformation of gravel surfaces exposed to constant traffic is well known. Such surfaces may not be satisfactory unless traffic compacted. Does this not prove that it is possible to compact such materials far better than by ordinary construction methods? What can be done by traffic can be accomplished by vibration equipment especially designed for the compaction of sandy and gravelly soils.

Considering that the cost of compaction is only a fraction of the cost of the material itself, particularly where long hauls are necessary, it is only logical to improve the material by compaction to the utmost limit. Manufacturers of construction equipment would do well to pay attention to this problem and to its partial solution already attained before the beginning of World War II (44).

Design of Concrete Pavements.—Colonel Stratton advances pertinent arguments for restricting the thickness of concrete pavements to about 8 in., and for providing the necessary support on poor subgrades by an appropriate increase in the thickness of the gravel base. Success of such a design will depend on the uniformity of subgrade support. When dealing with nonuniform support such as results from irregular subgrade conditions, soils subject to swelling and shrinkage, or nonuniform frost action, in spite of a good base at times there may be little support, and corners may break under the design load, or even less load. For such unfavorable subgrade conditions, the use of thicker slabs with steel reinforcement cannot be avoided. Warping stresses might be controlled by a sufficiently small size of the slabs, say, 12 ft by 12 ft for a pavement 12 in. thick. The wire mesh reinforcement will preferably be used in two layers, spaced about 3 in. from the top and bottom of the slab. Some additional reinforcement in the top of the pavement should be provided at the corners. In such a design the steel would actually participate in carrying stresses, instead of acting merely to tie broken slabs together.

In thick pavements even greater attention will have to be paid to the design of the joints for very heavy loads. The most desirable solution would be to make all joints alike, each one permitting a slight amount of expansion. A trapezoidal tongue and groove design could be used with premolded asphaltic boards, about 3/16 in. thick, containing enough air voids (for example, in form of admixture of cork particles) to permit a compression of about 1/16 in. Thus, the total expansion possible for 100 ft of pavement, assuming a 12-ft spacing of the joints, would be about 1/2 in. This allowance is ample in view of the uniform distribution of this expansion and the greater assurance that all joints will remain operative, as compared with the standard, wide expansion joints that have been known to become partly inoperative with time. Heavy, short steel dowels should be provided, painted with asphalt and having a thin compressible disk glued to each end, to allow for the necessary slight amount of movement. If vertical steel pins are used to hold the premolded boards in place (piercing the trapezoidal tongue and being driven into base or subgrade), the section inside the tongue may require a sleeve or cover to allow movement between this part of the pin and the concrete.

A uniform joint construction, distributing expansion uniformly over all joints, would aid greatly in solving many undesirable features in the concrete pavements of the past.

Insulation for Frost Protection.—One of the feasible methods for protection against frost action is the use of an insulating material beneath the pavement, of such thickness as to prevent frost penetration into the subgrade. Where cheap timber is available and gravel scarce, timber deserves serious consideration, because a thickness of 3 in. will offer the same protection as 18 to 30 in. of gravel.

Another material which the writer has seriously considered for this purpose, ever since he became acquainted with it in 1929, is a porous concrete made from a mixture of cement mortar and foam. This Danish development has been used in Europe and South America for heat insulation and other construction. Fracture surfaces have the appearance of slag. It can be manufactured in different unit weights and insulating characteristics, and has the important advantage that it can be mixed in the field and poured like concrete. This, or some other cheaply made material full of air voids and having a compressive strength of at least 300 lb per sq in. (which will not soak up water into the air voids when submerged), poured either in place or in pre-cast slabs, would solve the problem of frost action in cases where the necessary thickness of gravel would be prohibitive in cost. In areas of permanently frozen ground such insulating materials could serve to protect the frozen ground under pavements from melting.

Wheel Loads of Airplanes.—Airplane designers should realize that placing the entire weight of the heaviest planes on only two wheels, or on two pairs of dual wheels with little space between the dual tires, creates the necessity of very expensive airfield pavements and limits the number of airfields on which such planes can land safely. For the heaviest planes the load should be spread over four wheels, with the dual tires spaced at least 6 ft apart. For further increase in plane load to 150 tons, the writer foresees the necessity of spreading the load over two four-wheel carriages, resembling perhaps the wheel carriages of railroad cars. With such an eight-wheel landing gear, a 150-ton plane would have the enormous advantage that in an emergency it could land on an ordinary commercial airfield without danger of breaking through the pavement and of serious consequences for plane and passengers. Civil engineers specializing in airfield construction should be consulted in connection with the design of the landing gear of air liners.

Closing Remarks.—The writer takes this opportunity to express his appreciation of the broad application of soil mechanics to the design of earthworks, foundations, water-front construction, and now to airfields, which the Army Engineers have followed during the past decade, and in which Colonel Stratton has played a leading rôle. The Army Engineers have contributed more to the advancement of applied soil mechanics than any other governmental or private organization.

HARRY E. COTTON,[19] M. AM. SOC. C. E.—The Corps of Engineers, with which the two authors (Colonel Stratton and Mr. Hathaway) of the Symposium under discussion are connected, has contributed much to the various problems of airport design and construction—especially base and subgrade design, proper backfill for subdrains, and runoff criteria for the design of storm drains.

Backfill for Subdrains.—The design of base-coarse drainage in Fig. 8(*b*) of Colonel Stratton's paper shows the perforations placed up which is contrary to good practice especially if proper backfill (filter material) is used. This suggests that a discussion is in order on backfill, the proper design of the infiltration area (perforations), and the amount of infiltration area required.

The chief enemy of subdrains is clogging either of the pervious trench backfill or of the pipe itself or of both. This problem has been solved in an investigation undertaken by the U. S. Waterways Experiment Station at Vicksburg, Miss. (45). The function of a subdrain is to remove ground water from a water bearing zone. Such zones are of previous material—silts, fine sands, coarse sands, and gravel. To prevent the entrance of the fine soil particles into the trench backfill, the investigation shows that the backfill should be not coarser than that in Table 10.

TABLE 10.—MAXIMUM SIZE FOR FILTER MATERIAL

Standard A.S.T.M. sieve	Percentage passing
3/8 in.	100
No. 3	86
No. 4	68
No. 6	52
No. 8	41
No. 10	36
No. 16	26
No. 20	18
No. 30	0

A clean backfill having a grain-size curve between the curve of grading in Table 10 and that of a medium sand (No. 60 sieve) will be satisfactory. This is a radical departure from the usual state highway specification for material mostly between 2 in. and 1/2 in. Some states allow 0 to 10% passing a No. 4 sieve and some allow material as large as 6 in. The idea that a coarse backfill is needed with a pipe subdrain probably originated with the requirements for a "French" drain (without pipe) in which large voids are necessary for lateral flow. The vertical flow capacity of medium sand is ample for all ordinary subdrain conditions. The pipe provides lateral flow.

Such fine grained backfill is not only necessary to prevent clogging but, in many localities, more economical. However, it is necessary that the drain pipe be designed to eliminate the entrance of the backfill and yet allow for the entrance of water.

Because most of the recommended backfill material is of a smaller grain size than the perforation diameter, it drops through the holes by gravity action when the perforations are in the upper half of the pipe. This fact was demonstrated in the Vicksburg tests as well as in all other similar tests previously conducted. Therefore, instead of the perforations being up, as shown in Fig. 8(*b*), they should be down, or in the upper part of the lower half of the pipe. Also, to prevent the backfill from entering, due to side pressure, the upper row of holes should not be higher than 22.5° below the horizontal axis of the pipe as shown in Table 11.

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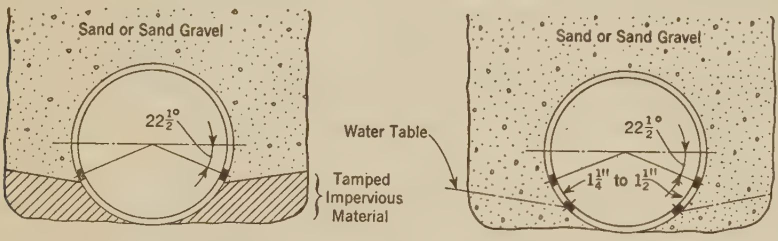
There seems to be an undue concern among engineers regarding the loss of water once it has been collected inside the pipe. There are few instances in subdrainage work where such concern is justified. For the usual condition of lowering a water table in a saturated soil (section (b), Table 11), the zone under the pipe is already saturated and therefore no loss of water will occur. However, for landslide and terrace bank drains, to collect seepage that may be flowing on top of a thin impervious zone, the bottom of the pipe may lie in a pervious soil and precaution should then be taken to prevent loss of water by methods such as those shown in section (a), Table 11. For these cases and for base drainage, the quantity of water to be collected is small and two rows of perforations are more than adequate. When drain pipes are installed adjacent to pavements in impervious soil zones for base drainage and also for intercepting

TABLE 11.—RECOMMENDED PERFORATION DESIGN


Sand or Sand Gravel
$22\frac{1}{2}°$
Water Table
Tamped Impervious Material
Sand or Sand Gravel
$22\frac{1}{2}°$
$1\frac{1}{4}''$ to $1\frac{1}{2}''$

(*a*) FOR BASE DRAINAGE AND LANDSLIDES

(*b*) FOR SUBDRAINAGE

Number of:	Section (a)	Section (b)
Rows of perforations (diameter, 3/8 ± in.)	2	4
Perforations per linear foot, averaged per section of pipe (minimum)	8	16

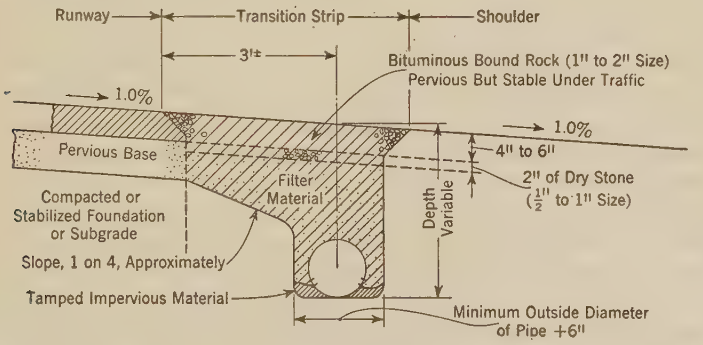

Runway
Transition Strip
Shoulder
3'±
Bituminous Bound Rock (1" to 2" Size)
Pervious But Stable Under Traffic
1.0%
1.0%
Pervious Base
4" to 6"
2" of Dry Stone
($\frac{1}{2}''$ to 1" Size)
Filter Material
Compacted or Stabilized Foundation or Subgrade
Depth Variable
Slope, 1 on 4, Approximately
Tamped Impervious Material
Minimum Outside Diameter of Pipe +6"

FIG. 38.—THE INTERCEPTING DRAIN APPLIED AS A COMBINED SURFACE DRAIN AND A BASE DRAIN

drains, an impervious trench bottom, as shown in section (a), Table 11, and Fig. 38, is advisable to insure complete removal of free water collected in the trench. The backfill or "filter material" over the top and sides should be a tamped mixture of sand and gravel not coarser than the grain-size curve of material in Table 10 but having a permeability sufficient for runoff.

Required Infiltration Area (Perforation Area).—The hydraulics of subdrainage is not an exact science. The engineer may be guided by research on ground-water flow, but the answer is largely based on judgment and experience. (Typical of such research are the runoff tests by the Engineering Experiment Stations of Iowa State College at Ames and the University of Minnesota at Minneapolis. Also the Darcy formula may be used:

$$Q = A\,k\,i \qquad (11)$$

in which k = coefficient of permeability, i = hydraulic gradient, and A = cross-section area of flow.) The ground-water flow is seldom sufficient to fill a 6-in. pipe within a 1,000-ft run. Even for the most severe seepage condition encountered in highway and railway cuts, 6-in. pipe has sufficient capacity in the first 400 to 500 ft of the run. A change of pipe size in the run (for example, 6 in. to 8 in.) is required to carry the accumulated interception but not because of increased ground-water flow to the drain per linear foot. Therefore, the larger drain pipes need no more infiltration area (perforations) than is required in a 6-in. pipe.

Little is known about the infiltration capacity of perforations in a pipe surrounded with sand or sand and gravel. One such test with a metal pipe gave an approximate answer of 0.009 cu ft per sec per sq in. of perforations (1/4-in. holes) for a condition in which little or no head was assumed above the top of the pipe. On this basis, a 6-in. pipe, on a 1.0% slope with perforations as specified in section (b), Table 11, could fill itself in less than 30 ft. When consideration is given to the fact that the area of perforations in a run of 16 ft as specified in section (b), Table 11, is equal to the end area of a 6-in. pipe, it is self-evident that the infiltration area is many times larger than is necessary; in other words, a large factor of safety is provided. When too many rows of perforations are specified in pipe, requiring the top row to be at or near the spring line (horizontal axis), solids are likely to enter. To prevent such entrance is one of the requirements of a well-designed subdrain pipe.

Impact.—Colonel Stratton's discussion of loading criteria leads the reader to conclude that impact is not a factor in the design of runway pavements and may even be construed to infer that impact does not occur. The subject of impact has probably received more discussion than any other one item in connection with airport design. The justification for designing runways for wheel loads without impact seems to be based mostly on the observation that runway pavements show less damage than apron pavements used for warming airplane engines. The writer is of the opinion that the apron failure is not due to increased load, but mostly to the failure of the foundation, which becomes puddled by vibration, creating a condition similar to that of beach sand when tapped with the foot.

It can be assumed that there is no impact at the instant of a perfect landing but, to this date, published data indicate definitely the existence of impact for the average landing. R. D. Evans, of the Goodyear Tire and Rubber Company, has stated (46):

> "In connection with this work [landing tests to determine the efficiency of tires in landings on a variety of soils and surfaces], we used two planes, one in the 3,000-lb. class and one in the 5,500-lb. class. They were equipped with a recording accelerometer whereby the actual impact factors developed during landing could be determined. As a result of this work, we arrived at the following conclusions: Ordinarily our typical landings made on hard surfaces developed impact factors ranging from 1.25 to 1.75 *g*. Anything less than 1.25 *g*. would be rated as an easy or soft landing and one above 1.75 or 2.0 *g*. would be called a hard landing. The hardest landing which we ever experienced in all this series of tests was 3.4 *g*. The design factor in both of these planes was, as I recall, 6.5 so that we still had quite a ways to go before approaching the limit of impact values which the planes were designed to withstand. In this connection, it may be of interest to point out that all types of airplane tires, if operated at recommended values of load and inflation will not 'bottom'; that is, deflect down to contact with the rim until the impact load reaches 3.5 to 4.0 times the static load xx."

In contrast to the Goodyear impact tests with light planes, the Douglas Aircraft Company conducted tests (47) on a DC-4 weighing 63,000 lb. The normal landing impact factor was 1.43 and the extreme landing impact factor was 3.5. Although there was a wide difference in the weight of planes used, the impact factors developed in both tests are in close agreement.

As a basis for the design of the primary airplane structure, the Civil Air Regulations (48) require, in addition to a factor of safety, depending on the part considered, a limit load factor for level landings and three-point landings. For planes as heavy as 2,000 lb gross weight this factor $n = 4.33$, whereas for planes heavier than 2,000 lb,

$$n = 2.8 + \frac{9,000}{W + 4,000} \qquad \text{(12)}$$

in which W = the gross weight in pounds. For a 74,000-lb plane (37,000-lb static wheel load) the factor n becomes 2.91, to which is added a factor of safety.

In summing up this impact question, it should be remembered that the ground reaction is the sum total of the forces causing tire deflection, shock absorber travel, and stresses in the chassis. Also, whenever the tire deflection or the shock absorber travel is greater than that under static load, impact must exist.

The California bearing ratio method is now used extensively for the design of airport pavements and pavement bases. This method is empirical, based on California highway experience, and no doubt contains an impact factor for highway traffic, the existence of which was so definitely proved by tests conducted at the Iowa Engineering Experiment Station. In these tests trucks moving as fast as 20 miles per hr were used, and transmitted loads were scale weighed. The resulting recommendation for the design of highway structures, including culverts, was that impact factors from 1.5 to 2 should be used. Therefore, it would seem preferable to state that impact is allowed for in the California method rather than that no impact is used, or to infer that normal landings create no impact.

The time factor is another item that has received considerable discussion. The contention by some engineers seems to be that a load applied and removed suddenly does not produce the same transmission effect on the pavement foundation or on a buried structure as would a static load or a slow moving load. Whether this is true, or to what extent the transmitted load may be lessened by a supersudden load, as compared to just a sudden load, is not known. Observation has shown that mud pumping occurs from slab deflection under fast moving trucks as well as under slow moving trucks. Also the deflection of a large flexible culvert under a heavy railroad loading with 3 ft of earth cover has been observed. In this case a distinct deflection occurs with the passing of each of the front and rear group of wheels and with a deflection recovery between the application of these loads, thus indicating the speed with which moving loads are transmitted through earth.

Surface Drainage.—Mr. Hathaway's criteria for surface drainage is based entirely on catch basins or inlets. Two methods have been adopted for the collection of surface water—the catch basin (or inlet) and the surface intercepting drain. On some airports a combination has been used. Experience has shown that both methods have their advantages and disadvantages.

The "surface intercepting drain" was devised because of the apparent difficulty of concentrating surface water at one point without the use of pronounced gutters or ditches, which are objectionable on airport surfaces. The idea is to intercept the surface water without changing the uniform sheet flow to a piled-up flow; and, when properly functioning, these drains do just that. The two chief difficulties have been silting of the top layer and displacement of the top layer by traffic. These difficulties are not the fault of the idea, but are due to improper use and construction. Silting will occur when the surface water carries silts and fine sands; and, therefore, when used as indicated in Fig. 38, all turf areas should slope away from the drain. Regarding displacement, the tendency is to use too little bitumen. The correct amount can be determined best by making tests with the particular stone to be used. Contrary to the opinion of asphalt technicians, the claim is sometimes made that if enough bitumen is used to stabilize the surface, porosity is lost. This problem needs and deserves much more research work.

Catch basins (or inlets) function more satisfactorily on sumped areas than any other type. Those of ordinary size are not efficient in collecting water from valley surfaces where warping is limited as on airports. This fact is recognized by those who use extra long grate-top inlets across valleys in runway pavements. These valleys are generally located in the outer part of the pavement as shown in Fig. 8(*c*) to collect the runoff from the runway and thus prevent erosion and softening of the shoulder. Catch basins are even less efficient in collecting surface flow in turf valleys. A combination of the catch basin and surface intercepting drain system as shown in Fig. 39 would insure much better interception. For this use, the intercepting drain backfill up to the surface should not be coarser than the grain-size curve of the filter material in Table 10. On newly graded fields, silting may, and probably will, occur, but, by using the recommended backfill, the silt and fine sands, carried by the surface water, will be stopped in approximately the upper 1 or 2 in. This can be bladed off and replaced with clean filter material. Under ordinary sod-growing conditions, such replacement will not be required more than once or twice.

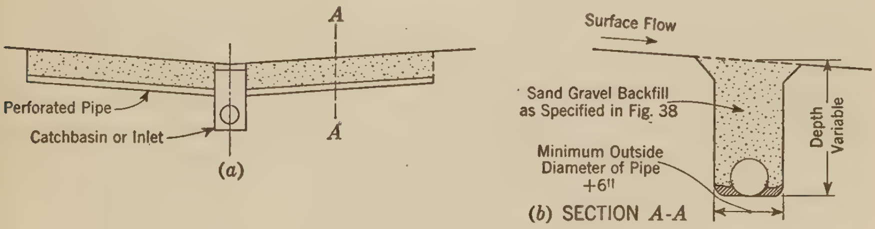

A
A
Perforated Pipe
Catchbasin or Inlet
(a)
Surface Flow
Sand Gravel Backfill as Specified in Fig. 38
Minimum Outside Diameter of Pipe +6"
Depth Variable
(b) SECTION A-A

FIG. 39.—COMBINED CATCH BASIN AND INTERCEPTING DRAIN IN TURF VALLEYS

Combined Surface and Subdrain (*or Base Drain*) *Systems.*—The combined system, shown in Fig. 38, has received criticism on the ground that moisture may be introduced into the subgrade, or water may be forced into the pervious base, if and when the intercepting drain becomes full of water. For storms greater than that for which the drain pipe is designed (frequencies of 1 year, 2 years, 5 years, etc.), the trench backfill may become full of water, but theoretically only once in the design frequency period and then only for a few minutes. In considering this question, one must remember that water follows the line of least resistance and will not enter the pervious base unless under a head—a condition so infrequent as to be negligible. As for the introduction of moisture into the subgrade by capillary action, the design provides for complete removal of free water in the trench, and the remaining moisture will probably be no more than is normally contained in the subgrade soils.

For economic considerations of backfill, etc., the accumulated water in the intercepting drain pipe is usually diverted into a main sewer when a pipe larger than 18 in. is required.

The realization of the need for pervious bases and base drainage is increasing. The proper place for a base drain is at the edge of the pavement. Also, to prevent damage to runway shoulders, the proper place to intercept surface water is at the edge of the pavement. A combined drain that performs both functions is usually more economical than a separate system. However, as in other engineering problems, the conditions must govern the decision as to whether the combined system or separate systems are to be used.

Runoff Computations.—The computation of runoff from various types of sizes of drainage areas has always been an unsatisfactory task. No two engineers working independently will arrive at the same answer. This fact is probably due more to the values selected for the variable factors than to the formula used.

Most runoff formulas contain a "runoff factor" which is expressed as a percentage of rainfall. The Horton formula (Eq. 2) applies this factor (σ) as the rainfall less detention. This seems to be a step in the right direction as it is self-evident that for like surface conditions (soil, vegetation, slope, etc.), the percentage of runoff would be greater for a 2-in. rainfall than for a 1-in. rainfall. On the other hand, the engineer is confronted with the task of assigning a value for hourly detention on which there seems to be little information.

Mr. Hathaway has done a commendable job of simplifying the Horton formula computations by the use of charts, but the solution is still more laborious than by most other formulas. However, after sifting out the essential steps, Mr. Hathaway's solution is less difficult than it appears at first.

The writer has not made a sufficient number of computations to support a definite statement on runoffs obtained by the Horton formula as compared with runoffs obtained by other formulas, such as the Burkli-Ziegler and the rational formulas. In many cases the answers are in close agreement provided the same runoff factors are used.

D. O. Murray,[20] Assoc. M. Am. Soc. C. E.—The designing engineer will find, in this Symposium, considerable valuable information, presented in convenient form. He should bear in mind, however, that differences in hydrological features of widely separated regions will necessarily modify some of the standards proposed.

For example, Australia lies closer to the equator than the United States and, although snow falls on the tablelands and mountains of the southeastern part of Australia, trouble from frost is rare. For the most part, airdromes are so located that loss of subgrade support due to moisture-content increase is not a probability. Some airdromes have been constructed on poor subgrades. One site, abandoned before the war by the civil authorities, was later used because of the exigencies of war.

In Australia, particular care is taken to avoid conditions that will assist capillary moisture to rise under the surface. This precaution is most essential because of the thin bituminous surfaces typical of Australian airfields. Difficulty in procuring supplies has been a sufficient cause in itself for this procedure. Also, during past years, the various road authorities have paid particular attention to the construction of thin bituminous coats. In prewar days, under favorable circumstances, shire engineers were able to obtain as much as a year's service from a surface that had received only a heavy primer. The same treatment has now been given to a few airdromes, generally those in Class III.

Because of the confidence of designers in Australia that there will be no trouble from moisture in the subgrades, the opinion is gaining favor that the combined thickness of pavement and base can be 80% of the values given by Colonel Stratton in Fig. 3.

Surface treatment requires the use of the heaviest primer that the surface will take. If necessary, trial lengths with different grades of primer may be laid down to determine the heaviest that can be used satisfactorily. The rate of application of primer is limited to a maximum of about 0.25 U. S. gal per sq yd. The binder is usually bitumen fluxed to 85–100 penetration, the rate of application being about 0.3 U. S. gal per sq yd. The surface is then covered with screenings not exceeding a 0.4-in. gage. Only a sufficient quantity is spread so that there will be a minimum of loose material. Some engineers favor the use of tar instead of bitumen for surfacing areas upon which gas or oil may be spilled because of the greater stability of tar in the presence of these oils.

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The rainfall intensity curves for rainfall on the eastern coast of Australia conform closely with the curves given by Mr. Hathaway in Fig. 15. Table 12(*a*), containing data presented by G. B. H. Sutherland (49), indicates the close conformity with the curves shown in Fig. 15 (compare with Table 12(*b*)).

TABLE 12.—COMPARISON OF RAINFALL INTENSITIES

Probability (years)	Curve No. (Fig. 15)	Length of Duration of Storm in Minutes							
		10	20	30	40	50	60	120	240
(*a*) Data Presented by G. B. H. Sutherland									
Maximum recorded	....	10.2	8.3	7.0	6.0	5.3	4.8	3.15	2.05
30	....	6.8	5.3	4.3	3.6	3.15	2.8	1.65	0.95
10	....	6.2	4.8	3.8	3.2	2.75	2.45	1.4	0.75
3	....	5.3	4.0	3.2	2.6	2.3	2.0	1.15	0.6
(*b*) Data Presented by Gail A. Hathaway (See Fig. 15)									
....	2.8	6.6	4.8	4.0	3.7	3.1	2.8	1.7	1.0
....	2.4	6.0	4.5	3.7	3.1	2.8	2.4	1.4	0.9
....	2.0	5.2	3.9	3.1	2.7	2.15	2.0	1.1	0.7

A common practice in municipal drainage design is to adopt the formula,

$$i = \frac{K}{\sqrt{t}} \qquad (13)$$

in which i = intensity of rainfall in inches per hour; t = time of concentration of rainfall (that is, the time taken by the rain water to reach the point under consideration from the extremity of the catchment area); and K = a constant so chosen as to give a curve corresponding to the desired probability.

It is a worth-while practice to calculate the value of K for heavy storms or to check with Fig. 15 so as to develop some idea of the relative intensities of various storms. In this way, a better appreciation of the appropriate number for the curve corresponding to a specific storm will be developed. The formula gives low results for storms of short duration and high results for storms of long duration.

Frequently, to drain an airdrome, the neighboring region will have to be serviced by the airdrome drainage system. If the area involves more than about 1,000 acres, the use of time contours will lead to economy in the design of the main drainage ditches. The total area will have to be subdivided according to its separate small catchment areas as determined by roads, taxiways, or natural or other features. Each of these subdivisions will have its own features which will control the design of its drainage. The places reached by the drainage water from each subdivision are marked on the map for equal lapses of time. Usually lapses of 10 or 20 min are suitable—the larger the areas the longer should be the time interval. In this way, it will often be found that drainage from one subdivision has passed before drainage from another subdivision arrives.

The time contours are not constant. They vary with the quantity of water. The quantity of water will govern the depth of flow, and the velocity of the flow will vary with the hydraulic radius of the ditch. The trial computations may be made with either greater or less intensities of design storms. The effects of the time taken for the storm to travel across the full catchment area in various directions may also be considered. Useful information will be obtained as to the possibility of an economical design for a higher design storm.

The use of time contours to allow for ponding is imperative and simple. The total discharge may simply be the addition of the various discharges from the contributing parts of the catchment area.

The need for further research is clearly indicated. However, the latter part of the Symposium dealing with the design of drainage facilities makes available an excellent tool for a diligent application of principles of hydraulics to these drainage problems. Thus, the designer can visit the works he has designed and find the system functioning reasonably well in conformity with his design.

THOMAS B. PRINGLE,[21] M. AM. SOC. C. E.—During 1941–1942 and 1943 the Corps of Engineers constructed many million square yards of airfield pavements. Now that the major construction program is complete, it becomes increasingly more evident that the problem of developing and maintaining the airfield system will be a tremendous task and that to complete this work will require careful planning. Colonel Stratton has emphasized the importance of the development of airfield pavement design to carry the loads in use since the beginning of World War II and has called attention to the importance of traffic and its effect on pavement service. He has also stated that it is too early to forecast the ultimate service behavior of airfields of recent construction. However, it is not too early to lay the groundwork and begin planning the airfield development of the future. This planning will require the collection of factual data which will be the working tools of the planner or forecaster.

The average lives of various pavement surface types can be estimated by carefully compiling data on every airfield in the United States and assembling these data into orderly statistical form, showing for each surface type the square yardage constructed each year and the yardage remaining at the end of the year. The various state highway departments in cooperation with the Public Roads Administration have made studies, known as "Road Life Studies," which have given very reliable data on average pavement lives of various surface types for the nation as a whole, for regions, and even for individual states (50). Although there is no reason to believe that the pavement life of airfields will be the same as that of highways, the procedure for estimating the life will be identical and the statistical records that must be compiled before any comprehensive study can be made will be similar. The method of design must be substantiated by the statistical records and known service behavior of pavements. All these data can be collected only through the orderly compilation of existing information, such as pavement condition, maintenance costs, and reconstruction. These statistics, when studied with traffic records from the time of construction to the death or retirement of the pavement, give factual data for the future development of airfields and for setting up maintenance and reconstruction programs, and they form a basis for handling many complex problems of airfield administration. The actual number of load repetitions is far more significant in the case of airfields than in the case of highways, since the highways must be designed for an unlimited number of load repetitions, whereas on airfields (even those used for capacity operation) the number of repetitions is much less than on heavy traffic highways. From the standpoint of economy it will be necessary that airfield designs be predicated on anticipated life, estimated traffic, and average maintenance costs. Anticipated life can be estimated with reasonable accuracy from the collection of necessary data concerning pavements and the maintenance of reliable records of service, installations, retirements, and inventories. Retirement data and survivor curves, similar to those developed by life insurance companies, will be the basis of determining the probable life of the pavement. This information, if used to guide the expert judgment of men who know pavements and their maintenance problems, should produce a reasonable answer. Purely statistical data will not give the correct solution to the individual problem. However, these data can be of inestimable value to the experienced paving engineer in determining construction and maintenance programs.

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Data on traffic volume and character will be required. These data should include the number and distribution by weight of airplanes using the field; and, for planning purposes of peacetime, airfields should predict the distribution for the hours of the day. These data will show with reasonable accuracy what facilities are required to handle present and future traffic requirements and will help establish priorities for improvement and predict maintenance requirements for the various pavements at an airfield.

About 1915 highway departments thought in terms of new construction and the day when their system would be complete. These systems have been completed to a considerable extent, and their maintenance is as big or an even bigger item than the original construction. The same problems of increased maintenance will apply to airfields as years pass, together with the problems of reconstruction due to obsolescence and deterioration.

It is vitally necessary that an effective planning organization be established to collect the necessary data on service behavior of pavements, including traffic records by type and character, which should contain not only design weights of airplanes but data on actual pavement loading and the maintenance costs for various pavement designs and surface types. The present and future needs of each installation must be studied carefully in order that development can proceed according to the most feasible and economical plan that will utilize existing facilities to the greatest extent and require the least new construction.

Airfield planning and development as discussed herein are mainly problems that will come with peace and the development of the airplane for army and civilian peacetime needs. However, it is important that these questions are not completely overlooked, even during the stress of wartime. The Corps of Engineers has undertaken a study known as an "Evaluation Survey" which will form a nucleus of data that will be invaluable not only in formulating and proving pavement designs but also in planning efficient layouts and in guiding maintenance programs, and especially in planning airfield development for the future. The results of these evaluations must necessarily be held confidential until after the war emergency. Since the evaluation survey is being conducted only at airports of military occupancy, it is, therefore, not considered desirable to disseminate this information until after it ceases to be of military importance.

Colonel Stratton has most ably discussed the design method employed by the Corps of Engineers for airfield pavement construction. The method of design as now employed by the department was not in use during the early construction period and knowledge of the effect of the heavy wheel loads to which pavements are now subjected was unknown from actual experience. Unfortunately, some of the pavement in the runways, taxiways, and aprons of the hundreds of airfields constructed by the Corps of Engineers and other agencies were not built according to the designs discussed in Colonel Stratton's paper.

When the tremendous construction program was almost complete, it became obvious that many airfields had been constructed whose actual carrying capacity was unknown. For the Army Air Forces to set up a training program which would utilize all airfields most efficiently and economically and insure the least interruption, due to failure of pavements under heavy loads, it was necessary to determine the safe carrying capacity of each field. The increasing weights of airplanes and the necessity for designating fields to be used by certain type of aircraft made it essential to obtain a record of the evaluation of each airfield based on the carrying capacities of the pavements as actually constructed.

The field work investigation and preparation of reports are being done under the supervision and direction of the U. S. Engineer Department. For the purpose of this evaluation two kinds of operation are considered—namely, capacity and limited:

1. Capacity Operation.—Capacity operation is defined as the maximum traffic that can possibly operate on an airfield for a period of approximately 20 years. Daily operation may be assumed as varying from 100 wheel loadings for very heavy airplanes to 1,500 wheel loadings for very light airplanes.

2. Limited Operation.—Limited operation is defined as a few wheel loadings a day for a period of approximately 20 years (about 10% capacity operation) or as the maximum traffic that can possibly operate for a period of 2 to 4 years. However. the use of a pavement rated for limited operation by the

maximum traffic that can possibly be operated for a period of 2 to 4 years may require greater yearly maintenance than would an airfield rated for capacity operation.

The over-all evaluation as made is for the airfield as a whole and the runway system as a whole. These evaluations are termed field and runway evaluation:

(*a*) Field Evaluation.—Each airfield is given an over-all field evaluation based on the carrying capacity of the pavements or sections of pavements. The field evaluation is considered as the gross plane weight that may be operated at the field for capacity operation or limited operation without any hindrance due to excessive maintenance or reconstruction of any major pavement.

(*b*) Runway Evaluation.—Runways are evaluated separately from the field as a whole. The mechanics of evaluation are the same for the runway pavements as for field evaluation. Runway evaluation specifies the gross weight of plane that may be operated at limited and capacity operation on any runway without hindrance due to excessive maintenance or reconstruction except possibly at the end of runways.

The evaluation of each pavement is studied carefully, using all available information, and is determined either by actual tests or by estimation of the carrying capacity from design and construction data or other information. This leads to a unit evaluation of each and every pavement.

The evaluation is based on the design criteria presented in Colonel Stratton's paper. Instead of making tests of soil and construction materials, determining the results expected, and thereby determining the design requirements, the evaluation tests the in-place values and from the in-place values assigns the carrying capacity of the pavements. The principles of evaluation are the same as those for design except that they are applied in reverse order. For the evaluation of flexible pavements, the in-place CBR (California bearing ratio) values of subgrade and base courses are determined. This eliminates certain factors that must be considered in design, such as construction control and the possibility that the materials in the prototype will not conform with design values. Similar procedure applies to rigid pavements. The actual k-value or modulus of soil reaction and the flexural strength of concrete in place are determined. Consideration is given to possible frost action in connection with the evaluation of both flexible and rigid pavements and load carrying capacity is reduced because of probable frost action where justified.

The evaluation will be the beginning, and the basis, of service behavior data on airfield pavements. All investigations will be conducted under uniform procedure. The in-place or as-built pavements can be watched through their life, and from the known service behavior a definite relation can be established between theoretical design, as now advocated by the Corps of Engineers and described by Colonel Stratton, and the actual pavement performance. The present empirical curves developed from the results obtained from traffic tests, covering the entire United States, can be verified and corrected, if necessary, from the record of actual service behavior of many pavements constructed under all conditions with a variety of construction materials.

JOHN R. HASWELL,[22] M. AM. SOC. C. E.—Under the heading, "Drainage of Airfields: Subsurface Drainage Systems," Colonel Stratton aptly states that under certain conditions, with a properly designed surface drainage system, no subsurface drainage is necessary. However, the ground-water table is not fixed and a site that may appear satisfactory in a dry year, or even over a cycle of several years of low precipitation, may later develop excess moisture. Therefore, a single line of drains on each side of a runway pavement somewhat as shown in Fig. 8(*b*) would be a valuable precaution in many instances. There will be some seepage through the pavement that should be considered and provisions should be made for its removal. In heavy soils a "perched" water table may develop in the upper few feet of soil when the real ground-water table is somewhat deeper.

The second type of subsurface system likened to "a farm tile drainage system" prompts this discussion. For many years agricultural engineers have been working away from the old costly uniform grid and endeavoring to put the most drains where the wettest condition is encountered in the subsoil. On a reasonably flat field one would not look for seepage to be intercepted by cross drains, but a slope may occur in the subsoil strata or springs may come up almost vertically through a tough clay. Gravel layers or lenses may be tapped and may serve as laterals in some favored places.

The general subject of planning and constructing agricultural drainage systems in the United States is briefly covered in "Drainage in the Humid Regions" (51) and many state agricultural colleges have publications covering local land drainage probems.

The maintenance, where possible, of the old agricultural drainage, as part of the airfield development, has been treated by H. H. Nicholson (52). After quoting the Ninth Report of the Select Committee on National Expenditures dealing with Air Services, Mr. Nicholson writes:

> "It is to be noted that soil and drainage are mentioned last of the factors to be considered in choosing a site, but there can be no doubt that these two points exert a telling influence on the operational efficiency of an aerodrome.
>
> "Some of the special difficulties and possible sources of trouble in this sphere of drainage work can easily be envisaged. The need for an even surface of considerable extent means that pre-existing ditches must be piped and filled in; ridges and furrows must be obliterated; excrescences must be removed and hollows filled in. Existing land drains may or may not be broken in the process, especially where buildings are erected or runways laid down. Whatever is done by way of such operations, surplus water will continue to move along the tracks which it has followed in the past, and wherever the drains are broken or the underground drift of the water is interrupted, there will trouble arise, unless adequate measures are taken to forestall it."

The necessity for placing "natural earth backfill" as shown in Fig. 8(*b*), or "impervious bedding" as shown in Fig. 37, under the tile drain may be questionable. There are trenching machines of the digging wheel type, capable of making a satisfactorily graded ditch and leaving the bottom slightly curved to receive the tile. If the trench is dug in a haphazard manner as to grade, there is ample chance for enough variation in depth of fill, in tamping, and in kinds of soil, to cause sufficient differences in settling to ruin the grade. For most airfields, the placing of drain tile on the undisturbed subsoil, properly prepared to receive it, would seem far safer than the placing of drain tile on an uncertain backfill.

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The use of "impervious bedding" would result in the loss of at least one half the diameter of the drain tile in the effective depth of the drains as related to the soil below the pervious fill. Surely some water may be expected to seep into the subsoil from the base course. The movement of such water is downward in the soil to about the tile level and then horizontal to the drain, rising up in the open joints and entering almost entirely at the under side. It is to be expected that in some cases more water will enter the drains shown in Fig. 8(*b*) through the bottom of the open joints than through the perforations on the upper side of the tile. The same practice persists in some highway standards where the engineer seems to consider the soil mass as so much impervious material and not as a structure with pores and various water-holding capacities.

W. J. Turnbull,[23] M. Am. Soc. C. E., and William H. Jervis,[24] Assoc. M. Am. Soc. C. E.—The problems encountered at the beginning of the war construction program undertaken by the U. S. Engineer Department are covered in this Symposium. Colonel Stratton and Mr. Hathaway are to be congratulated for their contributions. Rigid adherence to schedules set up for completion of the various airfields to fit in with military necessity made these problems much more critical than they would have been in normal times. The solutions described in the Symposium were remarkable, especially considering the speed with which they were made.

Since 1942 the California bearing ratio (CBR) as a tool for the design of flexible pavements has been the object of much discussion by many engineers connected with the design of airfields. The discussion of the CBR test has been about equally divided between favorable and critical comment. Those engineers who support the use of the CBR test as a design tool recognize the fact that it is comparatively new with the Engineer Department. However, these engineers feel that it is better than any previous method and further that at the present time there is no alternate procedure which is as good. The engineers who are critical of the test think that it is something new and untried and that the background of previous experience is a better criterion for designing and building military airfields.

The CBR test measures the shear strength of a soil and is not a classification test. Since the strength of any soil varies widely under different moisture and density conditions and often varies widely between the undisturbed and remolded states, any shear test, whether a CBR, triaxial, an unconfined compression, or a direct shear test, must be divided into two parts—the preparation of sample for test and the actual test. The secret of proper performance of any shear test depends largely on the proper preparation of the specimen so as to reproduce faithfully conditions in the field. If this is not done, the test results will be misleading and may lead to either underdesign or overdesign. With this in mind, the Engineer Department, through the Mississippi River Commission, U. S. Waterways Experiment Station, in Vicksburg, Miss., has been conducting an extensive investigation of the CBR test technique involved both in preparation of the specimen and in performance of the test. This investigation has been supplemented by studies from numerous other laboratories in the Engineer Department. As a result of these comprehensive studies, the CBR test technique has been developed to such an extent that reasonably accurate and uniform results can be obtained by the ordinary testing laboratory.

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Much of the adverse criticism originally directed against the CBR test as a whole was due, not to difficulties entailed by performance of the test, but to lack of knowledge of the effects on the test of certain variables introduced in preparation of test specimens. At the beginning of the investigation it was thought that the variables with the most effect on the CBR would be density, water content, and soil type. Later in the investigation it was learned that other variables existed, the principal one being water content at the time the sample is molded for test. The indications have been that this water content is extremely critical for the soaked CBR values determined on all materials except cohesionless soils and high swelling clays.

Several changes in the test technique as a result of the Engineer Department investigation have occurred since Colonel Stratton prepared his paper. As previously mentioned, many variables involved in the CBR test now seem evident and may be controlled and adjusted to such an extent that the CBR test can be used practically for design purposes. As with any other testing procedure, refinements and variations will be made as further studies in the laboratory and the field become available.

In connection with the development and continuation of the use of the CBR test, the laboratory design test and actual field behavior should be correlated. In other words, it is not known whether the structure of the soil specimen produced in the laboratory is similar to the structure produced in the field by the field compaction equipment generally in use. For a complete and intelligent use of the CBR test, it will be necessary to correlate laboratory and field results. This may be accomplished by either of the following two methods:

(1) Observations and laboratory and field investigations of airports which have been constructed; and
(2) A closely controlled test of field versus laboratory compaction supplemented by actual field observations at existing airports.

Either method should be equally good with the exception that the first one will take considerably longer and its success will depend to great extent on the available field records of water content and density at the time of construction.

As of 1944, a method of conducting field in-place CBR tests has been developed which appears quite satisfactory and which agrees reasonably well with laboratory tests on remolded and undisturbed samples when moisture, density, and soil structure effects are considered. The field in-place test has many advantages because it can be used directly for evaluation purposes without the necessity of considering the many variables which exist in the test on remolded samples for use in design. The main criterion of the field in-place test is that the engineer, using or making the test, must have knowledge of the condition of the subgrade or base-course material with respect to the probable ultimate condition of moisture and density which will exist over the period of the life of the airfield.

Under the heading, "Base Courses," Colonel Stratton states that "Base-course materials sometimes are stabilized with commercial admixtures such as portland cement, cut-back asphalts, emulsified asphalts, and tars." The stabilization of base courses by means of various commercial admixtures should be approached cautiously since it has been proved in several instances that essentially stable base-course materials have been weakened by the use of admixtures. The CBR test is an excellent tool for detecting any increase or reduction of strength in the natural material when admixed with any so-called stabilizing material.

Because improper drainage often may have been responsible for the failure of military airfields, the necessity for surface and subsurface drainage systems at airports cannot be overemphasized, birdbaths on the surface of runways, taxiways, and aprons, as a result of the combination of low transverse grades and consolidation of the base courses and subgrades, are believed to be extremely detrimental, since, if any cracks exist in the surface pavement, water will enter and a localized weakened condition in the base course and subgrade will result. Nonplastic base courses, compacted to a density equal to that obtained at optimum moisture by the AASHO Method, as modified by the Engineer Department, to a great extent, will eliminate detrimental consolidation.

The construction of military airfields has been a huge program and, due to the war emergency, has been completed in record time. In spite of the difficulties inherent in so hurried a program, the design methods which were adopted should in general prove themselves adequate. The CBR test, even though admittedly a new and unperfected tool, has contributed materially to the success of the military airfield program.

Ralph R. Proctor,[25] M. Am. Soc. C. E.—The description, by Colonel Stratton, of the failure of highway design and construction methods to meet the demands for increased subgrade loadings required for military landing fields is most interesting and has prompted this discussion.

Relationship Between Soil Loadings in Earth-Fill Dams and Landing Fields.—More use should be made of earth-fill dam design and construction methods in landing field construction; the soils in the foundation and lower part of a high earth-fill dam are subjected to greater pressures than any mentioned by Colonel Stratton and must withstand these pressures (about 1 lb per sq in. for each foot of height of the dam), without undue settlement or distortion. This condition of loading requires soil stability, secured from the coefficient of friction and cohesion, under greater loadings than those contemplated by Colonel Stratton.

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Uncertainty of Plate Bearing Test Results.—Plate bearing tests involve elements of uncertainty. Three separate soil conditions may be involved; the soil may be saturated at the start of the test, it may become saturated during the test, or it may not become saturated at all during the test. It is believed that no formula for evaluating the relationship of tests under these conditions can be developed for the three following reasons:

(*a*) A fully saturated soil has a modulus of elasticity determined by the soil and water during the time the water is being forced out of the soil by the applied load;

(*b*) An unsaturated soil is permanently deformed by consolidation, even under loadings of 5 lb per sq in., and develops some elasticity at the end of the consolidation period, but consolidates further upon application of a greater loading; and

(*c*) A soil that becomes saturated by consolidation passes through both condition (*a*) and condition (*b*) with an additional influence of compressed air in the voids of the soil when under loading.

These unpredictable variations in soil characteristics (conditions (*a*), (*b*), and (*c*)) led the writer to the conclusion that the best way to avoid such complications in earth-fill dam construction is to compact the soils used to dry weights and moisture contents that would permit full soil consolidation without involving saturation, with consequent reduction of the effective coefficient of friction from internal pressure.

Prediction of Soil Consolidation by Use of the Indicated Saturated Penetration Resistance.—Fig. 40 shows the mean measured soil consolidation, under various loadings, that was secured from 859 laboratory tests of compacted soils of practically all soil types. The solid lines represent the range of the tests and the dotted lines represent interpolated values. The consolidation of broken granite, from 1.5 in. to 2.5 in. in size, when hand placed in a layer 14 in. in diameter by 7 in. deep, is also shown; the compressive strength of the rock was 14,950 lb per sq in. It will be noted that the amount of soil consolidation to be expected from a given loading is shown in terms of the "indicated saturated penetration resistance" of the soil. Fig. 41 shows the method of determining this value. Point A shows an assumed moisture content and dry weight of the soil whose characteristics are shown by the dry weight and penetration resistance curves. Point B shows the moisture content at which the soil would become saturated at the dry weight shown at point A. Point D shows the indicated saturated penetration resistance of the soil at a dry weight represented by line AE. The indicated saturated penetration resistance is seen thus to be dependent on the relative dry weight of the soil.

Prevention of Internal Pressure Caused by Soil Consolidation.—This method of evaluating the relative density of a soil as compacted or as found in place has been used by the writer for the last 12 years. Fig. 40 has been in use (with revisions from time to time as more test results became available) since 1936 for predicting the consolidation of soils in earth-fill dams in order to provide room

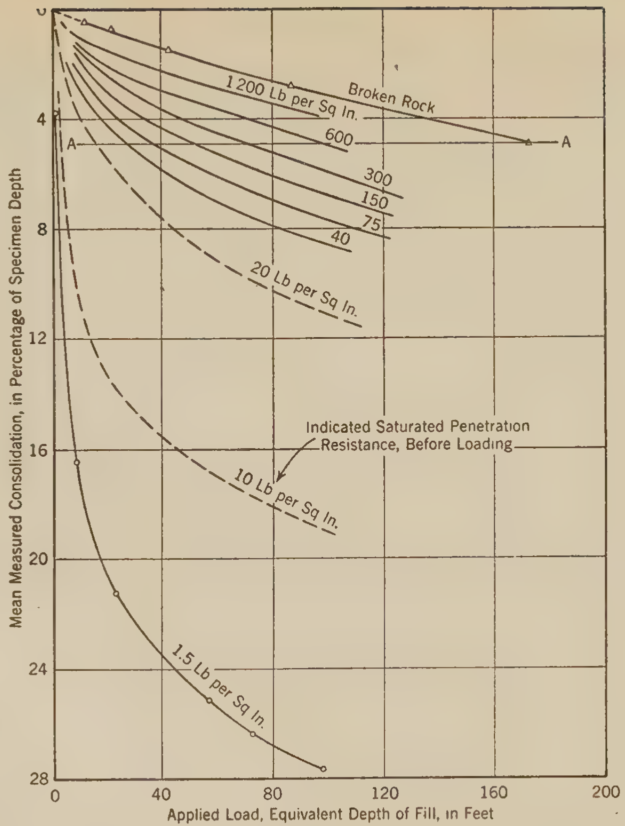

FIG. 40.—MEAN MEASURED CONSOLIDATION OF 859 SOILS WHEN PLACED AT VARIOUS INDICATED SATURATED PENETRATION RESISTANCES AND CONSOLIDATED UNDER VARIOUS LOADINGS

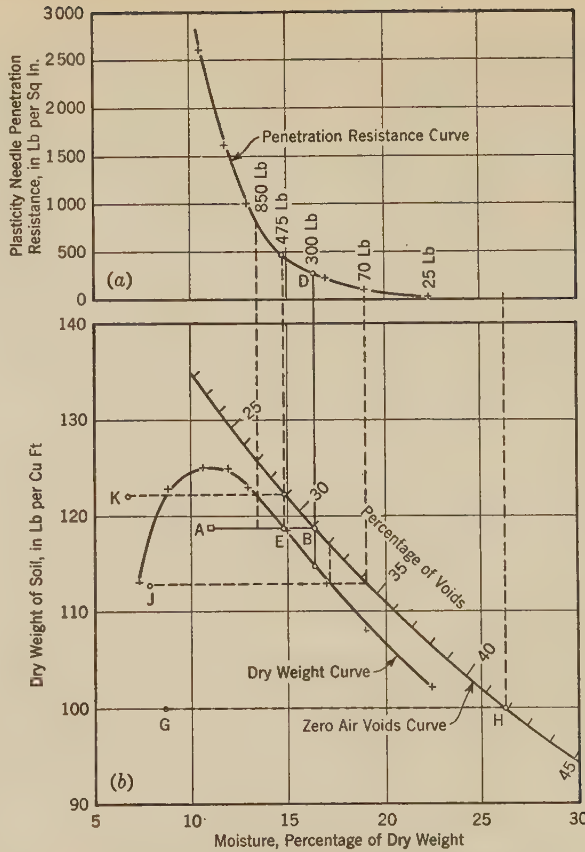

FIG. 41.—TYPICAL COMPACTION-PENETRATION CURVES

for soil consolidation, by proper control of the compacted soil dry weight and moisture content, without the consolidation causing the soil to become saturated. If the soil represented by Fig. 41 is compacted to the dry weight and moisture content represented by point E, subsequent consolidation, without any change in moisture content, will be represented by a vertical line in Fig. 41 from point E. When the dry weight of the soil represented by point E is increased by 3.5 lb per cu ft, complete saturation is reached. Fig. 40 indicates a required depth of fill of about 30 ft to cause this consolidation. However, this consolidation also involves compression of entrapped air in the soil. Therefore, for a 30-ft depth of fill loading, or 30 lb per sq in., it is better to compact this soil, in the case of the dry weight shown at point E, at a moisture content not higher than 13.5%, thus allowing for the aforementioned consolidation to take place with a resulting soil dry weight and moisture content that falls on the dry weight curve. If a soil becomes saturated by consolidation, part of the applied load is carried by water pressure, reducing the effective coefficient of friction of the soil. The writer's opinion is that no appreciable internal pressure will occur in the soil at this and lesser moisture contents when the consolidated dry weight and moisture content are not to the right of the right-hand side of the soil dry weight curve.

Variations in Density of Natural Soils.—Points G, J, and K of Fig. 41 show the variations in soil density of a particular soil encountered during the foundation excavation for a dam. The comparative consolidation to be expected from these soils can be found by noting the indicated saturated penetration resistance of each and comparing the consolidations shown in Fig. 40 at these resistances for some particular soil loading. Point G is a commonly found relative soil density and consequent saturated penetration resistance for soils one to two feet below the original ground surface. This indicates the need for extreme care in using natural soils for subgrade purposes.

Miscellaneous Notes on Soil Testing Methods.—The modified CBR (California bearing ratio) test described by Colonel Stratton has one serious defect—that is, the establishment of an "optimum" soil density as that presumably secured by dropping a 10-lb, tamper eighteen inches. The writer has not found the use of an "optimum" soil density, thus secured, to be reliable as a standard for soil compaction. However, this should be a considerable improvement over the standard CBR method, as a study of the foregoing discussion of Figs. 40 and 41 shows that the consolidation secured from tests after compressing a soil under 2,000 lb per sq in. would have no connection with the consolidation when the soil was compacted to lesser dry weights. A better method for performing this test would be to compact and test the soil at about three different densities, representing the range of variation in moisture content in a well-controlled compacted fill; that is, at penetration resistances of 500, 1,500, and 4,000 lb per sq in. penetration resistance. The lower value is selected in order that the effect of compacting while the soil is too wet may be determined. There is one important point in experimental soil compaction that is overlooked generally. Whenever soil is compacted the soil particles break down; the percentage of fines is increased, evidently by erosion of the larger sized particles. The experimental compaction method should not alter the soil characteristics in this respect to a greater extent than occurs during the actual compaction by the rollers. Repeated compaction of soil samples can change their characteristics enough to make the tests worthless. Another rather common practice in preparing compaction-penetration resistance curves should be avoided; that is, an entire soil sample should never be dried in an oven and then tested by adding various amounts of water to secure predetermined moisture contents. These curves are best prepared by dividing the soil sample in two parts at about 800 lb per sq in. penetration resistance and then progressively drying and compacting to secure the left half of the curves, and progressively wetting and compacting to secure the right half of the curves.

If a plate bearing test is made of this soil at the density and moisture content shown at point G, the resistance to penetration by the plate should increase as the penetrating load is increased, in accordance with the soil consolidation curves of Fig. 40, until the applied pressure becomes great enough to start actual penetration, at which time the deformation curve turns in the opposite direction, following a path similar to that shown in Fig. 4. However, if this soil is tested similarly when in a nearly saturated condition, most of the test load is carried almost immediately by water pressure and a resistance curve similar to Fig. 4 is secured. Fig. 5 indicates that the bearing plate deformation remains constant for a given unit pressure loading for plates larger than 30 in. This indicates that the laws of soil consolidation, rather than of penetration, should apply or that, during the tests that established this ratio, there was not sufficient depth of soil below the plates to determine the true relationship of load to deformation. The writer would expect to secure results similar to those of Fig. 5 by testing a 4-ft thick layer of compacted soil placed on a hard surface.

Fig. 40 shows the soil consolidation that may be expected from a confined soil; it is believed that the results shown in Fig. 40 more nearly portray the action to be expected in a subgrade from either compacted or natural soils than do the results of penetration tests, the plate bearing test is essentially a large penetration needle test. The results secured in Fig. 40 occurred over a period of about two weeks; however, an average of 75% of the consolidation occurred within a period of 5 min. The load repetition test merely determines the time required for a given load to consolidate the soil below the pavement sufficiently to cause failure by excessive slab deflection. For instance, if the duration of an aeroplane wheel loading is 0.1 sec, 75% of the soil consolidation shown in Fig. 40 should occur after 3,000 trips, if it is not necessary to force water from the soil to permit this consolidation.

Both Fig. 5 and Fig. 40 show an increased rate of subgrade resistance with increased deformation; this appears to indicate a serious error in any method of computing slab thickness that is based on the assumption of constant subgrade reaction.

Application of Laboratory Methods to Construction.—It appears that the use of the determined indicated saturated penetration resistances of compacted or natural soils, assembled from compaction-penetration curves made from soil samples taken from frequent field soil density tests as the subgrade is being constructed, should serve as a better guide to the bearing power of a subgrade than plate bearing tests. This is particularly true if the relationship between the indicated saturated penetration resistance of soils and bearing power of soils for slabs is determined by trial loading of test slabs. This testing procedure during actual construction may be illustrated best in connection with Fig. 42, which shows typical soil compaction results secured during one month's work on an earth-fill dam project involving 7,000,000 cu yd of compacted earth fill; 374,000 cu yd were compacted during this month, equivalent to two 7,000-ft runways, 200 ft wide, with 3 ft of compacted fill below the subgrade. A constant check of the addition of water to the soil, both at the dam and borrow pits, was maintained by use of the compaction cylinder and needle. A total of 238 compacted fill density and penetration resistance measurements was made and the results compared with results secured by immediate hand compaction of the removed soils. The mean of these results is shown in Fig. 42 as: Mean cylinder needle, 2,198; mean fill needle, 1,880; mean cylinder, 122.7; and mean fill, 120.7. The minimum compacted penetration resistance was 700 lb per sq in.; the maximum was 3,900 lb per sq in. From the 238 soil density measurements, 69 representative soil samples were selected for preparation of compaction-penetration resistance curves; 25 of the 69 specimens selected for test were taken from the area showing the lowest soil density secured for each day's work. The determination of the indicated saturated penetration resistance of the fill was made from these curves, 28 samples being selected from the 69 samples for which compaction-penetration resistance curves were prepared for consolidation-percolation tests. A corresponding number (28) of plate bearing tests to the consolidation tests would place one every 500 ft along the runways. The soil density and compaction-penetration resistance tests (69) would be spaced apart 200 ft and the compacted density hand-compaction comparison tests (238) would be spaced apart 60 ft. As all of these measurements and tests have been found necessary to control soil compaction in dams, they should be required for similar compaction control in subgrades and should also yield the data for determining pavement thickness as they are obviously much more representative of actual soil conditions than plate bearing tests ever could be. The variations found in actual measured consolidations of apparently similar samples of the same soil show that the mean of a large number of soil tests is more to be relied upon than that of a few tests. A soil density and compaction-penetration resistance test costs six man-hours. A consolidation test costs six additional man-hours. It is obvious that dozens of these tests could be made for the cost of a plate bearing test; furthermore, no heavy equipment need be provided, with consequent high hourly charges for its use. Inspection of the rates of consolidation of soil samples indicates that at least 24 hr should be allowed for each added increment of soil loading for a plate bearing test and, if five increments are added, a total time of six days would be required for each test, involving at least $1,500 of equipment rental and being equivalent in cost to about 100 consolidation tests. The aforementioned 24-hr increment assumption applies to soils that do not become saturated by consolidation; several weeks might be required for an accurate plate bearing test on dense, clayey soils.

Control of Expansion of Compacted Soils When Saturated.—The foregoing condition of compaction (see Fig. 42) has not, in fourteen years of experience of compacting soils by this method, resulted in a dry enough soil to permit subsequent expansion from the absorption of water. Conditions represented by point A of Fig. 41 may possibly permit such expansion; those represented by points G, J, and K of Fig. 41 are almost certain to permit expansion of lightly loaded clay soils.

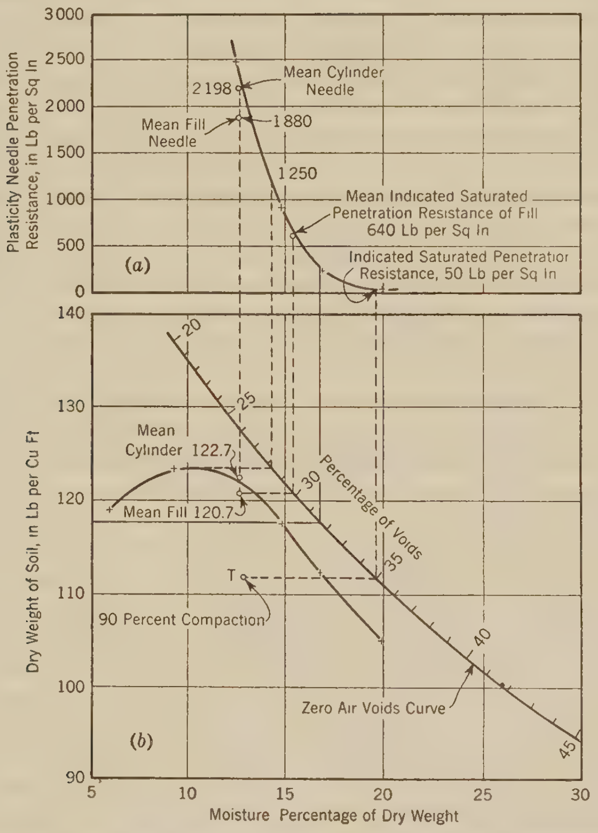

Fig. 42.—A Representative Compaction-Penetration Resistance Curve for 374,000 Cu Yd Compacted Earth Fill, Showing the Mean Values Secured from 238 Compacted Fill Density Tests and Hand-Compaction Tests

Reduced Bearing Power from 90% Compaction.—It is interesting to note the comparison of the results shown with the "90% compaction" standard. The relative consolidation under a given loading to be expected from the mean indicated saturated penetration resistance secured of 640 lb per sq in. may be compared with the 50 lb per sq in. indicated for the 90% compaction point T, Fig. 42. Incidentally, the degree of compaction shown in Fig. 42 is higher than is usually secured by the many variations of hand-compaction methods now in use. It has been the writer's practice for years to require hand compaction that secures a maximum dry weight of soil at about 2,500 lb per sq in. penetration resistance. All of the various "modifications" of the originally published methods for hand compaction (53), such as dropping various weight tampers from different heights, mechanical tampers, etc., were tried and discarded as too cumbersome and not worth while several years ago. No use is made of the actual peak dry weight; the measure of soil compaction used is the indicated saturated penetration resistance. Referring to Fig. 40, the consolidation under a loading of 10 lb per sq in. would be 2.9% for the 90% compaction and 1.3% for the mean compacted fill density shown. Also, referring to line AA of Fig. 40, it will be seen that a loading of 172 lb per sq in. is required to secure a consolidation of 4.9% for the broken rock, 110 lb per sq in. is required for the mean of the compacted fill, and 32 lb per sq in. is required for the 90% compaction. This indicates that, if the loading required to cause a given deflection of a bearing plate is to be used as an index of the bearing power of this soil, the bearing powers of the mean compacted fill (dry weight, 120.7 lb per cu ft) and the 90% compaction (dry weight, 112 lb per cu ft) will be 61% and 19%, respectively, of that indicated for the broken rock. Assuming the broken rock bearing power to correspond to the k of 800 lb per sq in. per in. of Fig. 1, the corresponding values of k for the mean compacted fill and 90% compaction will be represented in Fig. 1 by 490 and 152 lb per sq in. per in., respectively. Applying these values along the 75,000-lb wheel loading curve of Fig. 1(a) the following concrete thicknesses are secured: Broken rock, 5.5 in.; mean compacted fill, 7.2 in.; and 90% compaction, 9.7 in. It is thus seen that, for whatever wheel loading the 75,000-lb curve might represent under these assumptions, the mean compacted fill would require 2.5 in. less concrete than the 90% compaction when used in a landing field subgrade.

Bearing Power of Natural Soils.—The results of plate bearing tests on natural soils cannot be used safely because the process of preparing the subgrade and placing the concrete is almost certain to disturb or remold such soils enough to affect materially their bearing power in their natural state. Concrete, cement, or aggregate trucks in particular are almost certain to have this effect. This remolding of natural soils can also occur after the pavement is placed, particularly in the case of bituminous pavements or where concrete pavements become cracked or broken. It is the writer's opinion that the values for soil consolidation secured from Fig. 40 by use of the indicated rather than the actual saturated penetration resistance are the more reliable for natural soils.

Preparation of Compacted Fill Subgrade for Paving.—There are two practical points of construction that appear to have been overlooked in subgrade construction. One is that the soil should be compacted to at least a foot above the finished subgrade and then trimmed carefully to grade immediately ahead of the placing of the pavement. This is probably the most important point contained in this discussion. It has been found repeatedly in dam construction that soil density tests must be taken from a greater depth than 10 in. from the surface of compacted fills in order to secure a true check of the density of the compacted fill; samples taken closer to the surface show lower soil density in most cases. In pouring the concrete, placing machines have a tendency to leave large areas of rock pockets next to the subgrade. A half inch of mortar brushed on the freshly trimmed subgrade, from which all loose soil has been removed, just ahead of the pavement placing will increase greatly the contact area between concrete or bituminous pavement and subgrade. One landing field came to the writer's attention at which the sheepsfoot fill was of fair order but the loose material remaining after the sheepsfoot rolling was finished was compacted by a smooth-drum roller. The indicated saturated penetration resistance of this part of the fill was 50 lb per sq in. The actual penetration resistance at the time of construction was much higher; the subgrade appeared to be firm and hard. After the first good rain an aeroplane with a tire pressure of 60 lb per sq in. penetrated the 4-in. oiled macadam surfacing and the subgrade until the sheepsfoot compacted fill was reached, just as had been predicted from the soil density test made at the time of construction. It is almost certain that no trouble would have been experienced if the sheepsfoot fill had been trimmed as described.

The use of at least 3 ft of compacted fill subgrade below the trimmed surface, unless the natural soil has an indicated saturated penetration resistance of at least 400 lb per sq in., will eliminate much of the uncertainty of the bearing power of subgrades as the subgrade will at least be uniform in bearing power if the work is done properly. The cost of this work should not be more than for 1.5 in. of extra concrete pavement thickness.

Design and Use of Sheepsfoot Roller.—All of the elements of roller design found necessary to secure soil compaction results comparable to Fig. 42 may be combined in one machine. The materials used successfully for compacted fills in earth-fill dams include decomposed granite and diorite, sandstones and shales soft enough to be dug with a rooter, glacial moraines, alluvial soils, volcanic ash, glacial clay, and heavy California adobes. The roller consists of three drums, universally mounted abreast, 5 ft long outside dimension, by 42 in. inside diameter and fitted with six circumferential rows of thirteen teeth each, arranged so that no more than two teeth may bear on a plane surface at onetime, when directly below the axis of rotation of the drum. The teeth are 8.5 in. long and are fitted with removable shoes having areas of 6 sq in., 7 sq in., 8 sq in., and 10 sq in. The area used is dependent on the material, the choice of area being made to secure a final penetration of 3 in. to 5 in. after sixteen roller trips have been made. The weight, including frame, on each drum should be 10,500 lb, with provision for adding water in the drums to increase the weight to 13,000 lb per roller drum, or 39,000 lb for the entire unit. If required, water and sand can be placed in the roller drums, increasing the weight per drum to 15,500 lb. This roller, at 39,000 lb total weight, can be handled economically by the larger tractors, with a drawbar pull of 13,000 lb at 240 ft per min, up to 6,500 ft above sea level. It may be necessary to use two-drum rollers having a loaded weight of 20,000 lb each to compact some heavy clay soils; this type of roller has also been used successfully to compact decomposed granitic rocks. The aforementioned tractor should haul the three-drum roller, loaded to 20,000 lb per drum, at a speed of 176 ft per min. However, there may be some advantages to the higher speed of travel attained by the use of only two drums of this weight as the compacting load is applied more suddenly. The correct soil moisture content for use with such rollers will be that wherein a penetration resistance in the compaction cylinder from 2,000 lb per sq in. to 4,000 lb per sq in. is secured, the maximum moisture content should be that of an indicated saturated penetration resistance of 400 lb per sq in., with average results of about 600 lb per sq in. indicated saturated penetration resistance.

All of the dam construction experience of the writer indicates that the correct use of the type of roller described will secure substantially similar bearing power from all materials that can be compacted. Actually the only problem in a subgrade is to determine the proper slab thickness for a given loading after the soil is compacted by the roller available for use. All that technical supervision can do is to regulate the soil moisture content so the roller will get the maximum compaction it is capable of producing and to make sure the correct number of roller trips are made over all parts of the fill.

Emergency Construction Procedure.—For hasty or emergency construction the use of the three-drum roller described previously will automatically create high soil densities with no other precaution than controlling the soil moisture so the 7-sq-in. tooth penetration with 13,000-lb drums will be between 3 in. and 5 in. and making certain that sixteen roller trips are made over all parts of the fill. A similar roller, with 10,500-lb drums, was used to secure the results shown by Fig. 42. This method applies only to soils that need addition of water. Where most of the soils are too wet for use some of the previously mentioned decomposed sedimentary or igneous rocks, glacial moraines, and even alluvial soils may be found in place while so dense that, when saturated, the moisture content is low enough to permit compaction of the type described. Where excessive moisture cannot be avoided the use of larger shoes and 10,500-lb roller drums to prevent complete penetration of the roller teeth will secure the maximum soil density possible. The method of testing outlined will determine the indicated saturated penetration resistance of the resulting fill and, in itself, furnish data for determining the pavement thickness or will show which areas of the runways are representative of the fill in order that tests of any other type may be made at locations known to be representative of actual compacted fill conditions.

Concluding Remarks.—The foregoing discussion describes methods of soil compaction that should be especially applicable to hasty or emergency construction; that is, the work of preparing the subgrade could start without delay and the construction forces, by use of the methods described herein, could secure the best compacted subgrade possible with the equipment and materials at hand, after which the type and thickness of paving could be selected to fit the constructed subgrade. The success of emergency or deliberate construction depends on the functioning of the engineering force in charge; poor engineering will result in poor compaction. The engineering procedure for securing this type of compaction control, while exacting, is simple and does not involve complicated test methods; personnel for the control of dam construction have been assembled and trained for several different projects without difficulty. A carefully planned series of tests to determine the required thickness of pavement, landing field or highway, in terms of the indicated saturated penetration resistance of the subgrade would be most useful in that, at least for emergency construction, the data for determining pavement thickness could then be secured from the records of the engineering inspection of the compaction of the subgrade soils.

It appears from this discussion that subgrade soils should be compacted to relative densities represented by "Mean Cylinder 122.7" of Fig. 42 if possible, rather than holding the soil dry weight to a lower value as was done in connection with dam construction in this case to provide for soil consolidation without appreciable internal pressure. It appears that the close approach to saturation of such soils, with consequent internal pressure, will do no harm but, rather, the necessity of forcing water from the soils will slow the consolidation and result in a longer trip-life for the pavement if it is subjected to wheel loads that, if static, would break the pavement. The same reasoning indicates that subgrades compacted to lesser dry weights, because of the presence of too much water in the soil to allow full compaction, should be also compacted to the dry weights and moisture contents represented by the right-hand half of the dry weight curve, as close to saturation as possible. In the case of the slab being designed to withstand full soil consolidation under loading a longer period will elapse, more load trips, before the full slab deflection is reached, allowing the concrete to attain greater strength before being stressed fully.

D. M. Burmister,[26] Assoc. M. Am. Soc. C. E.—A most important and timely contribution to the design and construction of airports is given by this Symposium. The Corps of Engineers, U. S. Army, should be credited with developing the principles and construction practices for the unprecedented program of wartime airport construction. There are a few comments the writer wishes to make on Colonel Stratton's paper.

Subgrade Support.—It is evident from the design curves of Fig. 1 and Fig. 3 that subgrade support is of somewhat less importance for rigid concrete pavements than for flexible bituminous pavements. With the small deflections permissible in concrete slabs to keep the tensile stresses within safe limits, only a minor increase in subgrade support can be developed by deflection of the slab directly under the load. In other words, a rigid concrete pavement has a very considerable load-spreading effect on the pressures transmitted to the subgrade, the load being distributed more uniformly over an area, possibly as large as 10 ft or more in diameter, at correspondingly greatly reduced intensities. On the other hand, the subgrade support is far more important for flexible bituminous pavements, because the deflections are greater and the load-spreading effect is smaller. The greater concentration of stress on the subgrade and the greater curvature of the pavement under the load tend to cause greater deformations in the subgrade, which, under the action of repeated loadings, tend to accumulate to the point of failure of the pavement, unless kept to sufficiently small values. Whether this limitation can be expressed in terms of a maximum allowable settlement under a single load application for different types of soil conditions is an open question. The performance of existing airfields under traffic conditions should clarify this subject considerably.

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It is hoped that Colonel Stratton will be able to present some factual data on settlement or deflection limitations for design purposes.

Base Courses.—Colonel Stratton has emphasized that extremely thick concrete pavements can be avoided by the use of base courses to improve the subgrade support. Although the improvement of pavement support was not always evidenced by plate bearing tests, traffic tests and service performance have demonstrated the value of base courses. The importance of the restraints exerted by a concrete pavement on the supporting capacity of a base course has been investigated (54). The effect may well increase the effectiveness of the base course by 50% to 100%. The restraint effects are due (*a*) to the surcharge effect of the weight of the concrete slab; (*b*) to its rigidity and marked load-spreading effect, which prevents any tendency for upheaval resulting from the deflection of the slab; and (*c*) to the shearing restraint at the interface between the slab and the base course, which prevents lateral displacements in the base course.

In the case of flexible bituminous pavements, the base course furnishes the primary support, and the bituminous layer serves as a wearing course. In Fig. 3, which gives the design curves of required thickness of base course for flexible pavements, there is no distinction made as to the quality of the base course. It is evident, however, from the minimum values of the California bearing ratio (CBR) equals from 50% for light plane loads to 80% for the heaviest plane loads that there is a very considerable range of quality and strength. The supporting capacity depends on the kind of material and the grading, the construction procedures of placing and rolling, and the degree of compaction achieved. When performance under actual traffic conditions has been observed in sufficient detail, it may be possible to establish minimum standards of good construction practice for design and construction purposes. The type of base-course construction can then be selected with due regard for economic and practical considerations, local use of materials, and the thickness required to provide adequate support and length of service. Minimum

TABLE 13.—MINIMUM STANDARDS FOR BASE-COURSE CONSTRUCTION

Type	Quality	Degree of compaction	Relative Density (%)		Tentative thickness factor[a]
			From:	To:	
(*a*) CRUSHED STONE BASES					
B-1	Best	Maximum	90	100	0.50
B-2	Good	Good	70	90	0.60
(*b*) GRAVEL BASES					
B-3	Well graded	Maximum	90	100	0.75
B-4	Run of bank	Good	70	90	1.00

[a] Relative factors based on run-of-bank gravel as 100%.

standards of base-course construction are suggested in Table 13 as a basis for discussion (54).

Experience and performance records will indicate what the relative thickness factors should be. Relative density should be a more suitable measurement than percentage of compaction for the degree of compaction achieved in the field. This is illustrated in Fig. 43. Relative density is based on the loose, dry state as 0% and on the maximum compacted state as 100% as convenient references (laboratory), which give a clear picture of conditions with respect to degree of compactness and also supporting capacity. For example, 95% compaction is only 70% relative density for the material shown, and can be considered only fair to good, being on the border line between medium compact and compact. With the development of better rolling equipment and compacting procedures, it may be possible to obtain 95% to 100% relative density, and ultimately even 110% or more, based on laboratory densities.

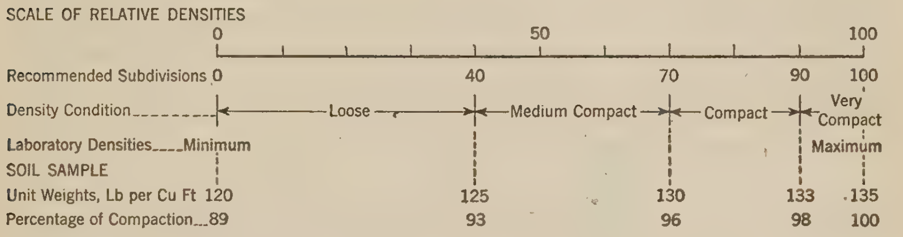

SCALE OF RELATIVE DENSITIES
0
50
100
Recommended Subdivisions 0
40
70
90
100
Density Condition
Loose
Medium Compact
Compact
Very Compact
Laboratory Densities
Minimum
Maximum
SOIL SAMPLE
Unit Weights, Lb per Cu Ft 120
125
130
133
135
Percentage of Compaction 89
93
96
98
100

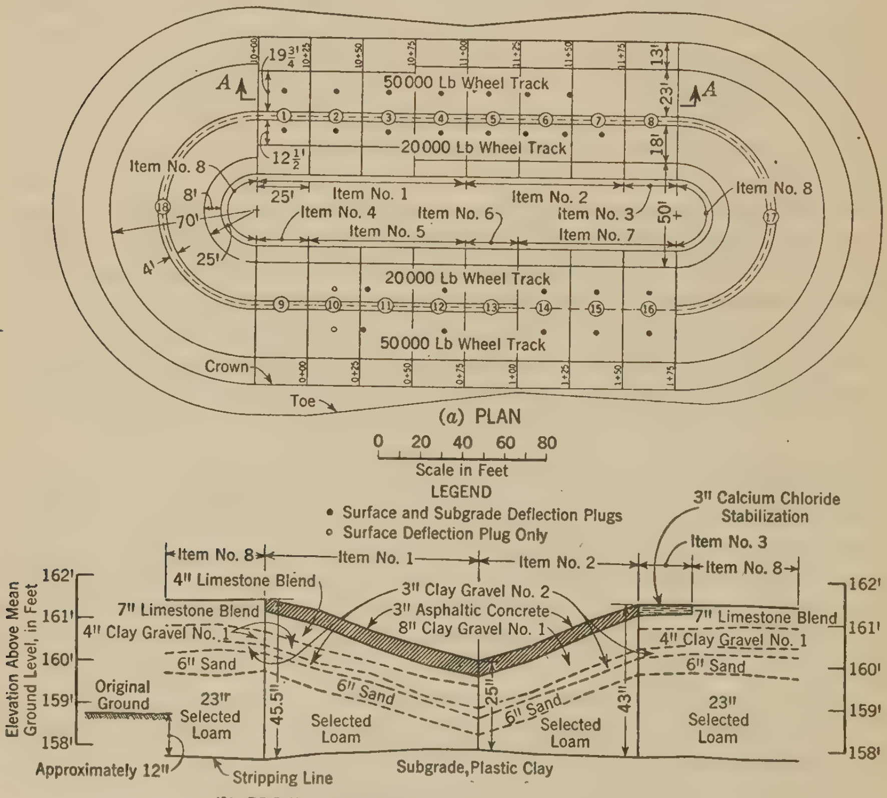
Fig. 43.—Comparison of Relative Density and Degree of Field Compaction

Good compaction is the most essential factor. The close packing and interlocking of the larger particles, whatever the grading may be, give high strength and resistance to deformations. To achieve this condition practically, especially in the upper layer directly beneath the pavement, where the stress concentrations are greatest, the methods used so successfully in the construction of "water-bound macadam" pavements may be used, omitting, however, the topping of fines with water. The principles are simple: A single-sized coarse layer (1¼ in. to 2½ in. in size) 4 in. thick is rolled to maximum compactness. This achieves the close packing and interlocking of the large particles. Choke or filler stone (½ in. to 1 in. in size) is spread and worked into the interstices of the base and rolled until the base is filled. This choke stone provides the proper bedding, so that the large particles cannot rock or shift. An almost ideal condition is achieved for the top course of the base. Where the heaviest wheel loads are anticipated, the best bases so far constructed should be used to avoid excessive thicknesses.

The service and performance records under traffic conditions of the airports constructed by the Corps of Engineers would be of inestimable value for future construction, when correlated with the design and construction procedures given in the Symposium.

Ralph Hansen,[27] Assoc. M. Am. Soc. C. E.—A valuable contribution to the design of airfield pavements is presented in this excellent paper by Colonel Stratton. As the author states (see heading, "Synopsis"), "* * * the design and construction procedures have been verified by accelerated traffic tests utilizing heavy, rubber-tired earth-moving equipment." The following is a description of one of the traffic tests on flexible pavements, initiated by the Chief of Engineers, the results of which confirm the design curves tentatively established for wheel loadings in excess of 12,000 lb.

Barksdale Field, near Shreveport, La., was selected for this accelerated traffic test as it was believed the subgrade at this site would be representative of plastic clay subgrades which might be encountered in the construction of flexible pavements in the southern part of the United States. The specific purpose of the Barksdale Field test was to determine the following for a 20,000-lb and a 50,000-lb wheel load on flexible pavements with a 3-in. asphaltic concrete wearing course:


50000 Lb Wheel Track
20000 Lb Wheel Track
Item No. 1
Item No. 2
Item No. 3
Item No. 4
Item No. 5
Item No. 6
Item No. 7
Item No. 8
$19\frac{3}{4}'$
$12\frac{1}{2}'$
13'
23'
18'
50'
25'
70'
8'
4'
A
Crown
Toe
0 20 40 60 80
Scale in Feet
LEGEND
• Surface and Subgrade Deflection Plugs
○ Surface Deflection Plug Only
3" Calcium Chloride Stabilization
4" Limestone Blend
7" Limestone Blend
4" Clay Gravel No. 1
3" Clay Gravel No. 2
3" Asphaltic Concrete
8" Clay Gravel No. 1
6" Sand
23" Selected Loam
Selected Loam
Original Ground
45.5"
25"
43"
Elevation Above Mean Ground Level, in Feet
162' 161' 160' 159' 158'
Approximately 12"
Stripping Line
Subgrade, Plastic Clay

(a) PLAN

(b) PROFILE, 50000 LB WHEEL TRACK, SECTION A-A

Fig. 44.—Plan and Profile of Test Section

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[27] Head, Soils and Pavement Section, U. S. Engr. Office, Little Rock, Ark.

1. The thickness of pavement and base necessary to prevent detrimental shear deformation in the subgrade;
2. The effect of the quality of base materials on the total specified thickness of pavement and base;
3. The quality of base materials required directly under the pavement;
4. The thickness of a soil cement base needed to prevent detrimental shear deformation in the subgrade; and
5. The adequacy of the compaction requirements as adopted by the Office of the Chief of Engineers to prevent detrimental consolidation of the base materials and subgrade.

Extensive tests on the subgrade and materials locally available for base-course construction were conducted to determine the CBR (California bearing ratio), compaction requirements, and other properties prior to design and construction of the test pavements. The subgrade was stripped level, compacted, and a test section consisting of two test tracks, one for the 20,000-lb wheel load and the other for the 50,000-lb wheel load, was constructed. Each test track comprised the following four different types of bases: (*a*) Layered base construction consisting of several layers of different materials with a high CBR material directly under the pavement; (*b*) layered base construction similar to the first type but with a lower CBR material directly under the pavement; (*c*) a high CBR material for the full thickness of the base; and (*d*) a soil cement base. The test section was oval in shape with two parallel paved straightaways joined at the ends by semicircular unsurfaced turnarounds, with the inner track for the 20,000-lb wheel load and the outer track for the 50,000-lb wheel load (Fig. 44(*a*)). The unsurfaced turnarounds and shoulders offered an opportunity to obtain data on the effect of repetition of heavy wheel loads on unsurfaced bases. Deflection plugs were installed in both tracks to measure the deflection, by means of electrical apparatus, of the pavement, base, and subgrade under the action of moving and static wheel loads and plate bearing loads. In Fig. 45 some typical oscillograms are shown for deflections under moving wheel loads at item 1, section 3, Fig. 44(*a*). The curves are recorded on a 60-cycle time base. The T gage measures the deflection of the surface of the pavement and the B gage measures the compression of the base materials. The difference between the two readings is the deflection in the subgrade. Hydrostatic and earth pressure cells were installed in both the 20,000-lb and 50,000-lb tracks. Loaded earth-

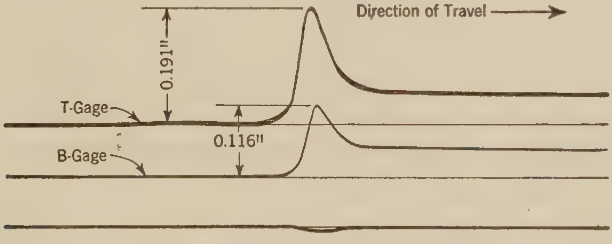
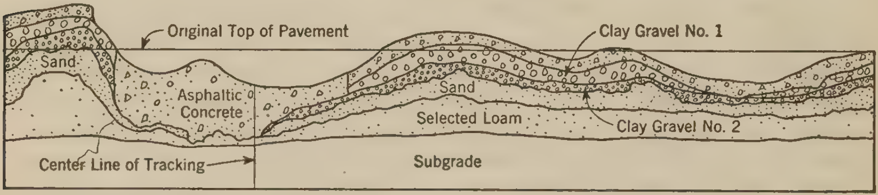
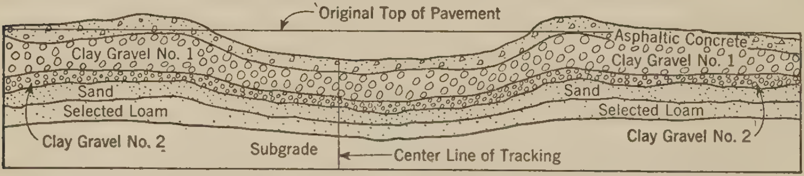


FIG. 45.—TYPICAL OSCILLOGRAMS FOR DEFLECTIONS UNDER MOVING WHEEL LOADS, 50,000-LB TRACK

moving equipment with tire contact areas approximately comparable to those of planes with similar wheel loads were used to produce the specified wheel loads. The loaded equipment was pulled around each test track by traction units, with one rear wheel of the loaded equipment traveling successively in three adjacent tracking paths, each path equal in width to the width of the tire imprint. Traffic was continued until 5,000 trips were made in each tracking path. Traffic was halted at various intervals for test measurements, maintenance and repair to the tracks, and observations. After completion of the traffic test, trenches were excavated in each test track to determine the lateral displacement and consolidation of the base materials and subgrade.


Original Top of Pavement
Clay Gravel No. 1
Sand
Asphaltic Concrete
Sand
Selected Loam
Clay Gravel No. 2
Center Line of Tracking
Subgrade

(*a*) STA. 11+45.7; 20000 LB TRACK; THICKNESS OF PAVEMENT AND BASE 22.5 IN.


Original Top of Pavement
Asphaltic Concrete
Clay Gravel No. 1
Clay Gravel No. 1
Sand
Sand
Selected Loam
Selected Loam
Clay Gravel No. 2
Subgrade
Center Line of Tracking
Clay Gravel No. 2

(*b*) STA. 11+01; 50000 LB TRACK; THICKNESS OF PAVEMENT AND BASE 22.5 IN.

FIG. 46.—TYPICAL CROSS SECTIONS, VERTICAL FACES, AFTER TRACKING

In Fig. 46 are included typical cross sections (item 2, Fig. 44(*a*)) of the lateral displacement of the base materials and subgrade, vertical deformation of the pavement, and curvature of the 3-in. asphaltic concrete pavement resulting from repetitions of wheel loads. Extensive field and laboratory tests were made on the asphaltic concrete pavement, all base materials, and the subgrade. After they have been assembled and analyzed and after correlation with the data from other tests, these data will be used in improving the design technique for airfield pavements.

H. V. PITTMAN,[28] M. AM. SOC. C. E.—Some of the more serious problems encountered in the construction of flexible pavement with clay-gravel base during late fall and winter are included in this discussion of Colonel Stratton's paper which deals particularly with "Construction of Concrete Pavements" and "Drainage of Airfields."

Construction of Concrete Pavements.—As stated by Colonel Stratton (see heading, "Concrete Paving"),

> "A well-prepared rigid pavement design may be vitiated by carelessness or improper construction, and emphasis is placed on the quality of

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[28] Prin. Engr., War Dept., U. S. Engr. Office, Little Rock, Ark.

> subgrade and base-course construction; protection of the prepared subgrade, base courses, and the concrete during the construction; the selection of proper equipment for construction; construction sequences; and adequate curing of the concrete."

These requirements are considered about the minimum essentials necessary to obtain satisfactory results. Experience has shown that, due to the exigencies of the war necessitating tight construction schedules which at times appeared almost impossible of execution within the time limit specified, the final results are considerably less than these minimum essentials or those which would obtain under more normal conditions. Therefore, the ultimate service behavior of some of the wartime airfields is certain to reflect some faults in construction and it should be evaluated accordingly. However, even under the very adverse conditions encountered, the results obtained, as attested by service use, are remarkably good.

From the writer's experience with airfield pavement construction, the subgrade and base-course construction was generally satisfactory. However, provisions for the proper protection of the subgrade, base course, and concrete during construction were often inadequate, due primarily to rapidity of construction, lack of carefully prepared construction schedules and sequences, etc., with resultant damage by traffic, rain, frost, and by drying out during the hot summer months. The specifications, in addition to providing for the exclusion of traffic from the prepared subgrade, should also provide for adequate protection from the elements. It might be possible to use some type of soil treatment that would waterproof the prepared subgrade. Such treatment would insure retention of the original moisture, exclusion of additional moisture, and permit easy disposal of accumulated rain water and continued operations with a minimum of delay. The spanning of one lane with a covering consisting of some type of substantial but lightweight housing of the low-trussed type is deemed worthy of consideration. Some type of protection similar to the foregoing is very much needed so that a minimum of from 800 to 1,000 lin ft of prepared subgrade can be adequately protected from the elements and kept in a state of readiness for pavement operations at all times.

More adequate protection and curing of concrete during construction should be provided, particularly during the hot summer months. Specifications for airfield pavement usually provide for the use of any one of five methods of curing concrete—namely, impervious membrane, cotton mat, ponding, wetted earth, and waterproof paper. Of these methods which serve the dual purpose of protecting and curing the concrete, contractors invariably select and use the impervious membrane curing method because of the ease in handling and applying the membrane and because of the resultant low cost. This type of curing does not adequately protect the concrete from the high summer temperatures which are known to cause severe warping of the pavement. The other methods specified, particularly the cotton mat method, would, if used, provide adequate protection and curing. However, contractors will continue to use the cheapest and most convenient method permitted by the specifications even though it is less effective.

Alternate and Successive Lane Construction.—Colonel Stratton's statements (see heading, "Construction of Concrete Pavements") that "* * * there is need for improvement in placement methods" and that

> "Alternate lane construction of multi-strip airfield pavements is conducive to serious disturbance to the subgrade by the operation of construction equipment and by ponding of rain water on the intervening subgrade between completed strips"

are in accord with observations of the writer. In order to meet design requirements fully the alternate lane method of construction should be eliminated, particularly where concrete is placed directly on a prepared subgrade. Experience has shown also that superior and satisfactory results can be obtained by using the successive lane method of construction of airfield runways progressively from the center line to shoulders by following a carefully prepared construction schedule, based on a proper order of work and operations sequence. This method results in improved drainage conditions on the shoulders and excludes traffic from the prepared subgrade. In addition, where all work of grading, subgrade preparation, and paving is properly coordinated, a minimum of time is lost in backtracking and in waiting until the concrete pavement has aged sufficiently to accommodate the necessary construction equipment.

Drainage of Airfields.—Referring to the section on "Drainage of Airfields," a serious drainage problem encountered during construction should be mentioned—namely, the design location of the pavement below ground surface at the shoulders, a problem not normally encountered in highway construction. Such location or depression of the pavement, in the writer's experience, has proved particularly troublesome during rainy weather and winter construction, has resulted in serious delays to progress and has necessitated excessive ditching at shoulders, pumping out excess rain water, and, in some cases, removal, replacement, and reworking of the subgrade. Whenever possible the pavement grade should be raised so that the base of pavement will correspond to ground elevation at shoulders after stripping, thus affording free surface drainage of the prepared subgrade from center line to shoulders and thence to the drainage disposal system. This procedure would increase the amount of shoulder fill where subgrade is in fill, and would reduce the amount of shoulder excavation where subgrade is in cut. In cases where the stripped natural ground would coincide with the base of elevated pavement, the excavation would be reduced to a minimum as the material in such cases requires only scarifying and compacting, and the operations of excavating, wasting, or hauling to fill to accommodate a depressed pavement are eliminated.

The elevation of pavement grades would also be advantageous at those fields where the terrain is extremely flat and where the drainage system slopes for disposal of surface and subsurface water to border ditches and beyond to natural drainage systems are at a very minimum. The combination of depressed pavements with successive lane construction should eliminate many of the difficulties, delays, and expenses in the construction of some airfields.

Later designs employed in some districts have provided grades which preclude most drainage difficulties encountered earlier in the program.

The location of the water table in rice-growing areas should be given more serious consideration than usual—particularly in late fall, winter, or early spring. In such areas, the cultivated topsoil is invariably underlain with a so-called clay hardpan—a dense, impervious material the existence of which determines the suitability of such land for rice growing. In many cases this hardpan occurs near the surface with a correspondingly high water table and very wet topsoil. In such cases, it is necessary to waste the otherwise suitable topsoil and to substitute borrow material having suitable moisture content for the compacted subgrade. This procedure often results in delay and additional cost while more soils investigations are made and in the necessity for longer hauls. In some cases it was necessary to use the only borrow available which was less desirable than that originally specified. In selecting a site for an airfield in a rice-growing area, it is very important to determine if the site contains sufficient suitable borrow for completion of the compacted subgrade in case seasonal conditions prohibit the normal use of the designated topsoil. Also, this determination of availability of suitable borrow should be made in advance of final selection of the site and, in any case, well in advance of construction.

Problems and Difficulties of Winter Construction.—The construction of concrete pavements can be successfully prosecuted during winter, even in regions of rather severe climatic conditions, without appreciable increase in cost or letdown in design requirements. The most serious problem is that encountered in the preparation and protection of a satisfactory subgrade—a problem common to all types of pavement. On the other hand, the construction of flexible pavement with a stabilized clay-gravel base under winter conditions in areas of frequent rainfall and freezing temperatures, such as prevail in latitudes of Little Rock, Ark., and Memphis, Tenn., is difficult, involves additional time for completion and an appreciable increase in costs, and, more often than otherwise, results in a pavement considerably below the usefulness contemplated by design requirements.

Among the most serious problems encountered are the following:

(*a*) Exposure to damage of large areas of prepared subgrade by all the vagaries of the weather and by construction equipment in delivering, spreading, and compacting the base course.

(*b*) Exposure of large areas of partly or completely prepared base course to like damage by weather and traffic while being constructed in two layers of equal thickness. Often the initial layer is subjected to damage by rain and frost action and requires reworking and compaction before the succeeding or final layer can be placed. When this condition occurs, excess moisture penetrates the subgrade, necessitating complete reworking and compaction of both the subgrade and the initial base course. This same condition may obtain during or after placement of the second or final base course and, at times, after application of the prime coat. Once moisture, appreciably in excess of the specified optimum, enters the base-course material, its removal is a difficult problem due especially to low temperatures and a very low rate of evaporation. Also, the clay-gravel base course is very susceptible to damage by freezing which destroys its stability, necessitating reworking of all areas so affected.

(*c*) Exposure of large areas of primed surface to the elements necessitated by the required minimum curing period of 48 hr and in order to provide sufficiently large primed surfaces for the economical prosecution of paving operations. Such exposure often results in serious damage to the base course, thereby requiring reworking, compaction, and repriming. Also, exposed primed areas hold excess surface water which must be dissipated before application of the wearing surface, resulting in additional delay.

TABLE 14.—NUMBER OF DAYS SUITABLE FOR WORK (SUNDAYS EXCLUDED) FOR PAVEMENTS WITH FLEXIBLE BASE, AND FOR CONCRETE PAVEMENTS

Month	FLEXIBLE BASE				CONCRETE			
	Average, 1937–1942		Actual Days per Month		Average, 1937–1942		Actual Days per Month	
	Days per month	Cumulative days	Minimum	Maximum	Days per month	Cumulative days	Minimum	Maximum
January	7	7	0	14	16	16	6	25
February	5	12	0	8	18	34	10	22
March	15	27	11	19	23	57	20	25
April	14	41	10	17	21	78	18	24
May	20	61	16	24	24	102	23	26
June	18	79	15	21	23	125	22	25
July	20	99	15	25	24	149	22	27
August	20	119	13	24	24	173	22	26
September	21	140	16	24	23	196	21	26
October	22	162	15	26	25	221	22	26
November	13	175	10	18	22	243	20	25
December	11	186	6	16	22	265	20	24

Table 14 shows, in summary, a comparison between the days suitable for work (Sundays excluded) on flexible base and concrete pavement, based on climatological data for Little Rock and vicinity for 1935 to 1942. Table 14 indicates that much time is lost in construction of flexible pavements with a clay-gravel base during the winter months and during periods of heavy rainfall, whereas the experience on concrete shows relatively short periods of lost time occurring during cold weather and following heavy rainfall. Table 14 should afford a good comparison of the effects of weather on construction progress for the two types of pavement.

Winter construction of flexible pavement with a clay-gravel base should be limited to areas of more favorable climatic conditions than exist in the vicinity of Little Rock and Memphis, if results comparable to those of concrete are to be obtained.

G. R. SCHNEIDER,[29] ASSOC. M. AM. SOC. C. E.—The rapid advances in airfield pavement and drainage design procedures are emphasized in this Symposium, and the authors are to be commended for the thorough manner in which the information has been presented. However, some of the unusual problems encountered in the field during the recent construction program in the United States are not mentioned, and it may not be out of place to note some of them in order to complete the record.

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[29] Lt. Col., Corps of Engrs., U. S. Army; Chf., Eng. Div., U. S. Engr. Office, Little Rock, Ark.

Many elements affected the location of the sites selected, and it was sometimes necessary to construct the fields in areas where the conditions were unsuitable from an engineering standpoint. When military airfields are to be constructed for training many thousands of fliers in a short time, they must be located far enough apart to permit construction of auxiliary or satellite fields from which fliers can train without interference with each other and with fliers from other fields. Obviously, a training field should not be located on an important airway. It should be placed where there is a reasonable amount of safe flying weather during the year, where air currents will not cause an undue hazard, and where it will not be subject to overflow by flood waters. Railroad and power line connections must be reasonably close because of critical labor and materials, and an adequate water supply must be readily obtainable. Nearness and accessibility of population centers must be considered because of the labor supply for construction and maintenance of the field and because of the social and recreational benefits. A location where the field would be useful after the war was preferred to one which would have little or no utility in peacetime. It was desirable to avoid hilly land because of the large amount of grading necessary, but flat land where little grading was involved made drainage difficult. Cultivated land was undesirable because the time and energy expended in working the soil to make it suitable for farming rendered it unsuitable as a pavement foundation. On the other hand, if the land was not cleared, the operations of clearing and grubbing, in addition to being time consuming and expensive, disturbed the original soil structure and often resulted in a poor foundation. It was usually desirable to have an all-over landing field in which case the soil had to be suitable for the growth of sod. The aforementioned limitations on site location meant that sites were selected which, in many cases, involved difficult foundation, drainage, and construction problems.

A further difficulty was the urgent need for speed in selection, planning, and construction. It was not possible to spend the desired amount of time on preparation of plans; and the time of construction could not be chosen so as to conform with favorable weather conditions. Because of the magnitude of the construction program, all sources of concrete aggregate and base materials that could be considered at all suitable were used, and, in some cases, materials that were not as good as might be desired had to be utilized. The railroads did an excellent job in hauling enormous quantities of concrete aggregates and base materials, and the Office of Defense Transportation distributed the available railroad cars and expedited their movement very efficiently. However, the pavement design was influenced in some instances by consideration of the difference in railroad haul requirements of the construction materials.

Airfields in the northern part of Louisiana and the eastern part of Arkansas were usually located in the flat alluvial areas where relief is slight and drainage problems are serious. In fact, some of the sites were selected by studying maps of areas that had been overflowed by major floods and then examining the narrow strips of relatively high land between the streams that had not been overflowed for the most suitable site. The soils generally are relatively impervious sandy clay silts, in some instances underlain by sand and in other instances by clay pan. Some of the latter areas were planted in rice or had been used for rice cultivation. Generally, these subgrade soils fall in the Public Roads Administration classification A-4-6. In their natural state they are loose and in the summer they dry out until they are quite hard, but they rapidly become saturated in the presence of water. When wet, the original structure of the subgrade soils is quickly broken down by the passage of construction equipment and their shear strength is largely lost.

Three important considerations in designing a pavement to be constructed on a subgrade of this type under any except the most favorable weather conditions are the depth of the clay pan below the finished subgrade, the limitation imposed by the subgrade on the wheel load of the construction equipment, and the selection of a type of pavement that can be constructed with a minimum of working of the subgrade. If the impervious clay pan is near the surface of the ground, water will collect readily and will saturate the subgrade making construction impossible unless the subgrade soil is removed to the clay pan and replaced with more stable material. Even though favorable weather conditions permit construction, there is the possibility of pavement failure during a wet season with field operations at or near capacity load. Consideration should also be given to the use of runway gutters and inlets, as shown by Fig. 8(*c*), to remove water from the pavement. Otherwise, the turf along the edge of the pavement will act as a dam in accumulating water which saturates and seriously weakens the subgrade at the edge of the pavement. This type of runway gutter is particularly desirable along taxiways if the field is to be used by planes in which the pilot's field of view is restricted so that he cannot see the edge of the pavement.

Subsurface drainage is generally ineffective with subsoils of this type, but if a subsurface drainage system is used special care must be taken in the design and construction of the filter surrounding the pipe to prevent the soil from being washed into the drains. It is essential, too, that the pipe used in the storm drainage system have absolutely tight joints or soil will be washed into the pipes, undermining the fill over the pipe and resulting in sinkholes.

This Symposium provides a sound rational basis for the design of airfields. However, so many factors are involved in the selection, design, and construction of an airfield that the engineer should study carefully all the local conditions that will affect the cost or utility of the field, in addition to familiarizing himself with the features described in the Symposium.

H. M. WILLIAMS,[30] ESQ.—A most acceptable method of airfield drainage design, which has proved useful during the present war emergency construction program, is presented in the paper by Mr. Hathaway. The suggested methodology, however, does not provide for the determination of runoff from lengths of flow greater than 600 ft. The selection of such a limiting value is basically correct since the phenomenon of sheet flow as expressed in the basic equations is normally not experienced on typical airfield covers for lengths greater than 600 ft. In fact, it may be limited to much shorter distances before channelization occurs. However, in many types of airfield layouts, other than that indicated in Fig. 26, the engineer is frequently confronted with conditions under which it is entirely feasible and desirable to permit considerably longer lengths of flow than are covered by Mr. Hathaway.

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In an effort to provide such an extension to the design criteria, consideration has been given to the development of a synthetic hydrograph procedure following, in general, the methodology suggested by Franklin F. Snyder, Assoc. M. Am. Soc. C. E. (29). For the necessary data on which to base the required hydrograph constants, recourse has been made to the data of the U. S. Soil Conservation Service for small, turfed watersheds having characteristics which, in so far as possible, were similar to those of typical airfield areas. The data for the various watersheds which have been studied are assembled in Table 15.

To provide a common basis of comparison for these various watersheds, use was made of the relationships expressed in Fig. 17 by means of which an effective length for $n = 0.40$ and $S = 1\%$ was determined for each area. In these determinations the effective length is subdivided into effective channel length and effective overland flow length. The length of flow in the channel was measured along the proposed collecting channel or swale for that section in which appreciable depth of flow may reasonably be expected to occur during the design storm. Length of overland flow is the average distance from the end of the effective channel to the edge of the drainage area measured in the direction of flow as indicated on the proposed grading plans.

From Mr. Snyder's concept of the unit hydrograph, the following equations may be obtained:

$$t_p = T_p - K_q (t_r - t_d) - \frac{t_r}{2} \qquad (14a)$$

and

$$q_u = \frac{37.5}{t_p} = \frac{Q_{pR}}{R\,A}\left[1 + \frac{K_q (t_r - t_d)}{t_p}\right] \qquad (14b)$$

Also,

$$Q_{pR} = \frac{37.5\,R\,A}{t_p + K_q (t_r - t_d)} \qquad (15)$$

In Eqs. 14 and 15, in addition to the notation of the Symposium:

K_t and K_q = factors which vary as functions of $\frac{t_r}{t_d}$;

Q_{pR} = peak rate of discharge for given area for given rainfall excess;

q_u = peak rate of discharge of unit graph, in cubic feet per second per acre;

R = rainfall excess, in inches;

T_p = time, in minutes, from beginning of rainfall excess to peak discharge;

t_p = lag, in minutes;
t_r = length of rainfall excess, in minutes; and
t_d = unit of duration of rainfall excess, in minutes $\left(\text{assumed equal to } \frac{t_p}{5.5}\right)$.

(This notation has, of necessity, been varied from that presented in Mr. Snyder's paper (29).)

Because of the possible wide range in duration of design rainfall, when using the procedure of constant rainfall intensity for critical durations, it was found necessary to change the assumption by Mr. Snyder that $K_q = K_t = \frac{1}{4}$. The addition of the dimensionless unit hydrograph, discussed subsequently, to

TABLE 15.—DETERMINATION OF

No.	Location of watershed	Area (acres)	Date of storm	LENGTH (FT)		*n*		*S* (%)		ADJUSTED *L* (FT)	
				Overland	Channel	Overland	Channel	Overland	Channel	Overland	Channel
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	Zanesville, Ohio.......	3.57	6-21-37	400	....	0.40	....	14.0	....	110	....
2			6- 6-37	400	....	0.40	....	14.0	....	110	....
3			8-10-37	400	....	0.40	....	14.0	....	110	....
4	Hastings, Nebr.........	3.40	6- 5-41	250	300	0.40	0.10	3.5	5.3	140	30
5			8- 7-42	250	300	0.40	0.10	3.5	5.3	140	30
6	Hastings, Nebr.........	3.74	6-20-42	300	100	0.40	0.10	4.0	5.0	170	20
7			10- 2-41	300	100	0.40	0.10	4.0	5.0	170	20
8			6- 2-41	300	100	0.40	0.10	4.0	5.0	170	20
9			6 -8-41	300	100	0.40	0.10	4.0	5.0	170	20
10			8-11-39	300	100	0.40	0.10	4.0	5.0	170	20
11	Hastings, Nebr.........	0.69	5-17-40	310	....	0.40	....	5.8	....	130	....
12			10- 2-41	310	....	0.40	....	5.8	....	130	....
13	Hastings, Nebr.........	0.69	6- 8-40	310	....	0.40	....	5.5	....	130	....
14			9-24-40	310	....	0.40	....	5.5	....	130	....
15			6- 5-41	310	....	0.40	....	5.5	....	130	....
16	Hastings, Nebr.........	0.69	6- 8-40	310	....	0.40	....	8.0	....	110	....
17			5-17-40	310	....	0.40	....	8.0	....	110	....
18	Edwardsville, Ill.......	49.95	3-15-38	500	1,500	0.40	0.06	2.0	2.0	360	160
19			3-22-38	500	1,500	0.40	0.06	2.0	2.0	360	160
20			5-27-38	500	1,500	0.40	0.04	2.0	2.0	360	110
21			6-21-42	500	1,500	0.40	0.04	2.0	2.0	360	110
22			2- 9-39	500	1,500	0.40	0.06	2.0	2.0	360	160
23			5-17-43	500	1,500	0.40	0.04	2.0	2.0	360	110
24			5-17-43	500	1,500	0.40	0.04	2.0	2.0	360	110
25			5-17-43	500	1,500	0.40	0.06	2.0	2.0	360	160
26			9- 2-41	500	1,500	0.40	0.06	2.0	2.0	360	160
27	Albuquerque, N. Mex...	40.5	8- 2-39	200	2,400	0.15	0.06	0.9	5.0	75	180
28			8-20-39	200	2,400	0.15	0.06	0.9	5.0	75	180
29	Colorado Springs, Colo..	35.6	8-10-38	400	1,500	0.40	0.10	7.5	3.1	150	200
30	Colorado Springs, Colo..	35.4	7-21-40	900	1,500	0.40	0.10	6.0	3.1	380	200
31	Vega, Tex.............	95.9	5-30-38	900	2,700	0.20	0.04	2.0	1.5	320	200
32			5-30-38	900	2,700	0.20	0.04	2.0	1.5	320	200
33			6-21-39	900	2,700	0.20	0.04	2.0	1.5	320	200
34			7-26-39	900	2,700	0.20	0.04	2.0	1.5	320	200

determine hydrographs for durations of rainfall in multiples of t_d indicated that K_q and K_t varied as functions of $\frac{t_r}{t_d}$. This relation is shown in Fig. 47.

Values of t_p and q_u shown in Table 15 were obtained for the various runoff occurrences by means of Eqs. 14, and, in turn, were plotted against the effective length of the corresponding drainage areas. From these data, the following relations were determined:

$$t_p = 0.466\ (L\ L_{CA})^{0.3} \qquad (16)$$

or, if L_{CA} is assumed to be $\frac{1}{2}$ L, then:

$$t_p = 0.378\ L^{0.6} \qquad (17a)$$

and

$$q_u = \frac{37.5}{t_p} \qquad (17b)$$

Synthetic Unit Hydrograph Constants

Total adjusted L (ft) (12)	t_r (min) (13)	T_p observed (min) (14)	t_p estimated (min) (15)	t_d estimated (min) (16)	Q_{pR} observed (cu ft per sec) (17)	Volume of runoff (in.) (18)	q_u (cu ft per sec) (19)	Q_{pR} reproduced (cu ft per sec) (20)	T_p reproduced (min) (21)	No.
110	11.0	14.0	7	1	7.4	0.66	4.52	9.2	14	1
110	7.5	9.0	4	1	3.3	0.21	7.02	3.5	11	2
110	3.0	5.0	4	1	8.0	0.43	5.98	8.8	8	3
170	6.0	13.5	9	2	2.8	0.19	4.67	2.7	12	4
170	3.0	10.5	9	2	3.4	0.21	4.75	3.2	10	5
190	16.0	21.0	10	2	3.4	0.47	2.81	4.8	20	6
190	11.0	16.0	9	2	1.6	0.17	3.27	2.1	16	7
190	12.5	20.0	12	2	1.0	0.12	2.71	1.4	17	8
190	10.0	17.5	11	2	0.8	0.10	2.48	1.3	15	9
190	5.0	11.5	9	2	3.0	0.30	2.71	4.7	12	10
130	29.0	29.0	8	1	0.21	0.16	4.53	0.23	31	11
130	13.0	16.5	7	1	0.23	0.11	4.65	0.26	16	12
130	6.0	12.0	8	1	0.44	0.21	3.34	0.65	11	13
130	10.0	14.5	8	1	0.35	0.18	3.66	0.48	14	14
130	5.0	11.0	8	1	0.34	0.17	3.08	0.57	13	15
110	10.0	12.0	5	1	0.50	0.19	6.23	0.54	13	16
110	11.0	14.0	6	1	0.46	0.20	5.19	0.54	14	17
520	17.0	24.0	13	2	24.6	0.32	2.04	30.6	26	18
520	18.0	30.0	17	2	16.3	0.18	2.21	16.8	27	19
470	11.0	17.0	10	2	87.0	0.75	2.89	84.0	21	20
470	16.0	21.0	10	2	89.0	0.66	3.92	67.0	25	21
520	10.0	25.0	19	2	18.0	0.20	2.48	21.8	22	22
470	7.0	16.0	11	2	76.0	0.74	2.43	89.3	19	23
470	10.0	16.0	10	2	62.0	0.65	2.14	75.0	20	24
520	40.0	46.0	17	2	20.7	0.39	1.66	27.7	44	25
520	5.0	19.0	18	2	21.9	0.19	2.39	22.1	19	26
255	5.0	11.0	8	1	17.6	0.14	3.35	19.7	13	27
255	8.0	13.0	8	1	12.1	0.12	2.77	15.5	15	28
350	10.0	15.0	9	1	83.3	0.66	4.43	59.9	19	29
580	10.0	22.0	16	2	37.5	0.39	2.88	25.6	27	30
520	13.0	20.0	12	2	136.0	0.73	2.40	144.0	24	31
520	13.0	21.0	13	2	96.0	0.66	1.83	131.0	24	32
520	63.0	67.0	20	2	62.0	0.60	2.36	53.9	67	33
520	23.0	32.0	15	2	64.0	0.50	1.91	48.8	31	34

A dimensionless unit hydrograph was adopted (Fig. 47), the peak of which could be determined for any drainage area by Eqs. 17. The length of the recession was assumed to be four times the rising side of the hydrograph or a total hydrograph base length of $5\left(t_p + \frac{t_d}{2}\right)$. With these instruments, several of the hydrographs were reproduced with a resulting degree of agreement which was considered good.

Runoff data obtained by the foregoing method have been incorporated in a reproduction of Fig. 23 for a comparison of results of the two methods, as shown in Fig. 48. For lower values of supply, the synthetic unit hydrograph produces a higher rate of peak discharge, and, also, for peak discharges reduced by any appreciable amount of ponding and with the more common supply values of between 1.0 and 2.0, the required drain-inlet capacity does not vary greatly for either method. However, because of the inherent differences in the basic principles of the unit hydrograph and the theory of overland sheet flow, it would be impossible to achieve complete agreement in the results obtained by both methods for a given area subject to a given rainfall intensity.

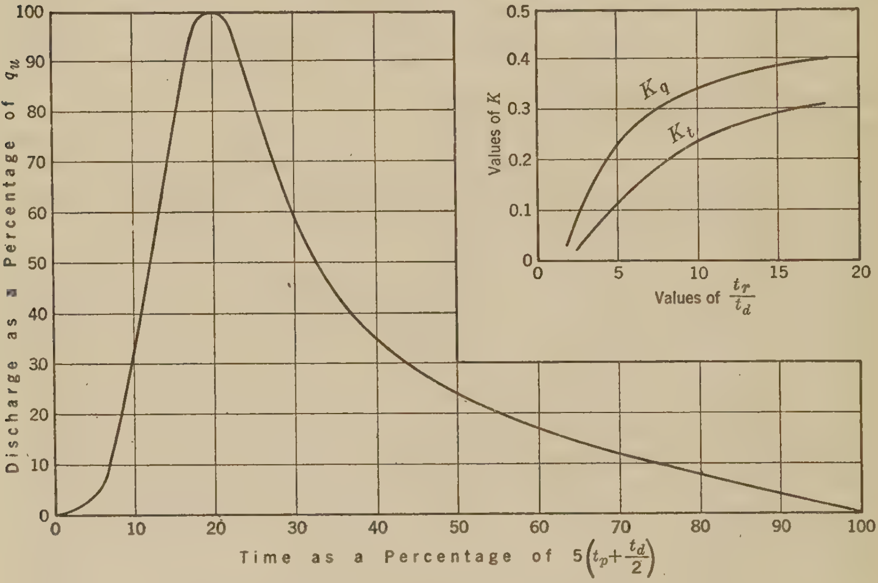

FIG. 47.—DIMENSIONLESS UNIT HYDROGRAPH AND RELATION OF K_q AND K_t TO $\frac{t_r}{t_d}$

For a further comparison of the two methods, the runoff from an assumed turfed airfield area 400 ft wide and 2,000 ft long was computed for a supply of 1-in. volume occurring in 20 min. Runoff from the area was assumed to be collected in a shallow swale with 1.5% transverse slope and 0.5% longitudinal slope along the long axis of the area. To conform with the limits of overland flow suggested by Mr. Hathaway, the area was subdivided into twenty subareas, 200 ft square. The hydrographs for the subareas, determined by Mr. Hathaway's method, were routed through the storage in the swale. It was observed that the resulting hydrograph for the total area was very similar to the synthetic hydrograph, both having a peak of 23 cu ft per sec, but the synthetic hydrograph lagged 6 min. Needless to say, the effort required to route the hydrographs for the subareas through storage greatly exceeds that required to develop the synthetic hydrograph.

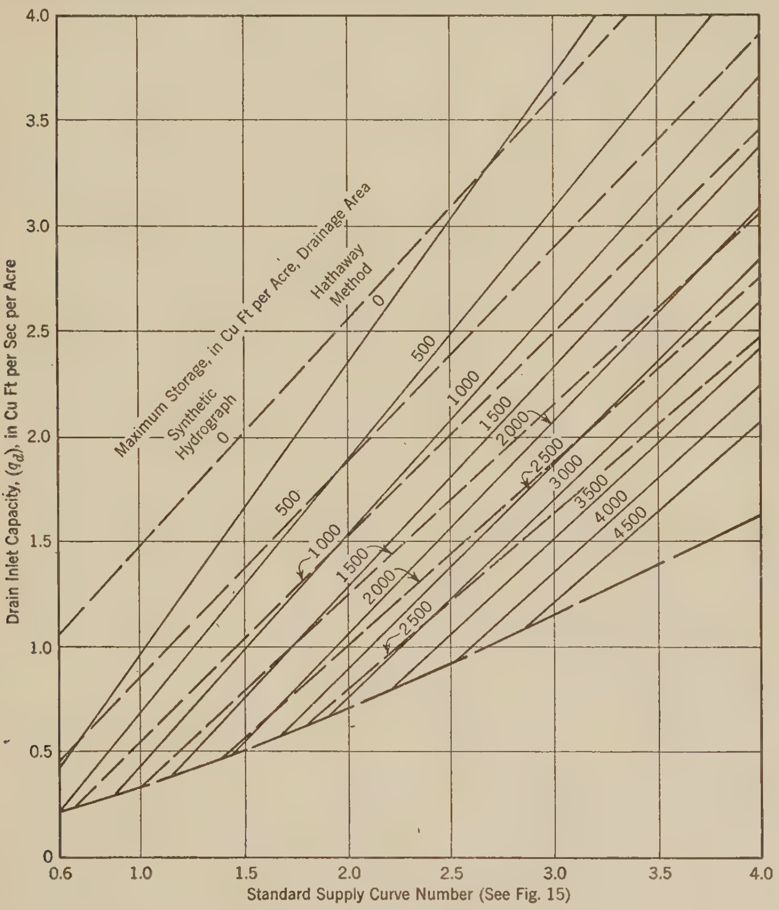

Fig. 48.—Comparison of Required Drain-Inlet Capacity by Mr. Hathaway's Method and by Synthetic Unit Hydrograph ($L = 400$ Ft, $S = 1\%$, and $n = 0.40$)

The records on which to base a unit hydrograph study of this nature are somewhat limited, but additional records are becoming available rapidly to amend these preliminary investigations. However, the study presented herein indicates a feasible method of extending the design criteria presented by Mr. Hathaway to the determination of runoff from larger airfield drainage areas.

Acknowledgment is made to Mr. Snyder for his helpful comments in connection with these studies and to the U. S. Soil Conservation Service for field data on small watersheds.

Ira B. Mullis,[31] Esq.—This Symposium is one that deserves to be widely read and discussed. Apparently Brigadier-General Stratton has found that the AASHO Method (55) is an inadequate one; otherwise, he would not have modified the method by utilizing a 10-lb rammer dropped from a height of 18 in. in five sets of twenty-five drops each (see heading, "Compaction Tests and Control"). In terms of compactive work applied, this is more than four times that of AASHO Method.

The writer has also found that the AASHO Method (55) is entirely inadequate, except in cases of well-graded sandy clay or other materials of a similar nature. Many failures over a widespread area have been observed where this standard of compaction was followed. Flexible pavements have failed in many cases under rolling operations during construction or later. Of course, the roadbed at the time of failure was wet enough to be plastic. To complete the work over wet spots that had failed, the pavement was removed and later patched when dry. Since patching does not alter the porous condition of the roadbed, other failures follow during wet periods. One project is recalled in which a concrete pavement, built under the general supervision of an ardent advocate of the AASHO Method (55), had settled and broken so that much of it has been raised by means of the mud pump to overcome the depressions.

Although General Stratton states that the service performance of airport pavements under severe operating conditions has been extremely gratifying, he makes no prediction as to final results to be expected except to state that the optimum moisture condition at which the subgrade and base courses are placed does not represent the moisture condition after the passage of time. Data collected by Bert Myers (56) tend to support this statement.

Apparently the water content at the time of compaction has no effect on controlling the future water content, though pore space does have a marked effect as shown by Mr. Myers' data (56*a*). The writer has many data (a considerable part of which has not been published) in support of the statement that limiting the pore space is a means for controlling the water content (57). One reason for the strong resistance of hard pan and consolidated sandy clay to loads and to the penetration of water is its very limited pore space.

Although many hold that certain static or impact forces can be used as standards for forecasting the ability of roadbed materials to resist traffic and weather, the writer knows no data that will support this belief. Indeed, if one should supplement pressure with high-frequency vibration, the method would not constitute an adequate standard of stability, although it might represent the maximum compaction obtainable by any method. Furthermore, work is not a measure of results to be expected; but skill in applying work, and the ease with which certain types of materials are formed, are items that must be considered. In view of this fact, the result obtained in making a roadbed resistant to traffic and water must be the sole standard of quality.

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Fig. 41 is a well-suited illustration of the relations of density, water content, voids, and resistance to force. It shows that resistance to force or penetration decreases uniformly with increase in water content from 2,500 lb to about 1,000 lb at which point resistance to force decreases rapidly. Above the 1,000-lb point the curve shows that the body of soil is rigid, but below this point the soil is plastic. The proportional limit having been passed, near the 1,000-lb point, none of the loads corresponding to the densities and moisture contents of about 121.5 lb per cu ft and 13%, respectively, can be considered safe for the load imposed unless confined.

If the moisture-density curve and the moisture-penetration curve were extended to the left as far as they are to the right, the moisture-penetration curve would begin to drop at a point in excess of 2,500 lb. The cause of the drop of the curve at this point would be due to an increase of air content. As the density of the mass and its water content are lowered, the air content increases and thereby changes from a state of rigidity at 2,500 lb to a state of plasticity due to excessive volumes of air.

If the dry weight curve were extended downward to the left to a weight of 100 lb, the voids would be 39.6% on the basis of an assumed specific gravity of 2.65; at 90 lb the voids would be 45.7%; and at 80 lb, 51.7% with little resistance to loads or the penetration of water. Consequently, the penetration resistance curve on the dry side (left) would decrease in a manner similar to the wet side of the curve.

Although engineers are aware of the softening effect of water on roadbeds, few seem to realize that rutting or plastic flow does not start until a certain amount of water is absorbed. Of course a small amount of water in pore spaces is desirable at all times, but when a critical amount is absorbed the material is plastic, as shown by Fig. 41 where a less than 2% increase of water changes the resistance from 850 lb to 475 lb. Fig. 42 shows a similar critical change near the 750-lb point.

Elsewhere, the writer (58) has presented many data concerning this critical change, which he had previously indicated in 1938 (57). Inasmuch as the presence of a critical amount of water in a clay body produces a sharp and definite change in its structural properties it conforms to Hooke's law and must be so regarded. On the one hand, a consolidated body of dry or moist clay that is overstressed fails by rupturing, but one that is critically wet fails through plastic distortion. That which marks the boundary between the rigid and plastic states is rupture, a property universally recognized by physicists. It would seem, therefore, that bodies of earthy sediment containing critical amounts of air or water, or both, should be expected to fail even under light loads.

The method of guarding against this form of weakness is discussed elsewhere (57)(58).

C. F. Izzard,[32] Assoc. M. Am. Soc. C. E.—A comparison of actual runoff hydrographs resulting from simulated rainfall with theoretical hydrographs computed by Eq. 5 for two types of surface—smooth pavement and dense turf—is contained in this discussion. The supporting data are taken from a cooperative project for which the Soil Conservation Service of the U. S. Department of Agriculture performed the experiments and the Public Roads Administration of the Federal Works Agency directed the project and analyzed the results. The equipment used and the method of performing tests are described in detail elsewhere (59). Fig. 49 shows the equipment in operation.

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[32] Senior Highway Engr., Public Roads Administration, Washington, D. C.

The "paved surface" was actually an asphaltic roofing paper with crushed slate particles on the surface, laid on a wooden platform covered with an asphaltic mastic. The surface conformed closely to a true plane, probably more accurately than the average surfaced runway. The turf surface was obtained by placing a dense bluegrass sod upon the paved surface. The soil was about 1.5 in. thick. Movement of water beneath the soil was checked by transverse wooden strips. Tests were made with the soil in a saturated condition so that infiltration was almost negligible.

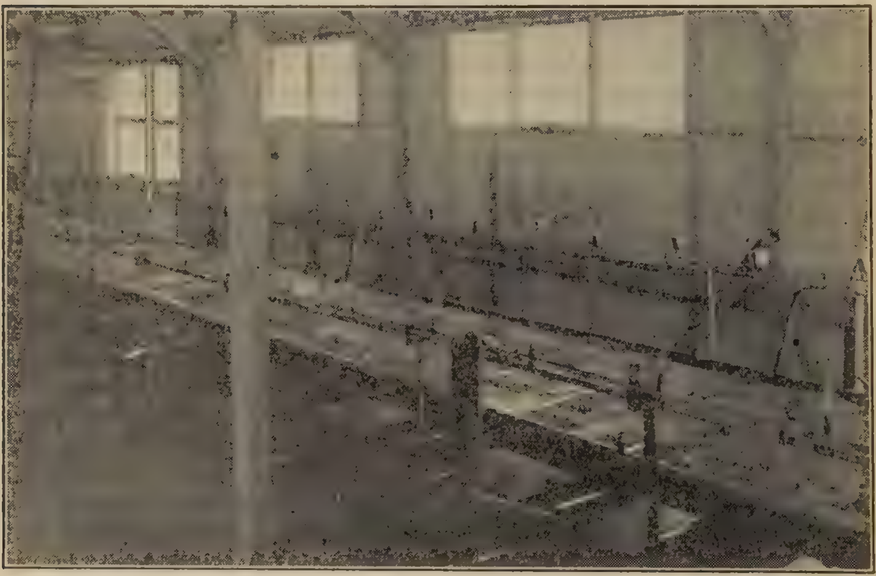

Fig. 49.—Rainfall Simulator in Operation on Bluegrass Turf Plot, 72 Ft Long; Rainfall Intensity, 3.81 In. per Hr; Slope of Plot, 1%; Runoff Rate Approaching Equilibrium

A typical hydrograph with paved surface on a 1% grade is shown as a solid line in Fig. 50. The rain began at zero minutes and continued beyond the time at which equilibrium was reached. The dashed line shows the theoretical hydrograph computed by Eq. 5 for the indicated rainfall intensity, length of overland flow, and slope, with $n = 0.02$ as recommended by Mr. Hathaway for smooth pavement. The dotted line shows the best fit obtainable with Eq. 5 by shifting the time axis and adjusting the value of n so as to fit the slope of the hydrograph at one third the maximum runoff intensity, which is the point of inflection on the theoretical curve. The time shift in this case was 0.52 min as shown in Fig. 50, and the adjusted value of n was 0.00603.

The average percentage deviation of the unadjusted computed hydrograph from the actual hydrograph was 18%, based on ordinates at 1-min intervals excluding the first minute. The average percentage deviation of the adjusted computed hydrograph from the actual hydrograph was only 1%.

Similar comparisons were made for typical runs at 3.6 in. per hr and 1.8 in. per hr on lengths of 12 ft, 24 ft, and 48 ft, and for 72-ft plots on slopes of 2% and 4%. Results were similar to those in Fig. 50, although the adjusted computed curve shown in this figure fitted better than any of the others. In all cases it was necessary to shift the computed hydrograph on the time axis and to change the value of n from the recommended value of 0.02 in order to obtain a reasonably good fit. For unadjusted computed hydrographs on 1% slope ($n = 0.02$) the average deviation was about 15%, generally decreasing with decrease in the length of plot. The deviation tended to increase as the slope of the plot increased. The computed hydrographs, adjusted and unadjusted, were always at or below the actual hydrograph, except during the first minute.

The adjusted computed hydrographs on the 1% slope averaged about 5% deviation from the actual hydrograph. The deviation appeared greatest for the 24-ft and 48-ft plots, and increased as the slope of the plot increased. The adjusted values of n varied from 0.005 to 0.010. The shift in the time axis to bring the adjusted hydrograph into position with the actual hydrograph at the one-third point ranged from 0.3 min to 0.8 min.

The average percentage deviation for both adjusted and unadjusted computed hydrographs as compared to the actual hydrograph was generally greater for the 1.8 in. per hr rainfall intensity than for the 3.6 in. per hr rainfall intensity.

For the turf plot the computed hydrograph shown in Fig. 51, based on the recommended value of $n = 0.80$ for dense turf at a rainfall rate of 3.81 in. per hr, deviated from the actual hydrograph by about 16%. The adjusted computed hydrograph deviated about 6%. The latter deviation was about the same for the other lengths of plot at the same rainfall intensity and occurred principally in the upper half of the hydrograph. The tendency for the actual hydrograph to differ from the theoretical curve was common to all turf runs and was observed in runs made under actual field conditions.

The unadjusted computed hydrograph for the 1.83 in. per hr rate of rainfall, on a 72-ft plot sloping 1%, deviated 39% from the actual hydrograph. The adjusted computed hydrograph for this case, with $n = 0.429$ and a time shift to the right of 1.33 min, deviated less than 2% on the average from the actual hydrograph.

The data so far available do not justify changes in the values of n recommended by Mr. Hathaway or the introduction of an arbitrary time displacement.

The tests here reported did not extend beyond a 3.8 in. per hr rainfall intensity on a 72-ft plot. Within this range the evidence is fairly conclusive (60) that discharge on both paved and turf surfaces is proportional to the third power of the depth instead of the second power, as indicated by the exponent

$M = 2$ in Eq. 5. When one attempts to use $M = 3$ in Eq. 4, the turbulence factor I becomes zero, and the factor K in Eq. 3 becomes infinite. Furthermore, the general formula, Eq. 2, was derived by Mr. Horton using an assumed value of $M = 2$ for which the integration of his differential equation resulted in the hyperbolic tangent function. For $M = 3$, integration results in a more complex function; and for other than whole-number values of M, integration becomes almost impossible. Mr. Horton states (18) that the equation here referred to as Eq. 2 is only quasi-rational for values of M other than 2.

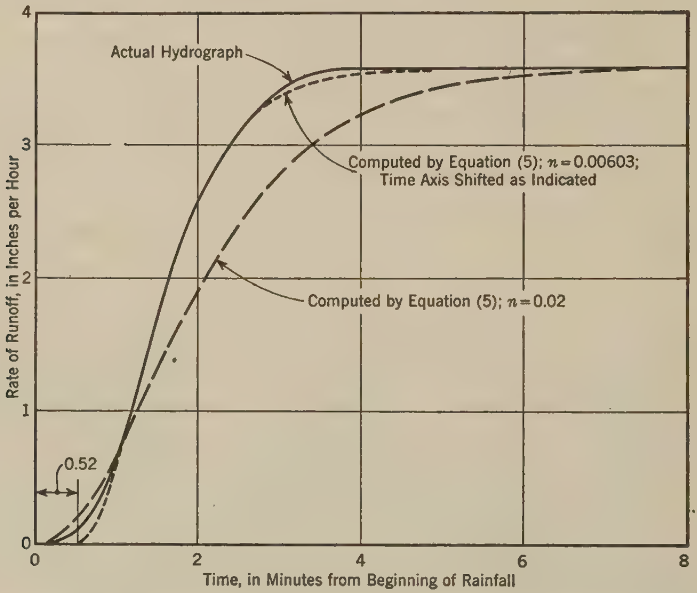

FIG. 50.—ACTUAL AND COMPUTED HYDROGRAPHS FOR PAVED SURFACE (LENGTH OF PLOT, 72 FT; SLOPE, 1%; AND RATE OF RAINFALL, 3.57 IN. PER HR)

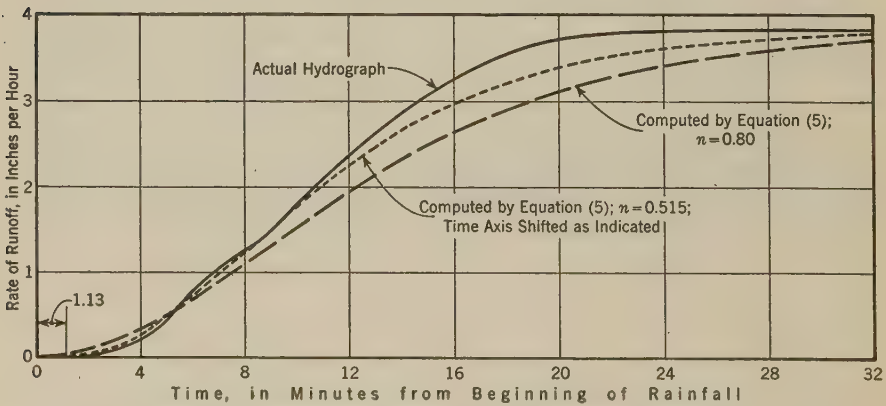

FIG. 51.—ACTUAL AND COMPUTED HYDROGRAPHS FOR TURF SURFACE (LENGTH OF PLOT, 72 FT; SLOPE, 1%; AND RATE OF RAINFALL, 3.81 IN. PER HR)

It is doubtful that the viscous flow condition, which apparently exists within the range covered by the tests reported herein, continues to exist for greater rates of discharge. A limited series of tests performed on the paved surface by admitting water at the upper end only (60) indicates that, for overland flow undisturbed by rainfall, the viscous type of flow terminates at about R = 2,000 (Reynolds' number) corresponding roughly to runoff at 4 in. per hr from a 72-ft plot. Beyond that range the data are inconclusive and incomplete; but, since flow in pipes becomes fully turbulent beyond R = 2,500 ±, it appears reasonable to assume that overland flow would also become fully turbulent in the same range.

Consequently, from a scientific viewpoint, it appears likely that, with rainfall at 4 in. per hr, discharge is proportional to the cube of the depth up to length of overland flow approaching about 80 ft, and becomes turbulent, with discharge proportional to depth to the five-thirds power for greater lengths. Further experiments are needed to establish what actually happens.

The concept of the factor of turbulence I, as used in Eq. 4, appears open to question although it is probable that Eq. 5 gives a rough approximation of actual hydrographs on lengths of overland flow exceeding 100 ft. There is some doubt in the writer's mind as to whether elaborate developments based on this equation are warranted, although it must be admitted that nothing better exists at the present time.

L. J. Mensch,[33] M. Am. Soc. C. E.—The student of highway and airfield pavements will gain much information from this excellent Symposium, especially if he compares the data presented by Brigadier-General Stratton with the specifications for concrete pavements available in the 1930's. This discussion will be confined mainly to the joint spacing and design of joints, as illustrated in Fig. 2.

In most cases longitudinal expansion joints may be dispensed with even in airfields 300 ft wide. Concrete pavements expand lengthwise from two causes—moisture variation and increase in temperature. The Arlington (Va.) tests of the Bureau of Public Roads (5*a*) show that, due to moisture variation alone, concrete pavements lengthen 0.0003 in. per in. in 4.5 years after construction. It is probable that the expansion may be as much as 0.0005 in. per in. after 10 years. The expansion due to an increase in the temperature of the slab depends on the difference between the temperature at which the concrete has set and the average temperature of the slab (between top and bottom surface) when the sun is shining on the pavement in June or July.

If this interval is 50° F, for example, the expansion from this interval will be approximately 0.00025 in. per in., or a total expansion of 0.0005 + 0.00025 = 0.00075 in. per in.; that is, 1.35 in. at each side shoulder in a pavement 300 ft wide. The shoulders will yield, and the only considerable resistance comes from the friction of the ground and the weight of the slab. Assume the slab to be 12 in. thick, weighing 150 lb per sq ft, and the coefficient of friction to be 2 (5). The total resistance in a slab 12 in. wide, 150 ft long, and 12 in. thick will then be 150 × 2 × 150 = 45,000 lb. This force divided by the section of the slab, 12 in. × 12 in., gives about 300 lb per sq in. as the maximum stress in the center line of the 300-ft-wide airfield. Such a compression helps the concrete as it diminishes any tensile stresses that may occur. Because of the time loading, it would shorten a pavement 150 ft long about 0.2 in. when the modulus of elasticity is assumed as 1.33×10^6, leaving a net increase in width at each shoulder of about 1.13 in. A drop in temperature of 100° F will shorten the 150-ft length about 0.0004 in. per in., or 0.72 in. This is expected to be distributed among the ten to fifteen longitudinal joints, so that there is a gap at each joint of from 0.048 in. to 0.072 in. Of course, if the dowels are not properly held in place, absolutely parallel to the direction of the movement (for example, if they should depart from the parallel as much as 1 in. in 16 in.), the dowels may freeze this movement in the joints and concentrate it in one joint. On the other hand, they may cause a longitudinal crack or at least a high tensile stress of the order of 300 lb per sq in. near the center of the pavement. When the designer has no confidence in the kind of load-transfer device he specifies or in the kind of workmanship he may obtain and when he fears that the dowels will not be parallel, then he is quite justified in providing a longitudinal expansion joint such as that shown in Fig. 2.

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The size of the pavement slabs between the various longitudinal and transverse joints is shown in Fig. 2 to vary between 10 ft by 12.5 ft and 15 ft by 25 ft. It is generally believed that this size should be limited by the stresses generated by moisture and temperature variation. Another important point has been overlooked by General Stratton, however. The individual slabs are subject to rocking and vibration due to the heavy concentrated wheel loads at the edges and ends. These wheel loads must be resisted by the weight of the slab acting with a moment arm from its center of gravity to the part of the subgrade so located as to relieve the weight of the wheel load and its shock. Well-doweled joints will reduce this rocking greatly, and, therefore, dummy joints without dowels have no place in airfields. It is well known (5) that the greatest stresses occur at the ends and edges of a slab, and this is another important reason that joints in airfield pavements should always have load-transfer devices. No reliance can be placed on the interlocking power of concrete aggregate a few years after the pavement is built (5). Hence, the greater the wheel load, the greater will be the size of the individual slab. Possibly a size of 12.5 ft by 18 ft should be a minimum.

The fear that pavements may be injured by irregular cracking, due to warping from moisture and temperature variation, when the individual slabs are 10 ft by 25 ft or larger is based on the absence of an accepted theory concerning such actions and, of course, on everyday experience. In desperation some engineers have advocated slabs 10 ft by 10 ft which are entirely unsuitable for airfields. Slabs 25 ft long crack mostly because of faulty load-transfer devices, unequal frost heaving of poor subgrades, or excessive wheel loads.

Approximate Theory of Stresses Produced by Warping.—Temperature warping has been investigated thoroughly by engineers of the Bureau of Public Roads (5*b*), who found that the highest temperature differential, Δ, between top and bottom surfaces occurs in the spring and early summer when the sun is shining on the slab. This value, in degrees Fahrenheit, is given approximately by

$$\Delta = 3.75\, d \quad \text{(18)}$$

in which d is the depth of the slab in inches, corresponding to a unit differential in the length of these surfaces of

$$\delta = 3.75 \times d \times 0.0000048 \doteq 0.000018\, d \quad \text{(19)}$$

For example, a temperature differential of 33° F was found for a 9-in. slab. A weightless slab would deform into the shape of a sphere of radius $r = \frac{d}{\delta} = \frac{d}{0.000018\, d} = 55{,}555$ in. The Bureau of Public Roads measured the warping of a 9-in. slab, 20 ft long, at a temperature interval of 23° F. The radius in this case would be $r = 55{,}555 \times \frac{33}{23} = 79{,}700$ in. To simplify the calculation and at the same time to arrive at safe and acceptable results, assume that the warped surface is a cylinder with its axis along the transverse center line of the slab, thus forming the arch ACB (Fig. 52(a)). The rise of this arch may be found by the well-known formula,

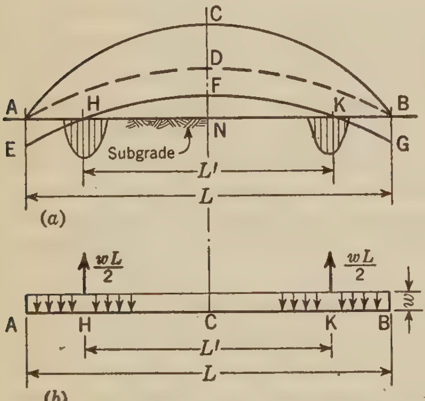

FIG. 52

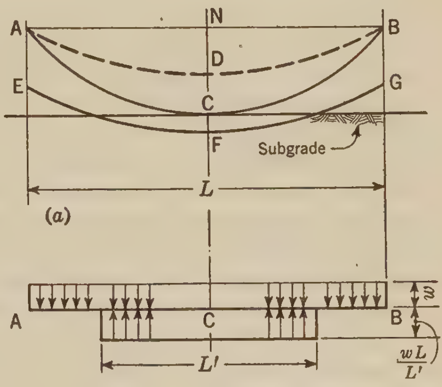

FIG. 53

$$f = \frac{L^2}{8\, r} = \frac{240^2}{8} \times 79{,}700 = 0.09 \text{ in.} \quad \text{(20)}$$

Under its own weight, however, the arch will become embedded in the subgrade near ends A and B, somewhat as shown by line EFG, depending on the soil modulus, the condition of the subgrade near the ends, and the moisture warping. The line ADB (Fig. 52(a)) has been drawn equidistant from line EFG to compare the deflection of the slab with the ideal condition shown by line ABC. The deflection CD is caused by the loading shown in Fig. 52(b); that is, by a uniform dead load of 113 lb per sq ft acting over the entire span of the arch and by two subgrade reactions (shaded areas) assumed concentrated at points H and K, symmetrical with the center line.

The deflection caused by the uniform dead load is easily found to be: $\frac{w\,L^4}{8 \times 16\,E\,I}$. The deflection of the end in relation to the center point C caused by the load $\frac{w\,L}{2}$ at point H is most easily found by the moment area method: $\frac{w\,L}{2} \times \frac{L'}{2} \times \frac{1}{2} \times \frac{L'}{2}\left(\frac{2}{3} \times \frac{L'}{2} + \frac{\qquad\prime}{2}\right) = \frac{1.688\,w\,L^4}{8 \times 16\,E\,I}$. The difference—$0.688\,\frac{w\,L^4}{8 \times 16\,E\,I}$—gives the deflection CD. Let $L = 240$ in.; $w = \frac{113}{12}$; $L' = 0.75\,L$; $E = 4 \times 10^6$; and $I = 729$. Then the deflection CD = 0.0578 in. The vertical distance between points E and F was measured as 0.035 in. (5*c*). This is equal to DN; and the total rise of the arch, CN = 0.0578 + 0.035 = 0.0928 in., as compared with the 0.09 in. found theoretically from the temperature interval. In other words, agreement between theory and practice is very good. The bending moment at the center of this arch, 1 ft wide, is found to be: $M = \frac{w\,L}{2} \times \left(\frac{L'}{2} - \frac{L}{4}\right) = 33{,}800$ in-lb, leading to a flexural stress of 209 lb per sq in. with tension at the bottom, as compared with a flexural ultimate strength in paving concrete of rarely less than 600 lb per sq in. Furthermore, this stress should be relieved considerably by the action in a transverse direction, which the writer has neglected, and by the continuity of good joints equipped with proper load-transfer devices. The Bureau of Public Roads estimated the largest temperature stress in this slab based on extensometer readings at 350 lb per sq in., but admitted that there might be some reservations concerning the assumed modulus of elasticity. For a differential of 33° F, a similar calculation and an assumption of $L' = 0.8\,L$ would result in a flexural stress of 250 lb per sq in.

Warping due to moisture variation was investigated for the Bureau of Public Roads by W. K. Hatt, M. Am. Soc. C. E. (61). He found the maximum differential change of length between the top and bottom surfaces of a wet subgrade on a dry summer day to be 0.000176 in. per in. in a slab 7 in. thick, in a beam 8 in. thick, and (it may be assumed) also for a 9-in. slab. A weightless slab would deform into a sphere, convex downward (see line ACB, Fig. 53(*a*)), with a radius $r = \frac{9}{0.000176} = 51{,}200$ in. and a rise (line NC) of $\frac{L^2}{8\,r} = 0.14$ in. for $L = 240$ in. Assuming again a cylindrical warping except that the dead load of the cylinder is supported by the subgrade in the middle part of the arch, these upward reactions may be considered to be nearly uniformly distributed on a length $L' = 0.6\,L$ without significant error. The deflection CD of the slab due to its uniform weight of w per square foot and the reactions $\frac{w\,L}{L'}$ per linear foot on the length L' may be found in a manner similar to that described for temperature warping: The deflection due to dead load $= \frac{w L^4}{8 \times 16 E I}$, when $w = \frac{113}{12}$; and the deflection due to the reactions $= \frac{w L}{L'} \times \frac{(L')^2}{8} \times \frac{1}{3} \times \frac{L'}{2} \times \left(\frac{3}{4} \times \frac{L'}{2} + \frac{L - L'}{2} \right) = \frac{0.408\, w L^4}{8 \times 16 E I}$. The deflection CD resulting from time loading $= (1 - 0.408) \frac{w L^4}{8 \times 16 E I} = 0.0982$ in., in which $E = 2 \times 10^6$. The measured vertical rise from the transverse joint edge to the center of the slab can be estimated (5*d*) at less than 0.04 in., giving the rise NC $= 0.0982 + 0.04 = 0.1382$ in. which is practically the same as 0.14 in. computed theoretically from length changes. The maximum bending moment per linear foot of width, $M = \frac{w L}{2} \left(\frac{L - L'}{4} \right) = 26{,}000$ in-lb, leading to a flexural stress of 168 lb per sq in. with tension on top, whereas the maximum stress from temperature variation was found to be 209 lb per sq in. at the bottom. Fortunately these stresses are not additive, except for the small effect of temperature warping at night.

The foregoing speculations seem to indicate that warping stresses increase somewhat less than the square of the length of the individual slabs, and, as the deflections increase with the fourth power of the slab length, soon a stage will be reached where the points D in Figs. 52(*a*) and 53(*a*) will reach the subgrade, which will probably occur in 10-ft-wide slabs when the length is about 25 ft; any increase of length will probably not increase the warping stresses to any extent. Much lower stresses occur in transverse directions at the center, and this fact should be taken into account when it is decided to use reinforcement steel to strengthen the slabs at the bottom. Reinforcing bars are advantageous if placed at the bottom in a longitudinal direction along the edges of the slab and in a transverse direction at the ends. Where top reinforcing is selected, the bars should be of high carbon steel and preferably ⅜ in. in diameter to avoid surface cracking. Such bars have been used by the writer with great success for pavements designed to support heavy trucks. General Stratton has treated the design of load-transfer devices rather meagerly. One gains the impression that he leaves to the discretion of the individual engineer the choice of positive holding devices or resilient fillers for expansion joints. The Bureau of Public Roads has shown (5) that ¾-in. dowel bars are not of much value for heavy loads, even if spaced 12 in. on centers. This has also been found true by the writer (62). It is quite correct to use a greater spacing of dowels for construction joints than for expansion joints, provided that such joints do not become expansion joints (as they have often been observed to become by the writer) while the expansion joints which have been provided become close. The writer has shown that dowels heavier than ¾ in. are needed for heavy wheel loads, especially such as occur on airfield pavements (62) and that it is of the utmost importance that the dowels be held positively, by other means than wire and stake supports, in the correct position parallel to the movement of the pavement. For airfield pavements with 150,000-lb wheel loads, dowels of large pipes or of similar sections will be needed, and the concrete around the dowels will have to be specially reinforced to prevent the edges of the slabs from cracking due to the concentrated bearing pressures exerted by the dowels.

Hibbert Hill,[34] Assoc. M. Am. Soc. C. E.—At the request of Brigadier-General Stratton, who has been assigned to overseas duty, the writer presents the following reply to the discussers of the first paper in the Symposium.

It would appear that the design of airfields constitutes simply the synthesis of known elements into a structure of new form. The problem is a tantalizing one in that respect, for close study shows that many of the elements are also new, although seemingly familiar. It was General Stratton's purpose to reveal this point. The discussions have amply assisted him.

Loads.—Brigadier-General Newman presents a graphic illustration of the magnitude of present large airplane wheel loads, and Mr. Stanton comments in similar vein. Mr. Rudolph asks when and where the limit will be. The answer to this question is not now discernible. It does appear that, if airplane design proceeds on the basis of two main wheels carrying substantially the total load, an economic balance of some kind between cost and value of runways will soon be reached. In one instance, an estimate of cost for a single runway and taxiways, proposed for a future heavy plane, was $7,000,000. Airplane designers can extend the limit of total load, so far as it is governed by paving costs, by spreading the weight on more wheels, as Professor Casagrande suggests. In doing so, it is necessary that soils engineers and pavement engineers be consulted so that there can be a thorough understanding of the effects of tire pressure, footprint area, and wheel spacing. The airplane designer's problem, however, is to minimize the dead weight of landing gear in the interest of pay load. Landing gear for the present heavy planes represents from 10% to 15% of the gross weight which, in terms of tonnage, is a very real consideration—increasingly so as gross weight increases. The solution of this problem is concentration of loads on few wheels, with resultant high pavement costs. It has been suggested by Col. Bartley M. Harloe that the great planes of the future might be compared to ocean liners, not expected to be landed under their own power, but rather to be warped carefully into position by tugs or by aerial equivalents. Under this conception, paved tracks would be substituted for wide runways. Some new thought on the subject is certainly in order. Mr. Rudolph has suggested the possibilities of catapults, and rockets may be used for the same purpose. Both catapults and rockets would project the planes into the air. Similar means for landing them may be found in reversible propellers.

In the same vein, the runway designer should be able to take all possible advantage of the peculiarities of airplane traffic distribution in designing pavements. Mr. Pringle has defined "capacity" and "limited" operation, as used by the Corps of Engineers, to rate pavement strength and life. Mr. Stanton has referred to the reduction in pavement strength permitted by the limited number of repetitions of load on runways.

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In his general discussion of the Symposium, the writer also points to possible variations in pavement thickness to meet the peculiar distribution of traffic on runways. There appear to be economies in this direction which have not been fully realized.

The reality of high impact loads, as discussed by Mr. Cotton, is not in doubt. The effect of these loads on the pavement, however, is another question. Numerous measurements of pavement deflections, and a number of base and subgrade pressure determinations, have been made under static and moving loads. These measurements, made in the course of accelerated traffic tests, all show a marked reduction in deflection under the moving load, and a marked reduction of subgrade pressure—that is, an apparent spreading of the load—under the moving wheel as compared with the static wheel. These results, coupled with the fact that no airfield pavement failures attributable to impact have been observed, have led the Corps of Engineers to conclude that allowance for landing impact stresses in the pavement design is not warranted.

Site and Layout.—It is unfortunately true, as Professor Haswell has noted, that soil and drainage seem often to be the last considerations in site selection. The swamp and power line criterion mentioned by General Newman may not actually be contained in site selection instructions, but the combination has occurred too often to be comfortable. Exigencies such as those discussed by Lieutenant-Colonel Schneider, or the need for a location within given boundaries, often force selections which are not the best. However, the economies to be realized, in initial construction as well as in maintenance, from good site selections are startling. The cost of the aforementioned runway—an estimated $7,000,000—became less than $1,000,000 when the location was changed to one having a suitable soil. In the case of military airfields, and particularly in the case of airfields for very heavy planes, sites can be sought over a wide area. An additional 100-mile flight is a bagatelle for such planes, whereas the savings of time in construction and in construction and maintenance costs, resulting from a choice location, are decisive. There is also the possibility of improved atmospheric conditions and, hence, of improved flying conditions, which may be gained from the selection of sites within a wide area.

General Newman has given a number of layout criteria for military airfields. The criteria he gives are for training fields. Under operational conditions, with trained pilots, these standards may be reduced. These criteria are not fixed. They are being studied in the light of past experience and in the light of the requirements of the newest planes. They will no doubt be continually revised as airplanes change and experience accumulates. The entire question of requirements for airfield sites and layouts appears to offer a fruitful field for continuous study.

Drainage.—The numerous comments on the critical importance of adequate drainage serve to emphasize a subject which General Stratton in no way meant to minimize. Captain Parmer cites a number of considerations of perhaps controlling interest, which indicate some of the difficulties of the subject. Mr. Feld states that pavements are not impervious at joints and cracks and that, as a consequence, positive subsurface drainage is necessary. Such drainage is automatic if base and subgrade are permeable. If base and subgrade are not permeable (the usual case), the damage is done before any feasible method of subsurface drainage becomes effective. Either the pavement must be designed to carry the load under the worst conditions, or it must be kept properly sealed. The latter is a feasible course. Mr. Greulich fears that moisture accumulation makes guesswork of plate bearing data and California bearing ratio (CBR) test information. The designs of the Corps of Engineers are based on the important assumption that base and subgrade will become saturated by capillary action and condensation which cannot be controlled by drainage. The validity of this assumption is proved in general by available data, although a serious doubt remains as to whether saturation occurs in arid and semiarid regions. In this respect, Mr. Murray makes a very interesting comment on the entire absence of any moisture accumulation under conditions in Australia.

In general terms, the function of drainage is to prevent the entrance of moisture into load bearing areas of soil. The removal of rainfall by proper surface drainage thus would be better practice, than, as suggested by Mr. McGuire, the removal of rainfall, as filtered water, by subsurface drains.

Mr. Cotton advocates a combined surface and subsurface drain. Experience with such drains has been unfortunate. If the drains are properly constructed to avoid infiltration of the surrounding soil, they quickly clog at the surface and thus become ineffective for removal of surface runoff. When drains are constructed of coarse material, their use has, in numerous cases, resulted in failure of the pavement edge because of infiltration of soil from the subgrade or base course. The drains, under these conditions, also are very likely to clog, with consequent saturation of the subgrade by backwater.

Mr. Cotton's discussion of backfill filters for subdrains is valuable. It should be emphasized that the material in Table 10 represents a maximum size. Finer gradations, containing materials passing the 150-mesh sieve, will be required for silt and clay soils. Filters should be designed to fit the foundation soils at the site. The design of satisfactory filters is discussed in detail in the *Engineering Manual* (23), and Karl Terzaghi, M. Am. Soc. C. E., has also presented a method (63) of design.

Messrs. Cotton, Greulich, Haswell, and Feld have criticized the design of drain shown in Fig. 8(*b*). Professor Haswell's question as to the need for the natural earth backfill beneath the drain is very pertinent. It is no doubt best practice to trench exactly to grade and to lay the pipe on undisturbed soil, thus avoiding tamping beneath the pipe and the excavation required for the fill. The placing of the perforations upward, criticized by Mr. Cotton, is the result of mature consideration. With upward perforations, the hydraulic characteristics of the pipe are believed to be improved; there is not the same opportunity for outward flow in areas where ground water is below the pipe that there is when the holes are downward; and placement of the backfill and filter is facilitated. Specifications require that the filter be designed to fit the local soil conditions, so that the filter material cannot drop through the perforations. If the filter material could drop through the perforations, it could also be pushed through the perforations by hydraulic pressure in Mr. Cotton's design. Mr. Feld raises a very practical point of construction. He is entirely correct for the drain design that he discusses, but he does not consider filled shoulders, which would introduce a somewhat different problem. His remarks also recall the problem of drainage during construction. This problem is intimately related to construction procedures and is of such critical importance that, in the writer's opinion, the necessary procedures should be incorporated in the plans and specifications by the designer, and should not be left to decision in the field under the stress of construction exigencies.

Mr. McGuire expresses a hope that a clear and usable analysis of subsurface drainage requirements, comparable to Mr. Hathaway's analysis of surface drainage, will be prepared, and Mr. Greulich comments in similar vein. The writer, unfortunately, can only echo the hope. Such an analysis is greatly needed. A beginning has been made in the *Engineering Manual* (23), on the basis of available information, and revisions of the *Engineering Manual* will be made as knowledge accumulates.

Base and Subgrade Requirements.—Specifications for base and subgrade materials are intimately concerned with moisture and drainage, as Mr. Cotton notes. Mr. Cotton's remarks on the value of pervious materials from this standpoint are reinforced by the information given by Messrs. Casagrande and Parmer on the strength and compaction of such materials. Field experience is rapidly accumulating to show that a wide variety of granular materials can be compacted satisfactorily and that when so compacted these materials form very strong bases, relatively or entirely free from moisture troubles. These properties are especially important in the foundations of runways to carry the very heavy planes. Such loads create important soil stresses to depths as great as 5 ft below the surface of flexible pavements and will cause consolidation to such depths, as well as high shearing stresses at somewhat lesser depths. Recent field experience indicates that sands not fully compacted will compact under traffic without shear failure and that moisture content is not critical. On the other hand, tests (unpublished) by staff members of the Flexible Pavement Laboratory, U. S. Waterways Experiment Station, Vicksburg, Miss., show that the shearing strengths of plastic soils, including those of soils of medium and low plasticity, are very likely to decrease severely upon further compaction under traffic and that, in such a case, water content is extremely critical. Ideally, of course, any base and subgrade should be initially compacted to at least the ultimate density it will acquire under traffic. Allowing for a margin of error, however, and in view of the great wheel loads involved, the use of granular base and subgrade materials provides a comforting factor of safety.

Mr. Mullis' remarks on the inadequacy of the standard AASHO test are certainly correct, for these conditions. The modified test used by the Corps of Engineers results in a much greater density than does the standard test. It was found in tests at Barksdale Field, La., with a 50,000-lb wheel load, that compaction of from 103% to 105% of modified AASHO density was required in the 17 in. of base material (to 20 in. below the pavement surface); the compaction requirements then decreased with depth to 90% of modified AASHO density in the subgrade at a depth of 53 in. below pavement surface.

It appears beyond question that the high densities and deep compaction required by airports will necessitate the development of new equipment for this purpose, including the vibratory equipment foreseen by Professor Casagrande.

The subject of frost effects, briefly discussed by Messrs. Casagrande, Feld, and Parmer, is most important. The reasonably satisfactory treatments that have been developed in highway practice require reexamination and refinement in the light of the deep and expensive base construction required for airfield loads. In this respect, again, the value of clean granular base courses, not subject to frost effects, is reiterated. The possibilities of such insulating materials as Professor Casagrande mentions, and the possible advantage that can be taken of relative heat transmission coefficients, as noted by Mr. Feld, are appreciated. In the past, reliance has been placed on natural granular materials—in large part because these materials have been available and furthermore because of a lack of information on the behavior of other materials. Curiously enough, in areas of permanently frozen ground mentioned by Professor Casagrande, the principal problem is most often one of avoiding interference with the natural ground-water regimen rather than one of maintaining a permanently frozen subsoil. For this purpose very coarse granular materials are best.

Construction Practices.—The discussion by Mr. Pittman emphasizes the fact that design and construction practices must be coordinated to obtain a satisfactory job. He and Mr. Feld agree that successive lane construction of concrete pavements is the best practice. In the writer's opinion, this method of construction is essential for good results, unless someone could guarantee that there would be no rain! The writer also agrees that the usual loss of bearing power under thickened edges, resulting from excavation of the compacted base material, is likely to offset the value of the thickened edge, as noted by Messrs. Feld and Parmer. The same excavation traps rain water during construction, thus also contributing to the loss of bearing. For these reasons, the Corps of Engineers has eliminated the thickened edge as standard practice, except at exterior edges and at the longitudinal expansion joint at the center of the runway. Emphasis has been placed on obtaining compaction and base support under these edges equal to the design assumptions.

Mr. Pittman calls attention to a very troublesome design feature found in those cases where the pavement is below ground level at the shoulders. Thus, subgrade and pavement have to be constructed in a broad shallow trench. If the soil is not pervious, this situation results in drainage difficulties during construction and quite possibly after construction. In this case, design and construction sequences must each be directed toward providing means of drainage during and after construction. Mr. Pittman gives one solution.

Mr. Pittman's remarks on winter construction arise from extensive experience. It is no doubt correct that concrete pavements can be constructed under conditions that would make construction of flexible pavements impossible. The writer believes, however, that, unless the need is pressing, as it has been during World War II, airfield construction should not be undertaken in seasons of unfavorable weather. In the case of such construction, extremely conscientious attention must be given to the quality of work, and delays must be accepted, to obtain desired results. Thus, an almost impossible burden is placed on contractors and inspectors. It may be remarked, also, that one disadvantage of concrete is that it can be placed on an unsatisfactory base, to look well on the surface but in fact to have nowhere near the load supporting value contemplated in the design. The base for flexible pavement must have strength enough, at least, to permit rolling of the pavement surface!

Mr. Proctor advocates compacting the soil to about a foot above grade, then trimming this foot off to reveal a fully compacted surface. In the last analysis, the question of how to obtain the desired compaction at the surface is an economic one. Subgrade planers are used to advantage to remove the top inch or so, in both highway and runway construction. However, it is the writer's opinion that the removal of 12 in., as proposed by Mr. Proctor, is not essential and is not economical. Mr. Proctor's remarks on the use of the sheepsfoot roller, and on emergency construction procedure, are very interesting, and grow out of long experience. These remarks center around the use of an "available" roller. To obtain the high compaction necessary in airfields for heavy planes, it will be necessary to specify the use of particular rollers, although admittedly the characteristics of those rollers are not yet fully determined.

The use of membrane curing for concrete pavements is permitted by specifications of the Corps of Engineers. These specifications require moisture retaining ability as determined by an established test procedure. This method of curing has great advantages in ease and speed of application, economy, and negligible maintenance cost. The material has no insulating or cooling properties, and it may be, as Mr. Pittman states, that its use in hot weather will permit dangerous thermal stresses in the concrete to occur, especially if the material is of dark color. Mr. Pittman's statement, however, is the first to that effect which has come to the writer's attention, and the writer has been unable to find any case in which damage to the concrete can be ascribed to such thermal stresses.

Design.—Messrs. Turnbull and Jervis, and Hansen have presented interesting background material on the development of the CBR method and on the method of accelerated traffic testing. Their statements with respect to the findings of these investigations are just sufficient to whet the reader's interest, but by no means to satisfy it. In continuation of their remarks, it may be stated that, since Mr. Hansen's discussion was written (September, 1944), the Barksdale tests have been made available to a limited distribution (64).

The application of the formulas introduced by H. M. Westergaard, M. Am. Soc. C. E., to design of pavements on marshy ground is questioned by Mr. Horonjeff. His doubt, it appears to the writer, is not in fact directed at the formulas but rather at the method of determining K which is used by the Corps of Engineers. In the writer's opinion, Mr. Horonjeff's question is justified. The plate bearing test for the determination of K has little to recommend it except that no other method is available and except that, within the usual range of subgrade soils, it gives results which are reasonably consistent with experience. There is no present method for extending this experience to soils such as Mr. Horonjeff describes. On such soils, the plate bearing test will give indeterminate results; yet such soils do have supporting values under rigid slabs.

Professor Casagrande's remarks on the difficulties of a theoretical approach to the design of flexible pavements are illuminating. In spite of these difficulties, engineers will not rest until such an approach has been accomplished. Present studies by the Corps of Engineers are aimed at computing the soil stresses, and specifying the necessary treatment of soils and bases to resist these stresses. The accomplishment of this goal is an indefinite distance in the future. Until that time, refinement of the CBR and similar empirical methods will not be neglected.

Messrs. Casagrande and Horonjeff both discuss the problem of load transfer between rigid slabs under heavy wheel loads. This problem becomes more pressing as loads and slab thicknesses increase. One absolute requirement for a successful joint is that the joint surely function. In this respect, the writer fears the doweled joints, suggested by Professor Casagrande, in which the dangers of improper installation would be increased by the large number of joints.

In the writer's opinion, there is no such value as an "allowable pavement deflection," on which Professor Burmister hopes to obtain factual information. Considering the pavement alone, the allowable deflection is a function of the curvature, and will be greater the longer the radius of curvature; hence, the allowable deflection will be greater the larger the tire imprint—other factors being equal. Allowable deflection will also vary with the soil and base constants. In the Barksdale tests, initial subgrade deflections, exceeding about 0.1 in. under moving wheel loads of 50,000 lb, increased under succeeding wheel passages to more than 0.2 in., and shear failure occurred. In similar tests with 40,000-lb wheel loads, but on a pavement laid over a fine sand subsoil, no failure occurred although the initial deflections were somewhat greater than at Barksdale. Considerable settlement did occur, however, as the subsoil compacted. In such cases, the differential, rather than the maximum, settlement is important. To the writer's knowledge, no flexible airfield pavement has been overstressed as the result of compaction, unaccompanied by shear failure, under traffic. T. A. Middlebrooks and R. M. Haines, Assoc. Members, Am. Soc. C. E., have presented the results (65) of numerous deflection measurements, and Mr. Stanton also contributes some information.

In Table 13, Professor Burmister introduces a thickness factor which, when applied, permits a good base material to be less thick than a poor one. The CBR method of design is predicated upon the hypothesis that a given total thickness of base and pavement is required over a given subsoil or base and that the total thickness is independent of the quality of the material as long as the material has sufficient strength. This hypothesis is based upon experience, and is definitely proved by the Barksdale and other similar tests. Professor Burmister thus argues erroneously that, because material with a CBR value of 50 is allowed immediately under pavements for light wheel loads, a lesser thickness of material with a CBR value of 80 would be suitable. These minimum CBR values are needed to resist stresses (shear principally) in the base material itself, and do not influence materially the stresses in the underlying soil, so far as available evidence shows. This statement must be limited to the usual base materials. Evidently, in the extreme, it would not apply to concrete.

The general adoption of relative density in place of percentage compaction, as suggested by Professor Burmister, is an end to be desired. It has special virtue in the case of sandy soils, since these soils undergo marked changes of strength in passing from loose to dense states. However, in order to use relative density as a measure, it is necessary to have some standard definition of the end points—that is, of the loose and dense states. Such a definition or test is not available.

Mr. Proctor, raises a number of interesting questions concerning soil compaction in earth dam construction. In reply to one of his first statements, it should be mentioned that methods of compaction and compaction control developed in earth dam practice are the bases for similar operations in pavement construction. It does not follow, however, that criteria which have been successfully applied in earth dam construction will suffice for successful pavement construction. The maximum unit loads found in earth dam construction greatly exceed the unit loads encountered in pavement construction, but this comparison does not consider the size of the loaded areas, the load distribution, or the repeated stresses found in pavement practice. Because of these factors, all present information leads to the conclusion that the permissible soil strain in pavement practice is only a fraction of that required to develop the ultimate strength of the soil under a single loading, and permitted in earth dam practice. It appears, therefore, that, for airfield pavements, relatively higher densities must be obtained than are commonly accepted in earth dam practice. It appears also that, possibly excepting sands, the bases and subgrade must be initially compacted to substantially such a density that no further compaction will occur under traffic. On the basis of the same information, the writer very much doubts Mr. Proctor's interpretation of Fig. 41, although all the information necessary to resolve this doubt is not given in Fig. 41.

To clarify several apparent misapprehensions which appear in Mr. Proctor's discussion, it may be stated, first, that the CBR method is not dependent upon an optimum density. This test evaluates the soil at any density and moisture content. The most desirable values of density have been discussed in previous paragraphs. On the basis of available tests, it appears that the most desirable water content is somewhat on the dry side of optimum for the compactive effort applied.

Secondly, Mr. Proctor very correctly points out the great reduction in strength of soil at 90% compaction, as compared to its strength at 100% compaction. It should be noted that 90% of modified AASHO density is still a rather high density by usual standards, but, more especially, that 90% compaction is allowed by the specifications of the Corps of Engineers only in fill, the top 6 in. of which are required to be compacted to 95% density. The 90%

criterion is not applied to base courses. The present hope is to determine the actual density and compacting moisture content necessary at a given depth, and to specify the values. In the aforementioned Barksdale tests, it was determined that 90% compaction was sufficient at certain depths, 95% was required at an intermediate depth, and from 103% to 105% was needed in the upper base materials.

In the same connection, Mr. Proctor stresses the need for control of moisture content at such a value that subsequent consolidation, from whatever cause, will not build up excess hydrostatic pressures in the soil. This is a very grave possibility in airport practice where the heavy wheel loads may cause consolidation under traffic, and, if such pressures occur, they are fatal to the pavement. This phenomenon was observed in one failure of a test pavement on a silty clay soil.

Messrs. Rudolph and Horonjeff each briefly raise the problem of reinforcing existing pavements to withstand heavier loads than those for which they were originally designed. The problem has already arisen at a considerable number of Army fields. The procedure has been first to evaluate subgrades, bases, and pavements as they actually exist, by field and laboratory tests—and, where applicable, by accelerated traffic tests—then to design the reinforcement. The thickness of flexible pavements is increased in accordance with the CBR requirements. This increase may involve base courses on top of the existing pavement. In such a design, careful consideration must be given to the effects of moisture in the new base; hence, closed crushed stone is preferred for this base. In the case of concrete pavements, the thickness of a slab required for the new and greater load is calculated, and the equivalent two-beam (one beam being the existing slab) thickness is determined. The general practice thus far has been to place a leveling course of sand asphalt, where necessary, or a thin bituminous bond breaking course, between the old and new slabs. This is done to relieve temperature stresses in the upper slab, but principally to avoid irregular bearing of the upper slab following movements resulting from temperature effects.

Present limited experience with such designs is satisfactory. When pavements are thus thickened, original grades and drainage may be so disturbed that the required changes in these elements result in an uneconomical over-all design. It may then be necessary to remove and reconstruct a part of the existing pavements to meet grades at buildings and drainage structures.

In conclusion, it is evident from the discussions, and from General Stratton's paper, that, although much progress has been made toward the rational and exact design of airfield pavements, many questions are at present unanswered, and much remains to be done. Mr. Pringle has outlined a program of periodic condition surveys, and other discussers have mentioned the need for correlation of test and laboratory results with field experience. A number of organizations are working on these problems. The Corps of Engineers has established three laboratories to work continuously on the correlation of field and laboratory experience and on the extension of knowledge through field testing. These laboratories are the Flexible Pavement Laboratory at Vicksburg, Miss., the Rigid Pavement Laboratory at Cincinnati, Ohio, and the Frost Effects Laboratory at Boston, Mass.

Gail A. Hathaway,[35] M. Am. Soc. C. E.—The comments and suggestions on drainage facilities contained in the discussions of the Symposium are appreciated. Several valuable modifications in procedure have been suggested, and recognized deficiencies in the method have been indicated.

Professor Howland presents a mathematical method for the determination of a runoff hydrograph for a rain of limited duration and compares the hydrograph so obtained with hydrograph No. 2, Fig. 18. The rising limb of hydrograph No. 2, Fig. 18, was computed by the general equation (Eq. 2) developed by Mr. Horton and utilized by Professor Howland, whereas the crest shape and recession limb of the hydrograph were derived from the curve of overland flow rates by utilizing the unit hydrograph procedure. The unit hydrograph method is recognized as an empirical procedure based on the analysis of actual runoff data. However, the writer has found that the method gives excellent results for small areas when reliable and accurate basic information is available. The limitations of the equation developed by Mr. Horton for lengths of flow greater than 600 ft are recognized, and alternate procedures are contained in Mr. Williams' discussion. In that connection, a few test runs by Mr. Izzard on the bluegrass turf strip, in which instance rainfall was stopped before equilibrium was reached, would have furnished valuable information on the shape of the crest and recession limb of the hydrograph. Professor Howland's contribution outlines problems as yet unsolved and emphasizes the need for actual runoff data from airfields.

The writer agrees with Mr. Jenkins' statement stressing the need for, and the practical value of, a table showing the range of infiltration capacities for different soils and soil covers. The data on infiltration capacities as shown in Tables 6 and 7, and in Mr. Jenkins' accompanying discussion, are very informative and valuable for design purposes. More information of a similar nature in the files of the Department of Agriculture should be made available to the engineering profession. Mr. Jenkins states that airfield turf is generally not well established until about the end of the first year, and in many cases not for two years, following completion of grading. It has been the writer's observation that many fields do not have a grass cover at the end of the third growing season although every effort has been made to secure a good turf. The points raised by Mr. Jenkins on the applicability of the unit hydrograph theory to the determination of runoff from graded airfield slopes 500 ft or 600 ft long have been covered to some extent by Mr. Williams.

With reference to Mr. Jenkins' comments on the relation of airfield grading requirements to drainage, it is gratifying to note that the importance and necessity of good drainage have resulted in modifications and improvements of grading criteria for Army airfields. The transverse slopes of runway shoulders and graded areas have been increased to 2%. Along the drain line, normally 250 ft from the runway center line, a 2% longitudinal grade is permitted with a maximum change in grade of 3%. In addition, where the foregoing criteria will not permit development of ponding areas, it is permissible to provide a small transverse ridge at least 100 ft downstream from the inlet, with the crest not more than 0.5 ft above the inlet.

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Mr. Jenkins suggests that the method of maintaining the hydraulic grade line near the top of the drain pipe be clarified. The design assumptions are based on a controlled outflow from each ponding basin. He also discusses the formula used by the Civil Aeronautics Administration (see Eq. 9). The utilization of this formula in the determination of runoff from airfield areas is questionable. For example, T (the duration of rainfall) is assumed to be 1 hour. The use of a value of 1 hour for T with no consideration for varying times of concentration, particularly for apron and runway design, is obviously incorrect as the computed runoff will be inadequate for numerous high intensity storms of short duration but with a frequency less than that of the design storm. The quantity t in Eq. 9 (time allowed for removal of rainfall after the end of the storm) assumed to be 2 hours is actually a storage correction; and, in many cases, the storage is not available. It has been observed that the apparent simplicity and ease of using Eq. 9, with T equal to 1 hour and t equal to 2 hours, have led to numerous cases in which storage reductions are assumed without the required storage being available or even being considered. Mr. Jenkins' thorough discussion covers several points not touched by the writer and clarifies and expands others in the paper.

The comments of Mr. Irwin on the need for infiltration data and the results of his experience with clogging are timely and informative. Much work remains to be done in obtaining accurate field data on roughness coefficients for varying types and degrees of grass cover. In that connection, the tests conducted by Mr. Izzard are of inestimable value.

It is of interest to note that Mr. Jenkins' computations for a typical layout using the modified rational method (Eq. 9) give runoff values considerably lower than those obtained by the procedures described by the writer, whereas the example presented by Mr. Irwin gives somewhat higher runoff values. The writer concurs in Mr. Irwin's statement that a careful study of critical rainfall distribution deserves investigation to determine critical conditions for the area under consideration.

Mr. Greulich and Mr. McGuire stress the great need for a type of analysis that can be utilized in the design and efficient adaptation of subdrainage pipe. More data are needed on this phase of airfield drainage, and every effort should be made to conduct field tests supplemented by the necessary laboratory research to provide a satisfactory method for rational design of subsurface drainage systems. It is desirable to maintain a low ground-water table in the airfield area, and lowering the ground-water table can be facilitated by providing proper surface drainage.

The theoretical purpose of the "surface intercepting drain" or modified "French drain" (namely, to intercept the surface water without changing the uniform sheet flow to a piled-up flow) is a good one; but unfortunately the intercepting drains fail to function, in many instances, under practical operating conditions. Not only do the surface intercepting drains become silted by fines but, in most instances, the assumed collecting capacity is in excess of the quantity of water that is intercepted by the drain. Instances have been recorded in which the pavement base-course material has moved into this type of drain, resulting in the subsidence and failure of the outer lanes of the runway. The suggested combined catch basin and intercepting drain for turfed valleys in Fig. 39 may have considerable merit for ponding areas, provided the intercepting drain does not permit water to have access to the foundation of the runway or apron pavement.

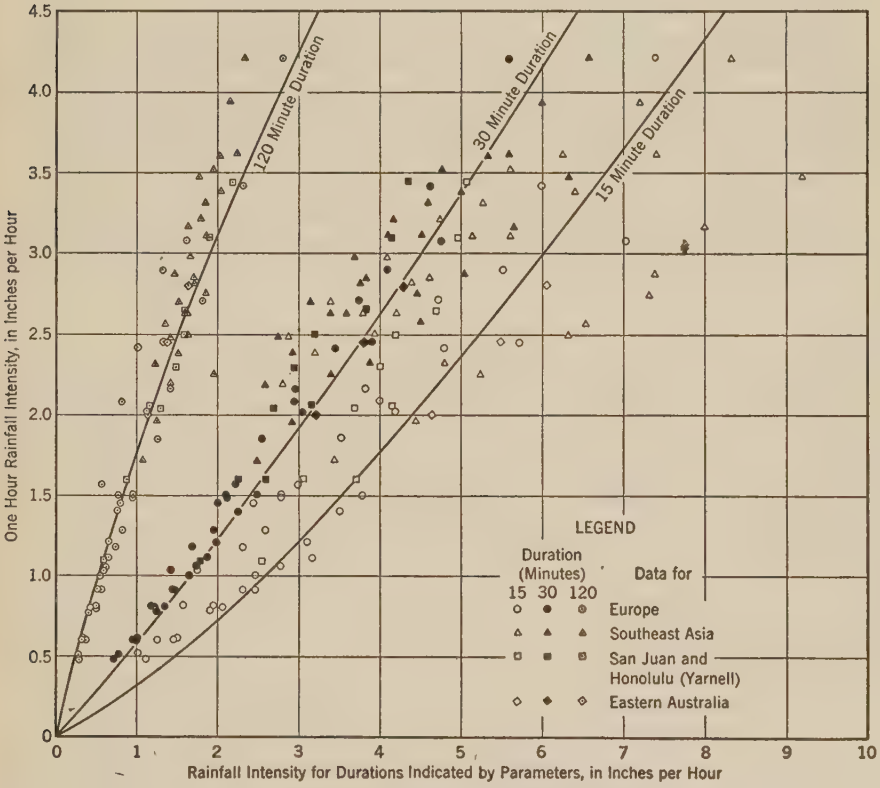

FIG. 54.—RELATION OF 1-HR RAINFALL INTENSITIES TO INTENSITIES FOR DURATIONS OF 15, 30, AND 120 MIN IN CERTAIN FOREIGN COUNTRIES

The statement by Mr. Murray that intensity curves for rainfall on the eastern coast of Australia conform closely to those in Fig. 15 is especially significant. Subsequent to the preparation of the curves in Fig. 15, rainfall intensity data were obtained for Europe and southeast Asia which conform closely to those for the United States. (Data for eastern Australia were taken from Table 12. In foreign countries, data for durations of 5 min and 240 min are not available.) These data have been plotted in Fig. 54 for comparison with the curves previously developed in Fig. 14 for the United States. In general, the foreign data for short-duration rainfall indicate a lower intensity—a condition which may possibly be attributed to a lack of recording gage records. However, the general agreement of the foregoing data does suggest a possible universal application of the design methods discussed herein.

The discussion by Mr. Williams is a valuable contribution to the paper, and the procedure he suggests will be found useful in extending the design criteria to the determination of runoff from larger airfield areas.

Mr. Izzard's comparison of actual hydrographs resulting from simulated rainfall with the theoretical hydrographs included in the paper provides a valuable check on the accuracy of the runoff computations. Although a shift in the time axis is necessary to bring the adjusted hydrograph into position with the actual hydrograph, the change would not seriously affect the ultimate drain-inlet capacity. Laboratory and field checks along the lines pursued by Mr. Izzard are of inestimable value in arriving at correct design procedures and in establishing confidence in new and better approaches to the solution of rainfall-runoff problems.

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Paper No. 2248

FLOOD FORMULAS BASED ON DRAINAGE BASIN CHARACTERISTICS

By H. B. Kinnison,[1] M. Am. Soc. C. E., and B. R. Colby,[2] Esq.

With Discussion by Messrs. Clarence S. Jarvis, L. R. Beard, M. J. Ord, Howard M. Turner, Byron O. McCoy, C. O. Clark, and H. B. Kinnison and B. R. Colby.

Synopsis

The determination of the magnitude and frequency of expected floods has been the subject of constant study by the engineering profession. The systematic collection of stream-flow records in every section of the United States has contributed immeasurably to a knowledge of the subject. The long-time records of precipitation assist in extending the frequency studies beyond the range of stream-flow records. Contributing to this knowledge also have been the disastrous floods since 1927, which in many instances exceeded the maximum expected, based on previous records.

In this paper, flood formulas are developed from stream-flow records of Massachusetts streams, for which fairly complete data are available. The formulas define the magnitude of flood peaks having probable recurrence intervals of 15, 100, and 1,000 years, and the maximum possible flood.

Volumes of runoff per unit area for the different classes of floods except the maximum were computed for streams having the longest stream-flow records, from frequency curves of observed floods and of observed precipitation. The peak discharge of each type of flood was computed for these streams by means of the unit hydrograph. These discharges were then correlated with known physical characteristics of these streams, and general equations were prepared which are applicable to both gaged and ungaged areas.

These formulas are based upon data for drainage basins in and adjacent to Massachusetts and therefore are conceived as applicable only to basins of that state or basins having similar physical and meteorological characteristics. The formulas might assume a different form in the case of drainage basins in other sections of the United States. The primary purpose of the paper is to present

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Note.—Published in March, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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an extensive analysis of flood experience which engineers may embody in their knowledge of floods and utilize with some benefit in problems involving flood magnitudes and frequencies.

INTRODUCTION

Numerous methods of predicting future flood flows have been developed and described in engineering publications. Prior to 1900 these methods were necessarily rather crude, because authentic records of stream flow were scanty and less accurate than later records. Practically all such studies were based on observations of flow in small areas or on single streams, and were usually summarized as formulas giving the expected rate of flow as a function of the drainage area, although some investigators added other variables. These early formulas were usually developed for maximum floods, the meager base data precluding any except the roughest qualitative consideration of frequency.

In 1913, the late Weston E. Fuller,[3] M. Am. Soc. C. E., published the first comprehensive study of statistical methods applied to floods in the United States. He developed a formula introducing the concept of a magnitude-frequency relationship, in this respect following a line of reasoning that was introduced into studies of river flow about the same time by the late Allen Hazen, M. Am. Soc. C. E. Mr. Hazen's application of statistical methods to engineering studies and Mr. Fuller's adaptation of these methods to flood prediction have had an important influence on engineering developments since that time.

Shortly after the Civil War, the late Maj. E. T. D. Myers, M. Am. Soc. C. E., developed a formula for computing the cross-sectional area of the waterway required to pass a limiting flood. The Modified Myers formula[4]—

$$Q = C_m \sqrt{M} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

—was developed from the original. In this formula, Q is the discharge in cubic feet per second, and M is the area in square miles. The base factor C_m is taken as 10,000, but for any particular area it is multiplied by a percentage which is referred to as the Myers rating. Eq. 1 has had wide application, and is discussed in recent engineering publications.[5,6] The reliability of the results obtained by it depends upon the choice of the coefficient C_m. The magnitude of the coefficient selected will be governed principally by the magnitude of the observed floods in the locality, and by the judgment and experience of the engineer.

Further studies have developed a different approach to the problem of flood prediction, by the so-called "rational" methods. These involve the relation of rainfall to the resulting runoff by an analysis of rainfall records and corresponding hydrographs, with due consideration being given to the influence of ground water, evaporation, transpiration, shape of drainage area, and other factors. This method includes studies of the frequency of intense rainfall and its distribution over large areas. Reports on the frequency and distribution of rainfall having relation to New England have been made by the Miami Conservancy District[7] and by the U. S. Weather Bureau.[8]

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[3] "Flood Flows," by Weston E. Fuller, *Transactions*, Am. Soc. C. E., Vol. LXXVII (December, 1914), pp. 564–617.

[4] "Flood Flow Characteristics," by C. S. Jarvis, *ibid.*, Vol. 89 (1926), pp. 985–1104.

[5] "Low Dams," by National Resources Committee, 1938, p. 32.

[6] "Physics of the Earth," Pt. 9, Hydrology, National Research Council, McGraw-Hill Book Co., Inc., 1942, p. 536.

As stream-flow records have become more extensive and reliable, the tabulations of extreme floods have increased in value and have made obsolete some of the earlier formulas based on very limited or deficient experience. It is inevitable, however, that the much more extensive experience now available should be analyzed with a view to expression by formulas of relationships between magnitude and frequency of floods as well as of the influence of physical characteristics of basins on the magnitude of floods. Thereby, the maximum utility may be given to the available data which, although much more plentiful than formerly, often have only a rather remote relation to the needs of a specific problem.

The "rational" method of approach has been extended in this paper. Investigations by the writers have shown that the magnitude of a flood of a particular frequency can be correlated with various physical characteristics of the drainage basins. By means of these characteristics, formulas have been developed which are applicable to drainage areas for which stream-flow records are not available or are inadequate.

The common practice in the past has been to use enveloping curves of maximum known discharge per square mile in the design of hydraulic structures. This practice ignores the element of frequency, which may have an important bearing on the risks involved. This paper undertakes to develop a basis for a more discriminative method of considering flood magnitudes in which consideration may be given both to probable frequency and to the influence of the various physical characteristics in the production of floods.

These formulas should not be expected to give exact or unquestionable results on ungaged areas. However, they do provide a convenient means of evaluating the risks involved in the design of hydraulic structures on streams in or near Massachusetts even though the drainage areas may have widely different drainage basin characteristics.

The writers are under no illusion as to the reliability of deductions that may be drawn as to frequencies of major or rare floods from a record which covers little more than 25 years. Nevertheless, it is desirable to make an attempt at such a deduction, and in this paper the results of a systematic, scientific attempt are presented. The writers conceive that the specific form of the formulas developed may not comprise a substantial or lasting contribution to engineering knowledge. However, they hope that the account of the studies and the results may be in such a form that other engineers may assimilate some worth-while ideas in their body of experience with consequent benefit to their handling of flood-frequency problems.

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[7] "Storm Rainfall of Eastern United States," *Technical Reports*, Miami Conservatory Dist., Pt. 5 (revised), 1936.

[8] "A Report on the Maximum Possible Precipitation over the Ompompanoosuc Basin Above Union Village, Vt.," U. S. Weather Bureau, 1940 (unpublished).

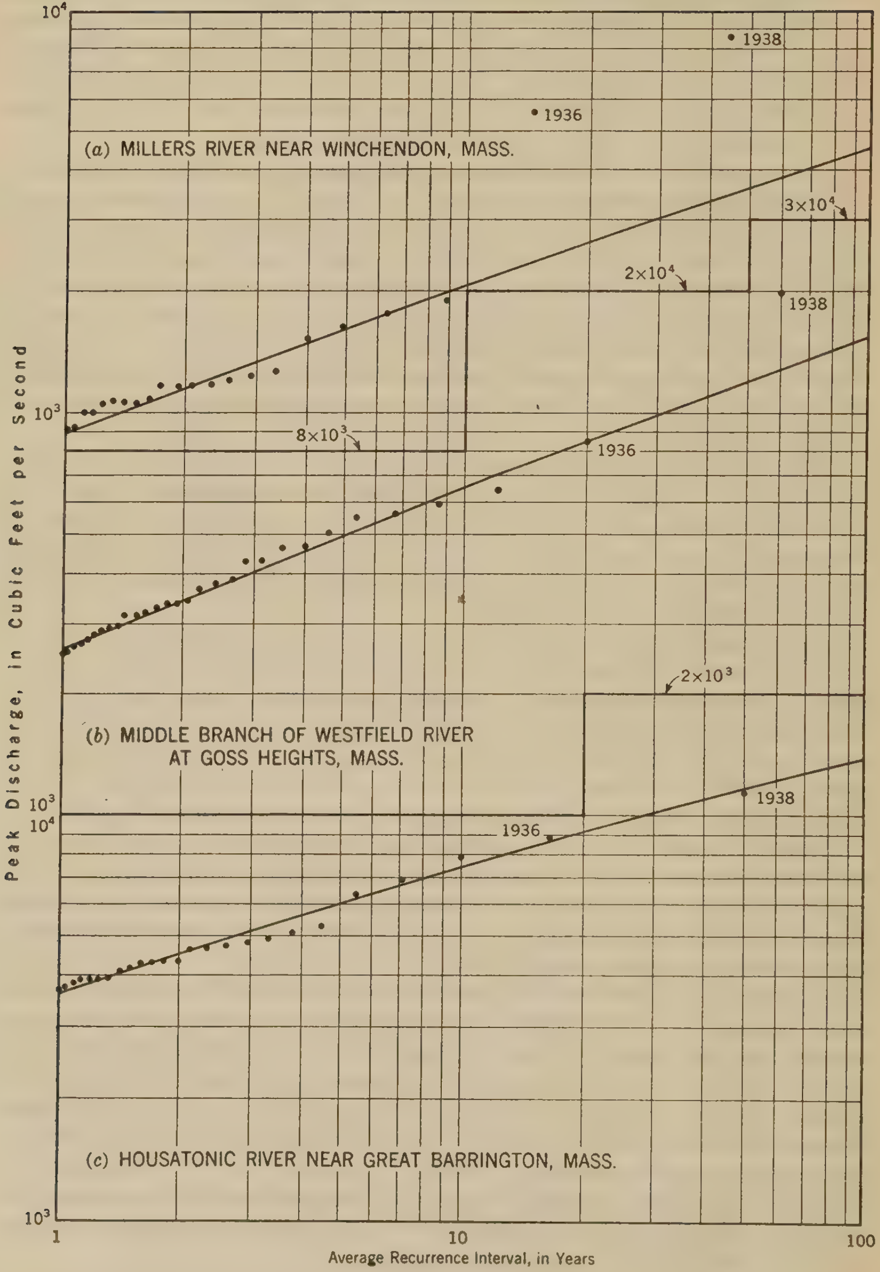


FIG. 1.—FREQUENCY OF FLOOD PEAKS

Frequency Curves of Peak Discharge from Observed Floods

The most logical method of determining the magnitude of expected floods is by a study of the frequency and magnitude of past floods. A frequency curve of instantaneous peak discharges was plotted for each gaging station having a sufficient length of record to justify its use. Fig. 1 shows frequency curves for Millers River near Winchendon, Mass., with 23 years of record, Middle Branch of Westfield River at Goss Heights, Mass., with 26 years of record; and Housatonic River near Great Barrington, Mass., with 25 years of record. In this diagram, the average recurrence interval in years is plotted against discharge. The average recurrence interval I is computed from the equation

$$I = \frac{N_y}{N_p - 0.5} \quad \text{(2)}$$

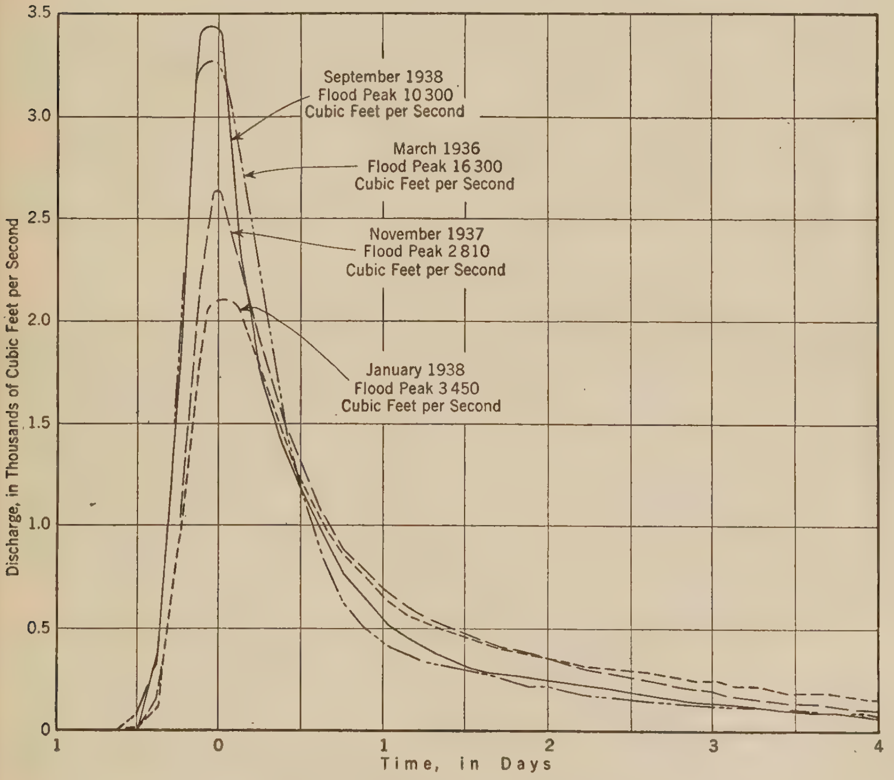


Fig. 2.—Six-Hour Unit Hydrographs Computed from Indicated Floods; North Nashua River, Near Leominster, Mass.

in which N_p is the number of the flood peak in order of size without regard to the time of its occurrence, and N_y is the number of years of record.

Widely different frequency curves can be drawn from the base data for any particular station, and extensions of the curves to recurrence intervals of 100 and 1,000 years are not defined because of insufficient length of record. If the floods of 1936 and 1938 had not occurred during the period of record, much different curves might have been drawn. However, by allowing for extreme severity of these floods in certain areas, the magnitude of a flood having an average recurrence interval of 15 years (called a 15-yr flood) can be determined with reasonable accuracy for many streams.

Because stream-flow records are not long enough to define the frequency curves of peak discharge beyond short recurrence intervals, peak discharges at recurrence intervals of 100 and 1,000 years were computed from a study of unit hydrographs, and depths and distribution of runoff.

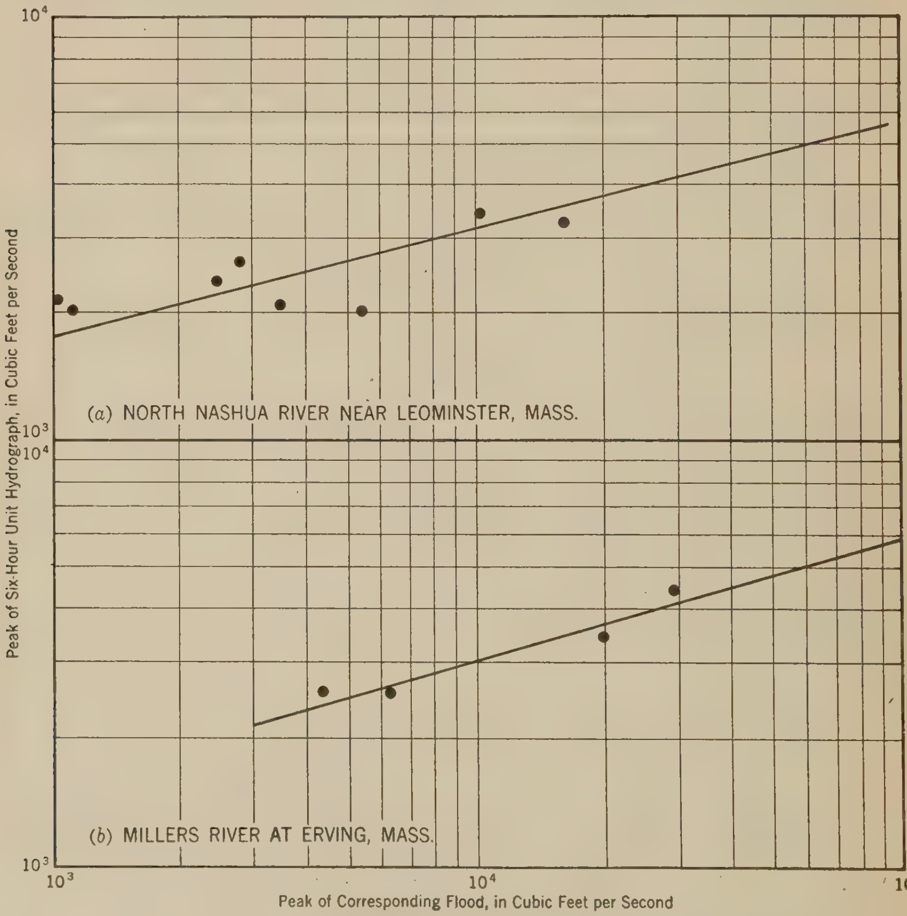


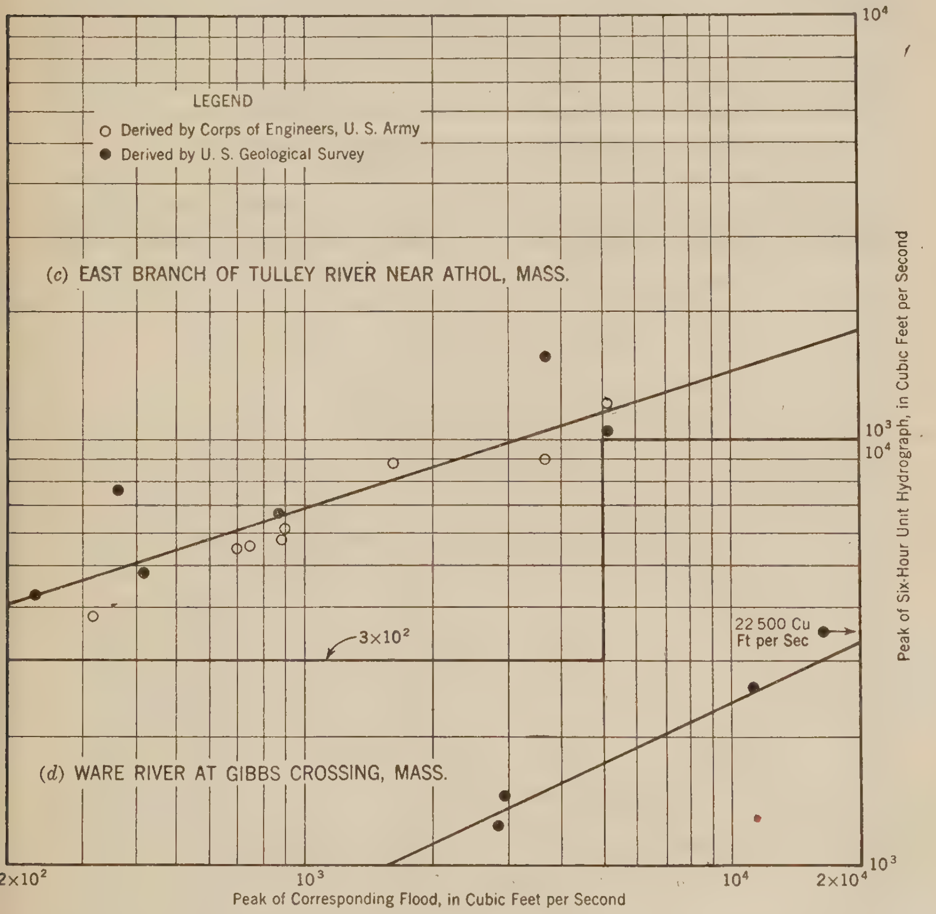
FIG. 3.—RELATION BETWEEN UN

UNIT HYDROGRAPHS

The unit hydrograph method was used to compute the discharge hydrographs of floods from runoff data. The peak discharges were then determined by these hydrographs.

As used in this paper, the unit hydrograph is the discharge hydrograph of 1 in. of direct runoff uniformly generated over the drainage area at a uniform rate. This definition is in common use, although it differs from that given by W. G. Hoyt, M. Am. Soc. C. E., in 1936,[9] by restricting the term to uniformly generated runoff of unit depth. The term, unit hydrograph, usually is qualified by stating the duration of the runoff that generated it. For example, a 4-hr unit hydrograph is the discharge hydrograph resulting from runoff generated uniformly during the period of 4 hours at a rate of 0.25 in. per hr.

The unit hydrograph theory usually presumes that the base and shape of the unit hydrograph for a particular point on a stream will be the same for all floods where the runoff generation is uniform over the area. Differences in the unit hydrographs derived from observed floods have been attributed



Hydrograph Peaks and Flood Peaks

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[9] "Studies of Relations of Rainfall to Run-Off in the United States," by W. G. Hoyt, *Water-Supply Paper No. 772*, U. S. Geological Survey, 1936, p. 124.

usually to the uneven distribution of runoff generation over the area. This is undoubtedly true of many unit hydrographs derived from observed floods, and such distortion must be expected. However, studies by the writers and others[10] have indicated a definite tendency for the peak of the unit hydrograph to increase with an increase in the magnitude of the flood from which it was derived. Several typical unit hydrographs for North Nashua River near Leominster, Mass., are shown in Fig. 2. These curves illustrate the difference between the peaks of the unit hydrographs derived from the higher floods and

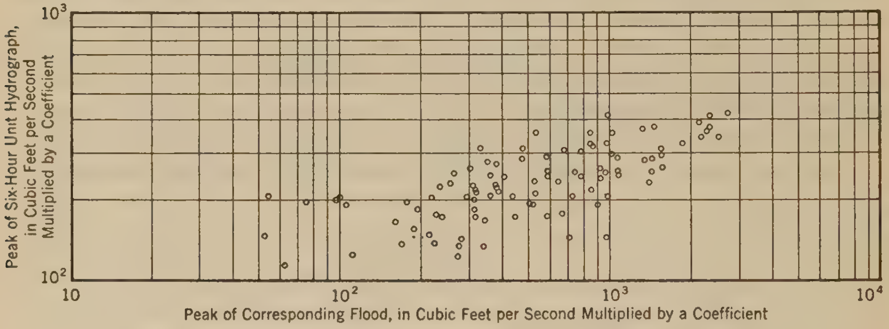

FIG. 4.—RELATION BETWEEN UNIT HYDROGRAPHS AND PEAK FLOWS, BOTH MULTIPLIED BY A COEFFICIENT

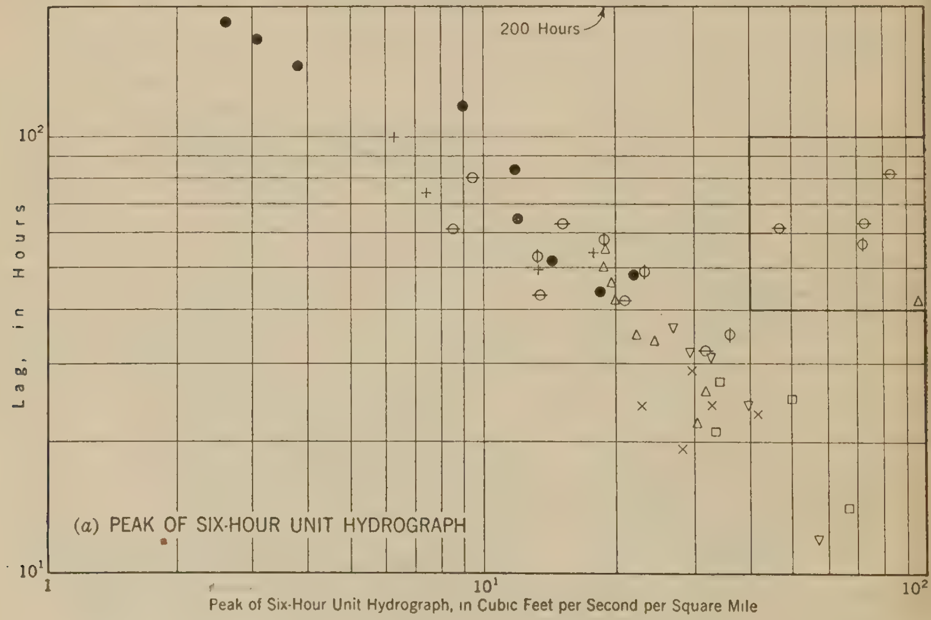

FIG. 5.—RELATION BETWEEN

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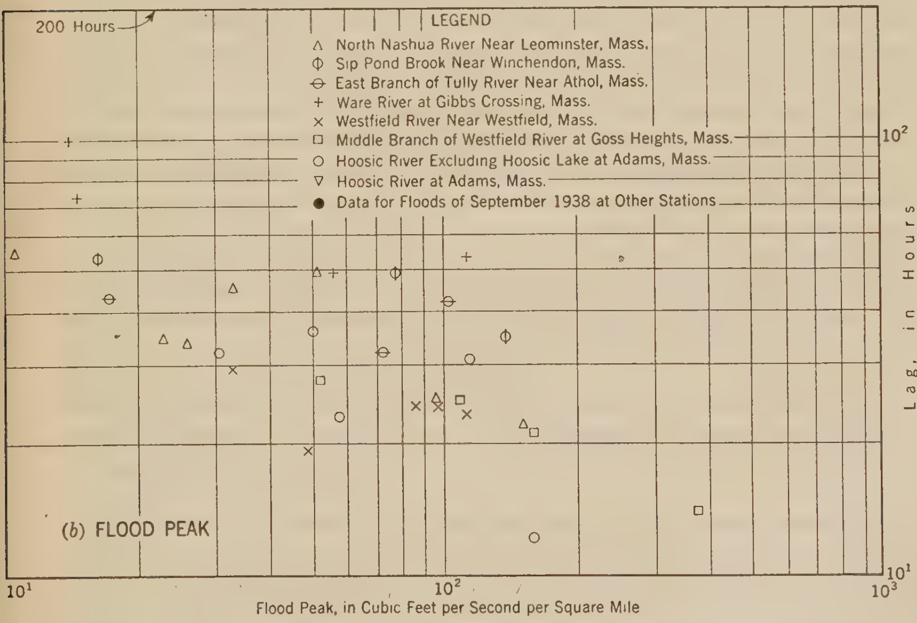
[10] "Application of Hydrology in Flood Control," by G. A. Hathaway, *Proceedings*, Hydrology Conference, State College, Pa., 1942.

those derived from smaller floods. This tendency is shown graphically in Fig. 3, in which the peaks of several floods are plotted against the peaks of the corresponding 6-hr unit hydrographs. The general increase of unit hydrograph peaks with increase in the corresponding floods is demonstrated also in Fig. 4, in which similar data from several stations are plotted on a composite diagram. Each coordinate has been multiplied by a coefficient which would make the trend line of each station pass through a point whose coordinates are 3,000 and 400. In spite of the scattering of the points on this figure, the general upward trend is indicated clearly.

The peaks of unit hydrographs for many gaging stations in and adjacent to Massachusetts were plotted in the manner similar to Fig. 3. The trend for some stations is uncertain and it was not defined at all if the observed floods did not cover a fairly wide range of discharge. The relation, if it could be completely defined, probably would not be a straight line on logarithmic paper either for a composite graph or for an individual station. The data are insufficient to define any trend other than a straight line. However, the data were sufficient to justify the adjustment of the peaks of the unit hydrographs for the types of floods considered in this study. Extensions of the relation curves for extremely large floods are inexact, but certainly are more accurate than results obtained by assuming no increase in unit hydrograph peaks with increase in the size of the flood.



Lag and Peak Flows

W. B. Langbein,[11] Assoc. M. Am. Soc. C. E., has shown that the peak of unit hydrographs, as expressed by the S-curve, increased with a decrease in the time by which the center of mass of discharge lagged behind the center of mass of runoff generation. "Runoff generation" as used in this paper is defined as the origination of direct runoff, and "direct runoff" is that part of the precipitation which flows more or less over the land surface. The lag time was computed from the recession curve by the method suggested by Mr. Langbein for different floods at each of several gaging stations, and for the flood of September, 1938, at several other stations. Fig. 5(*a*) shows the relation between the lag time and the peak of the unit hydrograph. In Fig. 5(*b*) the lag time, in hours, for different floods at seven gaging stations is plotted against the peak discharge of these floods. The tendency for this lag time to decrease with an increase in size of flood peak, in cubic feet per second per square mile, is shown both for the different floods at a single station and for floods at different stations. Thus, Fig. 5 confirms the increase in the peak discharge of the unit hydrograph with an increase in the peak discharge of the corresponding flood.

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[11] "Channel-Storage and Unit-Hydrograph Studies," by W. B. Langbein, *Transactions*, Am. Geophysical Union, Pt. II, 1940, pp. 620–627.

Frequency Curves of Depth of Runoff

In order to use the frequency curves of precipitation in a study of the frequency of peak discharges, the use of frequency curves of depth of runoff is desirable. Although a relationship exists between precipitation and peak discharge, a much closer relationship exists between precipitation and the volume of runoff which is usually expressed as a depth in inches over the basin. Once this correlation between precipitation and depth of consequent runoff is established, the frequency of a given depth of runoff can be computed from the frequency curve of precipitation. When the depth of runoff for a flood of certain frequency is known, the discharge hydrograph can be computed by means of the unit hydrograph.

To establish the relationship between precipitation and runoff, the amount of direct runoff resulting from each storm of 1.5 in. or more precipitation over each of three areas was plotted against the corresponding precipitation. The depth of runoff was determined from stream-flow records, and precipitation records published by the U. S. Weather Bureau were used. From these plotted points and a general consideration of the probable limits and distribution of runoff factors, a probability table was prepared. It showed the probability of a given depth of runoff being generated by any given depth of precipitation.

Frequency curves of precipitation used in this study are shown in Fig. 6. The frequency curve of 24-hr precipitation was included only as an aid in defining the curve of 72-hr precipitation, for which the data are less complete. These curves are based on data for precipitation at a point, from "Rainfall Intensity-Frequency Data"[12] and "Storm Rainfall of Eastern United States."[7] The upper ends of the frequency curves are based on the maximum precipitation as shown in the Miami Conservancy Report of 1936;[13] the maximum possible precipitation on an area of 100 sq miles as given in the Ompompanoosuc Report of the Weather Bureau in 1940;[14] and the maximum point rainfall of the northeastern United States from unofficial records as indicated by an extension of a curve in the Ompompanoosuc Report.[15] These maximum amounts of precipitation are plotted at arbitrary recurrence intervals of 10,000 or 100,000 years to provide some basis for an extension of the frequency curves.

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[12] "Rainfall Intensity-Frequency Data," by David L. Yarnell, *Miscellaneous Publication No. 204*, U. S. D. A., 1935.

[13] "Storm Rainfall of Eastern United States," *Technical Reports*, Miami Conservancy Dist., Pt. 5 (revised), 1936, Figs. 38 and 40.

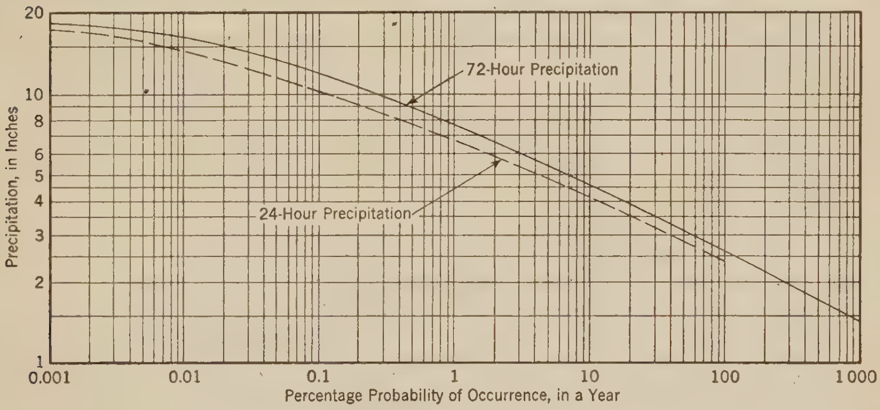

FIG. 6.—FREQUENCY OF SEVENTY-TWO HOUR PRECIPITATION FOR DRAINAGE BASINS IN MASSACHUSETTS

The frequency curve of precipitation in 72 hours was assumed to be applicable to drainage basins in Massachusetts, although certain assumptions had to be made in drawing it. The upper end of the curve was drawn to represent frequency of precipitation on drainage areas of 100 or 200 sq miles. At average recurrence intervals of 100 years or less, the curve is based more or less directly on observations of precipitation at a point, so that it does not apply exactly to precipitation over the actual drainage area. The difference

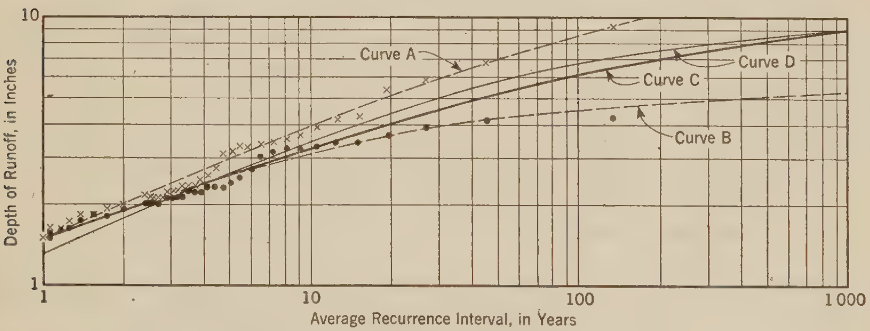

FIG. 7.—FREQUENCY CURVES OF DEPTH OF RUNOFF; COMBINED RECORDS FOR TAUNTON RIVER, PRIEST BROOK, AND THE MIDDLE BRANCH OF WESTFIELD RIVER

between point and areal precipitation probably is not great on drainage areas of less than a few hundred square miles and for average recurrence intervals not longer than 100 years. Probably the error from using the same curve of 72-hr precipitation for drainage basins of all sizes is partly compensated by the error in using the duration of 72 hours for precipitation on all basins.

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[14] "A Report on the Maximum Possible Precipitation over the Ompompanoosuc Basin Above Union Village, Vt.," U. S. Weather Bureau, 1940, Table VI (unpublished).

[15] *Ibid.*, Fig. 24.

The frequency curve of 72-hr precipitation was combined with the probability table which showed the probability of a given depth of runoff being

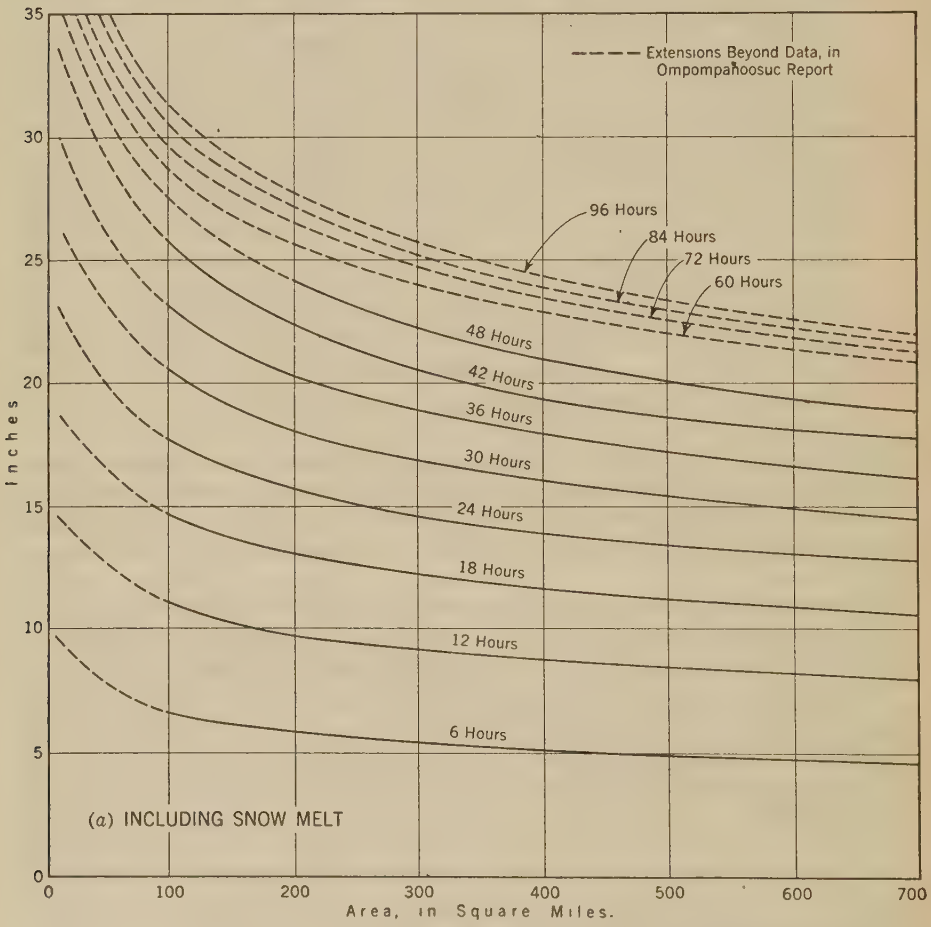
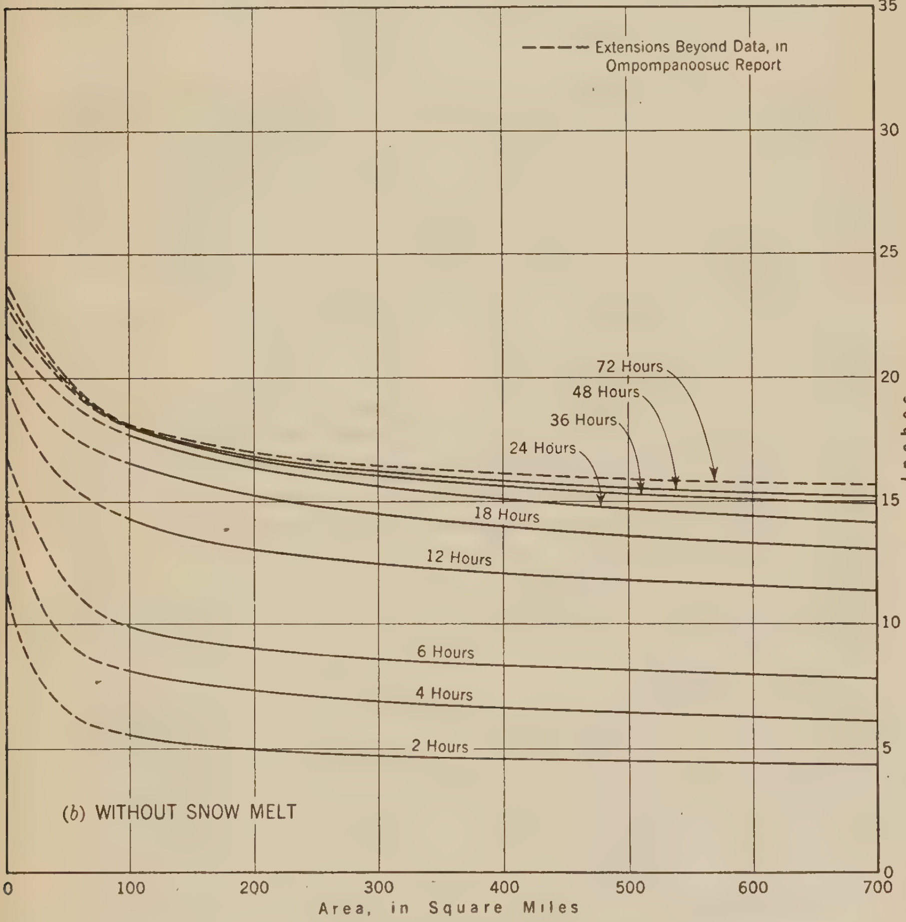


FIG. 8.—MAXIMUM RAINFALL BASED

generated by any given depth of precipitation, and a frequency curve of depth of runoff obtained. This curve is shown as curve D in Fig. 7.

Frequency curves of depth of runoff were prepared for ten streams in Massachusetts. They have the same limitations as frequency curves of discharge, owing to shortness of record. Three of these drainage areas are widely separated, and for this study the records may be combined and used as one continuous record of a length equal to the sum of the three individual records. These areas are Taunton River at State Farm, Mass., Priest Brook near Winchendon, Mass., and Middle Branch of Westfield River at Goss Heights, Mass., whose combined length of record is 69 years. Curve A in Fig. 7 is the frequency curve for this combined record. On the assumption that floods of the magnitude of those of March, 1936, and September, 1938, on the Middle Branch of Westfield River and Priest Brook might not have occurred during the period



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of record, these have been omitted in determining frequency curve B. The floods of 1936 and 1938 on Taunton River at State Farm were not exceptionally high.

Curves A and B differ widely for the larger recurrence intervals, and illustrate the possible influence of one or two extremely large floods in a comparatively short period of record. These curves represent the probable extremes of a true frequency curve. Curve D shows lower depths of runoff at short recurrence intervals because some winter storms having less than 1.5 in. of precipitation were followed by as much as 2 in. of runoff.

The depths of runoff at recurrence intervals of 15, 100, and 1,000 years were estimated from curves A, B, and D. These depths were used later, with assumed time distributions of the runoff, to compute flood peaks. After a study of these peaks, the depths of runoff were revised slightly, and curve C, Fig. 7, represents these revised depths.

Maximum Flood Runoff

In the report of the Ompompanoosuc Basin in Vermont,[8] maximum possible rates of precipitation during both summer and winter storms and the maximum possible rates of snow melt were computed. Fig. 8(*b*) was prepared from that report.[16] Extensions from data shown in that report are indicated by dashed lines. One extension is from rainfall on a drainage area of 100 sq miles to rainfall at a point. The estimate of maximum rainfall at a point was also taken from that report.[17] The extension from a duration of 48 hours to a duration of 72 hours was determined by logarithmic plotting.

Fig. 8(*a*) shows the maximum possible winter rainfall[18] plus the maximum possible snow melt.[19] Extensions beyond the data available are indicated by dashed lines. The maximum possible quantity of water accumulated as ice and snow was estimated to be 10 in. on a drainage area of 1,000 sq miles and 16 in. on a drainage area of 10 sq miles. The detailed basis of the estimates is not given herein; but, during the time that stream-flow records have been collected, as much as 6 to 9 in. of precipitation have accumulated as ice and snow on some drainage basins in Massachusetts during winters when the total snowfall was much less than the greatest observed seasonal snowfalls. However, if the maximum possible amount of accumulated precipitation were overestimated by as much as 6 in., the computed peak discharges for the maximum winter floods would be less than 10% too large for most of the drainage basins which were investigated.

To any one who has not studied snow melt and amount of precipitation accumulated as ice and snow on the basis of the maximum possible limits of occurrence, the limits used in preparing Fig. 8(*a*) may seem too large. By "maximum possible" is meant not the maximum that should be expected, but the maximum that could not be exceeded under any possible, even though very unlikely, combination of circumstances.

Determination of Flood Peaks for Minor, Major, Rare, and Maximum Floods

The peak discharges of certain defined types of floods have been determined for several streams for which adequate discharge records are available. This was accomplished by determining for each basin the depth of runoff in inches for each type of flood, assuming a distribution of this runoff, and computing the flood hydrograph by means of the unit hydrograph.

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[16] "A Report on the Maximum Possible Precipitation over the Ompompanoosuc Basin above Union Village, Vt.," U. S. Weather Bureau, 1940, Figs. 36 and 44 (unpublished).

[17] *Ibid.*, Fig. 24.

[18] *Ibid.*, Table VII.

[19] *Ibid.*, Fig. 9.

Minor Flood Peaks.—For the purpose of this paper, a minor flood is defined as an approximation of a flood which would be equaled or exceeded on an average of once in 15 years. The depth of runoff for a minor flood has been found to be fairly uniform for drainage basins in Massachusetts. However, the depths of runoff do vary with physical characteristics. The best correlation of depth of runoff is with mean altitude above sea level. This relationship is shown in Fig. 9 where depth of runoff for minor floods is plotted against mean altitude above sea level. The points were obtained from frequency curves of depth of runoff for drainage areas having about twenty-five years of record, except No. 10, which has eight years of record. To make this relatively short record comparable to the longer records at other stations, it has been adjusted by the proportion which the frequency curve based on the last eight-years of a longer record bears to a curve based on the complete record. Although the points scatter, and the relation below an altitude of 800 ft is not defined, the runoff definitely increases with an increase in altitude. The runoff for Swift River at West Ware, Mass., appears low in comparison with the other records, and was given very little weight in determining the correlation. The increase in runoff with an increase in the altitude of the basins may be attributed to factors other than the altitude, such as soil types, land slopes, and temperature, but these factors probably correlate fairly well with altitude.

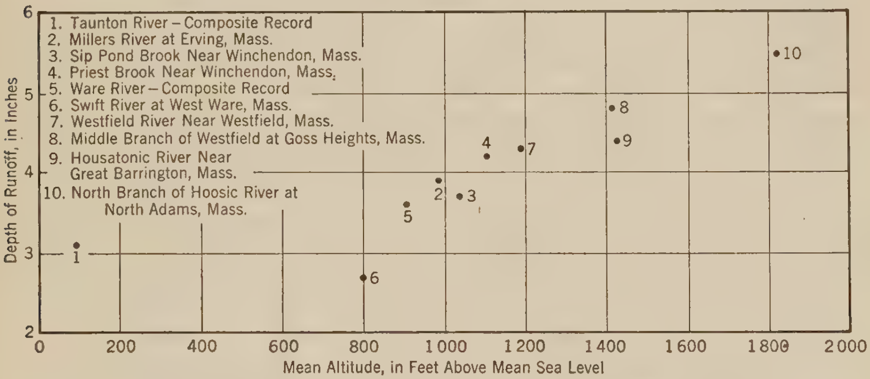

FIG. 9.—RUNOFF EQUALED OR EXCEEDED ON AN AVERAGE OF ONCE IN FIFTEEN YEARS

The surface area of lakes, ponds, and reservoirs within a basin correlated to some extent with depth of runoff. No other correlation between depth of runoff and physical characteristics was found.

From the foregoing data, and Fig. 7, the final equation for depth of runoff for a minor flood was determined to be:

$$D = 3.6 - 0.3\,a + 0.001\,E \qquad (3)$$

in which D is depth of runoff in inches; a is the water surface of lakes, ponds, and reservoirs expressed as a percentage of the drainage area; and E is the difference, disregarding negative quantities, between the average altitude of the basin in feet above mean sea level and 800 ft.

The depth of runoff for a 72-hr storm was distributed by successive 6-hr increments of 3%, 5%, 5%, 10%, 10%, 20%, 34%, 10%, and 3%. For some areas, the 12-hr period covered by the two largest 6-hr increments was subdivided into 2-hr increments, 2%, 6%, 12%, 24%, 8%, and 2% of the total depth. The assumed runoff pattern has been determined from a study of precipitation and discharge records of Massachusetts and near-by areas, and a study of the distribution used by the Corps of Engineers for design floods.

Peak discharges computed from the depth-of-runoff formula and the distribution for 72-hr runoff were in agreement with the peak discharges of 15-yr floods obtained from frequency curves of peak discharges for those streams where these curves were well defined. The runoff, however, should not be presumed to follow the assumed distribution for any particular storm; nor should the depth of runoff be considered a representative value for 15-yr floods. On drainage basins having sharp flood peaks, the depth of runoff occurring in 72 hours may have little significance, for the highest flood peaks may follow short, intense periods of rainfall. Depths and distribution of runoff, as used in this study, are interdependent. The depths of runoff which have an average recurrence interval of 15 years may not be the same as the average depths of runoff associated with peaks of 15-yr floods. The distribution used here is only a means, uniform for all basins, of converting depth of runoff to discharge hydrographs having peaks which may be expected to occur with the same frequency as the depth of runoff.

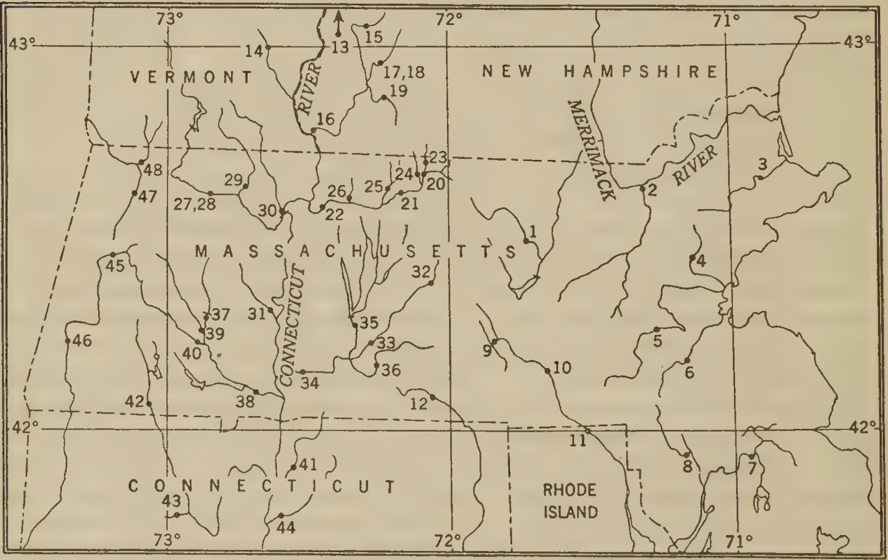
73°
72°
71°
43°
42°
VERMONT
NEW HAMPSHIRE
MASSACHUSETTS
CONNECTICUT
RHODE ISLAND
RIVER
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Fig. 10.—Key to Gaging Stations in Table 1

The discharge hydrograph for a minor flood on each stream was computed by applying unit hydrographs to the 6-hr increments of depth of runoff assumed for the storm, and a base flow of 1 cu ft per sec per sq mile was included. The peaks are listed in Table 1 (and Fig. 10) together with the peaks of other classes of floods and the physical characteristics of the basins.

Major Flood Peaks.—A major flood is defined as an approximation of a flood that would be equaled or exceeded on an average of once in 100 years. The final equation for the depth of runoff was determined to be:

$$D = 6.1 - 0.35\,a + 0.001\,E \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The effect of lake and reservoir surface on the major floods as compared with minor floods was assumed to be slightly greater as measured in inches but to be considerably less as measured in percentage of the depth of runoff, and the effect of mean altitude was assumed to be the same as for minor floods.

A slightly more concentrated runoff distribution was assumed for the major flood than for the minor flood. Successive 6-hr increments of 2%, 3%, 5%, 10%, 10%, 20%, 38%, 10%, and 2% of the depth of runoff were used. For some areas, the 12-hr period covered by the two largest 6-hr increments was subdivided into successive 2-hr increments of 2%, 6%, 12%, 25%, 10%, and 3%. The discharge hydrographs for major floods on all streams were computed by the unit hydrograph method, and the peaks are listed in Table 1, Cols. 13 to 16. A base flow of 1 cu ft per sec per sq mile is included.

Rare Flood Peaks.—A rare flood is defined as an approximation of a flood that would be equaled or exceeded on an average of once in 1,000 years. The depth of runoff for rare floods was computed from the equation:

$$D = 8.9 - 0.4\,a + 0.001\,E \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The first term on the right-hand side of Eqs. 4 and 5 was based on curve C of Fig. 7. The effect of lakes and ponds on rare floods was assumed to be slightly greater than their effect on minor or major floods; and the effect of altitude on rare floods was assumed to be the same as its effect on other classes of floods.

Successive 6-hr increments of runoff of 2%, 3%, 5%, 8%, 10%, 20%, 40% 10%, and 2% of the total depth of runoff were used, and for some areas the two largest increments were subdivided into successive 2-hr increments of 2%, 6%, 12%, 27%, 10%, and 3%. For rare floods the discharge hydrographs were computed by the unit hydrograph method in the same manner as for minor and major floods. The computed peaks are listed in Table 1 and are plotted in cubic feet per second per square mile in Fig. 11. A base flow of 1 cu ft per sec per sq mile is included.

The depths of runoff used in computing rare floods and the computed peaks of the rare floods were compared with the greatest depths of runoff and the highest flood peaks that have been observed on streams in Massachusetts since stream-flow records have been collected. The computed figures seem consistent with the observed depths of runoff and the observed peak discharges. During the flood of September, 1938,[20] the depths of runoff and the peak discharges at gaging stations in the Millers River Basin except Moss Brook, and the Ware River Basin except Swift River at West Ware, were approximately the same as the depths and peaks which were computed for rare floods.

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[20] "Hurricane Floods of September 1938," *Water Supply Paper No. 867*, U. S. Geological Survey, 1940, pp. 14–35.

TABLE 1.—PHYSICAL CHAR-

No.[a]	Outlet of drainage area	Years[b]	PEAK DISCHARGE[c]		PHYSICAL
			Cu ft per sec	Date	M (sq miles)
	(1)	(2)	(3)	(4)	(5)
1	North Nashua River near Leominster, Mass.	4	16,300	3-18-36	107
2	Concord River below River Meadow Brook, at Lowell, Mass.	3	3,790	7-29-38	405
3	Ipswich River near Ipswich, Mass.	10	2,610	3-15-36	124
4	Aberjona River at Winchester, Mass.	1	....	....	23.3
5	Charles River at Charles River Village, Mass.	2	3,170	3-19-36	183
6	Neponset River at Norwood, Mass.	1	....	....	35.2
7	Taunton River excluding Assawompsett Pond, at State Farm, Mass.	9	3,050	4-14-35	211
8	Wading River near Norton, Mass.	13	1,030	3-12-36	42.2
9	Blackstone River at Worcester, Mass.	15	2,520	3-18-36	31.3
10	Blackstone River at Northbridge, Mass.	1	....	....	139
11	Blackstone River at Woonsocket, R. I.	11	15,100	7-24-38	417
12	Quinebaug River at Westville, Mass.	1	....	....	93.8
13	Sugar River excluding Sunapee Lake, at West Claremont, N. H.	10	14,000	3-19-36	224
14	West River at Newfane, Vt.	14	52,300	9-21-38	308
15	Ashuelot River near Gilsum, N. H.	16	5,220	9-21-38	71.2
16	Ashuelot River at Hinsdale, N. H.	29	18,000	3-29-20	420
17	Otter Brook excluding Granite Lake, near Keene, N. H.	15	6,130	9-21-38	37.6
18	Otter Brook excluding Granite Lake and Ellis Reservoir, near Keene, N. H.	15	6,130	9-21-38	24.2
19	South Branch of Ashuelot River at Webb, near Marlboro, N. H.	18	5,960	9-21-38	36.4
20	Millers River near Winchendon, Mass.	22	8,500	9-22-38	83.8
21	Millers River at South Royalston, Mass.	1	....	....	186
22	Millers River at Erving, Mass.	24	29,000	9-22-38	370
23	Sip Pond Brook near Winchendon, Mass.	24	2,630	9-21-38	19.0
24	Priest Brook near Winchendon, Mass.	21	3,000	9-21-38	18.7
25	East Branch of Tully River near Athol, Mass.	22	5,140	9-21-38	49.9
26	Moss Brook at Wendell Depot, Mass.	23	1,540	3-19-36	12.2
27	Deerfield River at Charlemont, Mass.	26	56,300	9-21-38	362
28	Deerfield River excluding Harriman Reservoir, at Charlemont, Mass.	26	56,300	9-21-38	178
29	North River at Shattuckville, Mass.	1	....	....	88.4
30	Green River at Greenfield, Mass.	0	....	....	89.4
31	Mill River at Northampton, Mass.	1	....	....	53.3
32	Ware River at Cold Brook, Mass.	11	14,000	9-21-38	100
33	Ware River at Gibbs Crossing, Mass.	28	22,700	9-21-38	201
34	Chicopee River at Bircham Bend, Mass.	10	45,200	9-21-38	702
35	Swift River at West Ware, Mass.	28	7,590	3-19-36	188
36	Quaboag River at West Brimfield, Mass.	28	8,470	9-21-38	149
37	Westfield River at Knightville, Mass.	29	37,900	9-21-38	162
38	Westfield River near Westfield, Mass.	24	55,500	9-21-38	497
39	Middle Branch of Westfield River at Goss Heights, Mass.	30	19,900	9-21-38	52.6
40	West Branch of Westfield River at Huntington, Mass.	4	21,800	9-21-38	93.7
41	Scantic River at Broad Brook, Conn.	10	7,360	9-21-38	98.4
42	Farmington River excluding Otis Reservoir, near New Boston, Mass.	25	18,500	9-21-38	75.3
43	Burlington Brook near Burlington, Conn.	7	645	9-21-38	4.1
44	Hockanum River near East Hartford, Conn.	12	5,160	9-21-38	74.5
45	Housatonic River at Coltsville, Mass.	4	6,400	9-21-38	57.1
46	Housatonic River near Great Barrington, Mass.	25	11,500	9-22-38	280
47	Hoosic River excluding Hoosic Lake, at Adams, Mass.	8	5,080	9-21-38	31.4
48	North Branch of Hoosic River at North Adams, Mass.	9	9,980	11- ?-27	39.1

[a] See Fig. 10. [b] Years of record used in the study of flood peaks. [c] Maximum known peak discharge.

The maximum observed discharges and the computed peaks of rare floods are plotted in Fig. 11, which shows that on many streams the rare flood has not been remotely approached.

Maximum Flood Peaks.—A maximum flood is the computed flood that might be caused by the maximum possible precipitation, distributed according to the most destructive pattern, the losses from evaporation and infiltration being a

CTERISTICS AND FLOOD PEAKS

HARACTERISTICS			COMPUTED FLOOD PEAKS IN CUBIC FEET PER SECOND								No.[a]
			From Flood Formulas				From Unit Hydrographs				
L miles)	*s* (ft)	*a* (%)	Minor	Major	Rare	Maximum	Minor	Major	Rare	Maximum	
(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	
10.9	578	2.2	4,310	11,000	18,600	65,400	5,050	10,600	18,000	63,900	1
30.5	203	2.8	3,580	8,940	14,100	49,000	3,500	7,490	12,900	47,200	2
13.8	91	1.1	1,940	3,710	6,080	23,900	1,840	3,680	6,050	24,600	3
4.2	109	2.3	900	1,680	2,990	12,600	....	....	....	....	4
19.8	132	1.6	2,180	4,750	7,480	28,500	2,090	4,340	7,180	27,900	5
5.7	153	2.9	1,090	2,280	3,970	16,000	....	....	....	....	6
14.0	90	1.9	3,110	5,810	10,000	38,600	4,050	8,400	14,200	46,600	7
7.8	125	2.2	1,030	2,130	3,540	14,400	900	1,940	3,240	13,300	8
5.2	266	4.3	1,160	2,860	5,000	19,500	970	2,330	4,050	23,600	9
13.4	341	4.2	2,780	7,530	12,700	44,400	....	....	....	....	10
19.6	365	3.2	6,390	16,600	29,100	97,200	....	....	....	....	11
10.6	285	3.1	2,060	5,410	8,970	32,900	....	....	....	....	12
17.9	870	0.7	11,900	25,500	42,400	143,000	....	....	....	....	13
22.2	1,225	0.3	29,100	46,200	76,100	258,000	....	....	....	....	14
10.6	727	2.6	4,100	10,000	16,500	59,200	....	....	....	....	15
28.5	984	1.3	19,900	40,000	65,000	213,000	....	....	....	....	16
6.3	690	1.1	3,080	7,160	12,300	45,000	....	....	....	....	17
5.0	580	0.3	1,950	4,520	7,850	29,500	....	....	....	....	18
5.6	577	1.2	2,510	6,050	10,700	39,400	....	....	....	....	19
7.9	260	4.0	2,180	5,280	9,320	34,600	2,440	4,990	7,920	29,700	20
12.0	285	3.0	3,680	9,000	15,900	55,800	....	....	....	....	21
21.7	498	2.1	7,150	19,000	32,000	106,000	9,100	18,700	29,100	91,800	22
3.9	173	3.2	800	1,720	3,020	12,600	900	1,860	3,150	16,000	23
5.8	243	0.3	710	1,660	2,670	10,800	940	1,930	2,960	12,600	24
6.9	430	1.1	2,120	5,350	9,260	34,000	2,380	4,990	8,240	30,900	25
3.8	343	1.5	700	1,800	3,040	12,100	740	1,400	2,270	10,900	26
27.2	1,446	1.3	40,100	60,200	97,100	326,000	....	....	....	....	27
13.6	1,192	0.2	23,400	35,700	61,400	212,000	....	....	....	....	28
10.1	978	0.2	9,890	17,500	30,300	107,000	....	....	....	....	29
12.6	868	0.4	6,690	13,800	22,500	79,600	....	....	....	....	30
8.4	730	0.6	3,950	8,860	14,900	53,700	....	....	....	....	31
9.2	315	2.4	2,550	6,450	11,300	40,800	3,620	7,490	11,900	42,700	32
20.0	510	1.5	4,420	12,200	19,500	65,700	5,720	12,000	23,400	83,800	33
32.3	668	1.4	14,900	36,400	60,500	193,000	13,500	34,000	61,600	227,000	34
16.0	410	0.9	3,980	10,400	17,300	59,500	5,070	11,400	19,100	64,700	35
16.2	460	2.6	3,380	9,410	15,400	53,300	1,930	4,080	7,960	37,300	36
15.8	966	0.2	12,700	23,300	38,800	133,000	14,700	27,500	41,700	112,000	37
24.6	1,075	1.0	30,900	55,700	93,600	305,000	57,000	69,700	111,000	292,000	38
10.3	998	0.05	6,620	11,600	18,800	67,900	6,490	12,100	21,100	82,100	39
11.2	1,020	1.0	9,900	18,600	31,200	112,000	7,780	14,700	23,100	77,900	40
11.5	387	0.2	2,750	6,760	11,200	40,100	....	....	....	....	41
7.7	728	0.8	5,690	12,400	21,900	78,000	7,440	14,400	24,900	92,900	42
1.8	180	0.05	380	700	1,230	5,470	....	....	....	....	43
10.1	372	1.8	2,070	5,500	9,170	33,400	....	....	....	....	44
7.1	648	1.0	3,850	8,980	15,700	56,200	4,240	7,780	12,000	38,800	45
24.4	712	1.7	8,310	20,700	33,200	109,000	9,920	18,300	29,300	81,300	46
4.5	860	0.05	5,450	9,400	17,000	63,300	4,300	7,680	13,600	71,700	47
5.4	965	0.2	6,910	11,900	21,300	78,500	5,180	10,000	16,400	76,700	48

minimum, and the discharge occurring in accordance with a unit hydrograph having an extremely high peak. The maximum flood (if it occurs during the winter) is a similarly computed flood which is based on the additional assumptions of the largest possible water content in accumulated snow and ice, the fastest rate of melting, and the worst combination of snow melt and precipitation.

Increments of maximum possible precipitation during summer storms were taken from Fig. 8(*b*) by 6-hr periods for each drainage basin. The maximum 6-hr increment was subdivided into 2-hr increments for the four streams whose flood hydrographs have the sharpest peaks. The 6-hr increments were arranged in the order which would produce approximately the highest flood peaks. The largest increment was preceded in order by the second, fourth, sixth, eighth, tenth, and twelfth largest increments. The largest increment was followed in order by the third, fifth, seventh, ninth, and eleventh largest increments.

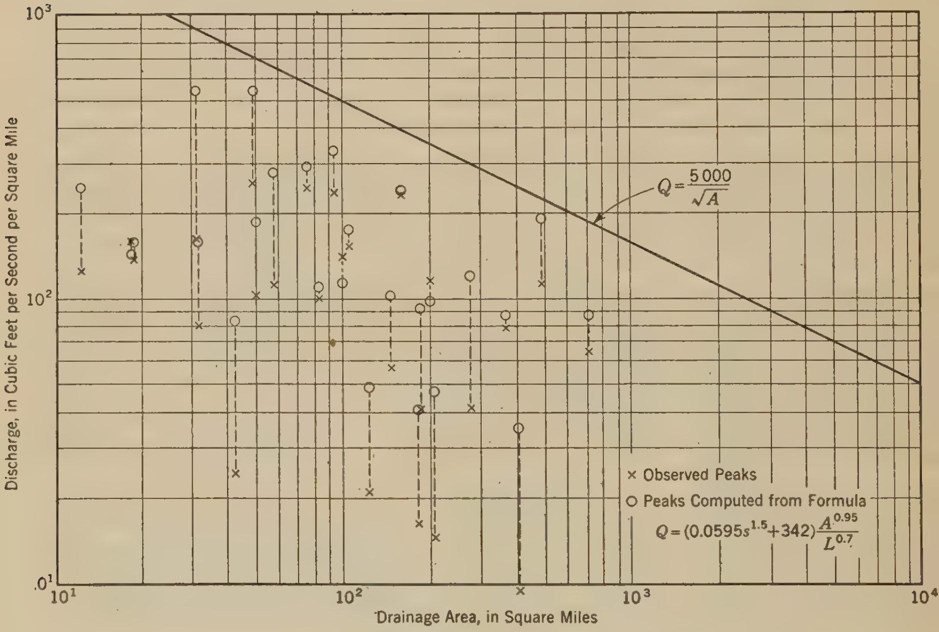

FIG. 11.—RARE FLOOD PEAKS

When precipitation of high intensity falls on saturated soil, the losses by infiltration probably are small. Also, the precipitation which is called the maximum possible may not contain sufficient factors of safety, or the assumed distribution of the precipitation may not be that distribution which would cause the highest possible peak discharge. For these reasons a runoff factor of 100% was used.

Increments of winter precipitation plus snow melt were taken from Fig. 8(*a*) by 6-hr periods for each drainage basin. These increments were arranged to precede and to follow the maximum increment in an order similar to that used for the maximum possible summer storms. A runoff factor of 100% was used for the drainage basins in the Providence, Thames, and Connecticut river valleys. Some appreciable loss by infiltration and evaporation must occur on the coastal basins, from which the discharge is slow. For these basins a runoff factor of 90% was assumed to be the maximum possible for winter floods.

Unit hydrographs were determined and the peaks of the maximum floods were computed in the same general way as those for the other classes of floods. Base flow of 2 cu ft per sec per sq mile was added to the computed net peaks of both the maximum winter and the maximum summer flood peaks. The lower one of the two computed peaks for each basin was discarded from further consideration, and the higher one was listed in Col. 16, Table 1. The maximum summer flood was higher or almost as high as the maximum winter flood for all drainage basins except the Concord River below River Meadow Brook, at Lowell, Mass., the Ipswich River near Ipswich, Mass., the Charles River at Charles River Village, Mass., the Taunton River excluding Assawompsett Pond, at State Farm, and the Millers River at Erving, Mass.

The maximum flood, as the term is used in this paper, is entirely different from a design flood or a maximum probable flood. It is that computed flood which is expected to equal or exceed the flood resulting from the worst combination of flood-producing conditions. It contains factors of safety for the inadequacy of past records and for ignorance both of relationships between cause and effect and the physical limits of natural hydraulic phenomena. Any design based on less than the maximum flood peak should be assumed to involve some risk. The remoteness of the probability that a flood peak even close to the size of the peak of the maximum flood will occur during the existence of any structure is indicated by the great difference between the computed peaks of the rare floods and those of the maximum floods.

The reason for the large size of the maximum flood peak can be shown best by contrasting that peak with a flood having the order of magnitude of a maximum probable flood. This flood might follow precipitation that was 80% of the maximum, the distribution might have an effect on the peak which was 85% of the maximum, the runoff factor might be 85%, and the unit hydrograph might have a shape which would have 75% of the maximum effect on the computed flood peak. This flood would then have a peak discharge which would be about 45% of the peak discharge of the maximum flood. A similar rough analysis applied to winter floods would show the peak of the maximum probable flood to be an even smaller percentage of the peak of the maximum winter flood.

PHYSICAL CHARACTERISTICS

The physical characteristics of more than one hundred drainage basins in New England were computed by the personnel of the Work Projects Administration under the supervision of the Boston district of the U. S. Geological Survey. Of these basins, forty eight were in Massachusetts, although not all of them were entirely separate basins because some overlapped. Except for the type of vegetal cover, which was computed from information supplied by the National Resources Committee, the following physical characteristics were computed from topographic maps published by the U. S. Geologic Survey:

Areas.—(1) Drainage area; (2) area of lake, reservoir, and pond surfaces; and (3) area of swamps.

Altitude.—(1) Maximum altitude; (2) minimum altitude (altitude at the outlet); (3) median altitude (altitude at which 50% of the drainage area is higher and 50% is lower); and (4) mean altitude.

Slopes.—(1) East-west land slope; (2) north-south land slope; (3) average land slope; (4) slope of principal streams (a principal stream being defined as a watercourse that drains more than 10% of the total area of the basin); and, (5) slope of tributary streams (tributary streams being defined as all watercourses contributing to a principal stream).

Configuration of Basin and Streams.—(1) Length of principal streams; (2) stream density (total length of all streams divided by the drainage area); (3) average distance uniformly distributed runoff must travel to the outlet; (4) length of basin (twice average distance water must travel); (5) width of basin (drainage area divided by length of basin); (6) form factor (length of basin divided by width of basin); and (7) branch index (length of stream channels, selected according to an arbitrary criterion, divided by length of basin).

General Type of Vegetal Cover.—(1) Percentage of cropped land; (2) percentage of wooded land; and (3) percentage of grass land not cropped.

Correlations with Flood Peaks

The peak discharge of a flood probably would be affected by any large change in any one of these basin characteristics even though all the other characteristics remained constant; but the change might be very slight. Also a change in one characteristic is usually associated with changes in other characteristics. Therefore, as the computed flood peaks are not exact and the relationships among the characteristics are complex, the effect of any particular characteristic usually cannot be determined accurately. Thus, the results of correlations may not really represent laws of cause and effect, so the formulas may not be applicable to regions other than Massachusetts or to ranges in any of the characteristics which are much beyond the limits included in this study. The correlations show, however, that changes in certain characteristics were associated in a definite way with changes in the size of the flood peaks.

Each computed physical characteristic was considered as a possible basis for correlation equations except the maximum altitude, the minimum altitude, and the east-west and north-south land slopes. Correlations between peak discharge and the physical characteristics for each type of flood were determined first by plotting on logarithmic paper the peaks of that type of flood against the drainage area. A straight line was used to represent the relation between peak discharge and drainage area. The ratio r_1 between each peak discharge and the peak discharge which would plot on the straight line was computed. Correlations of ratios r_1 with stream slopes, land slopes, and mean elevations above the outlet were investigated. Before any one of these characteristics was plotted against the ratios r_1, any correlation between the characteristic and drainage area was eliminated. To do this, the characteristic was plotted against the drainage area, and a curve showing the relationship was drawn. Ratios r_2 of the departure of the characteristic from this curve were computed. Then ratios r_1 were plotted against ratios r_2 and a curve was drawn to fit the plotted points. Ratios r_3 of the departure of the plotted points from this curve were computed. Next these ratios r_3 were correlated with another characteristic from which any correlation with a characteristic previously included in the correlation had been eliminated. When the remaining ratios no longer showed a correlation with a basin characteristic, the correlations already found were adjusted slightly to make them fit the base data as closely as possible.

The formulas representing the best correlations which were found are as follows: For minor floods—

$$Q = (0.000036\, s^{2.4} + 124)\, \frac{M^{0.95}}{a^{0.04}\, L^{0.7}} \quad \text{(6}a\text{)}$$

and for major floods—

$$Q = (0.0344\, s^{1.5} + 200)\, \frac{M^{0.85}}{L^{0.5}} \quad \text{(6}b\text{)}$$

for rare floods—

$$Q = (0.0595\, s^{1.5} + 342)\, \frac{M^{0.95}}{L^{0.7}} \quad \text{(6}c\text{)}$$

and for maximum floods—

$$Q = (0.128\, s^{1.6} + 1{,}800)\, \frac{M^{0.9}}{L^{0.7}} \quad \text{(6}d\text{)}$$

In these equations Q is the peak discharge in cubic feet per second; a is the percentage that lake, pond, and reservoir surface is to the total drainage area; s is the median altitude of the drainage basin in feet above the outlet; M is the drainage area in square miles; and L is the average distance, in miles, which water from runoff uniformly distributed over the basin must travel to the outlet. Thus M is a measure of the volume of water to be discharged; s is a measure of the fall of the water; and L is a measure of the distance that the water travels to the point of discharge. In addition, a in Eq. 6*a* is a rough measure of the effect of storage in lakes and reservoirs on the quantity of water discharged or on the shape of the hydrograph. The form of the equation indicates that a must not be zero. It should always be assumed to equal or exceed 0.05% even though the topographic map shows little or no water surface of lakes, ponds, or reservoirs. As a is not an entirely satisfactory measure of the storage, flood flows should be routed through the large lakes or reservoirs or those which have a large portion of the flow passing through them. The absence of a in the formulas for large floods (Eqs. 6*b*, 6*c*, and 6*d*) should not be interpreted to mean that lakes and reservoirs have no effect on the peak discharge of these floods. These floods should also be routed through the larger lakes or reservoirs. Perhaps no correlation was found because a correlates to some extent with either L or s, or the correlation is obscured by the effect of other basin characteristics or by inaccuracies in computed flood peaks.

The measures of basin characteristics, although the most practicable, are not entirely satisfactory. Average stream and land slopes, altitudes, and lake and swamp areas were used without determining the distribution of them in the basins. Steep slopes on a few tributaries may increase the average slope considerably, yet these slopes may have little effect on peak discharges. Steep slopes or abnormally high altitudes in that part of a basin upstream from a lake may affect the average land slope and mean elevation of the basin materially; but the lake may decrease the peak discharge so much that the peak outflows from the lake would be little different if the slopes and elevations upstream from the lake were greatly reduced. A lake on the headwaters of a flashy stream may have no noticeable effect on flood peaks, but a similar lake on the main channel near the outlet may greatly decrease the peak discharges. Consideration should be given to these facts in the study of any particular basin.

General type of vegetal cover did not correlate with peak discharge. Probably no correlation should be expected in Massachusetts where little of the land is either bare or in cultivated crops and where nearly all of the basins have a large proportion of woodland. Area of swamps seemed to correlate somewhat with stream slopes, with s, and with peak discharges. However, topographic maps based on surveys made at different times usually show different swamp areas. Median altitude usually is about the same as the mean altitude, and the differences between the two did not seem to be associated with a consistent variation in peak discharges. Average land slope is a fairly satisfactory measure of topography for use in correlation equations, but it did not correlate with peak discharges as well as mean altitude above the outlet. Slopes of principal streams and tributary streams are important basin characteristics, but the division between principal and tributary streams is arbitrary, and the average slope of either is not a very satisfactory measure of channel storage or of time of travel to the outlet. Stream density does not change much from place to place in Massachusetts, and the changes which do occur tend to correlate with land slopes. Neither the branch index nor the length of principal streams seemed to be as good a measure of the concentration characteristics of drainage basins as either the form factor or the average distance that water has to travel to the outlet. Length of basin, width of basin, form factor, and average distance water has to travel to the outlet are all interdependent, and as good correlations can be obtained with one as with any of the others. However, the average distance water must travel to the outlet has a simple and direct meaning; so it was used.

Mean altitude above sea level does not appear in the formulas, although it was included in the equations for depth of runoff. However, it correlates fairly well with s, the mean altitude above the outlet of the basin, so that by including s in the formulas, the effect of mean altitude above sea level is not neglected.

Minor, major, rare, and maximum flood peaks were computed with slide-rule accuracy for 48 basins from the formulas and are listed in Table 1 (Cols. 9 to 12). Table 1 also shows the physical characteristics that were used in the final formulas, the flood peaks computed from unit hydrographs, and the maximum observed or known flood peaks published by the U. S. Geological Survey.[21] No maximum known peak discharge is given if the period of record is short and no large flood discharge from the basin has been determined.

The points for which peak discharges were computed, usually sites of gaging stations operated by the U. S. Geological Survey, are shown in Fig. 10. Numbers on the map correspond to the numbers given in Table 1.

Frequency curves of peak discharge for three streams are given in Fig. 12. Curves such as these form the best available check on the computed peak discharge determined from the formulas. However, in determining the curve for Fig. 12(*b*), due consideration must be given to the extreme size of the flood of September, 1938. The frequency curves shown in Fig. 12 are not identical with those shown in Fig. 1 for the same stations, since the values obtained from the formulas are used in making the extensions beyond the observed data.

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[21] *Water-Supply Paper No. 871*, U. S. Geological Survey, 1939.

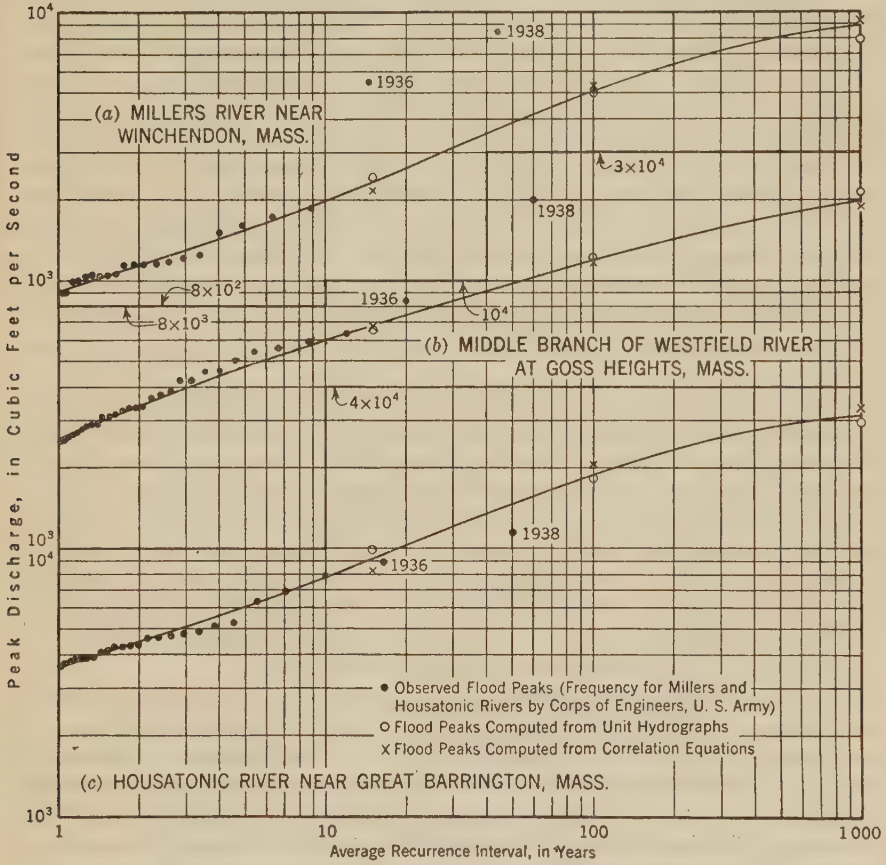

Fig. 12.—Frequency of Flood Peaks

Flood peaks computed from the formulas are too high for Quaboag River at West Brimfield, Mass., because of unusual valley storage in the basin. The slope of the river surface from West Brimfield upstream to Warren is steep, but for about 10 miles above Warren the stream is very flat. Large areas in this part of the valley are flooded at high stages, and the lake thus formed reduces the peak discharges at West Brimfield materially. Short periods of intense precipitation sometimes cause two distinct peak discharges at West Brimfield. The first one is a sharp peak caused principally by the runoff from the area downstream from Warren which is about 10% of the total. The second is a smooth, flat peak caused principally by runoff from the area above Warren.

The formulas should be used with care on streams like the Ashuelot River in New Hampshire and its tributaries. The South Branch of Ashuelot River near Marlboro, N. H., has no large amount of natural or artificial storage in the basin, and the computed peaks are consistent with the observed peaks. Otter Brook, another tributary of the Ashuelot River, near Keene, N. H., the Ashuelot River at Gilsum, N. H., and the Ashuelot River at Hinsdale, N. H., have observed peak discharges which are low in comparison with the computed peak discharges. Discharge from a small part of the Otter Brook tributary area is almost completely controlled by Granite Lake. Nearly half of the remaining drainage area above the gaging station near Keene is upstream from Ellis Reservoir. Storage in this reservoir materially decreases peak discharges at the gaging station. Hence, computed peak discharges based on the drainage area exclusive of the area tributary to Granite Lake are too high, and computed peak discharges based on the drainage area, exclusive of the areas tributary to Granite Lake and Ellis Reservoir, are too low. A lake or reservoir like Ellis Reservoir probably would not greatly reduce peak discharges if it were similarly located in a drainage basin having flat slopes, but Ellis Reservoir reduces the peaks greatly, and delays the discharge from the steep slopes of the area tributary to it.

Similarly, Ashuelot and other ponds, Millen Lake, and unusual quantities of storage at some places along the valley reduce the peak discharges of Ashuelot River at Gilsum. Peak discharges of the Ashuelot River near its mouth at Hinsdale are much below those which might be expected by comparison with the computed peak discharges of minor, major, and rare floods. Storage on the tributaries causes some of this difference, but much more of it is probably caused by the unusual amount of valley storage along the river in the vicinity of Keene where the profile of the water surface is very flat and flooding of marshes and low river banks forms a lake at high stages through which about 80% of the total flow of the river must pass.

The computed peak discharges are too large if the formulas are applied to basins like these which have unusually large amounts of storage, particularly if the basins have steep slopes and hence large average amounts of fall to the outlets. Flood discharges downstream from lakes, reservoirs, and abnormally large quantities of valley storage ordinarily should be considered as individual problems. The inflow should be computed and routed through the storage to the point downstream where discharges are required, allowance being made for inflow to the reaches through which the outflow is routed.

Peak Discharges for Ungaged Areas

To use the formulas in computing the different classes of flood peaks for a particular point on a stream, the basin characteristics, M, a, L, and s, must be computed with whatever degree of accuracy is required. The drainage area M can be readily determined by planimeter. Water-surface areas necessary for computing a can be obtained by measuring with a planimeter large surfaces and estimating small surfaces by comparison with a set of squares or rectangles of known area. Length L can be estimated directly from a map for small areas. Large areas, however, must be divided into small basins, for which the distance of water travel to the point of discharge from the main area can be measured, and L computed by weighting according to the area of each subdivision. The median altitude can be computed by preparing a grid, on transparent paper, of squares representing areas of one quarter of a square mile or other convenient unit, laying this grid over the topographic map on which the basin is outlined, and estimating the average altitude of the area within each unit or fractional unit of area. The weighted average of these units will be the median altitude of the total area. The altitude of the outlet of the basin is subtracted from the median altitude to determine s.

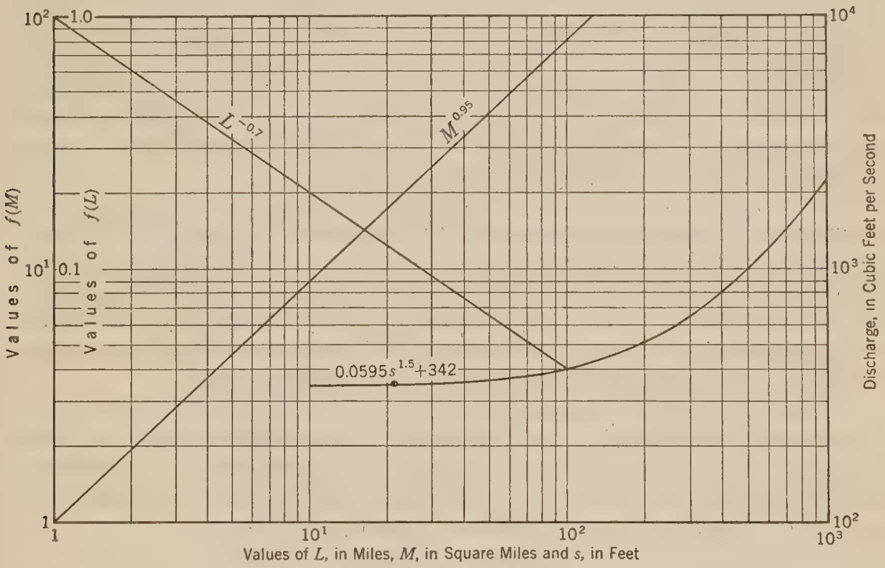

FIG. 13.—DIAGRAM FOR SOLVING EQ. 6c, THE FORMULA FOR RARE FLOODS

Fig. 13 is a diagram that can be used to determine peak discharges for rare floods. Similar diagrams can be prepared for other classes of floods, for convenience in computation.

CONCLUSIONS

The peak discharges of unit hydrographs increase as the peaks of the floods from which they were derived increase, and unit hydrographs developed from small floods should not be used without adjustment to compute the hydrographs of much larger floods.

Depths of runoff and the typical distribution of runoff can be combined with unit hydrographs to construct flood peaks that seem reasonably consistent with the frequency and magnitude of observed peaks. Depth of flood runoff does not vary appreciably with the size of drainage basin except for maximum floods, but it does increase with an increase in altitude, at least above 800 ft, and it decreases with an increase in the surface area of lakes and ponds.

Frequency curves of instantaneous peak discharges or of depths of runoff, even though they are based on 20 to 30 years of record, can be misleading, especially if the records contain one or two very large floods. The frequency curve of the Connecticut River based on 100 years of record was changed radically by the inclusion of the flood of 1936.[22] Studies of frequencies made from records prior to some of the disastrous floods of recent years would have indicated that such floods should never have occurred.

Flood peaks can be correlated sufficiently well with physical characteristics of drainage basins to determine the peaks for ungaged areas more accurately than they can be computed from general flood formulas which involve only the drainage area, or the two factors of drainage area and shape of basin. However, the formulas given in this paper are not applicable to all ranges of characteristics and they should not be used for drainage areas which differ widely in any important physical characteristic from the basins which were included in this study. The formulas should not be applied to a basin that contains a large amount of storage unless a careful study is made of the effects of such storage; nor should they be used for an area outside of Massachusetts without careful analysis of stream-flow data in that area.

Formulas of this type which contain the probable frequency as well as the magnitude of a flood provide a means of evaluating the chances that a hydraulic structure will fail. Any assumed or computed capacity of a hydraulic structure that will handle less than a maximum flood should be expected to involve some risk, however remote. No measure of probable frequency can forecast the particular time when an extreme flood will occur even though the average recurrence interval for this extreme flood were known accurately.

Stream-flow records are inadequate for computing major, rare, and maximum flood peaks directly, and extensions of these records cannot be made precisely. However, a reasonable basis has been developed for determining these peaks, and the computed peaks should be sufficiently accurate to be useful in the design of some hydraulic structures.

ACKNOWLEDGMENTS

The drainage basin characteristics, upon which much of the work in this study was based, were determined by the Work Projects Administration in the Boston office of the Geological Survey, by methods developed by the Division of Water Utilization of the Geological Survey, under the direction of R. W. Davenport, M. Am. Soc. C. E., chief.

The assistance of the Corps of Engineers, U. S. Army, in this study is gratefully acknowledged. This agency furnished many studies used in the investigation, including more than one hundred unit hydrographs, several frequency curves, and rainfall distribution and frequency data.

The material in this paper is drawn to a large extent from an unpublished report by the co-author, B. R. Colby, entitled "Flood Peaks on Massachusetts Streams." The study upon which this paper was based was conducted by Mr. Colby and others in the Boston office of the U. S. Geological Survey, under the direction of the co-author, H. B. Kinnison, district engineer. The writers are indebted to C. C. McDonald, Assoc. M. Am. Soc. C. E., for assistance in the preparation of this paper.

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[22] *Proceedings*, Boston Soc. of Civ. Engrs., January, 1942, Fig. 33, p. 106.

DISCUSSION

Clarence S. Jarvis,[23] M. Am. Soc. C. E.—If the many hours required for the writer to grope through the maze of interrelationships disclosed in this paper may be taken as an index of the authors' labor in exploring, analyzing, and then formulating the wealth of material in the paper, the profession has received the harvest of months if not years of earnest effort. Recognizing the practical value of the authors' masterly approach and the comparatively unique if not new relationships presented, the writer somewhat reluctantly undertakes to examine background or foundational features, to trace the derivations, and to analyze the adopted factors throughout the practicable range, thus introducing both alternate and divergent views regarding major as well as minor issues—all intended as constructive suggestions throughout.

Translation of the quantities listed in Cols. 3 and 9 to 12 in Table 1 into terms of percentage ratings according to the Modified Myers formula[4] (Eq. 1) readily discloses the following relationships:

1. Quantities listed in Table 2, proposed as a supplement to Table 1, form geometric series with the following characteristics—

(*a*) Percentage ratings for major peaks (Col. 10) are usually from 1.5 to 2.5 times those for minor peaks (Col. 9), averaging nearly 2.0.

(*b*) Corresponding ratios between percentages listed for rare peaks (Col. 11) and for major peaks (Col. 10) are usually from 1.5 to 2.0, averaging nearly 1.7.

(*c*) Between maximum peaks (Col. 12) and rare peaks (Col. 11), the ratios are usually from 3.2 to 4.2, averaging nearly 3.6.

(*d*) A notable degree of consistency pervades most of the series, manifested in a tendency to adhere either to average values or to above-average or below-average values throughout.

(*e*) Random selection of items for checking computations, mainly by log-log slide rule, resulted in satisfactory checks, within 3% or less, differences being partly attributable to the mechanics of the procedures and to cumulative errors.

(*f*) Considering all the factors concerned with flood frequencies and magnitudes, it seems advisable to re-examine all items in Table 1 with ratings according to the Modified Myers formula for "maximum" peaks (Col. 12) greater than 90%. Very few, if any, of these stations seem to deserve such high ratings according to the writer's interpretation of all available data pertinent to the locality involved.

2. Factors involved in Eqs. 6 have been platted in Fig. 14 on semilogarithmic paper throughout the practicable range for most rivers. To depict all the trends on a single drawing, it was necessary to convert the three fac-

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[23] Senior Civ. Engr., Intelligence Div., Office, Chf. of Engrs., Washington, D. C.

TABLE 2.—PERCENTAGE RATINGS FOR USE IN THE MYERS FORMULA, EQ. 1, TO SUPPLEMENT TABLE 1, COLS. 3, 9, 10, 11, AND 12, RESPECTIVELY

(Maximum Observed Flood Ratings Are Underlined)

No.[a]	Years[b]	M (sq miles)	COL. 9		COL. 10		COL. 11		COL. 12	
			Known peaks[c]	Minor peaks[d]	Known peaks[c]	Major peaks[d]	Known peaks[c]	Rare peaks[d]	Known peaks[c]	Maximum peaks[d]
1	4	107	....	4.2	....	10.6	15.7	18.0	....	63.1
2	3	405	....	1.8	1.9	4.4	....	7.0	....	24.3
3	10	124	....	1.7	2.3	3.3	....	5.5	....	21.5
4	1	23.3	....	1.9	....	3.5	....	6.2	....	26.1
5	2	183	....	1.6	2.3	3.5	....	5.5	....	21.1
6	1	35.2	....	1.8	....	3.8	....	6.7	....	27.0
7	9	211	....	2.1	....	4.0	....	6.9	....	26.6
8	13	42.2	....	1.6	....	3.3	....	5.4	....	22.2
9	15	31.3	....	2.1	4.5	5.1	....	8.9	....	34.8
10	1	139	....	2.4	....	6.4	....	10.8	....	37.6
11	11	417	....	3.1	7.4	8.1	....	14.2	....	47.6
12	1	93.8	....	2.1	....	5.6	....	9.3	....	34.0
13	10	224	....	8.0	9.4	17.0	....	28.3	....	95.5
14	14	308	....	16.6	....	26.3	29.8	43.3	....	147.0
15	16	71.2	....	4.9	6.2	11.8	....	19.6	....	70.0
16	29	420	8.8	9.7	....	19.5	....	31.7	....	104.0
17	15	37.6	....	5.1	10.0	11.7	....	20.1	....	73.3
18	15	24.2	....	4.0	....	8.5	12.8	15.5	....	60.0
19	18	36.4	....	4.2	9.9	10.0	....	17.7	....	65.3
20	22	83.8	....	2.4	....	5.8	9.3	11.7	....	37.8
21	1	186	....	2.7	....	6.6	....	11.6	....	40.9
22	24	370	....	3.7	....	9.9	15.1	16.6	....	55.0
23	24	19.0	....	1.8	....	3.9	6.0	6.9	....	28.9
24	21	18.7	....	1.6	....	3.8	....	6.2	6.9	25.0
25	22	49.9	....	3.0	7.3	7.6	....	13.1	....	48.2
26	23	12.2	....	2.0	4.4	5.2	....	8.7	....	34.6
27	26	362	....	21.1	29.6	31.6	....	51.0	....	171.0
28	26	178	....	17.5	....	26.7	42.2	46.0	....	159.0
29	1	88.4	....	10.5	....	18.6	....	32.2	....	114.0
30	0	89.4	....	7.1	....	14.6	....	23.8	....	84.2
31	1	53.3	....	5.4	....	12.1	....	20.4	....	73.5
32	11	100	....	2.6	....	6.5	....	11.3	14.0	40.8
33	28	201	....	3.1	....	8.6	....	13.7	16.0	46.3
34	10	702	....	5.6	....	13.7	17.0	22.8	....	72.8
35	28	188	....	2.9	5.5	7.6	....	12.6	....	43.4
36	28	149	....	2.8	6.9	7.7	....	12.6	....	43.7
37	29	162	....	10.0	....	18.3	29.8	30.5	....	104.4
38	24	497	....	13.8	....	25.0	....	42.0	....	136.8
39	30	52.6	....	9.1	....	16.0	....	25.9	27.4	93.5
40	4	93.7	....	10.2	....	19.2	22.6	32.2	....	115.8
41	10	98.4	....	2.8	....	6.8	7.4	11.3	....	41.5
42	25	75.3	....	6.6	....	14.3	21.3	25.2	....	113.8
43	7	4.1	....	1.9	3.2	3.5	....	6.1	....	27.0
44	12	74.5	....	2.4	6.0	6.4	....	10.6	....	38.7
45	4	57.1	....	5.1	8.5	11.9	....	20.7	....	74.3
46	25	280	....	5.0	6.9	12.4	....	19.8	....	65.1
47	8	31.4	9.1	9.7	....	16.8	....	30.3	....	113.0
48	9	39.1	....	11.5	16.0	19.0	....	34.0	....	125.4

[a] See Fig. 10.

[b] Years of record used in the study of flood peaks (see Col. 2, Table 1).

[c] Maximum known peak discharges correspond to values in Col. 3, Table 1.

[d] Peak values computed from flood formulas (see Cols. 9 to 12, Table 1); percentage $= \frac{Q}{100\sqrt{M}}$, in which Q = peak discharge in cubic feet per second; M = drainage area in square miles (solved from a single setting of a slide rule per item).

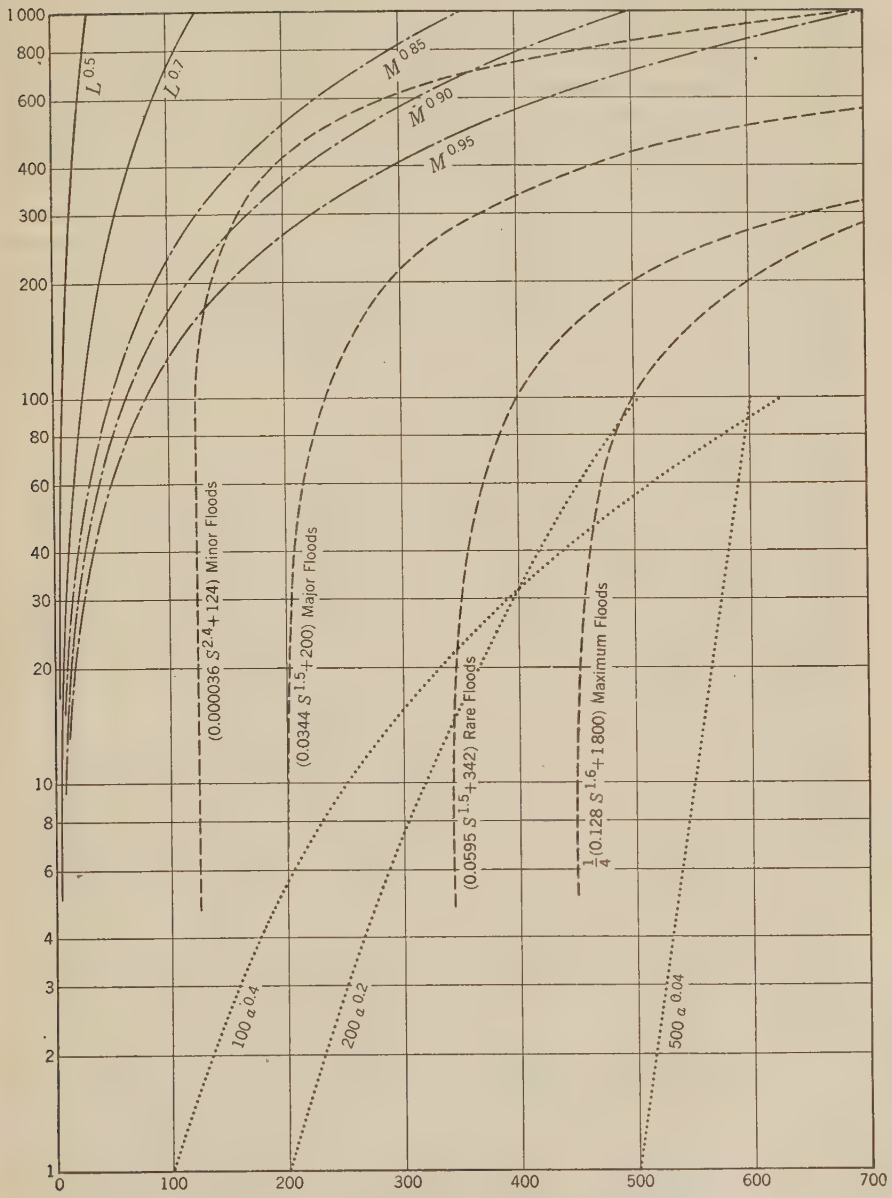

FIG. 14.—FLOOD FORMULA FACTORS

tors a, dealing with pondage to percentage values, by multiplying by 100. Furthermore, it was required to use another factor 5 for the authors' expression, to take best advantage of the chart space. The other two factors were suggested by the writer as more nearly expressive of the probable influence of pondage. It was also necessary to divide the quantities of the factor involving s in Eq. 6d by 4, to keep the right-hand curve within the limits of the drawing. The outstanding characteristics of curves relating to length of runoff travel, drainage area, and altitude above the outlet are increased curvature and divergence above 100.

3. The general form of Eqs. 6 with a fractional power of drainage area in the numerator and with a lesser power of runoff travel distance in the denominator seems to recall the derivation of the width[24] formula derived by C. R. Pettis, M. Am. Soc. C. E., directly from the Modified Myers formula,[4] Eq. 1. Through the introduction of a "compactness factor" Colonel Pettis obtained the expression,

$$Q = 328\,P\,W^{1.25} \qquad (7)$$

in which P represents the rainfall depth in inches for the critical or adopted period and W is the average width of the drainage basin in miles, or $\frac{M}{L}$.[25] (A review and summary of Colonel Pettis' paper is readily available.)

4. Within the ordinary range of observed drainage areas, where length is some multiple of the width, it is possible to convert the authors' fractional power of M divided by a lesser power of L to a power of W quite closely approaching that used by Colonel Pettis, with some advantage in the way of simplification, but with appropriate changes in coefficients.

5. With 681 station-years of record included for forty-eight items of Table 1, giving an average of 14.2 years per station and a maximum of 30 years per station, and with only four of the observed maxima exceeding the quantities listed in Col. 11 as "rare" (the amount of such excess being 25% or less), it is difficult to reconcile the idea of adding another 300% or more to attain the "maximum flood" as determined by the authors. Addition of 150% might well suffice.

6. Critically examining the various factors of Eqs. 6, graphically outlined in Fig. 14 (with such alterations as seemed essential to keep them within limits of the drawing), the following comments are submitted—

(*a*) The values of $L^{0.7}$ vary between 3.09 and 15.5 for 5 miles and 50 miles, respectively, or 77.5 for 500 miles. Thus, as the L-factor appears in the denominator of each formula, its influence would be such as to decrease flood peaks to 1/25 or 4% for such a range of variation. For the alternate Eq. 6b, with $L^{0.5}$ instead, the total range between 5 and 500 would be 10 instead of 25—a more readily defensible factor according to extensive records and observations.

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[24] "A New Theory of River Flood Flow," by C. R. Pettis, 1927 (privately published).

[25] "Floods in the United States; Magnitude and Frequency," *Water-Supply Paper No. 771*, U. S. Geological Survey, 1936, pp. 36–37.

(*b*) The s-factor for minor floods, with added constant, amounts to 126.27 for 100 ft of altitude, or 137 for 200 ft, 237 for 500 ft, or 694 for 1,000 ft of altitude above the outlet. Thus, because of the influence of altitude in Eq. 6*a*, the flood peak is multiplied by a factor 5, in consequence of increased median altitude from 200 to 1,000 ft. All three of the other s-factors and associated constants run off the drawing before attaining an altitude of 600 ft. Their trends magnify the influence of altitude of drainage area above the outlet in a greater degree at 1,000 ft than has been noted for minor floods.

(*c*) The pondage factor, a, as used by the authors shows but slight variation between 1% and 5%, attaining a value of only 1.067 or 1.169 at 50%. In other words, the regulation afforded by ponds covering fully half of the drainage area would decrease the peak to 1/1.169 or 85.6% of its magnitude with no pondage. If the fractional power were 0.4 instead of 0.04, the corresponding values of the a-factor would become 1.90, 3.90, and 1/3.90 or 25.7%. This is more nearly in accord with the writer's experience, although 0.2 might serve somewhat better, with values of 1.38, 2.19, and 1/2.19 or 45.8%.

The proper scope of the authors' equations and formulas in this paper lies mainly if not entirely below 1,000 for both M and s—drainage area and median altitude above the outlet, respectively. Similarly, the width formula derived by Colonel Pettis was intended to serve primarily for drainage areas ranging from 1,000 to 10,000 sq miles; it occasionally served fairly well outside those limits, but more often wandered beyond control.

For many years the writer has striven for simplification of expression within the hydrologic field. One of the prime essentials seemed to be a formula capable of comparing the floods of either the Mississippi River or the Amazon River with the peaks of torrential floods at their headwaters, regardless of areal extent. Investigations covering several years led to a measure of approval and acceptance by the hydrologic and engineering profession, as demonstrated by a wide use of the square root of drainage area as a convenient first approximation to the influence of areal extent for the entire range above 4 to 7 sq miles. Essentially the same conclusion was reached as the result of a later investigation conducted by a committee of the Boston Society of Civil Engineers. This report laid some of the foundations for later development in the field of unit hydrographs, as has been tacitly acknowledged by the originators of that procedure.[26] For the usual form of drainage area, with length considerably greater than average width, introduction of the expression $\sqrt{M}$ may be regarded as equivalent to adopting the width as the governing dimension with either a coefficient or a power greater than unity; or it might be construed as being the same as using the dimension of length in the denominator, with either a coefficient or a power somewhat less than 1. These developments have been presented in considerable detail in some of the references in the paper, as well as in other publications deserving mention.[27,28,29,30,31,32,33]

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[26] Report of Committee on Floods, *Journal*, Boston Soc. of Civ. Engrs., September, 1930, pp. 297 and 376–407.

In spite of the time-consuming mazes of complex computations involved in the authors' procedure and formulas, not to mention the occasional exceptions or divergences of opinion freely expressed by the writer as constructive suggestions, the entire investigation, together with its lucid presentation, is both profitable and enlightening to the profession. The concepts of basin characteristics for not only flood phenomena but also rainfall and runoff relations generally are not new. The late Emil Kuichling, M. Am. Soc. C. E., the late A. N. Talbot, Past-President and Hon. M. Am. Soc. C. E., Major Myers, and many other authors of formulas made special provision for accommodating the various factors throughout their probable range. The writer's modification of the Myers formula included extension of the range both above and below that originally suggested by Major Myers. The use of the Modified Myers formula is so simple and elementary as to deter its adoption by those who presumably seek the more abstruse and complicated forms. No matter how involved any preferred procedures, the results are readily translated into terms of Myers percentage ratings, as demonstrated by Table 2.

L. R. BEARD,[34] JUN. AM. SOC. C. E.—A phase of hydrology that deserves close attention from those interested in hydrologic design is treated in a practical manner in this paper. The Los Angeles (Calif.) office of the U. S. Engineer Department adapts the principles of the summation hydrograph (S-graph) as presented by Mr. Langbein[11] and Franklin F. Snyder,[35] Assoc. M. Am. Soc. C. E., for relating flood characteristics to drainage area characteristics. This relation is accomplished through the medium of "lag," which is a function similar to "time of concentration." Lag is related to the length and slope of the longest stream and the length of the stream up to the center of the drainage area. All other variations are considered negligible within a given runoff region.

The function of length used by the authors appears to be a much better measure of lag or other time functions than the length of the longest stream, which often is not a representative measure for the entire drainage system. It is interesting to note that the peak discharge of Eqs. 6*a*, 6*c*, and 6*d* is inversely proportional to the length raised to the 0.7 power. This result is in substantial agreement with Mr. Snyder's equation which relates the discharge inversely to the length raised to the 0.6 power, and is in slightly better agreement with studies made in the Los Angeles Engineer Office which relate the discharge to the length raised to the 0.76 power.

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[27] "Methods of Estimating Flood Discharge," by Robert L. Lowry, *Bulletin*, Colorado Soc. of Engrs., June, 1936.

[28] "Handbook of Applied Hydraulics," Calvin V. Davis, Editor, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1942, Sections 1 to 3, pp. 1–132.

[29] "Maximum Stream-Flow with Reference to Flood-Formulas," by C. S. Jarvis, *Transactions*, Am. Geophysical Union, 1937, pp. 409–419.

[30] "Great Floods in the United States," by C. S. Jarvis, *ibid.*, 1939, pp. 157–166.

[31] "Estimating Annual Yield from Ungaged Drainage-Basins and Effect of Land-Use Treatment on Surface Runoff," by C. S. Jarvis, *ibid.*, 1941, pp. 875–893.

[32] "Derivation of River Discharge from Measurements of Tributaries," by C. S. Jarvis, *ibid.*, 1942, pp. 517–533.

[33] "Gleanings from the Field of Hydrology," by C. S. Jarvis, *ibid.*, 1943, pp. 651–693.

[34] Associate Engr., U. S. Engrs., War Dept., Los Angeles, Calif.

[35] "Synthetic Unit-Graphs," by Franklin F. Snyder, *Transactions*, Am. Geophysical Union, Pt. I, 1938, pp. 447–454.

The function of slope used by the authors also appears well chosen and should prove to represent the slope factor better than more simply computed functions. Although the slope factor is omitted in the ordinary application of the summation hydrograph, it is almost mandatory that a slope function be introduced into formulas that relate runoff and drainage area characteristics if such formulas are to have wide application. Heretofore, application of the S-graph has been restricted within relatively small hydrologic regions, within each of which the slope function probably does not vary sufficiently to affect results and has therefore been omitted. The lag equation derived by Mr. Snyder for Appalachian areas shows the lag to be several times as long as lags developed for Pacific slope areas where stream slopes are much greater. The slope factor should account in a large part for these discrepancies.

Variation of the shape of the unit hydrograph with flood magnitude is usually overlooked or avoided although its existence is reasonable to assume. The curves of Fig. 3 showing the relation between the peak of the unit hydrograph and peak runoff are in substantial agreement with the findings of R. L. Gregory and C. E. Arnold,[36] Assoc. Members, Am. Soc. C. E., who expressed the time of concentration as inversely proportional to the discharge raised to the one-fourth power. As stated by the authors in substantiation of the data shown in Fig. 5(*a*), the peak of the unit hydrograph is inversely proportional to the time of concentration or to lag. (E. P. Bennett, who has reviewed this discussion, has stated that this relation does not hold strictly unless the unit-rain interval is proportional to the lag. However, this relation is approximate for cases in which the unit-rain interval is short compared to the lag.) Thus, the peak of the unit hydrograph, in accordance with the equations of Messrs. Gregory and Arnold, should be directly proportional to the discharge raised to the one-fourth power, as approximated by the curves in Figs. 3(*a*), 3(*b*), 3(*c*), and 4. (Fig. 5(*b*) does not show this relation because factors other than flood discharge have an effect.) The incorporation of a function of discharge in lag equations may improve the relationships considerably, but, before application of the results shown in Figs. 3, 4, and 5, it should be ascertained whether subtraction of subsurface flows from observed hydrographs has involved sufficient error to affect the peaks of the unit graphs.

Those who work with rainfall and runoff frequency determinations for the Pacific slopes area are aware of the great variations of over-all runoff coefficients that might prevail. The curves of Figs. 1 and 2 are presented for illustration of the great variation of runoff magnitude compared to the variation of rainfall magnitude in some regions (which indicates that accuracy in the determination of rainfall magnitude does not necessarily result in accuracy in the derivation of runoff magnitude). Fig. 15 shows a duration curve of peak runoff for Big Bear Lake Dam, and Fig. 16 shows a duration curve of daily rainfall for the Santa Ana River near Mentone, Calif., which is in the tributary area above Mentone. Inasmuch as these curves are representative of hundreds of rainfall and runoff curves for the region, they serve to illustrate that the largest of 50 annual maximum occurrences of rainfall for any duration is usually about 110 times the smallest, whereas the largest of 50 annual maximum runoff occurrences is usually about 1,000 times the smallest. It is questionable whether such curves could be correlated by the analysis presented in this paper with as much accuracy as the accuracy of the duration curve of runoff.

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[36] "Run-Off—Rational Run-Off Formulas," by R. L. Gregory and C. E. Arnold, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1058.

Regarding the great influence of rainfall-runoff relations and their erratic nature, an example will be cited. The most disastrous flood that ever struck Southern California was that of March, 1938. During that flood, hundreds of rainfall and runoff records were broken. In January, 1943, a storm occurred whose 24-hr rainfall set a record for the entire nation and which was from 30% to 50% greater than the 24-hr rainfall of the 1938 storm. However, mainly because of the difference in ground conditions, the greater storm of 1943 resulted in a comparatively small flood, which is not generally classed as a major flood.

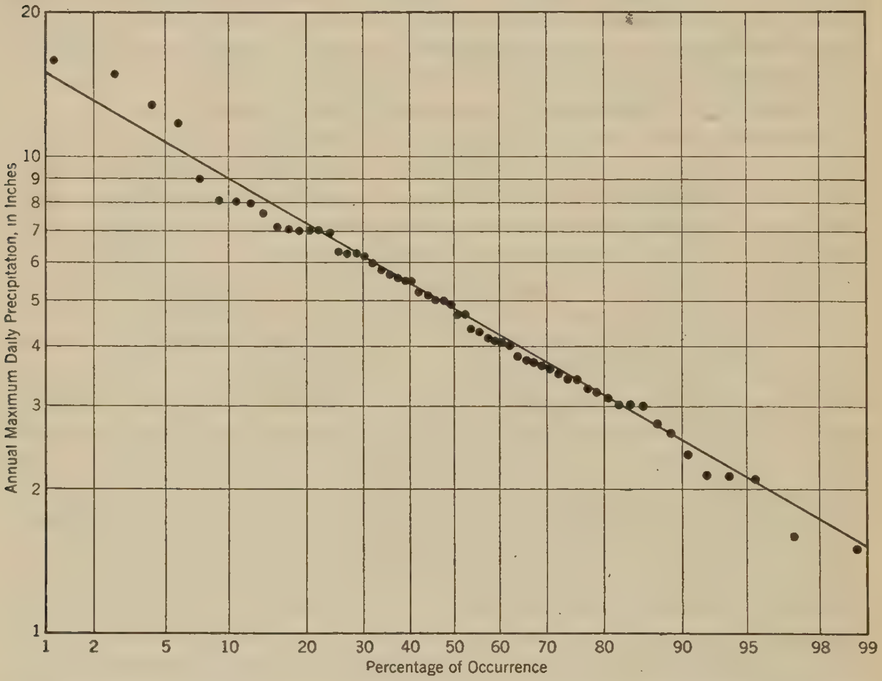

Fig. 15.—Duration Curve of Daily Precipitation, Big Bear Lake Dam; 1883–1944 (60 Years)

Of course, the over-all runoff coefficients in the Massachusetts area do not vary as much as those in Southern California, but it is possible that they do vary more than has become evident in the analysis and that curves C and D, Fig. 7, actually should agree more closely with curve A. It is not evident from the presentation just how reliable the precipitation curve of Fig. 6 may be, but it should be expected that errors of that curve and errors in rainfall-runoff relationships combine to produce the errors in curve D, Fig. 7. Curve C or D is not obviously more accurate than curve A. In fact, the occurrence, within a short period, of a flood in excess of the 1,000-yr value is so highly improbable as to cast serious doubt on the accuracy of curves C and D. The inaccuracy of curve B on this basis cannot be reasonably doubted. It would be helpful if the authors would include with their closing discussion the data that show the probability of a given depth of runoff being generated by a given depth of precipitation. Also, the criteria used in selecting 6-hr and 2-hr runoff amounts from the 72-hr volume have not been indicated.

The writer's only criticism of the analysis and results would be that the scope of the analysis might be too limited even for application within Massachusetts. Some manner of combining runoff records for the entire Atlantic coast of the United States and perhaps for some distance inland should yield results that not only would be more generally applicable but also would be more accurate for each small region as a result of the combination of many independent records. In the opinion of the writer, the advantage of resorting to rainfall-runoff analysis lies not in the superiority of rainfall data to runoff data but in that the results of such an analysis permit the combination and

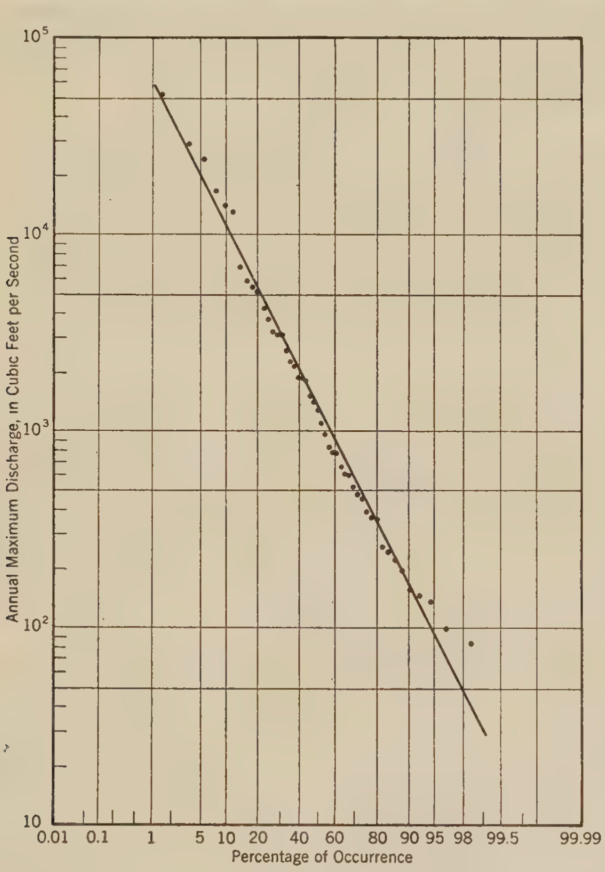

FIG. 16.—DURATION CURVE OF PEAK DISCHARGE, SANTA ANA RIVER NEAR MENTONE 1896–1943 (47 YEARS); 189 SQ MILES

correlation of runoff records. Extrapolation of individual runoff frequency curves by a study of rainfall is generally unsound because of the inadequate knowledge of the frequency variation of runoff factors and the effect of such variation on the runoff-frequency curve.

The analysis presented is straightforward and simple. It includes wisely chosen general expressions such as average elevation of an area (which includes the factors of soils, vegetal cover, temperature, and rainfall distribution) and size of area (which includes short-time variations in precipitation and diminution of average rainfall with size of area). The degree of "breakdown" into the various component factors can be too high, and often much simplified functions such as those used in this paper are adequate for application within a limited region.

M. J. ORD,[37] JUN. AM. SOC. C. E.—Flood characteristics of Massachusetts streams have been analyzed thoroughly in this paper. However, the writer questions the wisdom of presenting flood formulas based on what the authors admit to be inadequate basic data. Although limitations have been given to their formulas, once presented, such formulas are frequently used without considering their limitations. It might have been preferable to present the relationships in the form of curves together with plotted values to define the curves.

The minor, major, rare, and maximum floods given in Table 1 can be expressed as percentages of the major flood (see Table 3). In view of the fact that reasonably consistent and uniform rainfall and runoff factors were used in the development of the floods, the percentages for each magnitude flood developed by unit hydrographs should be expected to be reasonably uniform. The percentages for the minor and rare floods do show only a small variation—for the most part within 10% of the average percentage. The percentages for the maximum flood have a greater variation. The floods represented by the extreme variations should be questioned.

The average percentages for the minor, major, and rare floods (based on unit hydrographs) were plotted versus their probable recurrence intervals of 15, 100, and 1,000 years. The resulting curve is shown in Fig. 17. It is of interest to note that the percentage for the maximum flood has a recurrence interval of 1,000,000 years determined by a conservative extrapolation of the curve.

In Table 1 there are fourteen stations for which the maximum recorded flood exceeds the computed 100-yr flood. Such a condition is not reasonable considering the short period of record, without further evidence. More weight should have been given the 1936 and 1938 floods in determining the frequencies. Thus, the slope of the frequency curve should be increased. However, accepting the authors' probabilities of occurrences, Fig. 17 will give results as acceptable as those determined by the formulas presented by the authors, and it has the advantage of permitting the determination of a flood of any frequency once the 100-yr flood has been determined. The 100-yr flood can be determined by the authors' formula or, in the writer's opinion, preferably by unit hydrograph, assuming that the distribution of unit runoff amounts has been correctly defined.

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[37] Engr., U. S. Engr. Dist., Los Angeles, Calif.

For areas having unknown rainfall-runoff relationships, synthetic unit hydrographs can be determined by any one of several methods. The writer prefers a method, developed by the Los Angeles Office of the U. S. Engineer Department, which is similar to that proposed by Mr. Snyder[35] and modified by Mr. Langbein.[11]

TABLE 3.—FLOOD PEAKS FROM TABLE 1 EXPRESSED IN PERCENTAGES OF FLOOD PEAKS FOR MAJOR FLOODS

No.[a]	FROM FLOOD FORMULAS				FROM UNIT HYDROGRAPHS			
	Minor (9)	Major (10)	Rare (11)	Maximum (12)	Minor (13)	Major (14)	Rare (15)	Maximum (16)
1	39	100	169	593	47	100	170	602
2	40	100	158	548	47	100	172	632
3	52	100	164	645	50	100	164	670
4	54	100	178	750	....	....	....	....
5	46	100	158	600	48	100	165	643
6	48	100	174	701	....	....	....	....
7	53	100	172	664	48	100	169	556
8	48	100	166	676	46	100	167	686
9	41	100	175	681	42	100	174	1,012
10	37	100	169	590	....	....	....	....
11	38	100	175	585	....	....	....	....
12	38	100	166	608	....	....	....	....
13	47	100	166	562	....	....	....	....
14	63	100	165	558	....	....	....	....
15	41	100	165	592	....	....	....	....
16	50	100	162	532	....	....	....	....
17	43	100	172	628	....	....	....	....
18	43	100	174	654	....	....	....	....
19	41	100	177	650	....	....	....	....
20	41	100	177	655	49	100	159	596
21	41	100	177	620	....	....	....	....
22	38	100	168	557	49	100	156	490
23	46	100	175	733	48	100	169	860
24	43	100	161	650	49	100	153	652
25	40	100	173	635	48	100	164	618
26	39	100	169	671	53	100	162	778
27	66	100	161	542	....	....	....	....
28	66	100	172	594	....	....	....	....
29	56	100	173	612	....	....	....	....
30	48	100	163	576	....	....	....	....
31	45	100	168	606	....	....	....	....
32	40	100	175	633	48	100	159	571
33	36	100	160	538	48	100	195	697
34	41	100	166	531	40	100	181	668
35	38	100	166	572	44	100	167	566
36	36	100	164	567	47	100	195	912
37	54	100	167	572	53	100	152	407
38	55	100	168	548	82	100	159	418
39	57	100	162	584	53	100	174	680
40	53	100	168	601	53	100	157	528
41	41	100	165	592	....	....	....	....
42	46	100	177	628	52	100	173	645
43	54	100	176	781	....	....	....	....
44	38	100	166	606	....	....	....	....
45	43	100	175	626	54	100	154	498
46	40	100	160	527	54	100	160	444
47	58	100	181	673	56	100	177	934
48	58	100	179	658	52	100	164	767
Average	46	100	169	615	50	100	167	649

[a] See Fig. 10.

In this method "lag" is plotted against a factor $\frac{L \times L_{ca}}{S^{0.5}}$, in which L = length of the longest watercourse, in miles; L_{ca} = length along a watercourse, measured upstream, to a point opposite the center of area, in miles; and S = over-all slope of drainage area, in feet per mile. Lag is defined as the elapsed time (in hours) from the beginning of unit rainfall to the instant that the corresponding summation hydrograph reaches 50% of ultimate discharge. The summation hydrograph for a given stream at a given point is the hydrograph that would result from the application of the unit hydrograph for that location to a continuous uniform supply of effective rainfall over the tributary area. The **S**-graph, first introduced by Mr. Langbein,[11] is the summation hydrograph with the coordinates expressed in dimensionless form, with the discharge scale as a percentage of ultimate discharge, and with the time scale as a percentage of lag. With the lag curve and **S**-graph determined for a region, synthetic unit hydrographs can be developed easily for areas of unknown runoff characteristics from the factors L, L_{ca}, and S.

In the development of the lag relationship for mountain streams in Southern California, numerous physical characteristics were investigated, including values corresponding to M and L in the paper. The factor $\frac{L \times L_{ca}}{S^{0.5}}$ gave as satisfactory a correlation with lag as any combination of drainage basin characteristics and, in addition, had the advantage of simplicity of determination. For the mountain region of Southern California the improvement in the relationship resulting from introducing the slope factor S is small. The necessity for introducing the slope in the lag relationship is much more apparent in correlating the lag for areas of steep slope with those of gentle slope. A lag relationship applicable to all regions would probably be very complex. The factor of roughness n may be an important item, especially where there is a large variation in the hydraulic efficiency of the channels of different basins.

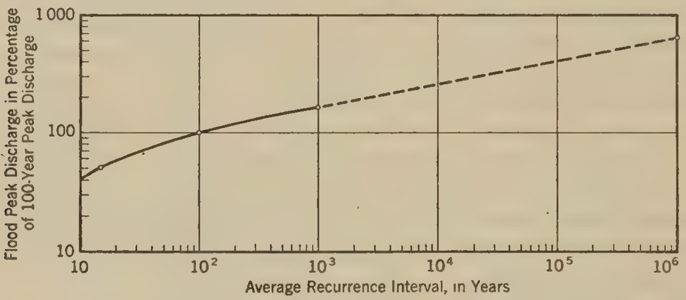

FIG. 17.—FREQUENCY CURVE OF FLOOD PEAKS, STREAMS IN MASSACHUSETTS

HOWARD M. TURNER,[38] M. AM. SOC. C. E.—The title of this interesting paper does not indicate the complete subject covered, because it omits the treatment of flood frequency which is a very interesting part. The authors have succeeded in deriving a set of formulas which take into account more characteristics of the drainage area than any others for river floods, and which show very good correlation with similar floods computed by unit hydrographs. These formulas should prove of great value for general use in regions similar to Massachusetts. Many problems arise, particularly on small drainage areas where there are not sufficient records for drawing unit hydrographs or where the problem does not justify a complete analysis. In such cases, formulas like these which take into account the factors that affect the shape of the unit hydrograph will be of great use.

The authors give the qualifications as to the use of these formulas in certain specific cases, emphasizing the fact that when there is a large storage the flood should be computed and then routed through the storage. It would be interesting to know whether they have tried to apply these formulas to any other rivers than those given in the paper and whether they consider the formulas limited to streams of less than 500 sq miles. It would seem probable that they may be generally applied to New England and similar regions, as well as Massachusetts.

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[38] Cons. Engr., Boston, Mass.; Lecturer Water Power Eng., Harvard Univ., Cambridge, Mass.

These formulas cover four flood magnitudes described as minor, major, rare, and maximum and further defined as of 15-yr, 100-yr, 1,000-yr frequency and a maximum possible flood. Methods of computing flood frequency are open to great question due to the difficulty of extending the frequencies considered far beyond the normal records of the past. It is true that referring to floods as minor or major, rare or maximum, brings in a frequency element, but this is entirely different from assigning a definite period of years to a given sized flood. The authors recognize the unsatisfactory results of the usual methods of computation and the wide variation in results obtainable. They introduce a method of estimating frequency on a somewhat different basis from the usual figures, which are based on past records of peak flow extended by plotting or by some mathematical process.

The major differences in the authors' methods are the use of precipitation records and flood runoff figures instead of stream-flow records of peak floods to obtain frequency curves. This is, in effect, an adaptation of the rational method used for small areas. The writer believes that this method is a definite advance in the studies of flood frequency and should give more consistent results. There is, however, the same difficulty in the extension of records to periods many times in excess of their actual length and the handling of the records of major storms occurring in a short period.

There are also certain objections to the authors' method. The use of flood runoff rather than peak discharge has certain disadvantages—chiefly the question of what is to be included in the runoff for different floods. An example of this is the 1936 flood in New England where there were two peaks due to two storms, in part superimposed and combined with a large amount of melting snow. In theory, the runoff should be a better value by which to express the magnitude of floods as it would make floods on rivers with different characteristics much more comparable. By means of the unit hydrograph, flood runoff can be converted satisfactorily to peak flow, but this presupposes a normal flood due to a single storm without large base-flow complication often affected by runoff from melting snow.

It seems to the writer, also, that the use of precipitation records for frequency computation of flood flows is open to question. It is true that precipitation records are longer and thus serve better to establish frequency relations; but the other factors bring in another unknown which may more than offset the advantage of the long record—that is, the further question of the condition of the ground during any given storm. For example, there is the case of the 1936 flood cited, where the ground was frozen and the runoff from the rainfall was combined with a large runoff from melting snow. There was the 1927 flood in which two weeks of previous rainfall had filled up the level of the water in the ground; and on the other hand there is the large storm of September, 1932, in New England, which produced no serious floods, as it came after a long period without rain. The necessity of allowing the additional factor to cover the question of how much of a given storm runs off introduces an uncertainty which, it seems to the writer, more than makes up for the advantage of using the longer storm records instead of peak flood flow records.

In their use of the unit hydrograph to convert from the flood runoff to peak flow, the authors point out an interesting relation, indicating that the distribution graph should show a larger peak for large floods than for small floods. It would be advisable to investigate this further. The authors have found it so from the rivers they studied but it should be noted that these are all small rivers—with one exception less than 500 sq miles. It would be interesting to know whether they have made any study to see whether the same relation holds for larger rivers, and also whether this tendency is not greater on the smaller rivers and tends to increase as the size of the drainage area increases. The distribution graphs given in U. S. Geological Survey *Water-Supply Paper No. 772* [39] do not show this; but these are mostly for larger areas.

The authors' frequency data lead them to classify floods with definite figures of 15, 100, and 1,000 years for the minor, major, and rare floods. This frequency of the 15-yr flood can be determined definitely, and that for the 100-yr flood, fairly satisfactorily in many cases. There is no difficulty in the use of some maximum figures, provided the maximum is defined as in this paper. There is clearly an important flood value between the "major" floods of 100-yr frequency and the maximum flood. The authors have used a "1,000-yr flood" computed by their method.

It would be better to leave out the definite figure of 1,000 years and merely call the flood "rare." Frequency computations may help in obtaining comparable floods on the different rivers for these formulas as the authors have done, but other methods for obtaining comparable floods on different streams could be used. One would be on the basis of flood runoff—a rare flood could be expressed as one which produces the equivalent of a definite flood runoff of, say, 8 in. on 500 sq miles, with corresponding adjustment for larger or smaller areas; or the rare flood could be expressed in a percentage increase over the 100-yr flood, without using the definite 1,000-yr figure.

The difficulty with the 1,000-yr frequency, expressed as such, is that there are so many ways of computing frequencies. There is the authors', as given in this paper, besides numerous others, all of which give widely different results, depending not only on the method of estimating but also on the length or the period of the record, which is much more difficult. Thus, when the term "1,000-yr flood" is used, it really should be combined with the statement of how it is computed. For example, one should say, to be definite, the 1,000-yr flood according to Fuller, based on a 25-yr period, 1918–1943; or the 1,000-yr flood according to Hazen, based on a 100-yr record, 1843–1943.

Something of the difficulties involved in finding these frequency relations is shown in the paper itself. The curves in Fig. 12 show frequencies for the 1938 flood as follows: Millers River near Winchendon, 680 years; Middle

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[39] "Studies of Relations of Rainfall and Run-Off in the United States," by W. G. Hoyt, *Water-Supply Paper No. 772*, U. S. Geological Survey, 1936.

Branch of Westfield River at Goss Heights, 1,000 years; and Housatonic River near Great Barrington, 25 years. This appears to be too wide a difference, even when the smaller rainfall on the latter area is considered.

Any one who has been through the work necessary to analyze the floods on any stream will realize the work that has gone into this paper. The authors are to be congratulated for their success in expressing the effect of the various characteristics of the drainage area on the flood peak in a formula of usable form. This should prove a valuable check on the other methods particularly where more or less special characteristics are involved.

BYRON O. MCCOY,[40] ASSOC. M. AM. SOC. C. E.—The formulas presented by the authors will be of much assistance to engineers in and around Massachusetts. A tremendous amount of time and effort has been expended on the method presented in this paper, and the engineering profession is fortunate in having this additional tool for its use. It would be desirable to make similar investigations in other states. Further correlation with other watershed characteristics may be disclosed in regions with soil, vegetal cover, temperature, and other conditions differing from those found in Massachusetts. As with all such empirical equations, it is well to observe the authors' caution that the formulas should not be applied where conditions are materially different from those on which the formulas are based.

The exponents applied to the various physical characteristics used in the formulas vary somewhat with frequency. To be entirely consistent, for a given watershed, the variation of magnitude with frequency should be a function of probabilities. It does not seem reasonable that the effect of drainage area or distance the water must travel should be less on the major flood than on the minor or rare floods.

The data on maximum possible floods are of much interest. As defined by the authors, the maximum flood is one (see heading, "Maximum Flood Runoff") "* * * that could not be exceeded under any possible * * * combination of circumstances." For all practical purposes it has a probability of occurrence of one only. As the authors state, a design based on (see heading, "Conclusions") "* * * less than a maximum flood should be expected to involve some risk * * *."

As an indication of the risk being assumed in present practice, Fig. 18 shows the probability of occurrence of the design flood for the Clarks Falls dam recently built on the Lamoille River in Vermont. The spillway drains an area, M, of 701 sq miles, with a median altitude above the dam, s, of 880 ft, and an average distance the runoff must travel, L, of 43 miles. It was assumed that the authors' formulas would apply, and the 15-yr, 100-yr, and 1,000-yr floods have been plotted in Fig. 18. A curve drawn through these three points and extended to become asymptotic to the maximum flood of 160,000 cu ft per sec at infinity indicates that the spillway design flood of 85,000 cu ft per sec has a probability of occurrence of about once in seven thousand years.

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[40] With Charles T. Main, Inc., Boston, Mass.

The authors' method of deriving magnitude-frequency relations should prove very valuable. Heretofore, probability studies based on past records for a particular stream have been inadequate because they did not take into account the probability of occurrence on that stream of the requisite meteorological and hydrological conditions to produce a given flood.

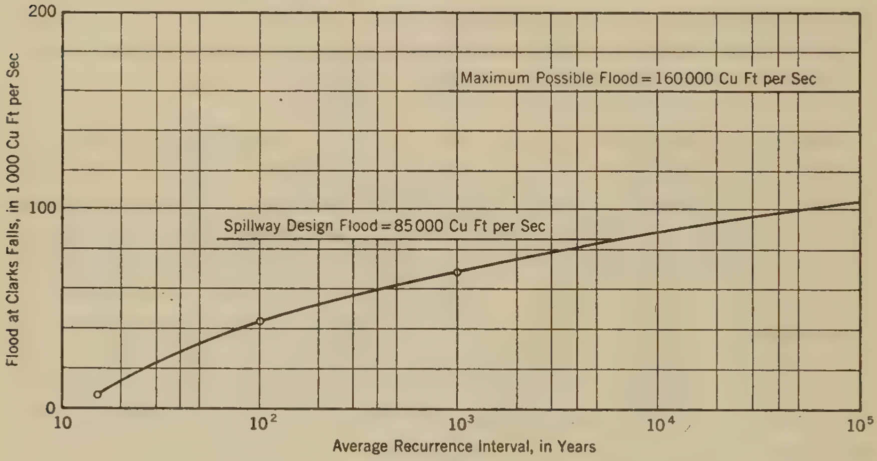

FIG. 18.—PROBABILITY OF OCCURRENCE OF CLARKS FALLS SPILLWAY DESIGN FLOOD BASED ON THE KINNISON AND COLBY FORMULAS

Magnitude-frequency relations are often used by the engineer to evaluate economic benefits of flood control projects; but, beyond a period of fifty years or so, the annual benefit becomes so small that additional expenditure for relief from such rare floods is usually difficult to justify on an economic basis. For floods of greater magnitude, the engineer becomes interested in the probability of occurrence—that is, the chances of occurrence, rather than frequency.

The authors have assumed that major floods and larger floods in Massachusetts will be caused by a 72-hr storm and that the chances of such a storm occurring over any drainage area in Massachusetts are one in one. They have also assumed that the chances are one in one that the distribution of runoff generation will follow the pattern they have used. Then, from the frequency, or probability, of a given rainfall and the probability of that rainfall producing a given runoff, the authors have obtained the probability of the resulting flood. The method is an important contribution in arriving at the true probability of occurrence of a given flood at a given station.

C. O. CLARK,[41] JUN. AM. SOC. C. E.—The expression of flood formulas in terms of drainage basin characteristics—a more comprehensive classification than drainage area—has been needed for many years. The authors' proposal that the unit hydrograph be used as a basis for the expression of flood experience is the soundest procedure yet available.

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[41] Hydr. Engr., U. S. Engr. Office, Winchester, Va.

The authors' emphasis on the element of frequency is important. The contemplation of maximum possible floods is appropriate in extending the vision of planners to the magnitude of possible floods; but to regard such floods as truly the maximum possible is to ignore their derivation from actual rainfall and climatological data. It seems probable that, as more records of phenomenal rainfall are collected, estimates of maximum possible rainfall will be revised upward. Excellent as it is, the technique of estimating maximum possible rainfall is an infant whose horizon will be extended as it matures.

As the understanding of the possible magnitude increases, the impracticability of structural provision for it will become increasingly apparent, but the alternative of economic protection by insurance will remain, and more planners will turn to this alternative. With increasing demand for flood insurance will come a greater demand for flood formulas which express the probability of floods of various magnitude.

In applying the formulas in the paper to streams of unusual shape in Virginia and North Carolina, the writer found better correlation with the formulas for runoff depth (Eqs. 3, 4, and 5), than with the flood flow formulas (Eqs. 6*a*, 6*b*, and 6*c*). Some of the universal quality which the authors incorporated into their data by the use of unit hydrographs and runoff depths may have been lost in the later attempt to express results in terms of so few variables. The data as presented may be wholly adequate for the expression of New England flood experience. The following comments are intended to suggest a technique by which the data could be presented in a manner more easily applicable to wider geographic areas, and by which some of the influences which the authors examined but were unable to analyze possibly could be expressed.

The authors' presentation of flood formulas in terms of drainage basin characteristics and probability (or frequency) of occurrence involves combining several logical elements in the analysis. Some of these elements are much stronger than others; one appears weak. Perhaps separation of the elements will allow a stronger recombination with less dependence on the weaker elements. Basically, the logical elements appear to be:

(*a*) The expression of relative flood size for various frequencies in terms of the frequency of runoff volume—

1. Assuming runoff to vary in proportion to the combined flood experience of some streams;
2. Assuming frequency of recorded point rainfall to be indicative of frequency of flood-producing rainfall on an area and indicative of runoff volume frequency;
3. Assuming a constant chronological sequence of this rainfall or runoff to be equally critical on all watersheds;
4. Assuming runoff volume to be uniformly distributed over the watershed; and

5. Assuming variation of the runoff volume with watershed characteristics.

(*b*) The expression of the flood characteristically produced by a watershed from a given amount of runoff, in the unit hydrograph, assuming that this element is not constant but that the peak flow per unit volume increases with total runoff volume.

(*c*) The expression of the flood peaks of probable expectancy as the product of elements (*a*) and (*b*).

(*d*) The expression of element (*c*) as a flood discharge formula in terms of characteristics of drainage area.

The writer is in accord with the basic elements (*a*), (*b*), and (*c*). Possibly these steps could be strengthened if dependence on some of their subelements could be increased and extended and that on others reduced. Thus, element (*a*)1 has a basis in provable flood experience which makes it a very strong factor, whereas element (*a*)5 is likely to be obscured by many other influences and to require considerable data to prove its nature conclusively. The writer proposes to extend these logical uses by discussion and examples.

Element (*d*) tries to correlate variation in element (*c*) with watershed characteristics. This attempt can be no more successful than an expression of the unit hydrograph in terms of watershed characteristics. The writer's attempts at the latter have been only moderately useful when utilizing a graphical expression of watershed shape and depressingly inadequate if confined to terms of slope, watershed area, and watershed length—the authors' selected variables. The authors have clearly indicated the difficulties of simple correlation of flood peaks and watershed characteristics.

The writer proposes to abandon element (*d*) and to express element (*c*) as a multiple of the peak of the unit hydrograph, leaving the user free to adapt the unit hydrograph of his particular stream. This proposal is based upon the writer's experience that unit hydrographs are secured rather easily from records if time and cost permit; that, even when developed from short records, such unit hydrographs are more consistently reliable than if developed by any method of synthesis known to the writer; and that, if the flood producing characteristic of a watershed must be expressed from topographic maps, there are means available which exceed in accuracy any expression possible in terms only of s, M, and L. However, it appears that the authors have collected considerable data on watershed characteristics and unit hydrographs. Direct correlation of unit hydrograph peak with watershed characteristics may be possible. Such correlation is highly desirable, and, if possible, it would help to solve both this problem and many related problems depending on synthesis of the unit hydrograph.

The proposed modified expression would parallel the foregoing logical theory, but would place slightly different emphasis on some elements, thus:

(*A*) Relative flood size for various frequencies would be expressed in terms of frequencies of runoff volumes equal, or equivalent in peak flow produced, to multiples of a uniformly distributed unit volume generated in unit time:

1. Although this definition does not exclude the use of the applicable assumptions of element (*a*) in the authors' procedure, it by-passes numerous controversies regarding the absolute truth of these assumptions by including equivalents as well as equalities.
2. There is an opportunity to include the authors' variation of peak of unit hydrograph as the equivalent of a larger volume of runoff, and thereby to avoid the controversies between those who attribute the variation in actual flood peaks to variation in the unit hydrograph and those who attribute it to intensification and concentration of runoff volume as compared to the assumption of uniformity in unit hydrograph theory. Regardless of the merit of either viewpoint, a distinction is not essential to the problem at hand.

(*B*) The flood peak characteristically produced by a watershed would be that resulting from uniformly distributed unit volume generated in unit time.

(*C*) This would be the same as element (*c*) in the authors' procedure.

In deriving element (*A*), the authors' use of combined actual stream-flow records may be extended to include more records of similar streams, because the peak flows may be known, even though the records may not be complete enough to define the actual flood volume. The question of similarity may be perplexing, and any one can propound elements of hypothetical dissimilarity, whose influence may be easily exaggerated. The writer's technique is to assume simplicity and similarity until the preponderance of evidence indicates otherwise. Thus, it may be desirable to combine all or most data available as a single record, assuming similarity, and later to analyze smaller groups of data of more definite similarity in a particular characteristic to ascertain if that particular characteristic is significant. Thus, a large group of streams all sheltered by mountain ranges over 2,500 ft in elevation might be found to experience records 25% lower than the mean of all data. However, it takes many records to prove this kind of variation conclusively. It must be remembered that the casting of three 7's in succession is not proof that dice are loaded.

In illustration of the simplified technique, an analysis of 834 flood occurrences is presented. The aggregate record totals 734 years, collected at 56 recording stream gaging stations in the adjacent watersheds of the Potomac, James, and Roanoke rivers. The stations are listed in Table 4. All these major watersheds drain some part of Virginia east of the Allegheny Mountains. The largest stream included is the Potomac River at Point of Rocks, Md., with a watershed of 9,651 sq miles, and the smallest is Georges Creek at Franklin, Md., with a watershed of 72 sq miles. This very large range in drainage area has been included in one analysis to bring out the possibilities of this form of analysis. The group of data is then subdivided into lesser groups

TABLE 4.—FLOODS OF THE MIDDLE ATLANTIC SLOPE RIVERS

Stream	Watershed				Unit flood peak[c] (cu ft per sec)	Flood Record			
	Station	Drainage area (sq miles)	Length[a] (miles)	Fall[b] (ft per mile)		Years	Flood Ratios[d]		
							Largest	Number[e]	Number per year
Shenandoah River	Millville, W. Va.	3,040	197	5.8	37,800	28.8	6.09	23	0.80
Meherrin River	Lawrenceville, Va.	553	67	3.2	6,700	11.5	5.66	14	1.22
Cacapon River	Great Cacapon, W. Va.	677	107	9.5	15,600	9.0	5.62	4	0.44
Potomac River	Hancock, Md.	4,073	179	6.5	61,000	19.3	5.57	16	0.83
Roanoke River	Weldon, N. C.	8,445	280	3.9	48,200	41.8	5.41	151	3.62
Goose Creek	Leesburg, Va.	338	33	11	8,330	3.9	5.38	1	0.26
Rivanna River	Palmyra, Va.	675	50	8.0	14,700	9.1	5.15	10	1.10
South Branch, Potomac River	Springfield, W. Va.	1,471	120	11	28,400	14.2	5.03	10	0.71
Willis River	Flanagans Mills, Va.	247	54	4.2	2,300	6.5	4.87	12	1.86
Potomac River	Shepherdstown, W. Va.	5,936	234	4.6	70,000	15.1	4.78	15	1.00
Banister River	Halifax, Va.	552	46	7.2	5,500	14.4	4.73	17	1.18
North Fork, Shenandoah River	Strasburg, Va.	772	96	12	21,700	12.1	4.60	5	0.41
Potomac River	Point of Rocks, Md.	9,651	257	4.4	105,000	48.3	4.54	40	0.83
North Branch, Potomac River	Cumberland, Md.	875	86	21	19,900	14.0	4.44	10	0.72
Passage Creek	Buckton, Va.	87	32	31	4,800	5.1	4.37	1	0.20
North River	Rockbridge Baths, Va.	329	45	27	7,700	14.4	4.28	22	1.52
South Fork, Shenandoah River	Front Royal, Va.	1,635	146	7.5	30,400	12.1	4.27	7	0.58
Sandy River	Danville, Va.	113	21	21	5,600	12.9	4.10	11	0.86
Roanoke River	Brookneal, Va.	2,420	154	6.1	32,600	13.7	3.99	14	1.02
Cow Pasture River	Clifton Forge, Va.	456	69	13.1	8,700	8.5	3.94	16	1.88
Appomattox River	Petersburg, Va.	1,335	122	2.9	7,200	16.0	3.88	33	2.06
Dan River	South Boston, Va.	2,730	179	5.1	21,100	13.8	3.84	27	1.95
Monocacy River	Frederick, Md.	817	62	5.9	13,500	13.5	3.78	31	2.29
North River	Lexington, Va.	487	58	21	11,000	15.2	3.63	16	1.05
South Branch, Potomac River	Petersburg, W. Va.	642	60	24	14,500	13.9	3.56	8	0.57
Falling River	Brookneal, Va.	228	23	11	6,600	6.7	3.48	10	1.49
North Fork, Shenandoah River	Cootes Store, Va.	215	26	44	14,500	5.0	3.45	2	0.40
Wills Creek	Cumberland, Md.	247	35	40	11,100	13.6	3.43	3	0.22
Roanoke River	Toshes, Va.	1,020	96	9.2	21,000	13.6	3.33	8	0.59
Mayo River	Price, N. C.	260	38	20	9,200	12.0	3.26	6	0.50
Pig River	Toshes, Va.	394	72	8.7	8,600	12.7	3.26	7	0.55
Otter River	Evington, Va.	325	34	18	8,600	6.5	3.21	7	1.07
Blackwater River	Union Hall, Va.	208	46	14	6,200	13.6	3.18	9	0.66
James River	Cartersville, Va.	6,242	285	4.6	58,100	43.3	3.17	72	1.66
James River	Lick Run, Va.	1,369	78	17	21,700	14.5	3.08	21	1.45
Hyco River	Omega, Va.	338	....	....	2,400	8.6	3.02	38	4.44
Roanoke River	Niagara, Va.	511	53	16	11,600	16.8	3.02	16	0.95
South Fork, Shenandoah River	Lynnwood, Va.	1,076	63	12	28,700	12.1	2.80	6	0.49
Slate River	Arvonia, Va	235	34	9.0	4,400	6.1	2.79	8	1.31
Roanoke River	Roanoke, Va.	388	48	17	10,500	6.0	2.78	3	0.50
South Fork, South Branch, Potomac River	Moorefield, W. Va.	283	54	25	8,250	3.3	2.78	3	0.91
North Branch, Potomac River	Bloomington, Md.	287	49	45	8,100	13.8	2.76	10	0.72
Goose Creek	Huddleston, Va.	187	33	14	6,400	12.7	2.68	10	0.79
Savage River	Bloomington, Md.	115	26	65	4,560	13.7	2.63	6	0.44
Jackson River	Falling Spring, Va.	409	47	22	9,500	8.5	2.62	9	1.06
Georges Creek	Franklin, Md.	72.4	18	78	3,320	13.6	2.56	4	0.29
Smiths River	Martinsville, Va.	374	48	13	15,500	13.6	2.52	8	0.59
North River	Burketown, Va.	375	18	37	17,500	3.9	2.46	3	0.77
Dan River	Francisco, N. C.	124	43	35	5,100	12.0	2.44	5	0.42
Antietam Creek	Sharpsburg, Md.	281	49	11	2,830	9.9	2.43	8	0.81
South River	Waynesboro, Va.	144	24	17	4,900	14.3	2.42[f]	8	0.56
Craig Creek	Parr, Va.	331	....	....	6,900	5.8	2.20	12	2.06
Conococheague Creek	Fairview, Md.	494	....	....	7,600	9.7	2.14	10	1.04
Middle River	Grottoes, Va.	360	54	14	12,600	4.2	1.67	3	0.70
Hardware River	Scottsville, Va.	116	22	18	2,600	4.4	1.33	4	0.92
Seneca Creek	Dawsonville, Md.	101	17	17	5,560	12.8	1.17	1	0.08

[a] Total length of longest low water flow path. [b] Uniform slope of profile along longest flow path which would have same area under profile and above low water elevation at gage. [c] Peak discharge for 1.0-in. runoff generated uniformly in 6 hours. [d] Discharges of actual floods in analyzed record in ratio to unit flood peak. [e] Number of floods analyzed in excess of 0.9 unit peak. [f] Prior to record analyzed, flood of September 30, 1870 (relatively the largest known flood at any of the stations analyzed), reached a flow estimated as 8.0 times unit flood peak.

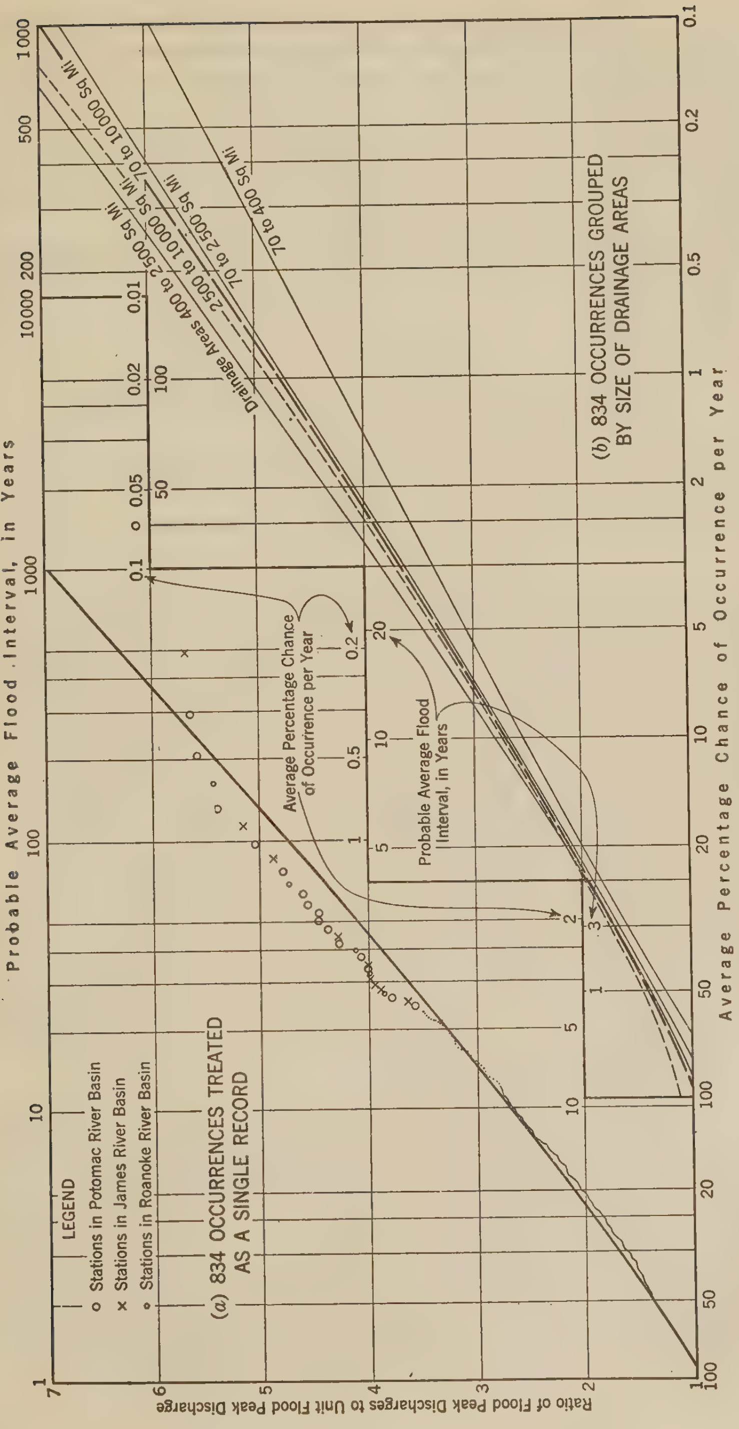

FIG. 19.—FLOOD FREQUENCIES, POTOMAC, JAMES, AND ROANOKE RIVERS, FOR DRAINAGE AREAS OF FROM 70 SQ MILES TO 10,000 SQ MILES

of more similar drainage area in an attempt to determine whether drainage area alone is a significant variable affecting relative frequency of floods.

In each analysis all peak flood discharge occurrences have been listed in order of the magnitude of the ratio by which they exceed the peak flow produced by 1.0 in. of runoff uniformly generated over the drainage area at a uniform rate during a period of 6 hours. These 834 flood ratios in descending order have been treated as a single record of 734 years in the determination of the frequency curve shown in Fig. 19(*a*).

The use of the frequency curve shown to determine flood discharge frequency on any particular small stream in the same region would involve several assumptions, namely:

I. That the frequency with which the unit flood discharge is exceeded on small watersheds is the same as that for the average of all watershed sizes in the region;
II. That the effects of other drainage area characteristics are the same as for the averages of all watersheds; and
III. That, in the region, the peak of the unit hydrograph of a single stream can be determined with suitable accuracy.

One should maintain an open mind regarding the accuracy of an hypothesis, such as assumption I, and demonstrate its correctness when the sample of data becomes large enough to be conclusive. The writer has never worked with a single flood record long enough to support a conclusive judgment. However, analysis of smaller groups of aggregate records from more assuredly similar watersheds may be qualitatively indicative. Size of drainage area alone was studied in five analyses, the results of which are represented in Fig. 19(*b*). In these analyses, the aforementioned record of 834 occurrences was divided into groups of more nearly similar watershed area as indicated in Fig. 19(*b*).

A spread in the curves results. This spread is no greater than that commonly experienced in the analysis of flood records for a single station. Furthermore, relative size in the rare frequency range does not increase or decrease consistently with increasing area, although a consistent trend does appear in the range of 1-yr to 5-yr floods. Until a contrary theory is demonstrated conclusively, the influence of watershed area would be regarded as negligible, except in the range of very frequent floods where flows would tend to be more nearly proportional to watershed area.

The accuracy of the similar assumptions with regard to other watershed characteristics might be checked similarly. An illustrative application to the aforementioned data is prohibited by the labor of evaluating the factors. However, since the authors have already evaluated more factors than they were able to report in their flood formulas, they may find re-examination of their data feasible.

Exact determination of the peak of the unit hydrograph of a single stream is not as simple as much published material might appear to indicate. As the authors have shown, different investigators using the same data will not agree, and investigators using different data from the same stream or different methods of analysis may reach quite different conclusions. Much of the variation is random in character and therefore tends to average in the analysis of a large number of streams.

The results of a general analysis like the foregoing, which is applied to a single stream, would embody any errors that might be involved in determining that particular unit hydrograph peak, and should be checked accordingly. If the record of the single stream were long enough to establish a reasonably accurate 10-yr flood, the relative size of rarer floods in proportion to the decennial flood might be used instead of the proportion of rarer floods to the unit flood. This would be tantamount to revision of the unit hydrograph peak by such an amount as would make the actual 10-yr flood and the computed 10-yr flood agree. Judgment would decide whether such an adjustment should be made. The writer knows no acceptable criterion for determining the amount of variation from mean flood experience which can occur without a stream being conclusively indicated as an exception to a stated probability law. Some records of 60 to 100 years show 10-yr periods deviating by 400% from the mean flood experience of the entire period; that is, four 10-yr floods when one would be normal for 10 years.

Some adjustment of the type mentioned in the preceding paragraphs is necessary when comparing conclusions of different investigators. Table 5 shows a comparison of frequency curves for the Middle Branch of Westfield River at Goss Heights; that is, Fig. 12 is compared with data that would develop from the frequency study of Fig. 19(*a*) (70 to 10,000 sq miles) adjusted to make the 10-yr values agree.

TABLE 5.—COMPARISON OF FREQUENCY CURVES FOR THE MIDDLE BRANCH OF THE WESTFIELD RIVER

No.	Description	Average Recurrence Interval, in Years							
		1	2	5	10	20	50	100	1,000
	Ratio of Flood Discharge; from Fig. 19(*a*):								
1	To unit flood	0.95	1.40	2.08	2.68	3.28	4.10	4.75	6.95
2	To decennial flood	0.34	0.52	0.78	1.00	1.22	1.53	1.77	2.60
	Discharge, in Cubic Feet per Second:								
3	From Fig. 19(*a*)	2,100	3,100	4,700	6,000	7,300	9,200	11,000	16,000
4	From Fig. 12	2,600	3,400	4,800	6,000	7,400	9,700	12,000	20,000

The agreement in lines 3 and 4, Table 5, is much closer than the writer had expected. However, this illustrates the fact that conclusions of many independent investigators may agree much more nearly than the techniques or assumptions that the investigators might use to reach the conclusions. For this reason the writer does not like to argue about the validity of many individual hydrological concepts and premises. Few conclusions depend on one such concept; most depend on the combination of many. Divergent theories about one premise are counterbalanced by opinionated variations on the opposite side. The writer belongs to a school of thought which denies the validity of unit hydrograph peak variation with volume of runoff and, without intending to criticize the authors' excellent analyses, he attributes such indications to an inherent fallacy in a very commonly used method for the derivation of unit graphs and to the common neglect of nonuniformity of runoff distribution in the derivation and use of unit graphs. On the other hand, his techniques for analyzing the nonuniformity of areal distribution and runoff factors are extensive. In any actual use, both techniques would be used together and seasoned with the advocates' judgment and experience. The results would be compared with a record. Presumably, neither the authors' nor the writer's analyses would agree with the record exactly, although both might be acceptably close.

Hydrology and the hydraulics of natural stream flow are complex, interrelated phenomena, too involved for practical use or exact understanding. As a result, most analyses are oversimplified explanations of an observed phenomenon in terms of a limited number of mentally conceived rational premises. The exact truths of the premises cannot be demonstrated, since only a combined result can be observed. Thus, the combination of rational concepts is the yardstick by which an individual calibrates his own judgment.

Papers of this kind present a yardstick for comparisons with other similar standards. If all yardsticks are the correct length, it is frequently presumptuous to argue the merit of the pieces. Sometimes, however, through such argument, one discovers that what he thought was an essential piece of his yardstick is either a useless piece hanging on the side, or an unnecessary piece that has been put in at one end and taken off at the other and thus adds only dead weight to the whole.

If this modified analysis is as appropriate when applied to New England stream flow records as when developed from records elsewhere, it will indicate that the same conclusions could have been reached without use of rainfall records or conversions of rainfall to runoff. It would then raise the question: "Are the premises which deal with these elements really part of the yardstick or are they computationally just extra pieces which went into the derivation at one place and out at another without really influencing the answer?"

H. B. KINNISON,[42] M. AM. SOC. C. E., AND B. R. COLBY,[43] ESQ.—The excellent discussions of this paper are deeply appreciated, and they strengthen the writers' belief that the study of the frequency and magnitude of floods should have the best efforts of many hydraulic engineers. Although records of stream flow are much more numerous than they were only a few years ago, they still are inadequate for most types of investigations; and, therefore, differences of opinion and divergent results are to be expected. The writers hope that the paper and the discussions are a constructive contribution to the field of flood studies.

The discussion by Mr. Jarvis presents a comparison between the Modified Myers formula and the writers' formulas, and states that "No matter how involved any preferred procedures, the results are readily translated into terms of Myers percentage ratings, as demonstrated by Table 2." The form of the equation, $Q = C_m \sqrt{M}$, makes it possible to convert the results of any procedure which arrives at a value of Q into a percentage rating, as the percentage rating becomes the only unknown. However, the very fact that the percentage ratings in Table 2 show so wide a variation for a flood of a given magnitude is an indication of the limitations of this type of formula. For instance, for "rare floods," the percentage rating varies from 5.4 to 51.0; and, in relation to the highest observed flood, the "rare flood" varies from 81% to more than 300% for those streams having more than 10 years of record. The engineer faced with the problem of choosing a reasonable percentage rating for a particular stream must seek another procedure rather than the Modified Myers formula to properly evaluate the flood-producing characteristics of that stream. Unit hydrographs of streams with the same drainage area show wide divergence in peak discharge, even though the basins may be adjacent to each other. The writers' formulas were developed in an attempt to evaluate the effect of those physical characteristics which cause so much divergence in the flood peaks.

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In Table 2, Mr. Jarvis has given the computed peaks of the minor, major, rare, and maximum floods in percentage ratings based on the Modified Myers formula. He concluded from this table that the maximum flood peaks are too high. Mr. Clark states that they are probably too low. The maximum flood peak as defined in the paper has an infinite recurrence interval, or a probability of occurrence approaching zero. It is a computed peak representing the upper limit of possible peaks. Because ratings computed from the Modified Myers formula have been derived almost entirely from observed flood peaks or for peaks of probable or design floods, ratings computed for maximum peaks will naturally seem to be very high. The maximum peaks in Table 2 that have percentage ratings greater than 90% are not inconsistent with the other maximum peaks. They are all of the same order of magnitude in that they have the same periodicity. The largest percentage for a maximum peak (right-hand half of Col. 12, Table 2) is only 8.1 times the smallest percentage for a maximum peak; but the largest percentage for a minor peak (right-hand half of Col. 9, Table 2) is 13.2 times the smallest percentage for a minor peak.

If Mr. Jarvis means that the maximum probable flood peak should be about 150% of the rare flood peak, the writers agree that this would be a reasonable assumption. As stated in the paragraph preceding the heading, "Physical Characteristics," the peak discharge of the maximum probable summer flood might be "about 45% of the peak discharge of the maximum flood."

Mr. Jarvis found from his analysis that the formulas should not be applied to basins having characteristics that differ widely in any important physical characteristic from the basins used in the study. This limitation agrees with the stated conclusions of the writers. They are indebted to Mr. Jarvis for stressing this fact.

In his criticism of the factors included in Eq. 6, Mr. Jarvis has misinterpreted the effect of different physical characteristics on computed flood peaks, and his criticism of the use of L and a in Eq. 6 is not valid. The effect of the characteristic L on computed flood peaks must be considered in relation to the changes in other characteristics which are normally associated with changes in L. Similarly, any variation in the pondage factor a is invariably associated with changes in the slope factor s, and usually with changes in L, at least in central New England where the formulas are considered applicable.

Entirely different formulas might have been developed on the basis of Mr. Jarvis' assumptions. However, the writers preferred to develop formulas from relationships which were indicated by the basic data, and which fit the data for this particular locality closely.

The original report of the study on which the paper was based (see "Acknowledgments") was prepared for use in Massachusetts and possibly in some adjoining areas. When the paper was prepared for wider circulation, some changes might have been made. As Mr. Ord suggests, the formulas, from the original report, might well have been shown by graphs. Also, a discussion of the floods of March, 1936, and September, 1938, should have been included, so that any one not familiar with these floods on Massachusetts streams would not have questioned the fact that these were greater floods than would normally occur during the short period covered by gaging station records.

Mr. McCoy's observation that the exponents and coefficients in the formulas should be consistent for different classes of floods is a logical one. In determining the effects of the various characteristics, each of the four classes of floods was considered separately, and correlations indicated by the basic data were used without adjustment. The writers felt that, because of the limited knowledge of the influence of each characteristic, complete dependence should be placed on the basic data.

Mr. Beard and others question the accuracy of curves C and D in Fig. 7. These curves are not well defined, but they are based on logical extensions of the available data. Discussion of the derivation of these curves was considered much too lengthy to be given in the paper. However, it should be understood that runoff coefficients were not confined to a narrow range. For example, precipitation in the range from 5.6 in. to 6.0 in. (the range was expressed to the nearest tenth of an inch) was found to have about one chance in seven of generating less than 1.0 in. of runoff. Under extreme conditions of precipitation falling on snow cover, precipitation was assumed to have less than one chance in one thousand of generating more than 10 in. of runoff.

Depth of runoff was not determined solely from curve D which was based directly on incompletely defined relations between rainfall and runoff. The basis for the depths of runoff that were finally used is stated in the paragraph preceding the heading, "Maximum Flood Runoff."

In Table 3, Mr. Ord lists the minor, rare, and maximum flood peaks in percentages of the major flood peaks. Fig. 17, showing a curve drawn through the average percentages for each class of flood peak, is interesting and could be used to compute flood peaks having other recurrence intervals after the major flood peak had been computed. However, the writers do not agree that flood peaks computed from this type of average curve would be as accurate as flood peaks computed from the frequency curves for individual drainage areas. Although most frequency curves for basins in the same region probably should be approximately parallel when drawn on logarithmic paper, each one is not necessarily parallel to all the others. Also, the use of an average curve seems to be inconsistent with the suggestion made by Mr. Ord that the floods of March, 1936, and of September, 1938, should have been given more weight in the study. Figs. 1 and 11 show how difficult it would be to draw parallel curves for the three basins if much emphasis were given to the floods of March, 1936, and of September, 1938.

Mr. Ord states, "The percentages for the maximum flood have a greater variation." He then adopts the doubtful assumption that those maximum peaks which show the extreme variations from the average should be questioned. The maximum flood peaks have a greater variation only because the maximum flood peaks are farthest from the major flood peaks, which Mr. Ord makes coincide. If the maximum flood peaks were made to coincide, the peaks of the minor floods would have had the greatest variation. Table 2 shows that the ratio of the highest to the lowest maximum flood peak is less than the corresponding ratio for any other class of flood peak.

Mr. Clark and Mr. Ord prefer to make the unit hydrograph a direct basis for computations of expected flood peaks. That is a logical procedure, and the study reported in the paper was begun on a similar plan, but that plan was soon abandoned. The computed unit hydrograph is an imperfect foundation on which to build. As Mr. Clark states, the unit hydrographs obtained by investigators using different methods may not agree. For flashy streams, the peaks of the unit hydrographs computed by the same person from several flood hydrographs may differ widely. Also (on Massachusetts streams at least), the peaks of computed unit hydrographs vary with the depth of runoff. Publication of depths of runoff to be used with unit hydrographs computed by other engineers might lead to very inaccurate computations of flood peaks. Besides convenience of application, the correlation of computed peaks for each class of flood with the physical characteristics of drainage areas has the advantage of insuring at least a consistent computation and the use of unit hydrographs. Also, the peaks of floods computed from unit hydrographs having sharp peaks are not as variable as the peaks of the unit hydrographs.

The writers were reluctant, as is Mr. Turner, to assign numerical recurrence intervals to the different classes of floods because of the accuracy which numerical intervals connote. However, adjective ratings alone are indefinite and would have a wide range of interpretation. Also, some assumption of recurrence intervals was needed for plotting. Hence, numerical ratings were used as a means of precise definition only. The computed flood peaks are approximations of the peaks of floods having the definite recurrence intervals. The writers hope that the flood peaks will be understood according to these definitions.

It should be recognized that the formulas have been derived from a study of runoff from areas of less than 500 sq miles and have not been tested on larger areas. They should not be applied to drainage areas having significant physical characteristics which differ materially from those on which this study has been made, or to localities affected by storms of a different character than those in Massachusetts. Computed flood flows should be routed carefully through storage in the basin if the storage is extensive or is so located that it has an appreciable effect upon the flood peak. Before the formulas are used for areas some distance from Massachusetts, an adequate study should be made to determine their applicability. It is hoped that the method can be used in the derivation of similar formulas for other areas and, possibly, eventually for the derivation of more satisfactory formulas for general application.

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ANALYSIS OF THE GENERAL TWO-DIMENSIONAL FRAMEWORK

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With Discussion by Messrs. Leon Beskin, Jaroslav Polivka, A. A. Eremin, and Yves Nubar.

Synopsis

A method is presented for the analysis of displacements and stresses in the general two-dimensional framework composed of rectilinear members, subjected to loading and displacement in its own plane. The joints may be articulated or rigid, or they may be a combination of such types.

The analysis is characterized by the division or decomposition of the frame into triangular elements by means of imaginary diagonals and the consideration of the axial deformations of the bars (given or imaginary) of the resulting network.

The technical literature contains descriptions of many excellent methods for the analysis of frames, some of which deal with specialized types. Most of them can be loosely classified as being based on one of the following concepts: Least work, slope deflection, fixed points, moment distribution, angle distribution, etc. The method developed in this paper has a semi-geometric basis. It is applicable to simple structures, as well as to complex systems of straight lines meeting at joints to form panels of arbitrary shape. Its value consists perhaps in the development of certain inherent properties of the general framework, helpful in the systematization of the analysis. This tends to unify, under one consistent procedure, the method of approach in a number of superficially unrelated problems, such as, for instance, the determination of deflections in a hinged truss and of moments in a rigid frame.

The application of the method is illustrated by several practical cases.

Introduction

General Description.—In this analysis, the frame is first decomposed completely into triangular elements by means of imaginary diagonals, in an arbi-

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Note.—Published in April, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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trary but simple fashion (see dotted lines in Fig. 1). These diagonals may be conceived as infinitely thin wires stretched between the joints, carrying infinitesimal forces, but undergoing finite axial strains as a result of the deformation of the frame. The axial strains of the given members and those of the imaginary diagonals are taken as the unknowns, with the understanding that the axial strains in given or imaginary members joining supports are equivalently written in terms of the linear movements of the supports. These elements determine the relative positions of all deflected joints and supports and lead to the solution of stresses. The number of these unknowns is greatly reduced in the case (frequent in problems on rigid frames) in which the axial deformations of the given members are assumed to be negligible and the linear movements of the supports are prevented.

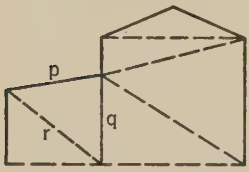

FIG. 1

The following is an outline of the procedure of the analysis:

A.—Since closed circuits must remain so after deformation, conditions of compatibility must be satisfied by the axial and lateral strains of the sides of all closed circuits. For triangular elements such as $p\ q\ r$ of Fig. 1, these conditions are written in a form involving the axial strains of the three sides and the difference $\delta_p - \delta_q$ (expressed as δ_{p-q} in this paper) between the lateral strains δ_p and δ_q of any two of them. The axial strains, adopted as unknowns in this analysis, may be interdependent; this follows from restrictions imposed by the compatibility conditions. These conditions lead to the following results:

(*a*) Expression of elements δ_{p-q} in terms of the unknowns;
(*b*) Determination of the relative displacement of any two joints of the frame in terms of the unknowns; and
(*c*) Reduction of the unknowns to a minimum number of them which are independent.

B.—Relationships between the dependent and independent unknowns are developed through a study of the topological properties of the general frame. (Topology is defined by R. Courant and H. E. Robbins[2] as "the study of the properties of geometrical figures that persist even when the figures are subjected to deformations so drastic that all their metric and projective properties are lost." In a framework, the numbers of members, joints, regions, etc., are thus topological elements, and relations between them represent topological properties of the framework.) Such a study discloses the degree of freedom of the frame in joint displacements and relates it to the number of independent axial strains.

C.—The boundary conditions are translated into relations which exclude the support reactions and involve the unknowns of this analysis.

D.—All end moments (and consequently tranverse end forces) are expressed, by means of recurrence formulas, in terms of δ_{p-q}; that is, in terms of the independent unknowns (see A(*a*) and A(*c*)).

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[2] "What is Mathematics?" by Richard Courant and Herbert E. Robbins, Oxford Univ. Press, London and New York, p. 235.

E.—The independent unknowns are determined. This results from the consideration of a conjugate frame obtained by removing all bars corresponding to dependent unknowns (see A(*c*)) and substituting hinges for all the joints. This conjugate frame is statically determinate; its solution yields the exact number of conditions for the determination of the independent unknowns. In general, the number of these unknowns is rather small; for instance, in the case of a one-story bent of n columns on rigid supports, with the axial deformations of given members assumed to be negligible, this number is 1.

F.—All stresses and joint displacements are now found using the values of the unknowns just determined.

NOTATION

The letter symbols in this paper are defined when they first appear and are assembled for convenience of reference in the Appendix.

ASSUMPTIONS

All assumptions leading to the validity of the well-known equation,

$$\frac{d^2y}{dx^2} = \mp \frac{M}{E\,I} \quad \text{(1)}$$

which relates displacements to applied loads, are adopted in this analysis. (In Eq. 1, x and y are the coordinates of a point on the deflected member, M is the bending moment, E the Young modulus, and I the moment of inertia—all referred to this point.) In particular, the assumption is made that all deformations are very small in comparison with the dimensions of the members.

It is further assumed that:

(1) The moment of inertia of each member is constant along its length, although not necessarily the same for all members;

(2) The joints in the frame are either perfectly rigid, free to rotate, or belong to both categories; in other words, flexible joints are not considered. No such restriction is imposed upon the supports; and

(3) For the purpose of this analysis, the given frame may be replaced by an idealized system formed by the gravity axes of the members.

These assumptions are common to most methods for the analysis of frames. With some modifications, the method presented here may be extended to remove restrictions (1) and (2).

COMPATIBILITY EQUATIONS

The axial strains and unit deflections of members resulting from the displacements of the joints are subject to conditions of compatibility derived in the following manner.

Let AB and A′B′ in Fig. 2 represent any member in a frame, respectively, before and after displacement. From point A′, draw a line A′B″ parallel to AB and equal to it in length. Point B‴, the foot of a perpendicular from B′ on A′B″, defines the change in length e_1 of member AB and the transverse movement Δ_1 of joint B. In Fig. 2, these elements are indicated in their positive directions, that is, elongation for e_1 and clockwise rotation for Δ_1.

Let δ_1 designate the unit deflection of the member, defined by $\delta_1 = \dfrac{\Delta_1}{l_1}$; ϵ_1 its axial strain, defined by $\epsilon_1 = \dfrac{e_1}{l_1}$; and let λ_1 be the inclination of AB on

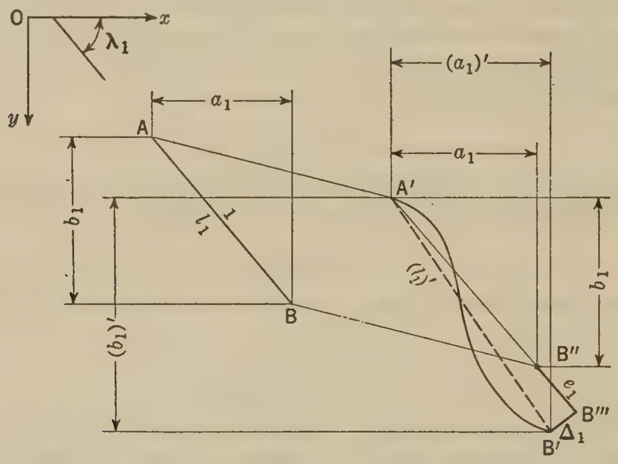

FIG. 2

the x-axis. The following relations may be written between the rectangular projections of AB and A′B′:

$$(a_1)' = a_1 + e_1 \cos \lambda_1 - \Delta_1 \sin \lambda_1 \ldots\ldots\ldots\ldots (2a)$$

and

$$(b_1)' = b_1 + e_1 \sin \lambda_1 + \Delta_1 \cos \lambda_1 \ldots\ldots\ldots\ldots (2b)$$

or, in terms of δ_1 and ϵ_1:

$$(a_1)' - a_1 = a_1\,\epsilon_1 - b_1\,\delta_1 \ldots\ldots\ldots\ldots (3a)$$

and

$$(b_1)' - b_1 = b_1\,\epsilon_1 + a_1\,\delta_1 \ldots\ldots\ldots\ldots (3b)$$

Eqs. 2 and 3 apply to any member of the frame, regardless of its position or orientation, provided its rectangular projections are taken with the proper signs in relation to the reference axes. Apply these relations to a closed circuit of bars (given or imaginary); denote by $\sum_o$ a complete summation around this circuit, continuously in a clockwise direction. Since, for a closed circuit, $\sum_o a = 0$; $\sum_o a' = 0$; $\sum_o b = 0$; and $\sum_o b' = 0$, the following relations may be derived from Eqs. 3:

$$\textstyle\sum_o(a\,\epsilon) - \sum_o(b\,\delta) = 0 \ldots\ldots\ldots\ldots (4a)$$

and

$$\textstyle\sum_o(b\,\epsilon) + \sum_o(a\,\delta) = 0 \ldots\ldots\ldots\ldots (4b)$$

These relations simply mean that the axial and lateral strains of the bars are compatible in maintaining a closed circuit. It seems appropriate, therefore, to call Eqs. 4 "compatibility equations."

Consider a closed circuit, ABCDA, Fig. 3(*a*), in which the sides may be some of the given members in a frame or simply imaginary diagonals connecting two joints. Designate any two consecutive sides by the letters p and q. Eqs. 4 may be written in a simpler manner if the differences $\delta_p - \delta_q = \delta_{p-q}$ and $\epsilon_p - \epsilon_q = \epsilon_{p-q}$ are introduced instead of the elements δ and ϵ. At the joint where members p and q meet, apply the orthogonal vectors δ_{p-q} and ϵ_{p-q} in the direction of the arbitrary axes x and y. Treat all other joints in the same manner, as indicated in Fig. 3(*a*). It will be shown that the resultant moment of all these vectors about point O is zero if the compatibility equations are satisfied.

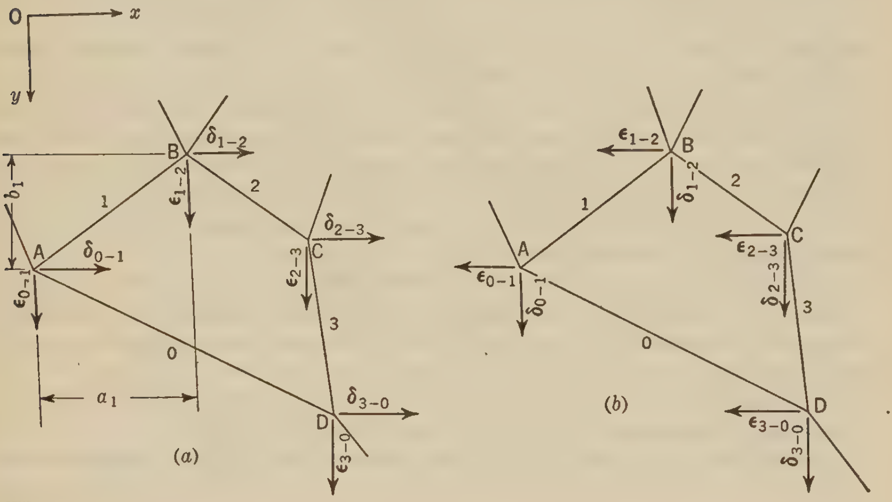
O
x
y
B
δ_{1-2}
1
2
ϵ_{1-2}
δ_1
A
δ_{0-1}
C
δ_{2-3}
ϵ_{2-3}
3
ϵ_{0-1}
0
a_1
δ_{3-0}
D
ϵ_{3-0}
(a)
ϵ_{1-2}
B
δ_{1-2}
2
1
A
C
ϵ_{2-3}
δ_{2-3}
ϵ_{0-1}
δ_{0-1}
3
0
(b)
D
ϵ_{3-0}
δ_{3-0}

FIG. 3

This moment is

$$M = x_a\,\epsilon_{0-1} + x_b\,\epsilon_{1-2} + x_c\,\epsilon_{2-3} + x_d\,\epsilon_{3-0} - (y_a\,\delta_{0-1} + y_b\,\delta_{1-2} + y_c\,\delta_{2-3} + y_d\,\delta_{3-0}) \ldots\ldots\ldots\ldots (5)$$

in which x_a, y_a, etc., are the coordinates of joints A, etc.; and ϵ_{0-1}, δ_{0-1}, etc., are equal, respectively, to $\epsilon_0 - \epsilon_1$, $\delta_0 - \delta_1$, etc. Similarly, if $x_a - x_d$, etc., are expressed as x_{a-d}, etc., Eq. 5 may be transformed as follows:

$$M = x_{a-d}\,\epsilon_0 + x_{b-a}\,\epsilon_1 + x_{c-b}\,\epsilon_2 + x_{d-c}\,\epsilon_3 - (y_{a-d}\,\delta_0 + y_{b-a}\,\delta_1 + y_{c-b}\,\delta_2 + y_{d-c}\,\delta_3) \ldots\ldots\ldots\ldots (6)$$

or again

$$M = \sum\nolimits_o (a\,\epsilon) - \sum\nolimits_o (b\,\delta) \ldots\ldots\ldots\ldots (7)$$

Eq. 7, which represents the moment about O of the vectors δ_{p-q} and ϵ_{p-q}, directed as in Fig. 3(*a*), is equal to zero by virtue of Eq. 4*a*.

If all these vectors are rotated bodily 90° clockwise, as in Fig. 3(*b*), it may be shown in a similar manner that their resultant moment about O reduces to the expression

$$M' = \sum\nolimits_o (b\,\epsilon) + \sum\nolimits_o (a\,\delta) \qquad (8)$$

which is also equal to zero because of Eq. 4*b*.

It is thus seen that, if a certain number of the joints of a frame are considered as forming a closed circuit (by drawing imaginary diagonals, if necessary), the two moments (about any point) of the vectors δ_{p-q} and ϵ_{p-q}, directed first as in Fig. 3(*a*), then as in Fig. 3(*b*), reduce to zero if the compatibility conditions are satisfied; and vice versa. In addition, these vectors satisfy the two self-evident relations, $\sum_o \delta_{p-q} = 0$; and $\sum_o \epsilon_{p-q} = 0$.

The system formed by these two moment equations and the relation $\sum_o \delta_{p-q}$ is equivalent to a more symmetrical system obtained by equating three resultant moments to zero. These moments are taken about three arbitrary points, of the orthogonal vectors δ_{p-q} and ϵ_{p-q}, directed along three arbitrary directions. In each moment operation, all δ_{p-q}-vectors are made parallel, pointing in the same direction. This applies similarly to vectors ϵ_{p-q}.

Compatibility equations, expressed in the form of a system of three moment equations as described herein, can be written conveniently when applied to triangular elements, the vertices of which are used as centers of moments (see Example 1).

The compatibility equations developed in this section apply to any frame, regardless of the nature of the joints—rigid, hinged, or flexible.

Example 1.—After a frame has been decomposed into its hypothetical triangular elements, the compatibility conditions are applied to one of these elements. Fig. 4(*a*) represents a triangle ABC, formed by any three joints of a frame. The orthogonal vectors δ_{p-q} and ϵ_{p-q} are applied at the respective joints as shown. Equate their moment about point A to zero; thus:

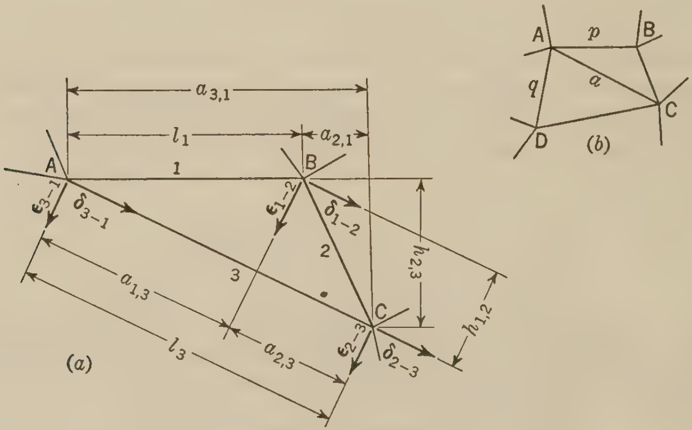

FIG. 4

$$M = h_{1,2}\,\delta_{1-2} + a_{1,3}\,\epsilon_{1-2} + l_3\,\epsilon_{2-3} = 0 \qquad (9)$$

or

$$\delta_{1-2} = \frac{l_3\,\epsilon_3 - a_{1,3}\,\epsilon_1 - a_{2,3}\,\epsilon_2}{h_{1,2}} \qquad (10a)$$

If these vectors, fixed at right angles to each other, are rotated to bring the δ_{p-q}-vectors parallel with line BA, and their moment about point B is equated to zero:

$$\delta_{2-3} = \frac{l_1 \epsilon_1 + a_{2,1} \epsilon_2 - a_{3,1} \epsilon_3}{h_{2,3}} \quad \text{.................} \quad (10b)$$

A third expression of a similar type is obtained by equating to zero the moment about point C of the vectors δ_{p-q} and ϵ_{p-q}, rotated so that δ_{p-q} becomes parallel to line CB.

It may be concluded that δ_{p-q} at any angle formed by two given members in a frame meeting at a joint may be written in terms of the axial strains of the members forming the angle and that of the diagonal opposite. When the members are not contiguous, such as AD and AB in Fig. 4(*b*), δ_{p-q} may be written as $\delta_{p-a} + \delta_{a-q}$, in which each term may be expressed using the axial strains ϵ of the lines forming the triangles ABC and ACD.

If one (or several) of the sides of a triangular element happens to be a bar joining two supports, the δ-terms and ϵ-terms of this side will represent, respectively, the unit deflection and the axial strain of the line of supports. Both of these elements can be expressed easily as functions of the linear movements of the supports. In the development of this analysis, it will be found expedient to assume that the unit deflection and axial strain of bars (given or imaginary) joining two supports have been replaced in terms of the support movements. As a result, the compatibility equations will contain these movements explicitly, when applied to panels adjacent to support lines.

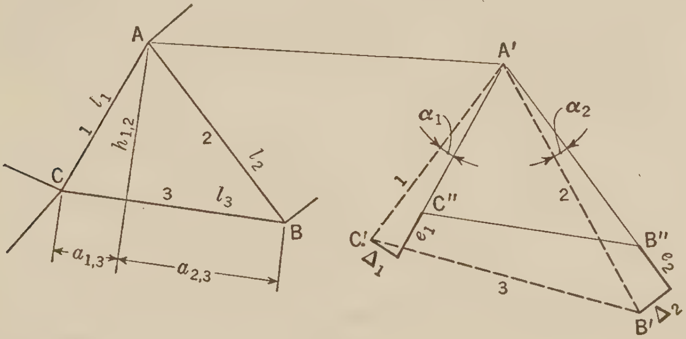

FIG. 5

A geometric representation of δ_{p-q} and ϵ_{p-q} may be drawn in terms of the deformation of a triangle. Let ABC, Fig. 5, be a triangular element in the frame; A′B′C′ is the triangle formed by joining the deflected positions of the vertices; and A′B″C″ the position of ABC translated until joint A coincides with joint A′. By definition,

$$\delta_{1-2} \; (= \delta_1 - \delta_2) = \frac{\Delta_1}{l_1} - \frac{\Delta_2}{l_2} \quad \text{..................} \quad (11)$$

but $\frac{\Delta_1}{l_1}$ and $\frac{\Delta_2}{l_2}$ are respectively equal to angles α_1 and α_2. Therefore, $\delta_{1-2} =$

$\alpha_1 - \alpha_2$ = angle C′A′B′ − angle CAB = ΔA, in which ΔA is the variation of the original angle at joint A compared to that formed by straight lines connecting the new positions of the joints. Again, by application of compatibility equations, ϵ_{1-2} may be written easily as

$$\epsilon_{1-2} = \frac{a_{1,3}\,\delta_{1-3} + a_{2,3}\,\delta_{2-3}}{h_{1,2}} \qquad (12)$$

If δ_{1-3} and δ_{2-3} are referred to their appropriate angles C and B (Fig. 5) as $-\Delta C$ and $+\Delta B$, respectively, Eq. 12 can be expressed as:

$$\epsilon_{1-2} = \frac{\Delta B}{\tan B} - \frac{\Delta C}{\tan C} \qquad (13)$$

Example 2.—An important function of the compatibility conditions consists in determining the relative displacement of two arbitrary joints in a frame in terms of elements δ_{p-q} and axial strains ϵ, assuming that the unit deflection δ of one of the bars in the frame is given. For instance, referring to Fig. 6, let it be required to find the displacement of joint D with respect to joint A. Two cases are distinguished:

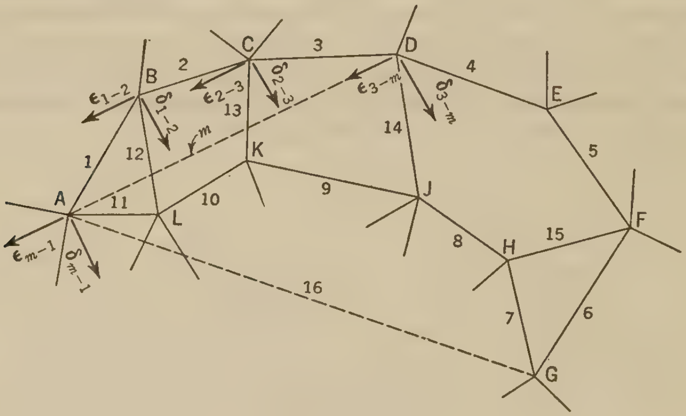

FIG. 6

Case 1.—The bar whose unit deflection δ is given is an actual member in the frame, say, member 8, Fig. 6. Draw line AD (or, as a subscript, simply m). At the joints of the closed circuit ABCDA apply the corresponding vectors δ_{p-q} and ϵ_{p-q}, with the ϵ_{p-q}-vectors parallel to AD, as shown. Equate to zero the moment of these vectors about joint A; ϵ_{3-m} will disappear and δ_{3-m} will be given as a linear function of the vectors applied between joints A and D, such as,

$$\delta_{3-m} \quad \text{or} \quad \delta_3 - \delta_m = F(\delta_{1-2}\ ,\ \epsilon_{1-2}\ ,\ \delta_{2-3}\ ,\ \epsilon_{2-3}) \qquad (14a)$$

In Eq. 14*a*, δ_3 may be expressed in terms of δ_8 (assumed given) and some δ_{p-q}-elements, as follows:

$$\delta_3 = \delta_{3-14} + \delta_{14-8} + \delta_8 \qquad (14b)$$

Substitute this value of δ_3 in Eq. 14*a* and obtain δ_m. The product $l_m\,\delta_m$ is then the movement of joint D at right angles to line AD. Rotating the vectors, in Fig. 6, 90° clockwise and again taking their moment about joint A, δ_{3-m} will disappear and ϵ_{3-m} (or, $\epsilon_3 - \epsilon_m$) will be found as a linear expression of the vectors between joints A and D. The movement of joint D along line AD is then given by $l_m \epsilon_m$.

Case 2.—The bar whose unit deflection δ is given is an imaginary diagonal, say, line 16, Fig. 6, drawn between two joints. Eq. 14*a* holds also for this case; Eq. 14*b* will have to be replaced by:

$$\delta_3 = \delta_{3-14} + \delta_{14-8} + \delta_{8-7} + \delta_{7-16} + \delta_{16} \quad \text{(14c)}$$

In Eq. 14*c*, δ_{16} is assumed known; all the terms in the second member, except δ_{7-16}, refer to actual angles in the frame, these being the only ones which will be necessary to consider in this analysis; δ_{7-16} can be found, and then eliminated in Eq. 14*c*, by locating the orthogonal vectors δ_{p-q} and ϵ_{p-q} at the joints of the closed circuit ALKJHGA (with the ϵ_{p-q}-vectors parallel to line AG) and equating to zero their moment about joint A. The movement of joint D along line AD is found as in case 1.

Corollary to Example 2.—Assuming that the given frame is completely decomposed into triangles by means of imaginary diagonals, the δ_{p-q}-elements at all angles in the frame may be written, as shown in Example 1, in terms of the axial strains of the bars (given or imaginary), and the linear movements of the supports. Therefore, as a corollary to Example 2, it may be concluded that the relative displacement of any two joints may be found in terms of the axial strains of the actual members, those of the imaginary diagonals, and the linear movements of the supports, knowing the unit deflection of any one bar in the frame (for this bar, one joining two supports may be used).

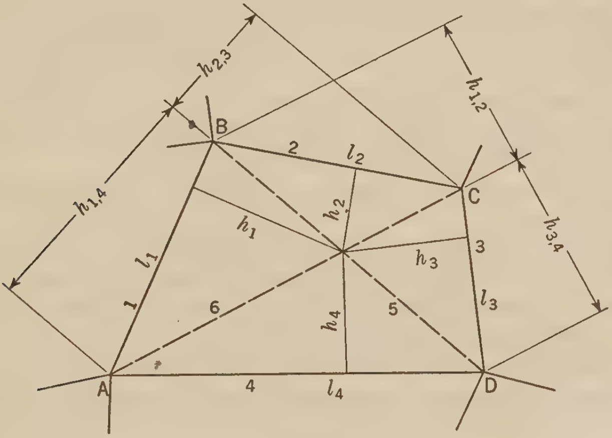

FIG. 7

Example 3.—Compatibility equations may be applied to quadrilateral elements, or polygons of any number of sides, either for the study of their deformation, or, more immediately, as a preliminary for the analysis of redundant polygonal panels. A quadrilateral element is shown in Fig. 7. Imagine it divided into the four triangles ABC, BCD, CDA, DAB. Express δ_{1-2}, δ_{2-3}, δ_{3-4}, and δ_{4-1} by applying the method of Example 1 to each of these triangles, and equate their sum to zero; the following relation is obtained after simplification:

$$2\,A_e\left(\frac{\epsilon_5}{h_{1,4}\,h_{2,3}}+\frac{\epsilon_6}{h_{1,2}\,h_{3,4}}\right)=\frac{l_1\,\epsilon_1}{h_1}+\frac{l_2\,\epsilon_2}{h_2}+\frac{l_3\,\epsilon_3}{h_3}+\frac{l_4\,\epsilon_4}{h_4}\,\ldots\ldots(15)$$

In Eq. 15, A_e represents the area of the quadrilateral and ϵ_5 and ϵ_6 the axial strains of the diagonals. If the axial deformations of the sides are negligible, it takes the form

$$\frac{\epsilon_5}{h_{1,4}\,h_{2,3}}+\frac{\epsilon_6}{h_{1,2}\,h_{3,4}}=0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

which correlates the axial strains of the diagonals in a quadrilateral with rigid sides.

Example 4.—The results of Example 3 will be used for an outline of the determination of the axial forces in the pin-connected truss with one redundant member, shown in Fig. 8. If the redundant member BD is replaced by a pair of opposite forces X, the symbol X being the unknown force in the member, the axial strains in each of the members 1, 2, 3, 4, and 5 will be of the form $\frac{T^1\,X+T^0}{E\,A}$; in this expression, A is the cross-sectional area of the member, and T^1 and T^0 are, respectively, the axial forces found, first, by assuming $X = 1$ and all external loads removed, and second, by assuming $X = 0$ and allowing the external loading to act. The axial strain in the redundant member itself is $\frac{X}{E\,A}$. Substitute these axial strains in the compatibility equation (Eq. 15) for panel ABCD, and solve for the unknown X.

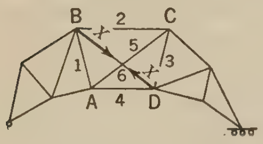

FIG. 8

It may be noted that only the members bounding the panel which contains the redundant member enter into the calculations.

The case in which several redundant members are present may be treated in a similar manner.

Example 5.—The procedure outlined in Example 2 will be applied to a statically determinate frame. Let it be required to find the vertical deflection of joint D with respect to joint A, in the Pratt truss of Fig. 9. (This truss has been analyzed elsewhere by means of Williot-Mohr diagrams.[3])

In this symmetrical truss, symmetrically loaded, $\delta_{11} = 0$. The procedure of case 1, Example 2, applied to the closed circuit ABCDA, reduced to a straight line in this case, gives

$$\delta_{8-m} = -\tfrac{1}{75}\,(25\,\delta_{2-4} + 50\,\delta_{4-8})\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

corresponding to Eq. 14*a*; in Eq. 17, the subscript m designates the imaginary line drawn between joints A and D. The elements ϵ_{p-q} of the circuit do not appear because their moment about A is zero. Also

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[3] "The Theory and Practice of Modern Framed Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, 9th Ed., John Wiley & Sons, Inc., New York, N. Y., Pt. I, p. 306; also Pts. II and III.

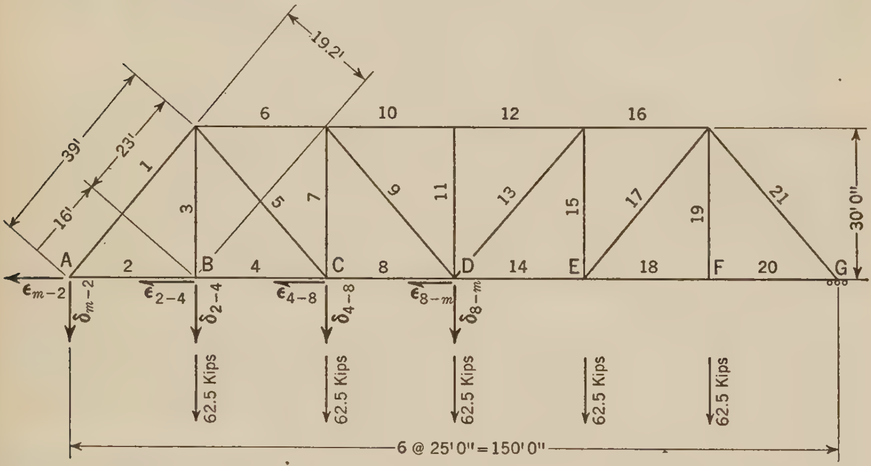
$$\delta_8 = \delta_{8-9} + \delta_{9-11} + \delta_{11} = \delta_{8-9} + \delta_{9-11} \qquad (18)$$

corresponding to Eq. 14*b*. Combining Eqs. 17 and 18,

$$\delta_m = \delta_{8-9} + \delta_{9-11} + \tfrac{1}{3}\,\delta_{2-4} + \tfrac{2}{3}\,\delta_{4-8} \qquad (19a)$$

or

$$\delta_m = \delta_{8-9} + \delta_{9-11} + \tfrac{1}{3}\,(\delta_{2-3} + \delta_{3-4}) + \tfrac{2}{3}\,(\delta_{4-5} + \delta_{5-7} + \delta_{7-8}) \qquad (19b)$$


19.2'
39'
23'
16'
1
2
3
4
5
6
7
8
9
10
11
12
13
14
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16
17
18
19
20
21
A
B
C
D
E
F
G
30'0"
ϵ_{m-2}
δ_{m-2}
ϵ_{2-4}
δ_{2-4}
ϵ_{4-8}
δ_{4-8}
ϵ_{8-m}
δ_{8-m}
62.5 Kips
62.5 Kips
62.5 Kips
62.5 Kips
62.5 Kips
6 @ 25'0"=150'0"

FIG. 9

All δ_{p-q}-terms in Eq. 19*b* are now expressed as functions of the axial strains of the members, using the method of Example 1 in the several triangles forming the truss; their expressions are:

$$\delta_{8-9} = -\frac{30\,\epsilon_7 - 30\,\epsilon_9}{25} \qquad (20a)$$

$$\delta_{9-11} = -\frac{25\,\epsilon_{10} - 25\,\epsilon_9}{30} \qquad (20b)$$

$$\delta_{2-3} = -\frac{39\,\epsilon_1 - 16\,\epsilon_2 - 23\,\epsilon_3}{19.2} \qquad (20c)$$

$$\delta_{3-4} = -\frac{39\,\epsilon_5 - 23\,\epsilon_3 - 16\,\epsilon_4}{19.2} \qquad (20d)$$

$$\delta_{4-5} = -\frac{30\,\epsilon_3 - 30\,\epsilon_5}{25} \qquad (20e)$$

$$\delta_{5-7} = -\frac{25\,\epsilon_6 - 25\,\epsilon_5}{30} \qquad (20f)$$

and

$$\delta_{7-8} = -\frac{39\,\epsilon_9 - 23\,\epsilon_7 - 16\,\epsilon_8}{19.2} \qquad (20g)$$

The substitution of these values in Eq. 19*b* will give δ_m; thus:

$$\delta_m = -0.68\,\epsilon_1 + 0.28\,\epsilon_2 + 0.28\,\epsilon_4 + 0.68\,\epsilon_5 - 0.56\,\epsilon_6 - 0.40\,\epsilon_7 + 0.56\,\epsilon_8 + 0.68\,\epsilon_9 - 0.83\,\epsilon_{10} \qquad (21)$$

In this problem, the axial forces T in the members can be found immediately by statics; the corresponding axial strains are of the form $\epsilon = \dfrac{T}{EA}$.

Substituting the numerical values[3] for the ϵ-terms, $\delta_m = \dfrac{27{,}820}{29{,}000{,}000} = \dfrac{9.55}{10{,}000}$. The vertical movement of joint D is then $75 \times 12\,\delta_m = 0.86$ in.

Topology of Frameworks

The given frame having been completely decomposed into triangular elements by drawing imaginary diagonals, the axial strains of these diagonals, those of the given members, and the linear movements of the supports (see Example 1 for the reason of their introduction), are used as elements for the determination of the displacements and stresses in the frame. Their immediate connection with some of the important aspects of the problem has been illustrated in preceding sections. Their essential rôle will be further disclosed as the analysis proceeds.

These elements—axial strains and support movements—may be interdependent. This may be noted, for instance, by considering the plane deformation of a quadrilateral panel with two diagonals; evidently, the axial strains of any five of the six bars will determine that of the sixth.

The reduction of these elements to a minimum number which are independent, the determination of this number, the exposition of other properties connected with the configuration of the frame (such as relations between number of regions, members, joints, etc.) and leading ultimately to the solution of the independent elements—all result from a study of the topology of the frame.

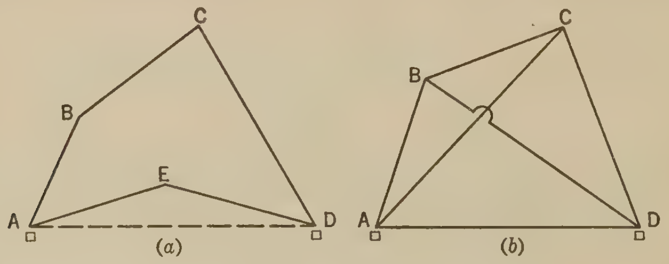

Fig. 10

A region in a framework is defined in this paper as the space within a closed circuit formed by some of the given members; or, within a closed circuit formed by some of the given members and an imaginary line joining two supports. For instance, the frame in Fig. 10(*a*) has three regions—ABCDEA, ABCDA, AEDA; that of Fig. 10(*b*) has five—namely, ABCA, ACDA, BCDB, ABDA, and ABCDA. A distinct region is defined as one whose boundary contains at least one bar (not joining 2 supports) which has not already been used in circumscribing other regions; on this basis, the first frame has only two distinct regions, and the second only three, among the several regions enumerated for each of them. It is immaterial which particular regions are considered as distinct among all others resulting from the configuration of the frame; for reasons of simplicity, it is well to choose those with the smallest number of sides. In this analysis, only distinct regions will be considered.

Let:

n_r = total number of distinct regions in a frame, exclusive of those with all their vertices at supports;

n_m = total number of given members, exclusive of those connected to a support at each end;

n_p = total number of joints, exclusive of the supports.

(The reason for the exclusion from this list of members joining two supports, is that their axial strains and unit deflections are assumed to be expressed in terms of the linear movements of the supports. Similarly, the supports are not included among the number of the joints for the reason that the equilibrium and displacement requirements at the supports will be disposed of in the establishment of boundary conditions. The limitation imposed upon the number of regions is due to similar conditions.)

Proposition (*a*).—Between the numbers of regions, members and joints as defined above there exists the following relation:

$$n_r = n_m - n_p \qquad (22)$$

(Compare Eq. 22 to Euler's equation[2] relative to the number of faces, edges, and vertices of polyhedra.)

Eq. 22 is, of course, evident in the case of a rudimentary system composed of one member and one support ($n_r = 0$, $n_m = 1$, $n_p = 1$). It will be sufficient to show that it remains true when successive members, joints, and supports are added to it to form an arbitrary frame. For instance, at the free end of this system attach a closed region of n members. Then $n_r = 1$, $n_m = n + 1$, $n_p = n$, and Eq. 22 is satisfied. Next, add one member between two of the joints, or between one of the joints and the support, or between one of the joints and a new support: There are now one more region and one more member; the number of the joints has not changed; and the relation still applies. Add a member between two supports: The number n_r does not change; neither does n_m, nor n_p; and the relation is again valid. It will remain valid after the addition of a new member connecting a joint or a support with some point on one of the members, thus creating a new joint at that point and converting one member into two (this will result in one more region, two additional members, and one new joint).

Similarly, it may be shown that Eq. 22 remains valid at each stage of growth resulting from any other type of addition to the frame.

For example, in Fig. 9, $n_m = 21$, $n_p = 10$, $n_r = 21 - 10 = 11$, these regions being the ten triangles subdividing the truss, and the space between the bottom chord and the line of supports (which coincides in this case with the bottom chord).

Proposition (*b*).—From Eq. 22, the total number of distinct triangular elements n_t (excluding those with all their vertices at supports) into which the frame is decomposed by diagonals is given by:

$$n_t = (n_m + n_{mt}) - n_p \ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

In this relation n_{mt} represents the total number of imaginary diagonals required for the complete decomposition of the frame into triangles, exclusive of those joining two supports.

After the frame has been divided into triangles, a total of $2\,n_t$, compatibility equations (Eqs. 4) may be written for the n_t triangular elements forming the transformed frame. These equations will be in terms of the unit deflections δ and δ_t, and the axial strains ϵ and ϵ_t of the given members and the imaginary diagonals, respectively. They will also involve the linear movements of the supports. Consider the $(n_m + n_{mt})$-elements δ and δ_t eliminated between these $2\,(n_m + n_{mt} - n_p)$ equations; and there will remain $2\,(n_m + n_{mt} - n_p) - (n_m + n_{mt}) = n_m + n_{mt} - 2\,n_p$ linear relations between the elements ϵ and ϵ_t and the linear movements of the supports; call n_i the number: $n_m + n_{mt} - 2n_p$.

Proposition (*c*).—As a result of Proposition (*b*), calling n_σ the number of the support movements, it may be concluded that a number n_i of the $n_m + n_{mt}$ axial strains ϵ and ϵ_t are dependent upon, and may be determined as functions of, the remaining $n_m + n_{mt} + n_\sigma - n_i = n_m + n_{mt} + n_\sigma - (n_m + n_{mt} - 2\,n_p) = 2\,n_p + n_\sigma$ elements (axial strains and linear movements of supports), considered as independent. This last number reduces to $(2\,n_p - n_m) + n_\sigma$ in the assumption of negligible axial strains for the n_m given members. In other words, the degree of freedom of a frame in joint displacement is $2\,n_p + n_\sigma$ in the general case, and $(2\,n_p - n_m) + n_\sigma$ if the axial deformations of the n_m given members are assumed to be negligible; (this is, of course, in addition to its displacement as a whole).

In most cases, these n_i-relations may be obtained easily by expressing the value of δ_{p-q} for each angle at an interior joint or interior support, in terms of the ϵ-elements and ϵ_t-elements of the neighboring bars and the linear movements of the supports (see Example 1), and writing the self-evident relation $\sum \delta_{p-q} = 0$ at these points.

An illustration is afforded by the frame in Fig. 13 (given subsequently), in which $n_m = 5, n_p = 3, n_t = 4, n_{mt} = 2$; number of regions $n_r = n_m - n_p = 2$; number of triangles $n_t = n_m + n_{mt} - n_p = 4$; $n_i = n_m + n_{mt} - 2\,n_p = 1$ ($\sum \delta_{p-q} = 0$ at support F, Fig. 13); degree of freedom in joint displacement, $2\,n_p = 6$, or $2\,n_p - n_m = 1$.

The application of this analysis to articulated frames leads to their classification as statically determinate, redundant, unstable, and redundant-unstable, and to criteria for such classification.[4]

BOUNDARY CONDITIONS

In preparing to find end moments in terms of the unknowns adopted in this analysis, boundary conditions will be expressed in a form excluding the reactions of the supports; their expression will involve only axial strains, linear and angular movements of supports, and end moments.

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[4] "Displacements and Stresses in the General Two-Dimensional Framework," by Yves Nubar; submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy, Faculty of Pure Science, Columbia Univ., New York, N. Y., 1941. On file for reference in Engineering Societies Library, New York, N. Y.

A support of a general type is shown at point a in Fig. 11. Let X_a, Y_a, and M_a be the reactions at the support and e_{xa}, e_{ya}, ω_a the corresponding linear and angular movements. At this support, boundary conditions in their general form may be represented by three expressions between these six elements, such as

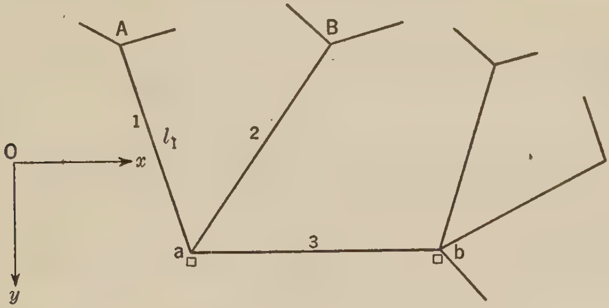

FIG. 11

$$\left.\begin{aligned} F_1(X_a, Y_a, M_a, e_{xa}, e_{ya}, \omega_a) &= 0 \\ F_2(X_a, Y_a, M_a, e_{xa}, e_{ya}, \omega_a) &= 0 \\ F_3(X_a, Y_a, M_a, e_{xa}, e_{ya}, \omega_a) &= 0 \end{aligned}\right\} \quad (24)$$

In practical cases, these expressions are of much simpler type; for instance, at a hinged, immovable joint, they reduce to: $e_{xa} = 0$, $e_{ya} = 0$, $M_a = 0$; and, at a support on rollers, they may be written as: $e_{ya} = 0$, $X_a = 0$, $M_a = 0$.

Reactions X_a, Y_a, and M_a can be eliminated from Eqs. 24 as follows: The moments and forces in each of the members (for instance, member 1) meeting at the support are of the form: Axial force, $E A_1 \epsilon_1$; end moments, M_{1a} and M_{1A}; transverse end force, $\dfrac{M_{1a} + M_{1A}}{l_1}$ (this expression will also involve lateral loads on the member, if any). Using these expressions, the equilibrium conditions at the support are:

$$X_a + \sum(E A_1 \epsilon_1)_x + \sum\left(\frac{M_{1a} + M_{1A}}{l_1}\right)_x = 0 \quad (25a)$$

$$Y_a + \sum(E A_1 \epsilon_1)_y + \sum\left(\frac{M_{1a} + M_{1A}}{l_1}\right)_y = 0 \quad (25b)$$

and

$$M_a + \sum M_{1a} = 0 \quad (25c)$$

Solve for X_a, Y_a, and M_a and introduce the result in Eqs. 24. These relations will represent the boundary conditions at support a, Fig. 11, but will involve only the linear and angular movements of the support, the axial strains and end moments in the members connected there.

If line 3, Fig. 11, is only an imaginary diagonal joining two supports, the forces and moments corresponding to it will naturally disappear from Eqs.

24 and 25. If it is an actual member, its end moments, transverse and axial forces may be assumed to have been written in terms of the angular and linear movements of the supports, using the slope deflection[5] equations of the member.

DETERMINATION OF END MOMENTS

The proposition will be established that the end moments in the members of a rigid frame can be found as linear functions of the δ_{p-q}-elements at the angles formed by the given members of the frame and the linear movements of the supports.

It was shown in Example 1 that, if the frame is completely decomposed into triangular elements, these δ_{p-q}-terms themselves may be expressed as functions of the axial strains ϵ and ϵ_t of the bars (given members and imaginary diagonals) and the linear movements of the supports. Consequently, the aforementioned proposition will result in a linear dependence of all end moments upon the unknowns, ϵ, ϵ_t, and the support movements.

When such a dependence is established, it will follow that the axial strains ϵ and ϵ_t, together with the linear movements of the supports, can specify completely the system of displacements and stresses in the general two-dimensional frame; this conclusion is valid, because:

(1) The axial force T in a member equals $E\,A\,\epsilon$;

(2) The transverse forces at the ends of a member may be found from the end moments by means of the equilibrium conditions of the member;

(3) The relative displacement of any two joints can be determined in terms of the axial strains ϵ and ϵ_t and the linear movements of the supports (see "Corollary to Example 2");

(4) The relative rotation of two joints may be found from the end moments of the members by application of slope deflection[5] equations (see, for instance, Eqs. 28, given subsequently);

(5) The boundary conditions, as written in the preceding section, furnish the angular and linear movements of the supports in terms of axial strains, and end moments.

The aforementioned proposition is demonstrated by first establishing the following property of rigid joints:

I.—If, at a joint of n members, there exists one linear relation between the end moments of each of $n - 1$ members (a total of $n - 1$ relations), another linear relation may be found between the end moments of the nth member. This expression involves the coefficients of the $n - 1$ linear relations and the terms δ_{p-q} at the joint.

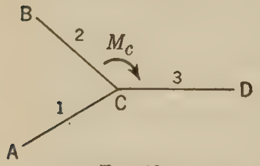

FIG. 12

A joint of three members will be considered for simplicity, the demonstration being similar for a joint of any number of members. Let M_{1A} and M_{1C} designate the moments exerted by member 1, Fig. 12, upon joints A and C, respectively; m_{1A} and m_{1C} designate the same moments if joints A and C do not rotate, and if member AC does not deflect laterally (fixed end moments);

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[5] "Analysis of Statically Indeterminate Structures by the Slope Deflection Method," by W. M. Wilson, F. E. Richart, and Camillo Weiss, *Bulletin No. 108*, Eng. Experiment Station, Univ. of Illinois, Urbana, 1918.

ω (with the subscript of a joint) represents the rotation of the joint common to all members connected to it. These moments and rotations are taken as positive in a clockwise direction.

For members 1 and 2, Fig. 12, assume given two linear relations:

$$M_{1A} + C_{1C}\, M_{1C} = D_{1C} \quad \text{(26a)}$$

and

$$M_{2B} + C_{2C}\, M_{2C} = D_{2C} \quad \text{(26b)}$$

C_{1C}, D_{1C}, C_{2C}, D_{2C} representing given coefficients.

It is proposed to derive a similar relation

$$M_{3C} + C_{3D}\, M_{3D} = D_{3D} \quad \text{(26c)}$$

for member 3—that is, to determine coefficients C_{3D} and D_{3D}.

With the adopted sign conventions, the slope deflection equations[5] for member 1 are

$$M_{1A} = -2\, E\, K_1\, (2\, \omega_A + \omega_C - 3\, \delta_1) + m_{1A} \quad \text{(27a)}$$

and

$$M_{1C} = -2\, E\, K_1\, (2\, \omega_C + \omega_A - 3\, \delta_1) + m_{1C} \quad \text{(27b)}$$

in which K_1 is the stiffness $\frac{I_1}{l_1}$ of member 1. Solving for ω_A and ω_C:

$$\omega_A = \delta_1 + \frac{[(M_{1C} - m_{1C}) - 2\, (M_{1A} - m_{1A})]}{6\, E\, K_1} \quad \text{(28a)}$$

and

$$\omega_C = \delta_1 + \frac{[(M_{1A} - m_{1A}) - 2\, (M_{1C} - m_{1C})]}{6\, E\, K_1} \quad \text{(28b)}$$

Consideration of members 2 and 3 will furnish the following additional expressions for ω_C:

$$\omega_C = \delta_2 + \frac{[(M_{2B} - m_{2B}) - 2\, (M_{2C} - m_{2C})]}{6\, E\, K_2} \quad \text{(29a)}$$

and

$$\omega_C = \delta_3 + \frac{[(M_{3D} - m_{3D}) - 2\, (M_{3C} - m_{3C})]}{6\, E\, K_3} \quad \text{(29b)}$$

Subtract Eq. 29b from Eq. 28b and Eq. 29b from Eq. 29a:

$$\begin{aligned} 6\, E\, \delta_{1-3} &+ \frac{1}{K_1}\, (M_{1A} - 2\, M_{1C}) + \frac{1}{K_3}\, (2\, M_{3C} - M_{3D}) \\ &= \frac{1}{K_1}\, (m_{1A} - 2\, m_{1C}) + \frac{1}{K_3}\, (2\, m_{3C} - m_{3D}) \quad \text{(30a)} \end{aligned}$$

and

$$\begin{aligned} 6\, E\, \delta_{2-3} &+ \frac{1}{K_2}\, (M_{1B} - 2\, M_{2C}) + \frac{1}{K_3}\, (2\, M_{3C} - M_{3D}) \\ &= \frac{1}{K_2}\, (m_{2B} - 2\, m_{2C}) + \frac{1}{K_3}\, (2\, m_{3C} - m_{3D}) \quad \text{(30b)} \end{aligned}$$

By Eqs. 26*a* and 26*b*:

$$\frac{C_{1C}+2}{K_1} M_{1C} = \frac{1}{K_3}(2\,M_{3C} - M_{3D}) + 6\,E\,\delta_{1-3} + \frac{D_{1C}}{K_1}$$
$$- \frac{1}{K_1}(m_{1A} - 2\,m_{1C}) - \frac{1}{K_3}(2\,m_{3C} - m_{3D}) \ldots\ldots\ldots\ldots (31a)$$

and

$$\frac{C_{2C}+2}{K_2} M_{2C} = \frac{1}{K_3}(2\,M_{3C} - M_{3D}) + 6\,E\,\delta_{2-3} + \frac{D_{2C}}{K_2}$$
$$- \frac{1}{K_2}(m_{2B} - 2\,m_{2C}) - \frac{1}{K_3}(2\,m_{3C} - m_{3D}) \ldots\ldots\ldots\ldots (31b)$$

If M_{1C} and M_{2C} from Eqs. 31 are substituted in the equilibrium equation of the joint—

$$M_{1C} + M_{2C} + M_{3C} + M_C = 0 \ldots\ldots\ldots\ldots\ldots\ldots (32)$$

in which M_C represents, for generality, an external couple applied at joint C, the following is the result:

$$2\,M_{3C}\left(\frac{K_1}{C_{1C}+2} + \frac{K_2}{C_{2C}+2} + \frac{K_3}{2}\right)$$
$$- M_{3D}\left(\frac{K_1}{C_{1C}+2} + \frac{K_2}{C_{2C}+2}\right) = -K_3\,U \ldots\ldots\ldots\ldots (33)$$

in which

$$U = +M_C + \frac{K_1}{C_{1C}+2}\left[6\,E\,\delta_{1-3} + \frac{1}{K_1}(D_{1C} - m_{1A} + 2\,m_{1C}) - \frac{1}{K_3}(2\,m_{3C} - m_{3D})\right]$$
$$+ \frac{K_2}{C_{2C}+2}\left[6\,E\,\delta_{2-3} + \frac{1}{K_2}(D_{2C} - m_{2B} + 2\,m_{2C}) - \frac{1}{K_3}(2\,m_{3C} - m_{3D})\right] \ldots\ldots\ldots\ldots (34)$$

Coefficients C_{3D} and D_{3D} in Eq. 26*c* are found by comparing this equation to Eq. 33; thus:

$$C_{3D} = -\frac{\dfrac{K_1}{C_{1C}+2} + \dfrac{K_2}{C_{2C}+2}}{2\left(\dfrac{K_1}{C_{1C}+2} + \dfrac{K_2}{C_{2C}+2} + \dfrac{K_3}{2}\right)} \ldots\ldots\ldots\ldots (35a)$$

and

$$D_{3D} = -\frac{K_3\,U}{2\left(\dfrac{K_1}{C_{1C}+2} + \dfrac{K_2}{C_{2C}+2} + \dfrac{K_3}{2}\right)} \ldots\ldots\ldots\ldots (35b)$$

This establishes the property stated in paragraph I.

The relations developed in this section may be considered to represent compatibility conditions in joint rotations, since they imply members rigidly connected to each other.

In case another member CE (numbered 4) is hinged at joint C of Fig. 12 (this member not shown), it is to be considered as nonexistent, so far as the operations of this section are concerned; its coefficients C_{4E} and D_{4E} are evidently zero.

The generalization of Eqs. 35 for application to joints of any number of members is obvious by inspection of the structure of these formulas. Coefficients C depend only upon the stiffness of the members. Therefore, they may be written once and used for all cases of loading and lateral deflections in the frame.

Eqs. 35 represent recurrence relations which make it possible to correlate end moments in a series of members by passage from one group of members to adjacent ones. The process of determining the successive coefficients C and D for consecutive members may be described as follows:

The determination of coefficients C and D may start at a member connected to a support. Two cases will be considered, depending on the type of this support, taken as the origin.

Case 1.—In the first case, the origin is a simply connected support; that is, only one member is connected to it. If this member is hinged at the origin, its coefficients C and D equal zero; if this is not the case, to find its coefficients C and D, consideration is given to Eq. 28*a* relative to this member. In this equation, the support rotation ω (if not zero) can be replaced by the end moments and the linear movements of the support, using the appropriate boundary condition therein which correlates these elements. The lateral deflection δ in Eq. 28*a* can also be written in terms of some adjacent δ_{p-q}-elements and some support movements (this is true for even more complex types of supports, such as support a of Fig. 11, in which $\delta_1 = \delta_{1-2} + \delta_{2-3} + \delta_3$). The resulting expression will be a linear relation involving the end moments of the member; its coefficients are the required values of C and D for this member; coefficients D will be in terms of the δ_{p-q}-elements and the support movements.

Case 2.—In the second case, the origin is a multiply connected support; that is, it is situated on the periphery of one or more closed circuits, such as joint a of Fig. 11. The rotation of the support ω (if not zero), common to all members meeting it, is maintained in equations such as Eq. 28*a* corresponding to each one of these members. Their lateral deflections δ can be replaced, the same as in case 1, using neighboring δ_{p-q}-elements and linear movements of adjacent supports. The resulting linear equations, involving the end moments, will again furnish the coefficients C and D for each member, except that, in this case, coefficients D will also be functions of the support rotation ω as a parameter, used for "opening up" the closed circuits at the origin. It will be eliminated in the remaining operations which will be described in the next paragraph. In case all members are hinged at the common origin, their coefficients C and D naturally vanish.

Coefficients C and D for one or several starting members having been found, operations may proceed continuously in one direction, using Eqs. 35. Successive sets of C and D coefficients for each span are written as functions of those preceding, all coefficients D in linear terms of δ_{p-q}-elements. Each set signifies one linear relation between the end moments of a member. A span will be reached finally where the end moments are already correlated by another equation resulting from a given boundary condition. This will mean two equations in terms of two end moments. Their solution will yield these moments which then can be carried backward toward the origin, for the determination of all end moments in preceding spans. For case 2, all these moments will contain the rotation ω of the origin as a parameter. This rotation can now be determined by writing $\sum M = 0$ at the origin; then it can be replaced in the expressions of the moments.

The operations are similar to the foregoing when both the origin and terminal support are multiply connected, or when the frame contains closed regions. In such cases, a joint or support rotation can again be maintained in the formulas for the purpose of "opening up" closed regions; later it can be eliminated. Several adjacent closed regions may be "opened up" by the use as a parameter of a single joint rotation.

The general approach presented herein may be greatly simplified in each individual case of practical application.

The principal conclusion to be drawn from this discussion is that the end moments in a frame can be found as linear functions of the δ_{p-q}-elements at the angles and the linear movements of the supports. This proves the proposition stated at the head of this section.

A geometric interpretation of the property of rigid joints stated in paragraph I is the following: If, at a joint of n members, the moment lines of all but one of the members pass through known fixed points ($n - 1$ in all), the moment line of the remaining member will also pass through a fixed point which may be located using the coordinates of the known points, and the δ_{p-q}-elements at the joint. Upon this interpretation, a generalized theory of fixed points may be based for the determination of end moments by means of semi-graphical operations.[4] (See also the work[6] of L. H. Nishkian, and D. B. Steinman, Members, Am. Soc. C. E.)

DETERMINATION OF AXIAL STRAINS AND THE LINEAR MOVEMENTS OF SUPPORTS

The essential function of the group of the axial strains ϵ and ϵ_t of the given and imaginary members and the linear movements of the supports, in specifying the system of displacements and stresses in a frame, has been described in the preceding part of this paper. The determination of this group of elements is the object of this section.

In some problems, the displacements of the joints and the supports are supplied as part of the data; in such cases, the analysis of the frame is comparatively simple: All δ_{p-q}-elements appearing in Eqs. 34 and 35 assume known values; and all end moments, axial forces, and shears are found at the conclusion of the operations of the preceding section. In other problems, all joints are hinged, and the axial forces in the members may be determined by simple statics; the axial strains are then available and can be used for the determination of all deflections (see "Corollary to Example 2" and "Example 5"). In general, however, the frame is rigid and the joints and supports are subject to unknown displacements. The determination of axial strains ϵ and ϵ_t and support movements is then necessary, so far as the present analysis is concerned.

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[6] "Moments in Restrained and Continuous Beams by the Method of Conjugate Points," by L. H. Nishkian and D. B. Steinman, *Transactions*, Am. Soc. C. E., Vol. 90 (1927), pp. 1-206.

At this stage of the analysis, end moments, hence transverse end forces in all given members, are assumed to have been written as functions of $n_m + n_{mt} + n_\sigma$, axial strains ϵ and ϵ_t, and support movements. More precisely, they may be assumed to be expressed in terms of the quantity $2\,n_p + n_\sigma$ of these elements considered as independent (see "Topology of Frameworks: Proposition (*c*)").

In the given frame decomposed completely into triangular elements by means of n_{mt} imaginary diagonals, the axial strains in n_i of the bars are considered to be dependent upon the others (see Proposition (*c*)). Remove from the resulting figure n_i bars judiciously chosen, and replace all joints and supports by perfect hinges. The structure thus obtained will be called a conjugate frame (see Fig. 13, given subsequently). This frame has the same n_p joints as the original; but the number of its bars is $(n_m + n_{mt}) - n_i = n_m + n_{mt} - (n_m + n_{mt} - 2\,n_p) = 2\,n_p$. At the joints of the conjugate frame apply the corresponding joint loads of the original, and, in addition, load them with the transverse end force exerted by each given member. So far as the forces in the bars are concerned, the conjugate frame is statically determinate since it has $2\,n_p$ bars and n_p joints, each furnishing two equilibrium equations, $\sum X = 0$ and $\sum Y = 0$. The solution of this frame will yield $2\,n_p$ axial forces as functions of the $2\,n_p + n_\sigma$ independent unknowns. Equate to zero the axial forces corresponding to the imaginary diagonals; equate to $E\,A\,\epsilon$ of the corresponding member, those belonging to the given members; the result is a system of $2\,n_p$ equations for the determination of $2\,n_p$ unknown axial strains in terms of n_σ support movements. These movements can then be found by the use of the corresponding boundary conditions.

In case the axial strains of the given members are assumed to be negligible—a very frequent case in rigid frames—the number of independent axial strains decreases from $2\,n_p$ to $2\,n_p - n_m$. It will then be sufficient to solve only for the axial forces in the $n_{mt} - n_i$ $(= 2\,n_p - n_m)$ imaginary diagonals and to equate them to zero. Often, a convenient way of doing this is to apply the equilibrium equation $\sum(M) = 0$ to a portion of the conjugate frame on one side of some line X–X, treated as a free body; if this line is drawn to intersect one of the imaginary diagonals, and if the center of moments is chosen with care, this equation may contain none (or only a few) of the axial forces in the members (these forces are of no immediate interest in this case). The result will be a relation involving only the loads at the joints of the conjugate frame, that is, a relation between the $(2\,n_p - n_m) + n_\sigma$ independent unknowns. By varying the line X–X and the center of moments, additional relations of the same type may be obtained (see Illustrative Problem, and Fig. 13, given subsequently).

The treatment of the case (mentioned at the beginning of this section), in which the joints of a frame are subjected to given displacements rather than loads, can now be amplified further. These displacements immediately specify all elements δ, δ_{p-q}, ϵ, ϵ_t. If the movements of the supports are also assumed given, all end moments, axial and transverse end forces throughout the frame will be known in terms of these displacements; the $2\,n_p$ equations of equilibrium at the joints of the conjugate frame will then furnish the $2\,n_p$ components X and Y of the n_p joint constraints corresponding to, and resulting in, the joint displacements impressed upon the frame.

APPLICATION TO THE GENERAL CASE

The method presented in this paper will be applied to the general case of a frame in which axial deformations are not negligible; the actual solution is best described by decomposing it into the following sequence. This will help to recapitulate the procedure of the analysis.

Step 1.—Divide the frame completely into simple triangular panels by means of imaginary diagonals. For the $2\,n_p + n_\sigma$ independent unknowns (see "Topology of Frameworks: Proposition (*c*)"), choose (in addition to the n_σ linear movements of supports) the axial strains of $2\,n_p$ of the given members, if $2\,n_p \leqq n_m$. Choose the axial strains of all n_m given members and $2\,n_p - n_m$ of the imaginary diagonals, if $2\,n_p > n_m$. Some judgment must be exercised in this choice, with a view to convenience in the operations of step 2.

Step 2.—Proceeding as in Example 1, express all δ_{p-q}-elements, one at each angle formed by given members, in terms of the independent unknowns. This is done conveniently by first writing these δ_{p-q}-values using the axial strains of all the bars in the vicinity, and then eliminating the dependent axial strains by means of the n_i-relations (see "Topology of Frame-work: Proposition (*c*)").

Step 3.—By application of the procedure described under the heading, "Determination of End Moments," express all end moments as functions of δ_{p-q}-elements (hence, as functions of the independent unknowns); if the end moments determined thus involve also the rotations at some of the multiply connected supports (or at joints on the periphery of closed regions), these rotations may be found and eliminated by the use of $\sum M = 0$ at such supports or joints.

Step 4.—Using these end moments, express all transverse end forces as functions of the independent unknowns; load the conjugate frame and solve for the $2\,n_p$ independent axial strains in terms of the n_σ linear movements of the supports (see heading, "Determination of Axial Strains and the Linear Movements of Supports").

Step 5.—Use the boundary conditions and determine the displacements of the supports. Of three boundary conditions at each support, one, relating to the support rotation and equivalent to $\sum M = 0$, will yield the rotation of the support, if not already found in step 3 of this sequence. The remaining two boundary conditions will furnish the linear movements of the support, and Eqs. 25 will give the reactions.

All stresses and displacements throughout the frame may now be found using the $2\,n_p + n_\sigma$ independent unknowns just determined.

The general solution outlined herein can be simplified appreciably if one chooses to analyze a rigid frame, as a preliminary step, under the assumption of negligible axial deformations in the members. Such an analysis (outlined under a subsequent heading, "Application to a Less General Case") will yield axial forces in the given members which, in general, are correct enough for a first approximation. The axial strains of these members may then be computed and used in the compatibility equations of step 2 (this sequence), which will then contain a small constant term (corresponding to these axial strains), in addition to the axial strains ϵ_t of imaginary diagonals and the movements of the supports. The solution can then proceed, yielding another set of axial forces, which may be used for a second cycle of the same operations, each cycle altering the small constant term in the compatibility equations to a still smaller extent. The procedure may be repeated as many times as desired. In general, one cycle is ample when dealing with most rigid frames.[4]

Application to a Less General Case

The analysis of a rigid frame in which the axial deformations of the n_m given members are assumed to be negligible proceeds along steps similar to those involved in the general case. The number of independent unknowns becomes $(2\,n_p - n_m) + n_\sigma$, comprising $(2\,n_p - n_m)$ axial strains (this is also the number of imaginary diagonals in the conjugate frame) and n_σ support movements; $2\,n_p$ must be greater than n_m, otherwise, the axial deformations of the given members may not all be assumed negligible. The determination of the independent unknowns is much simpler than in the general case, since, as has been noted before, it will be sufficient to write only that the axial forces in the imaginary diagonals vanish.

If they are of interest, the axial forces in the given members may be found by the solution of the conjugate frame, in the same manner as in any statically determinate frame.

Illustrative Problem

Although somewhat different from the ordinary type of structures, the following case was chosen in order to illustrate as many points of the preceding analysis as possible.

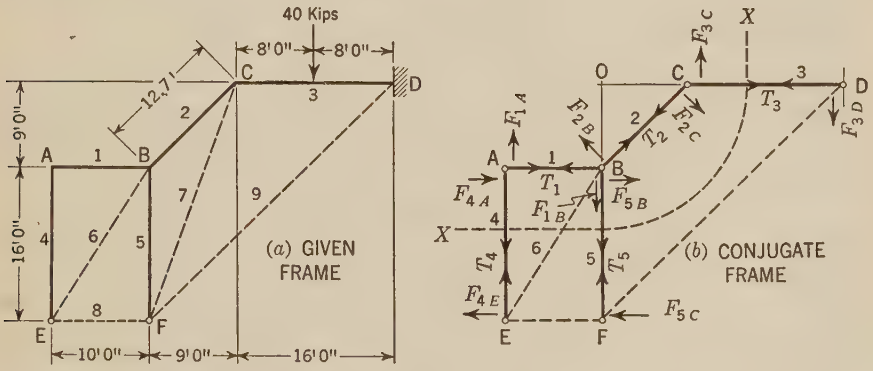

FIG. 13.—(STIFFNESS FACTORS: $K_1 = K_4 = 1$; $K_2 = K_5 = 2$; AND $K_3 = 3$)

The rigid frame of Fig. 13(a) is to be analyzed for moments and deflections. Linear movements of the supports are prevented, supports E and F are hinged, and support D fully restrained; axial deformations of the members are assumed to be negligible; units are in feet and kips. The steps in the procedure correspond to those in "Application to the General Case."

Step 1.—Draw imaginary diagonals 6, 7, 8, and 9. Due to the nature of the supports, $\delta_8 = \delta_9 = 0$, $\epsilon_8 = \epsilon_9 = 0$; and, because of negligible axial deformations, $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_4 = \epsilon_5 = 0$. The number of independent unknowns is $2\,n_p - n_m + n_\sigma = 2 \times 3 - 5 + 0 = 1$. The axial strain ϵ_6 of imaginary diagonal 6 will be taken as this unknown.

Step 2.—By application of Example 1, the expressions for the several δ_{p-q}-elements (using scaled dimensions) are as follows:

$$\left.\begin{array}{rl} \delta_{8-4} = \delta_{8-6} + \delta_{6-4} = & -\,2.22\,\epsilon_6 \\ \delta_{4-1} = & +\,2.22\,\epsilon_6 \\ \delta_{1-5} = \delta_{1-6} + \delta_{6-5} = & -\,2.22\,\epsilon_6 \\ -\,\delta_{8-5} = \delta_{5-8} = & +\,2.22\,\epsilon_6 \end{array}\right\}\ldots\ldots\ldots\ldots(36)$$

$$\left.\begin{array}{rl} -\,\delta_{5-9} = \delta_{9-5} = \delta_{9-7} + \delta_{7-5} = & -\,1.78\,\epsilon_7 \\ \delta_{5-2} = & +\,4.92\,\epsilon_7 \\ \delta_{2-3} = \delta_{2-7} + \delta_{7-3} = & -\,4.92\,\epsilon_7 \\ \delta_{3-9} = & +\,1.78\,\epsilon_7 \end{array}\right\}\ldots\ldots\ldots\ldots(37)$$

The number of n_i-relations is $5 + 2 - 2 \times 3 = 1$. This is the equation $\sum \delta_{p-q} = 0$ at the interior support F, or $\delta_{8-5} + \delta_{5-9} = 0$; that is, $-\,2.22\,\epsilon_6 + 1.78\,\epsilon_7 = 0$. Hence: $\epsilon_7 = 1.245\,\epsilon_6$. Substituting this value of ϵ_7 in Eqs. 37:

$$\left.\begin{array}{ll} \delta_{9-5} = -.2.22\,\epsilon_6; & \delta_{5-2} = +\,6.12\,\epsilon_6 \\ \delta_{2-3} = -\,6.12\,\epsilon_6; & \delta_{3-9} = +\,2.22\,\epsilon_6 \end{array}\right\}\ldots\ldots\ldots\ldots(38a)$$

and

$$\delta_{1-2} = \delta_{1-5} + \delta_{5-2} = 3.90\,\epsilon_6\ldots\ldots\ldots\ldots\ldots\ldots(38b)$$

Step 3.—The coefficients C and D in each successive member are determined by Eqs. 34 and 35, in which all δ_{p-q}-elements are replaced by their equivalents, Eqs. 36 and 38. Operations begin at joint E and proceed toward joint D $\left(\text{note that } m_{3C} = +\dfrac{16 \times 40}{8} = +\,80,\ m_{3D} = -\,80\right)$:

Since $M_{4E} = 0$—
in member 4, $C_{4A} = 0$; and

$$D_{4A} = 0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(39a)$$

in member 1, $C_{1B} = -\,0.25$; and

$$D_{1B} = -\,3.33\,E\,\epsilon_6\ldots\ldots\ldots\ldots\ldots\ldots\ldots(39b)$$

Since $M_{5F} = 0$—
in member 5, $C_{5B} = 0$; and

$$D_{5B} = 0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(40a)$$

in member 2, $C_{2C} = -\,0.305$; and

$$D_{2C} = -\,18.75\,E\,\epsilon_6\ldots\ldots\ldots\ldots\ldots\ldots\ldots(40b)$$

in member 3, $C_{3D} = -\,0.22$; and

$$D_{3D} = +\,30.4\,E\,\epsilon_6 + 52.8\ldots\ldots\ldots\ldots\ldots\ldots(40c)$$

Using the values for C_{3D} and D_{3D} from Eqs. 40*c* in the equation $M_{3C} + C_{3D} \times M_{3D} = D_{3D}$, for member 3:

$$M_{3C} - 0.22\, M_{3D} = +\,52.8 + 30.4\, E\, \epsilon_6 \tag{41a}$$

Making $\omega_D = 0$ in the slope deflection equation for member 3 (corresponding to Eq. 28*b*), a second relation is obtained between M_{3C} and M_{3D}:

$$M_{3C} - 2\, M_{3D} = +\,240 - 39.96\, E\, \epsilon_6 \tag{41b}$$

Solving Eqs. 41:

$$M_{3C} = +\,29.7 + 39.15\, E\, \epsilon_6 \tag{42a}$$

and

$$M_{3D} = -\,105 + 39.53\, E\, \epsilon_6 \tag{42b}$$

All end moments may now be found in terms of ϵ_6 by proceeding backward and using the C-values and D-values in Eqs. 39 and 40; thus:

$$M_{2C} = -\,M_{3C} = -\,29.7 - 39.15\, E\, \epsilon_6 \tag{43a}$$

and $M_{2B} + C_{2C}\, M_{2C} = D_{2C}$; or,

$$M_{2B} = -\,9.06 - 30.75\, E\, \epsilon_6 \tag{43b}$$

At joint B, Eq. 31*a* should be applied to member 5, giving $\frac{C_{5B} + 2}{K_5} M_{5B} = \frac{1}{K_2} (2\, M_{2B} - M_{2C}) + 6\, E\, \delta_{5-2}$; or,

$$M_{5B} = +\,5.79 + 25.81\, E\, \epsilon_6 \tag{44}$$

Continuing:

$$M_{1B} = -\,M_{2B} - M_{5B} = +\,3.27 + 5.21\, E\, \epsilon_6 \tag{45a}$$

$M_{1A} + C_{1B}\, M_{1B} = D_{1B}$; or,

$$M_{1A} = +\,0.82 - 2.03\, E\, \epsilon_6 \tag{45b}$$

and

$$M_{4A} = -\,M_{1A} = -\,0.82 + 2.03\, E\, \epsilon_6 \tag{45c}$$

Step 4.—The transverse forces exerted on joints A and B by member 1 will be designated by F_{1A} and F_{1B}, respectively. Similar symbols will be selected for the transverse end forces of other members. The positive directions of these reactions will be such as to create a clockwise couple. Then:

$$\left.\begin{array}{ll} F_{4E} = F_{4A} = -\dfrac{M_{4A}}{16}; & F_{1A} = F_{1B} = -\dfrac{M_{1A} + M_{1B}}{10} \\[2ex] F_{5F} = F_{5B} = -\dfrac{M_{5B}}{16}; & F_{2B} = F_{2C} = -\dfrac{M_{2B} + M_{2C}}{12.7} \\[2ex] \multicolumn{2}{c}{F_{3C} = -\dfrac{M_{3C} + M_{3D} + 320}{16}} \end{array}\right\} \tag{46}$$

Load the conjugate frame with these forces, as shown in Fig. 13(*b*), and equate to zero, the moment about point O of all forces on the free body above line X–X:

$$-\,10\, T_4 + 10\, F_{1A} - 9\, F_{4A} - 9\, F_{5B} + 6.35\, F_{2B} + 6.35\, F_{2C} - 9\, F_{3C} = 0 \tag{47}$$

The force T_4 may be eliminated by the condition $\sum Y = 0$ at joint A, giving $T_4 = F_{1A}$. Use Eqs. 42 to 46, inclusive, and, in Eq. 47, replace all F-elements by their equivalents in terms of ϵ_6; the following equation is the result:

$$+ 318.67 + 224.51\, E\, \epsilon_6 = 0 \qquad (48)$$

or, $E\, \epsilon_6 = - 1.37$.

The substitution of this value of ϵ_6 into the expressions of the several unknowns in terms of ϵ_6 will give all unknowns explicitly: For instance, $M_{3C} = + 29.7 + 39.15\ (- 1.37) = - 24.4$; and $M_{3D} = - 105 + 39.53\ (- 1.37) = - 159.5$.

All rotations and displacements of joints may now be found by using the end moments, the axial strains, and the unit deflections determined as a result of the preceding analysis. It is necessary, however, to use the actual K-values of the members; if member 4, for example, is a 10-in., 72-lb, H-section, its actual K is $\frac{420}{12^4 \times 16}$ ft³, while its relative K was assumed as 1. Using feet and kips, the actual value of ϵ_6 (Eq. 48), is therefore: $\epsilon_6 = -\frac{1.37}{30{,}000 \times 12^2} \div \frac{420}{12^4 \times 16} = -\frac{1}{3{,}994}$.

If the vertical displacement of joint C is desired, Eq. 38a yields: δ_{3-9} or $\delta_3 = -\frac{2.22}{3{,}994}$. Then: $\Delta_3 = -\frac{2.22 \times 192}{3{,}994} = - 0.11$ in. (clockwise about joint D) or, + 0.11 in. downward.

CONCLUSION

A complete analysis of the general two-dimensional framework is achieved through the consideration—and use as principal unknowns—of the system of the axial strains in the members and those in the imaginary diagonals which divide it into triangular elements. The paper brings into evidence the compatibility conditions and topological properties of the frame and establishes a correspondence between it and a conjugate frame which is statically determinate. These considerations lead to the determination of all unknowns.

The adaptability of the general theory to different types of problems, and its workability as a practical method of approach, are demonstrated by several illustrative applications.

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APPENDIX

NOTATION

The following symbols, used in this paper, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering, and Testing Materials,[7] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932. In the text, subscripts are used with these symbols, to relate them to the corresponding member or joint.

A = area; cross-sectional area of a member: A_e = the area of a quadrilateral element (Eq. 15); in Eq. 13, A, B, and C are used to designate the interior angles at joints A, B, and C of a triangle;

a = linear dimension in a triangle (see also b and h):
    a_1 = linear projection of the line AB (see Fig. 2) on the x-axis;
    $(a_1)'$ = linear projection of the line A′B′ (see Fig. 2) on the x-axis;
    $a_{1,3}$, etc. (see Fig. 4) = linear projection of member 1 on member 3, etc.;

B = (see angles A, B, and C in Eq. 13);

b = linear dimension in a triangle (see also a and h):
    b_1 = linear projection of the line AB (Fig. 2) on the y-axis;
    $(b_1)'$ = linear projection of the line A′B′ (Fig. 2) on the y-axis;

C = coefficient in the general linear relation between the end moments of a member; see Eqs. 26 (see also coefficient D); in Eq. 13, A, B, and C are used to designate the interior angles at joints A, B, and C of a triangle;

D = coefficient in the general linear relation between the end moments of a member; see Eqs. 26;

E = Young's modulus;

e = total axial deformation of a member, elongation being positive: In Eqs. 24, e_{xa} denotes the linear movement of support a in the x-direction and e_{ya} in the y-direction;

F = transverse end force in a member (also, where indicated, F = "function of"): F_{1A}, etc., in Eqs. 46, designate the transverse forces exerted by member 1 upon joint A, Fig. 13;

h = linear dimension of a triangle (see also a and b); in Eq. 9, $h_{1,2}$ is the perpendicular drawn on member 3 from the intersection of members 1 and 2;

I = moment of inertia;

K = stiffness ratio I/l;

l = length of a member from center to center of joints: l_1 is the length of member 1 = AB, Fig. 2; $(l_1)'$ is the corresponding length of the deformed member A′B′, Fig. 2;

M = moment: M_{1A}, etc. = the moment exerted by member 1 upon joint A, the clockwise direction assumed positive; M_a = the moment at support a, Fig. 11;

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[7] ASA—Z10a—1932.

m = moments similar to M when the end rotations and the lateral deflection of the member are zero;

n = a number:

  $n_i = n_m + n_{mt} - 2\, n_p$;

  n_m = total number of given members, excluding those connected to a support at each end;

  n_p = total number of joints, excluding the supports;

  n_r = total number of distinct regions in a frame, excluding those with all their vertices at supports;

  n_t = total number of distinct triangles resulting from the division of the frame by imaginary diagonals, excluding those with all their vertices at supports;

  n_{mt} = total number of imaginary diagonals required for the complete division of the frame into triangles, excluding those joining two supports;

  n_σ = total number of linear movements of the supports in a frame;

T = axial force in a member, positive for tension:

  T^0 = axial force found by assuming $X = 0$ and allowing the external loading to act (Fig. 8);

  T^1 = axial force found by assuming $X = 1$ and all external loading removed (Fig. 8);

U = a substitution factor used to simplify Eq. 33;

X = reaction: X_a, Y_a, and M_a are the horizontal, vertical, and moment reactions at support a, Fig. 11;

x = coordinate distance parallel to the x-axis; x and y are the coordinates of a point on a deflected member; x_a, y_a are the coordinates of joint A;

Y = (see X);

y = (see x);

α = angles shown in Fig. 5;

Δ = relative lateral displacement at one end of a member with respect to the other, measured at right angles to the member, positive if either end moves clockwise about the other;

δ = unit deflection of a member, defined by $\delta = \dfrac{\Delta}{l}$; δ_t = unit deflection of an imaginary diagonal; $\delta_{p-q} = \delta_p - \delta_q$, the difference in unit deflection between two adjacent members p and q;

ϵ = axial strain of a given member defined by $\epsilon = \dfrac{e}{l}$; ϵ_t = axial strain in an imaginary diagonal; $\epsilon_{p-q} = \epsilon_p - \epsilon_q$, the difference in axial strain between two adjacent members p and q;

λ = the angle of a given line referred to the x-axis;

ω = angular rotation of a joint or support measured from its position before deformation or displacement, positive when clockwise;

$\sum_o$ = sign of summation around a closed region continuously in a clockwise direction, including all joints (or all members) on the boundary.

DISCUSSION

Leon Beskin,[8] Assoc. M. Am. Soc. C. E.—To clarify this discussion, the method of framework analysis presented by the author is summarized as follows:

1.—The redundant unknowns are divided into two groups—(*a*) linear displacements of the joints; and (*b*) rotations at the joints, for which the moments at the joints can be substituted in the case of relatively simple structures.

2.—Group (*a*), Item 1, is represented by the relative variations of lengths (strains) of the sides of a statically determinate triangulated system which "maps" the actual structure according to a method well known in surveying.

3.—Group (*b*), Item 1, can be calculated in a relatively simple manner as a function of group (*a*) when the framework is of the "open" type; that is, when the computer progresses along the framework in a definite direction without intersecting hinges (free rotation) or fully restrained multiple supports (no rotation), no member can be encountered twice. In this special case, moments can be considered directly, instead of rotations. The procedure becomes increasingly complicated when multiply-connected frames are considered and it becomes necessary to introduce a third group of redundant unknowns, which must be added to the moments and which includes the rotations of some of the joints.

4.—The equilibrium of moments at the joints is obtained without considering the equilibrium of the longitudinal and transverse forces. These forces can be defined in terms of the moments at the joints. Then the structure can be replaced by an equivalent hinged structure and the conditions of stability and equilibrium of forces corresponding to that structure give the equations necessary to define the unknowns of group (*a*), Item 1.

A profitable comparison can be made between this paper and the simple and direct analysis presented by David M. Wilson,[9] M. Am. Soc. C. E., who admits "that no new method of analysis is proposed." For example:

(1) Divisions of the unknowns into two groups is the basis of both methods.

(2) For group (*a*), Item 1, Mr. Wilson uses rotations and Mr. Nubar uses strains. The two groups are similar, and only practical considerations can indicate the superiority of one group. Mr. Wilson limits himself to structures in which axial deformations are negligible, which is the "less general case" treated by Mr. Nubar; but the treatment of the more general case by successive approximations is well known; it is the conventional method for the analysis of trusses. A good example can be found in a study by K. A. Calisev,[10] who examines trusses, beams on continuous supports, and frameworks.

(3) For group (*b*), Item 1, Mr. Wilson uses rotations or moments, and applies slope deflection and moment distribution methods. Mr. Nubar uses

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[8] Structures Engr., Consolidated Vultee Aircraft Corp., Allentown, Pa.

[9] "Analysis of Rigid Frames by Superposition," by David M. Wilson, *Transactions*, Am. Soc. C. E., Vol. 110 (1945), p. 1489.

[10] "The Method of Successive Approximation for the Analysis of Statically Undetermined Systems," by K. A. Calisev, *Publications*, International Assn. for Bridge & Structural Eng., Vol. IV, 1936, Zurich, p. 199.

moments wherever possible and applies the method of fixed (conjugate) points. All these methods are well known and the special conditions of each problem or, more often, the habits of the analyst, dictate the choice between these methods. However, for multiply-connected circuits, the moment distribution method has the definite advantage of being well developed, available with numerous examples in any textbook. On the contrary, the fixed points method, developed many years ago in a straightforward manner for open circuits only, is far from being perfected (as shown by the author's statements) for use in analyzing multiply-connected circuits. The author indicates, in substance, that some rotations must be introduced as auxiliary unknowns and that the system of equations containing these unknowns must then be solved. This is known and is a condition that must be applied to any statically indeterminate structure. A method is new and useful essentially in the extent to which it presents a definite and straightforward manner of choosing unknowns and of solving systems with a high degree of indeterminacy.

(4) The determination of group (*a*), Item 1, when group (*b*) is defined in terms of group (*a*), is identical in the Nubar and the Wilson methods.

From the foregoing, it appears that the Nubar method follows closely the pattern of conventional methods. It remains to be decided whether the method introduces an element of newness and simplification over conventional procedures.

As indicated, the division of unknowns in two groups is conventional. Generally the unknowns of group (*a*) are: The horizontal displacements for multi-story frames; the vertical displacements for Vierendeel beams; and the axial loads (or strains) for triangulated trusses.

The author replaces unknowns that result from a choice adapted to the nature of each problem, by a system of arbitrary unknowns which are the same in all cases. This attitude can be compared to that of an analyst who would always use rectangular coordinates, even if other sets of coordinates (polar, cylindrical, spherical, etc.) are more adapted to the problem. Such an approach is utterly unsatisfactory; and sometimes the main difficulty in the solution of a problem is in the choice of a proper system of coordinates. In the present case, since it is always necessary to calculate the angular displacements δ of the straight lines connecting the ends of each member (called unit deflections by the author), it appears logical, wherever possible, to use the latter as principal unknowns. For instance, in Fig. 13 no advantage results from the use of ϵ_6 as an unknown, against the use of either the horizontal displacement of AB or the common rotation of EA, BF, and CD. Defining δ as this rotation, Eqs. 36, 37, and 38 become:

$$\left.\begin{aligned} -\delta_{8-4} = -\delta_{1-5} = \delta_{3-9} = \delta_{4-1} = \delta_{5-8} = \delta \\ \delta_{1-2} = \frac{16}{9}\delta = 1.777\,\delta \qquad\qquad\qquad \\ \delta_{5-2} = -\delta_{2-3} = (\delta_{3-9} + \delta_{1-2}) = 2.777\,\delta \end{aligned}\right\}\ldots\ldots\ldots\ldots(49)$$

The simplification is obvious and clearly demonstrates, by means of the example given by the author himself, that the method is of very little practical interest for frameworks, even of an unconventional type.

For a conventional framework, there is no doubt that the linear displacements of the various stories, defining the rotations at a glance or (even better) the rotations of each line of columns between stories, are the logical unknowns to be used (Fig. 14). If axial strains are considered, they are added to the rotations to form the first group of unknowns.

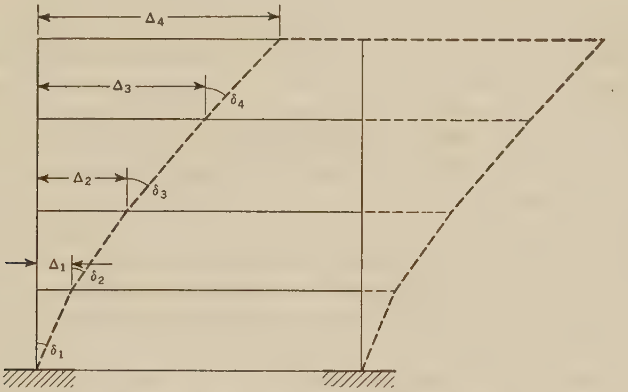

FIG. 14

His illogical choice of the unknowns for rectangular frames has forced the author to use, in all cases, what he called "equations of compatibility" which relate axial strains to angular displacements. These equations are well known under other names. Eqs. 4 are the equivalent of the Williot-Mohr construction. The condition expressed in terms of moments (Eq. 5) is the equation used for the calculation of deflections of curve beams, the sole difference being that the quantity $\sum x_k (\epsilon_{k,k+1} - \epsilon_{k-1,k})$ is replaced by the equivalent quantity $\sum \epsilon_{k,k+1} (x_{k+1} - x_k) = \sum \frac{P}{E\,A} (x_{k+1} - x_k)$, whereas the quantity $\sum y_k \left(\frac{\Delta_{k,k-1}}{L_{k,k-1}} - \frac{\Delta_{k-1,k}}{L_{k-1,k}} \right)$ is generally replaced by the integrated expression:

$$\int y_k \frac{d^2\Delta}{ds^2}\, ds = \int y \frac{M}{E\,I}\, ds \ldots\ldots\ldots\ldots\ldots\ldots (50)$$

Eq. 10*a* is well known, in English[11] as well as in French[12] literatures under the equivalent form (see Fig. 15):

$$\frac{\Delta_b}{b} - \frac{\Delta_c}{c} = \delta\,\alpha = \frac{\epsilon_b - \epsilon_a}{\tan\gamma} - \frac{\epsilon_c - \epsilon_a}{\tan\beta} \ldots (51)$$

Topology of Frameworks.—The purpose of the section entitled "Topology of Frameworks" appears to be the establishment of Proposition (*c*) which appears several times in the paper. However, this proposition is self-evident without any preliminary proof: The n_p-joints each

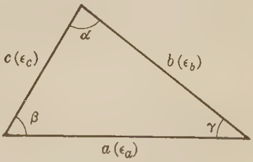

FIG. 15

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[11] "Airplane Structures," by A. S. Niles and J. S. Newell, Vol. I, 3d Ed., 1943, New York, N. Y., p. 402.

[12] "Résistance des Matériaux," by B. de Fontviolant. Vol. II, 1927, Paris, p. 148.

have two degrees of freedom in the plane; the entire framework in joint displacement has $2\,n_p$ degrees of freedom, plus n_σ degrees of the n_σ linear displacements of support, minus $n^r{}_m$ degrees for the $n^r{}_m$ rigid (nondeformable) members. Thus:

$$\text{Degree of freedom} = 2\,n_p + n_\sigma - n^r{}_m \qquad (52)$$

In Eq. 52, $n^r{}_m$ (in which r is an index and not an exponent) may vary from zero to n_m, the total number of members. It appears that this entire section is extraneous to the subject of the paper.

Boundary Conditions.—Since moments and axial strains are used as unknowns, and, perforce, they define the stresses and loads in the structure completely, it is obvious that the boundary conditions can be expressed in terms of such elements, and general Eqs. 24 do not clarify the problem for practical applications.

End Moments.—The method used to determine in succession the moments in a structure of the "open" type is the well-known method of "conjugate points" (English terminology), "focal points" (French terminology), "fixed points" (German terminology).

The development of Eqs. 35 can be presented in a simpler and more symmetric manner: Using the author's notation, and with r denoting any member, it can be shown that, if n members meet at a joint J and if, for $n - 1$ members, there exist relations of the type

$$M_{rR} + C_{rJ}\, M_{rJ} = D_{rJ} \qquad (53)$$

(R being the joint at the end of the member r), a relation of the same type holds for the nth member.

This fact is obvious from the general relations of linearity between moments and rotations. The coefficients C_{n-J}, D_{n-J} of the nth member can be related to the other coefficients. The equation of equilibrium of a joint is:

$$M_J + \sum M_{rJ} = 0 \qquad (54)$$

in which M_J is a couple applied at the joint. Expressing the rotation of any member at the joint in terms of the loads and couples,

$$\omega_J = \delta_r + \frac{1}{6\,E\,k_r}\,(2\,m_{rJ} - m_{rR} + M_{rR} - 2\,M_{rJ}) \qquad (55)$$

All rotations ω_J are equal. Replacing M_{rR} in terms of M_{rJ} by means of Eq. 53, the value of M_{rJ} can be expressed as follows:

$$M_{rJ} = -\,\omega_J\,\frac{6\,E\,k_r}{C_r + 2} + \frac{6\,E\,k_r\,\delta_r + 2\,m_{rJ} - m_{rR} + D_{rJ}}{C_r + 2} \qquad (56)$$

Substituting in Eq. 54:

$$M_J + \sum \frac{6\,E\,k_r\,\delta_r + 2\,m_{rJ} - m_{rR} + D_{rJ}}{C_r + 2} - \omega_J\,6\,E \sum \frac{k_r}{C_r + 2} = 0 \qquad (57)$$

When M_J, δ_r, m_r, and D_r are all equal to zero, there may be a rotation ω_J under the action of moments M_{rR}. This shows that the coefficient of ω_J is equal to zero (except for a completely restrained joint for which $\omega_J = 0$ and $C_r = -2$). Thus

$$\sum \frac{k_r}{C_r + 2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (58)$$

Also

$$M_J + \sum \frac{6\,E\,k_r\,\delta_r + 2\,m_{rJ} - m_{rR} + D_{rJ}}{C_r + 2} = 0 \ldots\ldots\ldots\ldots (59)$$

Eqs. 58 and 59 are absolutely symmetrical and are equivalent to Eqs. 34 and 35, provided it is noted that $C_{rR} = 1 : C_{rJ}$ and $D_{rR} = D_{rJ} : C_{rJ}$. The method developed from Eqs. 58 and 59 is a mere extension of the method of fixed points for continuous beams, adapted to "open" structures. In that form it was presented by Prof. M. Ritter[13] as early as 1936.

For multi-connected circuits, it is necessary to introduce the rotations of some of the joints, which cannot be eliminated directly. Very rapidly, the method becomes extremely complicated, due to the number of simultaneous unknowns, and loses its value, at least in its present stage. Moreover, the author limits himself to a simple suggestion of the method, without attempting to develop it into a practical procedure. For a conventional truss, the number of rotations to be introduced is the integer equal or immediately superior to one third of the number of the circuits. In Fig. 16(*a*), the "opened" joints are represented by hinges. For a conventional Vierendeel structure (Figs. 16(*b*) and 16(*c*)), the corresponding ratio is one half. The last result also applies to a one-span, multi-story portal, and to a two-story, multi-span

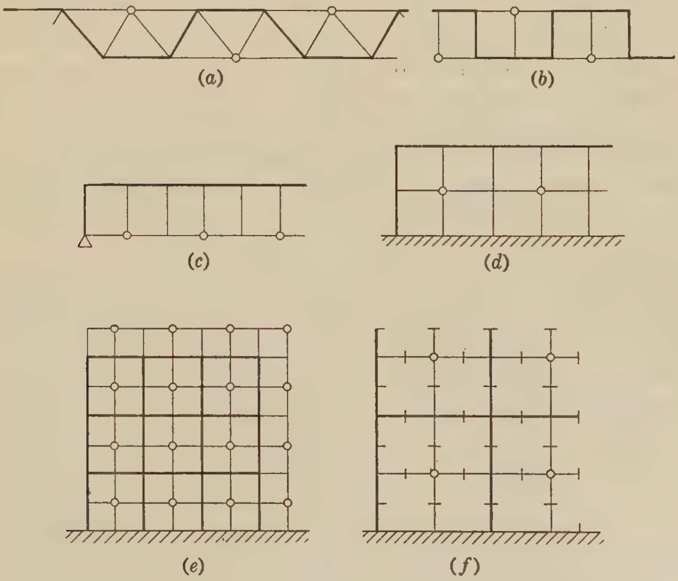

FIG. 16

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[13] "Continuous Beams on Compressible Supports," by M. Ritter, *Publications*, International Assn. for Bridge & Structural Eng., Vol. IV, 1936, Zurich, p. 509.

portal (Fig. 16(*d*)). For a multi-span, multi-story portal, the number will be approximately two thirds to one half of the order of multiplicity of the portal. For instance, the portal represented in Fig. 16(*e*) is of 42d order. It can be reduced to the 6th order as shown, and then to the "open" type as shown in Fig. 16(*f*). Thus a total of twenty-eight rotations are needed for a framework of the 42d order, and four rotations are required for a framework of the 6th order. It is obvious that in such cases only iteration methods are of practical interest, and in these methods it may be advantageous to use all the rotations of joints in a symmetric manner, avoiding the separation of any that may be considered as fundamental unknowns in order to calculate the remainder in terms of the segregated unknowns.

When the author states (see heading, "Determination of End Moments") that "The proposition will be established that the end moments in the members of a rigid frame can be found as linear functions of the δ_{p-q} elements * * * and the linear movements of the supports," he simply states that to solve a system of linear equations the following procedure can be followed:

Solve the equations in terms of the given constants, some of the unknowns being considered as fundamental unknowns. The remaining unknowns will be expressed as linear functions of the fundamental unknowns and the constants. Thus, the author's statement, presented as a new proposition, does not reflect any new light on the problem. Moreover, the proposition is incomplete, since the end moments are functions, not only of the δ-elements, but also of the rotations that must be introduced in multi-connected circuits. However, for "open" structures, that procedure, which could be designated as Ritter's method, is of great interest because: (1) The number of fundamental variables is small; and (2) the secondary variables can be expressed in a relatively simple manner in terms of the fundamental variables.

If similar methods were available for multi-connected circuits, a great step would be made toward the general direct analysis of structures of that type. It appears that such methods do not yet exist.

Determination of the Axial Stresses and Linear Movements of the Supports.—As indicated by Mr. Wilson,[9] the conventional procedure is to consider appropriate free bodies and to determine the transverse forces in the members. The conditions of stability and equilibrium of the totally hinged structure, or the conditions of equilibrium of several free bodies that are statically independent of one another, define all the conditions required to find the fundamental unknowns. For instance, in the simple case of the frame of Fig. 17, the hinged structure is unstable. Thus, the equilibrium of two free bodies must be examined: The condition of equilibrium of body I defines the load acting on body II. Then the equilibrium of body II can be expressed, defining the load acting on the vertical member.

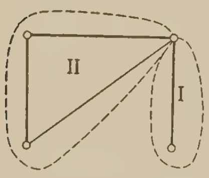

Fig. 17

In order to solve the same problem, the author introduces a fictitious diagonal and stipulates that the load in that diagonal must be equal to zero, which appears as a useless complication. Actually, the author, in Fig. 13, does not use that diagonal, and his method, somewhat artificial, does not give the required equation directly. It is possible to find the result directly. For example, projecting the forces as shown in Fig. 18, in order to eliminate the loads along 3 and 5:

$$F_{2B} + F_{2C} - (F_{4A} + F_{5B} + F_{3C})\sqrt{2} = 0 \qquad (60)$$

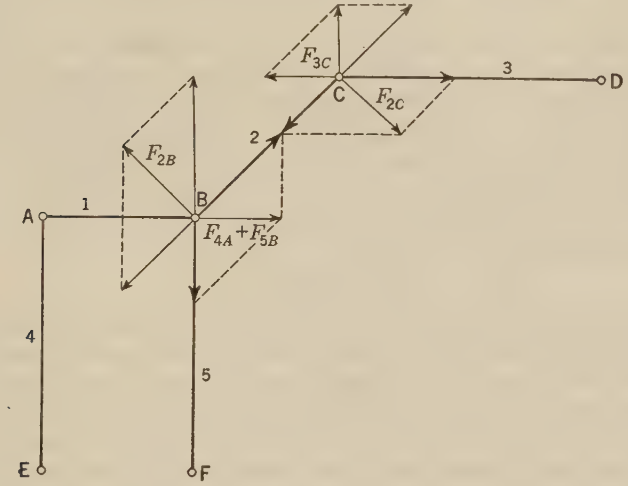

FIG. 18

which is strictly equivalent to Eq. 47 after the subsequent elimination of T_4 from the latter equation.

Conclusions.—The following conclusions deserve emphasis:

1.—The author does not present a substantially new method of analysis of redundant structures; he merely gives a somewhat different presentation of existing methods. The presentation shows a tendency to generalize the field of application of some methods. In the case presented, this approach is erroneous, since absolutely general methods are available (least work). The trend of future research must be toward the development of special methods well adapted to each category of problems, since absolutely general methods are impracticable in highly redundant structures, because of long numerical computations.

2.—The author introduces conditions that create undue complications in the presentation of simple and well-known processes, such as his introduction of conjugate frames and his treatment of the subject "Topology of Frameworks." He also presents well-known concepts with a changed and special terminology, such as compatibility equations, boundary conditions, the incomplete proposition relative to the determination of end moments as linear functions of the δ_{p-q}-elements, and the fundamental relation of the fixed-point method.

3.—Precisely at the point where the method seems to suggest new developments (in the introduction of a minimum of additional rotations), the author merely states that these quantities must be introduced and then eliminated.

4.—An incorrect interpretation is given to the number of the principal unknowns. Actually, to the unknowns defining the joint displacements must be added the unknowns defining the joint rotations in multi-connected frameworks.

5.—The author has repeated a summary in two places of his paper—first in the "Introduction" and again in the "Application to the General Case." The second summary is much more intelligible than the first but, when compared with the outline given by Mr. Wilson,[14] it appears that both summaries are practically identical step by step, as far as principles are concerned. From a practical standpoint, however, there is no doubt that Mr. Nubar's paper will prove much less valuable to the engineer who is desirous of finding a simple and rapidly applicable method.

Jaroslav Polivka,[15] M. Am. Soc. C. E.—An interesting and original general method of analyzing a two-dimensional framework, based on the superposition principle, is presented by the author. The method has the same advantages and disadvantages as some of the other methods of superposition, such as the method of moment distribution by successive approximation. Superposition methods require more accuracy because of the many calculations involved and extreme caution is necessary to avoid confusion in sign conventions.

A greater simplification is attained by methods permitting a better visualization of the elastic deformations under the given load, such as, for example, column analogy, the method of fixed points, and a method based on the ellipse of elasticity.

The author's method and the method of characteristic (fixed) points are similar in some ways. To demonstrate the common features of both methods, and for purposes of checking, the writer has solved the problem illustrated in Fig. 13, using the method of characteristic points, as shown in Fig. 19. The results were checked by the slope deflection method, involving the following equations:

$$\left.\begin{aligned} 7\,B + 2\,C - 18\,R &= 0 \\ 9\,C + B + 2\,E + 46\,R &= 0 \\ 5\,E + C + 5\,R &= 80 \\ 27\,B - 138\,C - 30\,E - 6{,}526\,R &= 11{,}520 \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (61)$$

Solving: $B = -\dfrac{605{,}460}{821{,}199}$; $C = +\dfrac{6{,}181{,}120}{821{,}199}$; $E = +\dfrac{13{,}570{,}720}{821{,}199}$; and

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[14] "Analysis of Rigid Frames by Superposition," by David M. Wilson, *Transactions*, Am. Soc. C. E., Vol. 110 (1945), p. 1491.

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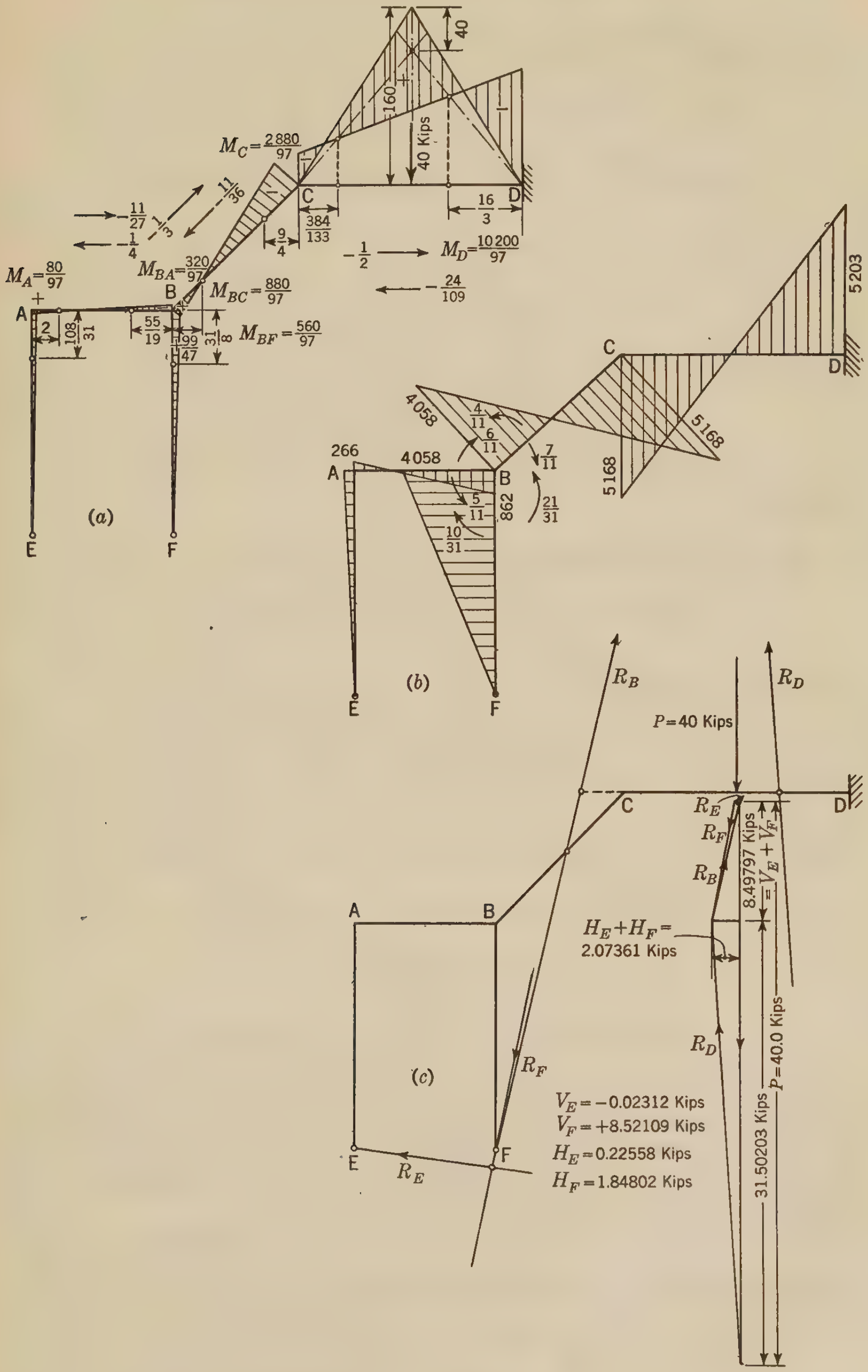
40
160
40 Kips
$M_C = \frac{2880}{97}$
$\frac{11}{36}$
$-\frac{11}{27}$
$\frac{1}{3}$
$-\frac{1}{4}$
$\frac{9}{4}$
$\frac{384}{133}$
$\frac{16}{3}$
$-\frac{1}{2}$
$M_D = \frac{10200}{97}$
$-\frac{24}{109}$
$M_A = \frac{80}{97}$
$M_{BA} = \frac{320}{97}$
$M_{BC} = \frac{880}{97}$
$M_{BF} = \frac{560}{97}$
2
$\frac{108}{31}$
$\frac{55}{19}$
$\frac{99}{47}$
$\frac{31}{8}$
A
B
C
D
E
F
(a)
5203
4058
$\frac{4}{11}$
$\frac{6}{11}$
$\frac{7}{11}$
5168
266
4058
862
$\frac{5}{11}$
$\frac{10}{31}$
$\frac{21}{31}$
(b)
R_B
R_D
$P = 40$ Kips
R_E
R_F
R_B
8.49797 Kips $= V_E + V_F$
$H_E + H_F =$ 2.07361 Kips
R_D
$P = 40.0$ Kips
31.50203 Kips
R_F
(c)
$V_E = -0.02312$ Kips
$V_F = +8.52109$ Kips
$H_E = 0.22558$ Kips
$H_F = 1.84802$ Kips
R_E

Fig. 19.—Check Solutions of Illustrative Problem (Fig. 13)

$R = -\dfrac{1{,}667{,}760}{821{,}199}$. The slope deflection equations are:

$$\left.\begin{aligned}
&M_{AE} = \frac{3}{4}B - \frac{9}{2}R;\quad M_{AB} = B + \frac{C}{2};\quad M_{BA} = C + \frac{B}{2};\\
&M_{BF} = 2\left(\frac{3}{4}C - \frac{9}{2}R\right);\quad M_{BC} = 2\left(C + \frac{E}{2} + 16\,R\right);\\
&M_{CB} = 2\left(E + \frac{C}{2} + 16\,R\right);\quad M_{CD} = 3\,(E - 9\,R) - 80;\\
&\qquad\text{and}\quad M_{DC} = 3\left(\frac{E}{2} - 9\,R\right) + 80
\end{aligned}\right\}\ldots(62)$$

The shear equations are:

$$\left.\begin{aligned}
&M_{EA} + M_{AE} = 16\,H_E;\quad M_{BF} + M_{FB} = 16\,H_F;\\
&M_{BC} + M_{CB} = 9\,(H_E + H_F) - 9\,(V_E + V_F);\\
&\text{and}\quad M_{CD} + M_{DC} = -\,16\,(V_E + V_F) + 8 \times 40
\end{aligned}\right\}\ldots\ldots\ldots(63)$$

Furthermore,

$$\left.\begin{aligned}
&\frac{16}{9}\,(M_{BC} + M_{CB}) = M_{AE} + M_{BF} + M_{CD} + M_{DC} - 320;\quad\text{and}\\
&\qquad 36\,M_{AE} + 36\,M_{BF} + 36\,M_{CD} + 36\,M_{DC}\\
&\qquad\quad - 64\,M_{BC} - 64\,M_{CB} = 11{,}520
\end{aligned}\right\}\ldots(64)$$

and the joint equations are:

$$\left.\begin{aligned}
&4\,M_{AE} + 4\,M_{AB} = 0;\quad 2\,M_{BA} + 2\,M_{BF} + 2\,M_{BC} = 0;\\
&\qquad\text{and}\quad M_{CB} + M_{CD} = 0
\end{aligned}\right\}\ldots\ldots(65)$$

The solution (computation machine values) yields:

$$M_{AE} = -\,M_{AB} = +\,\frac{2{,}964{,}000}{821{,}199} = +\,3.60;\qquad M_{BA} = +\,\frac{3{,}153{,}840}{821{,}199} = +\,3.84;$$

$$M_{BF} = +\,\frac{24{,}281{,}520}{821{,}199} = +\,29.56;\quad M_{BC} = -\,\frac{27{,}435{,}360}{821{,}199} = -\,33.40;$$

$$M_{CB} = -\,M_{CD} = \frac{20{,}045{,}760}{821{,}199} = -\,24.41;$$

$$\text{and}\quad M_{DC} = +\,\frac{131{,}081{,}520}{821{,}199} = +\,159.62.$$

The same results were obtained by using the method of ellipse of elasticity, substituting for the frame EABF an imaginary ellipse of support at joint B, and combining this ellipse of support with the ellipse of the section BCD. The graphic results of this analysis are shown in Fig. 19(*c*). Comparing both methods the following statements may be made:

(1) The values C in the author's paper are the carry-over factors of the individual members of the framework (Fig. 19(*a*)): $C_{1B} = -\dfrac{1}{4}$, $C_{1A} = -\dfrac{11}{27}$, $C_{2C} = -\dfrac{11}{36}$, $C_{3D} = -\dfrac{24}{109}$, etc.

(2) The coefficients of $E\,\epsilon_6$ in Eqs. 42, 43, 44, and 45 are the moments due to axial strain $E\,\epsilon_6$ of the imaginary member 6 (Fig. 13) and are proportional to the moments of the framework when subjected to a horizontal displacement (translation) along AB equal to Δ_B.

(3) The other values of Eqs. 42 and 45 are the bending moments of the framework subjected to the given load when the lateral displacement Δ_B is equal to zero.

A. A. Eremin,[16] Assoc. M. Am. Soc. C. E.—An interesting method for computing stresses and deformations in continuous frames is presented by Mr. Nubar. Like Prof. O. Mohr,[17] Mr. Nubar considers the geometric and loading factors as forces applied at the joints of the frame, although, regrettably, his illustrative example (Fig. 13) fails to demonstrate any superior simplicity in practical application. By graphical methods, the moments in the frame, shown in Fig. 13, can be determined with less effort and time.

Graphically, the writer would proceed in two steps:[18] The first step is to determine the physical properties of the frame by locating the characteristic points, termed "central points," "inflection points," and "elastic points." The relative location of the characteristic points is determined by constructing a "three-line polygon," as described briefly by the writer elsewhere.[19] The second step is to determine the moments produced by the given loading and the distortion of the frame.

Referring to Fig. 20, which is an analysis of the frame shown in Fig. 13: The central moments in member CD with a load of 40 kips are $M_1 = M_2 = 80$ kip-ft. If the joints are fixed horizontally, the moments in the frame are determined by drawing "cross lines," as in Fig. 20(c). A distortion diagram with a unit displacement at joint C is shown in Fig. 20(b). The basic bending moments due to distortion in member CD are

$$M_{CD} = N\,a\,k_1 = 42.3\,k_1 \qquad \text{(66a)}$$

and

$$M_{DC} = N\,b\,k_1 = 78.2\,k_1 \qquad \text{(66b)}$$

in which

$$N = \frac{600\,K\,\Delta}{L\,(L - a - b)} \qquad \text{(67)}$$

and k_1 is a distortion coefficient; a and b are the distances to the points of inflection (shown in the parentheses, Fig. 20(a)); Δ is transverse displacement of member ends (Fig. 20(b)); and K (= I/L) is the basic stiffness of member, shown in the circles in Fig. 20(a). The numerical factor 600 in Eq. 67 is introduced for convenience of computations. The basic moments in each member are determined by Eqs. 66 and 67 and distributed, as shown in Fig. 20(d).

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[16] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[17] "Secondary Stresses in Bridges," by Cecil Vivian von Abo, *Transactions*, Am. Soç. C. E., Vol. 89 (1926), p. 31.

[18] "Analysis of Continuous Frames by Graphical Distribution of Moments," by A. A. Eremin, Sacramento, Calif., 1943 Ed. Published by the author.

[19] *Transactions*, Am. Soc. C. E., Vol. 110 (1945), p. 1543.

As a condition of static equilibrium, the reaction at joints E, F, and D is:

$$H_E + H_F - H_D = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (68)$$

Using the moments in Figs. 20(*c*) and 20(*d*) to determine the values of H_E, H_F, and H_D in Eq. 68, a value of $k_1 = 0.48$ is computed.

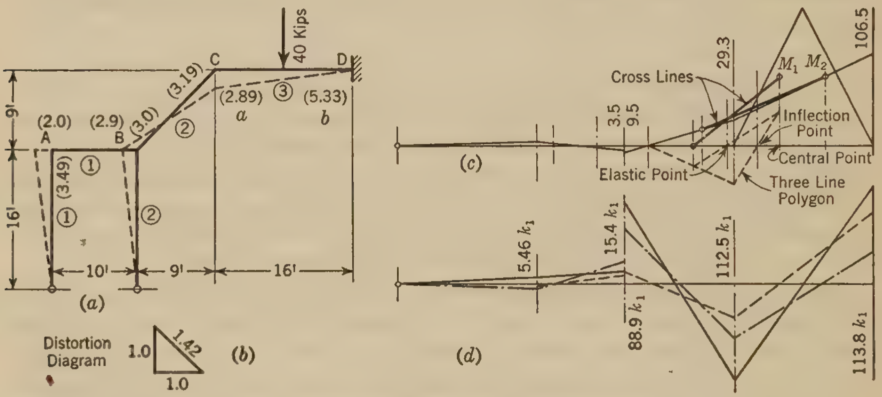
40 Kips
C
D
(3.19)
(2.89)
③
(5.33)
a
b
9'
(2.0)
(2.9)
A
B
(3.0)
②
①
(3.49)
①
②
16'
10'
9'
16'
(a)
Distortion Diagram
1.0
1.42
1.0
(b)
Cross Lines
29.3
106.5
M_1 M_2
3.5
9.5
Inflection Point
(c)
Central Point
Elastic Point
Three Line Polygon
5.46 k_1
15.4 k_1
112.5 k_1
88.9 k_1
113.8 k_1
(d)

FIG. 20.—GRAPHIC DISTRIBUTION OF MOMENTS

Superposing the moments in Fig. 20(*c*) and the moments in Fig. 20(*d*), the final bending moments in the frame are $M_{DC} = 161.5$ and $M_{BF} = 29.1$, which differ slightly from those computed by the algebraic formulas introduced by the author.

Contrary to Mr. Nubar's method, the operations in the graphic distribution of moments may easily be visualized. Therefore, the moments may readily be checked. Furthermore, graphics may be used in checking the analytical computations of moments. The constant C used by the author in Eq. 35*a* may be checked by the inflection point distance in the graphical method. Thus, for member CD, Fig. 20(*a*): $C_{3D} = \frac{a}{L - a} = 0.22$, which is the same as that computed by the author. Computation of the inflection point distances, *a* and *b*, in members with variable sections may be simplified by using the constants given by the writer elsewhere.[18]

YVES NUBAR,[20] M. AM. SOC. C. E.—Some of the comments reflect a misunderstanding—namely, that the primary object of the paper was the presentation of another method for the quick solution of special types of frames. Apparently with this thought in mind, Messrs. Polivka and Eremin recommend different procedures for such solutions. Mr. Beskin expresses a similar misunderstanding in his conclusion 5. The primary purpose of the paper, described in the "Synopsis," was an investigation of the basic properties of the general framework. Rapid solutions may be obtained as by-products, after the proper simplifications (many of them obvious) are made in the general relations to fit special cases. The writer has avoided presenting simplifications of limited application for the sake of concentrating on the general approach applicable to all cases.

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Some of Mr. Beskin's comments originate from his predilection (see his conclusion 1) for "* * * special methods well adapted to each category of problems, since absolutely general methods [such as least work] are impracticable * * *." This opinion is not generally accepted. Scientific progress might be promoted best by a search for methods elastic enough to apply to the most general systems (even if processes of approximation are necessary for numerical solutions) and at the same time immediate enough to permit visualization of the geometric relationships between component elements. The method of least work meets the first requirement, but not the second. The paper has attempted to meet both requirements. Preferences are a matter of opinion, however, and it is idle to argue about them.

More of Mr. Beskin's comments seem to reflect an almost personal reaction, arising either from the fact that a few of the results which can be easily derived from the writer's general relations as special consequences have been presented directly by others under one name or another (mostly in connection with the analysis of continuous beams) or simply from the fact that sometimes Mr. Beskin himself can bring a solution of limited application to one of the isolated phases of the problem. This attitude permeates his discussion. The fact is (and Mr. Beskin chooses to ignore it) that an integrated and coherent method of analysis, applicable to very general frameworks, has been presented in this paper, based on concepts which are new, to the best of the knowledge of the writer and of all those connected with the preparation and examination of the paper.

Mr. Beskin derives Eq. 60 from a simple consideration of Fig. 13; it is indeed equivalent to Eq. 47, established in a more roundabout way. Mr. Beskin uses this fact as an argument purporting to show the "useless complication" of the method. However, his approach is quite specially applicable to the frame of Fig. 18; for a more complex frame he would find it impossible to proceed by such simple analysis. On the other hand, in this case as in every other one treated in the paper, a formal approach has been adopted consistently in order to conform to the routine of the general procedure applicable to all cases.

Mr. Beskin advances Eqs. 50 and 51 as arguments for denying the originality of the writer's compatibility relations, Eqs. 4, and their consequences, Eqs. 7 and 8. Mr. Beskin's equations can indeed be found in many textbooks; but, far from being the equivalents of Eqs. 7 and 8, they represent only very special (and easily derivable) consequences of these compatibility relations applied to curved beams and to triangular frames, respectively. As a matter of fact, that is precisely the way Mr. Beskin establishes his Eq. 50. He then refers to the Williot-Mohr construction, which, as he correctly implies, also has been known for years; but this again is one more particular result of these same compatibility relations (Eqs. 7 and 8) applied to articulated frames. Example 5 of the paper shows clearly the analytical translation of this construction as an application of the method. Eqs. 4, 7, and 8 are general and they are original; by his comments, Mr. Beskin unwittingly furnishes additional proof that they are also fertile.

Mr. Beskin analyzes the frame of Fig. 16(*c*) quite correctly; it is indeed of the 42d order and a rather complex structure. As adverse criticism of the method, he advances the fact that this frame requires twenty-eight rotations to be considered in the general relations. In reply, the writer would state that, comparatively, the conventional slope deflection method requires sixty-four unknown rotations, plus seven story deflections, and that, with the method of least work, the analyst must deal with some 170 unknowns. The writer had no reason for preferring the use of "arbitrary unknowns which are the same in all cases," beyond the very simple reason that the number of unknowns is appreciably less here than that in other methods. Mr. Beskin's own example is proof. Naturally, for the actual numerical solution of frames of this character, any experienced engineer will resort to processes of approximation, whether he uses one of the many existing methods, or the writer's method.

Mr. Beskin's opinions of the section under the heading, "Topology of Frameworks," are based on the fact that he can establish Eq. 52 directly, instead of deriving it, as in Proposition (*c*) of the paper, from a general analysis. It is necessary to proceed with such an analysis in order to establish, among other important relationships, the n_i-equations required by this method for the elimination of dependent axial strains. In addition, a study of the topological properties of two-dimensional frameworks will be found a helpful steppingstone when a general analysis of space structures is attempted.

As regards Mr. Beskin's conclusions 3 and 4, the writer agrees that the unknown joint rotations that help to "open up" multi-connected frames may properly be included in the set of axial strains used by the writer as principal unknowns. However, in conformity with the general procedure of the method, these joint rotations themselves may be written in terms of the axial strains of the imaginary diagonals of some of the component parts of a frame, such as shown in Figs. 16(*e*) and 16(*f*). Therefore, the writer's introductory description of his method and the process of operations remain valid.

The method presented in this paper deals with the two-dimensional frame in its general, almost symbolized form; hence, a penalty must be paid (see Mr. Beskin's conclusion 5) in the form of more deliberate approaches for the inclusion of complex features which seldom appear in everyday frames. Mr. Beskin's own simple derivation of Eq. 60 for use in the structure of Fig. 13 proves that the method can be simplified quickly for the treatment of special cases. His several references to the excellent paper[9] by D. M. Wilson, M. Am. Soc. C. E., are irrelevant, since the writer's paper and Professor Wilson's paper have different aims.

The semigraphical method of characteristic (fixed) points described by Mr. Polivka has merit; the writer referred to an identical method in the concluding paragraph of the section under the heading, "Determination of End Moments." This method has been developed at great length as one of the steps in a previous analysis.[4] In this paper, an analytical, instead of semigraphical, approach was preferred as being simpler to present. The method described by Mr. Eremin is based on similar concepts. Either Mr. Polivka's or Mr. Eremin's method will lead to quick solutions in the case of simple frames.

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Paper No. 2250

UNUSUAL CUTOFF PROBLEMS—DAMS OF THE TENNESSEE VALLEY AUTHORITY

A SYMPOSIUM

PAGE

Deep Solution Channel, Kentucky Dam, Kentucky.
By James B. Hays, M. Am. Soc. C. E. 948

Flowing Water in Underground Channels, Hales Bar Dam, Tennessee.
By L. A. Schmidt, Jr., M. Am. Soc. C. E. 965

With Discussion by Messrs. Berlen C. Moneymaker, Portland P. Fox, J. K. Black, L. C. Glenn, E. H. Burroughs, James B. Hays, and L. A. Schmidt, Jr.

Note.—Published in November, 1943, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

DEEP SOLUTION CHANNEL, KENTUCKY DAM, KENTUCKY

By James B. Hays,[1] M. Am. Soc. C. E.

Synopsis

An unusual method of constructing a cutoff in a solution channel nearly 200 ft deep in the foundation at Kentucky Dam, together with an accurate picture of the results of grouting in the clay-filled and rock-filled channel, is described in this paper. The cutoff wall was constructed in the form of an arch dam except that it was built from the top down rather than starting from the bottom and working upward.

So little is known about grouting in clay that it is thought the results presented herein would be of interest to many engineers engaged in similar work.

Kentucky Dam, a Tennessee Valley Authority (TVA) project, is on the Tennessee River, in Kentucky, 22 miles from its confluence with the Ohio River at Paducah, and is one of the series of flood control, navigation, and power dams that will maintain a 9-ft navigation channel to Knoxville, Tenn.

The over-all length of the combined earth and concrete structure is 8,700 ft, and the maximum height of the concrete spillway section is 165 ft. The spillway section has twenty-four fixed roller gates 40 ft wide by 50 ft high, each in three sections. The normal head is 57 ft with a maximum of 73 ft. The lock chamber is 110 ft wide by 600 ft long. The power house will ultimately have five units. Turbines are of the adjustable-blade Kaplan type. A general plan of the dam is shown in Fig. 1.

The dam was constructed in three stages. The lock, built first, was completed and placed in temporary operation early in 1942. Because of the topography of the site, the lock was built in an open excavation on the east, or right, bank flood plain, surrounded by an earth-dike cofferdam to hold back flood waters. This permitted the starting of the second stage, the building of the cofferdam in the river for the power house and part of the spillway, without waiting for the completion of the lock. The third stage involves the remainder of the spillway.

Foundation drilling and exploration of several sites began in the summer of 1935, with the selection of the final site in 1937. Closer and more detailed investigations continued even after construction was started on July 1, 1939. The rock was known to be limestone of the Fort Payne formation, but very little was exposed at the site. A few outcrops occurred on the right bank above the flood plain. The overburden was deep. The shallowest depth, 20 to 30 ft, to rock was in the area covered by the lock. In the river section, rock was 50 ft below the bed of the river, and, on the left bank, under the long earth-fill section, the depth was from 85 to 110 ft.

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[1] New York, N. Y.; formerly Project Mgr., South Holston and Watauga Projects, TVA, Bristol, Tenn.

After considerable study of the cores and near-by formations, the geologists advised that most of the cavities in the foundation would probably be in the form of vertical solution channels along joints or fractures in the rock. For this reason considerable exploratory drilling was done by using 45° overlapping angle holes. This revealed many defects in the foundation that had been missed in the earlier vertical drilling. It later developed that the geologists' interpretations were generally correct, with a few exceptions.

At the time the construction department took charge, the general exploration had been nearly completed. It was necessary, however, to continue the drilling to fill in gaps and develop the conditions in more detail.

Many of the cavities or defects in the foundation were located before actual excavation had uncovered them. The drill records merely located the cavities but did not indicate their actual shape or the condition of such material as might be in them. The upper cavities were known to be generally filled with clay; those at lower elevations were often only partly filled or open.

The Fort Payne formation is a dark limestone interbedded with layers and large flat lenses of chert. The chert layers were generally from 2 in. to 6 in. thick with the limestone between varying from 4 in. to 12 in. The bedding varied from nearly flat, under a part of the lock, to a 6° dip toward the left bank. The limestone, in weathering, turned to a clay, but the chert layers remained unaltered, although generally found broken or cracked into small pieces. A large part of the area excavated to rock was covered with a thick layer of residual clay and chert varying from 8 ft to 10 ft in thickness. The clay and chert layers over solution channels had often slumped down, and the extent of this slumping generally indicated the extent of the cavity.

Several vertical solution channels in the lock had been located by the time excavation had reached full swing. The deepest and most extensive channel was found at the junction of the earth dam and the river wall of the lock just downstream from the lower lock gate. A study of the trend of the joint system in the rock, together with the earlier drilling records, indicated that this channel extended under the river and angled across the dam at about 45° to the axis and into the lower lock channel.

A line of drill holes was put down from the top of the lock cut on the side toward the river, with the idea of further developing the trend and extent of the channel. These holes were also grouted to prevent excessive leakage into the excavation. The drill records indicated layers of rock and clay with the deep cavities open or partly filled. Since the open cavities were water bearing, there was the danger that they might connect directly with the river and be a source of trouble. As a result of the grouting, there was practically no leakage through the rock or the solution channels into the lock excavation.

An intensive program of drilling was undertaken in the area of the lock foundation with holes on 20-ft centers each way to develop the extent of the solution channel (see Fig. 2). Two problems were involved: One was how the lock-wall would be supported over the channel; the other was what treatment would be required in the channel itself to make an adequate cutoff under the dam and prevent possible serious damage in the future. This paper deals with the second problem. All holes were grouted soon after drilling was completed..

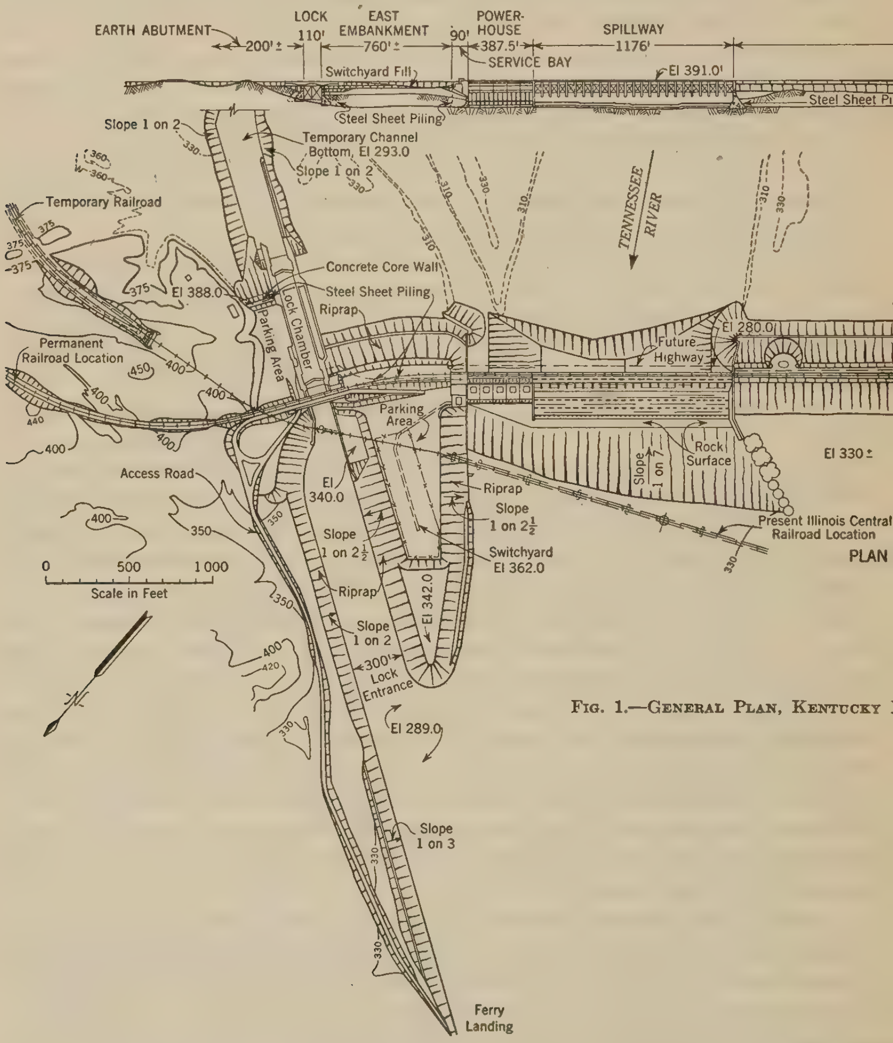
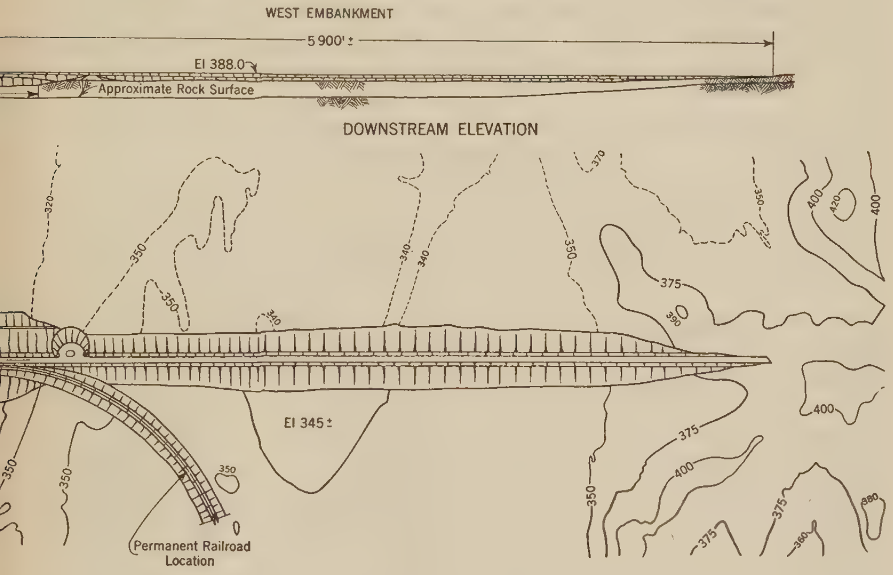

EARTH ABUTMENT
200'±
LOCK
110'
EAST EMBANKMENT
760'±
90'
POWER-HOUSE
387.5'
SERVICE BAY
SPILLWAY
1176'
Switchyard Fill
El 391.0'
Steel Sheet Pi
Steel Sheet Piling
Slope 1 on 2
Temporary Channel Bottom, El 293.0
Slope 1 on 2
330
360
Temporary Railroad
375
310
330
TENNESSEE RIVER
Concrete Core Wall
Steel Sheet Piling
Riprap
El 388.0
Lock Chamber
Parking Area
Permanent Railroad Location
450
400
440
El 280.0
Future Highway
Parking Area
Rock Surface
Slope 1 on 7
El 330±
Access Road
El 340.0
Riprap
Slope 1 on 2½
Present Illinois Central Railroad Location
PLAN
Switchyard El 362.0
350
Slope 1 on 2½
0
500
1 000
Scale in Feet
Riprap
El 342.0
Slope 1 on 2
420
300' Lock Entrance
N
El 289.0
Slope 1 on 3
Ferry Landing

Fig. 1.—General Plan, Kentucky

These preliminary drill records (Fig. 3) indicated that there was one wide channel and a near-by smaller one probably parallel to it. The main channel at the narrowest point was approximately 70 ft wide down to a depth of 100 ft below the floor of the lock, which was at El. 289 (sea-level datum). At that point it narrowed to about 30 ft and continued another 110 ft to El. 70, with a few small cavities on one side, as shown in Fig. 3. The narrow channel was not exposed at the rock surface under the river-wall of the lock, but there was evidence of it just downstream from the land-wall. The problem of supporting the lock-walls was solved by spanning the wide channel with deep, heavily reinforced-concrete slabs, shown in Fig. 4.


WEST EMBANKMENT
5 900'±
El 388.0
Approximate Rock Surface
DOWNSTREAM ELEVATION
320
350
350
340
340
340
350
370
375
380
350
400
420
400
El 345±
375
400
400
350
350
350
350
375
375
360
380
Permanent Railroad Location

The method of making a satisfactory cutoff in the channel was discussed at considerable length. The proposal of ordinary grouting of the clay-filled channel was considered. It was felt that this would probably prove satisfactory for a time, but there would remain the uncertainty that the clay might eventually soften and be washed out by water, or pushed out by the pressure, and produce a disastrous blow.

The part of the earth dam between the power house and the lock, built on original ground, had a sheet-pile cutoff driven to rock that was originally intended to tie in to the lock-wall just downstream from the solution channel.

The final decision was to construct a definite concrete cutoff by drilling a series of holes, 36 in. and 48 in. in diameter, along a circular arc as shown in the plan, Fig. 5. These holes would be used as shafts for successively excavating small chambers and backfilling with concrete, thus creating an arch dam underground with the convex side against the water pressure. The plan view in Fig. 5 is an assumed stage in the procedure of constructing the cutoff walls. The outline of the working slab is not shown. The location was chosen to permit the underground work to proceed without delaying the construction of the lock-walls. The 48-in. holes were located over the deepest part of the solution channel since this particular drill was equipped for deep drilling. A single 48-in. hole was located to cut off the narrow channel.

The area had been grouted as well as possible. The grouting record is shown in Table 1 and Fig. 3. Previous experience had proved that it was practically impossible to drill large holes and excavate in clay-filled channels or cavities without preliminary grouting. The grout, besides filling open cavities, has a tendency to force out soft clay and to compact any that remains. On the lower two thirds of the zone, grout pressures as high as 80 to 100 lb per sq in. were used. On the upper part, pressures of from 10 to 20 lb would often blow out due to lack of sufficient cover for resistance.

TABLE 1.—BORING RECORD, WEATHERED ZONE CUTOFF WALL, EAST EMBANKMENT

Boring No. (prefix RW; see Fig. 2)	CEMENT USED (CU FT)			Refusal (see Fig. 2)	Grouting time (hr)	Date (1940)
	Stage 1	Stage 2	Stage 3			
560B-105; 45°	0	....	....	Grouted through other holes		
594B-141	460½	....	....	Ref. @ 80#-30#	32	August 6
600B-146	....	....	837	Ref. @ 60#-50#-40#	28	October 29
605B-151	96	....	....	Comp. @ 5#	1	July 19
608B-155	....	....	828½	Ref. @ 50#-40#	32	October 29
611.5B-160	....	2,765	....	Ref. @ 10#	48	September 21
619B-166	....	....	1,386	Ref. @ 40#-50#	27	October 29
626B-171	5,250	....	....	Comp. @ 0#-80#	131	August 6
629B-176	....	....	1,503	Comp. @ 5#-100#	25	November 11
631.5B-180	8,040	....	....	Ref. @ 40#	107	September 7
636.5B-185	....	....	2,210	Ref. @ 20#	19	November 10
641.5B-190	....	2,790	....	Comp. @ 0#-80#	83	September 30
645B-190; 45°	....	367	....	Ref. @ 10#	14	October 28
646.5B-195	....	....	895	Comp. @ 5#-75#	19	November 5
651.5B-200	....	1,255	....	Ref. @ 10#	32	September 20
656.5B-205	....	....	519	Ref. @ 30#	20	November 10
661.5B-210	....	4,736	....	Ref. @ 40#	58	September 30
669B-216	....	....	4¾	Ref. @ 100#	6	November 4
676B-221	584	....	....	Ref. @ 40#	8	July 19
686B-231	6	....	....	Ref. @ 100#	2	January 22
691B-236	....	10	....	Ref. @ 50#	4	October 28
696B-241	16	....	....	Ref. @ 60#	1	August 5
742B-288; 45°	....	....	....	....	....	....
701B-246	....	740	....	Ref. @ 75#-10#	16	October 28
706B-251	945	....	....	Ref. @ 60#	15	July 23
710B-255	....	20	....	Ref. @ 75#	3	October 28
714B-260	359	....	....	Ref. @ 70#	8	August 2
747B-292	....	248	....	Ref. @ 75#-10#	21	October 29
Total	15,756½	12,931	8,183¼	36,870¾ cu ft of cement		

To give the drillers a suitable place to work and to furnish the preliminary support for the first sections of concrete backfill, a thick reinforced-concrete slab was placed over the area. The surface of this slab was at El. 265. Reinforcing steel stubs were driven down into the clay, or into jackhammer holes drilled through loose rock slabs or boulders directly over the proposed concrete cutoff, to be used for anchoring the lower concrete blocks in position and prevent them from settling into the soft clay or dropping into the next excavation below. Reinforcement inserted in jackhammer holes drilled in rock was packed with clay to prevent any bond to rock that might later need to be removed. As the work progressed downward there was never any certainty that rock in the floor of a chamber would or could be allowed to remain, since any such rock might be a loose slab or boulder that would have to be removed in the next operation. The side-walls of the channel were tested with jackhammer holes at frequent intervals.

One 48-in. and two 36-in. shot drills were set up on alternate holes. The plan, as shown in Fig. 6 (see step I), was to drill 5 ± ft below the concrete slab and excavate a chamber 6 ft wide normal to the line of the holes and halfway to the next regular hole on each side. As soon as each unit of excavation was completed, reinforcing steel stubs were driven laterally toward the two adjacent holes into the clay or into jackhammer holes drilled in boulders, and another set was driven into the floor of the chamber, lapped and tied to the steel extending down from the roof that had previously been driven into the clay below the heavy concrete mat and was exposed when the excavation was made (see dowels A, Fig. 6). The lateral steel was used to tie the blocks together and assist in arch action, since the vertical joints between blocks were quite irregular in shape. No steel was placed to come directly under the drill hole. The chamber was then filled with concrete and the drill moved to another location while the concrete hardened, to repeat the process, after which it was moved back to the first hole. The concrete that filled the first chamber was drilled through and a similar excavation was made directly under it (step II, Fig. 6). Reinforcing steel was again installed, as before, and the chamber again was filled with concrete. In filling each excavation, the concrete was placed to a depth of 2 or 3 ft in the shaft above the roof and vibrators used to force the concrete to fill the space completely.

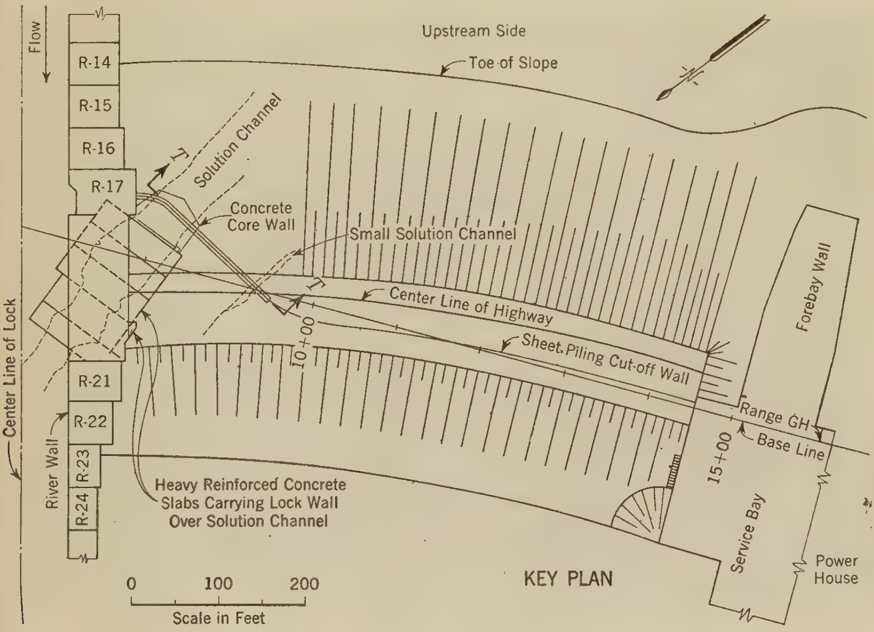
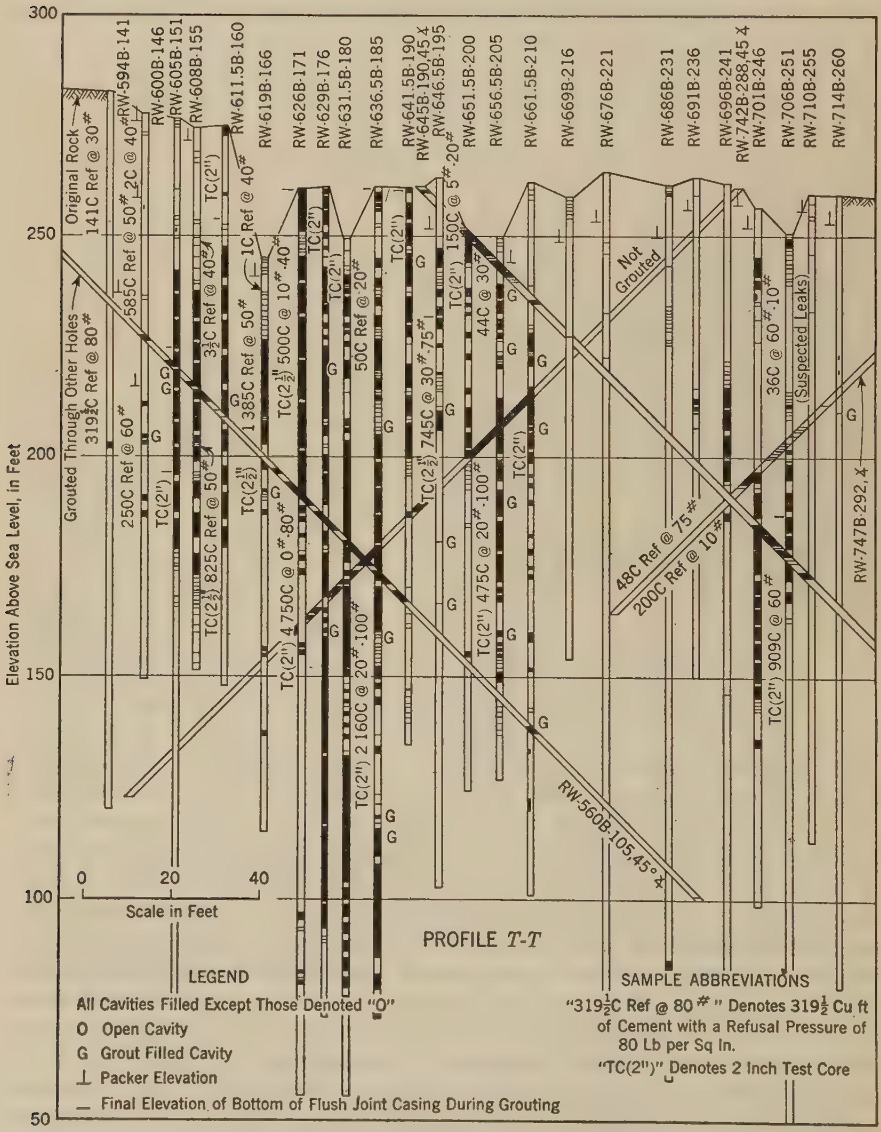

Flow
Upstream Side
Toe of Slope
R-14
R-15
R-16
R-17
Solution Channel
Concrete Core Wall
Small Solution Channel
Center Line of Highway
Sheet Piling Cut-off Wall
Forebay Wall
Range GH
Base Line
10+00
15+00
Center Line of Lock
River Wall
R-21
R-22
R-23
R-24
Heavy Reinforced Concrete Slabs Carrying Lock Wall Over Solution Channel
Service Bay
Power House
KEY PLAN
0
100
200
Scale in Feet

Fig. 2.—Key Plan of Weathered Zone Cutoff Wall, East Embankment

The cycle of operations was repeated (steps III, etc.), moving the drills from one hole to another to keep up a continuous operation for the drillers and permit the concrete to harden. In some clay pockets no drilling was done, in which case the drill was used only for hoisting out the muck that was excavated by hand into buckets. When contact was made with rock, a test hole was drilled by jackhammer in the center of the 36-in. or the 48-in. hole; then addi-



Elevation Above Sea Level, in Feet
300
250
200
150
100
50
Original Rock
141C Ref @ 30#
Grouted Through Other Holes
319½C Ref @ 80#
RW-594B-141
RW-600B-146
RW-605B-151
RW-608B-155
RW-611.5B-160
RW-619B-166
RW-626B-171
RW-629B-176
RW-631.5B-180
RW-636.5B-185
RW-641.5B-190
RW-645B-190,45°
RW-646.5B-195
RW-651.5B-200
RW-656.5B-205
RW-661.5B-210
RW-669B-216
RW-676B-221
RW-686B-231
RW-691B-236
RW-696B-241
RW-742B-288,45°
RW-701B-246
RW-706B-251
RW-710B-255
RW-714B-260
585C Ref @ 50# 2C @ 40#
250C Ref @ 60#
TC(2")
3½C Ref @ 40#
TC(2½) 825C Ref @ 50#
TC(2")
Ref @ 40#
1C Ref @ 50#
1 385C Ref @ 50#
TC(2½)
TC(2½) 500C @ 10#·40#
TC(2") 4 750C @ 0#·80#
TC(2")
TC(2")
50C Ref @ 20#
TC(2") 2 160C @ 20#·100#
TC(2")
TC(2½) 745C @ 30#·75#
TC(2") 150C @ 5#·20#
44C @ 30#
TC(2") 475C @ 20#·100#
TC(2")
Not Grouted
48C Ref @ 75#
200C Ref @ 10#
36C @ 60#·10#
TC(2") 909C @ 60#
(Suspected Leaks)
RW-747B-292,45°
RW-560B-105,45°
G
0
20
40
Scale in Feet
PROFILE T-T
LEGEND
All Cavities Filled Except Those Denoted "O"
O Open Cavity
G Grout Filled Cavity
⊥ Packer Elevation
— Final Elevation of Bottom of Flush Joint Casing During Grouting
SAMPLE ABBREVIATIONS
"319½C Ref @ 80#" Denotes 319½ Cu ft of Cement with a Refusal Pressure of 80 Lb per Sq In.
"TC(2")" Denotes 2 Inch Test Core

Fig. 3.—Preliminary Drill Records, Weathered Zone of Cutoff Wall, East Embankment

tional holes were drilled for dowels if the rock was found to be satisfactory. If a cavity was encountered, holes were drilled and dowels were extended into it, as before. After the first group of holes had been extended down four or five stages, the intermediate holes were started and carried down, alternating with the first set. The chambers excavated through the intermediate holes had concrete on two sides, with dowels projecting from the walls as well as from the roof. Frequently these chambers were narrower than the first set, due to overbreak on the latter. In the narrow channel, the depth of excavation in each section of the hole was dependent on the conditions indicated in Figs. 7 and 8.

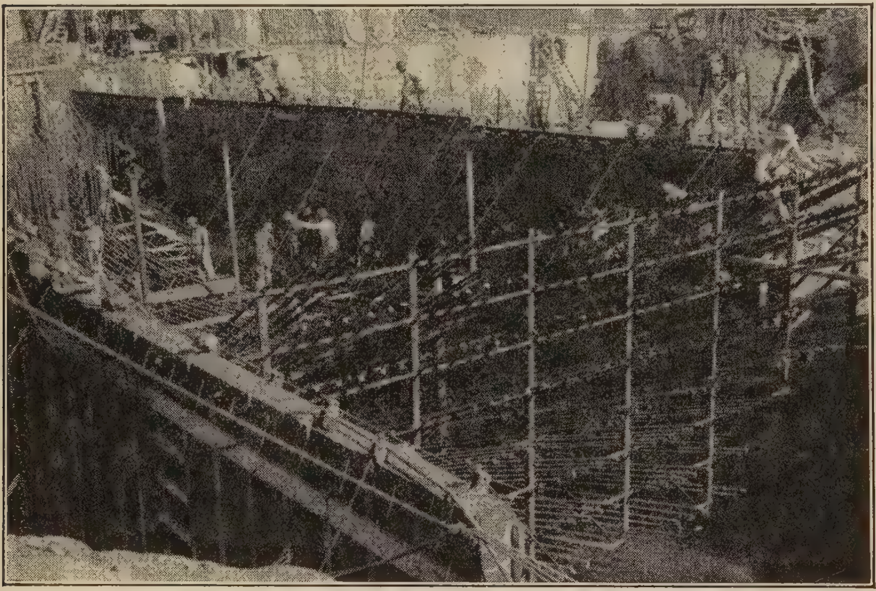


Fig. 4.—View of Heavy Reinforced Concrete Slab

This series of large drill holes was started on November 16, 1940, and hole No. 6 (Fig. 8), the last one to be completed, was finished July 2, 1941. The first concrete pours were made under the slab through the large drill holes, and the drilling, mucking, and concreting proceeded in sections 5 ft to 10 ft high until the bottom was reached. To complete this cutoff required 1,763.4 lin ft of large drill holes, including holes B and C, Fig. 8.

Water leakage into the excavated chambers, which was relatively small in volume, gradually increased as the work deepened. Part of the leakage came from the drills working on other holes which usually drained into the deepest hole. Small air-driven centrifugal pumps were adequate until the head became too great. To take care of this condition, a larger chamber was opened up in hole No. 6 between El. 108 and El. 90 to be used as a relay pumping station. Hole No. 12 had followed a narrow channel and below El. 140 had been drilled deeper than the usual 6 ft before concreting and was completed earlier. In the pump room in hole No. 6 the side-walls, along the line of the cutoff, were solid

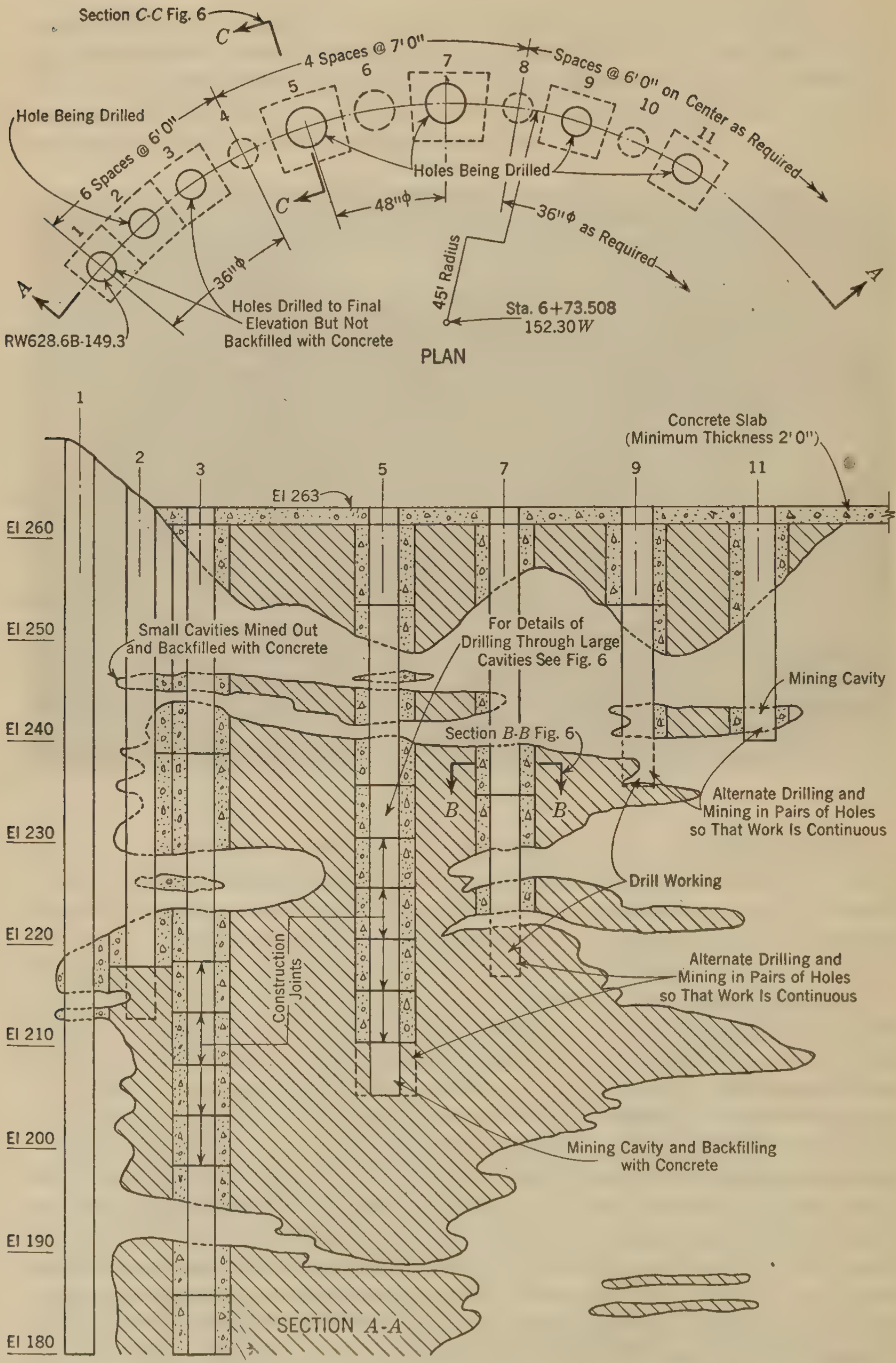

Section C-C Fig. 6
C
4 Spaces @ 7'0"
Spaces @ 6'0" on Center as Required
Hole Being Drilled
6 Spaces @ 6'0"
1
2
3
4
5
6
7
8
9
10
11
Holes Being Drilled
C
48"ϕ
36"ϕ as Required
36"ϕ
45' Radius
A
A
Holes Drilled to Final Elevation But Not Backfilled with Concrete
RW628.6B-149.3
Sta. 6+73.508
152.30W
PLAN
1
2
3
5
7
9
11
Concrete Slab (Minimum Thickness 2'0")
El 263
El 260
El 250
El 240
El 230
El 220
El 210
El 200
El 190
El 180
Small Cavities Mined Out and Backfilled with Concrete
For Details of Drilling Through Large Cavities See Fig. 6
Mining Cavity
Section B-B Fig. 6
B
B
Alternate Drilling and Mining in Pairs of Holes so That Work Is Continuous
Drill Working
Construction Joints
Alternate Drilling and Mining in Pairs of Holes so That Work Is Continuous
Mining Cavity and Backfilling with Concrete
SECTION A-A

Fig. 5.—Plan and Axial Section Showing Procedure

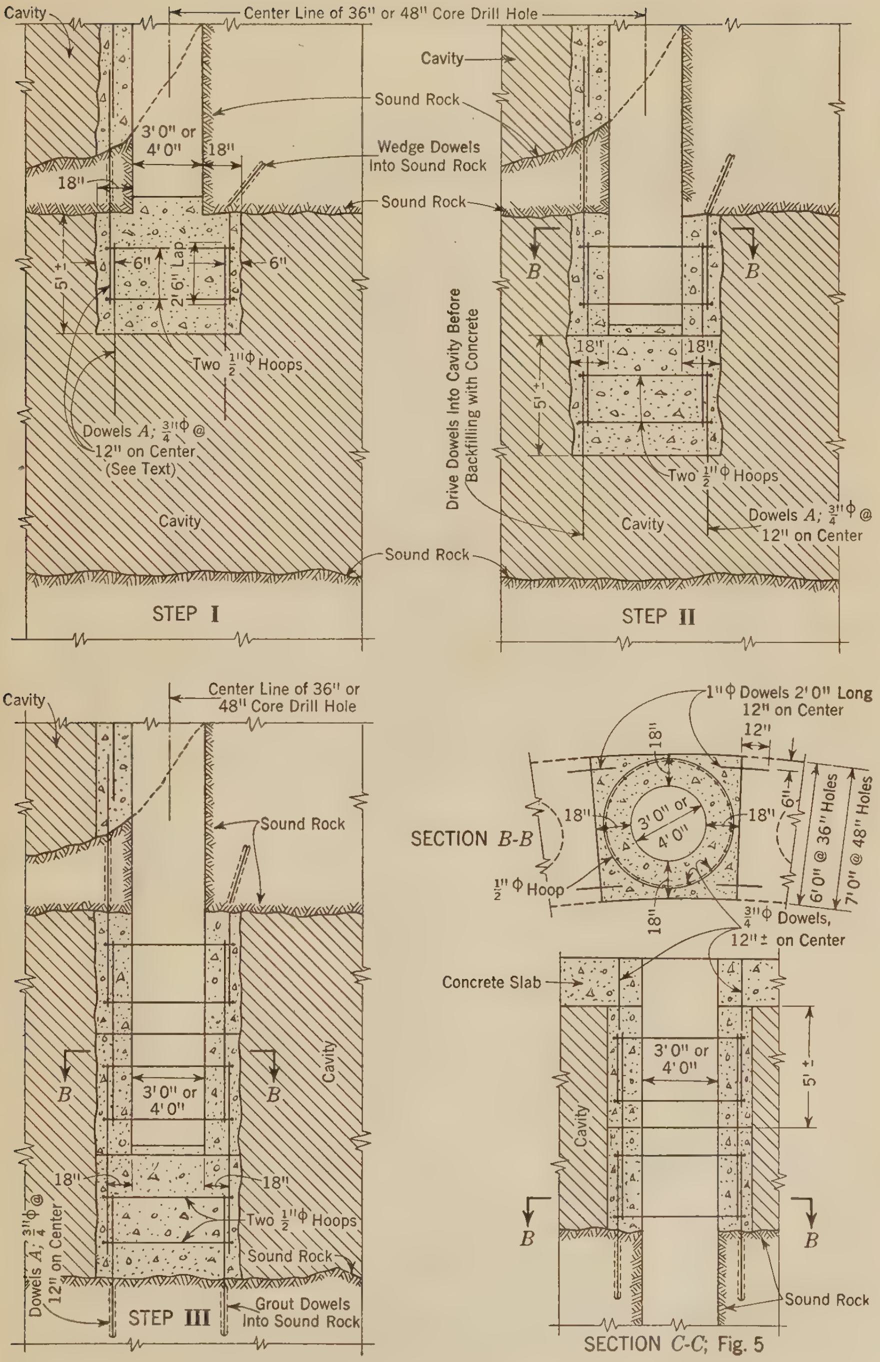

Cavity
Center Line of 36" or 48" Core Drill Hole
Cavity
Sound Rock
3'0" or 4'0"
18"
Wedge Dowels Into Sound Rock
18"
Sound Rock
6"
2'6" Lap
6"
5' ±
Two 1/2"φ Hoops
Dowels A; 3/4"φ @ 12" on Center (See Text)
Cavity
Sound Rock
STEP I
Drive Dowels Into Cavity Before Backfilling with Concrete
B
B
18"
18"
5' ±
Two 1/2"φ Hoops
Cavity
Dowels A; 3/4"φ @ 12" on Center
STEP II
Cavity
Center Line of 36" or 48" Core Drill Hole
Sound Rock
B
B
Cavity
3'0" or 4'0"
18"
18"
Two 1/2"φ Hoops
Sound Rock
Dowels A; 3/4"φ @ 12" on Center
STEP III
Grout Dowels Into Sound Rock
1"φ Dowels 2'0" Long 12" on Center
12"
18"
SECTION B-B
18"
3'0" or 4'0"
18"
6"
6'0" @ 36" Holes
7'0" @ 48" Holes
1/2"φ Hoop
18"
3/4"φ Dowels, 12" ± on Center
Concrete Slab
3'0" or 4'0"
5' ±
Cavity
B
B
Sound Rock
SECTION C-C; Fig. 5

FIG. 6.—STEPS IN THE PROCEDURE

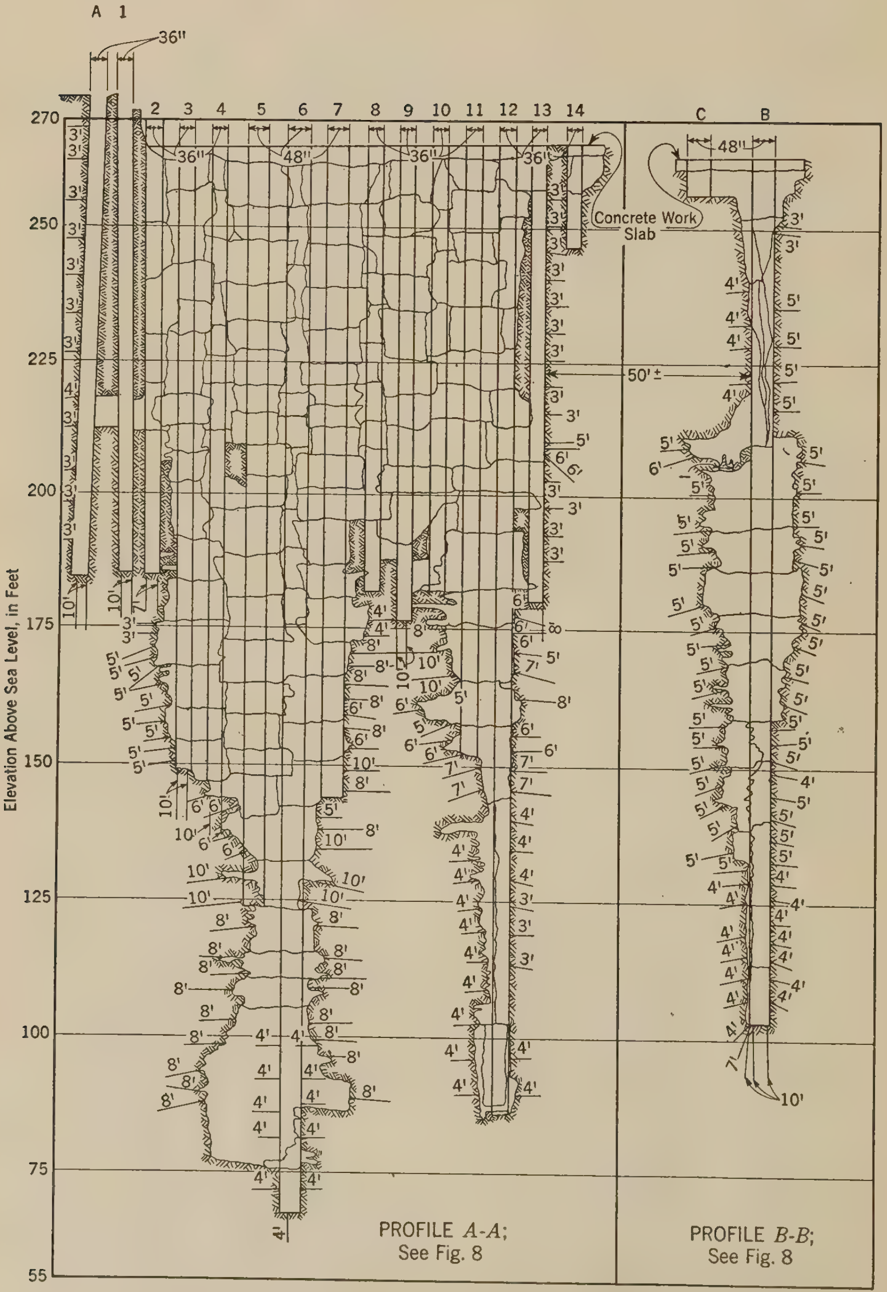

A
1
36"
2
3
4
5
6
7
8
9
10
11
12
13
14
36"
48"
36"
36"
C
B
48"
Concrete Work Slab
50'±
Elevation Above Sea Level, in Feet
270
250
225
200
175
150
125
100
75
55
10'
PROFILE A-A;
See Fig. 8
PROFILE B-B;
See Fig. 8

Fig. 7.—Outline of Excavation, Showing Concrete Pours

rock. The upstream and downstream walls exposed grouted residual clay and were first timbered and later concreted. Test drilling with jackhammers below

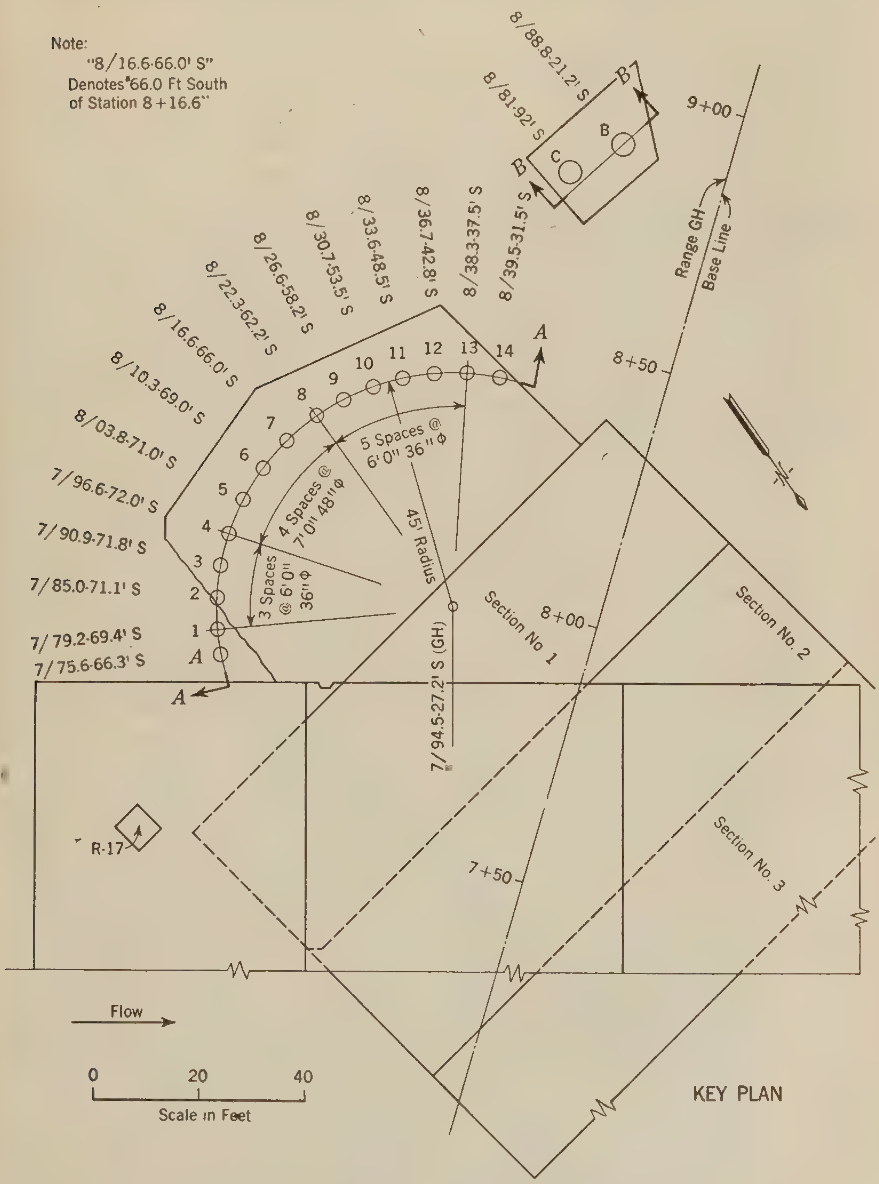

Note:
"8/16.6-66.0' S"
Denotes 66.0 Ft South
of Station 8+16.6"
8/88.8-21.2' S
8/81.9-2' S
B
B
C
B
9+00
Range GH
Base Line
8/39.5-31.5' S
8/38.3-37.5' S
8/36.7-42.8' S
8/33.6-48.5' S
8/30.7-53.5' S
8/26.6-58.2' S
8/22.3-62.2' S
8/16.6-66.0' S
8/10.3-69.0' S
8/03.8-71.0' S
7/96.6-72.0' S
7/90.9-71.8' S
7/85.0-71.1' S
7/79.2-69.4' S
7/75.6-66.3' S
A
14
13
12
11
10
9
8
7
6
5
4
3
2
1
A
A
8+50
5 Spaces @ 6'0" 36"Φ
4 Spaces @ 7'0" 48"Φ
3 Spaces @ 6'0" 36"Φ
45' Radius
Section No 1
8+00
Section No. 2
7/94.5-27.2' S (GH)
R-17
Section No. 3
7+50
Flow
0
20
40
Scale in Feet
KEY PLAN

FIG. 8.—KEY PLAN SHOWING POSITION OF SECTIONS IN FIG. 7

this point indicated that the channel was narrowing. By this time the work was nearly at the level at which diamond-drill holes had indicated the existence of a channel. During the cleaning out of a narrow channel in the floor, the

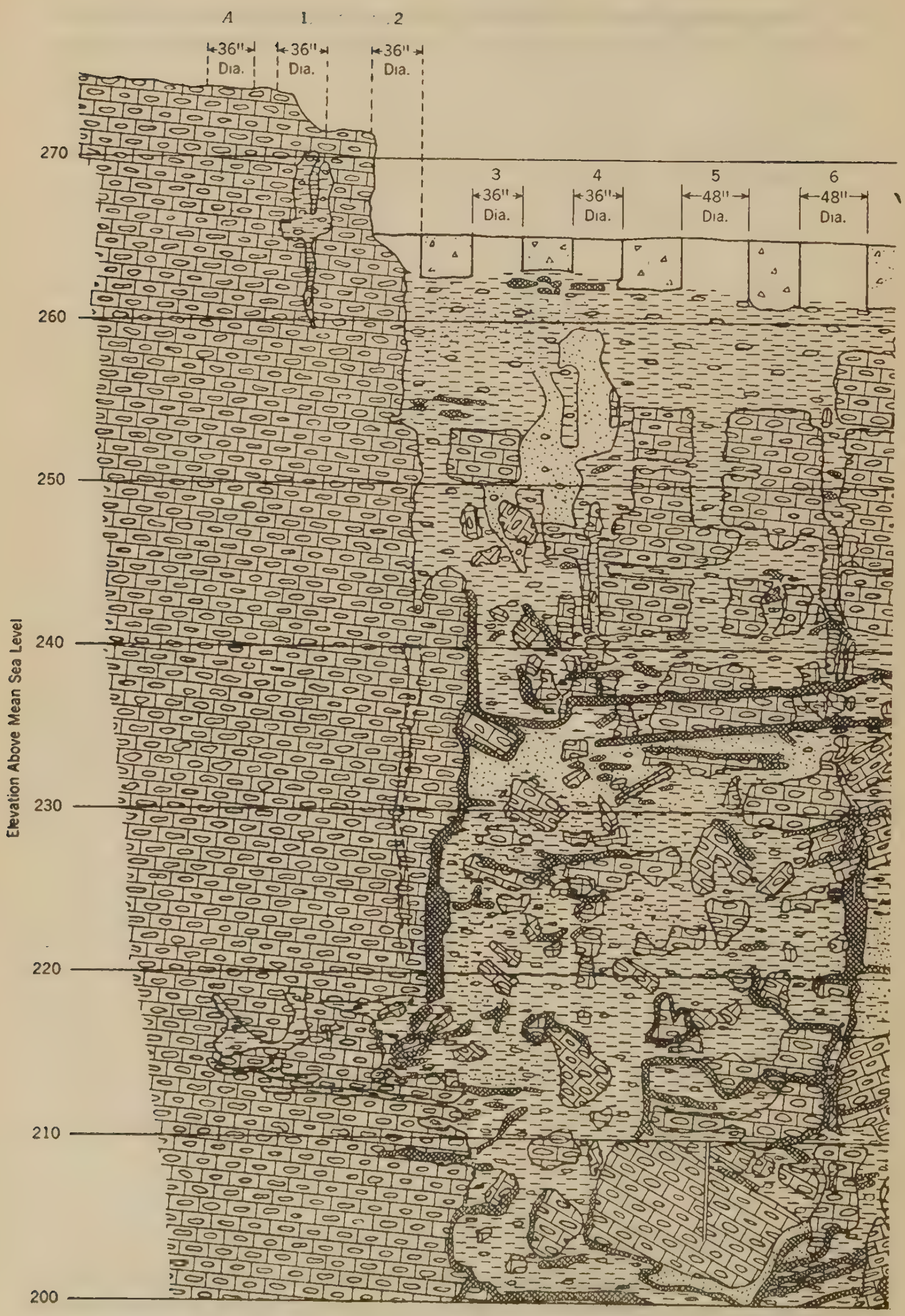
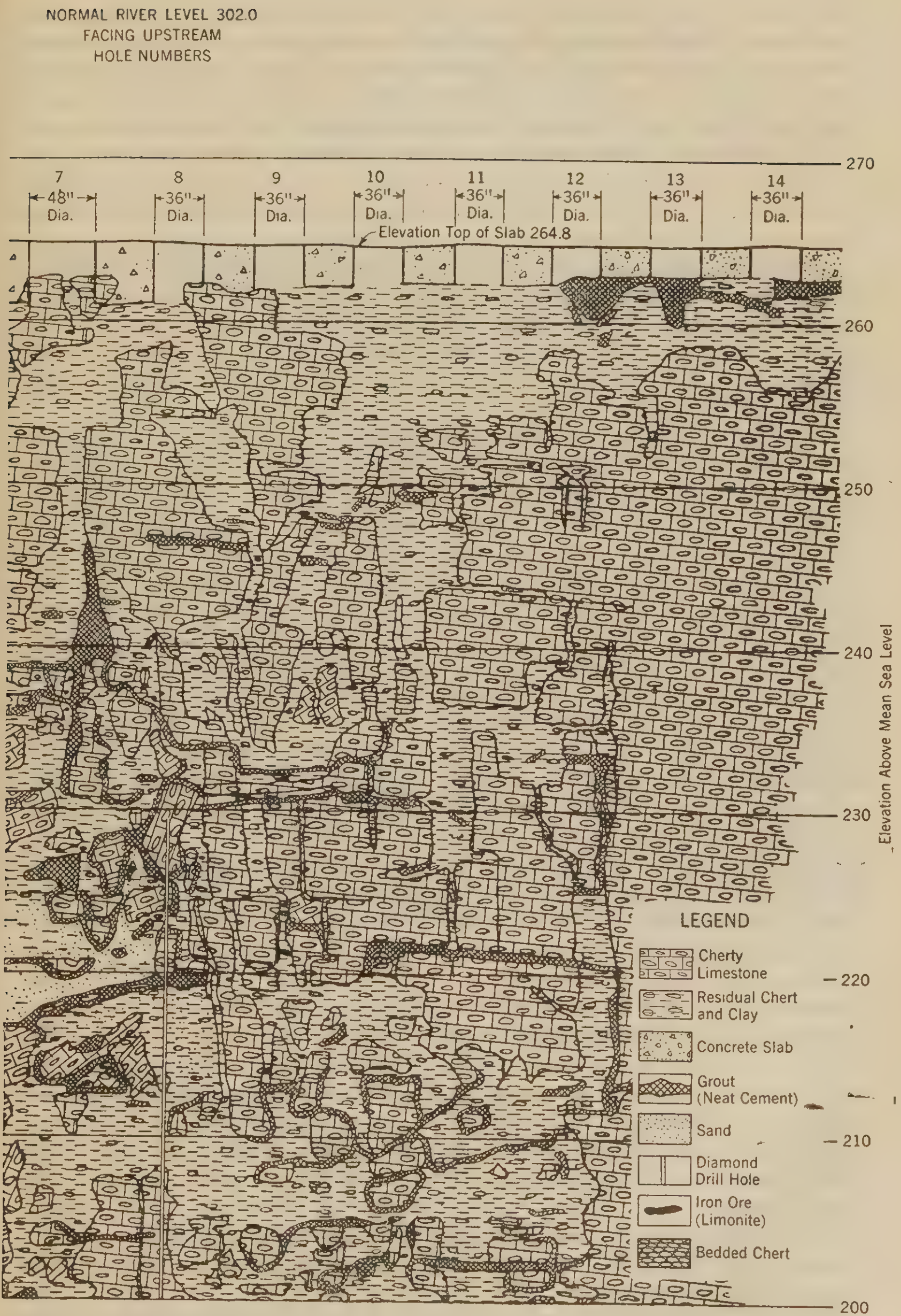

A
1
2
36" Dia.
36" Dia.
36" Dia.
3
4
5
6
36" Dia.
36" Dia.
48" Dia.
48" Dia.
270
260
250
240
230
220
210
200
Elevation Above Mean Sea Level

FIG 9.—GEOLOGICAL


NORMAL RIVER LEVEL 302.0
FACING UPSTREAM
HOLE NUMBERS
7
48" Dia.
8
36" Dia.
9
36" Dia.
10
36" Dia.
11
36" Dia.
12
36" Dia.
13
36" Dia.
14
36" Dia.
Elevation Top of Slab 264.8
270
260
250
240
230
220
210
200
Elevation Above Mean Sea Level
LEGEND
Cherty Limestone
Residual Chert and Clay
Concrete Slab
Grout (Neat Cement)
Sand
Diamond Drill Hole
Iron Ore (Limonite)
Bedded Chert

Section, East Embankment

clay began to push up from below. As the digging continued, the movement increased. Realizing that this might be dangerous, the contractor disconnected the pump and hoisted out, and the workmen then went up themselves, only a little ahead of a slowly rising column of water. The water soon flowed out of the top of the hole. The first measurement, made by pumping, gave a flow of 50 gal per min. Two hours later it had increased to 80 gal per min.

Before further work could be done on the cutoff through this hole, it was necessary to plug it and grout the lower part of the channel. To do this a 4-in. pipe was installed in the shaft, extending from the lowest point in the bottom of the 48-in. hole, at El. 87, to 3 ft above the top, at El. 268. The large chamber was filled with 3-in. crushed rock. A pump was installed with its suction inserted in the 4-in. pipe, and the water was lowered a few feet below the top of the hole to approximately El. 260. Concrete was placed over the crushed rock and well up in the 48-in. hole to seal it off, while the pump pulled the water through the 4-in. pipe.

A diamond-drill hole previously bored at approximately El. 130 between holes Nos. 4 and 5, to check and grout contact zones between blocks of concrete in the cutoff, was extended into the rock-filled chamber. Another diamond-drill hole was bored through the concrete plug to the top of the crushed rock to be used as a vent.

Grouting was started through the 4-in. pipe, from which the water had been flowing, using thick grout, 0.6 water-cement ratio by volume. In a short time the grout gate indicated no pressure. The 180-ft column of grout in the 4-in. pipe was heavier than the water pressure and created a vacuum. Eventually back pressure developed and grouting was continued until 2,297 bags of cement had been injected to a refusal pressure of 100 lb per sq in. The diamond-drill hole between holes Nos. 4 and 5 was next grouted and required 693 bags. The vent hole drilled through the concrete was deepened to El. 80 and grouted, using 3,493 bags of cement with 150-lb refusal pressure. A total of 6,483 bags of cement was used.

Drilling with the 48-in. equipment was resumed after the grout had set sufficiently. The core in the shaft had to be broken out with paving breakers, due to the 4-in. pipe which prevented breaking the core in the usual manner. The grouted broken rock below the concrete was found to be a solid mass. Jackhammer test holes indicated no pockets; hence, this material was not removed. Many of the stones were found coated with thin films of clay, but there were no continuous seams to cause leakage. The 48-in. hole was finally driven to the bottom and concreted. As a final check, diamond-drill holes were put down midway between each of the 36-in. or 48-in. holes and grouted. Fig. 7 shows the outline of each block of concrete placed in the cutoff. In the secondary channel, only one hole was drilled except where it was necessary to drill a second hole through the concrete slab at the top to fill a shallow clay-filled pocket.

The location of the sheet-pile cutoff was changed to follow directly over the underground curtain. The backfill was rolled earth and its width at this depth permitted the change.

During the process of constructing the cutoff, a detailed examination was made of the undisturbed walls and floor of each chamber just prior to backfilling with concrete. A sketch was made of the upstream face showing the rock with its dip and strike, the residual clay, sand pockets, grout, and other rock formations. The assembly of this information to El. 200 is shown in Fig. 9. The original drawing extends downward to El. 80. There were many very thin seams and stringers of grout that were too small to indicate, and many thin seams are exaggerated in thickness. Fig. 10 is a photograph of a typical face, with arrows indicating many of the grout seams. Where diamond-drill holes were encountered, they have been shown in Fig. 9.

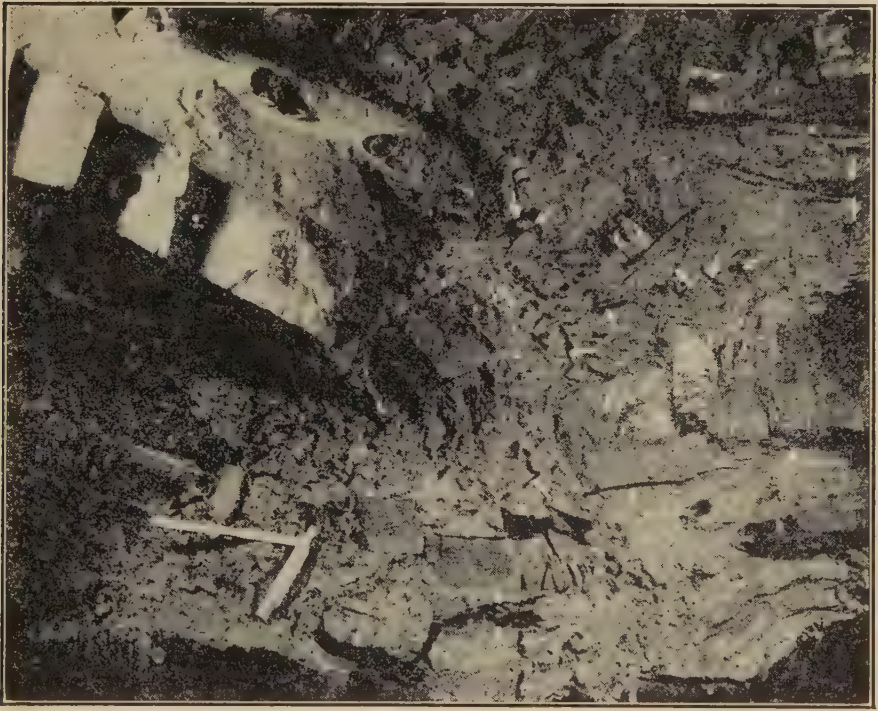


Fig. 10.—Face of Excavation on the River Side of the Line of Cutoff; Hole No. 6 at El. 120

By means of this chart it was possible to develop the geology of the deep vertical solution channel, to compare the actual conditions with the original interpretations of the diamond-drill holes, and also to illustrate the results of grouting in the clay and rock-filled channel.

There is no evidence of faulting. Diamond-drill holes had been driven to 320 ft below sea level on both sides of this channel about 700 ft to the south of the cutoff. The next formation that can be identified accurately at the contact is the Chattanooga black shale at El. − 307.

The rock that was exposed in the foundation excavation had numerous vertical joints, most of which had been recemented with calcite. The general trend of most of the weathered joints is north and south, with a secondary set at right angles. Weathered joints of all degrees of alteration have been found, and practically all of them start and develop in the same manner. Ground water readily entering the joint slowly dissolves various layers of the limestone, but does not affect the chert layers which at first remain as rigid shelves or projections, often carrying the weight of the clay above. The residual clay at first does not completely fill the voids in the limestone. Drill holes, encountering these openings, often develop flowing water at the surface if the area has not been grouted, indicating a connection with a higher source of water. As the load on the chert, or chert and limestone, shelves becomes greater, due to the continued weathering of the lower limestone and the widening of the channels, they break down and a general slumping occurs. Strata of chert that have sagged, due to slumping, often can be traced across the weathered channel. Since the limestone is variable in its solubility, some layers dissolve more rapidly than others and many narrow seams extend outward some distance into the solid rock. These, being only partly filled, carry active waters of solution. The diamond-drill records, shown in Fig. 3, indicated open cavities near the bottom or at the sides in narrow seams with several feet of sound rock above them. No open cavities were found in the upper areas due to the slumping action in the wide part of the weathered zone, which forced clay into the horizontal lateral cavities.

A study of the effect of grout in this material is interesting. The cavities were grouted in stages, from the bottom up. Seams in massive rock were generally grouted individually. In the main body of the clay and boulders, grouting was done through the bottom of the casing which was jacked up at intervals, a foot or two at a time, until within 20 to 30 ft from the surface, where it was generally impossible to use any pressure without a blowout and no spread of grout could be obtained. It will be noted that the grout generally was found in measurable quantities adjacent to rock surfaces with main feeders often extending in a horizontal direction from one boulder to another, frequently following the layers of undissolved chert in the clay. In hole No. 6, from El. 220 to El. 225, a column of grout from 8 in. to 18 in. in diameter was found with evidence of a filled drill hole in the center. Another was found in the same hole from El. 100 to El. 108. In hole No. 7 a similar condition was found between El. 240 and El. 246.

Thin seams and isolated bulbs of grout were found in many areas. Many of the larger pieces of excavated grout were found to have pockets of clay in them. There was ample evidence that the grout had solidified the clay and packed the seams sufficiently to permit the walls to stand vertical, and without slumping, at least during the construction period.

The total work with the large drills was listed as 1,763.4 lin ft. The drills had to penetrate each section of concrete, so that the actual drilling was about twice this value. The total volume of grout used in the immediate vicinity of the cutoff was 47,198 bags before the cutoff was made, and 2,657 bags afterward. This is in addition to the amount used to stop the aforementioned sand and water flows.

ACKNOWLEDGMENT

T. B. Parker, M. Am. Soc. C. E., was chief engineer of the TVA, and A. L. Pauls, M. Am. Soc. C. E., is chief construction engineer. At Kentucky Dam, George P. Jessup is project manager; B. S. Philbrick, M. Am. Soc. C. E., is construction superintendent; and the writer was construction engineer, until January, 1942, when he was transferred to another project.

FLOWING WATER IN UNDERGROUND CHANNELS, HALES BAR DAM, TENNESSEE

By L. A. Schmidt, Jr.,[2] M. Am. Soc. C. E.

Synopsis

Hales Bar Dam on the Tennessee River was built by private interests (1905–1913) and is one of the classics in the annals of early dam building in the United States. Since its completion it has presented a foundation leakage problem which remained unsolved at the time the property was acquired by the Tennessee Valley Authority (TVA) in 1939. Previous efforts to correct the leakage were varied and numerous and inevitably resulted in only temporary improvement. Leakage has been progressively greater until a flow of 1,700 cu ft per sec was measured as ungainfully passing just before the present work by TVA was begun in 1940. A cutoff wall was placed in the foundation leakage area just upstream of the dam to reduce flows, and the foundation was stabilized by placing concrete, cement, and asphalt grout in the foundation under the structure. Special equipment and methods were required and devised to do this work without interrupting the hydroelectric power operations.

Among the more unique core drilling projects in the United States to date is the foundation treatment given to Hales Bar Dam on the Tennessee River, thirty-three river miles downstream from Chattanooga, Tenn. Fig. 11 is a general view of the spillway, facing upstream from the top of the south control house on the river-wall of the lock.

On heavy construction core drills are frequently used for exploration and for determining excavation requirements both as to equipment and to yardage involved. At Hales Bar core drills were used for excavation, cutting out some 35,000 lin ft of 18-in. hole and removing several thousand cubic yards of badly leaking foundation rock. This material was replaced by concrete specially placed in a predetermined pattern to effect a cutoff wall under this historical river barrier.

Stories describing many previous attempts to stop leakage at Hales Bar are legend, and the colloquial narratives are sufficiently enigmatical to intimidate engineering efforts that challenge the flow of water in the underground channels. To understand this leakage problem one must delve back to 1905–1913, the time the dam was built by private interest for hydroelectric power and for navigation improvement in the Tennessee River to Chattanooga. Even then foundation leakage into cofferdams under only nominal differential heads was great enough to force the constructors to build a part of the dam by the use of pneumatic caissons. After the reservoir was filled, leakage in the foundation was detected from water boiling up on the downstream side of the dam.

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[2] Acting Constr. Engr., TVA, Hales Bar Dam, Chattanooga, Tenn.

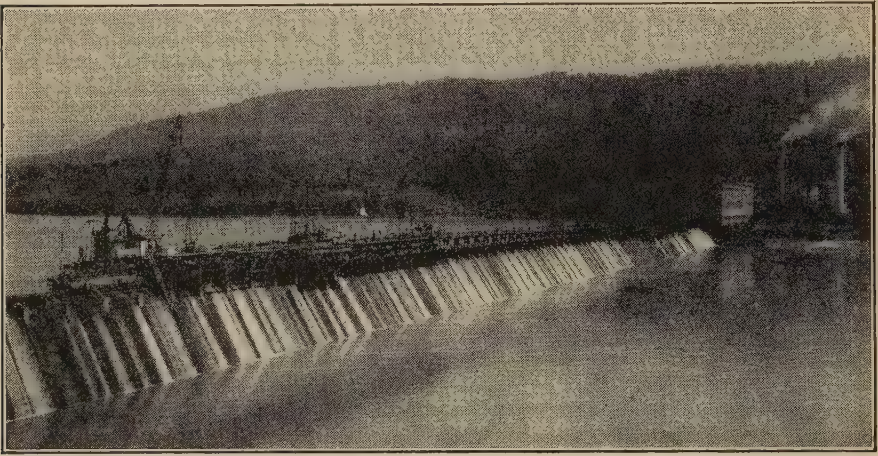

FIG. 11.—SPILLWAY OF HALES BAR DAM

The dam is built on the Mississippian limestone formations of this vicinity (see Fig. 12). Leakage at Hales Bar exists principally in the foundation under the dam. Noticeable points of suction have been found in the reservoir and the path of flow from these points to the visible boils downstream has been traced with dye. Leakage is caused by three principal foundation defects:

1. Washing out of the shales in horizontal bedding planes, causing horizontal openings (note, in Fig. 12, at El. 503.3 a piece of typical quartz with Golconda shale sticking to it was found where it had fallen from above);
2. Solution of calcites, forming vertical solution channels; and
3. Faulting.

Defect 3 probably caused very little leakage and was handled fairly easily. Defect 1 contributed considerable leakage, but when such planes were found they were readily corrected. Leakage from this cause also seemed to exist in serious quantities only in the upper foundation reaches from 2 ft to 30 ft below the foundation rock level. Defect 2 contributed greatly to leakage and through its far-reaching labyrinth of passages, some of which extend to considerable depth, large quantities of water eluded the engineer's plans and efforts to stop this waste of potential hydroelectric energy.

As early as 1915, L. C. Glenn,[3] wrote as follows:

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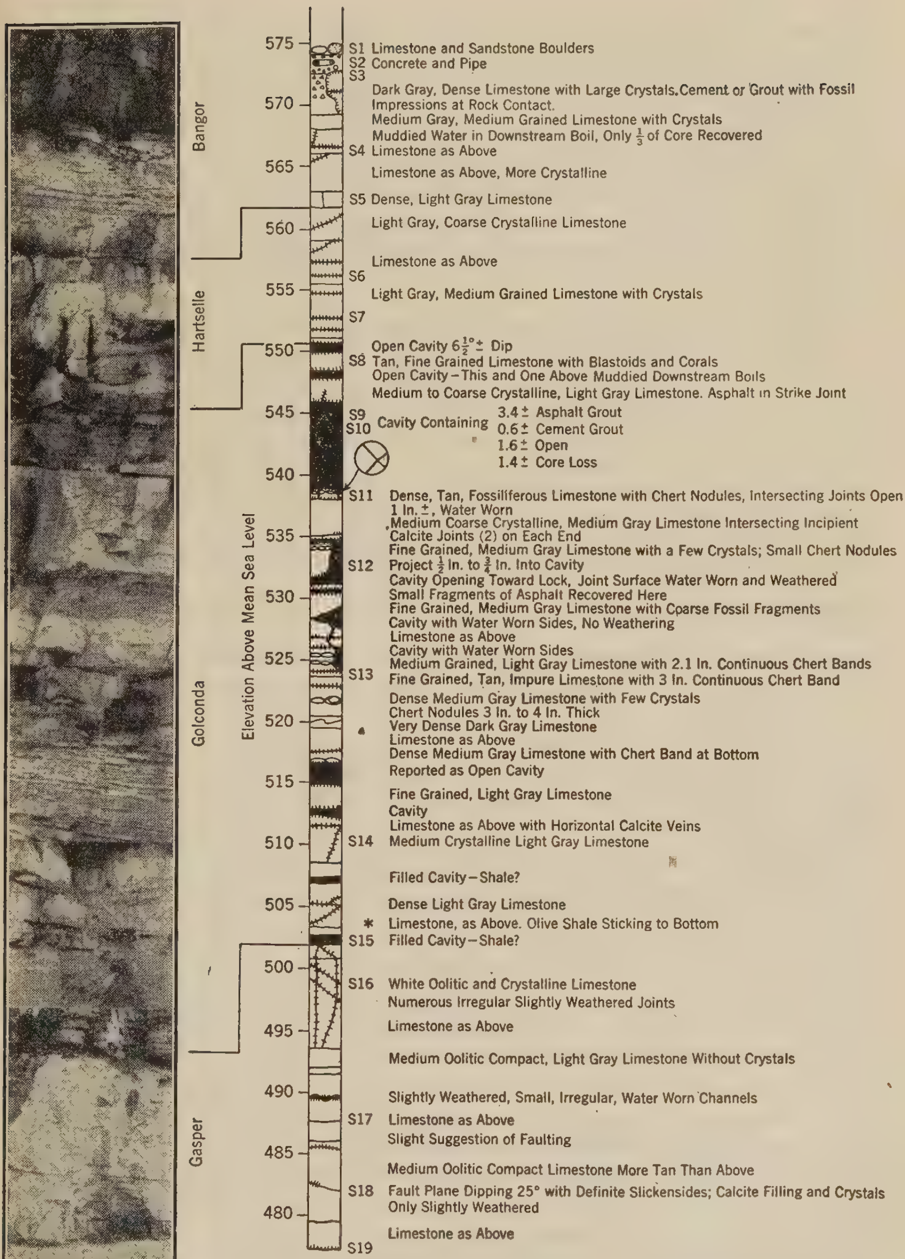
[3] "Geology Applied to Dams and Reservoirs," by L. C. Glenn, *Proceedings*, Eng. Assn. of the South, Vol. 26.


Bangor
Hartselle
Golconda
Gasper
Elevation Above Mean Sea Level
575
570
565
560
555
550
545
540
535
530
525
520
515
510
505
500
495
490
485
480
S1 Limestone and Sandstone Boulders
S2 Concrete and Pipe
S3
Dark Gray, Dense Limestone with Large Crystals. Cement or Grout with Fossil Impressions at Rock Contact.
Medium Gray, Medium Grained Limestone with Crystals
Muddied Water in Downstream Boil, Only 1/3 of Core Recovered
S4 Limestone as Above
Limestone as Above, More Crystalline
S5 Dense, Light Gray Limestone
Light Gray, Coarse Crystalline Limestone
Limestone as Above
S6
Light Gray, Medium Grained Limestone with Crystals
S7
Open Cavity 6½°± Dip
S8 Tan, Fine Grained Limestone with Blastoids and Corals
Open Cavity – This and One Above Muddied Downstream Boils
Medium to Coarse Crystalline, Light Gray Limestone. Asphalt in Strike Joint
S9
S10 Cavity Containing
3.4± Asphalt Grout
0.6± Cement Grout
1.6± Open
1.4± Core Loss
S11 Dense, Tan, Fossiliferous Limestone with Chert Nodules, Intersecting Joints Open 1 In. ±, Water Worn
Medium Coarse Crystalline, Medium Gray Limestone Intersecting Incipient Calcite Joints (2) on Each End
Fine Grained, Medium Gray Limestone with a Few Crystals; Small Chert Nodules
S12 Project ½ In. to ¾ In. Into Cavity
Cavity Opening Toward Lock, Joint Surface Water Worn and Weathered
Small Fragments of Asphalt Recovered Here
Fine Grained, Medium Gray Limestone with Coarse Fossil Fragments
Cavity with Water Worn Sides, No Weathering
Limestone as Above
Cavity with Water Worn Sides
Medium Grained, Light Gray Limestone with 2.1 In. Continuous Chert Bands
S13 Fine Grained, Tan, Impure Limestone with 3 In. Continuous Chert Band
Dense Medium Gray Limestone with Few Crystals
Chert Nodules 3 In. to 4 In. Thick
Very Dense Dark Gray Limestone
Limestone as Above
Dense Medium Gray Limestone with Chert Band at Bottom
Reported as Open Cavity
Fine Grained, Light Gray Limestone
Cavity
Limestone as Above with Horizontal Calcite Veins
S14 Medium Crystalline Light Gray Limestone
Filled Cavity – Shale?
Dense Light Gray Limestone
* Limestone, as Above. Olive Shale Sticking to Bottom
S15 Filled Cavity – Shale?
S16 White Oolitic and Crystalline Limestone
Numerous Irregular Slightly Weathered Joints
Limestone as Above
Medium Oolitic Compact, Light Gray Limestone Without Crystals
Slightly Weathered, Small, Irregular, Water Worn Channels
S17 Limestone as Above
Slight Suggestion of Faulting
Medium Oolitic Compact Limestone More Tan Than Above
S18 Fault Plane Dipping 25° with Definite Slickensides; Calcite Filling and Crystals Only Slightly Weathered
Limestone as Above
S19

* At Elevation 503.3 a Piece of Typical Quartz with Golconda Shale Sticking to It Was Found Where It Had Fallen from Above.

(*a*) COLUMNAR SECTION IN LOOKOUT MOUNTAIN SYNCLINE

(*b*) LOG OF TYPICAL DRILL HOLE, HALES BAR DAM

FIG. 12.—COMPARISON OF MISSISSIPPIAN FORMATIONS IN LOOKOUT MOUNTAIN AND AT HALES BAR DAM

> "No better illustration of the serious dangers from solution channels in limestones could be given than to cite the Hale's Bar dam on the Tennessee River near Chattanooga. The Bangor limestone, very pure, highly soluble and generally noted for its caves, forms the bed rock, and extends downward for some hundreds of feet beneath the river channel. This limestone was honey-combed with intercommunicating solution channels, some of which were choked with sand, gravel and cobbles, while others carried water under pressure. Excavation was carried some distance down into this rock and the foundation was thoroughly drilled and grouted, several hundred thousand barrels of cement being pumped into these openings. This honeycombed foundation caused a delay of several years in the completion of the project and increased by five or six million dollars, or more, the cost of construction.
>
> "The writer is by no means certain that the troubles there from leakage are ended, since geological examination shows the dam to be located on a syncline, or trough, in the Bangor limestone, and this means that the lower portion of this limestone there is the stretched member of the curved mass, and this tension, caused by the folding, undoubtedly opened joints in the lower part of the limestone and these filled with water under very considerable pressure would yield to solution more rapidly than the part up near the surface, so that the lower part is probably more open than the upper part that has been grouted, and the tendency will be, with the added head above the dam, for the water to work its way beneath the dam through these lower lying fissures and channels in constantly increasing quantities."

The reasons for correcting the leakage condition at Hales Bar were twofold:

1. To save water for power during periods of low flow; and
2. To protect the investment in this property against failure of the foundation.

The value of lost energy can be capitalized to determine a sum that may be expended economically, but economic considerations for the latter may range from the extreme of laissez-faire to that for a complete new project, in accordance with the individual opinions as to the urgency of this work.

From October 1, 1940, to October 1, 1941, there were 277 days during which river flow at Hales Bar was less than turbine capacity. Assuming that the demand for electrical energy existed during that period, a total of more than 26,500,000 kw-hr was lost. Similarly, between October 1, 1941, and October 1, 1942, the equivalent of nearly 21,000,000 kw-hr was lost. No one can gainsay but that, during these two years, the drought and then the war have demanded the conservation of every potential kilowatt-hour available.

Besides stopping leakage to increase the quantity of water available for power, it was essential that these large openings be filled to secure the bearing value of the rock foundation for permanent stability to the dam structure itself. This work was also a necessary step preparatory to raising the headwater level to El. 634, which will provide a 9-ft navigation channel to Chickamauga Dam in Tennessee, when the dam is raised after the war.

The first efforts to stop the flow in the foundation consisted of placing inert materials of many descriptions into the upstream suction inlets. These inlets were discovered when exploring for the cause of boils which appeared in the tailwater under full hydrostatic pressure as a result of filling the reservoir to dam-crest level; but any success in this effort was short lived since flow broke out elsewhere.

In the early 1920's, corrective work was attempted by pumping more than 11,000 bbl of asphalt into the foundation by way of the newly invented hot electric wire process on which a patent was later granted. This treatment was partly effective in stopping some of the flow, but was of doubtful value for bearing purposes. Leakage increased again later to the 1940 flows, so that evidently these leaks broke out in other places, probably by additional washing of clays and shales which the asphalt could not displace or penetrate. Some credence to this theory is available from the logs of the present drill holes. Invariably open cavities either above or below, or both, were found to exist when the old asphalt was encountered. It is now believed that, if an intensive cement grouting program had followed the asphalt grouting to effect consolidation of the foundation and to confine the asphalt, the leakage problem might have been solved at that time. The years that elapsed following the completion of this work enhanced the obstacles to be overcome in effecting a barrier to this leakage so that the methods used in the decade 1920–1930 no longer were effective, at least in some areas. The predominant leakage occurred in two areas as indicated in the elevation in Fig. 13. To some extent cavitation, as shown in this figure, is hypothetical since these cavities were projected from drill logs of holes all drilled in the same vertical plane, and since much of the predication of these cavities was based on core loss, probably due to drilling through vertical seams in many cases (see Fig. 14). However, the interlocking and water-carrying connections of these cavities are well established and the openings, as they are shown, are at least representative. The general pattern of cavitation is verified by 13-in. and 3-in. holes drilled later through the dam, and downstream from the upstream face.

The approach of the TVA to the problem has been based on placing a watertight diaphragm across the valley in the bedrock below, and just upstream from the upstream face of the dam. To secure this effect, it was proposed to drill and grout the foundation wherever feasible by conventional methods. Where the openings were not groutable, special grouts were developed, and, if these failed, recourse was had to a lined cutoff trench filled with concrete to cut off the through passage of water.

Water in the reservoir stands at about El. 628.5. Overburden is at approximately El. 583, foundation rock, 573, and apparent lowest cavities 481. To maintain a full reservoir for power, which is indispensable for war industries, it was necessary to devise some means of excavating for the cutoff through 55 ft of water and for depths as great as 92 ft below that point.

The first step was to sink NX size diamond-drill holes 20 ft on centers, and approximately 125 ft in solid rock, across the entire length of the dam. These holes were water tested, air tested, and dye tested by stages, using a $1\frac{1}{2}$-in. pipe extending down the hole, and a unique quick-breaking and quick-sealing packer at the bottom. This packer was developed on other TVA work and operated so satisfactorily that its adoption at Hales Bar was a natural consequence.

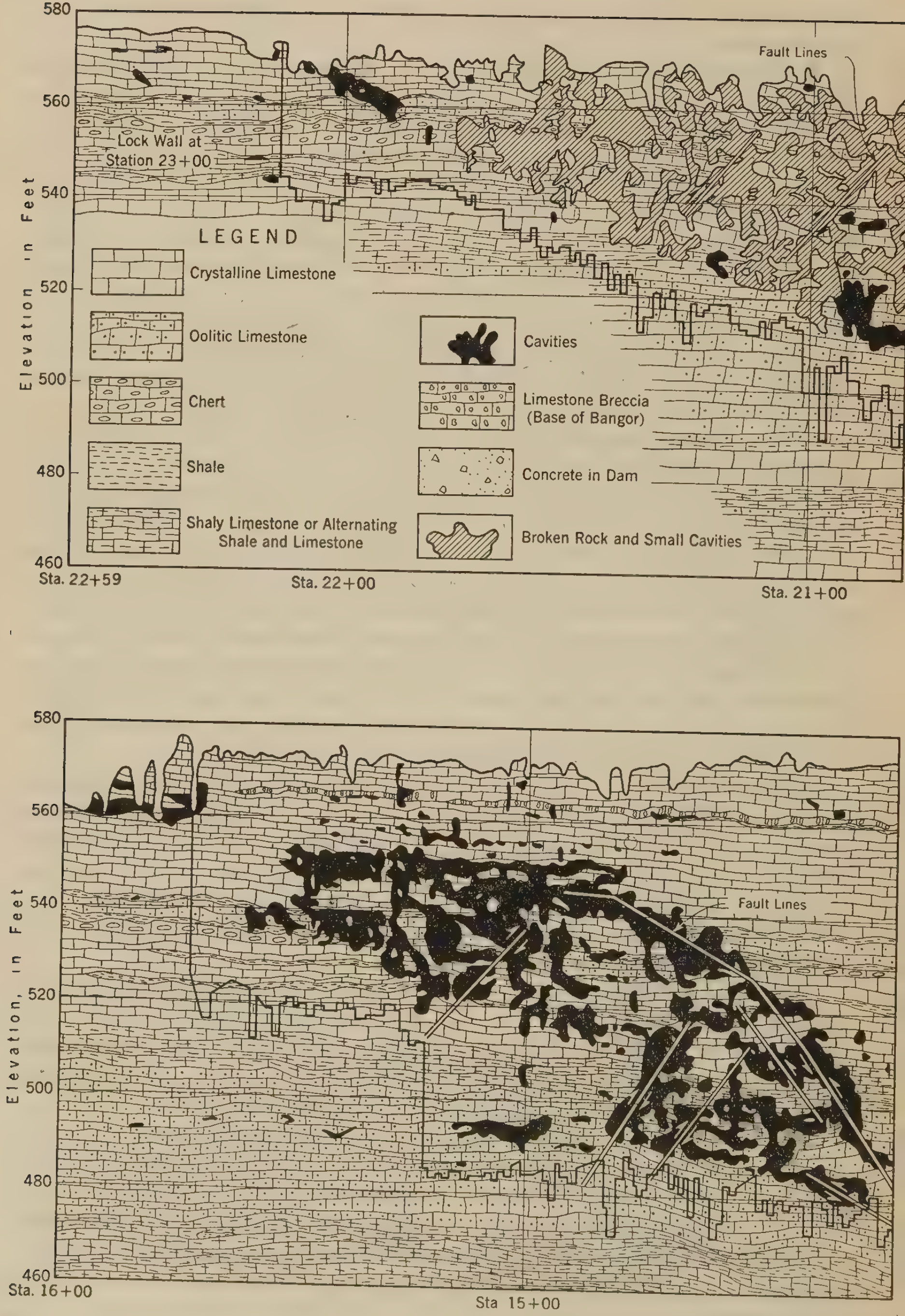
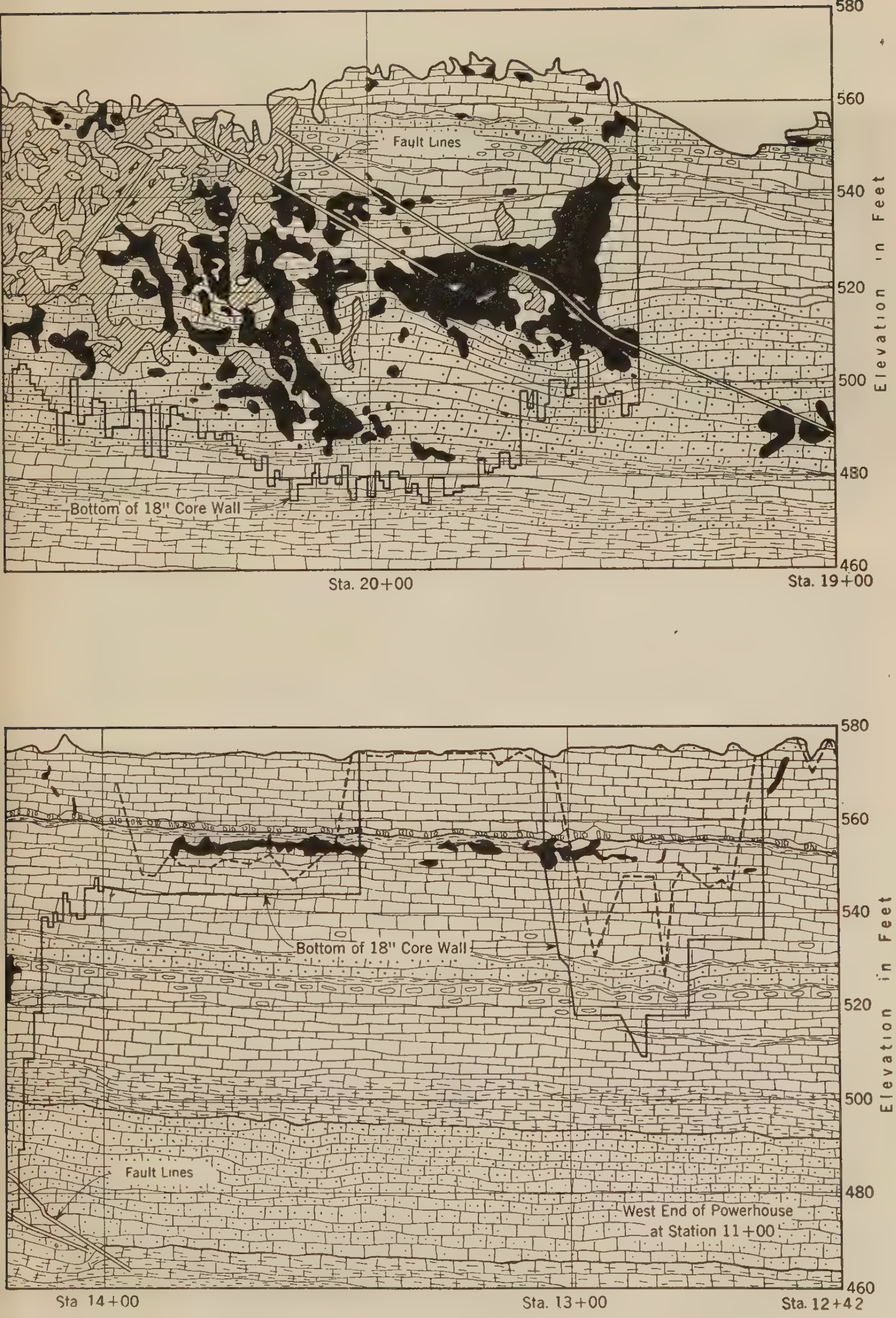

580
560
540
520
500
480
460
Elevation in Feet
Fault Lines
Lock Wall at Station 23+00
LEGEND
Crystalline Limestone
Oolitic Limestone
Chert
Shale
Shaly Limestone or Alternating Shale and Limestone
Cavities
Limestone Breccia (Base of Bangor)
Concrete in Dam
Broken Rock and Small Cavities
Sta. 22+59
Sta. 22+00
Sta. 21+00
Elevation, in Feet
Fault Lines
Sta. 16+00
Sta 15+00

FIG. 13.—CROSS SECTION THROUGH THE FOUNDATION


580
560
540
520
500
480
460
Elevation in Feet
Fault Lines
Bottom of 18" Core Wall
Sta. 20+00
Sta. 19+00
580
560
540
520
500
480
460
Elevation in Feet
Bottom of 18" Core Wall
Fault Lines
West End of Powerhouse at Station 11+00
Sta 14+00
Sta. 13+00
Sta. 12+42

UNDER HALES BAR DAM, FACING UPSTREAM

The water tests were made to determine the level to which grouting would be feasible and the type of grout to be used. Air and fluorescein dye tests established the paths of the underground connecting channels. Cavities, penetrated by holes at different levels, each connecting with different boils in the tailwater were found. The intensity of the coloring was a measure of the quantity of flow, and the travel time from the time the coloring was placed in the hole until it appeared downstream was noted carefully to establish a velocity index. Figs. 13 and 15 show the outlets and leakage paths established by these tests.

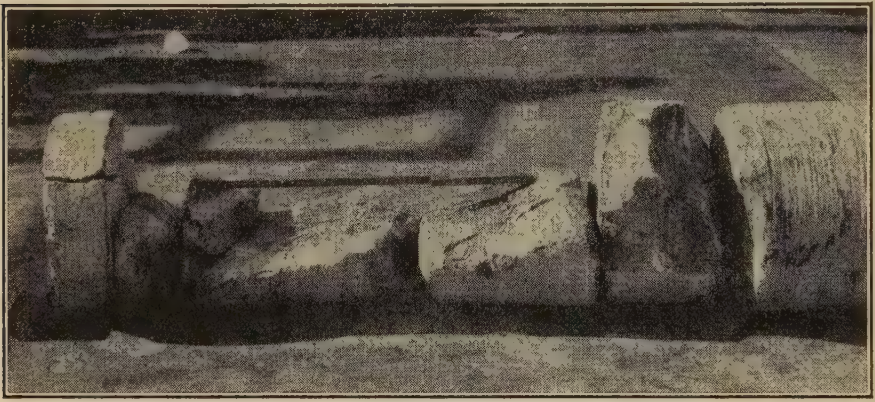


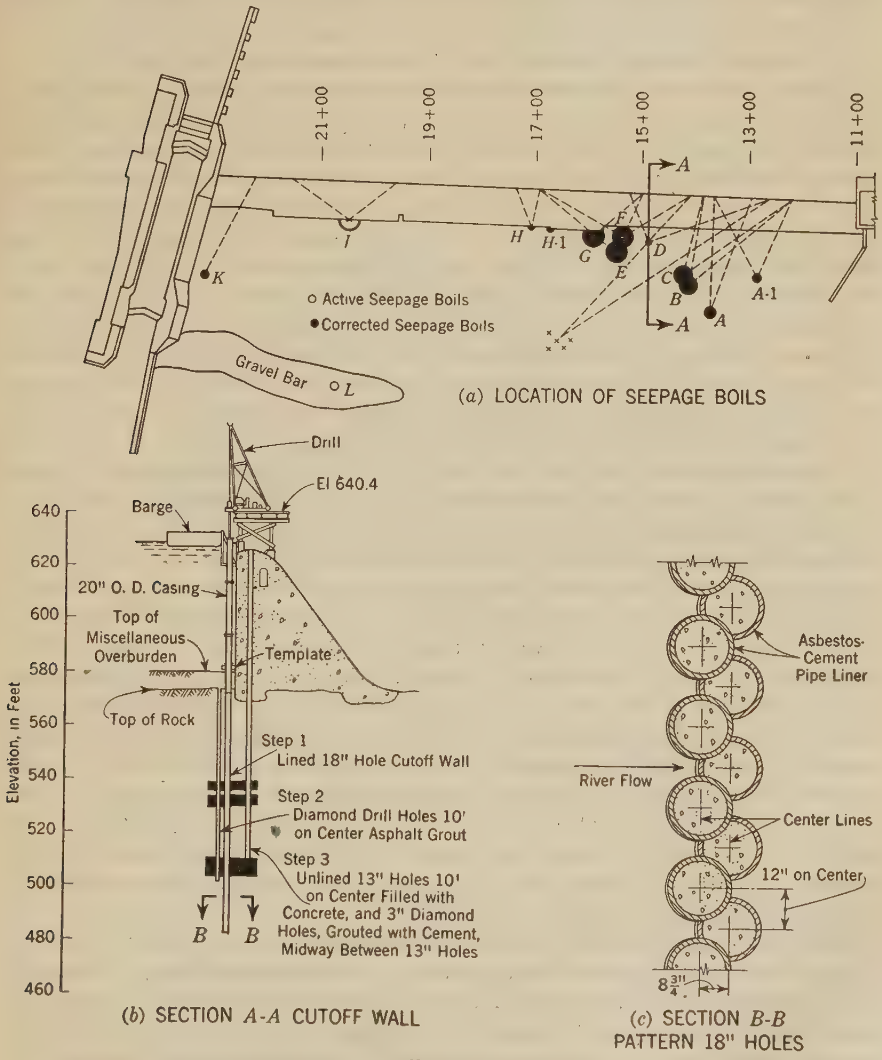
FIG. 14.—TYPICAL CAVITY, AS REVEALED BY 18-IN. DRILL HOLE CORES

These NX holes were then grouted to as high a level as feasible under pressure varying from 30 to 70 lb per sq in., depending on the depth of rock penetrated. Grouts of consistencies varying in water-cement content from 1.0 to 0.5 were used progressively, made from neat cement, neat cement mixed with as much as 6% calcium chloride by weight of cement to accelerate the set, grouts with cottonseed hulls added, grouts "sweetened" with quick-setting property admixtures, and asphalt, depending on the characteristics of the holes. Asphalt was used only when cement grouts proved to have been lost, and then only in seams carrying water and in holes that were apart from the cutoff trench areas where it would not be feasible to extend this cutoff trench to the seam penetrated. Wherever asphalt was used it was followed immediately by cement grout for confining and consolidating the asphalt, and adjacent holes were cement grouted to assure this effect. Excellent results have been achieved by hot asphalt grouting where stoppage of flowing water is the only consideration, bituminous material being very effective for this purpose.

Wherever holes could be grout-filled to the rock surface, secondary holes were drilled midway and grouted. Additional holes at still closer spacing were drilled and grouted, as required, until the area was tight. The average volume of grout taken by the foundation per foot of hole drilled for the various hole spacings in groutable areas was as follows:

Spacing of holes (ft)	Cu ft grout per ft drilled hole
20	6.25
10	5.20
5	3.60
2½	1.60

Diamond-drilling speeds varied greatly due to difficulties encountered in setting the casing and other features of drilling through 60 ft of water, many of which were not comparable with other jobs. In good operating, after the casing was set, 20 to 30 ft were drilled easily in one 8-hr shift. Cast-set bortz


21+00
19+00
17+00
15+00
13+00
11+00
A
A
K
I
H
H-1
G
F
E
D
C
B
A
A-1
○ Active Seepage Boils
● Corrected Seepage Boils
Gravel Bar
L
Drill
El 640.4
Barge
20" O. D. Casing
Top of Miscellaneous Overburden
Template
Top of Rock
Step 1 Lined 18" Hole Cutoff Wall
Step 2 Diamond Drill Holes 10' on Center Asphalt Grout
Step 3 Unlined 13" Holes 10' on Center Filled with Concrete, and 3" Diamond Holes, Grouted with Cement, Midway Between 13" Holes
B
B
Elevation, in Feet
640
620
600
580
560
540
520
500
480
460
Asbestos-Cement Pipe Liner
River Flow
Center Lines
12" on Center
8¾"

(*a*) LOCATION OF SEEPAGE BOILS

(*b*) SECTION *A-A* CUTOFF WALL

(*c*) SECTION *B-B* PATTERN 18" HOLES

FIG. 15

bits were used for the most part although some work was done with tungsten-carbide-faced blank bits. The bit cost on this job was high—approximately 60¢ per foot—but it was caused by drilling in old construction territory, where it was common experience to drill through reinforcing bars, old pipe, timber, bolts, caving holes, asphalt, concrete, and many other burdensome inert materials.

The only effective and permanent cutoff across the leakage areas was felt to be a continuous concrete wall in the foundation under the dam. It was desirable to do this work without lowering the reservoir so that some novel excavating process had to be devised to accomplish this desired result.

Numerous ideas were studied. In the first experimental drilling, the plan was to core drill large diameter holes and line them with fabric and rubber bags, line-drilling the overlapping holes; but numerous defects developed which led to the evolution of the method of drilling 18-in. holes 24 in. on centers, lining them with asbestos-cement pipe, and concreting inside the asbestos-cement pipe by tremie. This step was followed by drilling intersecting holes $8\frac{3}{4}$ in. upstream and midway between the other two, again lining and concreting, using the velocity of the water to draw the liner against the previously concreted holes for a seal (see Fig. 15(*c*)). Thus a continuous cutoff, either by conventional grouting or by concrete placed into lined holes excavated by core drills, was constructed to depths required as predetermined by the cores from NX size diamond-drill holes, water and dye tests, and grouting. This excavation varied in depth from 20 to 100 ft. With this cutoff in place, the entire foundation could then be consolidated as follows (see Fig. 15(*b*)).

After completing the 18-in. drilling and concreting, diamond-drill holes were driven upstream from the new cutoff on 10-ft centers and the upstream openings were grouted by cement or asphalt to perfect the seal. These were followed wherever necessary by similar treatment at closer spacing. Later 13-in. holes were drilled through the dam, 6 ft to 10 ft on centers tapping the cavities, and these were filled with concrete. Diamond-drill holes were then drilled through the dam midway between the 13-in. holes for final cement grouting. All these holes through the dam, both 13 in. and 3 in., demonstrate the effectiveness of the cutoff. The diamond-drill holes upstream from the lined cutoff were also dye tested before asphalt grouting and these indicated a marked decrease in dye intensity and an increased travel time which, in turn, indicated the reduction of leakage, to the magnitude of a seepage, by the construction of the cutoff wall. Some asphalt grouting through the dam in particularly troublesome areas was required. In general such grouting was required for localized seepage, and it was always followed by concrete or cement grout for effective consolidation.

Since the linear feet of cutoff wall represent an undetermined number of holes, it was desirable to drill holes of as large diameter as possible. However, since access into the hole to remove cores was impossible because of the underwater nature of the work, the diameter of the holes was restricted to a size that would permit breaking and removing cores by gravel-grouting the core into the core barrel. Eighteen-inch holes at depths prevailing on this job appeared to be the probable upper limit, as recommended by the drill manufacturers, and this size was adopted. The selection of this size seems to have been well justified since no insurmountable obstacles were encountered in pulling cores although the bottom pulls in good rock were usually difficult, thus assuring contact of the cutoff trench with good foundation rock, and confirming geologists' predictions. It is believed that 20-in. or even 24-in. holes probably could have been used down to the level of good rock; but material scarcity caused by the national emergency precluded changing tools and casings to try the larger diameter.

The selection of the correct drill for such highly specialized excavation was very important. Preliminary experimental drilling was done with a 35-hp gasoline-driven 36-in. shot core drill. The results of this test indicated the desirability of having a heavier and more rugged machine, and 48-in. machines were specified for the work. These were built from standard parts to TVA specifications for a machine particularly adapted to the Hales Bar work but also one that could be used elsewhere. Nine of these drilling units were used, and were equally adaptable for the 18-in. and 13-in. drilling which was done.

Water for feeding shot to the drill bit and washing up the sludge was supplied from two 60 gal per min, 250-lb electric-driven pumps set up on the trestles at opposite ends of the drilling operations. The pumps were connected by a pipe line running along the downstream edge of the trestle, and a sufficient number of take-offs were left in this line to make quick hose connections to any drill at any time.

Air for operating the air-driven hoists, grouting, cleaning out holes, and testing was furnished from two 315 cu ft per min portable compressors permanently placed on the left bank of the river upstream from the power house. These were replaced later by one electric-driven 1,500 cu ft per min stationary compressor. Air was piped from the bank along the downstream edge of the trestle with sufficient take-offs to permit hose connections wherever desired.

Electric power was brought to the trestle from a bank of three 250-kw 44,000/2,300-v transformers in a substation on the left bank of the river, downstream from the power house. A 2,300-v bus line was suspended on cross arms over the downstream side of the trestle, to two sets of three each, 112.5-kw 2,300/440/220 v and one set of three 150-kw 2,300/440/220-v transformers; whence power was delivered to the drills in conduit to steel outlet boxes near the upstream side of the trestle and conveniently arranged so that all drills could be fed within a minimum distance. Three-conductor cable with three-pronged plugs for plugging in at these boxes made quick connections possible whenever the drills were moved.

Where there was 50 to 70 ft of water on the surface, the casing had to be at least 60 ft long and sufficiently rigid to insure that the boring itself was absolutely plumb. For this purpose 0.5-in. wall, 20-in. steel pipe (outside diameter) with cast-steel flanged couplings was selected. These three-ton casings were rotated into place, which is a most effective procedure at slow speeds. Drilling in old asphalt also slowed up the speed of rotation where horsepower was required. When the workers were drilling through cavernous areas with only one side of the bit riding on rock, high speeds and slow feeds were required but momentary horsepower was also high.

For continuous and satisfactory drilling, it was desirable, therefore, that the driving motors on the drills have multi-speed control and constant horsepower. Four-speed electric motors, of 40 hp, were specified. The efficacy of using this much horsepower has been proved in drilling holes where velocities are high. For lifting tools one 3,500-lb line-pull electric hoist and one 2,000-lb line-pull air hoist were specified. The drill derrick was designed to move back for available hook work over the hole with outside live boom equipment. The manufacturers devised a unique eccentric arrangement for this purpose and also built the five-head block sheaves and two-lead sheaves into the crown block of the derrick, resulting in a maximum available headroom and flexibility for reeving lines to the two hoists. Hoists were arranged for individual mounting, one on each side of the drill, and the operator worked up in front where he had a clear unobstructed view of operations (see Figs. 16 and 17).

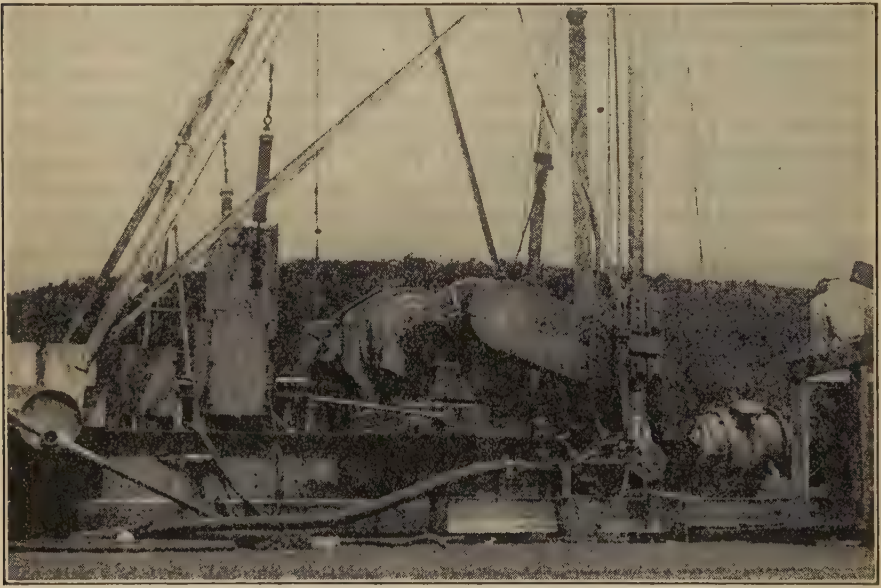


Fig. 16.—View of 48-In. Core Drill

Drill rods were of the 4-in. box and pin type since it would have been impossible to recover broken nuts or bolts of flanged type rods from a completely submerged hole. Oil-well type wrenches were used to break joints, the rods being supported on elevators and forks on the 20-in. casing. The cavernous condition of the foundation produced much pounding through the rods and the powerful 40-hp motors made these rods up very tight. Wrenches capable of 56,000 ft-lb of torque broke consistently, and it was necessary to specify casing tongs capable of more than 100,000 ft-lb of torque to overcome this difficulty (see Fig. 18). These tongs were activated from cables reeved through snatch blocks to air hoists, one of which was located centrally on each barge, and each hoist served several drilling units when pulling core.

For vertical holes, the drill tool was longer than 20 ft (see Fig. 19). It consisted of an open end piece on top to catch cutting sludge, a male plug designed to receive a pin type coupling from the drill rods, a core barrel, a renewable coupling between the core barrel and bit, the point of maximum wear, and the bit. This assembly proved to be very effective and, except for the bit, was built from standard 18-in. pipe with a $\frac{1}{2}$-in. wall. The bit was intended to

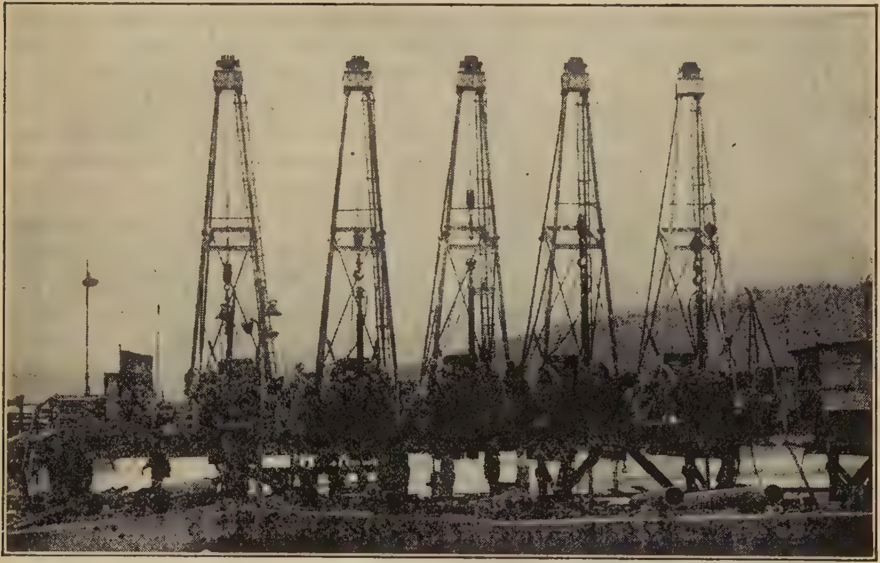


Fig. 17.—Battery of 48-In. Core Drills in Operation

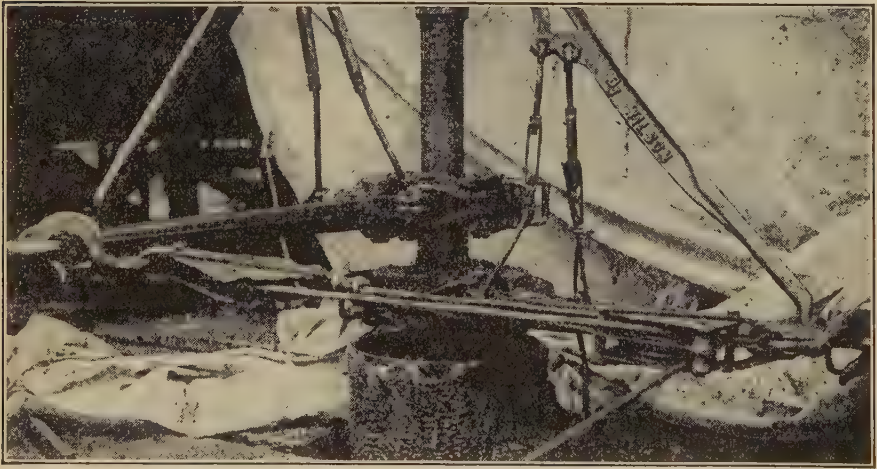


Fig. 18.—Breaking Out a 4.5-In. Tool Joint with Casing Tongs

be made from standard 18-in. pipe of $1\frac{1}{8}$-in. wall thickness but it was impossible to secure this type, and cast-steel bits and later even laminated bits from thinner wall pipe were used satisfactorily, although the latter did not wear as well as the solid section bits.

Since the overburden varied in thickness from 5 to 35 ft it was very desirable to remove as much of this as possible before drilling in any one area. Silt and mud accumulated in the reservoir at the heel of the dam and much trash from the original construction was also still in place. Old crib cofferdams, piles of rock excavation, old-style steel sheet piling, pipe, wrenches, concrete buckets, logs, and sundry items which are found, would be an impediment to drilling if not removed first. Therefore, a $1\frac{1}{2}$-yd clamshell digging bucket with the floating steam whirley crane was used to clean out an area 10 ft to 15 ft upstream from the upstream face of the dam before the drill operations were begun.

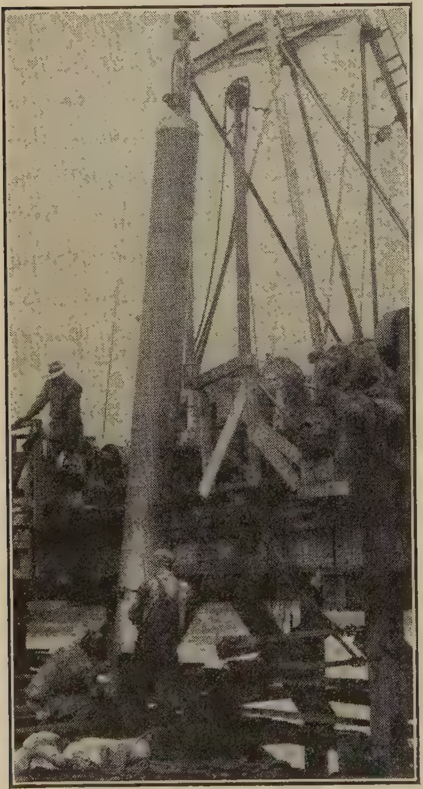


Fig. 19.—18-In. Drill Tool Assembly

Among the many other items that needed to be removed were four old closure gates that had never been salvaged. These 19-ton gates were supposedly standing in an upright position in front of the closure blocks but it remained for an 18-in. drill penetrating one corner of one of them to reveal that they had fallen forward and were now resting flatwise in the river bed at angles from 0° to 45°. Divers were dispatched to the bottom to determine their position and location more accurately. They hooked the gates to the crane which removed them from the river bottom where they had been for almost thirty years (see Fig. 20). The steel was cleaned and found to be in surprisingly good condition, and, because of the steel shortage, these gates were disassembled and all the structural shapes and plates were reused.

Holes were alined by the template shown in Fig. 21 which was fabricated from structural steel, and suspended at the line of overburden on cables from the deck of the trestle on the dam. This template had $20\frac{1}{2}$-in. holes 24 in. on centers, with guide skirts for entering the casing. After a template had been lowered and leveled, a 20-in. flush-joint casing 60 ft long was set into one end hole by a floating whirley crane. At the opposite end of the template a flush-joint casing was set similarly into the second hole from the end. Flush-joint casing was made up with $\frac{1}{4}$-in. square threads. A 1-in. plate, machined to take a pin type coupling, was bolted to a flange screwed to the top of the casing, and the drill rotated each of these end casings about 2 ft to 3 ft into good rock. These casings then served to hold the template in place for all intermediate drilling, and acted as guides for spacing the intersecting holes and setting adjacent templates. The steps in the use of the template were as follows:

First Template Setting, Position 1 (*Fig. 21*(*a*)).—The template was set with openings 1A and 2A facing upstream. The flush-joint casing was set in the template openings 1 and 2 and the casings were rotated 2 ft to 3 ft in rock. All the intermediate holes were drilled in the order shown, using flanged casings. The template was finally removed, leaving the flush-joint casings in place.

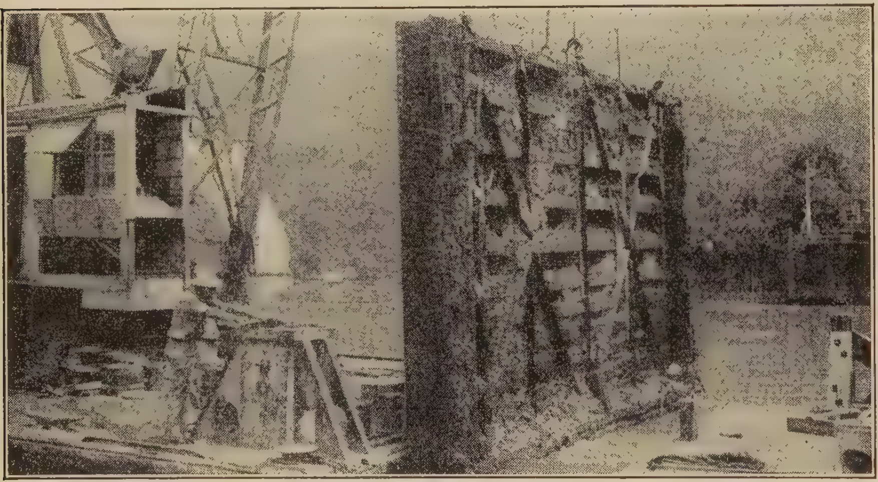
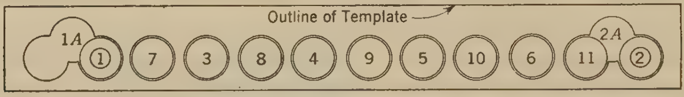


Fig. 20.—Recovery of an Old Closure Gate from the Reservoir at the Heel of the Dam

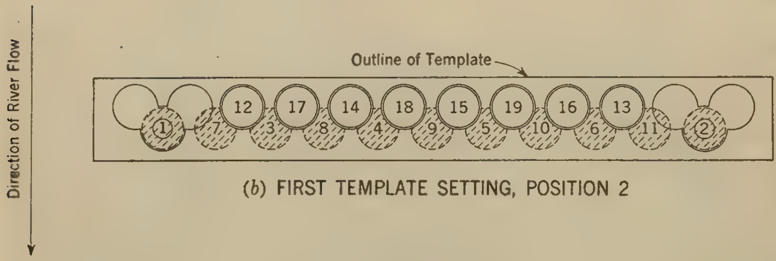
First Template Setting, Position 2 (*Fig. 21*(*b*)).—The template was then reversed by turning it end for end so that openings 1A and 2A (Fig. 21(*a*)) were placed over the flush-joint casing left in holes 2 and 1, respectively. The secondary holes were drilled in the order shown and the template was removed again, leaving all flush-joint casings in place (Fig. 21(*c*)).

Second Template Setting, Position 1 (*Figs. 21*(*g*) *and 21*(*h*)).—The template was moved to the left or to the right to extend the line of drill holes. As in the first setting, the template was placed with openings 1A and 2A facing upstream, the end opening being over the flush-joint casing in hole 1 when the template was moved to the left and over hole 2 if moved to the right. Flush-joint casing was set in template opening 20 and that primary hole was drilled. The intermediate primary holes were then drilled in the order shown.

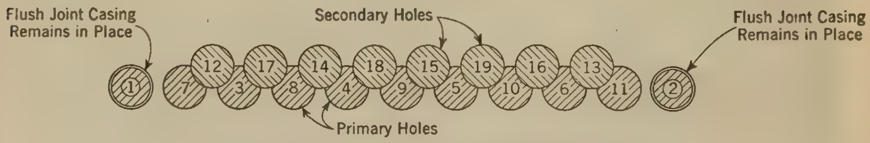
Second Template Setting, Position 2 (*Figs. 21*(*i*) *and 21*(*j*)).—This procedure was essentially the same as Fig. 21(*b*), template opening 1A being placed over the casing in hole 1 when the move was to the left and over the casing in hole 20 when to the right. Openings 2A were placed over the flush-joint casings in holes 20 and 2 left and right, respectively. Flush-joint casings were placed in template openings 30 and 31 and these secondary holes were drilled leaving the flush-joint casings in place temporarily. Next the flush-joint casing in holes 1 and 2 was removed. With flush-joint casings in holes 20,


Outline of Template
1A
2A
① 7 3 8 4 9 5 10 6 11 ②

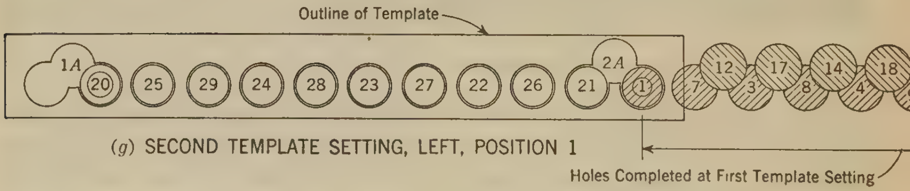
(*a*) FIRST TEMPLATE SETTING, POSITION 1


Direction of River Flow
Outline of Template
12 17 14 18 15 19 16 13
① 7 3 8 4 9 5 10 6 11 ②

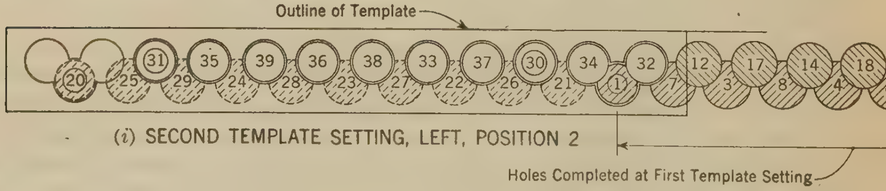
(*b*) FIRST TEMPLATE SETTING, POSITION 2


Flush Joint Casing Remains in Place
Secondary Holes
Flush Joint Casing Remains in Place
12 17 14 18 15 19 16 13
① 7 3 8 4 9 5 10 6 11 ②
Primary Holes

(*c*) HOLES COMPLETED AT FIRST TEMPLATE SETTING


Outline of Template
1A
2A
⑳ 25 29 24 28 23 27 22 26 21 ①
7 12 3 17 8 14 4 18
Holes Completed at First Template Setting

(*g*) SECOND TEMPLATE SETTING, LEFT, POSITION 1

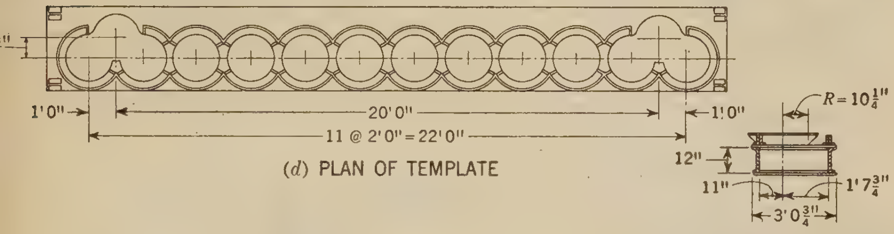

Outline of Template
㉛ 35 39 36 38 33 37 ㉚ 34 32
⑳ 25 29 24 28 23 27 22 26 21 ① 7
12 3 17 8 14 4 18
Holes Completed at First Template Setting

(*i*) SECOND TEMPLATE SETTING, LEFT, POSITION 2

Fig. 21.—Pattern of Cutoff Wa

31, and 30, all other secondary holes were drilled, using flanged joint casing. After the drilling of secondary holes in the order shown was completed all casings except that in holes 20 were removed and the templates again were moved forward into primary position.

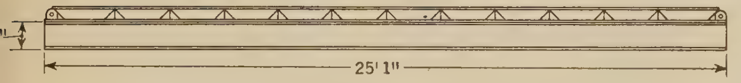
The flanged casings were 60 ft long and were placed by the crane in a manner similar to the flush-joint. The flanged casings were rotated to a depth of about 6 in. in solid rock. The bottom 20 ft of all casings, with which the drill tool was in contact when starting a hole, was always plumbed. For this work



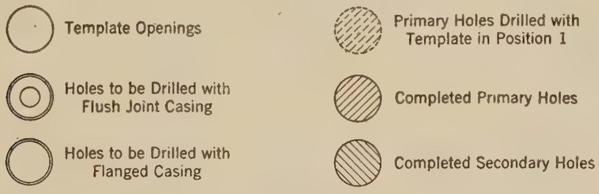
1'0''
20'0''
1'0''
11 @ 2'0'' = 22'0''
R = 10¼''
12''
11''
1'7¾''
3'0¾''

(*d*) PLAN OF TEMPLATE

END ELEVATION

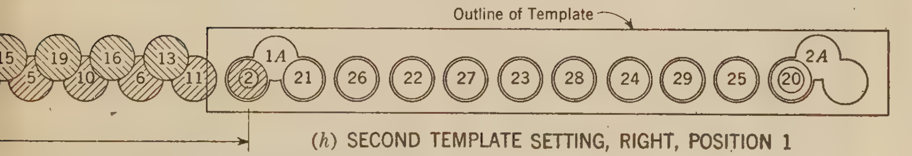


(*e*) ELEVATION OF TEMPLATE



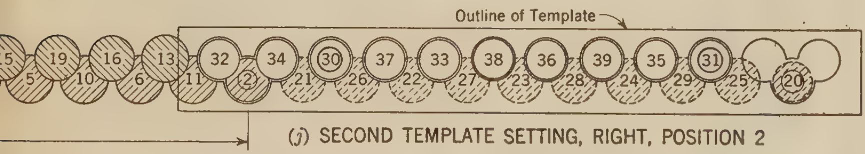
Template Openings
Holes to be Drilled with Flush Joint Casing
Holes to be Drilled with Flanged Casing
Primary Holes Drilled with Template in Position 1
Completed Primary Holes
Completed Secondary Holes

(*f*) SYMBOLS



Outline of Template
15
19
16
13
5
10
6
11
1A
2
21
26
22
27
23
28
24
29
25
2A
20

(*h*) SECOND TEMPLATE SETTING, RIGHT, POSITION 1

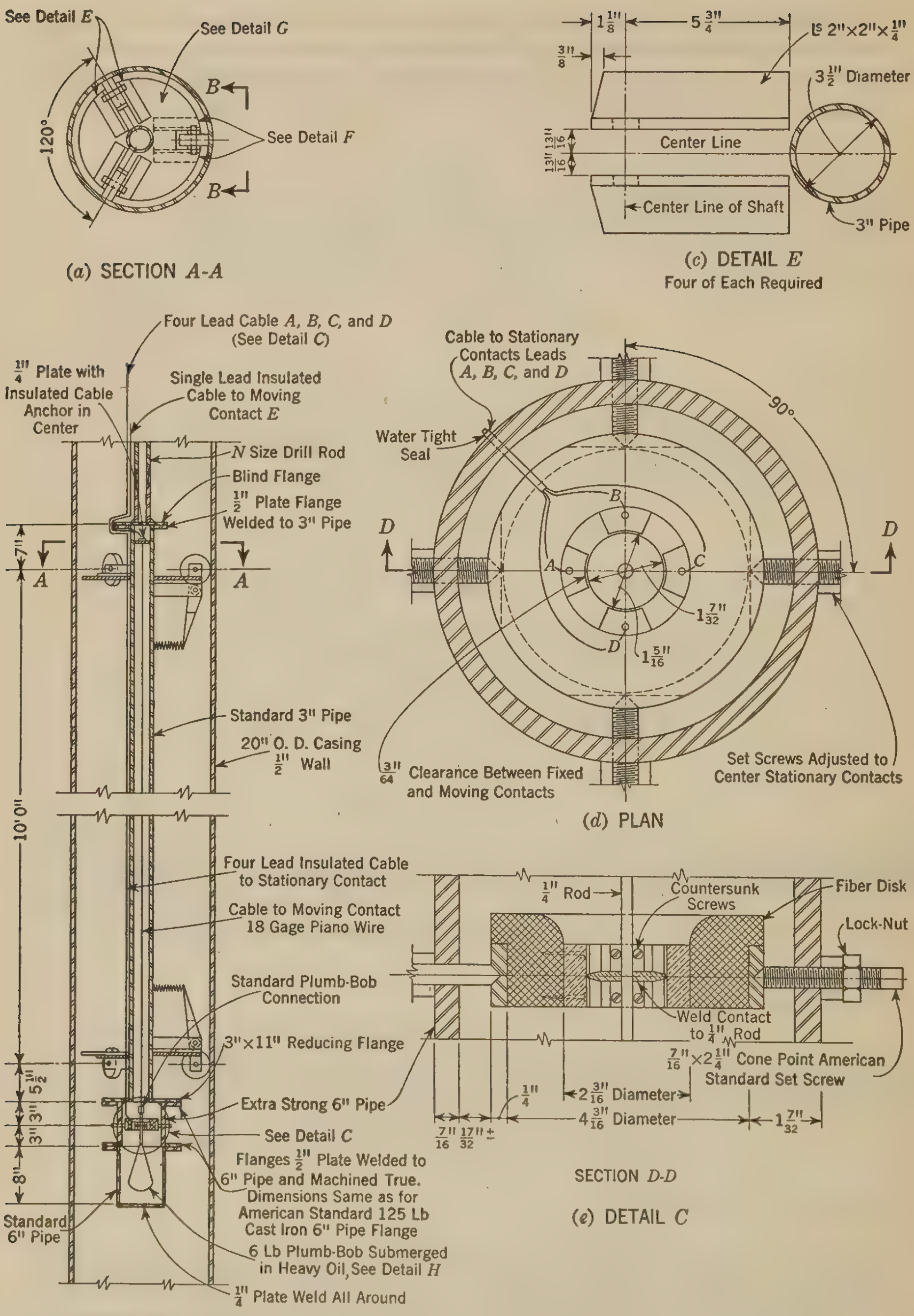


Outline of Template
15
19
16
13
5
10
6
11
32
2
34
21
30
26
37
22
33
27
38
23
36
28
39
24
35
29
31
25
20

(*j*) SECOND TEMPLATE SETTING, RIGHT, POSITION 2

Use of Drill Casing Template

the electric plumbing apparatus, shown in Fig. 22, was devised, consisting of a closed section of 3-in. pipe, 12 ft long, inside of which a 6-lb plumb bob was suspended on a piano wire and a ¼-in. rod. The bottom of the pipe had a flanged connection to a closed 6-in. well which was filled with oil, in which the


See Detail E
See Detail G
B
See Detail F
B
120°
$1\frac{1}{8}''$
$5\frac{3}{4}''$
$\frac{3}{8}''$
Ls $2''\times2''\times\frac{1}{4}''$
$3\frac{1}{2}''$ Diameter
$\frac{13}{16}''$ $\frac{13}{16}''$
Center Line
Center Line of Shaft
3" Pipe
Four Lead Cable A, B, C, and D (See Detail C)
$\frac{1}{4}''$ Plate with Insulated Cable Anchor in Center
Single Lead Insulated Cable to Moving Contact E
N Size Drill Rod
Blind Flange
$\frac{1}{2}''$ Plate Flange Welded to 3" Pipe
7"
A
A
Standard 3" Pipe
20" O. D. Casing $\frac{1}{2}''$ Wall
10'0"
Four Lead Insulated Cable to Stationary Contact
Cable to Moving Contact 18 Gage Piano Wire
Standard Plumb-Bob Connection
3"×11" Reducing Flange
$5\frac{1}{2}''$
3"
3"
8"
Extra Strong 6" Pipe
See Detail C
Flanges $\frac{1}{2}''$ Plate Welded to 6" Pipe and Machined True. Dimensions Same as for American Standard 125 Lb Cast Iron 6" Pipe Flange
Standard 6" Pipe
6 Lb Plumb-Bob Submerged in Heavy Oil, See Detail H
$\frac{1}{4}''$ Plate Weld All Around
Cable to Stationary Contacts Leads A, B, C, and D
Water Tight Seal
90°
D
D
B
A
C
D
$1\frac{7}{32}''$
$1\frac{5}{16}''$
$\frac{3}{64}''$ Clearance Between Fixed and Moving Contacts
Set Screws Adjusted to Center Stationary Contacts
$\frac{1}{4}''$ Rod
Countersunk Screws
Fiber Disk
Lock-Nut
Weld Contact to $\frac{1}{4}''$ Rod
$\frac{7}{16}''\times2\frac{1}{4}''$ Cone Point American Standard Set Screw
$\frac{1}{4}''$
$2\frac{3}{16}''$ Diameter
$4\frac{3}{16}''$ Diameter
$1\frac{7}{32}''$
$\frac{7}{16}''$
$\frac{17}{32}''\pm$

(*a*) SECTION *A-A*

(*c*) DETAIL *E*
Four of Each Required

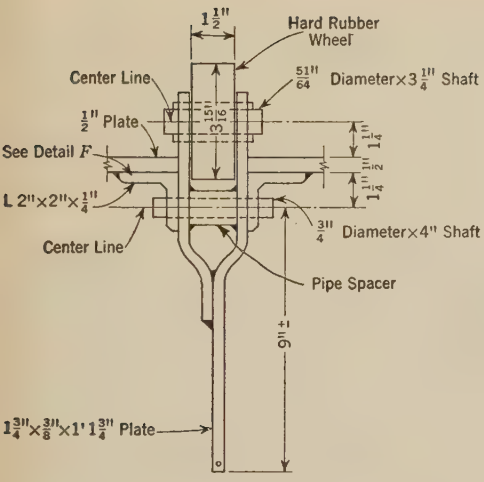
(*b*) SECTIONAL ELEVATION

(*d*) PLAN

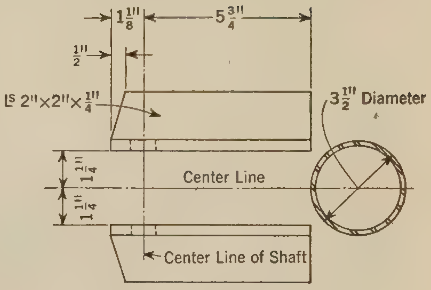
SECTION *D-D*

(*e*) DETAIL *C*

FIG. 22.—ELECTRIC DEVICE FOR

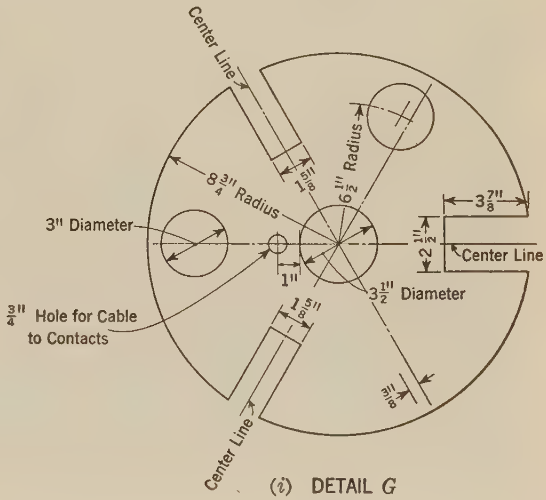

1½"
Hard Rubber Wheel
Center Line
51/64" Diameter×3¼" Shaft
½" Plate
3 15/16"
1¼"
½"
1¼"
See Detail F
L 2"×2"×¼"
Center Line
¾" Diameter×4" Shaft
Pipe Spacer
9"±
1¾"×⅜"×1'1¾" Plate

(*f*) SECTION *B-P*


1⅛"
5¾"
½"
Ls 2"×2"×¼"
3½" Diameter
1¼"
1¼"
Center Line
Center Line of Shaft

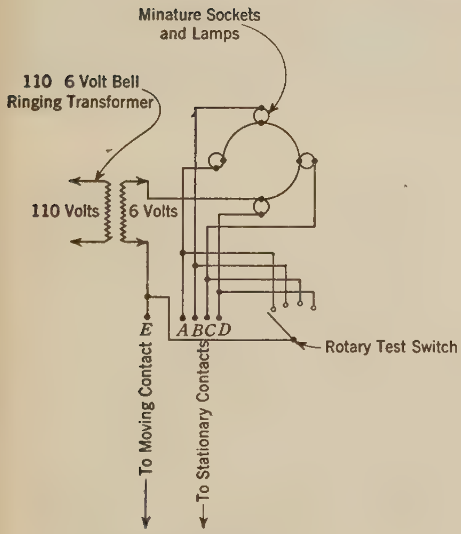
(*h*) DETAIL *F*

Two of Each Required

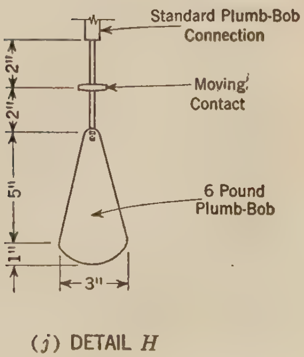

Center Line
1⅝"
6½" Radius
8¾" Radius
3" Diameter
3⅞"
2½"
Center Line
1"
3½" Diameter
¾" Hole for Cable to Contacts
1⅝"
⅜"
Center Line

(*i*) DETAIL *G*

Two Required


Minature Sockets and Lamps
110 6 Volt Bell Ringing Transformer
110 Volts
6 Volts
E
ABCD
Rotary Test Switch
To Moving Contact E
To Stationary Contacts

(*g*) WIRING DIAGRAM FOR CONTROL BOX


Standard Plumb-Bob Connection
Moving Contact
6 Pound Plumb-Bob
2"
2"
5"
1"
3"

(*j*) DETAIL *H*

Plumbing the 20-In. Drill Casing

plumb bob was suspended. Above the plumb bob a contactor was fastened to the ¼-in. rod, and this contactor was set at the same level as four copper contacts arranged in a circle and fastened to the inside of the 6-in. pipe. The contacts connected each to one wire of a four-conductor cable which led to the water surface where they were connected with four 6-v to 8-v lamps. The piano wire completed the circuit. A bell ringing transformer was used so that the system could be plugged into any 110-v light circuit.

The device had a set of three wheels on the outside, one of which was spring mounted both at the top and bottom. The two stationary wheels of each set were placed 120° apart on an arc of exactly 19 in., the inside diameter of the casing. The plumb-bob center was set with relation to these two wheels. The spring-mounted wheel insured that the two fixed wheels would always be in contact with the casing. It was also set at 120° with each of the other two.

This entire assembly was lowered to the bottom of the casing without pumping out. Movement of the plumb bob toward any contactor lighted the corresponding light at the surface and, by means of a jack at the center of the casing and two diagonal rods and turnbuckles, the casing could be moved in any desired direction until all lights were out, at which time the casing was plumb. These rods and jack were left in place to hold the casing during drilling, lining, and concreting. The lights were then checked by short circuiting to be sure that none had burned out. The clearance between the contacts and the contactor was $\frac{3}{64}$ in. all around.

Until a hole was 20 ft deep, a ¼-in. by 2-in. removable band in two pieces at the top of the sludge receiver was used on the cutting tool to obtain perfect vertical alinement by reducing the clearance between the drill tool and the guide casing. This band was made of two sets of two semicircular pieces of steel, the inner one projecting into the sludge collector. The inner band had four stud bolts which fastened to the sludge receiver. Therefore, the unit could be put on or taken off from any tool with a minimum of effort. Two pieces were required to form wash-out slots between the band sections.

A view of the template is shown in Fig. 23.

Studies on the drilling tool revealed that there was a point of maximum wear just above the bit; and in order to save on the other parts of the tool a renewable female coupling, 18 in. long, was placed between the bit and the core barrel. This concentrated wear on one short section of the tool, and this section was turned end for end when wear became noticeable but before the threads were worn down too far to be effective at the other end. Such a coupling was good for approximately 125 ft of drilling in this foundation. The 1⅛-in. wall bits have 15 in. of wearing surface and after 35,000 ft of drilling it appeared that bit life was about 115 lin ft of drilling. The solid section bits averaged closer to 150 ft of life but the over-all average was less due to enforced use of laminated bits because of material shortages. Because they were protected at the bottom by the coupling, core barrels averaged 420 ft of drilling, and usually could be cut off and rethreaded once.

Above the core barrel a male plug was used which had a useful life of approximately 900 lin ft of drilling. When this plug was placed into the drill tool, the expensive connection between the 18-in. tool and the drill rods was reduced materially since this connection was used with various replacements of other parts of the tools; whereas, if it had been a part of the tool, the connection would have been lost each time that part wore out. Experience at Hales Bar indicated that the drill rod connecting to the male plug crystallized after approximately 1,500 ft of drilling and invariably broke just above the pin type coupling. These rods were changed after 1,000 and 1,500 ft of drilling, eliminating considerable breakage at depths requiring costly "fishing" operations.

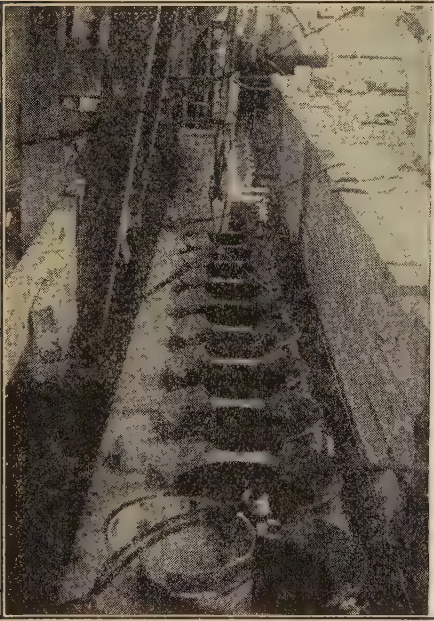


FIG. 23.—VIEW OF TEMPLATE PLACED IN SECONDARY POSITION

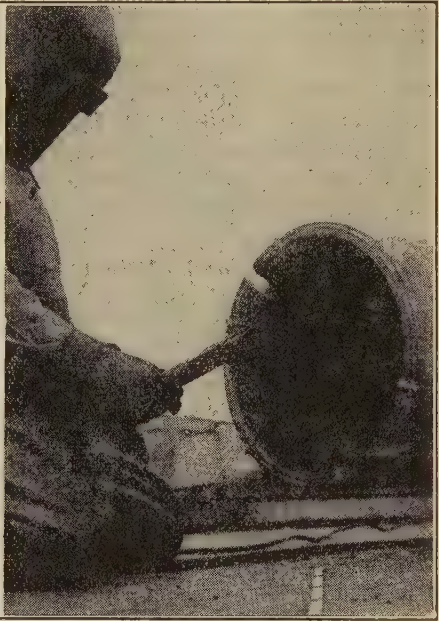


FIG. 24.—WELDER APPLYING ELECTRIC TUBE TUNGSTEN CARBIDE

The sludge collector above the plug was made with a left-hand thread fastening to the plug. The wear on this unit was light and its life was approximately 500 lin ft. These sections were generally reclaimed from worn core barrels, which diminished somewhat the period of their useful life.

All drilling was done from a trestle 11 ft above flashboard level. Three barges lined up in front of the trestle over a distance of 330 ft were used for laying out drill tools, rods, bits; removing core; and in general providing a platform and working space forward of the machines.

Where cavities were encountered which cause shot to be lost under the bits six tungsten carbide knobs were welded to the cutting face of the bit at equally spaced intervals (see Fig. 24). This material has a Brinell hardness of 700 and generally cuts to a point at which shot can again be held under the bit after one or several applications. If a full shot ring is cut, shot can be fed under the bit about as soon as the tungsten carbide is worn smooth without removing the tools, with the advantage that the same bits may be used for either shot or this diamond substitute drilling. Tungsten carbide is made in various screen sizes for acetylene or electric application. Experiments with various available screen sizes of the tube type rods revealed that the 20-mesh to 30-mesh particles were applied easiest and cut better footage than the coarser particles in the Hales Bar foundation rock.

Since drilling at Hales Bar offered no opportunity for differentiating between work done with shot and the tungsten carbide faced bits, it is necessary to state the amount of each being used as a joint figure. Approximately 11 lb of shot and 0.12 lb of tungsten carbide were used currently per foot of 18-in. drilled hole. This value seems high but it must be remembered that shot loss in cavities was high; also, shot and the diamond substitute were used for drilling in casing, which did not contribute any 18-in. footage drilled. Comparisons between 3-in., 13-in., and 18-in. drilling indicate the use of ⅔ lb of shot per inch of diameter of bit at Hales Bar.

Where large cavities occurred, the drills pounded very hard if given a slack line feed. This militated against plumb drilling and also caused repairs on the drills to run high. For such places, a turnbuckle was clamped into the hoist line, the bit was faced with extra tungsten carbide, and the drill was rotated at high speed and the drill tool fed very slowly with the turnbuckle. Although the speed of cutting was probably slower, the advantages gained by eliminating the tendency of the tool to deflect in the direction of the cavity, and by obtaining smoother operation, outweighed the disadvantage of decreased footage.

Drilling speeds varied from day to day, depending on the type of material encountered and on its state of solidity. Over-all rates for more than 35,000 ft of 18-in drilling which involved nine machines and twenty-seven different operators were: 5 in. per gross operating hour; and 6 in. per net operating hour. Thirteen-inch drilling was somewhat better, being about 9 in. and 11 in., respectively, per gross and net operating hours. The former included time for moving drills, drilling in casing, repairs, and other unavoidable delays except layoffs on account of high water—in other words, all workable shift hours. The latter is only net drilling time but includes pulling core. Monthly average rates per gross operating hour for 18-in drilling for the number of machines indicated have been as high as 7.50 in. per hr. Drilling the intersecting holes, however, reduced this rate, since the drilling was slower and more difficult. With nine drills running, 70 ft were drilled in one 8-hr shift, 199 ft in 24 consecutive shift hours, and 2,900 ft in one month of twenty-six working days.

A brief summary of the operating cost per foot for drilling 18-in. holes is as follows:

Description	Cost per foot
Labor	
Direct operations	$4.61
Repairs	0.39
Other expense	2.13
	$7.13
Drilling Tools	
Sludge collector	0.05
Plugs	0.10
Corebarrels	0.14
Couplings	0.19
Drill rods	0.14
Bits	1.07
Wrenches	0.11
	$1.80
Materials	
Lubricants	$0.16
Electricity	0.21
Air and water	0.38
Repair parts	0.28
Shot	0.42
Tungsten carbide	0.31
Sand and gravel	0.02
	$1.78
Other Costs	
Supervision	0.50
Drill depreciation	1.40
Miscellaneous	0.77
	$2.67
Totals	$13.38

It should be understood that these values do not include such job construction plant items as are customarily required in large construction work. Nor do they include such items as the trestle, templates, 20-in. casing, and other charges peculiar only to the Hales Bar project. The costs given are representative for 18-in. shot core drilling in foundation rock similar to that at Hales Bar, and, if used advisedly with due allowance for other locations and requirements, can furnish a satisfactory index of probable unit costs. It also is a satisfactory check-list of materials, equipment, and services involved in this type of operation. Application of these costs also must take into account the fact that nine drills were being used at Hales Bar, a fact which would alter labor, supervisory, and many of the other costs if less or more units are involved.

After the 18-in. hole had been drilled, in each case to a depth of 3 ft below the lowest known cavity, a $1\frac{1}{2}$-in. pipe was lowered into the hole to check the depth, thus determining if all core had been pulled. The grout pump was then hooked to this pipe and cement grout was pumped to the bottom of the hole to seal the liner. Asbestos-cement pipe, made up into one long piece from 13-ft sections, was then lowered into the hole on a 6-in. pipe screwed into a left-hand coupling which was concreted to a plug at the bottom of the asbestos-cement pipe. Sections of asbestos-cement pipe (inside diameter, 16 in.; wall thickness, $\frac{7}{8}$ in.; and lengths, 13 ft) were connected and lowered into the hole, and were supported by the 6-in. pipe on the 20-in. casing, using a fork and elevator. All these asbestos-cement pipe joints were fastened together progressively with No. 16 gage steel sheets 30 in. by 54 in., rolled to 16-in. diameter. The metal was fastened to one end of the pipe sections by six $\frac{1}{4}$-in. flat-head machine screws and nuts in the storage yard. The pipe was assembled over the hole in a vertical position, the end of the succeeding pipe being slipped over the protruding 15 in. of metal and fastened to the metal with two rows of four each, flat-head self-tapping sheet-metal screws (see Fig. 25). Across cavernous regions, a 3-in. mesh expanded metal was placed inside the asbestos-cement pipe for reinforcing, and a 1¾-in. round bar was placed inside the 6-in. pipe to hold it in a central position so that it would not interfere with drilling the intersecting hole. When sufficient liner and reinforcement had been assembled to line the hole completely, a cover was placed over the top of the asbestos-cement pipe, as shown in Fig. 26. This cover had outstanding angle legs tapped for stud bolts which fastened to the outside of the 6-in. pipe. On the underside, the cover was equipped with four rollers spaced to fit the inside diameter of the liner. These permitted the 6-in. pipe to turn without disturbing the liner. This cover, therefore, held the 6-in. pipe centered and also prevented any of the asbestos-cement pipe from breaking joints. Such breaks might occur in lowering sections to the bottom due to caving rock, tightening of the hole in previously placed asphalt areas, or friction developed by the differential head against the liner. In some instances, velocities of more than 6 ft per sec, measured by lowering a current meter into the open hole, were recorded. In some cases this differential head was sufficiently high to impede the rotation of the drill tool seriously, and, in lining these holes, the liners had to be forced down, obviously necessitating the use of the cover plate.

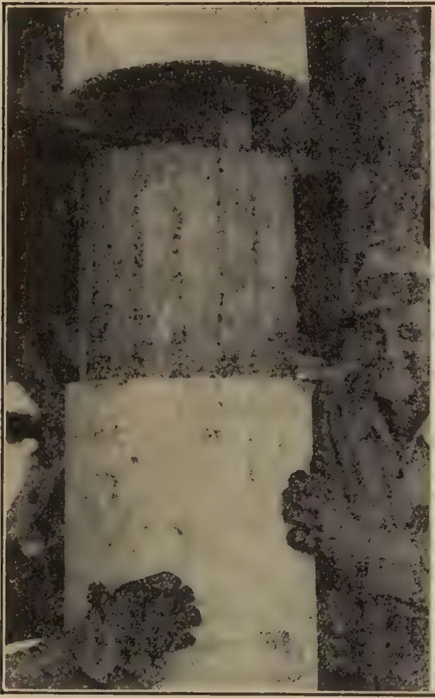


Fig. 25.—Making an Asbestos-Cement Pipe Joint

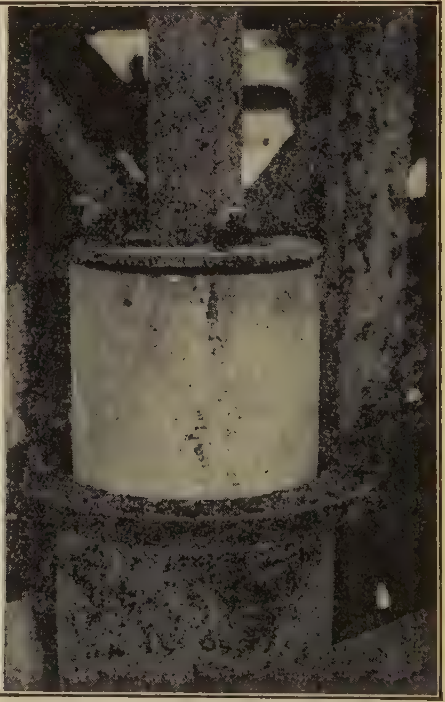


Fig. 26.—Cover or Hold-Down Plate at the Top of the Asbestos-Cement Pipe Assembly

Additional 6-in. pipe was added, permitting the assembled liner to reach bottom where it sank into the previously poured grout. The 6-in. pipe was then filled with heavy neat cement grout, and the left-hand thread at the bottom plug was unscrewed from the top. The grout dropped in the 6-in. pipe and filled the bottom of the asbestos-cement pipe until it found its level in both. Sufficient concrete to fill the liner was added through the 6-in. pipe which then acted as a tremie. The liner was overfilled by computed volume to effect a seal at the top of rock. It was important to gage this overfilling so as to be sufficient for the seal, but not so great to concrete in adjacent 20-in. casings in which other drills might still be operating. The remaining 6-in. pipe was then removed, and the 20-in. casing was pulled out and taken to the checking station where it was inspected for straightness. Small deflections were adjusted by shimming between flanges. The bottom of the casing was then prepared with a few tungsten carbide knobs and two water slots. If the casing was worn thin at the bottom it was cut off to a point where full wall thickness was available before applying the tungsten carbide. The casing was measured and marked and set aside for placement in the next projected drill hole. One casing was always placed in front of each drill, ready for drilling. Fig. 27 shows the method of lining and concreting, and details of the joints and bottom plug. To summarize the procedure:

(1) Fill the 6-in. pipe with neat cement grout through a 1.5-in. pipe to the bottom of the hole, by means of a grout pump, using a water-cement ratio of 0.4, and remove 1½-in. pipe;

(2) Unscrew the 6-in. pipe from the left-hand cap (see Fig. 27(*k*)); the grout rises inside the asbestos-cement pipe and drops in the 6-in. pipe to the point of head equalization for grout, establishing tremie action;

(3) Fill the 6-in. pipe with 1 : 2½ : 3 concrete using quick-setting cement;

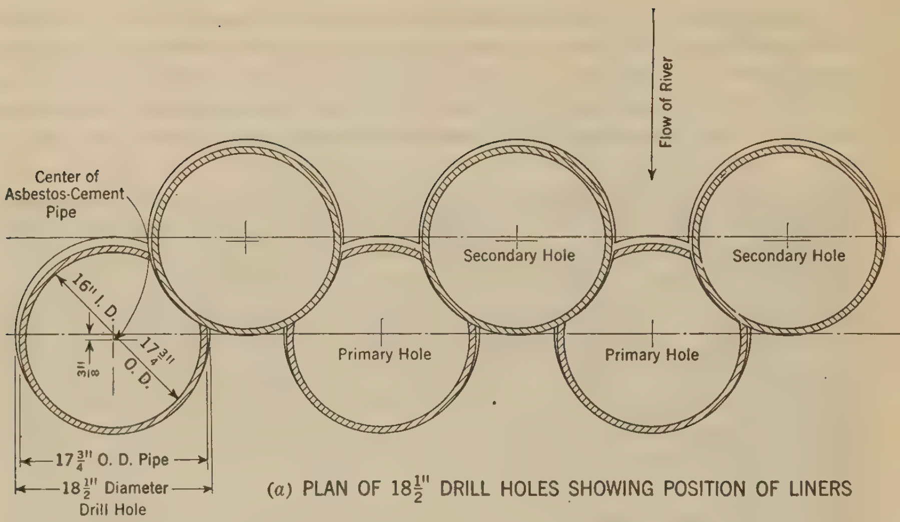
(4) Lift the 6-in. pipe and add concrete, maintaining the tremie action until the hole is full;

(5) Remove the 6-in. pipe; and

(6) Lift the 20-in. casing and thread it into the new template opening, after checking.

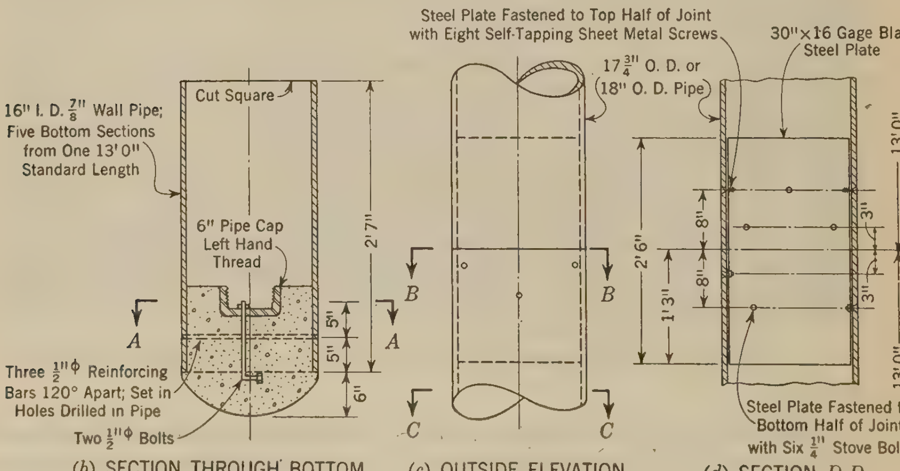
The 20-in. casing was handled in one piece by a floating steam whirley crane mounted on a barge. The crane had a lifting capacity of 5½ tons on a flat boom and 20 tons on a close boom. The boom was 75 ft long. The barge was equipped with one 56-ft and one 70-ft spud to reach bottom in the 50-ft to 60-ft depth of water upstream.

A small temporary plant consisting of a 10-sack concrete mixer on a barge also loaded with aggregate and cement was replaced by one of the TVA 2-yd floating concrete mixing plants, originally used during the construction of Wheeler Dam. Quick-setting cement, or standard cement with 2% calcium chloride added, was used for concrete since it was desirable to have the primary holes set up as quickly as possible so that secondary holes could be drilled. Except in a few exigencies, concrete in the primary holes was always 30 days old before secondary holes were drilled. All concrete materials except those in the mixer plant bins were normally stocked at the dock.



Flow of River
Center of Asbestos-Cement Pipe
16" I. D.
17¾" O. D.
⅜"
Secondary Hole
Secondary Hole
Primary Hole
Primary Hole
17¾" O. D. Pipe
18½" Diameter Drill Hole

(a) PLAN OF 18½" DRILL HOLES SHOWING POSITION OF LINERS

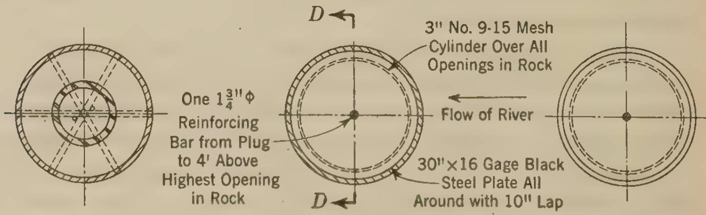


16" I. D. ⅞" Wall Pipe; Five Bottom Sections from One 13′0" Standard Length
Cut Square
6" Pipe Cap Left Hand Thread
2′ 7"
5"
5"
6"
A
A
Three ½"ϕ Reinforcing Bars 120° Apart; Set in Holes Drilled in Pipe
Two ½"ϕ Bolts
Steel Plate Fastened to Top Half of Joint with Eight Self-Tapping Sheet Metal Screws
17¾" O. D. or (18" O. D. Pipe)
B
B
C
C
30"×16 Gage Bla
Steel Plate
2′ 6"
1′ 3"
8"
8"
3"
3"
13′0"
13′0"
Steel Plate Fastened t
Bottom Half of Joint
with Six ¼" Stove Bol

(b) SECTION THROUGH BOTTOM SECTION OF HOLE LINER

(c) OUTSIDE ELEVATION OF JOINT

(d) SECTION D-D



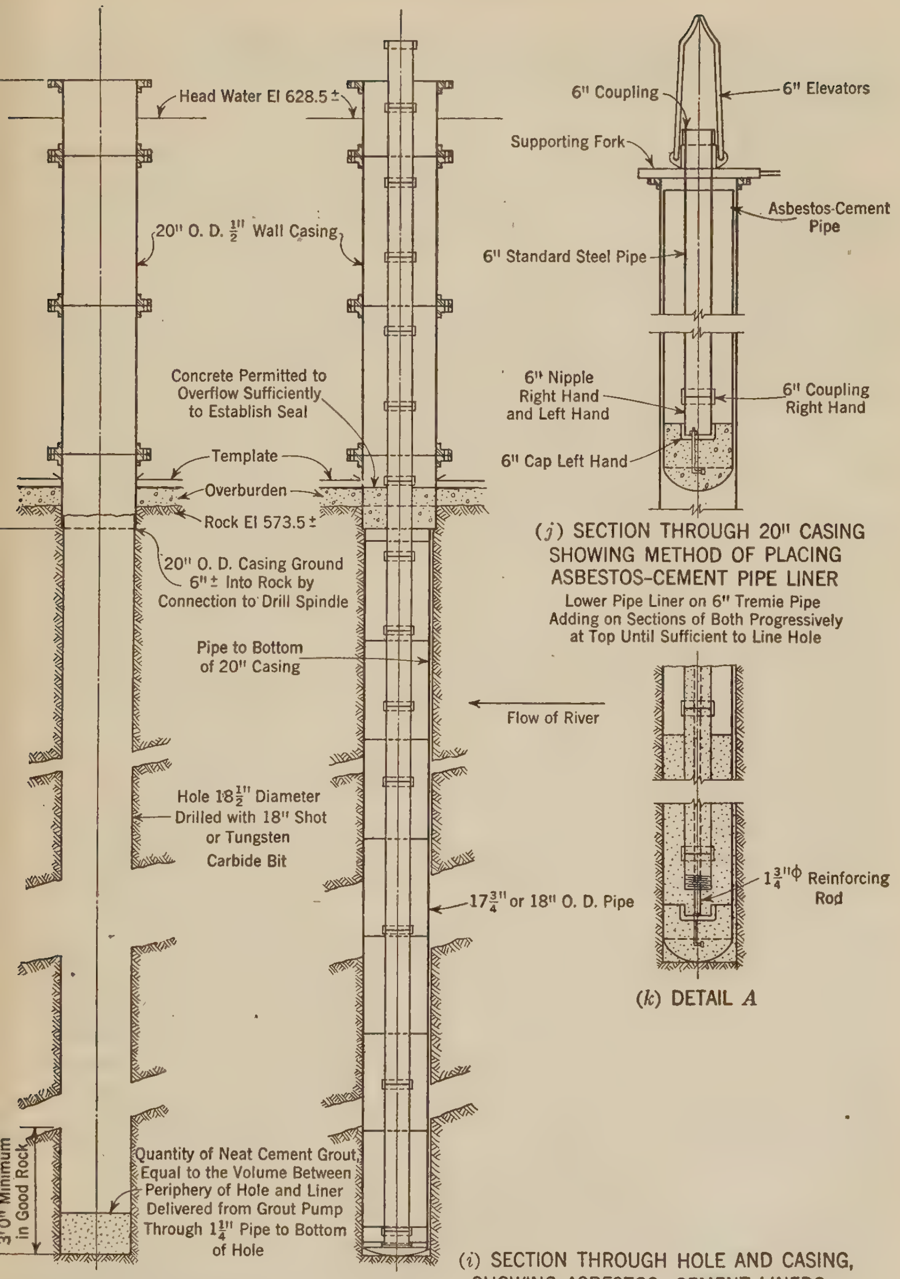
One 1¾"ϕ Reinforcing Bar from Plug to 4′ Above Highest Opening in Rock
D
D
3" No. 9-15 Mesh Cylinder Over All Openings in Rock
Flow of River
30"×16 Gage Black Steel Plate All Around with 10" Lap

(e) SECTION A-A

(f) SECTION B-B

(g) SECTION C-C

FIG. 2


Head Water El 628.5 ±
20" O. D. $\frac{1}{2}$" Wall Casing
Concrete Permitted to Overflow Sufficiently to Establish Seal
Template
Overburden
Rock El 573.5 ±
20" O. D. Casing Ground 6" ± Into Rock by Connection to Drill Spindle
Pipe to Bottom of 20" Casing
Hole 18$\frac{1}{2}$" Diameter Drilled with 18" Shot or Tungsten Carbide Bit
Flow of River
17$\frac{3}{4}$" or 18" O. D. Pipe
3'0" Minimum in Good Rock
Quantity of Neat Cement Grout, Equal to the Volume Between Periphery of Hole and Liner Delivered from Grout Pump Through 1$\frac{1}{4}$" Pipe to Bottom of Hole
6" Coupling
6" Elevators
Supporting Fork
Asbestos-Cement Pipe
6" Standard Steel Pipe
6" Nipple Right Hand and Left Hand
6" Coupling Right Hand
6" Cap Left Hand
1$\frac{3}{4}$"ϕ Reinforcing Rod

(*j*) SECTION THROUGH 20" CASING SHOWING METHOD OF PLACING ASBESTOS-CEMENT PIPE LINER

Lower Pipe Liner on 6" Tremie Pipe Adding on Sections of Both Progressively at Top Until Sufficient to Line Hole

(*k*) DETAIL *A*

) SECTION THROUGH HOLE
) CASING READY TO RECEIVE
LINERS AND CONCRETE

(*i*) SECTION THROUGH HOLE AND CASING, SHOWING ASBESTOS-CEMENT LINERS AND 6" TREMIE PIPE.

RETING DETAILS

A 22-ft by 100-ft steel work barge, on which 20-in. casing and 6-in. pipe were stored, asbestos-cement pipe was assembled, and reinforcing mesh and rods were placed, and the TVA towboat completed the marine equipment required for this work.

Concrete was proportioned from Tennessee River gravel and sand, the nominal mix being 1 : $2\frac{1}{2}$: 3 with a water-cement ratio of 0.5. This concrete tested consistently at 4,000 lb per sq in. in 28 days and the cement factor was 1.87 bbl per yard of concrete. This is high but was believed necessary due to the underwater nature of the work.

Twenty-four hours after concrete placement, the hole was sounded to determine the effectiveness of the concrete and if it was not as high as the anticipated level, it was brought up by placing cement grout or tremie concrete, after which it was sounded again after sufficient time had elapsed to permit hardening.

On October 15, the cutoff trench had been completed; the 13-in. drilling of 7,500 ft and concrete foundation consolidation were 90% complete; diamond drilling at 49,000 ft was 90% complete; cement and asphalt grouting were approximately 90% complete. Of the fourteen original boils showing downstream from the dam, all but two have been eliminated, and these two show marked improvement as a result of the impact of the work. Most of the remaining percentage of uncompleted work as of October 15 is in contact with these, and the completed work should show the same result here as has been effected on areas completed. Fig. 13 shows the boils which have been eliminated and those on which work is still being done. Circles representing these boils indicate by their diameter the relative sizes of the visible leakage as found at the time this work was begun.

All holes were logged and plotted in order that a complete record of the entire cutoff would be available at all times. A projection of the trend of the overlap in any series of holes indicated that the failure to secure perfect overlap should not be considered an impediment to securing satisfactory results since in cases where intersections were lost the drift of the holes was not sufficient to cause any serious harm and the later upstream asphalt placement was used to correct these minor defects. This was found to be correct in principle. After the cutoff was completely in place in the worst leakage area which connected with boils G, E, and F, Fig. 15, these boils had been reduced from bubbling as much as 18 in. above tailwater surface, to flat but still visible flows, indicating an estimated 75% effectiveness. Strategically placed asphalt holes upstream from this cutoff trench eliminated the visible balance of flow. Tremie concreting under the dam through the 13-in. holes, and cement and asphalt grouting through 3-in. holes, complete the operations. Job experience also indicates that, without the cutoff trench, the asphalt seal could not have been relied upon to secure the desired results because the flows were too large. The combination use of these methods, judiciously applied, is considered effective in cutting off flows of the magnitude encountered at Hales Bar.

During the period of high water in January, 1943, just after heavy general rains over the Tennessee Valley, some interesting phenomena occurred which had not been noticed during any previous high-water periods prior to completion of some parts of the cutoff walls.

At approximately Station 12+00 and about 150 ft upstream, where suctions had previously been indicated, and at Station 16+50 just upstream from the dam, boils appeared in the swollen headwater. Some of these carried mud and others were clear, but violent upheavals were in evidence more or less at 10-sec intervals.

At Station 19+50 and approximately 225 ft upstream, air bubbles in very large quantity effervesced continuously for three or four days during high water.

In the former instance an explanation is believed to be that the channels to higher ground water, as a result of the general rains, were open, but, since the cutoff obstructed easy passage to the downstream side of the dam, the differential head produced this boiling effect in the headwater. In the latter case it seems probable that large quantities of air became trapped in openings accessible to atmosphere during the long period of dry weather and that heavy rains running off into these subterranean channels entrapped this air and, due to hydrostatic pressure, shoved it through the openings to come up in the reservoir, the normal channel under the dam having been closed off.

A repetition of high water soon after the former high flows again brought out the water boils but not the air bubbles, which lends some weight to the assumptions.

Frequent checks of concrete in the adjacent holes have indicated that it remained well in the liner. Diamond-drill holes were drilled into this concrete to ascertain its condition, and soundings were made frequently through the intersecting holes as they were being drilled.

Many improvements over the originally conceived construction methods were made in the field. Notable among these were developments of the required core drill characteristics, which were used to specify requirements in acquiring this equipment. The drill tool used was modified from the originally proposed tool to take advantage of experiences gained on the wear of this part. The substitution of ½-in. wall casing for previously tried lighter sections, and the substitution of cast-steel instead of cast-iron flanges both on the 20-in. casing and on the 6-in. tremie pipe were important advances not originally foreseen. The development of the template and the underwater electric plumb bob stands out, perhaps, as the two most important improvements toward attaining the desired goal. The use of the tungsten carbide was essential. Not nearly enough importance can be attached to the wrenches and tongs which were acquired heavy enough to accomplish desired results after trial-and-error experiments with lighter tools. In the field of lining the holes, much work was done before the liner joints were satisfactory, and such handy adjuncts as the hold-down plate at the top of this liner were invaluable.

The original conception of a liner was that of a flexible fabric and rubber bag. In its trial stages this was found wanting. This type of liner was desirable because it filled the drilled hole completely. The very satisfactory handling of the rigid asbestos-cement liner, and the inability to secure the fabric and rubber products due to the war, precluded further experimentation with the flexible liner.

Summary

In retrospect, two improvements not evident without the experience gained on this work could have been made, and should be made if this plan is used elsewhere again:

1. After dredging upstream from the dam, a concrete block, tremied into the place of the removed overburden would have provided two advantages—
   (*a*) All casings could have been the same length; and
   (*b*) The templates could have been set directly on this block for drilling to begin.

Although it was thought that some overburden would be desirable for holding the casings in place, this was not found necessary.

2. Much importance was originally attached to overlapping of holes to secure a continuous cutoff. The amount of this overlap was determined to allow for some deflections in drilling to minimize the lack of it at lower levels. This overlapping has a definite theoretical advantage, but results at the job indicate that this is probably outweighed by practical consideration, such as:
   (*a*) Too deep overlaps cause caving of the concrete remaining in the primary holes, resulting in the necessity of drilling additional holes upstream from the secondary holes to correct this condition where it occurs. It is true that this condition obtains in only about 1% of the holes but it can be eliminated by decreasing the overlap.
   (*b*) The secondary hole does not drill as fast as the primary hole because of the overlap and consequent shot loss, particularly in holes lined with the rigid liner.
   (*c*) The time element, waiting for concrete to harden in holes adjacent to proposed intersecting holes, can be eliminated.

Therefore, to overcome these defects in application, it is felt that primary holes could have been spaced about the same, but that secondary holes should have been drilled farther upstream so that the periphery of the holes just touch instead of overlap. The disadvantage of this arrangement is that there would be slightly greater impairment of coverage of the leakage areas, but the record of overlap indicates the need for some asphalt grouting to perfect the seal anyhow, and therefore this disadvantage is outweighed by the aforementioned advantages.

Acknowledgment

The work was done by the Tennessee Valley Authority, for whom T. B. Parker, M. Am. Soc. C. E., later succeeded by C. E. Blee, M. Am. Soc. C. E., was chief engineer, and A. L. Pauls, M. Am. Soc. C. E., chief construction engineer. At the project, H. D. Towers was assistant superintendent in charge of the construction forces. He was assisted by B. W. Crowe, R. M. Winston, and L. E. Tidwell, shift foremen. A. H. Weber was in charge of engineering and inspection for the project. The writer also wishes to acknowledge the cooperation and assistance of Frank Miller and manufacturers supplying equipment and materials to the project for their part in the development of methods and specialized items required to do the work.

DISCUSSION

Berlen C. Moneymaker,[4] Assoc. M. Am. Soc. C. E.—An interesting and valuable record of how a serious and difficult foundation problem was solved successfully is presented in Mr. Hays' paper in this Symposium. Solution channels create foundation problems at nearly all limestone dam sites, but the large channel at Kentucky Dam presented a problem of unusual magnitude, involving both bearing strength and watertightness. So far as is known, this channel is the largest solution cavity ever to be encountered and treated in any part of a dam site. It lies entirely below the water table and is from 70 ft to 100 ft wide at the top (El. 289), decreasing in width with depth to El. 70±, where it disappears. The linear extent of the channel is unknown, but it is more than 1,000 ft long, and there is good evidence that it is more than 2,000 ft long. The channel extends across the lock and dam in the direction of N 02° W, but its trend changes to N 07° W a short distance downstream.

Immediately downstream from the lower lock gate, where the cutoff was effected by a subterranean arch dam 195 ft high, built from the top downward, the channel is 70 ft wide at the top. It extends downward without any appreciable decrease in width to El. 217, where it bifurcates, and from here the two prongs taper to extinction at about El. 70. The channel was nearly completely filled with residual chert and clay, sand, and blocks of limestone. In the upper region of the channel the limestone blocks are flat-lying and appear to be but little out of place. Although the channel was formed by the solution of the rock, it was never a large open cavity. Solution progressed along numerous closely spaced parallel joints, producing a series of vertical channels which subsequently were filled by material which slumped from the sides and from above. Solution, followed by cavity filling, progressed until nearly all of the rock in place was dissolved.

The deep solution channel was not found in the early stages of exploration; in fact, it was discovered little in advance of the actual excavation of the rock in the area involved. Although the Geologic Division of the TVA did not conduct the exploration activities at Kentucky Dam, it is appreciated that it was not practicable, however desirable it might have been, to drill closely spaced holes to within 100 ft of sea level over the entire dam site. As the dam alone is about 8,700 ft long, the total area under exploration was more than 10,000 ft long and several hundred feet wide, and ranged in elevation from 300 ft to nearly 450 ft. With the exception of two small outcrops, bedrock was covered by an overburden of alluvial and residual materials which varied in thickness from 20 ft to 110 ft. In view of the thickness and continuity of the overburden, the necessity of locating the exploration holes on fairly wide centers, the relatively narrow and linear character of the channel, and the presence of large and little disturbed blocks of limestone in the cavity filling down to El. 230, it is not surprising that the channel was not discovered in the course of the preliminary exploration.

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[4] Chf. Geologist, Geologic Div., TVA, Knoxville, Tenn.

The exploratory drilling revealed that, in places, the rock in the lock area was extensively dissolved, that the solution cavities were predominantly vertical channels, and that some of the cavities extended to great depths. Prior to January, 1939, only two holes, both relatively shallow, had been drilled into the channel. Both holes encountered numerous filled solution cavities alternating with layers (blocks) of undissolved limestone. As the same condition prevailed in narrow vertical solution channels, the vertical drill holes afforded no clue as to whether the solution channel was 1 ft wide or 100 ft wide. However, as additional holes subsequently were drilled, it became evident that rock was intensely and deeply dissolved in certain areas. Deep holes drilled into one of these areas at various angles disclosed that the deep and intense solution was confined mainly to a well-defined zone. Intensive drilling soon revealed the linear character of the zone, its width and its depth, and the extent of solution within the zone; but the details of geology as shown in the geologic section (Fig. 9) were not fully appreciated until excavation for the cutoff was undertaken.

Precisely how the cutoff of this large channel was effected is very clearly and concisely reported in Mr. Hays' paper. He is to be congratulated upon contributing to the engineering profession such an excellent record of the problem and its solution.

Mr. Schmidt deals with another unique cutoff problem. The excessive leakage at Hales Bar Dam has been referred to so frequently in engineering and geological publications that it has been accepted as the classic example of a dam built with insufficient foundation investigation, preparation, and remedial treatment.

When Hales Bar Dam was built (1905–1913), the science of engineering geology had not been developed. At that time, large masonry dams were few in number, were widely separated geographically, and were built with little or no geologic investigation. Even until 1933 most large masonry dams in the United States were built without more geologic investigation than a few cursory examinations by one or more consulting geologists. Until 1930 American geologists were almost unanimously of the opinion that solution cavities could not develop in limestones lying below the water table. It is not surprising, therefore, that Hales Bar Dam was built without an adequate geological investigation. Moreover, it is extremely doubtful whether a mere geologic investigation of the type current from 1905 to 1913, without extensive exploration with core drills, would have revealed in advance the serious foundation defects which caused trouble and delays during construction and appreciable leakage after completion of the project.

Hales Bar Dam is located on the Bangor (unrestricted) limestone of Mississippian age. The formation consists mainly of limestones with which are interstratified shaly layers of various thicknesses. The limestones, in the main, are quite pure, containing as much as 99% calcium carbonate. Structurally, the rock is almost horizontal and is cut by numerous joints and a few minor faults.

As stated by Mr. Schmidt, much of the rock under Hales Bar Dam is cavernous. The solution cavities, which are quite numerous in certain areas have developed mainly along joints, faults, and bedding planes. Some of them are large and open, and they are present down to El. 480, or more than 100 ft below the original river channel.

In effecting a cutoff under the dam, intersecting 18-in. holes had to be drilled and concreted to depths as great as 100 ft. The cavernous character of the rock rendered the task difficult at best, but the difficulties were enhanced appreciably by large volumes of water under considerable head, moving through the cavities at high velocities.

Mr. Schmidt's paper is timely and is of considerable interest to both engineers and geologists. In view of the widespread interest in the leakage problem at Hales Bar Dam, the paper is a valuable report on an entirely new method of cutting off excessive leakage.

In both papers of this Symposium, the value of detailed geologic work in the solution of the problems at hand is obvious although perhaps not sufficiently emphasized. At the Kentucky project the geologic work was done, successively, by Roger F. Rhoades, Portland P. Fox, Assoc. M. Am. Soc. C. E., and Leland F. Grant. The geologic studies involving the deep solution channel, discussed by Mr. Hays, were made first by Mr. Rhoades and later by Mr. Fox. At Hales Bar Dam the work was done first by Cecil B. McGavock, Jr., and later by John W. Frink, Juniors, Am. Soc. C. E. The geologic exhibits used in the two papers were adapted from drawings prepared by these men.

PORTLAND P. FOX,[5] ASSOC. M. AM. SOC. C. E.—Because it offers remedies for serious defects in dam foundations, this well-prepared paper by Mr. Hays should be of considerable interest to other engineers and geologists. The writer succeeded Roger F. Rhoades as geologist on the Kentucky project in April, 1941, and logged approximately the lower 50 ft of the deep channel. Although most of the exploration was done while Mr. Rhoades was geologist at the project, the writer had the opportunity to study the excavation at close hand.

The selection and exploration for the Kentucky dam site involved many unusual geologic problems and conditions.[6] Kentucky Dam is located at the northeastern edge of the Mississippi embayment and there are no exposures of bedrock on the left bank of the river for many miles either upstream or downstream. Bedrock outcrops near the right bank in the vicinity of the dam and consists of a few small, scattered, flat-lying limestone beds. It was impossible, therefore, to make any accurate detailed predictions of the bedrock conditions to be expected below the river bed from what could be seen on the surface. The territory surrounding Kentucky Dam is of very low relief; the highest hills, located from 1 to 6 miles away from the river, reach a maximum height of only 200 ft. Both abutments and most of the surrounding region are capped by from 10 to 100 ft of Cretaceous sand, clay, and gravel, which largely concealed most of the bedrock. Before and after the deposition of the sand, clay, and gravel, the limestone bedrock had been subjected to a long period of intense and deep weathering.[7] During the late Pleistocene period[8] the river cut its bedrock channel 70 ft below the present river bed and later filled the channel with more recent gravel.

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[5] Geologist, TVA, Fontana Dam, N. C.

[6] "Subriver Solution Cavities in The Tennessee Valley," by B. C. Moneymaker, *Journal of Geology*, Vol. XLIX, No. 1, January-February, 1941.

Between 1935 and 1937 seven dam sites on the lower Tennessee River, two on the Cumberland River, and four on the Ohio River were intensely investigated by numerous core drill holes. Most of the sites were found to be entirely unsatisfactory because of the absence of rock to depths of 200 to 300 ft below river bed or because of cavernous conditions below the top of rock.

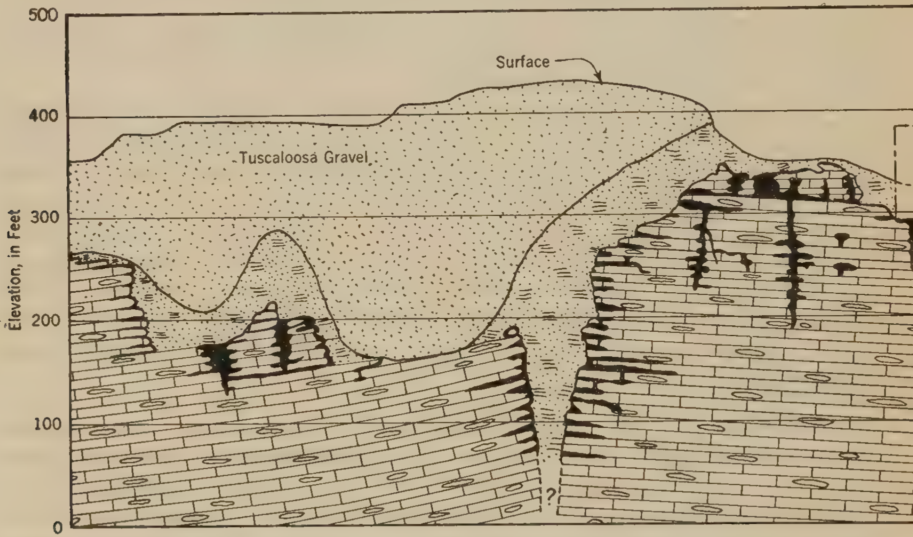

Fig. 28.—Geologic Section Diagonally Acros

During the exploration for the many dam sites, holes were drilled only 30 to 40 ft into bedrock regardless of the conditions encountered, because the first objective was to find a site where the top of the rock was high, and the second was to reduce the cost of drilling in case the site had to be abandoned. Two of these shallow core holes encountered a part of the large solution channel, but nothing unusual was thought of this condition as numerous other channels had looked much worse on the core record. The vertical holes did not indicate the width of the channels. This fact was realized early and many of the exploration holes were drilled on a 45° angle to discover the width of the channels. Of the 8,500-ft axis, 90% was covered with holes at 45°; but the large channel occurred near the right abutment not far from the only rock outcrops at the site, and in an area not penetrated by angle holes. The angled core drill holes revealed a slightly smaller but similar solution channel between Station 38+00 and Station 40+00, or about in the middle of the first bottom, and under the west earth embankment. Because there was a thick cover of nearly 100 ft of overburden, this area was treated by closely spaced grout holes.

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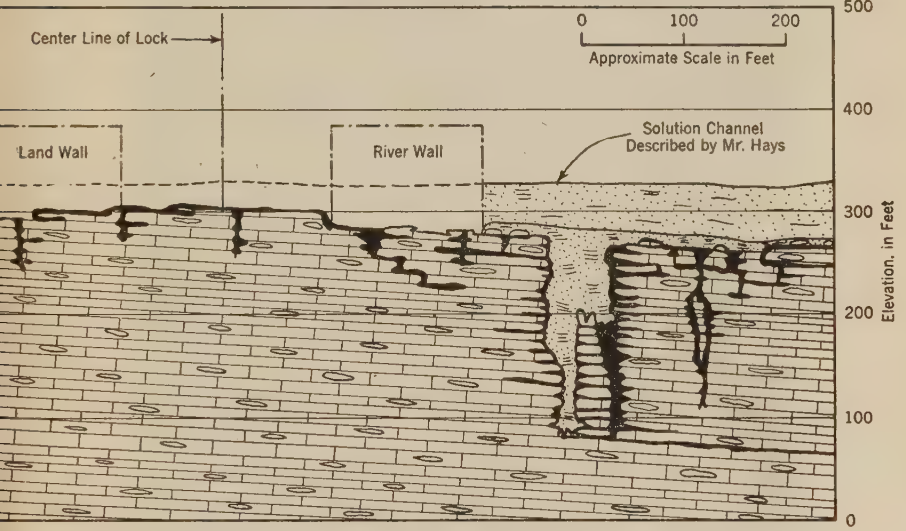
[7] "Relation of the Tuscaloosa Formation of Western Kentucky to a Pre-Existing Weathered Terrain," by R. F. Rhoades, *Bulletin*, Geological Soc. of America, Vol. LI, No. 12, Pt. 2, December 1, 1940, p. 1940.

[8] "Geologic Profile of the Lower Tennessee River," by Portland P. Fox, *Journal*, Tennessee Academy of Science, April, 1944.

Core holes drilled during the early exploration also disclosed a large escarpment in the bedrock which greatly influenced the location and design of the structure immediately upstream from the lock walls. This escarpment in the bedrock is approximately 100 ft high and has a north-south trend approximately parallel to the solution channel described by Mr. Hays. Recent exploration for grouting in the right abutment indicates that the escarpment is but one side of a larger solution channel that has formed in the crest of a breached anticline which is the dominant structure of the region (see Fig. 28).



ock and Right Abutment, Facing Upstream

This channel has been explored down to El. 130, or 301 ft below the surface, where it is approximately 40 ft wide.

The bedrock underlying the lower 60 miles of the lower Tennessee River is known as the Fort Payne formation of Lower Mississippian age. Although it is essentially flat lying, broad gentle folds having maximum dips of 10° are common but not obvious. Deep drill holes at the site have proved the formation to be 545 ft thick and to have a remarkably uniform lithology throughout this entire thickness. To make the geologic problem more complicated, the lower part of the overlying Warsaw and St. Louis formations in the vicinity have lithologic characteristics similar to the Fort Payne formation. The Fort Payne formation consists of alternating thick beds of dense, dark gray, relatively pure limestone and of thinner layers, lenses, and nodules of dense, glassy black chert (see Fig. 29).

The chert comprises about one third of the formation and is entirely insoluble, but, due to its brittle nature and numerous joints, it readily breaks up into small angular blocks when weathered. Some of the layers of chert are more than 1 ft thick and such layers frequently project nearly 1 ft from the receding soluble limestone beds. The limestone beds contain some impurities in the form of finely divided silt, but normally they are more than 70% soluble.

Because of the roughness of the cavity walls formed by the overhanging layers of chert, the residual clays never completely fill the spaces immediately under the chert. Thus, the limestone walls are exposed to constant solution and as the chert layers break off the face slowly recedes.

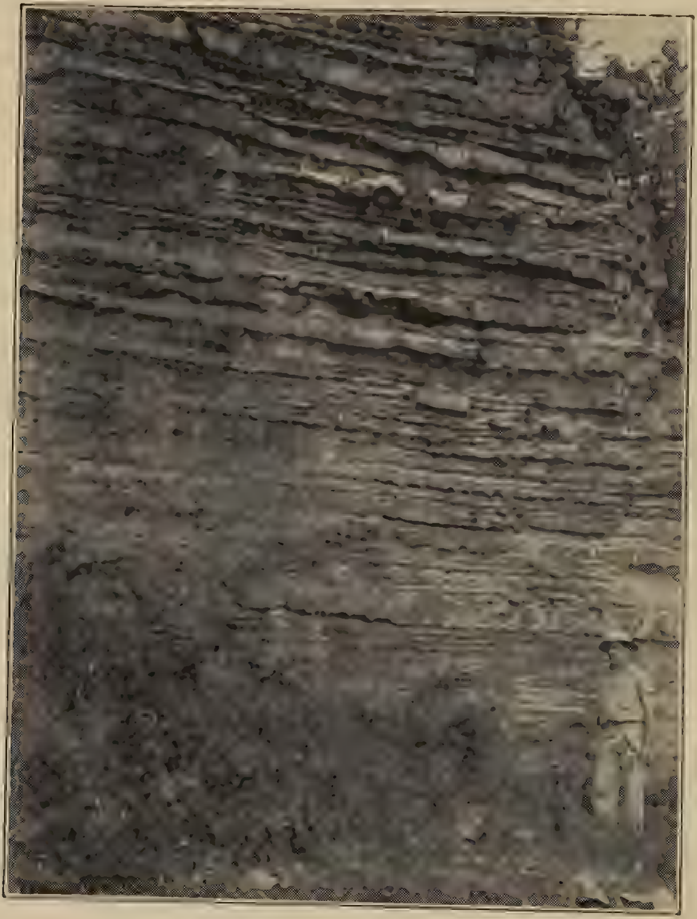


Fig. 29.—Typical Fort Payne Limestone and Chert

The larger solution channel described by Mr. Hays has formed along several large parallel strike joints. Because of the brittle character of the bedrock and a slight change in the dip of the strata, the rock had been literally torn apart to resemble a fault breccia at several places. The large tension joints and fractures disappear completely at the bottom of the solution zone on one of the more prominent bedding planes. If the joints had continued to a greater depth, so would have the solution. Thus, with a slight change in the dip of a uniform, brittle, cherty limestone, closely spaced joints can produce a major solution zone which perhaps is just as serious as a major fault.

During early exploration work considerable emphasis was placed on the superior value of angled core drill holes as compared with vertical holes, since the angle holes intercepted more of the vertical solution channels and they gave the width of the channels as well. These advantages are obvious and not too many angle holes were drilled, but, if more deep vertical holes had been completed first, several large continuous bedding plane seams would not have been overlooked. Even after the deviation of the angle holes had been measured by suspending hydrofluoric acid bottles in the holes and by measuring the angle with a transit sighted on a lowered flashlight in the hole, the deep holes curved so that both methods were inaccurate. As a result, the holes were never plotted correctly. Thus, the nearly horizontal seams were never recognizable from the staggered holes and they were not understood until construction was well under way. More deep vertical holes would have disclosed the bedding plane seams. The large solution channel described by Mr. Hays could have been recognized as easily from vertical holes since the high dips in the cores indicated tilted blocks, but a few angle holes would have given more information on the width of the channel.

In addition to the method described by Mr. Hays, this solution zone could have been treated by at least one other procedure. If the top of rock between the lock walls downstream from the lower gates had been cleaned off and a slab of concrete placed over the dissolved area, the solution channel could have been grouted extensively with clay where the cutoff was constructed. The concrete slab would have served to prevent too many grout leaks, and it would have prevented the washing out of the clay filling in the channel. With these precautions, and with the rather wide, deep cover of overburden upstream from the cutoff, the risk of possible leakage does not appear great.

Kentucky Dam[9] demonstrates the difficulties often encountered in limestone foundations that have long been exposed to intense weathering. A few closely spaced joints can make a tremendous difference. Even the most thorough exploration programs often leave much to be desired in cavernous limestones.

Mr. Schmidt's detailed account of the methods and equipment used to cut off the flow under Hales Bar Dam is a worthy contribution to engineering literature, but a more complete discussion of geologic conditions causing the cavitation would have been a welcome addition for a better understanding of the problems, since former literature on the foundation was similarly incomplete. The large amount of recent drilling should have afforded a much better picture of the geologic conditions than had been possible previously.

The "washing out of the shales in horizontal bedding planes, causing horizontal openings" is somewhat unusual and needs additional explanation. Shales are generally impervious and it is difficult to understand how the shales could wash out unless they were badly weathered or exposed in a cavity.

The photograph, Fig. 12(*a*), shows a considerable amount of cavitation following the bedding planes, but in Fig. 13 only a small amount of cavitation appears along the bedding of the limestone. This seems to be unusual and worthy of additional explanation. At Kentucky Dam nearly horizontal bedding plane seams, of from 0.1 ft to 0.5 ft thick, have been recognized for 4,000 ft down the slight dip.

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[9] "Foundation Exploration at Kentucky Dam Site," by A. V. Lynn and R. F. Rhoades, *Engineering News-Record*, Vol. 125, 1940, pp. 70–73.

In Fig. 13 both cavernous areas indicate a dip to the right approximately parallel to the reverse faults shown. Both cavernous zones seem to have developed in this pattern as a result of more general minor faulting and fracturing than is shown in Fig. 13. A probable minor fault could be assumed in Fig. 13 from Station 22+10, at the top of rock, to Station 19+85, at El. 485, where it becomes tight or dies out into bedding planes. Cavities do not need to exist everywhere along a fault, as the solid areas of rocks between the cavities represent pillars and there may be cavities only a few feet away behind these pillars. Once water has gained access to the limestone and a cavity has started to develop, solution of the limestone should take place in some part of the cavity away from the original fracture faster than along the opening.

If the cavernous areas are not the result of faulting and fracturing, why has not the entire river bed been affected uniformly since the limestone is essentially the same? Solution normally could not work its way below beds of impervious flat-lying shales unless they were broken. Small faults of only a few inches in limestones may afford just as good an opening to start solution as would a large fault. As shown in Fig. 13, curved fault planes are apt to produce additional small faults and fractures, which would aid solution. Indirectly, defect 3 (faulting) listed by Mr. Schmidt appears to have been the cause of both large cavernous areas which comprise at least 90% of the problem.

Geologic cross sections, such as those in Fig. 13, form an invaluable record, and, if a cross section had been prepared before construction of the dam, the seriousness of the problem would perhaps have been apparent. The problems of limestone foundations are not so difficult to treat when they are fully understood. Several errors seem to have been made at Hales Bar before the large leakage was finally cut off under Mr. Schmidt's direction. The first error was in misunderstanding the foundation problems or in not acknowledging their seriousness. The second error was in not having thoroughly grouted the rock for a considerable depth below the cofferdams before attempting to unwater.

Asphalt seems to have been used successfully with neat-cement grout, but the suitability of asphalt as a permanent material seems questionable. Mr. Schmidt noted that cavities frequently occurred above or around the former asphalt filling. He concluded that sand and clay had since been washed out to leave open cavities around the asphalt. Although this may be true in part, there are two other logical possibilities: (1) Gradual deformation and flow of the asphalt under the reservoir pressure, or slumping under its own weight if the cavities are not completely filled for some distance up and downstream, and (2) solution of the limestone since the asphalt was injected.

A perfect sphere of asphalt or pitch will flatten noticeably in one day under its own weight. In the course of several days the spheroidal shape of the asphalt is no longer noticeable. Columbus is supposed to have patched the leaks in his boats from the large asphalt lakes on the southwest corner of Trinidad; and vast quantities of the asphalt have been removed since Columbus visited the island, but the level of the asphalt remains approximately the same.

Asphalt is a semi-solid and its satisfactory behavior in cavities over a long period is doubtful. It is granted that a complete cutoff may be obtained in a large cavity without filling the cavity laterally for any great distance from the cutoff line. However, if the unfilled cavity has a slight downstream slope, there seems to be no reason why the asphalt seal will not be broken by slumping or slow flowing as a result of its own weight or the reservoir pressure.

If the asphalt is backed up with neat-cement grout as was done at Hales Bar Dam, this could be prevented. For the foregoing reason, up to 1940 the gradual increase in the leakage following asphalt grouting may have been due in part to a slow deformation of the asphalt.

During the construction of Watts Bar cofferdam, a large navigation dike made of limestone boulders was removed from the river channel. The dike had been constructed in 1912 by the Corps of Engineers, U. S. Army, and was removed in 1940, 28 years later. Most of the limestone boulders had been continuously under water and freely washed by the waters of the Tennessee River. After 28 years of exposure to solution the layers of limestone had receded by solution from one half to three fourths of an inch, and impure and cherty layers protruded the same distance. This example gives a very good idea of the normal rate of solution of a limestone in a free-flowing channel. This particular limestone was more dense and much tougher than the type occurring at Hales Bar Dam. Exposed to such rapid flow currents as existed under Hales Bar Dam, each wall of the solution channels might recede as fast as 1 in. in 25 years, or a narrow fissure only 1 in. wide might increase to 3 in. in width in this time. This factor may have contributed a noticeable amount to solution in the areas of broken rock and along the joints. For this reason a perfect cutoff is necessary in a limestone foundation.

The difficulties experienced with this and other dams constructed on limestone may cause many engineers to be dubious of a dam site on limestone. Regardless of this bad publicity, limestone foundations can be among the best if proper care is used in the selection of the site. Limestones have many good features not found in other rocks. They are usually soft and easy to drill, blast, and crush. Most limestones which have been subjected to great stresses have yielded by gliding within the calcite crystals; and fractures, shear planes, and joints are healed with cementation of calcite so that less difficulty is experienced than with many other rocks.

J. K. Black,[10] M. Am. Soc. C. E.—Complete and interesting descriptions of the cutoff programs at Kentucky and Hales Bar dams and records of procedures and results are contained in the papers of this Symposium. Both the problems and their solutions are exceptional even in a limestone formation where unusual conditions are expected. It is fortunate that the two papers were grouped as they were because the conditions which were corrected by Mr. Schmidt at Hales Bar Dam are a good illustration of what could have happened at Kentucky Dam if a cutoff by grouting alone had been considered acceptable.

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[10] Project Mgr., TVA, Fort Loudoun Dam, Lenoir City, Tenn.

Both papers illustrate the erratic changes that can occur in limestone formations, and emphasize the necessity for a great amount of exploratory drilling to secure complete information. They also demonstrate the need for, and the aid furnished by, the geologist in interpreting data, preparing drawings, and otherwise collaborating with the engineer on conditions and possible treatment of various formations.

A positive cutoff, in a badly weathered formation, at great depths below the water table, where rapid movement of large volumes of water occurs, is difficult to construct and requires the utmost study, skill, and perseverance. In addition to the difficulties encountered in handling the water, the engineer is confronted with the decision as to how far to go, especially in depth, in treating badly weathered formations under such extremely bad conditions. That decision is often very difficult to make.

At both Kentucky Dam and Hales Bar Dam, conditions were investigated thoroughly and completely and the objectionable features were treated and corrected in the same thorough and complete manner. There appears to be no doubt that the treatment was effective at both places. The methods adopted, the procedures followed, and the equipment selected evidence the thought and consideration that assured efficient execution and successful completion. The programs at both projects are to be commended.

L. C. Glenn,[11] Esq.—The emphasis in each of the papers in this Symposium is twofold: (1) Geological conditions, which, in general, were formerly considered as unusual, are described; and (2) the ingenious methods devised to cope with the problems are detailed. The extensive explorations, by the TVA, of dam sites in limestones have thoroughly discredited the old idea that circulation and consequent solution quickly cease below ground-water level. Solution under favorable circumstances may extend several hundred feet below ground-water level. Fortunately, solution does have a tendency to decrease in depth, so that solution channels may be expected to close up gradually if followed down with the necessary persistence. At Kentucky Dam the ordinary solutional activity below ground water has undoubtedly been considerably augmented by a depression of the water table in this region of some 200 ft or more in early Pleistocene time. The Ohio River gorge, only about 25 miles away, has been cut down to near sea level and has since filled with sand and gravel. In one of the very deep 48-in. holes at Kentucky Dam, pieces of chert were found near the bottom encrusted with limonite. The surfaces of the pieces of chert were covered with fine limonitic stalactites about an inch in length. These could have been formed only when the chert was above ground-water level. Since then, there has been a regional subsidence of some 200 to 300 ft at least. During the higher stand of the land there was a deeper-reaching solution.

Rocks at Kentucky Dam have been subjected to two periods of stress, each of which produced jointing or fracturing. The earlier fractures were later healed by deposition of coarsely crystalline calcite. The later fractures were not healed subsequently but have become the paths for the solution that has given trouble. Incidentally, the same two periods of fracturing also characterize the rocks of the Ozarks, the earlier fractures being healed by calcite, dolomite, and occasionally sphalerite, as is shown by core drilling for dams in that region. The regional structural history has been similar.

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[11] Cons. Geologist, Nashville, Tenn. (formerly Head, Dept. of Geology, Vanderbilt Univ., Nashville, Tenn.).

Fig. 9 shows the marked tendency of grout to follow the contact between limestone and residual clay and how difficult it is to obtain any worth-while penetration of the grout into the clay. Mining the clay out, as was done at Kentucky Dam, is really the only effective method of treatment.

It is hoped that Mr. Schmidt has written finis to the long-standing leakage troubles at Hales Bar. Water flowing in strong currents through honeycombed limestone beneath the dam was constantly enlarging its passageways. Also, the water velocity added much to the difficulty of successfully placing concrete to stop the flow and to consolidate the foundation. One is sometimes inclined to wonder how the dam has stood up at all, especially in recent years. Mr. Schmidt describes well the intricate details of the plan adopted for closure and the modifications necessary to meet conditions that developed during the work. He has shown himself highly ingenious and resourceful in handling a very difficult problem and in solving it in an original and effective way. Although some leakage may develop in the future, the troubles at Hales Bar seem essentially to belong to the past.

E. H. Burroughs,[12] M. Am. Soc. C. E.—The reader will be impressed by the bold scope and thorough execution of the deep cutoff construction on the Kentucky Dam, described in the interesting paper by Mr. Hays. The method used to provide an effective upstream water seal probably has salvaged an important dam site where the foundation materials would otherwise have precluded successful development of the relatively high-head structure to be supported.

The engineering and construction features of this structure should be emphasized, rather than the geology. A subterranean cutoff wall built by methods substantially the same as those used by the author will make valuable but defective reservoir sites suitable for development almost regardless of the geological conditions. Few changes are required to adapt this method to almost any type of difficult foundation, as practical experience has shown. The applicability of this method is not limited to cutoff walls built deeply into river beds where flowing ground water may be encountered, since the method is safe, rapid, and economical for almost any deep trench work, and the size of the working chamber can be reduced to an area in which the water inflow can be handled and sloughing controlled. The excavation principle is somewhat similar to the methods of middle-western contractors in the 1910's when they invaded the deep foundation field in New York, N. Y., with their so-called "Chicago method." With open caissons and heavy pumping equipment, plank-sheeted circular pits with steel-ring walers could be driven to great depths, for column footings, faster and cheaper in all soils than by the conventional methods of the pneumatic-caisson contractors.

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[12] Pres., Ambursen Eng. Corp., and Vice-Pres., Ambursen Dam Co., Inc., New York, N. Y.

As a last resort, where a water-logged material becomes too costly, slow, or dangerous to excavate, compressed air can be applied to the shaft until the difficult area has been traversed; but in no instance has it been needed.

In the case of the Hales Bar Dam, a cellular cutoff, built from the top downward, would have avoided heavy water losses and the eventual costly repair work described in Mr. Schmidt's paper. The writer can give similar instances, such as hydroelectric plants built under his direction on the porous limestone of the Shenandoah River in Virginia (one directly above the Luray Caverns) where solution channels swallowed enormous quantities of wasted grout.

It is difficult to conceive a cutoff wall or core-wall problem that will not be solved adequately by this type of construction. One of its greatest advantages, not mentioned by Mr. Hays, is that the men work without fear or restraint in the free air, as compared to conditions under the costly, laborious, and slow pneumatic-caisson method. Relatively little heavy equipment is needed. Safety for workmen against sudden inrushes of water or mud is assured by shafts of ample dimensions, where the undercut face of the excavation is only a few feet at most beyond shaft limits, as shown in Fig. 30. This strengthens the writer's view that the geological deficiencies of the foundation at Kentucky Dam are less important to the engineering profession than the designing and construction methods which repaired those deficiencies.

Once the initial drilling has determined the rock elevations at a dam site and has demonstrated suitability for the methods outlined, little is gained by further expenditure for an elaborate pattern of drilling, as any changes in the conditions are adequately met as the work progresses below ground.

Since 1928 the writer has applied the cellular cutoff wall design to four different projects, two actually having been built by that method. Each foundation material was entirely different from the others, and yet the advantages of the method applied with equal force to all four projects.

Designs were prepared under the writer's direction in 1930 for a high dam on the Kootenay River in British Columbia, Canada (later abandoned for financial reasons). The river bed was ledge rock but both sides consisted of rapidly rising, high banks of porous sand, gravel, and boulders, upon which large embankments were to be built as wings or abutments for the spillway. Core walls approximately 100 ft deep were required, but the cost of a sheet steel-pile core wall was prohibitive and driving it impossible. Comparable data for the cellular wedge type of core wall, built from the ground surface downward, showed a cost 46% of that of a concrete core wall of conventional type, constructed in a sheeted and shored open trench. This design reduced the top width of the excavation considerably and left adjoining material practically undisturbed. Any underground slumping of materials would be replaced by grouting as the core-wall construction progressed downward. Freezing was contemplated where necessary for controlling any water-bearing sand in the lower depths of the core wall, since the small confined area under each shaft was admirably suited to this method. Modern chemical soil consolidation would be equally helpful. The difficulty and danger of maintaining a deep sheeted and shored trench would have justified the method described even at an equal cost.

The writer believes the first example of the design and construction of a concrete cutoff wall built from the ground surface downward, making use of working shafts through the completed sections, was the Rodriguez Dam in Lower California, Mexico,[13] begun in 1928. This wall was constructed (without reaching bedrock at the bottom) to a final depth of about 300 ft below the working level in the stream bed to provide the proper creep ratio for the head of 240 ft above river bed created by a full pond. The total distance or head from pond level to bottom of cutoff wall was 475 ft. This work was done at low cost by a small force without accident or serious injury to any workman,

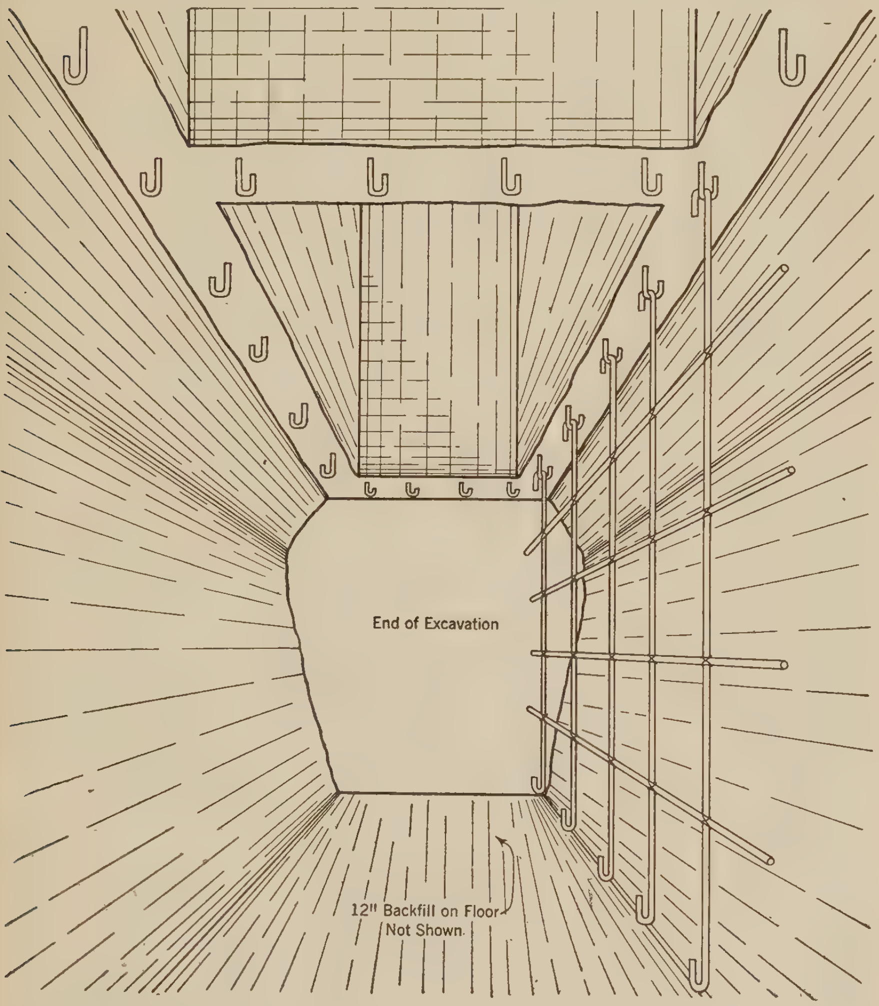

FIG. 30.—TYPICAL EXCAVATION FOR CELLULAR CUTOFF WALL BLOCK

against the full head of the Tijuana River. The structure was built through soft clay gouge, fugitive blocks of granite and rhyolite, areas of crushed foundation rock, and lenses or continuous beds of sand of varying texture, including quicksand.

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[13] "Foundation Treatment at Rodriguez Dam," by Charles P. Williams, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 295.

Mr. Hays is fortunate indeed to have had the opportunity of sketching the actual geology of the Kentucky Dam cutoff wall as the work progressed. His data afford an admirable check on the accuracy of the conclusions drawn from the wide network of borings. At the time the Rodriguez Dam site was selected for development in 1927, the Mexican Government was engaged in a very extensive irrigation program elsewhere in the republic and little money was available for a boring program. However, even had this government been able to make an advance subsurface survey as extensive as that made for the Kentucky Dam cutoff (which the writer understands cost about $500,000), the problem would not have been simplified. The site was the only one available, and the Mexican Government was committed to the development of this project for international and economic reasons. The writer cannot see how even the most elaborate pattern of borings would have affected, measurably, the ultimate design and construction of the cutoff wall and other foundation structures.

Only nine borings and fifty-two shallow test pits (which represented a cost of about $7,500) had been made by the National Irrigation Commission when the writer's company took over the design and construction of this project. Of the borings, five indicated satisfactory rock at depths not exceeding 50 ft, but no cores could be obtained from four, two of which indicated badly disintegrated materials. A main fault about 30 ft wide and a secondary fault about 10 ft wide existed, filled principally with clay gouge, but also with many pockets or layers of water-bearing sand. The area between the two faults was filled with crushed foundation rocks of varying sizes, bedded in clay and sand. The hanging wall of the main fault had been shattered for a considerable distance beyond the edge of the fault, necessitating special treatment, and similar conditions existed on the opposite side of the filled-in river channel. The main fault was assumed by the Mexican Government's consulting geologist, Frederic L. Ransome, to extend downward for a thousand or more feet.

Excavation for the Rodriguez Dam cutoff wall was begun in an open trench, curved in plan to secure the added strength of a subterranean arch without interfering with the connection at the top to the straight upstream deck of the 240-ft Ambursen type dam being built above it. Considerable sloughing occurred in the wet, broken formation resulting in the change to a wedge shape about 25 ft wide at the top and 15 ft wide at the bottom. Vertical shafts, mostly 5 ft wide and 10 ft long, were formed in this excavation and the entire trench plug was then concreted. The wedge shape was expected to aid in supporting the block against settlement, which would halt itself by consolidating the material on each side, but no measurable settlement ever occurred.

When the shaft excavation was well under way, construction began on the main dam itself, so that work on the dam was not delayed pending completion of the cutoff.

The vertical cutoff arch was projected into the fractured hanging wall by a system of grouting from the surface and from adits (as shown in Fig. 31), after other adits, forming a continuous vertical wall, had been cut into the hanging wall from the adjoining shaft to the distance considered economical by that method, as compared to the cost by drilling and grouting. Section C–C of

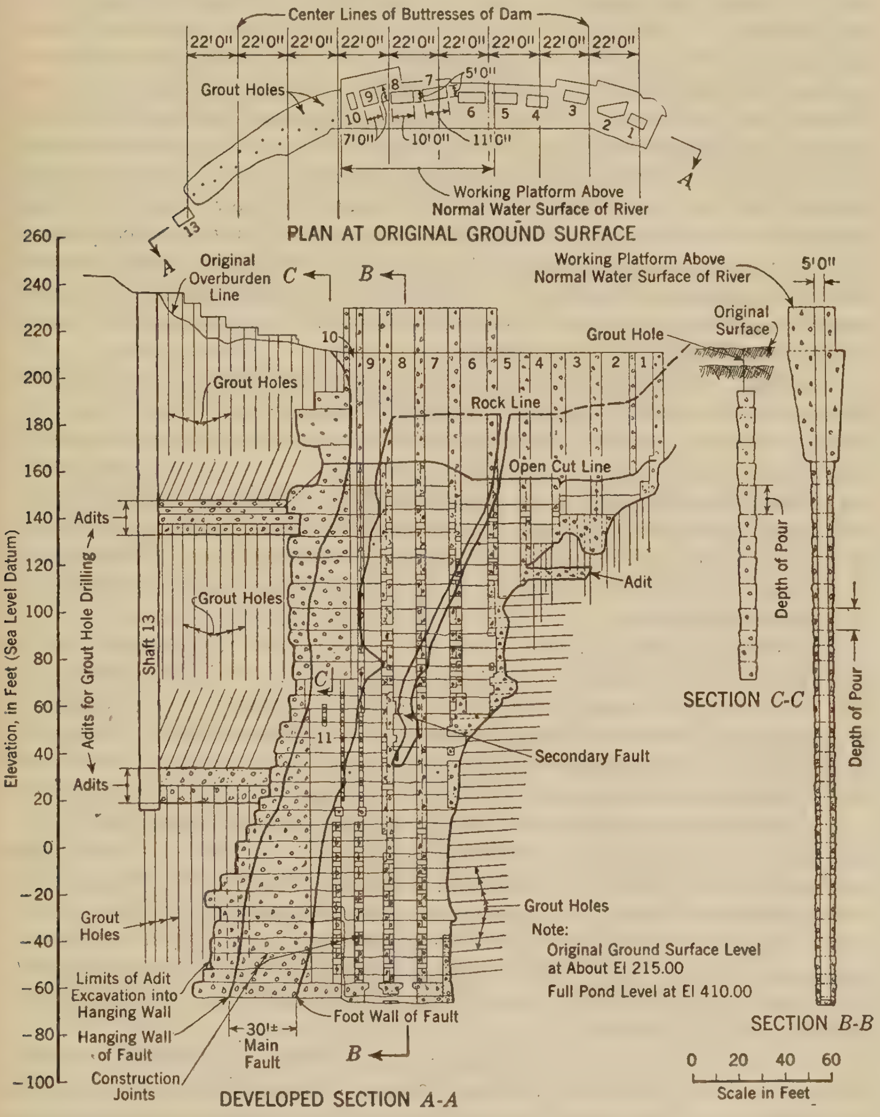
Center Lines of Buttresses of Dam
22'0" 22'0" 22'0" 22'0" 22'0" 22'0" 22'0" 22'0" 22'0"
Grout Holes
5'0"
7'0"
10'0"
11'0"
Working Platform Above Normal Water Surface of River
A
A
13
PLAN AT ORIGINAL GROUND SURFACE
260
240
220
200
180
160
140
120
100
80
60
40
20
0
−20
−40
−60
−80
−100
Elevation, in Feet (Sea Level Datum)
Original Overburden Line
C
B
Working Platform Above Normal Water Surface of River
5'0"
Original Surface
Grout Hole
10
9 8 7 6 5 4 3 2 1
Grout Holes
Rock Line
Open Cut Line
Adits
Adits for Grout Hole Drilling
Adits
Shaft 13
Grout Holes
Adit
Depth of Pour
SECTION C-C
C
11
Secondary Fault
Depth of Pour
Grout Holes
Grout Holes
Note:
Original Ground Surface Level at About El 215.00
Full Pond Level at El 410.00
Limits of Adit Excavation into Hanging Wall
Hanging Wall of Fault
Construction Joints
30'± Main Fault
Foot Wall of Fault
B
SECTION B-B
0 20 40 60
Scale in Feet
DEVELOPED SECTION A-A

FIG. 31.—RODRIGUEZ DAM: PLAN, AXIAL SECTIONS, AND VERTICAL SECTIONS, SHOWING PROCEDURE

Fig. 31 shows the novel method used for constructing these adits. The wedge-shaped adit at the highest elevation was first excavated and concreted. Then the adit next below was similarly excavated, using the bottom of the concrete next above as a roof, which made for very rapid and safe excavation. Much grout was taken by the drill holes in the hanging wall, which supports the right wing abutment of the main dam.

The writer doubts whether, on a project where the utmost economy might have to prevail, the cost of the author's method of redrilling 36-in. and 48-in. holes through the finished concrete blocks—to enable the cutoff wall to be deepened progressively below and concreted by the use of these very restricted openings as working shafts—would be justified. It has been the writer's experience that, by forming large rectangular shafts, plenty of room was obtained for the rapid evacuation of excavated material and for pipes, conduits, telephone cables, etc. More important still, where water-bearing sand and porous crushed rock under great pressure invited slumps and blowouts, the generous cross-sectional area of the working shaft (about 5 ft by 10 ft) enabled the men to work with greater confidence, and hence more effectively, than if the exit from the working chamber had been a circular shaft of small diameter. Wide steel rungs were set as a continuous ladder in every shaft from the top to the bottom —an emergency exit for the workmen.

The lengths of 6 ft along the axis, used for the Kentucky Dam cutoff wall blocks where the drill hole was 36 in., and 7 ft where it was 48 in. in diameter, called for mining out about 18 in. under the sides of each block—practically the average undercut for the Rodriguez Dam. However, the ultimate cost of the job is controlled more by the actual number of shafts and blocks than by their dimensions; hence, the writer believes that a given length of cutoff wall using long blocks with big shafts, similar to the Rodriguez cutoff, would cost much less than a length of wall with double the number of blocks half as long with their smaller shafts.

Attempts were always made to impart a wedge shape to the excavation for each block before concreting. The writer's preference for this procedure comes from the conviction that wedge-shaped pours are subject to less danger of settlement and that they form a tighter and stronger structure. They also create a longer and rougher path for percolation; but this was not taken into consideration in computing the creep ratio. These individual blocks are actually given support in four ways: (*a*) From a vertical reinforcing system; (*b*) by friction on the sides of the excavation; (*c*) by the horizontal steel reinforcing bonds, and (*d*) by bridging or beam action from the contacts with the adjoining blocks at the same level. The wedge-shaped configuration gives a fifth support. Faster progress was made in excavation by leaving a layer of 10 in. or 12 in. of loose material on the floor of the excavation and by pouring the concrete block on it, since excavation for the next lower block was made easier by starting from this soft loose layer.

The writer's experience was identical with that of Mr. Hays—by driving alternate shafts or blocks first and maintaining these ahead of the intermediate blocks, the width (thickness) of the latter could be considerably reduced and lifts could be extended deeper than for the adjoining blocks and with less pumping. Only two faces had to be maintained, as against four in the shafts which were pushed ahead first.

Straight bars were first used as the vertical reinforcing, and, as at the Kentucky Dam, were driven or drilled into the bottom of the excavation before the concrete was poured. Later, hooked bars were found more convenient. As shown in Fig. 30, the hooks made for better working conditions than a series of long bars projecting down into the area being excavated. The horizontal bars were driven from the open excavation laterally into the intermediate unexcavated area, as was the case at the Kentucky Dam.

At the beginning there was no certainty that this novel cutoff wall method could be followed to the bottom of all shafts, but, as ample compressor capacity was available, the job was always in readiness to switch to the pneumatic process, which never had to be adopted. Even areas of semifluid quicksand were usually controlled without undue delay by pushing adjoining shafts down past them. Drainage through the walls of these intermediate blocks soon eliminated the difficulties in the bad pockets.

It is not clear to the writer whether the 36-in. and 48-in. drill holes at the Kentucky Dam through which the excavation and concreting were conducted were eventually filled with concrete or spoil, or whether they were left open and merely capped at the top. Likewise, the details of the connection of the steel sheet-pile core wall of the dam to the cellular concrete core wall would be of considerable interest. Upon completion of the Rodriguez cutoff wall, all but two of the shafts were backfilled with concrete. However, it was considered that it might be desirable at some future time to extend the cutoff wall deeper, should seepage beneath the cutoff occur in dangerous quantities. Therefore, in shafts 8 and 9 a cylindrical form 48 in. in diameter was installed, the remainder of the shaft being filled with concrete. These two circular shafts were then plugged with concrete at the bottom, backfilled with sand and gravel, and topped with a specially designed watertight cover. In the future if it is considered advisable to deepen this cutoff, steel cylinders will be attached to the two circular shafts and excavation and concreting operations will be conducted through them from a point above the lake level.

It would be interesting to know how much grout the vertical joints between the Kentucky Dam cutoff blocks took and whether any attempt was made to grout the horizontal joints between the blocks. At Rodriguez Dam grout pipes were built into all contacts between concrete blocks and into the outside foundation material adjoining the cutoff wall. Practically no grout could be forced into any of the joints, indicating no subsidence of the blocks, probably due to their wedge-shaped formation and reinforcement. The pipes to the exterior of the blocks were capped on the inside when installed but were left open to the outside ground. When excavation began below, the caps were replaced by canvas hose, which led the outside drainage freely into the sump. This drained off much outside water that otherwise would have caused difficulties. To avoid building up pressure, the pipes were left open until several sections of the concrete had been placed below them, when grout was forced out into the foundation material on both sides of the wall, usually under a pressure of 100 lb per sq in. Every precaution was taken to reduce the head that could build up against any part of the excavation. Many slumps inevitably occurred behind the blocks as excavation progressed, but always were stopped by quick plank shoring. For many blocks, some sheeting and bracing were also necessary to restrict the amount of excavation. The sheeting and bracing were of the most rudimentary nature possible, usually merely a few short planks braced as might be most convenient. Most sheeting was left in place and, it is assumed, became subsequently embedded in the grout.

Contrary to Mr. Hays' experience on Kentucky Dam, where the water leakage increased as the work deepened, the Rodriguez Dam inflow at 200 ft to 300 ft below river level was no greater than at 50 ft, and as far as could be determined had no relation to the depth of the shaft. Rather, it depended on the texture of the material through which the wall was passing. The drainage program did much to ease the water control, and, by keeping as many drains open as possible above the working chamber, little or no trouble was experienced in maintaining satisfactory pumping from the sumps. Relay pumping stations were installed whenever needed, the water usually being pumped into a small tank with a high-head electric pump over it. The tank and pump were both placed in one side of the shaft where they would not disturb the hoisting of muck. Pipe lines, 4 in. in diameter, were originally installed in each shaft through which the water was pumped to the surface, but, as confidence was gained in the method being pursued, adjoining shafts were interconnected, which expedited the work materially and considerably reduced the pumping installations.

The over-all cost per cubic yard of work in the shafts was less than that for the adits cut into the hanging wall of the fault, and, as the hanging wall was departing from shaft 10 when the cutoff wall was about halfway down, shaft 11 was opened under one of the adits, which shortened all the remaining adits. Shaft 11 was interconnected with shaft 10 and was worked entirely from the latter shaft.

To maintain a continuous record under full reservoir conditions of the pressures at the bottom of the cutoff wall and under the dam itself, instruments including pressure gages and pipes for soundings were installed in the dam foundations and in the sides and bottom of the shafts before concreting. Frequent observations have indicated that such leakage as can be detected in the river bed below the dam does not come from beneath the cutoff wall, but through seams in the earthquake-shattered banks of the river which carry the side wings of the dam, below the limits of the grouting done in the bottom of the conventional relatively shallow cutoff wall where the dam rests on rock. No lateral or longitudinal flow can be detected along the base of the dam, and there is no evidence of any water passing under the cutoff wall at its bottom. At one point along the bottom of the cutoff wall the hydrostatic pressure is greater immediately downstream from the wall than immediately upstream.

The National Irrigation Commission felt that, by applying one of the conventional creep-ratio theories to the material in the fault, it would be safe to stop the cutoff wall at some depth between 200 ft and 250 ft. The decision to extend it to 300 ft was influenced by the low cost and relative ease of the cellular cutoff wall method.

The cellular cutoff wall method (including the features of building the cutoff wall from the top surface downward through shafts instead of from the bottom up) and also the adit construction method used in the firmer rock areas (of building a top adit first in wedge shape to form a safe and convenient working chamber below it for each succeeding wedge-shaped adit, thus constructing downward a continuous concrete curtain or cutoff) were developed by Spencer W. Stewart, M. Am. Soc. C. E. To the best of the writer's knowledge the Rodriguez Dam was the first project on which these principles were ever used.

A third project using designs and construction methods similar to those described in Mr. Hays' paper (except as to the drilled shafts) was the Bristol Dam on the Pemigewasset River in New Hampshire. This work was incidental to adding 30 ft to the height of the dam in 1931, to increase the head on the power plant from 50 ft to 80 ft—for which program the Ambursen type dam had been adopted ten years earlier. Deep core walls were constructed adjacent to the abutments on each side of the river, the maximum depth being 113 ft, without lowering the pond level or shutting down the power plant. By the method adopted the core walls were constructed with no difficulties, although they were extended 77 ft below the pond level.

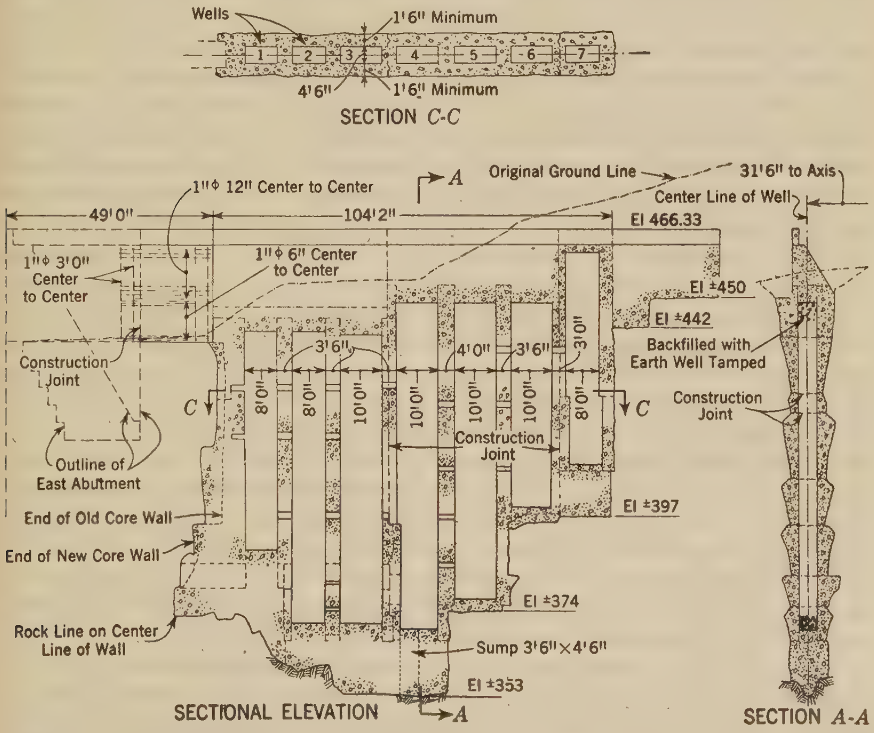
Wells
1'6" Minimum
1
2
3
4
5
6
7
4'6"
1'6" Minimum
SECTION C-C
1"ϕ 12" Center to Center
A
Original Ground Line
31'6" to Axis
Center Line of Well
49'0"
104'2"
El 466.33
1"ϕ 3'0" Center to Center
1"ϕ 6" Center to Center
El ±450
El ±442
Backfilled with Earth Well Tamped
3'6"
4'0"
3'6"
3'0"
Construction Joint
8'0"
8'0"
10'0"
10'0"
10'0"
10'0"
8'0"
C
C
Construction Joint
Construction Joint
Outline of East Abutment
End of Old Core Wall
End of New Core Wall
El ±397
El ±374
Rock Line on Center Line of Wall
Sump 3'6"×4'6"
El ±353
SECTIONAL ELEVATION
A
SECTION A-A

Fig. 32.—Bristol Dam: Plan, Axial Sections, and Vertical Sections, Showing Procedure

In constructing the Bristol core wall, an open trench 14 ft wide and 16 ft deep was first excavated (see Fig. 32). The sides were roughly sloped, so that the cellular masonry section would be partly supported by wedge action when material was excavated below it.

Because of experience gained on the Rodriguez Dam and in view of more favorable materials at the Bristol site, the shorter core wall in the right bank was built as a single block for its entire length, but contained three vertical shafts. The longer core wall behind the left abutment was built in three sections, two with three working shafts apiece and the other containing one. When the cellular concrete core wall was completed, a concrete bottom slab was poured in all shafts, and all were backfilled with spoil from the excavation.

Practically no grout was taken between the vertical and horizontal concrete surfaces of the blocks and very little was taken in the areas outside the walls.

Although no attempt was made to speed up the construction or to set a record of any kind, both core walls were built in 95 days, a demonstration of the advantages of this construction method.

Steel sheet piling could not have been driven to the necessary depth in the Bristol material. Estimates for conventional open-trench methods, including sheeting and pumping, indicated a total over-all cost for the excavation of $10 per cu yd, whereas the final cost on a comparable basis for the method followed was $4.15 per cu yd. One reason for the low cost was the freedom with which the men worked as compared with the caution necessary when working under adverse conditions in a deep heavily braced and shored trench.

The core wall for the Quabbin Dike Dam in Massachusetts, for which twenty-five bids were received based on the pneumatic-caisson method in 1933, affords persuasive comparative data as between the cellular core-wall free-air method (built from the ground surface downward) and the pneumatic-caisson method. Earth excavation through the caissons was estimated at 33,000 cu yd, and the depth averaged about 80 ft. The three lowest bids averaged $7.50 per cu yd; and all twenty-five bids, $12.67. Moreover, the heavy pumping and grouting costs were paid for under other items, and vastly more grouting was involved than with the cellular type of core wall. Rock excavation cost must always be higher because of the necessary use of expensive sand hogs instead of common labor, and concrete costs would also be somewhat higher under the pneumatic method.

The useful field for the method of cellular construction from the surface downward as described in Mr. Hays' paper is not limited to foundation materials of a soft or porous nature such as those at the Kentucky, Kootenay, Rodriguez, or Bristol dams. This method is likewise entirely suitable and surprisingly economical where shattered and seamy rock is to be penetrated, such as is the case at many California and other western sites. Naturally, it cannot be applied where the rock is well bedded or where conventional grouting will tighten the fractured material and restore sound bedding.

In 1930 when construction of the gravity section masonry dam for San Gabriel Site No. 3 was abandoned by the Los Angeles County Flood Control District because excavation showed the rock to be incapable of carrying a dam of that type safely, preliminary designs and estimates were prepared by the writer for an Ambursen type spillway dam, which, because of its relatively low base pressures, was entirely suited to the foundations. However, the problem of supporting and tightening the high rock banks on each side of the river was difficult to solve. The high rock banks were bad in their natural condition, and heavy blasting had further impaired their stability. An abutment was designed for each side of the spillway and three parallel cellular core walls were laid out for each, extending at right angles to the abutment. The upstream and downstream walls were to be sunk only enough in depth and in length to confine and support the shattered and seamy rock as found necessary, and the center core wall on each bank was to be constructed 200 ft down to cut off percolation through the banks. The cost by the method under discussion, starting at the top and working downward to the desired elevations, was found to be very much less than for core walls built in an open trench, which method probably could not have been used at all. As the subsequent choice of the district was for a different site, the cellular design was not projected to the construction stage, but its speed and low cost in poor rock formation were thoroughly established.

Mr. Hays' paper is a very instructive recital of the methods followed in solving a difficult engineering problem. It serves to emphasize that the cellular type of construction for cutoff walls and core walls has proved a practicable and efficient method of solving the problems incidental to providing an adequate water seal for poor foundations. The writer's experience with the aforementioned four projects shows also that this method can be used at low cost. Not only because of this economy, but because of the safety of the method itself, the cellular cutoff wall type of construction, from the ground surface downward, makes available for rapid and economical development many dam sites previously rejected as unsafe or uneconomical because of foundation difficulties. Mr. Hays has also added much knowledge to the scant information available to the engineering profession on the subject of compacting yielding material by grouting. The paper is a fine contribution to a relatively new and little understood field.

James B. Hays,[14] M. Am. Soc. C. E.—As stated by Messrs. Moneymaker and Fox, the preliminary exploration work on Kentucky Dam was not intended to disclose all the details of the underground conditions, but to select the best site from among those available. It was the practice of the Tennessee Valley Authority (TVA) to work out the details after the Construction Department took over from the Planning Division.

Mr. Fox gives an excellent description of the formations and explains how the solution occurred. The deep solution channel or fault shown on the left in Fig. 28 was grouted, but there is no certainty as to how thoroughly this was done. The leakage path through, or along, this channel was very long; hence, it was not considered a serious menace.

When the problem of making a cutoff in the channel at Kentucky Dam arose, the engineers were well aware of the conditions at Hales Bar Dam and realized only too well (as Mr. Black states) that "* * * the conditions * * * at Hales Bar Dam are a good illustration of what could have happened at Kentucky Dam if a cutoff by grouting alone had been considered acceptable."

Mr. Glenn has added more information on the general geology of Kentucky Dam and the surrounding territory. Mr. Burroughs' discussion is a welcome addition since it presents other methods for solving similar problems. It was part of the writer's purpose in presenting the paper to show the results obtained by grouting in the type of materials generally found in badly weathered zones of rock, particularly in limestone. In reply to Mr. Burroughs' question, the 36-in. and 48-in. core holes were all finally filled with concrete.

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[14] New York, N. Y.; formerly Project Mgr., South Holston and Watauga Projects, TVA, Bristol, Tenn.

Fig. 2 shows, in plan, the location of the steel sheet-pile cutoff wall. From about Station 10+00 to the powerhouse wall, it was driven through deep overburden to rock with grout pumped in along the base of the piles to further reduce the leakage. However, across the area between the lock (block R-17) and Station 10+00 (where all overburden had been removed), a concrete trench was built on the rock surface or concrete slab. The trench was filled with an asphaltic material. After the rolled earth fill had been placed to a depth of about 10 ft, steel sheet piling was driven through it into the asphaltic compound.

Mr. Burroughs' point about redrilling the shaft through each block of concrete is well taken and would be reconsidered if the job were to be undertaken again. In concrete that has not set too hard, shot drills penetrate rapidly. If the concrete is too green, the small aggregate is likely to ravel and interfere with the drilling. Previous experience at Chickamauga Dam (TVA) near Chattanooga, Tenn., had indicated the necessity of pregrouting an area similar to the one described herein, before attempting to drill large holes or to sink shafts in which men were expected to work.

When the writer was construction engineer at Chickamauga Dam, a thorough study of the construction methods followed at Hales Bar Dam was made, although at that time TVA did not own Hales Bar Dam. The general conditions were quite well known, and every effort was made to avoid a repetition of the earlier mistakes in building the much larger Chickamauga Dam. The writer has naturally followed Mr. Schmidt's work in repairing the foundation of Hales Bar Dam and offers his congratulations on the novel yet proper solution. The problem at Hales Bar Dam was one of stopping large flows of water with considerable velocity, whereas the problem at Kentucky Dam was one of stopping small flows and negligible velocities.

The writer and his staff at Kentucky Dam benefited greatly by the excellent practical advice of Roger F. Rhoades and Portland P. Fox, Assoc. M. Am. Soc. C. E., and their assistants who constituted the Geological Section on this project.

L. A. SCHMIDT, JR.,[15] M. AM. SOC. C. E.—The solution of a very difficult cutoff problem in the foundation of a dam under construction has been described by Mr. Hays. This solution reveals the ingenuity used to acquire the desired result, and sets this work apart as unique, effective, and unquestionably economical. Compared with the only other practicable method of attack—namely, open excavation and concrete backfill—the treatment accorded the foundation area under Kentucky Dam described by Mr. Hays is so logical that it is difficult to understand how any one could possibly criticize it. The successful use of large-diameter core drills for excavating purposes on this project appears to justify this type of equipment. In any case, the work done at

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[15] Engr., Chattanooga, Tenn.; formerly, Acting Constr. Engr., TVA, Hales Bar Dam, Chattanooga Tenn.

Kentucky Dam has prevented the development of a foundation problem similar to the one that existed at Hales Bar Dam, as aptly stated by Mr. Black.

Certainly Mr. Hays' application of core drills to excavation would have been the logical and surest method of securing the desired results at Hales Bar Dam had that project also been in the construction stage, or had it been possible or desirable to drain the reservoir. The Hales Bar Dam has been in operation since 1913. However, if these procedures had been used on the project they would have been inimical to the best interests of power production during the critical period before Pearl Harbor, and some other method capable of being executed without draining the reservoir was required at Hales Bar Dam. After considerable study the method described in the paper was evolved.

The writer is particularly grateful to Mr. Fox for his presentation of the geological characteristics inherent in the Hales Bar foundation. The part played by faulting in the development of the zones of weakness against leakage in the foundation was probably insufficiently emphasized. Unquestionably, the conditions that either permitted clays and shales from horizontal strata to be washed out by exposure to the differential water pressure, or permitted flows caused by solution of the limestone, were given their initial impetus by faulting as shown in Fig. 13.

The writer subscribes to Mr. Fox's contentions about asphalt, with some reservations. To some extent, Mr. Fox overlooks the effect of temperature on asphalt, and temperature is the relative medium by which the characteristics of asphalt may be judged. In a cavity from 60 to 160 ft below water surface, temperatures of about 56° F are prevalent in the Tennessee River. Thus, when asphalt is placed in the irregular labyrinths (which the channels form) at this low temperature, under sufficient pressure to compensate for shrinkage, it should be capable of long life. Supporting evidence was obtained in the form of asphalt cores removed in drilling, which indicated that asphalt placed twenty years previously had remained in place.

The 1923 placement of asphalt at Hales Bar Dam could have been the final "coup de grâce" to leakage. However, rather too much was expected of asphalt fill at that time since its possibilities had not been exploited to a maximum degree. In addition, an insufficient amount was placed in the widely spaced holes. Had closer spacing been used and had adequate cement grout under pressure been placed for consolidation, it is conceivable that the gains would have been stable.

That asphalt is not as desirable as cement is conceded, but, where cement cannot be placed, asphalt is far more desirable than nothing. Certainly backing up the asphalt with cement is correct and has no definable upper limit.

Mr. Moneymaker principally discusses the part the geologist can play in foundation problems, and the writer agrees with him that the geologist can contribute much in enabling constructors to avoid foundation errors. However, it should be emphasized that the geologist needs cores and data for analytical purposes, and without them his work cannot be effective. The writer wishes to acknowledge the accuracy of the predictions of geologic characteristics made by J. W. Frink, Jun. Am. Soc. C. E., who was stationed at Hales Bar Dam. The location of holes to supplement treatment was determined in many instances from data supplied by Mr. Frink. This type of service, although passive and secondary to the main objective, is often overlooked and Mr. Moneymaker rightly calls attention to its importance.

Final quantities at Hales Bar Dam were:

Description	Quantity
Drilling, in Linear Feet—	
18-in. diameter	34,221
13-in. diameter	7,448
3-in. diameter	55,264
Asbestos-cement liners, in linear feet	30,986
Concrete, in cubic yards	8,653
Reinforcement, in tons	70
Cement grout, in cubic feet	364,361
Asphalt, in cubic feet	135,017

The writer shares the hope expressed by Mr. Glenn that the work done at Hales Bar Dam this time may be more effective and permanent than was the previous work. The visible results at the completion of the job were gratifying. The placement of the final quantities certainly reduced any future potential leakage, and it is doubtful that leakage could again develop to such an extent as it had by 1940. The problem has been of such long standing, however, and the many attempts at its correction have been frustrated so often that the unequivocal assumption that additional maintenance in the future will be unnecessary seems too optimistic. Given careful, orderly inspection and maintenance if and when required, the problem should eventually become history.

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DESIGN CONSTANTS FOR BEAMS WITH NONSYMMETRICAL STRAIGHT HAUNCHES

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WITH DISCUSSION BY MESSRS. ROBERT O. JAMESON, JOHN SHERMAN, DALE H. REA, I. B. HOSIG, AND AUGUST L. AHLF.

SYNOPSIS

Tables are presented in this paper, with outlines of the procedure to be followed in their use, to assist the structural designer in determining the stiffness, carry-over factors, and fixed-end moments for various loadings of beams having straight haunches placed nonsymmetrically with respect to the center of the span. Although this work was performed to determine these factors for nonsymmetrically haunched beams, it is also applicable to beams having straight haunches at only one end and to symmetrical cases as well.

THE METHOD

Hardy Cross,[2] M. Am. Soc. C. E., and others[3] have shown that the column analogy method may be used to obtain the stiffness and carry-over factors and also the fixed-end moments for structural members having either constant or variable cross sections throughout their length.

The properties that must be obtained for the analogous column are the area, the location of the centroidal axis, and the second moment of the area about the centroidal axis. When these have been determined, the stiffness and carry-over factors may be obtained.

To determine the fixed-end moments, it is also necessary to obtain the area under the $\frac{M}{E\,I}$-curve corresponding to the loading and the dimensions of the beam as well as the location of the centroidal axis of the $\frac{M}{E\,I}$-diagram.

When the beam under consideration has haunches at the ends, which haunches are not symmetrical with respect to the center of the span, the

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NOTE.—Published in October, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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[2] "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan, John Wiley & Sons, Inc., New York, N. Y., 1932.

[3] "Theory of Modern Steel Structures," by L. E. Grinter, Vol. 2, Macmillan Co., New York, N. Y., 1937.

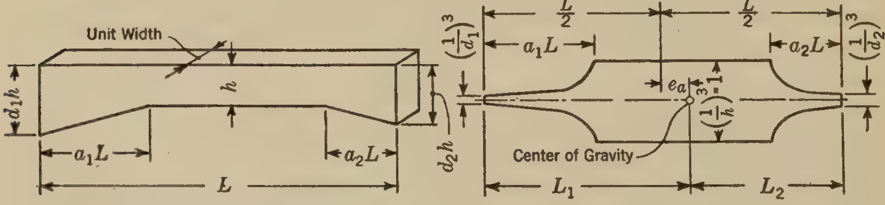
computations required to obtain the properties of the analogous column and its loading diagram may become laborious and time consuming, particularly if accurate results are desired.

The tabular values herein presented have been prepared to facilitate the computations. By their use, the properties of the analogous column and the $\frac{M}{E\,I}$-diagram may be obtained by simple arithmetical combinations.

Notation.—The letter symbols adopted for use in this paper are defined where they first appear, either in the text or by diagrams, and are assembled in an Appendix for convenience of reference.

The Analogous Column

A beam, having straight haunches placed nonsymmetrically with respect to the center of the span, and the corresponding analogous column are shown in Fig. 1. The properties of the analogous column were obtained by integrating the $\frac{1}{E\,I}$-diagram, which was divided into the four parts shown in Fig. 2. The properties of each segment were determined separately and then combined to obtain the desired properties at each end. The total area of the analogous column was found to be



(*a*) BEAM WITH NON-SYMMETRICAL STRAIGHT HAUNCHES

(*b*) ANAGOLOUS COLUMN

Fig. 1.

$$A = \frac{L}{E\,I_o}(C_1 + C_2) \quad \text{.......................(1)}$$

in which I_o is the moment of inertia of the central section of a beam. The dimensionless constants, C_1 and C_2, for this case, are the coefficients pertaining to ends 1 and 2, respectively. These constants (see Table 1(*a*)) are expressed by

$$C = \left(\frac{d+1}{2\,d^2} - 1\right) a + \frac{1}{2} \quad \text{.....................(2)}$$

Eq. 2 is made applicable to either end 1 or 2 by inserting the appropriate subscript on each symbol. This is true also of subsequent equations for C. Table 1(*a*) was prepared to yield values of C for various values of a and d, applying the appropriate subscript.

The total statical moment of the area of the analogous column about the center of the span was found to be

$$Q_I = \frac{L^2}{E\,I_o}(-C_1 + C_2) \quad \text{.....................(3)}$$

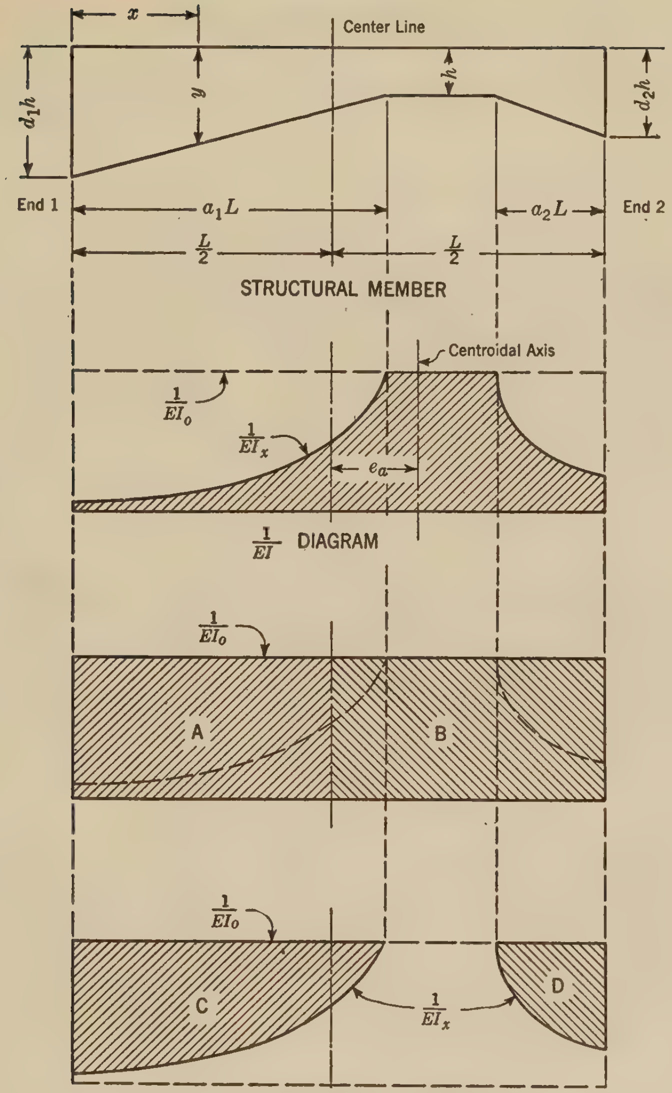

x
Center Line
d_1h
y
h
d_2h
End 1
a_1L
a_2L
End 2
$\frac{L}{2}$
$\frac{L}{2}$
STRUCTURAL MEMBER
Centroidal Axis
$\frac{1}{EI_o}$
$\frac{1}{EI_x}$
e_a
$\frac{1}{EI}$ DIAGRAM
$\frac{1}{EI_o}$
A
B
$\frac{1}{EI_o}$
C
$\frac{1}{EI_x}$
D

Fig. 2.

in which the factor C (with appropriate subscript) is expressed by:

$$C = + \left(-\frac{1}{2} + \frac{1}{4\,d} + \frac{1}{4\,d^2} \right) a - \left(\frac{1}{2\,d} - \frac{1}{2} \right) a^2 + \frac{1}{8} \quad \ldots\ldots\ldots (4)$$

Table 1(*b*) was prepared to yield values of C for various values of a and d in Eq. 4. The total second moment of the area of the analogous column about the center of the span is:

$$Q_{II} = \frac{L^3}{E\,I_o}(C_1 + C_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

TABLE 1.—VALUES OF THE DIMENSIONLESS CONSTANT *C* FOR BEAMS WITH STRAIGHT HAUNCHES

(Apply Subscripts 1 and 2 to Designate Ends 1 and 2, Respectively, of a Beam)

d	VALUES OF *a*:										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
(*a*) AREA OF THE ANALOGOUS COLUMN (EQ. 2)											
1.0	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
1.2	0.5000	0.4764	0.4528	0.4292	0.4056	0.3819	0.3583	0.3347	0.3111	0.2875	0.2639
1.4	0.5000	0.4612	0.4224	0.3837	0.3449	0.3061	0.2673	0.2286	0.1898	0.1510	0.1122
1.6	0.5000	0.4508	0.4016	0.3523	0.3031	0.2539	0.2047	0.1555	0.1063	0.0570	0.0078
1.8	0.5000	0.4432	0.3884	0.3296	0.2728	0.2160	0.1593	0.1025	0.0456	−0.0111	−0.0679
2.0	0.5000	0.4375	0.3750	0.3125	0.2500	0.1875	0.1250	0.0625	0.0000	−0.0625	−0.1250
2.2	0.5000	0.4331	0.3661	0.2992	0.2322	0.1653	0.0983	0.0314	−0.0355	−0.1025	−0.1694
2.4	0.5000	0.4295	0.3590	0.2885	0.2181	0.1476	0.0771	0.0066	−0.0639	−0.1344	−0.2049
2.6	0.5000	0.4266	0.3533	0.2799	0.2065	0.1331	0.0598	−0.0136	−0.0870	−0.1604	−0.2337
2.8	0.5000	0.4242	0.3485	0.2727	0.1969	0.1212	0.0454	−0.0304	−0.1081	−0.1819	−0.2576
3.0	0.5000	0.4222	0.3444	0.2667	0.1889	0.1111	0.0333	−0.0444	−0.1222	−0.2000	−0.2778
4.0	0.5000	0.4156	0.3313	0.2469	0.1625	0.0781	−0.0063	−0.0906	−0.1750	−0.2594	−0.3438
(*b*) FIRST MOMENT OF THE ANALOGOUS COLUMN (EQ. 4)											
1.0	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250
1.2	0.1250	0.1142	0.1047	0.0971	0.0911	0.0868	0.0842	0.0832	0.0839	0.0862	0.0902
1.4	0.1250	0.1070	0.0919	0.0797	0.0703	0.0638	0.0601	0.0593	0.0613	0.0662	0.0740
1.6	0.1250	0.1023	0.0833	0.0681	0.0566	0.0489	0.0448	0.0446	0.0481	0.0554	0.0664
1.8	0.1250	0.0988	0.0771	0.0598	0.0470	0.0386	0.0346	0.0351	0.0400	0.0494	0.0630
2.0	0.1250	0.0962	0.0725	0.0537	0.0400	0.0312	0.0265	0.0287	0.0350	0.0462	0.0625
2.2	0.1250	0.0942	0.0690	0.0491	0.0347	0.0258	0.0224	0.0243	0.0318	0.0447	0.0631
2.4	0.1250	0.0927	0.0662	0.0455	0.0307	0.0217	0.0185	0.0212	0.0298	0.0440	0.0643
2.6	0.1250	0.0914	0.0639	0.0426	0.0275	0.0185	0.0157	0.0190	0.0284	0.0440	0.0657
2.8	0.1250	0.0903	0.0621	0.0403	0.0249	0.0160	0.0134	0.0173	0.0276	0.0445	0.0677
3.0	0.1250	0.0894	0.0605	0.0383	0.0227	0.0139	0.0117	0.0161	0.0272	0.0450	0.0694
4.0	0.1250	0.0866	0.0556	0.0322	0.0162	0.0079	0.0069	0.0135	0.0275	0.0498	0.0781
(*c*) SECOND MOMENT OF THE ANALOGOUS COLUMN (EQ. 6)											
1.0	0.0417	0.0417	0.0417	0.0417	0.0417	0.0417	0.0417	0.0417	0.0417	0.0417	0.0417
1.2	0.0417	0.0365	0.0328	0.0303	0.0286	0.0275	0.0269	0.0263	0.0256	0.0244	0.0226
1.4	0.0417	0.0333	0.0274	0.0234	0.0209	0.0194	0.0185	0.0177	0.0166	0.0147	0.0116
1.6	0.0417	0.0312	0.0238	0.0190	0.0160	0.0143	0.0133	0.0125	0.0111	0.0086	0.0043
1.8	0.0417	0.0296	0.0212	0.0158	0.0126	0.0110	0.0100	0.0091	0.0075	0.0045	−0.0008
2.0	0.0417	0.0285	0.0194	0.0135	0.0102	0.0086	0.0077	0.0068	0.0049	0.0014	−0.0047
2.2	0.0417	0.0275	0.0179	0.0118	0.0084	0.0068	0.0060	0.0050	0.0031	−0.0010	−0.0078
2.4	0.0417	0.0268	0.0169	0.0105	0.0072	0.0056	0.0049	0.0039	0.0017	−0.0027	−0.0101
2.6	0.0417	0.0263	0.0159	0.0096	0.0061	0.0046	0.0040	0.0029	0.0006	−0.0042	−0.0123
2.8	0.0417	0.0258	0.0152	0.0087	0.0053	0.0039	0.0033	0.0023	−0.0002	−0.0052	−0.0138
3.0	0.0417	0.0254	0.0145	0.0081	0.0046	0.0033	0.0027	0.0016	−0.0009	−0.0063	−0.0155
4.0	0.0417	0.0242	0.0127	0.0061	0.0030	0.0017	0.0013	0.0003	−0.0028	−0.0092	−0.0204
(*d*) LOAD CONSTANTS FOR UNIFORM LOADS (EQ. 19)											
1.0	0.0417	0.0417	0.0417	0.0417	0.0417	0.0417	0.0417	0.0417	0.0417	0.0417	0.0417
1.2	0.0417	0.0413	0.0402	0.0385	0.0364	0.0339	0.0313	0.0286	0.0260	0.0232	0.0216
1.4	0.0417	0.0410	0.0391	0.0363	0.0327	0.0286	0.0242	0.0197	0.0154	0.0115	0.0082
1.6	0.0417	0.0408	0.0383	0.0346	0.0299	0.0246	0.0189	0.0132	0.0077	0.0028	−0.0012
1.8	0.0417	0.0406	0.0377	0.0333	0.0278	0.0216	0.0149	0.0083	0.0020	−0.0036	−0.0080
2.0	0.0417	0.0405	0.0372	0.0323	0.0261	0.0192	0.0118	0.0045	−0.0025	−0.0085	−0.0132
2.2	0.0417	0.0404	0.0368	0.0315	0.0249	0.0173	0.0093	0.0014	−0.0060	−0.0133	−0.0173
2.4	0.0417	0.0403	0.0366	0.0309	0.0238	0.0157	0.0073	−0.0010	−0.0088	−0.0154	−0.0204
2.6	0.0417	0.0403	0.0362	0.0303	0.0228	0.0143	0.0055	−0.0032	−0.0112	−0.0180	−0.0231
2.8	0.0417	0.0402	0.0361	0.0297	0.0220	0.0132	0.0041	−0.0048	−0.0131	−0.0201	−0.0252
3.0	0.0417	0.0401	0.0358	0.0293	0.0214	0.0123	0.0029	−0.0064	−0.0148	−0.0219	−0.0270
4.0	0.0417	0.0399	0.0351	0.0279	0.0189	0.0090	−0.0013	−0.0114	−0.0207	−0.0277	−0.0326

TABLE 1.—(*Continued*)

d	VALUES OF a:										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
(*e*) STATICAL MOMENT CONSTANTS FOR UNIFORM LOAD (EQ. 20)											
1.0	0.0078	0.0078	0.0078	0.0078	0.0078	0.0078	0.0078	0.0078	0.0078	0.0078	0.0078
1.2	0.0078	0.0076	0.0072	0.0066	0.0062	0.0058	0.0056	0.0054	0.0056	0.0058	0.0061
1.4	0.0078	0.0075	0.0068	0.0058	0.0051	0.0045	0.0042	0.0040	0.0044	0.0048	0.0053
1.6	0.0078	0.0074	0.0065	0.0053	0.0044	0.0036	0.0032	0.0032	0.0037	0.0043	0.0050
1.8	0.0078	0.0073	0.0063	0.0049	0.0038	0.0030	0.0026	0.0026	0.0033	0.0041	0.0049
2.0	0.0078	0.0073	0.0061	0.0046	0.0034	0.0025	0.0021	0.0022	0.0030	0.0039	0.0049
2.2	0.0078	0.0072	0.0060	0.0043	0.0030	0.0021	0.0018	0.0019	0.0028	0.0039	0.0050
2.4	0.0078	0.0072	0.0058	0.0041	0.0027	0.0018	0.0015	0.0017	0.0028	0.0040	0.0051
2.6	0.0078	0.0072	0.0057	0.0039	0.0025	0.0016	0.0013	0.0016	0.0027	0.0040	0.0052
2.8	0.0078	0.0071	0.0056	0.0038	0.0024	0.0014	0.0011	0.0015	0.0027	0.0041	0.0053
3.0	0.0078	0.0071	0.0056	0.0036	0.0022	0.0012	0.0010	0.0014	0.0027	0.0042	0.0054
4.0	0.0078	0.0070	0.0053	0.0032	0.0017	0.0007	0.0006	0.0013	0.0029	0.0046	0.0060
(*f*) LOAD CONSTANTS FOR TRIANGULAR LOAD WITH SMALL END ADJACENT TO HAUNCH (EQ. 21*a*)											
1.0	0.0365	0.0365	0.0365	0.0365	0.0365	0.0365	0.0365	0.0365	0.0365	0.0365	0.0365
1.2	0.0365	0.0362	0.0353	0.0340	0.0322	0.0301	0.0275	0.0249	0.0225	0.0195	0.0172
1.4	0.0365	0.0360	0.0345	0.0323	0.0290	0.0251	0.0220	0.0172	0.0132	0.0087	0.0052
1.6	0.0365	0.0358	0.0340	0.0310	0.0266	0.0214	0.0164	0.0110	0.0060	0.0010	−0.0044
1.8	0.0365	0.0357	0.0336	0.0300	0.0250	0.0190	0.0130	0.0068	0.0010	−0.0055	−0.0111
2.0	0.0365	0.0356	0.0332	0.0292	0.0235	0.0172	0.0102	0.0032	−0.0035	−0.0111	−0.0165
2.2	0.0365	0.0355	0.0329	0.0287	0.0225	0.0158	0.0080	0.0002	−0.0080	−0.0155	−0.0207
2.4	0.0365	0.0355	0.0327	0.0280	0.0216	0.0144	0.0058	−0.0022	−0.0110	−0.0190	−0.0241
2.6	0.0365	0.0354	0.0324	0.0275	0.0209	0.0133	0.0042	−0.0044	−0.0132	−0.0219	−0.0269
2.8	0.0365	0.0354	0.0323	0.0270	0.0202	0.0123	0.0030	−0.0060	−0.0150	−0.0234	−0.0290
3.0	0.0365	0.0353	0.0321	0.0268	0.0196	0.0115	0.0018	−0.0074	−0.0165	−0.0247	−0.0306
4.0	0.0365	0.0352	0.0315	0.0256	0.0175	0.0084	−0.0020	−0.0123	−0.0220	−0.0307	−0.0366
(*g*) STATICAL MOMENT CONSTANTS FOR TRIANGULAR LOAD WITH SMALL END ADJACENT TO HAUNCH (EQ. 22*a*)											
1.0	0.0064	0.0064	0.0064	0.0064	0.0064	0.0064	0.0064	0.0064	0.0064	0.0064	0.0064
1.2	0.0064	0.0063	0.0060	0.0056	0.0052	0.0047	0.0046	0.0047	0.0048	0.0049	0.0051
1.4	0.0064	0.0062	0.0056	0.0050	0.0044	0.0038	0.0034	0.0036	0.0039	0.0042	0.0048
1.6	0.0064	0.0061	0.0053	0.0046	0.0038	0.0031	0.0025	0.0028	0.0033	0.0039	0.0048
1.8	0.0064	0.0061	0.0050	0.0043	0.0034	0.0026	0.0020	0.0023	0.0031	0.0039	0.0049
2.0	0.0064	0.0060	0.0048	0.0040	0.0031	0.0022	0.0016	0.0020	0.0030	0.0040	0.0052
2.2	0.0064	0.0060	0.0047	0.0038	0.0028	0.0018	0.0013	0.0017	0.0030	0.0042	0.0055
2.4	0.0064	0.0060	0.0045	0.0036	0.0026	0.0016	0.0011	0.0016	0.0030	0.0044	0.0057
2.6	0.0064	0.0059	0.0044	0.0035	0.0024	0.0014	0.0010	0.0015	0.0030	0.0045	0.0058
2.8	0.0064	0.0059	0.0043	0.0034	0.0023	0.0012	0.0009	0.0015	0.0030	0.0046	0.0060
3.0	0.0064	0.0059	0.0043	0.0033	0.0021	0.0011	0.0008	0.0015	0.0030	0.0047	0.0062
4.0	0.0064	0.0059	0.0041	0.0030	0.0018	0.0007	0.0004	0.0013	0.0030	0.0049	0.0069
(*h*) LOAD CONSTANTS FOR TRIANGULAR LOAD WITH LARGE END ADJACENT TO HAUNCH (EQ. 21*b*)											
1.0	0.0469	0.0469	0.0469	0.0469	0.0469	0.0469	0.0469	0.0469	0.0469	0.0469	0.0469
1.2	0.0469	0.0464	0.0451	0.0430	0.0406	0.0377	0.0351	0.0323	0.0295	0.0269	0.0260
1.4	0.0469	0.0460	0.0437	0.0403	0.0364	0.0321	0.0264	0.0222	0.0176	0.0143	0.0112
1.6	0.0469	0.0458	0.0426	0.0382	0.0332	0.0278	0.0214	0.0154	0.0094	0.0046	0.0020
1.8	0.0469	0.0455	0.0418	0.0366	0.0306	0.0242	0.0168	0.0098	0.0030	−0.0017	−0.0049
2.0	0.0469	0.0454	0.0412	0.0354	0.0287	0.0212	0.0134	0.0058	−0.0015	−0.0059	−0.0099
2.2	0.0469	0.0453	0.0407	0.0343	0.0273	0.0188	0.0106	0.0026	−0.0040	−0.0090	−0.0139
2.4	0.0469	0.0452	0.0405	0.0338	0.0260	0.0170	0.0088	0.0002	−0.0066	−0.0118	−0.0167
2.6	0.0469	0.0451	0.0400	0.0331	0.0247	0.0153	0.0068	−0.0020	−0.0092	−0.0141	−0.0193
2.8	0.0469	0.0450	0.0399	0.0324	0.0238	0.0141	0.0052	−0.0036	−0.0112	−0.0168	−0.0214
3.0	0.0469	0.0449	0.0395	0.0318	0.0232	0.0131	0.0040	−0.0054	−0.0131	−0.0191	−0.0234
4.0	0.0469	0.0446	0.0387	0.0302	0.0203	0.0096	−0.0016	−0.0105	−0.0194	−0.0247	−0.0286

TABLE 1.—(*Continued*)

d	Values of a:										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
(*i*) STATICAL MOMENT CONSTANTS FOR TRIANGULAR LOAD WITH LARGE END ADJACENT TO HAUNCH (EQ. 22*b*)											
1.0	0.0092	0.0092	0.0092	0.0092	0.0092	0.0092	0.0092	0.0092	0.0092	0.0092	0.0092
1.2	0.0092	0.0090	0.0084	0.0079	0.0072	0.0068	0.0066	0.0065	0.0066	0.0068	0.0070
1.4	0.0092	0.0088	0.0079	0.0070	0.0060	0.0053	0.0050	0.0049	0.0052	0.0055	0.0058
1.6	0.0092	0.0087	0.0075	0.0063	0.0052	0.0042	0.0039	0.0038	0.0041	0.0047	0.0052
1.8	0.0092	0.0086	0.0071	0.0058	0.0045	0.0034	0.0031	0.0029	0.0035	0.0042	0.0048
2.0	0.0092	0.0085	0.0069	0.0053	0.0040	0.0028	0.0025	0.0025	0.0031	0.0039	0.0046
2.2	0.0092	0.0084	0.0067	0.0050	0.0036	0.0024	0.0022	0.0021	0.0028	0.0038	0.0045
2.4	0.0092	0.0084	0.0066	0.0047	0.0033	0.0021	0.0019	0.0018	0.0026	0.0036	0.0045
2.6	0.0092	0.0083	0.0065	0.0045	0.0031	0.0019	0.0017	0.0017	0.0025	0.0035	0.0045
2.8	0.0092	0.0083	0.0064	0.0044	0.0029	0.0017	0.0015	0.0016	0.0024	0.0034	0.0045
3.0	0.0092	0.0083	0.0063	0.0042	0.0028	0.0015	0.0013	0.0015	0.0023	0.0033	0.0046
4.0	0.0092	0.0082	0.0060	0.0037	0.0022	0.0007	0.0003	0.0010	0.0021	0.0032	0.0051

in which the factor C (with appropriate subscript) is expressed by:

$$C = \frac{1}{24} + \left[\left(\frac{1}{d-1} \right)^3 \left(\log_e d + \frac{2}{d} - \frac{1}{2\,d^2} - \frac{3}{2} \right) + \left(\frac{d-1}{2\,d^2} - \frac{1}{3} \right) \right] a^3 + \left(\frac{1}{2} - \frac{1}{2\,d} \right) a^2 + \left(\frac{d+1}{8\,d^2} - \frac{1}{4} \right) a \qquad (6)$$

Table 1(*c*) was prepared to yield values of C for various values of a and d in Eq. 6.

The location of the centroidal axis of the total area of the analogous column from the center of the span may be obtained by the relation,

$$e_a = \frac{Q_{\mathrm{I}}}{A} \qquad (7)$$

The eccentric distance e_a will be positive if the centroidal axis is toward end 2 from the center of the span and negative if it is toward end 1. The longer distance from the centroidal axis to the end of a beam (end 1, Fig. 2) is expressed by:

$$L_1 = -\frac{L}{2} - e_a \qquad (8a)$$

Similarly the remaining distance from the centroidal axis to the end of a beam (see end 2, Fig. 2) is

$$L_2 = \frac{L}{2} - e_a \qquad (8b)$$

If values of L_1 are always considered to be negative and values of L_2 are always considered to be positive, the signs in the column formula will follow automatically.

The total second moment Q_a of the area of the analogous column, taken about the centroidal axis, is

$$Q_a = Q_{\mathrm{II}} - e_a\,Q_{\mathrm{I}} \qquad (9)$$

STIFFNESS FACTORS

The "absolute stiffness factor" is defined herein as that bending moment required to rotate the hinged end of a beam through an angle of one radian while the opposite end remains fixed. It may be shown that the absolute stiffness factor for a member having a uniform cross section throughout its length is equal to $\frac{4\,E\,I}{L}$. However, if the member is not prismatic, but has a variable moment of inertia along its length, the coefficient of the absolute stiffness will be something other than 4. This factor will be different for the two ends unless the haunches at the two ends have the same dimensions.

When the modulus of elasticity, E, of the several members of the frame is the same and each of the members is a prismatic one, the quantity $4\,E$ will be constant and, therefore, can be eliminated from the computations. For this reason it has become common practice to regard the stiffness of a member as being equal to I/L. Herein this factor is called the relative stiffness factor. The relative stiffness factor may be obtained by dividing the absolute stiffness factor by $4\,E$.

When using the analogous column, the rotation of the structural member must be represented as a reaction or load and the bending moments are represented as fiber stresses. The stiffness factor at either end may be found by applying a unit load at that end and computing the extreme fiber stress at the corresponding end by the column formula,

$$f = \frac{P}{A} \pm \frac{P\,e\,C}{Q_a} \quad \text{(10)}$$

The load applied in this instance represents the unit load, P, of one radian; the fiber stress, f, represents the absolute stiffness factor; and the distance, C, is the distance from the center of gravity of the analogous column to the end of the beam where the stiffness factor is desired. The absolute stiffness at end 1 is, then:

$$K_1 = \frac{1}{A} + \frac{L^2_1}{Q_a} \quad \text{(11a)}$$

The relative stiffness at end 1 is

$$k_1 = \frac{K_1}{4\,E} \quad \text{(11b)}$$

Similarly, the absolute stiffness at end 2 is

$$K_2 = \frac{1}{A} + \frac{L^2_2}{Q_a} \quad \text{(12a)}$$

and the relative stiffness at end 2 is

$$k_2 = \frac{K_2}{4\,E} \quad \text{(12b)}$$

CARRY-OVER FACTORS

If the end of a member, which is upon unyielding supports at both ends, is rotated while the other end is held fixed, a moment will be induced into the fixed end. The induced moment is defined as the "carry-over moment," and that proportion of the applied moment which is carried over is defined as the "carry-over factor."

The "carry-over moment" induced at the far end of the member may also be computed by the column formula:

$$N = \frac{1}{A} + \frac{L_1 L_2}{Q_a} \quad \text{(13)}$$

The carry-over factor at end 1 may be obtained from:

$$r_1 = \frac{N}{K_1} \quad \text{(14a)}$$

and the carry-over factor at end 2 may be obtained from:

$$r_2 = \frac{N}{K_2} \quad \text{(14b)}$$

FIXED-END MOMENTS

The magnitude and point of application of the load placed on the analogous column to determine the fixed-end moment were obtained by integrating the $\frac{M}{E\,I}$-diagram shown in Fig. 3. The diagram was divided into the four parts shown. The load and statical moment about the center of the span were obtained for each part separately, and then combined to obtain the load and statical moment of the two ends. The load representing the total $\frac{M}{E\,I}$-diagram was found to be

$$P = \frac{W\,L^2}{E\,I_o}(C_1 + C_2) \quad \text{(15)}$$

The equations representing the values of these constants for the various types of loading are given subsequently under the discussion of the particular type of loading. Tables 1(*d*), 1(*f*), and 1(*h*) contain values of C for various values of a and d, and for several of the more common loadings, in the application of Eq. 15.

The following was found to represent the statical moment of the total $\frac{M}{E\,I}$-diagram about the center of the span:

$$Q_f = \frac{W\,L^3}{E\,I_o}(-\,C_1 + C_2) \quad \text{(16)}$$

The equations representing the values of these constants for the various types of loading are given subsequently under the discussion of the particular type of loading.

Values of C in Eq. 16, for various values of a and d for several of the more common types of loading, are given in Tables 1(*e*), 1(*g*), and 1(*i*).

The location of the centroidal axis of the total $\frac{M}{E\,I}$-diagram from the center

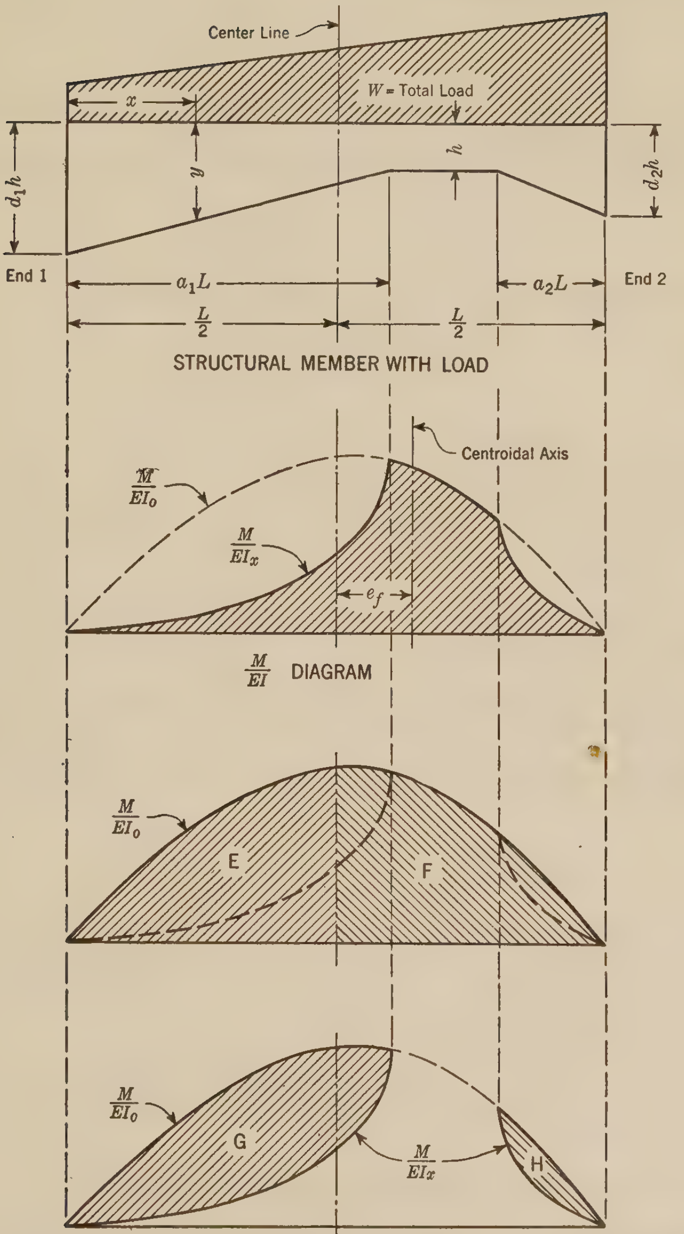

Center Line
W = Total Load
x
y
h
d_1h
d_2h
End 1
End 2
a_1L
a_2L
$\frac{L}{2}$
$\frac{L}{2}$
STRUCTURAL MEMBER WITH LOAD
Centroidal Axis
$\frac{M}{EI_0}$
$\frac{M}{EI_x}$
e_f
$\frac{M}{EI}$ DIAGRAM
$\frac{M}{EI_0}$
E
F
$\frac{M}{EI_0}$
G
$\frac{M}{EI_x}$
H

Fig. 3.

of the span may be obtained by

$$e_f = \frac{Q_f}{P} \qquad \text{(17)}$$

The value of e_f will be positive if the centroidal axis is toward end 2 from the center of the span and negative if it is toward end 1.

The fixed-end moment at end 1 may be computed from the column formula:

$$\text{FEM} = \frac{P}{A} + \frac{(e_f - e_a)\,P\,L_1}{Q_a} \qquad \text{(18}a\text{)}$$

Similarly, the fixed-end moment at end 2 may be computed by:

$$\text{FEM} = \frac{P}{A} + \frac{(e_f - e_a)\,P\,L_2}{Q_a} \qquad \text{(18}b\text{)}$$

Fixed-End Moment with Uniform Load

The value of the constant C necessary for determining the area of the $\frac{M}{E\,I}$-diagram for the case of a uniform load (Table 1(d)) was found to be

$$C = \frac{1}{24} + \left[\frac{\frac{d}{2} - 1 + \frac{1}{2\,d}}{2\,(1-d)^2} - \frac{1}{4}\right] a^2 + \left[\frac{1}{6} - \frac{2\,d - \frac{d^2}{2} - \frac{3}{2} - \log_e d}{2\,(1-d)^3}\right] a^3 \quad \text{(19)}$$

with subscripts 1 and 2, for symbols C, d, and a, to denote the appropriate end of the span. Similarly (see Table 1(e)), to find the statical moment of the $\frac{M}{E\,I}$-diagram for uniform load:

$$C = -\frac{1}{128} + \left[\frac{1}{8} - \frac{\frac{d}{2} - 1 + \frac{1}{2\,d}}{4\,(1-d)^2}\right] a^2 + \left[-\frac{1}{4} + \frac{-\frac{3\,d^2}{2} + 6\,d - \frac{9}{2} - 3\log_e d}{4\,(1-d)^3}\right] a^3 + \left[\frac{1}{8} - \frac{1 + \frac{3\,d}{2} - 3\,d^2 + \frac{d^3}{2} + 3\,d\log_e d}{2\,(1-d)^4}\right] a^4 \quad \text{(20)}$$

Fixed-End Moments for Triangular Loads

The constants C were found to have different values when the small end of the triangular load was at the end farthest from the centroidal axis (see end 1, Fig. 1) than when the larger end was so placed. When the small end of the triangular load was at end 1, the constants were found to be as follows:

For the area of the $\frac{M}{EI}$-diagram (see Table 1(f))—

$$\begin{aligned} C_1 = {} & \left[\frac{1}{3\,(1-d)^2} \left(0.5\,d - 1 + \frac{0.5}{d} \right) - \frac{1}{6} \right] a^2 - \left[\frac{1}{3\,(1-d)^4} (1 + 3\,d \log_e d \right. \\ & \left. + 1.5\,d - 3\,d^2 + 0.5\,d^3) - \frac{1}{12} \right] a^4 + \frac{7}{192} \qquad \text{(21}a\text{)} \end{aligned}$$

and (see Table 1(h))—

$$\begin{aligned} C_2 = {} & \left[\frac{2}{3\,(1-d)^2} \left(0.5\,d - 1 + \frac{0.5}{d} \right) - \frac{1}{3} \right] a^2 \\ & + \left[\frac{1}{(1-d)^3} (0.5\,d^2 - 2\,d + 1.5 + \log_e d) + \frac{1}{3} \right] a^3 \\ & + \left[\frac{1}{3\,(1-d)^4} (1 + 3\,d \log_e d + 1.5\,d \right. \\ & \left. - 3\,d^2 + 0.5\,d^3) - \frac{1}{12} \right] a^4 + \frac{3}{64} \qquad \text{(21}b\text{)} \end{aligned}$$

For the statical moment of the $\frac{M}{EI}$-diagram (see Table 1(g))—

$$\begin{aligned} C_1 = {} & \left[\frac{1}{6\,(1-d)^2} \left(0.5\,d - 1 + \frac{0.5}{d} \right) - \frac{1}{12} \right] a^2 \\ & + \left[\frac{1}{3\,(1-d)^3} (0.5\,d^2 - 2\,d + 1.5 + \log_e d) + \frac{1}{9} \right] a^3 \\ & - \left[\frac{1}{6\,(1-d)^4} (1 + 3\,d \log_e d + 1.5\,d - 3\,d^2 + 0.5\,d^3) - \frac{1}{24} \right] a^4 \\ & + \left[\frac{1}{3\,(1-d)^5} (0.5 - 4\,d + 4\,d^3 - 0.5\,d^4 \right. \\ & \left. - 6\,d^2 \log_e d) - \frac{1}{15} \right] a^5 + \frac{37}{5{,}760} \qquad \text{(22}a\text{)} \end{aligned}$$

and (see Table 1(i))—

$$\begin{aligned} C_2 = {} & \left[\frac{1}{3\,(1-d)^2} \left(0.5\,d - 1 + \frac{0.5}{d} \right) - \frac{1}{6} \right] a^2 \\ & + \left[\frac{7}{6\,(1-d)^3} (0.5\,d - 2\,d + 1.5 + \log_e d) + \frac{7}{18} \right] a^3 \\ & + \left[\frac{7}{6\,(1-d)^4} (1 + 3\,d \log_e d + 1.5\,d - 3\,d^2 + 0.5\,d^3) - \frac{7}{24} \right] a^4 \\ & - \left[\frac{1}{3\,(1-d)^5} (0.5 - 4\,d + 4\,d^3 - 0.5\,d^4 \right. \\ & \left. - 6\,d^2 \log_e d) - \frac{1}{15} \right] a^5 + \frac{53}{5{,}760} \qquad \text{(22}b\text{)} \end{aligned}$$

Symbols a and d carry the subscript 1 in Eqs. 21*a* and 22*a* and the subscript 2 in Eqs. 21*b* and 22*b*. These subscripts are omitted from Eqs. 21 and 22 to simplify the typography. Eqs. 21 and 22 can be adapted for the case when the large end of the triangular load is at end 1 by reversing the subscripts. Constants C_1 and C_2, then, for the area of the $\frac{M}{E\,I}$-diagram are read from Tables 1(*h*) and 1(*f*), respectively; and for the statical moment of the $\frac{M}{E\,I}$-diagram from Tables 1(*g*) and 1(*i*), respectively.

EXAMPLE 1

Find the stiffness and carry-over factors for the beam shown in Fig. 4.

Given: $L = 25.00$ ft; $a_1 L = 5.00$ ft; $a_2 L = 2.50$ ft; $h = 1.50$ ft; $d_1 h = 3.30$ ft; and $d_2 h = 3.00$ ft.

Calculate: $a_1 = \frac{5.00}{25.00} = 0.20$; $a_2 = \frac{2.50}{25.00} = 0.10$; $d_1 = \frac{3.30}{1.50} = 2.20$; and $d_2 = \frac{3.00}{1.50} = 2.0$.

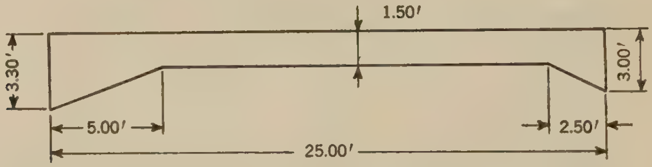


FIG. 4.

Area of Analogous Column.—From Table 1(*a*), using a_1 and d_1 to find C_1, and a_2 and d_2 to find C_2—$C_1 + C_2 = 0.3661 + 0.4375 = 0.8036$. Substituting this value in Eq. 1,

$$A = 0.8036 \frac{L}{E\,I_o} \qquad (23)$$

Centroid of Analogous Column.—From Table 1(*b*), using a_1 and d_1 to find C_1, and a_2 and d_2 to find C_2—$-C_1 + C_2 = -0.0690 + 0.0962 = 0.0272$. Substituting this value in Eq. 3,

$$Q_{\mathrm{I}} = 0.0272 \frac{L^2}{E\,I_o} \qquad (24)$$

By Eq. 7,

$$e_a = \frac{0.0272\,L^2}{E\,I_o} \div \frac{0.8036\,L}{E\,I_o} = 0.0338\,L \qquad (25a)$$

$$L_1 = -\frac{L}{2} - e_a = -0.5338\,L \qquad (25b)$$

and

$$L_2 = \frac{L}{2} - e_a = 0.4662\,L \qquad (25c)$$

Second Moment of Analogous Column.—From Table 1(*c*), using a_1 and d to find C_1, and a_2 and d_2 to find C_2—$C_1 + C_2 = 0.0179 + 0.0285 = 0.0464$. Substituting this value in Eq. 5,

$$Q_{II} = 0.0464 \frac{L^3}{E I_o} \qquad (26a)$$

and (Eq. 9)

$$Q_a = (0.0464 - 0.0009) \frac{L^3}{E I_o} = 0.0455 \frac{L^3}{E I_o} \qquad (26b)$$

Absolute Stiffness of End 1.—Substituting appropriate values in Eq. 11*a*,

$$K_1 = \frac{1.2444\, E\, I_o}{L} + \frac{6.2625\, E\, I_o}{L} = \frac{7.5069\, E\, I_o}{L} \qquad (27)$$

Relative Stiffness of End 1.—By Eq. 11*b*,

$$k_1 = \frac{7.5069\, E\, I_o}{L \times 4\, E} = \frac{1.8767\, I_o}{L} \qquad (28a)$$

Absolute Stiffness of End 2.—Substituting appropriate values in Eq. 12*a*,

$$K_2 = \frac{1.2444\, E\, I_o}{L} + \frac{4.7768\, E\, I_o}{L} = \frac{6.0212\, E\, I_o}{L} \qquad (28b)$$

Relative Stiffness of End 2.—Substituting appropriate quantities in Eq. 12*b*,

$$k_2 = \frac{6.0212\, E\, I_o}{L \times 4\, E} = \frac{1.5053\, I_o}{L} \qquad (28c)$$

Absolute Carry-Over Moment.—By Eq. 13,

$$N = \frac{1.2444\, E\, I_o}{L} + \frac{-5.4694\, E\, I_o}{L} = \frac{-4.2250\, E\, I_o}{L} \qquad (29)$$

Carry-Over Factors.—The carry-over factor away from end 1, by Eq. 14*a*, is $\frac{-4.2250\, E\, I_o}{L} \div \frac{7.5069\, E\, I_o}{L} = -0.563$; and, from end 2 (see Eq. 14*b*), is $\frac{-4.2250\, E\, I_o}{L} \div \frac{6.0212\, E\, I_o}{L} = -0.702$.

Example 2

Find the fixed-end moments for a uniform load totaling W pounds for the beam shown in Fig. 4. (Values for A, e_a, L_1, L_2, and Q_a are as taken from Example 1.)

Load on the Analogous Column.—From Table 1(*d*), using a_1 and d_1 to find C_1, and a_2 and d_2 to find C_2—$C_1 + C_2 = 0.0368 + 0.0405 = 0.0773$. Substituting this value in Eq. 15,

$$P = 0.0773 \frac{W L^2}{E I_o} \qquad (30)$$

Statical Moment of Load.—From Table 1(e), using a_1 and d_1 to find C_1, and a_2 and d_2 to find C_2—$-C_1 + C_2 = -0.0060 + 0.0073 = 0.0013$. Substituting this quantity in Eq. 16,

$$Q_f = \frac{0.0013\,W\,L^3}{E\,I_o} \tag{31}$$

Then Eq. 17 yields

$$e_f = \frac{0.0013\,W\,L^3}{E\,I_o} \div \frac{0.0773\,W\,L^2}{E\,I_o} = 0.0168\,L \tag{32a}$$

Finally,

$$e_f - e_a = (0.0168 - 0.0338)\,L = -0.0170\,L \tag{32b}$$

Fixed-End Moments.—Applying Eq. 18a to determine the fixed-end moment at end 1,

$$\begin{aligned} \text{FEM} &= \frac{0.0773\,W\,L^2}{E\,I_o} \div \frac{0.8036\,L}{E\,I_o} \\ &\quad + \frac{0.0773 \times -0.0170}{0.0455}(-0.5338)\,W\,L = 0.1116\,W\,L \end{aligned} \tag{33a}$$

Similarly, applying Eq. 18b to determine the fixed-end moment at end 2,

$$\text{FEM} = 0.0963\,W\,L + \frac{0.0773 \times -0.0170}{0.0455}\,0.4662\,W\,L = 0.0828\,W\,L. \tag{33b}$$

EXAMPLE 3a

Find the fixed-end moment for a triangular load totaling W pounds, having the small end of the load at end 1, for the beam shown in Fig. 4. (Values of A, e_a, L_1, L_2, and Q_a are as taken from Example 1.)

Load on the Analogous Column.—From Table 1(f), using a_1 and d_1 to find C_1; and from Table 1(h), using a_2 and d_2 to find C_2—$C_1 + C_2 = 0.0329 + 0.0454 = 0.0783$. With these factors known, Eq. 15 becomes

$$P = \frac{0.0783\,W\,L^2}{E\,I_o} \tag{34}$$

Statical Moment of Load.—From Table 1(g), using a_1 and d_1 to find C_1; and from Table 1(i), using a_2 and d_2 to find C_2—$-C_1 + C_2 = -0.0047 + 0.0085 = 0.0038$. With these factors known, Eq. 16 becomes

$$Q_f = \frac{0.0038\,W\,L^3}{E\,I_o} \tag{35}$$

Then, by Eq. 17,

$$e_f = \frac{0.0038\,W\,L^3}{E\,I_o} \div \frac{0.0783\,W\,L^2}{E\,I_o} = 0.0485\,L \tag{36a}$$

and

$$e_f - e_a = (0.0485 - 0.0338)\,L = 0.0147\,L \tag{36b}$$

Fixed-End Moments.—Applying Eq. 18a to determine the fixed-end moment at end 1,

$$\text{FEM} = \frac{0.0783\, W\, L^2}{E\, I_o} \div \frac{0.8036\, L}{E\, I_o}$$

$$+ \left(\frac{0.0783 \times 0.0147}{0.0455}\right)(-\,0.5338)\, W\, L = 0.0839\, W\, L \ldots\ldots (37a)$$

Similarly, applying Eq. 18b to determine the fixed-end moment at end 2,

$$\text{FEM} = 0.0974\, W\, L + \frac{0.0783 \times 0.0147}{0.0455}\, 0.4662\, W\, L = 0.1092\, W\, L \,..\, (37b)$$

EXAMPLE 3b

Find the fixed-end moment for a triangular load totaling W pounds, having the large end of the load at end 1, for the beam shown in Fig. 4. (Values of A, e_a, L_1, L_2, and Q_a are as taken from Example 1.)

Load on the Analogous Column.—From Table 1(h), using a_1 and d_1 to find C_1; and from Table 1(f), using a_2 and d_2 to find C_2—$C_1 + C_2 = 0.0407 + 0.0356 = 0.0763$. Eq. 15 then yields:

$$P = \frac{0.0763\, W\, L^2}{E\, I_o} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

Statical Moment of Load.—From Table 1(i), using a_1 and d_1 to find C_1; and from Table 1(g), using a_2 and d_2 to find C_2—$-\,C_1 + C_2 = -\,0.0067 + 0.0060 = -\,0.0007$. With these factors known, Eq. 16 becomes

$$Q_f = \frac{-\,0.0007\, W\, L^3}{E\, I_o} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

Then, as in Example 3a, Eq. 17 yields

$$e_f = \frac{-\,0.0007\, W\, L^3}{E\, I_o} \div \frac{0.0763\, W\, L^2}{E\, I_o} = -\,0.0092\, L \ldots\ldots (40a)$$

and

$$e_f - e_a = (-\,0.0092 - 0.0338)\, L = -\,0.0430\, L \ldots\ldots\ldots (40b)$$

Fixed-End Moments.—Applying Eq. 18a to determine the fixed-end moment at end 1,

$$\text{FEM} = \frac{0.0763\, W\, L^2}{E\, I_o} \div \frac{0.08036\, L}{E\, I_o}$$

$$+ \frac{(0.0763 \times -\,0.0430)}{(0.0455)}(-\,0.05338)\, W\, L = 0.1334\, W\, L \ldots\ldots (41a)$$

Similarly, applying Eq. 18b to determine the fixed-end moment at end 2,

$$\text{FEM} = 0.0949\, W\, L + \frac{0.0763 \times -\,0.0430}{0.0455}$$

$$\times (-\,0.5338)\, W\, L = 0.0613\, W\, L \ldots\ldots\ldots\ldots (41b)$$

SUMMARY

Published literature and texts present charts and tables whereby the design constants may be readily obtainable for structural members which have a haunch at only one end or which have haunches at each end that are symmetrical about the center of the span. Little information is available regarding these constants for beams which are nonsymmetrically haunched.

The usual method of obtaining the design constants for the nonsymmetrical case is laborious and time consuming.

This paper is presented to assist the engineer in obtaining the stiffness and carry-over factors for beams having nonsymmetrical straight haunches, and the fixed-end moments for uniform and triangular loads. The results were obtained by the use of the well-known column analogy. The labor required to compute the area, centroid, and moment of inertia of the analogous column, and the area and centroid of the $\frac{M}{E\,I}$-diagram has here been simplified so that they may be obtained by simple arithmetical combination of the constants presented in the tables. Examples have been included to illustrate the use of the tables and the procedure to be followed in obtaining the design constants.

The tables have not been extended to cover other loading conditions because of the great amount of work involved. The two cases covered are those most commonly encountered and therefore the most important loading conditions to be considered. The tables can be enlarged to cover any desired loading condition and the procedure would be equally adaptable.

ACKNOWLEDGMENT

The primary source from which the material was assembled which deals with the analogous column and the calculation of the stiffness and carry-over factors was an academic thesis by the writer, entitled "Stiffness and Carry-Over Factors for Beams with Nonsymmetrical Straight Haunches," submitted to the University of Colorado, in Boulder, in 1939, in partial fulfilment of the requirements for the Degree of Master of Science in Civil Engineering. The part of the paper pertaining to the fixed-end moments has been prepared since that date.

APPENDIX

NOTATION

The following letter symbols, used in the paper, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials,[4] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932:

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[4] ASA—Z10a—1932.

A = area; total area of an analogous column;

a = ratio of the total span length L (Fig. 1):

$a_1 = a$ for distances $a_1 L$ measured from end 1;

$a_2 = a$ for distances $a_2 L$ measured from end 2;

C = a dimensionless constant selected from Table 1 to suit various conditions of a problem, the subscripts 1 and 2 designating the end of the beam; in Eq. 10 it is the distance from the center of gravity of the column section to the point at which the fiber stress is computed;

d = coefficient of depth h to indicate total depth of haunch:

$d_1 = d$ for depths $d_1 h$ measured at end 1;

$d_2 = d$ for depths $d_2 h$ measured at end 2;

E = modulus of elasticity of the material;

e = eccentricity = distance from the center of the span to the center of elastic weights (in Eq. 10 it is the eccentricity of the applied load P):

$e_a = e$ for an analogous column;

$e_f = e$ for the problem of fixed-end moments;

f = the fiber stress in the column formula, Eq. 10;

h = uniform depth of a beam without haunches and the minimum depth of a beam with haunches;

I = moment of inertia:

I_o = inertia moment of the minimum section of a beam;

I_x = inertia moment at a distance x along a haunch (Figs. 2 and 3);

K = absolute stiffness factor (Eqs. 11*a* and 12*a*) with subscripts 1 and 2 denoting the end of the beam;

k = relative stiffness factor (Eqs. 11*b* and 12*b*) with subscripts denoting the end of the beam;

L = length of a beam:

L_1 = part of a beam to the left (end 1) of the gravity center of elastic weights (Fig. 2);

L_2 = part of a beam to the right (end 2) of the gravity center of elastic weights;

M = bending moment;

N = a carry-over moment induced at the far end of a member;

P = a load applied to an analogous column;

Q = statical moment of an area;

Q_{I} = first moment of the area of the analogous column about the center of the span (Eq. 3);

Q_{II} = second moment of the area of the analogous column about the center of the span (Eq. 5);

Q_a = total second moment of area of analogous column about the gravity axis (Eq. 9);

Q_f = statical moment, about the center of the span, of the load acting on an analogous column used in determining fixed-end moments (Eq. 16);

r = a carry-over factor (Eqs. 14), subscripts 1 and 2 being used to designate the ends;

W = total load on beam (Fig. 3);

x = variable distance from end 1 of a beam to a place on the haunch where the depth is y (Fig. 3);

y = depth of the haunch at any distance x from the left end of a beam (Fig. 3).

DISCUSSION

ROBERT O. JAMESON,[5] M. AM. SOC. C. E.—Evidently, considerable labor was involved in the preparation of this paper, and the author is to be congratulated for devising an ingenious method of solving a troublesome problem. A valuable addition to this paper would be the inclusion of data for calculating fixed-end moments due to a concentrated load at any point on a span; however, this would be a huge task. Such data could be presented on several charts showing influence lines for various combinations of haunch lengths. Comprehensive charts have been published,[6] showing properties of beams with parabolic haunches whose lengths are equal to one half of the span and with various ratios of end depths. With this exception, few data have been published on the subject of structural members with haunches of unequal dimensions. Although, in designing, the writer has seldom used the column analogy, due to its abstract qualities, this method has some advantages in the preparation of tables involving unsymmetrical haunch coefficients. Eqs. 1 to 14*b*, inclusive, have been verified. Values of stiffness and carry-over factors, as shown in Example 1, were checked by use of the fixed-point method and identical answers were obtained.

It is hoped that the following discussion, concerning beams having haunches of infinite moment of inertia, will add to the usefulness of this valuable paper.

In the design of rigid frames, including flat slab bridge and building frames, it is frequently necessary to calculate shape and load coefficients to satisfy the assumptions that (*a*) the haunches at each end of a member have unequal lengths, and (*b*) the value of the moment of inertia is infinite at the haunches. The effect of the latter assumption is to make the $\frac{M}{E\,I}$-value equal to zero for the length of the haunch. Based on the foregoing assumptions, shape coefficients, similar to those in Table 2, may be used in the design of members with symmetrical or unsymmetrical haunches or in the design of members that are haunched at one end only. This condensed table is offered as an illustration. In actual practice, of course, the table may be made as large as necessary.

Referring to Figs. 5(*a*), 5(*b*), and 5(*c*), the coefficients in Table 2 are defined as follows: p is the area of the $\frac{M}{E\,I}$-diagram for a unit moment at the left support of a unit span, with E and I equal to one; u is the distance of the centroid of p from the left support; q is the area corresponding to the definition of p, for a unit moment at the right support; v is the distance of the centroid of q from the right support; S_1 and S_2 are stiffness factors when a moment is applied at end 1 and end 2, respectively, while the opposite end of the member is fixed; and r_{12} and r_{21} are the percentages of moment transferred, respectively,

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[5] Negotiator, Southwestern Div., Corps of Engrs., Dallas, Tex.

[6] "Continuous Concrete Bridges," Portland Cement Assn., Chicago, Ill., 2d Ed., 1941, pp. 20–88.

TABLE 2.—SHAPE COEFFICIENTS FOR MEMBERS WITH HAUNCHES OF INFINITE MOMENT OF INERTIA

Shape Coefficients	VALUES OF a_2							
	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35
(a) $a_1 = 0.00$								
p	0.500	0.499	0.495	0.489	0.480	0.469	0.455	0.439
q	0.500	0.451	0.405	0.361	0.320	0.281	0.245	0.211
u	0.333	0.332	0.327	0.320	0.311	0.300	0.287	0.273
v	0.333	0.367	0.400	0.433	0.467	0.500	0.533	0.567
S_1	1.00	1.05	1.11	1.18	1.25	1.33	1.43	1.54
S_2	1.00	1.23	1.52	1.91	2.42	3.11	4.05	5.36
r_{12}	0.500	0.579	0.667	0.765	0.875	1.000	1.143	1.308
r_{21}	0.500	0.496	0.486	0.471	0.452	0.429	0.403	0.375
(b) $a_1 = 0.10$								
p	0.405	0.404	0.400	0.394	0.385	0.374	0.360	0.344
q	0.495	0.446	0.400	0.356	0.315	0.276	0.240	0.206
u	0.400	0.398	0.393	0.386	0.376	0.364	0.350	0.335
v	0.327	0.360	0.393	0.426	0.459	0.492	0.525	0.558
S_1	1.52	1.64	1.78	1.94	2.13	2.36	2.64	2.99
S_2	1.11	1.40	1.78	2.29	3.00	4.00	5.42	7.50
r_{12}	0.486	0.563	0.648	0.743	0.849	0.969	1.105	1.261
r_{21}	0.667	0.662	0.648	0.628	0.602	0.572	0.538	0.503
(c) $a_1 = 0.20$								
p	0.320	0.319	0.315	0.309	0.300	0.289	0.275	0.259
q	0.480	0.431	0.385	0.341	0.300	0.261	0.225	0.191
u	0.467	0.465	0.459	0.451	0.440	0.427	0.412	0.396
v	0.311	0.343	0.376	0.408	0.440	0.472	0.504	0.535
S_1	2.42	2.68	3.00	3.40	3.89	4.52	5.36	6.50
S_2	1.25	1.62	2.13	2.85	3.89	5.42	7.76	11.44
r_{12}	0.452	0.523	0.602	0.689	0.786	0.894	1.015	1.152
r_{21}	0.875	0.868	0.849	0.821	0.786	0.745	0.701	0.655
(d) $a_1 = 0.30$								
p	0.245	0.244	0.240	0.234	0.225	0.214	0.200	0.184
q	0.455	0.406	0.360	0.316	0.275	0.236	0.200	0.166
u	0.533	0.531	0.525	0.516	0.504	0.489	0.473	0.456
v	0.287	0.319	0.350	0.381	0.412	0.443	0.473	0.504
S_1	4.05	4.65	5.42	6.42	7.76	9.63	12.34	16.50
S_2	1.43	1.92	2.64	3.71	5.36	7.98	12.34	20.00
r_{12}	0.403	0.468	0.538	0.616	0.701	0.795	0.899	1.014
r_{21}	1.143	1.133	1.105	1.065	1.015	0.959	0.899	0.837

from end 1 to end 2 and from end 2 to end 1 when the far end of the member is fixed. Values of the foregoing coefficients are obtained from the following formulas:

$$p\,u = q\,v = \frac{(1-a_2)^2-(a_1)^2}{2} - \frac{(1-a_2)^3-(a_1)^3}{3} \tag{42}$$

$$p = \frac{(1-a_1)^2-(a_2)^2}{2} \tag{43a}$$

$$q = \frac{(1-a_2)^2-(a_1)^2}{2} \tag{43b}$$

$$u = \frac{p\,u}{p} \tag{44a}$$

$$v = \frac{p\,u}{q} \tag{44b}$$

$$r_{12} = \frac{v}{1-v} \tag{45a}$$

$$r_{21} = \frac{u}{1-u} \tag{45b}$$

$$k_1 = S_1\frac{I}{L} \tag{46a}$$

$$k_2 = S_2\frac{I}{L} \tag{46b}$$

In Eqs. 46, k_1 and k_2 are relative stiffnesses, as defined by the author, and values of S_1 and S_2 are:

$$S_1 = \frac{1-v}{4\,p\,(1-u-v)} \tag{47a}$$

$$S_2 = \frac{1-u}{4\,q\,(1-u-v)} \tag{47b}$$

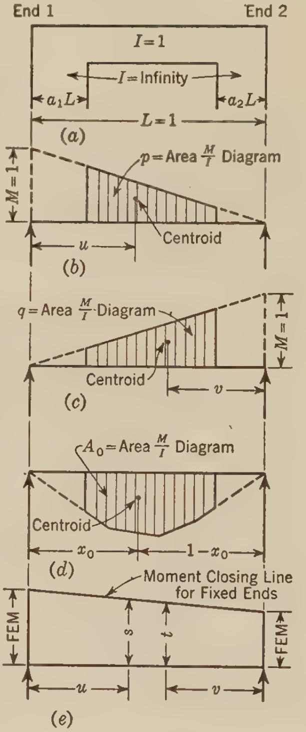
End 1
End 2
$I=1$
$I=$ Infinity
a_1L
a_2L
$L=1$
(a)
$p=$ Area $\frac{M}{I}$ Diagram
$M=1$
Centroid
u
(b)
$q=$ Area $\frac{M}{I}$ Diagram
Centroid
v
$M=1$
(c)
$A_0=$ Area $\frac{M}{I}$ Diagram
Centroid
x_0
$1-x_0$
(d)
Moment Closing Line for Fixed Ends
FEM
s
t
FEM
u
v
(e)

FIG. 5

Load coefficients for obtaining fixed-end moments and fixed-point ordinates are presented herein for: (*a*) Uniform loading across the span; and (*b*) a concentrated load placed at any point on the span.

Referring to Fig. 5(*d*), the symbols are defined as follows: A_o is the area of the $\frac{M}{E\,I}$-diagram for an external vertical loading of unity on a unit span when the values of E and I are equal to one; X_o is the distance from the left support to the centroid of A_o; and $1 - X_o$ is the distance from the right support to the centroid of A_o. Referring to Fig. 5(*e*), s and t are fixed-point ordinates or ordinates to the closing lines of the fixed-end moments for a conjugate beam. For any type of external vertical loading, the values of s and t are obtained from:

$$s = \frac{A_o\,(1-X_o)}{p} \tag{48a}$$

$$t = \frac{A_o\,X_o}{q} \tag{48b}$$

For a uniformly distributed load of unity on a unit span:

$$A_o\, X_o = \tfrac{1}{2}\left[\frac{(1-a_2)^3-(a_1)^3}{3}-\frac{(1-a_2)^4-(a_1)^4}{4}\right]\ldots\ldots(49a)$$

$$A_o\,(1-X_o) = \tfrac{1}{2}\left[\frac{(1-a_1)^3-(a_2)^3}{3}-\frac{(1-a_1)^4-(a_2)^4}{4}\right]\ldots(49b)$$

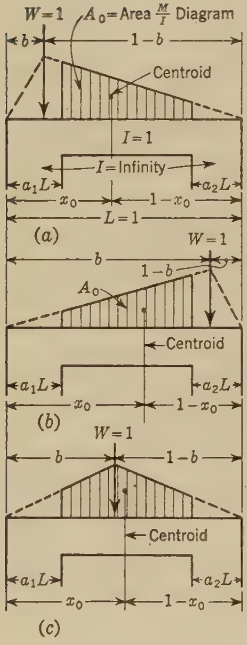

Fig. 6

Fixed-end moments for a total uniform loading, W, and span length, L, are:

$$\text{FEM}_1 = \left[s+\left(\frac{s-t}{1-u-v}\right)u\right]W\,L\,.(50a)$$

$$\text{FEM}_2 = \left[t-\left(\frac{s-t}{1-u-v}\right)v\right]W\,L\,.(50b)$$

Values of s and t, for uniform loading, may be determined by Eqs. 48 or by use of a chart[7] devised by Ulrik T. Berg, Assoc. M. Am. Soc. C. E. For a unit concentrated load on a unit span, the load coefficients are as follows:

For a load over the left haunch (Fig. 6(*a*))—

$$A_o\, X_o = b\left[\frac{(1-a_2)^2-(a_1)^2}{2}-\frac{(1-a_2)^3-(a_1)^3}{3}\right]$$
$$= b\,p\,u\ldots\ldots\ldots\ldots\ldots(51a)$$

$$A_o\,(1-X_o) = b$$
$$\left[(a_1)^2-(a_2)^2-a_1+a_2+\frac{(1-a_2)^3-(a_1)^3}{3}\right]$$
$$= b\,p\,(1-u)\ldots\ldots\ldots\ldots(51b)$$

For a load over the right haunch (Fig. 6(*b*))—

$$A_o\, X_o = (1-b)\left[\frac{(1-a_2)^3-(a_1)^3}{3}\right] = (1-b)\,(1-v)\,q\ldots(52a)$$

$$A_o\,(1-X_o) = (1-b)\left[\frac{(1-a_2)^2-(a_1)^2}{2}-\frac{(1-a_2)^3-(a_1)^3}{3}\right]$$
$$= (1-b)\,(p\,u)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(52b)$$

For a load not over a haunch (Fig. 6(*c*))—

$$A_o\, X_o = (1-b)\left[\frac{b^3-(a_1)^3}{3}\right]+b\left[\frac{(1-a_2)^2-b^2}{2}-\frac{(1-a_2)^3-b^3}{3}\right].(53a)$$

$$A_o\,(1-X_o) = (1-b)\left[\frac{b^2-(a_1)^2}{2}-\frac{b^3-(a_1)^3}{3}\right]$$
$$+\,b\left|\,(a_2)\,(1-a_2)+b^2-b+\frac{(1-a_2)^3-b^3}{3}\right]\ldots\ldots\ldots(53b)$$

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[7] "Structural Bending Moments Simplified," by Ulrik T. Berg, Polygraphic Co. of America, Inc., New York, N. Y., 1938, p. 95.

Values of s and t for the solution of Eqs. 51 to 53 are obtained by Eqs. 48, whereas fixed-end moments are determined by Eqs. 50, using W for total concentrated load.

A diagram showing influence lines for obtaining fixed-end moments, due to a concentrated load on a beam with symmetrical haunches of infinite moment of inertia, has been published elsewhere.[8]

JOHN SHERMAN,[9] ESQ.—Contrary to the author's opinion (see heading, "Summary"), considerable information is available regarding constants for the analysis of haunched beams. As applied to the properties of haunched beams, the tables prepared by Walter Ruppel,[10] Assoc. M. Am. Soc. C. E., from the earlier tables by A. Strassner,[11] contain all the information needed for the analysis of haunched beams (haunched at one end or both ends), not only by the method of conjugate points but by the moment area, slope deflection, three moment equation, column analogy, and moment distribution methods.

The Ruppel tables, being adapted from the Strassner tables (which made it possible to determine (*a*) the end rotation angles due to a unit couple applied at each end of a haunched beam, and (*b*) the end rotation angles due to a lateral load applied to the haunched beam), may be used to determine these end rotations. According to the principle of superposition, the end rotation angles of a haunched beam should be considered as composed of two parts: (1) The rotation angle due to a beam of uniform moment of inertia, and (2) the rotation angle due to the haunch. As stated, the properties of beams, haunched at one end, are given in the Ruppel tables; and with these values known it is possible, by the principle of superposition, to find (*a*) the properties of a beam symmetrically haunched, and (*b*) the properties of a nonsymmetrically haunched beam.

Mr. Ahlf also states (see heading, "Summary") that "The labor required to compute the area, centroid, and moment of inertia of the analogous column, and the area and centroid of the $\frac{M}{E\,I}$-diagram has * * * been simplified * * * in the tables." This statement is true for the cases presented in the paper; but, for the analysis of continuous bridges, the properties mentioned by Mr. Ahlf are used only by those who choose to ignore the properties of influence lines. It is possible to consider the various $\frac{M}{E\,I}$-diagrams as superfluous and to construct one influence line for each reaction by loading the conjugate beam with the $\frac{M}{E\,I}$-diagram due to this reaction—the $\frac{M}{E\,I}$-diagram being considered as equivalent to a system of closely spaced loads.

The Ruppel tables accomplish so much more than do the tables prepared by Mr. Ahlf that the latter may be considered of interest only in the very narrow range of the moment distribution and column analogy methods as applied to a fixed beam; and even in this specific case the use of the Ruppel tables is more simple and direct. In one sense, the entire subject of haunched beams is primarily a matter of academic interest as the use of haunched beams is the exception rather than the rule.

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[8] "Modern Developments in Reinforced Concrete, No. 10," Portland Cement Assn., Chicago, Ill., 1944, p. 5.

[9] Estimator-Designer, Virginia Bridge Co., Roanoke, Va.

[10] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 152.

[11] "Neure Methoden zur Statik der Rahmentragwerke," by A. Strassner, 2d Ed., Vol. 1, Ernst, Berlin, Germany, 1921.

Dale H. Rea,[12] Esq.—The problem of straight haunched beams has practically been removed entirely from the realm of tedious and laborious indeterminate analysis by this paper. To prove this conclusion the writer has undertaken to check the paper. The formulas are true expressions of column analogy. The formulas for C and the tables computed from them were not checked because of the enormous work involved. However, the column analogy solution presented herein will enable one to predict probable variations of C.

TABLE 3.—Solution of the Beam in

Section	Segment length	Properties of Haunched Beams						Uniform Load		
		y	I $(=y^3)$	A $\left(=\frac{3.38}{y^3}\right)$	x_0	Q_I $(=A\ x_0)$	Q_{II} $(=A\ x_0)$	x_1	M_s $\left(=\frac{x}{2}-\frac{x^2}{50}\right)$	P $(=M_s A$
1	1	3.12	30.4	0.111	−12	−1.33	16.0	0.5	0.25	0.03
2	1	2.76	21.0	0.161	−11	−1.77	19.5	1.5	0.70	0.11
3	1	2.40	13.8	0.245	−10	−2.45	24.5	2.5	1.13	0.28
4	1	2.04	8.5	0.398	−9	−3.58	32.2	3.5	1.50	0.60
5	1	1.68	4.75	0.712	−8	−5.70	25.6	4.5	1.85	1.32
6	1	1.50	3.38	1.000	−7	−7	49.0	5.5	2.14	2.14
7	1	1.50	3.38	1.000	−6	−6	36.0	6.5	2.41	2.41
8	1	1.50	3.38	1.000	−5	−5	25.0	7.5	2.62	2.62
9	1	1.50	3.38	1.000	−4	−4	16.0	8.5	2.81	2.81
10	1	1.50	3.38	1.000	−3	−3	9.0	9.5	2.94	2.94
11	1	1.50	3.38	1.000	−2	−2	4.0	10.5	3.05	3.05
12	1	1.50	3.38	1.000	−1	−1	1.0	11.5	3.10	3.10
13	1	1.50	3.38	1.000	0	0	0	12.5	3.13	3.13
14	1	1.50	3.38	1.000	1	1	1.0	13.5	3.10	3.10
15	1	1.50	3.38	1.000	2	2	4.0	14.5	3.05	3.05
16	1	1.50	3.38	1.000	3	3	9.0	15.5	2.94	2.94
17	1	1.50	3.38	1.000	4	4	16.0	16.5	2.81	2.81
18	1	1.50	3.38	1.000	5	5	25.0	17.5	2.62	2.62
19	1	1.50	3.38	1.000	6	6	36.0	18.5	2.41	2.41
20	1	1.50	3.38	1.000	7	7	49.0	19.5	2.14	2.14
21	1	1.50	3.38	1.000	8	8	64.0	20.5	1.85	1.85
22	1	1.50	3.38	1.000	9	9	81.0	21.5	1.50	1.50
23	1	1.58	3.95	0.856	10	8.56	85.6	22.5	1.13	0.97
24	1	2.10	9.28	0.365	11	4.00	44.0	23.5	0.70	0.26
25	1	2.70	19.7	0.172	12	2.06	24.8	24.5	0.25	0.04
Σ	....	....	....	20.02	....	16.79	717.2	....	....	48.23
Σ in terms of L			....	(0.801)	....	(0.027)	(0.046)	....	....	(0.0772)

The following rules may prove helpful in analyzing haunched beams: (*a*) The greater the haunch, the larger will be the stiffness factor, but the smaller the carry-over factor; (*b*) the product of the carry-over factor away from the end and the stiffness factor at the end is constant for a given beam; and (*c*) increasing the size of a haunch increases the fixed-end moment at that end.

Table 3 presents the data necessary for a solution of Examples 1, 2, and 3. In computing this table, the beam is divided into sections 1 ft long, and all variables refer to the midpoint of the sections.

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[12] Engr., U. S. Bureau of Reclamation, Denver, Colo.

In addition to the notation of the paper, the letter symbols in Table 3 are defined as follows: x_0 is the variable distance from the midspan of the beam to any given section; x_1 and x_2 are the distances from end 1 and end 2, respectively, of the beam to any given section; and M_s is the simple bending moment at any section.

Values in Table 3, when substituted in Eqs. 23 to 41, yield results that are quite comparable. Thus, for Example 1: $A\,E\,I_o = 0.801\,L$; $Q_I\,E\,I_o = 0.027\,L^2$; $e_a = 0.034\,L$; $L_1 = -0.534\,L$; $L_2 = 0.466\,L$; $Q_{II}\,E\,I_o = 0.046\,L^3$; $Q_a\,E\,I_o = 0.045\,L^3$; $K_1\,L = 7.58\,E\,I_o$; $k_1\,L = 1.89\,I_o$; $K_2\,L = 6.08\,E\,I_o$; $k_2\,L = 1.52\,I_o$; $N\,L = -4.28\,E\,I_o$; $r_1 = -0.564$; and $r_2 = -0.704$.

FIG. 4 BY THE COLUMN ANALOGY METHOD

	TRIANGULAR LOAD; SMALL END AT 1				TRIANGULAR LOAD; SMALL END AT 2			
Q_f $(=M_s\,A\,x_0)$	x_1	M_s $\left(=\frac{x}{3}-\frac{x^3}{1{,}875}\right)$	P $(=M_s\,A)$	Q_f $(=M_s\,A\,x_0)$	x_2	M_s $\left(=\frac{x}{3}-\frac{x^3}{1{,}875}\right)$	P $(=M_s\,A)$	Q_f $(=M_s\,A\,x_0)$
−0.4	0.5	0.17	0.02	−0.2	24.5	0.33	0.04	−0.5
−1.2	1.5	0.50	0.08	−0.9	23.5	0.91	0.15	−1.7
−2.8	2.5	0.82	0.20	−2.0	22.5	1.42	0.35	−3.5
−5.4	3.5	1.15	0.46	−4.1	21.5	1.87	0.74	−6.7
−10.6	4.5	1.45	1.03	−8.2	20.5	2.24	1.59	−12.7
−15.0	5.5	1.74	1.74	−11.2	19.5	2.55	2.55	−17.9
−14.5	6.5	2.02	2.02	−12.1	18.5	2.79	2.79	−16.7
−13.1	7.5	2.27	2.27	−11.4	17.5	2.97	2.97	−14.9
−11.2	8.5	2.50	2.50	−10.0	16.5	3.10	3.10	−12.4
−8.8	9.5	2.71	2.71	−8.1	15.5	3.18	3.18	−9.5
−6.1	10.5	2.88	2.88	−5.8	14.5	3.20	3.20	−6.4
−3.1	11.5	3.01	3.01	−3.0	13.5	3.19	3.19	−3.2
0	12.5	3.13	3.13	0	12.5	3.13	3.13	0
3.1	13.5	3.19	3.19	3.2	11.5	3.01	3.01	3.0
6.1	14.5	3.20	3.20	6.4	10.5	2.88	2.88	5.8
8.8	15.5	3.18	3.18	9.5	9.5	2.71	2.71	8.1
11.2	16.5	3.10	3.10	12.4	8.5	2.50	2.50	10.0
13.1	17.5	2.97	2.97	14.9	7.5	2.27	2.27	11.4
14.5	18.5	2.79	2.79	16.7	6.5	2.02	2.02	12.1
15.0	19.5	2.55	2.55	17.9	5.5	1.74	1.74	12.2
14.8	20.5	2.24	2.24	17.9	4.5	1.45	1.45	11.6
13.5	21.5	1.87	1.87	16.8	3.5	1.15	1.15	10.4
9.7	22.5	1.42	1.23	12.2	2.5	0.82	0.70	7.0
2.9	23.5	0.91	0.33	3.6	1.5	0.50	0.18	2.0
0.5	24.5	0.33	0.06	0.7	0.5	0.17	0.03	0.4
21.0	....	....	48.75	54.2	....	....	47.62	−12.1
(0.0013)	....	....	(0.0780)	(0.0035)	....	....	(0.0762)	(−0.0008)

Similar results for Examples 2 and 3 are given in Table 4. A comparison of the author's values with the foregoing shows close agreement. The maximum difference is about 2%. Other haunched beams analyzed by the writer in the past have been recomputed by the author's method and show even closer agreement. The difference of 2% is well within limits of design when one considers the many variables involved in rigid frame analysis.

TABLE 4.—CHECK COMPUTATIONS FOR EXAMPLES 2 AND 3

Notation	Example 2	Example 3*a*	Example 3*b*
$P\,E\,I_o$.....	$0.0772\,W\,L^2$	$0.0780\,W\,L^2$	$0.0762\,W\,L^2$
$Q_f\,E\,I_o$....	$0.0013\,W\,L^3$	$0.0035\,W\,L^3$	$-0.0008\,W\,L^3$
e_f.........	$0.017\,L$	$0.045\,L$	$-0.010\,L$
$(FEM)_1$...	$0.112\,W\,L$	$0.0872\,W\,L$	$0.1348\,W\,L$
$(FEM)_2$...	$0.083\,W\,L$	$0.1063\,W\,L$	$0.0603\,W\,L$

If the two triangular loads in Example 3 are placed upon the beam at the same time, the result is a uniform load of 2 W, which is twice as great as the uniform load W in Example 2. It follows that all other functions bear this same relation regardless of the type of beam used. This relation exists in Table 3 within the limits of slide-rule accuracy.

Adapting this same reasoning to Table 1, it is found that the sum of values in Tables 1(f) and 1(h) equals two times the corresponding values in Table 1(d), and that the three values will be of different magnitude. Therefore, Eqs. 30, 34, and 38 should never be of equal value. The stiffness of all members in a rigid frame, haunched or prismatic, should be expressed as either relative stiffness or as absolute stiffness and not intermixed. In Example 1, N is referred to as the "absolute carry-over moment." The word absolute appears to be extraneous to carry-over moment and should be used only in referring to "stiffness factor."

I. B. Hosig,[13] Esq.—The first reaction to the presentation of data such as those in this paper is the question: "Will engineers use the data in their work?" The writer is sufficiently acquainted with the labor connected with haunched beam problems to appreciate both the immense amount of time applied to the construction of the tables and the objectives sought in presenting the tables. If teaching the elements involved in the calculation of beam performance were as standardized as teaching trigonometry, for instance, the tables presented could, perhaps, be used as safely as are trigonometric tables because similarly fundamental relations based on geometric figures are presented. The tables are the best evidences that the day is approaching when such teaching will be standardized and that facility in the design of haunched and irregular beams will approach present facility in trigonometric calculations. In fact, tables will be necessary to make such facility common, as they are necessary in trigonometry, and these are the best known to the writer. However, he confesses that he would not use the tables except as a check to a problem solved by fundamental relations, but for such a purpose he considers the tables a very valuable addition to his "tool kit."

The second reaction following the general reading of the text accompanying the tables is the complete absence of the words, "curvature," "slope," and "deflection," and the rare use of the words, "rotation" and "reaction." The fact that the author has produced the end products, "stiffness," "carry-over factors," and "fixed-end moments" without the use of these words shows that the words are no longer necessary to him in his progress toward facility, and perhaps is again a preview of the shape of things to come when teaching the elements becomes standardized. The writer believes that such a trend is wrong, however, and that teachers must right it. An indication of the error resulting from the author's viewpoint (which would not result if the entire problem were considered as an interplay of slopes, rotations, etc.) is shown in the second sentence under the heading, "Carry-Over Factors," namely: "The induced moment is defined as the 'carry-over moment,' and that proportion of the applied moment which is carried over is defined as the 'carry-over factor.' "

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The fundamental concept of the "carry-over moment" presumes what is equivalent to a cantilever beam, the free end of which is first rotated through an angle of one radian by a couple at that end. Then the beam is restored to its original level by a force applied at the free end to eliminate the deflection that followed the rotation. If the couple alone were acting, there would be along the entire length of the beam a uniform moment with a value equal to that of the couple. The application of the force or reaction induces moments, varying from zero at the free end to the value of force times the span length at the fixed end. The net moment at the fixed end is the difference between the two and may well be called the "induced moment." In all cases investigated by the writer, however, the moment produced by the force is larger than that produced by the couple; hence, the net moment can be no part of the "applied moment." This explains why the carry-over moment is of opposite sign to the applied moment.

The writer has checked scattered values in the tables and found no errors. The computations were made following a procedure in which moments and reactions are applied to the free end of a similarly shaped and loaded cantilever beam to reduce the cantilever beam to a fixed or hinged beam as required. The derivations of the formulas for the constants were not checked by the writer, however, because of the difference between his viewpoint and that of the author, and because available time did not permit the complete analysis of the physical significance of these constants. In the case of a beam uniformly haunched from one end to the other, the method of using the tables is not obvious, and it is suggested that the author clarify the procedure in his closing discussion.

AUGUST L. AHLF,[14] JUN. AM. SOC. C. E.—The discussion by Mr. Jameson, dealing with beams having haunches of infinite depth, is a welcome addition to the paper. This phase of the analysis of haunched beams is somewhat simpler than the case of beams with varying depth of haunch in that one set of variables, d_1 and d_2, are eliminated, leaving only the length of the haunches, a_1 and a_2, to be considered.

The general equations of column analogy as presented in the paper may be made to apply to the case where the beam has haunches of infinite depth. The expressions representing the constants of Eqs. 2, 4, and 6, respectively, will then be as follows: In Eq. 2—

$$C = 0.5 - a \qquad (54a)$$

in Eq. 4—

$$C = 0.5\,(0.5 - a)^2 \qquad (54b)$$

and in Eq. 6—

$$C = 0.333\,(0.5 - a)^3 \qquad (54c)$$

The values of the expressions $(C_1 + C_2)$ in Eq. 15 and $(-C_1 + C_2)$ in Eq. 16 may be obtained by computing the area and locating the center of gravity of the area of the moment diagram which lies between the haunches. As the depth of the beam (and hence the moment of inertia) is constant throughout this reach, these values may be computed from the geometrical properties of the moment diagram without any difficult integration. For the case of a uniform load or a concentrated load, Eqs. 49, 51, and 52 may be used if desired.

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The author regrets that the scope of discussion did not permit Mr. Sherman to include a detailed illustration of how the Ruppel tables and the principle of superposition could have been used to simplify the solution of nonsymmetrical haunched beams. A comparison of the relative accuracy and the amount of labor involved would have been of interest.

Mr. Rea has demonstrated ably the procedure to be followed in the column analogy solution of haunched beams by dividing the member into a number of segments. The close agreement between the results obtained by this approximate method and those obtained from tabular values is gratifying. The great number of beam segments chosen and the care with which Mr. Rea made the computations no doubt explain this agreement.

The truth of the statement—"the product of the carry-over factor away from the end and the stiffness factor at the end is constant for a given beam"—may be demonstrated by rearranging Eqs. 14*a* and 14*b* and equating:

$$r_1 K_1 = N = r_2 K_2 \dots\dots\dots\dots (55)$$

The term, "absolute carry-over moment," was used with deliberation. The carry-over factor was determined by dividing the induced or absolute carry-over moment by the applied moment represented by the absolute stiffness factor. It is important that the absolute stiffness factor and not the relative stiffness factor be used. The word, "absolute," was applied to both cases so that the numerator and denominator would agree in terminology.

The energy expended in the preparation of this paper is amply repaid by Mr. Hosig's statement that he "considers the tables a very valuable addition to his 'tool kit'"—even though only as a check for the results obtained by other, more laborious methods. After several hours have been spent in setting up the fundamental equations involving the slopes and deflections of a member, such as in the illustrative example, and in evaluating the resulting integrals, the values in Table 1 should give a convenient, accurate, and useful check of the results.

A discussion of the slopes, deflections, and curvatures of the haunched beam was avoided purposely. The primary objective of the paper was to present a rapid and accurate method by which the structural designer could obtain the stiffness factors, carry-over factors, and fixed-end moments for the nonsymmetrically haunched beam. It was presumed that an academic discussion was not required as a designer qualified to perform this type of analysis must be acquainted with the meaning of these terms.

Mr. Hosig's statement that there is an indication of error resulting from the viewpoint held by the writer is incongruous with his later statement that scattering values were checked by the classical method and that no errors were found.

The writer prefers to consider the phenomena of the induced or carry-over moment as actually existing in a structural frame. In the method of distributing fixed-end moments, each joint of the frame is considered to be held rigidly against rotation while the frame is subjected to loading. As each joint is released successively from restraint, the ends of the members, which comprise the joint, will be rotated because the unbalanced moment has been released. The opposite end of each member, still rigidly held, will have moments induced or carried over to that end because of the rotation which the released end experiences. This procedure seems simpler than the consideration of each member as an isolated cantilever which is permitted to deflect and is then restored to its original position by the application of a force and a couple at the cantilevered end.

The statement that the induced moment can be no part of the applied moment is not consistent with structural theory. There are many charts and tables available whereby the design constants may be obtained for members with a haunch at only one end. The method presented is applicable, although not recommended, for this case. The procedure to be followed is the same as that outlined in Examples 1, 2, and 3. In this case, however, the length of the haunch at the small end should be considered equal to zero and the values of Table 1 under the heading of $a = 0$ must be used.

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Paper No. 2252

PRESSURE ON THE LINING OF CIRCULAR TUNNELS IN PLASTIC SOILS

By D. P. Krynine,[1] M. Am. Soc. C. E.

With Discussion by Messrs. M. A. Drucker, John M. Bird, and D. P. Krynine.

Synopsis

No attempt to develop a mathematical theory of pressure on the circular tunnel lining in plastic soils is made in this paper. Available field data are interpreted, using simple statics, arithmetic, and some geometry. The discussion is concerned mostly with the analysis of pressures on the lining of the Lincoln Tunnel, in New York, N. Y.

Introduction

Important observations of pressure on the lining of circular tunnels have been made in the United States since 1930. These observations represent a genuine contribution of American engineers to the world tunnel literature since, to the writer's knowledge, no such observations on full-sized circular tunnels have been made before. In this paper the available data concerning pressure on the lining of a circular tunnel are analyzed mostly as applied to the Lincoln Tunnel, New York City.[2] A comparison is made between pressures on this tunnel and the Central Avenue Pipe Tunnel,[3] in the City of Detroit, Mich. Finally, a few data referring to the Chicago (Ill.) Subway[4] are also cited. For the sake of brevity, these tunnels will be termed hereafter "Lincoln Tunnel," "Detroit Tunnel," and "Chicago Subway," respectively. The general data concerning these tunnels are as follows:

Diameter.—The outside diameter of the lining of the Lincoln Tunnel is 31 ft, that of the Detroit Tunnel about 13 ft, and that of the tubes of the Chicago Subway about 25 ft.

Note.—Published in May, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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[2] "Lincoln Tunnel. The Field Measurements and Study of Stresses in Tunnel Lining," by G. M. Rapp and A. H. Baker, The Port of New York Authority, July, 1937 (lithoprinted).

[3] "Earth Pressure on Tunnels," by W. S. Housel, published as part of the Symposium on "Earth Pressure and Shearing Resistance of Plastic Clay," *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 1037.

[4] "Shield Tunnels of the Chicago Subway," by Karl Terzaghi, *Journal*, Boston Soc. of Civ. Engrs., Vol. 29 (1942), No. 3, p. 163.

Lining.—The lining of both the Lincoln Tunnel and the Chicago Subway tubes consists of the conventional cast-iron segments bolted together. The Detroit Tunnel tube is of reinforced concrete. This is a thick pipe, its inside diameter being 9.5 ft.

Soil Conditions.—The average depth of the Lincoln Tunnel below the bed of the Hudson River is about 46 ft. Silt material (organic clayey silt), in which this tunnel is built, is quite similar to that found in the deltas of many other rivers flowing into the Atlantic. This is a plastic material with high natural moisture content. The silt layer in some parts of the Hudson River gorge is perhaps 400 ft thick.

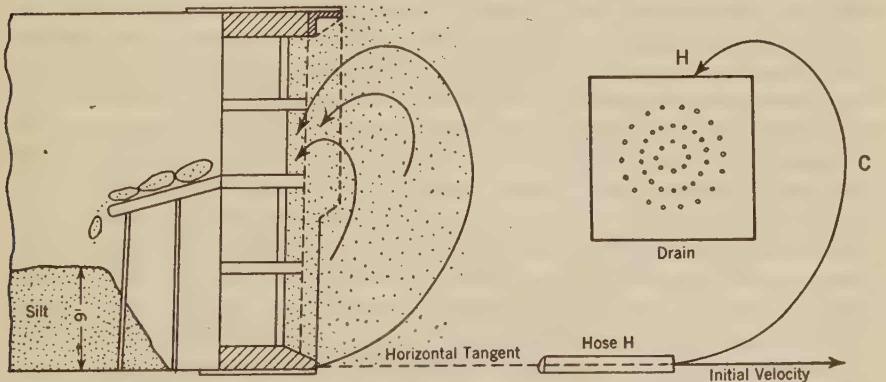
The soil, in which both the Detroit Tunnel and the Chicago Subway are built, is plastic clay. The Detroit Tunnel is located about 60 ft below the ground surface. There are about 12 ft of artificial fill and natural sand covering the soft and medium blue clay deposit, which is about 86 ft thick.

The Chicago soil profile is formed by a layer of silt, sand, and artificial fill, from 10 to 15 ft thick, underlain by a soft blue clay deposit (average thickness 70 ft). Hardpan and rock follow.

Methods of Construction.—Both the Lincoln Tunnel and that part of the Chicago Subway which is referred to in this paper have been constructed by the shield method. The presence of the second tube (in both cases) practically did not influence the results of measurements of the pressure on the lining. The Detroit Tunnel had an open-front face during the construction.

BEHAVIOR OF THE SOIL DURING CONSTRUCTION OF THE LINCOLN TUNNEL

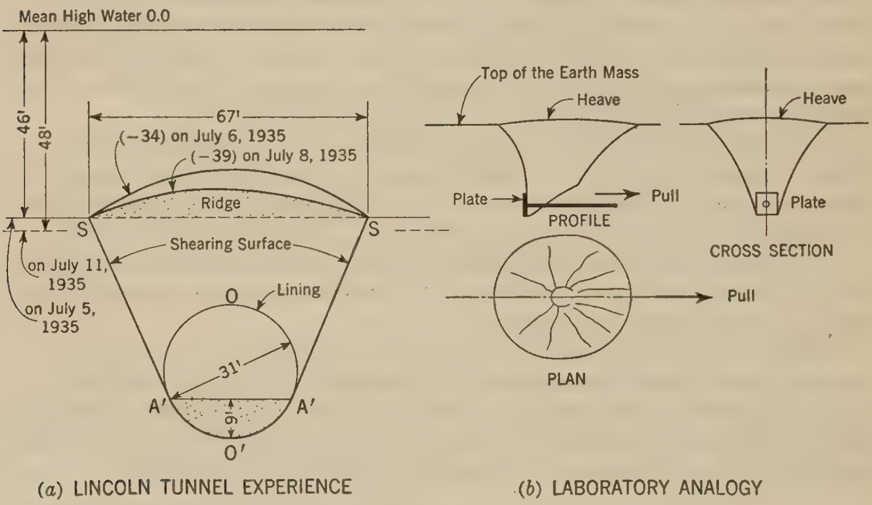
Material Above the Haunches and in Front of the Bulkhead.—There is a tendency for the earth in front of the advancing bulkhead to push forward horizontally; but it cannot move in that direction, because of the passive resistance of the earth mass. It must move somewhere, however, since the shearing strength of the material is overcome, and the bulkhead keeps moving forward. Shearing surfaces are developed in the earth material in front of the bulkhead, both below and above the top of its front face. The particles



(*a*) SECTION OF BULKHEAD (*b*) PLAN OF ANALOGOUS DRAIN

FIG. 1.—BEHAVIOR OF SOIL DURING CONSTRUCTION OF LINCOLN TUNNEL

from the lower part of the mass thus disturbed move upward, follow smooth curvilinear shearing surfaces, and enter the openings in the front face of the bulkhead approximately as shown in Fig. 1(*a*). The particles from the upper part of that disturbed mass push up the earth above them, creating bulges at the boundary of the earth mass similar to the one shown in Fig. 2(*a*). In the Hudson River above the Lincoln Tunnel the advancing shield formed a bulge about 12 ft high. The Chicago Subway is constructed in soft clay, the roof of the tunnel being located beneath a stiff clay crust which separates the soft clay from the superimposed stratum of sand and silt. The heaving of the street was rather slight, because the stiff crust was not broken. The width of this bulge was about twice the diameter of the tunnel. Silt entering through the openings in the bulkhead of the Lincoln Tunnel filled the lower 9 ft of the circular section of the lining which constitutes about 30% of the volume of the displaced silt. Assuming that the remaining 70% formed a parabolic ridge 12 ft high, its width should have been about 67 ft, again practically twice the diameter of the tunnel. Obviously, in each particular case the width of the ridge depends on the depth of the tunnel, other circumstances being equal. The foregoing can be further clarified by analogy: (1) For the movement of particles pushed toward the openings in the bulkhead front face; and (2) for the movement of particles pushed upward and forming the bulge.



(*a*) LINCOLN TUNNEL EXPERIENCE (*b*) LABORATORY ANALOGY

Fig. 2.—Material Displaced by Tunnel Driving

Analogy 1.—In the depressed center of the concrete floor of the writer's laboratory there is a drain for emptying water from a tank, through hose H, Fig. 1(*b*), into a sewer. Water leaving hose H follows a path C, Fig. 1(*b*), and enters the drain from the opposite side (point H). The curve in Fig. 1(*b*) is similar in shape to the curve in Fig. 1(*a*). In describing the shield tunnels of the Chicago Subway,[4] Karl Terzaghi, M. Am. Soc. C. E., states that "when watching the clay coming out of the openings, one got quite decidedly the impression that the clay flowed toward the openings from above." Obviously, the same phenomenon should be expected in the construction of the Lincoln Tunnel.

Analogy 2.—A vertical plate, provided with a horizontal rod and buried in earth, if pulled by the rod, produces both bulging and cracking at the earth surface (Fig. 2(*b*)).

Downward Motion of the Ridge and of the Silt Below It; Friction and Shear.—Formation of the two shearing surfaces A′S, Fig. 2(*a*), divides the silt mass into two individual masses: (*a*) Mass SA′O′A′S bounded by the shearing surfaces and the lining, and (*b*) the remainder of the silt. When the mass SA′OA′S moves up, its movement is resisted first by the shearing strength of the silt, and subsequently, as the latter is overcome, by friction directed downward (that is, against the direction of the movement). As soon as the ridge or bulge reaches its highest position, a downward motion begins due to the consolidation of the silt mass. The friction is now reversed and acts upward. As time passes, the silt along the shearing surfaces creates a healing process which restores the bond between the two masses. The restoration of bond probably is incomplete in the sense that the disturbed silt does not recover its initial shearing strength during the consolidation period. In this period of adjustment friction opposing the downward motion of mass SA′OA′S is gradually replaced by shearing stresses. After all, friction and shearing stress are phenomena of the same order. If two bodies in contact tend to move along the plane of contact with respect to each other, the force opposing the motion is friction; and, if a part of a body or of a mass tends to move with respect to the other part of the same body or mass, the motion is opposed by shearing stress.

For the sake of simplicity, shearing stresses will be assumed acting vertically, and not obliquely, which does not change the reasoning and the results. It is convenient to remember that shearing stresses acting along two mutually perpendicular planes are equal. In other words, existence of a vertical shearing stress, at a point of the mass, implies the existence at the same point of a horizontal shearing stress of the same magnitude.

Consistency of the Remolded Silt Material.—The silt material above the lining and in front of the bulkhead is remolded. As a first approximation, it will be assumed that it acts as a perfect fluid. This is the assumption made by G. M. Rapp and A. H. Baker, Assoc. Members, Am. Soc. C. E., in their report on the Lincoln Tunnel.[2] It implies that at a point of the surface of the lining there is a normal pressure and that both the vertical and horizontal pressures at the same point are numerically equal to that normal pressure. Messrs. Rapp and Baker state[2] that the tangential pressures along the periphery of the lining "were generally less than 2 lbs. per sq. in., and a large portion of them ranged between 0 and 0.5 [lbs. per sq. in.]." In other words, the measured tangential pressures in the silt surrounding the lining were negligible.

Material at the Lower Part of the Lining.—It follows from a simple visual inspection of Fig. 2(*a*) that at the upper part of the lining (above points A′) the moving bulkhead was remolding the silt material and pushing it forward against the passive resistance ("passive pressure") of the silt. In the lower part of the lining (below points A′) the moving bulkhead was making its way through silt which pressed on it from both sides. This is the "active pressure" of the silt on the lining.

It follows furthermore from Fig. 2(*a*) that the moving bulkhead formed a trough in the silt bounded by the line A′O′A′ (Fig. 2(*a*)). The material within this trough was remolded and, as stated, with a certain approximation may be considered as a liquid. This statement concerns a thin cushion of silt along the lower part of the lining A′O′A′. Conventionally and for the sake of brevity, this part A′O′A′ will be termed "bottom of the lining" in this paper.

Recapitulation.—The foregoing statements concerning the behavior of the silt material around the tunnel may be summarized as follows: The bulkhead pushes the silt forward. A part of it travels paths along curved surfaces and enters the openings in the shield. The remainder bulges upward to form a ridge as the shield driving continues. At the same time, the lower part of the lining makes its way through undisturbed silt. During this time a thin cushion of remolded silt is being formed in the trough A′O′A′ as a result of shoving. The lining, disengaged from undisturbed silt, "floats" in the liquid mud of the "trough" A′O′A′ (Fig. 2(*a*)).

External Forces Acting on the Lining and on the Adjacent Silt Mass

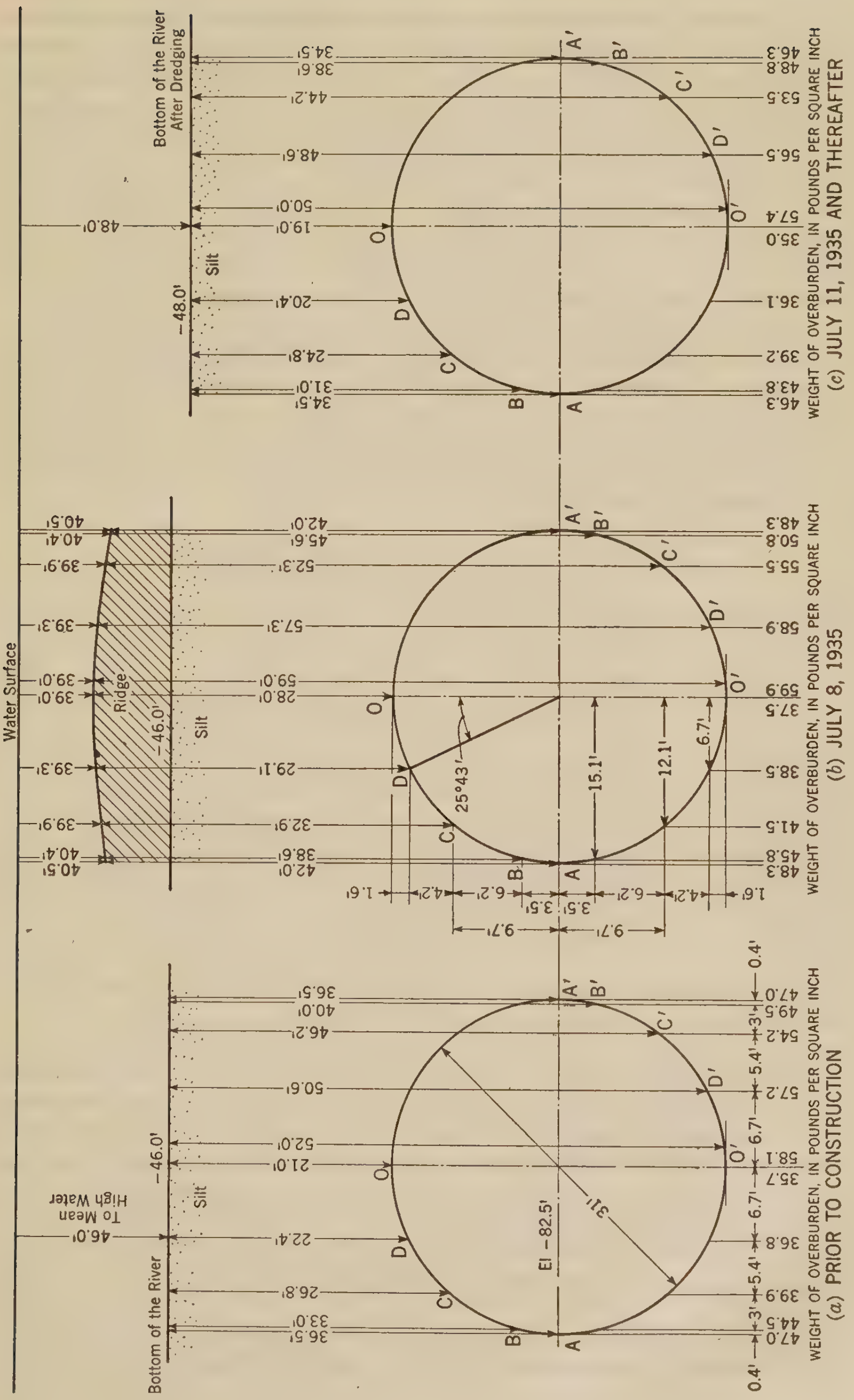
Weight of the Overburden.—During the construction of the Lincoln Tunnel the following facts were recorded:[2] Before driving, the elevation of the river bottom next to the test station was − 46, the datum plane being the mean high-water level in the Hudson River (Fig. 2(*a*)). On July 6, 1935, a ridge 12 ft high was produced by the moving bulkhead (elevation of the top of the ridge, − 34). Due to erosion and other circumstances, this ridge was 7 ft high on July 8 (elevation of the top, − 39). On July 11 the bottom of the river was dredged to elevation − 48.

On the basis of this information[2] (assuming the weight of water in the Hudson River at 64 lb per cu ft and the weight of saturated silt at 104 lb per cu ft), the weight of the overburden on July 8 and on July 11 and afterward is given in Figs. 3(*b*) and 3(*c*), in which the unit weight of the overburden, in pounds per square inch of the horizontal projection of the lining, is shown.

Pressure Records and Standard Loading.—A special silt pressure gage and plugs were designed to measure both normal and tangential pressures on the periphery of the Lincoln Tunnel at a special test station. Accuracy of measurements has been reported[2] to be within 1 lb per sq in. or 2 lb per sq in. The tangential pressures were generally less than 2 lb per sq in., and a large proportion of them ranged between 0 and 0.5 lb per sq in. as already stated. In this paper the tangential pressures at the periphery of the lining of the Lincoln Tunnel are neglected altogether. For comparison, the writer decided to reduce all record data to a standard loading as follows:

(*a*) The lining is loaded with the overburden as of July 11, 1935, the only exception being made for the record of July 8, 1935, in order to discount the effect of the ridge (see heading, "Weight of the Overburden");

(*b*) The total weight of the lining is 18 kips and the total weight of the inside silt deposit is 16 kips per running foot of the tunnel;


Water Surface
Bottom of the River After Dredging
Bottom of the River
To Mean High Water
Silt
Ridge
−48.0'
−46.0'
El −82.5'
31'
25°43'
15.1'
12.1'
6.7'
9.7'
3.5'
6.2'
4.2'
1.6'
0.4'
3'
5.4'
A
B
C
D
O
A'
B'
C'
D'
O'
48.0'
46.0'
34.5'
38.6'
44.2'
48.6'
50.0'
19.0'
20.4'
24.8'
31.0'
40.5'
40.4'
39.9'
39.3'
39.0'
42.0'
45.6'
52.3'
57.3'
59.0'
28.0'
29.1'
32.9'
38.6'
36.5'
40.0'
46.2'
50.6'
52.0'
21.0'
22.4'
26.8'
33.0'
46.3
48.8
53.5
56.5
57.4
35.0
36.1
39.2
43.8
48.3
50.8
55.5
58.9
59.9
37.5
38.5
41.5
45.8
47.0
49.5
54.2
57.2
58.1
35.7
36.8
39.9
44.5

WEIGHT OF OVERBURDEN, IN POUNDS PER SQUARE INCH
(*a*) PRIOR TO CONSTRUCTION

WEIGHT OF OVERBURDEN, IN POUNDS PER SQUARE INCH
(*b*) JULY 8, 1935

WEIGHT OF OVERBURDEN, IN POUNDS PER SQUARE INCH
(*c*) JULY 11, 1935 AND THEREAFTER

FIG. 3.—WEIGHT OF OVERBURDEN, LINCOLN TUNNEL

(*c*) There is no compressed air within the lining above the normal atmospheric pressure. If the situation is different from that described, corrections are introduced. First, the corrections for the compressed air will be computed.

Compressed air was in contact with clay through the openings in the shield only. Hence the expanding lining exerted active pressure on the surrounding silt. As a result, the silt exerted a numerically equal passive resistance, which must be deducted from the measured normal pressure (negative correction). In the case of the Detroit Tunnel, the pressure of the compressed air was transmitted through the pore moisture of clay at the front face and around the lining. This neutral stress decreased the measured pressure (positive correction).

Points on the periphery of the lining of the Lincoln Tunnel where pressure records are available are shown in Fig. 3. To identify different vertical sections of the lining, letters A, B, C, D, and O will be used for the upper part of the lining, whereas letters A′, B′, C′, D′, and O′ correspond to the lower part of the lining (Fig. 3(*b*)). Numerical values of pressures, their averages separately for the upper and the lower semicircumference of the lining, and the compressed air pressures are given in Table 1.

TABLE 1.—RECORD OF PRESSURE INTENSITIES, LINCOLN TUNNEL (LB PER SQ IN.)

Date[a]	DOWNWARD PRESSURE p_d AT:[b]				Average p_d	UPWARD PRESSURE p_u AT:[b]				Average p_u	Compressed air pressure
	Point O	Point D	Point C	Point B		Point B′	Point C′	Point D′	Point O′		
July 8, 1935:											
Recorded	41.5	40.5	43.0	45.5	42.6	48.0	47.5	46.5	46.5	47.1	16
Corrected	39.1	38.1	40.6	43.1	40.2	45.6	45.1	44.1	44.1	44.7	
July 16, 1935:											
Recorded	37.0	36.5	38.0	42.0	38.4	43.5	43.0	42.6	42.7	43 0	16
Corrected	34.6	34.1	35.6	39.6	36.0	41.1	40.6	40.2	40.3	40.6	
Aug. 8, 1935:											
Recorded	38.5	38.0	41.0	43.5	40.3	45.0	44.5	44.0	45.0	44.6	28
Corrected	34.2	33.8	36.8	39.3	36.1	40.8	40.3	39.8	40.8	40 4	
Aug. 29, 1935:											
Recorded	35.0	34.0	37.0	38.0	36.0	40.5	40.0	39.5	40.0	40.0	0
Corrected	....	....	....	....	....	....	....	....	....	....	

[a] "Recorded" values are the original pressures, not corrected for the compressed air pressures; "corrected" values are so corrected. [b] See Fig. 3.

The recorded pressure was corrected by dividing the difference of the average recorded pressures on two given days by the corresponding difference of compressed air pressures. The increase in compressed air pressure from 16 lb per sq in., on July 16, to 28 lb per sq in., on August 8, corresponds to an increase in recorded pressure on the upper part of the lining of $\frac{40.3 - 38.4}{12} = 0.16$ lb per sq in. per pound of compressed air pressure. The corresponding value for the lower part of the lining is $\frac{44.6 - 43.0}{12} = 0.13$ lb per sq in. The reduction in compressed air pressure from 28 lb per sq in., on August 8, to zero, on August 29, is computed as 0.16 lb per sq in. for both parts of the lining.

In view of these considerations all recorded pressures were corrected by deducting from them as an average, 0.15 lb per sq in. per pound of compressed air pressure on that day.

Analogy 3.—Before discussing the play of forces acting on both the lining and the adjacent silt mass, consider the following analogy:

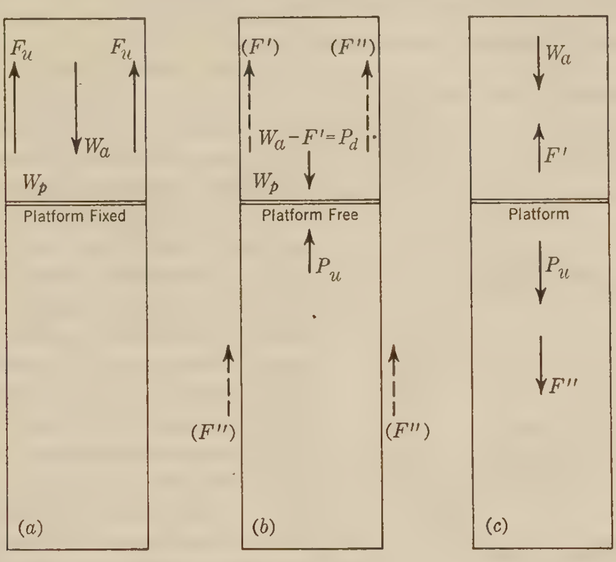


FIG. 4.—ELEVATOR SHAFT ANALOGY

In Fig. 4, a perfectly rigid platform is fixed at a certain position in a hypothetical elevator shaft of unit cross-section area. It supports its own weight W_p and the weight W_a of nonconsolidated silt in the shaft above the platform. Below the platform the shaft is completely filled with a perfectly consolidated silt. As the silt above the platform consolidates, friction forces F_u develop along the walls of the shaft, relieving the load on the platform to that extent. In other words (see Fig. 4(*a*)), the resultant downward force acting on the platform, except its weight, is

$$P_d = W_a - F_u \qquad \qquad (1)$$

and since the platform is wedged in position it reacts upward in the same degree in the silt load. Forces acting upward are distributed throughout the upper silt mass. The resultant of these forces is equal numerically to F_u. Since the platform is fixed (by brakes, say, or wedges), it does not exert any pressure on the silt below it; and since the latter is fully consolidated there is no movement and no friction below the platform.

If the platform is released, a consolidation process begins beneath it, and then the total force or load moving the platform downward is $P_d + W_p$. The platform moves downward following the consolidating silt mass; and this new movement is opposed by some new friction forces F' and F'' on the walls of the shaft and by the reaction of the mass P_u at the bottom of the platform (Fig. 4(*b*)). The weight of the mass below the platform has already caused complete consolidation of the material and need not be considered. Consequently, the mass under the platform should be assumed as being acted upon by a force P_u and vertical forces spread throughout the mass and having F'' for the resultant (Fig. 4(*b*)). The final distribution of the forces acting on the silt mass both above and below the platform is shown in Fig. 4(*c*).

Since the platform and the silt, on which it reposes, move, forces of inertia should be taken into consideration. The possible motion is very slow, however, and the forces of inertia will be disregarded.

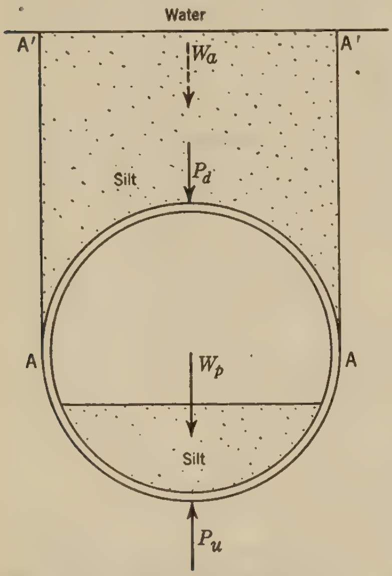


Fig. 5.—Vertical Forces Acting on the Lining

Forces Acting on the Silt Mass Above and Below the Lining.—Analogy 3, as illustrated by Fig. 4, can be applied to the example in Fig. 5, which shows the distribution of forces acting on the lining of the Lincoln Tunnel during its construction. As in Fig. 4, W_a is the weight of the overburden above the lining or "platform" (see Fig. 3); W_p is the total weight of the lining and the inside silt deposit (that is, silt which entered the lining as in Fig. 1(*a*)), P_d and P_u are forces computed from pressures measured during the construction (see Table 1). The total value of the weight W_p is 34 kips per running foot of the tunnel, 18 kips being the weight of the lining itself, and 16 kips being the weight of the inside silt deposit. The distribution of the weight W_p along the surface of contact with the silt mass at the bottom of the lining A′O′A′, in Fig. 2(*a*), is unknown. Two assumptions will be made: (*a*) The lining is perfectly rigid, and the weight W_p is uniformly distributed along the horizontal projection of the lining; and (*b*) the lining is perfectly nonrigid, and the weights of the lining and of the inside silt deposit are distributed along the horizontal projections of the lining proportionately to the respective thicknesses of the lining and of the inside silt deposit in a given vertical section of the tunnel. The results are shown in Fig. 6. The actual situation is between these two assumptions. It is to be noted also that, in the case of the lining (Fig. 5), all "friction forces," as shown in Fig. 4, are "shearing forces."

In Fig. 5 and in all subsequent figures and discussion capital letters designate total forces acting on a slice 1 ft long cut from the lining by two vertical planes perpendicular to its longitudinal axis. These forces are represented by the areas of corresponding diagrams (for instance, areas under curves W_a and P_d in Fig. 7). Unit pressures (or shearing stresses) acting on the lining in vertical planes parallel to its longitudinal axis are ordinates of the corresponding curves. The use of lower case letters for unit pressures was thus avoided except in Table 1.

Difference Between the Analogy (Fig. 4) and the Case of the Actual Lining.—In Fig. 4, the clay mass under the platform is assumed fully consolidated, and as soon as the "platform" is released, a new process of additional consolidation begins under the weight of the platform and the silt deposit above it. In the case of the lining of the Lincoln Tunnel the situation was different: First, in shoving the tunnel a cushion of silt along the bottom of the lining was remolded and became liquid; and, second, the overburden under which the silt along the bottom of the lining was consolidated was not increased, but was relieved as the lining passed through it. This can be disclosed by comparing the weight of the overburden at points O′, D′, C′, B′ (Fig. 3) and the values of $P_d + W_p$ (Table 1 and Fig. 6).

Interpretation of the Curve $W_a - P_d$.—In Fig. 7 the values of the weight of the overburden W_a and of the downward pressure P_d are plotted and their difference $W_a - P_d$ is represented in the form of curves. The difference $W_a - P_d$ is the relieved weight of the overburden at a given vertical section of the lining, as expressed in pounds per square inch on the horizontal projection of the lining. In computing these values, the difference $W_a - P_d$ at the center line of the lining (point O) was assumed to be zero.

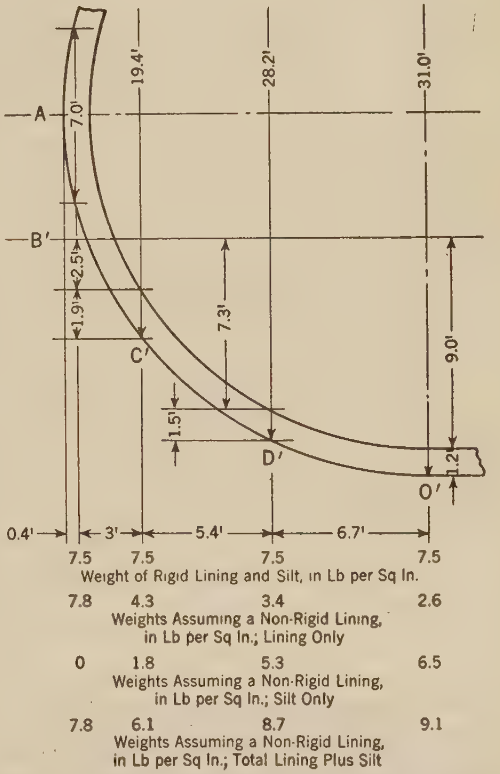

A
B′
C′
D′
O′
19.4′
28.2′
31.0′
7.0′
2.5′
1.9′
7.3′
9.0′
1.5′
1.2′
0.4′
3′
5.4′
6.7′
7.5 7.5 7.5 7.5
Weight of Rigid Lining and Silt, in Lb per Sq In.
7.8 4.3 3.4 2.6
Weights Assuming a Non-Rigid Lining, in Lb per Sq In.; Lining Only
0 1.8 5.3 6.5
Weights Assuming a Non-Rigid Lining, in Lb per Sq In.; Silt Only
7.8 6.1 8.7 9.1
Weights Assuming a Non-Rigid Lining, in Lb per Sq In.; Total Lining Plus Silt

Fig. 6.—Average Distribution of the Weight of the Lining, Lincoln Tunnel (Pounds per Square Inch)

Due to symmetry, there can be no horizontal shear and hence no vertical shear at point O (center line of the lining). In other words, there can be no relief in the weight of the overburden at that point. Another feature of the curve $(W_a - P_d)$, and of the curves in Fig. 8, is the horizontal tangent at the center line of the lining, obviously due to symmetry.

Conditions of equilibrium require that at any vertical plane between the center line of the lining and its sides (haunches), the total shearing force, acting upward at that plane, must balance the relieved weight between that center line and the vertical plane in question. For each of the vertical planes B, C, and D (Fig. 7) the relieved weight between that plane and the center line of the lining was computed. This is the "accumulated relieved weight in pounds," in Fig. 7. The area of each of the vertical planes B, C, D, above the lining was taken from Figs. 3(*b*) and 3(*c*). It equals numerically the height of the overburden above the points B, C, D, of the lining. The corresponding figures are to be found at the line "area of action of the shearing stress in square feet"

(Fig. 7). By dividing the accumulated relieved weight at each plane by the area of the latter, the average shearing stresses (in pounds per square foot) were computed. At no place is this shearing stress more than 156 lb per sq ft. The value of the shearing strength of the Hudson River silt may be estimated from comparison with other similar materials of the Atlantic Coast. For instance, the shearing strength of the silt in the Thames River at New London, Conn., which is more wet and hence less resistant than the Hudson River silt, is about 150 lb per sq ft as determined by direct shear test. Apparently the average shearing stress above the lining of the Lincoln Tunnel reached the value close to its shearing strength; but no data exist for judging what was the maximum value of that shearing stress. The ordinates of the curves of the "average upward shearing stress" beyond the sides (haunches) of the lining must decrease and must vanish at a certain distance from the tunnel.

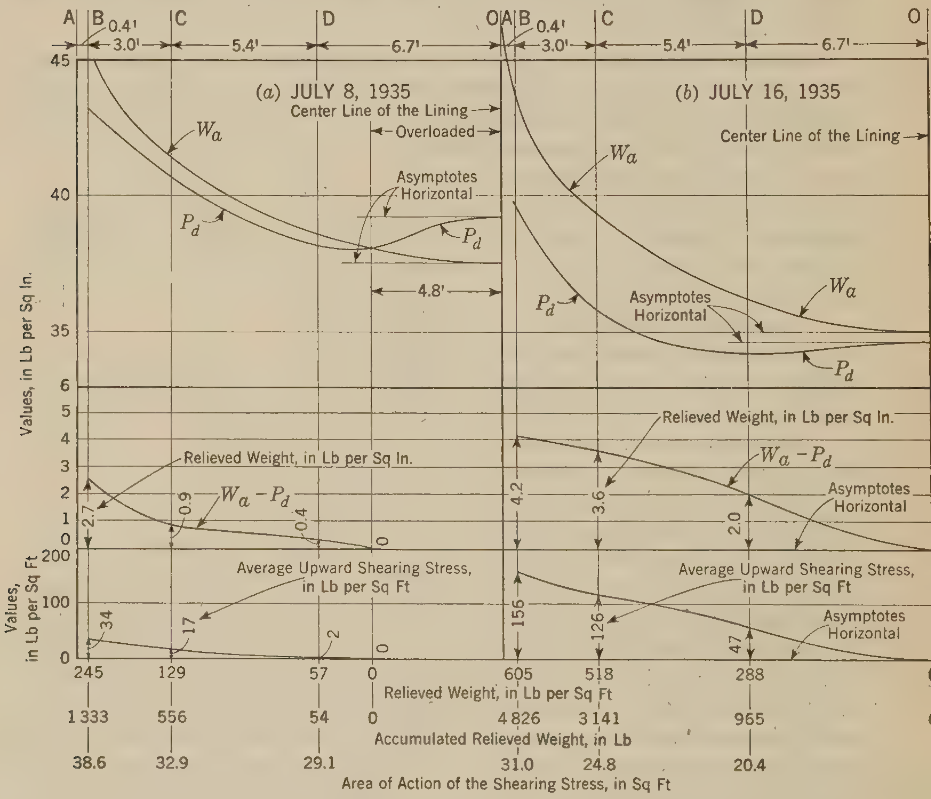


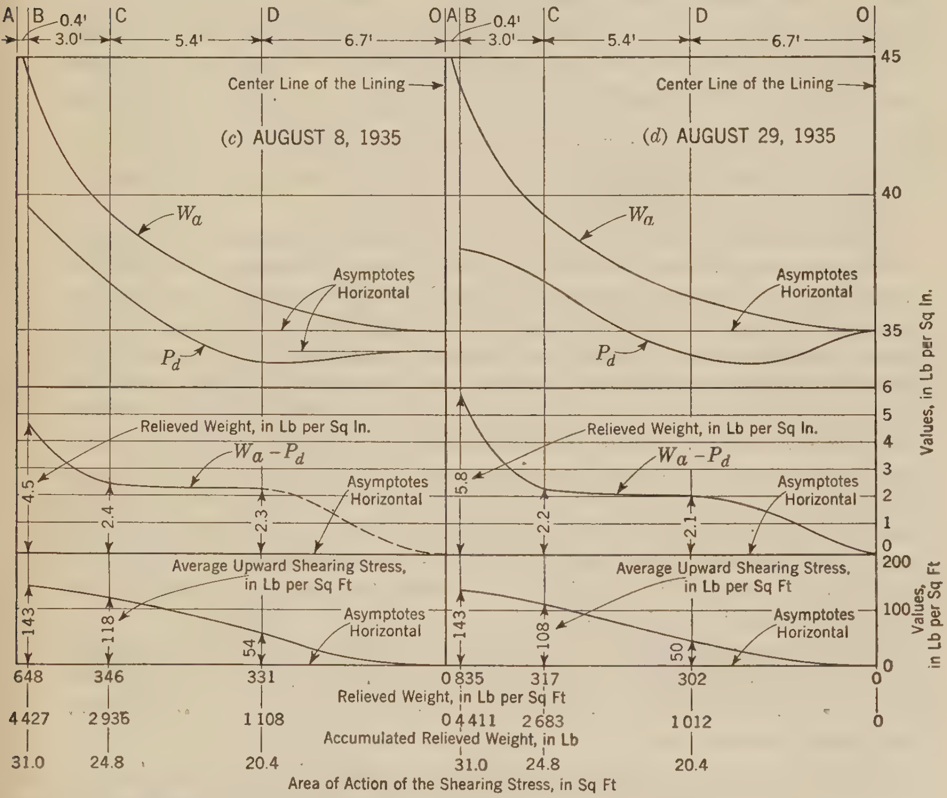
FIG. 7.—EXTENT OF RELIEF FROM TH

Comparison of the Vertical Upward and Downward Forces Acting on the Lining.—These forces are: The downward pressure P_d; the weight of the lining including the inside silt deposit (W_p in its entirety); and the upward pressure P_u. Their resultant,

$$F_d = P_d + W_p - P_u \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

is plotted in Fig. 8. The values of W_p are from Fig. 6, for rigid and nonrigid linings, respectively. Since the force $P_d + W_p$ acting downward must be balanced, and the force P_u is not large enough for this purpose, the area bounded by curve F_d, representing the difference of the downward and upward vertical forces acting on the lining shows the force balancing the system in question.

This force may be interpreted as: (*a*) A downward shearing force (such as force F'' in Fig. 4(*c*)); (*b*) a force tending to make the lining settle; or (*c*) items (*a*) and (*b*) combined. There are no data for estimating the value of the downward shearing forces; and some assumption should be made, as, for instance, that the upward and the downward shearing forces at each vertical plane are numerically equal. For comparison purposes, curve F has been plotted in Fig. 8. Under the approximate assumptions made, the difference between areas bounded by curves F_d and F corresponds to the value of the force that tends to make the tunnel settle. On July 8, 1935, this force ($F_d - F$) was considerable (Fig. 8(*a*)), due to the presence of the ridge of freshly remolded silt material. It reduced practically to zero after dredging (July 16, 1935), and afterward gradually increased until August 29, 1935.



WEIGHT OF OVERBURDEN, LINCOLN TUNNEL

Horizontal Pressure on the Lining.—To plot the horizontal pressure on the lining, Table 1 was used again. In Fig. 9 the recorded pressures corrected for compressed air are plotted horizontally at the points of measurement shown in Fig. 3.

Due to the pressure of the surcharge material forming the ridge, the ordinates of curve 1 are about 4 lb per sq in. larger than the ordinates of curves 2, 3, and 4, which are practically the same. The horizontal pressure at the sides (haunches) (point A) on August 29, 1935, was about 40 lb per sq in. Deducting from this value 21.3 lb per sq in. corresponding to the pressure of water on the bottom of the Hudson River, the difference would be $40.0 - 21.3 = 18.7$ lb per sq in. The vertical pressure at the same point and on the same date was $46.3 - 21.3 = 25.0$ lb per sq in. (points A and A′, Fig. 3(*b*)), which corresponds to a ratio of $\frac{18.7}{25} = 0.74$. In a perfect liquid this ratio would be 1. Using the value of 0.74 in the Rankine formula, the corresponding angle of friction of the silt material on August 29, 1935, was about 9°, which is a reasonable value for that kind of material.

The well-known Rankine formula was derived by its author as applying to undisturbed earth mass in which the unit pressure increases from the top downward, following a straight-line law (triangular pressure distribution). Hence, by no means can it be applied after disturbance to the bottom of the lining where the pressure should be practically constant. In this part of the lining the idealized horizontal pressure may be expressed by a vertical line (MN or M′N′ in Fig. 9). In the upper part of the lining (that is, practically above points B′) where the remolded material consolidates, a triangular pressure distribution may be expected, however (lines PN or P′N′, Fig. 9). The slope of the lines PN, P′N′ is too steep due to the fact that the curves, shown in Fig. 9, are constructed considering the remolded silt as a perfect liquid. In reality the pressure at point A may be correct whereas that at point O is exaggerated. Point P″, Fig. 9, is a corrected position of point P′ (for an "angle of friction" of silt equal to 9°).

Change in Average Pressure on the Lining after August 29, 1935.—Detailed data concerning the pressure distribution on the lining after August 29, 1935, are not available. The curve showing the average pressure on the lining[2] reveals the following facts:

(*a*) The average pressure was decreasing until December 24, 1935, when it reached its minimum (36 lb per sq in.). On that day the tie rods were erected. Afterward that pressure was increasing until April 13, 1936, when it was stabilized at a value of 37.6 lb per sq in.

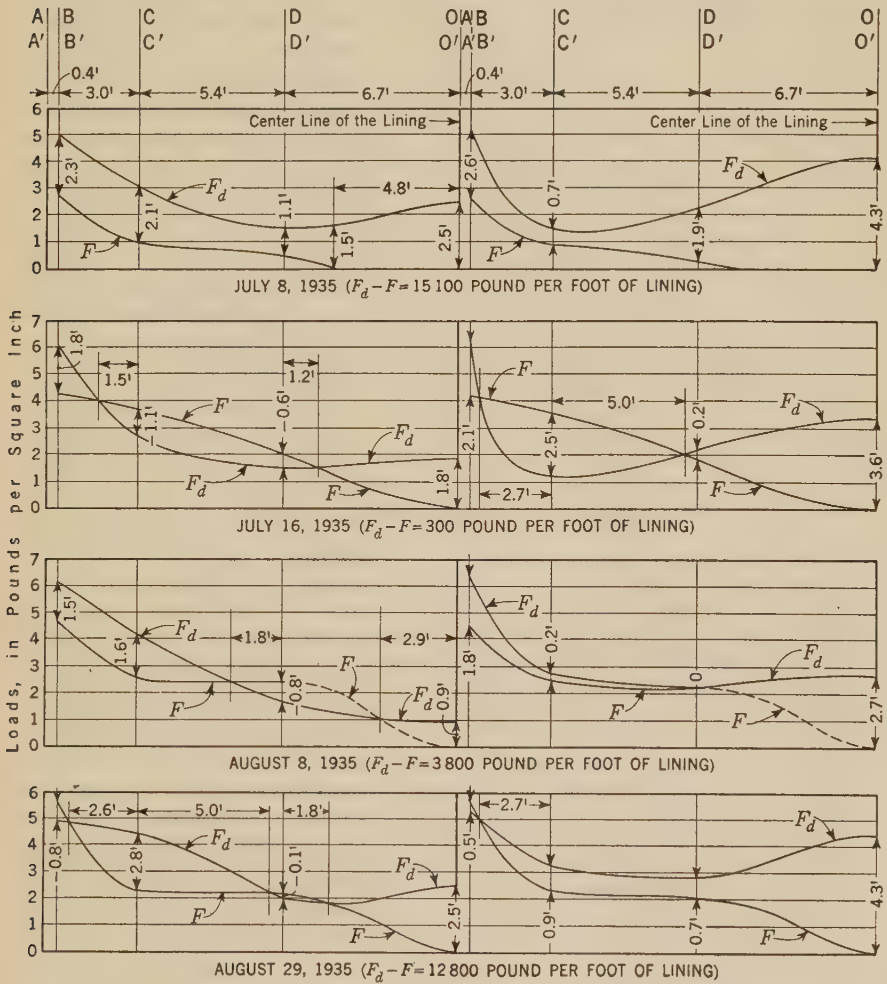
(*b*) On September 30, 1935, the inside silt deposit (weight 16 kips per running foot of the tunnel) was removed.

(*c*) Between December 24, 1935, and April 13, 1936, the roadway was completed, which increased the original weight of the lining (18 kips) to 40 kips per running foot of the tunnel.

A change in load of 1,000 lb inside the lining causes an average change in pressure on the lining equal to $\frac{1{,}000}{2 \times 31 \times 144} = 0.11$ lb per sq in. Corrections due to such changes are:

(1) On December 24, 1935, for the inside silt deposit, 0.11 × 16 = 1.8 lb per sq in., the corrected pressure being equal to 36.0 + 1.8 = 37.8 lb per sq in.

(2) On April 13, 1936, corrections are indicated for both the inside silt deposit and the additional weight of the roadway, these two corrections being of opposite signs. The final pressure, 37.6 lb per sq in., is decreased by 0.11 × (22 − 16) = 0.7 lb per sq in., yielding a value of 36.9 lb per sq in.



JULY 8, 1935 ($F_d - F$ = 15 100 POUND PER FOOT OF LINING)

JULY 16, 1935 ($F_d - F$ = 300 POUND PER FOOT OF LINING)

AUGUST 8, 1935 ($F_d - F$ = 3 800 POUND PER FOOT OF LINING)

AUGUST 29, 1935 ($F_d - F$ = 12 800 POUND PER FOOT OF LINING)

(*a*) RIGID LINING (*b*) NON RIGID LINING

Fig. 8.—Forces That Tend to Drive the Lining Down, Lincoln Tunnel

These results are plotted in Fig. 10, in which the solid line represents the average pressure on the lining as measured in the field, and the dotted line shows that pressure as reduced to the specified standard loading. The reduced pressure decreased systematically until April 13, 1936, when the field records reported a stabilization of the average pressure on the lining.

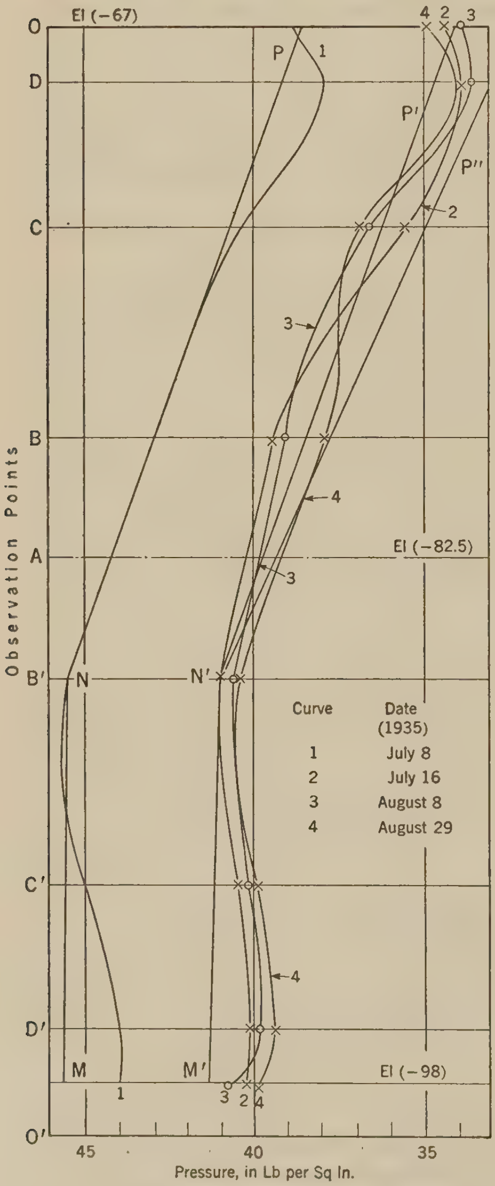


Fig. 9.—Horizontal Pressure on the Lincoln Tunnel Lining

The published material obtained during the construction of the Lincoln Tunnel² is not detailed enough and covers a rather short period (253 days). As an example of a more complete procedure, the Detroit Tunnel³ may be cited. Pressure observations at that tunnel covered the period from December, 1930, to January, 1941—about eleven years. In the ordinary case such a long period of observations may be unnecessary; but it seems advisable to observe pressures on any tunnel during a reasonably long-time interval, which may be estimated, for instance, at three years; this may involve but a small additional expense. It was found at the Detroit Tunnel that the vertical pressure increased and tended to return to its original (preconstruction) value, whereas the horizontal remained more or less constant during the entire period of observations. These facts may be explained by a partial or complete disappearance of shearing stresses, both vertical and horizontal. No definite conclusions as to the disappearance of the shearing stresses can be drawn on the basis of published reports on the Lincoln Tunnel.²

Decrease in the Horizontal Diameter of the Lincoln Tunnel

According to an official statement:²

"Measurements of the horizontal and vertical diameters of the lining showed distortions similar to those that have been observed in other tunnels in Hudson River silt. Immediately after erection the horizontal diameter began to shorten, and the vertical diameter began to lengthen by about the same amount. Nine days after the lining had been erected at the test station, the horizontal diameter had decreased and the vertical diameter had increased, each, by 1½ inches. This was the maximum distortion observed at the test station. From then on, the horizontal diameter increased and the vertical diameter decreased until they returned to their original values six months after the lining was erected [about December 24, 1935; see Fig. 10]."

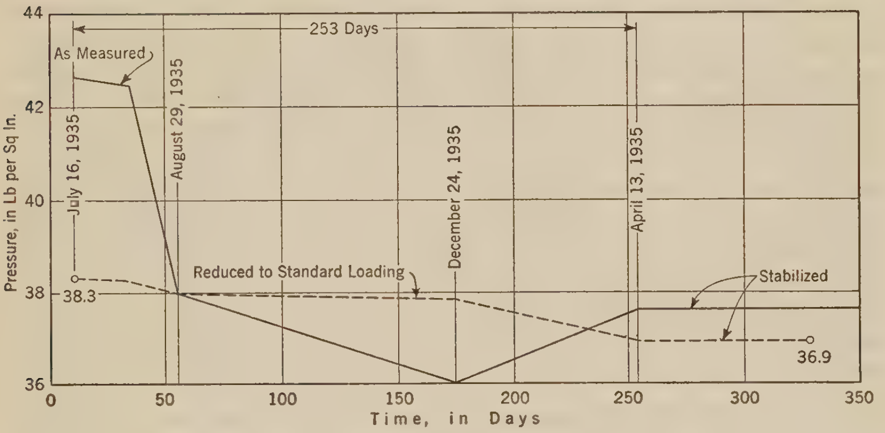


FIG. 10.—AVERAGE PRESSURE ON THE LINING, LINCOLN TUNNEL

A similar phenomenon was observed during the construction of the Chicago Subway. As stated by Professor Terzaghi:[4]

> "Within the first few days after the rings were assembled, the (horizontal) diameter of the rings steadily decreased. The greatest shortening ranged between 1 and 1½ in. * * * After the initial period of shortening the horizontal diameter of the tube increased and after about one year assumed a constant value, which was about 1½ in. in excess of the initial length of the diameter."

Elongation of vertical diameter and shortening of the horizontal diameter seem to be characteristic also of tunneling in North River (New York City) silt where the material is so unstable that the shield can be shoved "blind."

The foregoing examples include tunnels constructed by the shield method in plastic organic silts and plastic clays in which the ratio of the horizontal pressure at a point of the mass to the vertical pressure at the same point is considerable. However, in soils where this ratio is small, such as sandy soils, the horizontal diameter of the lining does not shorten. For instance, John H. Quimby, M. Am. Soc. C. E., in correspondence with the writer, stated:

> "It has been a maxim in constructing all of the East River shield tunnels from the time of the Pennsylvania R. R. tunnels to the present time, that the top iron in the ring should be kept high, i.e. that each ring should be constructed 'out of round' with the vertical diameter longer than the horizontal diameter, since it is practically the universal experience in the soils under the East River, and under the Harlem River as well, that the tunnel rings tend to settle down and elongate on the horizontal diameter."

It follows from this evidence that the shortening of the horizontal diameter of the lining is not characteristic of the shield method generally as is sometimes believed, but that it occurs in plastic soils only, in which the ratio of the horizontal pressure to the vertical is considerable.

It seems undeniable that, to shorten the horizontal diameter, the total horizontal force acting on the lining must be larger than the vertical; yet in the case of the Lincoln Tunnel a careful computation of these forces for all pressure records available since July 6, 1935, did not reveal this to be the case. It must be concluded therefore that the cause of the shortening is to be sought at the very beginning of the shoving. The following explanation may be offered. Before applying force to the shield any point of the bottom of the lining A′O′A′ (Fig. 2) is under the action of a vertical pressure caused by the overburden (silt and water in the river, Fig. 3(*a*)). The bulkhead relieves this vertical pressure (see heading, "Difference between the Analogy (Fig. 4) and the Case of the Actual Lining"); but the lateral pressure that exists in the natural silt mass still persists at the points A′, B′, C′, and D′ and squeezes the lower part of the lining, thus shortening the horizontal diameter. Since the lining is rigid the compressive force at the shield acts not only on the head section of the lining, but also on rear sections through a certain distance. This explains why the decrease in horizontal diameter lasts some days after the lining is set.

So far as its shape is concerned, the graph of lateral pressure distribution at the initial moment of shoving should be similar to line P′N′ in Fig. 9 and its continuation (Rankine or triangular pressure distribution). Due to the remolding of the silt in the bottom of the lining, this pressure is partly replaced by a uniform pressure (distributed approximately as shown by line N′M′ in Fig. 9). There is an opinion that the use of the Rankine formula in the design and analysis of tunnels should be discontinued. Evidence presented in this paper makes it advisable to think twice before doing so.

Action of Compressed Air

The downward vertical pressure P_d, as shown in the first line corresponding to each date in Table 1, may be separated into two parts: (*a*) The active pressure that was exerted by the overburden on the lining as shown in the second line corresponding to each date in Table 1, and (*b*) the pressure exerted by the lining on the overburden due to the action of compressed air during construction. This is the passive pressure of the overburden on the lining, and the pressure-measuring devices recorded the sum of the two. Of course, at no time was the active pressure of the overburden on the lining larger than the weight of the overburden (Fig. 3). Consider, for instance, the action of the compressed air on August 8, 1935. At an air pressure of 28 lb per sq in., the compression stress in the lining at its inside periphery was also 28 lb per sq in. At the outside periphery the average compression stress was only $0.15 \times 28 = 4.2$ lb per sq in., which means that the overburden was loaded approximately with average unit loads of that value acting upward. The resultant of these loads per running foot on half of the lining was $4.2 \times 144 \times 15.5 = 9{,}374$ lb. Upon dividing this value by the area of action of the shearing stress at the sides (haunches) of the lining (34.5 sq ft, see Fig. 3(*c*)), the average value of the downward shearing stress at the haunches caused by the compressed air was $\frac{9,374}{34.5} = 272$ lb per sq in. The shearing stresses in the silt mass caused by the weight of the overburden (Fig. 7) and those due to the action of the compressed air acted in opposite directions. Therefore, the compressed air not only restored the original preconstruction condition in the silt, but even created some excess of safety.

Conclusions

1. During the construction of the Lincoln Tunnel, the weight of the overburden above the tube was relieved by the upward vertical shearing stresses.

2. There were downward vertical shearing stresses below the lining of the Lincoln Tunnel.

3. A diagram of shearing stresses for different points of the periphery, plotted from a horizontal line, has a zero ordinate and a horizontal tangent at the center line of the lining. The ordinates of this diagram increase toward the sides (haunches) of the lining. Such diagrams should be extended beyond the sides (haunches) in order to satisfy conditions of equilibrium; but there are no available data as to the shape of such extensions.

4. The average normal pressure on the lining of the Lincoln Tunnel as reduced to the conditions of a standard loading decreased steadily until it apparently assumed a constant value about eight months after the erection of the lining.

5. The decrease in the average pressure on the lining of the Lincoln Tunnel as cited in conclusion 4 should indicate the gradual change of value in the shearing stresses acting on the mass. No definite conclusions on this subject can be drawn from the information published to date.

6. The decrease in horizontal diameter of the lining of the Lincoln Tunnel is a phenomenon that has been observed in a number of other cases in which the shield method was applied in plastic soils, such as organic silts or plastic clays, but not in sands or similar soils. It is caused apparently by the action of a considerable lateral pressure in the lower part of the lining at the initial stage of shoving, which action is combined with a simultaneous relief in vertical pressure due to the passage of the shield.

7. Diagrams of horizontal pressure on the lining of the Lincoln Tunnel do not sustain the belief that the Rankine formula must be abandoned in the design and analysis of tunnels.

Acknowledgment

The writer extends his thanks to the New York City Tunnel Authority, the Port of New York Authority, and to John H. Quimby, M. Am. Soc. C. E., and G. M. Rapp, Assoc. M. Am. Soc. C. E., for supplying and collating valuable unpublished data used in preparing this paper. James H. Dugan, M. Am. Soc. C. E., was very helpful to the writer in advising him on tunnel terminology.

DISCUSSION

M. A. Drucker,[5] Esq.—As a result of his analysis of the horizontal pressure on the Lincoln Tunnel, the author concludes that the Rankine formula should not be abandoned in the design of tunnels. It is not intended to dispute this statement; but the method indicated for deriving the angle of repose, as well as the angle obtained, seems questionable. In obtaining the angle of repose of the silt, the author considers a uniform horizontal water pressure equal to that at the bottom of the river extending downward through the silt and, in addition, pressure due to a mixture of water and solid particles for the depth of the silt—the total pressure at point A (Fig. 9) being equal to 40 lb per sq in. Two objections may be raised to this method. In the first place, the pressure due to the water is assumed to be divided into two parts, and, secondly, the silt is considered to be a fluid mixture having an angle of repose. That it is not a mixture is evident from the depth of clear water above the top of the silt. It is in fact a sediment having its own angle of repose in water.

Generally for structures in soil below water, the horizontal pressures are calculated on the basis of full water pressure plus the effect of the soil, whose angle of repose is either known or assumed and whose weight is considered to be reduced by the buoyant effect of the water. Applying this method to the case under consideration, it may be seen that the unit water pressure at point A would be $\frac{82.5 \times 64}{144}$ or 36.7 lb per sq in. This result leaves 3.3 lb per sq in., from the measured horizontal pressure of 40 lb, for the effective horizontal soil pressure. For the given weight of silt of 104 lb per cu ft, the effective weight of the solid particles, per cubic foot of silt, would be 40 lb. Using the Rankine formula for active earth pressure, the pressure at point A for a height of silt of 34.5 ft gave the following relationship: $3.3 = \frac{34.5 \times 40}{144} \times \frac{1 - \sin \phi}{1 + \sin \phi} = 9.6 \times \frac{1 - \sin \phi}{1 + \sin \phi}$. From this it was found that $\phi = 29° 20'$—a rather large angle of repose for the material in question. Pressure readings somewhat smaller than the actual pressures could account for this.

The paper contains information from which the angle of repose may be obtained more directly. As stated by Jacob Feld, M. Am. Soc. C. E., the vertical shear planes, discussed by the author, may be considered as approximately equal to the angle of repose of the material.[6]

Assuming that the shearing surfaces start from the horizontal axis of the tunnel and spread out so that they are 67 ft apart at the bottom of the river, which is 46 ft below water level (as given in the paper), the tangent of the angle they make with the vertical is $\frac{0.5\,(67 - 31)}{82.5 - 46} = \frac{18}{36.5} = 0.49$, giving an angle of 26°. A somewhat smaller angle is obtained by considering the bulge of soil over the tunnel. It is stated that, on July 6, this bulge was 12 ft high and 67 ft wide—apparently an unstable condition—and, on July 8, the bulge was down to 7 ft. Assuming the curve of the bulge to be a parabola, as does the author, its maximum slope would be $\frac{7 \times 2}{33.5} = 0.418$ which corresponds to an angle of 22° 40′. This agrees very closely with the assumed upper limit of 20° used in the design of the Holland tunnels in the same Hudson River silt. For such an angle of repose, the pressure due to the solid particles of the silt would be $9.6 \times \frac{1 - \sin 20°}{1 + \sin 20°} = 4.7$ lb. This pressure, added to the 36.7 lb due to the water pressure, would give a total at point A of 41.4 lb which is only 1.4 lb greater than the measured 40 lb and is within the 2-lb limit of accuracy of the pressure measurements as stated in the paper. Incidentally, in accepting the angle of repose of 9° obtained by the author, one would very likely use the general method for finding the horizontal pressure and would obtain a value of $36.7 + 9.6 \times \frac{1 - \sin 9°}{1 + \sin 9°} = 36.7 + 7.0$ or 43.7 lb per sq in. instead of the 40 lb from which the author obtained the 9° angle of repose.

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[5] Structural Civ. Engr.—in Chg., Board of Transportation City of New York, New York, N. Y.

[6] *Proceedings*, Brooklyn Engrs. Club, Vol. 22, No. 2, January, 1924, p. 33.

There is another item in the paper that is of unusual interest. This is the calculated reduction in the measured external pressures due to the internal air pressure which, according to the author, causes the lining to expand. The author finds this correction to be 0.15 lb per sq in. per lb of internal air pressure or 4.2 lb for a maximum air pressure of 28 lb. It is true that after making corrections on this basis the average pressures, p_d and p_u, for the several dates, are brought into close agreement, as shown in Table 1. However, the differences in the measured pressures may be due to entirely different causes or to inaccuracies in the pressure measurements. In support of this view, the writer would like to call attention to the measured horizontal and vertical distortions of the tunnel as given in the paper. It is stated that the increase in diameter in one direction was accompanied by an equal decrease in the diameter at right angles to it. Theoretical calculations show that equal increase and decrease in lengths of diameter at right angles to each are due to external loading.

If the internal air pressure induced passive pressures in the surrounding soil, it would be necessary for the tunnel to expand in all directions. This would cause a greater increase in length of diameter that elongated, due to the outer pressures, and a smaller decrease in the diameter that is shortened due to the same causes. For the case of a corrected pressure of 4.2 lb per sq in., the pressure per square foot would be about 600 lb. Although it is not known what the soil constant, in pounds per square foot per inch of soil compression, would be for the silt around the tunnel, an assumed value of 1,000 lb, for the purpose of discussion, would seem reasonable. Thus, the tunnel diameter would have to increase $2 \times \frac{600}{1{,}000} = 1.2$ in. to develop the passive pressures applied as corrections; also the decrease of the shortened diameter would be reduced by 1.2 in. However, the measurements show that the elongation in one direction was equal to the shortening at right angles and the changes in diameter were about 1.5 in. Had the tunnel stretched in accordance with the assumption of a 1,000-lb soil constant, the total elongation of one diameter would have been 1.5 in. + 1.2 in., or 2.7 in., whereas the net shortening of the diameter at right angles would have been 1.5 in. − 1.2 in., or 0.3 in. If a larger soil constant had been assumed, although the nature of the soil does not seem to warrant it, the necessary differences between the increase and decrease in diameters would have been proportionately less than 2.7 − 0.3, or 2.4 in., but still of a magnitude that would have shown up in the measurements. Even for a soil constant of 5,000 lb, the difference between the elongation of one diameter and the shortening of the one at right angles to it would have been about 0.5 in. Also, the tendency of the tunnel to stretch, caused by the tensile stresses set up by the internal air pressures, is offset by the ring shortening due to the compressive stresses caused by the larger external active pressure. Furthermore, as the internal air pressure did not exceed 28 lb per sq in., which was less than the minimum external water pressure, then, at any point on the tunnel, the active outer pressure was greater than the inner air pressure. It is difficult to understand, therefore, how the internal air pressure could induce a passive pressure in the surrounding soil.

John M. Bird,[7] Assoc. M. Am. Soc. C. E.—This paper is an excellent interpretation of field data collected, for the most part, from measurements taken during and after the construction of the Lincoln Tunnel under the Hudson River at New York City. In addition to analyzing these data, Professor Krynine has compared the pressures on the Lincoln Tunnel with those on the Central Avenue Pipe Tunnel in Detroit. A few data referring to the Chicago Subway are also cited by the author. All three tunnels were constructed in plastic soil. This paper is a definite contribution to the extremely limited literature on earth pressures on tunnels. The writer feels that, in interpreting these field data, Professor Krynine has revealed a very important pattern that the earth pressures on tunnels constructed in plastic soils appear to follow. A study of the field data for the Detroit Tunnel indicates a pressure pattern similar, in part, to that of the Lincoln Tunnel. A study of the earth pressures observed on other tunnels seems to substantiate the existence of a definite pattern that the earth pressures follow.

The writer will discuss conclusions 1 to 7 in three parts. Conclusions 1, 2, 4, and 5 will be discussed in Part 1, conclusion 3 in Part 2, and conclusions 6 and 7 in Part 3.

Part 1.—The author states (conclusion 1):

> "During the construction of the Lincoln Tunnel, the weight of the overburden above the tube was relieved by the upward vertical shearing stresses."

This fact appears to be unassailable if the field data are taken at face value. The relief in pressure during construction and continuing for a certain short period after construction has been completed, as brought out in conclusion 4, is the first phase in the pattern followed by earth pressures on the lining. Apparently, there are two stages of pressure variation that must be considered in an analysis of data concerning pressure on the lining of a tunnel whether the tunnel be subaqueous or subterranean. The first stage is one in which pressures exist due to the action of a disturbed soil mass that is in a condition of temporary equilibrium. During the second stage, pressures exist due to the action of a soil mass which is undergoing readjustment in reaching a final condition of equilibrium. The first stage takes place during construction and for a short time following completion of construction. During this first stage, the vertical pressures on the lining are decreasing. Pressure-variation records for the Detroit Tunnel indicate that the vertical pressure on the lining decreased for a period of approximately three months. Similar records for the Lincoln Tunnel show that the average vertical pressure, after having been reduced to a specified standard loading, decreased systematically for the 253 days of observation "until it apparently assumed a constant value" (see conclusion 4). It is evident that the equilibrium of the earth mass was disturbed by driving the lining through the soil. Also, the soil mass is in a temporary condition of equilibrium during this stage since another variation in pressure, indicative of a readjustment of the soil mass, must occur before the final equilibrium is reached. Since long-time pressure observations were made at the Detroit Tunnel (from December, 1930, to January, 1941), it is possible, by examining the pressure variation during this period, to find evidence of the two stages of pressure variation. These records indicate that a decrease in the vertical pressure on the lining prevailed for a period of approximately three months (stage 1) and thereafter the pressure was constantly increasing for several years (approximately five years) until a final and permanent condition of equilibrium was reached (stage 2). Since, after this five-year period, the vertical pressure appeared to remain fairly constant, it is not unlikely that a final condition of equilibrium was reached. The period of observation for the Lincoln Tunnel is short, and, as the author states, no definite conclusions can be drawn from the information published to date. It is likely, however, that the period during which a steady decrease in the vertical pressure was noted (253 days) corresponds to the first stage of pressure variation and that a second stage during which the vertical pressures increase steadily will follow. In the case of the Detroit Tunnel, the vertical pressure increased until it nearly equaled the static head at the elevation of the measured pressure. Probably, a similar stage occurred in the case of the Lincoln Tunnel. The author states, however, that a final stabilization of pressure was reached at the end of the stage of decreasing vertical pressure. This seems unlikely to the writer.

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As the author states in conclusions 1 and 2, the decrease in pressure is caused by vertical shearing stresses within the earth mass. It must follow that an increase in pressure, as was noted on the lining of the Detroit Tunnel, is due to the diminishing or vanishing of these shearing stresses. During the period of temporary equilibrium when the vertical pressures are decreasing,[8] "* * * maximum resistances in the mass are being mobilized to transfer weight away from the tunnel," and, during the period of increasing vertical pressures culminating in final equilibrium, there is a transfer of some pressure to the tunnel. The writer believes that the soil mass, upon being disturbed, tends to form an arch above the tunnel, transmitting the pressure of much of the overlying soil through the soil arch to the earth mass adjacent to the tunnel. This idea was presented by the late J. C. Meem, M. Am. Soc. C. E., in 1907.[9] The two ideas, one of arch action within the soil and the other the relief in pressure through vertical shearing stresses, are merely two aspects of the same phenomenon. For subaqueous tunnels, it may be considered impracticable to assume that a definite relief in pressure will result from such arch action. However, in a material such as the organic clayey silt of the Hudson River and the blue clay in which both the Detroit and Chicago tunnels are built, a soil arch is probably formed. In the case of the Lincoln Tunnel, the creation of the bulge or the ridge at the top boundary of the earth mass during the initial period of the soil's disturbance appears to indicate that this arch action is initially in effect. It is unlikely that the soil arch will endure for a long period of time; in other words, it is unlikely that the vertical shearing stresses set up to relieve the pressure will continue indefinitely. In some way, due to some cause, the arch action will break down and transmit back to the tunnel lining some, if not all, of the load of the overburden (preconstruction pressure). The writer likens this breakdown of the arch action in the soil mass to the action that would take place within an asphalt beam that was initially deflected upward and allowed to remain over its end supports for an indefinite time. Although the beam would maintain this deflection for a certain period, eventually, due to its own weight and possibly due to vibrations from external causes, a form of plastic flow would cause the beam to return to its original horizontal position.

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[8] "Earth Pressure on Tunnels," by W. S. Housel, published as part of the Symposium on "Earth Pressure and Shearing Resistance of Plastic Clay," *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 1045.

The nature of the stresses set up to relieve the vertical pressure on the tunnel lining is of great interest to the foundation engineer. Of much greater importance, however, is the necessity of answering the following three questions:

1. How much decrease in pressure can be expected from vertical shearing stresses, and for what period of time will this relief exist?
2. Why is not this relief in weight permanent, and during how long a period do the shearing stresses diminish?
3. Do the shearing stresses finally vanish or do they transmit, permanently, some part of the load from the tunnel lining to the adjacent earth mass?

Answers to these three questions will come only as a result of (1) many more field studies like those for the Lincoln and Detroit tunnels and (2) an interpretation of the data from these field studies like that in this paper.

To know fully how much decrease in pressure can be expected, more information as to the shape of the area on which the shearing stresses act must be secured. The straight-line shearing surfaces in Fig. 2(*a*) are but crude approximations. The exact shape of these surfaces is indeterminate and depends on the type and condition of the soil mass in which the shearing stresses occur. The period of time during which the relief in pressure exists is a variable dependent on local conditions. For the Detroit and Lincoln tunnels, the period lasted from three months to a year, approximately. The answer to the question as to why the arch action or the shearing stresses diminish lies, possibly, in the effects of vibrations set up as a result of externally applied forces. Even a slight vibration might cause sufficient readjustment of the soil particles to cause a redistribution of stress. The period of time for the transfer of load back to the tunnel is a variable dependent on local conditions. It may be a few months but, as was the case for the Detroit Tunnel, it may be a number of years. That some permanent or residual shearing stresses exist in the soil mass as a result of the disturbance to its equilibrium is indicated by the fact that the lateral pressures against the tunnel lining of the Detroit Tunnel remained fairly constant and were substantially less than the static head after final equilibrium was reached. W. S. Housel, M. Am. Soc. C. E., states[8] that an indication of a permanent change in the condition of equilibrium in the soil mass from its preconstruction state may be found in the fact that the tunnel structure weighed less than the displaced earth and yet, in final equilibrium, vertical stresses of approximately the same magnitude as the static head existed.

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[9] "The Bracing of Trenches and Tunnels with Practical Formulas for Earth Pressures," by J. C. Meem, *Transactions*, Am. Soc. C. E., Vol. LX (June, 1908), p. 1.

Fig. 10 shows the variation of the average vertical pressure with time. The writer has computed what may be termed the average static head by averaging the pressures due to the weight of the overburden at points O, D, C, and B in Fig. 3(*a*). A similar procedure was used by the author in determining the average p_d in Table 1. A value of 39.2 lb per sq in. is thus obtained for the average static head. A comparison of the average vertical pressure and the average static head should indicate that the corrected average pressure stabilized at 36.9 lb per sq in. on April 13, 1935, is still considerably below the average static head value of 39.2 lb per sq in. Probably some increase in the average vertical pressure has occurred since this date. It is possible that the pressure will become approximately equal to the static head, as was the case for the Detroit Tunnel.

Part 2.—The author states under the heading, "External Forces Acting on the Lining and on the Adjacent Silt Mass: Interpretation of the Curve $W_a - P_d$," that

> "Due to symmetry, there can be no horizontal shear and hence no vertical shear at point O (center line of lining). In other words, there can be no relief in the weight of the overburden at that point."

In conclusion 3, Professor Krynine states that

> "A diagram of shearing stresses for different points of the periphery, plotted from a horizontal line, has a zero ordinate and a horizontal tangent at the center line of the lining."

Unquestionably, due to symmetry, there can be no vertical shear at point O, but, if there is to be no relief in the weight of the overburden at that point, the rate of change of the shear $\frac{ds_s}{dx}$ is assumed to equal zero at point O. For example, if an element of the soil mass of unit thickness is taken directly above point O, as shown in Fig. 11, the shearing stress on a vertical section a distance

Δx from point O will be $\frac{ds_s}{dx}\,\Delta x$. The pressure at the base of the element is p pounds per square inch, which corresponds to the measured p_d shown in Table 1. Writing the equation $\sum F\,v = 0$ for the forces on the element:

$$\gamma\, h\, \Delta x - p\, \Delta x - \frac{ds_s}{dx}\,\Delta x\, h = 0 \ldots\ldots(3)$$

and

$$\gamma\, h - p = W_a - P_d = \frac{ds_s}{dx}\, h \ldots\ldots(4)$$

in which the weight of the overburden expressed in pounds equals $\gamma\, h\, \Delta x$; and γ = unit weight of the mass.

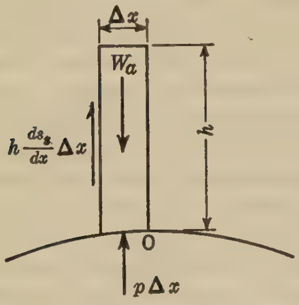
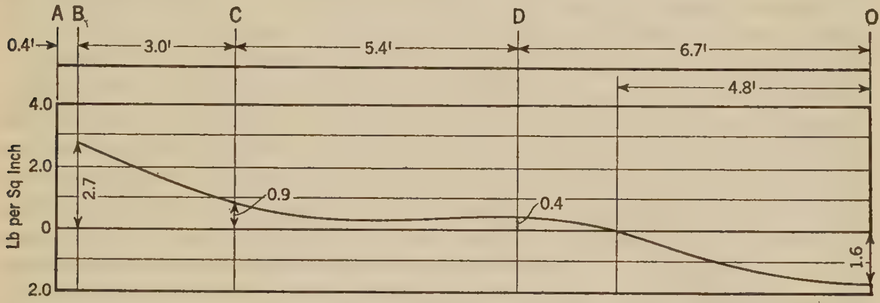
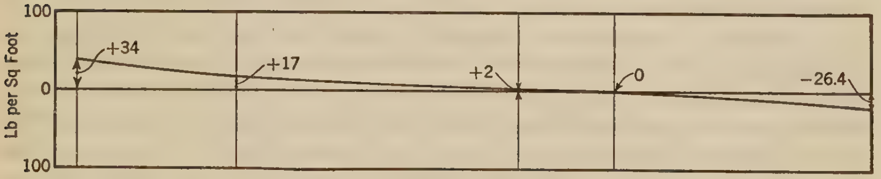


FIG. 11

To have no relief in weight at the center line, the rate of change of the shearing stress $\frac{ds_s}{dx}$ must equal zero at point O. The author apparently makes this assumption in conclusion 3 by stating that the diagram of shearing stress has a horizontal tangent at the center line. The writer admits that this may be the case. Evidence of such a condition will come only through experimental study. However, it does appear that the field data fail to support this theory, provided symmetry is assumed. A study of Fig. 7 shows that, on July 8, 1935, there is an overload at point O (P_d being greater than W_a) and also that downward vertical shearing stresses exist, although the values of these stresses have been omitted from Fig. 7. The writer has drawn the complete ($W_a - P_d$)-curve for this date and has determined the downward vertical shearing stress at point O in an approximate fashion (see Fig. 12). In making this rough computation of the vertical shearing stress at point O, the writer followed a method similar to that used by the author in his determination of the shearing stresses at the other points on the periphery. The area under the ($W_a - P_d$)-curve between the zero point (4.8 ft to the left of point O) and point O was computed on the basis that the curve was parabolic. This area, represented by $2/3 \times 4.8 \times (1.6 \times 144)$, equals 740 lb—the accumulated relieved weight between the two points in question. The area of action of the shearing stress is 28 sq ft, using the value of 28 ft for the height of the silt column above point O in Fig. 3(*b*). The approximate downward shearing stress equals $\frac{740 \text{ lb}}{28 \text{ ft}}$ or 26.4 lb per sq ft. On July 8, 1935, the vertical downward pressure P_d equaled 39.1 lb per sq in., the weight of the overburden W_a equaled 37.5 lb per sq in. (showing an overload of 1.6 lb per sq in.), and the vertical shearing stress (downward) was approximately 26.4 lb per sq ft. Fig. 7 shows that, on July 16, 1935, and on August 8, 1935, the weight of the overburden was relieved by the action of upward vertical shearing stresses. The field data show that, on August 29, 1935, no relief in weight, and thus no shearing stresses, existed at point O. Accepting the field data as exact, it must follow that on the first three dates symmetry was lacking and the rate of change of the shearing stress $\frac{ds_s}{dx}$ was not equal to zero at point O. If pressure intensities had been taken at corresponding points on each side of the center line of the lining, it would have been possible to determine if symmetry did exist. Reviewing Table 1, downward pressures, p_d, appear to have been taken only at points O, D, C, and B on the periphery; thus, it is impossible to tell whether there was a symmetrical condition about the center line on the aforementioned four dates. The deformations arising from the decrease in the horizontal diameter of the lining that commenced as the lining was being driven through the silt may have caused the symmetry to be broken.



(a) RELIEVED WEIGHT ($W_a - P_d$)



(b) AVERAGE SHEARING STRESS

(Upward) $-s_s$ (Downward)

FIG. 12.—COMPLETE ($W_a - P_d$)-CURVE FOR JULY 8, 1935

Part 3.—The decrease in the horizontal diameter commencing immediately after the erection of the lining is an extremely interesting phenomenon. The undeniable implication arising from this occurrence is that the horizontal pressure on the lining must exceed the vertical pressure. Such a relation between the horizontal and vertical pressure is not commonly expected in an undisturbed soil mass. The ratio has been found to be approximately 0.7 for plastic soil of the type of the Hudson River silt.[10] It appears logical that this phenomenon should occur only in soils that have a considerable ratio of the horizontal pressure to the vertical pressure in the mass. Since the field data for the Lincoln Tunnel show that the ratio of horizontal pressure to vertical pressure is less than unity (0.74 on August 29, 1935), an explanation of this phenomenon is necessary. The writer feels that the author's explanation—namely, that its cause must be sought at the very beginning of the shoving—is a probable answer.

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[10] "Design of Tunneled Sewers in Unstable Ground," by Robert G. Hennes, *Civil Engineering*, April, 1932, p. 248.

An examination of Fig. 9 showing the horizontal pressures on the tunnel lining indicates to the writer that the general shape of the curve from point O to point B′ (P to N) follows the straight-line law (triangular pressure distribution) as used by Rankine in his formula. This would lead to the conclusion that the Rankine formula may be applied without serious error to the determination of lateral pressures from point O to point B′. Conclusion 7 is excellent. It is logical that the Rankine formula should not be used for determining lateral pressure below point B′ since the pressure on this section of the lining is shown by Fig. 9 to be fairly uniform.

General Remarks.—Great care should be exercised in interpreting field data such as those for the Lincoln Tunnel. Fig. 7 shows that the maximum relief in weight, occurring at point B on the periphery, ranges from 2.7 lb per sq in. on July 8 to 5.8 lb per sq in. on August 29. Also, at the center line of the lining (point O) the "relief in weight" varies from an overload of 1.6 lb per sq in. on July 8 to a load of O lb per sq in. on August 29. In view of the fact that the accuracy of measurements made with the special silt gage and plugs was limited to the range, 1 lb per sq in. to 2 lb per sq in., it is entirely possible that much of the difference in W_a and P_d may have been caused by the inability of the gages to record true pressures. Certainly the overload at point O on July 8 could be attributed to this cause. It is within the realm of possibility that there was no relief in pressure. Although the writer acknowledges that this is not likely, he merely wishes to state that a slight inaccuracy in the pressure-gage measurements could easily account for a difference in pressure of 2.7 lb per sq in. In Fig. 3, the author presents the computed values of the overburden pressure W_a at ten points on the periphery, using in these computations a value of 64 lb per cu ft as the density of the Hudson River water and a value of 104 lb per cu ft as the weight of the saturated clayey silt. An error of as little as 4 lb per cu ft in estimating the density of the silt could produce a change in pressure on the lining equal to some of the values considered in this paper as relief in weight. To illustrate the effect of a variation in density of the silt on the lining pressure, a column of the silt mass 20 ft high and 1 sq in. in cross section, with a volume of 240 cu in. or 0.14 cu ft, has been selected. A variation in density of 4 lb per cu ft would cause a change in pressure of 0.56 lb per sq in. at the base of this column. Such an error in determining the density of the silt mass is certainly possible, since there is doubt in the writer's mind that a homogeneous, uniform material exists for 20 ft below the bed of the Hudson River. In addition, it does not seem likely that the bed of the Hudson River is a level ground surface. Therefore, it is difficult to justify using values for the height of the silt columns above the tunnel lining expressed to the nearest 0.1 ft. Certainly field measurements of this accuracy would be most difficult to make in the near fluid silt. A slight variation in the height of the overburden could cause substantial changes in the values of W_a and could be responsible for some of the difference between W_a and P_d hitherto attributed to relief in weight. Inaccuracies in gage readings, errors in estimating the density of the silt mass, and errors in determining the height of the silt column above the tunnel lining could produce results giving a pressure difference less than that which actually existed. The only purpose in emphasizing these possible sources of error is to caution against too strict an interpretation of the field data. Unless these data are exact (a fact which any engineer familiar with field and laboratory measurements will doubt), misleading results may be obtained. No criticism of the author's valuable interpretation of the field data on the Lincoln Tunnel is intended.

This excellent paper is a real contribution to the literature on earth pressures on tunnels. Many more similar studies are needed before definite conclusions can be reached as to the variations in pressure on tunnel linings constructed in plastic soils.

D. P. KRYNINE,[11] M. AM. SOC. C. E.—The possibility of the transfer of the weight of the overburden above a tunnel to the adjacent earth or rock mass is evident from the simple analogy shown in Fig. 13. If body A (weight W) is pressed between two plates B, it will not fall, provided the force of friction between body A and plates B balances the weight W. A system of shearing stresses acting upward is created in body A. Presumably, this system of shearing stresses extends into plates B. Another set of shearing stresses is created in plates B to balance the former system and to transmit the weight W to the earth. This analogy corresponds to the case of a tunnel in rock which may be in equilibrium without lining, or to the case of holes pierced in a masonry wall.

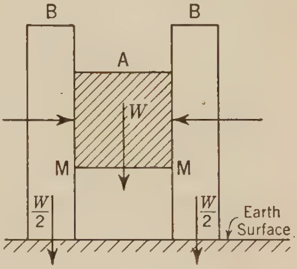

FIG. 13.—FRICTION ANALOGY

If friction between body A and plates B still exists but is not strong enough to prevent body A from falling, an upward force F, with a value less than W, may support body A in equilibrium. In this case, a system of shearing stresses with a resultant $W - F$ is set up in body A. Shearing stresses in plates B transmit part of the weight $W - F$ of body A to the earth. This situation corresponds to that in a tunnel provided with lining or to the redistribution of the weight of grain in a bin. The force F is analogous to the pressure absorbed by the lining of a tunnel or by the bottom of a bin, respectively, whereas the balance of the weight $W - F$ is transmitted to the mass surrounding the tunnel or to the walls of the bin, respectively. Secondary forces and stresses are introduced into the mass during construction of a tunnel so that the analogy of Fig. 13 may be complicated or altered.

Two extreme cases of circular tunnels in plastic soils are: (*a*) Circular holes in plastic rocks such as granite or basalt, when no lining is required because of the high shearing strength of the rock material; and (*b*) circular tunnels in plastic organic silt of restricted shearing strength of the Lincoln Tunnel type. Circular tunnels in weak plastic rocks or in plastic clays are classified between the two limits.

Shear Diagram for the Case of a Tunnel in Rock.—Fig. 14 represents a circular hole in rock, the radius of the hole being r and the height of the overburden above the level of the sides (haunches) being *h*. Because of symmetry, there is no vertical shearing stress along the center line OO′. The weight of the part of the overburden AOO′B, separated from the remainder of the mass by section AB, is carried by that section in cantilever action. The average shearing stress (in tons per square foot) acting along section AB equals area AOO′B (in square feet), divided by ordinate AB (in feet) and multiplied by the unit weight of the rock material (in tons per cubic foot). The shear diagram, at the bottom of Fig. 14, has been plotted for a shallow tunnel ($h/r = 2$) and a rather deep tunnel ($h/r = 6$). For plotting purposes, the weight of the rock material has been assumed at 1 ton per cu ft, the vertical scale of shearing stresses (tons per square foot) being the same as that of horizontal distances (in feet). To use this diagram, the ordinates must be multipled by the actual unit weight of the rock material (in tons per cubic foot), and due correction must be introduced for the actual ratio of the vertical and horizontal scales. From Fig. 14 it is evident that the average shearing stress at the sides (haunches) of a tunnel in rock without lining cannot be larger than $r\,\gamma$, in which r is the radius of the tunnel in feet and γ is the unit weight of rock material in tons (or in pounds) per cubic foot. Assume that the maximum shear on some oblique plane may be twice the vertical shear; then it may be concluded, as an example, that a circular tunnel (r = 30 ft) would stand without lining at any depth in

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[11] Research Associate in Soil Mechanics, Dept. of Civ. Eng., Yale Univ., New Haven, Conn.

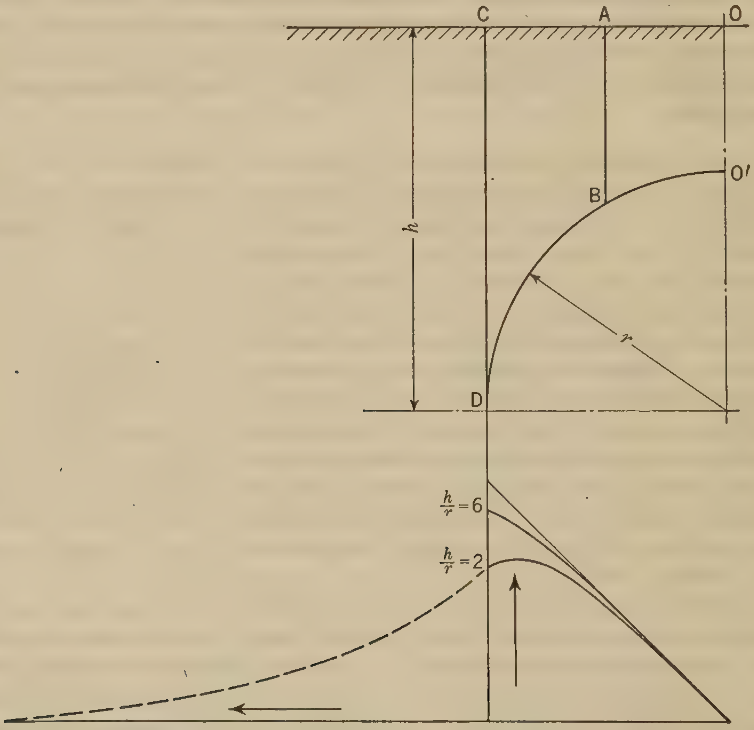


Fig. 14.—Shear Diagram for the Case of a Tunnel in Rock

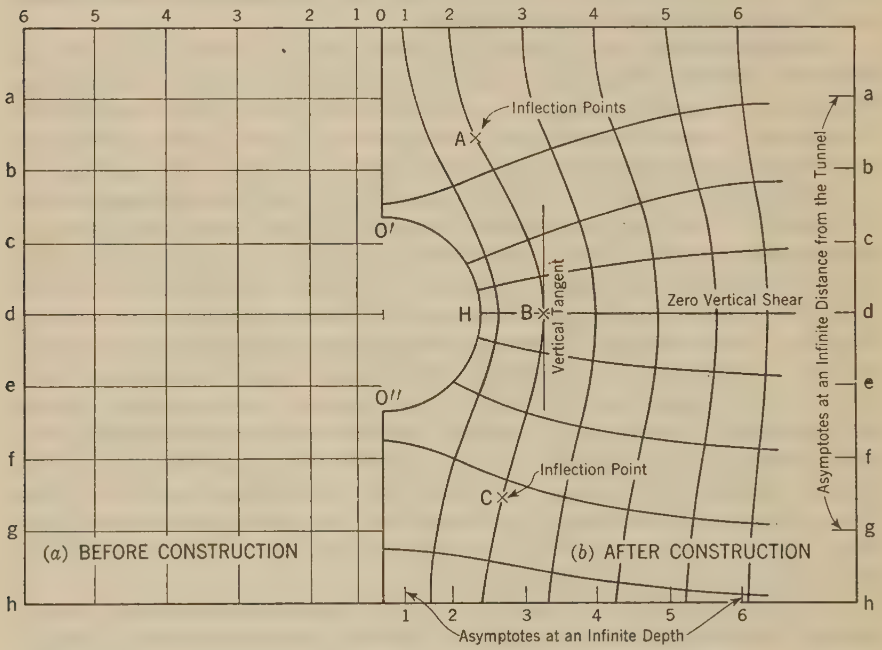
rock weighing 160 lb per cu ft, if the shearing strength of that rock is 2 × 30 × 160 = 9,600 lb per sq ft, or about 66 lb per sq in. Although the diagram in Fig. 14 shows that the average shearing stress at the sides (haunches) of a tunnel in rock, without lining, increases only slightly with the depth of the tunnel, geological conditions, such as fissures or cleavage planes, may change the foregoing computations substantially.

In a semi-infinite mass, of rock or of earth, there is a certain ratio between the horizontal and vertical pressures which is assumed constant. It is termed "coefficient of pressure at rest" and is often designated by the symbol K_o. As soon as a tunnel is constructed, the value of K_o next to the tunnel (and, theoretically, throughout the entire rock mass) decreases. The coefficient of pressure at rest preserves its original value K_o only at an infinite distance from the tunnel. Thus, the mass tends to move toward the tunnel; hence, the horizontal shearing stresses acting against the impending movement of the mass are directed from the tunnel. Shearing stresses in the overburden COO′D (Fig. 14) itself obviously act in the same direction (shown by an arrow in Fig. 14). Since there is no horizontal shear at infinity, the diagram of shearing stresses beyond the sides (haunches) of the tunnel must have the shape indicated by the dashed lines in Fig. 14. The shape of the shear-stress diagram for deep tunnels suggests the possibility that the maximum average shear may occur somewhere beyond the sides (to the left of the haunches in Fig. 14). It is needless to add, however, that both Figs. 14 and 15 are considered simple freehand sketches and that no attempt has been made to correlate Fig.14 with Fig. 15. Both of these illustrations are intended to furnish qualitative pictures of corresponding phenomena only.

The shear diagram in Fig. 14 refers to both horizontal and vertical stresses. It may be concluded that the weight of the rock mass close to the boundary is relieved by shearing stresses not only above the tunnel but also on both sides of the tunnel; and nowhere close to that boundary is the mass overloaded. To determine what part of the mass takes care of the relieved weight, the pattern of principal stresses around the tunnel must be considered.

Principal Stresses Around a Tunnel in Rock.—Before the construction of the tunnel, the principal stresses run vertically (the major principal stress) and horizontally (the minor principal stress). This original pattern is shown in Fig. 15(*a*). Since the piercing of a circular hole originates shears both vertically and horizontally, the directions of principal stresses change. However, a major principal stress (for instance, that emanating from point 2, Fig. 15(*b*)), after being distorted by the shearing stresses, tends to return to its original vertical line, reaching the line at an infinite depth below the hole. This is necessarily so, since at an infinite depth the hole exerts no influence. Since the major principal stress above and below the hole is distorted in two opposite directions, the shearing stresses above and below the tunnel act in opposite directions. Thus, the relieved weight close to the boundary of the mass is transmitted to that part below the sides (haunches) of the tunnel.

The vertical shearing stresses, which relieve the weight of the silt mass, become changed in sign at the horizontal line passing through the sides (haunches) of the tunnel H (Fig. 15) so that this horizontal line is the locus of zero vertical and horizontal shear. Mathematical analysis (which is beyond the scope of this paper) may show a more precise position of the locus. The principal stresses along the locus in question are vertical and horizontal; hence, the tangent to the distorted trajectory of the major principal stress is vertical.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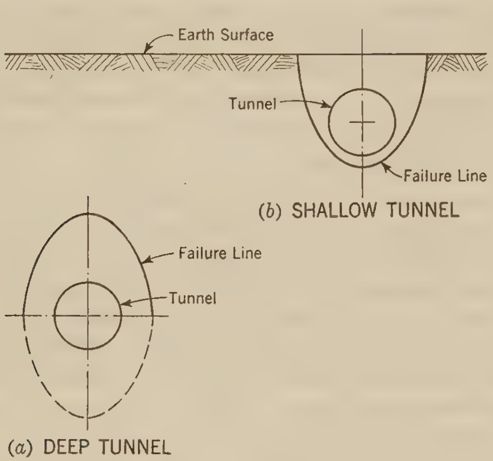
Inflection Points
A
O′
H
B
Vertical Tangent
Zero Vertical Shear
O″
Inflection Point
C
Asymptotes at an Infinite Distance from the Tunnel
(a) BEFORE CONSTRUCTION
(b) AFTER CONSTRUCTION
Asymptotes at an Infinite Depth

FIG. 15.—PATTERN OF PRINCIPAL STRESSES BEFORE (*a*) AND AFTER (*b*) CONSTRUCTION OF A TUNNEL IN ROCK, WITHOUT LINING

The trajectory of the major principal stress at the center line of the hole consists of two vertical straight lines and a semicircumference belonging to the hole. All other trajectories of major principal stresses (for instance, the trajectories emanating from point 2, Fig. 15(*b*)) consist of four distinct parts: (*a*) From the boundary to which the trajectory is normal to the inflection point A; (*b*) from the inflection point A to the intersection with the locus of zero vertical shear (point B); (*c*) back to the inflection point C, and (*d*) finally to infinity. The vertical shear along the trajectory increases from zero at the boundary to a maximum at the inflection point A. Another maximum value, but of the opposite sign, is found at the inflection point C. At the center line of the hole, O′ and O″ are considered inflection points.

Arching in Rock.—It has been suggested that an arch forms in the material above the tunnel. As far as rock or other homogeneous plastic material of considerable shearing strength is concerned, there is no such arch. The relief in pressure above the tunnel is due to the action of shearing stresses which transmit part of the weight of the rock mass close to the boundary to the lower strata. As the result of the action of the shearing stresses, straight trajectories of major principal stresses become distorted as in Fig. 15(*b*).

Case of Insufficient Shearing Strength of the Plastic Material Surrounding the Tunnel.—If a lateral support of a semi-infinite mass (such as a retaining wall) yields, a wedge separates from the remainder of the mass. In a similar way, if the lining of a tunnel is flexible and yields, there may be considerable settlement above the tunnel which is analogous to the yield of a lateral support.



(*b*) SHALLOW TUNNEL

(*a*) DEEP TUNNEL

FIG. 16.—SCHEMATIC DIAGRAMS OF FAILURE ZONES AROUND TUNNELS

In both cases, a failure line (sliding surface) is formed. In the case of a tunnel, this failure line bounds a closed zone in which rock or clay material tends to descend toward the opening. J. R. Dinsdale has shown[12] how the material falling from the roof and sides of a mine tunnel was gradually replaced with logs until the final closed failure zone was reached. Fig. 16 shows schematically a closed and an open failure zone, the latter taking place if the tunnel is a relatively short distance underground. R. Fenner makes some simplifying assumptions and, using abundant mathematics, even determines the ratio of the axes of the elliptical failure line.[13] This ratio is given as tan $\left(45° - \frac{\phi}{2}\right)$, ϕ being the angle of internal friction of the material in which the tunnel is built. According to Mr. Fenner, in the case of a shallow tunnel, the failure line is parabolic. Of course, the question of the theoretical shape of the failure zones above a tunnel is subject to discussion. The writer believes, however, that the basic idea of a closed failure zone above a deep tunnel, or of an open failure zone in the case of a shallow tunnel, is sound. The same considerations as those for a hole pierced in rock may be applied to a closed failure zone. Thus, the distorted trajectories of major principal stress run around the closed failure zone, and the weight of the material above the zone is transferred by the shearing stresses to sections at and close to the bottom of the failure zone. The lining is loaded by the weight of the material inside the failure zone only.

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[12] "Ground Pressures and Pressure Profiles Around Mining Excavations—1," by J. R. Dinsdale, *Colliery Engineering*, December, 1935, Fig. 8, p. 409.

[13] "Untersuchungen zur Erkenntniss des Gebirgedruckes," by R. Fenner, *Glueckauf*, Vol. 74, 1938, pp. 681 and 705.

Before applying these considerations to tunnels in silt of the Lincoln Tunnel type, it is necessary to determine whether a failure line due to downward motion of the silt material may be formed above a tunnel of this type. It should be realized also that, in a general case, shearing stresses above the tunnel are due to two causes—(*a*) consolidation of the silt layer of variable thickness, and (*b*) deflections of the lining. Even when no failure zone is formed, there are upward shearing stresses, the magnitude of which is controlled by the value of the vertical displacements due to causes (*a*) and (*b*).

Subaqueous Tunnels in Silt Constructed by the Shield Method.—The pressure on the lining of subaqueous tunnels in silt, constructed by the shield method, passes through the following stages:

(1) When the lining is shoved and the bulge is being formed, a frictional force along the shearing surfaces (Fig. 2(*a*)) acts downward, thus increasing the preconstruction weight of the overburden. This increase in weight lasts until the bulge starts moving downward. It is quite possible, therefore, that, on July 8, 1935 (see Fig. 7 and Table 1), there was an overload at the center line of the lining of 1.6 lb per sq in. (The lining is deflected during the shoving more than in the subsequent stages of the work when pressure and, hence, deflections are relieved by erosion, dredging, and upward shear and are perhaps somewhat redistributed.)

(2) Because of erosion and especially dredging of the bulge, the pressure on the lining decreases. At the same time two processes start—consolidation of the silt mass above the lining and healing of bonds along the sliding surfaces, as shown in Fig. 2. In stage 2, the pressure on the lining first decreases to its preconstruction value and afterward becomes less than that value. This statement is made with the understanding that a correction for compressed air has been introduced.

(3) Consolidation and healing of the mass proceed, and the pressure on the lining decreases. Examination of Fig. 3 shows that tunnels of the Lincoln Tunnel type are too shallow to develop a closed failure zone at this stage. In addition, it is possible that an open failure zone such as that shown in Fig. 16(*b*) cannot form since there are already sliding surfaces, as shown in Fig. 2. Examination of Fig. 7 reveals also that the shearing stress at the sides (haunches) of the tunnel practically equaled the shearing strength of the silt. Hence, although in a general case it is possible to decrease the pressure on the lining by making it more flexible, in the case of the Lincoln Tunnel there was little chance to do so. The relieving shearing stresses at this stage were mostly due to differential settlement (that is, to differences in the thicknesses of the consolidating silt layer at the center and at the sides (haunches) of the tunnel). The question "For what period of time will this relief exist?" may be answered only on the basis of the existing data—for about 253 days in the case of the Lincoln Tunnel and for about 90 days in the case of the Detroit Tunnel.

(4) The final stage, if any, could be characterized by the tendency of the earth mass to return to the preconstruction stress condition. The record period in the case of the Lincoln Tunnel was too short, and there is no published evidence of such return. The Detroit Tunnel data cannot be applied to the Lincoln Tunnel unconditionally since the method of construction in both cases

was not the same. The writer does not know whether or not, in a general case, the plastic mass, in which a circular tunnel is built, returns completely to its preconstruction-stressed state.

Vertical and Horizontal Silt Pressure on the Lining.—There was a relief in weight of the overburden in the case of the Lincoln Tunnel equal to about 10% at the sides (haunches) and to practically nothing at the center line, the average relief being about 6%. The diagram of actual vertical silt pressure may be obtained easily from Fig. 3(*c*) by introducing corrections from Fig. 7. The diagram in question is not a rectangle as is sometimes assumed.

In regard to the horizontal pressure on the lining, the writer believes that the Rankine formula is applicable—at least at the upper part of the tunnel. To be more precise, there is triangular lateral pressure distribution in the upper part of the lining. The question of lateral pressure is closely connected with that of the angle of internal friction in silt (or a similar value of the angle of repose). When the Holland Tunnel was designed, the angle of internal friction of 20° was assumed; and, using a certain conventional computation procedure, results very close to those measured during the construction of the Lincoln Tunnel were obtained. With an angle of internal friction of 9°, and using a somewhat different computation procedure, acceptable results may be obtained. In reality, neither 20° nor 9° is a correct value. The angle of internal friction of nonconsolidated plastic clays (and the Hudson River silt is very similar to such clay) is not far from zero. The writer believes firmly that this is true for the Lincoln Tunnel silt. Additional empirical formulas for the determination of the lateral pressure in the Hudson River silt may be proposed, for instance: "Compute the hydrostatic pressure at the given depth and add 10% for silt action." Such a recipe furnishes even better results than the formulas.

If another tunnel is ever built in the Hudson River silt, the lateral pressure may be computed in the foregoing conventional manner by assuming a 20° angle of friction, provided, however, that such a new tunnel is driven to the same depth and under the same loading conditions as the Lincoln Tunnel. In such a case, however, it is easier to scale the lateral pressure directly from Fig. 9.

Shearing Surfaces; Angle of Repose of the Silt.—As Mr. Bird rightly states, the straight-line surfaces in Fig. 2(*a*) are only crude approximations. The statement which Mr. Drucker attributes to Jacob Feld,[6] M. Am. Soc. C. E.—namely, that the angle which the shearing surfaces make with the vertical equals the angle of friction—is also perfectly correct, provided, however, that the soil under consideration is cohesionless sand with an angle of friction of 30°. However, this condition is not true for the Hudson River silt. Some idea of the "angle of repose" of the Hudson River silt is afforded by the following fact concerning the organic silt in the Thames River in Connecticut (which is very similar to that of the Hudson River). During the construction of the first bridge at New London, Conn.:

> "The dredging was * * * performed as an independent preliminary operation, with a clam-shell dredge, and carried down 18 to 23 feet below the river bottom, the material standing on a slope of one to one * * *."[14]

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[14] *The Engineering and Building Record and the Sanitary Engineer*, October 11, 1890, p. 296.

In other words, the angle of repose of the silt in question is 45°. Obviously, this value is only apparent and is due to the cohesion of the material. The writer has had opportunities to observe the behavior of the Thames River silt when excavation for the piers of the highway bridge (opened in 1943) was in progress. Large silt blocks extracted from the bottom of the river maintained their shapes—sometimes with practically vertical faces.

Tangent to the Shear Diagram at the Center Line of the Tunnel.—The writer originally assumed that this tangent to the diagram of the relieving shearing stresses at the center line was horizontal. Whether or not this assumption is true can be determined only from a considerable number of observations. In any case, the writer does not insist on this horizontality, especially because in the diagram, Fig. 14, the tangent in question is oblique. It should be explained, however, that the angle in Fig. 14 varies according to the change in ratio of the vertical and horizontal scales of the drawing.

Compressed Air Action, Passive Resistance, and Soil Constant.—If a circular metallic tube of a certain thickness is subjected to internal pressure, stresses develop in the tube, the radial stress at the outside surface of the tube being zero, because of boundary conditions. If the tube is buried in earth, the tube and the surrounding earth mass work as a unit. The combined system "tube plus soil mass" may be visualized as a tube of a very large (in some cases infinite) thickness made of a nonhomogeneous material; and, if the tube proper is subjected to internal pressure, the entire mass of surrounding soil is stressed, the radial stress being zero only at the boundary of that earth mass (in some cases at infinity). At the outside surface of the tube proper, there is a radial stress. Since action and reaction are equal, the radial stress equals the stress in the earth mass at the given point.

If the soil mass surrounding the tube in question is loaded for the first time, the use of the "soil constant" is fully justified; but, if the soil mass is already heavily stressed, an additional stress increment would produce only a very restricted displacement. Stress-strain diagrams of a heavily loaded soil mass illustrate this fact clearly when a further increment of load produces a surprisingly small strain. Thus, in the design and analysis of earth structures, the live load is not considered at all. The live load certainly produces stresses in the earth structure, but causes practically no deformations.

The so-called "soil constant" is a conventional "tool" used by some tunnel designers for convenience in computations. Deflection values computed using the "soil constant" are excessively large if the mass has been previously loaded. Therefore, the belief that passive resistance of a certain magnitude can be developed only by producing the displacement computed from the corresponding "soil constant" is decidedly erroneous.

Conclusion.—The writer greatly appreciates the valuable discussions by Messrs. Drucker and Bird which have offered the writer an opportunity to clarify several important points in the theory of tunnels. Many thanks are due to both discussers for their painstaking work. Many more field observations, analyses of data, and discussions by the profession are needed, however, before the problem of pressures on tunnels can be definitely solved.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2253

STABILITY AND STIFFNESS OF CELLULAR COFFERDAMS

By Karl Terzaghi,[1] M. Am. Soc. C. E.

With Discussion by Messrs. G. G. Greulich, Raymond P. Pennoyer, Dean P. Tsagaris, R. T. Colburn, A. F. Hedman, Adolph J. Ackerman, Savile Packshaw, Martin W. Lautz, J. S. Miller, Robert F. Legget, D. P. Krynine, J. J. Polivka, and Karl Terzaghi.

Synopsis

The incentive for preparing this paper was given by observations made during and after the construction of a cellular cofferdam which constitutes the side-walls and the middle wall of a shipway. A digest of these and various other observations on cellular cofferdams indicated that the theories on which the design of these cofferdams is usually based give the designer an inadequate conception of the factors that determine the stability of cofferdams.

The paper is divided into three parts. The first deals with past experiences with cellular cofferdams in general, the deflection of the crests of cofferdams, and the permeability of the sheet-pile enclosures. In the second part current methods for designing cellular cofferdams on rock, sand, and clay foundations are reviewed, examined, and revised. The third part contains conclusions. Discussion is specifically limited to cellular cofferdams whose cells are filled with sand or sand and gravel.

Part I. Practical Considerations

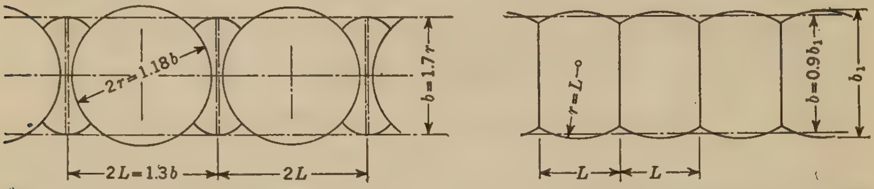
Experiences with Cellular Cofferdams.—The first cellular cofferdam was built in 1908 or 1909 at Black Rock Harbor, Buffalo, N. Y. (1).[2] It consisted of seventy-seven prismatic cells, whose horizontal dimensions were 30 ft by 30 ft. It rested on a rock foundation. Since all the walls were straight, it was inevitable that the longitudinal walls should bulge badly between the cross-walls. In one of the cells, the inner wall, with an unsupported height of about 30 ft, bulged 3.45 ft between cross-walls; yet it is reported that the inward movement of the crest of the wall at the cross-walls nowhere exceeded about 1 in.

Note.—Published in September, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Cons. Engr. and Lecturer, Graduate School of Eng., Harvard Univ., Cambridge, Mass.

[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix I).

In 1910 a cellular cofferdam was built in connection with the raising of the battleship *Maine* in the harbor of Havana, Cuba (2). It consisted of twenty cylindrical cells of the type shown in Fig. 1(a), interconnected by short arcs on the outer side of the dam. The subsoil consisted of a stratum of soft silt and mud resting on medium soft clay. The cells were filled with clay. It was expected that the dam would be self-supporting. However, while the salvage crews were pumping inside the cofferdam, the inward deflections increased so alarmingly that it was decided to increase the stability of the structure by adding an inner berm. During the last stage of excavation and pumping it was even considered necessary to brace the cofferdam against the hull of the *Maine.*



(*a*) CIRCULAR CELLS (*b*) SEMICIRCULAR CELLS

FIG. 1.—PRINCIPAL TYPES OF CELLULAR COFFERDAMS

In spite of the difficulties encountered, some engineers realized that the basic idea of the design of the cofferdam for the *Maine* was sound. Therefore, many cofferdams of a similar type have since been constructed. They are known as cellular cofferdams with circular cells.

In 1915 or 1916 the U. S. Army Engineers repeated the Black Rock Harbor experiment by building a similar cofferdam in Troy, N. Y. (3). Since nature gave a drastic hint at Black Rock Harbor that straight-length walls were not appropriate, the cross-walls were connected by arcs. The success of this dam initiated the development of cellular cofferdams with semicircular cells, Fig. 1(*b*).

Some engineers have expressed the opinion that cellular cofferdams are less reliable than other types because several important cellular cofferdams have failed. The design of such cofferdams on a base other than rock requires more judgment and experience than does the design of a double-wall cofferdam with a broad inner berm on a similar base. Hence failures of cellular cofferdams due to faulty design may be more frequent than similar failures of double-wall cofferdams; yet failures of this category cannot be ascribed to an inherent defect of the type of construction.

The only inherent weakness of the cellular cofferdams is the vulnerability of the locks. One or several cells of a cofferdam may burst while all the others remain intact. An individual cell may fail either because of a defect in the material of one of the locks or because a sheet pile was driven out of a lock without the incident being noticed; yet the number of failures due to both causes combined is so small, and, on an average, the costs of remedying the consequences of the incident are so unimportant that the hazard is far more than compensated for by the economic advantages. This statement is corroborated by the fact that large construction organizations have used the cellular cofferdam type many times.

Since cellular cofferdams are both economical and durable, they can also be used to great advantage as parts of permanent structures such as shipways, quay walls, breakwaters, and the like, provided the foundation conditions are satisfactory and suitable material for filling the cells can be secured at low cost.

Deflection of Cellular Cofferdams.—Experience indicates that the sheet-pile walls of cellular cofferdams can stand a horizontal deflection of the crest of the cofferdam equal to more than 10% of its height without injury to the locks. On the landward section of the cellular cofferdam, built about 1916 at the site for a lock at Brown's Landing on the Cape Fear River, North Carolina (4), the top of four cells moved in a distance of 7 ft 6 in. (25% of the free height of the cofferdam) because the pin connection of diagonal tie rods attached to the cross-walls failed; yet the published description of the incident indicates that the locks of these cells remained intact.

The deflection of a cellular cofferdam with given dimensions depends, quite obviously, to a large extent on the elastic properties of the fill in the cells. As a consequence, the relation between the unbalanced horizontal pressure on a cofferdam and the corresponding deflection is very similar to the relation between stress and strain in soils in general. The strain in soils increases roughly with the square of the stress and the deflection of the crest of the dams increases roughly with the square of the unbalanced horizontal pressure. Both the strain in soils and the deflection of the crest of cofferdams increase at slowly decreasing rates under constant stress or pressure conditions.

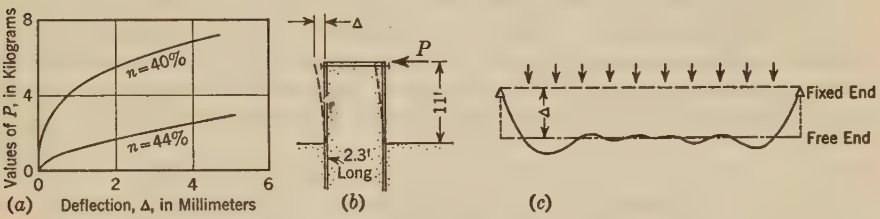


Fig. 2.—Deflection of Cellular Cofferdams

The decisive influence of the elastic properties of the fill on the horizontal deflection of a cofferdam was demonstrated experimentally by I. A. Rimstad with tests on a small-scale model of a double-wall, sheet-pile cofferdam (5). In one of the tests the space between the two walls was filled with loose sand (porosity $n = 44\%$) and in the second with the same sand in a compacted state ($n = 40\%$). In Fig. 2(*a*) the ordinates represent the unbalanced horizontal force and the abscissas the corresponding deflection of the crest of the dam.

In many instances the ends of a cellular cofferdam are tied to rigid structures or to cross-walls which prevent the horizontal pressure from producing a deflection of the ends. A cofferdam between two such ends may be considered as a thick, rectangular slab whose lower edge is fixed and whose vertical edges are freely supported. The upper edge is free. The theory of the horizontal deflection of such slabs under the influence of water pressure—which leads to the conclusion that the upper edge would deflect as indicated in Fig. 2(*c*)—is beyond the scope of this paper. Each half of the section between the two deflection maxima resembles the graphic representation of damped vibrations. The peculiar shape of the deflection curve is due to the resistance of the wall against bending in horizontal planes. If the distance between the end supports is not greater than a few times the height, the two maxima are absent, the elastic line is roughly parabolic, and the maximum deflection is very much smaller than the deflection Δ indicated in Fig. 2(*c*).

From the foregoing, it is evident that the deflection of a cofferdam with a given cross section acted upon by a given system of horizontal forces depends not only on the elastic properties of the cells but also, to a certain extent, on the ratio between the length and the height of the dam. Since a cellular cofferdam consists of two very different materials, steel and soil, its elastic properties can be compared to those of a composite construction material such as reinforced concrete. These properties are chiefly governed by those of the soil in the cells, but the steel walls are likely to stiffen the structure very considerably (see Part II).

The elastic properties of a given soil in a cell depend to a large extent on the method of filling the cell. There is no laboratory procedure that would yield reliable advance information on the state of the soil in the cell and on the corresponding elastic properties of the soil. Quantitative information regarding the stiffening effect of the sheet-pile walls also is not yet available. However, experience indicates that the crest of a cellular cofferdam really deflects approximately as theory leads one to expect. Fig. 3 represents the deflection of the crest of the cofferdam for the land section of the piers between West 44th Street and West 47th Street on the east side of the Hudson River in New York, N. Y. Similar curves were obtained from deflection observations on a very much wider cofferdam between West 47th Street and West 52d Street (6). In both instances the inside of the cofferdam was supported by a rock berm which was provided in the original design. In spite of the presence of this berm, the crest of both dams assumed the characteristic W-shape shown in Fig. 2(*c*) at an early stage of unwatering.

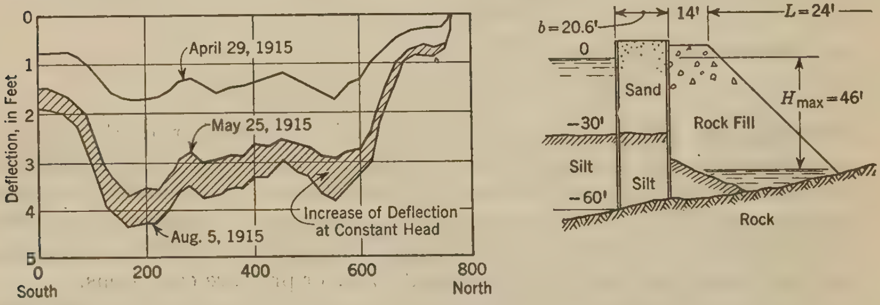


Fig. 3.—Deflection of Cofferdam Crest for the Land Section of Hudson River Piers (H = 46 Ft)

The same general tendency was noticed while the water was pumped out of two full-length submerged shipways whose walls consisted of self-supporting cellular cofferdams of the diaphragm type (7). The deflection of two of the longitudinal (east–west) walls is represented in Fig. 4. The reason for the excessive deflection near the west end of the wall in Fig. 4(*a*) will be discussed subsequently. The deflection of the two short walls was roughly parabolic, and, in accordance with theory, the greatest deflection of these walls was very much smaller than that of the long ones; yet there was no difference between the east–west sections and the north–south sections except in length.

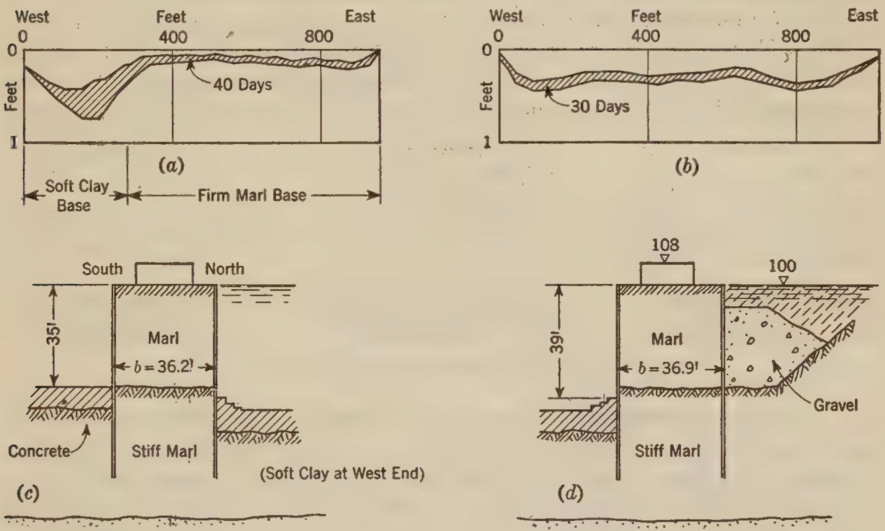


Fig. 4.—Deflection of a Steel Sheet Cofferdam for a Submerged Shipway

Permeability of Cellular Cofferdams and of Their Sheet-Pile Enclosures.—The water that percolates across a cellular cofferdam must pass through two different mediums—the rows of sheet piles and the soil between the sheet piles. The passage through each medium causes a loss of head, disclosed by a difference in the position of the line of saturation on either side of the obstacle against flow. If the cells of a cofferdam are filled with fairly clean sand and gravel, the loss of head is almost exclusively caused by the outer row of sheet piles. All engineers who have reported on experiences with cellular cofferdams filled with sand and gravel emphasized the fact that they had no trouble with water. Many single-wall cofferdams consisting of a row of steel sheet piles have leaked so badly that it was necessary to stop the flow through the locks by artificial means such as washing cinders or sawdust into the leaky locks. Hence, the consistently favorable reports on pumping between cellular cofferdams suggest that the relative watertightness of these dams is primarily due to the tension in the locks. Since there is tension only above the level of the base of the fill in the cells, that part of the rows of sheet piles below the level of the base is likely to be much more permeable than the upper part.

Favorable experiences with pumping from the space within cellular cofferdams filled with sand and gravel have led many engineers to believe that the practical implications of the permeability of the rows of sheet piles are negligible under any circumstances and that a cellular cofferdam with weep holes constitutes a mass of soil enclosed between one impermeable wall and one permeable wall. This conception may be justified under certain conditions, but it is unwarranted under others and it can lead to false conclusions.

Fig. 5 represents vertical sections through a cellular cofferdam, backed by water. The cofferdam rests on an impermeable base and the inner row of sheet piles is provided with weep holes. If the cofferdam is filled with clean, coarse sand (Fig. 5(*a*)), the quantity of water that leaks through the joints of the outer wall is very small. Hence, only the bottom of the cell will be covered with water. If the cells are filled with fine, silty sand (Fig. 5(*b*)), the quantity of water that flows through the outer wall is still smaller because the fill constitutes one more obstacle against the passage of the water. Nevertheless, the line of saturation in the cell should occupy a high position, because the permeability of the fill is so low that the discharge even of a small quantity of water requires a steep hydraulic gradient. Finally, if the cells are filled with clay (Fig. 5(*c*)), the loss of head caused by the passage of the water though the outer row of sheet piles is negligible compared to the loss of head in the clay. As a consequence, the water percolates through the dam as if the outer row were nonexistent.

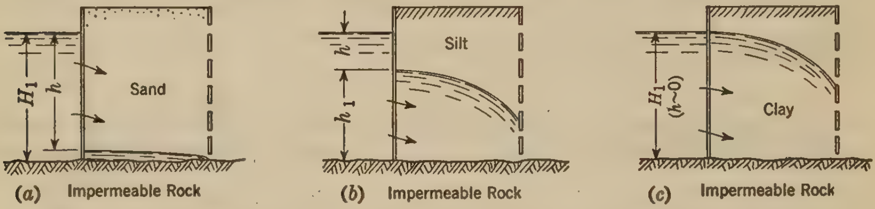
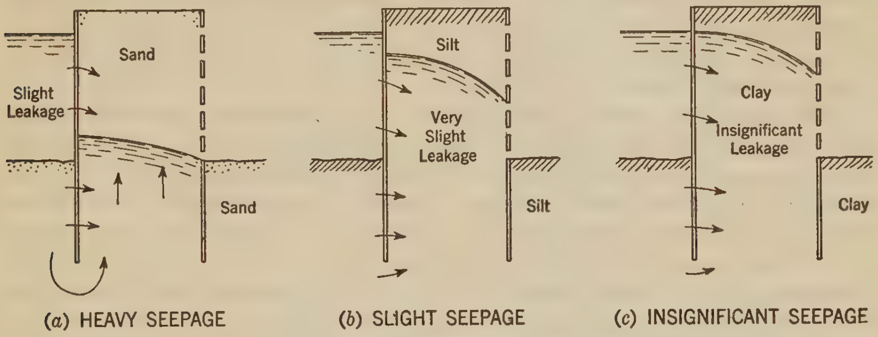


FIG. 5.—CELLULAR COFFERDAMS ON ROCK FOUNDATIONS

In practice the condition illustrated by Fig. 5(*a*) has rarely been encountered. The cells of the cofferdam for building the Troy dam and locks were filled with sand and gravel (3); yet the line of saturation started at the outside a short distance below water level. The average slope of the line was 1 (vertical) on 2 (horizontal). This observation suggests that the silt content in the fill of the cells was sufficient to establish conditions almost identical to those illustrated by Fig. 5(*c*). The cells of the cofferdam at Pickwick Landing (Tennessee Valley Authority) in Tennessee were filled with sand and gravel, and the inner wall of the cofferdam was provided with weep holes (8). Nevertheless, the line of saturation started at the outside of the dam only a few feet below the free water level, and it was almost horizontal over three fourths of the width of the cells. In spite of these experiences and observations, the customary methods of computing the tension in the locks of the sheet piles are based on the tacit assumption that the weep holes suffice to drain the fill in the cells almost completely, provided the fill consists of sand and gravel.

The cofferdams illustrated by Fig. 5 rest on an impermeable rock foundation. The seepage across a cofferdam with such a foundation merely affects the tension in the locks. If a cofferdam rests on a soil foundation, the effect of the seepage on the subsoil must also be taken into consideration. Fig. 6 shows three vertical sections through cofferdams resting on the same kind of soil as that with which the cells are filled. Because of the permeability of the subsoil, the water which seeps across the outer row of sheet piles combines with that which detours the buried part of the outer row and rises through the subsoil. This additional flow raises the line of saturation to a position slightly above that shown in Fig. 5. Otherwise the flow diagrams are identical.



(*a*) HEAVY SEEPAGE (*b*) SLIGHT SEEPAGE (*c*) INSIGNIFICANT SEEPAGE

FIG. 6.—CELLULAR COFFERDAMS ON A SOIL FOUNDATION

The conditions illustrated by Fig. 6(*c*) existed in the cofferdam around the hull of the *Maine* in Havana. Since the flow toward the pumps across the cofferdam with a total length of about 1,000 ft was less than 4 gal per sec, the engineers who worked on the dam were surprised by the high position of the line of saturation. They seem to have expected conditions like those illustrated by Fig. 6(*a*).

The water that passes through openings in the locks of the buried part of the inside row of sheet piles flows through the soil along the inside of this row in a vertical upward direction toward the surface adjoining the inside of the cofferdam. The mechanical effects of such a flow through clay are usually so insignificant that they cannot be noticed, because clay has a great resistance against scour. On the other hand, if the sheet piles were driven into fine silt, which has almost no resistance against scour, conditions are different. Because of the percolation through the locks, "boiling" may start inside the sheet-pile cutoff in silt at a small fraction of the head which would be required to produce the same effect in the same material on the inside of a perfectly impermeable cutoff. Hence, in connection with cofferdams on silt the permeability of the sheet-pile walls is a factor that must be considered.

PART II. DESIGN OF CELLULAR COFFERDAMS

General Considerations.—Cellular cofferdams may fail by bursting of the cells, by sliding on their base, by the shear deformation created by an inward tilt, or by a failure of their foundation.

Practically all the failures by bursting which occurred during or immediately after filling the cells can be traced back to the driving out of the lock. There seems to be no record to indicate that the cells of a cofferdam burst during application of the overturning moment, unless the cofferdam was in an advanced state of tilting, accompanied by a deflection of the crest of 20% or 25% of the height. There seems to be no record of a failure by sliding. All the failures by overturning that have come to public attention, such as the failures of several cellular cofferdams on the Mississippi River (6) and of the one at Grand Coulee (9) in the State of Washington have been preceded by boiling or piping. Since the foundation conditions have a decisive influence on design, the cellular cofferdams on rock, sand, and cohesive soil will be discussed under separate subheadings.

Notation.—The letter symbols in this paper are defined where they first appear, in the text or by diagram, and are assembled for reference in Appendix II. To simplify equations for the stability of cellular cofferdams, it is customary and justified to assume that the curved inner and outer rows of sheet piles are replaced by fictitious straight ones spaced a distance b apart (see Fig. 1). For cofferdams with circular cells (radius r) $b = 1.7\,r$, and for those with semicircular cells (outside width b_1) $b = 0.9\,b_1$. The lengthwise walls of the ideal substitute for cofferdams with circular cells (Fig. 1(a)) will be assumed to be interconnected by straight cross-walls. These substitute cross-walls are arranged in pairs between the cells. The space between the two walls of each pair is assumed to be empty, but very narrow. The distance between two consecutive pairs is $2\,L$. The cells are assumed to be filled with sand unless some other material is specified in the text.

Cellular Cofferdams on Rock.—The customary methods for designing cellular cofferdams on rock were well described in 1934 by Raymond P. Pennoyer, Assoc. M. Am. Soc. C. E. (10). Rules that can be considered representative of current methods have been published by the Carnegie-Illinois Steel Corporation (11). According to these rules, the sliding resistance on the base per unit of length of the dam is given by the formula,

$$F_b = b\,H\,\gamma\,\tan\rho \qquad (1)$$

and the corresponding factor of safety of a cofferdam, acted upon by a horizontal force P per unit of length, with respect to sliding is

$$G_s = \frac{b\,H\,\gamma\,\tan\rho}{P} \qquad (2)$$

Since the force which tends to displace the dam by sliding also produces an overturning moment, the unit pressure on the base of the dam increases from a minimum at the outer toe to a maximum at the inner one as indicated by the plain curve cd in Fig. 7(a). The area abdc is equal to the product $b\,H\,\gamma$ in Eq. 1. This equation is based on the assumption that the cofferdam does not fail by sliding until the shearing stress at every point of the base of the dam is equal to the normal stress at that point multiplied by the coefficient of friction, $\tan\rho$. Hence the distribution of the shearing stresses at the instant of failure should be given by the dash-dotted line ef whose ordinates are equal to those of curve cd times tan ρ. By the laws of mechanics it can be shown that the shearing stress at the outer toe must be almost equal to zero and at the inner toe it cannot be greater than the friction between the fill and the inner row of sheet piles per unit of area of the sheet piles at the elevation of the base. Hence, the shear-stress line is as indicated by the upper boundary of the shaded areas and the real sliding resistance is equal to the area abef $= b\,H\,\gamma$ tan ρ reduced by the shaded areas. However, the amount of reduction is not sufficiently great to require consideration.

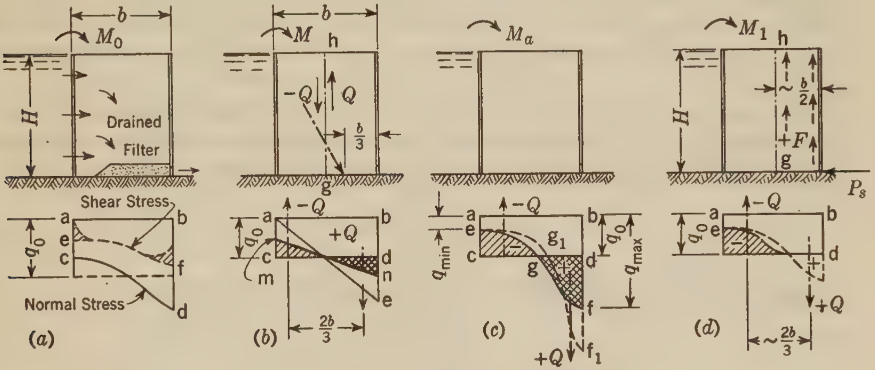


FIG. 7.—CELLULAR COFFERDAMS ON ROCK

Since γ in Eq. 2 represents not the submerged but the real unit weight of the fill in the cells, the equation is not valid unless the fill in the cells is almost completely drained. However, as stated, there is no record to indicate that this condition ever existed. If the line of saturation is at an average height H_s above the base of the dam, the corresponding sliding resistance F_{bs} is lower than F_b, Eq. 1.

The pressure that produces friction is equal to the difference between the total pressure and the water pressure on the base. Since the unit weight of the fill in a saturated state is but slightly greater than the unit weight γ of the fill in a moist state, the shearing resistance on the base of a fill saturated to a height of H_s above the base is:

$$F_{bs} = b\,(H\,\gamma - H_s\,\gamma_w)\tan\phi \qquad (3)$$

On the cofferdams at Troy and at Pickwick Landing the value H_s was almost equal to H. Nevertheless, the dams did not fail by sliding. Failure was probably prevented by the roughness of the rock surface which compensated for the effect of incomplete drainage. If the rock surface is rough, the coefficient of sliding of the fill on the base is much closer to the coefficient of internal friction of the fill than to the customary value of 0.5 which represents the coefficient of friction between sand and fairly smooth rock. The sliding resistance of a cofferdam on rough rock is further increased by the resistance against the sliding of the lower edge of the inner row of sheet piles on the rock surface. Since the sheet piles of the inner row are pressed down by wall friction, this supplementary resistance may be very considerable. However, if the rock surface is smooth, these factors do not apply, and hence the incomplete drainage of the fill may lead to failure by sliding. Therefore, if the rock surface is not very rough, Eq. 2 should not be used unless the designer takes care that the cells are really drained. This can be done by covering the bottom of the cells with a filter layer which communicates through weep holes with the space surrounded by the dam as shown in Fig. 7(*a*). The effect of the drainage provisions should be supervised on the job by means of telltale pipes.

In formulating the condition that the cofferdam should be safe with respect to overturning, it is generally assumed that the cofferdam can be regarded as a gravity wall. The moment required to overturn a gravity wall is

$$M_{\max} = \frac{b^2}{2} H \gamma \qquad (4)$$

and the factor of safety of the wall with respect to overturning is

$$G_s = \frac{M_{\max}}{M} \qquad (5)$$

If a perfectly elastic gravity wall with a rectangular cross section rests on a perfectly rigid base, the overturning moment M_a required to shift the point of application of the resultant pressure on the base from the center point to the inner boundary of the middle third is, approximately,

$$M_a = \frac{1}{3} M_{\max} = \frac{b^2}{6} H \gamma \qquad (6)$$

If the overturning moment is equal to M_a, the unit pressure on the base increases from zero at the outer edge to $2 H \gamma$ at the inner edge as indicated by the straight line ae in Fig. 7(*b*), and the corresponding factor of safety is

$$G_s = \frac{M_{\max}}{M_a} = 3 \qquad (7)$$

If a cofferdam is acted upon by an overturning moment M, Eq. 7 is satisfied, provided the average width b of the dam is equal to

$$b_m = \sqrt{\frac{6\,M}{H \gamma}} \qquad (8)$$

Because of Eq. 7, a cofferdam with width b_m (Eq. 8) is considered safe, provided it is safe with respect to sliding. This conclusion is open to serious objections.

According to the theory on which Eqs. 4 and 6 are based the distribution of the unit pressure over the base of a cofferdam is determined by a straight-line law as indicated by the straight line mn in Fig. 7(*b*). As soon as the overturning moment M becomes equal to M_a, Eq. 6, the unit pressure at the outer edge of the cofferdam should become equal to zero (pressure area abe); yet it is obvious that this condition cannot exist in a dam whose cells are filled with sand. Even if arching develops above the outer part of the base of the fill to the fullest possible extent, the pressure on the outer part of the base must have a considerable positive value $q_{\min}$, which cannot decrease in spite of an increase of the overturning moment. Hence, the most unequal distribution of the pressure on the base that can possibly develop under the influence of an overturning moment M_a, Eq. 6, must be like that shown by the curve ef in Fig. 7(*c*). Since $q_{\min}$ cannot decrease, an increase of the overturning moment and the corresponding increase of the force Q in Fig. 7(*c*) to Q_1 must cause the neutral point g to shift toward the right. The pressure line egf moves into the position eg_1f_1. The base of the cross-shaded pressure triangle dfg decreases, but its area increases from Q to Q_1. Hence the toe pressure $q_{\max}$ increases not in simple proportion but roughly with the square of the overturning moment, and the cofferdam would fail by overturning while the point of application of the resultant pressure on the base is still fairly close to the middle third, provided the locks could stand the tension due to the rapidly increasing side pressure at the toe.

However, before this failure occurs, the cofferdam is likely to fail from another cause which has never received any attention. To demonstrate the existence of this cause in simple terms, the customary assumption is made that the distribution of the pressure on the base of the dam is such as shown by the straight line mn in Fig. 7(*b*). The shearing stresses that act at the surface of contact between the fill in the cells and the sheet piles prior to the application of the overturning moment will be disregarded. In Fig. 7(*b*) the shaded areas indicate the stresses on the base due to the overturning moment M. If Q is the total force represented by each triangle, one can write $M = \frac{2\,b}{3}\,Q$; or

$$Q = \frac{3\,M}{2\,b} \qquad (9)$$

Equilibrium requires that the total shearing force on the neutral plane gh per unit of length of the dam should be equal to Q. The shearing resistance on this plane, S' per unit of length of the dam, is equal to the earth pressure P_e times the coefficient of internal friction $\tan\phi$ of the fill. The earth pressure per unit of area of a vertical section through the fill at depth Z is equal to the vertical unit pressure, $\gamma\,Z$, in the fill at that depth multiplied by an empirical constant C known as the coefficient of earth pressure

$$p_e = \gamma\,C\,Z \qquad (10)$$

The earth pressure on gh per unit of length of the dam is

$$P_e = \tfrac{1}{2}\,\gamma\,C\,H^2 \qquad (11)$$

and the shearing resistance S' is

$$S' = P_e \tan\phi = \tfrac{1}{2}\,\gamma\,C\,H^2 \tan\phi \qquad (12)$$

To the value S' must be added the friction in the locks, because no failure by shear along gh, Fig. 7(*b*), can occur without simultaneous slippage in the lock in this plane. If p_e, Eq. 10, is the unit earth pressure on gh at depth Z, the tension in the locks of an arc with radius r at depth Z is

$$t = r\,p_e = r\,\gamma\,C\,Z \qquad (13)$$

Total tension in a lock with length H is

$$T = \tfrac{1}{2}\gamma C H^2 r \dots\dots (14a)$$

and the total resistance against slippage in the lock is

$$T f = \tfrac{1}{2}\gamma C H^2 r f \dots\dots (14b)$$

wherein f is the coefficient of lock friction. Since the rows of sheet piles intersect the **Y**-piles at an angle of 120°, the tension in the locks of the cross-walls and the corresponding lock friction is equal to that in the arcs. The cofferdams with circular cells contain two cross-walls per cell and the length of each cell is $2L$; those of the diaphragm type contain one cross-wall per cell and the length of each cell is L. Hence, the resistance to shearing along gh contributed by the lock per unit of length of either type of cofferdam is

$$S'' = \frac{T f}{L} = \tfrac{1}{2}\gamma C H^2 \frac{r}{L} f \dots\dots (15)$$

To obtain the total average shearing resistance S on gh per unit of length of the cofferdam, S'' must be added to S', Eq. 12, whence,

$$S = S' + S'' = \tfrac{1}{2}\gamma C H^2 \left(\tan\phi + \frac{r}{L} f\right) \dots\dots (16)$$

The ratio $\frac{r}{L}$ is close to or equal to unity. Hence, approximately,

$$S = \tfrac{1}{2}\gamma C H^2 (\tan\phi + f) \dots\dots (17)$$

The corresponding factor of safety G_s with respect to failure by shear along gh is equal to the ratio between the shearing resistance S (Eq. 17) and the shearing force on gh which is equal to Q (Eq. 9) or

$$G_s = \frac{S}{Q} = \gamma \frac{b H^2}{3 M} C (\tan\phi + f) \dots\dots (18)$$

If one face of the cofferdam with height H is acted upon by water pressure, the overturning moment is

$$M_0 = \frac{1}{2}\gamma_w H^2 \frac{H}{3} = \frac{1}{6}\gamma_w H^3 \dots\dots (19a)$$

and

$$G_s = 2\frac{\gamma b}{\gamma_w H} C (\tan\phi + f) \dots\dots (19b)$$

The customary theories of cofferdams are based on the assumption that every vertical section through the fill in the cells is acted upon by the active Rankine pressure, P_A per unit of width of the section, involving

$$C = \tan^2\left(45 - \frac{\phi}{2}\right) \dots\dots (20)$$

To visualize the practical implications of Eqs. 19b and 20, it is assumed that $\phi = 30°$, $f = 0.3$, $\gamma = 110$ lb per cu ft, $\gamma_w = 62.5$ lb per cu ft, and $b = 0.85\,H$.

Evaluating Eq. 5 on the basis of these data it is found that the factor of safety with respect to overturning should be 3.8. In reality, it is certainly not greater than about 2. However, the factor of safety with respect to shear, Eq. 19*b*, is only $G_s = 2 \frac{110}{62.5} \frac{0.85\,H}{H} 0.577^2 (0.577 + 0.3) = 0.87$. In other words, the cofferdam would fail by shear along gh, Fig. 7(*b*), although even the real factor of safety with respect to overturning is still excessive for a temporary structure. Replacing $\phi = 30°$ by $\phi = 34°$ and $\phi = 40°$, the values of $G_s = 0.82$ and 0.74 instead of 0.87 are obtained.

The average width *b* of the cellular cofferdams with circular cells which were built on a rock foundation is not greater than about 0.85 *H*. The width of some of them was somewhat smaller; yet none of them failed by shear along gh, Fig. 7(*b*). The coefficient of friction on this plane cannot be greater than tan ϕ. Therefore, the normal pressure on this plane, and, as a consequence, the horizontal earth pressure in the entire fill in the cells, must be considerably greater than the active Rankine pressure involving

$$C > \tan^2\left(45° - \frac{\phi}{2}\right) \qquad (21)$$

Comparing Eq. 17 with Eq. 12, it can be noted that the lock friction increases the average coefficient of shearing resistance along gh (Fig. 7(*b*)) from tan ϕ to (tan $\phi + f$). The value of f is likely to be fairly close to 0.3. If $\phi = 30°$ and tan $\phi = 0.58$, the lock friction increases the factor of safety with respect to failure by shear by about 50%. Hence, the stabilizing influence of the lock friction is by no means negligible.

Because of the lock friction a cellular cofferdam may be as much as 50% more stable than a double-wall cofferdam with equal width and height, but without cross-walls. If the inner side of a cellular cofferdam is supported by a berm whose height is almost equal to the height of the dam, both the cross-walls and the inner row of sheet piles are acted upon by equal earth pressures from both sides. This condition practically eliminates the tension in the locks, but it also eliminates the lock friction. The lock friction becomes fully operative at a very much smaller deflection of the cofferdam than does the shearing resistance of the fill in the cells. Therefore, a cellular cofferdam with a free inside wall (self-supporting cellular cofferdam) is likely to deflect less than a cofferdam reinforced by an inner berm. In this connection, it may be mentioned that the crest of one of the two cofferdams with rock-fill berms between West 44th Street and West 49th Street in New York deflected more than 4 ft (Fig. 3) and the other one more than 6 ft. This is far in excess of the usual deflection of the crest of self-supporting cellular cofferdams.

On some cofferdams, the locks were lubricated to facilitate pulling the sheet piles. From a functional point of view, it would be preferable to increase the lock friction by artificial means.

Finally the possibility of a failure of cofferdams by a bursting of the locks must be considered. The tension per unit of length of a lock is given by Eq. 13. The greatest value which Z in this equation can assume is H. For

$Z = H$, t becomes equal to

$$t_1 = r \gamma C H \qquad \text{(22a)}$$

The standard method for computing the greatest tension per unit of length in a lock is based on the assumption that C in Eq. 22*a* is equal to the coefficient of the active Rankine pressure, $\tan^2\left(45° - \frac{\phi}{2}\right)$, whence

$$t_1 = r \gamma H \tan^2\left(45° - \frac{\phi}{2}\right) \qquad \text{(22b)}$$

The sheet piles are chosen so that t_1 is equal to or smaller than the allowable lock tension which in turn is equal to the failure tension divided by about 2.5. Hence, the factor of safety with respect to a failure by tension in the locks is assumed to be about 2.5. However, the discussion of the resistance of the cellular cofferdams against shear has led to the conclusion that the factor C in Eq. 22*a* is considerably greater than $\tan^2\left(45° - \frac{\phi}{2}\right)$. Hence, the factor of safety with respect to a failure of the locks by tension is smaller than 2. Furthermore, the method of computation represented by Eq. 22*b* fails to consider that the application of the overturning moment may increase the pressure in the fill at the inner toe. According to the standard concept, the application of the overturning moment should change the initial pressure area abdc, Fig. 7(*b*), for a cofferdam with the customary dimensions to about abe which involves an increase of the soil pressure at the inner toe by roughly 100%. This would reduce the factor of safety to less than unity and the cells would fail by bursting. In reality the lower boundary of the pressure area for the base of a cofferdam is not straight as indicated by the lines mn and ae in Fig. 7(*b*) but curved as shown by the line ef in Fig. 7(*c*). Hence the real increase of the lock tension due to the overturning moment is still more important.

Finally, Eq. 22*b* involves the tacit assumption that the fill in the cells is almost completely drained. If the line of saturation is close to the top of the fill as it was in the cellular cofferdams at Troy and at Pickwick Landing, the cells not only should burst; they should explode.

In contrast to this conclusion, experience has demonstrated the following fact: If the locks are strong enough to stand the lateral pressure of the fill before an overturning moment is applied, they are also strong enough to stand the subsequent changes produced by the overturning moment, provided the cofferdam is safe in every other respect. Hence, the increase of the lock tension due to the overturning moment cannot possibly be important. As a matter of fact, there is no evidence that the real factor of safety with respect to a failure by bursting of the cells of any one of the successful cellular cofferdams was appreciably smaller than 2; yet equilibrium requires that the overturning moment be fully counterbalanced by a couple on the base of the dam. Hence the question arises: "What is the nature of the forces which constitute this couple?"

To answer this vital and obvious question, the interaction between the fill in the cells and the adjoining sheet piles must be considered. If there were no initial shearing stresses at the surface of contact between the fill and the inner row of sheet piles, the vertical unit pressure in the fill adjoining the inner row at any depth Z below the top of the cells would be γZ, and at the inner edge of the base it would be γH. Any increase of the vertical pressure in the fill beyond these values involves a shortening of the height of the fill, and the shortening is resisted by the friction between the fill and the sheet piles. Since an insignificant slip along the surface of contact between fill and sheet piles is sufficient to mobilize the full sliding resistance along this surface, the unit pressure on the inner part of the base cannot increase beyond γH until the shearing stresses on the inner contact face become equal to the full value of the friction between sand and sheet piles. At that stage the pressure on the base of the fill is about as shown by the area abde in Fig. 7(*d*). On the inside (right-hand side) of the neutral plane gh the unit pressure on the base of the fill is still γH, but the vertical contact faces of the fill are acted upon by vertical friction forces, F_1 per unit of width of the sheet piles or Q per unit of length of the dam. Since the sand pressure per unit of width is equal to $\frac{1}{2} \gamma H^2 C$, the friction force F_1 is

$$F_1 = \tfrac{1}{2} \gamma H^2 C \tan \delta \qquad (23)$$

The total perimeter of one cell with length $2L$ of a cellular cofferdam with cylindrical cells, Fig. 1(*a*), on one side of the neutral plane gh, is $(2L + b)$ or

$$l = 1 + \frac{b}{2L} \qquad (24)$$

per unit of length of the dam. Hence,

$$Q = l F_1 = \tfrac{1}{2} \gamma H^2 C \left(1 + \frac{b}{2L}\right) \tan \delta \qquad (25a)$$

Since $\frac{L}{b}$ is roughly equal to 0.65 or $\frac{b}{2L} = 0.77$,

$$Q = 1.77 \times \tfrac{1}{2} \gamma H^2 C \tan \delta \qquad (25b)$$

The sum of all the vertical reactions produced by the overturning moment is equal to zero. Therefore, the sum of the forces acting on the fill on the outside (left-hand side) of the neutral plane gh must be equal to $-Q$ per unit of length of the dam. Assuming that the lever arm of the couple constituted by the forces $+Q$ and $-Q$ is roughly equal to $\frac{2b}{3}$, the overturning moment M_1 required to establish the state of stress on the base of the fill, indicated by the area abde in Fig. 7(*d*), is

$$M_1 = \tfrac{2}{3} b Q = 1.18 b \times \tfrac{1}{2} \gamma H^2 C \tan \delta \qquad (26a)$$

The overturning moment produced by the water pressure on one side of the dam is given by Eq. 19*a*:

$$M_0 = \tfrac{1}{6} \gamma_w H^3 \qquad (26b)$$

and the ratio m between the required moment M_1 and the acting overturning moment M_0 is

$$m = \frac{M_1}{M_0} = 3.54 \frac{b}{H} \frac{\gamma}{\gamma_w} C \tan \delta \qquad (27)$$

The average width of the base of cofferdams on rock which are built to withstand full water pressure is usually about 0.85 H. Assuming $\gamma = 110$ lb per cu ft, $\gamma_w = 62.5$ lb per cu ft, $C = 0.4$, and $\tan \delta = 0.4$, $m = 0.85$. This value, $m = 0.85$, indicates that about 85% of the total overturning moment M_0 is carried by the couple $+Q$ and $-Q$. In other words the unit pressure at the inner edge of the base of the fill does not begin to increase until about 85% of the total overturning moment M_0 is applied. The increase of the unit pressure due to the remaining 15% of the overturning moment is well within the margin of safety. This explains the empirical fact that the cells of a cofferdam with the customary dimensions burst either before the overturning moment is applied or not at all. However, once the overturning moment becomes equal to 1.2 M_1 or 1.3 M_1, a further increase of the moment must cause as rapid an increase of the soil pressure at the inner toe as it would from the very outset under the fictitious conditions illustrated by Fig. 7(*c*). Hence, after the overturning moment becomes greater than 1.2 M_1 or 1.3 M_1, the danger of a failure due to the bursting of the cells increases rapidly.

The maximum lock tension is further reduced by the forces which counteract an inward movement of the lower edge of the inner row of sheet piles. According to Eq. 22*a* the lock tension should increase in simple proportion to the depth below the top of the cells. Assuming that the elongation of the sheet-pile walls increases in simple proportion to the lock tension, the vertical section through the inner row of sheet piles should be fairly straight at any stage of filling and pumping, and its lower edge should move inward. However, experience indicates that the inner row of sheet piles bulges at an elevation $\frac{H}{4}$ or $\frac{H}{3}$ above the level of the base and that the lower edge of the sheet piles does not move. Such a deformation can occur only if the distribution of the tension along a lock is similar to that indicated by the area acc$_1$ in Fig. 8. Since there is little doubt that the earth pressure increases from the top of the cells toward the bottom as indicated by the pressure area aa$_1$c, equilibrium requires that the lower edge of the sheet piles be acted upon by a horizontal force P_s. The origin of this force is obvious. The force F_1 (Eq. 23) presses the lower end of the sheet piles to the rock surface and the pressure produces a frictional resistance, P_s. Fortunately for the cofferdam, the force P_s acts precisely at the point where the lateral earth pressure is greatest. Hence, the greatest lock tension is considerably smaller than that which would correspond to the soil pressure at the base. At the lower edge of the sheet piles where, according to theory, the lock tension should be greatest, it is equal to zero.

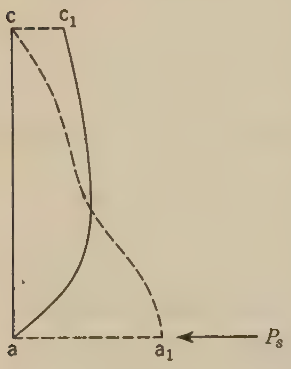


Fig. 8

To make the statement of the fundamental assumptions complete, it should be mentioned that throughout the discussion the empirical factor C in Eq. 10 was tacitly assumed to have the same value for every point in the fill of the cells. This is certainly not more than a very crude approximation. First, by analogy to the conditions in the backfill of a retaining wall it is very probable that the value C for the central part of the fill is much greater than that for the fill adjoining the inner row of sheet piles. Second, if C were a constant along every horizontal section through the inner row, the arcs should remain circular. However, the observations described subsequently show that this conclusion is not even approximately correct. The laws according to which the value C varies within the fill of the cells are still unknown. Hence, an attempt to evaluate the greatest lock tension on a purely theoretical basis would appear rather futile.

The preceding leaves no doubt that two of the three customary equations for the design of cellular cofferdams (Eqs. 8 and 22*b*) are inadequate. The third, Eq. 1, is based on the assumption that the fill in the cells is almost completely drained; yet this formula has been advocated without a statement of the important fact that weep holes alone may not suffice to drain the cells although they may be filled with sand or sand and gravel. So far the equations have not done any harm because they have fortunately been used on the basis of very conservative values for the soil constants. However, some day an ambitious designer may conceive the idea of determining the soil constants by laboratory tests and using the values furnished by the tests. The result could be fatal for the cofferdam.

The equations derived by the writer contain an empirical factor C (ratio between horizontal and vertical soil pressure in fill of cells) whose value could be determined only by actually measuring both the horizontal and the vertical soil pressures at different points in the fill of the cells in full-sized cellular cofferdams before and after the application of the overturning moment. So far no such observations have been made. On the basis of the known behavior of cellular cofferdams, the writer estimates that the value C for the middle part of the fill in the cells ranges between 0.4 and 0.5. Along the inner row of sheet piles it may be considerably smaller. In no event could C be determined by laboratory tests.

In the preceding paragraphs an attempt was made to analyze the conditions that determine the stability of cellular cofferdams on rock. The analysis disclosed the existence of a potential source of failure by shear along vertical planes (Eq. 19*b*) which has been ignored in the past, although it is much more likely to take place than failure due to overturning. It also demonstrated the decisive influence of the lock friction and of the friction between the fill and the walls of the cells on the strength and stability of the dam (Eqs. 17 and 22*a*). This opens up at least a theoretical possibility for increasing the stability or for reducing the dimensions of cellular cofferdams by artificially increasing the roughness of the surface of the steel. Since all the equations contain the empirical factor C whose value, for the time being, can only be guessed at, the equations cannot yet be used directly in connection with the design of cellular cofferdams. Pending further development, it seems safest to proceed on the basis of past experience as explained in the following paragraphs.

The average width b_0 of all the cellular cofferdams built to date on a rock foundation to sustain nothing but one-sided water pressure was about equal to 0.85 H. The water pressure that acts on the outside of the cofferdam per unit of length of the dam at the instant of overtopping will be designated by P_0 and the corresponding overturning moment is equal to M_0, Eq. 19*a*. Experience has shown that cofferdams with such dimensions are fully adequate; yet the preceding numerical computations have demonstrated that their factor of safety with respect to failure by shear along the neutral plane is certainly not greater than 1.5. Hence, the ratio, $\frac{b_0}{H} = 0.85$, may be considered an acceptable basis for design. To keep the factor of safety of a cofferdam with respect to sliding (Eq. 2) unaltered while increasing the unbalanced horizontal pressure from P_0 to $P = n_1 P_0$, it is necessary to increase its width from b_0 to $\frac{b_0 P}{P_0} = n_1 b_0$. To maintain the same condition with respect to a failure by shear along a vertical plane (Eq. 18), while increasing the overturning moment from M_0 to $M = n_2 M_0$, it is necessary to increase the width from b_0 to $\frac{b_0 M}{M_0} = n_2 b_0$. Hence, if a cofferdam with height H on a rock foundation is acted upon by a horizontal force $P = n_1 P_0$ producing an overturning moment $M = n_2 P_0$, it should have an average width,

$$b = n\, b_0 = 0.85\, n\, H \quad \text{(28a)}$$

in which n is the greater one of the two values n_1 and n_2.

According to the assumption illustrated by Fig. 1(*a*), the radius r of circular cells is $r = 0.59\, b$, whence,

$$r = 0.59\, b = 0.59\, n\, b_0 = 0.59 \times 0.85\, n\, H = 0.50\, n\, H \quad \text{(28b)}$$

The outside width b_1 of cellular cofferdams with semicircular cells (Fig. 1(*b*)) is 1.11 b; thus:

$$b_1 = 1.11\, b = 1.11\, n\, 0.85\, H = 0.94\, n\, H \quad \text{(28c)}$$

The distance L between the cross-walls of such dams is 0.65 b (±), whence,

$$L = 0.65\, n\, 0.85\, H = 0.55\, n\, H \quad \text{(28d)}$$

Since the arcs intersect the cross-walls at the Y-piles on an angle of 120°,

$$r = L = 0.55\, n\, H \quad \text{(28e)}$$

The maximum lock stress is given by Eq. 22*b*. The writer is not aware that any designer has ever used a value of much less than 0.4 for C in Eq. 22*a*. As a result of this wise practice, there have been no serious failures due to the bursting of cells. Therefore, the writer suggests that the greatest lock tension in the walls of cells filled with sand, or sand and gravel, be expressed by means of the empirical equation,

$$t = 0.4\, \gamma\, r\, H \quad \text{(28f)}$$

Eq. 28*f* does not create the illusion that it rests on a scientific basis and leaves no margin for misinterpretation. If the cells are filled with a soil with a high clay content, the resistance of the soil against sliding is likely to be deficient. If the surface of the rock is not very rough, adequate provisions should be made for draining the fill in the cells and the state of drainage should be ascertained at different stages of pumping within the cofferdam by measuring the elevation of the water level in telltale pipes whose lower ends are located at different depths below the crest—for instance, $\frac{H}{4}$, $\frac{H}{2}$, and $\frac{3\,H}{4}$. It is also advisable to observe the deflection of the crest of the dam during, and after, the time when the space between the cofferdams is being pumped out.

The cross-walls of some of the older cellular cofferdams, such as those of Troy (3), Cape Fear River (4), and St. Louis Intake (12) were reinforced by diagonal tension rods or tension beams. These members increased the stiffness of the cofferdams very considerably. This was demonstrated impressively on the Cape Fear River, where the failure of the pin connection of the tie rods in four cells caused the top of these cells to advance a distance up to 7.5 ft, whereas the deflection of all the other cells was unimportant. To utilize the tie-rod principle to full advantage, it would be necessary to make the pin connections strong enough to permit stretching of the tie rods well beyond the yield point. Otherwise, the pin connections fail before the shearing resistance of the fill in the cells is fully active.

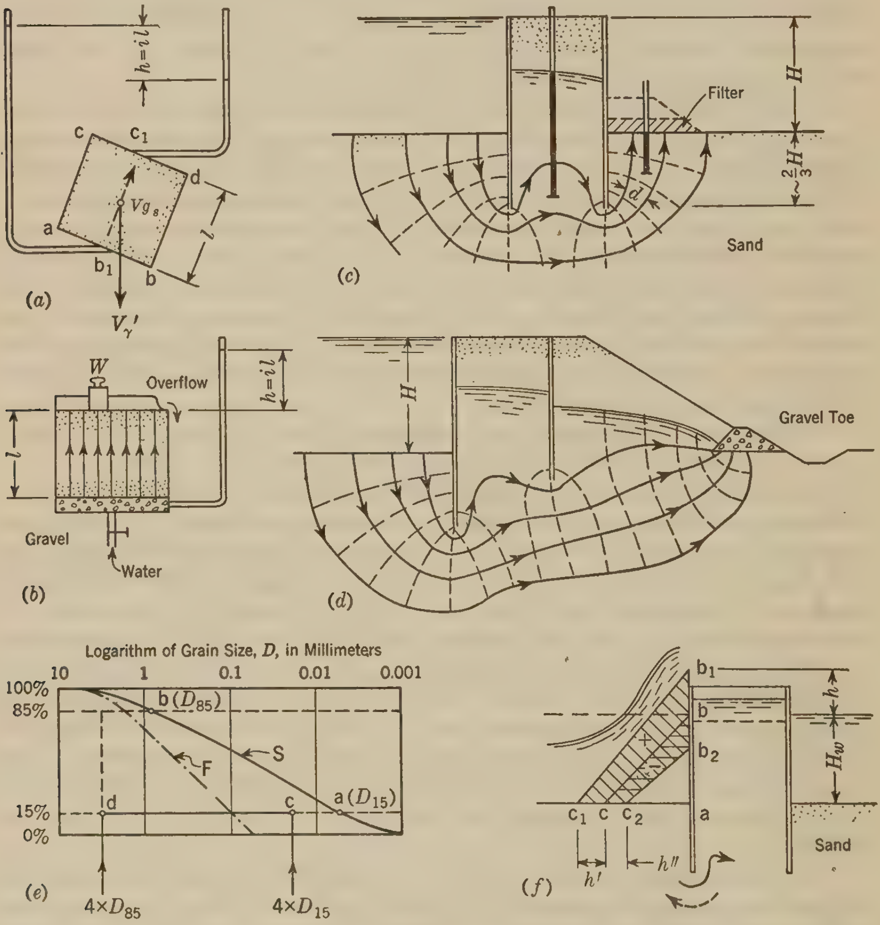
Lubrication of the locks may facilitate pulling the sheet piles, but it reduces, very considerably, the strength and stiffness of the cofferdam.

Cellular Cofferdams on Deep Sand Strata.—A review of current methods of designing cofferdams on sand reveals an amazing situation. The design of permanent storage dams on sand is governed by the necessity of eliminating the danger of the formation of boils at the downstream toe. All other requirements are of secondary importance and can easily be satisfied. In contrast to this sound practice, in all the theories pertaining to cofferdams on sand, such as those of H. Krey (13), K. Hager (14), Mr. Rimstad (5), and Bureau of Public Roads (15), the water is replaced by a continuous horizontal load equivalent to a water pressure on the outside of the dam and the other far more dangerous mechanical effects of the water are not even mentioned. The relative frequency of the failure of cofferdams on sand suggests that this unnatural mental attitude toward the problem is not limited to the writers of theories.

The mechanical effect of seepage on the stability of cofferdams had not received any adequate attention in engineering literature until it was treated by Lazarus White and Edmund A. Prentis, Members, Am. Soc. C. E. (6), in 1940. This work contains the first vigorous and well-documented protest against the prevailing indifference regarding the effect of seepage on the stability of cofferdams on sand. The following paragraphs contain a brief review of the mechanics of the formation of boils and of the means for preventing this dangerous phenomenon.

Fig. 9(*a*) is a vertical section through a prismatic element of a mass of sand with a volume V and a length l. The water percolates through the sand in the direction of the dashed arrow. Let γ' = submerged unit weight of the sand; γ_w = unit weight of the water; h = loss of hydraulic head associated with flow of water through distance l; and $i = \frac{h}{l}$, the corresponding hydraulic gradient.

In Fig. 9(*a*) the loss of head h is given by the vertical distance between the water levels in two standpipes whose lower ends are at the center of the entrance and the exit surface of the prismatic element. A loss of head h reduces the water pressure per unit of area of a plane section at a right angle to the direction of the flow by $\gamma_w h$. Hence, the percolating water adds to the submerged weight, $\gamma' V$, of the prismatic element (Fig. 9(*a*)), an unbalanced pressure $\gamma_w h$ per unit of area of its base ab acting in the direction of the dashed error. Since the length of the element in the direction of the flow is l, the unbalanced



h=il
c
c1
d
Vgs
a
l
b1
b
(a)
V'γ
Filter
H
~2/3H
Sand
(c)
W
Overflow
h=il
l
Gravel
Water
(b)
H
Gravel Toe
(d)
Logarithm of Grain Size, D, in Millimeters
10
1
0.1
0.01
0.001
100%
85%
15%
0%
b (D85)
S
F
d
c
a (D15)
(e)
4×D85
4×D15
b1
b
b2
h
Hw
a
c1 c c2
h'
h''
Sand
(f)

FIG. 9.—CELLULAR COFFERDAMS ON DEEP STRATUM

force is equal to

$$g_s = \gamma_w \frac{h}{l} = \gamma_w i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29a)$$

per unit of area and unit of length, in other words per unit of volume. This force represents the seepage force. If the water flows through a mass of sand vertically upward, the seepage force g_s acts in a direction opposite to that of gravity. If $g_s = \gamma_w i_1 = \gamma'$—

$$i_1 = \frac{\gamma'}{\gamma_w} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29b)$$

the resultant of the hydrostatic pressures counterbalances the force of gravity, and the sand should behave like a liquid (16). To verify Eq. 29*b*, a cylindrical vessel (Fig. 9(*b*)) was filled with sand, and water was allowed to percolate through the sand in an upward direction. The lines along which the water particles travel are marked by arrows. As long as the hydraulic gradient i was smaller than i_1, Eq. 29*b*, the sand grains were motionless and the permeability of the sand was unaltered. However, at the instant when i became equal to i_1, the sand started to boil and its permeability increased abruptly. Before the test a brass weight W was placed on the surface of the sand. As i approached the value i_1, the weight settled progressively, and when i became i_1 the weight disappeared in the boiling sand.

Fig. 9(*c*) is a vertical section through a cellular cofferdam on sand. The design of such a dam must satisfy three independent conditions: (*a*) The sand along the outer face of the cofferdam should be protected adequately against erosion; (*b*) the dam should be able to withstand the overturning moment produced by the water pressure on its outer face; (*c*) the subsoil should be able to carry the pressure on the base of the dam in spite of the tendency of the seepage forces to liquefy the sand at the inner toe. The practical implications of surface erosion have been presented expertly by Messrs. White and Prentis (6). In this paper only conditions (*b*) and (*c*) will be discussed; and for the sake of clarity they will be considered independently.

In the discussion of the stability of cofferdams on a rock foundation, illustrated by Figs. 7(*c*) and 7(*d*), it was shown that the inner row of sheet piles is pressed down by a friction force F_1, Eq. 23, per unit of length of the row. If the sheet piles move down under the influence of this force, the cofferdam is likely to fail by bursting of the locks. To prevent such a failure of a cofferdam on sand, the sheet piles of the inner row should be driven deep enough into the sand to withstand the vertical load F_1 (Eq. 23) per unit of length of the row of sheet piles without perceptible penetration. This condition is scarcely satisfied unless the sheet piles are driven to a depth of at least $\frac{2}{3} H$ into the sand.

To plan appropriate measures for the prevention of boiling, it is necessary to consider the flow of water through the subsoil of the cofferdam. In Figs. 9(*c*) and 9(*d*) the flow lines are marked by arrows. They were traced in such a manner that the quantity of water v_d which percolates between two flow lines is the same for every pair of flow lines. If this condition is satisfied, it is easy to realize that the spacing d between the flow lines indicates the distribution of the seepage force g_s (Eq. 29*a*) throughout the subsoil of the cofferdam.

The quantity of water that flows between two adjoining flow lines per unit of the distance between these flow lines is $\frac{v_d}{d}$. It is called the rate of percolation. According to Darcy's law the rate of percolation is equal to

$$\frac{v_d}{d} = k\,i \qquad (30)$$

in which k is a constant, the coefficient of permeability. Hence, $i = \frac{v_d}{d\,k}$ and

$$g_s = i\,\gamma_w = \frac{v_d\,\gamma_w}{d\,k} = \frac{1}{d} \text{ times a constant} \qquad (31)$$

According to Eq. 31, the seepage force g_s is inversely proportional to the spacing of the flow lines. As shown in Fig. 9(*a*), it acts at every point in the direction of the tangent to the flow line through this point.

The flow lines in Figs. 9(*c*) and 9(*d*) were constructed on the assumption that the rows of sheet piles are perfectly impermeable and that the sand is perfectly homogeneous. The short horizontal lines in Fig. 9(*c*) represent the lower ends of observation wells. The height to which the water would rise in each well is represented by a black column. Methods of constructing the flow lines and determining the height of the rise of the water in the wells have been discussed elsewhere (17). These methods also permit a determination of the seepage pressure g_s (Eq. 29*a*), for every point of the subsoil.

The water that percolates through the soil beneath the cofferdam escapes from the soil in a vertical upward direction as it does in the vessel shown in Fig. 9(*b*). The seepage velocity and the corresponding seepage force that tends to lift the sand are greatest next to the toe of the dam. Therefore, the floor of the excavation adjoining the toe constitutes an area of potential boiling. This will be called the danger zone. Boiling eliminates the resistance of the sand against an inward movement of the buried part of the inner row of sheet piles. It reduces the resistance of the inner row of sheet piles against penetration under the influence of the force F_1 (Eq. 23), and the cofferdam fails by toppling inward. The failure is either preceded or followed by a bursting of the cells. Several failures of this type have occurred on the Mississippi River (6).

The formation of boils can be prevented by two different methods: (*a*) By covering the danger zone with a loaded, inverted filter; and (*b*) by increasing the length of the path of percolation by means of a berm with a wide base. The inverted filter which constitutes the essence of method (*a*) is indicated in Fig. 9(*c*) by dashed lines. Since the filter material is much more permeable than the protected soil, the presence of the filter has no influence on the shape of the flow net, but the load that acts on the filter counteracts the vertical component of the seepage forces which tend to lift the sand beneath the danger zone.

Method (*b*) is illustrated by Fig. 9(*d*). The berm can be made of dredged material. Although the downstream ends of the flow lines in Fig. 9(*d*) are not vertical, they are spaced fairly closely, producing a tendency of the toe of the berm to slump due to backward erosion. This can be prevented by a gravel toe.

If an inverted filter made out of clean, coarse gravel or broken stone is used to cover the surface of the danger zone in a mass of sand, the sand is washed into the interstices of the gravel. One piece of gravel after another sinks into the sand and finally the filter ceases to function. Therefore, the filter material should satisfy two independent conditions. It should be coarse enough to permit free discharge of the seepage water, and, at the same time, its voids should be small enough to exclude the possibility of an invasion of finer soil particles from below. By experimenting with different subsoils and filter materials, the writer arrived at the empirical rule illustrated by Fig. 9(*e*). The abscissas of this diagram represent the logarithm of the grain size and the ordinates the percentage of the total weight composed of grains smaller than the size denoted by the abscissa. Curve S is the grain-size curve for the soil which should be protected. Abscissas of points c and d represent four times the grain size D_{15} and D_{85}, corresponding to the points a and b on the grain-size curve S. If the grain-size curve F for a filter material intersects the line dc, it can be considered suitable.

When building a cofferdam on an alluvial deposit, suitable filter material can usually be obtained by screening the soil which needs to be protected and by wasting the finest fractions. By wasting 50% of the soil represented by curve S in Fig. 9(*e*), one obtains the material represented by curve F which satisfies all the requirements. A filter bed on a flooded surface can be constructed only by means of one of the methods which are used for pouring concrete under water. If the filter material is simply dumped into the water, the coarsest particles settle first, and the result of the operation is a stratified layer which does not satisfy the essential requirements. The functioning of the filter should be supervised by means of observation wells whose lower ends are a short distance below the base of the filter. If the filter functions properly, the water should not rise in the wells more than a few inches above the base of the filter.

In 1923 the writer equipped the foundation for a permanent storage dam on sand and gravel with a filter designed on the basis of Fig. 9(*e*), and in 1925 he repeated the procedure on another dam on sand. Since both experiments were successful, he used the filter principle in the design of many dams and all the filters functioned as anticipated. On the basis of these experiences he recommends the design illustrated by Fig. 9(*c*) as an alternative to the customary berm type shown in Fig. 9(*d*), provided the rules for grading and constructing the filter are strictly adhered to and the functioning of the filter is conscientiously supervised.

The preceding comment furnishes a basis for the design of cellular cofferdams on sand. The width of the dam can be determined by Eq. 28*a*. The maximum lock tension is computed by Eq. 19(*b*). Boiling can be prevented by either one of the methods illustrated by Figs. 9(*c*) and 9(*d*). If a filter is used, the sheet piles should be driven to a depth $\frac{2\,H}{3}$ or until they bear on a hard stratum. The final choice between the two methods depends on the estimated cost and on the width of the space available for constructing the cofferdam. It is obvious that the unit price for the filter layer is much higher than that for a berm made of dredged material and the necessity for expert supervision during and after construction is a distinct liability in connection with temporary structures; but the berm requires much more space than the filter, as indicated in Figs. 9(*c*) and 9(*d*).

Figs. 9(*c*) and 9(*d*) represent two different ways for preventing boils by means of berms. If the local conditions do not permit the construction even of a narrow berm, the filter can be established at the base of the fill in the cells, in the position occupied by the filter in the cofferdam shown in Fig. 7(*a*). If an untried method is to be considered, it is necessary to investigate the mechanical efficacy of the proposed method by means of a flow net. The flow net can be constructed (17) or it can be obtained experimentally (6). Some engineers prefer the experimental method. The graphical method is cheaper, more expedient, and somewhat more accurate, but the experimental method carries more conviction with those engineers who are not accustomed to scientific reasoning.

The results obtained by either method are strictly valid only if the rows of sheet piles are perfectly impermeable and if the natural sand stratum is perfectly homogeneous. In reality neither one of the two conditions is satisfied and the resulting error can be very important. Hence, whatever method of investigation is used, it is imperative to discover by means of observation wells on the job whether the flow of seepage is at least approximately similar to that anticipated by the designer. If the discrepancy is on the unsafe side, supplementary measures of protection should be adopted during construction.

Fig. 9(*f*) is a vertical section through a breakwater. During storms breakwaters are subject to a pulsating water pressure. While the waves pound against the outside of the breakwater during a storm, the hydrostatic pressure on the outside of the breakwater varies between the values represented by the pressure triangles ab_1c_1 and ab_2c_2. The hydrostatic pressure at the outer toe oscillates between $[\gamma_w \, (H_w + h')]$ and $[\gamma_w \, (H_w - h'')]$, in which H_w is the depth of the water adjoining the breakwater before the storm. The values h', h'', and h in Fig. 9(*f*) depend on H_w, the wave length, and the height of the waves. A theory for estimating the wave pressure on the uppermost part of the exposed face of the breakwater was presented in 1935 by the late D. A. Molitor, M. Am. Soc. C. E. (18). A comparison between computed and observed values of wave pressure was made by T. L. Condron, M. Am. Soc. C. E. (19). Equations for computing the wave pressure over the entire exposed face were derived by M. Sainflou in 1928 (20). Since the excess hydrostatic pressure on the outside of the breakwater has the character of a pulsating force, it produces vibrations in both the structure and the subsoil. Because of the vibrations, the settlement is likely to be much more important than that produced by a static system of forces under similar circumstances (17). If the sand on which the breakwater rests is loose, the vibrations may cause a spontaneous settlement associated with temporary liquefaction of the sand whereupon the breakwater would suddenly sink into the sand as if the sand were a liquid. Thus, if the sand is loose, the construction of the breakwater should be preceded by compacting the sand beneath and on both sides of the breakwater by artificial means to a depth which is equal to at least 1.5 times the height H. This can be accomplished, for instance, by driving sand piles or by the "vibroflotation method" (21).

Cellular Cofferdams on Clay.—The only attempt to design a cofferdam on clay on a purely theoretical basis was made by H. Epstein (22), who computed stability on the assumption that the cofferdam can be regarded as a simple gravity wall. The highly controversial aspects of this assumption have already been discussed herein under the heading, "Cellular Cofferdams on Rock." In estimating the required depth of penetration of the sheet piles, the vertical forces transmitted from the fill in the cells through side friction to the sheet piles, and through the sheet piles to the foundation, have not been considered. The computations themselves involve an unwarranted generalization of the Rankine-Résal equations. For these reasons, the analysis fails to furnish a satisfactory basis for the design of a cofferdam on a clay foundation.

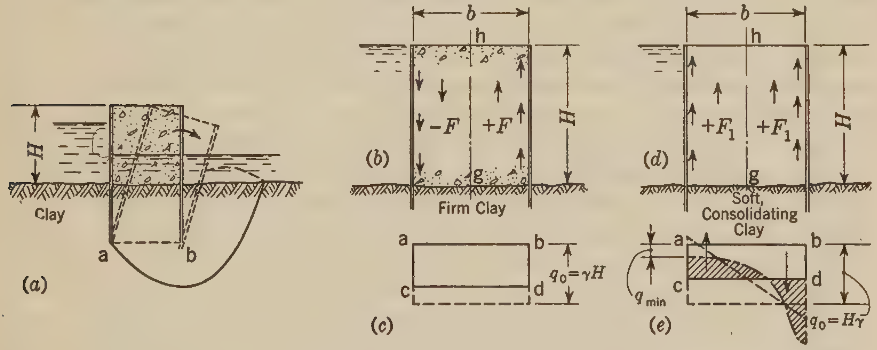


FIG. 10.—CELLULAR COFFERDAMS ON CLAY

The first requirement for the stability of a cofferdam on clay is that the clay stratum should be strong enough to sustain the weight of the cofferdam. The ultimate bearing capacity of an undisturbed, homogeneous clay stratum depends only on the average cohesion c of the clay. This statement has been confirmed repeatedly by experience—most recently by the failure of a large footing on soft clay (23) reported in 1942. Fig. 10(a) is a vertical section through a cellular cofferdam that rests on a homogeneous clay stratum of great depth. Before the cofferdam is acted upon by one-sided water pressure, it represents a continuous uniformly loaded footing whose base ab adheres to the supporting clay. If the upper surface of the clay were at the elevation of the base of ab, the failure load (in excess of the pressure which acted on ab before the construction of the cofferdam) would be equal to 5.7 c (17). The layer of clay above the level of ab in Fig. 10(a) increases the bearing capacity slightly. Hence, if the weight of the cofferdam, γH per unit of area, is equal to 5.7 c, the clay would be able to carry slightly more than the weight of the cofferdam. However, subsequent application of a one-sided water pressure would cause the cofferdam to fail by inward tilting as indicated by the dashed lines in Fig. 10(a). From the expression, $\gamma H = 5.7\ c$, the value,

$$H = 5.7 \frac{c}{\gamma} \quad \text{.........................} (32)$$

is obtained. The height of the cofferdam should not exceed 0.7 H. Otherwise, the cofferdam would fail on account of a failure of its foundation although it may be safe in every other respect. The value of γ is usually about 110 lb per cu ft, and the value of c for soft clay ranges between 400 and 1,000 lb per sq ft. The corresponding values of H are 20 ft and 50 ft. The danger of a foundation failure can be disregarded only if the clay is stiff or if the sheet piles are driven into a stratum of sand or stiff clay. A method of evaluating the average cohesion c of natural clay strata has been described elsewhere (24).

The stability of a cellular cofferdam itself depends to a considerable extent on the compressibility of the soil between the base of the fill in the cells and the level of the lower edge of the sheet piles. The following investigation of the influence of the compressibility of the base of the fill in the cells on the stability of a cellular cofferdam is based on the simplifying assumption that the fill obeys Hooke's law and that the intensity of the vertical friction forces which act on the rows of sheet piles is independent of the distance from the neutral plane, gh in Fig. 10(*b*). The second assumption is justified by the empirical fact that a very small displacement between sand and wall suffices for mobilizing the full frictional resistance (25). A further displacement has no more influence on the shearing stresses at the contact face. It is further assumed that the sheet piles are driven deep enough into the clay to sustain the vertical forces without noticeable vertical displacement. The fill in the cells is assumed to be completely drained, and the horizontal pressure of the fill on the walls of the cells is assumed to be determined by Eq. 10, regardless of the state of stress on the vertical boundaries of the fill.

The overturning moment M can be divided into two parts, M_s and M_f. One part, M_s, is carried directly by the sheet-pile enclosure of the cells without the assistance of the shearing resistance of the fill. The other one, M_f, is carried by the fill.

In the following computations the sheet-pile enclosures of the cells are considered prismatic boxes with lengths 2 L and widths b. In Fig. 1(*a*), these boxes are represented by dash-dotted lines. The lower edge of the boxes is fastened to the ground by the buried part of the sheet piles. The shearing strength of the walls along the locks is equal to the lock friction. Application of an overturning moment increases the pressure at the lower edge of the boxes on the inside of the neutral plane gh (Fig. 10(*b*)) by Q_p per unit of width of the sheet piles, and it reduces the pressure on the outside by the same quantity. The sum of all the vertical pile reactions on one side of the neutral plane of one cell is $Q_p\,(2\,L + b)$. At the instant of the first slip in the locks, this force must be equal to the friction $2\,T\,f$ (Eq. 14*b*) in the two locks located in the neutral plane of the cell. Hence, $Q_p\,(2\,L + b) = 2\,T\,f = 2 \times \frac{1}{2}\,\gamma\,H^2\,r\,C\,f$; or

$$Q_p = \tfrac{1}{2}\,\gamma\,H^2\,C\,f\,\frac{r}{L + 0.5\,b} \cdots\cdots\cdots\cdots\cdots (33)$$

The couple formed by the equal and opposite sheet-pile reactions on the two sides of the neutral plane of one cell is $2\,M_s\,L = 2\,L\,Q_p\,b + 2\,Q_p\,\frac{b}{2} \times \frac{b}{2}$;

whence,

$$M_s = Q_p \frac{b}{L} (L + 0.25\, b) = \tfrac{1}{2}\, \gamma\, H^2\, C\, r\, f\, \frac{b}{2}\, \frac{L + 0.25\, b}{L + 0.5\, b} \ldots\ldots (34a)$$

Introducing the customary values for cofferdams with circular cells, $b = 0.85\, H$, $L = 0.55\, H$, and $r = 0.50\, H$:

$$M_s = 0.3\, \gamma\, H^3\, C\, f \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34b)$$

The value M_s is independent of the compressibility of the base of the fill in the cells, because it depends only on the lock friction. On the other hand, the second part, M_f, of the resisting moment depends on the compressibility of the base, because the compressibility has a decisive influence on the stress conditions at the surfaces of contact between fill and sheet piles.

After a cell has been filled with sand all the vertical boundaries of the fill are acted upon by side friction in an upward direction, because the fill settles due to its own weight while the sheet piles are stationary. At that stage (that is, prior to the application of an overturning moment), the distribution of the pressure on the base of the fill is approximately uniform as indicated in Fig. 10(*c*) by the horizontal line cd and the total pressure is equal to the difference between the weight of the fill and the total side friction.

If the fill rests on an unyielding base such as firm clay, the application of an overturning moment reverses the direction of the friction forces on the vertical boundaries of the outer part of the fill, as indicated in Fig. 10(*b*), because the height of the outer part of the fill increases slightly while the height of the sheet-pile enclosure remains unchanged. The greatest value which the friction on the vertical boundaries can assume is F_1 per unit of length of the row of sheet piles. The value of F_1 is given by Eq. 23. The displacement between fill and sheet piles required to increase the friction forces F to their maximum value F_1 is very small. Hence the distribution of the pressure on the base of the fill remains approximately uniform until F becomes equal to F_1.

As long as the distribution of the pressure on the base of the fill is uniform, requiring $F < F_1$, the cofferdam can fail only by shear along the neutral plane. As soon as the unit pressure on the inner side of the base starts to increase ($F = F_1$), the cofferdam may also fail by a bursting of the locks of the inner row of sheet piles. In order to find out which one of these two types of failure is to be expected, compute the overturning moment required to produce a failure by shear and determine the corresponding friction value F. If the value thus obtained is smaller than F_1 the cofferdam fails by shear before the lock tension starts to increase appreciably.

The following computation is based on the assumptions illustrated by Figs. 10(*b*) and 10(*c*): The neutral plane passes through the center line of the crest of the cofferdam, the friction forces F which act on the vertical contact faces of the fill on either side of the neutral plane are equal and opposite, and the distribution of the pressure on the base of the fill is approximately uniform. On these assumptions the cofferdam fails as soon as the total friction force on the vertical boundaries of the fill on one side of the neutral plane, $F\,(2\,L + b)$ per cell of a cofferdam with circular cells, becomes equal to the total shearing resistance, $2\,L\,S'$, on the neutral plane of the fill in one cell. The value of S'

is given by Eq. 12. Since $2\,L\,S'$ must be equal to $F\,(2\,L + b)$,

$$F = \tfrac{1}{2}\,\gamma\,H^2\,C\tan\phi\,\frac{L}{L + \frac{b}{2}} \quad \text{(35a)}$$

The greatest value that F can assume is

$$F_1 = \tfrac{1}{2}\,\gamma\,H^2\,C\tan\delta \quad \text{(35b)}$$

The value $\tan\phi\,\frac{L}{L + \frac{b}{2}}$ in Eq. 35*a* is commonly smaller than the value $\tan\delta$ in Eq. 35*b*. Hence, the cofferdam is likely to fail by shear before the friction between fill and sheet piling is fully mobilized. The overturning moment for one cell at which failure occurs is $2\,L\,M_f = 2\,F\,L\,b + 2 \times F\frac{b}{2} \times \frac{b}{2}$. Hence,

$$M_f = F\,b\left(1 + \frac{b}{4\,L}\right) = \tfrac{1}{2}\,\gamma\,H^2\,C\,b\,\frac{1 + \frac{b}{4\,L}}{1 + \frac{b}{2\,L}}\tan\phi \quad \text{(36a)}$$

per unit of length of the cofferdam. Assuming $b = 0.85\,H$, $L = 0.55\,H$, and $\phi = 34°$, $\tan\phi = 0.675$,

$$M_f = 0.22\,\gamma\,H^3\,C \quad \text{(36b)}$$

The total overturning moment M_t required to produce the failure is equal to the sum of M_s (Eq. 34*b*) and M_f. Assuming f in Eq. 34*b* to be equal to 0.3 the value—

$$M_t = \gamma\,H^3\,C\,(0.09 + 0.22) = 0.31\,\gamma\,H^3\,C \quad \text{(37)}$$

—is obtained.

Since the shearing force on the outside contact faces of the fill in the cell is most likely smaller than that on the inside, the cofferdam must be expected to fail at an overturning moment of less than M_t. Whatever the critical value of the overturning moment may be, the stability of the cofferdam is higher than that of a cofferdam with similar dimensions on a rock foundation, because the outer row of sheet piles is anchored in the subsoil. The anchorage increases the stability of the dam. Therefore, it is safe to design the cofferdam on the basis of the same semiempirical rules that have been proposed for the design of cellular cofferdams on a rock foundation.

The sheet piles are acted upon by the sum of the vertical forces Q_p (Eq. 33) and F (Eq. 35*a*). The depth of sheet-pile penetration is determined by the condition that the sheet piles should be able to sustain these forces without noticeable displacement. The bearing capacity and the skin friction of the sheet piles can be determined by pulling tests on test sheet piles on the job.

If a cofferdam rests on a base that continues to yield under the influence of the weight of the fill in the cells while the overturning moment is applied, the fill slides along the sheet piles in a downward direction. Because of friction, all the contact faces of the sand are acted upon by a force F_1 per unit of width acting in an upward direction as indicated in Fig. 10(*d*) by arrows. The pressure on the base of the fill is equal to the difference between the weight of the fill and the full side friction (pressure area abcd in Fig. 10(*e*)). The overturning moment causes the cofferdam to tilt inward, but, since the sand continues to slide along the sheet piles in a downward direction, the tilt cannot alter the shearing stresses on the vertical contact faces. This fact constitutes a vital difference between the conditions for the stability of the cofferdams on a firm base (Fig. 10(*b*)) and on a consolidating base (Fig. 10(*d*)). Since the shearing stresses on the vertical boundaries of the fill in the cells have a constant value, no increase of the overturning moment beyond M_s (Eq. 34*b*) is conceivable without a corresponding change of the pressure on the base of the fill in the cells. In Fig. 10(*e*), the change of the pressure on the base, caused by the overturning moment, is indicated by shaded areas.

Hence as soon as the overturning moment becomes greater than M_s, Eq. 34*a*, the unit pressure on the inner part of the base increases rapidly and a failure due to bursting of a lock of the inner row of sheet piles becomes imminent, irrespective of the state of stress on the neutral plane. On account of the increase of the pressure on the inner part of the compressible base, an increase of the overturning moment beyond M_s should be associated with a conspicuous inward tilt of the cofferdam.

Only three cellular cofferdams on a soft, highly compressible base have come to the writer's attention. One of them is the cofferdam for raising the battleship *Maine*; the second is the western part of the middle wall of the shipways described by C. B. Jansen, M. Am. Soc. C. E. (7); and the third is a bulkhead adjoining one of these shipways.

Fig. 11(*a*) is a simplified vertical section through the cofferdam around the *Maine*. The cofferdam consisted of circular cells with a radius of 25 ft. The stratum between the base of the fill in the cells at El. − 35 and the surface of the clay at El. − 65 was described as a red loam with shells (26). The cells were filled with heavy clay excavated by means of a hydraulic dredge. The progressive consolidation of the clay beneath the base of the fill in the cells was demonstrated by the fact that the wooden drain boxes which were built into the fill of the cells settled 4 ft to 5 ft (26). As soon as the water level inside the cofferdam was lowered 15 ft, the inward tilt of the cofferdam in-

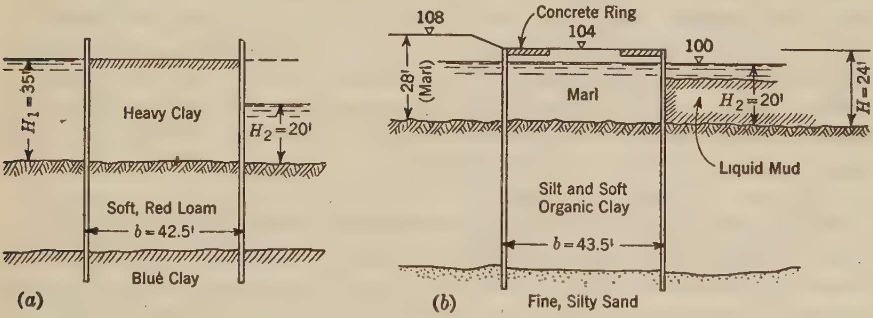


Fig. 11

creased so considerably that it was decided to increase the stability of the dam by constructing an inner berm. At that stage the overturning moment produced by the unbalanced water pressure was about 370 kip-ft per ft. To correlate the known behavior of the cofferdam with the results of the preceding analysis, the overturning moment M_s, which the sheet-pile enclosure of the cells can stand without the assistance of the shearing resistance of the fill in the cells, will be computed.

To determine the moment M_s, it is necessary to estimate the lock friction. If P_t denotes the difference between the horizontal pressure on the inside and that on the outside of the inner row of sheet piles of the cofferdam per unit of width of the sheet piles, the total friction in one lock is $P_t r f$. Using the same procedure which led to Eq. 34*b*, the formula,

$$M_s = P_t r f \frac{b}{L} \times \frac{L + 0.25\,b}{L + 0.50\,b} \quad \ldots\ldots\ldots\ldots\ldots\ldots (38)$$

is obtained. For the cofferdam around the *Maine*, the writer assumes $\gamma = 110$ lb per cu ft, $\gamma' = 50$ lb per cu ft, $\gamma_w = 63.5$ lb per cu ft, and $C = 1.00$. The corresponding value of M_s is 490 kip-ft per ft. The deflection of the cofferdam became conspicuous as soon as the overturning moment became equal to 371 kip-ft per ft $= 0.76\ M_s$.

In Fig. 4(*c*) is shown a section through the middle wall of the shipways described by Mr. Jansen (7). The cells were filled with a stiff, calcareous clay known as marl. The height H of this cofferdam is 35 ft, $r = L = 24.3$ ft, $\gamma = 120$ lb per cu ft, and $\gamma_w = 63.5$ lb per cu ft. Assuming $C = 1$, the value M_s (Eq. 38) is equal to 625 kip-ft per ft. When the shipway south of the wall was pumped out, the unbalanced water pressure on the north side of the wall produced an overturning moment $M = 456$ kip-ft per ft $= 0.73\ M_s$. At that stage, the average deflection of that part of the cofferdam which rested on stiff marl was less than 2 in.—which is negligible. However, the deflection of the westernmost part which rested on a soft, dark clay with a natural water content of about 70% was almost 8 in. as shown in Fig. 4(*c*). It increased so rapidly that it was decided to stop the movement by constructing a temporary support. This is a good illustration of the detrimental influence of a soft base on the stability of a cellular cofferdam.

Fig. 11(*b*) is a section through the cellular bulkhead adjacent to the aforementioned shipways. The cells are cylindrical and are filled with marl. The fill in the cells rests on a highly compressible stratum consisting of an irregularly stratified mass of silt, soft and dark, and organic clay with a very high water content. Evaluating Eq. 38 on the basis of $H = 24$ ft, $H_1 = 20$ ft, $\gamma = 120$ lb per cu ft, $\gamma' = 57$ lb per cu ft, $C = 1.00$, and considering, in addition, the side pressure of the liquid mud on the river side, with a submerged unit weight of 34 lb per cu ft, the value $M_s = 178$ kip-ft per ft was obtained.

The bulkhead is acted on by the active earth pressure of a mass of marl whose surface is at El. 108. The marl was excavated in a borrow pit by means of a hydraulic dredge, transported through pipe lines and deposited in such a manner that it displaced the river mud adjoining the land side of the bulkhead and forced part of the mud out into the river through a temporary gap in the bulkhead. The overturning moment produced by the active earth pressure of the marl was estimated on the assumption that the angle of internal friction of the marl is zero and that the cohesion is between 250 and 300 lb per sq ft. Thus, a moment M between 116 lb-ft per ft and 155 lb-ft per ft (0.65 to 0.88 M_s) was obtained.

During the construction of the backfill, the crest of the dam moved out to distances as great as 7 in. During the following 1.5 years the deflection increased by amounts as great as 15 in. which indicates that the overturning moment is fairly close to the maximum value which the cofferdam can safely stand. To prevent a distortion of the cells, a massive concrete ring was poured on the surface of the fill in every cell. Because of the consolidation of the fill and of the base of the fill, the concrete rings settled along the waterfront as much as 2 ft. On the land side the settlement is less than one half of that on the water side, which indicates that the pressure on the base increased from the land toward the water side. However, the overturning moment was still considerably smaller than M_s (Eq. 38).

By the theoretical analysis presented herein, the cofferdams represented in Fig. 11 should have shown no signs of distress until the overturning moment M became equal to M_s (Eq. 38); but all of them started to deflect quite alarmingly as soon as M became greater than about 0.75 M_s. Hence, it seems that the evaluation of M_s involves a considerable error on the unsafe side. The error may be due to the real lock tension being smaller than the estimated value, to the coefficient of lock friction being less than 0.3, or to both causes combined. Because of this error, cellular cofferdams on a soft base should be designed so that the computed resisting moment M_s is at least equal to 1.5 times the overturning moment.

If the construction of a cellular cofferdam on a clay foundation involves lowering the water level on one side of the cofferdam, the effect of the seepage pressure on the foundation of the cofferdam and on the subsoil of the unwatered area must be considered independently. In many instances it is possible to eliminate undesirable hydrostatic pressure conditions by simple drainage. The discussion of the drainage devices is beyond the scope of the paper.

Part III. Conclusions

Eleven conclusions seem indicated from the foregoing:

1. The traditional assumption that a cellular cofferdam can be regarded as a gravity wall leads to erroneous conclusions regarding the distribution of the soil pressure on the base of the dam.

2. Because of the friction between the fill and the sheet-pile walls of the cells, the increase of the soil pressure at the inner part of the base of a cellular cofferdam with conventional dimensions due to the overturning moment produced by the water pressure is negligible, provided the base is firm.

3. The lock friction in the cross-walls adds materially to the stability of the cofferdam, and it reduces very considerably the deflection of the crest. This fact constitutes an important advantage of the cellular cofferdam over the double-wall cofferdam.

4. The assumption that the weep holes in the inner wall of cells filled with sand suffice to drain the cells is not necessarily justified.

5. The lateral earth pressure of the fill in the cells on the sheet-pile enclosure of the cells is not even approximately equal to the Rankine pressure. Therefore, the greatest lock tension can only be estimated on a semiempirical basis.

6. The required depth of penetration of the sheet piles of cofferdams on a sand or a clay foundation is determined by the condition that the bearing capacity of the sheet piles per unit of width should be equal at least to 1.5 times the shearing force which acts per unit of width between the fill and the length rows of sheet piles at maximum overturning moment. Since the vertical forces that act on the sheet piles can be estimated in advance, the required depth of sheet-pile penetration can be determined by means of pulling tests on test sheet piles in the field.

7. Cellular cofferdams on a sand foundation require, in addition to adequate sheet-pile penetration, a protection against the formation of boils. Protection can be achieved with either a loaded, inverted filter or a plain sand berm with a broad base. If an inverted filter is used, its functioning requires supervision by means of telltale pipes.

8. Rows of steel sheet piles in contact with, or embedded in, fine silt or clay do not intercept the flow of seepage.

9. The principal dimensions of cellular cofferdams on stiff clay can be determined by means of the same semiempirical equations that have been proposed in this paper for the design of cellular cofferdams on a rock foundation.

10. If the fill in the cells of a cofferdam rests on a compressible stratum which continues to consolidate after the overturning moment has been applied, a relatively small moment suffices to produce a very unequal distribution of the pressure on the base of the fill in the cells. Such a cofferdam should not be expected to carry more than about 0.7 times the overturning moment M_s (Eq. 38). The degree of stability of the cofferdam is practically independent of the strength of the fill in the cells.

11. The shearing resistance on vertical sections through the fill in the cells and the lock friction can only be estimated. Since the stability of every cellular cofferdam depends to a large extent on these quantities, no theoretical refinements in the design of such dams are warranted and no equation pertaining to such dams can be used safely unless its approximate validity has been repeatedly demonstrated by construction experience. Therefore, the principal sources for increased knowledge on cellular cofferdams are accurate field observations.

APPENDIX I

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APPENDIX II

NOTATION

The following letter symbols, used in this paper, conform essentially with "Soil Mechanics Nomenclature,"[3] adopted by the Society's Board of Direction in 1941:

b = width; width of equivalent cofferdam with straight walls:

  b_1 = outside width of the real cofferdam (Figs. 1(a) and 1(b));

  b_0 represents an empirical value for the width b of equivalent cofferdams whose outside is acted upon by water pressure only;

  b_m (Eq. 8) = value of b for a cofferdam with a theoretical factor of safety G_s = 3 with respect to overturning;

C = coefficient of earth pressure, an empirical constant;

c = cohesion; average cohesion of clay per unit area;

D = diameter;

d = distance between flow lines (Fig. 9);

F = total frictional resistance on a surface of contact, per unit width:

  F_b = value of F on the base of a cofferdam (Eq. 1);

  F_{bs} = value of F at the base of a cofferdam which is completely saturated to a height H_s above the base;

  F_1 = resistance against sliding along surface of contact between fill and sheet piles, per unit of width of the row of sheet piles;

f = coefficient of lock friction;

G_s = factor of safety;

g = a force per unit of volume; g_s = seepage force in Eq. 29a;

H = height of a cofferdam:

  H_s = average elevation of line of saturation above base of a partially saturated cofferdam;

  H_w = depth of water adjoining a breakwater, between storms;

  H_1 = elevation of level of water adjoining the outside of a cofferdam, if the highest water level is below the crest of the dam (Fig. 5);

  H_2 = elevation of level of water adjoining the inside of a cofferdam, with reference to base of dam (Fig. 11);

  H_{max} = greatest vertical distance between level of water on two sides of a cofferdam (Fig. 3);

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[3] *Manual of Engineering Practice No. 22,* Am. Soc. C. E., 1941.

h = loss of hydraulic head that is caused by flow of water through distance l in a mass of sand (Fig. 9(a)) or through the locks of a row of sheet piles (Fig. 5); also maximum temporary excess head on wind side of a breakwater during a storm (Fig. 9(f)):

  h_1 = hydrostatic head on the inside of the outer row of sheet piles of a cofferdam (Fig. 5(b));

  h' = hydrostatic excess head at heel of breakwater while a wave breaks (Fig. 9(f));

  h'' = drop of head at heel of breakwater after a wave has spent itself;

i = hydraulic gradient h/l (Fig. 9(a)); $i_1 = \frac{\gamma'}{\gamma_w}$ (Eq. 29b) = hydraulic gradient in the case of upward seepage at the instant when boiling begins;

k = a constant = coefficient of permeability;

L = half the spacing of circular cells in cofferdams (Fig. 1(a)), or the distance between cross-walls in cellular cofferdams of the diaphragm type (Fig. 1(b));

l = length of a cubical element of a mass of sand (Fig. 9);

M = overturning moment per unit length per unit of length of a cofferdam:

  M_a = the moment required to move the point of application of the resultant pressure on the base of a gravity wall from the center to the inner boundary of the middle third (Eq. 6);

  M_f = the part of an overturning moment carried by the fill within the cells of a cofferdam on stiff clay;

  M_s = the part of an overturning moment carried by sheet-pile enclosures of the cells of a cofferdam on clay without the assistance of the shearing resistance of the fill;

  M_t = total overturning moment required to produce failure of a cofferdam on stiff clay;

  M_0 = overturning moment produced by the water pressure on the outside of a cofferdam after the outside water level has arrived at the level of the crest of the dam (Eq. 19a);

  M_1 = overturning moment required to increase the pressure on the inner part of the base of a cofferdam on a rock foundation (Fig. 7(d));

  $M_{\max}$ = moment required to overturn a gravity wall;

m = ratio between M_1 and M_0 (Eq. 27);

n = the greater of two values n_1 and n_2; $n_1 = \frac{P}{P_0}$ and $n_2 = \frac{M}{M_0}$;

P = resultant of all the horizontal forces which act on a cofferdam, per unit of length of the dam:

  P_A = active Rankine pressure on a row of sheet piles or on the neutral plane of a cofferdam, per unit of width;

  P_e = earth pressure on a row of sheet piles or on the neutral plane of a cofferdam, per unit of width;

P_s = horizontal force which prevents the inward movement of the lower edge of the inner row of sheet piles of a cofferdam on a rock foundation per unit of width of the row (Fig. 8);

P_t = difference between pressure which acts on inside and outside of inner row of sheet piles of cofferdam on clay, per unit of width of sheet piles;

p = pressure per unit of area; p_e = earth pressure per unit of area of a vertical surface, at depth Z below top of fill;

Q = equal and opposite vertical forces per unit of length of a cofferdam constituting a couple which is equal to the total overturning moment or to part of it; Q_p = vertical force per unit of width of sheet-pile enclosure of cells of a cofferdam on clay constituting a couple equal to the overturning moment M_s per unit of length of the dam;

q = pressure per unit of area of base of a cofferdam:

q_0 = average unit pressure, equal to weight of fill in the cells divided by area of its base;

$q_{\min}$ = smallest value which the unit pressure at outer edge of the base of cells filled with sand can assume;

r = radius of circular cells or arcs;

S = average shearing resistance per unit of width of neutral plane of a cofferdam:

S' = that part of S which is due to shearing resistance of fill in cells;

S'' = that part of S which is due to resistance against slippage in locks;

T = total tension in one lock;

t = lock tension per unit length at depth Z; t_1 = greatest unit tension in a lock (Eq. 22*b*);

V = volume;

v_d = quantity of water which flows through space between two adjoining flow lines per unit of time and unit of length of the dam;

W = weight;

Z = depths measured from the tops of cells;

γ = unit weight of moist fill in cells; γ' = submerged unit weight of fill in cells;

γ_w = unit weight of water;

Δ = deflection;

δ = angle of friction between substances such as the fill in a cell and sheet piles;

ρ = angle of friction for sliding; and

ϕ = angle of internal friction.

DISCUSSION

G. G. GREULICH,[4] M. AM. SOC. C. E.—Some new methods, and an expansion of some old ones, for the precise mathematical evaluation of certain phenomena relating to cofferdam design are offered in this paper. The phenomena have been noted before, and empirical allowances have been made for their design and construction. The new concepts of analysis presented in the paper should be useful in providing additional cross checks for the adequacy of a cofferdam design. However, the use of precise mathematical evaluations can result in misleading, erroneous, and dangerous conclusions in the hands of inexperienced individuals, when dealing with such uncertain quantities as the widely divergent materials in ordinary soils whose properties and resulting physical characteristics and effects can vary widely due to either man-made or natural changes or disturbances.

Deflection of Cellular Cofferdams.—In cellular cofferdams, the nature of the deflection depends largely not only on the elastic properties of the fill in the cells, but also on the method of depositing the fill. For example, fill deposited in cells inundated in water will not produce the immediate tamping or packing effect obtained by ordinary methods of dry filling. Tightening the interlocks by removing loose play, as well as by stretching the entire cell, aids materially in reducing the deflection. The analogy is similar to the comparison of an empty headless barrel or keg with dried-out staves and loose hoops to one with tight hoops and staves in solid contact. Even without fill, the tight barrel will stand considerable lateral pressure at the top without appreciable deflection when the bottom is fixed. The loose barrel, of course, will rack far out of position. Where cells have been built and filled in deep water, the deflections under load are less for subsequent unwaterings than for the first unwatering because the consolidation and settling effect from the first drainage of the fill in the cells is still felt during later cycles of filling and emptying the cofferdam. The deflection is also affected appreciably by the character of backfill material and the method of its deposition.

Permeability of Cellular Cofferdams and of Their Sheet-Pile Enclosures.—With reference to the permeability of sheet piling near the base of cells, tension in interlocks also prevails for a short distance below the base of fill. The softer the foundation material, the greater the depth of tension will be.

Resistance Against Sliding; Cellular Cofferdams on Rock.—Sheet piling is usually "toed in" on rock by driving so that it penetrates the soft upper layers of the rock. Where the rock is too hard to be penetrated by the sheet piling, mechanical methods such as dowels grouted into the rock have been proposed to prevent sliding. The method of grouting dowels into rock is highly objectionable due to intermittent supports and high stress concentrations at the points of support which would tend to rupture the sheet-pile interlocks. Another proposed method, which presents some very difficult construction problems, is the concreting of sheet piles in trenches cut into the rock.

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Prior to the early 1930's, cofferdam design methods were based largely on "rule of thumb" and on precedent. The customarily accepted Pennoyer method (10)(11) was offered as a rationalized, simplified procedure, and not as a perfected, all-inclusive scientific approach to the problem.

As Professor Terzaghi states (see text concerning Eqs. 1 and 2), the Pennoyer design method disregards some factors that may reduce the margin of stability. When these factors are omitted, greater sliding resistance is indicated than is really the case when the actual shear distribution and height of saturation are taken into consideration. Inasmuch as there seem to be no sliding failures on record, the Pennoyer method has apparently been adequate, even though the factor of safety was not as high as computed. A more precise approximation can easily be obtained by applying the author's method. However, as Professor Terazghi states (see paragraph introducing Fig. 3), there is at present no reliable method of determining in advance the state and properties of the soil in the fill; and quantitative data on the stiffening effect of the sheet-pile walls are not available. Therefore, during the present state of the art, any precise mathematical analysis can at best be considered only an approximation.

Resistance Against Internal Shear; Cellular Cofferdams on Rock.—The author is in error in stating (see paragraph introducing Eq. 9) that possible failure by shear along the vertical neutral plane in the cells has never received any attention. Colonel Pennoyer has called attention to the investigation of internal shear resistance and its relation to the safety of the cofferdam (10*a*)(10*b*).

Resistance Against Overturning; Cellular Cofferdams on Rock.—The preceding remarks on resistance against sliding are also applicable to the factor of safety against overturning. Since the factor of safety of 3.8 suggested by the method of Colonel Pennoyer when applied to the selected example (see text following Eq. 20) is based on premises that may not actually exist, the factor with the assumed values, as asserted by Professor Terzaghi, is still about 2, which should be adequate. The writer presumes that in making this assertion Professor Terzaghi considered that the limitations imposed by internal shear are largely offset by the benefits of soil compaction and of friction in the interlocks.

With respect to overturning, fortunately the cellular type of cofferdam can be widened at practically no additional cost. Extension of the width makes the spacing greater between cross-walls so that the amount of steel is not increased in circular cells and is increased very little on the straight diaphragm wall type. In any case, the extra fill is a minor item. Of course, the allowable interlock tension may limit the width for a given height. In general, the cofferdam should be made as wide as reasonable limits allow. This increases stability and also helps in reducing permeability.

The increased stiffness of cellular cofferdams, as compared with other types, has been recognized in many ways, and lock friction (see second paragraph following Eq. 21) has been assigned an important place as a factor in increasing the stiffness of the cells. Colonel Pennoyer has mentioned this fact (10*b*). Heretofore, it has been difficult to evaluate the combined effect of the shear resistance of the fill and of the steel interlocks. It is recognized (returning to the loose barrel-stave analogy) that deflection or racking of the top of the cofferdam soon tends to increase the sliding resistance of one piece of sheet piling against the other in the cross-walls and that a large percentage of the total resistance to shear may be developed by the steel before the earth fill is distorted enough to contribute its complementary shear resistance. Therefore, the true value may lie somewhere between the value for earth fill alone and the combined value, as determined in the paper. Professor Terzaghi has proposed a method of approximating the combined values which appears to be simpler and more rational than any that has previously been brought to the writer's attention. The author's method, however, does not take care of another factor introduced by the sheet piling. The added factor is the direct tension introduced in the sheet piling when the cells cant or lean inward. This movement crowds the interlocks together so that individual pieces can exert tremendous resistance to sliding or upward movement. Because of these phenomena, the diaphragm or cross parts of steel sheet-pile walls are capable of transmitting some direct tension from the inside top of the cell, diagonally down to the outside bottom of the cell. In other words, the effect is similar to the introduction of stay rods from the inner top to the outer base of the cell.

The fact that interlock tension failures, as observed by Professor Terzaghi, usually occur before overturning moment is applied to the cells (see second paragraph preceding Eq. 23) seems to indicate that internal pressures and shear resistance in the fill are considerably more than the Rankine active pressure. This is probably due to the tamping effect of dropping fill in cells, as well as to a squeezing and consolidating effect occurring as the partly filled cells weave and stretch during filling and drainage of water. These factors all contribute to the ultimate stiffness and stability of the cofferdam.

There could be considerable danger of interlocks failing by tension at the inner toe of a cofferdam on rock if the piling were not toed or bedded in the rock. Only the most foolhardy individual would permit such construction, especially on wide cells. It is generally recognized that driving on rock tends to damage the lower ends of the sheet piling and may produce incipient unlocking of adjacent piles. For this reason, manufacturers always recommend, and sound practice should dictate, the secure embedment of the lower ends on bare rock.

Even if the vertical reaction of the piles from overturning moment is theoretically sufficient to hold the piles in place as is repeatedly demonstrated by the shape of the bulge above the inner toe, the remarks regarding embedment of the lower ends still apply. Although, as Professor Terzaghi has emphasized (see third paragraph following Eq. 27), two of the three customary equations may be inadequate and the third is open to question, the fact remains that comparatively little trouble has been experienced with cellular cofferdams. As the author states, in the application of these equations, very conservative values have been used for soil constants. He expresses concern that an ambitious designer some day might use soil constants determined by laboratory tests, with fatal results. Fortunately, cofferdams usually represent the expenditure of considerable sums, and the final decisions regarding design and construction have been in the hands of mature, experienced individuals. The writer knows of no serious failures in which customary practices were followed by engineers, who generally allowed ample margins beyond the values obtained by the equations. Such practices may have been needlessly extravagant, and, admittedly, more exact methods of design are desirable to prevent future economic waste. There are also cases in which cofferdams with dimensions smaller than those indicated by the usual equations have been used without encountering difficulties; although in a few of such instances the factors of safety were so low that disaster could have followed any minor disturbance of soil or water level. In general, however, unanticipated forces or acts of nature have caused disasters which conceivably could have occurred regardless of any kind of theoretical analysis.

Penetration of Sheet Piling; Cellular Cofferdams on Deep Sand Strata.—Aside from consideration of uplift of sand due to percolation, it is believed that the depth for driving the sheet piling in sand need not be $\frac{2}{3}$ H (see second paragraph following Eq. 29*b*) but that $\frac{1}{2}$ H would be ample—because the sand in the cells can grip only one side of the piling, whereas, when driven, the piling is gripped on both sides.

Bearing and Extraction Values of Sheet Piling; Cellular Cofferdams on Deep Sand Strata.—Sheet piles in pairs may be driven and then loaded or extracted by means of hydraulic jacks to determine the forces required to shove down or pull out the piles (see second paragraph following Eq. 37). In similar soils, the values for a given depth of penetration are approximately uniform. It is easier to make an extraction or pulling test than a load test on the sheet piling, as anchor piles are not needed and the required pull may be exerted against cribbing on the ground. The bearing capacity may be safely assumed to be 15% greater than the extraction value, although, where piling is driven to rock, bearing value might be much greater than extraction value. The evaluation of vertical load capacities on sheet piling by means of hammer formulas is a very uncertain practice. The resistance of the interlocks when driving in a wall is a greatly varying factor—so great in some instances as to prevent driving to moderate depths. The values for the sheet piles in the soil must be determined on piles separate from a continuous wall.

Conclusions.—The conclusions are most excellently stated. However, conclusion 2 needs further clarification and possible expansion. With respect to conclusion 5, the reduced pressures seem to be due to arching or cohesive action, or to a combination of both, since the Rankine theory is conservative and deals with granular materials only. In conclusion 6, the depth of penetration of sheet piling can be dictated by requirements of watertightness which, in some cases, may indicate a greater depth than is needed for load capacity.

Cellular Versus Double-Wall Cofferdams.—The author's comments about the superiority of the cellular, as compared to the double-wall, cofferdam are true only when full consideration has been given to adequate foundation support and the reduction of permeability in the supporting soil. Inasmuch as double-wall cofferdams usually have inner, and often outer, berms, their service record has been about as good as that of the cellular type. There have been failures of individual cells due to boils or undermining from percolation which might not have occurred on a wide-bermed, double-wall installation.

General Remarks.—Professor Terzaghi has presented an analysis which should lead to further inquiring study and observation. He has provided another readily understood and practical series of steps for checking the design of any type of cellular cofferdam.

With this paper at hand, the designer may readily differentiate between an ultraconservative design for a permanent cofferdam or bulkhead and that for a temporary structure which, although it must be safe, may be allowed to have more deflection or movement while in use. If the movement becomes excessive in a temporary structure, it may be checked by one of the several known effective methods.

The author is to be commended for the thorough, thoughtful, and orderly presentation of a comparison between old and entirely new mathematical approaches on this subject. He deserves the wholehearted thanks of the engineering profession for assembling into convenient reference form much valuable historical and technical data on cofferdams, as well as for providing a most complete Bibliography on this difficult, complicated, and abstruse subject about which considerable quantitative information and experience must still be gathered.

Acknowledgment is made to J. R. Woods, sales engineer, Carnegie-Illinois Steel Corporation in Pittsburgh, Pa., for a constructive review of this discussion.

RAYMOND P. PENNOYER,[5] ASSOC. M. AM. SOC. C. E.—The basic computation to determine the necessary width of a cellular cofferdam, by bringing the intersection of the resultant of the moments created by two opposing forces (the lateral overturning force and the weight of the contents of the cells) within the middle third of the base, is expressed by Eq. 8. The author has stated that "This conclusion is open to serious objections." The only substitute offered in the paper for this basic determination is a statement that the "average width" of circular cells built on a rock foundation has been 0.85 H, in the past (see paragraph containing Eq. 27). All the remainder of the paper analyzes the theoretical internal stresses in a cell on the basis of this ratio.

Overturning Moment; Width of Cells.—The writer believes that 0.85 H has been more nearly the minimum width of completely excavated and unwatered cellular cofferdams than the average value. Furthermore, the proper and safe width of cellular cofferdams from which Professor Terzaghi has derived this questionable average has been determined, for years, by the very equation he states is open to serious objections.

It would seem axiomatic that, since the weight of the material contained in a cell determines its capability to resist the overturning moment, this material should receive careful consideration. Broken stone, gravel, sand, sand and clays, etc., have different weights, as have the undisturbed soils in which cells are built to various elevations. The analyses in the paper have all been based on sand weighing 110 lb per cu ft throughout the cell. No mention has been made of the possible buoyancy of water within the cells in so far as it will alter the weight of the contained soil (that is, the submerged weight); nor does Professor Terzaghi explain how he proposes to adjust the width of the cells to make the cells safe against overturning when there are almost infinitely variable factors affecting the weight of the contents.

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No method is proposed to determine the width of cell necessary to resist the lateral or overturning forces safely. The paper deals only with an analysis of a cofferdam built in water on bare rock, with no submerged soil overburden. This condition is extremely rare in practice and always requires that sheet piles be driven into empty cells on the rock—a major construction problem. The width of a cofferdam in salt water should be different from that in fresh water, even under such conditions. Likewise, there must be an adjustment in width to resist the lateral pressures of submerged soils, for various elevations, in combination with layers of natural soil or fill above such submerged soil. There must also be an adjustment to resist other possible lateral forces due to construction equipment, storage loads, railroads, derricks, wave action, ice, swiftly flowing water, etc. This basic consideration has been dismissed in the paper by stating that the average width of cellular cofferdams in the past has been 0.85 H, and further that this ratio may be considered an acceptable basis for the design of cofferdams in clear water on rock.

Eq. 7, with its supporting text, is offered under the assumption that a cellular cofferdam filled with soil has a factor of safety of 3 when the width is calculated to bring the resultant of the forces within the middle third of the base. This formula is in serious error since the soil within the cells must be assumed incapable of resisting any tension or uplift along the outside face (the heel) of the cofferdam. The width of a cell must be adjusted to bring the stress to zero at this point (neither tension nor compression) and, as a consequence, the factor of safety against overturning is actually unity, which means that no factor of safety exists. Professor Terzaghi then discusses the possible, theoretical distribution of pressures under the cells (Fig. 7) and decides that, since a sand-filled cofferdam having a width of 0.85 H actually has its theoretical factor of safety reduced from 3.8 to not more than 2, the internal shear becomes the critical consideration.

Internal Shear.—An attempt was made in 1934 to explain mathematically occasional leaning or distortion of the cells because of evident lack of internal shear resistance of the soil contained in the cells (10). At this time, the writer offered the following formula (10*c*) for vertical and horizontal shear, s, at a given point:

$$s = \frac{3\,P}{2\,b} \qquad (39)$$

in which, in addition to the notation of the paper, P equals the total lateral external load above the given point. Eq. 9, giving the total shear along the so-called neutral plane of the cell for a given moment, results in exactly the same value as Eq. 39 for maximum shear. Several successful bulkheads have been built which, upon analysis, were not more than 60% safe in internal shear, but which, at the same time, had a width sufficient to resist the estimated overturning based on the "middle third" method of determination. A mathematical theory of analysis will never serve as more than a guide based on precedent, because the assumption that the soil fails, along a plane of maximum shear, called the vertical axis, similar to a beam, seems fallacious.

What actually happens is that the entire mass of confined soil changes shape simultaneously. This action can best be described as a universal plastic flow, or distortion, which bears no relation to the strains set up in an elastic homogeneous mass that can resist shear, compression, and tension—with special emphasis on tension. When a cell leans inward, as a result of the simultaneous distortion, or flow, of all the contained material, the sheet piles in the internal, or diaphragm walls, remain parallel to each other and necessarily maintain their original length to lean into a parallelogram. Under this condition, all the interlocks in a cross-wall, or interior wall, slip the same amount at the same time, irrespective of their position in relation to the fictitious neutral axis.

As the writer stated previously (10), the resistance to lateral deformation of the bulkhead, through friction in the interlocks of the sheet piles, affords an indeterminate factor of safety in the calculation of internal shear. This still seems true despite the author's able mathematical analysis of this feature for a hypothetical cofferdam with a width of 0.85 H. With sufficient sand trapped and wedged into the interlocks, or sufficient penetration to develop frictional resistance against the sliding, the shear resistance of a cross-wall might be equal to that of a continuous plate of steel over the entire cross-wall. It is likewise possible that the assumption of the frictional value of an interlock as three tenths of the actual tension may be entirely too high. A cellular bulkhead built in undisturbed virgin soil, followed by excavation on one side, is a case in point. There might be no tension on the interlocks before the cells became distorted laterally.

Furthermore, the paper states (see sentence following Eq. 21) that the friction value f "is likely to be fairly close to 0.3" of the tension per unit of length on a sheet-pile interlock—with no explanation as to the basis for this seemingly arbitrary assumption. If this factor was assumed analogous to the coefficient of friction of steel on steel, the statement would seem fallacious. Interlocks of sheet piling are designed for the least frictional resistance when setting up, driving, and extracting, with emphasis on reducing the area of contact between surfaces to a minimum and on eliminating wedging action from forces tending to separate the interlocks. There is no relation between the frictional value of two pieces of steel in what amounts to line contact and the coefficient of friction of steel on steel times a tension force per unit of length, when no area of contact can be assumed. In the discussion of the bulkhead in Fig. 11, Professor Terzaghi has expressed some doubt as to the safety of using three tenths of the tension for the frictional value of an interlock in his analysis of the actual distortion of the cells.

Tension in Interlocks.—The author recommends that the method of approximating the possible maximum interlock tension by the so-called Rankine formula, using $\tan^2\left(45° - \frac{\phi}{2}\right)$ to arrive at the conversion factor between the vertical and horizontal pressure, be abandoned in favor of using 0.4 when the cells are filled with sand and gravel. He states that this method does not create the illusion of resting on a scientific basis. It is seldom, however, that a cell entirely filled with sand is encountered in actual practice. Consequently, the paper leaves the reader without a practical basis for analyzing the problem that generally confronts him. As a result, use of the factor 0.4 might be worthless or even dangerous.

Professor Terzaghi indicates that designers seldom, if ever, use a conversion factor of much less than 0.4 for the lateral pressure of the contents of a cell filled with sand or gravel, completely drained and founded on rock. With a working stress of 4,000 lb per lin in. in tension for the interlock of the sheet pile, cellular cofferdams would be limited to a height of approximately 48 ft and a maximum radius of cell of about 24 ft. With a working stress of 6,000 lb per lin in., the limitations would be a height of about 58 ft and a radius of approximately 29 ft. Since there are many successful examples of bulkheads in service with dimensions considerably beyond these limits and containing soils far less ideal than the assumed sand and gravel, this conversion factor of 0.4 does not agree with actual practice. A conversion factor as low as 0.17 has occasionally been used without any indication that the lateral pressures in the cells were underestimated. Obviously, the relieving effect of wall friction, even to the point where a complete arch is formed as described subsequently, is an important factor to be considered when establishing the conversion factor for use in any specific instance.

The paper offers no suggestions for approximating the possible interlock tension: (1) When the cells are partly full of water and submerged soil (see "Part I. Practical Considerations"); (2) when the cells have a layer of clay or other soils at the bottom or at various heights as part of the original soil on which the bulkhead is built; or (3) when possible surcharge loads are present. Another common condition not discussed in the paper is that in which the contained soil is not fill but is composed of various types of undisturbed soil into which the bulkhead is built before excavation on one side proceeds.

In his prior paper (10*d*)(10*e*), the writer illustrated a case in which external and internal earth pressures differ which included weep holes, submerged soil, and water levels at various heights. In the light of the practical problems illustrated and discussed, and considering the complexity of conditions encountered in practice, the writer feels that Professor Terzaghi has oversimplified the problem by assuming a cell width of 0.85 H and a conversion factor of 0.4 to develop the bursting pressures.

Probably no one realizes more clearly than the writer that, at best, the use of the Rankine conversion factor is a very approximate assumption. He will continue to use it, however, until something better is available, especially since water and submerged soil seldom are the critical considerations in overturning forces, weights of material, and bursting pressures. In the light of many precedents, the use of this method is a guide which helps the experienced engineer to determine the border line between conditions in which trouble with broken interlocks is likely to be encountered, on the one hand, and the likelihood that construction will be completed successfully, on the other hand. Mathematics is only one consideration in reaching this decision, because the final verdict must take into account the strength of the sheet piling as manufactured, precedent, the variety of conditions into which the sheet piling is set up and driven, and the experience, ability, and equipment of the constructer. Interlocks can be ruptured by sand trapped in the bowls during the pile driving; builders can fail to set and drive the sheet piling accurately; or the final closure pile can be used to pull the remainder of the wall into its proper place. In all such cases, indeterminate stresses are created in the steel, or an interlock is ruptured in the process. Certainly, mathematics is futile under such conditions.

Sand and gravel, or rock, may actually arch and form a complete bottom in cells above the base of the sheet piling. About 1932 the writer had occasion to inspect a bulkhead composed of a series of diaphragm-type cells, about 35 ft by 27 ft between cross-walls and 35 ft high, the sheet piling being driven into sand and gravel along a river bank. With the bulkhead in place, the sand and gravel on the river side were excavated to bare rock and used to fill the cells to the top. As excavation proceeded, it was found that the sheet piling had been driven through a layer of boulders to about 12 ft from the bottom and had been twisted and bent, the interlocks being ruptured in many places. Water coming through the bulkhead had washed away the original soil through the gaps in the piling up to the top of the breaks, some 12 ft off the rock, leaving three or four of the cells completely empty to about that elevation. The cells were standing, however, and were resisting the maximum lateral load from the river bank as intended. Above the 12-ft elevation, the sand and gravel had formed into about as perfect a dome arched between the side-walls, front-walls, and back-walls, as could have been built deliberately in brick or stone. There was an occasional sloughing off of sand in the dome, to be sure; yet, after the cells has been repaired, the superintendent reported, a week or ten days later, that a bucket actually had to be used to break the dome, so that the fill could settle to the bottom.

An analogy may be found in a pail filled with water. Its resistance to overturning depends upon its weight, height, and cross section. All shear is taken by the steel shell of the pail as water has no shear or frictional resistance value. Other material, such as sand and gravel, does have a high shear and frictional value and consequently possesses the inherent ability to arch as described in the aforementioned example. Thus, the shear stresses in a steel bulkhead would be relieved by the soil or fill material, whereas in the pail of water the total shear would have to be transmitted by the walls of the pail without relief.

If the foregoing theory is correct in principle, it would indicate that perhaps the "middle third" limitations on width are conservative and do have a factor of safety because the "bottom" is actually able to resist some uplift at the heel. This may be one reason why mathematical analyses of internal shear seem at wide variance with the actual resistance of cells to leaning or distortion in some cofferdams. Calculations of interlock tension of moist (not submerged) soil would thus become a study in the theory of pressures in bins. This may account for the fact that the maximum pressure seems to be at the zone of greatest bulge in a cell, ordinarily about halfway up the height, although it has been observed that this varies slightly for different soils. The maximum bulge may occur at the level of the first actual "bottom." Of course, such a theory cannot be used when there is water in the cells, but the bursting pressure from the soil alone might safely be ignored under ideal conditions below the point of maximum bulge. It is difficult to imagine a condition for which the pressure of the water in the soil at the bottom could be critical, especially if the bulkhead had weep holes.

Furthermore, if the foregoing theory is correct, the distribution of pressures at the surface of the rock (Fig. 7) would differ from the true conditions, because all the pressure at the toe would have been transmitted to the steel by friction and arching. Observers have long noted that the overturning moment does not create any consequential added bursting pressure in the interlocks of the sheet piling because no consequential added deflection of the steel has been perceived. The arching theory suggested herein may account for that phenomenon also, since interlocked sheet piling is a fully supported column.

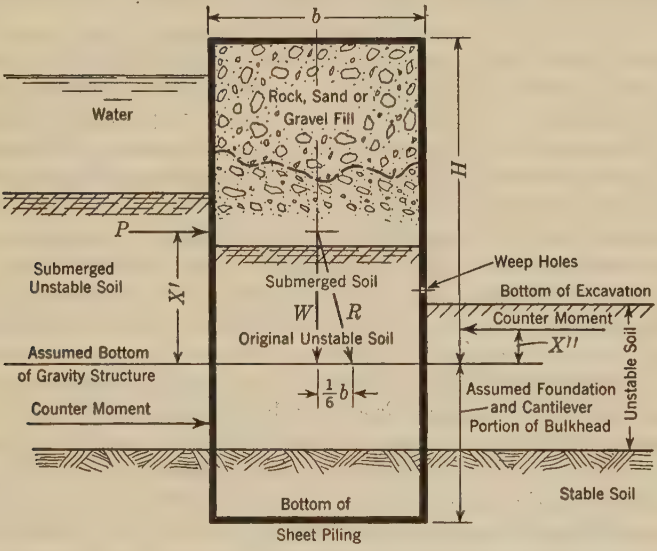
b
Rock, Sand or Gravel Fill
Water
H
P
Submerged Unstable Soil
X'
Submerged Soil
W
R
Original Unstable Soil
Weep Holes
Bottom of Excavation
Counter Moment
X''
Unstable Soil
Assumed Bottom of Gravity Structure
Counter Moment
$\frac{1}{6}b$
Assumed Foundation and Cantilever Portion of Bulkhead
Stable Soil
Bottom of Sheet Piling

FIG. 12

In general, it has been difficult for the writer to be completely satisfied with any of the theories as to why cellular bulkheads function successfully without adding the thought that arching of the fill actually forms a "bottom" or "bottoms" to the "pail" or cell. When a material like plastic clay or mud is used to fill the cells, the action becomes uncertain, variable, and completely different from the action caused by materials like rock, sand, or gravel. It may be that the incapacity of such plastic material to arch accounts for the difference.

The writer has watched the action of many cellular cofferdams since observing the sand dome in the broken cells as described herein, and as yet has found nothing to disprove his surmise. On the contrary, when such an actual bottom is assumed, the guides obtained from the application of the basic mathematical analyses are reassuring when compared with the characteristics and action of cofferdams in service.

Cellular Cofferdams in Clay.—In his former paper (10), the writer did not attempt to discuss cellular cofferdams founded on anything except rock, because he considered there were sufficient intangibles involved in that problem. Even with Professor Terzaghi's contribution, it would seem that the proper design of cellular cofferdams in clay (or even through overlying strata of clay or silt on top of stable strata), especially when submerged, is still largely dependent on making mathematics coincide with what experience, observation, and practice dictate to be adequate construction.

The determination of the proper width of a cellular structure, founded on a stratum of unstable soil, is still a problem largely dependent upon judgment based on precedent and experience. It is axiomatic that, as soon as the sheet-pile structure extends into soil, it becomes partly a cantilever structure. If it extends deep enough and has insufficient width (weight of contents), it no longer is a gravity structure but a cantilever structure. So far as the writer knows, no attempt has been made, even theoretically, to design a cellular sheet-pile structure acting entirely as a cantilever. When all the intangible factors are taken into consideration, the problem would seem to be indeterminate.

Fig. 12 illustrates a design problem quite usual in practice, to the solution of which the paper has rendered little of tangible value. In this figure, in addition to the notation of the paper, R is the resultant of all applied forces, vertical and horizontal; and X' and X'' are moment arms as shown. Successful practice has indicated that a bulkhead can be considered as a gravity structure down to an assumed elevation, or "bottom." The selection of this fictitious bottom is a matter of judgment, taking all factors into consideration, including the resulting width of the cell. This elevation is not selected where the "countermoment" due to the passive resistance of the soil balances the overturning moment (because the calculated width of the cell would indicate a greater width than experience dictates is necessary) but at a somewhat higher elevation. One cannot take into consideration, either, the countermoment on the outside (heel) of the structure, because in this case the structure would be assumed capable of resisting the support stresses of a cantilever and there is no certainty that the structure is capable of transmitting the internal stresses properly. That part of the structure below the assumed "bottom" is thus considered as a foundation only for the gravity structure above this elevation, transmitting the compression forces due to the overturning moment and weight of cells into the soil below. Such resistance as that of this foundation—a cantilever against overturning, tension at the heel, and shear resistance against lateral distortion—may be assumed to be an indeterminate, but added, factor of safety.

The structure above this elevation of the assumed "bottom" is then analyzed for proper width, based on the width necessary to bring the intersection of the resultant of the forces represented by the weight of the contents of the cell and the overturning moment within the middle third of that bottom. The size of the cells must also be brought within a proper radius so as not to exceed the safe interlock tension in the sheet piles. Sliding is no longer a consideration and theoretical figures for shear resistance are balanced with precedent.

Professor Terzaghi states that the familiar "middle third" method of approximating the proper width of cell "is open to serious objections" (see sentences following Eq. 8); but he has proposed no alternate solution. The problem of how to determine the proper width of a cell under the aforementioned conditions, which are subject to variation on every project, has been dismissed with the very doubtful statement that the average width of such structures on rock has been 0.85 H and with the further note that internal shear is usually the governing factor. Obviously, the shear resistance of the contents of a structure under the aforementioned conditions could be much greater than the shear resistance of the contents of a structure founded on rock, which does not have the cantilever supporting value that the "foundation" gives to the sheet piling, with special reference to the internal walls. Eq. 32 giving the vertical maximum height H for a given bearing value of clay is of questionable practical value because the distribution of the pressure at the toe of the cofferdam, due to friction between the soil and the sheet piling, has been disregarded; the deeper the sheet piling is driven, the greater the distribution will be. Safe construction, as it concerns possible settlement at the toe, has always been assured when the sheet piling was driven to high resistance against further penetration, provided, of course, that there was no underlying unstable stratum below the bottom of the sheet piles.

After reaching the conclusion expressed by Eq. 32, the author proceeds to analyze mathematically the internal stresses in cellular cofferdams on a compressible base, using the "customary" values of $b = 0.85\ H$, etc., which, in Part I, he stated were the average ratios for sand-filled cells built upon rock. In practice, there is no relation between the two conditions; nor is the ratio of $b = 0.85\ H$ applicable, except by chance after the "middle third" method determines the proper width based on the assumed overturning moment and the weight of material in the cells.

Also, the author has analyzed the division of the total overturning moment on a cellular bulkhead (see Fig. 11) between the shear resistance of the fill and the sheet piling. He attributes the fact that the bulkhead began to deflect alarmingly (in spite of theoretical analysis) to the probability that the real lock tension (friction) might actually have been smaller than calculated. The writer doubts whether the actual overturning moment could be determined as accurately as outlined, since a few of the uncertain factors involved include the assumption as to lateral forces, the arm of the overturning moment or point of rotation, the fact that the structure is partly a cantilever and partly a gravity structure, the amount of cantilever support dependent on the lean of the cells, and the supporting value of the soil (both under and on the sides of the cells at the base).

Cellular Cofferdams on Deep Sand Strata.—Professor Terzaghi states (see "Part II. Design of Cellular Cofferdams") that a review of current methods of designing cofferdams reveals an "amazing situation," inferring that the seepage of water, possible boils in the bottom, and loss of stability of the foundation soil inside a cofferdam have not received consideration. The writer does not know of any phase in the design of cofferdams which has received more attention from designers and contractors. In 1935, the writer presented a discussion (29) of this hazard. This article offered some rule-of-thumb ratios between the hydrostatic head and depth of penetration necessary for the sheet piling in single-wall cofferdams when driven into various types of soil. These ratios were based on long observation of successful practice in preventing bottoms from boiling seriously or "blowing up." The ratio given for water-bearing sand was a penetration of 30% to 40% of the unbalanced hydrostatic head, provided the cofferdam is properly pumped, as against the 67% mentioned in the paper.

Also, the author recommends a penetration of $\frac{2}{3}$ H into sand as being necessary to prevent possible settlement at the toe of the cofferdam, due to vertical load on the sheet piles. It is commonly known that sand is probably the soil most resistant to the penetration of sheet piling and, under most conditions encountered in practice, the writer doubts if sheet piling could be forced into the depth of sand, indicated as necessary in the paper, without jetting, without rupturing interlocks, or without encountering absolute refusal. The writer has test results before him giving safe bearing loads of sheet piles, which show that a single pile tested to 47 tons when driven 25 ft into sand and when interlocked with two other sheet piles tested to 120 tons. When one considers that for even a 60-ft height of cofferdam the load on the sheet piles at the toe could hardly exceed 7 tons per foot width of sheet pile, $\frac{2}{3}$ H, or about 40 ft of penetration, seems at wide variance with the actual need.

In the discussion of filters and gravel toes for taking care of seepage, the author has omitted to mention the usual methods of stabilizing the soil inside cofferdams by pumping from sumps and the remarkable achievements of the so-called well-point systems in this respect.

The discussion of breakwaters founded on sand includes a recommendation that the sand should be compacted to a depth of at least 1.5 H in some situations. Under the conditions in Fig. 9(*f*), showing water going over the top of the breakwater, the critical consideration for stability is the fact that the turbulent water scours the bottom on the harbor side until the breakwater fails, unless protected by a stone berm. Several articles on breakwaters published between 1932 and 1934 (30)(31)(32)(33) are enlightening with respect to the author's treatment.

Conclusion.—In general, the author deserves commendation for attempting to place the design of cellular cofferdams on a more precise mathematical basis. He has demonstrated that few problems have been overlooked in the past by the engineers and contractors who have had experience with the practical considerations necessary to design and build such structures properly and who have tried to reduce some of the intangible factors to some semblance of a precise basis for analysis. The paper is largely confined to a mathematical analysis of a cellular bulkhead of hypothetical size and weight of fill under a condition that is not common in practice. The attempt to apply the analysis, however, serves to indicate a wide variance between theory and practice. Dismissal of the old "middle third" method of approximating the proper width of a bulkhead under the wide variety of conditions met in practice by the substitution of the "average," "acceptable," or "usual" ratio of width equal to 0.85 H is not only erroneous but dangerous.

In estimating the interlock tension, a conversion factor of 0.4 for the computation of the lateral pressure in a sand fill would be too high under some conditions and dangerous in others. No method is proposed whereby the critical interlock tension can be determined or the maximum size of cells established to meet the wide variety of conditions encountered in practice. The conclusion that consideration of the shear value of the soil in cellular cofferdams is more critical than the determination of the proper width and maximum size of cells is fallacious.

Dean P. Tsagaris,[6] M. Am. Soc. C. E.—Comparatively little has been written or published on the rational design procedure and the behavior exhibited by this type of cellular cofferdam construction. Therefore, assumptions, analyses, and final judgment must be based on actual experience and observation. The paper constitutes a valuable and significant contribution to the engineering profession on this subject. As the author states, tests should be made, but results should be used with discretion, since they have often proved inconclusive.

The following comments and discussion are confined to cofferdams in which piling is driven to rock through an overburden varying from several feet to 50 ft in depth. Experience is largely drawn from the many cofferdams designed and built by the Tennessee Valley Authority (TVA) for the purpose of enclosing construction operations.

The inherent economic advantages in using circular cells with connecting arcs are due primarily to simplicity of construction and re-use on subsequent stages and projects. Furthermore, cells can be filled independently without interruption, and possible interlock failure will be localized in one cell. As a general rule, if a cofferdam does not exceed 50 ft in height and is carefully designed for sliding and interlock, allowing for saturation of fill within the cells, the resulting construction will be satisfactory and safe.

Designing on an "equivalent width" basis (and a length of unity) was found to be sufficiently accurate after a comparison was made with the results obtained from using a typical cell as a unit. This expedient obviously simplified calculations greatly.

Because of the great height of the Kentucky cofferdam (100 ft above rock) and of the magnitude of this undertaking as a whole, very careful consideration was given to the following factors: (*a*) Saturation of fill material within the cells; (*b*) interlock tension; and (*c*) internal resistance to shear. In view of this concern, the writer cannot agree with the statements made on the subject of internal shear, which read (see second paragraph following Eq. 8): "* * * before this failure [by overturning] occurs, the cofferdam is likely to fail from another cause which has never received any attention" and (see second paragraph preceding Eq. 28*a*) "* * * a potential source of failure by shear along vertical planes (Eq. 19*b*) which has been ignored in the past.* * *."

(*a*) *Saturation of Fill Material Within the Cells.*—On all cofferdams built by TVA, a mixture of sand and gravel with little silt content was used to obtain as much free drainage as possible. Filling was done by the hydraulic process, except for a few cells built in the dry, such as tie-ins. When the first cofferdams were constructed, the need for dependable information on saturation was realized. For purposes of study, several rows of pipes were provided within the cells, and observations were taken, after unwatering, on the level of water resulting from both a rising and receding river.

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In general, results disclosed a rather flat gradient across the cells, beginning from a point somewhat below the outside river elevation. No complete drainage was evidenced in spite of the customary weep holes on the dry side. In any case, much depends on the care exercised in the field and the extent of the provisions made to ameliorate saturation.

A saturation line with an approximate slope of 1 on 2 starting from the outside water elevation was considered satisfactory, and slopes as steep as 1 on 1 were adopted where special care was taken to keep cells well drained. Hence, all calculations were made accordingly on subsequent jobs, using 110 lb per cu ft for material above the saturation line, and 65 lb per cu ft for material below the saturation line. As a result, all cofferdams required a berm on the dry side to satisfy the aforementioned requirements. This does not necessarily mean that cells cannot be made wide enough to avoid berms, provided they check in every other respect and are economical. A horizontal line, at an elevation so chosen as to represent the average expected condition of saturation, should serve just as well and at the same time simplify computations.

(*b*) *Interlock Tension.*—This is one of the most important features in cellular cofferdam design and should be kept within safe limits. The practice of computing pressures on a straight-line variation all the way to rock is fallacious since cells almost invariably exhibit a characteristic bulge about one quarter to one third of the way up from the base. Driving apparently forces the steel sheet piling into rock sufficiently to prevent any appreciable movement at this point. When berms are used a similar condition exists, except that the overburden or the berms serve to restrain the piling. In the light of this phenomenon, the writer believes that pressures from which interlock tension is computed should be considered maximum at three quarters of the way down from the top in cases without berms. If berms are used, however, maximum pressures will occur at a somewhat higher position.

With the improvement of piling and the ability of the manufacturers to guarantee higher strengths, interlock failures have been very few and are due primarily to construction operations such as difficult driving. In such instances "brooming" and opening of interlocks have resulted near the bottom, but repairs can be made easily, especially in cases where no berms are used. All possible conditions should be investigated to determine maximum interlock tension, particularly during filling operations when the river elevation is low, because this condition most often controls.

The author does not mention the amount of tension contributed by the connecting arcs to the existing main cell tension. This has been found considerable, and should be neither neglected nor computed as a free force acting on the main cell through the tees or wyes. The neglect cannot be excused, and computation as a free force does not consider the passive resistance of the fill within the connecting arcs, which resists a large percentage of the tension before it can find its way into the main cell.

(c) *Internal Resistance to Shear.*—The possibility of shear failure has been recognized by TVA engineers concerned with cofferdam design, and investigations in this connection have been thorough. No rigorous mathematical analysis is justified, however, when friction coefficients for fill material, the actual pressure against the inside of the piling, and the resistance offered by interlocks are so variable. Very ably, Professor Terzaghi explains the implications involved in computing actual shear and evaluating the resistance to this shear along a selected critical plane.

Relatively small horizontal deflections are capable of mobilizing high pressures within the fill, especially against the inside surface of the piling, with equally large resistances. This tendency will continue as the lateral load increases provided the cofferdam is still safe against interlock failure, sliding, and overturning. Furthermore, if slippage occurs between sand and gravel particles, the new configuration of material may conceivably offer a greater resistance to shear than the results of computations indicate. Readjustment in each case is associated with planes of weakness, but other contact surfaces not so highly stressed will come into play to relieve the situation until horizontal movement due to slippage stops. This behavior transcends the realm of hypothesis; otherwise the paradox of nonfailure for cases with factors of safety less than unity cannot be explained. Since all cofferdams built by TVA were temporary, a factor of safety of 1.25 for shear, sliding, and interlock tension was considered quite adequate.

Briefly, the method used to compute total shear in any vertical plane, and the resistance, followed the general procedure described by the author, except that it was recognized that active pressure within the material was augmented by the effect of external horizontal forces tending to compact the fill confined in the cells. A friction coefficient of 0.55 was used. To the foregoing, the resistance against interlock slippage was added, using a coefficient of friction of 0.3, which gave the total effective resistance to shear. Under the most unfavorable conditions, the smallest factor of safety was 1.2. It is felt, however, that the coefficients of friction for both internal shear and slipping of interlock were quite conservative.

Recommendations.—To summarize, the writer would like to offer six recommendations, as follows:

1. Circular or clover-leaf type cells with connecting arcs should be used;
2. Selected free draining fill material, such as sand and gravel, will be found best;
3. Computations should be based on saturated fill below a line with an approximate slope of two horizontal to one vertical (for simplicity a horizontal line may be adopted at an elevation which represents average conditions);
4. Shear resistance should be considered and computed on a logical and rational basis;
5. Interlock tension must be kept within safe limits and the modifying effects of the characteristic bulge in sheet piling should be recognized; and
6. Grouting of the foundation rock will result in a tighter and safer cofferdam.

R. T. Colburn,[7] M. Am. Soc. C. E.—It is gratifying to find that the subject of the design of cellular cofferdams is being given the consideration it deserves. This paper is a welcome and comprehensive addition to the data previously existing on this subject, and points the way to possible future developments through field observations on actual structures. The Tennessee Valley Authority (TVA) has designed and built fourteen sheet-pile cofferdams since 1936, and the paper would have been extremely useful in this connection had it been published at an earlier date.

The TVA cofferdams were all constructed of standard 15-in. straight-web piling, with $\frac{3}{8}$-in. web thickness and weighing 38.8 lb per ft. They were driven through varying depths of overburden to solid rock and the cells were of either the clover-leaf type or the circular type with connecting arcs. About 90% of the cells were filled by the hydraulic method and in all cases the material was sand and gravel. Where the rock foundation was unsound grouting was used to consolidate it, to prevent undermining, and to reduce the flow of water under the cells and stop "boils."

The twelve cofferdams used at Pickwick, Guntersville, Chickamauga, Watts Bar, and Fort Loudoun dams, vary in height from 30 ft to 55 ft. The diaphragm or cross-wall type of cell was not used by TVA. The reasons for the selection of the circular and clover-leaf type cells deserve consideration. The designers found by analysis that the saving in amount of piling by use of the diaphragm type decreases as the height increases, and for heights of 40 or 50 ft no appreciable saving results. For the circular cell type of cofferdam for any given length, the amount of piling is practically constant regardless of the diameter of the cell. The circular and clover-leaf cells have a distinct advantage over the diaphragm type if they are filled hydraulically as was the case at TVA. In the diaphragm type the fill in one cell cannot be more than 5 ft or 10 ft higher than the fill in the adjoining cell without distorting the cells. This is especially inconvenient when the cells are filled with a hydraulic dredge. The principal reason for deciding on this type of cell, however, was that any failure due to a split interlock or other cause could be localized in the cell in which the failure occurred. Every cell is self-supporting. In the diaphragm type a serious failure in one cell will cause partial or progressive failure in several adjacent ones.

Both stages of the cofferdams built at Kentucky Dam were 100 ft high and presented a problem of a magnitude on which practically no data existed. Whereas the other twelve cofferdams built by TVA were designed in accordance with the usual current practice for (*a*) bursting of the cells, (*b*) sliding, and (*c*) overturning, it was at once discovered that the factor of vertical shear became extremely important on this high cofferdam. This fourth factor was carefully checked in the Kentucky Dam design and, in fact, it determined the height of the berms inside the cofferdam. The writer agrees with Professor Terzaghi that shear is a factor that should be considered.

To give the greatest interlock strength possible the piling at Kentucky Dam was specified to have a guaranteed ultimate strength of at least 16,000 lb per in. instead of the standard 12,000 lb per in. The increase in value was obtained by more careful inspection and selection of piling being rolled and no change was made in the standard piling section. The cells were driven to solid rock through 40 ft or 50 ft of overburden, which consisted of sand and gravel containing only a small amount of silt and fines and no boulders. The connecting arcs were made with fabricated 30° Y-connections instead of the standard tees and with a longer more flexible arc. This gave much more flexibility and allowed for deviations and inaccuracies in driving the main cells through the heavy overburden. If the usual T-connections had been used the arcs would have been shorter, and this important feature would not exist and undoubtedly many more interlocks in the connecting arc would have been split in driving. It was also felt that by using a 30° Y instead of the standard T-connection the increase in interlock stress in the adjacent piles in the main cell would be less.

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Many extensive laboratory tests were made on the fill material to determine the basic characteristics and provide a basis for determination of the design coefficients and assumptions. TVA engineers can readily testify to the accuracy of the author's statement that laboratory tests are inadequate. Several series of tests were made, the best of these showing ranges of 24° to 35° for the angle of internal friction and 0.45 to 0.70 for the coefficient of internal friction. Some tests showed these values to be even higher. The assumed value of this coefficient makes a great difference in the design, especially in the vertical shear strength. The values assumed at Kentucky Dam were 28° 50′ for the angle of internal friction with a corresponding coefficient of 0.55, which are extremely conservative; but in the absence of more conclusive tests a higher design value did not seem justified. The actual design assumptions for the Kentucky cofferdam were as follows:

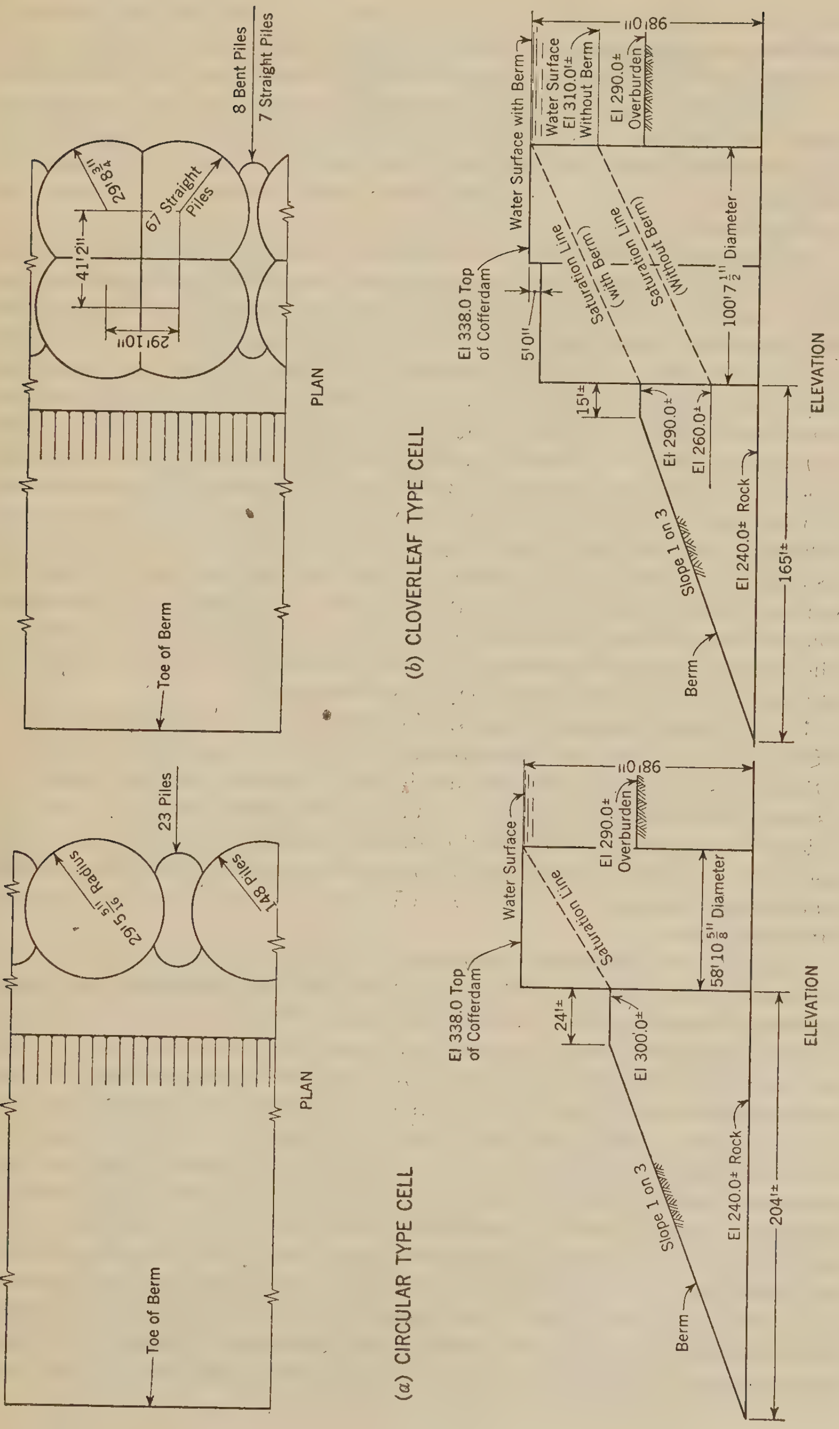
No.	Description	Quantity
1	Fill material: Sand and gravel, with a very small quantity of silt	...
2	Overburden: Sand and gravel—in some places a layer of cemented gravel and in one place a 20-ft layer of blue clay	...
	Weight of Material, in Pounds per Cubic Foot—	
3	Dry	110
4	Submerged	65
	Coefficients of Friction—	
5	Gravel on rock (sliding)	0.5
6	Gravel on steel	0.4
7	Steel on steel, in piling interlock	0.3
8	Internal friction	0.55
9	Angle of internal friction for dry and saturated material	28° 50′
10	Slope of berm	1 on 3
	Stresses in Piling, in Kips per Square Inch—	
11	Allowable interlock strength	12
12	Ultimate interlock strength	16
13	Allowable tensile strength in the web	32
14	Ultimate interlock strength in the web	70

No.	Description	Quantity
15	Factor of safety for sliding, vertical shear, and in the interlock	⋝ 1.25
16	Water pressure: Considered the full hydrostatic pressure, to rock	...
17	Active earth pressure: Considered triangular, with the resultant acting at $h/3$ above the base	...
18	Reactive earth pressure: Considered parabolic, with the resultant acting at $h/2$ above the base	...
19	Saturation line: Assumed as shown on the section	...

The plan and section of the Kentucky cofferdam is shown in Fig. 13. The design assumed the cell, with its fill material, to act as a homogeneous mass similar to an earth dam except that its shape was restricted and held in position by the sheet piling. The resistance to sliding, overturning moments, and shear stresses were calculated on this basis, taking into account the effect of the berms, overburden, and the shear resistance of the fill and the interlocks. On this basis the limiting factor in some cases was found to be vertical shear of the material in the center of the cell or at the inside face of the cofferdam. The stress in the sheet piling was calculated from the internal pressure or bursting effect of the material as it was restricted by the sheet piling, plus the stress introduced by the connecting arcs through the Y-pile connections. The effect of complete saturation during the hydraulic filling was considered. On this basis a cell 60 ft in diameter is about the maximum size that can be built without overstressing the interlocks. This was the determining factor in fixing the size of the cells. Obviously, this was too small to take care of the required 100-ft head. The additional support necessary was then provided by a berm of sufficient height and slope to back up the cells with 100-ft head of water on the outside. It can be seen that the overburden itself provided the major part of the berm which was undisturbed material and this section was used in locations where there was sufficient room for a berm. On the outstream arm, however, the berm on the inside had to be removed for construction reasons. This left a 100-ft-high cofferdam with no berm or overburden on the inside resting on a fairly flat rock surface, or in the second stage on the spillway apron. This is the section where the clover-leaf cell was used which provided a total width of approximately 100 ft. It might be mentioned here that even with this type it was felt necessary to stipulate that the berm could only be removed if the height of the water was 25 ft or more below the top of the cofferdam during the months that the berm was removed. This was done in the dry season and the water remained at a low level so that no harmful results were experienced.

The TVA has had an opportunity to make various tests on their cofferdams and has made several which are of value in that they give actual results under field conditions.

Saturation tests were made at Pickwick and are summarized in Fig. 14. As stated by Professor Terzaghi, the saturation line is almost horizontal except on the outstream third which rises sharply to within a few feet of the water level. Subsequent tests made at Guntersville and Kentucky dams con-


PLAN
Toe of Berm
29' 5 5/16" Radius
23 Piles
148 Piles
Water Surface
El 338.0 Top of Cofferdam
Saturation Line
El 290.0± Overburden
98' 0"
58' 10 5/8" Diameter
24'±
El 300.0'±
Slope 1 on 3
Berm
El 240.0± Rock
204'±
ELEVATION
PLAN
Toe of Berm
41' 2"
29' 10"
29' 8 3/4"
67 Straight Piles
8 Bent Piles
7 Straight Piles
Water Surface with Berm
Water Surface El 310.0'± Without Berm
El 290.0± Overburden
98' 0"
El 338.0 Top of Cofferdam
5' 0"
Saturation Line (with Berm)
Saturation Line (Without Berm)
100' 7 1/2" Diameter
15'±
El 290.0±
El 260.0±
Slope 1 on 3
Berm
El 240.0± Rock
165'±
ELEVATION

(a) CIRCULAR TYPE CELL

(b) CLOVERLEAF TYPE CELL

Fig 13.—Plan and Elevation of TVA Kentucky Cofferdam

firm these Pickwick tests and indicate the fallacy of assuming that a cell will ever become nonsaturated. It also shows the extreme importance of proper drainage. All TVA cofferdams were drained by weep holes on the inside face.

At Kentucky Dam 4-in. pipes were inserted in the cofferdam and driven several feet into the cells. Perforations in the pipe provided drainage and the pipes drained into a flume on top of the berm or to a system of corrugated pipe headers laid on the original river bed to keep the water from seeping into the berm itself.

At Chickamauga Dam an attempt was made to determine the stress in the piling by use of a strain gage. It so happened that several cells were built in the dry and filled before they received a water load on one side. Gage points were established at regular intervals on two cells and their connecting arcs and at points about 12 ft apart vertically. Careful readings were taken as the cell was filled and before and after the outside was flooded. The readings were so inconsistent and erratic, however, that no curves could be plotted and no conclusions could be made. Inadequate testing methods might account for this, but the actual conditions in the cells are probably the main reason. For example, it is probable that the piling does not interlock evenly over its full length, and the tension across the face of the piling might easily vary on this account. In fact, if the interlock contact was poor on a certain length of the pile, the tension across the face might be very low at that point with corresponding higher stresses adjacent to it. The only value of this test is that it indicates the very uncertain nature of the stresses in this type of structure and the danger of relying too heavily on a theoretical design.

The TVA also conducted some tension tests on interlock strength of piling which had already been used twice and a third use contemplated. The purpose of the test was to determine how much loss in interlock strength was to be expected after use. The sections of piling were interlocked at 3° and 22° angles. The specimens were tested to destruction and the results are not very conclusive; but they indicate that, with a certain amount of inspection, piling could be used several times. On TVA construction piling has been used as many as six times.

Deflection measurements were taken at Kentucky cofferdam. The water never came within 25 ft of the top of the first stage and therefore no full height test was available. The measurements taken, however, show so slight a deflection within the range of change in water level of the river as to be impossible to plot with any uniformity. As a matter of fact, one cell for some unexplainable reason showed a slight deflection outward with rising river.

In the second stage cofferdam, however, somewhat better although not conclusive measurements were made. This cofferdam was subjected to water pressure to within a few feet of the top of the cofferdam during the winter and was flooded in April, 1944. The flooding started on March 31, and the following day the water pressure inside the cofferdam equalized the tailwater pressure. On April 2 the upstream arm was overtopped. The maximum river elevation was reached on April 6 when it was several feet above the upstream cofferdam arm. Unwatering started on April 12 and continued through the month. Readings were taken on every other cell between November 6, 1943, and

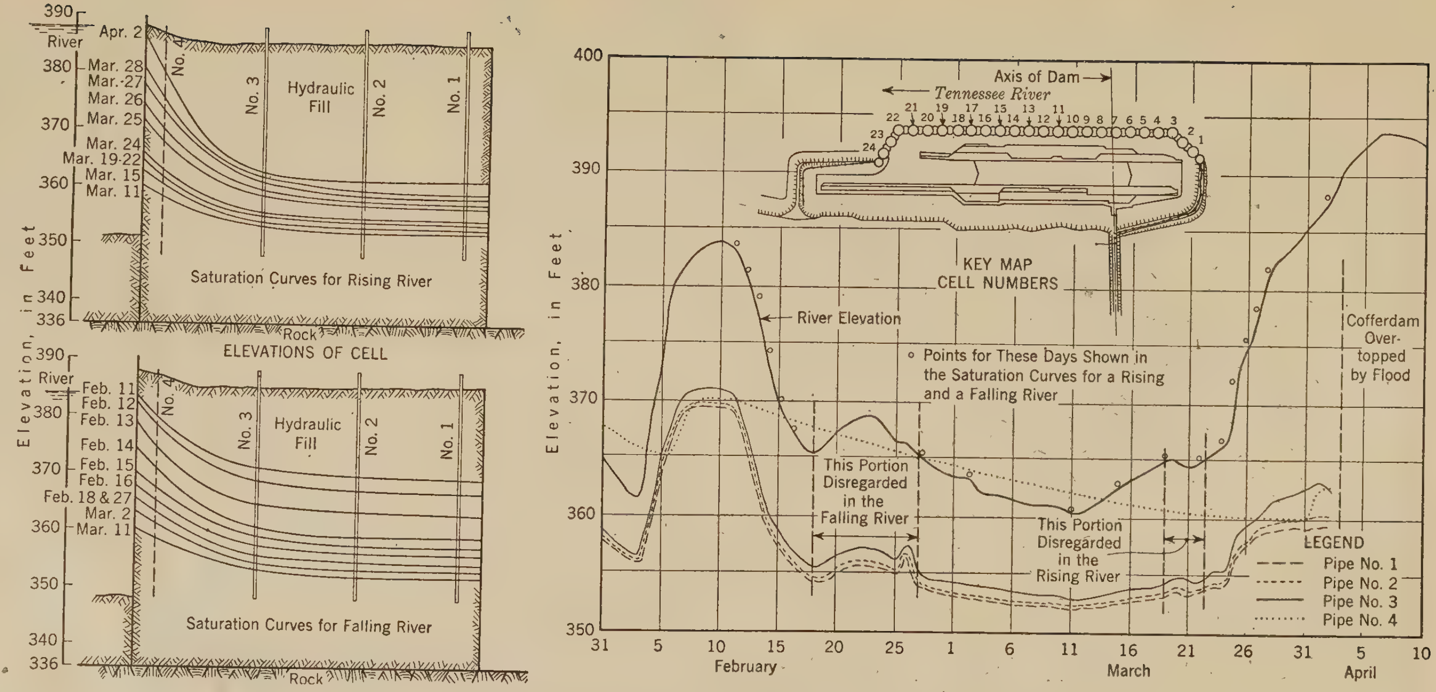


FIG. 14.—SATURATION TESTS; PICKWICK COFFERDAM

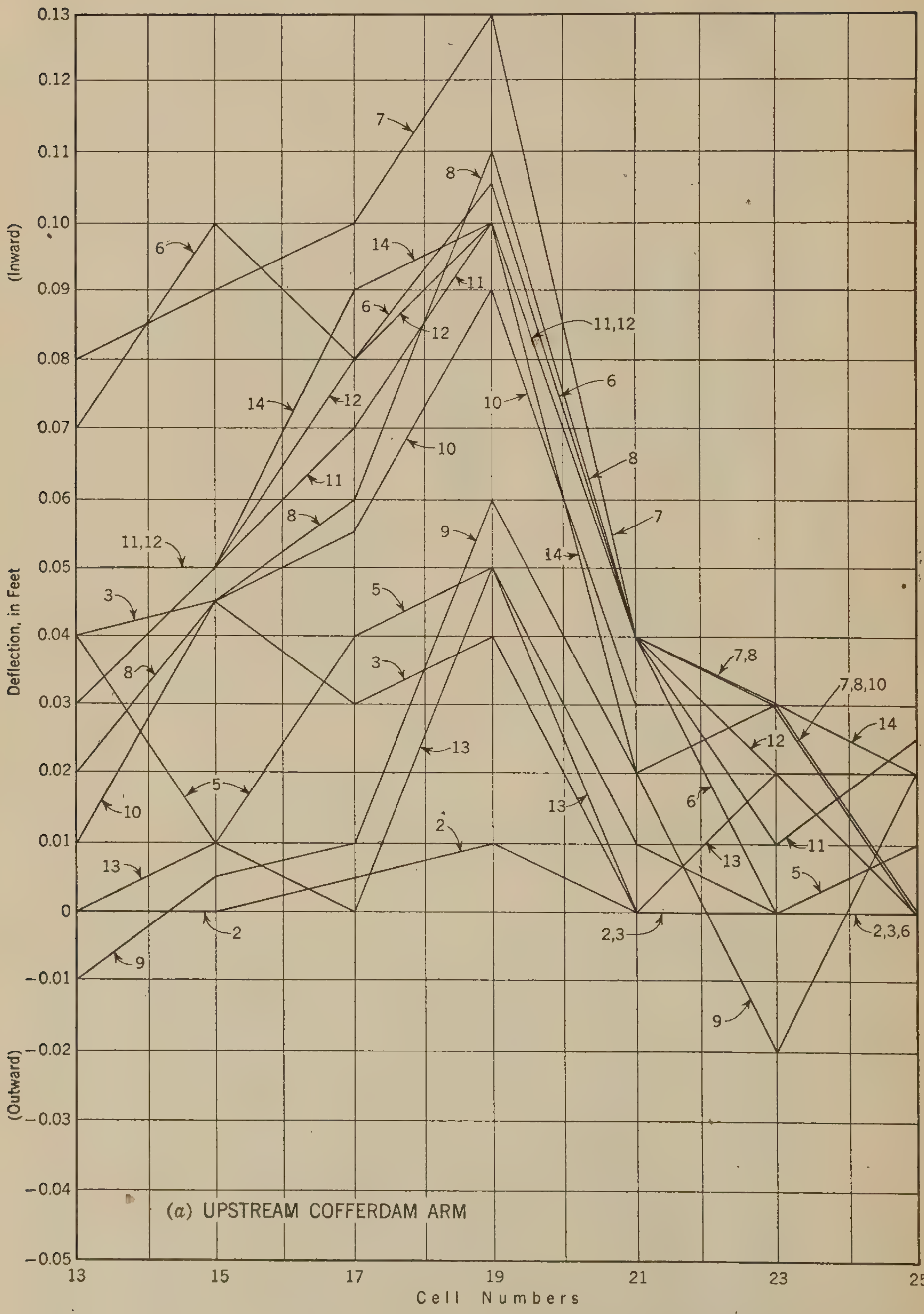
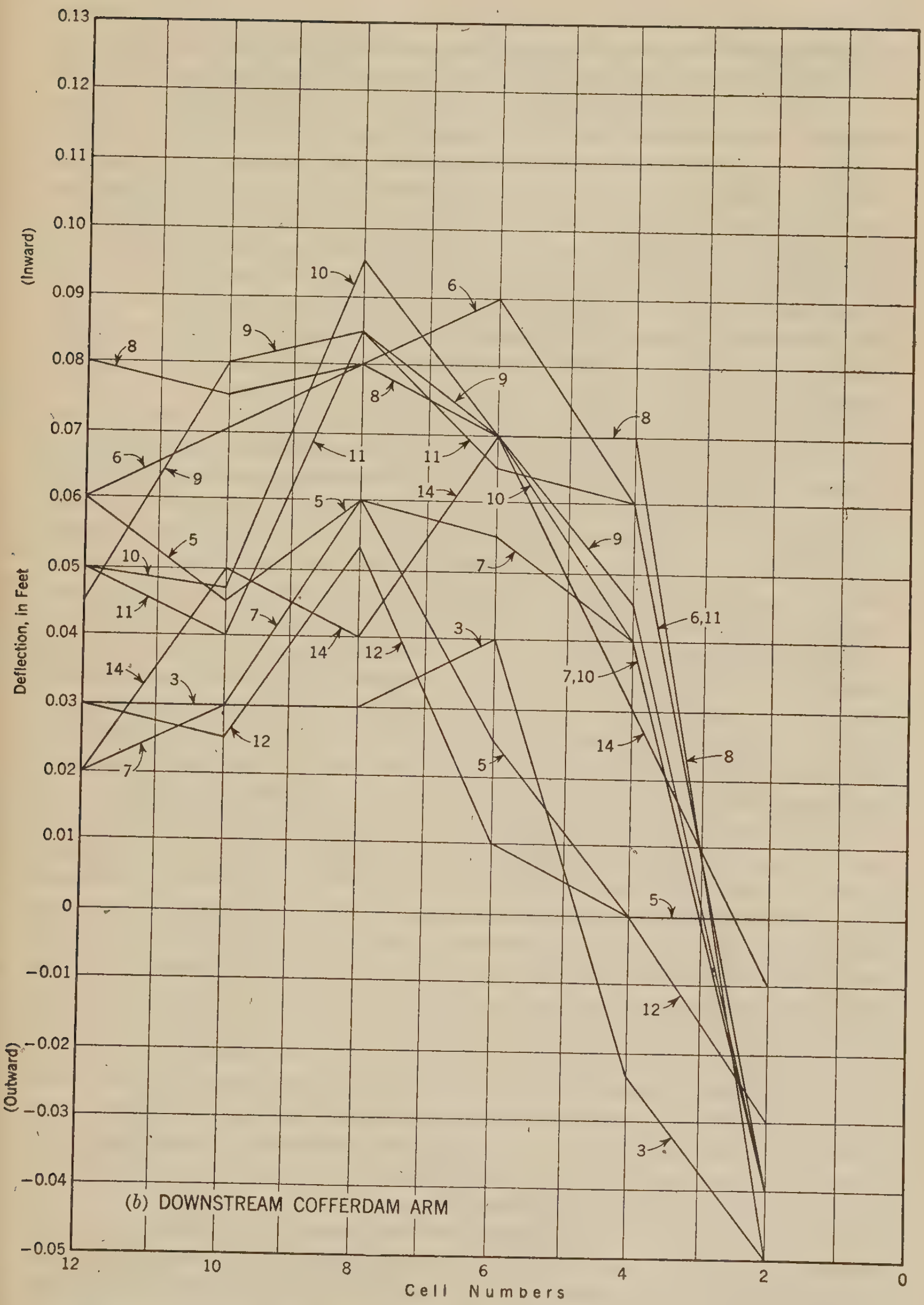


FIG. 15.—HORIZONTAL MOVEMEN



of Cells, Kentucky Cofferdam

April 24, 1944, to determine the horizontal movement of the top of the cells at various river stages. These readings indicated that the cofferdam cells deflected slightly inward with a rising river, the maximum being near the center of each arm of the cofferdam. The maximum amounted to only 0.13 ft on the upstream arm and 0.095 on the downstream arm. In general, the deflection was less near the shore where the overburden was highest. On the downstream arm the cell nearest the shore deflected outward. Fig. 15 shows the horizontal deflection of the cells in the upstream and downstream cofferdam arms plotted for each cell measured. The deflections decreased after the flooding of the cofferdam, as evidenced by comparing curves Nos. 11 and 12. When, later, the tailwater had dropped below the water inside the cofferdam the cells in the downstream arm deflected outward as indicated by curve No. 13. After most of the water within the cofferdam had been pumped out the cells again deflected inward, as shown in curve No. 14.

The best full-load test on TVA cofferdams occurred on the first one that was built at Pickwick Landing Dam. This was a cofferdam with cells 60 ft in diameter, 55 ft high, and with no berm. It withstood a head of a full 55 ft without any visible signs of distress. At this time the fill was completely saturated, as evidenced by the discharge from weep holes near the top. The only plausible reason why this cofferdam did not fail by vertical shear is that (*a*) the actual coefficient of friction of the material is much higher than laboratory tests indicate, and (*b*) the internal pressure is much higher than active earth pressure, as mentioned by Professor Terzaghi. During filling a certain interlock tension is developed in the piling. It is quite likely that this tension may be maintained by the development of reactive pressure in the cell fill. If the cells are filled by hydraulic dredge when the river level is low the piling tension may be quite large and consequently the reactive pressure quite large, which is a definite advantage in favor of hydraulic filling.

In the design of the Kentucky cofferdam the shear resistance in the fill was computed on this basis rather than by using the active earth pressure.

In computing the maximum interlock stress—that is, the interlock stress in the circular cell plus the additional stress due to the connecting arc—it was assumed that the maximum interlock stress which occurred during filling remained even after the cell fill above the saturation line is drained out. In computing the interlock resistance to shear, however, active pressure was used.

Several of the clover-leaf type cells were built in the dry. They constitute the tie-in connection for the second-stage cofferdam and were built inside the first-stage cofferdam and rested on the flat concrete apron. No hard driving was necessary and each pile was seated in a recess left in the apron to act as a key. This is an ideal condition and did not exist for most of the cells which required hard driving.

In some locations at Kentucky Dam the piling was driven through a stratum of cemented sand and gravel. This together with the 50-ft depth produced conditions which caused the interlocks to split in several cases. When the berm was removed these failures were exposed and reinforced with heavy steel bars as the berm was removed.

This interlock failure due to hard driving is the only type of failure experienced on any TVA cofferdam and is easily repaired. This leads to the conclusion that the driving conditions and interlock strength are the danger points in a properly designed cellular cofferdam.

The following general conclusions are possible in connection with steel sheet-pile cofferdam design based on TVA experience:

(1) Experience at Kentucky Dam indicates that driving through 40 or 50 ft of overburden produces excessive stresses in the interlock of a nature which cannot be calculated and that broken interlocks must be expected under such conditions. Dredging to reduce the amount of driving is recommended. To take care of conditions of deep driving, such as at Kentucky Dam, a suggestion is offered to piling manufacturers to design a pile with additional weight and strength in the interlock. It is not enough under these conditions for it to equal the strength of the web in tension. It should be considerably higher in interlock strength to take care of unpredictable stresses encountered in driving into heavy material.

(2) Proper provision for draining the cells should be made and a degree of saturation coinciding with actual tests should be used in the design. Cells are actually saturated to a much greater extent than usually assumed.

(3) Determination of the coefficient of internal friction and other coefficients of friction of the material should be carefully made by tests if there is doubt concerning its suitability. Actual field tests as suggested by Professor Terzaghi are preferable to laboratory tests. If, as in most cases, only laboratory tests are available, they should be used with judgment and conservatism.

The greatest difficulty in the path of a more exact design procedure is the lack of sufficient field test data. This lack is due in some degree to the fact that it is almost impossible to obtain a series of tests or measurements which show any degree of uniformity. It is to be hoped that better and more extensive testing methods can be made on future installations.

The writer wishes to acknowledge the valuable assistance of A. F. Hedman engineer, TVA, in the preparation of this discussion.

A. F. Hedman,[8] Esq.—It is with the greatest interest that the engineers of the Tennessee Valley Authority (TVA), who have been connected with the planning and the design of fourteen steel sheet-pile cofferdams on the Tennessee River, have read Professor Terzaghi's logical interpretation of cofferdam behavior. The importance of the paper is not so much that it develops new formulas, as that it endeavors to explain what is happening in a cofferdam, and thus gives the designer a better conception of the factors that will affect the design.

The writer agrees with the explanation of cofferdam behavior, in general, but takes exception to the author's statement about what has and has not been done in the past when designing cofferdams.

Experiences with Cellular Cofferdams.—Admittedly, cellular cofferdams are as reliable as other types and in the writer's opinion are even more reliable.

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[8] Engr., Constr. Plant Div., TVA, Knoxville, Tenn.

None of the cells in the TVA cofferdams has ever burst or has ever shown any sign of being near failure. With the exception of the cofferdams at Kentucky Dam, no apparent weakness has been discovered in the interlocks even though some piling was used as many as five and six times. At Kentucky Dam steel sheet piling was driven through 50 ft of overburden consisting of sand, gravel, and, under some of the cells, a 10-ft to 20-ft layer of blue clay. After the cofferdam had been unwatered, the overburden was excavated completely to rock at some of the cells, and it was discovered that some of the piling had been driven out of interlock. The failures all occurred near the bottom of the cell and were repaired by tying the piling together, welding heavy steel straps across the break. Where the overburden remained and acted as a berm, such a failure should not be serious because the interlock stress due to pressure from cell fill should be very small or nonexistent.

Deflection of Cellular Cofferdams.—Practically all cells in the TVA steel sheet-pile cofferdams were filled by pumping sand and gravel from the overburden or from near-by deposits in the river. In no case was any appreciable deflection noticed. No doubt this was partly due to the method of filling the cells which caused compaction of the fill material, thus changing the elastic properties. The hydraulic fill also caused higher interlock stresses and therefore greater interlock resistance, which, as the author states, should stiffen the structure. An attempt was made at Kentucky Dam to measure the horizontal deflection for various heights of water pressure after the cofferdam had been unwatered. The deflections were very small, the maximum deflection being only 0.14 ft. In contrast to the cofferdams described by the author, the crest of the cofferdams at Kentucky Dam did not assume the W-shape after unwatering. This was possibly due to the capability of the ends to deflect.

Permeability of Cellular Cofferdams and of Their Sheet-Pile Enclosures.—As stated by the author, observations were made at Pickwick Dam to determine the saturation line in the cell fill. The TVA has collected some data on the location of the saturation line in several of the cofferdams constructed. Pipes were installed in the cells to serve as observation wells, and frequent readings were taken during rising and falling stages of the river. To provide drainage, weep holes 1.5 in. in diameter were burned on 3-ft centers in every third or fifth pile of the inside row of piling. At Kentucky Dam two rows of nine holes, 4 in. in diameter, were burned in each cell, including the connecting cell, one row at the river bed and one at the top of the berm.

Observation wells in one cell of the lock cofferdam at Pickwick Landing Dam indicated that the saturation line was practically horizontal across the cell. The height of the saturation line above the base varied with the rise and fall of the river. The highest observed level of the saturation line, when the river was almost up to the top of the cofferdam, was approximately 0.71 times the height of river above base of cell. This ratio decreased later, possibly due to either cleaning or adding weep holes.

At Guntersville Dam where seven of the lock cofferdam cells were observed, the saturation line was nearly level across the cells (but only a few feet above the base of the cells), and there was very little fluctuation in this saturation line for variations in the river elevation.

Neither the Pickwick cofferdams nor the lock cofferdam at Guntersville Dam had a berm. The cells in these cofferdams were filled with hydraulic fill consisting of sand and gravel with only a small percentage of silt. In leakage through the cofferdam a comparatively large loss of head occurred at the outside piling. Only a small head was required for flow through the sand and gravel fill. A further loss of head occurred at the inner piling wall. This was quite large in the case of the cell at Pickwick Landing Dam and was probably due to the flow carrying what silt there was in the fill across the cell and depositing it near the weep holes, thus reducing the discharge. At Guntersville Dam there may have been less silt in the fill or the weep holes may have been kept functioning better, possibly by driving rods into them to prevent plugging.

At Chickamauga Dam auger holes were drilled in the third-stage cofferdam cells along the inner piling wall. The saturation line was near the top of the berm used in this cofferdam to back up the cells.

An elaborate drainage system was devised at Kentucky Dam. The cell fill around the 4-in. weep holes was removed and replaced with 3-in. gravel, and the weep holes were then connected by short pipes, welded to the piling, to a flume at the top of the berm and to a system of corrugated pipes at the level of the river bed. Great care was taken to keep all pipes discharging. Observation wells were placed in all cells and the saturation line remained very constant at an elevation of from about 4 or 5 ft below the top of the berm (which was 10 ft above the original river bed), although there was a 32-ft variation in the river level.

All these observations seem to show that it is possible to keep the saturation line quite low, but that it should not be so assumed for design purposes unless great care will be taken to insure that the cell-drainage system functions properly. Where a berm is used, the saturation line should not be assumed to be below the top of the berm. The tighter the interlocks, the less the leakage will be. Therefore, leakage will be affected by the method of filling the cells and by whether piling is new or used.

The author's statement (see heading, "Part I. Practical Considerations: Permeability of Cellular Cofferdams and of Their Sheet-Pile Enclosures"), "The seepage across a cofferdam with such a foundation merely affects the tension in the locks," is contradictory to his subsequent statements that the amount of saturation affects the sliding resistance of a cell.

Cellular Cofferdams on Rock; Sliding Resistance.—The average width of TVA cofferdams has been 1.60 r. The lower value is due to the use of smaller connecting cells. As stated by Professor Terzaghi, the sliding resistance at the base of the cell should be found by taking into account the saturated state of the fill. He implies, however, that this should be done only where the rock is smooth. The writer does not agree that the unit weight of the fill in a moist state should be used in Eq. 3. The weight per cubic foot of saturated fill at Kentucky Dam was approximately 18 lb more than that for the same volume of moist fill. Where H_s is large, a considerably higher value for F_{bs} will result if the weight of saturated fill is used. As correctly stated by Professor Terzaghi, the saturated state of the fill was not considered at Pickwick Dam when investigating for sliding. On subsequent jobs saturation was taken into account and a berm added on the inside not only to obtain sufficient resistance to sliding, but also to keep the resultant within the middle third. At Pickwick Dam the rock surface was fairly smooth, and considerable resistance against sliding was apparently obtained by the steel sheet piling being driven into rock.

Cellular Cofferdams on Rock; Resistance Against Internal Shear.—Contrary to the author's assumption, the possible failure of a cofferdam due to excessive vertical shear through the fill and the piling has been considered by others. Lengthy discussions on this and other related subjects have taken place among TVA engineers especially before the final design of the Kentucky cofferdam was begun. The writer feels that a summary of the methods used by TVA engineers when designing the cofferdams at Kentucky Dam will show that:

1. Uplift was considered to be based upon an assumed saturation line;
2. Vertical shear was computed and resistance to vertical shear was calculated by assuming an internal pressure as stated later (the interlock resistance was taken into account); and
3. The interlock stresses were computed first by considering the pressure that existed during hydraulic filling, and second, after unwatering, by including the water pressure within the cell corresponding to the height of the saturation line.

The Kentucky cofferdam which consisted of both circular and clover-leaf type cells has been described fully elsewhere (34)(35). A brief summary of the design assumptions used, and the design procedure follows.

Fig. 16 shows a typical cross section through the upstream or the downstream arm of the cofferdam. Circular cells with diameters of 58 ft 10 in. were selected. Piling was driven to rock and the cells were filled hydraulically. Before unwatering, the overburden was dredged to leave a berm as indicated, and an additional 10 ft of fill was added to bring the top of the final berm within 38 ft of the top of cofferdam. An equivalent cofferdam width was assumed equal to the combined areas of one circular cell and one connecting cell, divided by the distance between main cells. The saturation line for two conditions of berm drainage is shown in Fig. 16. Other design assumptions were:

No.	Description	Quantity
1	Water pressure, in feet, assumed acting on the full height of the entire structure	98
	Weight of Material, in Pounds per Cubic Foot—	
2	Dry or drained	110
3	Submerged	65
	Coefficients of Friction—	
4	Gravel on rock	0.5
5	Gravel on steel (corresponding to $\delta = 21° 50'$)	0.4
6	Steel on steel	0.3
7	Coefficient of internal friction, tan ϕ, for dry or saturated material ($\phi = 28° 50'$)	0.55

No.	Description	Quantity
	Allowable Stresses in Sheet Piles—	
8	Interlock tension, in pounds per linear inch	12,000
9	Web tension, in pounds per square inch	32,000
10	Section modulus (feet^3) based on the average width of cell	391
11	Factor of safety against interlock failure, sliding, and vertical shear (factor of safety against overturning not computed)	1.25

The active earth pressure was computed by the Coulomb formula,

$$P_{e,a} = \frac{C_1 \gamma H^2}{2} \qquad (40)$$

For level surfaces, $\phi = 28° 50'$; $\delta = 21° 50'$; and $C_1 = 0.29$. The pressure distribution for active earth pressure is triangular, the resultant acting at a point distant $H/3$ above the base.

The reactive earth pressure, for broken surfaces, was computed graphically. The pressure distribution is parabolic, with the resultant acting at a point distant $H/2$ above the base.

Computation of Interlock Stresses.—Maximum stress occurred during filling by hydraulic dredge when the fill was saturated to the top of the cell. Tension was computed at the top of the overburden on the basis of triangular distribution of pressure. Actually, maximum tension will occur at a higher elevation due to the restraint of the overburden. The increase in interlock stress due to the connecting arc was considered (discussed subsequently).

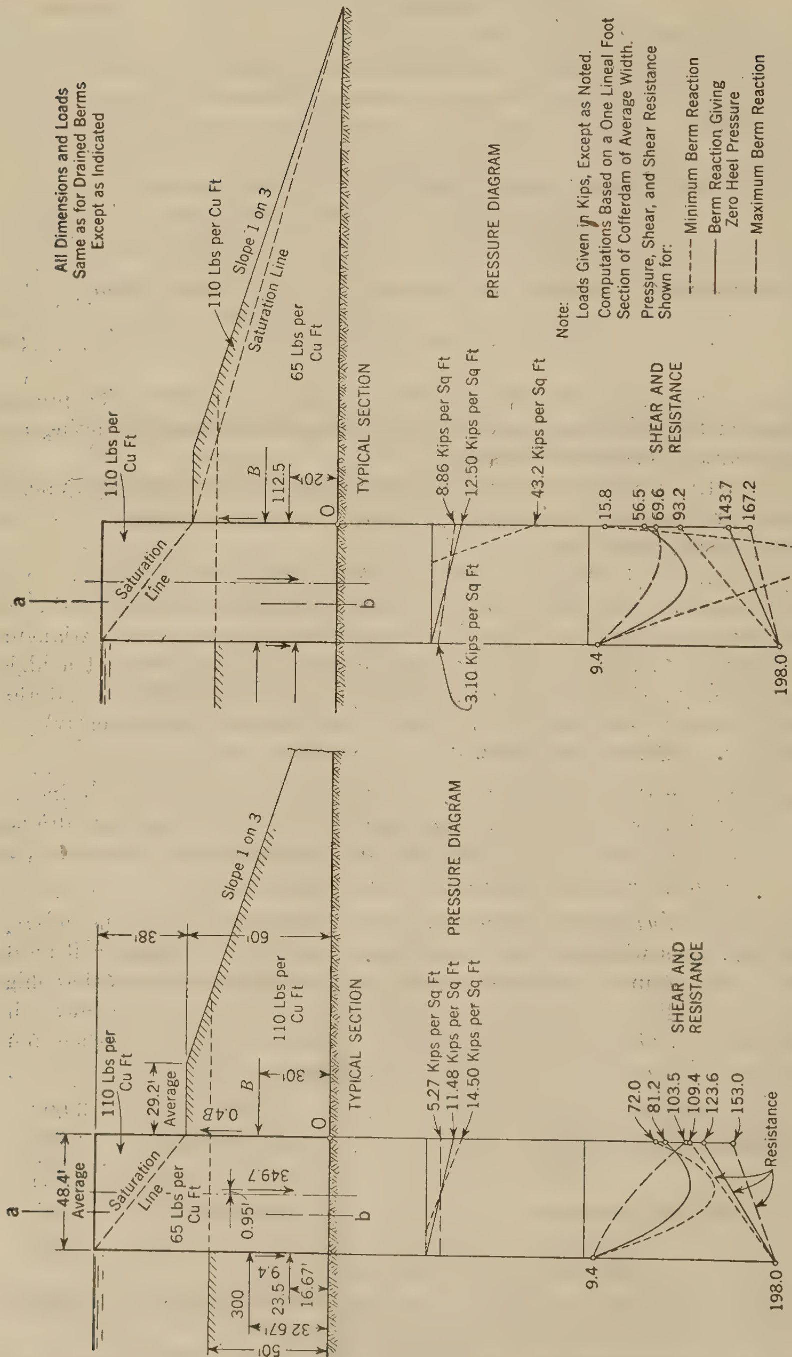
Stability of Cells.—The following cases were analyzed:

1. Circular cells: (*a*) Berm saturated; and (*b*) berm completely drained.
2. Clover-leaf cells: (*a*) Berm saturated; (*b*) berm completely drained; (*c*) berm removed; (*d*) clay seam at the base with berm saturated; and (*e*) clay seam at the base with berm removed.

Résumé of Procedure for Determining Interlock Stresses.—Only the design of the circular cells with the berm saturated or completely drained will be discussed in this résumé. When investigating the interlock stresses that might occur during filling, it was assumed that the river would not be lower than El. 305 (see Fig. 17). During filling, the pressure adjacent to steel sheet piling at El. 290 (in pounds per square foot) would be from:

Water pressure within the cell	$62.5 \times 48 =$	3,000
Submerged fill in the cell	$0.29 \times 65 \times 48 =$	904
		3,904
Outside water pressure	$-62.5 \times 15 =$	-937
		2,967

In the main cell alone the interlock stress will be $\frac{2{,}967}{12} \times 29.44 = 7{,}280$ lb per lin in.; and in the connecting cell it will be $\frac{2{,}967}{12} \times 9.57 = 2{,}365$ lb per lin in.


All Dimensions and Loads
Same as for Drained Berms
Except as Indicated
110 Lbs per Cu Ft
Slope 1 on 3
Saturation Line
65 Lbs per Cu Ft
110 Lbs per Cu Ft
Saturation Line
a
b
B
112.5
20'
O
TYPICAL SECTION
PRESSURE DIAGRAM
8.86 Kips per Sq Ft
12.50 Kips per Sq Ft
43.2 Kips per Sq Ft
3.10 Kips per Sq Ft
Note:
Loads Given in Kips, Except as Noted.
Computations Based on a One Lineal Foot
Section of Cofferdam of Average Width.
Pressure, Shear, and Shear Resistance
Shown for:
Minimum Berm Reaction
Berm Reaction Giving Zero Heel Pressure
Maximum Berm Reaction
SHEAR AND RESISTANCE
15.8
56.5
69.6
93.2
143.7
167.2
9.4
198.0
a
48.4'
Average
110 Lbs per Cu Ft
Saturation Line
65 Lbs per Cu Ft
29.2'
Average
0.4B
349.7
0.95'
b
38'
60'
Slope 1 on 3
110 Lbs per Cu Ft
B
30'
O
TYPICAL SECTION
300
23.5
9.4
16.67'
32.67'
50'
PRESSURE DIAGRAM
5.27 Kips per Sq Ft
11.48 Kips per Sq Ft
14.50 Kips per Sq Ft
SHEAR AND RESISTANCE
72.0
81.2
103.5
109.4
123.6
153.0
Resistance
9.4
198.0

(a) COMPLETELY DRAINED BERM

(b) SATURATED BERM

FIG. 16.—DESIGN ANALYSIS, KENTUCKY COFFERDAM

The pressure of the water and the submerged cell fill, of course, changed the shape of the cells into the familiar barrel form. As the water drained out of the cell fill, the cell probably maintained most of its changed shape, the reactive earth pressure being substituted for the water pressure as the cell tended to contract. For this reason it was felt that the interlock tension would not be reduced appreciably as the cell fill drained. Therefore, when computing the maximum interlock tension, the value obtained in the main cell alone was used, to which was added the effect of the connecting arc. To reduce this effect a 30° wye was used instead of the more common 90° tee.

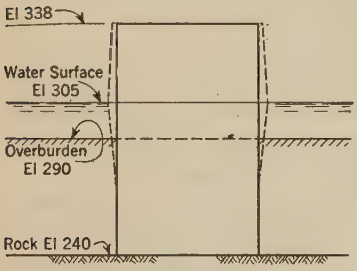


Fig. 17.—Section Through Cell During Filling

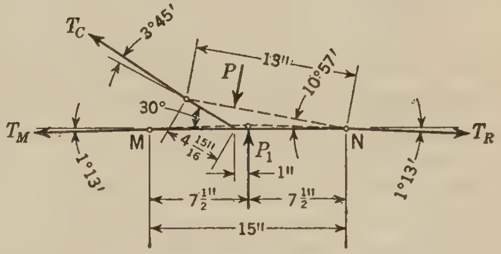


Fig. 18.—Detail Data for a 30° Wye

The forces acting upon the wye are shown in Fig. 18. Considering a section of piling, 1 in. high, the following forces were known: $T_c = 2{,}365$ lb; $T_m = 7{,}280$ lb; and $P_1 = 1.25 \times 2{,}967 \times \frac{1}{12} = 309$ lb.

By projecting the forces on line MN, Fig. 18, and on a line perpendicular to line MN, it was found that $P = 1{,}292 \text{ lb} > \frac{2{,}967}{12} 1.08$, indicating that reactive pressure was developed. Knowing P, it was then possible to find the maximum interlock stress T_r. This stress was found to be equal to 9,500 lb—more than 50% higher than would have been the case if the author's suggested empirical formula, Eq. 28*f*, had been used. The method of filling the cell and the river level at time of filling should definitely affect the interlock tension, making it impossible to establish and use Eq. 28*f* as suggested by Professor Terzaghi. The interlock stress on all TVA steel sheet piling cofferdams (including the ones at Pickwick Landing Dam) was computed by assuming water pressure and saturated fill within the cell—for the full height of the cell—counteracted by the water pressure (low river) acting on the outside of the cell. The factor C (the ratio between horizontal and vertical pressure) in most cases was close to 0.7.

The interlock tension was also computed for the condition after unwatering, and was found to be smaller than the foregoing value.

The average width for the aforementioned cofferdams was approximately 0.89 H for the first ones designed (cells assumed drained and no berm used) and varied between 0.50 H and 0.79 H for those with berms (saturation and vertical shear considered).

Professor Terzaghi states, in the second paragraph following Eq. 21, that

> "If the inner side of a cellular cofferdam is supported by a berm whose height is almost equal to the height of the dam, both the cross-walls and the inner row of sheet piles are acted upon by equal earth pressures from both sides. This condition practically eliminates the tension in the locks, but it also eliminates the lock friction."

This statement is true if the cofferdam is filled while the river is near the top of the cells, which apparently was the case for the cofferdams in New York, mentioned by the author. However, if the cells are driven and filled hydraulically with the river at a low level (which is generally the case where dams are constructed), a large pressure is created in the cell fill, resulting in a high interlock stress. This pressure and the corresponding interlock tension are likely to be maintained by the reactive pressure induced in the cell fill by any tendency of the piling to contract. A berm added afterward on the inside of the cofferdam will exert a pressure far smaller than the pressure within the cell. With rising river the berm may develop reactive pressure, and it is possible that the interlock tension may be somewhat reduced.

Résumé of Procedure for Determining the Stability of Cells.—A saturation line was selected extending from the top of the cell on the river side and sloping down to the top of the berm on the dry side. Perhaps it might have been just as well to select a horizontal saturation line about 20 ft below the top of cell. Observations made after the cofferdam was completed indicated that the saturation line at the inner piling wall was actually below the top of the berm (see also "Part I. Practical Considerations: Permeability of Cellular Cofferdams and of Their Sheet-Pile Enclosures"). Not knowing where the saturation line in the berm might be, the TVA engineers decided to investigate the cell with the berm being either saturated or completely drained.

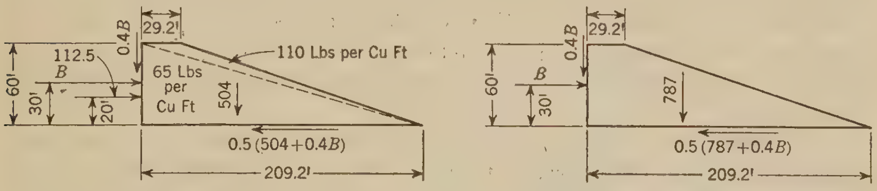
The weight of the cell and the berm, as well as the forces acting on the cell, was then determined. When computing the weight of the cell and the berm, uplift, as measured by the distance from the assumed saturation line down to rock, was taken into account by using the submerged weight of fill. The following results for a 1-ft length of cofferdam were obtained:

No.	Description	Quantity
	Weight of Material, W, in Thousands of Pounds, or Kips—	
1	Cell fill (piling not included), W_c	349.7
2	Saturated berm, W_b	504.0
3	Completely drained berm, W_b	787.0
4	Center of gravity of the cell fill (distance in feet from the center line of the cell)	0.95
	Forces, in Kips, Acting on the River Side of the Cell—	
5	Water pressure on the full height of the cell ($P_1 = 0.5 \times 0.0625 \times 98^2$)	300.0
6	Pressure from overburden; saturated ($P_{e,a} = 0.5 \times 0.065 \times 0.29 \times 50^2$)	23.5
7	Vertical friction on the cell; downward ($F_1 = 23.5 \times 0.4$)	9.4

No.	Description	Quantity
	Forces, in Kips, Acting on the Dry Side of the Cell; Saturated Berm—	
8	Water pressure ($P_2 = 0.5 \times 0.0625 \times 60^2$)	112.5
9	Maximum sliding resistance of berm; see Fig. 19(*a*) [$F_b = 0.5\ (504 + 0.4\ F_b)$]	286.5
10	Reactive pressure by graphical construction	362.0
11	Maximum vertical friction on the cell ($F_2 = 0.4\ F_b = 0.4 \times 174$)	69.6
	Forces, in Kips, Acting on the Dry Side of the Cell; Drained Berm—	
12	Water pressure	0
13	Sliding resistance of berm; see Fig. 19(*b*) [$F_b = 0.5\ (787 + 0.4\ F_b)$]	492.0
14	Reactive pressure by graphical construction	561.0
15	Maximum vertical friction on the cell ($F_2 = 0.4\ F_b = 0.4 \times 492$)	196.8

The maximum reactive pressure of the berm was found, by graphical solution, to be greater than the sliding resistance of the berm. The latter, therefore, became the controlling factor, and was the maximum resisting force that could be developed if necessary. The horizontal forces, water pressure, and overburden pressure, which act on the river side of the cell will be resisted by the friction between the cell and the rock and by the berm. In the case of a saturated berm, water pressure between the cell and the berm will help to counteract any movement or deflection of the cell.

Next, it was assumed that full friction was developed under the cell, and that the berm then had only to develop a resistance equal to the difference between the horizontal forces on the river side and the friction F_c under the cell, where $F_c = 0.5\ (W_c + F_1 - F_2)$. This resistance was found to be considerably smaller than the sliding resistance F_b of the berm. The resistance, B (Figs. 16 and 19), was applied at one half the height of the berm. The vertical force $F_2 = 0.4\ B$ was computed, and, by taking moments of all the forces about point O at the toe of the cell, the pressure at the base of the cell was calculated. This pressure was assumed to vary as a straight line. Next, the vertical shear in the cell was determined. On any vertical plane in the cell this shear would



(*a*) SATURATED BERM (*b*) COMPLETELY DRAINED BERM

FIG. 19.—FORCES ACTING ON THE BERM

be equal to the resultant of all vertical forces to the left or the right of the plane considered—that is, the sum of F_1 plus the weight of cell fill W_c (partly dry and partly saturated) less the pressure on the base. In the computation of the shear resistance of the cell fill, the pressure within the fill was assumed to vary as a straight line between the pressure of water and overburden, $P_1 + P_{e,a}$, on the river side to the resistance from berm and water, $B + P_2$, on the dry side. By multiplying the pressure by 0.55, the assumed coefficient of internal friction, the resistance to vertical shear was obtained. The resistance to failure in vertical shear which exists in the piling interlock was added to the shear resistance of the cell fill. Although it was felt that the high interlock stress, which occurred during the hydraulic filling of the cells during low water in the river, would remain after unwatering and subsequent draining of the cell fill regardless of variations in river elevations, it was decided to be conservative because there was no proof that such an assumption had been made before. The resistance of the steel sheet piling to vertical shear was consequently based on the interlock stress produced by active pressure. Another reason for doing this was the possibility that, with maximum water pressure on the river side and the corresponding resistance from the berm, the cell fill would possibly be squeezed together somewhat and thus would reduce the initial interlock stress obtained during hydraulic filling. The resistance of the piling to shear was determined by multiplying the interlock stress by the coefficient of friction in the interlocks. By combining the resistance in the cell fill and the resistance in the piling interlock, the total resistance for the cell was obtained.

If this resistance is less than the shear in any vertical plane a-b through the cell (see Fig. 16), the cell will fail in vertical shear and will deflect farther toward the berm. This movement will increase the resistance of the berm until equilibrium is obtained and the horizontal movement is stopped. The increased berm pressure will decrease the vertical shear and increase the resistance to vertical shear as indicated by the curves in Fig. 16. The resistance B created in the berm cannot be greater than the sliding resistance F_b of the berm. It cannot be greater than the sum of the horizontal forces acting on the river side; nor can it be expected to be so great that the heel pressure is greater than the toe pressure.

For the cofferdam at Kentucky Dam the minimum resistance of the berm, required to prevent sliding, was not sufficient to prevent failure in vertical shear, if the berm should be saturated. The maximum resistance $B = 174.0$ kips (plus 112.5 kips of water pressure) for the saturated berm and a resistance $B = 259$ kips for the drained berm (which gave uniform pressure at the base of the cell) both proved to produce sufficient resistance to failure in vertical shear. Smaller berm resistances—203 kips for the drained berm and 142 kips (plus 112.5 kips of water pressure) for the saturated berm—were required to give zero heel pressure.

A similar procedure followed when designing the clover-leaf cells is not discussed herein.

Cellular Cofferdam on Rock; Without Overburden and Without Berm.—To form an idea of the stability of a cofferdam built on rock, where no overburden exists and no berm is used to support the cells, the writer investigated the possibility of a cell in such a cofferdam failing by sliding on the rock or in vertical shear. The following assumptions were made:

(*a*) The cell would be filled hydraulically with sand and gravel, water being present outside the cell not higher than halfway up the cell;

(*b*) The pressure produced inside the cell would remain, due to the tendency of the cell to contract and create reactive pressure, and no additional increase would occur after the water on the outside had risen to the top of the cell (after unwatering of cofferdam);

(*c*) The interlock tension during filling would remain after unwatering and subsequent draining of the cell, regardless of outside water pressure;

(*d*) The saturation line after unwatering would not be higher than halfway up the cell; and

(*e*) The coefficient of friction against sliding on rock and the coefficient of internal friction would be 0.50 and 0.55, respectively (both values conservative). The coefficient of friction in the interlock was assumed to be 0.3.

The following safety factors were found for various dimensions of cells:

Width of cell	Against failure by sliding	Against failure in vertical shear
0.85 H	1.19	1.29
1.00 H	1.40	1.79

If the cells are filled during a period when the water level is lower than that for the foregoing assumption, the cofferdam should be still safer. The opposite would be true if the water level were higher.

Had the cells in the cofferdam been filled with dry material, the internal pressure during filling would have been considerably smaller. After completion of the cofferdam and unwatering, the outside pressure would possibly increase the internal pressure somewhat and thus the resistance to shear, but a cell built in this manner would always be weaker than one filled hydraulically.

As stated by Professor Terzaghi, the internal pressure in the cell is not known definitely. Several assumptions are possible. Whether the cell is filled hydraulically or with dry fill, the horizontal pressure on the inner and the outer walls can be determined. If, next, the pressures on the base and at various other levels above the base, due to overturning moments, were computed, another horizontal pressure diagram could be determined. Should the greater value found by these methods be used when investigating vertical planes on either side of the center of the cell? Does the external water pressure increase the pressure in the fill at the outer wall as assumed in the Kentucky cofferdam design? Answers to these questions would do much to clarify the designer's problems.

The writer is grateful to R. S. Blomfield, associate civil engineer, Construction Plant Division, TVA, for his valuable assistance in the preparation of this discussion.

ADOLPH J. ACKERMAN,[9] M. AM. SOC. C. E.—This paper represents an important milestone in the history of retaining wall structures composed of sheet piling cells filled with sand and gravel. According to past practice, it has been customary to regard such structures as gravity walls which were designed under two very elementary considerations. First, the width of the wall should be approximately equal to from 0.83 to 1.0 times the head of water, in feet, which the wall is to support; and second, the cylindrical or hoop stress in the cells and in the interlocks should not exceed a designated acceptable value. Many sheet-pile cellular cofferdams have been constructed under these principles, and experience has demonstrated that such cellular structures, when filled with sand and gravel, are reasonably successful for cofferdams and other temporary types of construction.

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[9] Director of Eng., Dravo Corp., Neville Island, Pittsburgh, Pa.

In spite of the fact that the foregoing statements provided sufficient criteria for the design of sheet-pile cellular walls, there was, nevertheless, in practically every case, a large amount of scientific-appearing computation work performed to make it seem that the structure had the benefit of a careful design analysis. In many cases, however, the assumptions, under which the mathematical analyses were developed, were at complete variance with the actual conditions surrounding the structure. The reassuring thing with respect to past practice has been the fact that, in the end, if the width was approximately equal to from 0.83 to 1.0 times the height of the wall, if the structure stood on a suitable foundation, and if it was filled with a stable granular material, the wall could be considered safe and the computations a success.

Professor Terzaghi's paper summarizes the following significant points:

1. It is incorrect to assume for design purposes that a sheet-pile cellular wall acts like a solid gravity wall;
2. The cellular wall is a composite construction of two different materials—steel and soil (the stresses in each material and the interaction of the two materials are analyzed);
3. The distribution of the stresses in the cell fill material is clarified in terms of basic principles of soil mechanics;
4. The frictional stress of the earth fill on the inside of the steel cell is defined;
5. The principal stability of the structure is obtained from the cell fill, with the steel shell acting as an important stiffening influence;
6. High frictional resistance in the interlocks is desirable;
7. In some cases the vertical shear in the cell fill may be a point of critical weakness (in the past, this generally has been ignored in the design);
8. Percolation under cellular walls resting on soil foundations requires close attention;
9. The permeability in sheet-pile cellular walls is clarified; and
10. It is a fallacy to assume that cells with weep holes in the back-wall drain to the bottom of the weep holes. (This assumption in the past has been intended to assure reduced interlock stresses, and maximum weight of cell fill for stability.) The buoyancy effect which results when cells are not adequately drained reduces the stability of the cells.

Professor Terzaghi states that a cellular cofferdam consists of two very different materials—steel and soil—and that its elastic properties can be compared to those of a composite construction material such as reinforced concrete. Although the composite construction can be described readily in this manner, it appears preferable to identify the sheet-pile cellular wall as a "plastic structure," to which the usual laws of elastic structures do not apply except in special instances where a cementing action in the cell fill material might result in the development of typical characteristics of elasticity. For example, when the writer was studying means for increasing the stiffness of a cellular wall which started to deflect excessively, he considered the possibility of welding together all the interlocks in a zone about 10 ft high in the upper part of the diaphragm cross-walls. However, in discussing this subject with the writer, Raymond P. Pennoyer, Assoc. M. Am. Soc. C. E., promptly showed that this would be a mistake because it would introduce a rigid section into a plastic structure. In the event of further deflection, the upper section of the diaphragm could not adjust itself with respect to the body of the structure, and there would be a localized building up of stresses in the welded area which might eventually result in sudden failure.

Professor Terzaghi has presented theories and formulas which are intended to be more rational than the methods of analysis which have been used. Fortunately, in the past, good judgment has prevailed sufficiently so that the mathematical analyses were not relied upon to an extent that might have been harmful. However, the paper is timely because the possibilities in connection with cellular structures are rapidly expanding and several recent installations in so-called permanent structures have opened up new fields for the utilization of sheet-pile cellular construction. For such structures, it is important to have more precise and dependable means of design.

A few engineers with considerable experience in this class of structure have advocated the use of the middle third theory of stability analysis. In the hands of these engineers this may be an acceptable check or yardstick, but it is a fair presumption that considerably more reliance probably is placed on judgment than on computation. The application of the middle third theory to plastic structures reminds one very much of the housewife who has learned to determine when a flatiron is hot enough for ironing by moistening the fingers and for an instant tapping the hot spot. With the requisite skill and judgment, such a test serves its purpose, but in less experienced hands it may lead to burning the fingers!

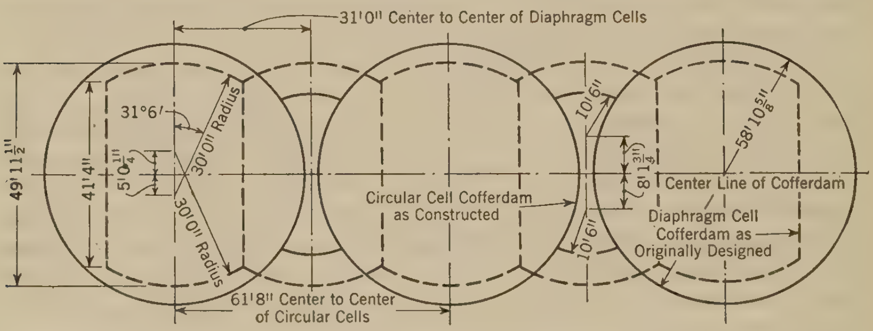
The chief objection to the arbitrary use of the middle third theory is that it tends to classify structures of the cellular cofferdam type with the rigid type of concrete structures, such as gravity dams. For most engineers the design of a cellular structure is an infrequent incident, and, when such an assignment develops, there is a natural tendency, particularly among the younger designers, to apply, automatically, their usual training in the theories of elasticity and gravity structures without developing a proper concept of the distribution of stresses as they actually occur in a cellular cofferdam. On a number of occasions the writer has inspected design computations for the stability of cellular structures which disclose this tendency of blissfully accepting the middle third theory for stability. In some cases design assumptions were set up for applying the middle third theory in a manner which indicated a complete lack of concept regarding the actual distribution of stresses. It may be that some of this faulty reasoning accounts for the fact that as recently as 1944 it is still possible to hear about a failure of a cellular sheet-pile structure.

Such failures, of course, will continue to persuade conservative engineers to regard cellular type of construction as useful purely for temporary purposes such as cofferdams. It will only be through such attempt as that presented in this paper, in developing a more rational concept of stress distribution in such structures, that broader acceptance will be accorded this type of construction for so-called permanent structures.

In view of the potentially larger use of steel sheet piling, once this type of construction wins proper acceptance for permanent structures, a paper such as this should be a sign of encouragement to the manufacturers of sheet piling. The author's painstaking effort to remove some of the mystery, and to develop a clear explanation of the principles of soil mechanics as they apply to this type of structure, deserves strong encouragement.

In the past, the best-known reservoirs of accumulated knowledge in this field have been with the manufacturers of sheet piling. Much valuable guidance and advice are available from such sources, but it seems only proper to point out that engineers and builders with limited experience in this field will find themselves in an untenable position, in the event of failure, if they assume

TABLE 1.—COMPARISON OF SHEET PILING FOR COF


31'0" Center to Center of Diaphragm Cells
31°6'
30'0" Radius
30'0" Radius
49'11½"
41'4"
5'0¼"
10'6"
10'6"
58'10⅝"
8'1¾"
Center Line of Cofferdam
Circular Cell Cofferdam as Constructed
Diaphragm Cell Cofferdam as Originally Designed
61'8" Center to Center of Circular Cells

Plan

Item	Length (ft)	SHEET PILES (TONS)		EARTH FILL (CU YD)		ESTIMATED COSTS,			
						Piles, at $56 Per Ton		Fill	
		Total	Unit	Total	Unit	Total	Unit[a]	Total	Unit[a]
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	123.33	341.83	2.772	10,965	88.908	19,142.48	155.21	3,618.45[f]	29.34[f]
2	124.00	340.45	2.746	10,832	87.355	19,065.20	153.75	4,116.16[g]	33.19[g]

[a] Dollars per longitudinal foot of cofferdam. [b] At $10.00 per ton. [c] Maximum compression, in pounds
$0.33 per cu yd. [g] At $0.38 per cu yd.

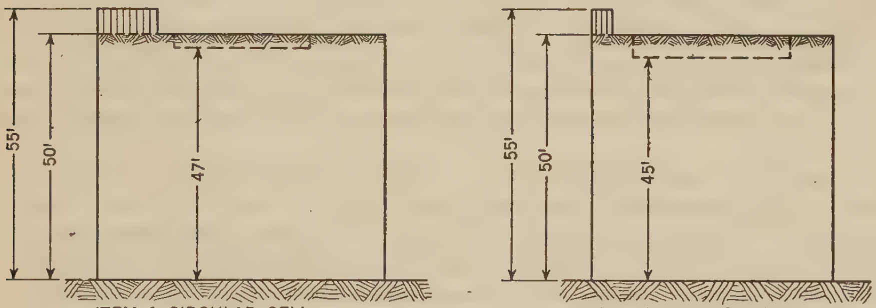
that the manufacturer accepts any responsibility for the design of the structure, or any obligations or liability beyond those covered by the manufacturer's guarantee in the fabrication of piling. Under the circumstances, any cellular sheet-pile structure of importance should have the benefit of the best obtainable professional engineering advice. This is particularly so for cases where there is the added factor of a difficult foundation condition.

The most elaborate design theories are of little use, however, if proper construction methods are ignored. Since a considerable portion of most cell structures is driven below ground or below water, where it cannot be seen, there is a serious possibility of wrecking even the best design during the process of construction and before the structure is put into service. Under the circumstances it seems in order to call attention to some of these factors which, in themselves, may influence the ultimate design.

Whenever cells are to be driven through a stratum of mud or soft clay, which is unsuitable for cell fill materials because of its low shearing resistance, it is advisable to excavate and remove such objectionable material before driving the cells. Where this is not feasible, the mud inside of the cells must be replaced by sand and gravel by a carefully controlled process of displacement to avoid collapsing the cells due to external pressures.

Sheet-pile cellular walls should be treated as living structures. It is always

ferdam Cells; Pickwick Landing Dam, Tennessee



ITEM 1 CIRCULAR CELL

ITEM 2 DIAPHRAGM CELL

Sections Through Cofferdam

in Dollars				Maximum Compression[c]	Maximum Resisting Overturning Moment[d]		Maximum Friction at the Base[e]		Item
Driving[b]		All Items							
Total (10)	Unit[a] (11)	Total (12)	Unit[a] (13)	(14)	Total (15)	Unit (16)	Total (17)	Unit (18)	
3,418.30	27.72	26,179.23	212.27	11,000	958,746,500	7,771,600	16,281,380	132,015	1
3,404.50	27.46	26,585.86	214.40	11,000	803,589,000	6,480,500	16,084,640	129,715	2

per square foot, when there is no uplift. [d] In foot-pounds. [e] In pounds; coefficient assumed as 0.5. [f] At advisable to maintain field observation with respect to deflection, water pressure, and saturation lines, in order to understand all of the forces acting on the structure, and to secure data for future designs.

Some unusual experiences were encountered in driving piles at Madden Dam (in the Isthmus of Panama) through a gravel formation containing many boulders which were not disclosed in the original exploratory borings. This pointed to the advisability of driving a test cell on large projects, ahead of the main undertaking of cofferdam construction; such a cell can be excavated to rock and the conditions of the interlock observed as well as the penetration into the rock. If such a procedure is followed with a careful recording of the type of hammer and energy required to drive the piling, formation through which driving occurs, amount of ground water encountered, and similar data, the cost of such a test cell is quickly recovered in the better procedures that can be adopted for driving and subsequent pulling of the main cofferdams. Such a test cell was employed to advantage at Pickwick Landing Dam and others built subsequently.

Pile driving and construction procedures have a vital bearing on the successful application of a design in which sheet-pile cellular walls are used.

Often the driving of sheet piling is considered an ordinary operation, and the highly specialized techniques are frequently ignored. This results in typical "butchery" of the sheet piling, making it practically useless for re-driving or salvage. A sheet pile, deflected by a boulder, can curl underground in such a way that, if driving is continued, the entire pile may be dislocated. Furthermore, driving out of the interlock is a possibility which must be controlled properly. Where there is a well-developed understanding of the formation through which piles are to be driven and where there is competent and experienced pile driving supervision with proper recognition of the design objectives and limitations, a cellular wall structure of high structural capacity can be secured.

Pickwick Landing Dam was the first of the large projects on the Tennessee River requiring sheet-pile cellular cofferdams. In view of the fact that a number of important projects had previously been constructed with the diaphragm type of cofferdams, under heads corresponding to those encountered at Pickwick, the diaphragm type was adopted at the outset. This was also in accordance with general advice offered by sheet-pile manufacturers. This plan required the minimum tonnage of steel and presumably the manufacturers considered that they were acting in the best interests of the customer by saving steel. However, in the process of studying the construction procedures and other details more carefully, the writer decided to inspect three cofferdams, then in operation, in the eastern part of the United States, at Gallipolis Dam on the Ohio River Tygart Dam in West Virginia, and on the Fear River in South Carolina. He was impressed with the fact that in each case the circular cell type of cofferdam had been used and certain practical construction advantages were evident. This experience led him to abandon the original plans for Pickwick and to adopt the circular construction, which was used subsequently on all the other Tennessee River projects. A comparison of the two types of cells is shown in Table 1.

The good fortune of this decision was emphasized later when the job forces perfected a portable type of template which could be lifted out of a completed cell and set up alongside for assembly of the next cell. This in itself was a major contribution toward reducing the cost of constructing this type of cofferdam. With a diaphragm type of construction, it has often been necessary to drive timber piling and introduce internal bracing to hold the diaphragm straight while the filling is carefully placed on each side of the diaphragm, with only a slight difference in elevation of the fill from the full cell down to the empty cell, ranging in steps over eight or ten cells. With circular cells, it is feasible to fill each cell to the top. Furthermore, in the event of damage to, or failure of, a circular cell, the trouble remains localized and can be repaired more readily. The use of the diaphragm type of cellular wall is justified in special cases where there is no problem of filling the cells, or holding the empty cells in place until they can be filled. The simplest way to fill circular cells is by means of a suction dredge and here again, if there is an objectionable percentage of fines in the materials, the suction dredge tends to churn up the material and throw the fines into suspension so they can be drained off, as the heavier and sounder materials settle into the cells.

It is of interest to note in Table 1 that, for cofferdams higher than 35 ft, there is little difference in the sheet-pile tonnage in the circular cell or the diaphragm type cell, but there are great savings in the cost of constructing the circular type of cofferdam where it is possible to employ a cheaper method of filling the cells.

No doubt the art of design in the type of structure discussed herein would be advanced considerably if a precise record of stress measurements could be made on a full-sized structure to determine actual stress distribution in the steel cell, as well as in the fill and in the supporting foundation. It would appear that the promotion of such a research project deserves further consideration.

Savile Packshaw,[10] M. Am. Soc. C. E.—The design of cellular cofferdams is treated as a problem in soil mechanics in this paper, and the author has made an analysis of the forces and stresses acting on the dam. As a result, most of the obscurities existing until now have been swept away; where uncertainties remain the author has indicated the boundaries within which they are confined.

Among the previously neglected aspects is the investigation of stability with respect to shear along a plane through the center of the dam. The author shows that the horizontal earth pressure within the fill near the center of the dam must be considerably greater than the active Rankine pressure. Otherwise there would be no explanation for the stability of cellular dams that have been built from time to time, and for the fact that the corresponding factor of safety then decreases with increasing values of ϕ—the opposite of what one would visualize. On the other hand, the pressure in the vicinity of the inner wall, which has to withstand the greatest lock tension, is not much in excess of the active Rankine pressure, as confirmed by the author's recommendation that, with sand or gravel filling, an empirical pressure coefficient of 0.4 should be used.

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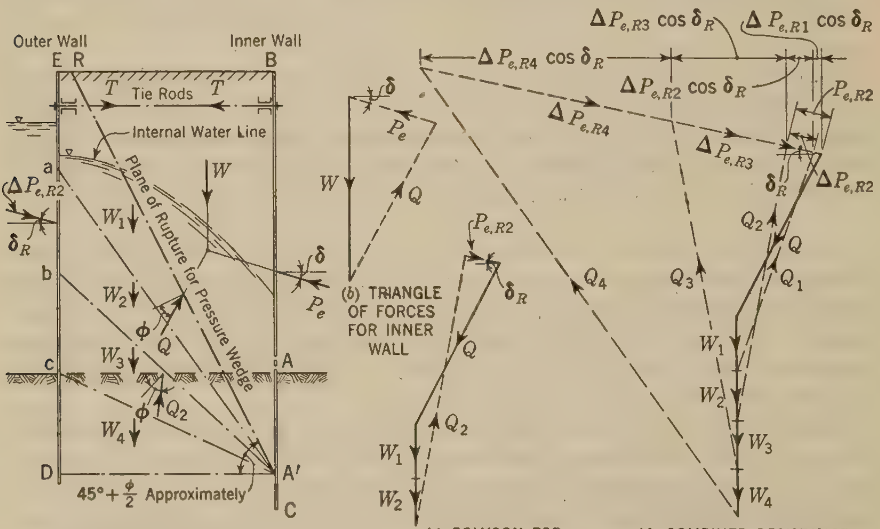
[10] With The British Steel Piling Co., Ltd., London, England.

The variation in the state of stress within the filling is even more marked in double-wall cofferdams than in cellular cofferdams, as the sheet piling in the former is subjected mainly to bending stresses and to horizontal deformations. Although there is a good deal of difference between the two types, a brief description of the methods commonly employed in the design of double-wall dams may be useful for purposes of comparison. The basic principles can be summarized by stating that the inner and outer walls of the dam are treated as a retaining wall and an anchor wall, respectively; the problem therefore resolves itself mainly into evaluating the forces, pressures, and resistances which the two walls must withstand or from which they derive their stability.

In this type of dam the two longitudinal walls of sheet piling are driven along straight lines and not in a series of circular arcs, and are connected together by one or sometimes two sets of tie rods near the top. The inner wall thus acts as a vertical beam spanning between the tie rods at the top and the reaction from the ground at the bottom. It behaves like an ordinary sheet-pile bulkhead and must withstand the bending stresses caused by the pressure of the filling. Therefore, a high section modulus is desirable. On the other hand, interlock strength against axial tension is of no importance. The same criterion applies to the outer wall, although the forces and pressures acting on it are more complex. The frequent cross-walls which form an essential part of a cellular cofferdam are not necessary in a double-wall dam, although occasional cross-walls at intervals of from 150 to 200 ft might be provided to facilitate placing the filling and for convenience in construction. The cross-


Outer Wall
E R
Inner Wall
B
T Tie Rods T
Internal Water Line
Plane of Rupture for Pressure Wedge
a
$\Delta P_{e,R2}$
δ_R
b
c
D
A
A′
C
W
W_1
W_2
W_3
W_4
ϕ
Q
Q_2
δ
P_e
45° + $\frac{\phi}{2}$ Approximately
W
P_e
Q
δ
$P_{e,R2}$
δ_R
Q
Q_2
W_1
W_2
$\Delta P_{e,R3}\cos\delta_R$
$\Delta P_{e,R1}\cos\delta_R$
$\Delta P_{e,R4}\cos\delta_R$
$\Delta P_{e,R2}\cos\delta_R$
$P_{e,R2}$
$\Delta P_{e,R4}$
$\Delta P_{e,R3}$
δ_R
$\Delta P_{e,R2}$
Q_2
Q
Q_1
Q_4
Q_3
W_1
W_2
W_3
W_4

(a) RUPTURE PLANES FOR PRESSURE AND RESISTANCE

(b) TRIANGLE OF FORCES FOR INNER WALL

(c) POLYGON FOR WEDGE A′bER

(d) COMBINED RESISTANCE POLYGON FOR OUTER WALL

Fig. 20.—Earth Pressure and Resistance in Double-Wall Cofferdams

walls do not add much to the stability, as the deep-arch or Z-sections which are necessary for this type of dam have little strength against tension or compression in the plane of the sheeting. The author shows that in a cellular dam the cross-walls add something like 50% to the stability of the dam. However, this is obtained at the expense of approximately the same amount of additional sheet piling.

The design of the inner wall, Fig. 20, is quite straightforward when the level and shape of the saturation line have been estimated in accordance with the author's suggestions (see "Part I. Practical Considerations: Permeability of Cellular Cofferdams and of Their Sheet-Pile Enclosures"). The earth pressure is derived from the formation of the unstable wedge A′RB, the failure line A′R being assumed to be straight even if wall friction is taken into account. The point A′ would be at A for a dam fixed in a rock trench and at C for a wall freely supported by the soil; if the sheeting acts as a beam fixed in the ground A′ is likely to be at about the point where the deflection line crosses the axis of the wall, so that $\overline{AA'}$ is approximately $0.75\,\overline{AC}$. After allowing for the reduction in soil density due to the subsoil water within the dam, the pressure on the inner wall can be obtained from a triangle of forces consisting of the weight W of the wedge, the earth pressure P_e, and the reaction Q along the plane of rupture. The earth pressure is equal to the active Rankine pressure when δ is assumed to be zero. The error caused by neglecting wall friction is small and adds to the factor of safety. As a rule the width of the dam is sufficient to permit the formation of the complete pressure wedge. The approximate distribution of pressure and the resistance for a dam on cohesionless soil are shown in Fig. 21, although the sharp changes in pressure or resistance, of course, are hypothetical. The diagram as drawn does not indicate the probable redistribution of pressure on a flexible wall, leading to a reduction of bending moment in the sheeting. The design of the sheet piling and the determination of the tie-rod load T follow the usual practice and call for no comment.

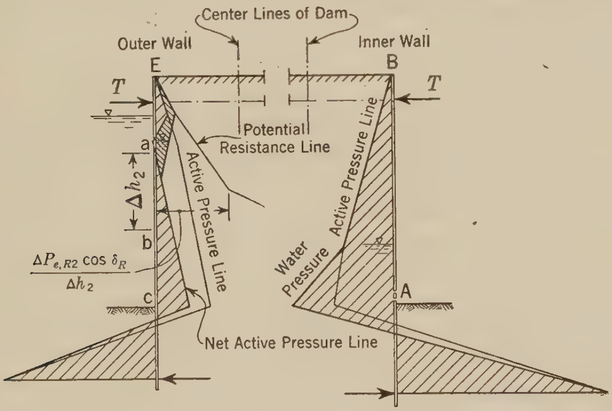


FIG. 21.—PRESSURE AND RESISTANCE DIAGRAMS

The outer wall must withstand an inward pressure due to the difference in water levels outside and inside the dam and is also subjected to the tie-rod load T imposed on it by the inner wall. To balance these forces earth resistance is developed by the soil mass A′DER, Fig. 20. The magnitude and distribution of this resistance are obtained from the assumption that the unstable soil wedge A′RB does not affect the resistance, except for the effect of the force Q, which is similar to that of a super-load applied to the plane of rupture A′R. This also implies that there is a sharp division between the expanding soil in the pressure wedge to the right of A′R and the contracting soil to the left. Actually there would have to be an intermediate zone of soil at rest.

It has been shown (36) that for any point such as point a or point b on the outer wall the least resistance will be developed by failure along a plane coinciding nearly enough with aA′ or bA′. It is permissible, therefore, to divide the area A′DER into a series of wedges radiating from point A′. Then the equilibrium of the first wedge, aA′RE, is governed by its weight W_1, the force Q from the pressure wedge, the reaction Q_1 along aA′ acting at an angle ϕ from the normal to aA′, and the reaction or earth resistance $P_{e,R1}$ along aE. The first two forces are known in magnitude and direction, the remaining two in direction only; $P_{e,R1}$ acts at the angle of wall friction δ_R to the normal to aE, provided the sheet piling is held by the soil and therefore is able to resist uplift. A polygon of forces can be drawn to determine $P_{e,R1}$, therefore, and the construction can be repeated for successive wedges so as to obtain the resistance due to each. For the sake of clearness, only four wedges have been shown. The potential resistance line, which shows the resistance that the soil can develop at various depths, is indicated in Fig. 21. It can be drawn by taking the resistance that can be derived from, say, an element ab of the wall and dividing it by the length of the element, Δh_2. This gives the average resistance along ab, assumed to occur midway between points a and b.

The resistance line starts from the ground level at point E or from the intersection of the failure line with the outer wall when the dam is rather narrow. It is usually found that, with the normal width of dam equal to about 2/3 H or 3/4 H, there is little resistance developed in the top quarter of the wall. Below that it increases rapidly and as a rule only a portion needs to be called into action. The least pressure on the sheeting, however, is the active earth pressure of the filling, with due allowance for the reduction in weight due to the buoyancy of the soil below the internal water level. This may be partly balanced, or even more than balanced, by the inward water pressure caused by the difference in water levels inside and outside the dam; but, if the net diagram obtained in this manner is taken as the basis for the design of the wall, it will be found that the reaction at the upper support—that is, the tie-rod load—does not exactly correspond to the pull T from the inner wall. Most probably the reaction at the outer wall will be smaller than T, so that the effect of the excess tie-rod load will be to draw the wall in toward the filling and thus mobilize some of the potential resistance until the necessary increase in pressure behind the wall has been obtained. For design purposes the extra pressure area would correspond roughly to that shown by the close-hatched area in Fig. 21. It would tend to concentrate as much as possible in the vicinity of the tie rods and one or two trials may have to be made before satisfactory agreement with the tie-rod load T is obtained. The fundamental conditions of equilibrium—

that the algebraic sums of forces and of moments must be equal to zero—are taken into account by the usual methods of bulkhead design.

The development of cellular cofferdams in the United States can be ascribed partly to the production of straight-web sections of steel sheet piling with a high interlock strength. On the other hand, these sections have a relatively low moment of resistance to bending. In Europe the emphasis since the beginning of the century has been mainly on the production of deep-arch sections with a good strength-to-weight ratio and a high section modulus. These encouraged the development of double-wall dams but precluded the construction of cellular dams owing to their very low interlock strength. A comparison of the technical and economic merits of cellular and double-wall cofferdams seems to be outside the scope of this discussion. It might be suggested, however, that, since Z-sections and other types with a high strength modulus have now been available in the United States for some time, double-wall dams may find occasional application in the future.

Martin W. Lautz,[11] Assoc. M. Am. Soc. C. E.—Before 1910, the subject of this paper began to interest engineers. Since that time many cellular cofferdams and bulkhead walls have been constructed. The proportions of a few of them were based on experience only; but most of the cofferdams were subjected to design analysis. Resistance against overturning, sliding on the base, and tension in the interlocks were determined, and in later years the shearing resistance of the confined earth was recognized as a most important factor. The added stability afforded by the cross-walls, including the resistance of the sheet-pile interlocks against sliding, was also stressed; in fact, consideration was given to increasing this resistance by welding.

The paper has outlined an approach to the subject of cellular cofferdams. The author's exposition of methods for determining the vertical shear, shearing resistance in the earth fill, and the sliding resistance of the piling interlocks is of great interest, and should prove a most important basis for further discussion and consequent increased knowledge.

In regard to the shearing resistance of the fill on a central vertical plane, the writer believes that the normal pressure on the plane is increased substantially beyond the active Rankine pressure by the horizontal load imposed on the outside of the structure. How much external load is exerted on this plane is problematical—perhaps as much as 50%. This theory merits discussion and may account for the high resistance against shear present in actual installations mentioned by the author.

The following three examples of cellular sheet-pile structures are presented because they have features that are somewhat different from those found in most installations.

Example 1. Cellular Dock Wall at an Eastern Shipyard.—Fig. 22 is an example of a permanent cellular type sheet-pile wall constructed in 1941 at an eastern shipyard. The following reasons for the selection of this type of dock wall are significant.

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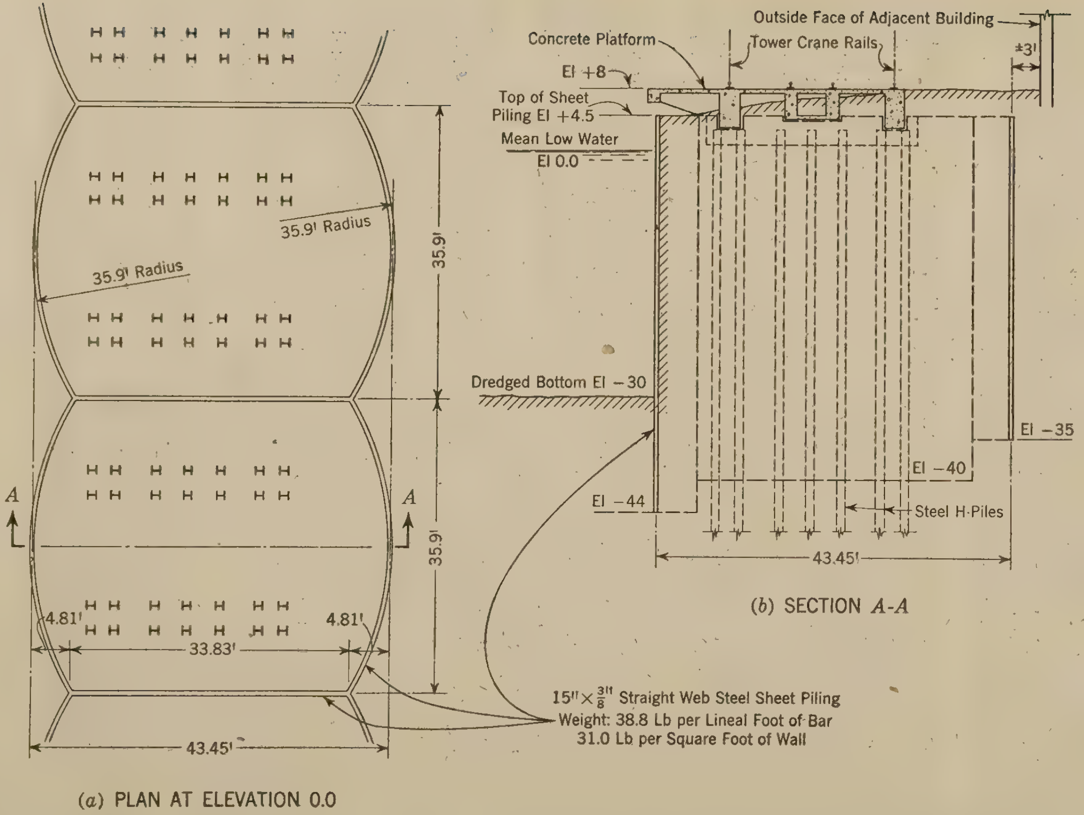
[11] Sales Engr., Bethlehem Steel Co., Bethlehem, Pa. (Mr. Lautz died on April 23, 1945.)


H
A
35.9' Radius
35.9' Radius
35.9'
35.9'
4.81'
4.81'
33.83'
43.45'
Outside Face of Adjacent Building
Concrete Platform
Tower Crane Rails
±3'
El +8
Top of Sheet Piling El +4.5
Mean Low Water
El 0.0
Dredged Bottom El −30
El −35
El −40
El −44
Steel H-Piles
43.45'

(a) PLAN AT ELEVATION 0.0

(b) SECTION A-A

15" × $\frac{3}{8}$" Straight Web Steel Sheet Piling
Weight: 38.8 Lb per Lineal Foot of Bar
31.0 Lb per Square Foot of Wall

+9.8
+4.8
El 0.0
−5.2
−15.2
−24.2
−28.2
−33.2
−38.2
−42.2
−47.2
−52.2

Materials	No. of Blows per Ft
Fill	
Loam and Clay	4
Fine Sandy Silt	4
Very Soft Mud	1
Silt	4
Medium Brown and Gray Clay	8
Coarse Brown Sand and Gravel	40
Hard Brown and Gray Clay	30
Hard Brown and Gray Clay	50
Compact Fine Brown Sand	50
Very Hard Gray Clay Some Fine Sand	150

(c) TEST BORING

FIG. 22.—CELLULAR SHEET-PILE DOCK WALL AT AN EASTERN SHIPYARD

The location of the left-hand face of the dock was determined by plant layout considerations. Buildings on the right-hand side were on adjacent property and could not be disturbed by the dock-wall construction. Thus, only a strip of ground not more than 48 ft wide could be used for the proposed construction. A platform type of marginal wharf with a sheet-pile wall along the inner edge was eliminated by the fact that brace piles could not be driven where desired due to interference from the adjacent buildings which were about 30 ft high. A straight-wall bulkhead, consisting of a row of sheet piles at the left-hand face tied back to an anchorage, was also rejected because it would have been necessary to place the anchorage at least 60 ft back of the sheet piling—well into the adjacent property under the adjoining buildings.

A cellular type sheet-pile structure did not have any of the foregoing disadvantages, and its cost compared favorably with the two other designs considered. A disturbing factor, however, was the unstable character of the earth down to about 24 ft below water level as is indicated on the test boring record shown in Fig. 22(*c*), which was typical of others made within the area to be occupied by the proposed structure. Dredging this poor material and replacing it by sand fill would have cost a prohibitive amount because of the necessity of maintaining the adjacent buildings. The dock wall was built as shown in Fig. 22(*b*), leaving the original ground in place and driving the sheet piling well into the firm underlying strata. The average width of the cells as built was about 39 ft as compared with a height above dredged bottom of about 35 ft —a ratio of 1.15:1. Vertical loads on the top of the dock wall, including a movable tower crane, were supported on a concrete platform resting on steel H-piles driven to practical refusal at about El. +50. There was no connection between this structure and the sheet piling. The sheet-pile cross-walls, which were driven with their tops at El. +4.5, were notched away by flame-cutting to allow the reinforced concrete beams to extend through without contact with the sheet piling. It was considered undesirable for any vertical loads to be carried by the sheet piling.

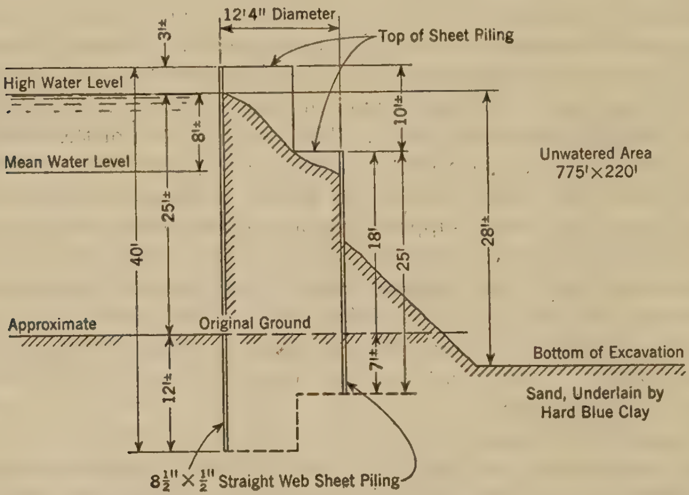
An indication of the stability of the soil below El. −28 was the fact that as many as 400 blows per foot from a single-acting steam hammer (capacity, 15,000 ft-lb) were required to obtain penetration of the sheet piling to the elevations shown in section A–A, Fig. 22(*b*). (The numbers in the right-hand column of Fig. 22(*c*) indicate the number of blows required to drive a sample pipe 1 ft, using a 140-lb weight with a drop of 30 in.)

After installation of the sheet piling and concrete platform with bearing pile supports, the slip along the left-hand face was dredged to El. −30. At that time a slight outward movement of the piling on the water side was noted. This resulted from the unbalanced earth load, which caused take-up in the piling interlocks. After 3.5 years of service, there were no visual signs of further movement. The crane rails on the dock wall have maintained alinement throughout their length.

To summarize—this structure owes its stability to: (*a*) A large ratio between width and height; (*b*) the penetration of sheet piling into firm material; and (*c*) an unyielding base. These conditions have increased the safety factor sufficiently to overcome the weakness inherent in the unstable soil confined within the piling walls.

Example 2. Cellular Cofferdam, Hastings, Minn.—The cellular type cofferdam shown in Fig. 23 was used in the construction of a lock and dam in the Mississippi River in 1929. This cofferdam formed part of the enclosure around an area of 775 ft by 220 ft and consisted of eighty-six circular cells, each having a diameter of 12 ft 4 in., connected in the usual manner by short piling walls. The cells were filled with sand dredged out in making the excavation for the proposed structure. The section of the cofferdam in Fig. 23 is interesting because it shows the average width to be less than half the height and yet this structure served its purpose without failing.

One explanation of the stability of this wall could be that it acted in part as a cantilever fixed at the bottom by the passive resistance of the soil into which the sheet piling was driven. This favorable factor has not received sufficient attention in design analyses, but may be worthy of further consideration in cases where the sheet piling penetrates firm, resistant soil below the base of the cofferdam.



12'4" Diameter
Top of Sheet Piling
3'±
High Water Level
Mean Water Level
8'±
10'±
Unwatered Area
775'×220'
25'±
40'
18'
25'
28'±
Approximate
Original Ground
Bottom of Excavation
7'±
12'±
Sand, Underlain by
Hard Blue Clay
8½"×½" Straight Web Sheet Piling

FIG. 23.—MAXIMUM SECTION OF CIRCULAR CELL COFFERDAM; U. S. GOVERNMENT LOCK AND DAM AT HASTINGS, MINN.

Example 3. Cellular Cofferdam for Intake System, Detroit, Mich.—The cofferdam shown in Fig. 24 was built in 1929 to enable the construction of two shafts and an intake structure in the Detroit River near Detroit. The ground material was plastic clay of the same general character as that found in the Detroit area—decidedly not an ideal kind of ground material on which to build a cellular cofferdam.

The lower part of the soil enclosed within the cell walls was clay in its original position, with clay fill above (see Fig. 24(*d*)). The inner sheet piling was longer than the outer walls and diaphragms to provide resistance against

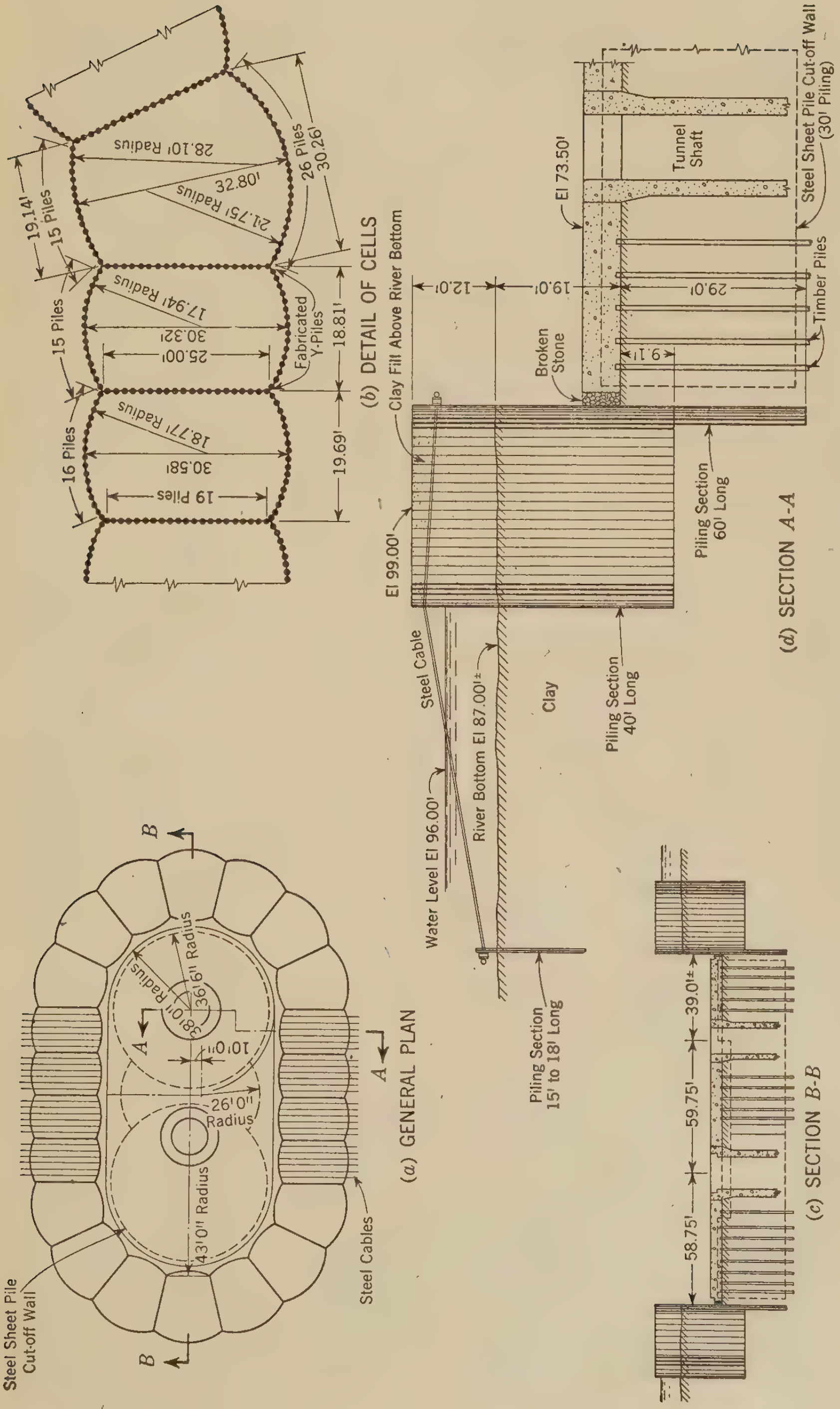

Steel Sheet Pile Cut-off Wall
B
B
A
A
38'0" Radius
36'6" Radius
10'0"
26'0" Radius
43'0" Radius
Steel Cables
(a) GENERAL PLAN
19.14'
15 Piles
15 Piles
16 Piles
28.10' Radius
32.80'
21.75' Radius
26 Piles
30.26'
17.94' Radius
30.32'
25.00'
Fabricated Y-Piles
18.81'
18.77' Radius
30.58'
19 Piles
19.69'
(b) DETAIL OF CELLS
Clay Fill Above River Bottom
El 99.00'
Steel Cable
Water Level El 96.00'
River Bottom El 87.00'±
Clay
Piling Section 15' to 18' Long
Piling Section 40' Long
Piling Section 60' Long
12.0'
19.0'
29.0'
9.1'
Broken Stone
El 73.50'
Tunnel Shaft
Steel Sheet Pile Cut-off Wall (30' Piling)
Timber Piles
(d) SECTION A-A
39.0'±
59.75'
58.75'
(c) SECTION B-B

FIG. 24.—CELLULAR COFFERDAM FOR INTAKE SYSTEM AT DETROIT, MICH.

settlement at the toe, penetration below the general excavation level being almost equal to the height of the cofferdam. The average width, if taken as 90% of 30.58 ft, would be about 27.5 ft, or 89% of 31 ft—the height of the dam.

After construction was begun, it was believed that excessive deflections might occur and that additional stability should be provided. Therefore, the tieback system shown in Fig. 24(*a*) was installed to afford a greater factor of safety. At the semicircular ends it was considered that arch action would make the tiebacks unnecessary. The sheet piling used was a 15-in. web section, now obsolete, having a minimum ultimate interlock strength of 9,500 lb per in.

No excessive movement of the cells occurred after inside excavation and unwatering were completed. This result probably would not have been obtained if the tiebacks had been omitted.

J. S. MILLER,[12] ESQ.—Professor Terzaghi stated in his paper that the most important criterion for determining the proper width of a cellular cofferdam structure has grown out of past experience. Under the circumstances it seems proper to discuss some of the practical applications of steel sheet piling and cellular structures as employed on projects with which the writer has been familiar.

Credit for the circular cell type of cofferdam must be given to Maj.-Gen. Harley B. Ferguson, M. Am. Soc. C. E., Corps of Engineers, U. S. Army (Retired), and to the late C. S. Boardman, M. Am. Soc. C. E. General (then Captain) Ferguson used the circular steel cell method in the raising of the battleship *Maine* in Havana harbor in 1910 and 1911, and this application stimulated much interest in the use of cellular cofferdams for other projects.

There are several advantages in the circular cell type of cofferdam for dam construction. The most important advantage concerns the very common situation in which the upper arm of the cofferdam must be extended across the river against the current. The current, of course, becomes more rapid as the structure is extended and the channel becomes correspondingly constricted. Furthermore, there is always the danger of freshets; therefore, it is distinctly advantageous to fill and make each main cell safe as soon as it is in place. Another advantage of circular cells is that the connecting arcs can be omitted until all the main cells are driven, thus providing partial relief of hydrostatic head while the main cells are being installed.

The cells should be filled with sand, or preferably sand and gravel, as these materials compact almost as soon as placed, especially if they are well saturated and if the cells must resist distortion and provide the greatest frictional resistance to sliding. Generally, it pays to dredge the area on which the cofferdam is to be built to within 10 ft, or better still to within 4 ft or 5 ft, of rock before driving piles. Thus, the chance of the sheets encountering boulders and injuring the interlocking joints is reduced.

Most frequently, the diaphragm-wall type of cofferdam is used in protected areas (in this case, parallel with the current and relatively close to the protected shore), whereas the circular cell type is used when fighting currents such as are encountered on dam construction. Care must be taken in placing the filling in this single connecting-wall type of cofferdam. The filling must be stepped down from cell to cell to avoid distortion of the cross-walls. After being completely filled, however, the resulting cofferdam is very satisfactory in every respect.

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The decision as to whether a circular cell or single connecting-wall cellular type of coffer be used also depends on the depth and character of the material through which the steel piles are driven. The single connecting-wall coffer should not be used where the sheets are driven through boulders as the sheets may drive out of the interlock. It is difficult to repair this type of coffer because of the necessity of removing and replacing a large amount of fill. Furthermore, the damaged sheets may be in the cross-wall where the break, of course, is concealed. In the case of driving through boulders, it is preferable to use circular cells as the filling of one cell can be removed for necessary repairs without affecting the adjoining cells, and with less expense and delay.

The single connecting-wall type of coffer, however, has the advantage of stiffening the coffer against deflection, especially in cases where the sheet piles are not on rock and are driven below the excavation line. The decision as to which of these two types should be used is largely a matter of judgment. It is important that the sheets be long enough to permit driving into firm material to practical refusal.

The writer's firm has conformed very closely to a width of cofferdam approximately equal to the height. If the walls were parallel, the width should be approximately 83% of the height. However, because of the loss of width due to curvature of the cells, an over-all width has been fixed arbitrarily that is equal to their height for cellular construction. A 6-in. concrete cap is customarily placed on top of the cells, on which are mounted the whirler cranes for the permanent construction work. In spite of the concrete cap, some damage to the concrete, and a small amount of washing out of the filling material, may occur during floods, and requires careful watching and prompt replacement.

In many cases the sheet-pile interlock would probably have sufficient strength to allow full hydrostatic head to the top of the sheets on the inner side of the cofferdam. Nevertheless, the writer and his associates do not like to take this chance, and consider it very good insurance to burn holes through the webs of enough sheet piles on the inner side of the cofferdam to insure the drainage of each cell to within at least 5 ft of the bottom of the excavation. This is one point that is insisted on because the writer has known of one or two instances where a cellular cofferdam failed with river stages just about to the top. Probably, the cell first filled with water and one or more of the interlocking joints were not sufficiently strong to withstand this hydrostatic head, with the result that the inner wall failed first; and, of course, the outer wall also failed after the rupture of the inner wall permitted the coffer filling to become displaced. The draining of the cells on the inside does not ordinarily result in an increased leakage because the outer row of sheeting should exclude the water from the cofferdam area.

The writer takes exception to the statement (see heading, "Part I. Practical Considerations: Permeability of Cellular Cofferdams and of Their Sheet-Pile

Enclosures") that "As a consequence, the water percolates through the dam as if the outer row [of sheet piles] were nonexistent." Admittedly, a certain quantity of seepage escapes through a sheet-pile wall—either through the interlocking joints or under the sheets—but the leakage through the interlocking joints is practically nonexistent except in coarse material. One of the reasons for using long sheeting is to prevent water from descending on the outside, passing under and coming up on the inside of a sheet-pile wall, and thus tending to loosen the material into which the sheet piles are driven, resulting in danger of settlement or loss of bearing power on the inner sheet-pile wall.

The author has made the further statement (see heading, "Part III. Conclusions") that "The assumption that the weep holes in the inner wall of cells filled with sand suffice to drain the cells is not necessarily justified." Wherever possible, in the writer's experience, sand and gravel are used for filling cellular cofferdams. As the cofferdam is unwatered, holes are burned in the inner row of a certain number of sheet piles in each cell and the progress of dewatering the cofferdam is limited by the progress of draining the individual cells. When the cells do not drain fast enough, the dewatering of the cofferdam is retarded.

It is easy to determine whether a cell is draining because, if the cell is not draining properly, moisture will appear at the joints in the sheet pile; and, if this moisture line is too high, additional holes need to be burned in the cell or the previously placed drain holes have become clogged. The important thing is to eliminate hydrostatic pressure against the inner row of sheet piles.

Although it is obvious that the water in the fill material on the back side of the cells is doubtless standing at a higher elevation than the water on the inner side, a certain length of time is required to lower the water table in the cells; hence, the cofferdam should be unwatered slowly enough to prevent undue hydrostatic head. In the design of cellular cofferdams, the writer and his associates usually provide a safe resistance against bursting of the cell for full hydrostatic head to an elevation one third of the height from the bottom to which the cofferdam will be unwatered.

The author's reference to boils on the inner side of cofferdams is most emphatically commended. Unless they come from a deep porous stratum, these boils can usually be greatly reduced by the use of long sheet piles and careful drainage of the inside walls of the cellular structure. Drainage is exceedingly important for the simple reason that it usually tends to reduce the hydrostatic head on any boils that may develop on the inside of the cofferdam.

To prevent boils inside the cofferdam, it is sometimes advisable to place a berm or blanket on the outside of the cofferdam; thus water cannot descend and pass under the outer row of sheet piles. Fundamentally, the outer row of sheet piles is the line of defense at which water should be stopped and held. The inner wall of the cofferdam should be considered as the stability side of the structure, and for this reason the same length of piles should be used on the inside as on the outside. In case of trouble due to boils, a berm should be provided on the outside because it usually results in reducing or eliminating the boils.

From many tests of the strength of interlock of the various types of steel piling the writer has found that sections M-107, M-108, M-112, and M-113 of the Carnegie piling, and their counterparts which are manufactured under other designations by Bethlehem, Inland, Weirton, and other steel companies, are the sections best adapted for cellular construction. In general, the strength of these sections lies in the interlocking joint; and, as a general rule, a closely fitting interlock is stronger than an interlock with wide grooves and narrow tongues. In addition, one can predict the strength of an interlock almost exactly by measuring the difference between the widths of the tongues and the widths of the grooves of the individual sheets involved. Obviously, the stronger the interlocked joint is, the better (even if the sheet pile may fail in the web rather than in the interlock), for the very simple reason that the interlock is liable to be injured during driving; and the heavier the interlocked joint is made, the less likelihood there is of damage during driving.

Another danger is the occurrence of too much carbon in the steel. There should never be more than 0.35% carbon, and preferably the carbon content should be just as close to 0.30% as possible, except that, in the case of sections that have to be bent, the carbon content should be very little more than 0.25%. In one case, three cells of a cofferdam split open from top to bottom right through the web of the sheet due to the fact that there was too much carbon in the steel, and large ice cakes pounding over the top of the cofferdam caused failure.

On a few occasions, trouble has been experienced due to lamination in the web, and even in the steel-pile interlock, in spite of careful inspection both at the mill and at the site of the work. Such failures, although quite disconcerting, are very rare and do not warrant condemnation of the use of steel piling in cellular construction. Extreme care should always be taken, however, with the interlock to insure that it is properly rolled and that there are no laminated sheets.

As yet, much is to be learned about the uses that can be made of steel piling. There is ample room for improvement in the design, and also in manufacture and installation, of cellular steel-pile structures. Progress in this field since 1925 has been most gratifying and there is every reason to believe that it will continue at a rapid pace.

ROBERT F. LEGGET,[13] M. AM. SOC. C. E.—The use of steel sheet piling has increased so rapidly in the past twenty years, for both temporary and permanent civil engineering works, that the written records of the profession still contain all too few discussions of the many complex factors that must be considered in the design of structures consisting of this versatile rolled steel section. Professor Terzaghi's paper, therefore, has a double value: (*a*) It assembles much useful information on actual experience with cellular cofferdams in connection with a very thorough design study; and (*b*) it inevitably mentions some of the general characteristics affecting the use of steel sheet piling for other types of structures. Although discussion of these wider implications of the paper is not in order at this time, it is to be hoped that critical study of the paper will direct increased attention to these other points and thus lead, even if indirectly, to improvement of the art.

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[13] Associate Prof., Civ. Eng., Dept. of Civ. Eng., Univ. of Toronto, Torónto, Ont., Canada.

Professor Terzaghi is unusually forthright in condemning the application of the results of laboratory soil tests to the design of cellular steel pile structures, and he emphasizes the corresponding importance of observations on actual structures. The soundness of this empirical approach, especially in view of the many variable factors, will not be questioned at this stage in the development of steel piling design. However, some simple laboratory soil testing may provide useful and practical design information—such as how to determine the type of fill material and the degree of its conformity with the assumptions as to its properties made by Professor Terzaghi, as well as how to determine the friction between fill material and the surface of the steel piling.

Since this friction (admittedly an important factor in design) is a variable, the statement (see heading, "Part I. Practical Considerations: Deflection of Cellular Cofferdams") that the "* * * elastic properties [of a cellular cofferdam] can be compared to those of a composite construction material such as reinforced concrete" is somewhat puzzling. Possibly, the author intended such a comparison to demonstrate the differences between the two combinations of materials. Some elaboration of the statement would be welcome.

Another frictional force, that developed in the interlocks of the steel sheet piling, is clearly one of the most important factors involved in the stability of cellular structures. It cannot be calculated on the basis of any theory in view of the many practical variables—the type of lock, the limits of clearance between the jaws of locks in contact, the nature of the material (if any) plugging the locks, and the relative position of "mating" locks (as determined by the way they are held by either the soil or the rock into which they have been driven). In view of the emphasis given in the paper to lock friction (see conclusion 3), it would be of great interest to know whether any experimentally determined values of lock friction have been developed for any type of steel pile or, alternatively, whether the author believes that such values could be obtained.

Lock friction is even more important than this paper would suggest; for, in the case of straight walls of deep-arch, steel sheet piling which retain lateral loading, the degree to which lock friction is developed determines whether the piles must be considered acting as individual units or, alternatively, whether the wall can be regarded as made of "interlocked units"—the lock friction exceeding the shear stress developed longitudinally in the lock. In some countries other than the United States, deep-arch piles have been developed which do give an "interlocked section modulus." In consequence, double-wall cofferdams have been used generally there instead of the cellular type of structure, which appears to be confined almost exclusively to American practice. Although it is probably outside the province of the paper, an economic comparison of the two types of piles would be of interest.

In view of this wider significance of lock friction, the author's condemnation of the practice of lubricating the interlocks of steel sheet piling will be most heartening to all who have had to argue about this matter with enthusiastic pile driving foremen. Unfortunately, his reasoned arguments will provide little useful "ammunition" for such on-the-job discussions; but they should go far toward clarifying not only this point but also many others in the minds of all those responsible for the design of structures of steel sheet piling.

D. P. KRYNINE,[14] M. AM. SOC. C. E.—Few formulas are as accurate as the Rankine formula as applied to cohesionless masses, such as cohesionless sand or gravel. If there are claims that actual pressures in a cohesionless mass are not in accordance with the Rankine formula, such claims possibly may be based on the fact that the formula in question has been used incorrectly. In particular, the value of C given by Eq. 20 is the ratio between the minor and the major principal stresses at a point of a cohesionless mass just immediately preceding the failure of that mass by shear, but, in a general case, the value of C given by Eq. 20 is not the ratio between the horizontal and the vertical pressure at the point in question.

Internal Shear in the Fill of a Cofferdam on Rock.—If the shear failure is expected to occur along a vertical plane, such as plane gh in Fig. 7(*b*), this direction is not a principal one because, by definition, there are no shears along principal axes. Hence, to compute the horizontal pressure on the plane of expected failure such as section gh in Fig. 7(*b*), Eq. 20 cannot be applied.

The well-known Mohr's circle can be applied for finding the ratio C between the horizontal and vertical pressures at a point on plane gh (Fig. 7(*b*)) at a depth Z from the top of the fill, just immediately prior to failure by shear. Simply draw a horizontal line and a circle of arbitrary radius with its center O on that horizontal line (Fig. 25). Draw a line tangent to a circle that will describe an angle ϕ intersecting the horizontal at point E, as shown in Fig. 25. Furthermore, draw both the diameter through the point of tangency B and the vertical line AEC. Then the ratio of the distances EF and EG is the ratio of the principal stresses at the given point as determined by Eq. 20. Distance AB is proportional to the normal pressure on the plane of failure (section gh in the given case, Fig. 7(*b*)), and distance CD is proportional to the pressure on the plane perpendicular to that plane of failure—that is, in the given case, the horizontal plane. Hence, the ratio of both distances AB and CD multiplied by γZ (which is the vertical pressure at the given point), is the horizontal pressure on plane gh at the given point. Simple algebra shows that the value of C for the plane of expected failure gh is

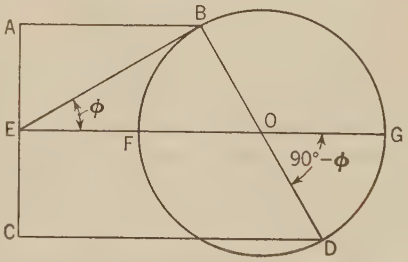

FIG. 25.—RANKINE FORMULA REPRESENTED GRAPHICALLY

$$C = \frac{\cos^2 \phi}{2 - \cos^2 \phi} \qquad (41)$$

If the angle of friction equals 30°, the value of C given by the Rankine formula, if correctly interpreted, is 0.6, and not $(0.577)^2 = 0.33$, as determined by Eq. 20.

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Recomputation of the Selected Example.—The factor of safety with respect to shear in the example immediately following Eq. 20 is $G_s = 0.87$. In the numerical formula for this factor of safety the value of $(0.577)^2$ must be replaced by 0.6, as explained in the preceding paragraph, which furnishes a corrected value of the factor of safety $G_s = 1.59$. Neglecting the friction in the locks—that is, dropping the value of 0.3 in the numerical formula in question—the factor of safety would be $G_s = 1$.

Perfect Safety of Plane gh *(Fig. 7(b)) with Respect to Shear.*—It will be assumed that Eq. 19*b* for the factor of safety G_s, with respect to shear, is correct except for the value of C which should be computed by Eq. 41. Consider a fill material weighing $\gamma = 110$ lb per cu ft with the angle of friction ϕ varying from 30° to 40°. Let $\gamma_w = 62.5$ lb per cu ft and $f = 0.3$. Then for $\phi = 30°$:

$$G_s = 2 \times \frac{110}{62.5} \times \frac{b}{H} \times 0.6 \times (0.577 + 0.3) = 1.85 \frac{b}{H} \quad \ldots\ldots (42a)$$

Similarly, for $\phi = 40°$,

$$G_s = 1.65 \frac{b}{H} \quad \ldots\ldots (42b)$$

Since Eq. 42*b* is less favorable than Eq. 42*a*, compute the values of the safety factor for $\phi = 40°$, as follows:

Ratio $\frac{b}{H}$	Safety factor G_s
0.85	1.4
1.00	1.65

Hence, under the given assumptions, plane gh is always safe with respect to shear. It is no miracle and no paradox, therefore, that many cofferdams wide enough, but with factors of safety of 0.6 or 0.7 incorrectly calculated, stand safely. Their true factor of safety with respect to shear certainly is greater than 1.

These considerations do not mean, however, that a shear failure cannot occur in a cellular cofferdam. They mean simply that under the given assumptions shear cannot occur along plane gh (Fig. 7(*b*)).

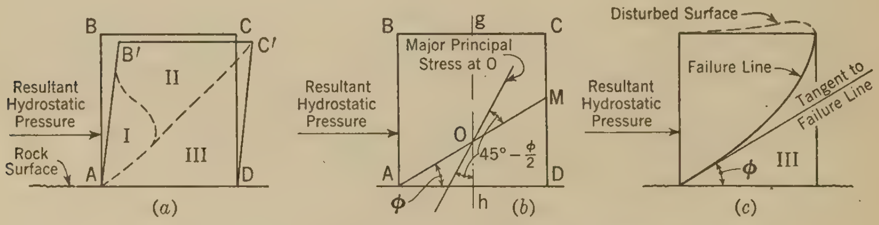
B
C
B′
C′
II
I
III
Resultant Hydrostatic Pressure
Rock Surface
A
D
(a)
B
g
C
Major Principal Stress at O
M
Resultant Hydrostatic Pressure
O
$45° - \frac{\phi}{2}$
A
D
ϕ
h
(b)
Disturbed Surface
Failure Line
Tangent to Failure Line
Resultant Hydrostatic Pressure
III
ϕ
(c)

FIG. 26.—INTERNAL SHEAR IN A CELLULAR COFFERDAM

Phenomenon of Internal Shear in a Cofferdam on Rock.—In 1935, R. P. Pennoyer, Assoc. M. Am. Soc. C. E. (10*c*), called the attention of the profession to the possibility of internal shear along the vertical plane gh (Fig. 7(*b*)).

Professor Terzaghi also considers this plane dangerous with respect to shear. The writer, however, visualizes the possibility that internal shear can occur in a still different way. When the cofferdam begins to tip, it comes from the initial position ABCD into some other position AB′C′D (Fig. 26(*a*)). As the tipping of the cofferdam progresses, the density within the compressed zone I (and perhaps the size of this zone) increases until a certain critical density has been reached. Zone I then bursts and pushes zone II upward. To bring this picture back to the Rankine formula, it should be remembered that at any point O of a mass (Fig. 26(*b*)) there are actually two directions of maximum shear—not one. Each of these makes an angle $45° - \frac{\phi}{2}$ with the direction of the major principal stress. It follows that, if point O (Fig. 26(*b*)) is located in plane gh, there is, at that point, another possibility of shear along plane AOM making an angle ϕ with the horizon. The hydrostatic pressure pushes the mass up along a plane AM making an angle ϕ at point O (Fig. 26(*b*)) with the horizon, to meet the inside wall of the cofferdam, MC. The actual failure probably occurs as shown in Fig. 26(*c*). An observer standing at the top of the cofferdam possibly would report that, in the case of shear failure, all sand grains and all rocks and pebbles of the mass were moving. In reality there is probably a zone of quiet marked III in Fig. 26(*a*) and 26(*c*). A simple inspection of Fig. 26(*c*) shows that, to offer due resistance, the failure line must be sufficiently long. In other words, a guarantee against possible shear failure is the widening of the cofferdam.

Overturning of the Cofferdam on Rock.—Unless a shear failure in the fill occurs, the cofferdam is exceedingly stable with respect to overturning. If there is no water outside the cofferdam the fill material is pressed against the outside wall by the active pressure $\frac{1}{2}\gamma C H^2$, the value of C being smaller than that at the plane gh (Fig. 7(*b*)). To be conservative, it will be assumed that C may be computed using Eq. 20. If there is water outside the cofferdam, the pressure of the fill material, in addition to active pressure, is applied against the outside wall by the full hydrostatic pressure due to flexibility of the sheet piles. To start overturning, friction between the fill material and the outside wall must be overcome. The situation is very similar to that of a man who wants to lean to the right but cannot do so because his left leg is deep in a sticky swamp; the first thing for him to do is to pull his leg out of that swamp. In the case of the cofferdam, the friction is tantamount to a vertical force acting downward at the outside wall. Neglecting the weight of the cofferdam entirely, the factor of safety with respect to overturning would be:

$$G_s = \frac{(\frac{1}{2}\gamma C H^2 + \frac{1}{2}\gamma_w H^2)\, b \tan\delta}{\frac{1}{6}\gamma_w H^3} \ldots\ldots\ldots\ldots\ldots (43a)$$

in which the term $\tan\delta$ denotes the coefficient of friction between fill and sheet-pile enclosure. Assuming, furthermore, that the width b is some fraction of the height H, such as $b = a\,H$, and the value of $\tan\delta = 0.4$, Eq. 43*a* may be rewritten thus:

$$G_s = 1.2\, a \left(\frac{\gamma C}{\gamma_w} + 1\right) \ldots\ldots\ldots\ldots\ldots\ldots (43b)$$

For current values of a, C, γ, and γ_w, this safety factor is always greater than 1.4. It should be recalled that Eqs. 43 were developed neglecting the weight of the fill inside the cofferdam completely, so that at least a value of 1.4 should be added to the value of the factor of safety computed as recommended in the paper. In the example selected to illustrate this procedure a total of $2 + 1.4 = 3.4$ should be computed, which is very close to 3.8 as determined by the usual gravity wall method.

Sliding of the Cofferdam on Rock.—Fig. 7(a) is of rather academic interest and perhaps should have been omitted. It is questionable whether the boundary corrections should be introduced in this diagram since discontinuity of stresses (but not of strains) is acceptable. Corrected as it is, the area shown in Fig. 7(a) is less than the value of F as given by Eq. 1. This fact implies the necessity of correcting the ordinates at the middle of the diagram and thus changing the given value of the angle of friction. Finally, boundary corrections, if any, should be extended through the diagram to meet somewhere at the middle of the curve and not be connected to the curve at some arbitrary points. What is important, however, is not the diagram but that "There seems to be no record of a failure by sliding" as Professor Terzaghi states (see heading, "Part II. Design of Cellular Cofferdams: General Considerations"), and this apparently settles the question of sliding.

Conclusion.—The designer of a cellular cofferdam on rock must ensure close cooperation between the sheet-pile enclosure and the inside fill, making them work together and preventing their possible separation by internal shear. Attention should be paid to: (a) A good packing of the gravel and sand fill inside the cofferdam in order to increase the horizontal pressure and (b) a good interlock arrangement. The size of sheet pile should be selected properly and the cofferdam should be given ample width, the latter item being of extreme importance. To design and analyze such structures good engineering judgment and vast experience in the given field combined with clear understanding of statics are needed. A great number of cellular cofferdams have been constructed successfully in the United States—incomparably more than in other countries. It is obvious therefore that American engineers have demonstrated fully their ability to design and to build these structures, both permanent and temporary.

J. J. POLIVKA,[15] M. AM. SOC. C. E.—In the field of cellular cofferdams this excellent paper is a valuable contribution to engineering knowledge. In July, 1944, the writer had the opportunity to make some preliminary studies on cellular cofferdams in the Kaiser Shipyards at Richmond, Calif. These cofferdams had been constructed in February–March, 1945, for the U. S. Maritime Commission, preliminary to deepening the existing concrete-lined basins. In many respects the basins resembled graving docks, 100 ft by 590 ft in plan, and originally were built behind the protection of a cofferdam made with part of the rock excavation (37).

Semicircular walls were built consisting of a series of cellular cofferdams, each 40 ft in diameter and 51 ft in average depth, connected by short arcs of

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9-ft radius (Figs. 27 and 28). Since the cofferdams rest on a rock foundation, a fill approximately 10 ft deep was placed before driving sheet piles (Fig. 29).

Studies and tests were made to determine:

(*a*) The relation between the overturning moment and the weight of the fill for different ratios H/r;

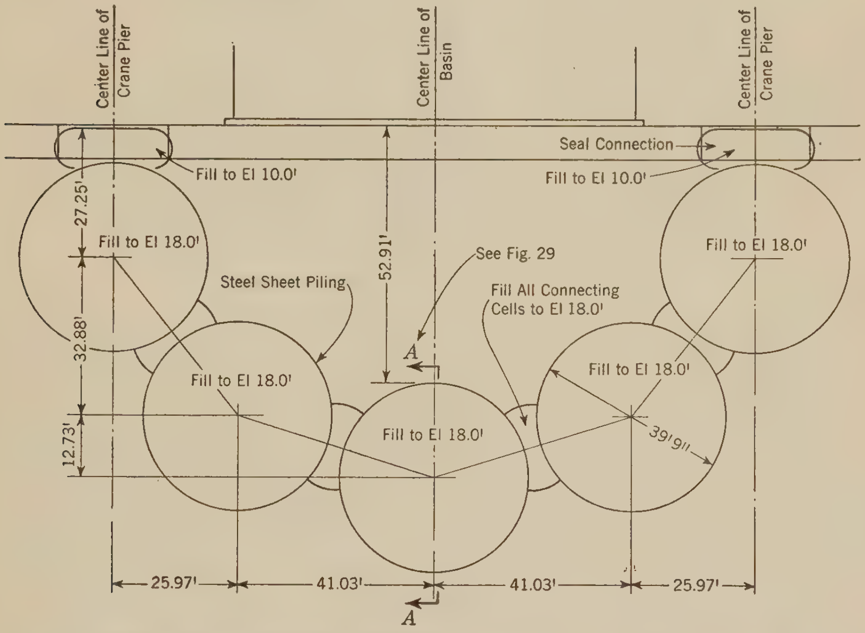

Center Line of Crane Pier
Center Line of Basin
Center Line of Crane Pier
Seal Connection
Fill to El 10.0'
Fill to El 10.0'
27.25'
Fill to El 18.0'
Fill to El 18.0'
Steel Sheet Piling
52.91'
See Fig. 29
Fill All Connecting Cells to El 18.0'
32.88'
A
Fill to El 18.0'
Fill to El 18.0'
12.73'
Fill to El 18.0'
39'9''
25.97'
41.03'
41.03'
25.97'
A

FIG. 27.—PLAN OF CELLULAR COFFERDAM, KAISER SHIPYARD, RICHMOND, CALIF.

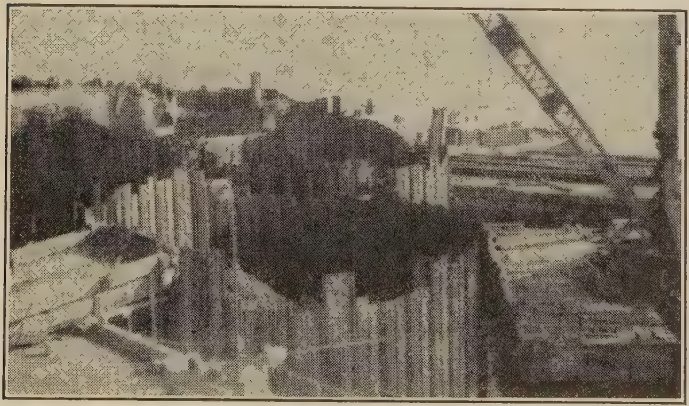


FIG. 28.—VIEW OF CELLULAR COFFERDAM

(*b*) The effect of the arch-web sheet piles on the safety of a cellular cofferdam; and

(*c*) The effect of the arch-shaped wall of the cofferdam cells upon the stability.

(a) Relation Between the Overturning Moment and the Weight of the Fill for Different Ratios H/r.—The results of the investigations under item (*a*) are in good agreement with the author's shear theory of the cofferdam presented in this paper. Fig. 30 shows some of the test installations. The standard formula for the pressure on a solid retaining wall, assuming that the full weight of the fill is acting, cannot be applied to a cofferdam consisting of a steel shell and containing more or less cohesive, plastic fill, with a variable degree of water saturation. This fact can be demonstrated by simple and inexpensive tests on small specimens made of thin sheet metal, thus simulating the cofferdam by a continuous smooth shell without irregularities of interlock. The minimum interlock strength of the sheet piles in the prototype is from 8,000 lb per lin in. to 12,000 lb per lin in. which, in the case of a $\frac{3}{8}$-in. web, is equal to from 21,333 lb per sq in. to 32,000 lb per sq in. In practical cases the stress ordinarily does not reach 50% of the latter values so that the effect of the interlock can be neglected. In the writer's investigations, the specimens simulating the cofferdams were made of plain sheet metal, 0.004 in. thick (see Fig. 30).

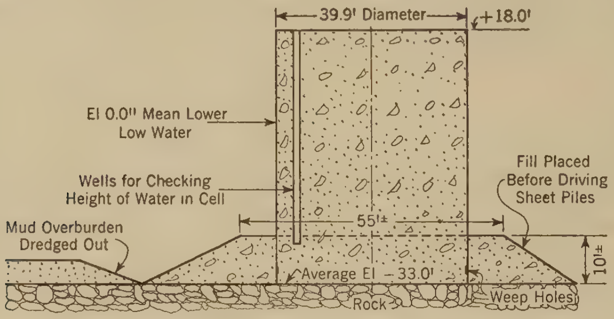

39.9' Diameter
+18.0'
El 0.0'' Mean Lower Low Water
Wells for Checking Height of Water in Cell
Mud Overburden Dredged Out
55'±
Fill Placed Before Driving Sheet Piles
10'±
Average El −33.0'
Rock
Weep Holes

Fig. 29.—Section A-A of Cellular Cofferdam (See Fig. 27)

TABLE 2.—Relation Between the Overturning Moment and the Weight of the Fill for Different Ratios *H/r*

Specimen No.	Radius *r* (in.)	Ratio *H/r*	Weight *W* (lb)	Overturning moment *M* (lb-in.)	*M/W*	Overturning moment[a]	*V* (lb)
1	3.5	1.72	12.0	17.7	1.5	17.5	3.2
2	3.0	2.00	8.8	12.5	1.4	12.6	2.7
3	2.5	2.40	6.1	8.1	1.3	8.2	2.1
4	2.0	3.00	3.9	5.0	1.3	4.9	1.5
5	1.5	4.00	2.2	2.6	1.2	2.6	1.1
6[b]	1.5	4.00	2.4	3.1	1.3	3.2	1.4

[a] Computed by Eqs. 5 and 6. [b] Special shape; see text.

With height (*H*) equal to 6 in. and with radii varying from 1.5 in. to 3.5 in., most of the ratio values of *H/r* occurring in practice were represented. The greater stiffness of the shell can possibly affect the result of test to a certain extent. A very fine sand was used as fill, for which, approximately, $\phi = 26.5°$ and $\gamma = 90$ lb per cu ft.

Table 2 shows the values of the weight and overturning moment of individual specimens as determined by measurements. It also shows the overturning moments computed from the total average shearing resistance exerted by the fill on the shell.

The shear resistance against overturning can be computed from the frictional resistance obtained by measuring the vertical force, V, necessary to overcome the friction between the fill and the inner surface of the cylindrical shell. The shear or sliding force acting upon the inner surface of the cylindrical shell can also be computed on the basis of a theory of arching as presented by the author in 1943 (17*a*). Applying this theory (which was originally developed for the yielding of a rectangular base) to a segment of a circle, representing the base of a circular cofferdam, the vertical stress σ_v on a horizontal section at the depth z below the surface of the fill is

$$\sigma_v = \frac{\frac{r}{2}\left(\gamma - \frac{2\,c}{r}\right)}{K \tan \phi}\,(1 - e^{-n}) + q\,e^{-n} \ldots\ldots\ldots\ldots\ldots (44)$$

in which

$$n = \frac{z}{r} K \tan \phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45)$$

and, in addition to the notation of the paper: K is a coefficient $= \sigma_h/\sigma_v = 1$; q is the additional unit load upon the fill; e is the base of Napierian logarithms; and c is cohesion in Coulomb's equation. The horizontal pressure exerted by the fill upon the unit length of the cofferdam is

$$\sum\nolimits_h = \int_0^H \sigma_h \, dz \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (46)$$

and the force, v, which it is necessary to apply in a vertical direction to overcome the friction between the fill and the inner surface of the circular cofferdam, per unit length of the circumference, is

$$v = \sum\nolimits_h \times \tan \delta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (47)$$

in which δ is the angle of the wall friction. The total force V is then

$$V = 2\,\pi\,r\,v \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (48)$$

It has been found that the vertical force V computed from Eqs. 44, 46, and 47 agrees with the results of tests. For sand as fill material, the cohesion, c, is equal to zero, and Eq. 46 has the simplified form

$$\sum\nolimits_h = \sum\nolimits_v = r\,\gamma \int (1 - e^{-z/(2r)})\, dz \approx r\,\gamma\,A \ldots\ldots\ldots\ldots (49)$$

assuming that $K \tan \phi = \frac{1}{2}$.

The term A can be computed by Simpson's rule or by substituting the irregular area with the base $b = (1 - e^{H/(2r)})$ and the height H by a triangle and a parabolic segment:

$$A = \frac{b\,H}{2} + \frac{2}{3} H\,b_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (50)$$

in which $b_0 = b' - \frac{b}{2}$, the dimension b' being the value of b for $z = \frac{H}{2}$.

This method of determining the value of V will be demonstrated for a case in which the ratio H/r is equal to 1.72 (Table 2). Assuming further that the fill material is cohesionless ($c = 0$), the angle of internal friction $\phi = 26.5°$ ($\tan \phi = 0.5$), the wall friction (friction between the fill and the sheet piles of the cofferdam) $\tan \delta = 0.4$, the unit weight of the fill $\gamma = 0.052$ lb per cu in., and no additional load upon the fill ($q = 0$), the following values are obtained: $b = 0.57876$; $b' = 0.34929$; and $b_0 = 0.34929 - 0.28938 = 0.05991$. The term $A = \frac{5.665 \times 0.1 \times 6}{1.72} = 1.976$, or, from the simplified Eq. 50, $A = 0.57876 \times \frac{6}{2} + 4 \times 0.05991 = 1.976$. The exponential values can be arranged as shown in Table 3.

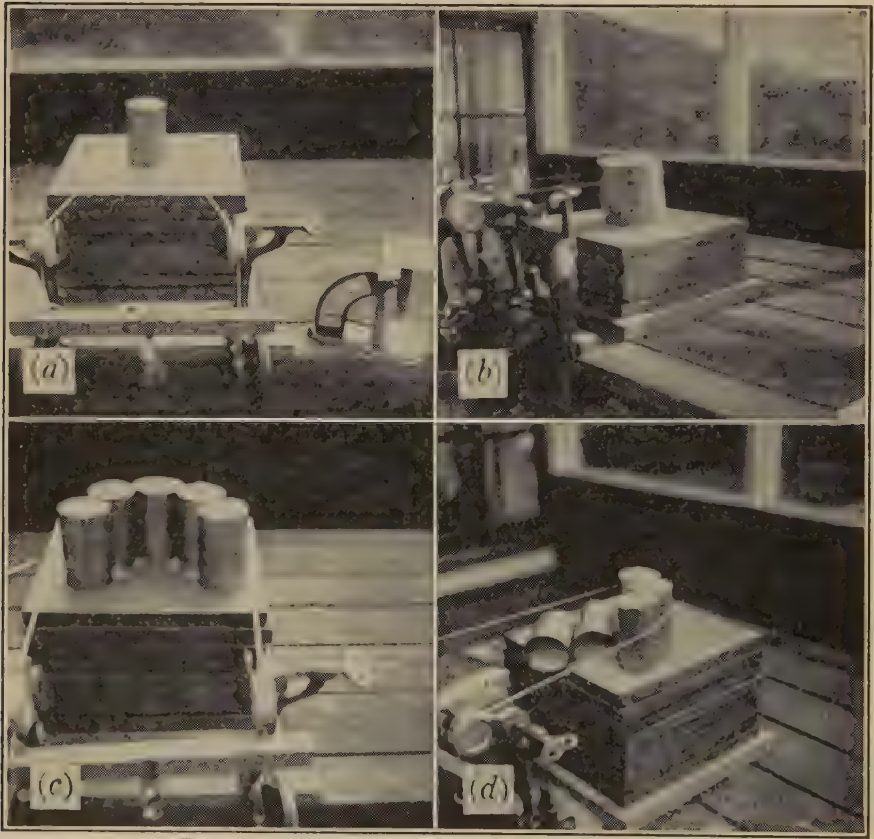


Fig. 30.—Laboratory Devices for Studying Cellular Cofferdams

The total horizontal pressure of the fill on the linear unit of the circumference of the cofferdam, by Eq. 49, is $\Sigma_h = 3.5 \times 0.052 \times 1.976 = 0.3596$ lb; the elemental vertical force (Eq. 47) is $v = 0.3596 \times 0.4 = 0.1438$ lb; and the total vertical force is $V = 2 \times 3.1416 \times 3.5 \times 0.1438 = 3.16$ lb. The measured value was 3.2 lb, so that the agreement with the test is good. The values of V for other specimens were computed similarly (see Table 2).

TABLE 3.—Computations for Arching Effect in the Fill

z/r	n (Eq. 45)	e^{-n}	$e^{-n} z$	$1 - e^{-n}$	$(1 - e^{-n}) z$	A (Eq. 50)
tan ϕ	0.5	0.95123	0.475	0.04877	0.025	....
0.1	0.05	....	....	....	....	....
0.5	0.25	0.77880	3.925	0.22120	0.575	....
1.0	0.50	0.60653	7.371	0.39347	2.129	2.111
1.5	0.75	0.47237	10.053	0.52763	4.447	....
2.0	1.00	0.36788	12.143	0.63212	7.357	7.353
2.5	1.25	0.28650	13.771	0.71350	10.729	....
3.0	1.50	0.22313	15.039	0.77687	14.461	14.437
3.5	1.75	0.17377	16.026	0.82623	18.474	....
4.0	2.00	0.13534	16.795	0.86466	22.705	22.521

The small differences between the computed and measured values may be explained as follows:

(1) Inaccuracy in the measuring devices;

(2) A slightly higher value of the coefficient of friction between the fill and the wall ($\tan \delta = 0.41 - 0.43$) as introduced in the analysis (0.4);

(3) Variable value of the coefficient K, assumed in the analysis to be equal to 1, but increasing to $K = 1.5$ for higher elevations (38).

If an external couple created by external hydrostatic pressure tends to overturn the circular cell, this external couple will be resisted by the skin friction (friction between the fill and the shell). At the critical instant the internal couple of the force V in the formula

$$M_i = V d_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (51)$$

will balance the external couple; that is, $M_e = M_i$. Thus the overturning moment of a cellular cofferdam can be determined readily if the values V and d_0 are known. The value V is given by Eqs. 47, 48, and 49, and the lever arm d_0 of the internal forces V can be determined under the assumption that the intensity of the friction between fill and shell follows a straight line, as shown in Fig. 31. In that case,

$$d_0 = r \frac{\int_0^{\pi} \sin^2 \phi \, d\phi}{\int_0^{\pi} \sin \phi \, d\phi} = \frac{\pi r}{2} = 1.57 \, r \ldots\ldots\ldots\ldots\ldots (52)$$

The values of the overturning moment computed by Eqs. 51 and 52 differ only slightly from the measured values (see Table 2). Table 2 contains also a specimen with folded shell (Fig. 32) which was included in the tests in order to determine the effect of the increased surface (circumference) of the shell upon the overturning moment. Such a case occurs when sheet piles with arched webs are used. The base of specimen No. 6, Table 2, was 7.6 sq in., and its circumference was 10.5 in.—12.5% greater than that of specimen No. 5.

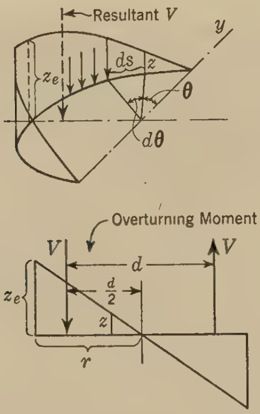


FIG. 31

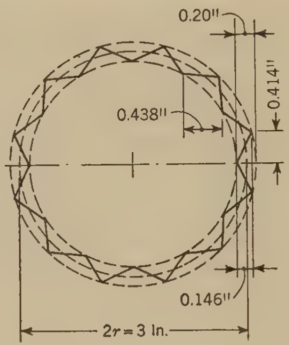


FIG. 32.—FOLDED SHELL TYPE OF COFFERDAM

It might be of interest to compare the values of overturning moment which resulted from the tests with those obtained by applying Eq. 17 to a continuous shell. Such a continuous shell simulates a sheet-pile wall without any slippage in the interlock joints. The use of Eq. 17 for this special case may be questioned, because it is necessary to substitute $f = 1$ in Eq. 17 in order to express the condition of unyielding interlocks of the cofferdam. Eq. 17 becomes then

$$S = \tfrac{1}{2} \gamma \, C \, H^2 (\tan \phi + 1) \ldots\ldots\ldots\ldots (53)$$

In accordance with Eq. 21, the coefficient of earth pressure, C, is greater than $\tan^2 (45° - \phi/2)$. Substituting the values of the tested specimens: $\gamma = 0.052$ lb per cu in., $\phi = 26.5°$, $H = 6$ in., and assuming $C > 0.383 = 0.4$, the average shearing resistance becomes $S = \frac{1}{2} \times 0.052 \times 0.4 \times 6^2 \times 1.5 = 0.562$ lb per inch of width of the neutral plane. According to the author, the critical overturning moment may be expressed as an internal couple of the shearing resistance S; that is:

$$M_e = S \, (2\, r) \, d'_0 \ldots\ldots\ldots\ldots (54)$$

The lever arm d'_0 will be smaller than $\frac{2}{3}\, b$ (Figs. 7(*b*) and 7(*d*)). Assuming that $d'_0 = \frac{4\, r}{\pi}$ (which is the distance of centroids of two semicircles), the overturning moment equals

$$M_e = \frac{8}{\pi} S \, r^2 = 1.43 \, r^2 \ldots\ldots\ldots\ldots (55)$$

Table 4 shows the values of overturning moment computed from Eq. 55 as compared with test results and values obtained from Eqs. 51 and 52. A better agreement between the measured overturning moments and their values calculated from Eq. 17 (Col. 3, Table 4) would be obtained by a variable coefficient of earth pressure, C, decreasing with an increasing ratio H/r, as shown in Col. 4, Table 4.

(*b*) *The Effect of Arch-Web Sheet Piles Upon the Safety of a Cellular Cofferdam.*—It should be emphasized that all investigations by the author regarding the shearing resistance on vertical sections through the fill in the cells and the lock friction are valid only for cellular cofferdams with straight-web sheet piles. If arch-web sheet piles are used, the total tension in a lock, expressed by Eq. 14*a*, will produce considerable deformation in the cofferdam shell. In the lower parts of the cofferdam the deformation may be beyond the elastic limit, resulting in an increase of 10 in. or more in the diameter of the cell. The bulge in such a case is quite noticeable. Any deformation of a structure beyond the elastic limit can be classified as failure, although several cofferdams of this type did not collapse, and served the purposes for which they were built. The total resistance against slippage in the lock will increase with bulging of the shell, since the total resistance against slippage increases with the increasing radius, as expressed by Eq. 14*b*.

TABLE 4.—COMPARISON OF OVERTURNING MOMENTS

Specimen No.	OVERTURNING MOMENT IN INCH-POUNDS, OBTAINED FROM:			C
	Tests (1)	Writer (2)	Author (3)	(4)
1	17.7	17.5	17.5	0.40
2	12.5	12.6	12.9	0.39
3	8.1	8.2	8.9	0.37
4	5.0	4.9	5.7	0.35
5	2.6	2.6	3.2	0.33

In the case of the cofferdam in the Kaiser Shipyards, the maximum tension in the locks, computed by Eq. 13, is $t_{\max} = r\,\gamma\,C\,H = 20 \times 110 \times 0.5 \times 41 = 45{,}100$ lb, or in sheet piles with $\frac{3}{8}$-in. web: $\frac{45{,}100}{0.375 \times 12} = 10{,}020$ lb per sq in. If arch-web sheet piles were used (for example, M112), the eccentricity of the interlock will produce bending of the sheet piles in a horizontal direction. The additional stress computed from the standard formula is so high $\left(\frac{45{,}100\,(1\frac{11}{32} - \frac{3}{16})}{0.28} = 185{,}531 \text{ lb per sq in.}\right)$ that the eccentricity ($1\frac{11}{32}$ in. $- \frac{3}{16}$ in.; see e in Fig. 33) will gradually decrease as the arch-web sheet piles stretch. In any case, the combined stress in the sheet piles will reach the yield point and cause permanent deformation as described previously.

The answer to the questions as to the approximateness of Eq. 13, and the nature of the two-dimensional stress conditions, can be attempted theoretically, considering the cellular cofferdam as a thin shell restrained at the bottom along its circumference. The results of any theoretical investigation of a problem, in which so many uncertain and indeterminate values are involved, must be approached with extreme caution. The analytical solution indicates deformation and stress distribution very similar to observations on actual cofferdams.

The horizontal force $P_s = Q_0$ (see Fig. 8) which is applied at the lower edge of the sheet piles in order to maintain equilibrium, and the restraining moment M_0 may be calculated (39) from the equations:

$$Q_0 = -\frac{\gamma\, r\, H\, h_2}{\sqrt{12\,(1-\nu^2)}}\left(2\,\beta - \frac{1}{H}\right) \qquad (56a)$$

and

$$M_0 = \frac{\gamma\, r\, H\, h_2}{\sqrt{12\,(1-\nu^2)}}\left(1 - \frac{1}{\beta\, H}\right) \qquad (56b)$$

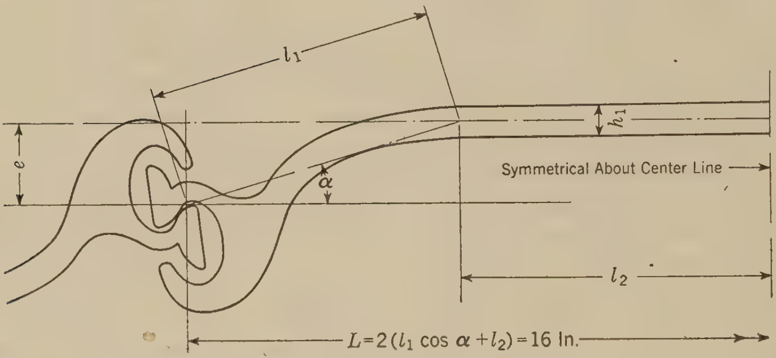

l_1
h_1
e
α
Symmetrical About Center Line
l_2
$L=2(l_1\cos\alpha+l_2)=16$ In.

FIG. 33

in which h_2 is a "theoretical thickness" of sheet piles derived from the section modulus S; thus,

$$h^2_2 = \sqrt{\frac{6\,S}{12}} = 0.7071\,\sqrt{S} \qquad (57)$$

In Eqs. 56, furthermore, ν is Poisson's ratio for steel $= 0.25$; and β is computed from

$$\beta^4 = \frac{3\,(1-\nu^2)\,h_1}{r^2\,h^3_2} \qquad (58)$$

In Eq. 58, h_1 is the actual web thickness of sheet piles. The values $N_{z'}$ and $M_{z'}$ for any depth $z' = H - z$ are expressed (39) by:

$$N_{z'} = -E\,\frac{h_1\,w}{r} = \gamma\, r\, H\left[1 - \frac{z'}{H} - \theta\,(\beta\, z') - \left(1 - \frac{1}{\beta\, H}\right)\zeta\,(\beta\, z')\right] \qquad (59a)$$

and

$$M_{z'} = -D\,\frac{d^2w}{dz^2} = \frac{\gamma\, r\, H\, h_2}{\sqrt{12\,(1-\nu^2)}}\left[-\zeta\,(\beta\, z') + \left(1 - \frac{1}{\beta\, H}\right)\theta\,(\beta\, z')\right] \qquad (59b)$$

in which

$$\theta\,(\beta\, z') = e^{-\beta z'}\cos\beta\, z' \qquad (60a)$$

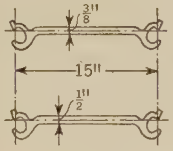
and

$$\zeta\,(\beta\, z') = e^{-\beta z'}\sin\beta\, z' \qquad (60b)$$

The preceding values can be applied only for straight-web sections (see Table 5).

For arch-web sections, such as Carnegie Steel sections M112 and M113, or Inland sections (see Fig. 33) a modification of the formulas is necessary, expressing greater elastic displacements in circumferential direction due to bending of sheet piles. This effect can be attained theoretically by introducing an imaginary modulus of elasticity, E_1, for calculating elastic elongations in a circumferential direction, which is considerably smaller than the modulus of elasticity, E_2, expressing the stress-strain ratio in vertical direction of the sheet pile.

TABLE 5.—CHARACTERISTICS OF STRAIGHT-WEB SHEET PILES

No.	Section	Cross-section area A (sq in.)[a]	Section modulus S (in.^{3})[a]	Weight (lb)[a]	h_1 (in.)	h_2 (in.)	β (in.$^{-1}$)
A		9.1	3.0	31	0.375	1.225	0.0563
B		10.3	3.1	35	0.5	1.245	0.0598

[a] Per linear foot of wall.

The ratio $\frac{E_1}{E_2}$ $\left(\text{equal to } \frac{1}{100} \text{ approximately for section M112}\right)$ can be calculated under the assumption that the elastic displacement (expansion) of an arch-web sheet pile considered as an elastic bent is equal to the elongation of an imaginary straight-web sheet pile having a modulus of elasticity, E_1.

The computations confirm the fact that arch-web sheet piles must be avoided for cofferdams having a lateral (horizontal) tension (from two-dimentional stress condition). For cellular cofferdams only straight-web sheet piles should be used (see Table 5).

In all literature pertaining to cellular cofferdams to which the writer has had access, there was no statement to the effect that, for cellular cofferdams where a circumferential tension can occur, it is inadvisable and even dangerous to use arch-web sheet piles. Examination of the literature has shown that in very many cases the use of the arch-web sheet piles resulted in bulging or failure.

KARL TERZAGHI,[16] M. AM. SOC. C. E.—The discussion of this paper has been eminently constructive; any one who faces the problem of designing a cellular cofferdam will find in the discussions an inexhaustible source of information and guidance. Therefore, the profession as a whole is indebted to the discussers for the generosity with which they contributed from the store of their experience.

One of the principal theses of the paper is that the customary stability computations of cellular cofferdams based on the gravity concept need to be supplemented by an inquiry into the stress conditions in the fill constituting the body of the cofferdam. This conclusion was forced upon the writer by the results of observations made on a large cofferdam during 1941 and 1942. According to the gravity concept, the dam should have been stable; yet the observations left no doubt that the dam would be unable to stand the unbalanced horizontal pressures unless the shearing resistance of the earth in the cells was increased by artificial methods. These methods were adopted and their effect actually demonstrated the efficacy of the procedure.

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[16] Cons. Engr. and Lecturer, Graduate School of Eng., Harvard Univ., Cambridge, Mass.

Some of the discussions revealed the fact that the engineers on the staff of the Tennessee Valley Authority (TVA) arrived at similar conclusions at an earlier date. The TVA engineers checked all their designs for what they called "internal shear," and for this purpose they used a method similar to the one proposed by the writer. This is more than a mere coincidence. It indicates that the time was ripe for this new development, and the writer acknowledges with pleasure that the credit for the corresponding modification of the design procedure belongs to the TVA engineers.

Mr. Greulich expresses the opinion that the inward deflection of a cellular cofferdam is associated with an important increase of the resistance against sliding in the interlocks of the cross-walls, equivalent to the introduction of stay rods from the inner top to the outer base of the cell. The lock friction as such, because of the hoop tension in the walls of the cells, has the mechanical effect of stay rods, because it counteracts the distortion of the cross-walls. However, it seems doubtful that the inward deflection of the cofferdam is associated with an increase of the lock friction. If such an increase occurs, it can scarcely be evaluated rationally.

Commenting on the depth to which the sheet piles for cofferdams on deep sand strata should be driven, Mr. Greulich contends that $H/2$ would be ample. His argument is that the sand in the cells can grip only one side of the sheet piles whereas when driven into the sand the piling is gripped on both sides. This argument can be accepted for the outer row, but it does not apply to the inner row. On the inner side of the inner row the water percolates through the sand in a vertical upward direction (see Fig. 9(*c*)). This process reduces the skin friction very considerably. An extreme possibility for this reduction was demonstrated by the following incident. An excavation was made by dredging in dense sand between sheet piles for a small dry dock in the vicinity of Hamburg, Germany. Prior to pumping, 1,500 wood piles were driven between 12 ft and 15 ft into the sand. No deeper penetration by driving could be obtained without breaking the piles. The pile cutoff was a short distance above grade. When the water level was lowered within the sheet-pile enclosure by about 8 ft, the piles began to rise one by one; yet the seepage pressure was not important enough to cause the sand to boil. It merely reduced the total skin friction of the pile to less than the hydrostatic uplift.

The total seepage pressure which acts on the sand adjoining the inner side of the inner row of sheet piles depends on details of stratification which are unknown. Hence, in some instances, a penetration of $H/2$ may be more than ample and in others not enough.

At the end of his discussion, Mr. Greulich requests an amplification of conclusion 2 of the paper and he also comments on conclusion 5. Conclusion 2 refers to the distribution of the soil pressure over the base of cellular cofferdams with conventional dimensions. According to the results of the analysis given in the paper, the application of an unbalanced horizontal force on such a cofferdam has very little, if any, effect on the distribution of the pressure on the base. In other words, the pressure distribution remains practically uniform. The entire overturning moment is transferred through skin friction on to that part of the sheet piles which is located below the base of the cofferdam. In conclusion 5 the writer implied that the ratio between the horizontal and the vertical pressure in the fill in the cells is greater than the Rankine value. Hence, the Rankine value is not on the conservative side as suggested by Mr. Greulich.

In several paragraphs of his discussion Colonel Pennoyer expresses the opinion that the paper involves an attempt to supersede the design method based on the gravity concept by proposing a standard ratio between width and height. Thus, he states:

> "* * * the writer feels that Professor Terzaghi has oversimplified the problem by assuming a cell width of 0.85 H and a conversion factor of 0.4 to develop the bursting pressures."

This, and many other similar passages in Colonel Pennoyer's discussion, indicate that the paper was not explicit enough to eliminate the possibility of misunderstandings in the minds of at least some readers. To clarify the issue, the following synopsis of the basic reasoning is submitted.

The stability and stiffness of cellular cofferdams depend on a great number of factors including the foundation conditions and the shearing resistance of the fill occupying the cells. Even under the simplest conditions, involving a cofferdam filled with sand and gravel resting on a solid foundation, theory alone cannot be relied upon because one of the most important factors—the ratio between horizontal and vertical pressure in the soil which occupies the cells—cannot be computed. This fact has been known ever since the first measurement of the side pressure in grain bins was made. Therefore, the designer depends to a certain extent on data derived from experience.

During the twenty years since 1925 a great number of cellular cofferdams have been built which have the following features in common: The sheet piles are driven through sand to bedrock; the cells are filled with sand or sand and gravel; the cofferdams are acted upon by one-sided water pressure; and the average (not the maximum) width of the cofferdams is roughly equal to 0.85 H. Since all these cofferdams gave satisfactory service, they constitute useful precedents and prototypes. To apply the experiences incorporated in the prototype to the design of cellular cofferdams to be built under different conditions of foundation and loading, the writer investigated the influence of several commonly encountered deviations from the prototype on stiffness and stability. If these influences are clearly recognized, the dimensions and proportions of the prototype can be changed in accordance with the findings. No claim is made that all the conceivable deviations from the prototype are covered, because such an attempt would obviously transcend the scope of a single paper. For the same reason, the discussions are limited strictly to cellular cofferdams filled with sand. As a matter of fact, cellular cofferdams filled with other soils are rather exceptional, as demonstrated by the construction record of the Dravo Corporation and the TVA.

Cellular cofferdams are commonly considered major structures and the uncertainties involved in their design are generally recognized. Hence, it goes without saying that no responsible engineer will consider the design of such cofferdams a routine problem under any except very simple conditions. Every scrap of available information should be utilized; and every known theoretical approach, including the stability computation based on the gravity concept, should be made before final decisions are reached. The design of the Kentucky cofferdam described by Messrs. Tsagaris, Colburn, and Hedman is an outstanding example. If two different approaches lead to different results, the more unfavorable one will be accepted.

Colonel Pennoyer comments on the writer's assumption that the soil fails along a vertical plane and calls the assumption fallacious. He further states:

> "What actually happens is that the entire mass of confined soil changes shape simultaneously. This action can best be described as a universal plastic flow, or distortion * * *."

The writer's concept of the process of failure is essentially identical with that described by Colonel Pennoyer, and the equations which were derived by the writer are merely mathematical expressions of the stress conditions required for producing this type of failure. Since the original explanation of the physical meaning of the equations was apparently not explicit enough to avoid a misunderstanding, the following discussion is presented in the hope of clarifying the issue. Fig. 34 is a vertical section through a mass of soil confined between two straight rows of sheet piles, not interconnected by cross-walls. The soil is assumed to adhere to the piles, and the resistance of the sheet piles against bending at ground level in the plane of the drawing is assumed negligible. As soon as a horizontal force P is applied at an elevation h above the ground surface, the outer row of sheet piles is pulled by a force $-S$, and the inner row is pushed down by the same force S. The force S is equal to the overturning moment $M = P\,h$ divided by the distance b between the two rows. The horizontal force P produces a change of shape as described by Colonel Pennoyer. As soon as the horizontal force exceeds a certain critical value, the distortion increases at a constant rate and the cofferdam fails by tilting forward more and more—whereby the cross section of the dam changes from a rectangle into a parallelogram with equal area. Since there are no cross-walls, the failure by plastic flow starts simultaneously at every point in the confined mass of soil. The failure occurs by shear along an infinite number of vertical planes. According to the theory of plastic materials, failure occurs

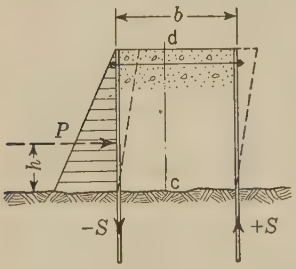


Fig. 34

as soon as the total shearing force on any vertical section such as cd becomes equal to the shearing resistance on this plane; and, according to the laws of mechanics, the shearing force is equal to S.

If the dam shown in Fig. 34 is reinforced by cross-walls consisting of sheet piles, the presence of these walls complicates the process of failure to some extent. Instead of starting at every point at the same time, the plastic-flow spreads from the neutral plane toward the walls. However, the laws that determine the conditions of failure are identical with those cited previously.

Colonel Pennoyer maintains that, if the coefficient of lock friction of 0.3

> "* * * was assumed analogous to the coefficient of friction of steel on steel, the statement would seem fallacious. Interlocks of sheet piling are designed * * * with emphasis on reducing the area of contact between surfaces to a minimum * * *. There is no relation between the frictional value of two pieces of steel in what amounts to line contact and the coefficient of friction of steel on steel times a tension force per unit of length, when no area of contact can be assumed."

The value of 0.3 assumed by the writer really is nothing more than the coefficient of friction of steel on steel. According to sources available to the writer (40), the coefficient of friction for metal on metal, dry, ranges from 0.15 to 0.20 and for metal on metal, wet, it is 0.30. To these values may be added the results of some tests performed by the writer who found that, for steel on smooth glass, dry, f equals from 0.20 to 0.22 and that, for steel on smooth glass, wet, f equals from 0.32 to 0.47 (16). In the walls of a cofferdam, the locks are wet or moist. Tests have shown that a trace of moisture is sufficient to produce the full "antilubricating" effect of water. Therefore, it seemed justified to assume $f = 0.3$.

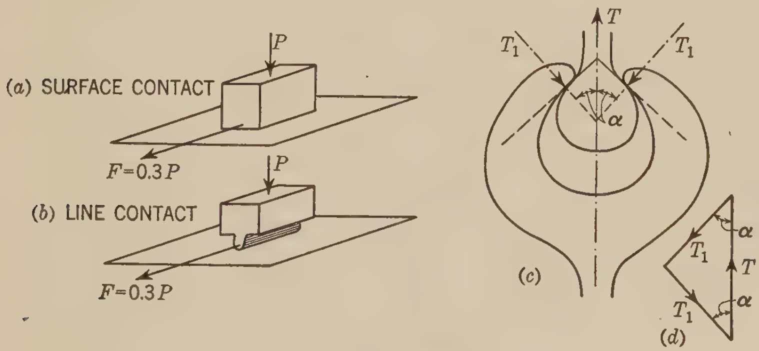


FIG. 35

Figs. 35(a) and 35(b) represent two different test arrangements for making a friction test. One involves a surface contact and the other what appears to the unaided eye as a line contact. Experience has shown that both test arrangements lead to the same result. Even if the two surfaces touch each other at what appears to be only a point, the result is no different. In other words, the coefficient of friction, $f = F/P$, is independent of the area of contact. This is quite generally regarded as one of the fundamental laws of physics.

Fig. 35(*c*) illustrates the influence of the shape of the lock on lock friction. It represents a section through a fictitious lock. If T is the lock tension per foot of length, the pressure on the two contact lines combined is equal to $\frac{T}{\cos \alpha}$, and the lock friction is $F = \frac{0.3\,T}{\cos \alpha}$ per foot of length. For $\alpha = 45°$, $F = 0.42\,T$. If $\alpha = 0°$, the lock friction assumes the smallest value that can be obtained without the aid of a lubricant—that is, $0.3\,T$. There seems to be no fallacy involved. The computation of the lock friction for the Kentucky cofferdam was made on the basis of a coefficient of friction of 0.3. Considering the customary shape of the locks, the value of 0.3 seems to be conservative.

Colonel Pennoyer describes an arching phenomenon which he observed in 1932 on a cellular cofferdam of the diaphragm type. After the sand was washed out under the fill of the cell, the fill was found to arch over the cavity. This observation led Colonel Pennoyer to conclude that arching may quite generally lead to the formation of a "bottom," or of "bottoms," in the cell, and he discusses the mechanical implications of the phenomena. Since the writer has noticed similar phenomena in grain and coal bins, he was interested in the mechanical prerequisites for such arch formation. Therefore, in 1918, he made several experiments involving the pressure conditions in bins filled with sand (41). The bottom of the bin was mounted on a platform scale. The side pressure of the sand on the walls of the bin was measured by horizontal friction tapes installed at different depths below the top. The tests were made with both dense and loose sand. The bin was filled while the bottom was held in its original position; and, after the bin was filled, both side and bottom pressures were measured. The next step was to lower the bottom of the bin by increments and to observe the effect of this operation on the pressures. The test results are illustrated by Fig. 36. The abscissas of the curve C_1 represent the unit pressure on the side-walls at different depths below the top of the bin. The unit pressure increased steadily from the top down. As soon as the bottom of the bin was allowed to descend, the pressure on the bottom and on the lower part of the side-walls decreased, whereas the pressure on the upper part of the side-walls remained unchanged. Finally, the pressure distribution on the side-walls became that shown by curve C_2. At this stage a slight amount of cohesion would have sufficed to keep the sand in the bin without any bottom support. This was the action that occurred in the cofferdam cells inspected by Colonel Pennoyer. The yield of the bin bottom in the experiment was produced in nature by the escape of the sand which originally occupied the space beneath the fill. However, under normal conditions the base of the fill remains intact; and, as a consequence, the distribution of the pressure on the walls of the cells is shown in Fig. 36 by curve C_1 and not by curve C_2.

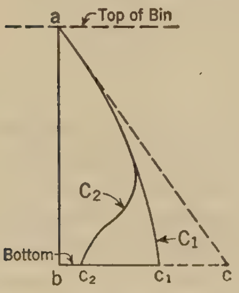


Fig. 36

Colonel Pennoyer remarks that he will continue to use the Rankine conversion factor until something better is available. Since the cells of a cellular cofferdam are essentially bins, the following data may be of interest. In 1920 the writer determined, by small-scale experiments, the ratio between the horizontal pressure in dense sand and the corresponding vertical pressure. In these tests the friction tape method was used and a value of 0.42 was obtained. The Rankine conversion factor for this sand was 0.22 (16). In 1929 direct measurement was made of the pressure exerted by loose sand on to the walls of a prismatic reinforced concrete container 7 ft high with a base 14 ft by 14 ft. The results of the observations furnished a value of 0.405 against a Rankine conversion factor of 0.31 (25) for the ratio between horizontal and vertical pressure. The lateral pressure exerted by completely submerged glacial till with an effective grain size of 0.019 mm was found equal to the sum of the full hydrostatic pressure of the water and the lateral pressure of the fill, computed on the basis of the submerged unit weight and a conversion factor of 0.38. The Rankine conversion factor for this material, computed on the basis of the results of friction tests, was 0.22 (42). These test results suggest that the value of 0.4 proposed by the writer may be much closer to the real ratio between the horizontal and the vertical pressure in sand enclosed in the cells of a cellular cofferdam than the Rankine conversion factor.

In discussing cofferdams on deep sand strata, Colonel Pennoyer refers to his own paper published in 1935 (29). Since this paper had escaped the writer's attention, he appreciated the reference and studied the paper with interest. The paper contains empirical values for the required sheet-pile penetration, together with other information. For water-bearing sand, the value is given as 30% to 40% of the unbalanced hydrostatic head. The writer has observed that the values in Colonel Pennoyer's paper are quite generally accepted in practice. However, he also noticed that blowouts are quite common, which induced him, about 1920, to make an inquiry into the causes.

The first step was to determine the hydraulic head required to produce a blowout under a straight row of sheet piles embedded in perfectly homogeneous sand. Both theory and experiment showed that the blowout does not occur until the hydraulic head approaches a value somewhat less than three times the depth of sheet-pile penetration. The grain-size characteristics of the sand are irrelevant (16); yet several instances have come to the writer's attention in which the sand began to boil on the inner side of single sheet-pile cofferdams at a head which ranged between 1½ and 2½ times the depth of sheet-pile penetration. Lazarus White, M. Am. Soc. C. E. (6), even describes a serious blowout in an open cut near the intersection between Canal Street and West Street in New York, N. Y. In this case the depth of sheet-pile penetration was equal to the full depth of the cut.

When several records of this nature were assembled, the reasons for the apparent incompatibility between test results and experience became quite plain. A blowout occurs as soon as the upward "drag" exerted by the water percolating through the sand along the inner face of the buried part of the sheet piles becomes equal to the submerged weight of the sand through which it percolates. The tests were made with perfectly homogeneous sand and the theory was developed for similar materials. However, in nature, sand strata are never even approximately homogeneous. By theory it can be shown that the drag depends to a large extent on the variations of the permeability of the sand in both horizontal and vertical directions, other factors being equal. By properly changing the permeability pattern for a sand adjoining a row of sheet piles, one can regulate the critical head within very wide limits. Hence, in nature too, the hydraulic head required to produce a blowout at a given sheet-pile penetration may vary between wide limits, and the real value cannot be ascertained by any practicable procedure before the sheet piles are driven and an unbalanced head is established.

A second factor that needs to be considered is the effect the seepage pressure or upward "drag" has on the bearing capacity of the inner row of sheet piles and on the resistance of the sand against a lateral displacement of the buried, lower part of the piles. Colonel Pennoyer mentions an instance in which a single sheet pile, driven 25 ft into sand, sustained a load of 47 tons. The piles previously mentioned in connection with Mr. Greulich's discussion would probably have sustained a load of 80 or 100 tons each because they were driven into hard sand. However, while the water level within the cofferdam was being lowered 8 ft by pumping, the sand loosened its grip on the piles to such an extent that the piles floated out of the sand. Hence, when evaluating the bearing capacity of the sheet piles of the inner row, the reducing effect of the "drag" on the skin friction must be considered. The results of load tests can be very misleading.

The mechanical effects of the "drag" must also be considered in the computation of the passive resistance of the sand adjoining the inner face of the buried part of the inner row of sheet piles. Lowering the water level adjoining the inner face of the cofferdam reduces this resistance very considerably as illustrated by Fig. 37, representing two vertical sections through a single-wall cofferdam. In Fig. 37(*a*), the water level is at the same elevation on both sides of the sheet-pile enclosure. On this assumption, the passive resistance of the sand on the right-hand side of the buried part of the sheet piles can be computed by Rankine's equation:

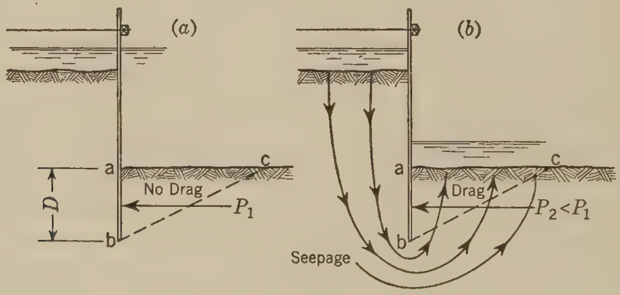


FIG. 37

$$P = \tfrac{1}{2} D \gamma' \tan^2 \left(45 + \frac{\phi}{2}\right) \qquad (61)$$

in which γ' is the submerged unit weight of the sand. If the water level within the cofferdam is lowered by pumping the water down to the level indicated in Fig. 37(*b*), the sand within the sliding wedge abc is acted upon not only by the simple hydrostatic uplift associated with submergence in still water but also by the seepage pressure or upward "drag" of the water which percolates through the wedge from below. This drag reduces the passive resistance of the sand considerably. The loss of resistance due to drag can be estimated by the flow-net method (17). In Colonel Pennoyer's computation of the passive resistance of the sand adjoining the inner face of a sheet-pile cofferdam (29), the drag has been disregarded. Therefore, the real passive resistance is smaller than the resistance indicated by Colonel Pennoyer's equations.

Considering all these facts and the inevitable uncertainties resulting from the unknown details of stratification of natural sand layers, the writer felt compelled to recommend a sheet-pile penetration of $2\,H/3$ instead of the customary $0.3\,H$ to $0.4\,H$.

In two places Colonel Pennoyer calls the reader's attention to various omissions in the writer's paragraphs on breakwaters. The design of cellular breakwaters has been treated so ably and comprehensibly in older publications that there was no incentive for discussing any aspect of this subject except the one that had not received any attention—the possibility of a base failure due to spontaneous liquefaction of underlying strata of loose sand. The writer's remarks on this subject were prompted by the spontaneous and complete subsidence of a breakwater on sand off the Atlantic coast of South America during a heavy storm. There are so many failures due to sponteneous liquefaction of masses of loose sand on record that this danger should not be ignored.

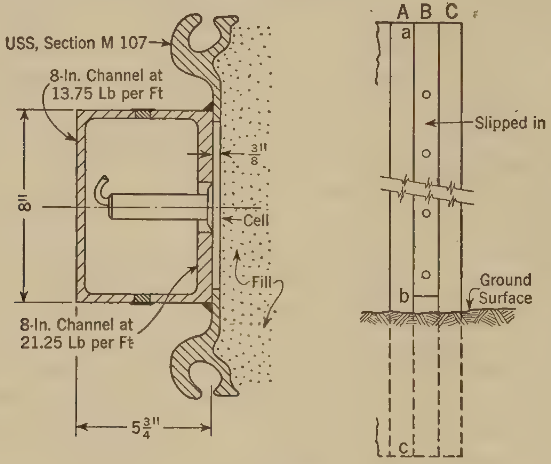
Mr. Tsagaris presents a brief review of the methods used in the design of the cellular cofferdams for the TVA. He points out that the possibility of a failure due to internal shear has received serious consideration, and comments on the writer's neglect of the tension contributed by the connecting arcs to the tension in the locks of the main cells. The reason for this omission will be given in the writer's comments on Mr. Hedman's discussion.

Mr. Colburn describes the Kentucky cofferdam, the highest cellular cofferdam which has ever been built. His account of the drainage provision on this and some of the other TVA cofferdams deserves special attention. An attempt to measure the tension in the sheet-pile enclosure of two of the cells of the Chickamauga cofferdam by strain gages was unsuccessful. The observations disclosed a very erratic distribution of the tensile stresses over the sheet piles.

More reliable information on the pressure acting on the walls of the cells can be obtained by pressure cells inserted into some of the sheet piles as shown in Fig. 38(*a*). A box composed of two channels is welded to a pile. The cells are $\frac{3}{8}$ in. thick and have a diameter of about 7 in. They fit into circular holes in the sheet piles and are seated on the outside of the web of the inner channel. The strain gage chambers project into the box and the cables are also installed in the box. The minimum requirement for obtaining fairly consistent results is the equipment of three sheet piles with five cells each.

These piles should be placed at the three-quarter points of the main arc on the inside of the cofferdam at the junction of the cofferdam with the connecting arcs.

Sheet piles to which cells are attached should not be driven because the impact of the hammer is likely to injure the cells. In order to incorporate a cell pile (unit B in Fig. 38(*b*)) into the sheet-pile enclosure of a cell, the pile is made of two pieces. First the lower part bc is driven to the required depth by means of a follower; then the next pile (unit C) is driven to full depth; the upper part of unit B is pulled; and finally the section containing the cells is slipped into place.



(*a*) (*b*)

FIG. 38

The cells register the sum of the earth and the water pressure which acts on the face of the cell. The water pressure alone is measured independently. If the fill consists of sand, reliable values can be obtained by means of observation wells with a small diameter.

Deflection observations on the Kentucky cofferdam showed that one of the cells deflected slightly outward with a rising river. This observation calls attention to the fact that the movement of every dam due to a change of the unbalanced head is equal to the difference between two movements in opposite directions. If the head increases, all the cells tilt inward with reference to their base on account of the increase in the overturning moment. At the same time, the base tilts outward due to the increase of the water load on the area outside the cofferdam. If the water is retained by a flexible cofferdam on a relatively unyielding foundation, the inward tilt prevails. On the other hand, a fairly rigid cofferdam on a yielding base must be expected to tilt outward. For similar reasons, the rise of the water level behind a relatively rigid storage dam on a less rigid foundation causes a tilt in an upstream direction. Such an upstream tilt was expected, and subsequently observed, on a concrete dam resting on a thick stratum of clay in northeastern Russia (43).

Before construction began, it was computed that filling the reservoir would cause a 0.75% tilt of the dam and powerhouse in an upstream direction. Since the turbine shafts had to be installed before the reservoir was filled, they were mounted with a 0.75% tilt toward the downstream side. While the reservoir was being filled for the first time, the shafts moved into an almost vertical position.

The Kentucky cofferdam rested on a limestone base. Hence, the outward tilt due to a rise of the river level undoubtedly was very small compared with the inward tilt of most of the cells; yet it might have been important enough to produce a measurable outward deflection of those cells which, for some reason, retained their original position with reference to the base.

Mr. Colburn expresses the opinion that the reactive pressure of the fill in the cells prevents a relaxation of the lock tension even though the bursting pressure may decrease. There is no doubt in the writer's mind that this assumption is justified.

Mr. Hedman gives a detailed account of the methods used in the design of the Kentucky cofferdam. He remarks that the writer's statement regarding the effect of seepage on the tension in the locks is contradictory to subsequent statements regarding the effect of seepage on sliding resistance. The apparent contradiction is due to the fact that the writer's first statement was inadequately qualified. Furthermore, Mr. Hedman does not agree "that the unit weight of the fill in a moist state should be used in Eq. 3." This objection is justified. The effect of the air content on the unit weight of a drained soil can be disregarded only if the effective grain size is equal to, or smaller than, about 0.05 mm.

Mr. Hedman presents a résumé of the procedure for determining the lock stresses:

> "This stress was found to be equal to 9,500 lb [per lin in.]—more than 50% higher than would have been the case if the author's suggested empirical formula, Eq. 28*f*, had been used."

The major part of the difference is due to the fact that the TVA computation was based on the assumption of complete saturation of the fill in the cells, whereas the writer's equation presupposes fairly complete drainage. The fill in the cells of most or of all the TVA cofferdams was deposited by the hydraulic method during a low-water stage. On this condition, Eq. 28*f* should be modified in such a way as to comply with the state of full saturation. The modified equation furnishes, for the interlock stress in the main cells, a value of about 8,120 lb per lin in. against the value of 7,280 lb per lin in. given by Mr. Hedman.

To the value of 7,280 lb per lin in. was added 2,220 lb per lin in. for the tension contributed by the connecting areas. This supplementary force was computed on the assumption that the sand pressure on the connecting arcs increased like a hydrostatic pressure in direct proportion to depth. However, considering the slenderness ratio of the columns of sand adjoining the connecting arches, the bin effect must be very important. In 1918 when the writer made his bin tests, he tried to destroy the bin effect by various means, such as pounding the sides of the container, but he did not succeed. He also poured the sand into water and afterward drained it through the perforated bottom of the bin; yet the bin effect persisted. Hence, it can be counted upon. If the bin action is taken into account, the contribution of the connecting arches to the tension in the main cells will probably be found to be rather unimportant and well within the range of error associated with the estimate of the bursting pressure exerted by the sand in the main cells. There is no doubt, however, that the junction between the main cells and the connecting arches is a delicate detail, and the attention which is received in the TVA design was fully justified.

Commenting on the effect of a berm on the lock tension in the cross-walls, Mr. Hedman points out correctly that the writer's statements concerning this effect require qualification. These statements are not valid unless the cells are filled while the water level in the river is fairly close to its highest position. The influence of the river stage when the cells are filled is strikingly demonstrated by the difference between the deflections of the Kentucky cofferdam which was filled at low water and those of the cofferdams on the west shore of the Hudson River in New York City which were filled while the water level was close to the high-water mark.

Considered as a whole, the procedure for designing the Kentucky cofferdam, as described by Mr. Hedman, is an outstanding example of sound engineering. This procedure discloses a thorough grasp of all the theoretical principles involved combined with mature judgment, and it deserves the attention of every engineer who faces the problem of designing a cellular cofferdam.

Mr. Ackerman presents a significant résumé of the practical considerations that enter into the selection of the type of cellular cofferdams under given conditions of foundation and exposure. Attention is called to his remarks on the benefits derived from the use of portable templates. The writer agrees with Mr. Ackerman that it might be preferable to refer to the cellular cofferdams as plastic and not as composite structures because the reactions of these cofferdams to external forces are decidedly those of plastic materials.

For comparison, Mr. Packshaw discusses the conditions required for the stability of double-wall sheet-pile cofferdams. The stability computation which he presents conveys a clear conception of the factors that determine the stability of these structures. Since, even in the United States, the double-wall type of cofferdam is far more frequently used than the cellular type, his contribution will be welcomed by many readers.

Mr. Lautz assumes that the excess of the normal pressure over the Rankine pressure on the vertical shear plane or planes is chiefly due to the horizontal load imposed upon the outside of the cofferdam. The bin tests which were described in the writer's comments on Colonel Pennoyer's discussion suggest that this excess is chiefly due to the fact that the fill is deposited within a laterally confined space. According to the results of bin tests, the normal pressure on every vertical section through the cells is much greater than the active Rankine pressure even before horizontal forces have been applied.

Example 1 in Mr. Lautz's discussion illustrates the application of the cellular cofferdam method to the construction of a dock-wall. The cells of this wall contain soft silt and clay in an undisturbed state. In this state clays and silty clays possess a remarkable degree of rigidity although their water content may be very high. Quite recently the writer examined a stratum of clay with a liquid limit of about 40% of the dry weight. The water content of the clay in its natural state was about 60%. Nevertheless, the clay contained hair cracks and was decidedly brittle. By kneading, it was transformed into a thick, viscous liquid. The rigidity of undisturbed clay, combined with an increase of the lock friction in the cross-walls and the low intensity of the unbalanced horizontal forces, accounts for the absence of noticeable deflections. The cells of the cofferdam for the intake system in Detroit, Mich. (Example 3), also chiefly contained soft clay in an undisturbed state. However, in contrast to the dock-wall, the cofferdam was acted upon by clay and water pressure combined. Hence, in spite of its ample dimensions, it required supplementary support.

Mr. Miller's discussion ranks among the most noteworthy contributions to this subject because it is based on extensive and varied experience in the design and construction of cellular cofferdams. Mr. Miller takes exception to the writer's statement concerning the influence of the outer row of sheet piles on the percolation of water through cells filled with clay or other soils with a low permeability. This influence has been the topic of several good-natured verbal controversies between Mr. Miller and the writer with the result that both parties remained utterly unconverted. The writer's arguments in favor of his thesis appear in "Part I. Practical Considerations" under the heading, "Permeability of Cellular Cofferdams and of Their Sheet-Pile Enclosures." Mr. Miller's confidence in the efficacy of weep holes is somewhat at variance with the results of the observations of the TVA engineers described in the discussions of Messrs. Tsagaris, Colburn, and Hedman.

Professor Legget comments on the writer's mildly skeptical remarks regarding the practical value of laboratory tests in connection with the design of cellular cofferdams. The principal benefit to be derived from these tests is that they give the engineer a firsthand knowledge of the character and the properties of the materials involved in the design. The benefits derived from such knowledge are important enough to justify a considerable amount of testing.

In the course of his discussion, Professor Legget requests an amplification of the writer's comparison of cellular cofferdams with structures made of composite materials. When making this comparison, the writer merely wished to emphasize that the distribution of the external forces acting on the two constituents of the dam—the steel and the soil—is governed by the stress-deformation characteristics of the two materials involved. The analogy cannot be extended further.

Commenting on lock friction, Professor Legget presents a list of all those variables which are likely to influence the friction value, and he raises the question as to whether or not reliable values could be obtained by experimental methods. In the writer's opinion, the lock friction can be resolved into two parts. One part is determined by the coefficient of friction of metal on metal and by the orientation of the surfaces of contact with reference to the direction of the pull. It has been referred to as the basic part of the friction. The second part is caused by the effect of one of several of the factors enumerated by Professor Legget. All these factors tend to increase the lock friction to some extent. Hence, the basic value determines the lower limit for the lock friction. The writer estimated that the basic value for the coefficient of lock friction is $\frac{0.3}{\cos \alpha}$ (see Fig. 35(*c*)). The data and considerations which led to this value are treated previously in the writer's comments on Colonel Pennoyer's discussion.

A more accurate basic value could be obtained by friction tests with specimens cut out of new steel sheet piles. Some information on the difference between the real and the basic values of the lock friction for the buried part of sheet-pile enclosures could be obtained by friction tests on specimens cut from sheet piles after they have been pulled. To preserve the state of contamination of the lock with soil particles, two sheet piles should be clamped together securely and pulled in a single operation. When cutting the test specimen from the pair, special precautions should be taken to prevent a slippage in the locks prior to the test.

Professor Legget also raises the question as to how much reliance can be placed on the lock friction in straight rows of sheet piles composed of deep arch sheet piling. In such rows, the tension in the locks is at least theoretically equal to zero. Hence, the basic value for the lock friction is also equal to zero and the resistance against slippage in the locks is entirely due to those factors which produce lock friction in excess of the basic value in the driven part of the sheet-pile enclosures of cellular cofferdams. Nevertheless, in Germany the manufacturers of deep arch sheet piling with locks in the neutral plane claimed that the weakening effects of the lock can be disregarded, and the moments of resistance contained in their catalogs were computed on the assumption that the locks do not exist. In design practice, it was customary to distinguish between the sheet piles driven into sand and those driven into clay. For sheet piles in sand, it was considered safe to ignore the presence of the locks, whereas the moment of resistance of sheet piles in clay was reduced by one third of the value obtained for jointless piling. The writer has never had much confidence in this procedure; yet it is conceivable that the mere process of driving sheet piles into the soil may create various sources of considerable friction. Hence, if in those parts of the steel skin of a cellular cofferdam which is located below the original ground surface or dredging line, the real lock friction may be greater than the basic value.

Professor Krynine furnishes convincing proof that shear failure on vertical planes is inevitably preceded by, and associated with, a considerable increase of the normal pressure on these planes. At the instant of failure the normal pressure is much higher than the value computed on the basis of the Rankine conversion factor $\tan^2\left(45 - \frac{\phi}{2}\right)$. This is a noteworthy contribution to the clarification of the theoretical issues involved in the cofferdam problem. The Mohr diagram which is used to support the argument serves its purpose to perfection.

Professor Krynine also calls attention to the possibility of a failure along shear planes other than vertical ones. However, the writer doubts whether these other planes can possibly be oriented as indicated in Fig. 26(*c*). A failure along such planes requires that the major principal stress be oriented as indicated in Fig. 26(*b*) and in Fig. 39(*a*). Such orientation of the major principal stress would be associated with a distortion of the body abcd in Fig. 39(*a*) into the shape a_1b_1cd. A deformation of this kind involves an outward tilt which is incompatible with the nature of the problem. To account for the inward tilt of the cells, it is necessary to assign the orientation indicated in Fig. 39(*b*) to the shear planes. One set of shear planes is vertical whereas the other slopes at an angle ϕ toward the inside of the cells. Failure occurs simultaneously on the vertical and the inclined planes. Hence, for estimating purposes, it seems justifiable to evaluate the factor of safety with respect to failure by internal shear on the basis of the stress conditions on vertical planes.

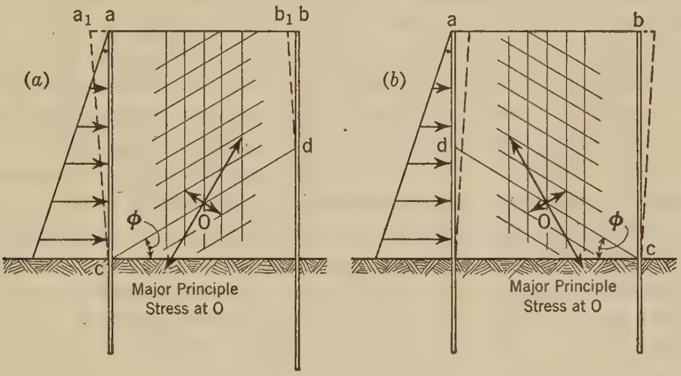


FIG. 39

Mr. Polivka describes the small-scale model tests made preliminary to the design of a cellular cofferdam in the Kaiser Shipyards at Richmond, Calif. The model cells were no more than 6 in. high. They consisted of cylinders made of plain sheet metal, 0.004 in. thick, filled with very fine sand. The results of tests on such cells cannot be correlated with the author's theory of internal shear, because the walls were incapable of distortion comparable to that in cells with walls containing locks. This difference is crucial.

In his mathematical interpretation of the test results on the basis of the general bin theory, Mr. Polivka assigned a value of 26° 30′ to the angle of internal friction ϕ of the sand and a value of $K = 1$ to the ratio between the horizontal and the vertical pressure in the sand. The value of ϕ appears to be rather low. Reliable values can be obtained only by an appropriate shear box apparatus or a triaxial compression device. The smallest value that was ever obtained by either one of these devices ranges from 27° to 28° 30′ (44)(45). The tests were made on Ottawa standard sand—a very uniform sand consisting of almost perfectly spherical quartz grains with a smooth surface. The normal values for loose sand range between 32° and 34°. The value $K = 1$ was derived from trap-door experiments and is not applicable to the case under consideration. The experimental values obtained from bin tests are mentioned previously in the writer's comments on Colonel Pennoyer's discussion. They are approximately equal to 0.4.

To establish an equation for the resistance of the model cells against shear along vertical sections, Mr. Polivka introduced a value of $f = 1$ into Eq. 17. This does not seem permissible, because the equation thus obtained applies to walls with locks, if the coefficient of lock friction is equal to unity. If the walls are continuous (such as is the case for model cells), Eq. 17 cannot be used.

In the last part of his discussion Mr. Polivka describes the practical implications of making the walls of cellular cofferdams of arch-web sheet piles such as the Carnegie M112 and M113 sheet piles and their equivalents. In cofferdams local stressing of the steel beyond the yield point can scarcely be considered a failure. However, the excessive increase of the diameter of the cells which results from the low value of the "equivalent modulus of elasticity," E, of arch-web sheet piles is a very undesirable feature. Since it involves an important lateral stretching of the fill in the cells, it is associated with an equally important decrease of the horizontal pressure in the fill, which in turn reduces the lock friction in the cross-walls and the resistance of the fill against deformation. Hence, the writer agrees with Mr. Polivka that the straight-web sections are preferable.

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TRANSLATORY WAVES IN NATURAL CHANNELS

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With Discussion by Messrs. Harold A. Weggel, W. M. Lansford, M. A. Churchill, and J. H. Wilkinson

Synopsis

The phenomena of translatory wave travel in natural channels have been studied for many years without entirely satisfactory results. Published studies have been largely theoretical, supplemented by laboratory tests in some instances, but with few observed data on waves in natural channels. Stream flow regulation by storage reservoirs in the Tennessee River Basin during the past few years has developed numerous translatory waves of various magnitudes. Under such conditions, waves can be selected for study which travel considerable distances without being appreciably affected by tributary inflows.

This paper presents data on natural channel translatory waves on the Clinch and the lower Tennessee rivers, and on Wheeler Reservoir in Alabama for a considerable range of flow in each type of channel. Theoretical velocities are computed for these waves by formulas and are compared with observed velocities. Results indicate that wave velocities on the Clinch River can be computed with reasonable accuracy, and that velocities for the rising face of a wave on the lower Tennessee River can be approximated. Computed velocities for waves in Wheeler Reservoir are in poor agreement with observed velocities.

Translatory Waves on the Clinch River

Regulated discharges at Norris Dam in Tennessee have developed a number of well-defined translatory waves which were not affected appreciably by tributary inflow in the reach of the Clinch River between Norris Dam and Wheat stream gage 65.3 miles downstream (see Fig. 1). The Clinch River at Norris Dam drains a mountainous area and has an average annual flow of 4,600 cu ft per sec. Below the dam the river drains a hilly country and has

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Note.—Published in June, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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an average slope of about 1.6 ft per mile in the reach from the dam to the Wheat gage.

Continuous stage recorders located at intervals throughout this reach provide records from which gage-height graphs in Fig. 2 are plotted. Norris and Wheat gages have been rated by the U. S. Geological Survey, and ratings for Clinton and Edgemoor gages are developed from these ratings. Discharge ratings and corresponding channel cross-section areas for these stations are given in Table 1. The gage-height relation between Wheat gage and Gallahers Ferry gage is also given. As discharge measurements for rating Wheat gage are made at Gallahers Ferry, data on the channel cross section at Wheat gage are not available. Pertinent data at these gages are given in Table 2.

TABLE 1.—RATINGS, CHANNEL AREAS, AND

Gage height Z (ft)	NORRIS GAGE			CLINTON GAGE			EDGEMOOR	
	Discharge Q (cu ft per sec)	Channel area A (sq ft)	Velocity c_1 (miles per hr)	Discharge Q (cu ft per sec)	Channel area A (sq ft)	Velocity c_1 (miles per hr)	Discharge Q (cu ft per sec)	Channel area A (sq ft)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0	....	420	....	100	1,260	0.9	120	1,350
1	11	750	....	520	1,500	1.6	870	1,690
2	790	1,150	2.1	1,120	1,750	1.8	2,030	2,040
3	2,210	1,550	2.8	1,850	2,010	2.1	3,680	2,400
4	4,050	1,950	3.5	2,710	2,270	2.4	5,980	2,770
5	6,310	2,360	4.0	3,690	2,540	2.6	8,500	3,150
6	9,000	2,790	4.4	4,790	2,830	2.8	11,050	3,550
7	11,860	3,220	....	6,010	3,130	3.0	13,600	3,950
8	14,760	3,650	....	7,350	3,440	3.2	16,150	4,360
9	17,660	4,100	....	8,810	3,750	3.4	18,700	4,780
10	20,560	4,550	....	10,390	4,050	3.6	21,250	5,220
11	23,500	5,000	....	12,090	4,360	3.8	23,800	5,670
12	26,400	5,450	....	13,910	4,670	4.0	....	....
13	29,500	5,900	4.5	15,850	4,990	4.2	....	....
14	32,800	6,350	....	17,910	5,310	4.4	....	....
15	35,900	6,800	....	20,090	5,640	4.6	....	....
16	....	....	....	22,390	5,980	4.7	....	....
17	....	....	....	24,810	6,320	....	....	....
18	....	....	....	27,350	6,680	4.8	....	....

[a] Gage heights at Norris, see Fig. 1.

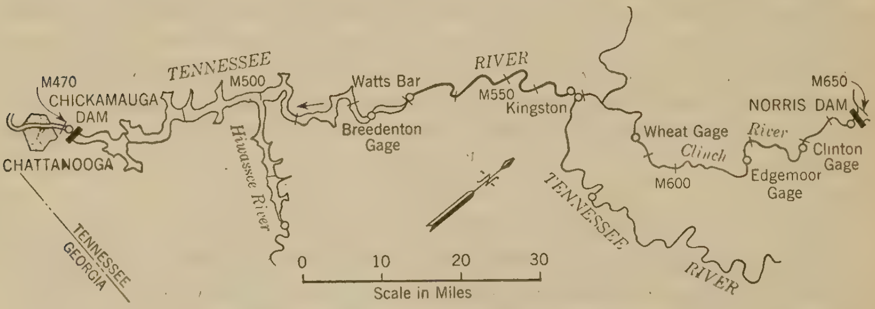
M470
CHICKAMAUGA DAM
CHATTANOOGA
TENNESSEE
GEORGIA
TENNESSEE
M500
Hiwassee River
Watts Bar
Breedenton Gage
RIVER
M550
Kingston
0
10
20
30
Scale in Miles
TENNESSEE
RIVER
Wheat Gage
Clinch
River
M600
M650
NORRIS DAM
Clinton Gage
Edgemoor Gage

FIG. 1.—CLINCH RIVER AND UPPER TENNESSEE RIVER

Velocity of a Translatory Wave.—To determine the velocity of a translatory wave from observed data, it is necessary to identify identical points on the wave at successive stations. In this study (1) the beginning-of-rise and (2) the midpoint-of-rise are used for this purpose on rising waves, and (3) the beginning-of-fall and (4) the midpoint-of-fall on falling waves. Velocities indicated by the points of fully developed rise and fully developed fall were investigated but were found to be unsatisfactory, partly because these points could not be identified accurately and partly because these points are modified considerably by other influences.

VELOCITIES, CLINCH RIVER STATIONS (SEE FIG. 1)

Gage	Wheat gage discharge Q (cu ft per sec)	Gallahers Ferry		Wheat gage velocity c_1 (miles per hr)	Norris Gage to Wheat Gage[a]		Gage height Z (ft)
Velocity c_1 (miles per hr) (10)	(11)	Relative gage height Z (ft) (12)	Channel area A (sq ft) (13)	(14)	Wave velocity c_1 (miles per hr) (15)	Water velocity[b] V (miles per hr) (16)	(1)
1.1	....	....	....	....	....	....	0
2.0	....	....	....	....	....	....	1
2.8	0	2.05	1,030	....	2.0	0.4	2
3.7	590	3.52	1,460	1.5	2.7	0.8	3
4.5	1,780	4.93	1,890	2.2	3.2	1.2	4
4.5	3,300	6.27	2,310	2.6	3.6	1.5	5
4.3	4,900	7.58	2,730	2.9	3.8	1.8	6
4.1	6,610	8.75	3,110	3.1	3.9	2.1	7
4.0	8,360	9.88	3,500	3.2	4.0	2.3	8
....	10,160	11.01	3,880	3.4	4.1	2.5	9
3.9	12,000	12.07	4,250	3.5	4.15	2.7	10
3.8	13,940	13.16	4,630	....	4.2	2.8	11
....	15,940	14.21	5,010	3.6	....	....	12
....	17,940	15.25	5,380	....	....	....	13
....	20,040	16.28	5,770	3.7	....	....	14
....	22,140	17.31	6,150	....	....	....	15
....	24,340	18.30	6,530	3.8	....	....	16
....	....	....	....	....	....	....	17
....	....	....	....	....	....	....	18

[b] V = velocity, mean (Q/A).

TABLE 2.—CLINCH RIVER STATIONS

Station	River mile	Elevation[a]	Drainage area (sq miles)
Norris Dam........	79.8	....	2,912
Norris gage........	78.7	819.11	2,913
Clinton gage.......	59.0	778.54	3,056
Edgemoor gage.....	47.6	768.08	3,089
Wheat gage........	14.5	717.40	3,385
Gallahers Ferry.....	13.2	713.53	3,387
Mouth............	0	....	4,413

[a] Elevation of gage zero above Mean Gulf Level.

The following formulas[2] are used for computing theoretical velocities from channel characteristics:

$$c_1 = \frac{0.682\, dQ}{b\, dZ} \quad \ldots.(1a)$$

and

$$c_2 = \frac{0.682\,(Q_2 - Q_1)}{A_2 - A_1} \quad ..(1b)$$

in which c_1 is the velocity of a wave traveling on a stream at a given stage; c_2 is the average velocity of the wave, taking into account the channel discharge

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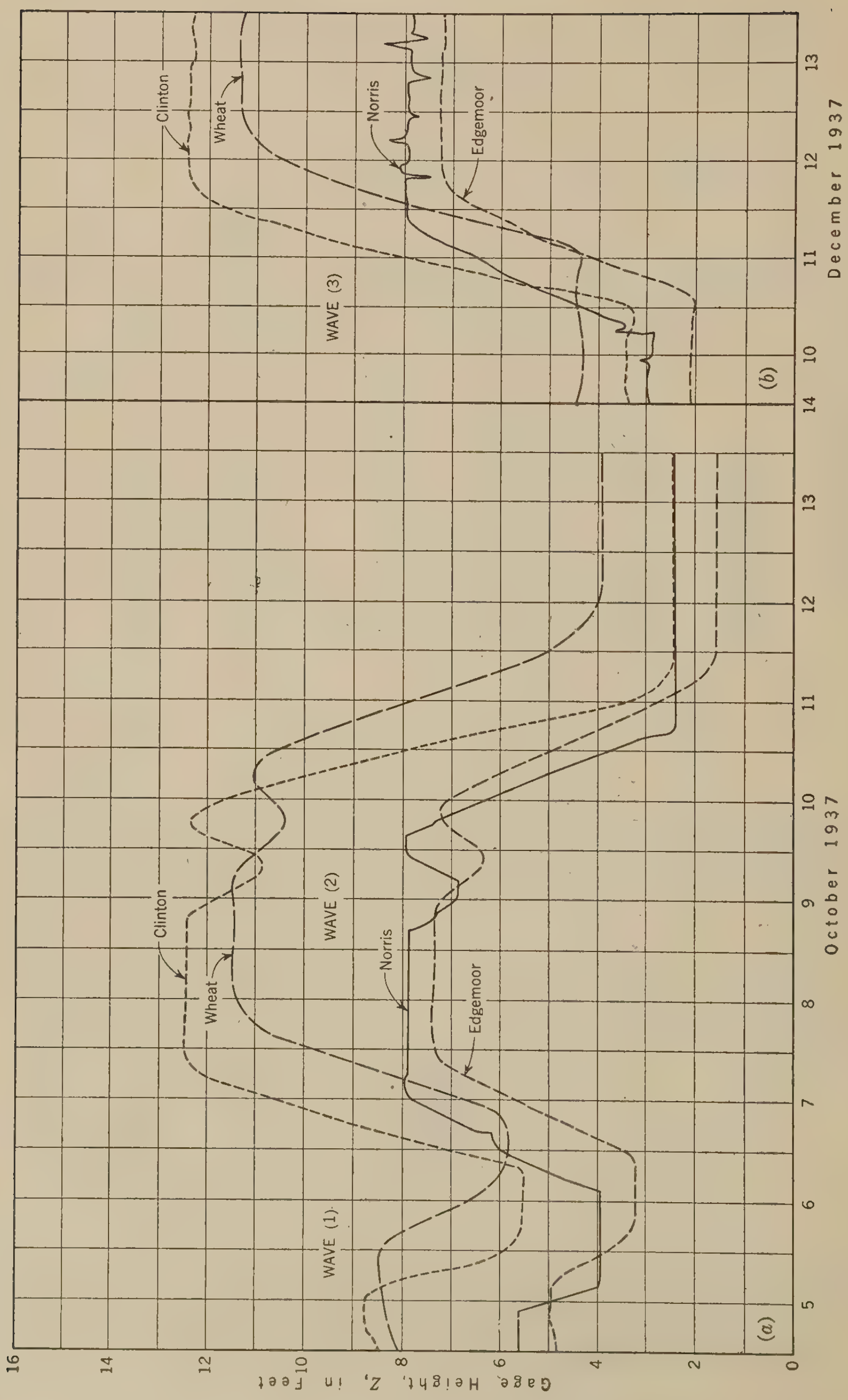
[2] "The Hydraulics of Flood Movements in Rivers," by Harold A. Thomas, *Engineering Bulletin*, Carnegie Inst. of Tech., 1934, pp. 18–19.


Gage Height, Z, in Feet
0
2
4
6
8
10
12
14
16
(a)
WAVE (1)
WAVE (2)
Clinton
Wheat
Norris
Edgemoor
5
6
7
8
9
10
11
12
13
14
October 1937
(b)
WAVE (3)
Clinton
Wheat
Norris
Edgemoor
10
11
12
13
December 1937

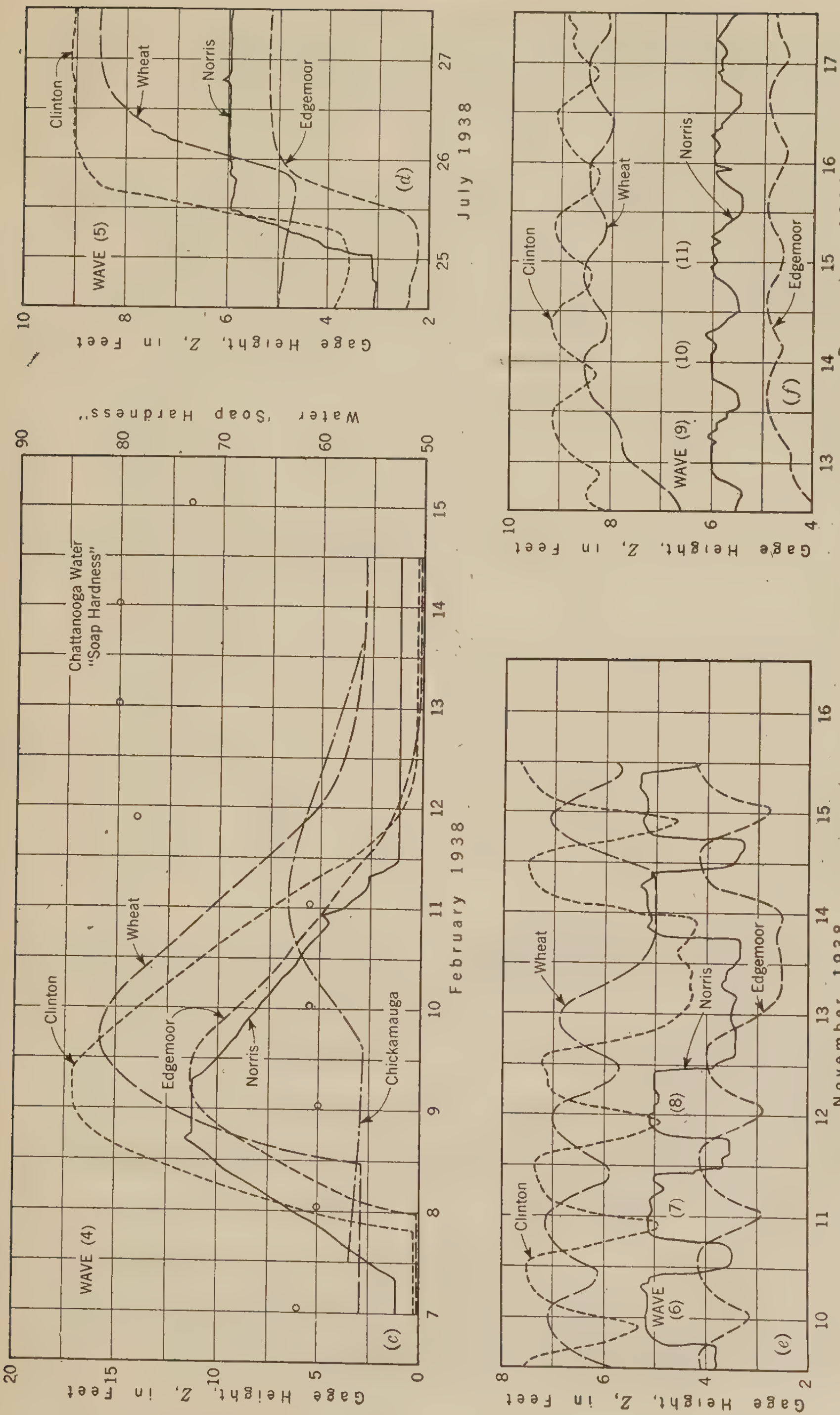


FIG. 2.—GAGE HEIGHTS, CLINCH RIVER WAVES

characteristics throughout the entire rise of the wave; Q_1 and Q_2 are the discharges corresponding to gage heights Z_1 and Z_2, respectively; dQ is an increment of discharge; dZ is a selected increment in a gage height Z; b is the width of the channel; and A_1 and A_2, respectively, are the cross-section areas below stages Z_1 and Z_2.

Eq. 1*a* is applicable only for determining the velocity of a wave traveling on a stream at a given stage. This becomes evident when the derivation of the equation is examined. Eq. 1*b* is applicable for determining the average velocity of the wave, as it takes into account the channel discharge characteristics throughout the entire rise of the wave. Results from both equations will be the same when the discharge characteristic of the channel, that is, $\frac{dQ}{b\,dZ}$, is constant throughout the range in stage covered by the wave. It will be noted that c_1 is a function of the slope of the rating curve at a given stage, whereas c_2 is a function of the slope of a secant to the rating curve between two stages, or the average slope of the rating curve. The constant 0.682 in Eqs. 1 is inserted to convert to miles per hour.

Using Eqs. 1, wave velocities are computed for the four identifiable points on each wave at each of the four stage recording stations. These computed velocities are assumed to exist half the distance to adjacent stations in determining the theoretical time required for the wave to traverse each of the three reaches between stage recording stations. The average velocity between Norris gage and Wheat gage is then computed from the total time required for the wave to traverse the three intermediate reaches.

TABLE 3.—COMPUTATION OF c_1 (EQ. 1*a*) FOR BEGINNING-OF-RISE

Station (1)	Observed Z_1 (2)	b (ft) (3)	dQ (cu ft per sec) (4)	dZ (ft) (5)	c_1 (Eq. 1*a*) (6)	Reach, L (miles) (7)	Time, t (hr) (8)
Norris	4.0	417	210	0.1	3.43	9.85	2.87
Clinton	5.5	282	110	0.1	2.66	15.55	5.85
Edgemoor	3.2	369	210	0.1	3.88	22.25	5.72
Wheat	5.8	....	139	0.1	2.92	16.55	5.67
Gallahers	7.3	323	....	....	....	....	....
Average c_1 (Norris to Wheat)					3.19	64.2	20.11

Col. 1.—Stations at which data on gage heights are available.
Col. 2.—The river stage at the beginning-of-rise, Z_1, is taken from the gage-height graph for each station in Fig. 2(*a*). As data on the channel at Wheat are not available, the stage at Gallahers corresponding to the observed stage is determined from the gage-height relation in Table 1 for computing the velocity at this station.
Col. 3.—Channel widths b at these stages are determined from the rate of change in channel cross-section area at stage Z_1 from Table 1.
Col. 4.—The rates of change in discharges, dQ, per 0.1 ft change in stage, at these stages are determined from the discharge rating curves in Table 1.
Col. 5.—0.1 ft change in stage.
Col. 6.—c_1 is computed by substituting values in Cols. 3, 4, and 5 in Eq. 1*a*.
Col. 7.—The reach length L in miles is the distance over which the computed velocity c_1 at each station is assumed to exist.
Col. 8.—The number of hours required for the wave to traverse this distance if traveling at a velocity c_1. The average velocity c_1, from Norris to Wheat, is then equal to 64.2 miles divided by the total number of hours required for the wave to traverse these four reaches.

The method of applying Eqs. 1 is illustrated in Tables 3 and 4, using data from Wave 1, October 6, 1937.

As the velocity c_1 in Eq. 1*a* at a given station is a function of the stage, curves for values of c_1 can be developed for each station as shown in Fig. 3. A curve for the average velocity c_1 for the Norris-to-Wheat reach can also be developed by determining stages at successive stations corresponding to a selected stage at Norris gage by mean gage-height relations and by computing the average velocity c_1. Such a curve indicates average values of c_1 plotted against Norris stages for the Norris-to-Wheat reach under average conditions of local inflow. It gives the same value of c_1 as computed in Table 3. This occurs only when average conditions of local inflow exist and the stages at all stations agree with the mean gage-height relation used for constructing this curve.

TABLE 4.—COMPUTATION OF c_2 (EQ. 1*b*) FOR THE MIDPOINT-OF-RISE

Station (1)	Z_2 (2)	Z_1 (3)	Q_2 (4)	Q_1 (5)	A_2 (6)	A_1 (7)	Q_2-Q_1 (8)	A_2-A_1 (9)	c_2 (Eq. 1*b*) (10)	L (miles) (11)	t (hr) (12)
Norris	7.8	4.0	14,180	4,050	3,560	1,950	10,130	1,610	4.30	9.85	2.28
Clinton	12.5	5.5	14,860	4,210	4,830	2,680	10,650	2,150	3.38	15.55	4.61
Edgemoor	7.4	3.2	14,620	4,080	4,110	2,470	10,540	1,640	4.38	22.25	5.08
Wheat	11.3	5.8	....	....	....	....	....	....	3.24	16.55	5.12
Gallahers	13.5	7.3	14,540	4,590	4,736	2,642	9,950	2,094	....	....	....
Average c_2 (Norris to Wheat)									3.75	64.2	17.09

Col. 1.—Stations at which data on gage heights are available.
Col. 2.—Gage height Z_2 at end-of-rise from Fig. 2(*a*) (observed).
Col. 3.—Gage height Z_1 at beginning-of-rise (observed).
Col. 4.—Discharge Q_2 corresponding to gage height Z_2 determined from Table 1.
Col. 5.—Discharge Q_1 corresponding to gage height Z_1.
Col. 6.—Channel cross-section area A_2, corresponding to gage height Z_2 determined from Table 1.
Col. 7.—Area A_1 corresponding to gage height Z_1.
Col. 8.—Q_2-Q_1 from Cols. 4 and 5.
Col. 9.—A_2-A_1 from Cols. 6 and 7.
Col. 10.—Wave velocity c_2 computed by substituting Cols. 8 and 9 in Eq. 1*b*.
Cols. 11 and 12.—c_2, L, and t (Cols. 10, 11, and 12) are determined in the same manner as corresponding values in Table 3.

For the purpose of comparison, a curve showing the water velocity in the Norris-to-Wheat reach is also shown in Fig. 3 (water velocity is the velocity indicated by a current meter). This curve is developed from the discharge rating and channel cross-section areas at each station for stages determined by gage-height relations, assuming that each station velocity exists halfway to adjacent stations. The water velocity is about half the wave velocity at ordinary stages and less than half at low stages.

In the computation of c_2 by Eq. 1*b*, in Table 4, an inconsistency in discharges Q_2 and Q_1 at successive stations will be noted. This can be caused by either an error in the station ratings, errors in observed stages, or irregularities in flow. It is believed that this inconsistency will not affect results materially as long as the area difference, $A_2 - A_1$, corresponds to the discharge difference, $Q_2 - Q_1$.

Theoretical velocities c_1 and c_2 for the beginning and midpoint of rising waves, and for the beginning and midpoint of falling waves shown in Fig. 2, together with the velocity c_o, corresponding to the observed time of travel from Norris to Wheat, are shown in Table 5.

Comparison of Observed and Computed Wave Velocities.—Fig. 4 shows velocities c_o, c_1, and c_2 plotted against the gage heights observed at Norris gage, with mean curves drawn for each. Mean curves of computed velocities for the midpoint-of-rise (Fig. 4(*b*)) and the midpoint-of-fall (Fig. 4(*d*)) agree very well with the mean curves for observed velocities. It will be noted that the range of gage heights covered by these data is small, and, the gage heights being midpoints, do not extend to either extremely low or high stages. Results for the beginning-of-rise (Fig. 4(*a*)) and the beginning-of-fall (Fig. 4(*c*)) do not agree nearly as well, computed velocities being smaller than the observed in both cases. In the latter plottings, velocities c_2 are computed from the total

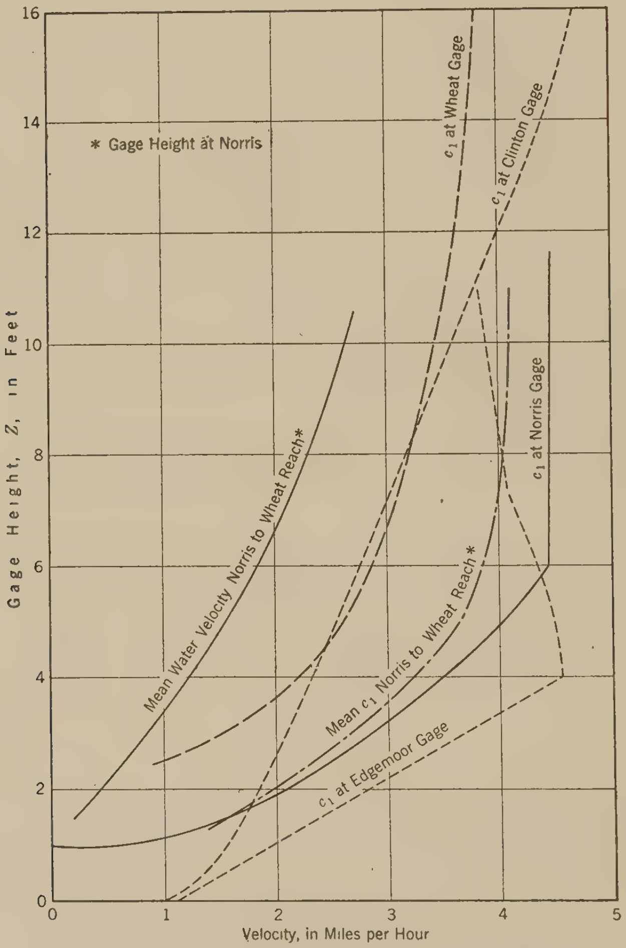


FIG. 3.—VELOCITY CURVES, NORRIS DAM TO WHEAT GAGE

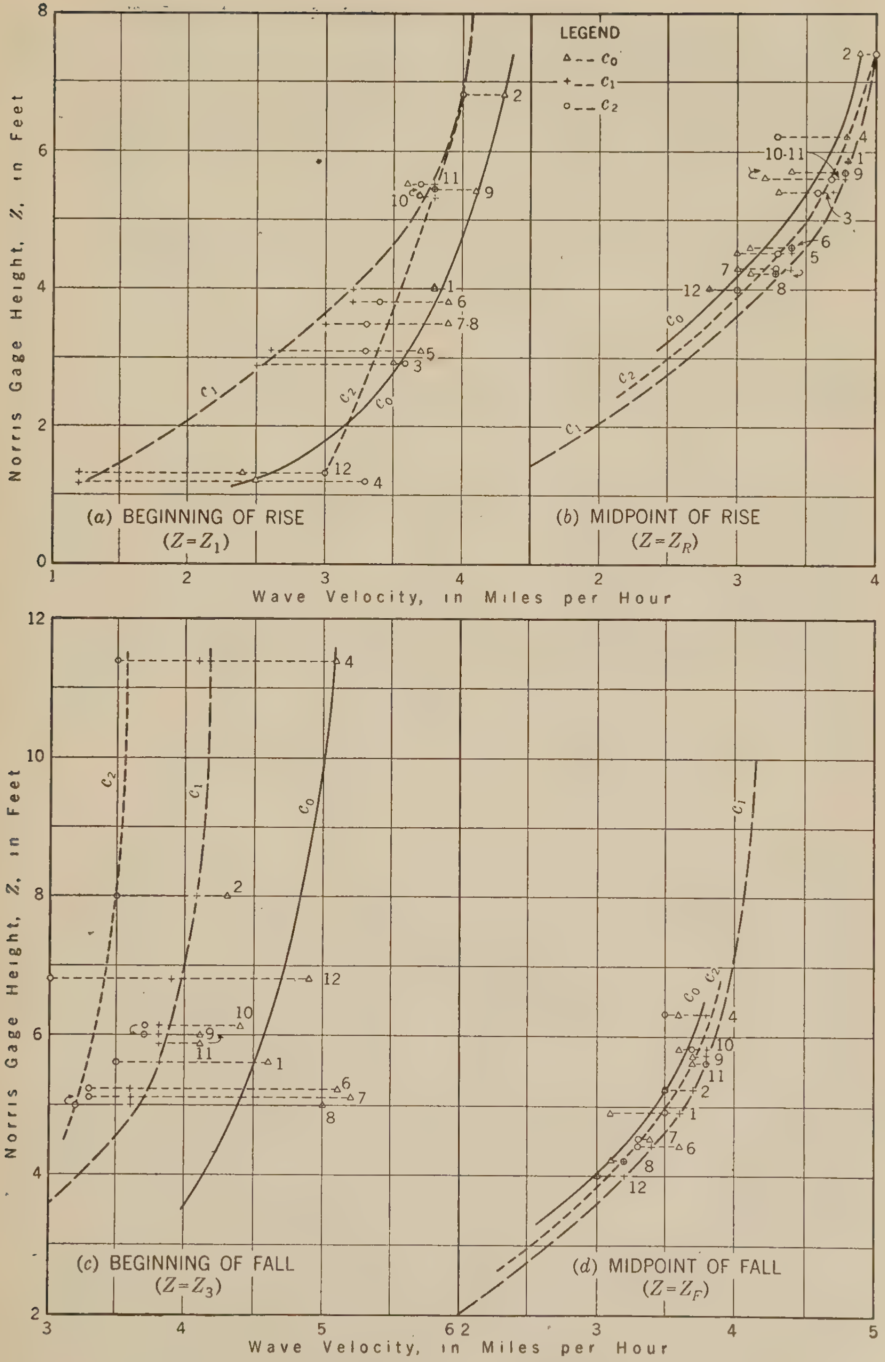


FIG. 4.—VELOCITY, CLINCH RIVER WAVES

rise or fall of the wave and are plotted against Z_1 or Z_2, respectively. Mean curves for observed velocities for both the beginning-of-rise and the beginning-of-fall are higher at a given stage than observed velocities for the midpoints.

Wave of February 7, 1938.—The wave of February 7, 1938 (Wave 4, Table 5), developed when the quantity released at Norris Dam was increased during a comparatively low and steady flow in the Clinch and upper Tennessee rivers.

TABLE 5.—OBSERVED AND COMPUTED WAVE VELOCITIES BETWEEN NORRIS AND WHEAT GAGES, CLINCH RIVER WAVES, IN MILES PER HOUR

Wave No.	Date	RISING WAVES								FALLING WAVES							
		Norris		Beginning-of-Rise[a]		Norris	Midpoint-of-Rise			Norris		Beginning-of-Fall		Norris	Midpoint-of-Fall		
		Z_1	Z_2	c_0	c_1	Z_R	c_0	c_1	c_2	Z_1	Z_2	c_0	c_1	Z_F	c_0	c_1	c_2
	1937																
1	Oct. 6	4.0	7.8	3.8	3.2	5.9	3.8	3.8	3.8	5.6	4.0	4.6	3.8	4.9	3.1	3.6	3.5
2	Oct. 9	6.8	7.9	4.3	4.0	7.4	3.9	4.0	4.0	8.0	2.4	4.3	4.0	5.2	3.5	3.7	3.5
3	Dec. 10	2.9	8.0	3.5	2.5	5.4	3.3	3.7	3.6	...	...	...	...	...	...	...	...
	1938																
4	Feb. 7	1.2	11.3	2.5	1.2	6.2	3.8	3.8	3.3	11.4	1.2	5.1	4.1	6.3	3.6	3.8	3.5
5	July 25	3.1	5.9	3.7	2.6	4.5	3.0	3.4	3.3	...	...	...	...	...	...	...	...
6	Nov. 10	3.8	5.2	3.9	3.2	4.5	3.1	3.4	3.4	5.2	3.5	5.1	3.6	4.4	3.6	3.4	3.3
7	Nov. 11	3.5	5.1	3.9	3.0	4.3	3.0	3.4	3.3	5.1	3.6	5.2	3.6	4.4	3.4	3.4	3.3
8	Nov. 12	3.5	5.1	3.9	3.0	4.3	3.1	3.3	3.3	5.1	3.4	5.0	3.6	4.2	3.1	3.2	3.2
9	Dec. 13	5.4	6.0	4.1	3.8	5.7	3.4	3.8	3.8	6.0	5.4	4.1	3.8	5.7	3.7	3.8	3.7
10	Dec. 14	5.4	6.0	3.7	3.8	5.7	3.2	3.8	3.7	6.0	5.5	4.4	3.8	5.8	3.6	3.8	3.7
11	Dec. 15	5.5	6.0	3.6	3.8	5.7	3.2	3.8	3.7	6.0	5.4	4.1	3.8	5.7	3.7	3.8	3.8
	1940																
12	Feb. 8	1.2	6.7	2.4	1.2	4.0	2.8	2.8	3.0	6.8	1.2	4.9	3.9	4.0	3.0	3.2	3.0

[a] c_0 is the wave velocity corresponding to the observed time of travel.

The release was gradually increased from 30 cu ft per sec to 24,300 cu ft per sec in 33 hr, was held about steady for 14 hr, and then was gradually reduced to about the original release in 48 hr. Gage-height graphs of the resulting wave at Clinch River stations and at Chickamauga Dam site, near Chattanooga, Tenn., are shown in Fig. 2(*c*), together with the "soap hardness" of water samples from the Chattanooga City Pumping Plant intake. This intake is about six miles below Chickamauga Dam site and the water travels this distance in about 3 hr. Prior to the arrival of the wave from Norris Dam, soap-hardness tests at Chattanooga showed a fairly constant value of 60. However about 48 hr after the arrival of the wave the hardness increased to a value of 80 for a 2-day period. It has been found that the hardness of Norris Reservoir water is fairly constant at 95 ppm, whereas the hardness of Tennessee River water at Chattanooga averages about 65 ppm, Norris water being considerably harder than water from most of the tributaries above Chattanooga. There is a marked similarity between the pattern of the points for soap hardness and the shape of the Chickamauga Dam gage-height graph during the arrival of the wave from Norris Dam, the peak of hardness occurring about two days later than the peak flow. It is unfortunate that data are not available on the hardness of the water in the several tributaries coincident with the passage of the wave.

If the hardness peak indicates the arrival of actual water released from Norris Dam, it requires four days, corresponding to a velocity of about 2 miles per hr, for water to travel 183.3 miles from Norris Dam to Chattanooga, compared with 53 hr (3.5 miles per hr) for the wave. The flow in the river at Chattanooga before the arrival of the wave was 23,000 cu ft per sec. This would indicate that the actual water in the wave when it arrived at Chickamauga Dam site was in the river in front of the original wave developed at Norris Dam.

TRANSLATORY WAVES IN WHEELER RESERVOIR

Translatory waves in Wheeler Reservoir (Fig. 5) have quite different characteristics from those in tributary streams, such as the Clinch River. The channel has greater depth and width, together with a flatter water-surface slope. Even with abrupt changes of considerable magnitude in the rate of inflow, translatory waves are quickly smoothed out.

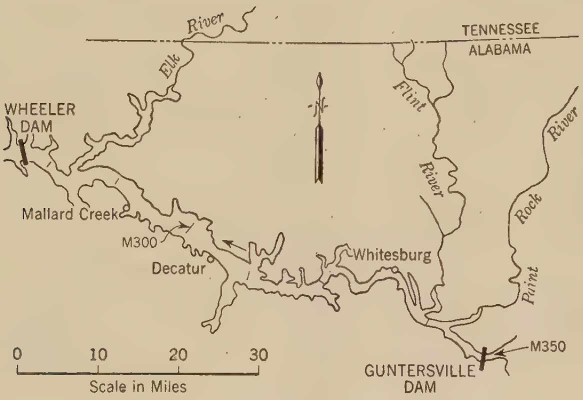


FIG. 5.—WHEELER RESERVOIR

Continuous stage recorders located in Alabama at Guntersville lower lock gage (mile 349.0), Whitesburg (mile 333.7), Decatur (mile 305.1), and Mallard Creek (mile 293.4), together with hourly observations at Wheeler Dam (mile 274.9), provide data from which elevation graphs in Figs. 6 and 7 were prepared. The accompanying hydrographs are prepared from computed discharges at Guntersville and Wheeler dams. The drainage area above Guntersville Dam is 24,450 sq miles with a mean annual discharge of 41,400 cu ft per sec, whereas the drainage area above Wheeler Dam is 29,590 sq miles with a mean annual discharge of 50,000 cu ft per sec.

Theoretical Formulas.—Changes in discharge at Guntersville Dam, which developed these waves, were made quickly enough to be considered as sudden for the purpose of analysis. H. W. King,[3] M. Am. Soc. C. E., has given a

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[3] "Handbook of Hydraulics," by H. W. King, 3d Ed., 1939, pp. 420–421.

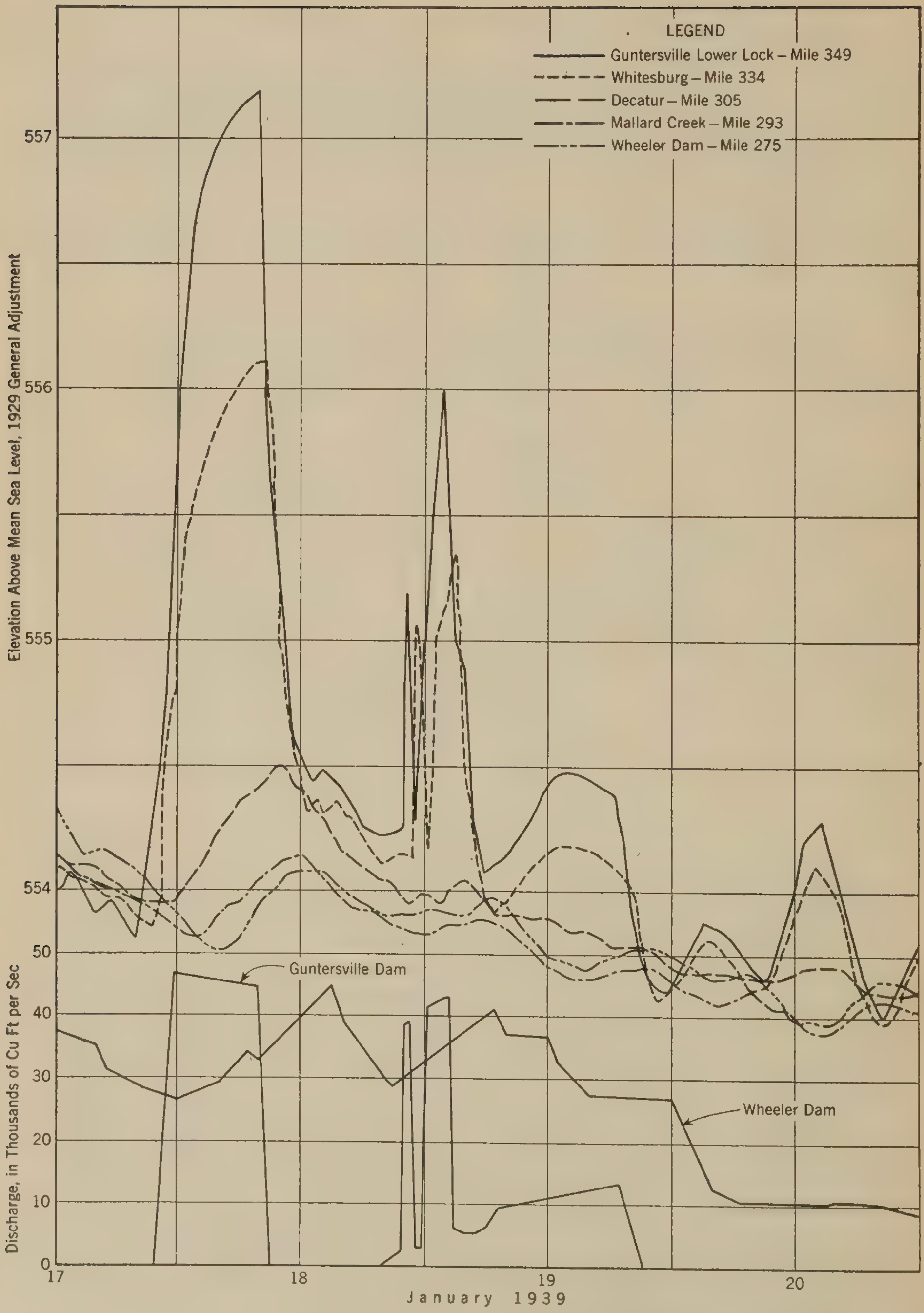


FIG. 6.—GAGE HEIGHTS AND DISCHARGE,

formula that is applicable for computing velocities of such waves:

$$c_3 = \left[V_1 + \sqrt{g \frac{A_2 \bar{y}_2 - A_1 \bar{y}_1}{A_1 \left(1 - \frac{A_1}{A_2}\right)}} \right] 0.682 \ldots\ldots\ldots\ldots (2a)$$

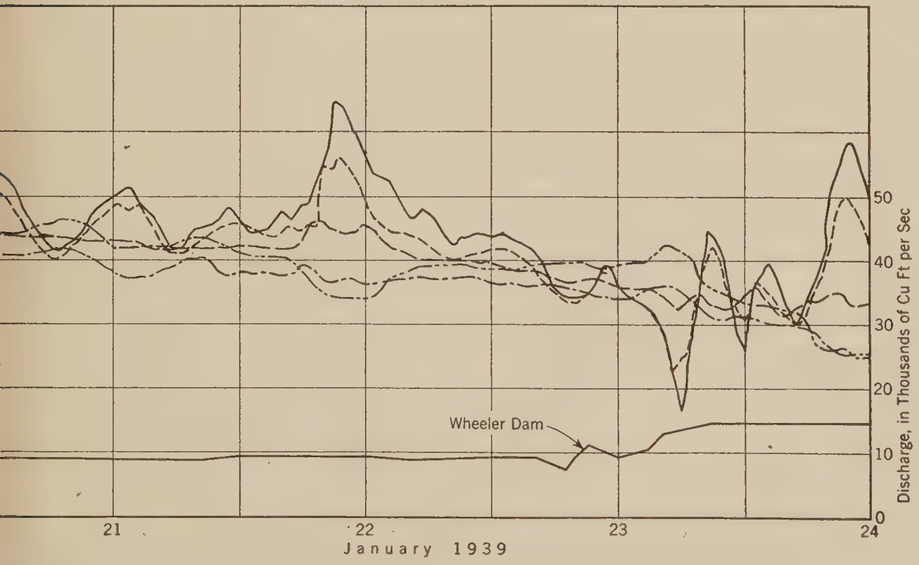
An approximation of this formula, based on the assumption that the difference between A_1 and A_2 is negligible, and also that the channel is of uniform cross section, is given by Professor King[3] as

$$c_4 = [V_1 + \sqrt{g\, y_m}]\, 0.682 \ldots\ldots\ldots\ldots\ldots\ldots (2b)$$

in which c_3 is the velocity of a wave created suddenly, the effect of the difference between A_1 and A_2 being taken into consideration; V_1 is initial water velocity; g is the acceleration of gravity; $\bar{y}_1$ and $\bar{y}_2$ are the depths to the center of pressure of A_1 and A_2, respectively; A_1 and A_2 are channel cross-section areas occupied by the wave at beginning and end, respectively; c_4 is the velocity of a wave created suddenly, the difference between A_1 and A_2 being considered negligible; and y_m is the mean depth of water. The constant 0.682 is inserted to convert to miles per hour.

The times of travel of observed translatory waves in reservoirs are determined from the beginning-of-rise in rising waves and the beginning-of-fall in falling waves. Observed velocities, c_o, and velocities c_3 and c_4, computed by Eqs. 2, are shown in Table 6 for each of four rising and four falling waves, for each of the four reaches between intermediate gages, and also for the overall reach between Guntersville and Wheeler dams.

Velocities of Translatory Waves in Wheeler Reservoir.—Velocities computed by Eqs. 2 (see Table 6) agree fairly well with the observed velocities in some



HEELER RESERVOIR WAVES IN JANUARY, 1939

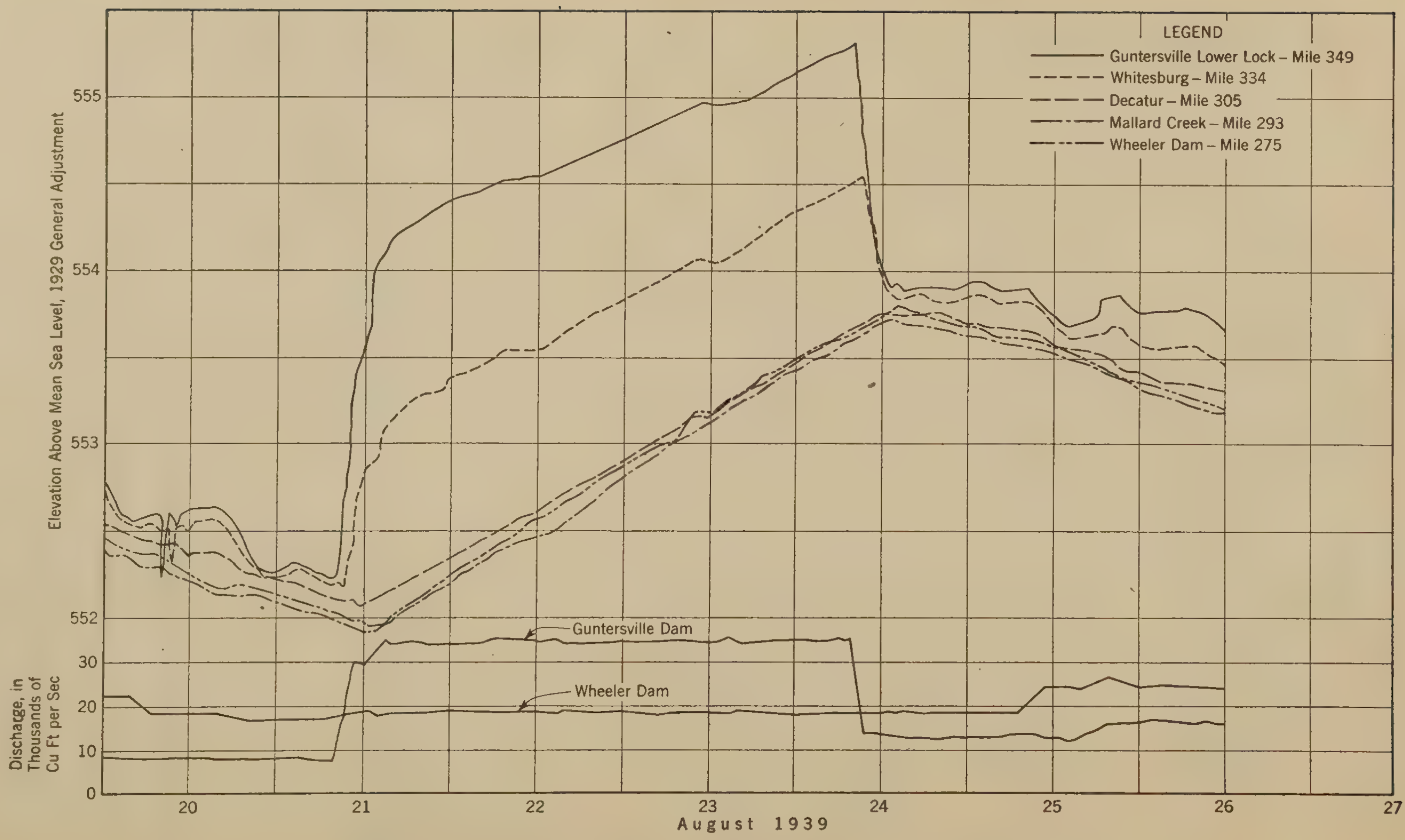


FIG. 7.—GAGE HEIGHTS AND DISCHARGE, WHEELER RESERVOIR WAVES IN AUGUST, 1939

cases and differ widely in others. For rising waves in the Guntersville Dam to Whitesburg reach, computed velocities by Eq. 2*a* and Eq. 2*b* both agree closely with the observed velocities except for the wave of January 17. Observed velocities for this wave are lower in all reaches than observed velocities for the other three waves. In this reach computed velocities c_3 and c_4 are practically the same for all waves.

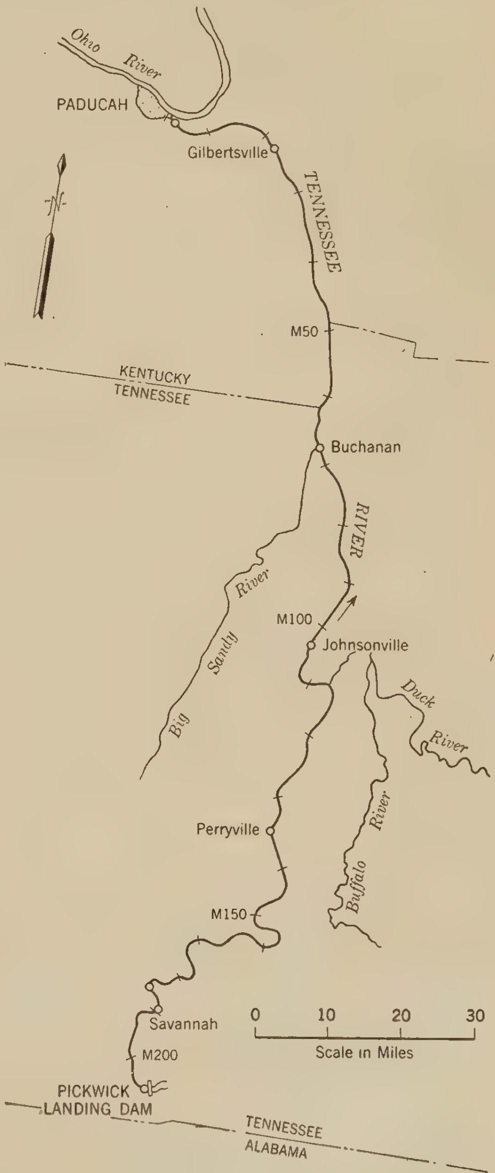

Ohio River
PADUCAH
Gilbertsville
N
TENNESSEE
M50
KENTUCKY
TENNESSEE
Buchanan
RIVER
Big Sandy River
M100
Johnsonville
Duck River
Perryville
Buffalo River
M150
Savannah
M200
PICKWICK LANDING DAM
TENNESSEE
ALABAMA
0 10 20 30
Scale in Miles

Fig. 8.—Lower Tennessee River

For rising waves in the Whitesburg to Decatur reach, computed velocities c_4 are about the same as for the previous reach, whereas computed velocities c_3 are consistently lower. The average of velocities, c_3, is practically the same as the average of c_o. For rising waves in the Decatur to Mallard Creek reach, both computed velocities give about the same results, which are higher than observed velocities in all cases. The average of c_o is only a little more than half the observed velocity c_o for the previous reach. For rising waves in the Mallard Creek to Wheeler Dam reach, both computed velocities again give about the same results, but are about double the average of the observed velocities. In the over-all reach from Guntersville Dam to Wheeler Dam, rising wave computed velocities c_3 average slightly lower than computed velocities c_4, but are about one third larger than the observed velocity. Observed and computed velocities for falling waves are similar to those for rising waves.

Considerable difficulty is encountered in determining the effective average channel cross section for the application of Eqs. 2 in the Whitesburg-Decatur and the Decatur-Mallard Creek reaches. Channel cross sections

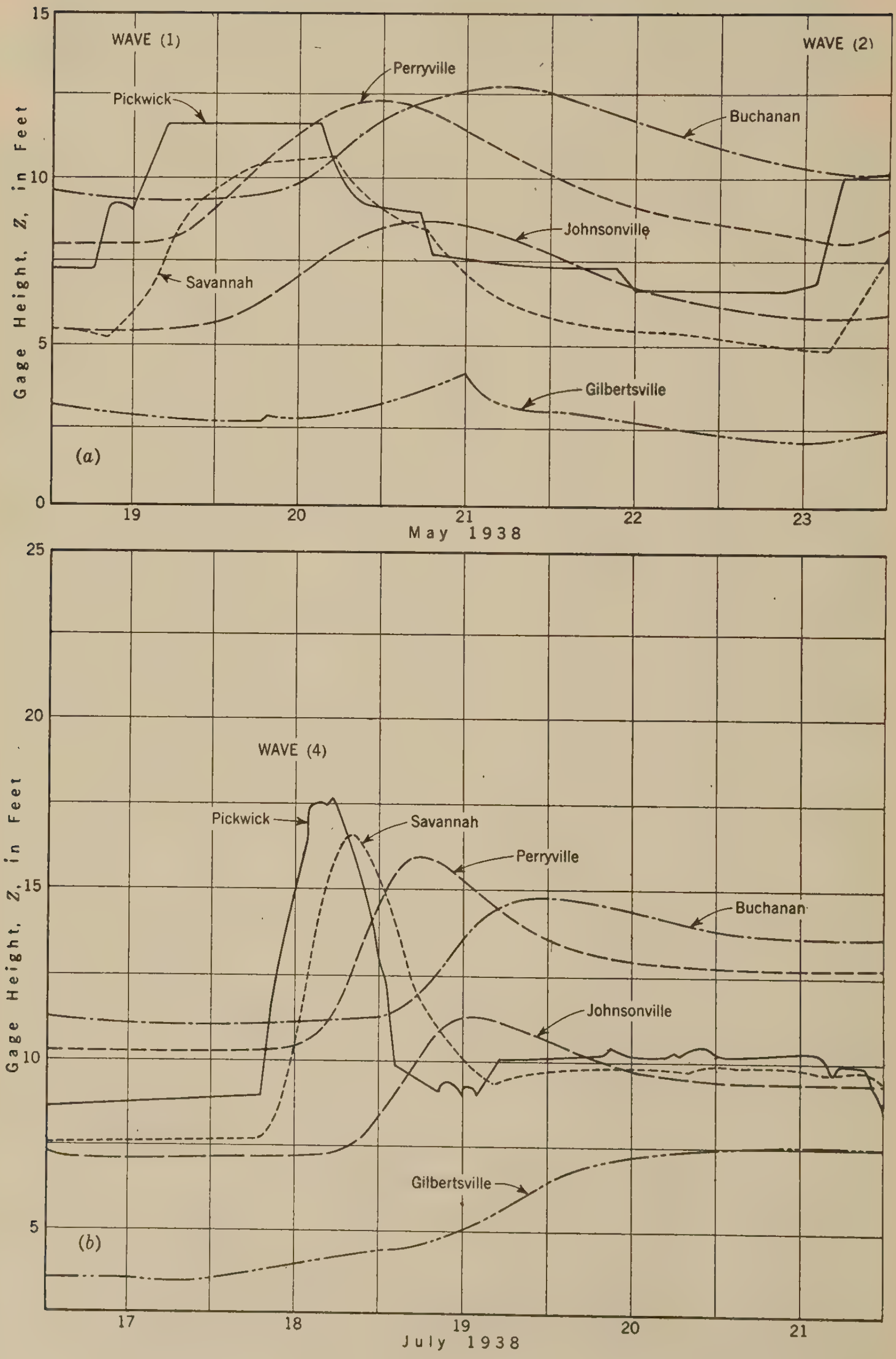
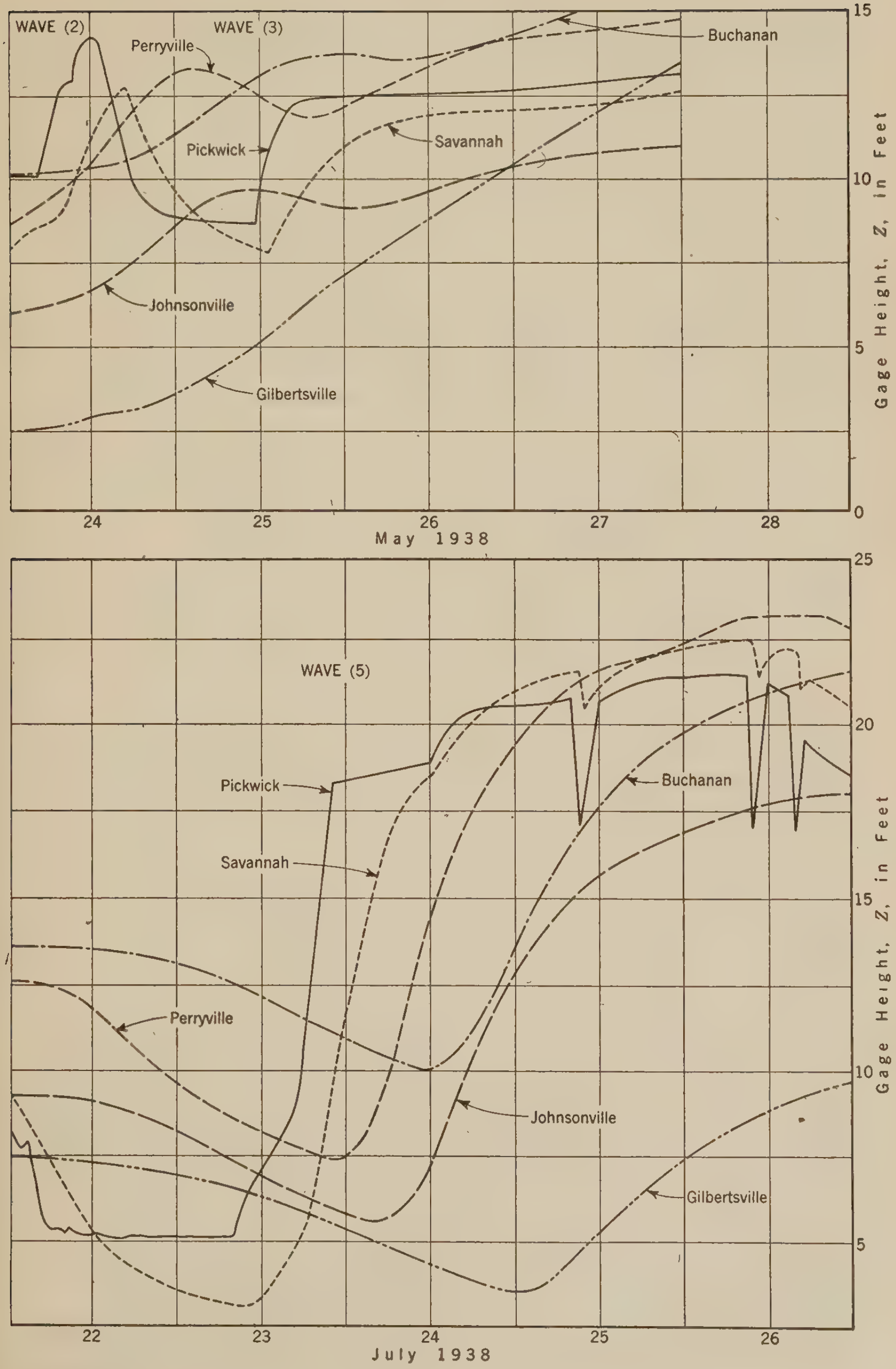


FIG. 9.—GAGE HEIGHTS,



Tennessee River Waves

taken at about one-mile intervals are available throughout the length of Wheeler Reservoir. Upstream from Whitesburg the reservoir is largely within the old river channel and has fairly regular cross sections, while downstream from Mallard Creek the reservoir covers the entire river bottom. Between these two gages there is a gradual transition from river channel to river bottom sections. This may account in part for the wide discrepancy between observed and computed velocities in these two intermediate reaches, but would not account for the even wider discrepancy in the results for Mallard Creek to Wheeler Dam reach. It is probable that when the reservoir widens, the lateral spread of the wave reduces its downstream velocity. Another probable source of error is the inability to determine discharges at intermediate stations. Discharges at Guntersville and Wheeler dams are determined from gate settings and should be reasonably accurate. Discharges at intermediate stations are estimated from these discharges taking into account local inflow from tributaries.

TABLE 6.—VELOCITIES IN MILES PER

Date of wave, 1939	GUNTERSVILLE DAM TO WHITESBURG L = 15 MILES				WHITESBURG TO DECATUR L = 29 MILES				DECATUR TO L = 12	
	Observed		Computed		Observed		Computed		Observed	
	t	c_0	c_3	c_4	t	c_0	c_3	c_4	t	c_0
									(a) RISING	
Jan. 17	1.50	10.0	19.7	19.1	2.50	11.6	17.6	19.9	2.00	6.0
May 2	0.75	20.0	20.8	20.4	1.50	19.3	17.4	20.9	1.00	12.0
June 19	0.75	20.0	19.0	19.1	1.25	23.2	16.8	20.1	1.25	9.6
Aug. 21	0.75	20.0	19.4	18.6	1.75	16.6	18.3	19.5	0.92	13.0
Average	....	17.5	19.7	19.3	....	17.7	17.5	20.1	....	10.2
									(b) FALLING	
Jan. 18	0.83	18.0	20.7	20.2	1.92	15.1	18.3	20.5	1.25	9.6
May 3	1.00	15.0	20.1	20.7	1.50	19.3	17.1	20.8	1.00	12.0
June 19	0.83	18.0	21.3	20.0	3.17	9.2	19.1	20.3	1.00	12.0
Aug. 24	0.83	18.0	19.1	19.7	2.67	10.9	17.9	20.2	1.00	12.0
Average	....	17.2	20.3	20.2	....	13.6	18.1	20.4	....	11.4

In general, observed velocities in the reaches upstream from Decatur where the pool is largely within the original river channel are materially higher than in the reaches below Decatur where the pool submerges the original river banks. Two bridge embankments at Decatur restrict the flow to the original channel at these points. When a wave reaches the upstream embankment a portion of it rebounds to form an upstream wave that reduces the magnitude of the wave at downstream points.

Observed velocities determined from beginning-of-rise and beginning-of-fall points do not differ materially in a given reach, both being about 17 miles per hr upstream from Decatur and about 12 miles per hr downstream.

Surges in Wheeler Reservoir.—It will be noted in Fig. 6 that, when the release at Guntersville Dam was reduced suddenly and then held steady for a time, a series of surges occurred at fairly regular intervals of about 11 hr. This interval is about the time required for a wave to travel from Guntersville Dam to Wheeler Dam and return. Usually the peaks at Guntersville Dam coincide with valleys at Wheeler Dam. These surges have a much smaller magnitude at Decatur than at stations upstream or downstream. Water-surface profiles of the reservoir plotted for elevations observed at successive hours show that the reservoir water surface appears to pivot on Decatur. The water surface falls at upstream stations when it is rising at downstream stations.

TRANSLATORY WAVES ON THE TENNESSEE RIVER BELOW PICKWICK LANDING DAM

Translatory waves on the Tennessee River below Pickwick Landing Dam (see Fig. 8) are typical of waves developed by regulated releases in a natural river channel having a considerable discharge capacity and a comparatively flat slope. The mean annual discharge of this section of the river is about 60,000 cu ft per sec, and the fall is 65 ft in 207 miles, 0.3 ft per mile.

HOUR, WAVES IN WHEELER RESERVOIR

MALLARD CREEK MILES		MALLARD CREEK TO WHEELER DAM L = 18 MILES				GUNTERSVILLE DAM TO WHEELER DAM L = 74 MILES				Date of wave, 1939
Computed		Observed		Computed		Observed		Computed		
c_3	c_4	t	c_0	c_3	c_4	t	c_0	c_3	c_4	
WAVES										
13.4	14.0	2.00	9.0	24.7	23.0	8.00	9.3	18.4	19.1	Jan. 17
17.2	14.6	1.50	12.0	23.8	23.6	4.75	15.6	19.2	19.9	May 2
14.7	14.3	1.00	18.0	21.6	23.3	4.25	17.4	17.7	19.3	June 19
13.8	13.4	1.83	9.8	23.5	22.7	5.25	14.1	18.5	18.5	Aug. 21
14.8	14.1	....	12.2	23.4	23.2	....	14.1	18.5	19.2	Average
WAVES										
13.9	14.3	1.50	12.0	27.2	23.2	5.50	13.5	19.4	19.6	Jan. 18
18.0	14.9	1.50	12.0	20.2	23.5	5.00	14.8	20.4	20.0	May 3
....	14.3	1.00	18.0	16.4	23.1	6.00	12.3	....	19.5	June 19
16.6	14.0	1.25	14.4	25.3	23.0	5.75	12.9	19.3	19.3	Aug. 24
16.2	14.4	....	14.1	22.3	23.2	....	13.4	19.7	19.6	Average

Waves for which gage-height graphs are shown in Fig. 9 are selected to cover a considerable range of flow conditions and have discontinuities that can be identified at successive downstream stations. Pertinent data on lower Tennessee River gaging stations are given in Table 7. Gage-height graphs are shown for Pickwick lower lock (Tenn.), Savannah (Tenn.), Perryville (Tenn.), Johnsonville (Tenn.), Buchanan (Tenn.), and Gilbertsville (Ky.) gages whenever the records for these stations are available for the wave selected. Stages at Gilbertsville are affected by Ohio River stages at all times, and high stages on the Ohio River affect stages as far upstream as Johnsonville.

Wave Velocities.—Wave velocities for the reach from Savannah to Buchanan are computed by the same formulas as are used for the Clinch River waves. These results, together with observed velocities determined from the beginning-of-rise, midpoint-of-rise, and beginning-of-fall points, are listed in Table 8.

TABLE 7.—LOWER TENNESSEE RIVER STATIONS

Station (see Fig. 8)	River mile	Elevation[a]	Drainage area (sq miles)
Pickwick lower lock	206.4	350.0	32,820
Savannah	189.9	341.61	33,140
Perryville	134.9	325.27	34,500
Johnsonville (near)	100.5	319.61	38,520
Buchanan	66.2	303.0	39,720
Gilbertsville	21.7	299.56	40,200
Ohio River	0	....	40,910

[a] Elevation of gage zero above Mean Gulf Level.

Discharge ratings, area, and channel width curves for each station are given in Table 9. The Pickwick lower lock to Savannah reach was excluded from the velocity comparison because the channel and discharge character at neither of these stations are representative of the intervening reach, and the Buchanan to Gilbertsville reach was excluded because points on very few waves can be identified satisfactorily at Gilbertsville due to the effect of the Ohio River. The method of applying formulas for wave velocities is similar to that previously described for waves on the Clinch River.

Computed wave velocities c_1 and c_2 for the beginning-of-rise point are generally lower than the observed velocity c_0. On the average c_0 is about 1.5 times c_1 and 1.4 times c_2. However, the variation is too large to be considered as a consistent relation. For the midpoint-of-rise, computed velocities in general agree much better with the observed. There is also better agreement between c_1 and c_2 than in computed velocities for the beginning-of-rise points. For beginning-of-fall points, the agreement between computed and observed velocities is about the same as for the midpoint-of-rise.

A study of velocities of midpoint-of-fall shows no agreement between observed and computed velocities, the computed velocities being much higher than the observed. When a falling wave travels down the lower Tennessee River, its shape is gradually modified until it finally becomes a recession that follows the characteristic curve of water draining out of the channel.

Factors Affecting Results.—During periods of rapidly changing stage, such as exist during the passage of a wave, the slope of the water surface and the discharge differ considerably from the steady flow conditions. Due to the flat slopes that normally exist on the lower Tennessee River, these variations have a relatively greater effect on the discharge than on the Clinch River which has a steeper slope. This factor accounts in part for results being less satisfactory for the lower Tennessee River than for the Clinch River, and would be most pronounced on the beginning-of-rise where the poorest agreement was found.

Rising waves on the lower Tennessee River have a tendency to flatten and lengthen at successive downstream stations in contrast to the tendency of Clinch River rising waves to maintain their height and to shorten. Falling waves in both rivers have a tendency to recede on a characteristic pattern which depends on the volume of the river channel.

The wide variation and lack of consistency in observed velocities are due partly to inability to identify selected points on the wave with any degree of certainty, particularly at the two downstream stations. The midpoint-of-rise can be determined with the most certainty when the wave consists of one continuous rise; but unfortunately this occurs in very few waves.

TABLE 8.—TRANSLATORY WAVES IN THE TENNESSEE RIVER FROM SAVANNAH (MILE 189.9) TO BUCHANAN (MILE 66.2)

WAVE		SAVANNAH		WAVE VELOCITY			BUCHANAN	
No.	Date	Z_1	Z_2	c_0	c_1	c_2	Z_1	Z_2
	(a) VELOCITIES FOR BEGINNING-OF-RISE POINTS							
	1938							
1	May 19	5.2	10.6	6.0	3.9	4.2	9.5	12.8
2	May 23	4.8	12.5	6.1	4.0	4.5	10.3	13.8
3	May 25	7.7	11.7	5.4	4.2	4.6	13.6	15.5
4	July 18	7.7	16.4	6.8	4.3	4.8	11.3	15.0
5	July 23	3.1	22.2	4.5	3.7	4.9	10.0	21.5
6	Dec. 16	4.1	6.7	6.4	3.7	3.9	8.0	9.0
	1939							
7	Jan. 11	9.1	12.9	7.2	4.6	5.1	13.4	14.2
8	Jan. 13	6.2	16.4	7.2	4.5	5.1	14.5	22.1
	(b) VELOCITIES FOR MIDPOINTS-OF-RISE							
	1938							
1	May 19	5.2	10.6	4.4	4.4	4.2	9.5	12.8
5	July 23	3.1	22.2	4.3	5.0	4.9	10.0	21.5
6	Dec. 16	4.1	6.7	4.2	3.8	3.9	8.0	9.0
	1939							
8	Jan. 13	6.2	16.4	4.2	5.0	5.1	14.5	22.1
	(c) VELOCITIES FOR BEGINNING-OF-FALL POINTS							
	1938							
1	May 20	10.6	4.8	5.1	4.6	4.3	12.8	10.3
2	May 24	12.5	7.8	3.9	4.8	4.9	13.8	13.6
4	July 18	16.3	9.4	4.5	5.0	5.1	15.0	13.8
6	Dec. 17	6.7	4.1	3.9	4.0	3.9	9.0	8.1
	1939							
7	Jan. 11	12.9	6.2	5.1	4.8	5.0	14.2	14.0
8	Jan. 16	17.4	10.4	8.7	5.4	5.2	22.1	17.0

CHARACTERISTICS OF TRANSLATORY WAVES

When a rising wave is developed by a fairly uniform increase in discharge, as is the case in most of the Clinch River waves, the rising face of the wave tends to retain the initial rate of increase in discharge at successive downstream stations. When the wave develops on an empty channel, as in Fig. 2(*c*), the beginning-of-rise continues to be abrupt at successive stations; but when the wave develops on a considerable initial flow, as in Fig. 2(*a*), a fillet develops at the beginning-of-rise which enlarges as the wave travels downstream.

A similar fillet occurs as the wave approaches fully developed rise. As the wave progresses downstream these two fillets continue to enlarge until the upper limb of the beginning-of-rise fillet meets the downward limb of the end-of-rise fillet. Although these fillets are enlarging, the duration of rise at the initial rate becomes shorter and finally disappears. On the Clinch River (see Fig. 1) this seems to occur at some point above Wheat, 65 miles below Norris, and on the lower Tennessee (see Fig. 8) at a point above Perryville, 71 miles below Pickwick.

TABLE 9.—RATINGS, CHANNEL AREAS, AND VELOCITIES,

Gage height Z (ft)	Pickwick			Savannah			Perryville	
	Discharge Q (cu ft per sec)	Channel area A (sq ft)	c_1 (miles per hr)	Discharge Q (cu ft per sec)	Channel area A (sq ft)	c_1 (miles per hr)	Discharge Q (cu ft per sec)	Channel area A (sq ft)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
2	6,000	3,620	2.2	14,000	10,200	3.4	....	....
4	12,800	5,800	2.5	21,000	11,500	3.8	11,900	18,000
6	21,500	8,000	2.8	28,750	12,700	4.1	18,200	19,200
8	31,500	10,300	3.1	37,500	14,000	4.5	25,600	20,400
10	42,400	12,500	3.3	46,500	15,300	4.8	34,200	21,800
12	54,400	14,910	3.6	55,500	16,700	4.9	43,600	23,000
14	66,400	17,200	3.8	65,500	18,100	5.0	54,000	24,300
16	78,400	19,570	3.9	75,500	19,500	5.0	64,700	25,700
18	90,600	21,900	4.0	86,000	21,000	5.0	76,000	27,000
20	102,800	24,290	4.1	97,000	22,400	5.0	87,600	28,400
22	115,200	26,800	4.2	108,000	23,800	4.9	100,000	29,800
24	127,600	29,060	4.3	119,000	25,300	4.9	113,000	31,200

In the case of a falling wave, a characteristic recession tends to develop regardless of the initial rate of reduction in discharge. This recession is retained at successive stations on the Clinch River until fillets at the beginning-of-fall and the end-of-fall enlarge to cover the entire falling face of the wave. This again appears to occur by the time that waves reach Wheat or Perryville.

When the period of maximum discharge at the crest of a wave continues for only a short period, as in Fig. 2(*c*), the spreading fillets at the end-of-rise and at the beginning-of-fall gradually shorten this period as the wave progresses downstream. Eventually the two fillets will overlap and the discharge will begin to fall off before the full rise has been developed. When this occurs flattening of the wave begins and the magnitude of rise becomes progressively smaller. A valley between two rises is affected in a similar manner, resulting in a gradual reduction in the depth of the valley as is shown in Figs. 2(*e*) and 2(*f*).

CONCLUSIONS

1. Velocities of translatory waves, both rising and falling, can be approximated with either Eq. 1*a* or Eq. 1*b* in natural channels such as the Clinch and

lower Tennessee rivers when rated stations at representative sections are located at suitable intervals throughout the reach.

2. Midpoints of rise or fall are best suited for determining the velocity of an observed wave.

3. Eqs. 2*a* and 2*b* give similar results for velocities of translatory waves in Wheeler Reservoir, but these results are consistently higher than observed velocities.

OWER TENNESSEE RIVER STATIONS (SEE FIG. 8)

	JOHNSONVILLE			BUCHANAN			Gage height Z (ft)
c_1 (miles per hr)	Discharge Q (cu ft per sec)	Channel area A (sq ft)	c_1 (miles per hr)	Discharge Q (cu ft per sec)	Channel area A (sq ft)	c_1 (miles per hr)	
(10)	(11)	(12)	(13)	(14)	(15)	(16)	(1)
....	12,500	11,000	2.7	....	....	....	2
3.4	21,250	13,200	3.2	7,000	16,300	....	4
4.0	31,750	15,200	3.6	14,500	18,200	2.8	6
4.5	44,000	17,300	3.9	23,500	20,000	3.2	8
4.9	57,000	19,400	4.2	33,000	21,900	3.7	10
5.2	70,000	21,600	4.4	44,500	23,800	4.0	12
5.4	84,000	23,800	4.5	56,500	25,800	4.3	14
5.6	98,000	25,900	4.7	69,500	27,800	4.6	16
5.8	112,000	28,100	4.9	83,000	29,900	4.7	18
6.0	126,000	30,200	5.0	96,000	31,800	4.9	20
6.2	142,000	32,500	5.1	112,000	33,900	5.0	22
6.3	158,000	34,800	5.1	127,000	36,000	4.9	24

4. Within the range of these data the volume of flow appears to have little effect on the velocity of translatory waves in a reservoir.

ACKNOWLEDGMENTS

Acknowledgment for assistance in the preparation of this paper is made to A. J. Cooper, Jr., and T. J. Nordenson, Juniors, Am. Soc. C. E., as well as to the other members of the River Forecasting Section, Hydraulic Data Division, Tennessee Valley Authority.

DISCUSSION

HAROLD A. WEGGEL,[4] JUN. AM. SOC. C. E.—A noteworthy detailed study of translatory waves is presented in this paper; yet in a positive sense the conclusions enumerated do not seem to compensate for the effort expended. It would be desirable to go one step further than simply to prove or disprove the validity of the use of certain theoretical formulas. Some of the conclusions have been known since 1842–1843 when John Scott Russell presented the "Report on Waves"[5] to the British Association for the Advancement of Science; and later experimenters have observed the remaining conclusions. However, very few observers have been able to apply corrections for change in slope of water surface, friction, and channel storage. The data in the paper seem to indicate that the channel characteristics could be determined for the particular waves studied so that corrections could be applied to the theoretical formulas.

The velocity of the wave is affected by the slope of the original water surface, being greater for steep channels and less for channels of flat slope. In the limiting condition of steep slope, the velocity of a small wave is twice the mean velocity of the channel, and, in the other extreme of a flat slope, the velocity of the wave is equal to the surface velocity of the channel.[6,7]

Theoretically, the wave velocity in quiet water is $\sqrt{g\,y_m}$ but experiments by A. Zeitlinger[8] at the Hydraulic Laboratory in Graz, Austria, showed that, in shallow water, friction caused a decrease in the velocity of the wave to

$$c = 0.91 \sqrt{g\,y_m} \qquad (3)$$

Perhaps a correction factor similar to Eq. 3 would make the results of Eq. 2*b* comparable with, instead of consistently higher than, observed velocities.

In a similar manner, Eq. 1*b* should be changed to

$$c_2 = V_1 + \frac{(V_2 - V_1)\,A_2}{A_2 - A_1} \qquad (4)$$

Eq. 4 separates the superimposed wave from the initial velocity V_1 so that a correction factor for wave friction may be applied. It is apparent from Figs. 4(*b*) and 4(*d*) that this friction factor must be relatively constant over a wide range for the midpoint-of-rise and for the midpoint-of-fall. In the solitary wave of translation these points are the only parts of the wave that really travel as a wave front or movement of constant flow or wave height.

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[4] Associate Structural and Hydr. Engr., Havens and Emerson, Cleveland, Ohio.

[5] "Report on Waves," by J. Scott Russell, *Report of the Fourteenth Meeting of the British Association for the Advancement of Science*, London, 1844, p. 311.

[6] "A Study in Flood Waves," by Elmer E. Moots, thesis presented to the State University of Iowa in Iowa City in February, 1927, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

[7] Letter to the editor of *Civil Engineering* from L. K. Sherman, discussing a paper, "Translatory Waves in Open Channels," by H. W. King, *Civil Engineering*, August, 1933, p. 473.

[8] "Dam-Break Waves," by Armin Schoklitsch, Mathematisch-Naturwissenschaftliche Classe, Vol. 126, 2A, 1917, pp. 1489–1514.

In the case of translatory waves on rivers of flat slope, such as the lower section of the Tennessee River, it is realized that some of the difficulties of the problem are practically insurmountable due to channel storage. Nevertheless, it may be possible to obtain some empirical coefficient which would give closer agreement between formulas and observed data on translatory waves.

Perhaps a study of the reflected waves in the Decatur section (see Fig. 5) may be helpful in an approximate determination of a factor to be applied to the velocity of the channel, since a difference of 5 miles per hr was noted for the wave travel (see heading, "Translatory Waves in Wheeler Reservoir: Velocities of Translatory Waves in Wheeler Reservoir"). The time of surges in Wheeler Reservoir, about 11 hr over a distance of 2 × 74 miles, or 148 miles, gives 13.5 miles per hr, or the average velocity downstream only. Do the surges travel upstream at the same rate or at some rate differing by a factor of channel velocity, such as that indicated by the equation:[9,10]

$$c = K \sqrt{g\, y_m} \pm \left(\tfrac{1}{4}\, V_1 + \tfrac{3}{4}\, V_2\right) \qquad (5)$$

A solution of these perplexing problems would do much to enhance the knowledge of the variations in velocities of translatory waves.

W. M. LANSFORD,[11] M. AM. SOC. C. E.—The problem of translatory waves is of much importance to hydraulic engineers, and the author has presented interesting and valuable data on the subject.

A few years ago the writer conducted some experiments on translatory waves in a rectangular channel in the Hydraulics Laboratory at the University of Illinois at Urbana. The channel used was made of concrete 5 ft wide and 4.5 ft deep and had a straight horizontal length between head gate and tail gate of 147 ft. The tail gate was used to control the depth and the head gate was used to help control the discharge. A vertical steel plate ½ in. thick which fitted into grooves cut in the sides of the channel was dropped across the channel to stop the flow, thereby causing a wave to travel upstream. This gate was placed 7 ft upstream from the tail gate.

The velocity of the wave was measured over a 114.6-ft length of the channel by means of floats connected by electric contacts to a pen which recorded on a revolving drum. The water used was taken from a standpipe from which approximately 6,500 gal per min could be supplied at a constant head of about 65 ft.

The observed wave velocities were compared with the theoretical values as computed from the formula proposed by Professor King[12,13] for abrupt waves in

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[9] "Wave Movements," by Hubert Engels, in "Handbuch des Wasserbaues," I Band, W. Engelmann, Leipzig and Berlin, 1914, pp. 125–138.

[10] "Traveling Waves," by Alexander Koch and Max Carstanjen, in "Bewegung des Wassers," Julius Springer, Berlin, 1926, pp. 132–150.

[11] Associate Prof., Theoretical and Applied Mechanics, Talbot Laboratory, Univ. of Illinois, Urbana, Ill.

[12] "Translatory Waves in Open Channels," by H. W. King, *Civil Engineering*, June, 1933, Eq. 8, p. 320.

[13] "Handbook of Hydraulics," by H. W. King, 3d Ed., 1939, pp. 421–422.

open channels. Professor King's formula is

$$c_w = \pm \sqrt{\frac{g\,\bar{y}_1}{2\,\bar{y}_2}(\bar{y}_2 + \bar{y}_1)} + V_1 \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

TABLE 10.—TRANSLATORY WAVES IN A RECTANGULAR CHANNEL

A.—DISCHARGE CONSTANT					*B.*—INITIAL VELOCITY CONSTANT					*C.*—DEPTH CONSTANT				
Test No.	Depth $\bar{y}_1$ (ft)	Velocities (Ft Per Sec)			Test No.	Discharge (cu ft per sec)	Depth $\bar{y}_1$ (ft)	Wave Velocities (Ft Per Sec)		Test No.	Discharge (cu ft per sec)	Velocities (Ft Per Sec)		
		Flow (initial)	Wave									Flow (initial)	Wave	
			Observed	Computed				Observed	Computed				Observed	Computed
(*a*) Q = 3.5 Cu Ft Per Sec					(*a*) V_1 = 0.50 Ft Per Sec					(*a*) $\bar{y}_1$ = 2.10 Ft				
71	1.2	0.58	6.19	6.13	40	3.5	1.40	6.22	6.66	81	3.00	0.29	8.02	8.20
72	1.4	0.50	6.55	6.66	41	4.0	1.60	6.93	7.01	82	3.50	0.34	8.13	8.20
73	1.8	0.39	7.14	7.59	42	4.5	1.80	7.40	7.52	83	4.00	0.38	7.90	8.18
74	1.7	0.40	7.55	7.30	43	5.0	2.00	7.52	7.88	84	4.50	0.43	7.90	8.15
75	2.0	0.35	7.78	7.96	44	5.5	2.20	8.40	8.35	85	5.0	0.48	7.75	8.11
76	2.2	0.32	8.12	8.33	45	6.0	2.40	8.79	8.69	86	5.5	0.52	7.87	8.13
77	2.3	0.30	8.25	8.56	46	6.5	2.60	8.76	9.04	87	6.0	0.57	8.03	8.10
78	2.4	0.28	8.61	8.73	47	7.0	2.80	9.14	9.40	88	6.5	0.62	8.07	8.17
79	2.6	0.27	9.15	9.00	48	7.25	2.90	9.50	9.55	....	....	....	....	....
80	2.8	0.25	9.00	9.45	49	7.5	3.00	9.43	9.71	....	....	....	....	....
(*b*) Q = 5.0 Cu Ft Per Sec					(*b*) V_1 = 0.60 Ft Per Sec					(*b*) $\bar{y}_1$ = 1.50 Ft[a]				
50	1.40	0.71	6.68	6.74	30	6.88	1.28	6.32	6.42	11	3.83	0.51	6.86	6.84
51	1.60	0.63	7.11	7.08	31	4.90	1.63	7.05	7.19	12	4.48	0.60	6.83	6.79
52	1.80	0.56	7.43	7.43	32	5.42	1.81	7.37	7.61	13	4.90	0.65	6.72	6.85
53	2.0	0.50	7.80	7.89	33	6.1	2.03	7.85	7.97	14	5.42	0.71[a]	6.78	6.79
54	2.2	0.45	8.29	8.29	34	6.6	2.20	8.32	8.37	15	6.10	0.80[a]	6.78	6.74
55	2.4	0.42	8.53	8.73	35	7.2	2.40	8.44	8.62	16	6.55	0.87[a]	6.73	6.81
56	2.5	0.40	8.95	8.91	36	7.8	2.60	8.92	8.98	17	6.77	1.11	6.70	6.65
57	2.6	0.39	8.81	9.08	37	8.4	2.80	9.13	9.34	18	7.55	1.01	6.70	6.75
58	2.7	0.37	9.37	9.29	38	8.7	2.90	9.43	9.65	19	8.10	1.08	6.64	6.72
59	2.9	0.35	9.55	9.54	39	9.0	3.00	9.43	9.70	20	8.60	1.15	6.77	6.72
60	3.0	0.33	9.55	9.81	....	....	....	....	....	....	....	....	....	....
(*c*) Q = 7.25 Cu Ft Per Sec					(*c*) V_1 = 0.70 Ft Per Sec					(*c*) $\bar{y}_1$ = 1.30 Ft				
3	1.07	1.36	6.04	6.56	61	4.2	1.20	5.94	6.22	21	3.50	0.54	6.32	6.23
4	1.39	1.04	6.74	6.65	62	4.9	1.40	6.62	6.60	22	4.53	0.70	6.40	6.40
5	1.76	0.82	7.62	7.57	63	5.3	1.50	7.10	6.80	23	5.30	0.81	6.31	6.31
1	2.06	0.70	8.14	8.14	64	5.6	1.6	6.92	6.99	24	5.81	0.90	6.33	6.36
6	2.19	0.66	8.22	8.27	65	5.95	1.7	7.20	7.20	25	6.18	0.95	6.33	6.34
7	2.42	0.60	8.75	8.85	66	6.3	1.8	7.52	7.51	26	6.90	1.06	6.32	6.31
8	2.59	0.56	9.05	9.14	67	7.0	2.0	8.02	7.92	27	7.30	1.12	6.29	6.26
9	2.91	0.50	9.46	9.65	68	7.7	2.2	8.03	8.21	28	7.70	1.19	6.26	6.30
10	3.07	0.47	10.32	9.93	69	8.4	2.4	8.54	8.60	29	8.10	1.25	6.20	6.23
....	....	....	....	....	70	9.1	2.6	8.65	8.94	....	....	....	....	....

[a] Exceptions: Test No. 14, $\bar{y}_1$ = 1.52; Test No. 15, $\bar{y}_1$ = 1.52; and Test No. 16, $\bar{y}_1$ = 1.51.

in which c_w is the velocity of the wave, in feet per second; V_1 is the initial velocity of the water, in feet per second; and $\bar{y}_1$ and $\bar{y}_2$ are the depths in feet before and after the wave, respectively. The results of these tests are given in Table 10.

The results given in Table 10*A* are for tests in which the discharge was maintained at a constant value. In Table 10*B* the initial velocity was kept constant, and in Table 10*C* the depth $\bar{y}_1$ was kept constant. In all the tests the percentage difference between observed and computed values of the wave velocities is small except in extreme cases of very low or very high values. The effect of the depth $\bar{y}_1$ seems more pronounced than it should because of the limited range over which these tests were made.

Although the data presented by the author in Table 9 show a rather wide variation between observed values of wave velocity and values computed by use of Eq. 2*a*, the data in Table 10 seem to indicate a fairly good agreement between these values. As mentioned by the author, the widening of the stream very likely accounts for some of the discrepancies. Within the limits of the tests herein presented Eq. 6 seems to yield reliable results.

M. A. CHURCHILL,[14] ASSOC. M. AM. SOC. C. E.—The Society is indebted to the author for the presentation and interpretation of a wealth of observed, not theoretical, data on translatory waves in natural channels. In an attempt to make the paper of still greater value, the writer presents herewith some supporting data on observed waves in natural channels and in reservoirs, together with corresponding data on observed rates of water travel.

TABLE 11.—SPEED OF TRAVEL AS OBSERVED, IN 1943, ON THE LOWER HOLSTON RIVER IN TENNESSEE

STATION		Distance, in miles	(*a*) WAVE TRAVEL, FALLING FACE; NOV. 7			(*b*) WAVE TRAVEL, RISING FACE; NOV. 8			(*c*) WATER TRAVEL; NOV. 12 AND 13[a]	
Location	Mile		Clock time at midpoint of fall or rise	Time in hours	Observed rate[b]	Clock time at midpoint of fall or rise	Time in hours	Observed rate[b]	Time in hours	Observed rate[b]
Below Cherokee Dam	52.0		12:10 a.m.			12:01 a.m.				
		18.8		4.50	4.17		5.33	3.52	8.25	2.28
Nances Ferry	33.2		4:40 a.m.			5:20 a.m.				
		16.2		4.67	3.47		3.83	4.22	9.00	1.80
Straw Plains	17.0		9:20 a.m.			9:10 a.m.				
		11.6		2.50	4.63		3.00	3.87	5.50	2.11
Highway 70 Bridge	5.4		11:50 a.m.			12:10 p.m.				
		46.6		11.67	(4.00)[c]		12.16	(3.83)[d]	22.75	(2.05)

[a] Average flow was 7,900 cu ft per sec. [b] In miles per hour. [c] Theoretical wave travel rate, 3.88 miles per hr. [d] Theoretical wave travel rate, 3.95 miles per hr.

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[14] Hydr. Engr., TVA, Knoxville, Tenn.

Wave Travel in Lower Holston River.—In connection with a determination of the time required for water to travel 46.6 miles from Cherokee Dam on the Holston River to a point near the head of Fort Loudoun Reservoir above Knoxville, Tenn., the velocity of a negative wave of translation was also observed. The negative wave was produced by reduction for a period of 24 hr in the discharge of Cherokee Dam on November 7, 1943, from 7,530 cu ft per sec to 3,840 cu ft per sec, and then an abrupt increase to 7,740 cu ft per sec. This negative wave was observed at four stations on the Holston River below Cherokee Dam as shown by Fig. 10(*a*) (stages) and Fig. 10(*b*) (discharges). Stages were observed at all four stations, but discharges are known only at the station immediately below Cherokee Dam and at Straw Plains. (Stages below Cherokee Dam and at Straw Plains are from continuous recorder charts. Stages at Nances Ferry and at Highway 70 Bridge are from staff gage readings at intervals of 2 hr or less.) Discharges at Nances Ferry and at Highway 70 Bridge were approximated. Table 11 gives pertinent data on this negative wave. The theoretical rates of travel (footnotes (^{c}) and (^{d}), Table 11) were computed by Eq. 1*b*, based on discharges and areas at the two gaging stations in the reach. It is recognized that the relatively good agreement between observed and computed wave velocities, when the computed velocities are based on only two, relatively short, rated river reaches, is "more luck than sense." The

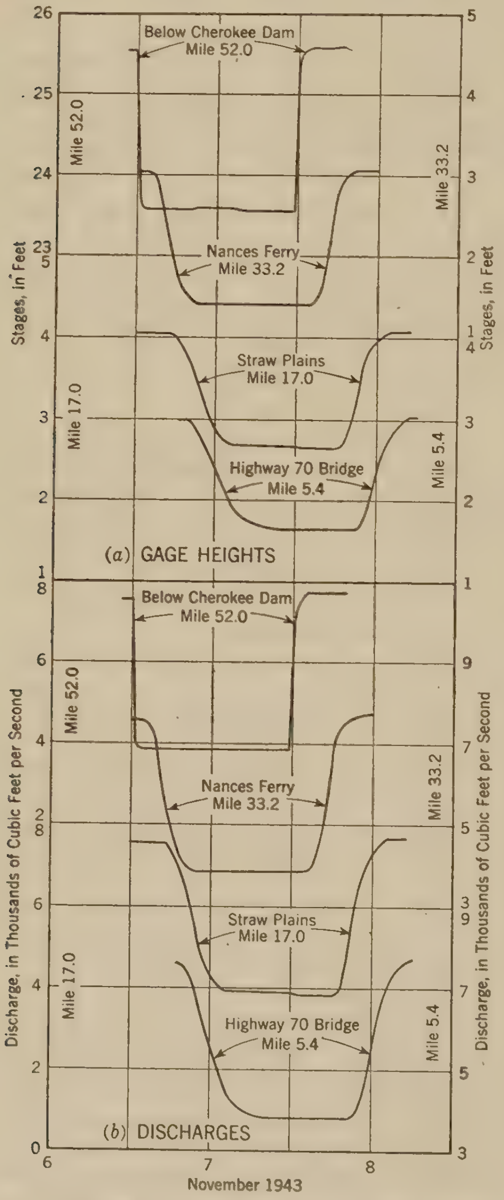

FIG. 10.—STATION OBSERVATIONS ON THE LOWER HOLSTON RIVER, TENNESSEE, NOVEMBER 7 AND 8, 1943

author necessarily based his comparisons of observed and computed rates on similar data and so it is rather remarkable that his results on Clinch River show such good agreement with the theory.

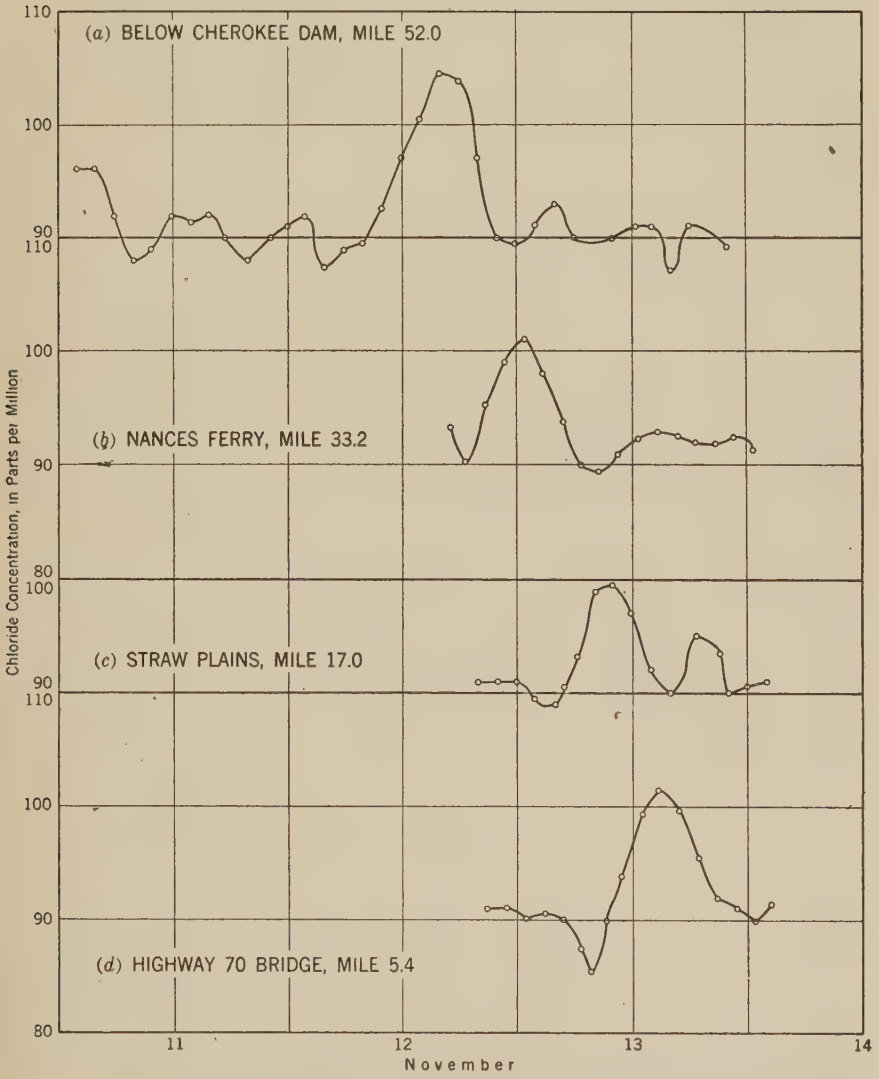

Fig. 11.—Chloride Concentrations at Observation Stations on the Lower Holston River, November 12 and 13, 1943

Water Travel in Lower Holston River.—The rate of water travel, as distinguished from wave travel, through the same reach of river was observed between November 12 and 14, 1943, during which time the release from

Cherokee Dam was nearly the same as both before and after the negative wave generated on November 7, 1943. (The release from Cherokee Dam on November 12 increased gradually from 7,770 cu ft per sec at 1:00 a.m. to 8,070 cu ft per sec at midnight.) To determine the rate of water travel, samples were collected at 2-hr intervals from the same four points at which the wave was observed. These samples were analyzed chemically for chloride concentration, with results as shown in Fig. 11. The presence in the river of an industrial waste of varying chloride concentration made possible the use of the chloride concentrations for identifying a particular mass of water at the successive downstream stations. Table 11(*c*) gives pertinent data on the rate of water travel.

Ratio of Wave Velocity to Water Velocity, Lower Holston River.—A comparison of the rate of wave travel with the rate of water travel in the lower Holston River indicates that the rate of wave travel is slightly more than twice the rate of water travel, for the particular rates of discharge observed. This ratio tends to support the author's contention (see heading, "Translatory Waves on the Clinch River: Velocity of a Translatory Wave") that "The water velocity is about half the wave velocity at ordinary stages * * *." In fact, the data presented in this discussion on the relation of wave to water velocities are probably better proof of this statement (as it applies to the general case) than the data presented by the author, since the author used water velocities at only four points in the 64.2-mile reach of the Clinch River between the Norris and Wheat gages.

Wave and Water Travel, Watauga and South Holston Rivers.—Observations have been made of the time required for water and for waves of translation to travel through a reach of 40 miles on the Watauga and South Holston rivers. The waves were produced by variations in release from a power dam at the head of the reach. Discharges varied from 200 to 1,300 cu ft per sec. The average time for a series of waves was determined to be 16 hr. Water travel over the same reach required 91 hr.

Comparison of wave-travel and water-travel rates shows that, for the particular rates of discharge existing, the waves traveled 5.7 times as fast as the water.

This ratio of wave to water velocity is rather high for a natural river channel, particularly considering the rather wide range of flow between crests and troughs of the various waves. As the author states, however, the ratio will be higher for low discharges, and during these observations the average rate of flow was about one third of the mean annual rate. The existence of a relatively large number of long, deep pools in the lower Watauga and South Fork Holston rivers could explain the high ratio. In any case, the wide difference in the relation of wave to water velocities found here, as compared with the relation found on the lower Holston River, indicates the possibilities of error in trying to use wave velocities as a basis for computing water velocities without rather complete data on physical conditions of the reaches involved.

Wave and Water Travel, Fort Loudoun Reservoir.—Observations of wave travel through Fort Loudoun Reservoir (42.8 miles) showed an over-all wave velocity of 21.4 miles per hr. The theoretical rate of wave travel was computed by Eq. 2*b*. The mean depth of the reservoir was found to vary essentially as a straight line from 14 ft at the upper gage to 48 ft at the dam, when the pool elevation is 811.5. The wave velocity, except for the influence of the water velocity, can be expressed as $\sqrt{g\,y_m} = \frac{dL}{dt}$; or:

$$dt = \frac{dL}{\sqrt{g\,y_m}} \qquad \text{(7)}$$

Therefore,

$$t = \int_0^L \frac{dL}{\sqrt{g\,y_m}} = 0.176 \int_0^{226{,}000} \frac{dL}{\sqrt{14 + 0.0001504\,L}} \qquad \text{(8)}$$

and $t = 0.176 \left(\frac{2\sqrt{14 + 0.0001504\,L}}{0.0001504} \right)_0^{226{,}000} = 7{,}460 \text{ sec} = 2.08 \text{ hr.}$

A total time of 2.08 hr represents an average velocity through the 42.8-mile (L = 226,000 ft) reach of reservoir of 20.6 miles per hr. Due to thermal stratification and a density underflow, water velocities on the surface existed only in the upper end of the reservoir. Therefore, V_1 in Eq. 2*b* was essentially zero. The theoretical wave velocity is thus computed to be 20.6 miles per hr as compared with the observed velocity of 21.4 miles per hr.

Measurements of actual water travel by chemical identification methods indicate that a period of 190 hr is required through the 42.8 miles of reservoir. This rate of travel is 0.22 mile per hr. Had the moving water occupied the full depth of the reservoir throughout the entire length of the pool, the water velocity would have been even slower than the observed value.

The wave velocity was 95 times as fast as the water velocity, over-all. This ratio explains why the author's conclusion is true that the water velocity has little influence on the wave velocity in a reservoir.

Basic Concepts of the Relation of Wave to Water Velocity.—In an attempt to get a clear understanding of exactly how a translatory wave could run ahead of the water which originally formed it, the writer formulated the following relations several years ago:

Assume a smooth, straight natural channel in which the flow is gradually increased from a steady flow of Q_1 to a higher flow Q_2. Under such conditions, a profile of the channel and water surface would appear as shown in Fig. 12. Since the hydraulic radius is greater for the higher flow Q_2, the velocity V_2 is greater than V_1, assuming that Q_2 is less than bankfull discharge. Since this is the case, water moving at mean velocity V_2 tends to overtake water moving at mean velocity V_1. This action produces a result similar to an express train crashing into the rear end of a slow freight. In that case the rear cars of the

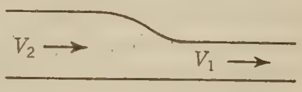

FIG. 12.—TRANSLATORY WAVE IN A CHANNEL OF UNIFORM WIDTH

freight are telescoped by the express. In the river, the slower moving water is "telescoped" by that moving at velocity V_2. The quantity of water per second so "telescoped" is expressed mathematically by $(V_2 - V_1) A_1$, in which A_1 is the cross-sectional area corresponding to V_1. This quantity of water is actually squeezed out of its original position in the river channel and is forced into that portion of the channel represented by $A_2 - A_1$. This quantity of water is thereby added each second in front of that advancing at the velocity V_2. Therefore, the rate of advance of the wave front is $\frac{(V_2 - V_1) A_1}{A_2 - A_1}$, relative to V_2. The actual rate of advance of the face of the wave is $V_2 + \frac{(V_2 - V_1) A_1}{A_2 - A_1}$. From this expression it is obvious that the wave velocity is always greater than the water velocity, provided the channel has an initial flow in it, since the second term is always positive. By simple algebra, this expression for the wave velocity can be shown to be equal to the familiar expression, Eq. 1*b*, except for the constant used to convert feet per second to miles per hour.

Wave-Height Reduction.—The author explains what happens to a translatory wave produced by a release of short duration, when its height is reduced (heading, "Characteristics of Translatory Waves"), without explaining the reasons for these happenings. The author states that "* * * the spreading fillets at the end-of-rise and at the beginning-of-fall gradually shorten this [crest] period as the wave progresses downstream." It seems to the writer that, in addition to any so-called "channel-storage" effects, the basic reasons why the crest of a wave of short duration is gradually lowered are as follows: As the wave moves downstream, it pushes the water already in the channel ahead of it, as explained previously herein, and forces this water into that area of the channel represented by $A_2 - A_1$. However, this pushed water is not spread uniformly over the area $A_2 - A_1$, but rather it tends to resist movement and stay near the lower edge of this area, where the channel width is the narrowest. This nonuniformity tends to push the beginning-of-rise point out farther and farther ahead of the crest of the rising wave. As the nose runs farther ahead, the rising face of the wave is thereby flattened; only a little water is added to the rising face of the wave, at and near the crest.

Although little water is added on to the forward end of the crest, a considerable quantity is removed from the rear. Since water at the crest stage moves at a velocity higher than that at the lower stage to the rear, a quantity of water equal to $(V_2 - V_3) A_3$ is removed from the rear face of the wave each second. This constant drain on the rear face of the wave eats it away and thus shortens the crest length until finally the original crest is all gone. Then the remaining crest is still further flattened until finally no wave exists.

An additional reducing influence is the fact that, due to the increased slope of the rising face, the water composing it tends to run faster than the falling face where the slope is flatter. Thus the wave tends to stretch out in length at the base, and consequently is lowered at the crest.

J. H. Wilkinson,[15] M. Am. Soc. C. E.—The comments and suggestions presented in the discussion of this paper are appreciated greatly. Mr. Weggel suggests that a correction factor provides a method for increasing the agreement between observed and computed velocities. This should be useful for predicting wave travel from data available on a given stream.

The writer is not entirely certain that, referring to the midpoint-of-rise and the midpoint-of-fall,

> "In the solitary wave of translation these points are the only parts of the wave that really travel as a wave front or movement of constant flow or wave height."

In connection with Wave 3, Fig. 2(*b*), if the increase in release at Norris Dam had stopped at 10:00 a.m., on December 12, a wave with $Z_2 = 6.0$ ft would have developed at the Norris gage. This phenomenon occurred in the case of Wave 5. Below the 6.0-ft stage, Wave 3 is practically identical to Wave 5. The portion of Wave 3 below the 6.0-ft stage travels as a wave front as much as does the portion of Wave 3 below the 8.0-ft stage. In a natural channel, the height of a wave varies with the characteristics of the river channel. Wave 3 rose 5 ft at the Norris gage, 5 ft at the Edgemoor gage, and 9 ft at the Wheat gage. At each station, the depth increased to that necessary to discharge the increased flow. Wave heights are constant only in a uniform channel.

When the midpoint-of-rise is used to determine the velocity of a wave in a natural channel, it is assumed that this is an identical point on the wave as it passes successive stations—that half the rise represents the same increase in discharge at successive stations. This is seldom exactly true.

Professor Lansford presents experimental results which are in excellent agreement with computed velocities by Eq. 6 for the entire range of depths and flows. These experiments eliminate the uncertainties of variable channel areas and slopes and of local inflow. The results show that the approximate Eq. 2*b* cannot be depended upon for accurate results.

Mr. Churchill presents some interesting data on wave and water velocities for the lower Holston River. The high chloride concentration provides an excellent means of identifying the wave at successive stations and gives much more dependable results for water velocity than does the process of estimating the wave from measurements at four gaging stations as was the case on the Clinch River. Fig. 10 presents observed stages and discharges throughout both the falling and rising waves. The stage-discharge relation which exists at a gaging station under normal flow does not hold during periods of rapidly changing stage such as exist during the passage of a wave. Discharges can be approximated by the Jones formula,[16] but it seems impracticable to attempt to determine discharge throughout a wave.

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[15] Senior Hydr. Engr., TVA, Knoxville, Tenn.

[16] "A Method of Correcting River Discharge for a Changing Stage," by B. E. Jones, *Water-Supply Paper No. 375–E*, U. S. Geological Survey, 1915, pp. 117–130.

The 5.7 ratio of wave to water velocity for the Watauga and South Holston rivers with a flow one third of the mean annual flow is slightly higher than that for the Clinch River, where the computed ratio for a corresponding flow is 3.8. Probably, the water velocity at gaging stations is somewhat higher than average for this flow.

Conversion of the wave velocity equation into the form of Eq. 7 simplifies the computation. The writer was fortunate in selecting a reservoir having a uniform variation in mean depth and results indicate that he was able to estimate, accurately, the effective mean depth of the reservoir.

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TRANSACTIONS

Paper No. 2255

CONSTRUCTION OF THE FLOW NET FOR HYDRAULIC DESIGN

BY H. ALDEN FOSTER,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. J. W. HOWE, CLARENCE S. JARVIS, AND H. ALDEN FOSTER.

SYNOPSIS

The general principle of the flow net has been known for many years, and its application to problems in hydraulics, electricity, and heat transmission has been described in various publications. The construction of a flow net by customary methods, however, is very tedious and the results are frequently far from precise.

In this paper, certain graphical and analytical methods are presented by means of which the construction of the hydraulic flow net in any particular case can be facilitated greatly, and much greater precision can be attained in the determination of velocities and pressures. The methods are applied both to conditions of fixed boundaries and to cases in which one of the boundaries is exposed to the atmosphere. Their application to certain cases of three-dimensional flow is also explained, together with a specific example.

INTRODUCTION

The usefulness of the flow net in hydraulic design has long been recognized, and graphical methods for constructing such a net have been described (1).[2] The flow net is of particular importance in any design problem involving the determination of pressures occurring under curvilinear flow. Under such conditions, the pressure variation does not follow the hydrostatic law (that pressure increases directly in proportion to depth of water), since the velocity is not uniform across the section. To compute the pressure by Bernoulli's formula, the velocity at any point must be known, and in many problems the only way in which the velocity can be determined with any degree of precision is by a model test or by construction of a flow net. If a flow net can be con-

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NOTE.—Published in May, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Designing Engr., Parsons, Brinckerhoff, Hogan & Macdonald, New York, N. Y.

[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix I).

structed accurately in conformance with certain fundamental conditions, the velocities determined from the flow net should conform closely to those in the actual structure, provided friction and impact losses are of a minor nature.

In the flow-net method as generally presented, emphasis is placed on constructing the net by freehand sketching, with gradual adjustment and correction until the streamlines and potential lines intersect at right angles and form curvilinear squares. This is a laborious process; and even when accurately done the resulting velocity determinations will not be very precise if the streamlines are close together, or if the streamlines are only slightly curved and the variation in velocity across the section is rather gradual. A graphical method of checking the spacing of the streamlines, which also permits a more precise determination of the velocities, is described in this paper. The basis of this method is given by Hunter Rouse, M. Am. Soc. C. E. (2), and by R. L. Daugherty (3). However, so far as the writer knows, the detailed application of the method has not been made generally available to engineers.

NOTATION

The letter symbols used in this paper are defined where they first appear and are assembled for convenience of reference in Appendix II.

TWO-DIMENSIONAL FLOW WITH FIXED BOUNDARIES

Consider first the case of two-dimensional flow within fixed boundaries, as shown in Fig. 1(*a*). A section of the water passage one unit in depth (normal to the plane of the paper) is considered. The radii of curvature of the fixed boundaries are r_s and r_c; these need not be concentric. The width of the water passage, measured along the curved potential line, APC, is d. Assuming that the streamlines have been plotted, as in Fig. 1(*a*), the radius of curvature of the streamline at any point along the line APC can be shown as a curve of r versus y, line $A_1P_1C_1$ in Fig. 1(*c*), y being measured from the outer curved boundary toward the center of curvature as indicated in Fig. 1(*a*). Then the basic theory of the flow net, with uniform energy distribution along the water passage (no friction or impact losses) requires that

$$\frac{V}{r} = \frac{dV}{dy} \qquad \text{.............................(1)}$$

in which V = velocity at any point, P, in the water passage, and r = radius of curvature of the streamline at point P.

A curve, A_2C_2, showing V versus y can be constructed in Fig. 1(*c*) as follows:

A value of unity is assumed for the velocity, V_s, at point A_2, where $y = 0$. Starting at this point, the slope (dV/dy) of the velocity curve will be V_s/r_s, according to Eq. 1. This slope is laid off graphically, as shown on the diagram, by swinging point A_1 along a circular arc of radius r_s, until it reaches the position A_3 on the axis of y. The straight line A_3A_2, extended a short distance beyond A_2 in the direction of y, gives the tangent of the velocity curve at point A_2, in accordance with Eq. 1. For all practical purposes, this piece of tangent may be taken as the true curve for a short distance, such as A_2D_2.

In this way, a new value of V is obtained at point D_2; at which point the process is repeated, using the value of r given by the point D_1 on A_1C_1, directly above point D_2 (construction not indicated in Fig. 1(c)). Thus, a new determination

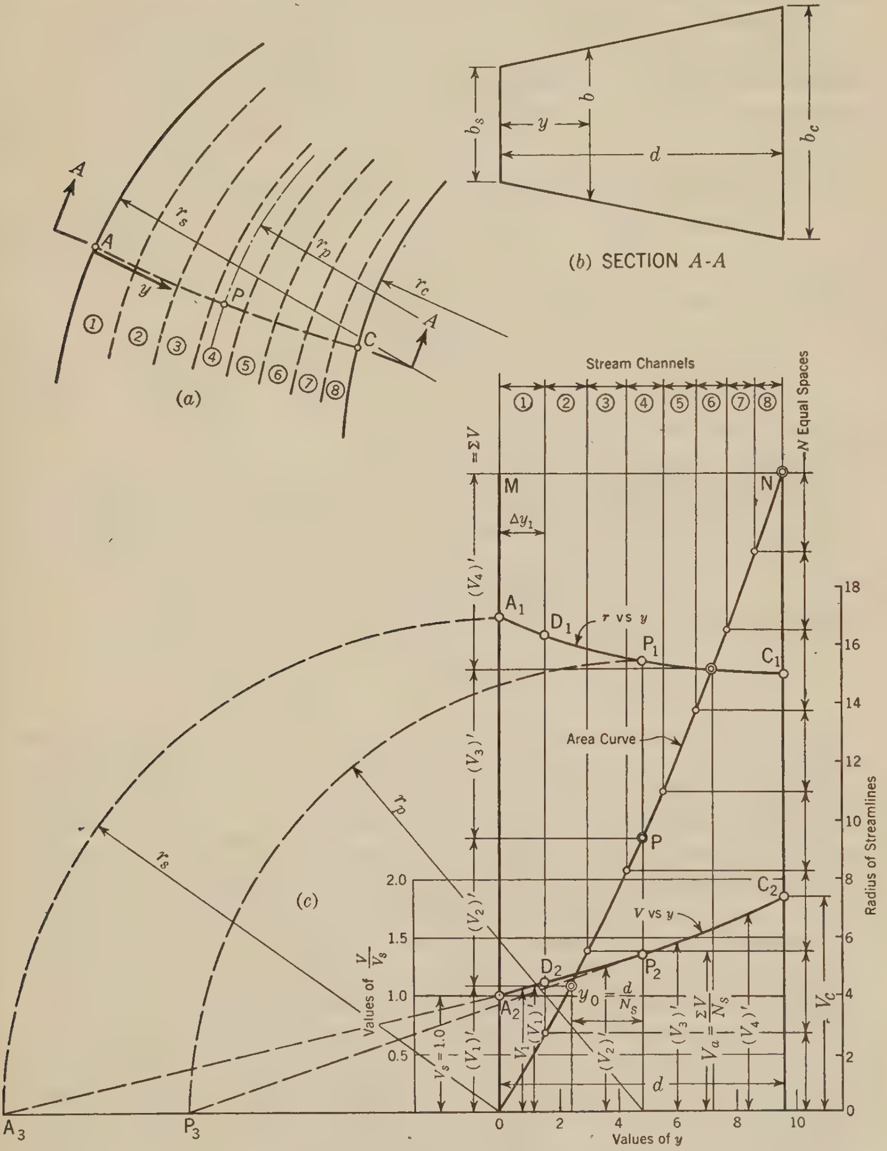

A
A
y
P
C
r_s
r_p
r_c
A
① ② ③ ④ ⑤ ⑥ ⑦ ⑧
(a)
b_s
y
b
d
b_c
(b) SECTION A-A
Stream Channels
① ② ③ ④ ⑤ ⑥ ⑦ ⑧
N Equal Spaces
$=\Sigma V$
M
N
Δy_1
$(V_4)'$
A_1
D_1
r vs y
P_1
C_1
Area Curve
$(V_3)'$
r_p
r_s
P
(c)
2.0
1.5
1.0
0.5
Values of $\frac{V}{V_s}$
$(V_2)'$
C_2
V vs y
D_2
P_2
A_2
$y_0 = \frac{d}{N_s}$
$V_s = 1.0$
$(V_1)'$
V_1
$(V_1)'$
$(V_2)'$
$(V_3)'$
$V_a = \frac{\Sigma V}{N_s}$
$(V_4)'$
d
V_c
18
16
14
12
10
8
6
4
2
0
Radius of Streamlines
0
2
4
6
8
10
Values of y
A_3
P_3

Fig. 1.—Flow Within Fixed Boundaries

of the tangent is obtained at point D_2, and the velocity curve is again extended a short distance beyond point D_2. The process is repeated at short intervals until the velocity curve is extended across the entire width of the water passage.

The ordinates of the velocity curve, A_2C_2, so determined, give values of V/V_s at any point in the line APC of Fig. 1(*a*). Note that, in this construction, the same scale must be used for r and y in Fig. 1(*c*), but this does not need to be the same as the scale of the flow net in Fig. 1(*a*).

The spacing of the streamlines in Fig. 1(*a*) is based on the condition that the discharge, q, of any stream channel between successive streamlines must be constant for all channels. Let the width of any stream channel be Δy, and the average velocity of the channel be V; then

$$q = V\,\Delta y \quad \text{(2a)}$$

Let Q = the total area under the curve A_2C_2; that is, the total discharge:

$$Q = \int V\,dy \quad \text{(2b)}$$

If there are N stream channels (eight in Fig. 1(*a*)),

$$q = \frac{Q}{N} = \frac{\int V\,dy}{N} = V\,\Delta y \quad \text{(2c)}$$

and Δy, for any stream channel, will equal $\frac{Q}{N}\frac{1}{V}$.

Divide the total width, d, into N_s equal segments (four in Fig. 1(*c*)), and note the value of V at the midpoint of each segment, $(V_1)'$, $(V_2)'$, $(V_3)'$, and $(V_4)'$. Lay off these values on the vertical line through $y = 0$, as OM. Since the width of each segment is $\frac{d}{N_s} = y_0$, the area under the velocity curve is (approximately) equal to $\overline{\text{OM}} \times y_0$ or $\Sigma V\,y_0$; that is,

$$\Sigma V\,y_0 = \int V\,dy = Q \quad \text{(3)}$$

The successive values, $(V_1)'$, $(V_1 + V_2)'$, $(V_1 + V_2 + V_3)'$, etc., are then projected to the right side of their respective segments, giving the points marked with double circles, to form the curve OPN, the ordinates of which represent the summation of the areas of the respective segments of the velocity curve, divided by y_0. If this area curve is then cut into N parts by horizontal lines equally spaced, and the points so located are projected vertically to the velocity curve, the latter will be subdivided into N segments of equal area. In other words, these N segments (indicated at the top of Fig. 1(*c*)) will give the required spacing of the streamlines to correspond with the originally assumed variation in the radius of curvature of the streamlines.

This graphical check should be made for several cross sections of the water passage, and the streamlines plotted as in Fig. 1(*a*). In general, the resulting lines will differ in spacing from those originally assumed. The radii of curvature at the two boundaries, of course, will remain the same, but the shape of the curve of r versus y may require modification. In that case, the velocity curve should be replotted, and a revised spacing of the streamlines determined.

From Eq. 3, the average velocity, V_a, will equal Q/d; or

$$V_a = \frac{Q}{N_s\,y_0} = \frac{\Sigma V}{N_s} = \frac{\overline{\text{OM}}\,V_s}{N_s} \quad \text{(4)}$$

and $V_a/V_s = \overline{\mathrm{OM}}/N_s$. The ratio V_a/V_s can therefore be determined directly from the "area curve," as shown in Fig. 1(*c*). Then,

$$V_s = \frac{V_a}{V_a/V_s} \tag{5a}$$

and

$$V_c = V_s \times \frac{V_c}{V_s} \tag{5b}$$

The ratio V_c/V_s may be read directly at the right end of the velocity curve, where $y = d$.

Free Vortex.—If the two boundary surfaces are cylindrical and concentric, and all the streamlines have the same center of curvature, $dy = -dr$. From Eq. 1, $\frac{dV}{V} = \frac{dy}{r}\cdot$ Hence,

$$\frac{dV}{V} + \frac{dr}{r} = 0 \tag{6}$$

Integrating Eq. 6, $\log V + \log r = \log C$, or $\log (V r) = \log C$, in which C is a constant. Hence, $V r = C = V_s r_s$; or

$$V = \frac{V_s r_s}{r} \tag{7}$$

This is the condition existing in a "free vortex" (2*a*); but

$$Q = \int_0^d V\,dy = -\int_{r_s}^{r_c} \frac{V_s r_s\,dr}{r} = V_s r_s \times \log_e \frac{r_s}{r_c} \tag{8}$$

Since $Q = V_a d$,

$$V_a = \frac{V_s r_s}{d} \log_e \frac{r_s}{r_c} \tag{9}$$

and

$$\frac{V_a}{V_s} = \frac{r_s}{d} \log_e m \tag{10}$$

in which $m = \frac{r_s}{r_c}\cdot$ Accordingly, with two-dimensional flow and parallel cylindrical walls for the water passage, so that "free-vortex" conditions apply, the velocity distribution can be obtained directly by Eqs. 7 and 10, and the graphical construction previously described can be omitted.

THREE-DIMENSIONAL FLOW WITH FIXED BOUNDARIES

The basic formula for the flow net (Eq. 1), as derived by Professor Rouse (2*b*)(2*c*), depends on the variation of the velocity in the direction y, taken normal to the streamline tangent at the particular point considered. In three-dimensional flow, this tangent may not remain in the same plane as one progresses along a given streamline in space. Likewise, the direction of y may not remain in the same plane. However, the mathematical relation shown by Eq. 1 will remain valid.

The previous discussion is applied to two-dimensional flow, or where the water passage is rectangular in section—that is, of constant depth b, perpendicular to the plane of the paper. For three-dimensional flow, or where the depth of the water passage is variable, the same methods can be used to determine the relative velocities, V/V_s, provided that the radius of curvature in space of the streamline at a given point remains approximately equal to what it would be for two-dimensional flow. This will be true if the cross section of the stream passage remains constant in the direction of flow; and, even if there is some degree of convergence in the water passage, the curvature of the streamlines may not change to any extent (3). Experimental proof of the assumption that the two-dimensional methods may be applied directly to three-dimensional flow is noted subsequently in this paper.

Assuming that the velocity distribution obtained by the two-dimensional method will also apply to three-dimensional flow, the actual velocities may be determined as follows:

Trapezoidal Section.—Assume that the cross section of the water passage is a trapezoid, as in Fig. 1(*b*). Divide the water passage into N strips of width Δy; the widths Δy may be equal or variable. Then total discharge is

$$Q = \sum_{y=0}^{y=d} V\, b\, \Delta y = \sum_{y=0}^{y=d} \frac{V}{V_s} V_s\, b\, \Delta y \ldots\ldots\ldots\ldots (11a)$$

If the values of Δy are equal,

$$Q = \Delta y\, V_s \sum \left(\frac{V}{V_s} b \right) \ldots\ldots\ldots\ldots (11b)$$

and

$$V_s = \frac{Q}{\Delta y \sum \left(\frac{V}{V_s} b \right)} \ldots\ldots\ldots\ldots (12)$$

If the values of Δy correspond to the streamlines as obtained for the two-dimensional condition, $V\, \Delta y$ = constant = $V_1\, \Delta y_1$, in which V_1 is the average velocity in stream channel 1, Fig. 1(*a*), and Δy_1 is the width of that channel. Hence,

$$Q = V_1\, \Delta y_1\, \Sigma b \ldots\ldots\ldots\ldots (13)$$

$$V_1 = \frac{Q}{\Delta y_1\, \Sigma b} \ldots\ldots\ldots\ldots (14a)$$

and

$$V_s = \frac{V_1}{V_1/V_s} \ldots\ldots\ldots\ldots (14b)$$

In either case, $V_c = V_s\, (V_c/V_s)$. In using Eqs. 11 to 14, the values of V/V_s, V_c/V_s, and V_1/V_s are taken from the velocity curve of Fig. 1(*c*). The variable depth, b, is to be computed from the geometry of the water passage. It should be noted that the value of V_a/V_s, shown in Fig. 1(*c*), is of no significance in this case because of the variable depth b.

For the "free-vortex" condition, since $b = b_s + \frac{y}{d}(b_c - b_s) = \frac{b_s\, d + y\, (b_c - b_s)}{d}$,

$r = r_s - y$, and $V = \dfrac{V_s r_s}{r}$:

$$Q = \int_0^d V\, b\, dy = V_s r_s \left\{ \frac{\log_e m}{d} [b_s d + r_s (b_c - b_s)] - (b_c - b_s) \right\} \quad \ldots (15)$$

Since $Q = V_a \dfrac{(b_c + b_s)\, d}{2}$ and $V_a = \dfrac{2\,Q}{(b_c + b_s)\, d}$,

$$\frac{V_a}{V_s} = \frac{2\, r_s}{(b_c + b_s)\, d} \left\{ \frac{\log_e m}{d} [b_s d + r_s (b_c - b_s)] - (b_c - b_s) \right\} \quad \ldots (16)$$

When $b_c = b_s = 1.0$, as in two-dimensional flow, $\dfrac{V_a}{V_s} = \dfrac{r_s}{d} \log_e m$ (see Eq. 10).

Circular Section.—If the cross section of the water passage is circular, with a diameter d (Fig. 2): $b = d \sin \theta$, $y = \dfrac{d}{2}(1 - \cos \theta)$, and $dy = \dfrac{d}{2} \sin \theta\, d\theta$. Then

$$Q = \int_0^d b\, V\, dy = \int_0^\pi d \sin \theta\, V \frac{d}{2} \sin \theta\, d\theta$$

$$= \frac{d^2}{2} \int_0^\pi V \sin^2 \theta\, d\theta \quad \ldots (17)$$

Let V_a = average velocity = $\dfrac{4\,Q}{\pi\, d^2}$. If the semicircle is divided into N_s segments with equal angles $\Delta\theta$,

$$V_a = \frac{4}{\pi\, d^2} \frac{d^2}{2} \sum_0^\pi V_s \frac{V}{V_s} \sin^2 \theta\, \Delta\theta$$

$$= \frac{2\, V_s}{\pi} \sum_0^\pi \frac{V}{V_s} \sin^2 \theta\, \Delta\theta \quad \ldots (18)$$

and

$$\frac{V_a}{V_s} = \frac{2\, \Delta\theta}{\pi} \sum_0^\pi \frac{V}{V_s} \sin^2 \theta \quad \ldots (19)$$

By constructing the velocity curve (V versus y) as explained herein, values of V/V_s can be determined for the several values of the angle θ, and substituted in Eq. 19 to obtain V_a/V_s. Then V_s is determined as $\dfrac{V_a}{V_a/V_s}$; and $V_c = V_s \dfrac{V_c}{V_s}$.

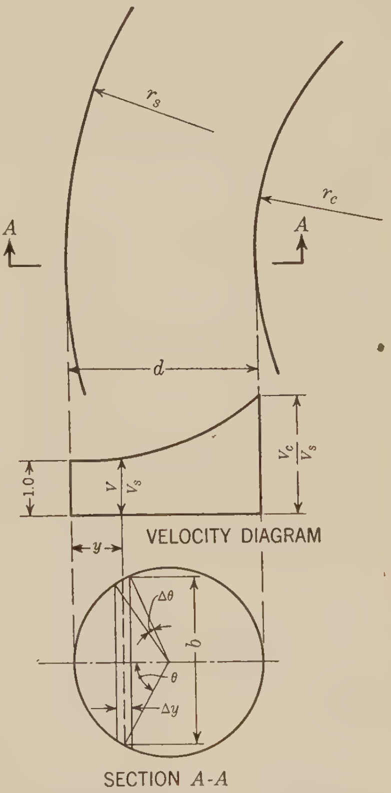


FIG. 2.—CIRCULAR WATER PASSAGE

For a standard bend in which $d = r_s - r_c$, if the streamlines are concentric the "free-vortex" condition may be assumed. Then, from Eq. 7,

$$V = \frac{V_s r_s}{r_s - y} = \frac{2\, V_s r_s}{(2\, r_s - d) + d \cos \theta} \quad \ldots (20)$$

and

$$\frac{V_a}{V_s} \doteq \frac{4\,r_s\,\Delta\theta}{\pi} \sum_{0}^{\pi} \frac{\sin^2\theta}{(2\,r_s - d) + d\cos\theta} \quad \text{(21)}$$

Actually in a standard pipe-bend, the streamlines will follow a spiral course instead of being two-dimensional curves. Nevertheless, experiments have shown that the velocity distribution approximates that of the free vortex, although the actual value of V_c/V_s is somewhat smaller than that indicated by the foregoing theory (4*a*)(5*a*)(6*a*).

Two-Dimensional Flow with Boundaries not Fixed

As shown in the preceding section, the graphical check of the flow net can be made without difficulty when the boundaries of the water passage are fixed. If one or more of the boundaries are exposed to the atmosphere, as in discharge over a straight spillway weir, the construction of the flow net becomes more difficult. However, the same graphical check can be applied after the flow net is laid out approximately.

It is first necessary to sketch in the streamlines of the flow net freehand, as described elsewhere (1). The flow net must satisfy the following requirements:

(*a*) The radius of curvature of the streamlines must vary in conformance with the aforementioned graphical method $\left(\frac{V}{r} = \frac{dV}{dy}\right)$;

(*b*) The drop in the water surface at any section, measured below the reservoir level, must equal the velocity head of the water flowing at the surface $V^2_s/2\,g$; and

(*c*) The average velocity multiplied by the depth, measured along any potential line, must equal the rate of discharge per foot of crest length.

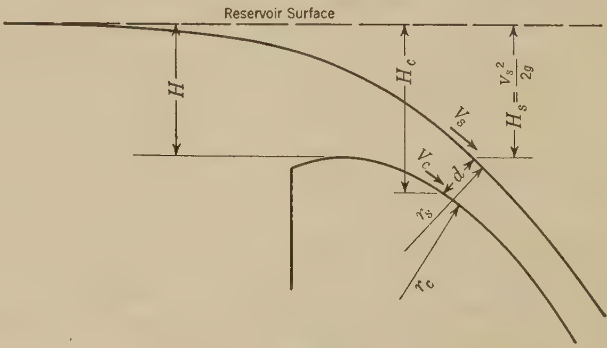


Fig. 3

Since the rate of discharge per foot of crest is not known in advance, this must be assumed. (In the case of a weir, it may be estimated by the use of known weir coefficients.) The flow net should then be sketched in freehand (see Fig. 3). When the aforementioned checks are applied, it may be impossible to make the second and third conditions agree with the first condition. This is an indication that the assumed rate of discharge is incorrect, and a new value of discharge must be assumed. There is only one value of discharge that will permit all three conditions to be satisfied simultaneously.

In the case of a weir, the correct discharge should be determined before attempting to extend the flow net downstream from the crest. The completion of the flow net then is much simplified, as the surface velocity becomes more nearly equal to the average velocity. Where the depth of flow is relatively small, compared with the radius of curvature of the streamlines, the latter may be assumed constant; that is, $r = r_c = r_s$. Under these conditions, the theory gives $\frac{dV}{dy} = \frac{V}{r}$ in which r is constant. Then $\frac{dV}{V} = \frac{dy}{r}$; and $\log_e V = \frac{y}{r} + C$. When $y = 0$, $V = V_s$; $C = \log_e V_s$; $\log_e \frac{V}{V_s} = \frac{y}{r}$; $\frac{V}{V_s} = e^{y/r}$:

$$Q = \int_0^d V\,dy = V_s\,r\,(e^{d/r} - 1) \qquad (22)$$

The average velocity equals $V_a = Q/d$; hence,

$$\frac{V_a}{V_s} = \frac{r}{d}(e^{d/r} - 1) \qquad (23)$$

By assuming a value for d at any given section, and taking r as the radius of the concrete surface, V_a/V_s can be computed by Eq. 23. Then

$$V_s = \frac{V_a}{V_a/V_s} = \frac{Q/d}{V_a/V_s} \qquad (24)$$

This value of V_s can be checked by the condition that $V^2{}_s/2\,g$ must agree with the elevation of the water surface at the given section. Having determined V_s, the next step is to compute V_c: Since $V = V_s\,e^{y/r}$,

$$V_c = V_s\,e^{d/r} \qquad (25)$$

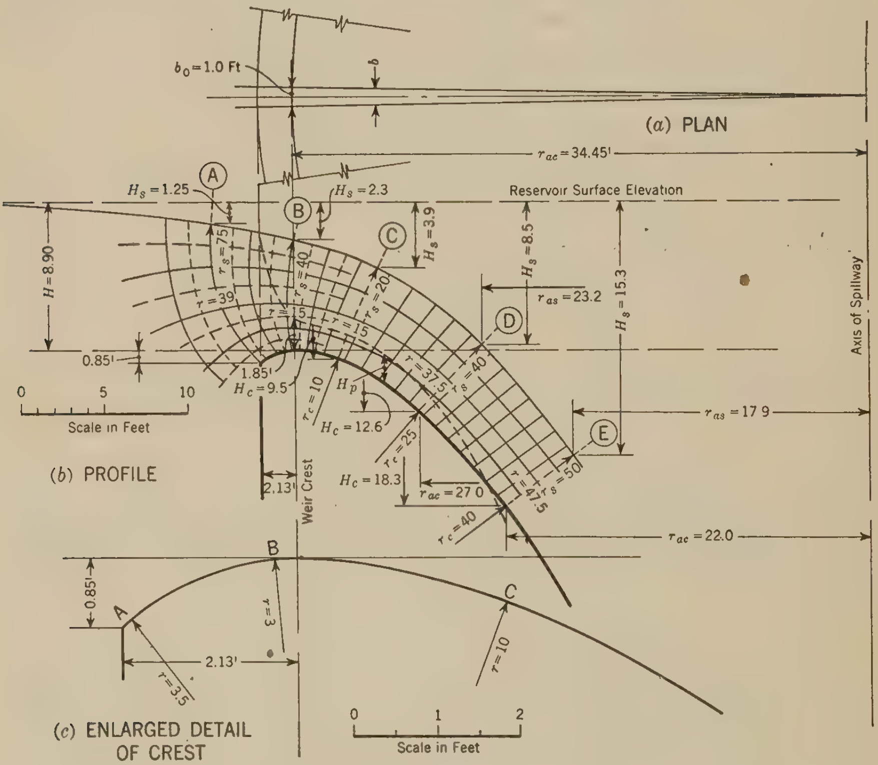
Check with Experimental Weir Nappes.—The writer has checked the methods described herein for testing a flow net by applying them to the case of a sharp-crested weir with aerated lower nappe. Curves for the upper and lower surface of the freely-discharging jet can be plotted by means of the table prepared by E. W. Lane, M. Am. Soc. C. E. (7). At any section through the jet, the elevations of the upper and lower water surfaces may be determined with respect to the reservoir surface (H_s and H_c in Fig. 3). Since atmospheric pressure prevails on both surfaces, $V_c/V_s = \sqrt{H_c/H_s}$.

By measuring the thickness of the jet and the radius of curvature of the upper and lower nappes at the given section, the value of V_c/V_s may also be computed by the graphical or analytical methods described herein. The results, as obtained by the writer, give a very close check with the values of V_c/V_s obtained from the velocity heads.

THREE-DIMENSIONAL FLOW WITH BOUNDARIES NOT FIXED

As in the case for fixed boundaries, the methods described herein for two-dimensional flow can be extended to three-dimensional flow with one or more boundaries exposed to the atmosphere. An example of this condition is the "morning-glory" type of spillway, or a sharp-crested weir which is circular in plan (8).

The flow net in this case is to be constructed in a manner similar to that described for two-dimensional flow, except that, in determining the actual velocities, the shape of the water passage must be taken into account, as described herein for fixed boundaries.



b_0=1.0 Ft
b
(a) PLAN
r_{ac}=34.45'
H_s=1.25
A
H_s=2.3
B
C
H_s=3.9
Reservoir Surface Elevation
H_s=8.5
H_s=15.3
r_{as}=23.2
D
Axis of Spillway
H=8.90
r_s=75
r_s=40
r=39
r_s=20
r=15
r=15
0.85'
1.85'
H_c=9.5
r_c=10
H_p
r_s=37.5
r=40
H_c=12.6
r_c=25
r_{as}=17 9
E
0 5 10
Scale in Feet
(b) PROFILE
2.13'
Weir Crest
H_c=18.3
r_{ac}=27 0
r=47.5
r_s=50
r_c=40
r_{ac}=22.0
B
0.85'
A
r=3
C
2.13'
r=3.5
r=10
0 1 2
Scale in Feet
(c) ENLARGED DETAIL OF CREST

Fig. 4.—Flow Net for a Morning-Glory Spillway

The writer has applied such methods to the case of a sharp-crested weir of circular plan, such as described by C. E. Camp, Assoc. M. Am. Soc. C. E., and J. W. Howe, M. Am. Soc. C. E. (8), whose work presents the results of tests made at the University of Iowa, Iowa City, including curves for the upper and lower nappes of freely discharging weirs with various radii of curvature. The test was applied to a weir with a radius of 36.6 ft and a weir head equal to 10.45 ft. After plotting the upper and lower nappe curves, the values of H_c and H_s were scaled for several sections through the jet and V_c/V_s was computed ($= \sqrt{H_c/H_s}$). The graphical and analytical methods previously described were then used to compute values of V_c/V_s for the same sections, using values of the radii of curvature and thickness of the jet scaled from the plotted

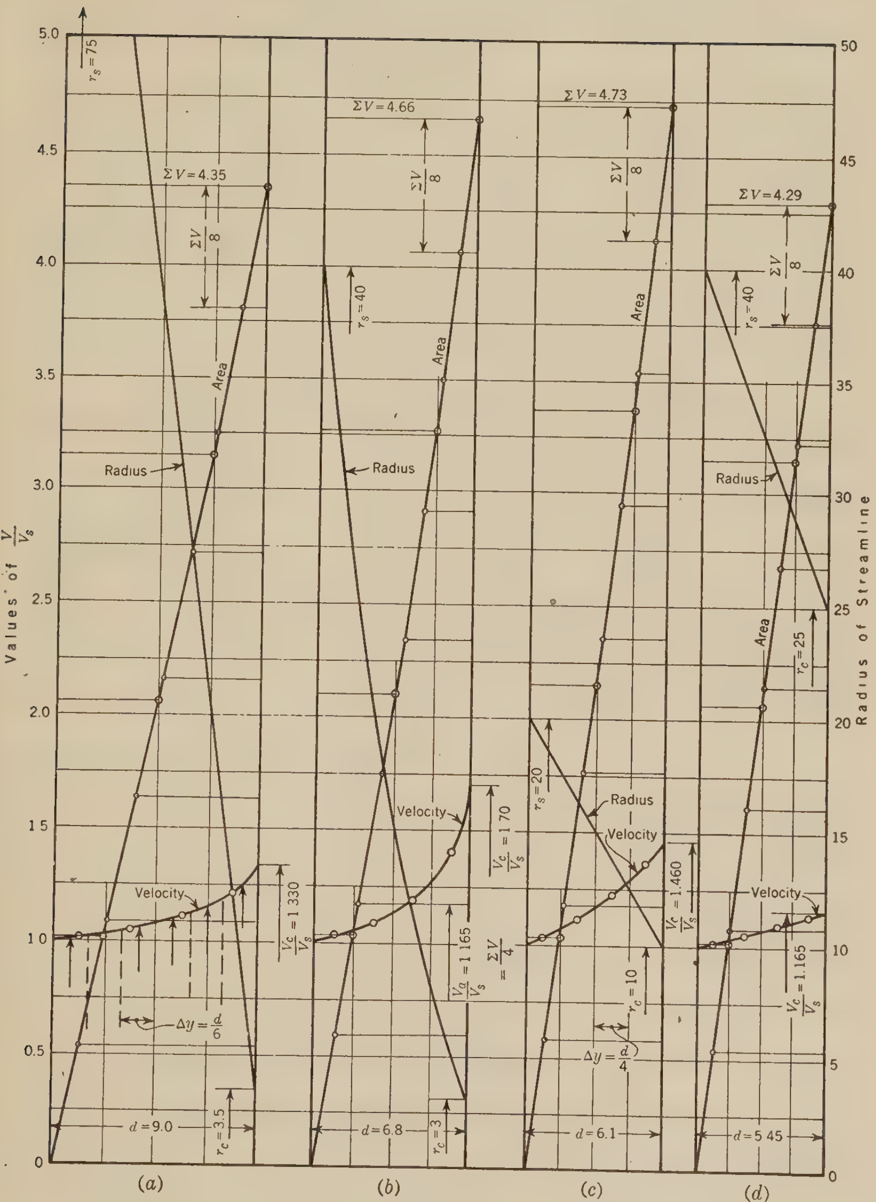

Values of $\frac{V}{V_s}$
Radius of Streamline
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
50
45
40
35
30
25
20
15
10
5
0
$r_s = 75$
$\Sigma V = 4.35$
$\frac{\Sigma V}{8}$
Area
Radius
Velocity
$\frac{V_c}{V_s} = 1.330$
$\Delta y = \frac{d}{6}$
$d = 9.0$
$r_c = 3.5$
$\Sigma V = 4.66$
$\frac{\Sigma V}{8}$
$r_s = 40$
Area
Radius
Velocity
$\frac{V_c}{V_s} = 1.70$
$\frac{V_a}{V_s} = 1.165$
$= \frac{\Sigma V}{4}$
$d = 6.8$
$r_c = 3$
$\Sigma V = 4.73$
$\frac{\Sigma V}{8}$
Area
$r_s = 20$
Radius
Velocity
$\frac{V_c}{V_s} = 1.460$
$r_c = 10$
$\Delta y = \frac{d}{4}$
$d = 6.1$
$\Sigma V = 4.29$
$\frac{\Sigma V}{8}$
$r_s = 40$
Radius
Area
$r_c = 25$
Velocity
$\frac{V_c}{V_s} = 1.165$
$d = 5.45$
(a)
(b)
(c)
(d)

FIG. 5

TABLE 1.—ILLUSTRATIVE EXAMPLE; BASIC DATA

(Flow Q = 99 Cu Ft Per Sec)

Symbol	Sections (See Fig. 4)				
	A	B	C	D	E
r_s	75	40	20	40	50[a]
r_c	3.5	3	10	25	45[a]
d	9.0	6.8	6.1	5.45	5.1
Δy	1.50	....	1.53	....	....
r_{ac}	....	....	....	27.0	22.0
r_{as}	....	....	....	23.2	17.9
d/r	....	....	....	0.1455	0.1073
b_c	....[b]	1.0	....[b]	0.784	0.639
b_s				0.674	0.520

[a] Average $r = \frac{1}{2}(r_s + r_c) = 47.5$. [b] See Table 2.

TABLE 2. —COMPUTATIONS FOR VARIABLE DEPTH

Segment No. (Fig. 4)	Section A				Section C			
	r_a	$b = \frac{r_a}{34.45}$	$\frac{V}{V_s}$ Fig. 5(*a*)	$b \times \frac{V}{V_s}$ (Eq. 12)	r_a	$b = \frac{r_a}{34.45}$	$\frac{V}{V_s}$ Fig. 5(*c*)	$b \times \frac{V}{V_s}$ (Eq. 12)
1	39.5	1.145	1.005	1.150	29.8	0.865	1.03	0.890
2	39.4	1.142	1.035	1.182	30.4	0.882	1.12	0.988
3	39.0	1.130	1.055	1.192	31.0	0.900	1.22	1.092
4	38.5	1.120	1.085	1.215	31.7	0.920	1.37	1.260
5	37.9	1.100	1.140	1.252	....	....	....	....
6	37.0	1.072	1.230	1.320	....	....	....	....
Total	....	....	....	7.311	....	....	....	4.230

TABLE 3.—COMPUTATION OF FLOW NET VALUES

Section (Fig. 4)	$\frac{V_c}{V_s}$ (Fig. 5)	V_s	$\frac{V_a}{V_s}$ (Fig. 5)
A	1.33	$\frac{99}{1.5 \times 7.31} = 9.02$ (Eq. 12)	....
B	1.70	$\frac{14.58}{1.165} = 12.52^b$ (Eq. 5*a*)	1.165
C	1.460	$\frac{99}{1.53 \times 4.23} = 15.4$ (Eq. 12)	....
D	1.165[a]	$\frac{2 \times 99}{(1.165 \times 0.784 + 0.674)\ 5.45} = 22.9$ (Eq. 27)	....
E	1.113	$\frac{2 \times 99}{(1.113 \times 0.639 + 0.520)\ 5.1} = 31.6$ (Eq. 27)	....

[a] Approximate computation: $\frac{d}{r} = 0.1455$; $\frac{V_c}{V_s} = e^{d/r} = 1.156$.

curves. As with the straight weir, the agreement with the velocity head computations was very close, particularly in the region of the jet where the streamlines have large radii of curvature. Near the weir crest the agreement was less satisfactory, but probably within the limits of accuracy of the curves used by Messrs. Camp and Howe (8) which are drawn at a rather small scale.

EXAMPLE

To illustrate the methods outlined herein, the case of a morning-glory spillway weir will be given. The profile of the crest is shown in Fig. 4, the high point of the weir being located in plan on a circle whose radius r_{ac} is 34.45 ft, and whose circumferential length is 217 ft. The discharge capacity is assumed as 21,500 cu ft per sec, or 99 cu ft per sec per foot of crest. The analysis is made for a sector 1.0 ft wide at the crest.

Because of the circular plan of the spillway, a section through the water jet, such as section D in Fig. 4, will be essentially trapezoidal in shape, the depth of the section at any point (normal to the paper) being equal to $r_a/34.45$, in which r_a is the radius measured from the axis of the spillway.

Preliminary studies indicate that the head over the spillway crest required to discharge 99 cu ft per sec per ft will be about 8.90 ft. Using this head, the flow net is sketched in as shown in Fig. 4. The radii of curvature of the water surface, the mid-streamline, and the concrete surface are scaled at various sections. For this purpose, a template on tracing paper showing curves of various radii is convenient.

The graphical check for section A, Fig. 4, is shown in Fig. 5(a), in which the area curve is divided into eight stream channels ($N = 8$). The computations for V_s and V_c are given in Tables 1, 2, and 3, the width d being divided into six equal segments ($N_s = 6$). As a check on the accuracy of the flow net, it will be noted that $V^2{}_s/2\,g$ should equal H_s. The pressure head at the concrete surface, H_p, is computed as equal to $H_c - \frac{V^2{}_c}{2\,g}$, and is plotted in Fig. 4 vertically from the concrete surface.

($Q = 99$ CU FT PER SEC)

$\frac{V^2{}_s}{2\,g}$	V_c (Eq. 5b)	$\frac{V^2{}_c}{2\,g}$	H_s (ft) (Fig. 4)	H_c (in feet) (Fig. 4)	H_p (in feet) $H_c - V^2{}_c/2\,g$	Section (Fig. 4)
1.26	$9.02 \times 1.33 = 12.0$	2.24	1.25	9.75	$9.75 - 2.24 = +7.51$	A
2.4	$12.52 \times 1.7 = 21.3$	7.05	2.3	8.90	$8.90 - 7.05 = +1.85$	B
3.7	$15.4 \times 1.460 = 22.5$	7.9	3.9	9.5	$9.5 - 7.9 = +1.6$	C
8.2	$22.9 \times 1.165 = 26.7$	11.1	8.5	12.6	$12.6 - 11.1 = +1.5$	D
15.5	$31.6 \times 1.113 = 35.2$	19.2	15.3	18.3	$18.3 - 19.2 = -0\;9$	E

[b] $V_a = \frac{99}{6.8} = 14.58.$

Section B is approximately vertical, and hence rectangular, with constant depth, $b = 1.0$. The graphical check is given in Fig. 5(*b*); and the computations are shown in Tables 1 to 3. In this case, V_a/V_s and V_c/V_s are obtained from the graphical check.

The computations for section C are made in the same manner as for section A, with d divided into four equal segments. The graphical check is given in Fig. 5(*c*).

At section D the graphical check is given in Fig. 5(*d*). Here the velocity curve is practically a straight line and the cross section of the stream passage is a trapezoid. The discharge may then be computed as

$$Q = (V_c\, b_c + V_s\, b_s)\, \frac{d}{2} \quad \dotfill (26)$$

or

$$V_s = \frac{2\,Q}{\left(\dfrac{V_c}{V_s}\, b_c + b_s\right) d} \quad \dotfill (27)$$

V_c/V_s being read from Fig. 5(*d*). The approximate method, using a constant radius of curvature for the streamlines, may also be used, in which case $V_c/V_s = e^{d/r}$. As shown in the computations, this gives an error of less than 1% in the value of V_c/V_s, even though there is an appreciable difference between r_c and r_s.

At section E the streamlines have practically a constant radius, and the approximate method is used to compute V_c/V_s. Then V_s is computed by Eq. 27 as in section D; thus:

$$\begin{aligned} \log_{10} e &= \ 0.43429 \\ &\ \ \times\ \underline{0.1073} \qquad \left(= \frac{d}{r};\ \text{see Table 1}\right) \\ \log e^{d/r} &= \ 0.04660 \\ e^{d/r} &= 1.113 = \frac{V_c}{V_s} \qquad (\text{see Eq. 25}) \end{aligned}$$

The graphical checks for sections A to D, inclusive, give the theoretical spacings of the streamlines, which are used to check the construction of the flow net. If there is any appreciable change to be made in the curvature of the streamlines, the graphical check should be revised accordingly. In the preliminary construction of the flow net it may be found that the computed $V^2_s/2\,g$ at any section does not agree with H_s shown on the flow net. The position of the water surface must then be shifted until these values agree.

In this example, considerable time is required to determine the correct head (H) over the spillway crest, for the given value of discharge. Several trial arrangements of the flow net between section A and section C must be laid out, using different values of H, until the values of H_s and $V^2_s/2\,g$ are brought into agreement. The flow net is not extended beyond section C until the correct value of Q has been determined.

This example was selected intentionally to illustrate the complications of a three-dimensional problem with water surface exposed to the atmosphere. A two-dimensional problem, such as a weir with straight crest, or a problem involving fixed boundaries for the water passage, would be appreciably easier to construct.

SUMMARY

The writer has found by experience that the methods outlined herein greatly facilitate the construction of a flow net, and permit much more precise velocity determinations than can be obtained merely by freehand sketching of the streamlines. The experience of other investigators (1), and the check computations, made by the writer, on the results of experiments on straight and curved sharp-crested weirs, lead to the conclusion that, when a flow net is carefully constructed and checked by the methods presented herein, the resulting computed velocities and pressures will be a reasonable indication of those to be expected in the prototype structure. However, it should be recognized that the flow-net method is not applicable in cases where friction and impact or eddy losses may be expected to have a controlling effect on velocities and pressures.

APPENDIX I

BIBLIOGRAPHY

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(3) "**Hydraulic Turbines,**" by R. L. Daugherty, McGraw-Hill Book Co., Inc., New York, N. Y., 1920, p. 251.

(4) "**Flow of Water Around Bends in Pipes,**" by D. L. Yarnell and Floyd A. Nagler, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1018. (*a*) p. 1037.

(5) "**Design of a High-Head Siphon Spillway,**" by Elmer Rock, *Transactions*, Am. Soc. C. E., Vol. 105 (1940), p. 1050. (*a*) p. 1065.

(6) "**Model. Tests on Siphon Spillways,**" by A. H. Gibson, T. H. Asprey, and F. Tattersall, *Minutes of Proceedings*, Inst. C. E., Vol. 231 (1930–1931), p. 203. (*a*) p. 272.

(7) "**Handbook of Applied Hydraulics,**" by Calvin Davis, Editor, McGraw-Hill Book Co., Inc., New York and London, 1942, p. 338.

(8) "**Tests of Circular Weirs,**" by Cecil S. Camp and J. W. Howe, *Civil Engineering*, April, 1939, p. 247. (*a*) Eq. 3, p. 248.

APPENDIX II

NOTATION

The following letter symbols, used in the paper, conform essentially to American Standard Letter Symbols for Hydraulics,[3] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942:

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[3] ASA—Z10.2—1942.

b = breadth of flow (Fig. 1(b)), b_s and b_c, respectively, being breadths at the outside and inside of the curved stream channel;
C = a constant (see Eq. 6);
d = depth, or total maximum width of a water passage measured along a curved potential line (see Fig. 1);
e = base of Napierian logarithms (see Eq. 8);
g = acceleration of gravity;
H = elevation of reservoir surface above weir crest (see Fig. 4);
H_c and H_s = respectively, the elevation of the lower and upper surfaces of the water at any section of a jet, measured downward from the reservoir surface:
H_p = pressure head on masonry surface (see Fig. 4);
m = the ratio r_s/r_c (Eq. 10);
N = total number of hypothetical channels in a stream (Fig. 1(a));
N_s = number of hypothetical segments for velocity curve, Fig. 1(c);
Q = total discharge, or total area under the curve A_2C_2, Fig. 1(c);
q = discharge in any given stream channel, or Q/N (Eq. 2c);
r = radius of curvature of a stream, the radii for individual channels (Fig. 1(a)) being designated by appropriate subscripts; the radii of the boundaries of the jet are denoted by r_s and r_c (Fig. 1(a));
r_a = radius of "morning-glory" spillway measured from the axis of the spillway shaft to a point in any section of a jet, the radii to the lower and upper surfaces at any section being designated by r_{ac} and r_{as}, respectively (see Fig. 4);
V = velocity of flow at any point P, Fig. 1(a), in the water passage, the velocity for individual channels (Fig. 1(c)) being designated by appropriate subscripts; average velocity is designated V_a, and the velocity at the inner and outer surfaces of any section is designated V_c and V_s, respectively:
V_1 = average velocity in stream channel 1 (see Fig. 1(c));
$(V_1)'$ = value of V at the midpoint of segment 1 of the velocity curve;
y = a part of d measured from the outer curved boundary toward the center of curvature (see Fig. 1(a));
y_0 = width of any single segment of area curve, Fig. 1(c)), equals d/N_s;
Δy_1 = width of stream channel 1 (Eq. 13);
θ = angle measured at the center of a circular channel (Fig. 2):
$\Delta\theta$ = central angle of the arc subtended by a segment of the velocity curve (Fig. 1(c)).

DISCUSSION

J. W. Howe,[4] M. Am. Soc. C. E.—The amplification of a graphical method for checking the location of streamlines in a flow net, as presented by the author, offers detailed working directions which should prove useful to any one desiring to make such a check. Experience, patience, and no little skill are essential in drawing a correct network, and even with these the labor may be considerable. The graphical checking of the tentative location of streamlines is also time consuming and probably offers little advantage to an experienced hand. To the inexperienced, however, such methods, no doubt, will provide a somewhat more rapid approach to a solution, since an essentially correct location at one point in the network is of material aid in shaping the pattern for a considerable distance in either direction.

The flow net has proved useful in many situations involving two-dimensional flow which is essentially irrotational. Its application to three-dimensional flow is subject to question, however. The writer is of the opinion that the accuracy of such an application would vary directly with the departure from two-dimensional conditions.

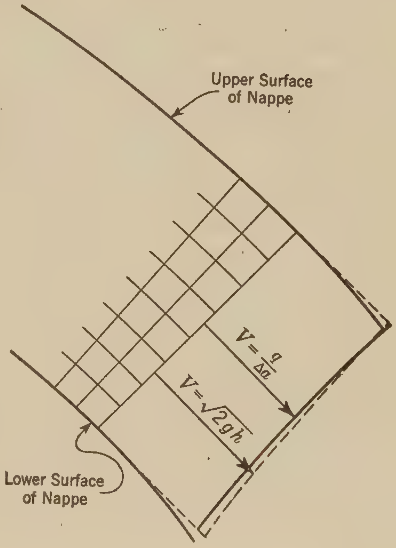

Fig. 6

In the experimental work done by Cecil S. Camp, Assoc. M. Am. Soc. C. E. (8), only the discharge and the position of the nappe surfaces were determined; pressure measurements in the nappe were not made. Because of the radial convergence of stream filaments in passing over the circular weir there must be pressure in the nappe even in regions where the streamlines of the two-dimensional flow net are parallel.

The writer applied the graphical method to a section downstream from a circular weir (head-diameter ratio, 0.14) at a point where the streamlines were practically parallel (such as point E, Fig. 4). The boundaries of the nappe were taken from Professor Camp's experimental data, and the location of streamlines was verified by the author's graphical method. The cross-sectional areas of the stream filaments were determined, and from these and the discharge, the mean velocities were computed. If the flow were two-dimensional, no pressure could exist in the nappe at this section and the velocity at any point would equal $\sqrt{2\,g\,h}$. A comparison of the latter quantity with the velocity obtained from the flow net is shown in Fig. 6. There is an indication of some pressure in the nappe although it does not reduce to zero at the underside as it should. Since the degree of convergence was quite small in this example, the indicated pressure may be of about the right magnitude. However, experimental substantiation of this possibility would be required to establish the validity of the flow net in such a case.

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[4] Prof. and Head, Dept. of Mechanics and Hydraulics, State Univ. of Iowa, Iowa City, Iowa.

Clarence S. Jarvis,[5] M. Am. Soc. C. E.—This presentation of the flow-net problem by Mr. Foster is original in approach and concept. Too often, however, a tendency toward abstract expression is confusing, and a more elementary presentation might prove acceptable to nonspecialists. In the structural feature employed by the author, for example, it seems preferable to concentrate on: (*a*) Access capacity over the "morning-glory" spillway rim; (*b*) outlet capacity of the stem or throat, leading to the open drainage channel below the dam; and (*c*) important phenomena associated with normal operation.

The usual behavior of such structures—laboratory models and also ordinary funnels—indicates that considerable turbulence and entrapment of air will occur where the converging filaments of swiftly flowing water crowd the outlet stem. Perhaps hydrostatic head will be built up as required to dispose of the changing overflow at the upper rim. Pressures resulting from the turbulence and the entrapped air doubtless would exceed the total pressures subjected to variations due to changing velocities of flow over concrete or some other structural surface.

Surely the first investigations should concern the discharge capacity over the rim, in accordance with the author's presentation. Thus:

$$Q = 3.28\,L\,H^{3/2} \qquad (28a)$$

or for a full circle,

$$Q = 10.36\,D\,H^{3/2} \qquad (28b)$$

For the example under consideration, in which the diameter is 68.9 ft, Eq. 28*b* yields

$$Q = 714 \times H^{3/2} \qquad (28c)$$

For $H = 9$ ft in Eq. 28*c*, Q becomes 19,300 cu ft per sec; or, for $H = 16$ ft, $Q = 45{,}700$ cu ft per sec.

Until the engineer can compute the vibration due to the inevitable shock of the readjustment of the section by the descending sheet of water and the motion for steady discharge in a nearly horizontal conduit before the discharge escapes into an open channel, it seems unimportant to determine the slight changes in velocity and related pressures, while totally neglecting friction, impact, and eddy losses. According to the author's statement in the concluding sentence of the "Summary," "* * * the flow-net method is not applicable in cases where friction and impact or eddy losses may be expected to have a controlling effect on velocities and pressures." Surely friction, impact, and eddy losses are not negligible, or the author's estimate for the discharge capacity of spillway lip under a head of about 8.90 ft could not be taken as 21,500 cu ft per sec—more than 10% in excess of the quantity derived (8*a*) from the use of Eq. 28*a*.

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[5] Senior Civ. Engr., Intelligence Dept., Office, Chf. of Engrs., Washington, D. C.

Considerations of economy would probably so limit the dimensions of the spillway stem and horizontal outlet conduit that it would be questionable whether a flow of 45,500 cu ft per sec could be accommodated. If the conduit capacity were somewhat less, it would become the controlling factor, causing a relatively quiet pondage over the entire morning-glory area except near the center, where the tendency to circular motion might establish the well-defined vortex so familiar to observers of eddies in turbulent stream surfaces. Obviously, the neglected factors of friction, impact, and eddy losses would control to such an extent that the length as well as the cross section of the outlet conduit would have to be considered in the determination of the actual capacity of the spillway under a given head of water in the reservoir and also under the total head above the downstream end of the outlet conduit.

Such theoretical studies and analyses should be encouraged, but the results should be reduced to a practical working basis.

H. ALDEN FOSTER,[6] M. AM. SOC. C. E.—The fact that so little discussion has been submitted on the paper might seem to indicate that there is not much general interest in the subject of flow nets as applied to hydraulic design. However, as noted by Professor Howe, the flow net has been used to good advantage in solving problems of two-dimensional flow. In the writer's opinion, the graphical method described in the paper does materially assist in the construction of the flow net; and, in many cases, the method should permit a much more precise determination of the velocity distribution, which, as a rule, is the primary purpose for which the flow net is constructed.

The extension of the graphical method to problems of three-dimensional flow has seemed to the writer to be logical, provided that the radius of curvature of the streamlines, in space, is not appreciably different from that for two-dimensional flow. Such an assumption is subject to verification by experiment. Unfortunately, there are few experimental data available on which to base a reliable test. The application of the graphical method to the experiments (8) by Cecil S. Camp, Assoc. M. Am. Soc. C. E., as noted by Professor Howe, indicates at least an approximate agreement when applied to a three-dimensional problem.

Essentially, a flow net for any particular hydraulic structure would be constructed only as a substitute for a model test. There is nothing to be determined by the flow net that cannot be learned with greater accuracy from a model test. However, model tests are often expensive and time consuming; and, in general, it should be possible to prepare a flow net at a considerable saving in cost and time. Even if a model test is to be conducted, it may be worth while to make a preliminary flow-net study as a guide to the construction and testing of the model. Any device that would facilitate the preparation of the flow net should be worth consideration by the designer.

Mr. Jarvis seems to be under the impression that the writer's primary intention was to present a method for the design of "morning-glory" spillways.

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[6] Designing Engr., Parsons, Brinckerhoff, Hogan & Macdonald, New York, N. Y.

Actually, this problem was discussed only for the purpose of illustrating the application of the flow-net method to the more complicated conditions of three-dimensional flow when one boundary of the water passage is exposed to the atmosphere.

The writer is in general agreement with most of Mr. Jarvis' comments on morning-glory spillways. However, this subject is entirely beyond the scope of the paper. Consequently, it does not appear advisable to discuss the questions raised by Mr. Jarvis. In the writer's opinion, no adequate treatment of the complete theory of the design of such spillways has yet been published. He hopes, however, that the present paper may prove useful in the design of morning-glory spillways as well as in that of many other types of hydraulic structures.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2256

CHARACTERISTIC REDUNDANTS USED FOR ANALYZING STATICALLY INDETERMINATE STRUCTURES

By John B. Wilbur,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Charles A. Ellis, Ralph W. Stewart, Myle J. Holley, Jr., M. W. Huggins, Leon Beskin, Frederick S. Merritt, and John B. Wilbur.

Synopsis

Orthogonal functions are commonly used in the solution of vibration problems. To a lesser degree such functions have been applied to some of the more complicated static analyses, as, for example, to plate analysis and to the determination of beam deflections by trigonometric series. In this paper, forces acting on relatively simple structures are reduced to characteristic forces which are orthogonal, and methods of analysis based on these characteristic forces are discussed.

The heart of the multiply indeterminate solution may be interpreted by means of the determination of so-called characteristic forces. After these forces have been found, redundant stresses corresponding to any condition of loading or distortion may be evaluated without the further solution of simultaneous equations, regardless of the degree to which the structure may be statically indeterminate.

It is not claimed that the methods suggested are either more quickly applied or more easily understood than other methods which are commonly used. It is hoped, however, that they will be of interest because they differ so radically from the usual methods. Being so different, it is possible that other applications of the principles involved may lead to methods of solution which constitute an improvement on the methods outlined herein.

Notation

The symbols used in this paper are defined where they first appear, either by text or diagram, and are assembled for convenience of reference in the Appendix.

Note.—Published in June, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Prof., Structural Eng., Mass. Inst. Tech., Cambridge, Mass.

1. Characteristic Forces

Suppose an elastic structure to be acted upon by a number of concentrated forces $P_a, P_b, \cdots, P_n$, which are applied at points a, b, $\cdots$, n, respectively, as shown in Fig. 1. Let:

Δ_a = total deflection at point a in the direction of P_a due to forces $P_a, P_b, \cdots, P_n$;

Δ_b = total deflection at point b in the direction of P_b due to forces $P_a, P_b, \cdots, P_n$;

. . . .

Δ_n = total deflection at point n in the direction of P_n due to forces $P_a, P_b, \cdots, P_n$.

Let values of $P_a, P_b, \cdots, P_n$ be determined in such a manner that

$$\left.\begin{aligned} \Delta_a &= \kappa P_a \\ \Delta_b &= \kappa P_b \\ &\cdots\cdots \\ \Delta_n &= \kappa P_n \end{aligned}\right\} \cdots\cdots\cdots\cdots (1)$$

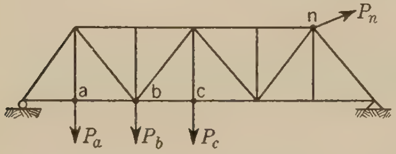

Fig. 1

in which κ is a proportionality factor having the same value in each of Eqs. 1. A set of forces which satisfy Eqs. 1 will be called "characteristic forces."

2. Determination of Characteristic Forces

If δ_{mn} is the deflection at any point m in the direction of P_m, due to a unit value of P_n, then, by superposition:

$$\left.\begin{aligned} P_a\,\delta_{aa} + P_b\,\delta_{ab} + \cdots + P_n\,\delta_{an} &= \Delta_a \\ P_a\,\delta_{ba} + P_b\,\delta_{bb} + \cdots + P_n\,\delta_{bn} &= \Delta_b \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ P_n\,\delta_{na} + P_b\,\delta_{nb} + \cdots + P_n\,\delta_{nn} &= \Delta_n \end{aligned}\right\} \cdots\cdots\cdots\cdots (2)$$

Substituting values of $\Delta_a, \Delta_b, \cdots, \Delta_n$ from Eqs. 1, Eqs. 2 may be rewritten as

$$\left.\begin{aligned} P_a\,(\delta_{aa} - \kappa) + P_b\,\delta_{ab} + \cdots + P_n\,\delta_{an} &= 0 \\ P_a\,\delta_{ba} + P_b\,(\delta_{bb} - \kappa) + \cdots + P_n\,\delta_{bn} &= 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ P_a\,\delta_{na} + P_b\,\delta_{nb} + \cdots + P_n\,(\delta_{nn} - \kappa) &= 0 \end{aligned}\right\} \cdots\cdots\cdots\cdots (3)$$

Since the right-hand side of Eqs. 3 is zero, the numerator determinant will be zero. Hence, in order for the forces $P_a, P_b, \cdots, P_n$ to have values other than zero, it is necessary that the denominator determinant formed by the coefficients to $P_a, P_b, \cdots, P_n$ on the left side of these equations be also equal to zero. This condition is represented by

$$\begin{vmatrix} \delta_{aa} - \kappa; & \delta_{ab}; & \cdots; & \delta_{an} \\ \delta_{ba}; & \delta_{bb} - \kappa; & \cdots; & \delta_{bn} \\ \cdots & \cdots & \cdots & \cdots \\ \delta_{na}; & \delta_{nb}; & \cdots; & \delta_{nn} - \kappa \end{vmatrix} = 0 \cdots\cdots\cdots\cdots (4)$$

The expansion of the determinant of Eq. 4 leads to an nth degree algebraic equation in terms of κ. If n forces are being considered, there will be n values of κ which will satisfy this equation. These n values of κ will be denoted by $\kappa_1, \kappa_2, \cdots, \kappa_n$, taken in order of descending magnitude.

Each of these values of κ_m may now be substituted separately into Eqs. 3. The simultaneous solution of any $n - 1$ equations of Eqs. 3 will then permit the expression of any $n - 1$ of the forces $P_{am}, P_{bm}, \cdots, P_{nm}$ in terms of any one of these forces such as P_{am}. The nth equation of Eqs. 3 may be used to check both κ_m and the solution of the $n - 1$ equations. Since there are n values of κ_m, n sets of characteristic forces exist, which are determined by the foregoing procedure. The complete set of characteristic forces corresponding to κ_m will be denoted by P_m.

There will result then, the following n sets of characteristic forces:

$$\left.\begin{array}{l}
\text{For } \kappa_1, \\
\qquad P_{a1} = P_{a1};\quad P_{b1} = A_{b1}\,P_{a1};\quad \cdots;\quad P_{n1} = A_{n1}\,P_{a1} \\
\text{for } \kappa_2, \\
\qquad P_{a2} = P_{a2};\quad P_{b2} = A_{b2}\,P_{a2};\quad \cdots;\quad P_{n2} = A_{n2}\,P_{a2} \\
\qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\text{and, for } \kappa_n, \\
\qquad P_{an} = P_{an};\quad P_{bn} = A_{bn}\,P_{an};\quad \cdots;\quad P_{nn} = A_{nn}\,P_{an}
\end{array}\right\}\ldots\ldots\ldots(5)$$

In Eqs. 5, A_{mn} is a factor by which P_{an} is multiplied to obtain P_{mn}, values of A_{mn} having been determined by the simultaneous solution of $n - 1$ equations of Eqs. 3.

3. Normalizing of Characteristic Forces

In Section 2 methods were presented whereby each set of characteristic forces P_m could be expressed in terms of any one force of the set, such as P_{am}. The relations of Eqs. 5 do not define P_{am}, however, which may be given any value without altering the characteristic aspect of the loads. As a matter of mathematical convenience in the development to follow, P_{am} is chosen arbitrarily so that:

$$(P_{am})^2 + (P_{bm})^2 + \cdots + (P_{nm})^2 = +1 \ldots\ldots\ldots\ldots\ldots(6)$$

To satisfy Eq. 6, substitute values of P_m from Eqs. 5 and solve for P_{am}:

$$P_{am} = \frac{1}{\sqrt{1 + (A_{bm})^2 + \cdots + (A_{nm})^2}} \ldots\ldots\ldots\ldots\ldots(7)$$

When values of P_{am} are determined in this manner, a set of characteristic forces is said to be "normalized." In the following treatment, normalized characteristic forces will be denoted by Q. Thus Q_{mn} represents the normalized characteristic force at m corresponding to $\kappa = \kappa_n$. Similarly, Q_m represents the complete set of normalized characteristic forces for points a, b, $\cdots$, n, corresponding to $\kappa = \kappa_m$. Using this new notation, Eqs. 7, 5, and 6, respectively, should be rewritten as

$$Q_{am} = \frac{1}{\sqrt{1 + (A_{bm})^2 + \cdots + (A_{nm})^2}} \quad \text{(8)}$$

$$\left.\begin{array}{l} \text{For } \kappa_1, \\ \qquad Q_{a1} = Q_{a1}; \quad Q_{b1} = A_{b1}\,Q_{a1}; \quad \cdots; \quad Q_{n1} = A_{n1}\,Q_{a1} \\ \text{for } \kappa_2, \\ \qquad Q_{a2} = Q_{a2}; \quad Q_{b2} = A_{b2}\,Q_{a2}; \quad \cdots; \quad Q_{n2} = A_{n2}\,Q_{a2} \\ \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \text{for } \kappa_n, \\ \qquad Q_{an} = Q_{an}; \quad Q_{bn} = A_{bn}\,Q_{an}; \quad \cdots; \quad Q_{nn} = A_{nn}\,Q_{an} \end{array}\right\} \quad \text{(9)}$$

and (see Eq. 6):

$$(Q_{am})^2 + (Q_{bm})^2 + \cdots + (Q_{nm})^2 = +1 \quad \text{(10)}$$

4. Orthogonality of Normalized Characteristic Forces

Suppose that a structure acted upon by one set of normalized characteristic forces Q_m is subjected to a virtual distortion corresponding to the deformation that occurs when a second set of normalized characteristic forces Q_n is applied to the structure. The external virtual work done by the forces Q_m may be expressed by

$$W_{mn} = Q_{am}\,\Delta_{an} + Q_{bm}\,\Delta_{bn} + \cdots + Q_{nm}\,\Delta_{nn} = \kappa_n\,(Q_{am}\,Q_{an} + Q_{bm}\,Q_{bn} + \cdots + Q_{nm}\,Q_{nn}) \quad \text{(11)}$$

Now suppose that the same structure, when acted upon by the forces Q_n, is subjected to a virtual distortion corresponding to the deformation that occurs when the forces Q_m are applied to the structure. The external virtual work done by forces Q_n may be expressed by

$$W_{nm} = Q_{an}\,\Delta_{am} + Q_{bn}\,\Delta_{bm} + \cdots + Q_{nn}\,\Delta_{nm} = \kappa_m\,(Q_{an}\,Q_{am} + Q_{bn}\,Q_{bm} + \cdots + Q_{nn}\,Q_{nm}) \quad \text{(12)}$$

Betti's law[2] states that the virtual work done by a group of external forces R_m during a distortion due to a group of external forces R_n is equal to the virtual work done by the forces R_n during a distortion due to the forces R_m. Hence, W_{mn} and W_{nm} can be equated from Eqs. 11 and 12, respectively, leading to

$$\kappa_n\,(Q_{am}\,Q_{an} + Q_{bm}\,Q_{bn} + \cdots + Q_{nm}\,Q_{nn}) = \kappa_m\,(Q_{an}\,Q_{am} + Q_{bn}\,Q_{bm} + \cdots + Q_{nn}\,Q_{nm}) \quad \text{(13)}$$

Since the parenthetical expressions on the two sides of Eq. 13 are identical, and, since $\kappa_n \neq \kappa_m$, it follows that the equation can be satisfied only when

$$Q_{am}\,Q_{an} + Q_{bm}\,Q_{bn} + \cdots + Q_{nm}\,Q_{nn} = 0 \quad \text{(14)}$$

Thus, for any two sets of normalized characteristic forces, the sum of the cross products of forces applied at the same point vanishes. Thus the two sets of normalized characteristic forces are orthogonal; and orthogonality does not depend on normalizing.

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[2] "Theory of Statically Indeterminate Structures," by W. M. Fife and J. B. Wilbur, McGraw-Hill Book Co., Inc., New York, N. Y., 1937, p. 39.

5. First Virtual Work Theorem

Because of the orthogonality of normalized characteristic forces, it follows from Eq. 11 that if $n \neq m$,

$$W_{mn} = \kappa_n(0) = 0 \quad\text{(15)}$$

Thus the external virtual work done by any one set of normalized characteristic forces Q_m, during a distortion caused by any other set of normalized characteristic forces Q_n, is equal to zero. This conclusion will be called the first virtual work theorem for normalized characteristic forces. Its validity does not depend on normalizing.

6. Second Virtual Work Theorem

Assuming that $n = m$, it follows from Eq. 11 that

$$W_{mm} = \kappa_m\left[(Q_{am})^2 + (Q_{bm})^2 + \cdots + (Q_{nm})^2\right] = \kappa_m \quad\text{(16)}$$

since the forces Q_m are normalized. Thus the virtual work done by a given set of normalized characteristic forces Q_m, during a distortion caused by the same set of normalized characteristic forces Q_m, is equal to κ_m. This conclusion will be called "the second virtual work theorem for normalized characteristic forces." Its validity does depend on normalizing.

7. Expression of Actual Forces in Terms of Normalized Characteristic Forces

Referring to Fig. 1, let $R_a, R_b, \cdots, R_n$ be any set of actual forces applied at points a, b, $\cdots$, n, respectively, along the lines of action of the characteristic forces $P_a, P_b, \cdots, P_n$, respectively. These actual forces may be expressed in terms of normalized characteristic forces by superposition as follows:

$$\left.\begin{aligned} R_a &= B_1 Q_{a1} + B_2 Q_{a2} + \cdots + B_n Q_{an} \\ R_b &= B_1 Q_{b1} + B_2 Q_{b2} + \cdots + B_n Q_{bn} \\ &\cdots\cdots\cdots\cdots \\ R_n &= B_1 Q_{n1} + B_2 Q_{n2} + \cdots + B_n Q_{nn} \end{aligned}\right\} \quad\text{(17)}$$

in which $B_1, B_2, \cdots, B_n$ may be called the participation factors for the first, second, $\cdots$, and nth sets of normalized characteristic forces, respectively, with respect to forces $R_a, R_b, \cdots, R_n$. If $R_a, R_b, \cdots, R_n$ are known, the participation factors may be evaluated as follows:

With the forces $R_a, R_b, \cdots, R_n$ acting on the structure, suppose the structure to undergo a virtual distortion due to the further application of the mth set of normalized characteristic forces Q_m. The virtual movements of points a, b, $\cdots$, n will be given respectively by $\Delta_{am} = \kappa_m Q_{am}$; $\Delta_{bm} = \kappa_m Q_{bm}$; $\cdots$; $\Delta_{nm} = \kappa_m Q_{nm}$. The virtual work done by forces $R_a, R_b, \cdots, R_n$ is given by

$$W_{Rm} = R_a \Delta_{am} + R_b \Delta_{bm} + \cdots + R_n \Delta_{nm} = \kappa_m (R_a Q_{am} + R_b Q_{bm} + \cdots + R_n Q_{nm}) \quad\text{(18)}$$

Substituting the relations of Eqs. 17 into Eq. 18 and simplifying:

$$\begin{aligned} W_{Rm} = + \kappa_m \{ & B_1 (Q_{a1} Q_{am} + Q_{b1} Q_{bm} + \cdots + Q_{n1} Q_{nm}) \\ & + B_2 (Q_{a2} Q_{am} + Q_{b2} Q_{bm} + \cdots + Q_{n2} Q_{nm}) \\ & + \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \\ & + B_m [(Q_{am})^2 + (Q_{bm})^2 + \cdots + (Q_{nm})^2] \\ & + B_n (Q_{an} Q_{am} + Q_{bn} Q_{bm} + Q_{nn} Q_{nm}) \} \end{aligned} \tag{19a}$$

From the relations of Eqs. 14 and 10 it follows that

$$W_{Rm} = \kappa_m B_m \tag{19b}$$

Equating Eqs. 18 and 19*b* for W_{Rm} and solving for B_m,

$$B_m = R_a Q_{am} + R_b Q_{bm} + \cdots + R_n Q_{nm} \tag{20}$$

With values of B_m determined by Eq. 20, the superposition of Eqs. 17 is completely defined.

8. Reciprocal Relations

Letting B_m as defined by Eq. 20 be substituted into the *m*th equation of Eqs. 17, and simplifying,

$$\begin{aligned} R_m = & R_a (Q_{a1} Q_{m1} + Q_{a2} Q_{m2} + \cdots + Q_{an} Q_{mn}) \\ & + R_b (Q_{b1} Q_{m1} + Q_{b2} Q_{m2} + \cdots + Q_{bn} Q_{mn}) \\ & + \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \\ & + R_m [(Q_{m1})^2 + (Q_{m2})^2 + \cdots + (Q_{mn})^2] \\ & + R_n (Q_{n1} Q_{m1} + Q_{n2} Q_{m2} + \cdots + Q_{nn} Q_{mn}) \end{aligned} \tag{21}$$

Eq. 21 must hold irrespective of the values assigned to $R_a, R_b, \cdots, R_n$. Assigning a value to R_m but making all other values of R equal to zero:

$$(Q_{m1})^2 + (Q_{m2})^2 + \cdots + (Q_{mn})^2 = 1 \tag{22}$$

If R_n be assigned a value, but all other values of R be taken as zero:

$$Q_{n1} Q_{m1} + Q_{n2} Q_{m2} + \cdots + Q_{nn} Q_{mn} = 0 \tag{23}$$

In a sense, Eqs. 22 and 23 are reciprocal expressions to Eqs. 10 and 14.

9. Use of Normalized Characteristic Forces in Computing Deflections

Referring to Section 7, let it be required to compute δ_m, the total deflection of point m in the direction of R_m, due to known actual forces $R_a, R_b, \cdots, R_n$. By Eqs. 17 these actual forces may be replaced by the characteristic forces $B_1 Q_1, B_2 Q_2, \cdots, B_n Q_n$. The deflection of point m due to the normalized characteristic forces Q_1 is given by $\Delta_{m1} = \kappa_1 Q_{m1}$. Hence, by superposition,

$$\delta_m = B_1 \kappa_1 Q_{m1} + B_2 \kappa_2 Q_{m2} + \cdots + B_n \kappa_n Q_{mn} \tag{24}$$

10. Choice of Characteristic Redundants

Let it be required to find the redundant stresses $X_a, X_b, \cdots, X_n$ in an indeterminate structure such as that shown in Fig. 2(*a*), due to any condition of external loading or imposed strains such as those caused by temperature changes or pier settlements. These redundant stresses may be considered as external loads acting on a statically determinate primary structure, as shown in Fig. 2(*b*), the values of $X_a, X_b, \cdots, X_n$ being so determined that $\delta_a, \delta_b, \cdots, \delta_n$, which represent the relative movements of points a, b, $\cdots$, n, respectively, due to all causes, are in each instance equal to zero.

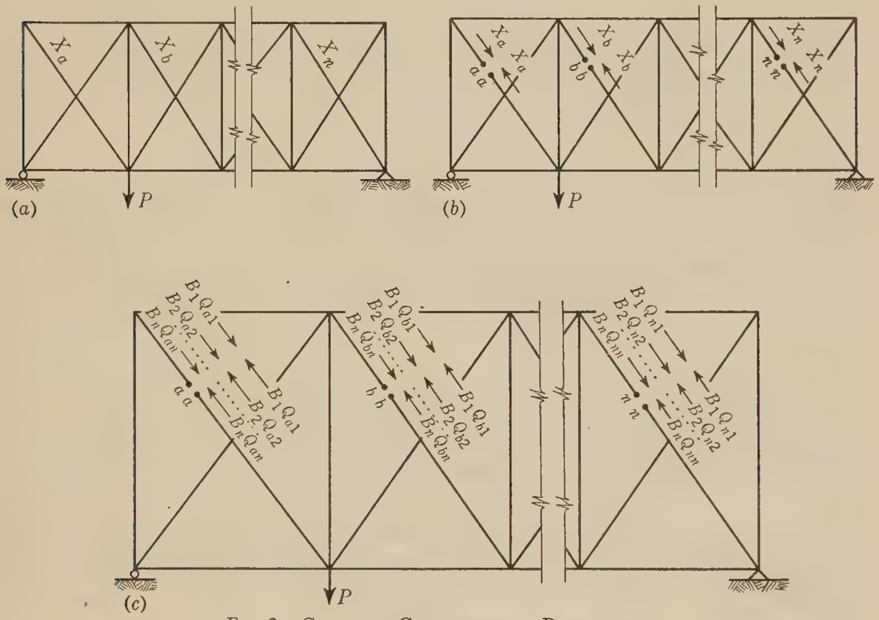

FIG. 2.—CHOICE OF CHARACTERISTIC REDUNDANTS

The redundants $X_a, X_b, \cdots, X_n$, however, may be expressed in terms of normalized characteristic forces, in accordance with Eqs. 17 as follows:

$$\left.\begin{array}{l} X_a = B_1 Q_{a1} + B_2 Q_{a2} + \cdots + B_n Q_{an} \\ X_b = B_1 Q_{b1} + B_2 Q_{b2} + \cdots + B_n Q_{bn} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ X_n = B_1 Q_{n1} + B_2 Q_{n2} + \cdots + B_n Q_{nn} \end{array}\right\} \quad \cdots\cdots\cdots\cdots (25)$$

Thus one may replace the redundant forces shown in Fig. 2(*b*) with the characteristic forces shown in Fig. 2(*c*), wherein, if the normalized characteristic forces $Q_1, Q_2, \cdots, Q_n$ have previously been determined, the participation factors $B_1, B_2, \cdots, B_n$ are the unknowns that must be determined in order that $X_a, X_b, \cdots, X_n$ may be computed from Eqs. 25.

11. EVALUATION OF PARTICIPATION FACTORS

Since $X_a, X_b, \cdots, X_n$ are not immediately known, values of $B_1, B_2, \cdots, B_n$ cannot be computed by Eq. 20. One may proceed as follows, however:

To evaluate B_m, apply the *m*th set of normalized characteristic forces Q_m, consisting of $Q_{am}, Q_{bm}, \cdots, Q_{nm}$ at points a, b, $\cdots$, n, respectively, as virtual forces that act on the primary structure during its distortion due to all causes combined. Note that Q_{am} consists of a pair of equal and opposite forces applied at points a, etc.

The distortion of the primary structure may be considered as due to three classes of causes:

(*a*) Distortion due to external forces, temperature changes, pier settlements, etc., including all causes except the redundant stresses themselves;

(*b*) Distortion due to the redundant $B_m\,Q_m$ (which consists of forces $B_m\,Q_{am}$, $B_m\,Q_{bm}, \cdots, B_m\,Q_{nm}$); and

(*c*) Distortion due to all other redundants—that is, $B_1\,Q_1, B_2\,Q_2, \cdots, B_n\,Q_n$, but not including $B_m\,Q_m$.

Let $\delta_{ao}, \delta_{bo}, \cdots, \delta_{no}$ be the relative deflections of points a, b, ···, n, respectively, due to the first class of causes, these deflections being taken as positive when they act in the directions of $X_a, X_b, \cdots, X_n$, respectively, as shown in Fig. 2(*b*). Then, assuming the virtual forces Q_m to act in the directions shown in Fig. 2(*c*), the external virtual work done by the virtual forces Q_m during this part of the distortion is given by

$$W_1 = Q_{am}\,\delta_{ao} + Q_{bm}\,\delta_{bo} + \cdots + Q_{nm}\,\delta_{no} = \sum(Q_m\,\delta_{mo}) \ldots\ldots \tag{26a}$$

Due to redundant $B_m\,Q_m$, the external virtual work done by the virtual forces Q_m is seen by the second virtual work theorem (Section 6) to be given by

$$W_2 = B_m\,\kappa_m \ldots\ldots \tag{26b}$$

Due to each of the other redundants, the external virtual work done by the virtual forces Q_m is seen by the first virtual work theorem (Section 5) to be given by

$$W_3 = B_m(0) = 0 \ldots\ldots \tag{26c}$$

The total external virtual work done by the virtual forces Q_m is given by $W_1 + W_2 + W_3$. This total must equal zero since points a, b, ···, n undergo no relative movements when all causes are considered. Hence $\sum(Q_m\,\delta_{mo} + B_m\,\kappa_m) = 0$; and

$$B_m = -\frac{\sum(Q_m\,\delta_{mo})}{\kappa_m} \ldots\ldots \tag{27}$$

For structures in direct stress, if F_{nm} is the stress in any bar of the primary structure due to a unit value of Q_{nm}, and if ΔL is the change of length of any bar of the primary structure due to external loads or temperature changes,

$$\begin{aligned} \sum(Q_m\,\delta_{mo} &= Q_{am}\,\delta_{ao} + Q_{bm}\,\delta_{bo} + \cdots + Q_{nm}\,\delta_{no}) \\ &= Q_{am}\sum(\Delta L\,F_{am} + Q_{bm})\sum(\Delta L\,F_{bm} + \cdots + Q_{nm})\sum(\Delta L\,F_{nm}) \\ &= \sum[\Delta L\,(Q_{am}\,F_{am} + Q_{bm}\,F_{bm} + \cdots + Q_{nm}\,F_{nm})] \ldots\ldots \end{aligned} \tag{28a}$$

but $Q_{am}\,F_{am} + Q_{bm}\,F_{bm} + \cdots + Q_{nm}\,F_{nm}$ equals the stress in any bar of the primary structure due to all the virtual forces Q_m acting simultaneously, and may be denoted by F_m, whence

$$\sum(Q_m\,\delta_{mo}) = \sum(\Delta L\,F_m) \ldots\ldots \tag{28b}$$

Considering only the action of external forces, ΔL is given by

$$\Delta L = \frac{F_o L}{A E} \quad \text{(29)}$$

in which F_o is the stress in any bar of the primary structure due to external forces only. Therefore,

$$\sum (Q_m \delta_{mo}) = \sum \left(\frac{F_o F_m L}{A E} \right) \quad \text{(30)}$$

and Eq. 27 becomes

$$B_m = -\frac{1}{\kappa_m} \sum \left(\frac{F_o F_m L}{A E} \right) \quad \text{(31)}$$

Thus, $B_1, B_2, \cdots, B_n$ may be evaluated by Eq. 27 or 31. Then the redundant stresses $X_a, X_b, \cdots, X_n$ may be determined by Eqs. 25.

It may be noted that, although the determination of characteristic loads involves the solution of Eqs. 3, the evaluation of redundant stresses corresponding to any condition of loading or distortion involves no further solution of simultaneous equations, regardless of the degree to which the structure is statically indeterminate.

12. Illustrative Solution

The use of characteristic redundants in analyzing a statically indeterminate structure is illustrated by an example evaluating the stresses X_a, X_b, and X_c in the structure shown in Fig. 3(*a*). For convenience, assume that for each member, $\frac{L}{A E} = 25$.

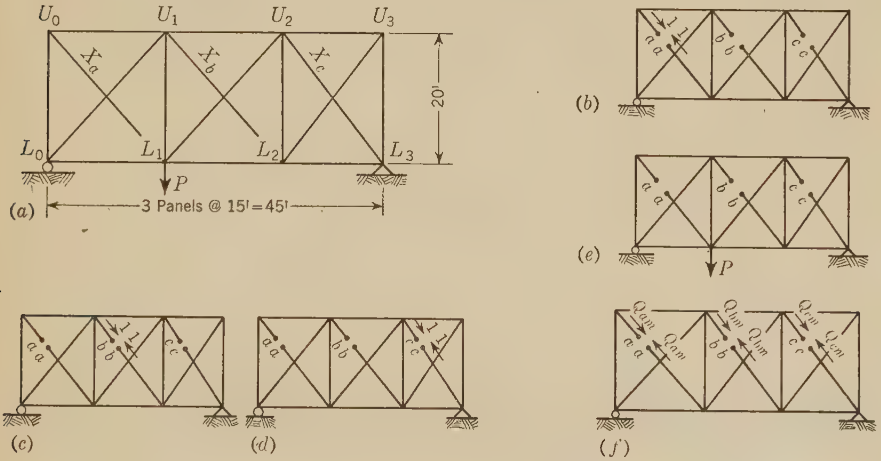

FIG. 3.—ILLUSTRATIVE SOLUTION

Determination of Normalized Characteristic Loads.—Four steps are required to normalize the characteristic loads: (*a*) Evaluation of δ_{mn}; (*b*) evaluation of κ_m; (*c*) determination of characteristic loads; and (*d*) normalizing.

(*a*) Evaluation of δ_{mn} (By Method of Virtual Work).—Let F_{am}, F_{bm}, and F_{cm} be the stresses in any bar due to the condition of loading shown in Figs. 3(*b*), 3(*c*), and 3(*d*), respectively. The computations for δ_{mn} are arranged as shown in Table 1. For example, $\delta_{aa} = \sum\left[\frac{(F_{am})^2\,L}{A\,E}\right] = \sum 25\,(F_{am})^2 = +\,100.$

TABLE 1.—EVALUATION OF δ_{mn}

Bar	$5\,F_{am}$	$5\,F_{bm}$	$5\,F_{cm}$	$25\,(F_{am})^2$	$25\,(F_{bm})^2$	$25\,(F_{cm})^2$	$25\,F_{am}\,F_{bm}$	$25\,F_{am}\,F_{cm}$	$25\,F_{bm}\,F_{cm}$
L_0L_1	−3	0	0	+ 9	0	0	0	0	0
L_1L_2	0	−3	0	0	+ 9	0	0	0	0
L_2L_3	0	0	−3	0	0	+ 9	0	0	0
U_0U_1	−3	0	0	+ 9	0	0	0	0	0
U_1U_2	0	−3	0	0	+ 9	0	0	0	0
U_2U_3	0	0	−3	0	0	+ 9	0	0	0
U_0L_0	−4	0	0	+16	0	0	0	0	0
U_1L_1	−4	−4	0	+16	+16	0	+16	0	0
U_2L_2	0	−4	−4	0	+16	+16	0	0	+16
U_3L_3	0	0	−4	0	0	+16	0	0	0
U_0L_1	+5	0	0	+25	0	0	0	0	0
L_0U_1	+5	0	0	+25	0	0	0	0	0
U_1L_2	0	+5	0	0	+25	0	0	0	0
L_1U_2	0	+5	0	0	+25	0	0	0	0
U_2L_3	0	0	+5	0	0	+25	0	0	0
L_2U_3	0	0	+5	0	0	+25	0	0	0
Σ =	....	....	....	+100	+100	+100	+16	0	+16

Similarly $\delta_{bb} = +\,100$; and $\delta_{cc} = +\,100$. Also, $\delta_{ab} = \delta_{ba} = \sum\left(\frac{F_{am}\,F_{bm}\,L}{A\,E}\right) = \sum 25\,F_{am}\,F_{bm} = +\,16$; $\delta_{ac} = \delta_{ca} = \sum\left(\frac{F_{am}\,F_{cm}\,L}{A\,E}\right) = \sum 25\,F_{am}\,F_{cm} = 0$; and $\delta_{bc} = \delta_{cb} = \sum\left(\frac{F_{bm}\,F_{cm}\,L}{A\,E}\right) = \sum 25\,F_{bm}\,F_{cm} = +\,16.$

(*b*) Evaluation of κ_m (Using Eq. 4).—Eq. 4 becomes—

$$\begin{vmatrix} +\,100 - \kappa; & +\,16 \quad ; & 0 \quad ; \\ +\,16 \quad ; & +\,100 - \kappa; & +\,16 \quad ; \\ 0 \quad ; & +\,16 \quad ; & +\,100 - \kappa; \end{vmatrix} = 0$$

which reduces to

$$(100 - \kappa)\,(+\,9{,}488 - 200\,\kappa + \kappa^2) = 0 \ldots\ldots\ldots\ldots (32)$$

The roots to Eq. 32 are: $\kappa_1 = +\,122.63$; $\kappa_2 = +\,100.00$; and $\kappa_3 = +\,77.37$.

(*c*) Determination of Characteristic Loads (Using Eqs. 3).—For κ_1,

$$\left.\begin{aligned} &(+\,100.00 - 122.63)\,P_{a1} + 16\,P_{b1} = 0; \\ &P_{b1} = +\,\frac{22.63}{16}\,P_{a1} = 1.414\,P_{a1} \doteq \sqrt{2}\,P_{a1}; \qquad (A_{b1} = +\,\sqrt{2}) \\ &+\,16\,P_{b1} + (+\,100.00 - 122.63)\,P_{c1} = 0; \\ &P_{c1} = +\,\frac{16\,P_{b1}}{22.63} = \frac{P_{b1}}{\sqrt{2}} = \frac{\sqrt{2}\,P_{a1}}{\sqrt{2}} = P_{a1}; \qquad (A_{c1} = +\,1) \end{aligned}\right\} \ldots (33a)$$

For κ_2,

$$\left.\begin{aligned}
&(100.00 - 100.00)\,P_{a2} + 16\,P_{b2} = 0; \\
&P_{b2} = \frac{0}{16}\,P_{a2} = 0; \qquad\qquad (A_{b2} = 0) \\
&+16\,P_{a2} + (100.00 - 100.00)\,P_{b2} + 16\,P_{c2} = 0; \\
&P_{c2} = -\frac{16}{16}\,P_{a2} = -P_{a2}; \qquad\qquad (A_{c2} = -1)
\end{aligned}\right\}\ldots(33b)$$

For κ_3,

$$\left.\begin{aligned}
&(+100.00 - 77.37)\,P_{a3} + 16\,P_{b3} = 0; \\
&P_{b3} = -\frac{22.63}{16}\,P_{a3} = -\sqrt{2}\,P_{a3}; \qquad\qquad (A_{b3} = -\sqrt{2}) \\
&+16\,P_{b3} + (+100.00 - 77.37)\,P_{c3} = 0; \\
&P_{c3} = -\frac{16}{22.63}\,P_{b3} = -\frac{P_{b3}}{\sqrt{2}} = -\frac{-\sqrt{2}\,P_{a3}}{\sqrt{2}} = +P_{a3}; \qquad (A_{c3} = +1)
\end{aligned}\right\}\ldots(33c)$$

Hence the characteristic loads are defined by:

$$\left.\begin{aligned}
&P_{a1} = +P_{a1}; \quad P_{b1} = +\sqrt{2}\,P_{a1}; \quad P_{c1} = +P_{a1} \\
&P_{a2} = +P_{a2}; \quad P_{b2} = 0 \qquad\quad ; \quad P_{c2} = -P_{a2} \\
&P_{a3} = +P_{a3}; \quad P_{b3} = -\sqrt{2}\,P_{a3}; \quad P_{c3} = +P_{a3}
\end{aligned}\right\}\ldots\ldots\ldots(34)$$

(*d*) Normalizing of Characteristic Loads.—Substituting in Eq. 8, $Q_a = \frac{1}{\sqrt{1+2+1}} = +\frac{1}{2}$. Similarly, $Q_{a2} = +\frac{1}{\sqrt{2}}$; and $Q_{a3} = +\frac{1}{2}$. Hence, by Eq. 9, the normalized characteristic loads are given by:

$$\left.\begin{aligned}
&Q_{a1} = +\tfrac{1}{2} \quad ; \quad Q_{b1} = +\frac{\sqrt{2}}{2}; \quad Q_{c1} = +\tfrac{1}{2} \\
&Q_{a2} = +\frac{\sqrt{2}}{2}; \quad Q_{b2} = 0 \quad ; \quad Q_{c2} = -\frac{\sqrt{2}}{2} \\
&Q_{a3} = +\tfrac{1}{2} \quad ; \quad Q_{b3} = -\frac{\sqrt{2}}{2}; \quad Q_{c3} = +\tfrac{1}{2}
\end{aligned}\right\}\ldots\ldots\ldots(35)$$

Evaluation of Redundants.—The first step is to determine the value of B_m. Let F_o and F_m be the stress in any bar due to the condition of loading shown in Figs. 3(*e*) and 3(*f*), respectively. Note that the F_m-stresses may be obtained from the F_{am}, F_{bm}, and F_{cm} stresses given in Table 1 by superposition. The arrangement of the computation is shown in Table 2, which supplies the remaining values to be substituted in Eq. 31, thus:

$$\begin{aligned}
B_m &= -\frac{1}{\kappa_m}\sum\left(\frac{F_o\,F_m\,L}{A\;E}\right) = -\frac{1}{\kappa_m}\sum\left(\frac{120\,F_o\,F_m}{120}\right)\times 25 \\
&= -\frac{5}{24\,\kappa_m}\sum(120\,F_o\,F_m)\ldots\ldots\ldots\ldots\ldots\ldots(36)
\end{aligned}$$

Hence, $B_1 = \dfrac{-5\,(-34\,P + 18\sqrt{2}\,P)}{24\,(+122.63)} = +0.0145\ P$. Similarly, $B_2 = +0.4890\,P$ and $B_3 = +0.1601\,P$. The redundants are then determined by substitution in Eqs. 25, thus:

$$X_a = +0.0145\,P\,(+\tfrac{1}{2}) + 0.4890\,P\left(+\frac{\sqrt{2}}{2}\right) + 0.1601\,P\,(+\tfrac{1}{2}) = +0.4330\,P.$$

Similarly, $X_b = -0.1030\,P$ and $X_c = -0.2586\,P$.

TABLE 2.—EVALUATION OF B_m

Bar	$12\,F_o$	$10\,F_1$	$10\,F_2$	$10\,F_3$	$120\,F_oF_1$	$120\,F_oF_2$	$120\,F_oF_3$
L_0L_1	$+6\,P$	-3	$-3\sqrt{2}$	-3	$-18\,P$	$-18\sqrt{2}\,P$	$-18\,P$
L_1L_2	$+3\,P$	$-3\sqrt{2}$	0	$+3\sqrt{2}$	$-9\sqrt{2}\,P$	0	$+9\sqrt{2}\,P$
L_2L_3	0	-3	$+3\sqrt{2}$	-3	0	0	0
U_0U_1	0	-3	$-3\sqrt{2}$	-3	0	0	0
U_1U_2	$-6\,P$	$-3\sqrt{2}$	0	$+3\sqrt{2}$	$+18\sqrt{2}\,P$	0	$-18\sqrt{2}\,P$
U_2U_3	$-3\,P$	-3	$+3\sqrt{2}$	-3	$+9\,P$	$-9\sqrt{2}\,P$	$+9\,P$
L_0U_0	0	-4	$-4\sqrt{2}$	-4	0	0	0
L_1U_1	$+8\,P$	$-4-4\sqrt{2}$	$-4\sqrt{2}$	$-4+4\sqrt{2}$	$-32\,P-32\sqrt{2}\,P$	$-32\sqrt{2}\,P$	$-32\,P+32\sqrt{2}\,P$
L_2U_2	$-4\,P$	$-4\sqrt{2}-4$	$+4\sqrt{2}$	$+4\sqrt{2}-4$	$+16\,P+16\sqrt{2}\,P$	$-16\sqrt{2}\,P$	$+16\,P-16\sqrt{2}\,P$
L_3U_3	$-4\,P$	-4	$+4\sqrt{2}$	-4	$+16\,P$	$-16\sqrt{2}\,P$	$+16\,P$
U_0L_1	0	$+5$	$+5\sqrt{2}$	$+5$	0	0	0
L_0U_1	$-10\,P$	$+5$	$+5\sqrt{2}$	$+5$	$-50\,P$	$-50\sqrt{2}\,P$	$-50\,P$
U_1L_2	0	$+5\sqrt{2}$	0	$-5\sqrt{2}$	0	0	0
L_1U_2	$+5\,P$	$+5\sqrt{2}$	0	$-5\sqrt{2}$	$+25\sqrt{2}\,P$	0	$-25\sqrt{2}\,P$
U_2L_3	0	$+5$	$-5\sqrt{2}$	$+5$	0	0	0
L_2U_3	$+5\,P$	$+5$	$-5\sqrt{2}$	$+5$	$+25\,P$	$-25\sqrt{2}\,P$	$+25\,P$
$\Sigma =$	....	....	....	....	$-34\,P+18\sqrt{2}\,P$	$-166\sqrt{2}\,P$	$-34\,P-18\sqrt{2}\,P$

It will be noted that the additional computation required to investigate a second condition of loading is not very great, since modifying the F_o-stresses will change only a part of the computation for B_m and consequently only a part of the final step.

13. ALTERNATE METHOD OF ANALYZING STATICALLY INDETERMINATE STRUCTURES BY MEANS OF CHARACTERISTIC FORCES

If the number of independent components of displacement for a structure is less than the degree to which the structure is statically indeterminate, it may be desirable to modify the details of the analysis.

Characteristic forces which correspond to the independent components of distortion may be determined and normalized. For the actual external loading condition to be investigated, participation factors for the normalized characteristic forces may be evaluated. Deflections of the points of application of these forces may then be computed, following which, the evaluation of stresses in the structure is straightforward.

Suppose, for example, it is required that the end moments in the frame of Fig. 4(*a*) be determined, due to the load P acting as shown. Corresponding to the three independent components of distortion θ_A, θ_B, and d, there will be considered as a set of characteristic forces the external moments X_a, X_b, and the external force X_c, respectively, acting as shown in Fig. 4(*b*).

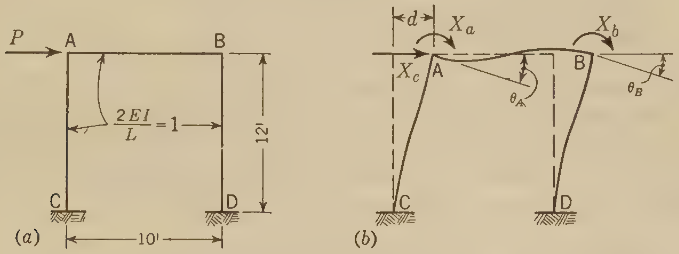

FIG. 4.—ALTERNATE METHOD OF ANALYSIS

By statics, the following three equations may be written:

$$\left.\begin{aligned} +12\,X_e + M_{AC} + M_{CA} + M_{BD} + M_{DB} &= 0\\ +\ X_a\ - M_{AB} - M_{AC} &= 0\\ +\ X_b\ - M_{BA} - M_{BD} &= 0 \end{aligned}\right\}\ldots\ldots\ldots\ldots(37)$$

Expressing end moments by the slope deflection equations, these reduce to

$$\left.\begin{aligned} 12\,X_e + 3\,\theta_A + 3\,\theta_B - d &= 0\\ 4\,X_a + 16\,\theta_A - 4\,\theta_B + d &= 0\\ 4\,X_b - 4\,\theta_A - 16\,\theta_B + d &= 0 \end{aligned}\right\}\ldots\ldots\ldots\ldots(38)$$

By Eq. 1, since X_a, X_b, and X_e are characteristic forces, $d = \kappa\,X_e$; $\theta_A = \kappa\,X_a$, and $\theta_B = \kappa\,X_b$, whence

$$\left.\begin{aligned} 3\,\kappa\,X_a + 3\,\kappa\,X_b + (12-\kappa)\,X_e &= 0\\ (4-16\,\kappa)\,X_a - 4\,\kappa\,X_b + \kappa\,X_e &= 0\\ -4\,\kappa\,X_a + (4-16\,\kappa)\,X_b + \kappa\,X_e &= 0 \end{aligned}\right\}\ldots\ldots\ldots\ldots(39)$$

Eqs. 39 correspond to Eqs. 3, and Eq. 4 becomes

$$\begin{vmatrix} +3\,\kappa & ; & +3\,\kappa & ; & +12-\kappa\\ +4-16\,\kappa & ; & -4\,\kappa & ; & +\kappa\\ -4\,\kappa & ; & +4-16\,\kappa & ; & +\kappa \end{vmatrix} = 0\ldots\ldots\ldots\ldots(40)$$

the roots to which are: $\kappa_1 = +17.23$; $\kappa_2 = +0.3333$; and $\kappa_3 = +0.1990$. Successively substituting these values of κ into Eqs. 39, the characteristic forces are determined as follows:

$$\left.\begin{aligned} X_{a1} &= +X_{a1}; & X_{b1} &= +X_{a1}; & X_{c1} &= +19.77\,X_{a1}\\ X_{a2} &= +X_{a2}; & X_{b2} &= -X_{a2}; & X_{c2} &= 0\\ X_{a3} &= +X_{a3}; & X_{b3} &= +X_{a3}; & X_{c3} &= -0.1012\,X_{a3} \end{aligned}\right\}\ldots\ldots\ldots(41)$$

Using Eq. 8 to normalize the characteristic forces,

$$Q_{a1} = \frac{1}{\sqrt{1+(+1)^2+(+19.77)^2}} = +0.0505.$$

Likewise, $Q_{a2} = +0.7071$ and $Q_{a3} = +0.7053$. The normalized characteristic forces are, therefore,

$$\left.\begin{array}{lll} Q_{a1} = +0.0505; & Q_{b1} = +0.0505; & Q_{c1} = +0.9975 \\ Q_{a2} = +0.7071; & Q_{b2} = -0.7071; & Q_{c2} = 0 \\ Q_{a3} = +0.7053; & Q_{b3} = +0.7053; & Q_{c3} = -0.0714 \end{array}\right\} \qquad (42)$$

The participation factors to be applied to each of these sets of normalized characteristic forces in order that they may correspond to the loading shown in Fig. 4(*a*) are now computed by Eq. 20: $B_1 = P\,Q_{c1} = +0.9975\,P$; $B_2 = P\,Q_{c2} = 0$; and $B_3 = P\,Q_{c3} = -0.0714\,P$. The independent components of distortion for this condition of loading are now computed by Eq. 24:

$$\left.\begin{array}{rl} \theta_A = & +0.9975\,P\,(+17.23)\,(+0.0505) \\ & \quad -0.0714\,P\,(+0.1990)\,(+0.7053) = +0.857\,P \\ \theta_B = & +0.9975\,P\,(+17.23)\,(+0.0505) \\ & \quad -0.0714\,P\,(+0.1990)\,(+0.7053) = +0.857\,P \\ d = & +0.9975\,P\,(+17.23)\,(+0.9975) \\ & \quad -0.0714\,P\,(+0.1990)\,(-0.0714) = +17.14\,P \end{array}\right\} \qquad (43)$$

Therefore, $\psi = \dfrac{d}{12} = +1.428\,P$ and the end moments in the frame are as follows:

$$\left.\begin{array}{l} M_{AB} = 2\,\theta_A + \theta_B = 3\,(+0.857\,P) = +2.571\,P = M_{BA} \\ M_{AC} = 2\,\theta_A - 3\,\psi = 2\,(+0.857\,P) - 3\,(+1.428\,P) = -2.571\,P = M_{BD} \\ M_{CA} = \theta_A - 3\,\psi = +0.857\,P - 3\,(+1.428\,P) = -3.428\,P = M_{DB} \end{array}\right\} \quad (44)$$

14. INTERDEPENDENCY OF CHARACTERISTIC FORCES

For the general case, Eqs. 3 consist of n simultaneous equations with n unknowns. The solution then requires the evaluation of the nth order determinant of Eq. 4 and, for each of the n roots κ thus obtained, the simultaneous solution of $n - 1$ equations of Eqs. 3.

Under certain circumstances, however, the points of application and the directions of characteristic forces may be such that the n equations of Eqs. 3 may break down into two or more groups, each of which may be solved by itself. For example, one might obtain for Eqs. 3, for five forces P_a, P_b, $\cdots$, P_e:

$$P_a\,(\delta_{aa} - \kappa) + P_b\,\delta_{ab} = 0 \qquad (45a)$$

$$P_a\,\delta_{ba} + P_b\,(\delta_{bb} - \kappa) = 0 \qquad (45b)$$

$$P_c\,(\delta_{cc} - \kappa) + P_d\,\delta_{cd} + P_e\,\delta_{ce} = 0 \qquad (45c)$$

$$P_c\,\delta_{dc} + P_d\,(\delta_{dd} - \kappa) + P_e\,\delta_{de} = 0 \qquad (45d)$$

and

$$P_c\,\delta_{ec} + P_d\,\delta_{ed} + P_e\,(\delta_{ee} - \kappa) = 0 \qquad (45e)$$

This condition would arise, for example, if neither of the forces P_a or P_b caused displacements at points c, d, or e along the lines of action of any of the forces

P_c, P_d, or P_e, respectively. In Eqs. 45 there are two separate groups. Eqs. 45*a* and 45*b* may be solved together, involving only a second-order determinant. From these two equations two sets of characteristic forces are found, for each of which P_a and P_b have values, whereas P_c, P_d, and P_e are zero. Eqs. 45*c*, 45*d*, and 45*e* may then be solved together, involving a third-order determinant. From these three equations three sets of characteristic forces are found, for each of which P_c, P_d, and P_e have values, whereas P_a and P_b are zero.

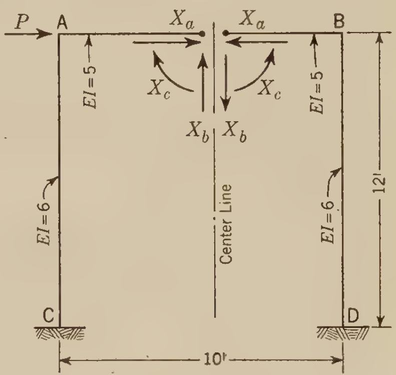

FIG. 5

When Eqs. 3 break down in this manner, a considerable saving in computation is possible, as is illustrated with reference to the structure of Fig. 4(*a*), by taking the redundants as shown in Fig. 5. Values of δ_{mn} may be determined by the usual methods of structural analysis, leading to

$$\left.\begin{array}{lll} \delta_{aa} = +192; & \delta_{ab} = \quad 0\;; & \delta_{ac} = +24 \\ \delta_{ba} = \quad 0; & \delta_{bb} = +116.6; & \delta_{bc} = \quad 0 \\ \delta_{ca} = +\;24; & \delta_{cb} = \quad 0\;; & \delta_{cc} = +\;6 \end{array}\right\} \qquad (46)$$

Eqs. 3 become:

$$P_a(+192 - \kappa) + P_c(+24) = 0 \qquad (47a)$$

$$P_b(+116.6 - \kappa) = 0 \qquad (47b)$$

$$P_a(+24) + P_c(+6 - \kappa) = 0 \qquad (47c)$$

First, solving Eq. 47*b* directly, $\kappa_2 = +116.6$ and $P_{b2} = +P_{b2}$. Next, solving Eqs. 47*a* and 47*c* simultaneously,

$$\begin{vmatrix} +192 - \kappa; & +24 \\ +\;24\quad; & +6-\kappa \end{vmatrix} = 0$$

which reduces to $\kappa^2 - 198\,\kappa + 576 = 0$, from which $\kappa_1 = +195$ and $\kappa_3 = +2.95$. For $\kappa_1 = +195$,

$$\left.\begin{array}{l} P_{a1} = +P_{a1}; \quad P_{c1} = +0.127\,P_{a1} \\ \text{and, for } \kappa_3 = +2.95, \\ P_{a3} = +P_{a3}; \quad P_{c3} = -7.88\,P_{a3} \end{array}\right\} \qquad (48)$$

By Eq. 8, $Q_{a1} = +0.992 \left(= \dfrac{1}{\sqrt{1 + (0.127)^2}} \right)$; $Q_{b2} = +1.000$; and Q_{a3}

$= +0.126$. Thus the normalized characteristic forces are:

$$\left.\begin{array}{ll}
\text{For } \kappa_1 = +195, & \\
& Q_{a1} = +0.992; \quad Q_{c1} = +0.126 \\
\text{for } \kappa_2 = +116.6, & \\
& Q_{b2} = +1.000 \\
\text{and, for } \kappa_3 = +6.00, & \\
& Q_{a3} = +0.126; \quad Q_{c3} = -0.992
\end{array}\right\} \dots\dots\dots\dots\dots (49)$$

By the usual methods of structural analysis, the following deflections due to load P may be obtained: $\delta_{ao} = +96\,P$; $\delta_{bo} = -60\,P$; $\delta_{co} = +12\,P$. Then, by Eq. 27:

$$\left.\begin{aligned}
B_1 &= -\frac{1}{195}\left[+0.992\,(+96\,P) + 0.126\,(+12\,P)\right] = -0.496\,P \\
B_2 &= -\frac{1}{116.6}\left[+1.000\,(-60\,P)\right] = +0.514\,P \\
B_3 &= -\frac{1}{6.00}\left[+0.126\,(+96\,P) - 0.992\,(+12\,P)\right] = -0.063\,P
\end{aligned}\right\} \quad (50)$$

and, by Eqs. 25:

$$\left.\begin{aligned}
X_a &= -0.496\,P\,(+0.992) - 0.063\,P\,(+0.126) = -0.500\,P \\
X_b &= +0.514\,P\,(+1.000) = +0.514\,P \\
X_c &= -0.496\,P\,(+0.126) - 0.063\,P\,(-0.992) = 0
\end{aligned}\right\} \dots (51)$$

The end moments in the members of the frame may now be computed by statics, and will be found to equal those obtained in Section 13.

15. Use of Characteristic Redundants Acting at Elastic Center

In the analysis of a fixed end arch, it is well known that the redundants may be advantageously taken as acting at the elastic center of the arch. The resulting solution may be interpreted as one wherein the redundants are characteristic forces, chosen in such a manner that Eqs. 3 break down into three independent equations. To illustrate the foregoing, consider the structure of Fig. 4(*a*) by taking the redundants as acting at the elastic center as shown in Fig. 6, and applying the procedures developed in this paper. For this structure, the elastic center lies on the vertical axis of symmetry for the structure, and 4.00 ft below the girder AB.

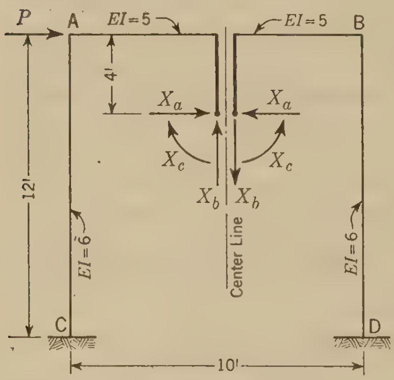

Fig. 6

Values of δ_{mn} may be determined by the usual methods of structural analysis, leading to:

$$\left.\begin{array}{lll}
\delta_{aa} = +96.0; & \delta_{ab} = 0\;; & \delta_{ac} = 0 \\
\delta_{ba} = 0\;; & \delta_{bb} = +116.6; & \delta_{bc} = 0 \\
\delta_{ca} = 0\;; & \delta_{cb} = 0\;; & \delta_{cc} = +6.00
\end{array}\right\} \dots\dots\dots (52)$$

Eqs. 3 become:

$$\left.\begin{aligned} P_a\,(+96.0-\kappa)&=0\\ P_b\,(+116.6-\kappa)&=0\\ P_c\,(+6.00-\kappa)&=0 \end{aligned}\right\}\qquad(53)$$

Solving these equations separately:

$$\left.\begin{aligned} \kappa_1&=+116.6; & P_{b1}&=+P_{b1}\\ \kappa_2&=+\ \ 96.0; & P_{a2}&=+P_{a2}\\ \kappa_3&=+\ \ 6.00; & P_{c3}&=+P_{c3} \end{aligned}\right\}\qquad(54)$$

Application of Eq. 8 leads to the following sets of normalized characteristic loads: $Q_{b1}=\frac{1}{\sqrt{1}}=+1$; $Q_{a2}=\frac{1}{\sqrt{1}}=+1$; and $Q_{c3}=\frac{1}{\sqrt{1}}=+1$. By the usual methods of structural analysis, the following deflections due to load P may be obtained: $\delta_{ao}=+48\,P$; $\delta_{bo}=-60\,P$; and $\delta_{co}=+12\,P$. Then, by Eq. 27:

$$\left.\begin{aligned} B_1&=-\frac{1}{116.6}(+1)(-60\,P)=+0.514\,P\\ B_2&=-\frac{1}{96.0}\ (+1)(+48\,P)=-0.500\,P\\ B_3&=-\frac{1}{6.00}\ (+1)(+12\,P)=-2.000\,P \end{aligned}\right\}\qquad(55)$$

and, by Eqs. 25,

$$\left.\begin{aligned} X_a&=-0.500\,P\,(+1)=-0.500\,P\\ X_b&=+0.514\,P\,(+1)=+0.514\,P\\ X_c&=-2.000\,P\,(+1)=-2.000\,P \end{aligned}\right\}\qquad(56)$$

The end moments in the members of the frame may now be computed by statics, and will be found to equal those obtained in Section 13.

16. Remarks Concerning Solution of Eqs. 3

In Section 2, it was assumed that all values of κ were real and separate. In all solutions made by the writer, values of κ have been real. Should imaginary values of κ be encountered, it is believed that the physical conditions stated by Eqs. 3 are impossible of fulfilment, and that as a result the analysis cannot properly be applied. It is considered doubtful that this condition will arise.

It is possible for two or more values of κ to be equal numerically, however, although this condition is infrequent. Suppose that of the n values of κ, a given value κ' appears m times. It will then be found that only $n-m$ equations of Eqs. 3 are independent when κ' is substituted, so that only $n-m$ values of P can be expressed in terms of the remaining m values of P. This leaves $m-1$ degrees of freedom for each value of κ', so that $m\,(m-1)$ further conditions must be imposed. These conditions must lead to m separate sets of characteristic forces, which must be mutually orthogonal. Within the limitation of these requirements, arbitrary conditions may be imposed, since

Eqs. 3 will be satisfied. The foregoing may be illustrated by the following example: Consider the equations

$$\left.\begin{array}{r} (10.62 - \kappa)\, P_a - 2.25\, P_b - 2.25\, P_c + 0.88\, P_d = 0 \\ - 2.25\, P_a + (10.62 - \kappa)\, P_b + 0.88\, P_c - 2.25\, P_d = 0 \\ - 2.25\, P_a + 0.88\, P_b + (10.62 - \kappa)\, P_c - 2.25\, P_d = 0 \\ + 0.88\, P_a - 2.25\, P_b - 2.25\, P_c + (10.62 - \kappa)\, P_d = 0 \end{array}\right\} \ldots\ldots (57)$$

The determinant formed by the coefficients to forces P reduces to: $(\kappa - 16.00)(\kappa - 9.74)^2 (\kappa - 7.00) = 0$. The roots are: $\kappa_1 = + 16.00$; $\kappa_2 = + 9.74$; $\kappa_3 = + 9.74$; and $\kappa_4 = + 7.00$. When the double root $\kappa_2 = \kappa_3 = + 9.74$ is substituted into Eqs. 57, only two independent equations are formed, which reduce to

$$P_a = - P_d \quad \text{and} \quad P_b = - P_c \ldots\ldots (58a)$$

Since Eqs. 58*a* do not define the relations between P_a and P_b, any assumption may be made regarding this relation, and Eqs. 57 will be satisfied. Suppose that for one set of characteristic forces, one such assumption is made arbitrarily, such as $P_b = C\, P_a$, in which C is a numerical factor. This arbitrary assumption defines the following set of characteristic forces:

$$P_a = + P_a; \quad P_b = + C\, P_a; \quad P_c = - C\, P_a; \quad \text{and} \quad P_d = - P_a \ldots (58b)$$

For the second set of characteristic forces, a second assumption such as $P_b = D\, P_a$ may be made, in which D is another numerical factor which cannot, however, be chosen arbitrarily. This defines the second set of characteristic forces as:

$$P_a = + P_a; \quad P_b = + D\, P_a; \quad P_c = - D\, P_a; \quad \text{and} \quad P_d = - P_a \ldots (58c)$$

The two sets of characteristic forces defined by Eqs. 58*b* and 58*c* must satisfy the orthogonal requirements of Eq. 14, whence

$$\begin{array}{l} (+ P_a)(+ P_a) + (+ C\, P_a)(+ D\, P_a) \\ \quad + (- C\, P_a)(- D\, P_a) + (- P_a)(- P_a) = 0 \ldots\ldots (59) \end{array}$$

and $D = - 1/C$. Thus for κ_2, for example, one may choose C as $- 1$ arbitrarily; that is, one may select $P_b = - P_a$, thus defining the first set of characteristic forces as:

$$P_{a2} = + P_{a2}; \quad P_{b2} = - P_{a2}; \quad P_{c2} = + P_{a2}; \quad \text{and} \quad P_{d2} = - P_{a2} \ldots (60a)$$

Then for κ_3, D must be computed from the relation $D = - 1/C = - 1/- 1 = + 1$, thus leading to the second set of characteristic forces:

$$P_{a3} = + P_{a3}; \quad P_{b3} = + P_{a3}; \quad P_{c3} = - P_{a3}; \quad \text{and} \quad P_{d3} = - P_{a3} \ldots (60b)$$

Since these two sets of characteristic forces are both associated with the same numerical value of κ, the distinction as to which shall be given the subscript 2 and which the subscript 3 is arbitrary. A different assumption as to the value of C would have led to different sets of characteristic forces, but it is believed that the over-all solution would not be modified thereby.

17. ALTERNATE BASIS FOR ORTHOGONALITY

In the treatment to this subject, sets of characteristic forces have been used, which were orthogonal with respect to each other. It is not necessary that the orthogonality exist between sets of forces; an alternate basis for procedure would be to operate on sets of forces so chosen that the qth set of forces is orthogonal with respect to the deflections caused by the rth set of forces. This condition may be illustrated by the equation

$$S_{aq}\,\Delta_{ar} + S_{bq}\,\Delta_{br} + \cdots + S_{nq}\,\Delta_{nr} = 0 \qquad (61)$$

In Eq. 61, the force applied at point m, of the qth set of such forces, is denoted by S_{mq}; Δ_{mr} denotes the total deflection at point m due to the rth set of such forces.

If n such sets of forces, each composed of n forces, are to be determined, there are in all n^2 forces to which values must be assigned. For the first, second, $\cdots$, nth sets of forces one may arbitrarily assign $n, n-1, n-2, \cdots, 1$ forces, respectively, the only limitation in assigning these values being that each set of forces shall be independent with respect to the other sets. The remaining forces, which consist of $0, 1, \cdots, n-1$ forces for the first, second, $\cdots$, nth sets of forces, respectively, must be computed on the basis of Eq. 61; this will necessitate the respective solution of $0, 1, \cdots, n-1$ sets of simultaneous equations. By the judicious choice of the $\frac{n}{2}(n+1)$ arbitrary values to be assigned (such as by assigning values of $+1$ and 0), the amount of computation in arriving at the $\frac{n}{2}(n-1)$ dependent values may be greatly reduced.

In a truss statically indeterminate to the nth degree, let the redundant stresses $X_a, X_b, \cdots, X_n$ be expressed in terms of such forces by superposition, as follows:

$$\left.\begin{array}{l} X_a = C_1\,S_{a1} + C_2\,S_{a2} + \cdots + C_n\,S_{an} \\ X_b = C_1\,S_{b1} + C_2\,S_{b2} + \cdots + C_n\,S_{bn} \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ X_n = C_1\,S_{n1} + C_2\,S_{n2} + \cdots + C_n\,S_{nn} \end{array}\right\} \qquad (62)$$

in which $C_1, C_2, \cdots, C_n$ are participation factors for the 1st, 2d, $\cdots$, nth sets of forces, respectively. Let the set of forces $S_{aq}, S_{bq}, \cdots, S_{nq}$ be denoted by S_q; let the total stress in any bar due to S_q acting on the primary structure be denoted by F_q. Let F_o be the total stress in any bar due to external loads acting on the primary structure.

Suppose the primary structure, acted upon by the external loads and by all the redundants $C_1\,S_1, C_2\,S_2, \cdots, C_n\,S_n$, be subjected to a virtual distortion due to an increase in the value of C_m by ΔC_m. Then the redundant $C_m\,S_m$ will do external virtual work which may be expressed by

$$\begin{aligned} W_1 = {} & C_m\,F_{am}\,(\Delta C_m\,\Delta_{am}) + C_m\,F_{bm}\,(\Delta C_m\,\Delta_{bm}) + \cdots \\ & + C_m\,F_{nm}\,(\Delta C_m\,\Delta_{nm}) = \Delta C_m\,C_m \sum (F_m\,\Delta_m) \qquad (63a) \end{aligned}$$

The other redundants will do no external virtual work in accordance with Eq. 61. The external loads will do external virtual work, which may be expressed by

$$W_2 = \Delta C_m \sum\left(\frac{F_o F_m L}{A E}\right) \qquad \text{(63}b\text{)}$$

Thus the total external virtual work, which equals zero, is given by $W_1 + W_2$, whence

$$C_m = -\frac{\sum\left(\frac{F_o F_m L}{A E}\right)}{\sum(F_m \Delta_m)} \qquad (64)$$

Hence if n sets of forces satisfying Eq. 61 can be obtained, Eqs. 64 and 62 lead to the evaluation of $X_a, X_b, \cdots, X_n$. The manner in which these n sets of forces may be obtained, as well as the application of this entire procedure, may be illustrated by a consideration of the structure of Fig. 3(a), for which $\frac{L}{A E} = 25$ for each member. By the usual methods of structural analysis (see Section 12) the following values of δ_{mn} are obtained:

$$\left.\begin{array}{lll} \delta_{aa} = +100; & \delta_{ab} = +16; & \delta_{ac} = 0 \\ \delta_{ba} = +16; & \delta_{bb} = +100; & \delta_{bc} = +16 \\ \delta_{ca} = 0; & \delta_{cb} = +16; & \delta_{cc} = +100 \end{array}\right\} \qquad (65)$$

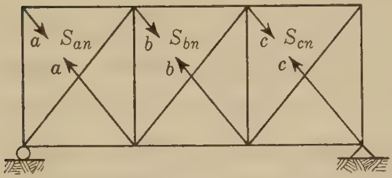

FIG. 7

Referring to Fig. 7, the first set of forces will be arbitrarily chosen as: $S_{a1} = +1$; $S_{b1} = 0$; and $S_{c1} = 0$. Then

$$\left.\begin{array}{l} \Delta_{a1} = +1\,(\delta_{aa}) = +100 \\ \Delta_{b1} = +1\,(\delta_{ba}) = +16 \\ \Delta_{c1} = +1\,(\delta_{ca}) = 0 \end{array}\right\} \qquad (66)$$

The second set of forces, by inspection, is: $S_{a2} = 0$; $S_{b2} = 0$; and $S_{c2} = +1$. For these forces: $\Delta_{a2} = +1\,(\delta_{ac}) = 0$; $\Delta_{b2} = +1\,(\delta_{bc}) = +16$; and $\Delta_{c2} = +1\,(\delta_{cc}) = +100$ so that Eq. 61 is obviously satisfied. For the third set of forces, $S_{b3} = +1$ is selected arbitrarily but S_{a3} and S_{c3} must be computed so as to satisfy Eq. 61. Considering the first and third sets of forces: $S_{a3}\,(+100) + 1\,(+16) + S_{c3}\,(0) = 0$; and $S_{a3} = -0.160$. Considering the second and third sets of forces: $S_{a3}\,(0) + 1\,(+16) + S_{c3}\,(+100) = 0$; and $S_{c3} = -0.160$. For the S_3 forces:

$$\left.\begin{array}{l} \Delta_{a3} = -0.160\,\delta_{aa} + 1.000\,\delta_{ab} - 0.160\,\delta_{ac} = 0 \\ \Delta_{b3} = -0.160\,\delta_{ba} + 1.000\,\delta_{bb} - 0.160\,\delta_{bc} = +94.88 \\ \Delta_{c3} = -0.160\,\delta_{ca} + 1.000\,\delta_{cb} - 0.160\,\delta_{cc} = 0 \end{array}\right\} \qquad (67)$$

To summarize, the three sets of forces are:

For S_1,

$$S_{a1} = +1; \quad S_{b1} = 0; \quad S_{c1} = 0$$

for S_2,

$$S_{a2} = 0; \quad S_{b2} = 0; \quad S_{c2} = +1$$

and, for S_3,

$$S_{a3} = -0.160; \quad S_{b3} = +1; \quad S_{c3} = -0.160 \qquad (68)$$

The total deflections due to these forces are, respectively:

For S_1,

for S_2,

and, for S_3,

$$\left.\begin{aligned} &\Delta_{a1} = +100; \quad \Delta_{b1} = +16\ \ ; \quad \Delta_{c1} = 0 \\ &\Delta_{a2} = 0; \quad \Delta_{b2} = +16\ \ ; \quad \Delta_{c2} = +100 \\ &\Delta_{a3} = 0; \quad \Delta_{b3} = +94.88; \quad \Delta_{c3} = 0 \end{aligned}\right\} \ldots\ldots\ldots (69)$$

Values of $\sum\left(\frac{F_o F_m L}{A E}\right)$ are obtained from Table 3 wherein values of F_o may be obtained by statics. Values of F_m are also obtainable by statics; they may be obtained by superposition from stresses due to unit values of X_a, X_b, and X_c, which will already be available, since they will have been used (see Section 12) in evaluating δ_{mn}. For substitution into Eq. 64: $\sum\left(\frac{F_o F_1 L}{A E}\right) = -1.667\,P\,(25) = -41.7\,P$; $\sum\left(\frac{F_o F_2 L}{A E}\right) = +1.101\,P\,(25) = +27.5\,P$; and $\sum\left(\frac{F_o F_3 L}{A E}\right) = +0.390\,P\,(25) = +9.75\,P$. Further,

TABLE 3.—EVALUATION OF C_m; ALTERNATE BASIS

Bar	F_o	F_1	F_2	F_3	$F_o F_1$	$F_o F_2$	$F_o F_3$
L_0L_1	+0.500 P	−0.600	....	+0.096	−0.300 P	....	+0.048 P
L_1L_2	+0.250 P	....	....	−0.600	....	....	−0.150 P
L_2L_3	....	....	−0.600	+0.096	....	....	....
U_0U_1	....	−0.600	....	+0.096	....	....	....
U_1U_2	−0.500 P	....	....	−0.600	....	....	+0.300 P
U_2U_3	−0.250 P	....	−0.600	+0.096	....	+0.150 P	−0.024 P
L_0U_0	....	−0.800	....	+0.128	....	....	....
L_1U_1	+0.667 P	−0.800	....	−0.672	−0.534 P	....	−0.448 P
L_2U_2	−0.333 P	....	−0.800	−0.672	....	+0.267 P	+0.224 P
L_3U_3	−0.333 P	....	−0.800	+0.128	....	+0.267 P	−0.043 P
U_0L_1	....	+1.000	....	−0.160	....	....	....
L_0U_1	−0.833 P	+1.000	....	−0.160	−0.833 P	....	+0.133
U_1L_2	....	....	....	+1.000	....	....	....
L_1U_2	+0.417 P	....	....	+1.000	....	....	+0.417 P
U_2L_3	....	....	+1.000	−0.160	....	....	....
L_2U_3	+0.417 P	....	+1.000	−0.160	....	+0.417 P	−0.067 P
Σ =	....	....	....	....	−1.667 P	+1.101 P	+0.390 P

$$\left.\begin{aligned} \sum(F_1\,\Delta_1) &= +1\,(+100) + 0\,(+16) + 0\,(0) = +100 \\ \sum(F_2\,\Delta_2) &= +0\,(0) + 0\,(+16) + 1\,(+100) = +100 \\ \sum(F_3\,\Delta_3) &= -0.160\,(0) + 1\,(+94.88) - 0.160\,(0) = +94.88 \end{aligned}\right\} \ldots (70)$$

Hence, by Eq. 64:

$$C_1 = -\frac{-41.7\,P}{+100} = +0.417\,P; \quad C_2 = -\frac{+27.5\,P}{+100} = -0.275\,P;$$

and

$$C_3 = -\frac{+9.75\,P}{+94.88} = -0.103\,P.$$

Finally, by Eqs. 62:

$$\left.\begin{aligned} X_a &= +0.417\,P\,(+1) - 0.275\,P\,(0) - 0.103\,P\,(-0.160) = +0.433\,P \\ X_b &= +0.417\,P\,(0) - 0.275\,P\,(0) - 0.103\,P\,(+1) \qquad\qquad = -0.103\,P \\ X_c &= +0.417\,P\,(0) - 0.275\,P\,(+1) - 0.103\,P\,(-0.160) = -0.259\,P \end{aligned}\right\} \quad (71)$$

A further condition could have been imposed with respect to each set of S forces:

$$S_{aq}\,\Delta_{aq} + S_{bq}\,\Delta_{bq} + \cdots + S_{nq}\,\Delta_{nq} = +1 \ldots\ldots\ldots\ldots (72)$$

This would amount to normalizing each set of forces with respect to the deflections caused by that set of forces. The detail of the solution would then change slightly; if T replaces S to indicate that Eq. 72 has been observed, and if F_m is interpreted as the stress in any bar due to T_m acting on the primary structure, Eq. 63*a* becomes $W_1 = \Delta C_m\,C_m$ and Eq. 63*b* remains unchanged. The denominator $\sum(F_m\,\Delta_m)$ then drops out of Eq. 64. In Eqs. 62, S should now be replaced by T.

This type of solution is possible for any degree of indeterminacy of a structure, although admittedly the labor involved in determining sets of forces which satisfy Eq. 61 would increase rather rapidly with the number of unknowns. Once such sets of forces have been established, however, the redundant stresses may be determined without further recourse to simultaneous equations. This is the same advantage that is gained by use of the elastic center, which, however, is limited in its direct benefits to structures with three or fewer redundants.

18. Conclusion

The purpose of this paper has been to present some of the principles involved in making use of orthogonal forces in the analysis of statically indeterminate structures. Illustrative examples demonstrate these principles, but the structures analyzed were chosen because of their suitability for illustrating principles rather than because their solution by the methods discussed was advantageous.

Structural engineers have been using orthogonal forces for some time in various analyses, but, in some instances, perhaps, without realizing that they were doing so. It is hoped that the presentation of this paper will stimulate further consideration of the possible advantages that may be obtained by their use.

APPENDIX

Notation

The following letter symbols, adopted for use in the paper, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials[3] and American Standard Letter Symbols for

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[3] ASA—Z10a—1932.

Mechanics,[4] prepared by committees of the American Standards Association, with Society representation, and approved by the Association in 1932 and 1942, respectively:

A = a factor in Eq. 5; A_{mn} is a factor by which P_{an} is multiplied to obtain P_{mn}; also (see Eq. 30), A = cross-section area of a structural member;
B = a participation factor in Eq. 17; $B_1, B_2, \cdots, B_n$ are participation factors for the first, second, and nth sets of normalized characteristic forces, $R_a, R_b, \cdots, R_n$, respectively;
C = a numerical factor in Eq. 58*b*; also, $C_1, C_2, \cdots, C_n$ are participation factors (Eq. 62) for the sets of forces $S_1, S_2, \cdots, S_n$, respectively;
D = a numerical factor in Eq. 58*c*;
d = lateral distortion of the top of a column (Fig. 4(*b*));
E = modulus of elasticity;
F = total internal stress in a member; thus, F_{nm} is the stress in any bar of the primary structure due to a unit value of Q_{nm}; F_o is the stress in any bar of the primary structure due to external forces only;
I = moment of inertia;
L = length of a structural member;
m = a number of times that the value κ' appears in an equation; also used as a general integer;
n = a number in a series; thus, $1, 2, \cdots, n$; or $a, b, \cdots, n$;
P = an external load (Eq. 2);
Q = a normalized characteristic force; thus, Q_{am} (Eq. 8) represents the complete set of normalized characteristic forces for point a, corresponding to κ_m;
q = a particular set of forces S (Eq. 61);
R = resultant external forces (Eq. 18);
r = a particular set of forces S (Eq. 61);
S = set of forces; S_{mq} is the force applied at point m, of the qth set of such forces;
T = set of forces to replace S in Eq. 72; T_m is a load applied to the primary structure which produces a stress F_m;
W = external virtual work; W_{mn} is the external virtual work done by the forces Q_m when a structure acted upon by one set of forces Q_m is subjected to a virtual distortion corresponding to the deformation that occurs when a second set of forces Q_n is applied to the structure;
X = redundant total stress;
Δ = total deflection; Δ_n is the total deflection at point n in the direction of P_n due to forces $P_a, P_b, \cdots, P_n$;
δ = deflection due to unit load; δ_{mn} is the deflection at any point m in the direction of P_m, due to a unit value of P_n;
θ = angular distortion, the subscript denoting point of rotation;
κ = a proportionality factor; κ' is a given value of κ that occurs m times for every n occurrences of κ;
ψ = a rotation factor in Eq. 44.

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[4] ASA—Z10.3—1942.

DISCUSSION

CHARLES A. ELLIS,[5] M. AM. SOC. C. E.—The author expresses the hope that the methods presented in his paper will be of interest because they differ so radically from the usual methods. That they differ no one will deny. Whether they will stimulate interest is a matter of individual opinion.

The great amount of labor involved in giving birth to the concept of his paper is a monument to the ingenuity and patience of the author, who, in his "Theory of Statically Indeterminate Structures,"[6] has shown a decided preference for the principles of least work and virtual work which, although sound in theory, are mathematical abstractions. They permit neither physical conception nor space perception which are so helpful in the understanding and application of any principle or method.

Had the author simplified or clarified these two principles he would have done the profession some service; but, on the contrary, his method requires seven pages and thirty-one formulas or equations for its development. Two applications of the method are made to a truss having redundant members. Three pages, seven equations, and two tables are required for the first application in section 12; and three pages, thirteen equations, and one table, for the alternate application in section 17—and all of it is fairly "tough meat."

The writer prefers the principle of deflections in solving all types of statically indeterminate structures. There are at least four methods of application, including area moments, expression of moments in terms of angular rotations (commonly called the slope deflection method), and moment distribution, all of which are well known to structural engineers. The fourth method—the method of Williot equations—is not so well known. In 1934 he published the first application of his method to the towers of Golden Gate Bridge;[7] and in the same year he derived the Williot equations for the three most common types of quadrilateral units to be found in parallel and inclined chord trusses and trestle bents.[8] Each of these three units has two diagonal members. The equations are derived from the geometrical properties of a Williot diagram for each unit, and give the relation between the strains of the six members in each unit.

A four-panel parallel chord truss having two diagonals in each panel is solved by the methods of least work, virtual work, and Williot equations, showing a comparison of the labor involved in each solution. This method will be used in solving the stresses in the author's similar redundant structure shown in Fig. 3(*a*), and the following equation, stated in words, is applicable:[9]

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[5] Prof., Structural Eng., Purdue Univ., West Lafayette, Ind.

[6] "Theory of Statically Indeterminate Structures," by W. M. Fife and J. B. Wilbur, McGraw-Hill Book Co., Inc., New York, N. Y., 1937.

[7] "Williot Equations for Statically Indeterminate Structures," by Charles A. Ellis, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 580.

[8] "Simplified Analysis of Indeterminate Frames," by Charles A. Ellis, *Engineering News-Record*, April 26, 1934, p. 534.

[9] "Simplified Analysis of Indeterminate Frames," by Charles A. Ellis, *ibid.*, April 26, 1934, p. 534, Eq. 2.

The sum of the strains in the two diagonals multiplied by cosec θ equals the sum of the strains in the two horizontal members multiplied by cot θ, plus the sum of the strains in the two vertical members, where θ is the angle between the diagonal and the horizontal member. ..(73)

Let the quantities, $5\,a$, $5\,b$, and $5\,c$, represent, respectively, the assumed tensile stresses in the three redundant members U_0L_1, U_1L_2, and U_2L_3 caused by the load P at L_1. The computed stresses are as follows:

Member	Stress
L_0L_1	$\frac{1}{2}P - 3\,a$
L_1L_2	$\frac{1}{4}P - 3\,b$
L_2L_3	$-3\,c$
U_0U_1	$-3\,a$
U_1U_2	$-\frac{1}{2}P - 3\,b$
U_2U_3	$-\frac{1}{4}P - 3\,c$
U_0L_0	$-4\,a$
U_1L_1	$\frac{2}{3}P - 4\,a - 4\,b$
U_2L_2	$-\frac{1}{3}P - 4\,b - 4\,c$
U_3L_3	$-\frac{1}{3}P - 4\,c$
U_0L_1	$+5\,a$
L_0U_1	$-\frac{5}{6}P + 5\,a$
U_1L_2	$+5\,b$
L_1U_2	$\frac{5}{12}P + 5\,b$
U_2L_3	$+5\,c$
L_2U_3	$\frac{5}{12}P + 5\,c$

The author assumes a constant value for $\frac{L}{A\,E} = 25$ for each member. In any ordinary truss of the same material throughout, E is constant and $\frac{L}{A}$ a variable. In this case the stress in each member should be multiplied by $\frac{L}{A}$ for the member, to give a quantity proportional to the strain. In this problem each stress is proportional to the strain, and the factor 25, which would appear in all terms of the Williot equation, may be canceled. Substituting in Eq. 73 (in which cosec $\theta = \frac{5}{4}$ and cot $\theta = \frac{3}{4}$):

For the left panel—

$$\left(-\tfrac{5}{6}P + 10\,a\right)\tfrac{5}{4} = \left(\tfrac{1}{2}P - 6\,a\right)\tfrac{3}{4} + \tfrac{2}{3}P - 8\,a - 4\,b \quad\ldots.(74a)$$

for the middle panel—

$$\left(\tfrac{5}{12}P + 10\,b\right)\tfrac{5}{4} = \left(-\tfrac{1}{4}P - 6\,b\right)\tfrac{3}{4} + \tfrac{1}{3}P - 4\,a - 8\,b - 4\,c \quad..(74b)$$

for the right panel—

$$\left(\tfrac{5}{12}P + 10\,c\right)\tfrac{5}{4} = \left(-\tfrac{1}{4}P - 6\,c\right)\tfrac{3}{4} - \tfrac{2}{3}P - 4\,b - 8\,c \quad\ldots.(74c)$$

Equs. 74 reduce to

$$100\,a + 16\,b = \tfrac{50}{6}P \quad\ldots\ldots\ldots\ldots(75a)$$

$$16\,a + 100\,b + 16\,c = -\tfrac{9}{6}P \quad\ldots\ldots\ldots\ldots(75b)$$

and

$$16\,b + 100\,c = -\tfrac{33}{6}\,P \quad \ldots\ldots (75c)$$

whence—

$$\begin{aligned} a &= 0.0866\,P, & 5\,a &= 0.433\,P \\ b &= -\,0.0206\,P, & 5\,b &= -\,0.103\,P \\ c &= -\,0.0517\,P, & 5\,c &= -\,0.2585\,P \end{aligned}$$

—which agrees with the author's solution for the stresses in the redundant members.

Eqs. 75 have one peculiarity in common with slope deflection equations in that the coefficient of one unknown quantity in each equation is large in comparison with the coefficients of the other unknowns in the same equation. The unknown values in equations having this peculiarity converge rapidly when solved by a series of approximations. The solution of slope deflection equations by approximations is analogous to the approximations made in moment distribution. However, when only a few equations are involved, the ordinary methods of solution are quicker.

The rectangular arch which the author shows in section 13 may be solved much more quickly by any one of the first three methods of the principle of deflections previously mentioned.

The author is aware, of course, that his solutions are long and considerably involved in comparison with usual methods. He states that the purpose of his paper is to present some of the principles involved in using orthogonal forces. What the writer would very much like to see is a problem or two for which this method is shorter and therefore superior to usual methods; or perhaps a problem, for which other methods are impotent, that may be solved by this method alone.

Ralph W. Stewart,[10] M. Am. Soc. C. E.—The bending moments in the symmetrical portal frame analyzed in this paper (see Fig. 4) were computed from a full page or more of equations and formulas containing three X-values, two θ-values, one d-value, one κ-value, and nine Q-values. These moments can be obtained from two readings at one setting of a slide rule and a little easy mental arithmetic by sketching a traverse of the elastic curves of this frame (Fig. 8). This traverse diagram shows at a glance that three sevenths of the sum of the moments in either column goes to the top of the column and four sevenths to the bottom. From symmetry and the fact that the sum of all column moments must equal the lateral force multiplied by the frame height:

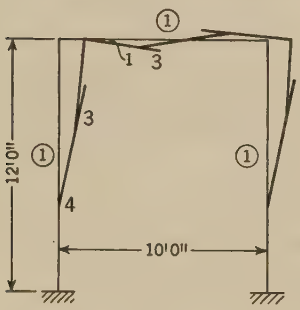

Fig. 8

$$\begin{aligned} \text{Moment at A} &= 3/7 \times 6\,P = 2.5714\,P \\ \text{Moment at C} &= 4/7 \times 6\,P = 3.4286\,P \\ \hline \text{Total (check)} &= 6.0000\,P \end{aligned}$$

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[10] Bridge Engr., City of Los Angeles, Los Angeles, Calif.

This one-minute solution uses two constants of flexure, one moment balance, and one traverse closure. The author's solution requires three simultaneous equations with sixteen symbols.

Unless a type of problem can be presented for which the method of characteristic redundants is superior to other methods, the writer feels that the paper presents a method which has some value as a mathematical exercise but is not of value as a practical method of computing stresses.

MYLE J. HOLLEY, JR.,[11] ESQ.—A new method for the solution of statically indeterminate structures is suggested in this paper. In addition, an entirely new line of reasoning is introduced which may yield valuable concepts and methods beyond those illustrated by the author. Practical application of the method, along the lines suggested, should prove it more advantageous than conventional methods for some special problems, and, in addition, from such application may develop even more potent analytical methods. For this reason, it is to be hoped that some members of the profession will be encouraged to apply the method to practical problems.

The method of characteristic redundants for the solution of statically indeterminate trusses offers a reduction in the effort of solving simultaneous equations. This gain results from (*a*) the fact that only one set of simultaneous equations need be solved, regardless of the number of different loadings for which the structure is to be analyzed, and (*b*) the fact that there is one less equation in the simultaneous set that must be solved than there would be if a conventional approach were used.

The disadvantages of the method are that it requires the solution of a polynomial equation for the determination of characteristic forces, and that it also requires a substantially larger amount of tabular work for the determination of the participation factors. With these relative advantages and disadvantages in mind one might use the following criteria in deciding whether the author's method would be worth while in a particular case:

1. If the structure is indeterminate to a small degree, the new method will not prove advantageous, even if there are many loadings to be investigated.
2. If the structure is indeterminate to a large degree but only one, or at most a very few, separate loadings must be considered, the method will not prove advantageous.
3. If the structure is indeterminate to a large degree, and there are several separate loadings to be investigated, the method may prove advantageous.

The writer realizes that the foregoing criteria are very crude, but if the analyst follows some such reasoning before starting he may avoid an unfair test of the author's method and may also save himself considerable effort. Of course, only the experience resulting from actual analyses can refine these criteria to a point where they will give critical combinations of the degree-of-indeterminacy and number-of-loadings.

Although the writer can visualize no case in which it would be practicable to use characteristic redundants to determine deflections, it may be of academic interest to point out that this could easily be done. If the deflection of a point at which there is no load is desired, or if the deflection at a loading point in a direction other than that of the actual load is desired, the characteristic set would include a characteristic load at the point in question, in the direction of the desired deflection, as well as a characteristic load at each loading point in the direction of the actual load. The characteristic load at the deflection point will be removed by the participation factors in the superposition process which results in the actual loading. The writer believes, however, that such a procedure would be far more laborious than any of the conventional methods of determining deflections. This is in no way a criticism of the paper, which (as its title indicates) presents a method for analyzing indeterminate structures (not a method for finding deflections) and, for this purpose, the writer believes that the method holds real promise.

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[11] With S. Morgan Smith Co., Rutland, Vt.

M. W. Huggins,[12] Esq.—The skill displayed in producing this intricate and interesting solution to a complex problem is deserving of high praise. The aim of the author, which was to find what he calls "the heart of the multiply indeterminate solution" (see "Synopsis") and by means of which the analysis of any indeterminate frame is reduced to the solution of one system of simultaneous equations for all loadings, is a laudable one. Unfortunately, the solution obtained has not simplified the analysis of such frames. On the contrary, for most cases, it has added to the complexity of the analysis. It seems quite within the realm of possibility, however, that some future development of the method may serve to simplify the steps.

It would appear that the Wilbur method is in much the same category as the "relaxation method,"[13] proposed by Prof. R. V. Southwell. As an extension of the principles of moment distribution to the solution of multiply redundant trussed frames, Professor Southwell's method was first developed to simplify the analysis of dirigible frames. In it, stresses are determined by a series of approximations, but, unlike the method of moment distribution, the approximations do not converge readily. In fact, considerable skill and patience are required in solving even some of the simpler frames. In spite of this, the method seems to hold promise and has the important advantage that it eliminates, completely, the solution of simultaneous equations. It may be used to analyze any problem which normally involves the solution of simultaneous equations. As with the method proposed by Professor Wilbur, it has its greatest advantage when used to analyze one frame for several different, separate types of loading.

Leon Beskin,[14] Assoc. M. Am. Soc. C. E.—An attempt is made in this paper to apply to statically indeterminate structures a method of calculation which is highly successful in various fields of applied mathematics. The fundamentals of this method, as presented by the author in Sections 1 to 11 and 16 and 17, can be found in various good mathematical treatises;[15,16,17,18] the material in Section 16, however, lacks precision inasmuch as Professor Wilbur does not apply to his problem the classical properties of definite quadratic forms which constitute the basis of the method. This attempt to transpose into the field of statically indeterminate structures a method which is essential in other fields is not successful, due to the fact that the problems to be solved in the case of statically indeterminate structures are considerably simpler than those for which the method has been developed. Only in some special cases, most of which are already well known, does the method present definite advantages. Similar developments were presented in 1907 by S. Müller[19] and in 1944 W. M. Kincaid and V. Morkovin[20] presented a straightforward method of "orthogonalization" limited to the case of rigid frames.

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[12] Asst. Prof., Civ. Eng., Univ. of Toronto, Toronto, Canada.

[13] "Relaxation Methods in Engineering Science," by R. V. Southwell, Clarendon Press, Oxford 1940.

[14] Structures Engr., Consolidated Vultee Aircraft Corp., San Diego, Calif.

To define the type of problem examined by the author, it must be remembered that the strain energy of a system is a quadratic function of redundants, and can be represented by an ellipse (two redundants), a quadric (three redundants), or a generalized quadric in a hyperspace (more than three redundants), these curves or surfaces corresponding to a determinate value of the strain energy.

The quadratic function is minimum at the center of the quadric, the coordinates of which represent the actual values of the redundants. The solution of a system of linear equations deduced from the condition of minimum strain energy is tantamount to the determination of that center.

Actually, the author examines two methods of determining that center. The first is the study of the principal axes of the quadric, which is of interest when these principal axes are needed as such—for instance, in problems of vibration, in which these axes represent the periods of vibration. When these quantities have no special interest, this method is much too complicated, since it requires the solution of an algebraic equation, the degree of which is equal to the number of redundants. The author has devoted more space to this method than it deserves.

The second method, which is specifically called orthogonalization of linear equations, is the study of an arbitrary set of conjugate axes of the same quadric—a much simpler problem than the first. Once these axes are defined, it is extremely simple to find the coordinates of the center, which are given by independent linear equations, containing one unknown each. This method, to be an improvement, must be shorter than the conventional method involving the direct solution of linear equations. It substitutes for the solution of n linear equations with n unknowns the solution of the sequence, $n - 1$ equa-

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[15] "A Course of Modern Analysis," by E. T. Whittaker and G. N. Watson, Cambridge, England, 1927, p. 224 et seq.

[16] "Die Differential- und Integral gleichungen der Mechanik und Physic," by P. Frank and R. von Mises, F. Vieweg & Sohn, Braunschweig, 1930; American lithographed edition, Mary S. Rosenberg, New York, N. Y., Vol. I, 1943, Chapter II, pp. 44–119.

[17] "Methoden der Mathematischen Physik," by R. Courant and D. Hilbert, Julius Springer, Berlin, 2d Ed., Vol. I, 1931, Chapters I and II, pp. 1–50.

[18] "Mathematical Methods in Engineering," by Theodor von Kármán and Maurice A. Biot, McGraw-Hill Book Co., Inc., New York, N.Y., 1940, Chapter V, pp. 162–216; Chapter VIII, pp. 323–364.

[19] "Zur Berechnung mehrfach statisch unbestimmter Tragwerke," by S. Müller, *Zentralblatt der Bauverwaltung*, 1907, p. 23.

[20] "An Application of Orthogonal Moments to Problems in Statically Indeterminate Structures, by W. M. Kincaid and V. Morkovin, *Quarterly of Applied Mathematics*, Vol. I, No. 4, January, 1944, pp. 334–340.

tions, $n - 2$ equations, . . . 1 equation. A comparison of the number of operations shows that the method is advantageous only for four unknowns or less. For these small numbers of unknowns, the additional work required by the preparation of the method offsets the theoretical advantage, except when simple formulas can be established for all problems related to a given type of structure (for example, the centroid and principal axes of a beam section or the elastic center in a rigid frame).

Moreover, the most efficient provision for solving simultaneous equations is the Gauss method, a remarkably simple application of which was presented by P. D. Crout[21] in 1941. The Gauss method is a systematic method of eliminating unknowns,[22] which is at the same time a method of orthogonalization. This method is so expeditious that other methods are never justified except when special physical interpretations make them easier to understand, or when further computations require some intermediate steps which the Gauss method by-passes. A few examples are offered to demonstrate the special conditions under which orthogonalization along a definite physical pattern is useful and also to indicate when research involving the principal axes is justifiable.

The problem involving combined bending and direct load is a case in point. When moments are taken with respect to the centroid, the property of orthogonalization will appear; that is, moments then produce only rotation and direct loads produce only translation.

In the general case of bending, the principal axes of inertia have the property of orthogonality—namely, that bending around an axis produces rotation around that axis. In this case, principal directions are defined, thus requiring the solution of an equation of second degree, although, in the previous condition (combined bending and direct load), principal directions are not defined because of their lack of physical significance. In combined bending and direct load, an orthogonal system of the third order is defined.

A more complex case of orthogonalization, introduced in the analysis of open cylindrical shells (aircraft structures and cylindrical roofs of the Zeiss-Dywidag type), is that of torsion bending. When these structures are subjected to twisting forces, transmission of torque produces axial stresses, which can also be produced by externally applied direct forces with no resultant and no resulting couple. These direct forces constitute a mechanical entity which the writer has called a "bicouple."[23] The distribution of stresses under arbitrary forces is determined by a method of orthogonalization, which defines, successively, the direct resultant, two couples at the centroid (along principal axes), and a bicouple in reference to the shear center. The orthogonalization is of the fourth order in this case.

A well-known case of the third order is that of frames. This problem is generally treated by the use of an elastic center, but sometimes it is presented as a case of orthogonalization (complex bending and direct load), and is then called the beam-column analogy.

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[21] "A Short Method for Evaluating Determinants and Solving Systems of Linear Equations with Real or Complex Coefficients," by Prescott D. Crout, *Transactions*, A.I.E.E., Vol. 60, 1941, pp. 1235–1240.

[22] "Methoden der Mathematischen Physik," by R. Courant and D. Hilbert, Julius Springer, Berlin, 2d Ed., Vol. I, 1931, p. 3.

[23] "Stress Analysis of Open Cylindrical Membranes," by Leon Beskin, *Journal of the Aeronautical Sciences*, Vol. 11, No. 4, October, 1944, pp. 343–355.

The writer has also examined the case of the two-story frame, leading to an orthogonalization of the sixth order,[24] which reduces to a fourth-order and a second-order problem, when the frame is symmetrical.

Incomplete orthogonalization is also used. The three-moment equation for continuous beams is an example: All moments at supports are orthogonal, except moments at three consecutive supports; the two-moment equation (conjugate point method[25]) is an improvement on the three-moment theorem, since only two consecutive moments are not orthogonal to each other. The method can be applied a second time, and new moments can be defined, which are entirely orthogonal to one another.

In all these problems, orthogonalization is justified because it has a physical significance. Incomplete orthogonalization is a compromise which greatly reduces the numerical work necessary to solve simultaneous equations, without introducing additional complications in the preparation of these equations. It is obtained by a rational choice of unknowns which, once established, does not require either any additional computation (continuous beams), or very little additional computation (centroid, elastic center), every time the method is applied.

Sometimes, also, the choice of unknowns facilitates handling equations by reducing the values of the rectangle terms (such as δ_{ab} in Eqs. 2) as compared with those on the first diagonal (δ_{aa}, δ_{bb}, $\cdots$). Then, the solution of the simultaneous equations is less sensitive to small errors in the determination of coefficients δ and in the subsequent numerical computation. This makes it possible to use a slide rule instead of a computing machine. The redundants are chosen in such a manner that the regions of the structure on which they act simultaneously are reduced to a minimum. For instance, considering a structure on elastic supports of various rigidities, it is inadvisable to use as an unknown the reaction of a very flexible support, which greatly increases all the coefficients of the simultaneous equations so that the coupling, represented by $\frac{\delta_{ab}}{\sqrt{\delta_{aa}\,\delta_{bb}}}$, is nearly equal to 1.

The foregoing comments show the broad advantages and limitations of orthogonalization. Details of the method presented by the author will next be examined.

As has already been indicated, Sections 1 to 13 are devoted to a method of determining natural frequencies. This method is also used, when the elastic stability of a structure is considered, to determine critical loads; these loads correspond to zero frequency. The author applies this method to the condition in which all masses and mass moments of inertia are equal to 1. Then the coefficients κ are the reciprocals of the squares of the frequencies ω:

$$\kappa_i = \frac{1}{(\omega_i)^2} \qquad \text{(76a)}$$

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[24] "Rings for Two Decked Airplanes," by L. Beskin, *Aircraft Engineering*, London, England (publication pending).

[25] "Moments in Restrained and Continuous Beams by the Method of Conjugate Points," by L. H. Nishkian and D. B. Steinman, *Transactions*, Am. Soc. C. E., Vol. 90 (1927), pp. 1–206.

Eq. 76*a* shows that the set of conditions, expressed by Eqs. 1, is unduly restrictive, and that the actual set should be

$$\Delta_{ji} = \frac{\kappa_i}{m_j} P_{ji} \qquad \text{(76b)}$$

in which m_j is a generalized mass at point j (mass if Δ_{ji} is a translation and P_{ji} is a force; and mass moment of inertia, if Δ_{ji} is a rotation and P_{ji} is a couple).

Then, n solutions are found in exactly the same manner as the author's solutions, the latter corresponding to $m_j = 1$. It is obvious that, when couples and forces are used simultaneously (Section 13), the condition $m_j = 1$ leads to coefficients κ which have indefinite units, since they are $F^{-1} L$ for translations and forces and $F^{-1} L^{-1}$ for rotations and couples. Thus, the author's solution is not homogeneous and changes with the choice of the unit length. For instance, a cantilever loaded at the end by a load P and a couple C has a deflection δ and a rotation ϕ:

$$\delta = \frac{P L^3}{3 E I} + \frac{C L^2}{2 E I} = \kappa P \qquad \text{(77a)}$$

and

$$\phi = \frac{P L^2}{2 E I} + \frac{C L}{E I} = \kappa C \qquad \text{(77b)}$$

which gives:

$$\frac{\kappa E I}{L} = \frac{1 + \frac{L^2}{3} \pm \sqrt{1 + \frac{L^2}{3} + \frac{L^4}{9}}}{2} \qquad \text{(78)}$$

If coefficients κ with zero dimension are chosen—for instance,

$$\delta = \frac{\kappa L^2 P}{3 E I} \qquad \text{(79a)}$$

and

$$\phi = \frac{\kappa L C}{E I} \qquad \text{(79b)}$$

—then:

$$\kappa = 1 \pm \frac{\sqrt{3}}{2} \qquad \text{(80)}$$

Remarks Concerning Solution of Eqs. 3.—Some of the statements in Section 16 must be examined. For example, concerning the first paragraph of that section, it is well known that the strain energy of an elastic solid is always a definite positive quadratic function, either of the forces or of the displacements. This means that all coefficients κ are real and, moreover, positive. In other words, all vibration modes are real. Thus, an elastic system of the type examined is always stable. This statement is true when the strain energy is of the form $\frac{1}{2} \Sigma(P \Delta)$. Then, the effect of the change of configuration under load is neglected, which is not the case when problems of elastic stability are examined. It is understood that such cases are not considered by the author, since he uses Eqs. 2 to define displacements.

For the case of equal roots Professor Wilbur's examination of κ is incomplete. When there are two equal roots, the determinant that yields κ can be reduced to the second order by eliminating other roots, thus:

$$(\kappa - a_{11})\,(\kappa - a_{22}) - (a_{12})^2 = 0 \qquad (81)$$

Eq. 81 has equal roots only if

$$(a_{11} - a_{22})^2 = -\,(a_{12})^2 \qquad (82)$$

which requires that $a_{11} = a_{22}$ and that $a_{12} = 0$.

The physical significance of Eq. 82 is that the two corresponding frequencies must be equal. This will happen in any system, if masses are chosen in a correct manner, and will occur for a definite configuration when the dimensions have determinate values.

For instance, in Fig. 9, let a mass be placed at the elastic center, which coincides with the extreme end, E, of the beam. The deflections are:

$$\delta = \frac{P\,L^3}{E\,I}\,\frac{1 + \sqrt{2}}{3\,\sqrt{2}} \qquad (83a)$$

and

$$\phi = \frac{C\,L}{E\,I}\,(1 + \sqrt{2}) \qquad (83b)$$

If the radius of gyration of the end mass is $\bar{r} = \dfrac{L}{\sqrt{3\,\sqrt{2}}}$ ($\bar{r} = 0.486\,L$), the two frequencies are equal. By the author's method, the two values of κ coincide when $\bar{r} = 1$, making $L = \sqrt{3\,\sqrt{2}} = 2.06$. This value is independent of the unit of length. Thus, if the computation is in inches, the cantilever equal to 2.06 in. has the exceptional property that its coefficients κ are equal. If the computation is in yards, a cantilever 74 in. long has the same exceptional property. It is obvious that this is a characteristic that has nothing to do with the structure, but merely with the choice of units. This effect can be eliminated by choosing other units. In other words, the condition imposed by multiple roots is generally of no significance for the problem examined by the author, despite the fact that it has a physical significance in the problem of natural frequencies.

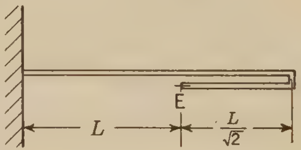

FIG. 9

When consistent units are used, multiple roots may have a physical significance. For instance, if the problem of general bending and axial force is examined in connection with a beam having equal moments of inertia in two directions, there are no principal directions; or, more exactly, all sets of mutually perpendicular axes can be assumed to be principal directions. It is simple and logical, in that case, to choose the principal axes of references as principal axes. The author examines two solutions (see Eqs. 59 and 60)—P_{a2}, $C\,P_{a2}$; and P_{a3}, $-\dfrac{1}{C}P_{a3}$—which define two perpendicular directions in the case of a beam. Factors C and $-1/C$ are the tangents of the angles of the principal directions with the initial axes. The logical choice in that case is P_{a2}, 0; and

0, P_{b2}. Then, instead of defining orthogonal systems by such complex formulas as Eqs. 60, the orthogonal system can be defined by:

$$P_{a2} = P_{a2}; \qquad P_{b2} = 0; \qquad P_{c2} = 0; \qquad P_{d2} = -P_{a2} \ldots\ldots (84a)$$

and

$$P_{a2} = 0; \qquad P_{b2} = P_{b2}; \qquad P_{c2} = -P_{b2}; \qquad P_{d2} = 0 \ldots\ldots (84b)$$

Characteristic Forces.—The first sentence in Section 13 is incorrect, unless the author has in mind a special definition of independent components of displacements. The latter are defined conventionally as the differences between the total number of displacements and the number of equations of statics. Moreover, for the example shown in Fig. 4, the author indicates (see third paragraph, Section 13) that there are three independent components of distortion, and it is well known that a built-in frame is redundant to the third order. Thus, the example is not consistent with the first sentence of Section 13, the meaning of which is obscure.

Alternate Basis.—The approach used in Section 17 is much simpler than that used in the remainder of the paper, and (as has been indicated) is the correct method for nearly all the problems considered here. It is correct to state that a judicious choice of arbitrary values simplifies the determination of dependent values; but simplification is generally obtained not by using values 0 or +1, as stated by the author, but by finding orthogonal groups from physical considerations, such as symmetry, antisymmetry, and polar multiaxial symmetry. In Section 14, the author examines such cases of simplification, but does not connect them with Section 17.

It is useful to clarify the connection between the two methods, seemingly independent in the author's presentation. This connection is very simple when the author's nonhomogeneous special relations (Eqs. 1) are replaced by Eq. 76*b*. Then, if a system is orthogonalized in an arbitrary manner by the second method, there will be a system of values m_j which will satisfy n systems of Eq. 76*b*, written successively for the n independent sets of values obtained by the method of orthogonalization. In Figs. 3 and 7, the values $1/m_j$ can be chosen equal to 1, 1, 0, and the values ω^2 can be taken as 1/100, 0, 1/100. In this case, there are indeterminate quantities 0/0, which can be defined by continuity from orthogonal sets differing from the solution selected, by infinitely small deviations.

These considerations show why the second method is simpler than the first: The system of masses m_j can have arbitrary values in the second method, whereas these masses are equal to 1 in the first. This determination of masses necessitates the solution of an equation of degree n, otherwise useless.

The advantage claimed in the last paragraph of Section 17 is purely fictitious. Experience in manipulating simultaneous equations reveals that, once the unknowns have been eliminated for a given set of loads (second members of the equations), the supplementary work necessary to find the redundancies in other loading conditions is extremely slight. When many loading conditions are contemplated (more than the order of redundancy), the use of unit conditions for the second members reduces the work in great proportions, and, if Mr. Crout's method is applied, this supplementary work is negligible.

FREDERICK S. MERRITT,[26] JUN. AM. SOC. C. E.—Although the purpose of this paper was to present some of the principles involved in utilizing orthogonal forces, the value of the paper as a contribution to structural theory suffers because:

(1) The terminology involved is awkward;
(2) The visualization of the physical processes signified by each step in the calculations is difficult, if not impossible;
(3) In addition to requiring the computation of constants which are used in other well-known methods, this theory requires the solution of simultaneous equations plus the determination of the roots of an nth-degree algebraic equation; and
(4) The one advantage claimed (after the characteristic forces have been found, redundant stresses may be evaluated without the further solution of simultaneous equations) is not exclusive.

The idea underlying advantage (4), however, is of extreme importance to structural theory and deserves further elaboration.

Whether this method of analyzing statically indeterminate structures, using orthogonal forces, can be improved so that it can stand on equal terms with procedures such as the "Dummy Unit Load Method" is questionable. On the other hand, the principles outlined in this paper may be a start in the right direction, especially if the subject of characteristic redundants is approached from another angle than that of orthogonal forces.

(1) A change in viewpoint may improve the terminology. The use of such words as "characteristic forces," "normalizing," and "orthogonality," although mathematically expressive, is discouraging from a structural viewpoint, because certain physical properties are implied which, unfortunately, have no connection with the structure being analyzed. Actually, an abstract mathematical operation is involved. If these terms were more closely related to structural action, their use would not be so awkward.

(2) A different approach may correlate, to a greater extent, the conception of characteristic redundants with the structural action involved. Such correlation is a desirable goal in all structural problems since it provides a continuous check on computations; but it is a goal that cannot always be achieved. As the method is presented, a mass of computation is required to determine A_{mn}, a quantity which, although it is essential to this method, has no immediately perceivable relation to any physical action in the structure. If a relation can be established, the principle involved in using orthogonal forces would be simplified considerably. After a study of the formulas for normalizing and orthogonalizing, the question naturally arises as to whether the concept of vectors and tensors could establish such a desirable relationship. For example, Eq. 10 indicates that Q_{nm} could be the component of a unit vector $\mathbf{Q}_\mathrm{m}$, and, thus, Eq. 14 is the vector equation, $\mathbf{Q}_\mathrm{m} \cdot \mathbf{Q}_\mathrm{n} = 0$, $n \neq m$. If $\mathbf{Q}_\mathrm{m}$ could be shown to have some meaning structurally and if the participation factors are related vectorially to $\mathbf{Q}_\mathrm{m}$, a clearer picture of the principle involved in this method would result.

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[26] With *Engineering News-Record,* New York, N. Y.

(3) By approaching the problem in a different manner, the solution of simultaneous equations may possibly be avoided or the number of equations may be considerably decreased, much as moment distribution methods have succeeded in doing for continuous, statically indeterminate beams and frames. A vectorial approach, involving the selection of suitable axes to simplify the resolution into components of a vector such as Q_m, may be indicated. However, there are other possibilities in methods not involving characteristic forces.

(4) A change in the method of analysis will not spoil the advantage that, once characteristic forces have been found, redundant stresses may be evaluated without the further solution of simultaneous equations. Almost all the well-known methods possess this advantage because the equations may generally be written so that the coefficients of the unknowns depend solely on the structural framework and are independent of loads or distortions.

However, this advantage is not to be belittled. For example, the popular method of fixed-end moment distribution (which, in general, does not require the solution of simultaneous equations) requires that whenever the load is changed either influence computations must be made in the first place or the structure must be analyzed again. In schoolroom problems, this may not be much of a handicap, but in practical design, where the stresses resulting from dead load, live load, wind forces, longitudinal and lateral forces, earth pressure, earthquake, temperature, shrinkage, and settlement must be determined separately, reanalysis is a big handicap and characteristic redundants are definitely advantageous.

The characteristic redundants need not be orthogonal forces, or even forces at all. This is suggested in the paper by the alternate basis for orthogonality, in which forces and deflections are involved. Deflections alone could be used. Whether or not in the general case the use of deflections would simplify the problem will have to be determined. Such a possibility is indicated by the use of end rotations in the solution of problems involving continuous, statically indeterminate beams and frames.

For example, in the method of moment distribution, by means of moment ratios, advantage is taken of the fact that the ratio of the moment induced at one end of an unloaded span of a continuous structure to the moment "carried over" to the other end is a constant independent of the magnitude of the moments. This ratio is a characteristic redundant in the same sense as the characteristic forces described by Professor Wilbur. By using it, the solution of simultaneous equations is reduced to a minimum or entirely eliminated. Offhand, the moment ratio in a given span might be considered a sort of characteristic force, but, actually, it is a function of the end rotations of the unloaded span considered as simply supported but subjected at either end to a unit torque.

The distribution of an unbalanced moment at a joint to the several connecting members is a function of the moment ratios. Also, in a loaded span, the end moments produced by the loads are a function of the moment ratios and of the end rotations due to the loads when the span is considered as simply supported. Thus, all the elements necessary for the solution of a continuous beam or frame problem are present, when the moment ratios are known.

Are there characteristic redundants in the form of deflections that can be used, in a similar manner, to determine the distribution of a force to each of several structural members which support it at a joint? If so, the problem of analyzing statically indeterminate structures should be approached from a viewpoint using these characteristic redundants.

JOHN B. WILBUR,[27] M. AM. SOC. C. E.—The discussion of this paper has been appreciated. The writer believes that the proposed method would have been more attractive to some if it had been interpreted in terms of the method of least work. The derivation in the paper might have ended with Section 2. The n characteristic forces $P_{a1}, P_{a2}, \cdots, P_{an}$ as defined in Eq. 5 then could have been taken as the n unknowns for a least work solution.

Let F_n denote the stress in any member of a truss due to the nth set of characteristic forces acting on the primary structure, and F_o equal the stress in any member of a truss due to the applied loads acting on the primary structure; then the n simultaneous equations resulting from a least work solution would be as follows:

$$\left.\begin{aligned} \frac{\partial W}{\partial P_{a1}} &= \sum (F_o + F_1 + F_2 + \cdots + F_n) \frac{L}{A\,E} \frac{\partial F}{\partial P_{a1}} = 0 \\ \frac{\partial W}{\partial P_{a2}} &= \sum (F_o + F_1 + F_2 + \cdots + F_n) \frac{L}{A\,E} \frac{\partial F}{\partial P_{a2}} = 0 \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ \frac{\partial W}{\partial P_{an}} &= \sum (F_o + F_1 + F_2 + \cdots + F_n) \frac{L}{A\,E} \frac{\partial F}{\partial P_{an}} = 0 \end{aligned}\right\} \quad \ldots\ldots (85)$$

Because the characteristic redundants are orthogonal, however, $\sum \frac{F_m\,L}{A\,E} \frac{\partial F}{\partial P_{an}} = 0$ when $n \neq m$; whence, Eqs. 85 reduce to

$$\left.\begin{aligned} \sum \frac{F_o\,L}{A\,E} \frac{\partial F}{\partial P_{a1}} + \sum \frac{F_1\,L}{A\,E} \frac{\partial F}{\partial P_{a1}} &= 0 \\ \sum \frac{F_o\,L}{A\,E} \frac{\partial F}{\partial P_{a2}} + \sum \frac{F_2\,L}{A\,E} \frac{\partial F}{\partial P_{a2}} &= 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots & \\ \sum \frac{F_o\,L}{A\,E} \frac{\partial F}{\partial P_{an}} + \sum \frac{F_n\,L}{A\,E} \frac{\partial F}{\partial P} &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (86)$$

Moreover, Eqs. 86 are not simultaneous in character since the first equation involves only P_{a1} as an unknown; the second equation, only P_{a2}; etc.

To illustrate this procedure, consider the analysis of the truss in Fig. 3(*a*) acted upon by the load. The characteristic loads as defined by Eq. 34 are computed as shown in the paper. Now, letting P_{a1}, P_{a2}, and P_{a3} equal X, Y, and Z, respectively, the F_o, F_1, F_2, and F_3 stresses are computed by statics, leading to the results shown in Fig. 10. Since $\frac{L}{A\,E}$ has the same value for all members, the working equations reduce to the following:

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$$\left.\begin{aligned}\sum F_o \frac{\partial F}{\partial X} + \sum F_1 \frac{\partial F}{\partial X} &= 0\\ \sum F_o \frac{\partial F}{\partial Y} + \sum F_2 \frac{\partial F}{\partial Y} &= 0\\ \sum F_o \frac{\partial F}{\partial Z} + \sum F_3 \frac{\partial F}{\partial Z} &= 0\end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots (87)$$

The required summations are obtained in Table 4.

TABLE 4.—EVALUATION OF SUMMATIONS OF EQS. 87

Bar	F = TOTAL STRESS =				$F_o \frac{\partial F}{\partial X}$	$F_1 \frac{\partial F}{\partial X}$	$F_o \frac{\partial F}{\partial Y}$	$F_2 \frac{\partial F}{\partial Y}$	$F_o \frac{\partial F}{\partial Z}$	$F_3 \frac{\partial F}{\partial Z}$
	F_o +	F_1 +	F_2 +	F_3						
L_0L_1	$+0.500\,P$	$-0.600\,X$	$-0.600\,Y$	$-0.600\,Z$	$-0.300\,P$	$+0.360\,X$	$-0.300\,P$	$+0.360\,Y$	$-0.300\,P$	$+0.360\,Z$
L_1L_2	$+0.250\,P$	$-0.849\,X$	0	$+0.849\,Z$	$-0.212\,P$	$+0.720\,X$	0	0	$+0.212\,P$	$+0.720\,Z$
L_2L_3	0	$-0.600\,X$	$+0.600\,Y$	$-0.600\,Z$	0	$+0.360\,X$	0	$+0.360\,Y$	0	$+0.360\,Z$
U_0U_1	0	$-0.600\,X$	$-0.600\,Y$	$-0.600\,Z$	0	$+0.360\,X$	0	$+0.360\,Y$	0	$+0.360\,Z$
U_1U_2	$-0.500\,P$	$-0.849\,X$	0	$+0.849\,Z$	$+0.424\,P$	$+0.720\,X$	0	0	$-0.425\,P$	$+0.720\,Z$
U_2U_3	$-0.250\,P$	$-0.600\,X$	$+0.600\,Y$	$-0.600\,Z$	$+0.150\,P$	$+0.360\,X$	$-0.150\,P$	$+0.360\,Y$	$+0.150\,P$	$+0.360\,Z$
U_0L_0	0	$-0.800\,X$	$-0.800\,Y$	$-0.800\,Z$	0	$+0.640\,X$	0	$+0.640\,Y$	0	$+0.640\,Z$
U_1L_1	$+0.667\,P$	$-1.933\,X$	$-0.800\,Y$	$+0.333\,Z$	$-1.290\,P$	$+3.735\,X$	$-0.533\,P$	$+0.640\,Y$	$+0.222\,P$	$+0.111\,Z$
U_2L_2	$-0.333\,P$	$-1.933\,X$	$+0.800\,Y$	$+0.333\,Z$	$+0.645\,P$	$+3.735\,X$	$-0.267\,P$	$+0.640\,Y$	$-0.111\,P$	$+0.111\,Z$
U_3L_3	$-0.333\,P$	$-0.800\,X$	$+0.800\,Y$	$-0.800\,Z$	$+0.267\,P$	$+0.640\,X$	$-0.267\,P$	$+0.640\,Y$	$+0.267\,P$	$+0.640\,Z$
U_0L_1	0	$+X$	$+Y$	$+Z$	0	$+1.000\,X$	0	$+1.000\,Y$	0	$+1.000\,Z$
L_0U_1	$-0.833\,P$	$+X$	$+Y$	$+Z$	$-0.833\,P$	$+1.000\,X$	$-0.833\,P$	$+1.000\,Y$	$-0.833\,P$	$+1.000\,Z$
U_1L_2	0	$+1.414\,X$	0	$-1.414\,Z$	0	$+2.000\,X$	0	0	0	$+2.000\,Z$
L_1U_2	$+0.417\,P$	$+1.414\,X$	0	$-1.414\,Z$	$+0.590\,P$	$+2.000\,X$	0	0	$-0.590\,P$	$+2.000\,Z$
U_2L_3	0	$+X$	$-Y$	$+Z$	0	$+1.000\,X$	0	$+1.000\,Y$	0	$+1.000\,Z$
L_2U_3	$+0.417\,P$	$+X$	$-Y$	$+Z$	$+0.417\,P$	$+1.000\,X$	$-0.417\,P$	$+1.000\,Y$	$+0.417\,P$	$+1.000\,Z$
$\Sigma =$	....	....	....	....	$-0.142\,P$	$+19.63\,X$	$-2.767\,P$	$+8.000\,Y$	$-0.991\,P$	$+12.38\,Z$

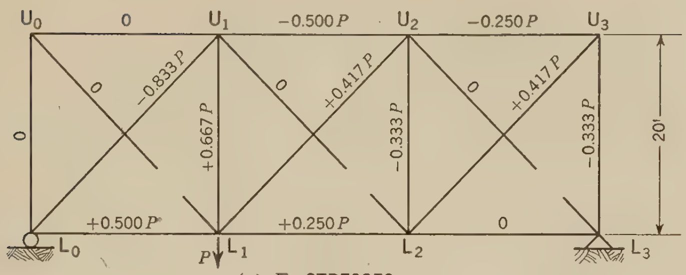
The equations to determine X, Y, and Z are;

$$\left.\begin{aligned} -\,0.142\,P + 19.63\,X &= 0; & X &= +\,0.007\,P\\ -\,2.767\,P + 8.000\,Y &= 0; & Y &= +\,0.346\,P\\ -\,0.991\,P + 12.38\,Z &= 0; & Z &= +\,0.080\,P\end{aligned}\right\} \ldots\ldots\ldots (88)$$

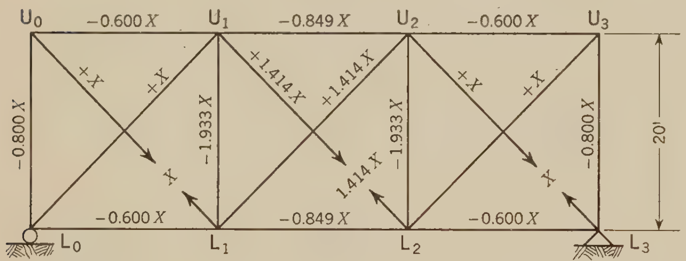
Hence, the basic unknowns, X_a, X_b, and X_c (see Fig. 3(*a*)), are obtained by superposition from the characteristic redundants as follows:

$$\left.\begin{aligned} X_a &= +\,0.007\,P\,(+\,1) + 0.346\,P\,(+\,1) + 0.080\,P\,(+\,1) = +\,0.433\,P\\ X_b &= +\,0.007\,P\,(+\sqrt{2}) + 0.346\,P\,(0) + 0.080\,P\,(-\sqrt{2}) = -\,0.103\,P\\ X_c &= +\,0.007\,P\,(+\,1) + 0.346\,P\,(-\,1) + 0.080\,P\,(+\,1) = -\,0.259\,P\end{aligned}\right\} . (89)$$

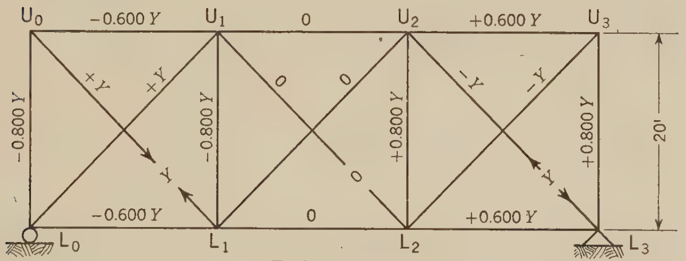
In a minor way, the foregoing solution may answer the objections of Messrs. Ellis, Stewart, and Merritt that the method is too mathematical and too tedious to be of value, but the writer cannot actually hope to answer this criticism. The paper was presented in the belief that ideas that are new to structural engineers should be offered for discussion, even though their direct and immediate practical application may not be apparent—provided there is some hope that these ideas may eventually lead to worth-while methods. Messrs. Huggins and Holley have accepted the efforts of the writer from this viewpoint.


U0
0
U1
−0.500 P
U2
−0.250 P
U3
0
−0.833 P
0
0
+0.667 P
0
+0.417 P
−0.333 P
0
+0.417 P
−0.333 P
20'
+0.500 P
+0.250 P
0
L0
P
L1
L2
L3

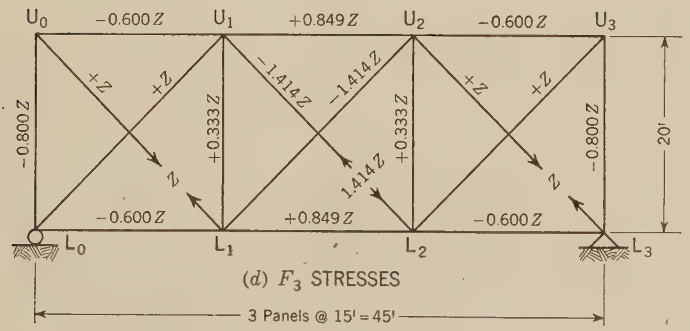
(a) F_0 STRESSES


U0
−0.600 X
U1
−0.849 X
U2
−0.600 X
U3
+X
+X
−0.800 X
X
−1.933 X
+1.414 X
+1.414 X
1.414 X
−1.933 X
+X
+X
X
−0.800 X
20'
−0.600 X
−0.849 X
−0.600 X
L0
L1
L2
L3

(b) F_1 STRESSES


U0
−0.600 Y
U1
0
U2
+0.600 Y
U3
+Y
+Y
−0.800 Y
Y
−0.800 Y
0
0
0
+0.800 Y
−Y
−Y
Y
+0.800 Y
20'
−0.600 Y
0
+0.600 Y
L0
L1
L2
L3

(c) F_2 STRESSES


U0
−0.600 Z
U1
+0.849 Z
U2
−0.600 Z
U3
+Z
+Z
−0.800 Z
Z
+0.333 Z
−1.414 Z
−1.414 Z
1.414 Z
+0.333 Z
+Z
+Z
Z
−0.800 Z
20'
−0.600 Z
+0.849 Z
−0.600 Z
L0
L1
L2
L3
3 Panels @ 15' = 45'

(d) F_3 STRESSES

Fig. 10

Mr. Beskin has contributed a learned and critical discussion which adds much to the general picture of the use of orthogonal functions in structural analysis, although he holds little hope for practical applications of this method in the analysis of statically indeterminate structures. He stresses the relation between the proposed method and methods for computing periods of vibration —actually the line of reasoning that led to the proposed method. An understanding of the proposed method of computing characteristic loads may help structural engineers wishing to extend their interests into the field of vibrations. Contrary to Mr. Beskin, the first sentence in Section 13 is correct, since the number of independent components of displacement is not necessarily equal to the number of independent redundant stress components, although equality happens to exist in the example chosen. By independent components of displacement, the writer simply refers to joint and chord rotations as commonly used in the analysis of rigid frames by the slope deflection method.

Summarizing, the writer believes that, once the characteristic forces corresponding to the redundants in an indeterminate structure have been determined, the remainder of the analysis, in terms of these characteristic forces, is simplified to an important degree. As indicated by the writer, the labor involved in determining the characteristic forces in the first place is such that the over-all method is not very promising in its present state of development. Perhaps it is possible to develop a simple method of determining characteristic forces, or perhaps, as Mr. Beskin suggests, these forces should be determined, in some instances, by inspection from physical considerations.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2257

HYDRAULIC MODEL INVESTIGATION OF MITER GATE OPERATION

By Maurice N. Amster,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Edward Soucek, Isaac DeYoung, F. W. Edwards, William A. Tripp, and Maurice N. Amster.

Synopsis

The miter gate is a commonly used type of closure for the chambers of navigation locks. It is a two-leaf arrangement, comparatively simple in design and adaptable to convenient means of operation. In the open position the two leaves are recessed in opposite lock walls with their faces parallel to the walls. For closure each leaf is swung about a vertical axis near its downstream (quoin) end so that the upstream (miter) ends of the leaves contact at the center line of the lock. The bottoms and the quoin ends of the leaves are equipped with sealing arrangements that contact floor and lock walls when the leaves reach their closed positions. In supporting a water load, the gate acts as a three-hinged arch, with the two quoin bearings and the miter bearing forming the hinges and the lock walls acting as abutments for the end thrust. The quoin end of each leaf is supported at the bottom by a pintle assembly and at the top by a gudgeon assembly. Frequently used types of operating machines are: Hydraulic cylinder and piston, cable and drum, and crank and strut.

In the design of machinery for the operation of lock miter gates, it is essential to know, as accurately as possible, the power requirements and the loads that will occur on the operating mechanism throughout the cycle. For lack of better data, designers in the past have had to resort to empirical methods usually based upon arbitrary assumptions as to hydraulic resistance. Mechanical difficulties in operation generally have been avoided by the use of large safety factors. In the design of the operating machinery for the gates of the Third Locks of the Panama Canal, the size of the leaves indicated that the forces to be encountered would exceed those in any similar structures in the past. Because of the large number of machines required, the use of high

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Note.—Published in March, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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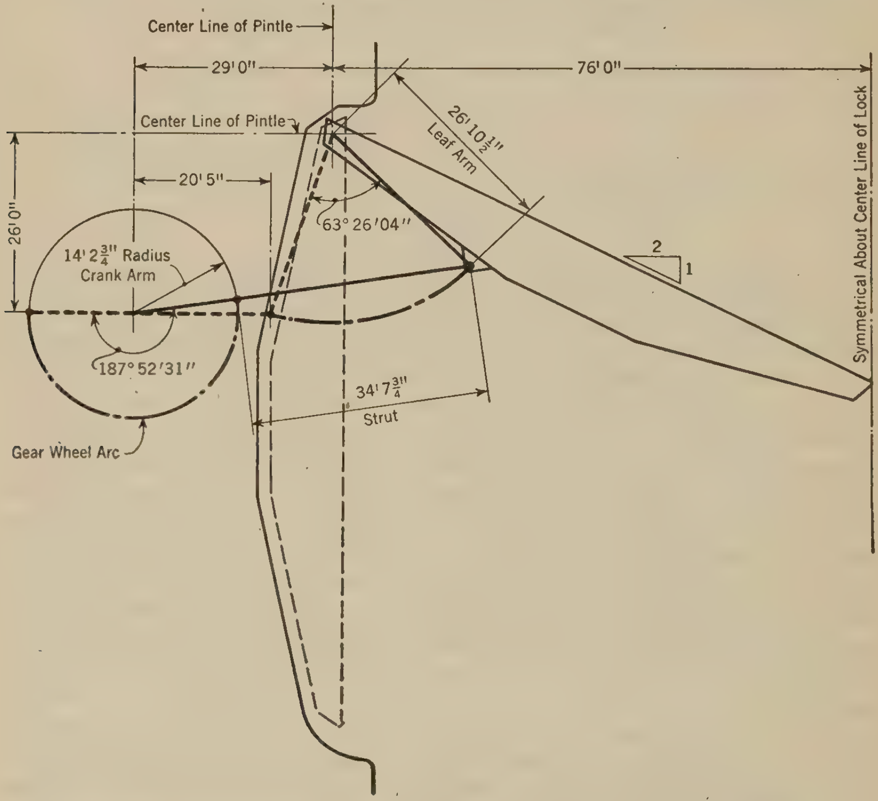
safety factors would have been extremely uneconomical. Accordingly, it was decided to conduct model tests to investigate the resisting forces that might be expected in the operation of the Third Locks gates.

THE PROTOTYPE

The mechanism designed for operating the prototype gates is of the crank-and-connecting-rod type, having kinematic characteristics similar to those used in the existing Panama Canal Locks but differing in mechanical details and arrangement. A large gear wheel, driven by two electric motors through trains of speed-reduction gears, rotates a vertical shaft at practically uniform velocity. A crank is keyed on the lower end of the shaft, and a strut is linked to the end of the crank and to the face of the gate leaf. The operating linkage is shown schematically in Fig. 1. The crank arm is driven through an arc of 187°52′31″ to move the gate leaf through an angle of 63°26′04″. The dead-center positions of the crank relative to the strut coincide with the extreme



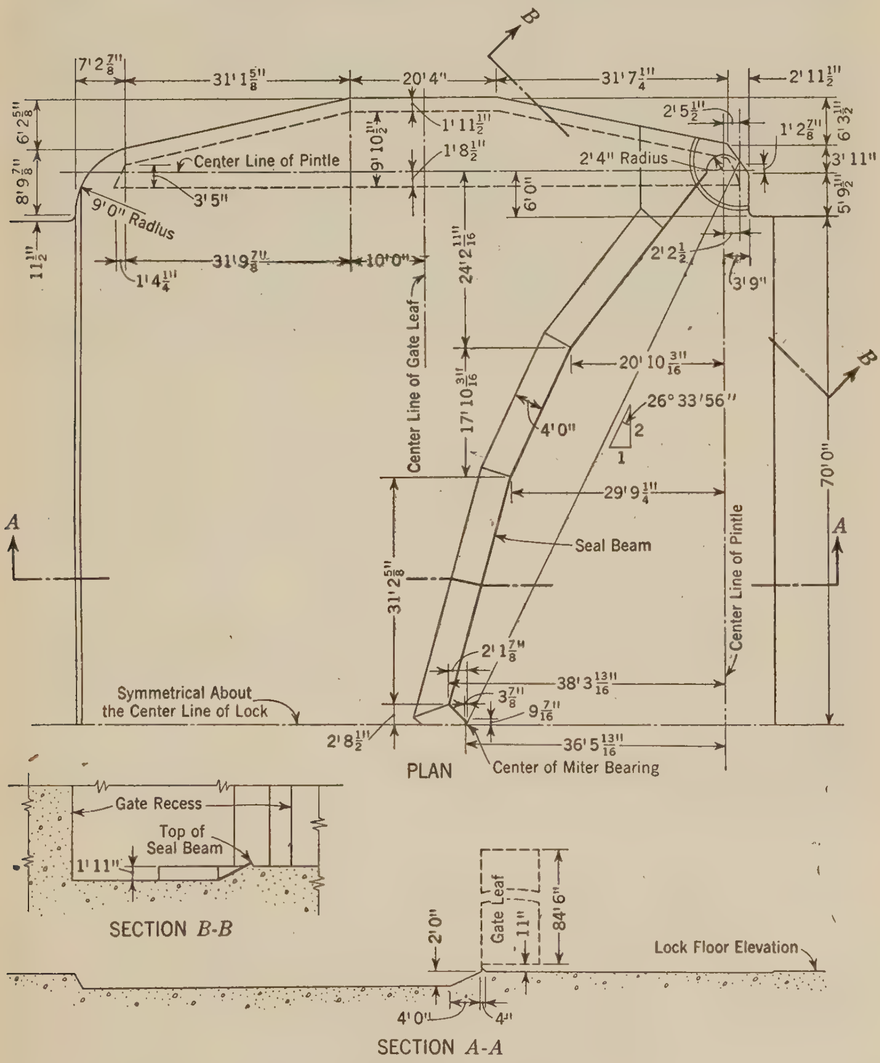
Center Line of Pintle
29'0"
76'0"
Center Line of Pintle
26'10½"
Leaf Arm
20'5"
26'0"
14'2¾" Radius
Crank Arm
63°26'04"
2
1
187°52'31"
34'7¾"
Strut
Gear Wheel Arc
Symmetrical About Center Line of Lock

FIG. 1.—LINKAGE OF OPERATING MACHINE (DIMENSIONS ARE PROTOTYPE VALUES; SCALE, 25 : 1)

positions of the gate leaf, effecting an infinite theoretical mechanical advantage at those positions.

THE MODEL

The model lock, constructed of concrete to a scale of 1 to 25, consisted of a gate bay located in a flume of sufficient length to permit simulation of the lock



B
7' 2 7/8"
31' 1 5/8"
20' 4"
31' 7 1/4"
2' 11 1/2"
6' 2 5/8"
8' 9 7/8"
2' 5 1/2"
1' 2 7/8"
6' 3 1/32"
1' 11 1/2"
9' 10 1/2"
1' 8 1/2"
2' 4" Radius
3' 11"
Center Line of Pintle
3' 5"
6' 0"
5' 9 1/2"
9' 0" Radius
11 1/2"
31' 9 7/8"
10' 0"
1' 4 1/4"
24' 2 11/16"
2' 2 1/2
3' 9"
Center Line of Gate Leaf
20' 10 3/16"
17' 10 3/16"
26° 33' 56"
2
1
4' 0"
70' 0"
29' 9 1/4"
A
Seal Beam
31' 2 5/8"
Center Line of Pintle
2' 1 7/8"
38' 3 13/16"
3 7/8"
9 7/16"
Symmetrical About the Center Line of Lock
2' 8 1/2"
36' 5 13/16"
PLAN
Center of Miter Bearing
Gate Recess
Top of Seal Beam
1' 11"
SECTION B-B
Gate Leaf
11"
2' 0"
84' 6"
Lock Floor Elevation
4' 0"
4"
SECTION A-A

Fig. 2.—Prototype Gate Bay, Recess and Sill

chamber for distances of 1,125 ft upstream and 625 ft downstream. Details of the gate bay are shown in Fig. 2. The gate leaves were constructed of an angle-iron framework faced with galvanized sheet metal. Holes bored in the tops and the bottoms of the leaves permitted water to flow easily into or out of the leaves during changes of the water-surface elevation in the flume. The weight of the prototype leaves was not simulated.

The model leaves were operated by the mechanism shown in Fig. 3. The position of each linkage relative to its leaf was displaced 180° about the vertical pintle axis for convenience in the model. An electric motor drove two speed reducers with crank arms keyed to their vertical shafts. Each model strut was pin-connected at one end to a crank arm and at the other end to an arm keyed to a vertical alloy-steel shaft $1\frac{1}{4}$ in. in diameter. This shaft transmitted the driving torque to the leaf.

The torque applied to one of the leaves during operation was determined by measuring the twist produced in a reduced section of the drive shaft. The diameter of the reduced section of each shaft was 0.6 in. Details of the measuring apparatus are shown in Fig. 4. A hollow cylinder was keyed to the drive shaft just below the reduced section, and a horizontal arm was bolted to a flange on the top of the cylinder. On another horizontal arm, which was keyed to the drive shaft just above the reduced section, were mounted two dial gages graduated to indicate linear deflections to the thousandth of an inch. The gage plungers were in contact with the arm attached to the cylinder and thus were actuated by the relative movement of the two arms. A value obtained with this arrangement was not exactly a measure of the twist arc, but the difference between a reading and the actual length of the corresponding twist arc was so slight that calibrations of the reduced section over a range considerably greater than that of the tests did not deviate appreciably from straight-line relationship. The twist in the section of the shaft below the point of attachment of the cylinder was negligible.

An extension of the arm attached to the cylinder indicated the angular position of the gate leaf on a segmental scale. Instantaneous values of leaf angle and dial-gage reading were obtained during an operating cycle with a motion-picture camera attached to the drive shaft above the measuring apparatus.

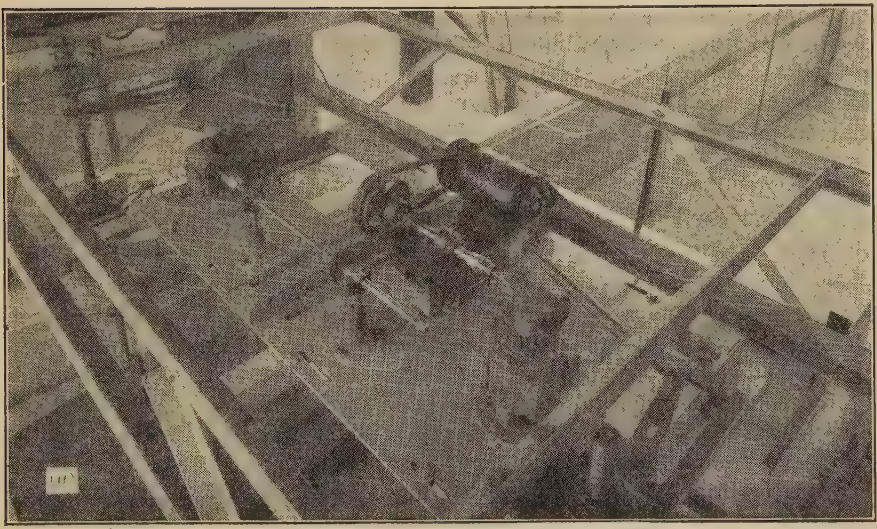
Operating dimensions of the model were derived from prototype values on the basis of Froude's law, as follows

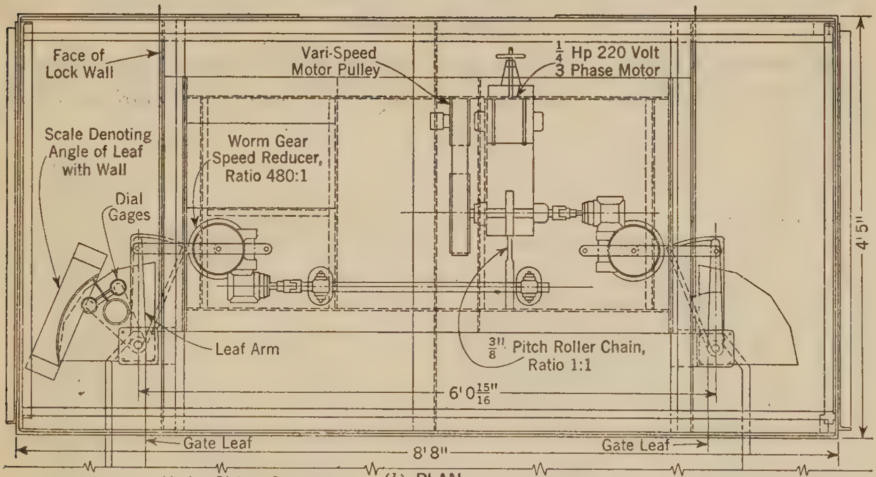
Dimension	Ratio, prototype to model
Length and depth	25 : 1
Volume, weight, and force	15,625 : 1
Time and velocity	5 : 1
Torque	390,625 : 1

Numerical values stated in this paper are in prototype dimensions.

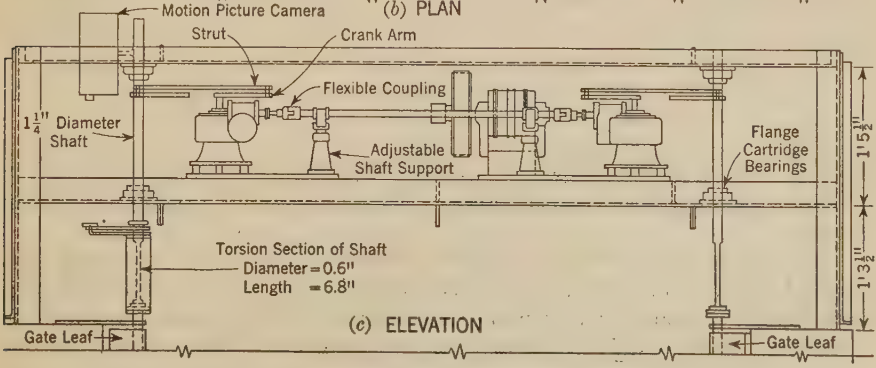
TESTING PROCEDURE

The water level in the flume was adjusted to submerge the gate leaves to the desired depth. Fresh water, only, was used in the tests. The operating speed was adjusted by means of the vari-speed pulley arrangement to complete an operating cycle in the required time. It was established by numerous trials




Face of Lock Wall
Vari-Speed Motor Pulley
$\frac{1}{4}$ Hp 220 Volt 3 Phase Motor
Scale Denoting Angle of Leaf with Wall
Worm Gear Speed Reducer, Ratio 480:1
Dial Gages
Leaf Arm
$\frac{3}{8}$" Pitch Roller Chain, Ratio 1:1
6' 0$\frac{15}{16}$"
4' 5"
Gate Leaf
8' 8"
Gate Leaf

(b) PLAN


Motion Picture Camera
Strut
Crank Arm
Flexible Coupling
1$\frac{1}{4}$" Diameter Shaft
Adjustable Shaft Support
Flange Cartridge Bearings
1' 5$\frac{1}{2}$"
1' 3$\frac{1}{2}$"
Torsion Section of Shaft
Diameter = 0.6"
Length = 6.8"
Gate Leaf
Gate Leaf

(c) ELEVATION

FIG. 3.—MECHANISM FOR OPERATING THE MODEL

that, for a fixed vari-speed pulley setting, the duration of the cycle varied less than 1 sec, other conditions remaining unchanged. The dial gages were set at zero, the motion-picture camera was started, and finally the operating mechanism was started. When the gate reached its terminal position, the mechanism was stopped by a limit switch, and, after allowance of time for the subsidence of eddy currents or surges, the operation was repeated for the reverse direction.

TESTS

The conditions believed to involve maximum hydraulic resistance in the operation of the prototype gates were: Synchronous operation of the two leaves, submergence of 82.5 ft, upstream chamber length of 1,105 ft, and downstream chamber length of 200 ft. These, together with an operating time of 2 min (which had been selected tentatively) were used as base conditions. The following conditions were tested to determine their individual effects upon the resisting torque: Nonsynchronous operation (differences in starting time of the two leaves from 0 to 20 sec), submergences from 45 to 82.5 ft, upstream chamber lengths from 300 to 1,105 ft, operating times from 1.75 to 2.75 min, two gate-leaf weights—2,000 tons and 2,400 tons, and single-leaf operation.

TEST RESULTS

In all tests, a clearly defined peak torque value was reached in the vicinity of the mitered position of the gate. For each set of operating conditions, peak opening torque was considerably greater than peak closing torque. (This does not include the single-leaf tests, which produced only moderate torque peaks.)

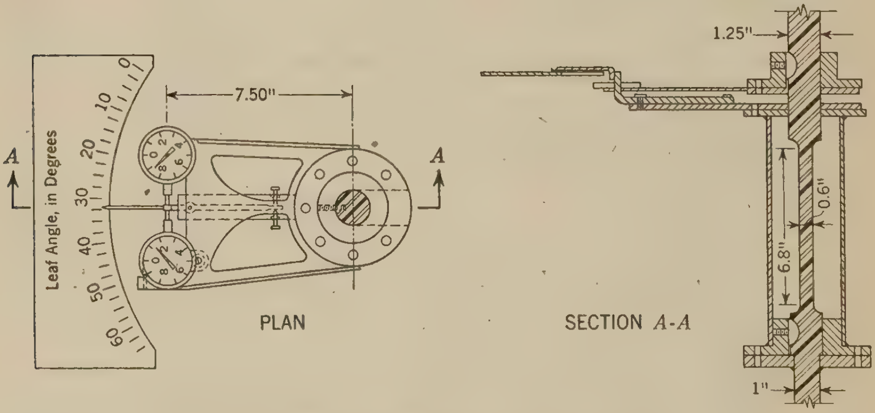


Fig. 4.—Measuring Apparatus

The results of base tests repeated throughout the testing program are compared in Fig. 5. Operating conditions were maintained identical for the base tests: Operating time, 2 min; submergence, 82.5 ft; upstream length of chamber, 1,105 ft; and downstream length of chamber, 200 ft. The extreme deviation from average is about 1,500,000 lb-ft or 8.1% in the opening operation and about 500,000 lb-ft or 3.8% in the closing operation.

The effect of speed variation is shown in Fig. 6(*a*), and the effect of variable submerged depth is shown in Fig. 6(*b*). With the relatively short chamber on one side of the gate, variation of the length of the chamber on the other side had no appreciable effect on the torque for lengths greater than 625 ft. Curves, similar to those in Fig. 6, for these variations indicated that the effects of chamber length were of minor importance compared with those of speed and submergence.

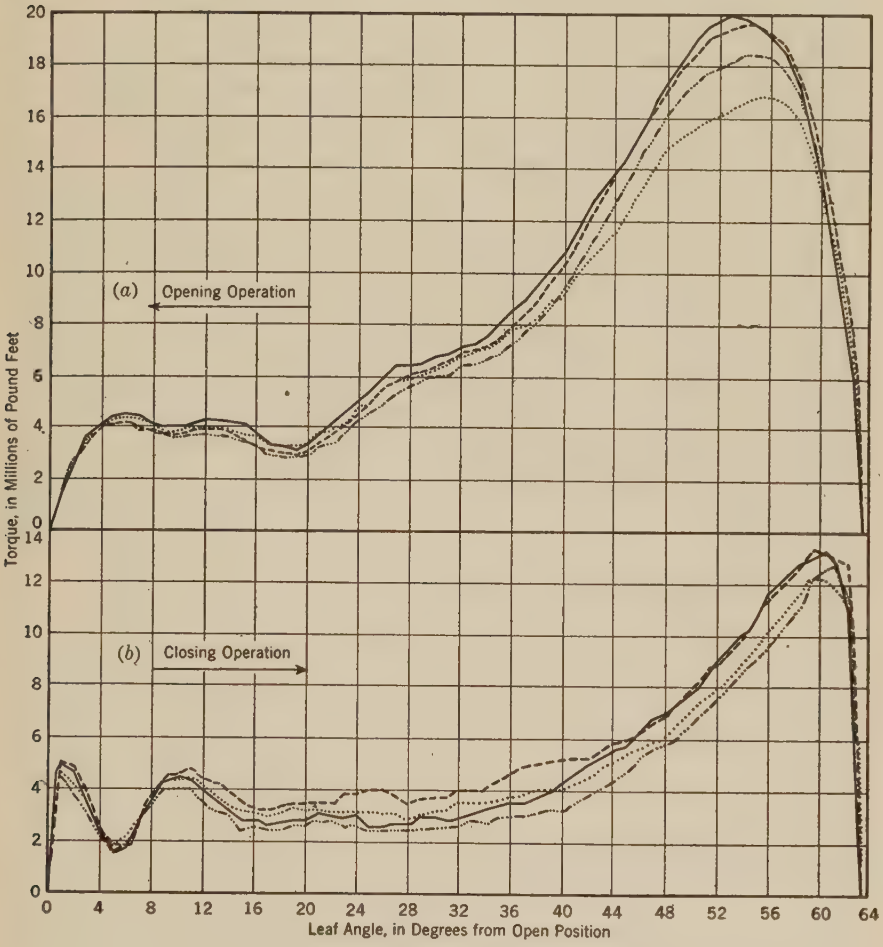


FIG. 5.—MODEL CONSISTENCY, BASE CONDITIONS (OPERATING CONDITIONS CONSTANT)

Likewise, the effect of an increase of 20% in leaf weight was not sufficient to be revealed by the model tests. The addition of weight increases the mechanical frictional resistance and the inertial resistance to acceleration, but generally the forces required to overcome bearing friction and inertia are quite small relative to the total resisting force. The angular acceleration of the

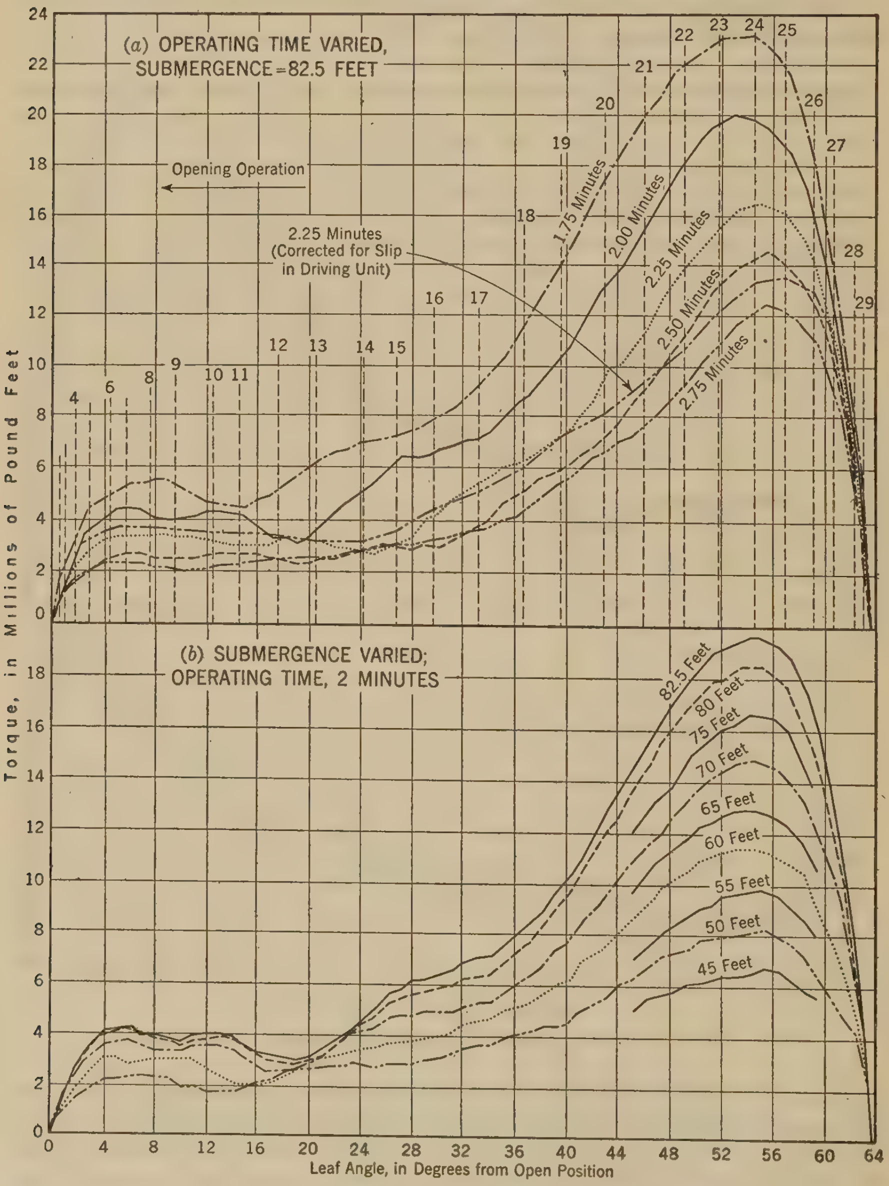


Fig. 6.—Effect of Variable Operating Conditions (Opening Operation)

prototype gate leaf was calculated to be about 0.00019 radians per sec per sec at the time of indicated peak torque, and the moment of inertia about the axis of rotation was estimated to be about 208,000,000 slug-ft². Therefore, the torque necessary to accelerate the leaf at peak-torque position would be 208,000,000 × 0.00019 = 39,500 lb-ft. Since the peak torque values in the opening operation of the base tests ranged from 17,000,000 to 20,000,000 lb-ft (Fig. 5(*a*)), the inertial component is only about 0.2% of the total.

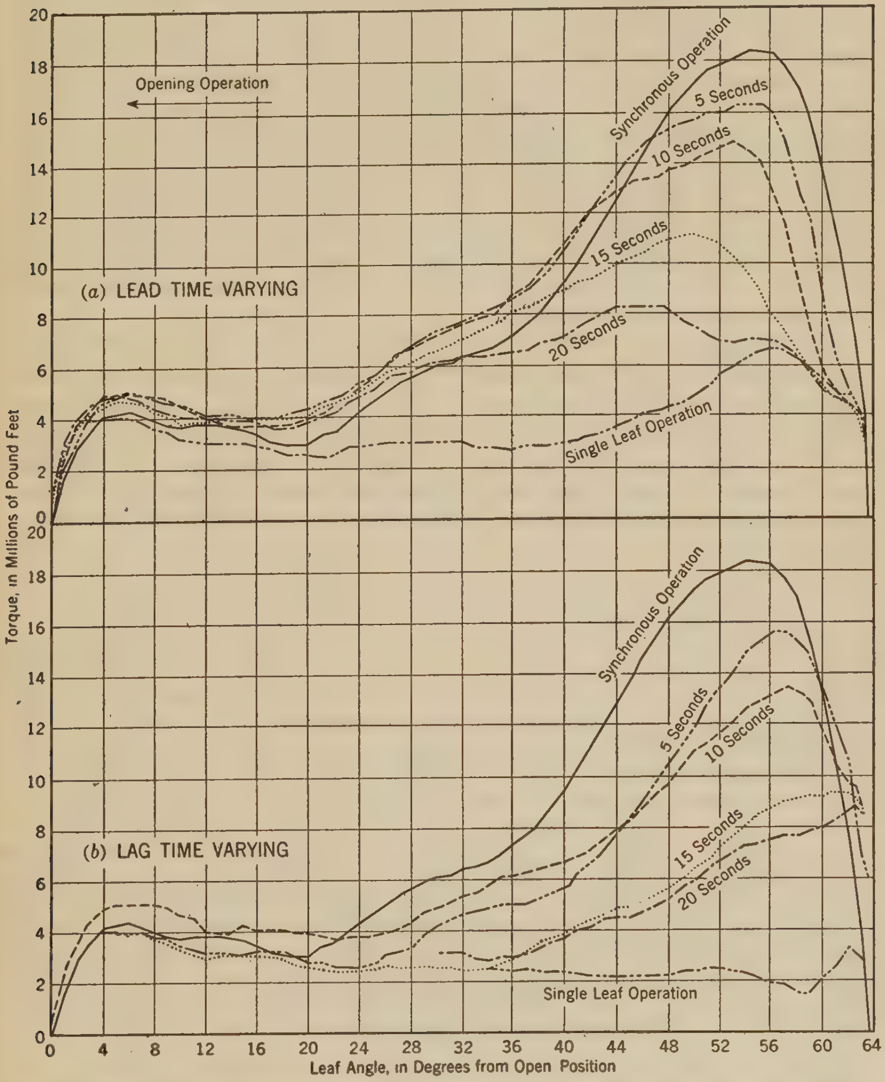


FIG. 7.—EFFECT OF NONSYNCHRONOUS OPERATION OF LEAVES (OPENING OPERATION, BASE CONDITIONS)

Fig. 7 demonstrates that the torque was lowered considerably in the vicinity of the peak values by delaying the starting of one leaf. It is observed that a 20-sec delay reduced the peak values about 50%. Comparison of Figs.

7(*a*) and 7(*b*) indicates that the leaf farthest from the mitered position always required the greater peak of operating torque.

VERIFICATION TESTS

Prior to the initiation of the model tests just described, tests had been conducted on a miter gate of the existing Panama Canal Locks to obtain data from which approximate load values for Third Locks gates could be derived for preliminary design purposes. Because of the availability of these data, the model constructed for tests on the Third Locks gates was used, with certain necessary alterations, for verification tests on the operation of the gate of the existing locks. The gate pintles in the Third Locks, as designed, are farther apart than those in the existing locks, but to simplify the alterations, the locations of the pintle bearings in the model were not changed, and a scale ratio of 1 to 18.813 was obtained for the verification tests. Space limitations prevented model reproduction of the lock-chamber lengths upstream and downstream from the prototype gate.

The tests conducted on the prototype gate of the existing locks consisted of measurements of power input to the driving motor of one of the gate machines during opening and closing operations. The characteristics of the motor were determined separately by means of Prony-brake tests. By taking into account the gear reduction, the mechanical advantage of the crank-and-strut linkage at the recording intervals, and the probable mechanical efficiency, the power-input readings were converted into values of torque about the axis of rotation of the gate leaf.

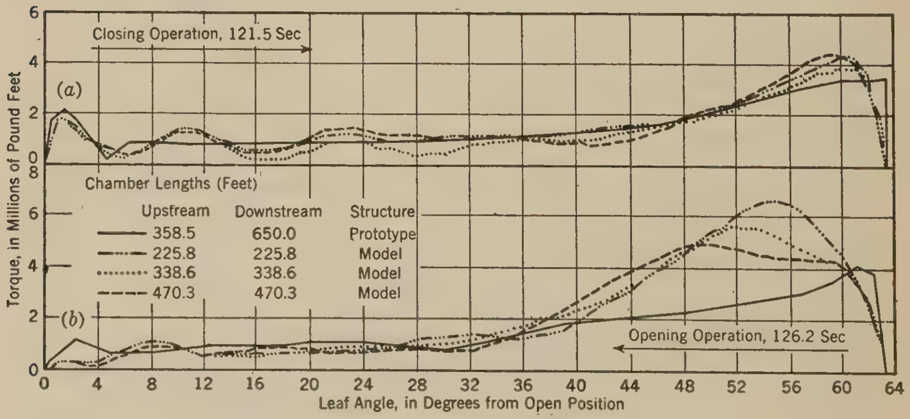


FIG. 8.—VERIFICATION TESTS (SUBMERGENCE, 73.16 FT)

The results of model and prototype tests are shown in Fig. 8. The prototype data represent the average of five test runs, and the model data represent a single run. In all cases, the peak torques observed in the model tests were larger than those recorded in the prototype tests and occurred farther from the mitered position of the gate leaf. The verification is reasonably close for the closing operation but not for the opening operation. There are several factors that could have contributed to the disparity:

(*a*) The driving motor of the model machine had negligible slip. The slip of the prototype motor was found to be very high at the position of peak resisting torque, and the consequent deceleration would tend to reduce the resisting torque.

(*b*) In the prototype tests, it was observed that the two gate leaves did not start or finish their movements simultaneously, even though the operating machines were started simultaneously. The full importance of this condition, which is probably attributable to a difference in the characteristics of the two motors, was not known at the time of the prototype tests, but its influence was demonstrated by the results of the model tests on the Third Locks gate with unsynchronized operation of the leaves (Fig. 7). Delay in starting a leaf in the opening operation reduces the peak load and delays its occurrence.

Conclusions from Tests

Three conclusions are derived from the foregoing:

1. When the two leaves are operated together, peak resistance occurs in the early part of the opening cycle;
2. The peak load can be reduced considerably by delaying the starting of one leaf, but the best load distribution for simultaneous operation of the leaves, whether synchronized or not, can be obtained by the use of an operating cycle involving low speeds at the extremes of travel—especially near the mitered position; and
3. Peak torque varies directly as operating speed and submerged depth of the gate.

Application of Test Results to the Prototype Design

Factors not represented in the model tests were: Hydraulic drag, mechanical friction, transient surge phenomena, salinity heads, and speed variations in the driving unit.

The hydraulic drag and the mechanical friction forces were assumed to be negligible relative to the total resisting force. Opposing surge heads that might exist at the time a gate is operated and the effect of waves produced by external influences such as wind and ship movement were not considered to be practical of reproduction in the model tests. The determination of the probable magnitude of salinity heads and the correction of test results for the effect of slip in the driving units will be discussed herein.

Notation.—The following letter symbols, used in the paper, conform essentially to American Standard Letter Symbols for Hydraulics,[2] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942:

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[2] ASA—Z10.2—1942.

c = distance of the culvert roof from the lock floor (see Fig. 9);
F = force per unit length of gate:
  F_f = force on the fresh-water side of the gate;
  F_n = net unbalanced force;
  F_s = force on the salt-water side of the gate;
T = torque:
  T_a = torque about the axis of rotation;
  T_g = torque at the gear input shaft;
y = depth:
  y_d = downstream, or salt-water depth to the lock floor;
  y_f = depth of water to the equal-pressure plane on the fresh-water side of the gate;
  y_s = depth of water to the equal-pressure plane on the salt-water side of the gate;
  y_u = upstream, or fresh-water depth to the lock floor;
γ = specific weight:
  γ_f = fresh water;
  γ_s = salt water.

Forces Due to Salinity Head.—When the specific gravity of the water on the two sides of a closed lock gate is not the same—a situation encountered at ocean locks supplied with lockage water from a fresh-water source—the attainment of flow equilibrium in the connecting culvert system is very unlikely to balance the total hydrostatic forces on the two sides of the gate. When the fresh or comparatively fresh water is on the upstream side of a closed miter gate, the direction of the unbalanced hydrostatic force is such as to oppose the opening of the gate, as illustrated in the following analysis.

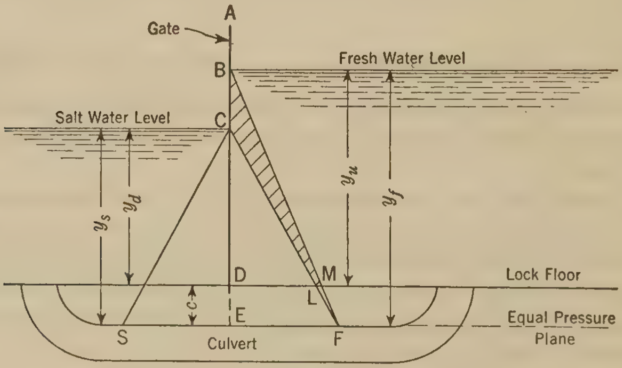

FIG. 9

Fig. 9 is a diagrammatic section parallel to a lock chamber, with the gate represented by the line AD and the pools on the two sides of the gate connected by a culvert running below the level of the gate bottom. For equilibrium to exist and flow to cease, there must be a plane of separation between salt water and fresh water at which the pressure is balanced. Since salt water is heavier than fresh water, the lower part of the culvert will be filled with salt water at least up to the level of the culvert roof. It will be assumed tentatively that the equal-pressure plane is at the same level as the culvert roof.

The pressure diagrams for the two sides of the gate are represented by lines CS and BF, with $\overline{\text{ES}}$ equal to $\overline{\text{EF}}$. The area BCLM represents unbalanced pressure on the fresh-water side of the gate.

By definition (see notation):

$$F_f = \frac{\gamma_f y^2_u}{2}\text{; and } F_s = \frac{\gamma_s y^2_d}{2} \qquad \text{(1)}$$

The net unbalanced force per unit length of gate equals

$$F_n = \frac{\gamma_f y^2_u - \gamma_s y^2_d}{2} \qquad \text{(2)}$$

Since $y_u = y_f - c$ and $y_d = y_s - c$ (see Fig. 9), Eq. 2 becomes:

$$F_n = \frac{\gamma_f(y_f - c)^2 - \gamma_s(y_s - c)^2}{2}$$

$$= \frac{\gamma_f y^2_f - 2\,\gamma_f y_f c + \gamma_f c^2 - \gamma_s y^2_s + 2\,\gamma_s y_s c - \gamma_s c^2}{2} \qquad \text{(3)}$$

For the condition of equilibrium,

$$\gamma_f y_f = \gamma_s y_s \qquad \text{(4)}$$

Substituting Eq. 4 in Eq. 3 reduces the force equation to

$$F_n = \frac{\gamma_s y_s (y_f - y_s) - c^2(\gamma_s - \gamma_f)}{2} \qquad \text{(5)}$$

With the value of y_s known, y_f can be determined from Eq. 4 for assumed values of γ_f and γ_s. With the substitution of $\frac{\gamma_s y_s}{\gamma_f}$ for y_f, Eq. 5 reduces to

$$F_n = \frac{\gamma_s y^2_s(\gamma_s - \gamma_f)}{2\,\gamma_f} - \frac{c^2(\gamma_s - \gamma_f)}{2} = \frac{\gamma_s - \gamma_f}{2}\left(\frac{\gamma_s y^2_s}{\gamma_f} - c^2\right) \qquad \text{(6)}$$

When the connecting culvert is above the level of the gate bottom, Eq. 6 applies without change. If the equal-pressure plane is at the level of the gate bottom, c of Eq. 6 equals 0, and

$$F_n = \frac{\gamma_s y^2_s(\gamma_s - \gamma_f)}{2\,\gamma_f} \qquad \text{(7)}$$

In the application of the developed formulas to the prototype, several factors ordinarily would enter into the decision as to which gate would be used as a criterion and into the assumption of salinity values. If no factual data were available, it would be logical to base the computations on the gate that will have the greatest operating submergence and to assume sea water with a specific weight of 64.0 lb per cu ft on one side and fresh water with a specific weight of 62.5 lb per cu ft on the other side. However, at the existing Panama Canal Locks, which correspond closely with the designed prototype locks and which connect the same bodies of water, information was available as to the location at which the greatest difficulty is encountered from salinity pressures, and the maximum salinity head measured at that location is about 0.7 ft. At the corresponding location in the Third Locks, the maximum salt-water depth to the lock floor on the downstream side of the gate is 68.9 ft, and the lowest culvert roof is 9 ft below the floor. The water on the downstream side is assumed to have a specific weight of 64.0 lb per cu ft. If the salinity of the water on the upstream side were assumed to be uniform for the full depth to the culvert roof, Eq. 6 would apply, but, for a given head difference, the worst condition obviously would be represented by the assumption of sea-water salinity on the upstream side from the lock floor down and a uniform lower salinity from the lock floor up, since pressure difference below the floor does not act on the gate. With this assumption, $y_s = 68.9$ ft and $y_f = 68.9 + 0.7 = 69.6$ ft. Therefore, $\gamma_f = 63.3$ lb per cu ft.

As indicated by this result, entirely fresh water probably will not be found on one side of a gate with undiluted sea water on the other side. Such a condition is prevented by the fact that every time the gate is opened, the sea water on one side displaces a large portion of the fresh water on the other side because of the difference in weight.

The prototype gate leaf is about 84 ft long from miter end to axis of rotation. Neglecting the small area from the axis of rotation to the quoin end and substituting in Eq. 7, the total force on the gate leaf is computed as 141,000 lb. The moment arm of this force about the axis of rotation of the leaf is equal to half the length of the leaf. Therefore, the moment resisting opening of the leaf is $\frac{1}{2} \times 84 \times 141{,}000 = 5{,}920{,}000$ ft-lb.

Correction of Test Results for Slip in Driving Unit.—The model machine was driven by a relatively high-powered motor that produced a practically constant speed for the reduction gear and the operating crank. In the designed prototype machine, however, power is to be transmitted from motors to reduction gears through hydraulic couplings. Therefore the speed variations to be expected in the prototype machine must be taken into account in application of the model test results.

Although most of the model tests were conducted with an operating time representing 2 min in the prototype, the operating time later selected for the prototype was 2.25 min. A tentative selection was made of motors, hydraulic couplings, and reduction gears such that, with the motors operating at rated full-load speed through the entire operating cycle, and taking into account the slip of the couplings, an operating time of approximately 2.25 min would result. The motor and the hydraulic coupling were of standard design, and their slip characteristics were available from manufacturers. For the purpose under discussion, the two slip characteristics were combined. Fig. 10 shows the speed of the output shaft of the hydraulic coupling for the torque range from no load to the breakdown load of the coupling, with the slip of the motor included.

The travel of the gate leaf between recessed and mitered positions was divided for convenience into thirty parts corresponding to thirty equal divisions of the arc of crank movement. The relation of these divisions with respect to angular position of the leaf is shown by the vertical lines in Fig. 6(*a*).

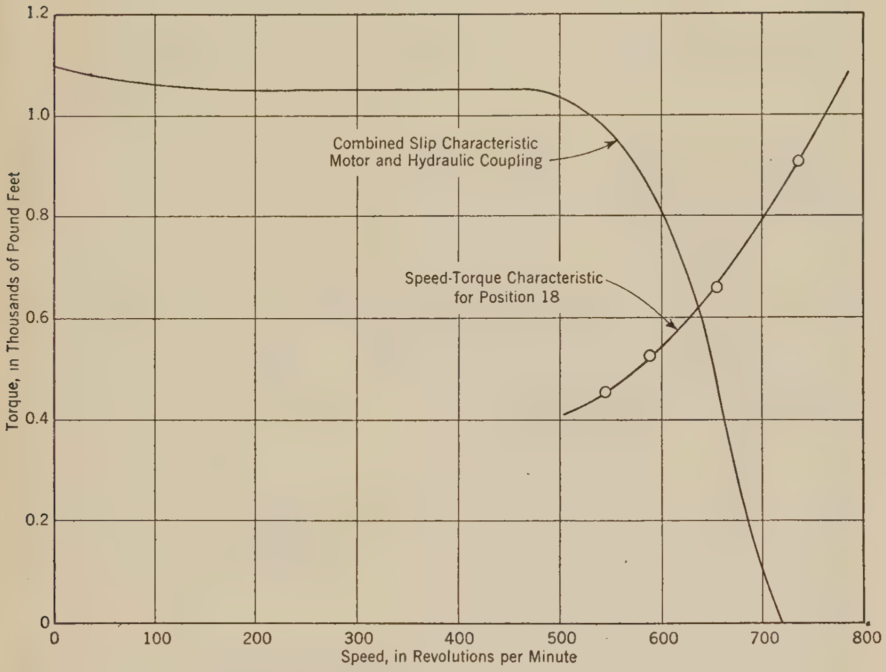

FIG. 10.—CORRECTION OF TEST DATA FOR SLIP IN DRIVING UNIT

Thus, for each position, the resisting torque about the axis of rotation of the leaf is indicated for five different operating times. On the assumption that the crank rotates at uniform speed throughout an operation, each operating time corresponds to a certain speed at the input shaft of the reduction gear. By computing trigonometrically the mechanical advantage of the crank-and-strut linkage for each position, taking into account the total gear reduction, and assuming a mechanical efficiency, the resisting torque at the input shaft of the reduction gear can be derived for each operating speed.

Leaf Position 18 will be taken as an illustration. For this position the mechanical advantage of the crank-and-strut linkage was calculated to be 1.92. The total gear reduction was 5,660 to 1, and the over-all mechanical efficiency, derived from assumed efficiencies for individual parts of the machine, was 89.4%. Therefore, the torque required at the gear input shaft to produce a given torque about the axis of rotation of the leaf was obtained by dividing by the factor (1.92 × 5,660 × 0.894 =) 9,715. With T_a representing the torque about the axis of rotation and T_g the corresponding required torque at the gear input shaft, the values in Table 1 were derived for the opening movement from the torque values shown in Fig. 6(*a*) for Position 18. The speed-torque relationships are plotted in Fig. 10 to the same coordinates as the hydraulic-coupling characteristic. Since the output torque of the hydraulic coupling must be the same as the resisting torque at the gear input shaft, the intersection of the curve for each leaf position with the hydraulic-coupling characteristic represents corrected torque and speed for that particular leaf position. Thus the corrected torque at the gear input shaft is 620 lb-ft, and the corrected speed is 637 rpm. The torque at the gear input shaft may be multiplied by the conversion factor 9,715 to obtain a corrected value of torque about the axis of rotation. This gives 6,020,000 lb-ft.

TABLE 1.—DATA FROM SPEED-TORQUE TESTS, OPENING MOVEMENT

No.	Description	OPERATING TIME, IN MINUTES				
		2.75	2.50	2.25	2.00	1.75
1	Speed at gear input shaft (rpm)	545	589	655	736	818
	Torque, in Pound-Feet:					
2	T_a, about axis of rotation (millions)	4.4	5.1	6.4	8.8	11.7
3	T_g, at the gear input shaft	453	525	659	905	1,205

This method was used to derive torque and speed values for each of the thirty positions for both the opening movement and the closing movement, and, from these values, crank-and-strut loads and motor horsepower were computed. The corrected values of torque about the axis of rotation for the opening movement are shown in Fig. 6(*a*) for comparison with the uncorrected model test results.

To determine the probable operating time, the average speed of the crank between each pair of consecutive positions was assumed to be equal to the average of the corrected speeds at the two positions. The total operating time calculated on this basis differed only a negligible amount from the desired 2.25 min, and the tentatively selected driving unit was adopted for the machine.

The correction method used is an approximate one, of course. The basic assumption was made that the speed-torque relationships indicated by the model test results for any given leaf position would hold true independently of speed variations at other positions. This assumption cannot lead to very accurate results, but the degree of accuracy probably is sufficient for the purpose of the computations.

Capacity of Machines.—All gates of the prototype will be operated by identical machines designed for the operating conditions at the gate with the greatest potential load. This load was assumed to be represented by the test results shown in Fig. 6(*a*). It was unnecessary to make additional allowances for salinity forces; at the gate where salinity is most likely to affect operation, the submergence is only about 70 ft, and, since the machines are designed to operate a gate submerged 82.5 ft, an ample reserve capacity is available (see Fig. 6(*b*)). The hydraulic coupling acts as a load-limiting feature, and, if salinity pressure exceeds expectations, the coupling slip will increase automatically, decelerating the opening movement until the salinity head has been dissipated by flow between the gate leaves. The operating strut is equipped with a shock absorber to dampen the effect of any transient loads such as waves. The shock absorber contains a limit switch that will stop the driving motors before the strut can be overloaded.

Application of Test Results to Other Miter Gates

The data obtained from the model tests probably could be applied to other miter gates having the same kinematic cycle. This would require correlation of scalar ratios with the relationships derived from the tests. The length of a gate leaf would determine the linear scale ratio with respect to the leaf of the test gate, which is 84 ft long. For example, if a gate leaf is 42 ft long, the linear scale would be 1 to 2. If the submergence is 30 ft, the curve in Fig. 6(*b*) representing a 60-ft submergence would be applicable with all torque values divided by 2^4, or 16. If the operating time is greater or less than 2 min, the appropriate curve of Fig. 6(*a*) is used and corrections are made for a number of leaf positions in accordance with the ratios in Fig. 6(*b*) between the torque values at a 60-ft submergence and those at an 82.5-ft submergence.

If it is necessary to correct for slip in the driving machine, new speed-torque curves should be derived from those of Fig. 6(*a*) on the basis of the scale relationships and the submergence ratio, and the correction method should be applied as outlined herein.

In this application, differences in chamber lengths are neglected. However, unless such differences are great, their effect may be relatively unimportant.

Acknowledgment

At the time of the tests described in this paper the Special Engineering Division of The Panama Canal was directed by T. B. Larkin, M. Am. Soc. C. E., supervising engineer. The investigation was conducted by the Hydraulic Section of the Division under the direction of F. W. Edwards, M. Am. Soc. C. E., senior hydraulic engineer, and later, E. Soucek, Assoc. M. Am. Soc. C. E., senior hydraulic engineer. The model operating mechanism was designed by G. D. Smith, engineer, of the Mechanical-Electrical Section. The writer was in charge of the investigation.

DISCUSSION

Edward Soucek,[3] Assoc. M. Am. Soc. C. E.—Prior to adoption of the test program so ably described by Mr. Amster, a study was made of the methods which had been used previously in design of miter gate moving machines. The usual basis for the hydraulic load is a differential head on the leaves acting in a direction opposing the motion. The magnitude of the head is commonly fixed by judgment—in some cases supplemented by observations at existing installations. The reliability of comparative studies for use in design is limited principally by uncertainty in allowing for differences in gate length, in submergence depth, and in kinematics of the gate motion; sill clearances are also believed to affect the hydraulic loads but probably to a lesser extent.

Some designers compute the varying differential head upon the leaves by consideration of the rates of change of the areas separated by the gates and the discharge past the gates. This method gives results that are much too small because the loads are principally due to water-level variations which are transient in character and local in extent—surges induced by the gate movement—and therefore not related in any simple manner to the adjacent water-surface areas.

During the search for design data, a paper that dealt with loads on moving miter gates was found (written in German). It appeared very promising because it contained many equations. Upon translation, however, all significant equations were found to contain that elusive term—the unknown and varying differential head—which had to be evaluated before the equations could be used in design. Owing to the lack of reliable design data, use of model tests was, as the author states, the only alternative to the use of extremely liberal safety factors for the design of a considerable number of large machines.

This situation not only gives Mr. Amster's paper a "scarcity value" which insures that it will be widely used, but it also requires that most careful consideration be given to the limitations stated by the author and to any other conditions that might affect the applicability of the test results to the design of gate moving machines in general. Operation of the model and interpretation of the test data in accordance with Froude's law are believed correct since the phenomena associated with the hydraulic loads are predominantly inertial in character. Elasticity of the gates, effect of flow through the lock culverts, and reduction in effective chamber length caused by the nearness of a large ship might be mentioned as factors not represented in the model additional to those listed by the author. Their effects are relatively small but should not always be considered negligible.

Under "Application of Test Results to Other Miter Gates," the author outlines a method for application of the test data to the design of dissimilar prototypes. The writer believes that the problem is not so simple as this section of the paper implies. If gate moving machines are not geometrically similar, the gate operating time is not a sufficient measure of the kinematic characteristics of the gate movement. If the model gates were operated at a uniform angular velocity, rather than with slow movement near the limits of travel, the hydraulic load for the same operating time would be increased many times. For this reason, it would be preferable to consider only the first 15 or 20 degrees of the movement for the critical opening operation rather than the entire opening period in some cases.

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However, even dismissing the questions of kinematic characteristics (other than operating time), leaf angle when fully open, sill clearance, and other factors which were not investigated, it is not clear that the proposed adjustment for operating time is generally valid. As the author explains, the ratio of prototype leaf length to model leaf length establishes the scale ratio. Then from Fig. 6(*b*), a torque curve can be selected that is correct for a particular prototype operating time (determinable from the scale ratio and the model operating time). It may, but probably does not, correspond to that at which the prototype will operate. Fig. 6(*a*) shows the effect of varied operating time only for a gate with leaf length approximately equal to the submerged depth. The effect of operating time upon peak torque indicated by the curves is shown in Table 2. It does not follow from these data that Fig. 6(*a*) is a valid basis for adjustment of torque curves in terms of operating time when the leaf length is much greater than the submerged depth—a frequent case in barge locks. Pending the presentation of further data or a conclusive analysis, it would appear that the relationships suggested should be used only when the leaf length approximately equals the submerged depth.

TABLE 2.—EFFECT OF OPERATING TIME ON PEAK TORQUE

Description	TIME, IN MINUTES				
	1.75	2.00	2.25	2.50	2.75
Torque, in millions of foot-pounds	22.3	20.0	16.5	14.5	12.5
Percentage time increase	0.0	14.4	28.5	43.0	57.5
Percentage torque decrease	0.0	10.5	26.0	35.0	44.0

The treatment of the forces due to density differences covers the ideal case, a static condition with the plane separating fresh and salt water in a known position. In reality, determination of the elevation of this plane is quite difficult. It may be of interest to point out that a series of tests on the existing Panama locks, corroborated by a small transparent model, indicated that the excess closing force produced by difference in density cannot be eliminated by increasing the elevation of the culvert outlets. Consideration of this problem in detail is beyond the scope of this discussion. It may be suggested, however, that designers of miter gate moving machines should question the efficacy of elevated outlets in reducing the loads upon the machines.

In the Panama Canal Third Locks Project, the terminal gates which are most subject to the density effect will be less deeply submerged than most of the other gates so the machines of the terminal gates will have a substantial overload capacity. At all gates, the "overtravel" of water in the lock culverts produces a transient opening force which can be used to reduce the critical opening load, either by swinging the gates at or near the time of maximum overtravel or by closing the culvert valves and "trapping" the favorable water level differential. Of course a falling tide eliminates the latter possibility at the terminal gates.

The writer's remarks are intended solely to clarify certain design problems and to emphasize the limitations upon general applicability of the test data. Many significant conclusions can be drawn from the paper in addition to those modestly claimed by the author. His efforts would be justified if the paper did no more than call attention to the large reduction in operating forces and peak power loads which can be attained by reducing the operating speed of the gates and by nonsynchronous operation of the leaves. Since the gate operating time is a relatively small part of the time required to perform a lockage, these methods frequently can be adopted with scarcely noticeable adverse effects upon over-all lockage efficiency.

ISAAC DEYOUNG,[4] M. AM. SOC. C. E.—Tests for determining the force required to swing the mitering gates of the locks at St. Marys Falls Canal in Michigan were made in 1941. Although the method of testing was not highly refined, the results, in some respects, are similar to those determined by the more elaborate tests conducted with the model mechanism of the Panama Locks.

The gate engines were not tested as usual, because of the difficulty of inserting a measuring device for determining the power to move these gates. The gates tested are operated by cables reaved over spiral drums with the cables attached to the gates near the toe of the gates and close to the center of gravity of the mass of water through which the gates swing.

Power for swinging the gates was obtained from adjacent capstans, and the tension in the cable was measured on a hydraulic weighing machine.

One of the tests was made on the intermediate lower gates of the Davis Lock at St. Marys Falls Canal. The tests were made on a single-leaf operation with the other leaf fixed in its recess. The time of a complete swing of the leaf was 1½ min, about the usual time for actual operation. The operating speed of the capstan through which the power was applied was constant, resulting in a uniform intake of rope which was attached to the gate fastening. The depth of submergence was 34 ft.

The width of the Davis Lock is 80 ft. The leaves of all the operating gates are 44.56 ft wide, and the intermediate gates are about 52 ft high. The gate swings through an arc of 68° 12′, and the operating cable is attached to the leaf at 39.62 ft from the center of rotation. The weight of each leaf is about 173 tons.

The results of the tests are shown in Fig. 11, representing the average of three test runs, for the opening of gate only. The maximum pull is about 61° from the position of the gate in the recess and is in close agreement with that obtained in the model tests at this point. Since the opposing leaf is in its recess, the movement of water seems to be throttled between the sill and the sill cushion of the leaf tested. There should be liberal depths under the gate in the pit over which the gate swings, as well as in the gate recess in order to give a more effective path for the ingress and egress of the water back of the gate.

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[4] Consultant to City Mgr., Sault Ste. Marie, Mich.

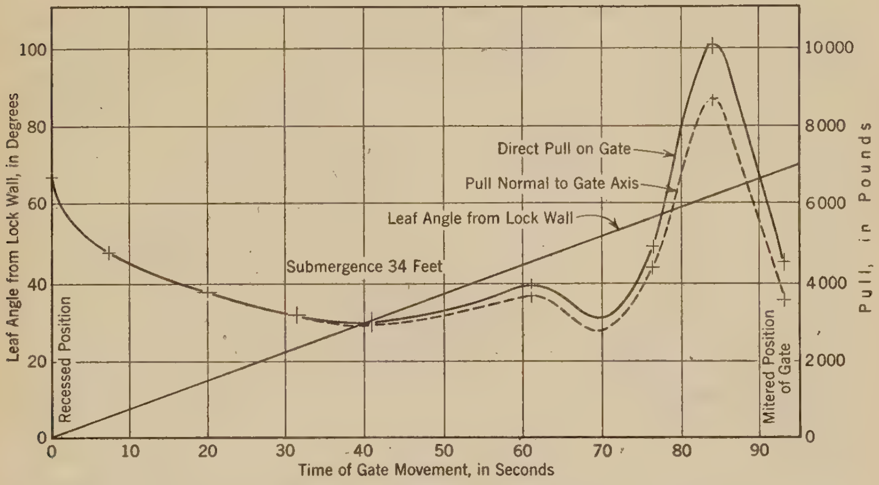

FIG. 11.—TYPICAL OPERATING GRAPH, DAVIS LOCK GATES, ST. MARYS FALLS CANAL

The clearance under the gate of the Davis Lock when in its recess is 3.5 ft and that between back of gate and recess wall is about 5 in. The operating graph of the Davis Lock gate shows that the force required to swing the gate from the open position is much larger than that required for the model tests. The low torque required in the model tests to start from the open position is surprisingly small and indicates an effective way for the water to reach the back of the gate in the recess.

Mr. Amster computed the mechanical advantage of the crank-and-strut linkage by trigonometry. The dead center positions of the crank coincide with the extreme positions of the gate, effecting theoretically an infinite mechanical advantage at those positions. The type of graph in Fig. 12 shows more clearly the relative torque applied to the gate at the different positions of its swing.

In the model and prototype the crank arm and the connecting rod form a straight line in both open and closed positions of the lock gate. With constant speed of gear input shaft, the rate of angular movement of the gate leaf swinging in either direction begins at zero, accelerating to a maximum at about 42° from the recessed position and then reduces to zero at the end of the swing. What is measured in the model tests is the torque required at the gate leaf which is taken as being directly proportional to the rate of swing, giving results of tests that would not apply if the gate were swung by an operating mechanism having different kinematic conditions between power input to shaft and force applied to the gate leaf.

The method used to measure the torque in the gate is applicable to model study of any lock gate of similar type. The results obtained, however, are applicable only to the model and prototype tested. For model tests of other lock gate installations, having different clearances between the gate leaf and sill, lock floor and recess, the model-driving mechanism would have to be redesigned to reproduce the kinematics of the prototype and model under the changed conditions.

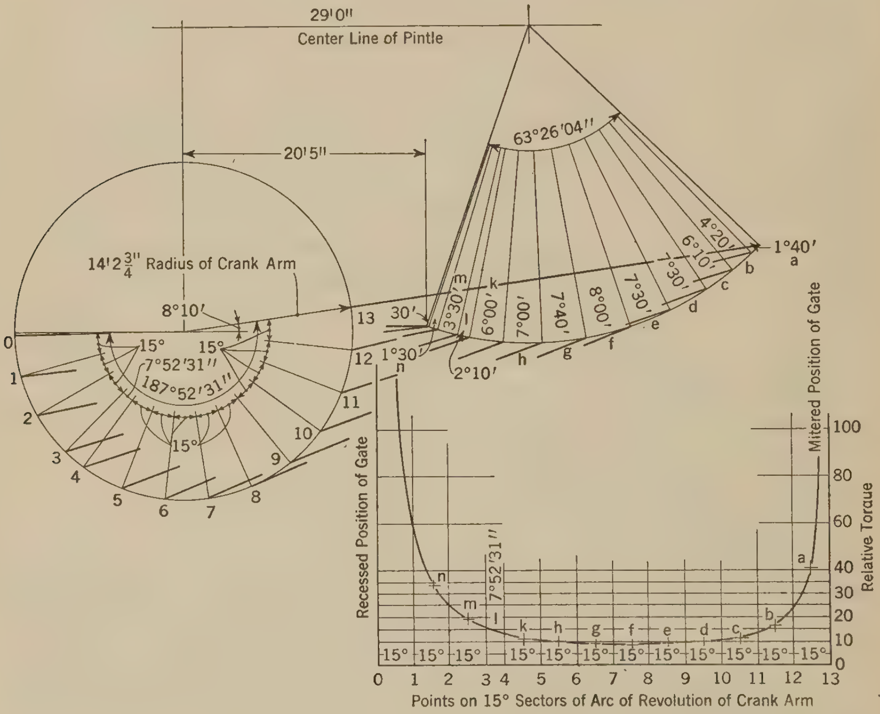
29'0"
Center Line of Pintle
20'5"
14'2 3/4" Radius of Crank Arm
8°10'
15°
15°
7°52'31"
187°52'31"
15°
0
1
2
3
4
5
6
7
8
9
10
11
12
13
30'
1°30'
2°10'
63°26'04"
3°30'
6°00'
7°00'
7°40'
8°00'
7°30'
7°30'
6°10'
4°20'
1°40'
a
b
c
d
e
f
g
h
k
l
m
n
Recessed Position of Gate
Mitered Position of Gate
Relative Torque
100
80
60
40
30
20
10
0
7°52'31"
15°
0 1 2 3 4 5 6 7 8 9 10 11 12 13
Points on 15° Sectors of Arc of Revolution of Crank Arm

Fig. 12.—Relative Torque Applied to Gate at Different Positions of Swing

It is interesting to note, from Fig. 2, that the seal between the gate and sill is near the upstream face of the gate leaf and that clearance between bottom of gate and top of seal is something less than 11 in. As a consequence, the gate in opening from the closed position is swinging over the sill with very little clearance for water to pass under the gate from the upstream to the downstream side. Consequently, water on the upstream side must be pushed into the lock chamber by the gate until the space between leaves at the miter end becomes large enough to equalize levels on upstream and downstream sides. This would account for the high peak torque at approximately 53° in the opening operation.

F. W. Edwards,[5] M. Am. Soc. C. E.—The use of an hydraulic model for the solution of a new type of problem is demonstrated in this paper. The data have been presented in a way which should be particularly valuable to designers. Mr. Amster deserves credit for his effort toward this end.

The curves in Fig. 6 are interesting from a design standpoint. An increase in gate operating time reduces greatly the peak torque near the mitered position, but has less effect on the peak near the recessed position. This is true for the smaller submergences as well as for the base test conditions. Where reduction in operating time is not an important factor, the time should be increased until the maximum torques near the recessed position and near the mitered position are equal. It is unfortunate that the tests were not extended to include gate operation for periods longer than 2.75 min. However, from extrapolation of data in Fig. 6(*a*), it appears that an operating time of about 4 min would reduce the peak torque to the practical minimum at both critical positions of the gate. This maximum torque for 4-min operation would be less than 10% of the peak for an operating time of 1.75 min and would have a pronounced effect on the required capacity of the gate operating machinery.

The curves in Fig. 7 illustrate a similar effect on the peak torque at the two positions for nonsynchronous operation. In this case, an increase in lead time causes a reduction of peak torque. However, it appears from the single-leaf operation that the peak torque near the mitered position could not be reduced to a value corresponding to the maximum obtained near the recessed position. Apparently, a 25-sec lead time, just outside the range of the tests, would reduce the torque to the practical minimum.

It is not entirely accurate to designate the prototype tests as verification tests. Prototype tests which are conducted specifically for verification of a model should be made with special care. Although the general conditions were indicated by the author, it should be emphasized that the tests on the existing lock gates were very rough. They were not intended originally for verification of the model. It is surprising that the results check as well as Fig. 8 indicates. The model, rather than the prototype, results are believed to represent the correct values.

The author states (see heading, "Conclusions from Tests"),

> "* * * but the best load distribution for simultaneous operation of the leaves, whether synchronized or not, can be obtained by the use of an operating cycle involving low speeds at the extremes of travel—* * *."

This part of conclusion 2 is not based entirely on the model results. In fact, the variation of the kinematic characteristics (except speed) of the gate operation was not tested. It would appear from the torque curves that, for the base operation time of 2 min, a faster movement of the gate could be used near the recessed position without affecting the required capacity of the gate operation machinery. This could be accomplished by establishing the end position of the crank arm away from dead center.

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William A. Tripp,[6] Esq.—The model tests described by Mr. Amster do not include two factors, present in the actual operation of the locks of the Panama Canal, which have a substantial effect on the operating loads. These are: (1) The influence of the 200-ft chamber on the effective length, or free surface, of the chambers; and (2) the use of the culvert in equalizing water levels. Without proper evaluation of these factors, it is felt that the tests do not establish a sound criterion for predicting the forces on the prototype and are of doubtful value for application to other installations.

An analysis of the effect of these factors made by the writer indicates the factors to be much more important than has been recognized. According to this analysis, failure to introduce the 200-ft chamber properly into the problem appears to produce values of torque that are approximately 100% too large, and failure to include the effect of the culvert appears to produce values that are 50% too large. The combination of these two factors indicates that the tests may vary by as much as 200% from a correct prediction of the normal operating torques of the prototype. The method of analysis used by the writer is believed to be sound as it has been checked against sample cases taken from both the model and the prototype tests on the Panama Canal, yielding results which are reasonably close.

The basic conditions selected for the model tests include an upper chamber of 1,105 ft and a lower chamber of 200 ft. The 200-ft chamber is only an intermediate chamber between two 1,105-ft chambers; and the gates at both the upper and lower ends of this short chamber are operated simultaneously. This method of gate operation makes the short chamber act hydraulically much like a pool with an infinite free surface, and materially reduces the hydraulic resistance offered to the operation of each gate, at least during the opening cycle—the maximum duty cycle.

The free surface of the lock chambers has an important influence on the forces produced in the gate machinery. Evidence in support of this fact is presented in Fig. 8, in which the test curves for the opening cycle show progressively higher torques with shorter chamber lengths. Mr. Amster discusses this condition briefly under the heading, "Test Results." He states that increasing the length of the larger chamber beyond some 625 ft did not have a material effect on the torque. It should be noted, however, that the 200-ft chamber by then would be the dominating influence in the model. The torque required to move a gate is caused largely by the differential head produced by the movement of the gate. In effect, the gate is a wall between the two chambers on each side of it. Movement of the gate is equivalent to a change in the sizes of the two chambers, and, starting with a given quantity of water in each chamber, a differential head will be set up until sufficient water can flow to equalize the water levels. With a gate of a given size, a given movement will represent a definite volumetric change, which must be distributed over the free surface of the lock chambers, raising the water level in the chamber above the gate and lowering it in the chamber below the gate. The larger the free surface of each chamber, the smaller will be the differential head produced by the quantity of water involved. Thus, the chamber area will have a direct influence on the machinery torques for a given gate and gate drive. By the same reasoning, a gate at the other end of one of the chambers which follows the movement of the first gate, and causes a displacement of water compensating for the displacement of water caused by the first gate, will substantially prevent any change in the water level in the intermediate chamber. This is what takes place in the 200-ft chambers in the prototype at the Panama Canal.

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[6] Elec. Engr., Chas. T. Main, Inc., Boston, Mass.; formerly Senior Engr., Mech.-Elec. Section, Special Eng. Div., The Panama Canal, Diablo Heights, Canal Zone.

Of course, the method of operation assumed for the model could be encountered in the prototype if something happened to prevent the "following" gate from operating, or if a gate were operated in this manner while being serviced. However, failure of the "following" gate to operate would be noted readily by the operator and, in any event, would impose only a short overload on the drive machinery. Such a failure would slow down the gate operation, thereby partly relieving the condition. Servicing operations could, and ordinarily would, be taken care of by nonsynchronous operation of the gate leaves. A substantial reduction in design requirements can thus be effected, by basing the size of the power unit on normal operation, and making the drive mechanism physically strong enough to take any momentary overloads that might be encountered with this smaller power unit.

With regard to the culvert, the operating practice at the Panama Canal is to keep the culvert valves open while swinging a set of gates. The importance of this procedure is evident from the discussion of the differential head produced by the movement of the gate. The significant fact is that the head is relieved by an equalizing flow of water between the chambers. Three channels are available for this flow—(*a*) the opening between the gate leaves, (*b*) the opening over the sill, and (*c*) the culvert. The first two start at zero for an opening cycle, whereas the third is constant. The importance of the culvert will depend upon its dimensions in relation to other geometrical features for the particular case. In the existing prototype at the Panama Canal, the culvert area is greater than the combined area of the other two channels for about the first 5% of the arc of gate swing in opening. It is understood that these relationships are about the same in the new locks. It is particularly significant, therefore, that the tests indicate that the peak of torque occurs within about the first 8% of the arc of swing. Of course, the rate of water flow per unit of cross-sectional area will be less for the culvert than for the gate opening, but the relative magnitude represents an important factor during a critical part of the cycle of operation. As is generally well known (and the model tests prove this), with lock gates following the kinematics of the Panama Canal type and with well-designed wall recesses, the torques determining the design of the gate machinery occur during this early part of the opening cycle. Anything that can be introduced to decrease torques during this period will have a substantial influence in reducing design requirements. Culverts of the relative size of those at the Panama Canal are of sufficient importance to warrant full and careful evaluation.

The magnitudes of the influences of the two factors discussed herein will vary for different physical relationships and operating conditions. They appear to be about the same for both the old and the new locks at the Panama Canal. Their relative importance indicates that these magnitudes should be introduced into any analysis or tests involving similar arrangements.

The writer wishes to acknowledge and express his appreciation of the interest and assistance of B. O. McCoy, Assoc. M. Am. Soc. C. E., in the study and analysis of this problem.

Maurice N. Amster,[7] Assoc. M. Am. Soc. C. E.—Quite appropriately, Mr. Soucek emphasizes the necessity for giving full consideration to the limitations involved in applying the test results to the design of other gate operating machines. More than one of the discussers overlooked the limiting nature of the writer's statement (see heading, "Application of Test Results to Other Miter Gates") that the test data "probably" could be applied to other miter gates having "the same kinematic cycle."

A specific limitation mentioned by Mr. Soucek is the fact that no tests were made of the effect of speed variation at low submergences. In the case of a miter gate of the same length, operated on the same kinematic cycle but with a submergence lower than that of the "base conditions," the only strictly applicable test results would be found in Fig. 6(*b*), and these would apply only for an operating time of 2 min. The effect of geometric disproportionality is small, however, and the speed-torque relations shown in Fig. 6(*a*) can be applied with reasonable reliability provided that the ratio of length of gate leaf to submergence is not too far from unity.

The test results presented by Mr. DeYoung in Fig. 11 are interesting and provide a basis for estimating loads on other gates operated with similar kinematic cycles. The angular position at which peak load occurs is close to that at which peak load was observed in the model tests of single-leaf operation. However, the kinematic cycle of the cable-and-drum mechanism described by Mr. DeYoung is so unlike that of the crank-and-strut mechanism used in the Panama Canal locks that the significance of the agreement between results in the two cases is doubtful. The diagram presented by Mr. DeYoung in Fig. 12 is quite convenient for some purposes.

Professor Edwards presents interesting extrapolations of the test results. The range of operating times tested with the model, like other phases of the model tests, was confined to the contemplated range of prototype values. It is regrettable that the model-test schedule could not be extended to cover more generalized studies.

As Professor Edwards states, the model study did not include any variation of the kinematic cycle except speed. The writer's conclusion that low speeds are best at the extremes of travel was based on the fact that peak loads occur near those positions. It is quite evident that a uniform speed throughout the leaf movement, for example, would produce even greater peak loads. The ideal load would be a uniform one, and this would be approached, not by increasing the relative speed near the recessed position, where a slight peak already exists, but by decreasing the relative speed near the mitered position. The degree to which this can be achieved in practice is limited by mechanical and structural design features and by space limitations. Dead-center recessed and mitered positions are desirable because they prevent gate-leaf overtravel in case of limit-switch failure.

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[7] Engr., Air Products, Inc., Chattanooga, Tenn.

Mr. Tripp contends that the model-test results are substantially in error because of improper consideration of the chamber lengths and the influence of the lock culverts. Mr. Tripp's statements regarding the effect of gate movement show clearly that he has applied steady-flow formulas to transient phenomena, which fact alone invalidates his views. Mr. Soucek describes accurately the nature of the gate loads when he states that they "are principally due to water-level variations which are transient in character and local in extent—surges induced by the gate movement—and therefore not related in any simple manner to the adjacent water-surface areas." However, since Mr. Tripp obtains results differing so greatly from those reported, and in consideration of the danger that his method of computation might be adopted by others, a detailed examination of the differences and the reasons therefor appears advisable. Accordingly, it will be shown that:

1. The gate loading cannot be computed on the basis of surface areas;
2. The effect of the 200-ft downstream chamber length is relatively small;
3. The culverts have negligible effect upon the critical gate load;
4. The loads for the "base conditions," although somewhat severer than the normal operating loads, may occur frequently in the prototype locks; and
5. The "base conditions," as applied, are logical criteria for design of the prototype operating machines.

Fig. 13 shows the effect of variation in the upstream chamber length between the limits of 300 ft and 1,105 ft for both the opening operation and the closing operation. The peak torque reached in the opening operation is critical for design. In these tests, the downstream chamber length was held constant at 200 ft, but it is theoretically evident, and was shown by similar tests, that the effect of length changes is about the same for an upstream chamber as for a downstream chamber. These tests show clearly that changes in the chamber length had no significant effect for lengths exceeding 625 ft and therefore that the gate loading cannot be computed on the basis of surface area. The theoretical basis for this result is quite simple. The wave velocity is expressed by

$$c = \sqrt{g\,y} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

in which c is the wave velocity; g is the gravitational acceleration; and y is the depth. Thus, $c = \sqrt{32.2 \times 82.5}$, or about 50 ft per sec. The peak opening torque occurs about 20 sec after operation begins; therefore, in order for the peak torque to be affected by the waves generated at the instant operation begins, these waves must encounter a reflecting surface no farther than about 500 ft from the gate. Since the torque remains at or near its peak value for a few seconds, it was stated conservatively (and it was shown by the tests) that lengths exceeding 625 ft have no effect upon the peak torque.

The fallacy in computing the operating loads on the basis of surface area, as suggested by Mr. Tripp when he mentions "* * * a definite volumetric change, which must be distributed over the free surface of the lock chambers * * *," is evident when it is considered that consistent application of that theory would indicate zero loads in the operation of a gate separating two areas of infinite extent.

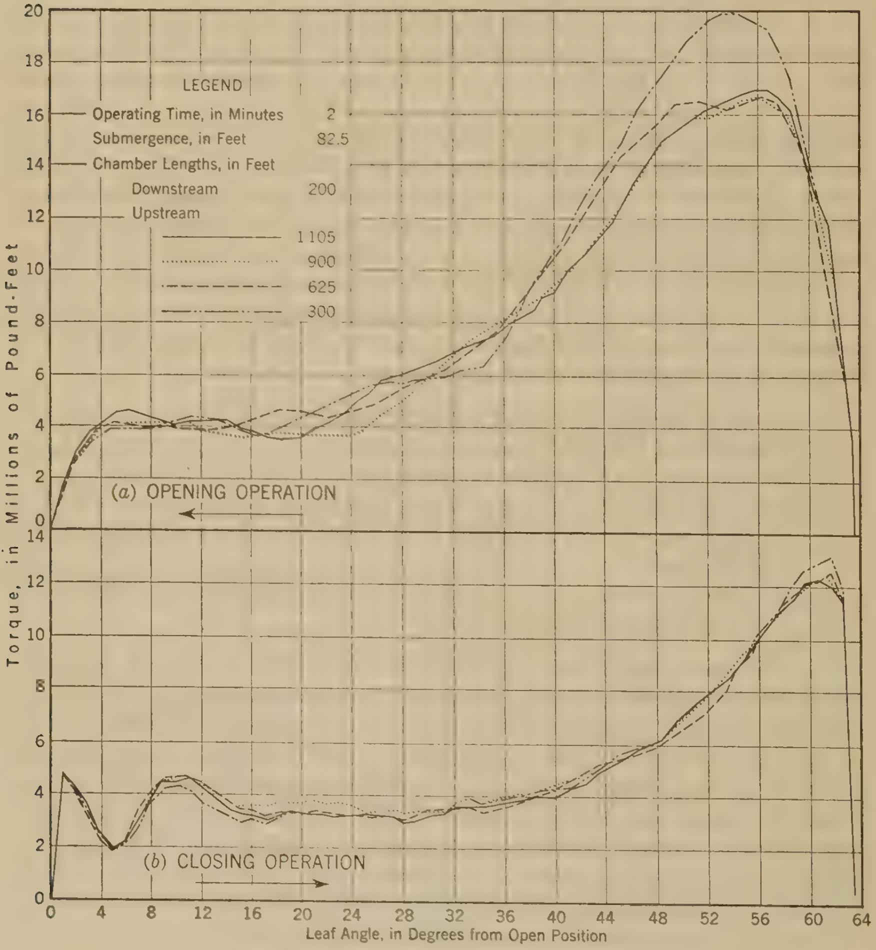

FIG. 13.—VARIATION OF CHAMBER LENGTH

As shown in Fig. 13, a reduction in upstream chamber length from 625 ft (the approximate limit of significant length effect) to 300 ft increased the peak opening torque from about 17,000,000 lb-ft to about 20,000,000 lb-ft; an increase of less than 10% in torque corresponds to a decrease of 52% in upstream chamber length within the effective range. Any reasonable method of extrapolating these results—even Mr. Tripp's computation procedure—would show that a decrease of 68% in chamber length would not increase the torque to the extent indicated by Mr. Tripp in his discussion. Even when one of the two chambers is only 200 ft long, it requires about 8 sec for a wave to travel the length of the short chamber and back. Thus, the reflected wave that is effective at the instant of peak torque originates from a slow gate movement occurring early in the cycle and produces a correspondingly small effect.

In making a substantial allowance for the effect of culvert discharge upon the gate operating load, Mr. Tripp again ignores a transient phenomenon that makes the effect of the culverts negligible. Tests of an intermediate gate at one of the existing locks showed that the duty cycle was absolutely unaffected by the position of the culvert valves (that is, whether they were closed or open). Since this result differs so greatly from Mr. Tripp's belief, some explanation of the reason is necessary. If a gate were operated under the base conditions—that is, with a 1,105-ft length of chamber upstream and a 200-ft length downstream—the throttling effect of the small port-and-lateral capacity in the short chamber would so restrict the culvert flow that little effect would be expected. Therefore, a condition more favorable to free flow through the culverts will be considered—opening of a single gate with chamber lengths of more than 1,000 ft both upstream and downstream and with culvert valves open. Since the gate leaves are accelerated continuously during the interval prior to occurrence of peak torque—a time of about 20 sec—the conditions will not be affected by reflections from the chamber ends and there will be an approximately uniform slope upward toward the gate from a point about 1,000 ft upstream and an approximately uniform slope downward toward the gate from a point about 1,000 ft downstream. At the instant of peak torque, the culvert discharge due to the average head on the system would be about 25% of that due to consideration of the head at the gate. However, the actual discharge is much smaller. It is impossible to determine the "equivalent unthrottled length" of the manifold system without considering the theory of hydraulic manifolds, but since only an approximate answer is sought, it will be assumed that the manifold system of the two 1,000-ft chambers has an "equivalent unthrottled length" of only 1,000 ft rather than the total culvert length of 2,000 ft. It is evident that the length cannot be assumed much shorter, because a shorter length would include so few ports and laterals that these rather than the culvert area would govern the rate of flow. With an "equivalent unthrottled length" of 1,000 ft, even if head losses that oppose the flow are ignored, the acceleration of the culvert flow would be approximately 0.016 ft per sec per sec at the instant of peak torque. The approximate mean acceleration from the start of gate operation to the instant of peak torque would be 0.008 ft per sec per sec, and the velocity of flow would be approximately 0.16 ft per sec. It appears that the velocity computed on the basis of steady flow corresponding to the head at the gate, disregarding friction losses, would be about 8.0 ft per sec—fifty times as great as the effect obtained by consideration of transient conditions. Of course, one simplifying assumption after another has been used in the preceding evaluation of the effect of the culverts for the transient conditions; however, any reasonable adjustment of these assumptions may be made without effect upon the principal conclusion—that the observed lack of appreciable effect of culvert discharge is consistent with theoretical considerations. A short loop culvert or an opening in the gate, of course, would have a pronounced effect.

It was not intended to convey the impression that the results for "base conditions" represent the average operating load. It is true that when two consecutive gates 200 ft apart are swung simultaneously the load is distributed to two sets of gate machines and is not critical. However, such operation will not be as general as would be inferred from Mr. Tripp's comments; it will occur only where it is desired to maintain two barriers against the upper pool. In other locations, it is contemplated that a single gate will be operated, with chamber lengths of more than 1,000 ft both upstream and downstream. Whether culverts are open or closed, the peak opening torque will be some 2,000,000 or 3,000,000 lb-ft less than the value obtained with the "base conditions" used in the model tests when no extraneous factors enter the problem. However, one normal condition, the presence of a large ship in either of the chambers between which the gate is located, will create a peak torque closely approximating that of the "base conditions." Occurrence of unexpected surges is another factor that makes it necessary that the design load exceed the usual operating load by a reasonable amount. Since the main purpose of the paper was to illustrate the application of model test data in design, it was not considered necessary to explain in detail all factors affecting selection of the base conditions, any more than it is felt necessary to defend the selection of a conservative unit stress in the design of an important structure subject to variable and uncertain loads. However, Mr. Tripp's claim that the base conditions were unduly conservative makes it clear that some of the additional factors considered in establishing the base conditions should have been explained for the benefit of those who are not familiar with the actual operating conditions.

The peak-torque values found in the model tests for "base conditions" were used as criteria for the design of the mechanical components of the gate operating machines because, as stated in the preceding paragraph, it is entirely possible and very likely that these values will occur in normal operation of the lock gates. However, the power of the driving motors was not controlled by the peak-torque requirements. During the part of the gate movement at which the peak torque occurs, the motors are loaded to approximately 180% of their full-load torque rating. It was also intended that, in the event of motor failure, one of the two motors for the machine of each gate leaf be capable of operating the gate leaf under otherwise favorable conditions, although the single motor would be considerably overloaded.

In conclusion, it is interesting to observe that transient conditions closely analogous to those affecting the peak opening torque exist in the field of electrical engineering. This is true not only of the wave conditions in the lock chambers but also of the inertial effects that make the discharge through the culverts insignificant.

The writer wishes to thank Messrs. Soucek and G. D. Smith for their assistance in the preparation of the text of this closure. The interest taken by all discussers is also appreciated.

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Paper No. 2258

SIMULTANEOUS CONSOLIDATION OF CONTIGUOUS LAYERS OF UNLIKE COMPRESSIBLE SOILS

By Hamilton Gray,[1] Jun. Am. Soc. C. E.

With Discussion by Messrs. Edward S. Barber, Jacob Feld, George W. Glick, and Hamilton Gray.

Synopsis

The results of an analysis of a condition which is sometimes encountered in foundation investigations—the consolidation of two contiguous layers of clay possessing different consolidation characteristics—are presented in this paper. A special case of such a phenomenon exists when one of two contiguous layers is relatively incompressible and functions merely to impede the outflow of water from the compressible stratum. When two adjoining compressible strata are made to consolidate under an applied load, the behavior of each stratum is influenced by the presence and action of the other. The extent of the mutual interference or influence of the layers depends on the physical properties (compressibility and permeability) of the materials comprising each layer. The analysis determines quantitatively how these physical properties govern the simultaneous consolidation of the two layers.

The details of the analysis have been placed on file for reference in Engineering Societies Library in New York, N. Y. They have been omitted from this paper, since they are long and do not influence the application of the results. The basic assumptions upon which the analysis is predicated are given in this paper, together with the appropriate differential equations and the final results. The user is thus able to apply these solutions with an understanding of the restrictions under which they have been secured. Graphs are presented illustrating the results secured for specific conditions. The nature of these solutions is in entire agreement with what "physical intuition" might anticipate, and a comparison is made between various approximations that have been made heretofore, and the "rigorous" solutions herein outlined.

Note.—Published in February, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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The results are purely theoretical and must be so considered until comparison with observed phenomena in the field confirms or denies their applicability to engineering problems. They are presented in the expectation that such comparisons may be more numerous if the nature of the theoretical approach is known.

INTRODUCTION

The original theory of the consolidation of clay deposits[2] was offered by Karl Terzaghi, M. Am. Soc. C. E., in 1925. Considerable interest has been shown recently in extensions of this theory to the more complicated practical problems involving three-dimensional flow and compression[3, 4, 5, 6] and stratified clay deposits.[7] Definite progress has been made in developing solutions for certain of these problems. A solution of the one-dimensional consolidation problem involving two contiguous layers of unlike clays has not appeared heretofore, although the technique required for obtaining it has been available for some years[8, 9] and somewhat analogous problems have been treated in the theory of heat flow.

The purpose of this paper is to draw attention to this solution and to indicate the nature of the results obtained. It must be realized that the application of these more recent developments would be justifiable only in the case of relatively large and expensive structures, since the analyses appear rather forbidding and at best necessitate considerable time and effort. Nonetheless, in connection with important structures it is frequently desirable to apply such advanced theories to secure one of perhaps a number of rough estimates of the possibilities inherent to the situation. Today engineers in increasing numbers are drawing extensively on the results of mathematical analysis as an aid in their work. In some cases the answer so obtained may be criticized as little better than a guess, but in any event it is a relatively "intelligent" or "conditioned" guess and, if actual behavior in the field is observed properly, one will ultimately be in a position to decide whether the "guess" was worth the effort or not. It is well to emphasize that experimental verification of the theories currently utilized for predicting the settlement of structures, perhaps by models, but more particularly by observing full-size structures, has not been as common as is desirable and that unwarranted confidence is sometimes placed in estimates that are based on theory unsubstantiated by field observation. Frequently actual conditions invalidate some of the assumptions on which a theory is based, and failure to consider this possibility may then result in large discrepancies between predicted and observed behavior. The following developments are offered as "tools," the accuracy of which remains to be determined, of which the engineer concerned with foundation design may avail himself at his discretion.

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[2] "Erdbaumechanik auf bodenphysikalischer grundlage," by K. Terzaghi, F. Deuticke, Vienna, 1925.

[3] "General Theory of Three-Dimensional Consolidation," by Maurice A. Biot, *Journal of Applied Physics*, Vol. 12, 1941, No. 2, p. 155.

[4] "Consolidation Settlement Under a Rectangular Load Distribution," by M. A. Biot, *ibid.*, No. 5, p. 426.

[5] "Consolidation Settlement of a Soil with an Impervious Top Surface," by M. A. Biot and F. M. Clingan, *ibid.*, No. 7, p. 578.

[6] "Bending Settlement of a Slab Resting on a Consolidating Foundation," by M. A. Biot and F. M. Clingan, *ibid.*, Vol. 13, 1942, No. 1, p. 35.

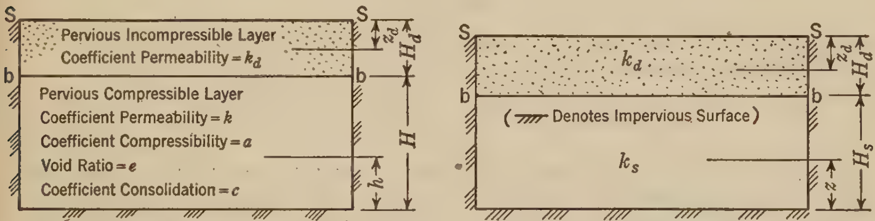
[7] "Factors Affecting the Accuracy of Settlement Rate Forecasts," by G. P. Tschebotareff, *Proceedings*, 18th Annual Meeting, Highway Research Board, Pt. II, 1938, p. 120.

[8] "The Cooling of a Solid Sphere with a Concentric Core of a Different Material," by H. S. Carslaw, *Proceedings of the Cambridge Philosophical Society*, Vol. XX, 1921, p. 399.

[9] "Bromwich's Method of Solving Problems in the Conduction of Heat," by H. S. Carslaw, *London, Edinburgh and Dublin Philosophical Magazine and Journal of Science*, Series 6, Vol. 39, 1920, p. 603.

A Special Case of Impeded Drainage

By way of introduction the relatively simple problem of a compressible stratum with impeded drainage will be considered. This is actually a special case of the more general problem treated herein, but is introduced here because of the relative ease with which it can be analyzed. The method has long been known;[10] it represents an extension of the usual attack by means of differential equations and Fourier series. Two practical applications are found when one investigates (*a*) the error involved in the assumption of "free outflow" from the surface of a clay sample being compressed in the laboratory under known conditions, and (*b*) the case of a relatively thin but compressible soil layer overlain by a thick bed of incompressible and relatively impervious soil.


S
S
Pervious Incompressible Layer
Coefficient Permeability = k_d
b
b
Pervious Compressible Layer
Coefficient Permeability = k
Coefficient Compressibility = a
Void Ratio = e
Coefficient Consolidation = c
z_d
H_d
H
h
S
S
k_d
b
b
(⫽⫽ Denotes Impervious Surface)
k_s
z_d
H_d
H_s
z

(*a*) TRUE DIMENSIONS (*b*) REDUCED DIMENSIONS

Fig. 1

The problem is shown schematically in Fig. 1, which serves also to define certain of the letter symbols introduced in the analysis. The remaining symbols are defined as they appear in the text and the complete list is presented, for the convenience of the reader, in the Appendix. The "reduced" dimensions of Fig. 1(*b*) were introduced for mathematical simplicity and are related to the true dimensions by the equations:

$$H_s = \frac{H}{1 + e} \quad \text{. . . . . (1a)}$$

and

$$z = \frac{h}{1 + e} \quad \text{. . . . . (1b)}$$

In accordance with the usual custom in dealing with the consolidation problem, it is assumed that the voids of both compressible and incompressible layers are completely filled with an incompressible liquid (water). It is further assumed that, before the consolidation process is initiated, the pressure, u_z, in the void liquid varies according to the law

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[10] "An Elementary Treatise on Fourier's Series and Spherical, Cylindrical, and Ellipsoidal Harmonics," by W. E. Byerly, Ginn & Co., Boston, 1893, pp. 117–121.

$$u_z = \gamma\, z_d \quad \ldots\ldots (2)$$

in which γ is the density of the liquid, and that at all times the liquid pressure is atmospheric when $z_d = 0$. A normal stress of intensity, p, is abruptly applied to the surface S-S, resulting in an upward flow of water associated with the compression or void reduction of the lower layer under the influence of this added load. If u_d (commonly referred to as "hydrostatic excess pressure") denotes the fluid pressure, in excess of that given by Eq. 2, induced by p in the incompressible layer, the flow of liquid that occurs in this incompressible layer is governed by LaPlace's equation:[11]

$$\frac{\partial^2 u_d}{(\partial z_d)^2} = 0 \quad \ldots\ldots (3a)$$

Similarly, the excess fluid pressure, u, in the compressible layer varies in accordance with the formula[2]

$$c\,\frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} \quad \ldots\ldots (3b)$$

in which c, the coefficient of consolidation of the compressible material, is defined by the equation

$$c = \frac{k}{a\,\gamma\,(1 + e)} \quad \ldots\ldots (4)$$

$-\,a$ is the slope of the pressure, void-ratio curve; $+\,a$ is known as the coefficient of compressibility; and t is the time. The solution of the problem consists of two expressions—one for u_d, as a function of z_d and t, and the other for u, as a function of z and t. These expressions must satisfy the differential Eqs. 3 and, in addition, the following "boundary conditions":

When $z = 0$—

$$\frac{\partial u}{\partial z} = 0 \quad \ldots\ldots (5a)$$

when $t = 0$—

$$u = p \text{ (a constant)} \quad \ldots\ldots (5b)$$

when $z_d = 0$

$$u_d = 0 \quad \ldots\ldots (5c)$$

and, when $z_d = H_d$ and $h = H$—that is, at the common boundary—

$$u = u_d; \qquad \text{and} \qquad k\,\frac{\partial u}{\partial h} + k_d\,\frac{\partial u_d}{\partial z_d} = 0 \quad \ldots\ldots (5d)$$

Since $h = z\,(1 + e)$, Eq. 5d may be written

$$\left[k_s\,\frac{\partial u}{\partial z}\right]_{z=H_s} + \left[k_d\,\frac{\partial u_d}{\partial z_d}\right]_{z_d=H_d} = 0 \quad \ldots\ldots (6)$$

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[11] "The Flow of Homogeneous Fluids Through Porous Media," by M. Muskat, 1st Ed., McGraw-Hill Book Co., Inc., New York, 1937.

in which $k_s = \frac{k}{1+e}$. By differentiation and substitution it is readily shown that—

$$u = 2\,p \sum_{n=1}^{n=\infty} \epsilon^{-TN} \left[\frac{\sin r_n \cos r_n \frac{z}{H_s}}{r_n + \frac{1}{2}\sin 2\,r_n}\right] \qquad (7a)$$

and

$$u_d = p\,\frac{z_d}{H_d} \sum_{n=1}^{n=\infty} \epsilon^{-TN} \left(\frac{\sin 2\,r_n}{r_n + \frac{1}{2}\sin 2\,r_n}\right) \qquad (7b)$$

—are two functions which, respectively, satisfy the differential Eqs. 3 and the prescribed boundary conditions, provided r_n is a root of the equation—

$$r \tan r = \frac{H_s}{H_d}\frac{k_d}{k_s} = R \qquad (8a)$$

In Eqs. 7, ϵ is the base of natural logarithms and the symbols in the exponent

$$N = (r_n)^2 \qquad (8b)$$

and

$$T = \frac{c\,t}{(H_s)^2} \qquad (8c)$$

are used solely to simplify typography. In Eq. 8*c* the symbol T is a dimensionless quantity commonly termed a "time factor."

The degree of consolidation at any point in the compressible stratum is defined as

$$U_p = 1 - \frac{u}{p} \qquad (9)$$

and the average degree of consolidation for the entire stratum is then given by

$$U = \frac{1}{H_s}\int_0^{H_s} U_p\,dz = 1 - \frac{1}{p\,H_s}\int_0^{H_s} u\,dz \qquad (10)$$

The degree of consolidation, U, equals the ratio of current subsidence or compression to the ultimate values of these quantities under the existing pressure provided the relation between void ratio and intergranular pressures in the soil is linear. Substituting the expression Eq. 7*a* for u under the integral sign in Eq. 10, and evaluating:

$$U = 1 - 2\sum_{n=1}^{n=\infty} \frac{\epsilon^{-TN}}{r_n}\,\frac{\sin^2 r_n}{r_n + \frac{1}{2}\sin 2\,r_n} \qquad (11)$$

The degree of consolidation is frequently expressed as a percentage and written as $U\%$. The values of $U\%$ then are one hundred times the magnitudes of U given by Eq. 11. As k_d approaches infinity (corresponding to unimpeded or free outflow from the upper surface of the compressible layer), R likewise becomes infinite and the roots of Eq. 8*a* become simply odd multiples of $\frac{\pi}{2}$, and may be represented by

$$r_n = (2\,m + 1)\,\frac{\pi}{2} \qquad (12)$$

in which m takes on successive integer values beginning with $m = 0$. The corresponding equation for U is

$$U = 1 - \frac{8}{\pi^2} \sum_{m=0}^{m=\infty} \frac{1}{(2\,m + 1)^2}\,\epsilon^{-TN'} \qquad (13)$$

which coincides with the familiar solution obtained when the boundary condition Eq. 5*d* for the compressible layer is $u = 0$ when $z = H_s$. In Eq. 13, the exponent N' is substituted for

$$N' = \frac{(2\,m + 1)^2\,\pi^2}{4} \qquad (14)$$

to simplify typography. For fixed values of H_s, H_d, k_s, and k_d, R is determined, and the roots of Eq. 8*a* may be found by the method of successive approximations. Typical curves such as those illustrated in Fig. 2, showing the variation of U with T for different values of R, may then be plotted.

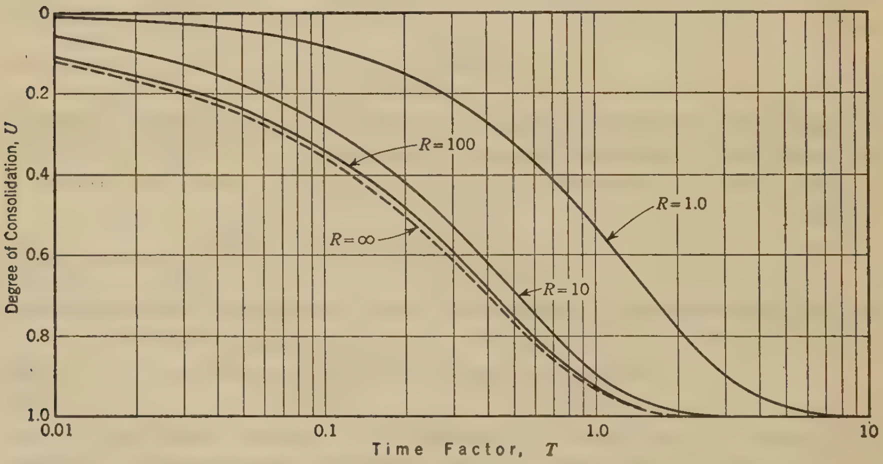

FIG. 2.—CURVES OF U VERSUS T

For large values of T, only one term of the series for U is needed for sufficient accuracy in plotting the curves. Hence, to find the approximate value of $T > 0.5$ required for a given value of $U > 0.76$ when R is known, it is sufficient to use the equation:

$$U = 1 - \frac{2\,\sin^2 r_1}{r_1 + \frac{1}{2}\sin 2\,r_1}\,\frac{\epsilon^{-NT}}{r_1} \qquad (15)$$

Solving for T (as in Eqs. 7, $N = (r_n)^2$, the value of n being 1 in this case):

$$T = \frac{1}{(r_1)^2} \log \frac{2\,\sin^2 r_1}{(1 - U)\,r_1\,(r_1 + \frac{1}{2}\sin 2\,r_1)} \qquad (16)$$

in which r_1 is the smallest root of $r \tan r = R$ (see Eq. 8*a*).

In determining the value of k from the results of a consolidation test it is necessary to utilize the relationships:

$$c = (H_s)^2 \frac{T}{t} \qquad (17a)$$

and

$$k = a \gamma c (1 + e) \qquad (17b)$$

Eq. 17b may also be written

$$k_s = a \gamma c = a \gamma (H_s)^2 \frac{T}{t} \qquad (17c)$$

In these formulas a is determined from the pressure, void-ratio diagram, and H_s by actual measurements. It is customary to compare the shape of an empirical time-consolidation curve of the tested soil with that of the theoretical curve, $R = \infty$, of Fig. 2. These two curves are usually rather similar in shape, but the empirical curve is plotted to scales of elapsed time, t, and measured compression of the soil sample, whereas the curve, $R = \infty$, of Fig. 2 is plotted to the scales of the dimensionless quantities T and U. It is generally possible to determine, for most empirical curves, the approximate value of elapsed time, t, which is required to attain some specified portion, say 60% or 90%, of the ultimate compression under the prevailing load.[12, 13] To be specific, the elapsed time required to reach 90% of the ultimate compression, when drainage is unobstructed, will be denoted by t_9. The corresponding value of U is 0.9, and of T, 0.848. Then

$$k_s = a \gamma (H_s)^2 \frac{0.848}{t_9} \qquad (18a)$$

and

$$k = k_s (1 + e) \qquad (18b)$$

in which e is the average void ratio under the load increment which resulted in the empirical time-consolidation curve, and a is determined from the pressure, void-ratio diagram.

This value of k will be accurate only if the actual value of R is infinite. If $R \neq \infty$, consolidation will occur in accordance with Eq. 11 rather than Eq. 13. The empirical curve should then be compared with a theoretical curve for $R \neq \infty$ rather than with the theoretical curve for $R = \infty$. However, the correct value of R is unknown since it depends upon the value of k to be determined.

It is possible to avoid this dilemma as follows: Assuming that $R \neq \infty$, let t_a denote the observed time which corresponds to 90% of the ultimate compression given by the empirical curve. Since drainage is impeded, $t_a > t_9$. Computing as before by using $T = 0.848$:

$$k_{sa} = a \gamma (H_s)^2 \frac{0.848}{t_a} \qquad (19)$$

in which k_{sa} is the "apparent" permeability of the soil. Since $t_a > t_9$,

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[12] "Principles of Final Soil Classification," by Charles Terzaghi, *Public Roads*, Vol. 8, May, 1927, p. 41.

[13] "Improved Soil Testing Methods," by G. Gilboy, *Engineering News-Record*, May 21, 1936, p. 732.

$k_{sa} < k_s$. Had the actual value of R been known, a magnitude of T greater than 0.848 could have been used, this magnitude being found by means of Eq. 16, in which $U = 0.9$. Use of this correct value of T (denoted by T_a) would have given the result, k_s, at once. Since $\frac{T_a}{0.848}$ may be determined as a function of R, when $U = 0.9$, by means of Eq. 16, the ratios $\frac{k_s}{k_{sa}} = \frac{t_a}{t_9} = \frac{T_a}{0.848}$ may also be found in terms of R (Eq. 8a). If $M = \frac{H_s}{H_d} k_d$, $\frac{M}{k_{sa}} = R \frac{t_a}{t_9}$. For any chosen value of R, the ratios $\frac{t_a}{t_9}$ and $\frac{k_s}{k_{sa}}$ can be found, and thence, $\frac{M}{k_{sa}}$. A plot of corresponding values of $\frac{k_s}{k_{sa}}$ versus $\frac{M}{k_{sa}}$ is shown in Fig. 3.

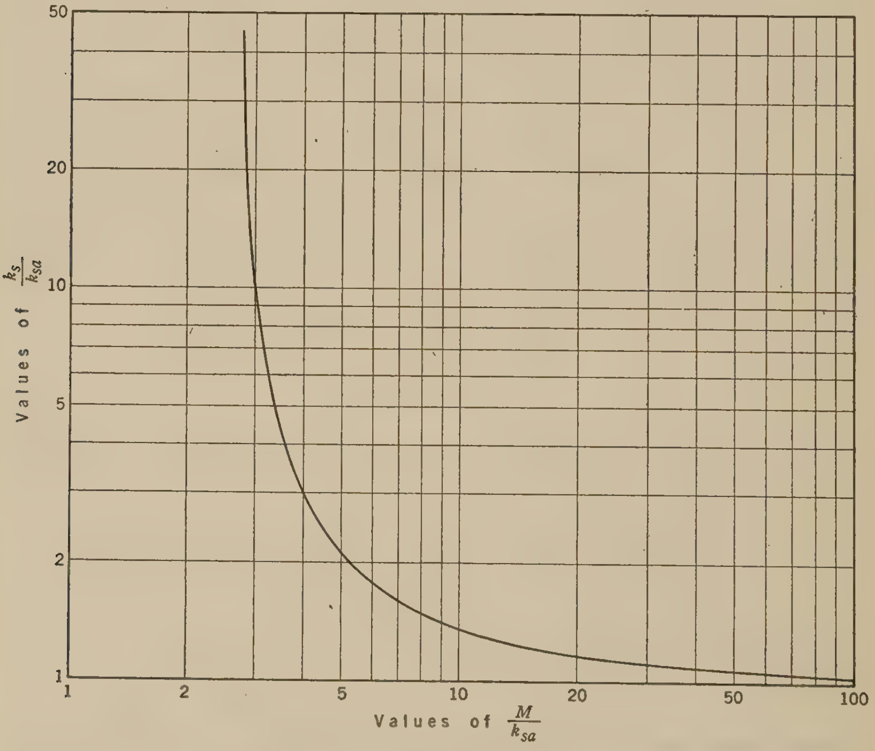


FIG. 3.—THE RATIO OF TRUE PERMEABILITY TO APPARENT PERMEABILITY AS A FUNCTION OF $\frac{M}{k_{sa}}$ (FOR $U = 0.9$)

From this curve it appears that, when $R = 100$, the ratio of $\frac{k_s}{k_{sa}}$ is only 1.025 so that it may be concluded that the drainage medium exercises a negligible influence if the value of R exceeds 100. Ordinarily the drainage media used in laboratory testing are of such a nature that R is of the order of magnitude of 10^5. If, in any instance, the value of R should fall below 100, the curve of Fig. 3 would permit proper correction to be made in determining the value of k_s.

Another application of the results expressed in Eqs. 8*a* and 9 is involved in the subsidence of a load placed upon a bed of relatively incompressible soil of low permeability, such as a cohesionless silt, which overlies a stratum of compressible clay. It is assumed that the load exerts a uniform pressure on the soil and that its lateral extent is so great as to afford no appreciable reduction in pressure increase between the upper surface of the silt and the bottom of the clay, which is assumed to rest on impervious rock; that is, a load of 1 ton per sq ft applied at the ground surface creates an additional pressure of nearly the same magnitude at the rock surface, and at all intermediate elevations.

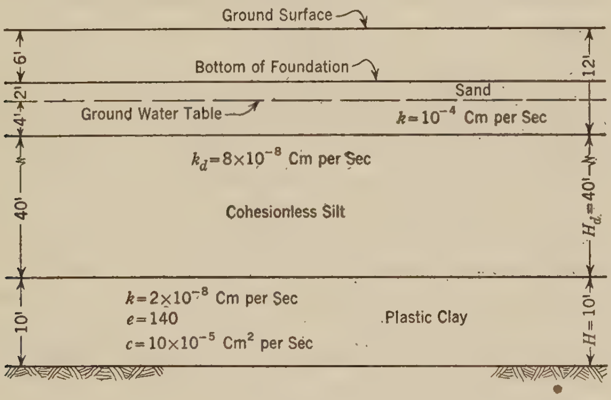

Ground Surface
Bottom of Foundation
Sand
Ground Water Table
$k=10^{-4}$ Cm per Sec
$k_d=8\times10^{-8}$ Cm per Sec
Cohesionless Silt
$k=2\times10^{-8}$ Cm per Sec
$e=140$
$c=10\times10^{-5}$ Cm^2 per Sec
Plastic Clay
6'
2'
4'
40'
10'
12'
$H_d=40'$
$H=10'$

FIG. 4

The consolidation phenomenon then is one-dimensional for all practical purposes. Suppose that subsoil and laboratory investigations have established data shown in Fig. 4. Eq. 8*a* yields

$$R = \frac{k_d}{k_s}\frac{H_s}{H_d} = \frac{80 \times 10^{-9}}{8.33 \times 10^{-9}}\frac{4.16}{40.0} = 1.0.$$

The time required to reach any fraction of the ultimate subsidence of approximately 4 in. may be secured by means of the curve $R = 1.0$ in Fig. 2. For example, 3 in. of settlement corresponds to $U = 75\%$ and $T = 1.85$ according to this curve. The time in years is obtained from

$$t = T\frac{(H_s)^2}{c} \times \frac{1}{3.16 \times 10^7} \qquad (20)$$

in which 3.16×10^7 is the number of seconds in a year. Thus,

$$t = 1.85\frac{(30.48 \times 4.16)^2}{10 \times 10^{-5}} \times \frac{1}{3.16 \times 10^7} = \frac{1.85}{3.16}(3.048 \times 4.16)^2 = 9.5 \text{ yr.}$$

Had the permeability of the silt been of the order of 10^{-4} cm per sec the value of R would have been

$$\frac{10^{-4}}{8.33 \times 10^{-9}} \frac{4.16}{44} = \frac{41.6}{44} \times \frac{10}{8.33} \times 10^{3} = 1.137 \times 10^{3} = 1{,}137.$$

Using the curve $R = \infty$ in Fig. 2, the value $T = 0.48$ corresponds to $U = 75\%$; whence $t = 9.5 \times \frac{0.48}{1.85} = 2.46$ yr.

It should be realized that the drainage conditions affect only the time rate of compression and have no influence on the total amount of compression. Therefore consideration of the drainage conditions is required only when the settlement rate is of importance.

Another application of the solution given by Eq. 7*a* consists of the computation of the variation, brought about by consolidation, of the shearing strength of a clay. In some instances it is feasible to schedule construction operations for earth dams or embankments in such a manner that a weak foundation, originally incapable of supporting the finished structure, may be gradually loaded and compressed during the construction period so that its strength is never exceeded, but gradually increases as the void ratio is reduced. Thus, ultimately, it is able to support the finished structure safely. In such circumstances it is necessary to know not only the shearing, but also the consolidation characteristics of the foundation material. If the soil, which is to be strengthened by consolidation, does not have free drainage, the usual consolidation theory based on the assumption of unimpeded drainage will yield results that are on the unsafe side.

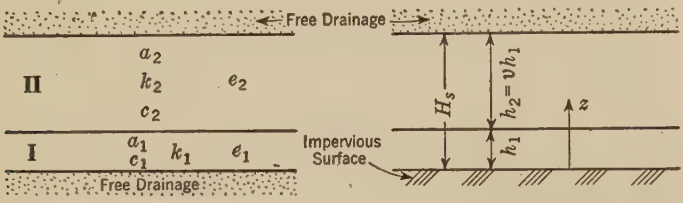
In considering the variation in shearing strength one may deal with the average strength of the weak stratum, in which case curves similar to those of Fig. 2 are useful, or one may deal with the weakest zone in the consolidating stratum. The shearing resistance at any point is a function primarily of the void ratio which may be found by means of the pressure, void-ratio diagram used in conjunction with Eq. 7*a*. At any instant of time the value of u is greatest at the bottom of the consolidating stratum, see Fig. 1, where $z = 0$. At this elevation the void ratio is greater and the shearing strength is less than for values of $z > 0$. Setting $z = 0$, Eq. 7*a* becomes

$$u = 2\,p \sum_{n=1}^{n=\infty} \frac{\sin r_n}{r_n + \frac{1}{2} \sin 2\, r_n}\, \epsilon^{-TN} \quad \text{.................} \quad (21)$$

If the thickness and the coefficient of consolidation of the consolidating stratum are known, $T = \frac{c\,t}{(H_s)^2}$ (Eq. 8*b*), and u may be expressed as a function of the time lapse following the application of a load increment, p. Therefore, one is in a position to determine the variation of shear strength with the passage of time at the elevation where the stratum is weakest. In this connection it is convenient to plot u as a function of T in accordance with Eq. 21.

THE GENERAL CASE OF TWO ADJACENT COMPRESSIBLE STRATA

In analyzing the problem of two contiguous layers of consolidating soils, it is expedient to consider the layers in a manner similar to that in the preceding text. This problem is depicted in Fig. 5, there being two distinct cases to consider—(*a*) free drainage top and bottom, and (*b*) drainage at top only, as shown. In Fig. 5 all the dimensions shown are "reduced." The layers of soil are numbered I and II, and the corresponding soil coefficients are identified by appropriate subscripts, 1 and 2. For each material,



(*a*) FREE DRAINAGE AT TOP AND BOTTOM

(*b*) FREE DRAINAGE AT TOP ONLY

FIG. 5

$$k = a\,c\,\gamma\,(1 + e) \qquad (22)$$

In addition, the following transformations are convenient:

$$k_{s1} = \frac{k_1}{1 + e_1} \qquad (23a)$$

$$k_{s2} = \frac{k_2}{1 + e_2} \qquad (23b)$$

and the introduction of four dimensionless numbers—

$$\mu^2 = \frac{c_1}{c_2}; \qquad \sigma = \frac{1}{\mu}\frac{k_{s1}}{k_{s2}}; \qquad \nu = \frac{h_2}{h_1}; \qquad \text{and} \qquad T = \frac{c_1}{(h_1)^2}\,t \qquad (24)$$

—is useful. The last (T) is the "time factor" for this case.

It is possible to solve the two basic cases simultaneously, and, in summarizing, the boundary conditions have been arranged in Table 1 so as to emphasize the similarity. For the lower layer the law of fluid flow may be expressed by

$$c_1\,\frac{\partial^2 u_1}{\partial z^2} = \frac{\partial u_1}{\partial t} \qquad (25a)$$

and for the upper layer the corresponding equation is

$$c_2\,\frac{\partial^2 u_2}{\partial z^2} = \frac{\partial u_2}{\partial t} \qquad (25b)$$

in which u_1 and u_2 denote the fluid pressures in excess of hydrostatic in the lower and upper layers, respectively.

Boundary condition 5, Table 1, is more restrictive than is necessary, since solutions can be obtained in cases where u is any function of z when $t = 0$. It may be shown by direct substitution that, for the case of drainage at both top

TABLE 1.—BOUNDARY CONDITIONS

No.	Boundary	Drainage top and bottom	Drainage top only
1	$z = 0$	$u_1 = 0$	$\frac{\partial u_1}{\partial z} = 0$
2	$z = H$	$u_2 = 0$	$u_2 = 0$
3	$z = h_1$	$u_1 = u_2$	$u_1 = u_2$
4	$z = h_1$	$k_{s1} \frac{\partial u_1}{\partial z} = k_{s2} \frac{\partial u_2}{\partial z}$	$k_{s1} \frac{\partial u_1}{\partial z} = k_{s2} \frac{\partial u_2}{\partial z}$
5	$t = 0$	$u = p$ (a constant)	$u = p$ (a constant)

and bottom, the two functions,

$$u_1 = \sum_1^\infty C_n \, \epsilon^{-TN''} \sin A_n \frac{z}{h_1} \sin \mu \, \nu \, A_n \ldots\ldots\ldots\ldots (26a)$$

for the lower layer, and

$$u_2 = \sum_1^\infty C_n \, \epsilon^{-TN''} \sin A_n \sin \left(1 + \nu - \frac{z}{h_1}\right) \mu \, A_n \ldots\ldots (26b)$$

for the upper layer, satisfy the respective differential equations and boundary conditions if A_n is a root of the equation—

$$F(A) = \sigma \cos A \sin \mu \, \nu \, A + \sin A \cos \mu \, \nu \, A = 0 \ldots\ldots\ldots (27)$$

and if

$$C_n = 2 \, p \, \frac{\sigma \sin \mu \, \nu \, A_n + \sin A_n}{\sigma \, A_n \sin^2 \mu \, \nu \, A_n + \mu \, \nu \, A_n \sin^2 A_n} \ldots\ldots\ldots\ldots (28)$$

In Eqs. 26, $N'' = (A_n)^2$ is used to simplify typography. The corresponding expressions for the average degree of consolidation in the lower and upper layers are

$$U_1 = 1 - 2 \sum_1^\infty \frac{\sin \mu \, \nu \, A_n \, (\sigma \sin \mu \, \nu \, A_n + \sin A_n)}{(A_n)^2 \, (\sigma \sin^2 \mu \, \nu \, A_n + \mu \, \nu \sin^2 A_n)} (1 - \cos A_n) \, \epsilon^{-TN''} \ldots (29a)$$

and $U_2 =$

$$1 - \frac{2}{\mu \, \nu} \sum_1^\infty \frac{\sin A_n \, (\sigma \sin \mu \, \nu \, A_n + \sin A_n)}{(A_n)^2 \, (\sigma \sin^2 \mu \, \nu \, A_n + \mu \, \nu \sin^2 A_n)} (1 - \cos \mu \, \nu \, A_n) \, \epsilon^{-TN''} \ldots (29b)$$

For the case of drainage at the top only, the equations for hydrostatic excess pressure are:

$$u_1 = \sum_1^\infty C_n \, \epsilon^{-TN''} \cos A_n \frac{z}{h_1} \sin \mu \, \nu \, A_n \ldots\ldots\ldots\ldots (30a)$$

and

$$u_2 = \sum_1^\infty C_n \, \epsilon^{-TN''} \cos A_n \sin \mu \, A_n \left(1 + \nu - \frac{z}{h_1}\right) \ldots\ldots (30b)$$

in which A must be a root of the equation—

$$G(A) = \sigma \sin A \sin \mu \nu A - \cos A \cos \mu \nu A = 0 \quad \ldots\ldots\ldots (31)$$

—and,

$$C_n = \frac{2\, p \cos A_n}{\sigma A_n \sin^2 \mu \nu A_n + \mu \nu A_n \cos^2 A_n} \quad \ldots\ldots\ldots\ldots\ldots (32)$$

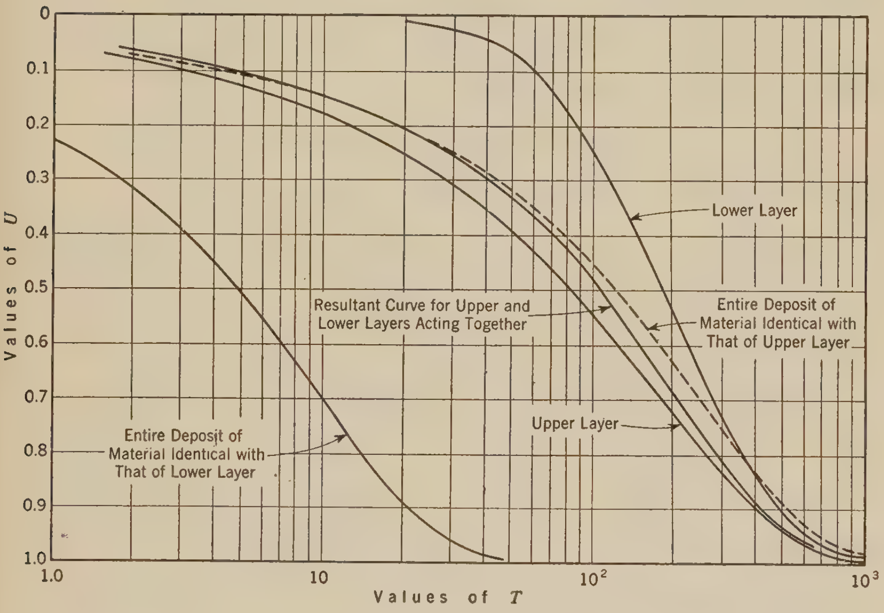


FIG. 6

The corresponding degrees of consolidation are:

$$U_1 = 1 - 2 \sum_{1}^{\infty} \frac{\sin A_n \cos A_n \sin \mu \nu A_n}{(A_n)^2 (\sigma \sin^2 \mu \nu A_n + \mu \nu \cos^2 A_n)} \epsilon^{-TN''} \quad \ldots (33a)$$

and

$$U_2 = 1 - \frac{2}{\mu \nu} \sum_{1}^{\infty} \frac{\cos^2 A_n (1 - \cos \mu \nu A_n)}{(A_n)^2 (\sigma \sin^2 \mu \nu A_n + \mu \nu \cos^2 A_n)} \epsilon^{-TN''} \quad \ldots (33b)$$

A convenient key to the application of these formulas is presented in Table 2. Results such as these are most readily comprehended if illustrated by graphical means, and the results of a particular case are shown in Fig. 6. In this situation it is assumed that there is drainage at the top only, with the following relationships between the coefficients and dimensions: $\nu = 4$; $e_1 = e_2$; $k_{s1}/k_{s2} = 10$; $a_1/a_2 = 0.4$; $\frac{c_1}{c_2} = \frac{k_{s1}}{k_{s2}} \frac{a_2}{a_1} = 25$; $\mu = \sqrt{\frac{c_1}{c_2}} = 5$; $\sigma = \frac{k_{s1}}{k_{s2}} \frac{1}{\mu} = 2$;

TABLE 2.—KEY TO DESIGN EQUATIONS

Definition	Upper Layer		Lower Layer	
	Drainage at top only	Drainage at top and bottom	Drainage at top only	Drainage at top and bottom
Hydrostatic excess pressure, u	Eq. 30*b*	Eq. 26*b*	Eq. 30*a*	Eq. 26*a*
Degree of consolidation, U	Eq. 33*b*	Eq. 29*b*	Eq. 33*a*	Eq. 29*a*
Coefficient, C_n	Eq. 32	Eq. 28	Eq. 32	Eq. 28

and $\mu\,\nu = 20$. Consequently, by Eq. 31,

$$G(A) = 2 \sin A \sin 20\,A - \cos A \cos 20\,A = 0 \quad \text{(34)}$$

The first eight roots of $G(A) = 0$ are given in Table 3.

TABLE 3.—ROOTS OF $G(A)$

Root No.	Root Symbol	Root Magnitude	
		Degrees	Radians
1	A_1	4° 5½′	0.0713
2	A_2	12° 19⅓′	0.2150
3	A_3	20° 39′	0.3604
4	A_4	29° 06′	0.5079
5	A_5	37° 39′	0.6571
6	A_6	46° 16⅔′	0.8076
7	A_7	54° 58′	0.9593
8	A_8	63° 41⅔′	1.1117

Fig. 7 shows the distributions of excess fluid pressures, u, in the soil layers at two different times corresponding to $T = 100$ and $T = 200$. As is to be expected from the assumed relation between the soil coefficients, the lower layer consolidates as rapidly as the upper will permit, the variation in u through the lower layer at any instant being very small. In the upper layer the variation is similar to that found in the case of a layer of homogeneous material with free drainage at one surface. In Fig. 6 the curves of U versus T are plotted for the upper and lower layers, together with a "resultant" curve computed from

$$U_r = \frac{U_1\,h_1 + U_2\,h_2}{h_1 + h_2} = \frac{4\,U_2 + U_1}{5} \quad \text{(35)}$$

Also shown are curves of U versus T which would result if the entire deposit of compressible soil were of material identical with that of (1) the upper layer, and (2) the lower layer.

Fig. 8 shows the "resultant" curve of Fig. 6, together with a curve obtained by averaging the coefficients of the two layers in the following manner, as suggested by Professor Terzaghi:[14]

$$a_{\text{aver}} = \frac{a_1\,h_1 + a_2\,h_2}{h_1 + h_2} = \frac{a_1\,h_1 + 2\tfrac{1}{2}\,a_1\,4\,h_1}{h_1 + 4\,h_1} = \frac{11}{5}\,a_1 = 2.2\,a_1 \quad \text{(36a)}$$

$$k_{\text{aver}} = \frac{h_1 + h_2}{\dfrac{h_1}{k_{s1}} + \dfrac{h_2}{k_{s2}}} = \frac{h_1 + 4\,h_1}{\dfrac{h_1}{k_{s1}} + \dfrac{4\,h_1}{k_{s1}}\,10} = \frac{5\,h_1}{\dfrac{41\,h_1}{k_{s1}}} = 0.122\,k_{s1} \quad \text{(36b)}$$

---

[14] "Sampling, Testing and Averaging," by K. Terzaghi, *Proceedings*, the Purdue Conference on Soil Mechanics and Its Applications, September 2 to 6, 1940, p. 151.

and

$$c_{\text{aver}} = \frac{k_{\text{aver}}}{\gamma\, a_{\text{aver}}} = \frac{0.122\, k_{s1}}{2.2\, a_1} = 0.0555\, \frac{k_{s1}}{a_1} = 0.0555\, c_1 \ldots\ldots (36c)$$

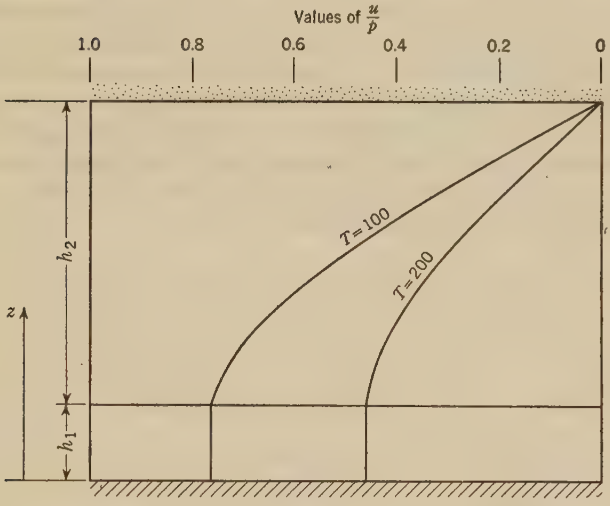

Values of $\frac{u}{p}$
1.0
0.8
0.6
0.4
0.2
0
T=100
T=200
h_2
h_1
z

FIG. 7

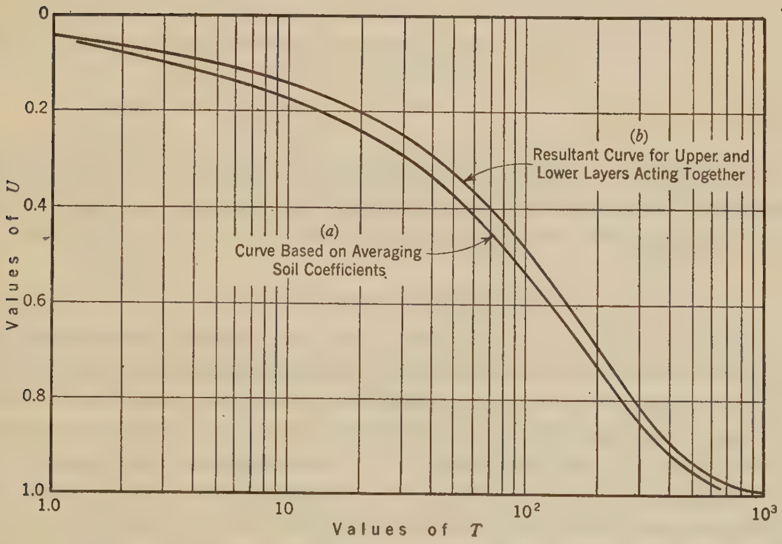


FIG. 8

Naturally, curve (a), Fig. 8, secured by the method of averaging, is obtained with far less effort than the theoretically correct "resultant" curve which it approximates. It is evident from Fig. 8 that the discrepancy between the two is not excessive. Fig. 9 shows these two curves plotted with T on an arithmetic scale. The difference between the two at any given value of T does not exceed 5% of the ultimate theoretical subsidence (that is, $U = 0.05$). The difference in T for a given value of U does not appear to exceed 20%. It is thought that the indicated agreement is adequate for practical purposes. Whether the agreement will be appreciably less close for other probable conditions can only be ascertained by "cut-and-try" methods. Such trials are time consuming, but, once values have been established, the degree of accuracy of the simpler method can be determined. This degree of accuracy need not be large, inasmuch as other variables, which enter the problem from sampling operation to estimates of the increase in load on the soil, all tend to influence the results in a more or less erratic manner.

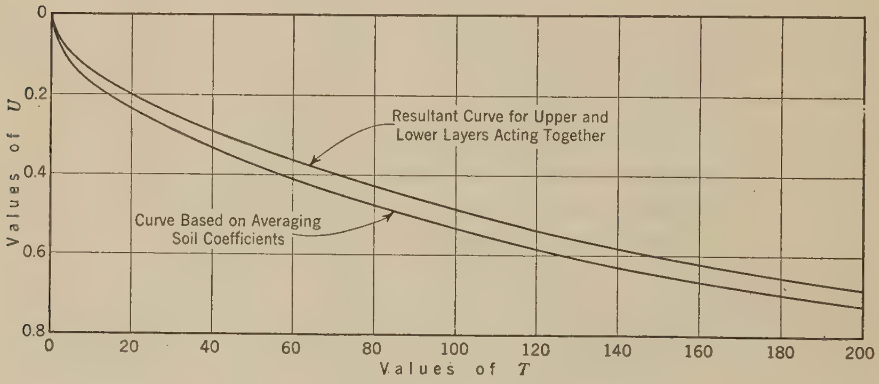


FIG. 9

For the case just considered, it is interesting to note that the theoretically correct curve lies between the curve secured by averaging, and that obtained by assuming the entire deposit to have the properties of the upper layer.

ACKNOWLEDGMENT

In times past the writer has had numerous interesting discussions with George W. Glick of the firm of Moran, Proctor, Freeman and Mueser, New York, N. Y., relative to various technical subjects, among them the consolidation of clay deposits. These discussions prompted the writer to investigate matters further and led ultimately to the preparation of this paper. The writer is indebted to Mr. Glick for the stimulation derived from these conversations and for pointing out the existence of the solution[10] used in analyzing the influence of the incompressible drainage medium. This solution and a part of that pertaining to two consolidating clay beds was developed while the writer was in the employ of the firm, Moran, Proctor, Freeman and Mueser, which has consented to the use of Figs. 2 and 3.

APPENDIX

NOTATION

The letter symbols used in this paper conform, in so far as possible, with those listed in "Soil Mechanics Nomenclature."[15] The units of the quantity represented by each symbol are given herein, in terms of force (F), length (L), and time (T). Subscripts 1 and 2 are used to indicate when a symbol refers to soil layers designated respectively by I and II.

Symbol	Definition	Unit
A	= dimensionless number in Eqs. 27 and 31: A_n denotes a root of Eqs. 27 and 31;	....
a	= coefficient of compressibility of soil;	$L^2 F^{-1}$
C_n	= dimensionless coefficient defined by Eqs. 28 and 32;	....
c	= coefficient of consolidation of soil;	$L^2 T^{-1}$
e	= void ratio of soil;	....
F	= "function of";	....
G	= "function of";	....
H	= thickness of soil layer:	L
	H_d = thickness of drainage layer;	
	H_s = reduced thickness as defined by Eq. 1*a*;	
h	= a vertical dimension in Fig. 1(*a*);	L
k	= coefficient of permeability of soil:	$L T^{-1}$
	k_a = "apparent" coefficient;	
	k_d = coefficient for the drainage layer;	
	k_s = reduced coefficient (Eq. 6);	
M	= a substitution constant = $\frac{H_s}{H_d} k_d$ (see Fig. 3);	$L T^{-1}$
m	= an integer indicating a succession of values (Eq. 14) beginning with $m = 0$;	....
N	= a substitute exponent (see Eq. 8*b*), equal to $(r_n)^2$, used in formulas to simplify typography;	....
N'	= a substitute exponent (see Eq. 14) used in formulas to simplify typography;	....
N''	= a substitute exponent (see Eqs. 26), equal to $(A_n)^2$, used in formulas to simplify typography;	....
n	= an integer indicating a succession of values (Eq. 9);	....
p	= pressure intensity;	$F L^{-2}$
R	= a ratio (Eqs. 8) = $\frac{H_s}{H_d} \frac{k_d}{k_s}$	....
r	= a ratio: r_n denotes a root of Eqs. 8;	....
T	= time factor: T_a = a correct value of T to determine a given value of k_s;	....

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[15] *Manual of Engineering Practice No. 22*, Am. Soc. C. E., 1941.

Symbol	Definition	Unit
t =	elapsed time:	T
	t_a = "apparent" time;	
	t_9, etc. = time corresponding to $U = 0.9$, etc.;	
U =	average degree of consolidation of a stratum: U_p = degree of consolidation at a point;	....
u =	hydrostatic excess pressure in soil: u_d = hydrostatic excess pressure in the drainage layer;	$F L^{-2}$
u_z =	hydrostatic pressure at point z, Fig. 1 and Eq. 2;	
z =	a reduced dimension (Fig. 1);	L
γ =	density of the liquid (the definition of density $F L^{-3}$ differs from the one commonly employed. It is used here to avoid the introduction of the gravitational constant);	$F L^{-3}$
ϵ =	base of natural logarithms;	....
μ =	the ratio $\sqrt{\frac{c_1}{c_2}}$ (Eq. 24);	....
σ =	a dimensionless ratio (Eq. 24);	....
ν =	a dimensionless ratio (Eq. 24).	....

DISCUSSION

Edward S. Barber,[16] Esq.—A timely analysis of the consolidation of contiguous layers of foundation soils having different consolidation characteristics is offered in this paper. The mathematically correct approach provides opportunity to evaluate the results furnished by the less laborious but approximate methods. An approximation that provides a reasonably simple solution for the consolidation of any number of contiguous layers and takes into account an influencing factor not considered in the author's method of average coefficients—namely, the relative position of the consolidating soil layers and the drainage faces—is outlined in this discussion. This approximation produces results that agree more closely with the exact theoretical analysis than do those by the method of average coefficients.

By considering the average gradient in the incompressible layer, H_d, shown in Fig. 1(a), required to carry away the water squeezed out of the consolidating layer, H, at any degree of consolidation, U, an approximate formula is obtained for the curves in Fig. 2, as follows:

$$U_R = \frac{U_\infty}{1 + \frac{U_\infty}{T\,R}} \qquad \text{(37)}$$

in which U_R = the value of U for a particular value of R, and U_∞ = the value of U when R is infinite, a theoretical condition assumed if it were possible to have the coefficient of permeability of the drainage layer, k_d, large enough to provide unrestricted flow.[17] The results furnished by Eq. 37 are good except for the combination of large values of T and small values of R.

The dimensionless coefficients, R, σ, μ, ν, and c used in Professor Gray's formulas may be simplified by expressing them in terms of the strain per unit increase in pressure, s, rather than a. The strain, s $\left(\text{which equals } \frac{a}{1+e}\right)$, can be obtained directly from the dial readings in a consolidation test. Actual thicknesses and permeabilities may be used instead of their reduced values and the void ratio becomes unnecessary.

Thus:

$$R = \frac{H/k}{H_d/k_d} \qquad \text{(38a)}$$

$$\sigma = \sqrt{\frac{k_1\,s_1}{k_2\,s_2}} \qquad \text{(38b)}$$

$$\mu\,\nu = \frac{H_2/k_2}{H_1/k_1}\,\frac{1}{\sigma} \qquad \text{(38c)}$$

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[16] Associate Highway Engr., Public Roads Administration, Washington, D. C.

[17] "The Theory of Soil Consolidation and Testing of Foundation Soils," by L. A. Palmer and E. S. Barber, *Public Roads*, March, 1937, p. 15.

and

$$c_v = \frac{H^2\,T}{t} = \frac{k}{\gamma\,s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38d)$$

Reference is now made to the comparison between the results furnished by the method of average coefficients and those by the exact analysis.

TABLE 4.—Properties of Layers in Illustrative Example

Layer	s	H	k	H/k	$s\,H$
II	1	1	1	1	1
I	0.4	5	10	0.5	2
Total	....	6	....	1.5	3

It appears that the consolidation of several layers should be averaged on the basis of the settlements rather than the thicknesses, as shown in Eq. 35. In such case, the agreement between the exact and average methods would be even closer than that shown in Fig. 8. The agreement of the two methods discussed by the author may not always be as close as that indicated in Fig. 8. To illustrate, assume two contiguous consolidating layers with one drainage face and properties as given in Table 4.

Using the method of average soil coefficients outlined by Eqs. 36 and taking $\gamma = 1$: $k = \frac{\sum H}{\sum H/k} = 4$; $s = \frac{\sum s\,H}{\sum H} = 0.5$; $c_v = \frac{k}{\gamma\,s} = 8$; and $t = \frac{H^2}{c_v}\,T = 4.5\,T$

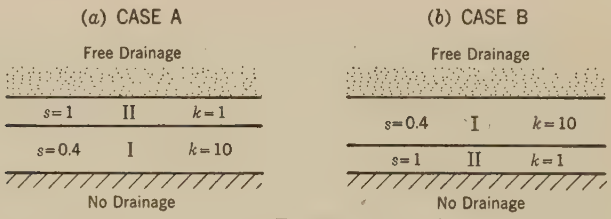

Fig. 10

Using Professor Gray's exact method, there are two cases (see Fig. 10): Case A, impervious face next to layer I, and case B, impervious face next to layer II. The constants are:

Constant	Case A	Case B
c_v	25	1
t/T	1	1
σ	2	0.5
$\mu\,\nu$	1	1

The time-consolidation curves resulting from these two methods of calculation are shown in Fig. 11. Also, in Fig. 11 are shown the curves obtained by means of the approximation introduced by the writer (which modifies the method of average coefficients to take into account the effect of the relative position of the boundaries and thus agrees more closely with the exact method).

The effect of this approximation is illustrated by the example for three contiguous layers outlined in Table 5. The time is computed as in the method of average coefficients. Thus, using the relationship,

$$s\,H = \frac{\delta}{p} \qquad (39)$$

the elapsed time t is expressed by

$$t = T \sum \frac{\delta}{p} \sum \frac{H}{k} \qquad (40)$$

which becomes 10 T for $p = 1$. However, the actual layers are replaced by their "equivalent thicknesses" H/k (Col. 4, Table 5) and, instead of using the same percentage load transfer in each layer, the part of the load transfer in each part of the total thickness[18] is used as shown in Fig. 12 and Table 6.

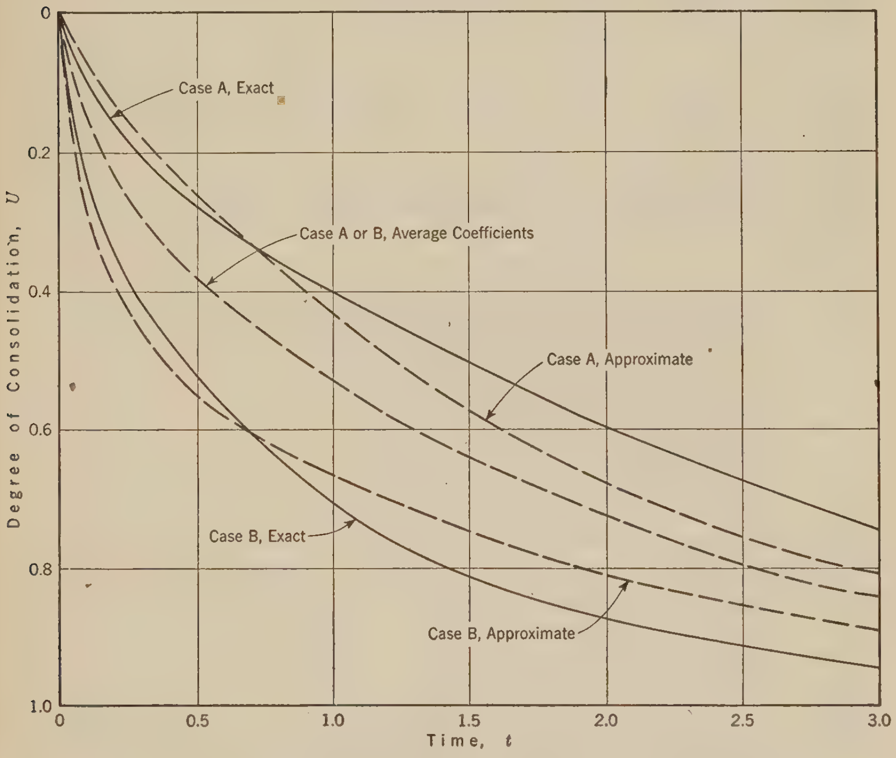

FIG. 11.—COMPARISON OF COMPUTATIONS OF TIME-CONSOLIDATION IN COMBINED LAYERS

In Table 5, Cols. 8 to 11, the settlement in a layer is computed by

$$\delta_p = \delta \frac{\Delta U_p}{H_p} \qquad (41)$$

---

[18] "Notes on Analytic Soil Mechanics," by J. R. Bruggeman, C. N. Zangar, and J. H. A. Brahtz, *Technical Memorandum No. 592*, Bureau of Reclamation, U. S. Dept. of the Interior, 1939, Fig. 34.

in which values of ΔU_p are taken from Table 5 as the difference between the U_p-values at the limits of the layer. Thus, for layer 2 (see Col. 10, Table 5), when $U = 0.60$ the value of ΔU_p (for $H_p = 0.30$ to 0.80) is equal to $0.440 - 0.210 = 0.230$ and δ_p (see Eq. 41) is equal to $1 \times \frac{0.23}{0.50} = 0.46$.

TABLE 5.—EXAMPLE OF THREE CONTIGUOUS LAYERS

Given				Computation				Inches of Settlement in Layer (Eq. 41) for the Following Degrees of Load Transfer U			
Layer No.	Final settlement δ (in.)	Thickness of layer H (ft)	Coefficient of permeability k	Equivalent thickness H/k	Equivalent part of total thickness H_p	Limits of Layer (Total H_p) From:	Limits of Layer (Total H_p) To:	0.20	0.40	0.60	0.80
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	3	6	1	6	0.30	0	0.30	0.96	1.67	2.10	2.50
2	1	20	2	10	0.50	0.30	0.80	0.03	0.19	0.46	0.74
3	2	12	3	4	0.20	0.80	1.00	0.88	1.38	1.60	1.80
....	6	38	....	20	1.00[a]	....	....	1.87	3.24	4.16	5.04

[a] This is for drainage above and below. For an impervious lower boundary make this value 0.50.

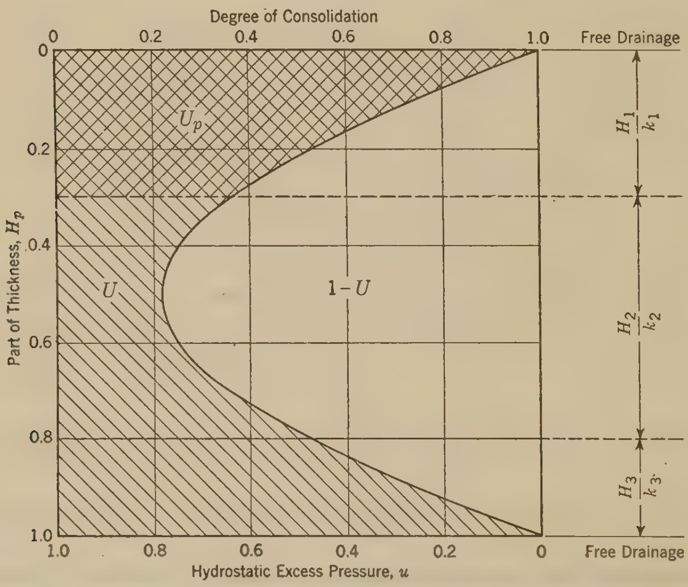

FIG. 12.—APPROXIMATE ANALYSIS OF TIME-CONSOLIDATION IN COMBINED LAYERS

The writer has assumed that the increase in pressure applied to the consolidating layers is uniform throughout their total thickness. To include the effect of a trapezoidal distribution of the initial excess pressure, values of T may be taken from Table 7, which is based on the following interpolation formula:[19]

$$U = U_1 + \frac{f-1}{f+1}(U_1 - U_0) \qquad (42)$$

in which T is constant and the subscripts are values of f.

TABLE 6.—CALCULATION OF TIME-CONSOLIDATION IN COMBINED LAYERS

(Degree of Load Transfer in Layer, U_p)

Part of total thickness, H_p	DEGREE OF LOAD TRANSFER IN TOTAL THICKNESS, U:								
	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.05	0.028	0.037	0.044	0.046	0.047	0.048	0.048	0.049	0.049
0.10	0.042	0.062	0.077	0.084	0.087	0.091	0.093	0.095	0.098
0.15	0.046	0.081	0.103	0.113	0.123	0.129	0.134	0.140	0.145
0.20	0.048	0.088	0.121	0.138	0.149	0.160	0.171	0.180	0.191
0.25	0.049	0.095	0.133	0.156	0.174	0.189	0.205	0.219	0.236
0.30	0.049	0.096	0.139	0.167	0.190	0.210	0.231	0.250	0.274
0.35	0.050	0.098	0.144	0.180	0.210	0.236	0.264	0.292	0.322
0.40	0.050	0.100	0.147	0.188	0.224	0.258	0.294	0.328	0.365
0.45	0.050	0.100	0.149	0.194	0.236	0.278	0.320	0.360	0.405
0.50	0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.450
0.55	0.050	0.100	0.151	0.206	0.264	0.322	0.380	0.440	0.495
0.60	0.050	0.100	0.153	0.212	0.276	0.342	0.406	0.472	0.535
0.65	0.050	0.102	0.156	0.220	0.290	0.364	0.436	0.508	0.578
0.70	0.051	0.104	0.161	0.233	0.310	0.390	0.469	0.550	0.626
0.75	0.051	0.105	0.167	0.244	0.326	0.411	0.495	0.581	0.664
0.80	0.052	0.112	0.179	0.262	0.351	0.440	0.529	0.620	0.709
0.85	0.054	0.119	0.197	0.287	0.377	0.471	0.566	0.660	0.755
0.90	0.058	0.138	0.223	0.316	0.413	0.509	0.607	0.705	0.802
0.95	0.072	0.163	0.256	0.354	0.453	0.552	0.652	0.751	0.851
1.00	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900

TABLE 7.—VALUES OF TIME FACTOR T; EFFECT OF BOUNDARIES ON TIME-CONSOLIDATION

f[a]	DEGREES OF CONSOLIDATION U								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.049	0.100	0.154	0.217	0.29	0.38	0.50	0.66	0.95
0.2	0.027	0.073	0.126	0.186	0.26	0.35	0.46	0.63	0.92
0.4	0.016	0.056	0.106	0.164	0.24	0.33	0.44	0.60	0.90
0.6	0.012	0.042	0.092	0.148	0.22	0.31	0.42	0.58	0.88
0.8	0.010	0.036	0.079	0.134	0.20	0.29	0.41	0.57	0.86
1.0	0.008	0.031	0.071	0.126	0.20	0.29	0.40	0.56	0.85
1.5	0.006	0.024	0.058	0.107	0.17	0.26	0.38	0.54	0.83
2.0	0.005	0.019	0.050	0.095	0.16	0.24	0.36	0.52	0.81
3.0	0.004	0.016	0.041	0.082	0.14	0.22	0.34	0.50	0.79
5.0	0.003	0.013	0.034	0.069	0.12	0.20	0.32	0.48	0.77
10.0	0.003	0.011	0.028	0.060	0.11	0.18	0.30	0.46	0.75
∞	0.002	0.009	0.024	0.048	0.09	0.16	0.28	0.44	0.73
....[b]	0.002	0.008	0.018	0.031	0.05	0.07	0.10	0.14	0.21

[a] Pressure at drainage face divided by pressure at impervious face = f. [b] Values of time factor for two drainage faces.

JACOB FELD,[20] M. AM. SOC. C. E.—By clearly indicating how the limitation of actual field conditions affects the theoretical solutions, the author has provided a valuable method for determining the range of values, not only in magnitude but also in rate of occurrence. This procedure will not mislead the engineer who looks to such tools for guidance in the determination of expected consolidation in soil fills and under foundations. All theoretical solutions are based on assumed end and surface conditions. Unless such ideal conditions occur, the formulas can only approximate true values and probably only give a range within which results may be expected. Only when a prophecy of consolidation, both as to extent and rate, is made on the basis of theoretical conditions alone, is no correlation found between actual occurrence and expected values, and one loses faith in all quantitative soil studies.

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[19] "The Theory of Soil Consolidation and Testing of Foundation Soils," by L. A. Palmer and E. S. Barber, *Public Roads*, March, 1937, p. 18, Eqs. 53, 54, and 55.

[20] Cons. Engr., New York, N. Y.

Sudden clear-cut changes in soil layers are not often encountered in nature —even in rock formations. There is usually a conglomerate, mixed skin separating unlike soil layers. Such a condition is difficult to evaluate in theoretical solutions, and thus subjects of this nature fall in the class of the practical art rather than in that of the theoretical science.

The same problem occurs in building up road and airport surfaces on sub-base materials. When a layer of pervious material is placed directly on a clay subgrade, apparently the ideal condition of contact in soil layers occurs. Actually, experience indicates that the two materials mix and form an intervening skin, which is similar to neither of the parent layers. An accelerated field demonstration of this phenomenon (probably not new) occurred during the construction of some twenty-five miles of pavements on extremely fine and impervious Seneca County (New York) clays in 1942. An examination of all existing roads in that area, and experience on other projects built in the previous year, showed that the fine clay worked up through the run-of-bank gravel sub-base, through penetration macadam, and even through bituminous mix surfacing. In one year, under rather light traffic, the clay had come through a total thickness of 8 in. of gravel sub-base, 4 in. of crushed dolomite base, and a penetration surfacing.

However, inspections, after almost two years of use, on the roads built under the writer's supervision showed no vertical intrusion of the clay into the road construction. In these cases, a fine bank sand containing leachings from limestone deposits had been placed about 2 in. deep on the excavated roadbed. Over the sand 6 in. to 8 in. of run-of-bank gravel, 4 in. of crushed stone base, and 2½ in. of tar mix surfacing were then placed. Similarly, some test roads built directly on top of a 2-in. or 3-in. weathered clay "topsoil" containing some grass roots were also satisfactory. Apparently the introduction of a fine sand screen, rolled into the clay and forming a conglomerate skin, changes all the consolidation characteristics normally encountered in the clay.

A consolidation forecast of filled materials must also take into account the possible altering of the exposed surface with the formation of a crust which will entirely change the expected free drainage surface conditions. In the construction of a large airport, grading operations included the removal of glacial till hills and the filling in of valleys to depths of almost 60 ft. The expected shrinkage and consolidation did not occur. When the paving was placed, about a year later, it was found necessary to remove about 1 ft of the fills to bring the surface to subgrade elevations. This top crust was very hard and of high density, but was floating on an almost liquid material which had not consolidated. The result could be prophesied by the author's methods if it had been assumed that there was practically no surface evaporation. However, if that assumption is not made, an erroneous result is found from the use of any consolidation formula.

In placing fills over completed subway and similar structures in densely populated areas, the most important requirement is rapid consolidation. Otherwise, there is not sufficient time to replace the subsurface utility structures in such a manner that failures will not occur after the streets are repaved. Too little has been done in experimenting with artificially treated backfill materials, when ideal porous filling materials are not available. The necessity for such treatment can be determined from the methods developed by the author, and by proper assumptions the relative depths of treated and untreated fills can also be obtained. In one personal experience, the use of river silt taken from a subaqueous tunnel operation was found unsatisfactory because of the slow drainage characteristics. However, when the tunnel operation ran into some difficulties and an appreciable amount of hay was used for "mudding up" the heading of the tunnel, the silt material became excellent backfill material and consolidated very rapidly. Once it was consolidated there was little danger of future volume changes because of the restrictions of subsurface structures, foundations, and impervious street paving. Of course, this idea is ancient in origin, dating back to biblical, or earlier, days when bricks were manufactured by the addition of straw to clay.

The author is to be congratulated for the presentation of workable formulas which can be checked in the field and which will give consistent and true results if, and only if, consistent and true field conditions are evaluated (as he so carefully explains) before using these formulas.

GEORGE W. GLICK,[21] ESQ.—The analysis by Professor Gray of consolidation in a two-layer system gives the answer to a most difficult problem and is a very worth-while contribution to the study of soil mechanics. As has been shown, the application of the formulas is long and tedious; yet, within limits, certain approximations can be used.

The problem of drainage through an incompressible porous medium is related to the problem of consolidation of a two-layer system with single drainage. This can be seen readily when the thickness of the outer layer becomes very thin, as the solutions for the consolidation of the inner layers become identical.

This type of problem was presented first a number of years ago during the testing of silty soils in the usual consolidation devices. At that time, it was realized that stones of high permeability were desirable, but the magnitude of the error that could occur from improperly chosen drainage stones was unknown until Professor Gray solved the problem in 1938.[22]

The author's remarks on the influence of the drainage medium on the shearing strength of consolidating soils are most pertinent.

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[21] Staff Engr., Moran, Proctor, Freeman & Mueser, New York, N. Y.

[22] "Research on Consolidation of Fine-Grained Soils," by Hamilton Gray, thesis presented to the faculty of the Graduate School of Eng., Harvard Univ., Cambridge, Mass., in partial fulfilment of the requirements for the degree of Doctor of Science in the field of Soil Mechanics, April, 1938.

The solution of the problem of the two-layer system with double drainage is most interesting and raises some unusual speculations. When one considers the solution as applied to two layers of homogeneous soil, it can be concluded that this is a solution of the familiar single-layer problem with the time factor referenced to the thickness of one of the two layers. In other words, the time factor for a single-layer system is multiplied by the square of the ratio of the total thickness to one of the layers to secure the time factor for that layer.

For homogeneous layers, the values of both σ and μ are unity. When only σ has the value of unity, the system might be considered pseudo-homogeneous as a stretching transformation can be applied to one of the layers to make a homogeneous system. In this manner, the time-consolidation coefficients for a single-layered homogeneous system can be used, thereby saving the labor of solving the equations for the two-layer system. When σ is unity, any number of layers can be combined by using their transformed thicknesses. The same type of reasoning can be applied to the two-layer system with single drainage.

From the solution of the two-layer system with double drainage, a solution for a single layer having one side with restricted drainage and one side with free drainage can be deduced by assuming one layer to be very thin.

When the value of σ differs from unity, the use of a stretching transformation introduces an error of unknown magnitude. It is a safe assumption that, when σ is less than 2, the error will be less than that for the author's case in which the time factor is about 13% too small at 90% consolidation. It would be interesting to know whether the writer's suggested transformation is applicable to other stress distributions.

The author is to be congratulated for his efforts in solving one of the most common, yet annoying, problems in this new science.

HAMILTON GRAY,[23] ASSOC. M. AM. SOC. C. E.—The discussions may be classified into two types. Those in type 1 suggest certain approximate solutions that can be applied with less expenditure of time than the rigorous solutions and for which the degree of approximation may be fairly high, at least within certain limits. Type 2 involves the application of the solutions to various foundation problems. Both types of discussion are valuable contributions to the subject.

There is no reason for applying involved and time-consuming analyses to engineering problems unless no simpler mode of attack will suffice. Exact methods of analysis may be useful from either of two standpoints: (*a*) As the only available rational attack on a problem, or (*b*) as a check on the degree of approximation in simpler methods of analysis.

Mr. Barber illustrates that, if preferred, compressions may be expressed in terms of strain rather than in terms of void-ratio changes. This is wholly a matter of personal taste since some engineers prefer pressure, void-ratio curves whereas others favor curves showing percentages of compression as functions of the pressure. In either case the results of the analysis are identical.

Mr. Barber's statement regarding the computation of the resultant consolidation on the basis of compressions rather than of thicknesses merits comment. Computation on the basis of thicknesses requires the use of the formula:

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[23] Asst. Prof., Civ. Eng., New York Univ., New York, N. Y.

$$U_r = \frac{U_1 + \nu\, U_2}{1 + \nu} \quad \text{(43a)}$$

which indicates the extent to which the transfer of stress from fluid to solid phase has occurred. Eq. 43*a* may be termed the "resultant degree of consolidation" by virtue of the definition of "degree of consolidation" given in Eqs. 9 and 10. Computation on the basis of compressions utilizes the expression:

$$U_r = \frac{U_1 + \frac{\mu\,\nu}{\sigma} U_2}{1 + \frac{\mu\,\nu}{\sigma}} \quad \text{(43b)}$$

Eq. 43*b* indicates what portion of the resultant ultimate compression has occurred, based on the assumption that the pressure, void-ratio curves are linear in the pertinent pressure range. This may be called the "resultant relative compression." From a practical standpoint, Eq. 43*b* is required for estimating the actual settlements. Only if $\frac{\mu}{\sigma} = \frac{a_2}{a_1} = 1.0$ will Eqs. 43*a* and 43*b* give identical results. The distinction between these two formulas should have been treated in the paper. For the example in the paper Eqs. 43 do not give very different results. This is illustrated in Fig. 13 which also reveals that, if the compressibility of layer I were to be ignored completely, a fairly close approximation to the exact result would be obtained.

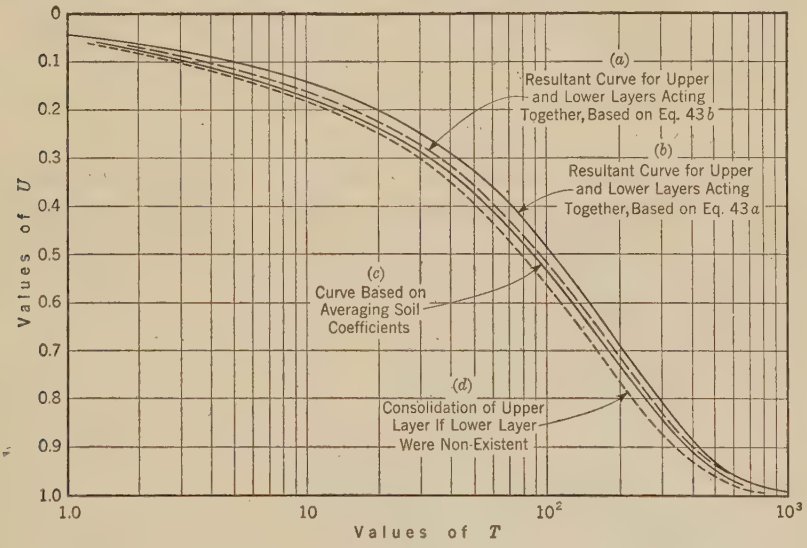


FIG. 13

Fig. 14 illustrates the situation in which the positions of the two layers which were considered in the paper are interchanged. Under these conditions, the agreement between Eq. 43*a* and Eq. 43*b* is less close, and neither of the two approximate solutions suggested in connection with Fig. 13 seems to yield as reliable results. Table 8 is included to clarify the differences that are reflected by the curves in Figs. 13 and 14. Fig. 15, based on case D, is analogous to Fig. 5 which is based on case C.

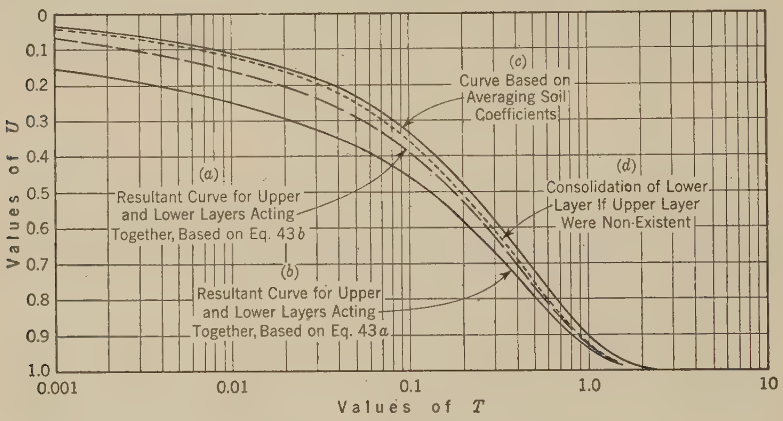


Fig. 14

Table 8 also illustrates typical numerical values for the pertinent physical constants. In this connection, it is of interest to note that the actual time required to achieve 50% of the ultimate resultant consolidation or settlement

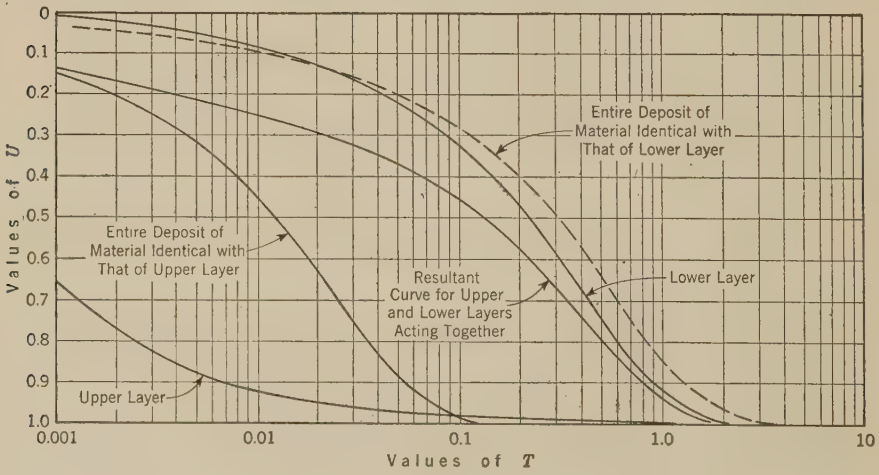
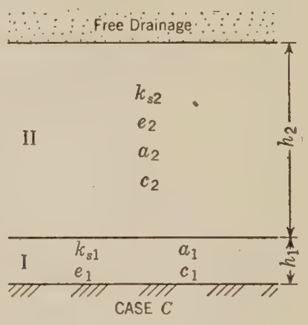
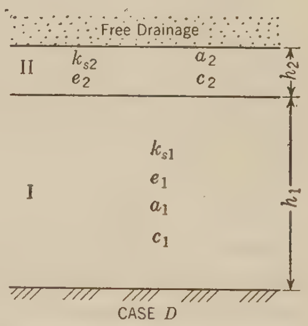


Fig. 15

is greater for case C than for case D—in case C consolidation of layer I is retarded by layer II as indicated in Fig. 7, whereas in case D layer II drains relatively rapidly and in addition exerts only a minor influence on layer I. If the thinner layer in each case is neglected, a moderately close approximation to the actual behavior of the two layers is obtained. This phenomenon seems to be due to the great difference in the consolidation characteristics of the two layers. In cases A and B discussed by Mr. Barber, layers I and II would each consolidate at the same rate if they were not contiguous, since the ratio, t/T, is the same for each. Consequently, ignoring one or the other of these layers will not give a close approximation of the exact behavior when the layers are contiguous.

TABLE 8.—TYPICAL VALUES OF PHYSICAL CONSTANTS

Description	Case C	Case D	Key
Layer II—			
k_{s2} (cm per sec)	1×10^{-8}	1×10^{-7}	 Free Drainage k_{s2} e_2 a_2 c_2 II h_2 I k_{s1} e_1 a_1 c_1 h_1 CASE C
h_2 (ft)	20	5	
e_2	1.5	1.5	
a_2 (cm² per g)	5×10^{-4}	2×10^{-4}	
c_2 (cm² per sec)	2×10^{-5}	5×10^{-4}	
Layer I—			
k_{s1} (cm per sec)	1×10^{-7}	1×10^{-8}	
h_1 (ft)	5	20	
e_1	1.5	1.5	
a_1 (cm² per g)	2×10^{-4}	5×10^{-4}	
c_1 (cm² per sec)	5×10^{-4}	2×10^{-5}	
Ratios—			 Free Drainage II k_{s2} e_2 a_2 c_2 h_2 k_{s1} e_1 a_1 c_1 I h_1 CASE D
$\nu = h_2/h_1$	4.0	0.25	
k_{s1}/k_{s2}	10.0	0.10	
a_1/a_2	0.40	2.5	
c_1/c_2	25	0.04	
μ	5.0	0.20	
$T = t\, c_1/(h_1)^2$ (yr)	0.68 t	0.0017 t	
t	1.47 T	590 T	
t_{50} (yr)	138	103	
σ	2.0	0.5	

As Mr. Glick states, when $\sigma = 1$ it can be shown that, if the thickness, h_2, of layer II be multiplied by $\mu = \sqrt{\frac{c_1}{c_2}}$ so as to yield $(h_2)' = \mu\, \nu\, h_1$, it is possible to obtain the following expressions for the resultant consolidation:

$$U_r = \frac{U_1 + \mu\,\nu\,U_2}{1 + \mu\,\nu} = 1 - \frac{8}{\pi^2} \sum_{n=0}^{n=\infty} \frac{1}{(2\,n + 1)^2}\, \epsilon^{-N'T'} \quad \ldots\ldots\ldots (44a)$$

and

$$U_r = \frac{U_1 + \mu\,\nu\,U_2}{1 + \mu\,\nu} = 1 - \frac{8}{\pi^2} \sum_{n=0}^{n=\infty} \frac{1}{(2\,n + 1)^2}\, \epsilon^{-N'''T'} \quad \ldots\ldots\ldots (44b)$$

in which

$$N'''_{\,} = (2\,n + 1)^2\,\pi^2 \qquad (45a)$$

and

$$T' = \frac{c_1\,t}{(1 + \mu\,\nu)^2\,(h_1)^2} \qquad (45b)$$

and N' is given by Eq. 14. Since the right-hand sides of Eq. 44*a* and Eq. 44*b*, respectively, are identical with the expressions for the degree of consolidation of homogeneous layers (thickness, $h_1\,(1 + \mu\,\nu)$, consolidation coefficient, c_1, and drained, respectively, at one face and at both faces), the transformation, $z' = \mu\,z$, applied to dimensions, Z, in layer II makes it possible to consider the two layers as a single homogeneous stratum. When μ is equal to unity, there does not appear to be any analogous transformation that may lead to a simplification of the general problem.

For practical applications, the various approximate solutions or modifications outlined by Messrs. Barber and Glick are very valuable, provided one knows the extent to which the approximate results differ from those which are theoretically exact. In some cases two or more approximate approaches are suggested, and it then becomes important to know which method is best—from the standpoint of accuracy as well as from that of time. In many cases an inspection of the relative consolidation characteristics of the pertinent strata will give some indication of the reliability to be expected from one or another of the suggested methods of approach.

Mr. Feld has described numerous cases, both natural and man made, to which the methods outlined in the paper may be applied. It is of great help to the engineer to be able to analyze actual field conditions in terms of an idealized situation, especially if, at the same time, he does not lose sight of the differences between actual conditions and assumed conditions. Mr. Feld emphasizes this fact, which the writer had in mind when he suggested that the results obtained in the paper should be verified by observations before being used with confidence. In many instances it is possible to explain discrepancies between a theoretical approach and the observed behavior on the basis of differences between actual and assumed conditions. The writer would disagree with one of Mr. Feld's statements since he believes that the influence of surface evaporation upon the consolidation of a soil deposit can be analyzed by appropriate methods.

The writer wishes to express his thanks to the discussers for their worthwhile comments on this paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2259

LOCK MANIFOLD EXPERIMENTS

BY EDWARD SOUCEK,[1] ASSOC. M. AM. SOC. C. E., AND
E. W. ZELNICK,[2] JUN. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. J. PIETRKOWSKI, JOHN S. MCNOWN, F. W. EDWARDS, W. M. LANSFORD, ROBERT F. KREISS, RALPH G. BERK, RALPH W. POWELL, AND EDWARD SOUCEK AND E. W. ZELNICK.

SYNOPSIS

In connection with the design of the Third Locks of the Panama Canal, a series of experiments dealing with the hydraulics of manifolds was performed in The Panama Canal Hydraulics Laboratory during 1939 and 1940. This paper presents a summary of the test results. Certain introductory and theoretical material which appeared desirable has been included. A method of predicting the performance of a manifold by the use of single port test data is presented and the method is checked by comparison with manifold tests. Some of the data are presented in abridged form but transcripts of original and more complete records have been filed with the Engineering Societies Library.

INTRODUCTION

Manifolds are important elements in the design of side-filling or bottom-filling locks. There is no generally accepted basis for their design, however. Discussions of a paper presented in 1940 by R. D. Gladding,[3] M. Am. Soc. C. E., and the absence of the subject from hydraulics textbooks are noteworthy in this connection. With few exceptions model tests used in the design of most recently built locks have been limited[4] to models of the entire lock. Consequently, there is little basis for designing the hydraulic system of locks that must meet special requirements or operate under unusual conditions. The size and importance of the new Panama Canal locks seemed to justify more fundamental treatment. The tests described herein were adopted to this end.

NOTE.—Published in October, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Washington, D. C.

[2] With The Panama Canal, Balboa Heights, Canal Zone.

[3] "Loss-of-Head Determination in Uniformly Tapped Pipes," by R. D. Gladding, *Engineering News-Record*, November 21, 1940, p. 697; also discussions by F. W. Edwards, F. C. Zeigler, W. E. Howland, R. W. Powell, W. M. Langsford, and the author, *ibid.*, January 16, 1941, p. 78; April 10, 1941, p. 522; and May 22, 1941, p. 801.

[4] "The Manifold Problem in Lock Design," by Martin E. Nelson, *Bulletin 20, Proceedings of Hydraulics Conference*, Studies in Engineering, Univ. of Iowa, Iowa City, March, 1940, p. 160.

It is not easy to predict the time required to fill or empty a lock through a manifold system. It is customary to use a differential equation based on the orifice formula with an estimated coefficient, to compute the time required to fill or empty a lock under a falling head. Owing in part to complications resulting from inertial effects and to difficulty in allowing for time consumed by valve operations, but principally to lack of a satisfactory theory of manifold hydraulics, these attempts at simplification often have been unsuccessful.

Inequality in port discharges tends to defeat the purpose of a lock manifold.[5] These differences in discharge are produced by changes in pressure and velocity along the manifold tube. Elimination of the differences, or compensation for the differences, is possible by various methods, discussion of which is beyond the scope of this paper. It is evident that the use of such expedients as tapered culverts, varied port sizes, and varied port spacing is highly speculative without a specific model test of each design or some knowledge of what takes place within a manifold.

Purpose

The purpose of the tests was to provide data that would expedite the design of lock manifolds. Specifically, it was desired to give mathematical expression to the phenomenon of manifold flow in order to facilitate the original design and the modification of model manifolds. Since a single port can be built and tested quite easily and quickly, determination of a means of predicting the performance of a manifold on the basis of single port tests and verification of the technique by tests of a manifold were the principal phases of the program.

Outline

The first part of the study, "Theoretical Considerations," describes the mathematical approach adopted for the analysis. It is perhaps regrettable that several simplifying assumptions were necessary in order to obtain tangible results without unduly extending the test program. Substantial correctness for design purposes rather than precision was the reason for simplifying the theory to the greatest possible extent. Since disturbances within a lock chamber during emptying operations are usually (perhaps always) negligible in comparison with those that occur during filling, more attention was given to the discharge ports. The difference between filling and emptying conditions is due to the lower head-to-depth ratio which prevails during emptying operations and to the absence of what might be called jet action. The basic discussions deal with what will be called "discharge manifolds," "intake manifolds" being covered by brief supplementary statements.

The second part of the study, "Single Port Tests," covers the results of tests of seven different ports, each attached to a 6-in. square conduit. The ports were tested as both "discharge ports" and as "intake ports."

The third and final part of the study, "Verification Tests," describes tests of a manifold containing five identical ports which were duplicates of one of those tested as a single port. A 6-in. square conduit again served as the manifold tube. Observations are compared with calculations based on single port characteristics which had been determined previously.

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[5] "Manual on Lock Valves," *Manual of Engineering Practice No. 3*, Committee on Lock Valves, Waterways Div., Am. Soc. C. E., 1930.

NOTATION

The following letter symbols, adopted for use in this paper, conform essentially to American Standard Letter Symbols for Hydraulics,[6] prepared by a Committee of the American Standards Association, with Society participation, and approved by the Association in 1942:

A = area:
  A_c = cross-sectional area of conduit, in square feet;
  A_p = throat area of the port, in square feet;
C = discharge coefficient in orifice formula with subscript d or i denoting discharge ports or inlet ports, respectively;
g = gravitational acceleration, in feet per second per second;
H = effective head on a given port, in feet, with subscript d or i denoting discharge ports or inlet ports, respectively;
k = a dimensionless coefficient to be studied experimentally, with subscript d or i denoting discharge ports or inlet ports, respectively;
n = Manning's roughness coefficient;
P = a ratio of discharges:
  $P_d = Q_3/Q_1$;
  $P_i = Q_3/Q_2$;
Q = discharge, in cubic feet per second:
  Q_1, in the conduit upstream from a port;
  Q_2, in the conduit downstream from a port;
  Q_3, through a port;
V = mean conduit velocity, in feet per second:
  V_1, upstream from port;
  V_2, downstream from port;
Z = elevation of the hydraulic gradient in feet; Z_1 = upstream elevation and Z_2 = downstream elevation; ΔZ = difference between upstream and downstream elevations;
γ = specific weight of water, in pounds per cubic foot.

THEORETICAL CONSIDERATIONS

It is evident that any fundamental study of the hydraulics of manifolds must be based upon an analysis of the behavior and effects of a single lateral port attached to a tube. Preliminary study made it clear that two problems were of primary importance—namely, (*a*) the pressure change which occurs in a tube in the vicinity of a lateral port and (*b*) the discharge through a lateral port and the factors that modify it. Solution of these two problems permits analysis of the hydraulics of a manifold by a simple computation procedure which resembles somewhat the calculation of a backwater curve. Mutual interference between closely-spaced consecutive ports is the principal limitation on the applicability of this procedure which otherwise appears quite general.

(*a*) *Pressure Changes.*—Consider a horizontal frictionless tube to which a discharging lateral port is attached. The port outlet is submerged in still water. The elevation of the water surface over the port outlet is called the

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[6] ASA—Z10.2—1942.

"submergence level." The tube is connected to a source of supply which is at some elevation greater than the submergence level. Downstream from the port, the tube is throttled in such a manner that the hydraulic gradient past the port is above the submergence level. The port is of such a length that the discharge through it occurs substantially at right angles to the axis of the manifold tube. Since the tube is frictionless, the pressure gradients both upstream and downstream from the port are horizontal but at different elevations. In the immediate vicinity of the port, there is an abrupt pressure rise in the direction of flow.

Consider the rate of change of momentum that occurs between two sections on the horizontal pressure gradients upstream and downstream, respectively, from the port. Neglecting nonuniform velocity distribution, the rate of change of momentum has the value $\frac{Q_2}{g} \gamma (V_1 - V_2) + \frac{Q_3}{g} \gamma V_1$. The forces producing this effect are the pressure rise in the conduit and the resultant of unbalanced pressures within the port. The latter force is unknown and undeterminable in any practical manner except as a residual. The unbalanced port pressure is intimately related to the momentum change of Q_3 and, therefore, it seems likely that any allowance for its effect can be applied to the term representing that momentum change.

It is considered that k_d is that fraction of the momentum change of Q_3 which is produced by a pressure rise in the conduit. Conversely, $(1 - k_d)$ is that fraction of the momentum change of Q_3 which is produced by unbalanced pressure within the port. This line of reasoning leads to an expression involving the pressure rise, an equation which is not obtainable without some simplifying assumption similar to the one which has been explained. The equation obtained is:

$$\gamma A_c (Z_2 - Z_1) = \frac{Q_2}{g} \gamma (V_1 - V_2) + \frac{k_d (Q_3 \gamma V_1)}{g} \dots\dots\dots (1)$$

By use of the law of continuity and some algebraic manipulation, Eq. 1 may be reduced to the form,

$$\frac{\Delta Z}{(V_1)^2/2 g} = 2 P_d (1 + k_d) - 2 (P_d)^2 \dots\dots\dots\dots\dots (2)$$

Certain published data indicate that, for a given port and conduit, the ratio on the left side of Eq. 2 is a function of P_d only. For this to be true, k_d must be either a constant or a function of P_d only. Fig. 1(*a*) is a comparison of tests by the U. S. Bureau of Reclamation[7] as interpreted by R. W. Powell,[8] M. Am. Soc. C. E., tests by John A. Oakey,[9] Assoc. M. Am. Soc. C. E., and the conclusions, supported by tests, of M. L. Enger, M. Am. Soc. C. E., and M. I. Levy.[10] The approximate diameter ratios, lateral to conduit, for the

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[7] "Model Studies of Penstocks and Outlet Works," *Bulletin 2*, Pt. VI, Boulder Canyon Project Final Repts., Bureau of Reclamation, U. S. Dept. of the Interior, 1938.

[8] "Hydraulics of Sprinkling Systems for Irrigation," by J. E. Christiansen, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 221.

[9] "Hydraulic Losses in Short Tubes Determined by Experiments," by John A. Oakey, *Engineering News-Record*, June 1, 1933, p. 717.

[10] "Pressures in Manifold Pipes," by M. L. Enger and M. I. Levy, *Proceedings*, A.W.W.A., May, 1929, p. 659.

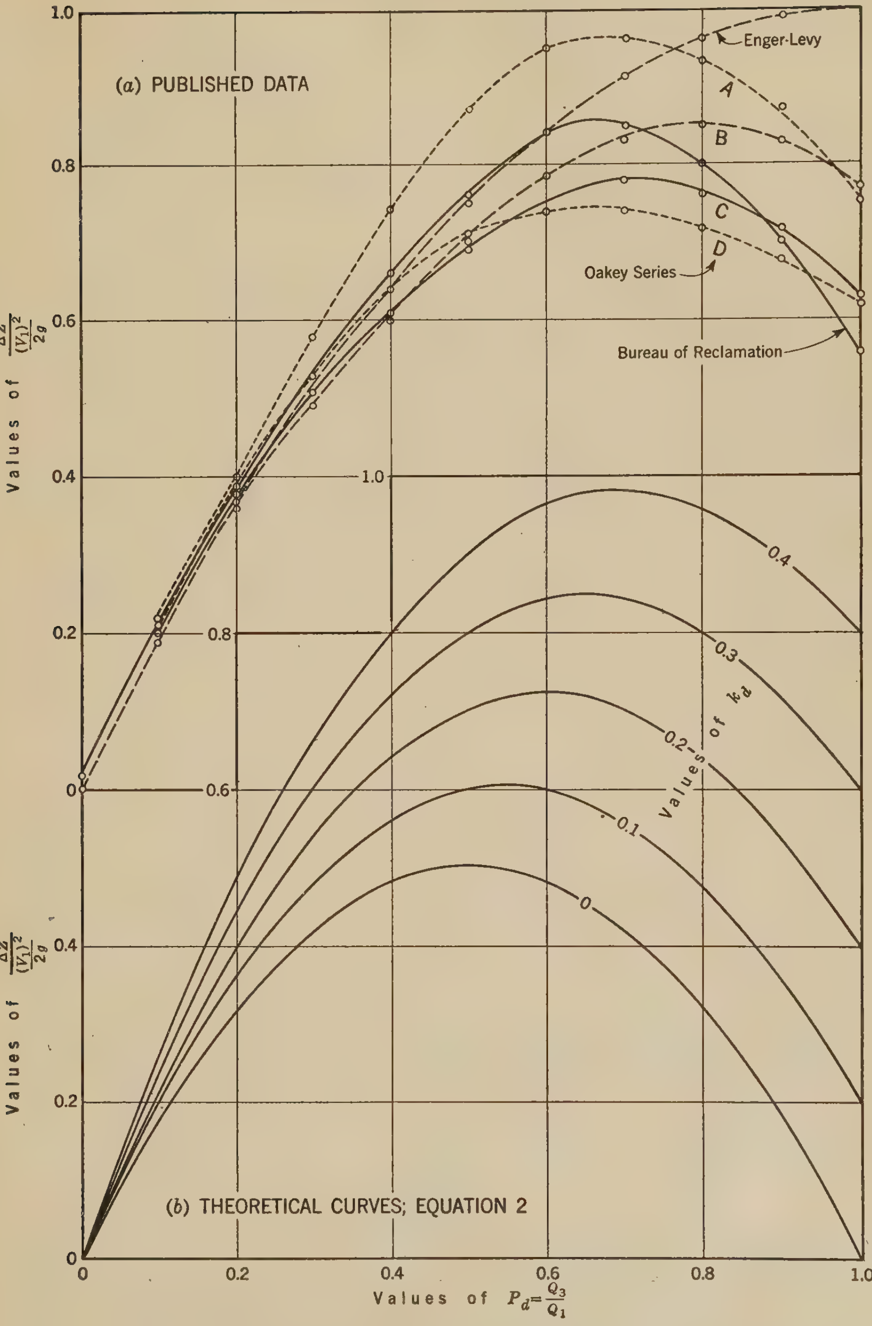


FIG. 1.—PRESSURE RISE; THEORETICAL EQUATIONS AND OBSERVED DATA

tests in Fig. 1(a) are as follows:

Series (see Fig. 1)	Diameter ratios
A	1:4.24
B	1:2.82
C	1:1.82
D	1:1.21
Enger and Levy	1:5.3
Bureau of Reclamation	1:2.3

In Fig. 1(b) are plotted graphs of Eq. 2 with various constant values assigned to k_d. It is evident that this comparison tends to support the analysis.

The published data exhibit a maximum relative pressure rise in the vicinity of P_d equal to 0.7, corresponding in that respect to the theoretical curve for k_d equal to 0.4. However, the relative pressure rises are of lower magnitude than those indicated by the theoretical curve for k_d equal to 0.4.

It should be noted that the graphs of relative pressure rise plotted against P_d are simply a convenient means of evaluating the effect of a residual unknown —the resultant of unbalanced port pressures. The graphical method could have been evolved by some other line of reasoning. However, the method appears to represent an approximately correct interpretation of the mechanics of pressure changes in manifolds. Of course, the effects of nonuniform approach velocity distribution and of angularity of port discharge are reflected in the relations between the relative pressure rise and P_d.

Since no real conduit is frictionless, a method had to be devised for measuring the pressure rise for a real manifold so as to minimize the effect of friction. The gradients in a real manifold are not level but, beyond the immediate effects of the port, they are straight lines. In analysis and application of pressure rise data, it is considered that the pressure change is sudden and occurs opposite the port center line. In practice then, the upstream gradient is extended downstream and the downstream gradient is extended upstream, both at their respective friction slopes, until they intersect a vertical erected at the center of the port. The intercept on this vertical, between the extended gradients, is considered to represent the pressure rise. As a matter of fact, the pressure rise is exceedingly abrupt. The described method of interpreting the pressure rise makes the calculated gradient for a manifold consist of a series of sloping lines with a step at each port. Fig. 2 illustrates these relations, the discharges, in cubic feet per second, being as follows:

Flow	Discharge Port	Intake Port
Q_1	2.631	1.844
Q_2	2.222	2.245
Q_3	0.409	0.401

An intake port is analyzed similarly. For an intake port, the elevation of the water surface over the port inlet is called the "submergence level." The conduit pressure gradient is below, instead of above, the submergence level. The port discharges into the conduit rather than out of it. A drop in pressure, in place of a rise, occurs in the direction of flow. The result of reasoning identical to that used for a discharge port is a similar equation,

$$\frac{\Delta Z}{(V_2)^2/2\,g} = 2\,P_i\,(1 + k_i) - 2\,(P_i)^2 \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

It is evident from physical considerations that k_i will be larger than k_d. Since water enters the port from a quiescent pool, the unbalanced port pressure is much smaller in an intake port. Accordingly, the pressure change in the conduit (in this case a drop) must account for substantially all the momentum change.

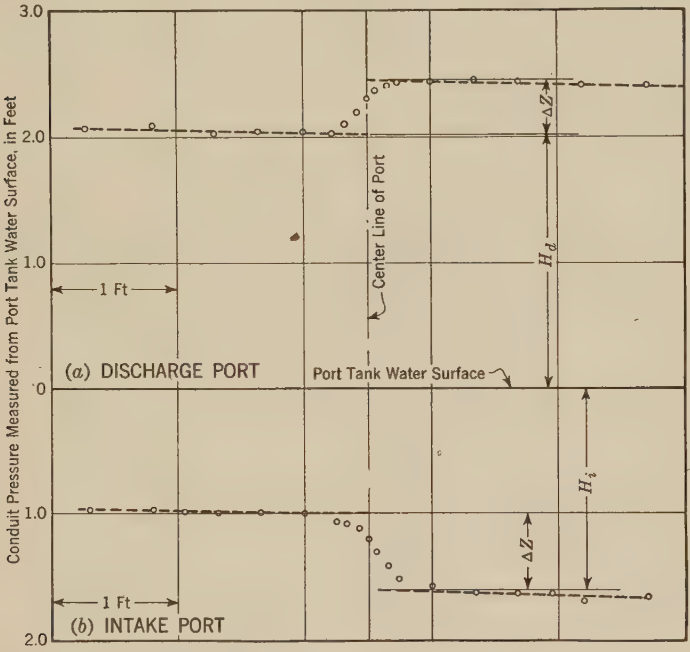
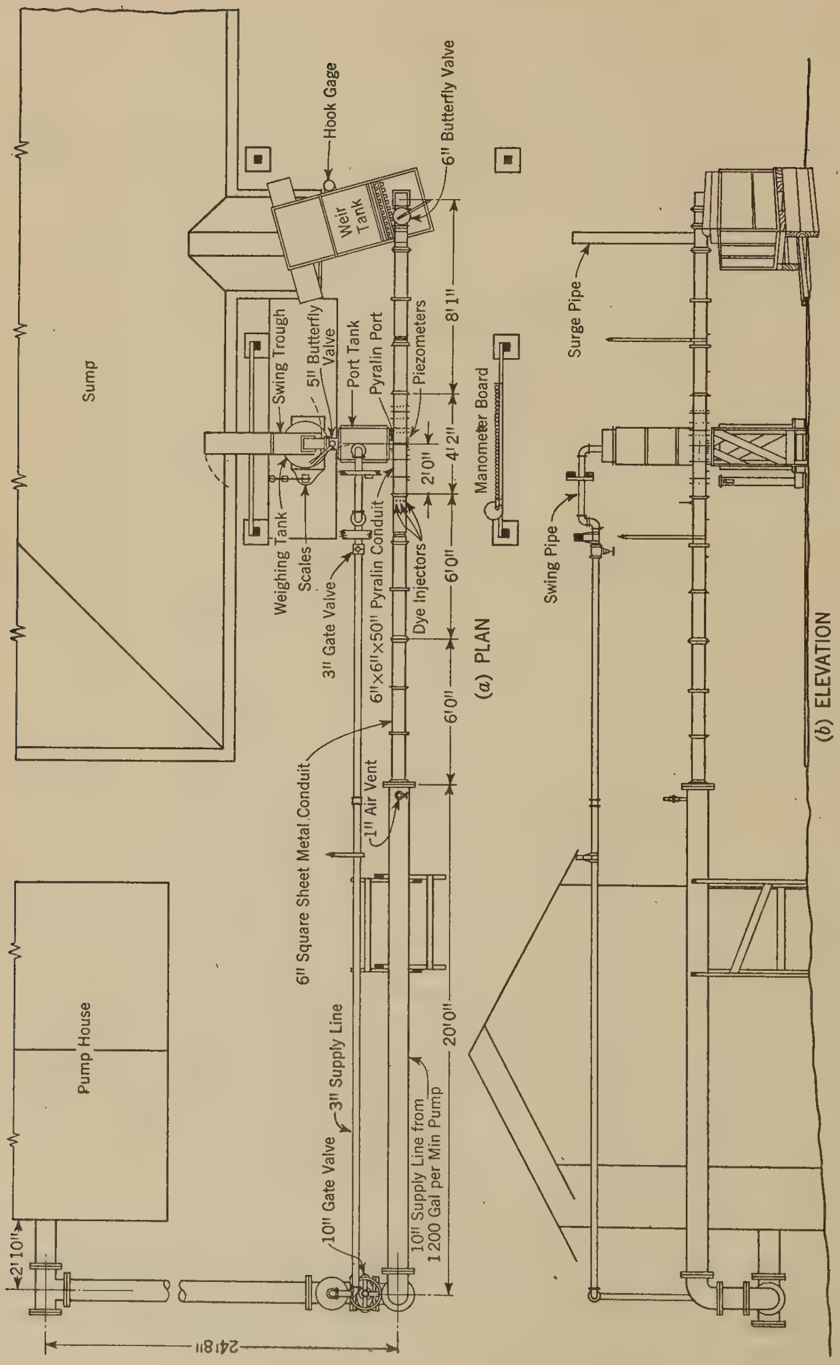


FIG. 2.—TYPICAL GRADIENTS; SINGLE PORT TESTS

Certain similarities exist between the equations derived and the formulas used for obtaining gradients in side channel spillways. If k_d in Eq. 2 or k_i in Eq. 3 is taken as unity (this is equivalent to considering that the unbalanced port pressure is zero), the value of pressure change obtained—actual, not relative—will equal twice the difference between the velocity heads upstream and downstream from the port. It is evident, of course, that Bernoulli's theorem is not applicable to the phenomenon under consideration, although a modified energy equation might be used.

(b) *Port Discharge.*—After considering various methods, it was decided to attempt to relate the port discharge to some head measurement by the use of the simple orifice equation ($Q_3 = C\,A_p\,\sqrt{2\,g\,H}$), and then to study variations in the resulting "discharge coefficient." Although it could not have been predicted definitely that this approach would be successful, study of published data[4,9,10] indicated that it was promising.


Sump
Hook Gage
6" Butterfly Valve
Weir Tank
Swing Trough
5" Butterfly Valve
Port Tank
Pyralin Port
Piezometers
8'1"
4'2"
2'0"
Manometer Board
Weighing Tank
Scales
3" Gate Valve
6"×6"×50" Pyralin Conduit
Dye Injectors
6'0"
6'0"
6" Square Sheet Metal Conduit
1" Air Vent
Pump House
3" Supply Line
10" Gate Valve
10" Supply Line from 1200 Gal per Min Pump
20'0"
2'10"
24'8"
Surge Pipe
Swing Pipe

(a) PLAN

(b) ELEVATION

Fig. 3.—Single Port Test Apparatus

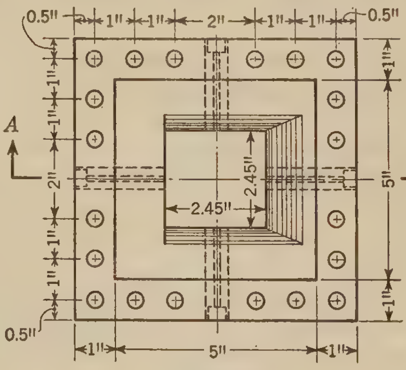
The hydraulic gradient in the vicinity of a port changes so rapidly that there is no natural method of defining the "head on the port." Since the choice is arbitrary, the head is defined in a manner which seems definite, convenient, and logical. The elevation difference between the submergence level and the intersection of the upstream gradient with a vertical line erected opposite the center of the port (the same intersection previously used in evaluating pressure change) is defined as the effective head, H_d, on a discharge port. This relationship is illustrated in Fig. 2.

For an intake port, the effective head, H_i, is defined as the elevation difference between the submergence level and the intersection of the downstream gradient with the intersection of a vertical line erected opposite the center of the port. This relationship is also shown in Fig. 2.

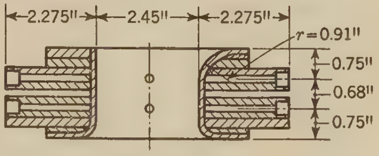
It might appear desirable to add the head corresponding to the mean velocity of approach to the effective head for a discharge port and to make a corresponding adjustment to the effective head for an intake port. This was not done for several reasons. The method of measuring the effective head and the discharge equations are rather arbitrary so it seemed desirable to use simple definitions. The principal effect of high conduit velocity approaching the port, considering now a discharge port, would appear to be such as to reduce the port discharge. Accordingly, it seemed illogical to insert a velocity term into the discharge equation in a manner that would indicate the contrary. The effect of conduit velocity is considered separately in studying the variation in discharge coefficients. Certain minor complications which are due to the manner in which the effective head is defined are discussed in another section of the paper.

SINGLE PORT TESTS

Apparatus.—The arrangement of test apparatus is shown in Fig. 3. The equipment was designed to simulate the range of heads, velocities, and port and culvert areas which ordinarily occur in lock manifolds. Port VII, shown in Fig. 4, was placed in the conduit with square corner vertical and the large radius opposite the square corner on the upstream side of the port for the discharge tests. The large radius was on the downstream side of the port for the intake tests. All verification tests were made with Port VII. Radii that are not shown in Fig. 4 are 0.36 in.



ELEVATION



SECTION *A-A*

Fig. 4.—Port VII

Ports I to VI, inclusive (not illustrated), had the dimensions shown in Table 1. The ports were equipped with flanges that fitted into openings in the conduit and port tank, so arranged that the radii of the rounded ports were tangential to the tank and conduit faces; also, the faces of the sharp-cornered ports were in the planes of the tank and conduit faces.

TABLE 1.—DIMENSIONS OF PORTS I TO VI

Port	Length (in.)	Height and width at throat (in.)	Top and bottom at ends of port	Sides at ends of port
I	3.50	3.50	Square	Square
II	3.50	2.75	Square	Square
III	3.50	2.00	Square	Square
IV	3.50	3.50	0.75[a]	1.25[a]
V	3.50	2.75	0.59[a]	0.98[a]
VI	3.50	2.00	0.43[a]	0.71[a]

[a] Radius of edge fillet, in inches.

In order to obtain parallel flow upstream from the port, a set of vanes was placed in the conduit as shown in Fig. 3. To simulate a discharge port, all the flow was supplied by the 10-in. line. To simulate an intake port, the port discharge was supplied by the 3-in. auxiliary line. Discharges and pressures were controlled by the valve on the 10-in. pipe, the valve on the 3-in. swing pipe, the butterfly valve on the port tank outlet, and the butterfly valve at the end of the conduit.

Particular care was taken in the construction of the pyralin conduit to prevent bulging under pressure. All piezometers except those in the port tank were placed in the pyralin sections where greater care in installation and maintenance was possible.

Flow patterns were observed by injecting dye through the piezometer connections. For some of the tests, velocities at various cross sections were obtained with a pitot tube, inserted through special openings provided for that purpose.

The level in the port tank was determined from the piezometers connected to the bottom of the tank at several points. The baffle shown in Fig. 5 was placed in the port tank to prevent excessive turbulence and entrainment of air during the intake tests. Port discharge measurements were made by diverting the flow into a 50-gal weighing tank. Conduit discharges were measured with a calibrated 90° V-notch weir.

Test Procedure.—For each port, intake and discharge tests were made with effective heads, previously defined, ranging from 0.3 to 3.0 ft. In addition to the variations in head, the conduit discharges were varied from zero to 2.5 cu ft per sec downstream from the port.

The hydraulic gradient in the culvert, the level in the tank, and the pressures in the port were obtained by reading forty-five manometers ten times during the course of each test and averaging the results for each manometer. In discharge port tests, discharge measurements for the port and the culvert were made at the beginning of each test, during the test, and at the conclusion of the test. For the intake port tests, discharge measurements were made

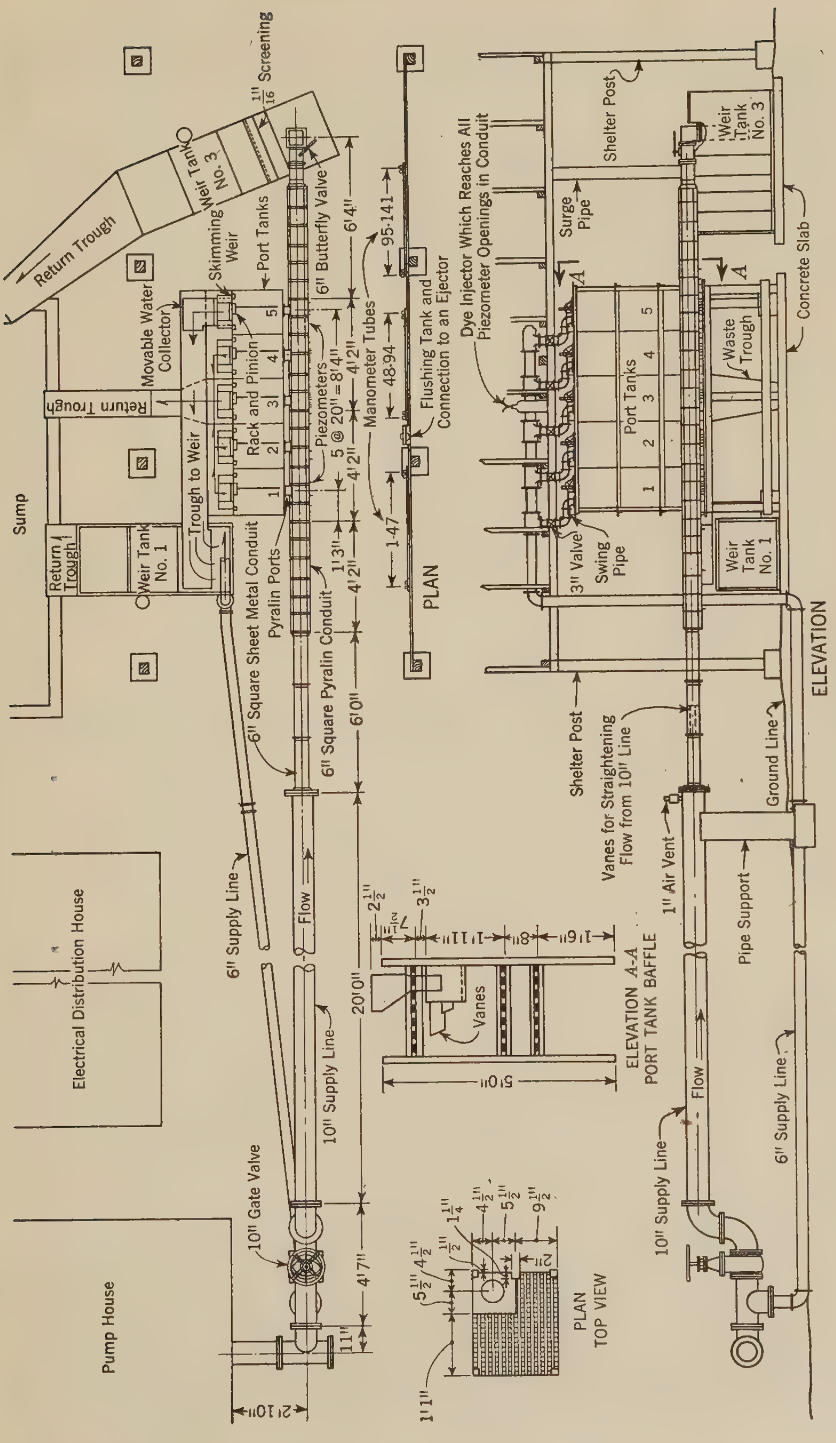

Pump House
Electrical Distribution House
Sump
Return Trough
Return Trough
Return Trough
Weir Tank No. 1
Weir Tank No. 3
Trough to Weir
Movable Water Collector
Skimming Weir
Rack and Pinion
Port Tanks
1
2
3
4
5
$\frac{1}{16}$" Screening
6" Square Sheet Metal Conduit
Pyralin Ports
6" Butterfly Valve
Piezometers
5 @ 20" = 8'4"
6" Square Pyralin Conduit
6" Supply Line
10" Supply Line
10" Gate Valve
Flow
2'10"
11"
4'7"
20'0"
6'0"
4'2"
1'3"
4'2"
4'2"
6'4"
Manometer Tubes
1·47
48·94
95·141
Flushing Tank and Connection to an Ejector
PLAN
$2\frac{1}{2}$"
$3\frac{1}{2}$"
$\frac{1}{2}$"
1'11"
8"
1'9"
5'0"
Vanes
ELEVATION A-A
PORT TANK BAFFLE
$5\frac{1}{2}$"
$4\frac{1}{2}$"
$\frac{1}{2}$"
$1\frac{1}{4}$"
$4\frac{1}{2}$"
$5\frac{1}{2}$"
$9\frac{1}{2}$"
$\frac{1}{2}$"
1'1"
PLAN
TOP VIEW
Dye Injector Which Reaches All Piezometer Openings in Conduit
Shelter Post
Shelter Post
Surge Pipe
A
A
Port Tanks
1
2
3
4
5
3" Valve
Swing Pipe
Weir Tank No. 1
Weir Tank No. 3
Waste Trough
Concrete Slab
Vanes for Straightening Flow from 10" Line
1" Air Vent
Ground Line
Pipe Support
10" Supply Line
Flow
6" Supply Line
ELEVATION

Fig. 5.—Manifold Test Apparatus

only at the beginning and at the end of each test, since the port discharge had to be diverted for measurement, thus disturbing the test conditions.

Several tests were made at various times to determine and check the roughness of the pyralin conduit. With undisturbed flow, friction slopes could be approximated closely by using an n of 0.009 in Manning's formula.

Analysis.—It proved possible to reduce the data, along the lines suggested under "Theoretical Considerations," to four characteristic curves for each port. Fig. 6 shows the characteristic curves for Ports I to VI, inclusive. To save space, these curves are shown to a very small scale and observation points are omitted. The complete data for Ports I to VII, to be used by interested investigators in plotting the complete set, are filed with the Engineering Societies Library. Fig. 7 shows the characteristic curves for Port VII to a larger scale with observations indicated. The curves for Port VII are indicative of the agreement between curves and observation points for Ports I to VI, inclusive. Since each observation point represents a ratio of two observed quantities—some of which are very small—considerable "scatter" is to be expected.

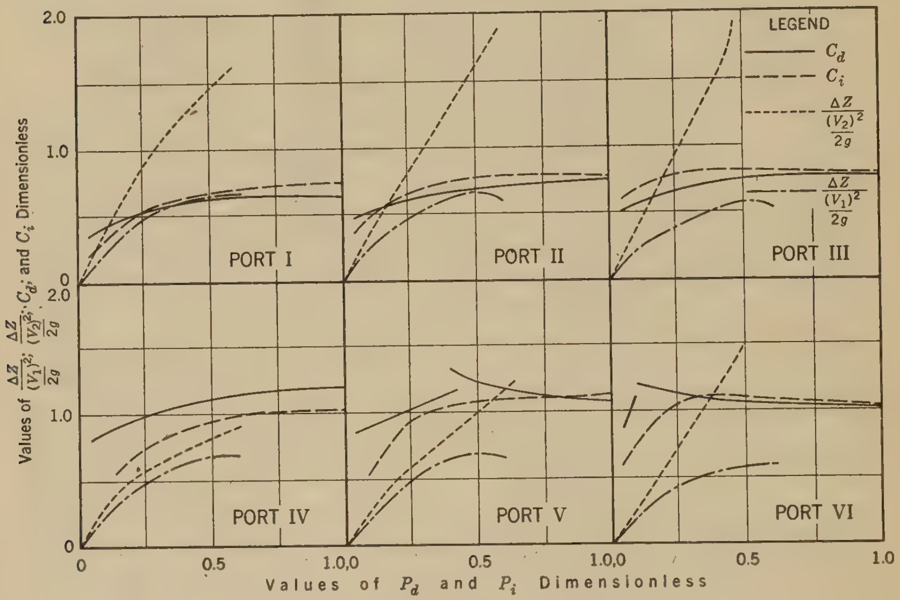


FIG. 6.—PORT CHARACTERISTICS, PORTS I TO VI, INCLUSIVE

This method of analysis was adopted for a number of reasons. The tentative nature of the theory did not justify a complex treatment. The relative pressure changes and discharge coefficients showed no significant dependence upon variables other than P_d and P_i. The time available for testing was limited; the need for tangible results great. The amount of data required for the method of analysis adopted was relatively small. The range of conditions under which lock manifolds operate makes certain compromises

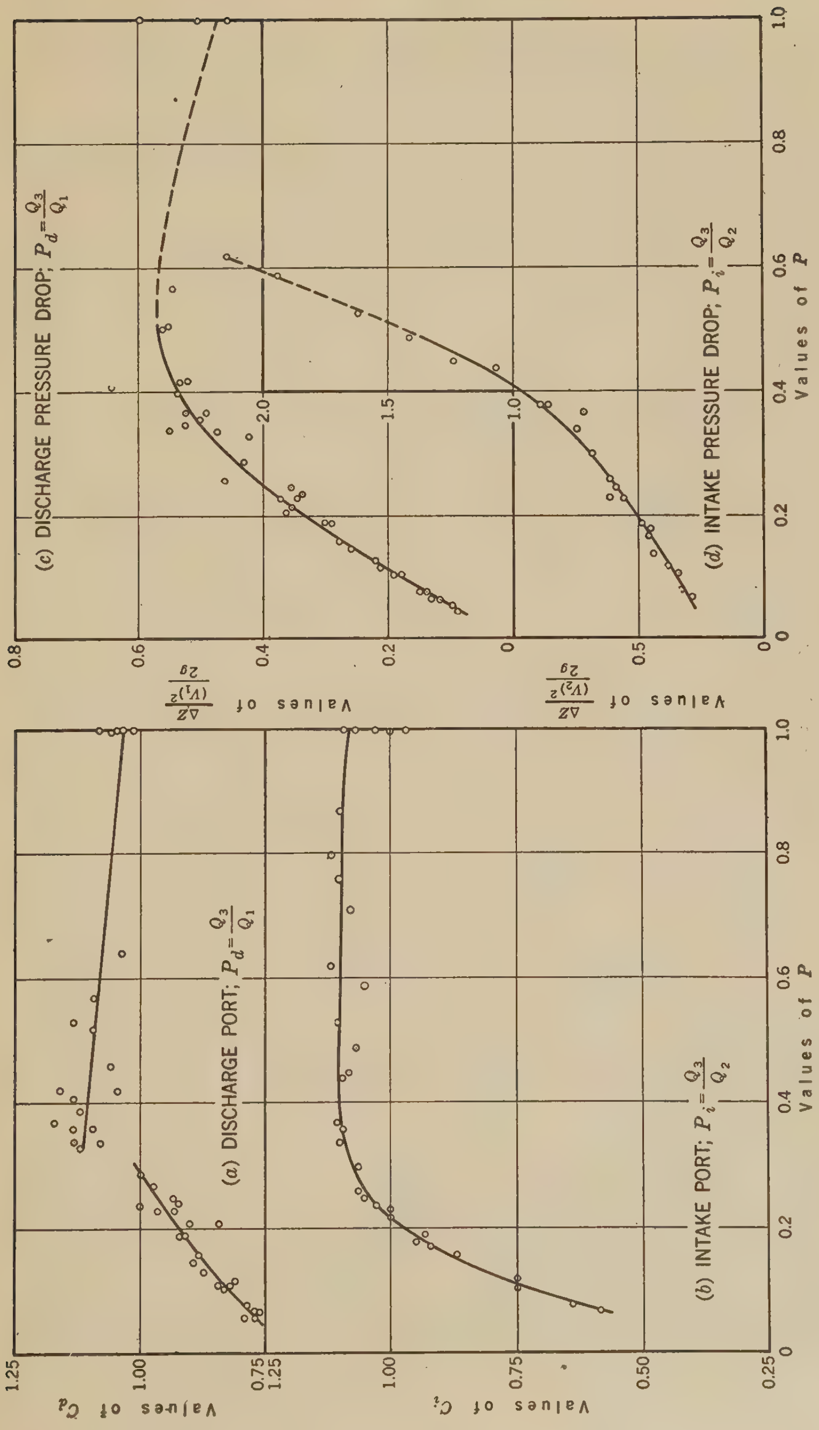


Fig. 7.—Port Characteristics, Port VII

in design inevitable. Inertial effects make real precision nearly impossible to attain. The dimensionless treatment makes it possible to solve many problems by proportion and simplifies model-prototype ratios. Finally, and most important, there is a reasonable physical basis for the relations used. Similar flow patterns and, therefore, similar relations between physical quantities might be expected to prevail at given values of P_d and P_i for a particular port, regardless of the absolute magnitude of the quantities involved.

The port discharge is a factor that occurs to the first power in both coordinates of the coefficient curves. Therefore, some other combination of variables, omitting the port discharge as a factor, could have been used. This apparent simplification did not prove advantageous so the more significant combinations were retained.

Discussion of Results.—The limitations of the apparatus made it impossible to obtain large values of ΔZ simultaneously with large values of P_d or P_i. Only the farthest downstream port in a discharge manifold and the farthest upstream port in an intake manifold have values of P_d and P_i, respectively, exceeding about 0.5. The pressure in the closed end is not required in analysis. It was not justifiable, therefore, to build the apparatus so as to reduce observational error in the pressure change when P_d or P_i exceeded about 0.5. The observed values in the unreliable range are only a few hundredths of a foot in magnitude and large percentage errors are inevitable. The use of ratios in the graphs does not show the absolute magnitude of the pressure changes and, when the pressure change itself is small, a very small observational error produces a large effect on the relative pressure change. For this reason, the pressure-change curves are not considered to be established beyond P_d and P_i exceeding 0.6. However, the data filed with the Engineering Societies Library show the observations beyond the range of the curves.

The rounded ports, particularly the smaller sizes, exhibit a discontinuity in the relations between C_d and P_d. The largest rounded port (Port IV) probably has a similar but minor discontinuity at a value of P_d equal to about 0.75. From observation of dye streams, the discontinuity is believed to mark the value of P_d at which a change in the character of the flow through the discharge port occurs. An eddy which existed along the upstream side of the port for values of P_d less than that corresponding to the discontinuity was not evident for higher values. Tests in the vicinity of the discontinuity were difficult to establish and conditions tended to be unstable. Fortunately, the magnitude of the break in the relation was not sufficient to produce serious error in the calculations.

With the exceptions to which attention has been called, the characteristics of the data are fairly consistent and regular. Although the tests are evenly divided between effective heads of 0.3, 0.6, 1.0, 1.5, 2.0, 2.5, and 3.0 ft, the data for each port were found to be represented with fair accuracy by the dimensionless curves. The effect of rounded port entrances is evident and as expected. The effect of port size or, by inference, port area to culvert area ratio is reasonably well established over a large range and probably can be extrapolated somewhat.

Limitations.—Before explaining the application of the data to manifold calculations, it seems desirable to call attention specifically to certain limitations. All characteristic curves for the seven ports are based on single port observations. Approach conditions were normal for a smooth conduit. Downstream from the port, the velocity distribution was irregular. Although the pressure change was abrupt, as shown by the typical gradients of Fig. 2, the velocity distribution remains irregular for some distance downstream from a port. The slope of the hydraulic gradient was found, in the single port tests, to reach a value which was normal for the mean velocity within a very short distance from the port. This condition does not necessarily indicate a normal velocity distribution. A gradual reduction in the velocity head coefficient might obscure a tendency toward a greater than normal slope produced by the abnormal flow pattern. Although it is frequently assumed that the excess energy due to a distorted velocity distribution is not recoverable, the implication of a normal friction slope cannot be accepted without reservations. Accordingly, although it is believed that the observed pressure changes correctly represent the displacement between upstream and downstream gradients for values of P_d and P_i below about 0.5, it is not claimed that the disturbed flow pattern may not affect the performance of an adjacent port. On the other hand, the relatively small rate of change in discharge coefficient with P_d or P_i indicates that the effect of velocity distribution upon port discharges will not be large.

It has been observed that some discharge may occur through a port when the head, as defined, is negative. Negative heads could have been avoided by introducing a velocity head term into the effective head and this may ultimately prove to be the best solution. The discharges accompanying negative heads, as herein defined, tend to be irregular and of small magnitude. In general, it is felt that this condition can and should be avoided in design. For very low values of P_d and P_i, however—below the range covered by the tests—the method of analysis would cause the coefficient curves to rise rapidly as the zero value of P_d or P_i is approached. The curve of discharge coefficients should not be extrapolated to low values of P_d or P_i.

The data are based on tests of ports of particular sizes, shapes, and lengths—attached to a conduit of a particular size and shape. In view of the absence of data on other shapes and sizes, it is likely that some extrapolation from these tests will be attempted. If done carefully, such extrapolation should give better results than a guess or empirical rule. Strictly, however, the results are applicable only to geometrically similar designs. Any other application should be made with the realization that substantial errors are possible.

Application.—The design of a manifold is an indirect process. Characteristic curves for the ports used must be established by single port tests or otherwise. A manifold is assumed and analyzed. If the result is unsatisfactory, the design must be modified. This procedure is repeated until the required relations are obtained. Only the analysis of a manifold with a series of identical ports will be considered. Other applications are obvious.

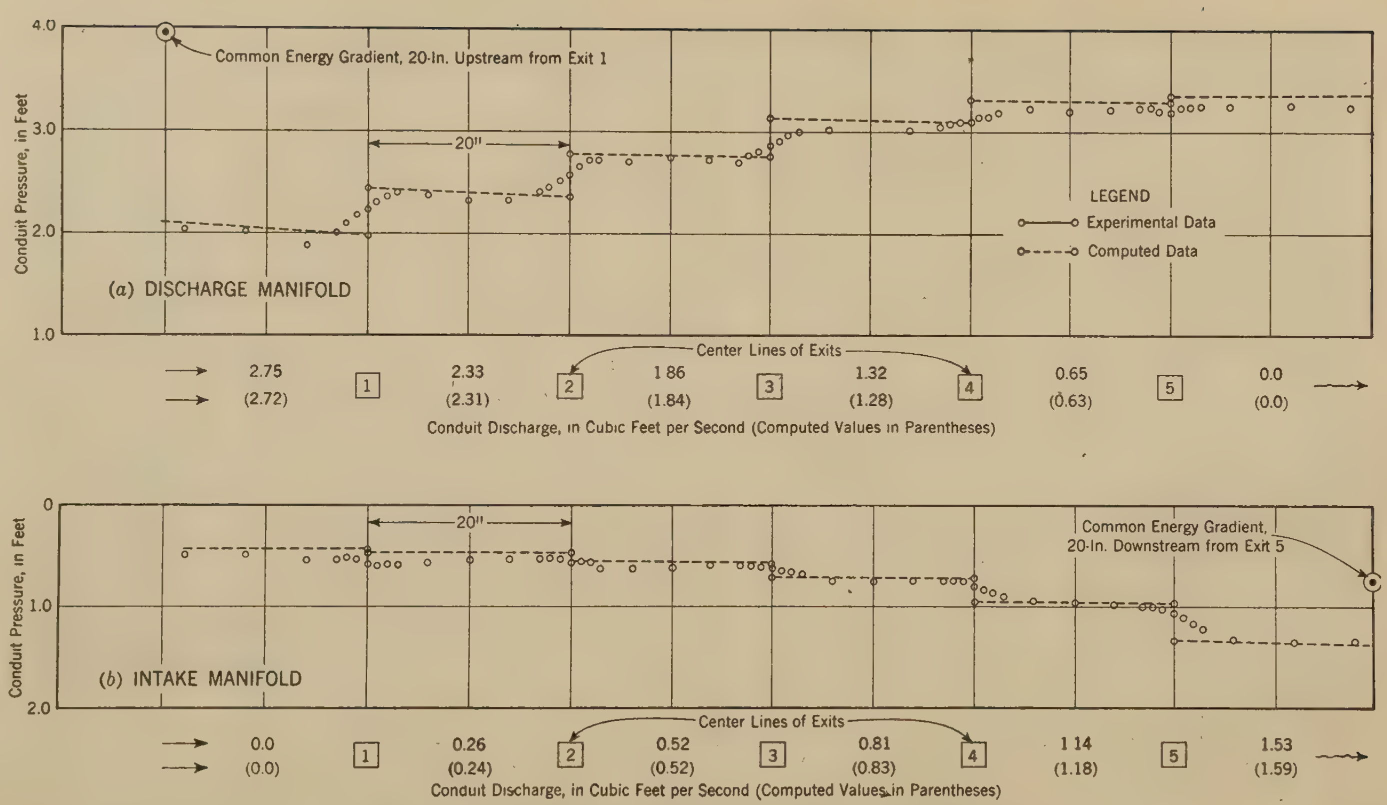


Fig. 8.—Comparison of Conduit Pressures and Discharges, Port VII
(Pressure Measured from the Water Surface in the Port Tank)

Assume a discharging manifold. The farthest downstream port has a value of P_d equal to unity. Any convenient discharge through this port may be assumed and the head calculated, the value of C_d being known. The friction loss to the next port can be calculated, since the discharge is known, and added to the previously determined head, the result being the head immediately downstream from the next port. The discharge through the second port from the downstream end is then assumed and the heads upstream and downstream are calculated, P_d being determinable for any assumed discharge. Only one discharge can be found which is consistent with the established discharge through the end port and the resulting gradient. The calculation proceeds up the manifold by trial until the farthest upstream port is passed. Since friction losses may be considered to vary with the square of the mean velocity, the distribution of the total discharge is the same, regardless of the port discharge originally assumed. Accordingly, the calculation made for one discharge may be adjusted to any other head or discharge by the following rules: (1) All heads vary in proportion to the over-all head applied to the manifold; and (2) all discharges and velocities vary as the square root of the applied head. An example of this adjustment is shown under "Verification Tests."

If the calculation indicates a negative head upstream from any port, the method fails. However, this result indicates that the port area is too great for good distribution. This may be corrected by reducing the size, number, or efficiency of the ports or by increasing the size of the conduit. The choice between these adjustments is governed by the desired relation between the applied head and the corresponding discharge and by the required distribution.

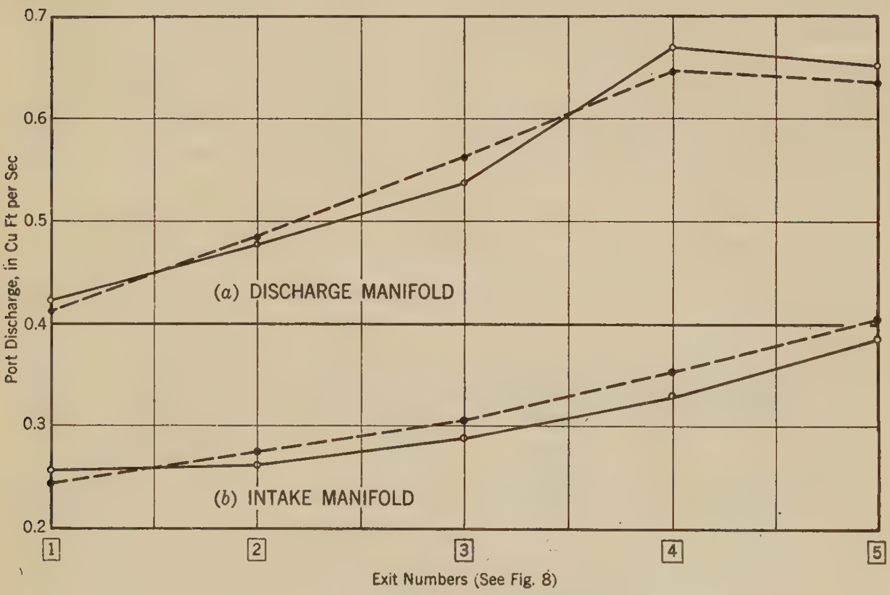


FIG. 9.—COMPARISON OF PORT DISCHARGE, PORT VII (SOLID LINES DENOTE EXPERIMENTAL DATA)

Methods of adjusting the distribution of discharge along the length of the manifold by varied port size or spacing are obvious.

For an intake manifold, the procedure is similar to that which has been explained. The calculations begin with the port at the upstream end of the manifold and proceed downstream.

It is possible to make the calculation without assigning a numerical value to the port discharges but this method has not been found advantageous. Examples of the calculations are presented under "Verification Tests."

VERIFICATION TESTS

Apparatus, Tests, and Procedure.—The apparatus used for the verification tests is shown in Fig. 5. Ports type VII were connected to the conduit at five points at a 20-in. spacing, each port leading to a separate tank. The water levels in the tanks were equalized by adjustment of slide valves for discharge tests and by regulating the inflow for intake tests. A small 90° V-notch weir was used to measure port discharges. For the intake tests, each port was supplied by a 3-in. swing pipe connected to a 6-in. auxiliary line. The test procedure was substantially as described for the single port tests.

Results.—Figs. 8 and 9 show typical comparisons of experimental pressures and discharges with calculated values. The basis of comparison is a common energy gradient 20 in. upstream from the farthest upstream port—for the discharge manifold—and 20 in. downstream from the farthest downstream port for the intake manifold. The number of significant figures used in the tabular calculations is not justified by the nature of the data and is used merely to illustrate better the method of making friction adjustments. Table 2 shows the calculations for the discharge manifold, based on a port discharge of 0.5 cu ft per sec for the farthest downstream port. The energy head calculated for this case—$H_1 + \frac{(V_1)^2}{2\,g} + H_f$—was $1.25 + 1.15 + 0.07 = 2.47$ ft. In Figs. 8(*a*) and 9(*a*), the tabulated results have been adjusted to an energy head of 3.95 ft, which is the experimental value. For the intake manifold the computed energy head—$H_2 - \frac{(V_2)^2}{2\,g} + H_f$—was $1.43 - 0.66 + 0.04 = 0.80$.

TABLE 2.—TYPICAL CALCULATIONS, DISCHARGE MANIFOLD

Exit No.	Q_2 (1)	Q_3 (2)	Q_1 (3)	$\bar{P}_d$ (4)	$\frac{\Delta Z}{\frac{(V_1)^2}{2\,g}}$ (5)	ΔZ (6)	C_d (7)	H_d (8)	H_f (9)	$H_d + H_f$[a] (10)	$H_d + \Delta Z$[a] (11)
5	0.000	0.500	0.500	1.000	....	....	1.032	2.086	0.004	2.090	....
4	0.500	0.509	1.009	0.505	0.568	0.143	1.090	1.947	0.016	1.963	2.090
3	1.009	0.444	1.453	0.305	0.455	0.238	1.010	1.718	0.033	1.751	1.956
2	1.453	0.377	1.830	0.206	0.330	0.274	0.923	1.481	0.053	1.534	1.755
1	1.830	0.326	2.156	0.151	0.251	0.289	0.871	1.247	....	....	1.536

[a] When the value in Col. 11 of any row substantially corresponds with the value in Col. 10 of the preceding row, the assumed discharges are verified.

As before, Figs. 8(*b*) and 9(*b*) show the tabulated results adjusted to an energy head of 0.78 ft, to agree with the experimental value. For this case Table 3, contains the corresponding computations, starting with a discharge of 0.25 cu ft per sec for the extreme upstream port. Since the theory indicates constant distribution of total discharge between the ports for any applied head and, as all experiments confirmed this result, tests at other heads are compared with calculated values only on the basis of total discharges and energy heads. These comparisons are shown, for the discharge and intake manifolds, in Fig. 10.

TABLE 3.—TYPICAL CALCULATIONS, INTAKE MANIFOLD

Exit No.	Q_1 (1)	Q_3 (2)	Q_2 (3)	P_i (4)	$\frac{\Delta Z}{\frac{(V_2)^2}{2g}}$ (5)	ΔZ (6)	C_i (7)	H_i (8)	H_f (9)	H_i+H_f[a] (10)	$H_i+\Delta Z$[a] (11)
1	0.000	0.250	0.250	1.000	....	....	1.080	0.479	0.001	0.480	....
2	0.250	0.283	0.533	0.531	1.580	0.107	1.102	0.588	0.005	0.593	0.481
3	0.533	0.317	0.850	0.373	0.870	0.156	1.095	0.748	0.011	0.759	0.592
4	0.850	0.362	1.212	0.299	0.685	0.250	1.078	1.008	0.023	1.031	0.758
5	1.212	0.418	1.630	0.256	0.600	0.394	1.046	1.427	....	....	1.033

[a] When the value in Col. 11 of any row substantially corresponds with the value in Col. 10 of the preceding row, the assumed discharges are verified.

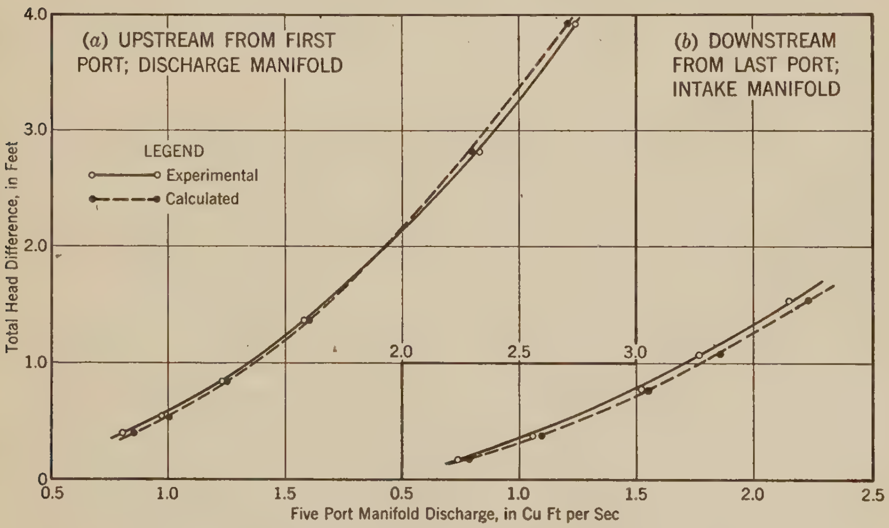


FIG. 10.—COMPARISON OF COMPUTED AND OBSERVED MANIFOLD DISCHARGES

CONCLUSION

The tests indicate that the application of single port test data to the design of manifolds for lock hydraulic systems is feasible. The use of dimensionless characteristic curves is a convenient generalization which expedites the analysis and application of single port test data. The theory presented appears to provide a reasonably correct basis for the analysis of problems in manifold hydraulics; its imperfections and its merits are due largely to its simplicity.

ACKNOWLEDGMENT

F. W. Edwards, M. Am. Soc. C. E., preceded the senior writer as Chief of the Hydraulic Section of the Special Engineering Division. He initiated the manifold tests, took an active interest in the work, and reviewed the text of this paper. S. O. Steinborn, Jun. Am. Soc. C. E., conducted the verification tests. P. S. O'Shaughnessy reviewed the text. Several other members of the Hydraulic Section were employed on the tests and calculations. The cooperation of all is gratefully acknowledged.

The Special Engineering Division was directed by Maj. Gen. T. B. Larkin, Supervising Engineer, and Brig. Gen. Hans Kramer, Assistant Supervising Engineer, Members, Am. Soc. C. E. E. E. Abbott, Designing Engineer, and J. E. Reeves, Assoc. M. Am. Soc. C. E., Assistant Designing Engineer, were in charge of design.

DISCUSSION

J. PIETRKOWSKI,[11] ESQ.—Elsewhere[12] the writer has developed equations for open channel flow, with $k_d = 1$, which were based on the same logic as Eq. 1. The water surface was found to rise in downstream direction corresponding to the pressure increase in the closed conduit treated by the authors.

Experience has shown that the water surface in the channel in the region of the side spillway is not horizontal but that it slopes toward the outlet. The effect of this phenomenon cannot be computed theoretically but must be appraised by the experimental determination of coefficients such as C_d in the conduit treated by the authors. Both the investigation reported by the authors for closed conduits and that reported by the writer for open channels can be interpreted best by instruction sheets, outlining steps in a computation, rather than by formulas.

Two differences between open channel flow and conduit flow should be emphasized:

(1) To be of any value, the spillway crest must be much longer than the corresponding dimension of the port; therefore the friction loss in the channel along the weir must be considered.

(2) The critical depth $d_c = \frac{V^2}{g}$ may occur in the channel, adjacent to the side spillway, with important consequent results upon the discharge over the weir.

It is noteworthy that the authors report the pressure change with $k_d = 1$, would be twice the difference between velocity heads upstream and downstream from the port. They conclude that Bernoulli's theorem is not applicable to problems of this kind. Similarly, the writer could show that the slope of the energy line in an open channel, computed by the momentum equation, is twice as great as the slope computed by the energy equation. Again Bernoulli's theorem is not applicable.

Incidentally, Bernoulli was the last man to expect too much from his famous theorem; he was the first to understand that conditions at the outlet of a vessel, or conduit, can be described adequately only by considering momentum.[13] L. Föppl has contributed a valuable statement[14] regarding Bernoulli's theorem —namely, that this theorem is valid either along a streamline even if curl occurs or between different streamlines when the curl is zero. The lock manifold and the side spillway are surely phenomena involving different streamlines with curl.

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[11] Asst. Drainage Engr., Public Works Dept., Jerusalem, Palestine.

[12] "Lateral Overflow Weirs," by J. Pietrkowski, *Journal*, Assn. of Engrs. and Archts. in Palestine, October 1942, p. 7.

[13] "Hydraulik," by Philipp Forchheimer, B. G. Teubner, Leipzig and Berlin, 1930, Eq. 182b, p. 337.

[14] "Vorlesungen über Technische Mechanik," by L. Föppl, 6th Ed., Vol. IV, Leipzig and Berlin, 1921, p. 385.

JOHN S. MCNOWN,[15] JUN. AM. SOC. C. E.—There are two general methods of approach to a design problem of the type outlined in this paper—the use of model studies and the application of a fundamental analysis or comprehensive research study. Only the first method can be applied in the absence of a general treatment, but this method possesses the disadvantages both of being more expensive and of furnishing results that may be of little use in related studies. In other words, the lock manifold study presented by the authors is an excellent example of specific model testing, but will not greatly reduce the work necessary for the design of a dissimilar lock port except in so far as it completely defines the experimental procedure to be followed. It is unfortunate, therefore, that the experimental work could not have been extended to include an investigation of the theory presented. The continued absence of a basic study is surprising because such a basic study would have many applications in the field of hydraulic engineering.

An attempt has been made herein to describe the general aspects of the problem and to present a specific analysis of the flow conditions at a point of lateral discharge or intake. Consideration has been given to the dimensional aspects of the problem and to the rôle of the energy and momentum equations. The analysis represents in part an extension of that given by the authors under the heading, "Theoretical Considerations," and in part an alternate approach to a general solution. Parallel developments have been carried out based on the momentum and energy principles. These are compared and combined in an attempt to obtain as much information from the analysis as possible.

Experiments have indicated that the effective area in a lateral may be considerably less than the total area of the lateral for large conduit velocities, and also that the lateral velocity is dependent upon the piezometric head within the conduit rather than upon the total head. Thus, the variation of the discharges from successive openings depends upon (*a*) the magnitudes of the velocities, and (*b*) the magnitude of the friction head between openings as compared to the increase in piezometric head at the openings. As a result, the characteristics of multiple lateral flow from a conduit vary markedly as the relative size and spacing of the lateral openings are changed.

For a conduit with closely spaced, relatively small openings, the velocity and friction heads in the conduit are negligible, and the pressure along the pipe and the discharge through the various openings are practically constant. For somewhat larger openings the velocity head becomes significant, but for close spacing the pressure recovery at the openings more than compensates for friction loss in the conduit. Consequently, the pressure and lateral discharge increase in the direction of flow, as in the lock manifold. The friction head and pressure recovery may compensate for some intermediate condition of outlet spacing and relative diameters, and the pressure and discharge may again be relatively constant for a few adjacent openings. As the spacing is further increased, pipe friction becomes predominant, and the pressure and lateral discharge then decrease in the direction of flow.

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Throughout the entire range, the flow conditions in the vicinity of the opening and those between the openings make up two independent parts of the problem. The pipe friction on the one hand and the pressure recovery and efflux on the other are essentially unrelated unless by an effect of close spacing of the openings. Thus, it should be possible to obtain a systematic representation of the flow at the openings over the entire range of discharge without regard to friction losses in the conduit. As the authors have shown, tests on a single opening can then be applied to a multiple system, once the component parts of the problem have been separated.

A single tube in the side of a circular pipe is perhaps the simplest case of a conduit with lateral discharge, and an analysis of this case should indicate the basic relationships involved in the general problem. These relationships might then be adjusted to describe additional effects such as changes in geometrical form and close spacing of the openings. Although the orifice or port in the side of a conduit discussed by the authors is somewhat simpler because the jet is unconfined, the uncertainty regarding the conditions in the direction of the jet complicates the application of the momentum equation to the extent of an additional variable.

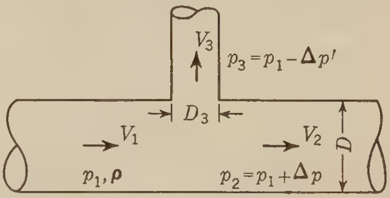

FIG. 11.—DEFINITION SKETCH FOR A SINGLE MANIFOLD PORT

The variables involved in the simple case, indicated in Fig. 11, are the pressures and velocities at the three points in question, the diameters of the pipe and opening, and the density of the liquid. Before investigating the proper grouping of these terms, it should be noted that the three pressures can be replaced by two pressure differences—say, $p_2 - p_1$ and $p_1 - p_3$—since the absolute magnitude of the pressures has no effect on the flow. Thus, the number of variables is reduced by one. The nomenclature may also be simplified somewhat by representing these two pressure differences as Δp and $\Delta p'$, respectively. Once the two diameters and fluid density have been established, only two of the flow variables can be determined arbitrarily; that is, if the two velocities, V_1 and V_2, are established by manipulation of the controls, the remaining variables, V_3, Δp, and $\Delta p'$, are also determined. Thus, five of the eight variables are independent, whereas the remaining three are dependent. Although practical considerations limit the dependent variables to the pressure and velocity terms, any three of these may be selected. Functionally, three independent equations are indicated, each expressing one of the dependent variables in terms of the five independent variables. Arbitrarily selecting V_1 and V_2 as independent variables, the dimensionless form of one such set of equations is as follows:

$$\frac{\Delta p}{0.5\,\rho\,(V_1)^2} = \phi_1\left(\frac{V_2}{V_1}, \frac{D_3}{D}\right) \qquad \text{(4a)}$$

$$\frac{V_3}{V_1} = \phi_2\left(\frac{V_2}{V_1}, \frac{D_3}{D}\right) \qquad \text{(4b)}$$

and

$$\frac{\Delta p'}{0.5\,\rho\,(V_1)^2} = \phi_3\left(\frac{V_2}{V_1}, \frac{D_3}{D}\right) \qquad (4c)$$

Eq. 4*a* describes the flow within the pipe, Eq. 4*b* reduces to the equation of continuity, and Eq. 4*c* expresses the pressure drop experienced by the water as it passes into the lateral tube. This particular set of functional relationships was selected because Eq. 4*a* has the form of Eq. 2. The independent dimensionless ratio V_2/V_1 in Eq. 4 is related to the discharge ratio P_d in Eq. 2 by the simple expression,

$$P_d = 1 - \frac{V_2}{V_1} \qquad (5)$$

Relationships of the type indicated by Eq. 4*a* can be obtained by the application of either the momentum or the energy equation between sections 1 and 2. In a manner similar to the authors' procedure, the momentum equation can be obtained in either of the following forms:

$$\frac{\Delta p}{0.5\,\rho\,(V_1)^2} = 2\left[1 - \left(\frac{V_2}{V_1}\right)^2\right] - \frac{2\,F}{\rho\,Q_1\,V_1} \qquad (6a)$$

or

$$\frac{\Delta Z}{(V_1)^2/2\,g} = 4\,P_d - 2\,(P_d)^2 - \frac{2\,F}{\rho\,Q_1\,V_1} \qquad (6b)$$

in which F is the residual force exerted on the water in this region. Eq. 6*b* differs from Eq. 2 only in that F has been used rather than k_d. A comparison of the two equations yields the relationship,

$$1 - k_d = \frac{1}{P_d}\,\frac{F}{\rho\,Q_1\,V_1} = \frac{F}{\rho\,Q_3\,V_1} \qquad (7)$$

which is in agreement with the authors' description of the ratio k_d. It should be noted that the effect of the velocity distribution on the momentum at the downstream section may be appreciable, and the neglect of this fact implies that k_d or F includes the effect of the velocity distribution as well as that of the unbalanced pressures. Also of interest is the fact that Eq. 4*a* indicates that $\frac{\Delta p}{0.5\,(V_1)^2}\left(\text{or } \frac{\Delta Z}{(V_1)^2/2\,g}\right)$ is a function of $\frac{V_1}{V_2}$ (or P_d) and of boundary conditions only, thus bearing out the authors' assumption that k_d was either a constant or a function of P_d for the particular geometrical form studied.

The energy equation written between sections 1 and 2 may take the form,

$$\frac{\Delta Z}{(V_1)^2/2\,g} = 2\,P_d - (P_d)^2 - \frac{H_f}{(V_1)^2/2\,g} \qquad (8)$$

A comparison of Eqs. 2, 6*b*, and 8 yields the following expressions for the head-loss term:

$$\frac{H_f}{(V_1)^2/2\,g} = (P_d)^2 - 2\,k_d\,P_d = (P_d)^2 - 2\,P_d + \frac{2\,F}{\rho\,Q_1\,V_1} \qquad (9)$$

For the special case of zero head loss, k_d has the value of 0.5 P_d. At the other extreme, $k_d = 0$, and the corresponding maximum value of the head loss is $\frac{(V_1 - V_2)^2}{2g}$, significantly identical to the loss for an abrupt enlargement. With these two limits as guides, it appears logical to substitute curves of $c = k_d/P_d =$ constant $(0 \leqq c \leqq 0.5)$ for the k_d (= constant)-curves in Fig. 1. This set of curves, defined by the formula,

$$\frac{\Delta Z}{(V_1)^2/2g} = 2P_d - 2(P_d)^2(1 - c) \quad \ldots . (10)$$

makes up part of the family of curves in Fig. 12, the remainder of which is obtained from the analysis of inflow manifolds.

As the upper limit for k_d has been shown to be 0.5 P_d and, as P_d varies from 0 to 1, k_d cannot exceed 0.5. If k_d were to exceed 0.5 P_d, an increase in unit energy in the direction of flow would be indicated—obviously an impossible situation. The authors' mention of the case $k_d = 1.0$, therefore, has no practical significance. It follows that an unbalanced force exists for all cases of lateral discharge. The authors' subsequent statement regarding the inapplicability of Bernoulli's equation needs clarification. For a discharge port it is possible to write an energy equation along a streamline or tube which may either continue along the conduit or pass out through the port, and each of the resulting equations has the essential characteristics of Bernoulli's theorem. For the intake port, the energy relationship may well be more complicated. The authors' remarks concerning the Bernoulli theorem and a "modified energy

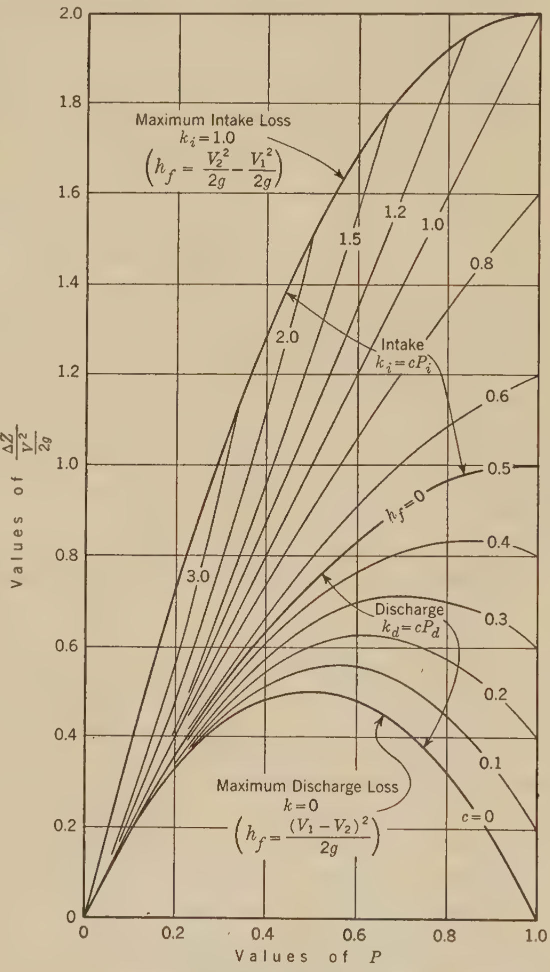

FIG. 12.—THEORETICAL CURVES FOR DISCHARGE AND INTAKE MANIFOLDS: $c = k/P =$ CONSTANT

equation" might be construed to refute the existence of Eq. 6, which to the writer is an unmodified energy equation, or Bernoulli's equation with a head-loss term. It is hoped that the closing discussion will amplify the brief statement in the paper.

The value of the energy approach lies in the determination and interpretation of the limiting curves for P plotted against $\frac{\Delta Z}{(V_1)^2/2\,g}$ as in Figs. 1 and 12. The Enger-Levy data coincide almost exactly with Eq. 6 for $H_f = 0$, or for $k_d = P_d/2$, and the substitution of $c\,P_d$ for k_d in Eq. 2 ($c < 0.5$) is equivalent to assigning a value to the friction head term as

$$\frac{H_f}{(V_1)^2/2\,g} = (P_d)^2\,(1 - 2\,c) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

A comparison of the c (= constant)-curves with the experimental data in Fig. 1 shows good agreement not only with the Enger-Levy curve at the upper limit, but with three of the curves from the Oakey series as well. The writer is unable to explain the discrepancy indicated by the position of the A-curve from the Oakey series above the upper theoretical limit of $c = 0.5$. Although the curves presented by the authors are not defined over a wide enough range to compare with the c (= constant)-curves, it is noteworthy that they tend to fall between the aforementioned limits.

The case of zero head loss for this type of flow perhaps can be visualized more easily for a special condition in which the tubes are extended into the flow, as shown in Fig. 13. If the tubes were closely spaced and carefully streamlined to prevent eddy formation at the downstream side, the friction losses might well be negligible, and the unbalanced force F would be the force exerted on a given tube. In this special case, the origin of the force is more easily visualized than in the case of a flush entrance. For the flush entrance, the unbalanced force must result from local pressure effects in the region of the tube—that is, nonuniform pressure distribution on either side of the opening, probably caused by eddy formations.

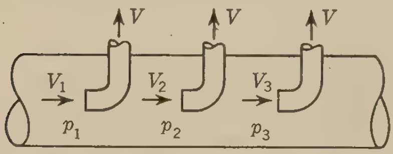

FIG. 13.—IDEALIZED MANIFOLD FOR ZERO ENERGY LOSS

The energy equation, applied in the case of the inward projecting tubes, shows that the efflux velocity would be the same for all tubes since

$$\frac{V^2}{2\,g} = \frac{(V_1)^2}{2\,g} + \frac{p_1}{\gamma} = \frac{(V_2)^2}{2\,g} + \frac{p_2}{\gamma} = \cdots \ldots\ldots\ldots\ldots\ldots (12)$$

This result is expected to differ from that for a tube with a flush entrance, since, even if the flow within the pipe for the latter case occurs with negligible head loss, the efflux could not be expected to do so. The effect of a high velocity in the pipe is to decrease the rate of lateral discharge because of larger losses and reduced effective area rather than to increase the flow because of the additional kinetic energy.

The foregoing remarks have been restricted to the case of a discharging lateral or manifold. To complete the analysis it is necessary to compare and contrast the foregoing conclusions with those for an intake system. The derivation and results in the two cases have a marked similarity. The values of k_i and P_i are implicitly different from those of k_d and P_d; that is:

$$k_d = 1 - \frac{F}{\rho\, Q_3\, V_1} \quad \text{(13a)}$$

and

$$k_i = 1 - \frac{F}{\rho\, Q_3\, V_2} \quad \text{(13b)}$$

and, also,

$$P_d = \frac{Q_3}{Q_1} = 1 - \frac{V_2}{V_1} \quad \text{(14a)}$$

and

$$P_i = \frac{Q_3}{Q_2} = 1 - \frac{V_1}{V_2} \quad \text{(14b)}$$

With these logical changes the momentum equation for intake flow is

$$\frac{\Delta Z}{(V_2)^2/2\,g} = 4\,P_i - 2\,(P_i)^2 - \frac{2\,F}{\rho\, Q_2\, V_2} \quad \text{(15a)}$$

corresponding to Eq. 3, and the energy equation is

$$\frac{\Delta Z}{(V_2)^2/2\,g} = 2\,P_i - (P_i)^2 + \frac{H_f}{(V_2)^2/2\,g} \quad \text{(15b)}$$

The head loss may again be expressed in terms of P_i and k_i or c:

$$\frac{H_f}{(V_2)^2/2\,g} = P_i\,(2\,k_i - P_i) = (P_i)^2\,(2\,c - 1) \quad \text{(16)}$$

With the proper substitutions, Eqs. 15 reduce to the intake counterpart of Eq. 10, and, for the special case of zero head loss, $k_i = 0.5\,P_i$, just as $k_d = 0.5\,P_d$ for outflow under the ideal condition. The upper limit for inflow is given by $k_i = 1.0$, which yields a maximum value for the head loss of $\frac{(V_2)^2 - (V_1)^2}{2\,g}$.

Because of the similarities indicated in the foregoing paragraph, it is possible to represent both discharge and inflow conditions on a single dimensionless plot, as shown in Fig. 12. The curves for inflow are a part of the same family of c (= constant)-curves which characterize the discharge manifold. The family is bounded by the practical limits of $k_d = 0$ and $k_i = 1.0$, these values representing conditions of maximum loss for the two different cases. The curve for zero head loss ($c = 0.5$) is the same for both types of flow, as indeed it must be for the ideal condition. With a redefinition of the principal variables, both types of flow could be represented by a single relationship. However, the foregoing analysis and the curves of Fig. 12 have been based, in so far as possible, on the terminology presented by the authors.

The agreement of the curves $c \leqq 0.5$ with experimental data for discharging flow has already been discussed. The curves $c \geqq 0.5$ are similar to those presented by the authors for the various inflow manifolds, although the agreement between individual curves is only fair. It is to be expected that the system of curves in Fig. 12 is only one step toward a complete solution of the general problem of lateral flow. Many uncertainties remain, and modifications doubtless will be necessary as more information is obtained. Much systematic experimental work will be required before the problem is solved adequately.

Throughout this discussion the effect of nonuniform velocity distribution has been neglected although it has been stated that the unbalanced force term may include this effect if appreciable. The distribution immediately downstream from a tube is certain to be irregular, but this irregularity is probably relatively unimportant unless the spacing of the outlet tubes is quite small. However, the upstream velocity distribution may affect the flow in a quite different and significant manner. The lateral discharge undoubtedly comes from a region of the pipe in which the velocity is below average, particularly for relatively small values of the lateral discharge. Hence, the subtraction of this quantity of water possessing below-average kinetic energy increases the average kinetic energy of the quantity continuing in the pipe. In this way it is possible to explain the tendency of several of the curves, aside from the Oakey A-curve, to exceed slightly the upper limit of zero head loss for small values of P_d.

Although the foregoing remarks have for the most part applied to the flow within the pipe or manifold, the lateral discharge is of primary importance in many problems. For the elementary case herein analyzed, the discharge is immediately determined from the value of V_2/V_1 by the equation of continuity, which can be written in the form of Eq. 4b:

$$\frac{V_3}{V_1} = \left(\frac{D}{D_3}\right)^2 \left(1 - \frac{V_2}{V_1}\right) = P_d \left(\frac{D}{D_3}\right) \ldots\ldots\ldots\ldots (17a)$$

for outflow, and

$$\frac{V_3}{V_2} = \left(\frac{D}{D_3}\right)^2 \left(1 - \frac{V_1}{V_2}\right) = P_i \left(\frac{D}{D_3}\right)^2 \ldots\ldots\ldots\ldots (17b)$$

for inflow. However, the functional relationship, Eq. 4c, between the pressure difference $\Delta p' = (p_1 - p_3)$ and the independent variables can be determined only by experiment. The method used by the authors is one of several that might be followed. Although $\Delta p'$ and V_3 do not occur in any one of the three relationships in Eqs. 4, the complete determination of the form of Eq. 4*b* permits the use of Eq. 4*b* in combination with Eq. 4*c* so that a simple discharge equation such as that proposed by the authors could be utilized. It should be emphasized, however, that, for an orifice or port, Eq. 17*a* is complicated by a contraction coefficient which is likely to be a function of D_3/D as well as of P. The variation of the coefficient with the velocity and diameter ratios and the outlet spacing must also be determined by experiment.

In conclusion, it is hoped that these remarks will serve to clarify the analytical background of multiple lateral flow, and at the same time will be of specific assistance in an experimental investigation of any phase of the problem. The effectiveness of dimensional analysis has been demonstrated through the determination of the number of independent equations which can be written and the functional form of each. The seeming contradiction between the momentum and energy equations has been examined in some detail, and an explanation presented. In this analysis, as in investigations of the sudden expansion of a pipe line, the Borda re-entrant tube, and the hydraulic jump, the simultaneous application of these two basic equations yields more information than would either equation used alone. The question as to which equation is applicable to a given problem is of no significance. In some cases, one is more easily applied than the other, and hence more useful; but both are applicable to all such problems.

F. W. EDWARDS,[16] M. AM. SOC. C. E.—Results of a successful effort to develop a method for designing lock manifolds are presented in this paper. The authors have prepared an excellent summary of the investigation and analysis and have called attention to the limitations of the procedure. These limitations, however, do not detract from the study because the results justify the method.

Although greater refinement was not necessary for the specific purpose of the experiments described in the paper, further studies based on the authors' method should yield valuable results for other fields as well as for lock design. Several factors require further investigation:

(1) A discrepancy in the discharge from a port under negative head (negative according to the definition adopted for head) is not important in designing lock manifolds, but it does show that the definition for head on the port is not precise. Although it does not appear logical to add the full velocity head to the pressure head to obtain the total head on the port, it might be more accurate to add all or part of the pressure rise to the head (as defined) for the discharging port. Using the same reasoning, all or part of the pressure drop would need to be subtracted from the head for an intake port. A trial plotting of discharge coefficients against P_d was prepared on this basis, but no significant change in design results was indicated.

(2) The relation between the coefficient of discharge and the head on the port should be studied. The tests gave a slight indication that separate curves

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[16] Associate Prof., Dept. of Civ. Eng., The Pennsylvania State College, State College, Pa.

of C_d versus P_d might be drawn for each head. However, many more tests would have been required to define such curves. An average curve which permitted simplification, while keeping the error for lock design within narrow limits, was used for all heads.

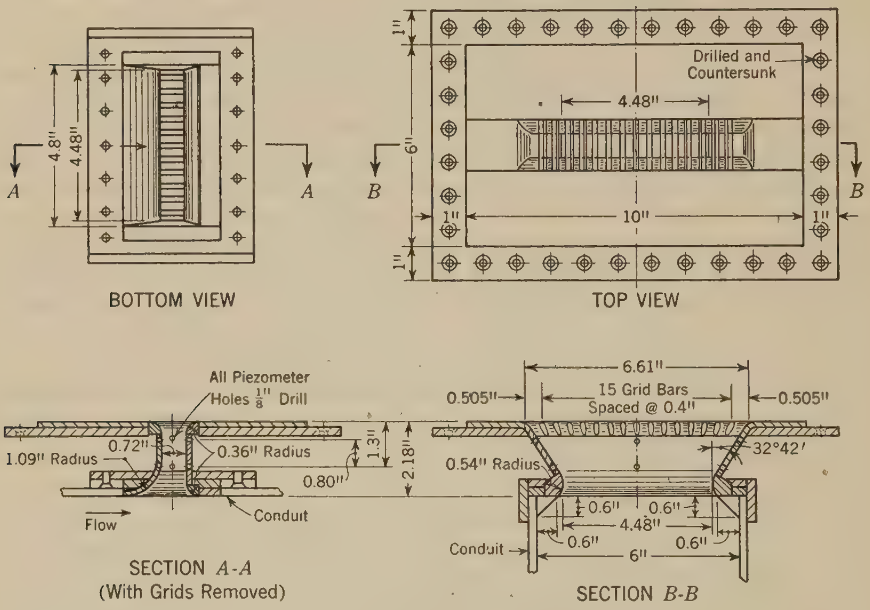
A
4.8"
4.48"
A
BOTTOM VIEW
1"
B
6"
1"
1"
Drilled and Countersunk
4.48"
10"
1"
B
TOP VIEW
All Piezometer Holes 1/8" Drill
0.72"
1.09" Radius
0.36" Radius
1.3"
2.18"
0.80"
Flow
Conduit
SECTION A-A
(With Grids Removed)
6.61"
0.505"
15 Grid Bars Spaced @ 0.4"
0.505"
0.54" Radius
32°42′
0.6"
0.6"
4.48"
0.6"
0.6"
Conduit
6"
SECTION B-B

Fig. 14.—Ports VIII and VIIIA

(3) The sudden change in the relationship of C_d and P_d for the ports with rounded entrance probably depends on several factors, including radius of curvature on the upstream edge of the port, shape of the port cross section, length of the port, and ratio of port area to conduit area. The authors present the test points for Port VII as being representative of the agreement between curves and observation points for Ports I to VI, inclusive. However, the test points for Ports I to III, inclusive (square-cornered ports), show less scatter than do those for Port VII. In general, the curve for a square-cornered port and the portion of the curve on the left of the discontinuity point for a round-cornered port are similar. This fact supports the belief that eddies along the upstream edge of the port entrance cause the discontinuity, as eddies always exist in the entrance of a square-cornered port.

It should be emphasized that the experiments described in the paper were made before the lock hydraulic system was designed and before the complete system was tested by hydraulic models. This study not only furnished a method of designing the manifold systems for the Third Locks, but also greatly reduced the revisions required on the large, more expensive models of the complete system. It is the writer's belief that many useless model tests are made because of inadequate preliminary studies and analyses.

The equipment described in the paper was used to test several types of ports before the final design was determined. In addition, tests of connections between the main tunnels in the lock walls and the laterals in the lock floor were conducted and analyzed by the authors' method.

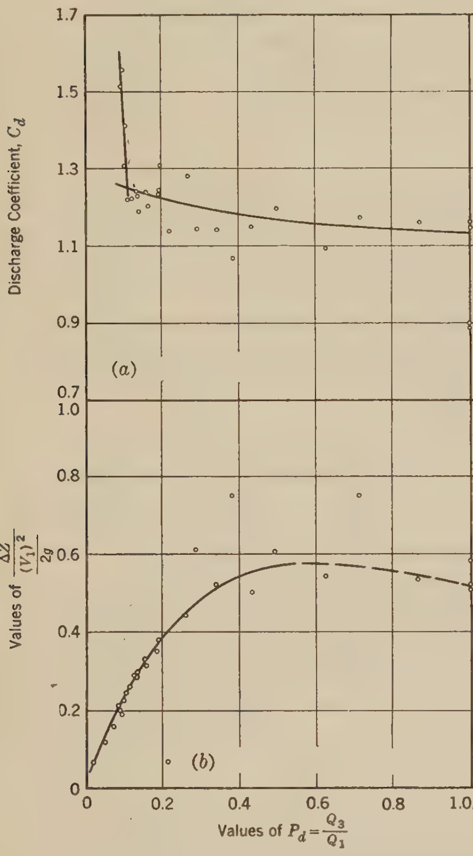

Fig. 15.—Port Characteristics, Port VIII

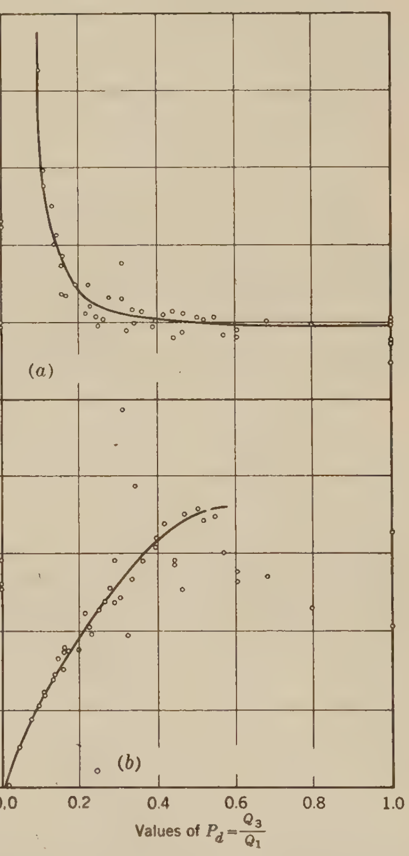

Fig. 16.—Port Characteristics, Port VIIIA

The test port developed for the design of the Third Locks (see Fig. 14) has several special characteristics. The port throat is long and narrow. The long dimension is perpendicular to the direction of flow in the conduit. The shape caused the flow to emerge from the port very nearly at right angles to the direction of flow in the conduit. The port was designed so that a grid could be used at the outlet to prevent objects from entering the hydraulic system.

The long dimension of the port is flared from the throat to the lock floor. The flare serves two purposes:

(*a*) It increases the area of the port so that the grid obstructs the flow as little as possible, and makes the operation of the hydraulic system practicable either with or without the grids in place; and
(*b*) It spreads the flow along the length of the lock.

The characteristic curves for this special port, with and without grid, are shown in Figs. 15 and 16. Curves for discharge tests only are shown. It is interesting to note that the Panama tests furnished the basis for the adoption of a similar port, without grid, in the MacArthur Lock at Sault Sainte Marie, Mich.

W. M. LANSFORD,[17] M. AM. SOC. C. E.—A problem that is difficult to analyze, not only as applied to canal locks but also as applied to piping systems such as those used in sand filters and sprinkling systems, is presented in this instructive paper.

In 1932–1933 the writer made a study of two manifold orifices in the side of a 3-in. brass pipe. The arrangement of the pipe and orifices is shown in Fig. 17. Both orifices had the same diameter for each series of tests. Four series of tests were made using orifices, 1/4 in., 3/8 in., 1/2 in., and 3/4 in. in diameter. The head on the orifices, as measured by the manometer upstream from each orifice, varied from 5.97 ft to 0.50 ft.

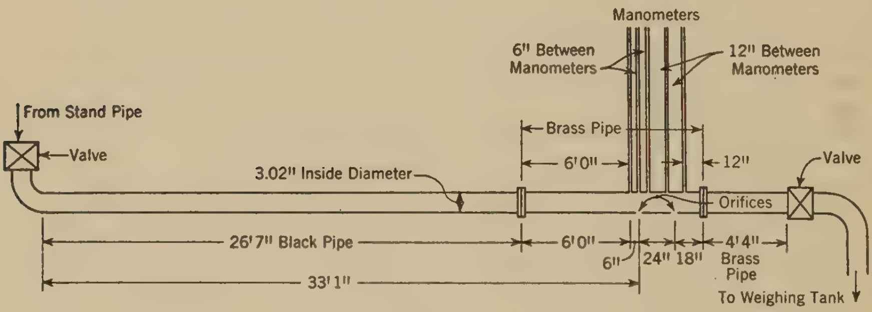

FIG. 17.—SKETCH OF APPARATUS

The water was taken from a constant level standpipe which maintained a static head of approximately 60 ft at the upstream valve in the supply line. The quantity of water discharged from each orifice and from the pipe was determined by simultaneous weighing operations. The maximum velocity obtained in the pipe preceding the upstream orifice was about 16.25 ft per sec.

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By adjusting the two valves in Fig. 17, it was easy to obtain several widely different velocities in the pipe while maintaining a constant head on the orifice. During early preliminary tests, an approach pipe shorter than the one in Fig. 17 was used. The shorter pipe caused the upstream valve to affect the flow from the orifice. The approach pipe, shown in Fig. 17, which was longer than 132 diameters, placed the valve far enough upstream that no disturbance from the valve was noticed at the orifice.

While the data presented by the authors seem to be in agreement with results obtained by the use of Eq. 2, the data herein presented by the writer are of such values that other means of analyzing and presenting the data had to be used.

The rise in the hydraulic gradient, ΔZ, could not be used as a criterion of the orifice flow in the analysis of these tests because of the very small value of ΔZ. The results of some of these tests are shown graphically in Fig. 18 by a relation between $\frac{(V_1)^2/(2\,g)}{Z_1}$ and $\frac{Q_3}{Q_1}$, in which $\frac{(V_1)^2}{2\,g}$ is the kinetic energy in the pipe; Z_1 is the potential energy in the pipe at a point preceding the orifice; Q_3 is the quantity discharged from the orifice; and Q_1 is the quantity flowing in the pipe preceding the orifice. The curves in Fig. 18 indicate the great effect the ratio of the kinetic energy to the potential energy has upon the flow through a manifold orifice in a pipe.

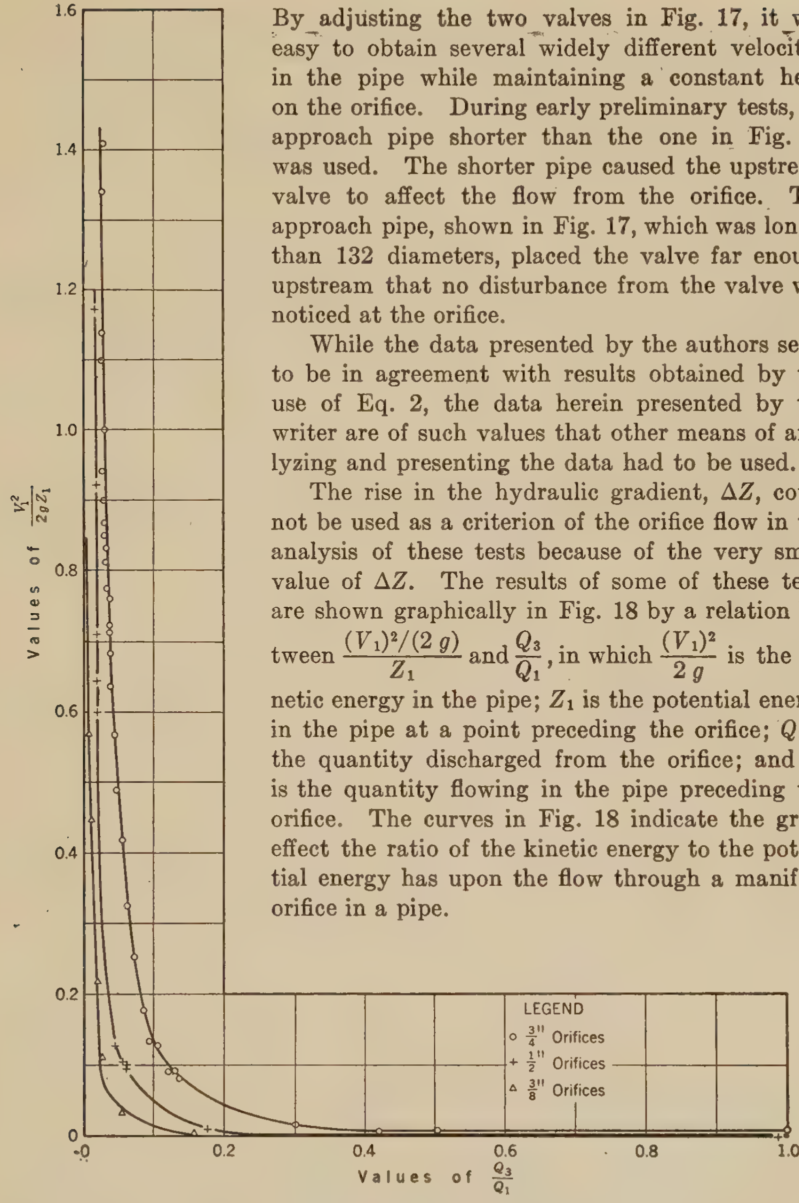

Fig. 18.—Relation Between $\frac{(V_1)^2}{2\,g\,Z_1}$ and $\frac{Q_3}{Q_1}$

Robert F. Kreiss,[18] Jun. Am. Soc. C. E.—Manifold tests whose results are in good agreement with the discharge distribution and with the over-all coefficients computed from calibrations of a single port confirm the usefulness of

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[18] Engr. (Hydr.), U. S. Engr. Sub-Office, Hydraulic Laboratory, Iowa City, Iowa.

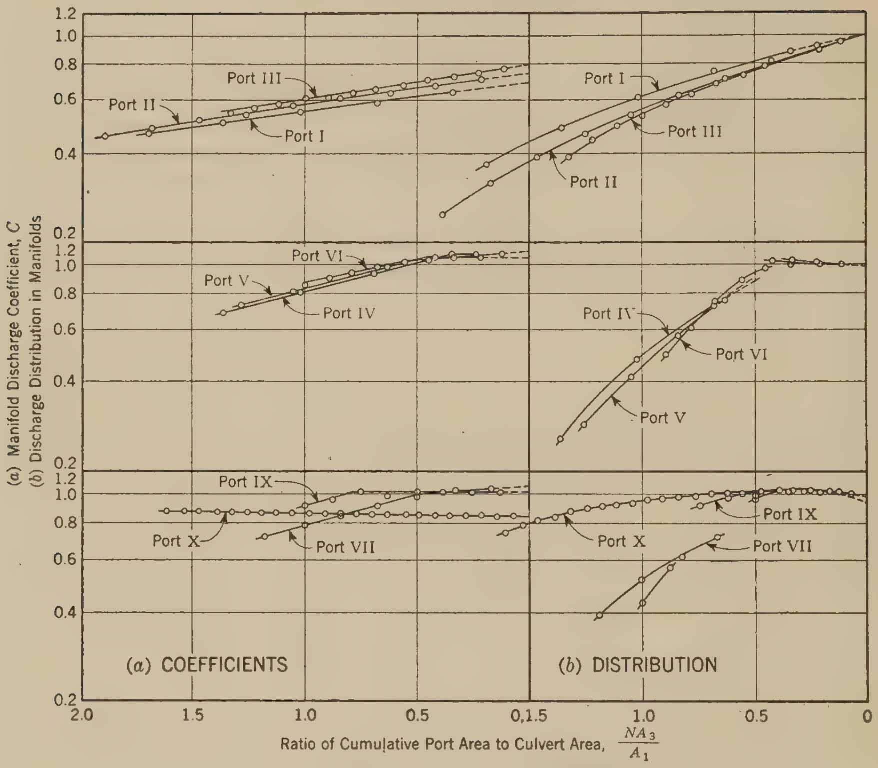

FIG. 19.—MANIFOLD DISCHARGE CHARACTERISTICS

TABLE 4.—VALUES OF C' AND C'' FOR FRICTIONLESS CONDUITS

Port	TYPE OF PORT DISCHARGE							
	Port Flowing Full				Port Flowing Partly Full			
	C'	C''	Range of Area Ratio		C'	C''	Range of Area Ratio	
			Minimum	Maximum			Minimum	Maximum
I	....	....	....	....	0.70	0.10	0	1.7
II	....	....	....	....	0.75	0.11	0	1.9
III	....	....	....	....	0.80	0.12	0	1.3
IV	1.09 +	0 +	0	0.34	1.27	0.19	0.34	1.4
V	1.07 ±	0 ±	0	0.47	1.33	0.20	0.47	1.3
VI	1.12	0.07	0	0.55	1.26	0.16	0.55	1.0
VII	1.06	0.05	0	0.5	1.28	0.21	0.5	1.2
IX	1.00	0.00	0	0.8	1.34	0.16	0.8	1.0
X	0.86	− 0.06	0	1.6	....	....	....	....
XI	0.96	0.06	0	1.3	....	....	....	....
XII	0.77 ±	0 ±	0	1.3	....	....	....	....

port calibrations. Inspection of the coefficient curves of Fig. 6 suggests that a plot of port coefficients against the ratio of average port velocity to conduit velocity would be significant in interpolating test results to ports of other sizes. Rather than present graphs of such relations, the writer has computed the characteristics of discharging manifolds from the calibrations on Ports I to VII. Two characteristics are shown as functions of the ratio of cumulative port area to conduit area in the semilogarithmic plots of Fig. 19. The results, which are the practical objectives of the single port calibrations, indicate that for ports of similar design the relative distribution of discharge and the over-all coefficients for manifolds are nearly single valued functions of the area ratio (compare results for Ports I to III and for Ports IV to VI). The discontinuity in the coefficients of the single port calibrations is apparent in the coefficients for the manifold, the discontinuity for Ports IV to VI occurring at an area ratio of about 0.4.

The computations for the manifold have been made essentially as outlined by the authors. The over-all coefficients assume a manifold acting as a unit such that the head producing discharge Q_1 through N ports of area A_3 is $H_d + \frac{(V_1)^2}{2\,g}$, or

$$C = \frac{Q_1}{N\,A_3\,\sqrt{2\,g}\sqrt{H_d + \frac{(V_1)^2}{2\,g}}} \quad \text{(18)}$$

the discharge, velocity, and head being those immediately upstream from the most upstream port. The results in Fig. 19 represent characteristics for a frictionless conduit. The discharge distribution in Fig. 19(*b*) refers to a unit discharge either at an area ratio approaching zero as a limit (Ports I to III) or at the downstream port (Ports IV to X).

An additional observation, worthy of note, is that the over-all coefficients, shown in Fig. 19(*a*), bear a relation to the area ratio as follows:

$$C = \frac{C'}{10^a} \quad \text{(19)}$$

in which, to simplify typography:

$$a = C''\,\frac{N\,A_3}{A_1} \quad \text{(20)}$$

In Eqs. 19 and 20, C' and C'' are constants dependent on the port design and on the two types of discharge mentioned by the authors. Computed manifold characteristics for ports of four other designs further check the validity of Eq. 19. Results for Ports IX and X are shown in Fig. 19. The values of the coefficients C' and C'', the range of area ratios to which the coefficients apply, and the type of discharge from the ports are given in Table 4 for Ports I to X. The tabulated values are subject to the limitations given by the authors (see heading, "Single Port Tests: Limitations"). It is evident that the discharge distribution and the efficiency of the manifold are dependent on the type of ports in the manifold.

Port IX is similar to Port VII (see Fig. 4) except that the dimensions of the opening are 1.52 in. (parallel to the conduit center line) by 2.97 in. Port X is a tapered flow-line port with one face extending from the floor of the conduit. Ports XI and XII extend downward from the conduit floor and then horizontally, the horizontal leg being of constant shape in Port XI and divergent in Port XII. (Details of design and port calibrations have been presented by Sydney O. Steinborn,[19] Jun. Am. Soc. C. E., and are on file in the Engineering Societies Library.)

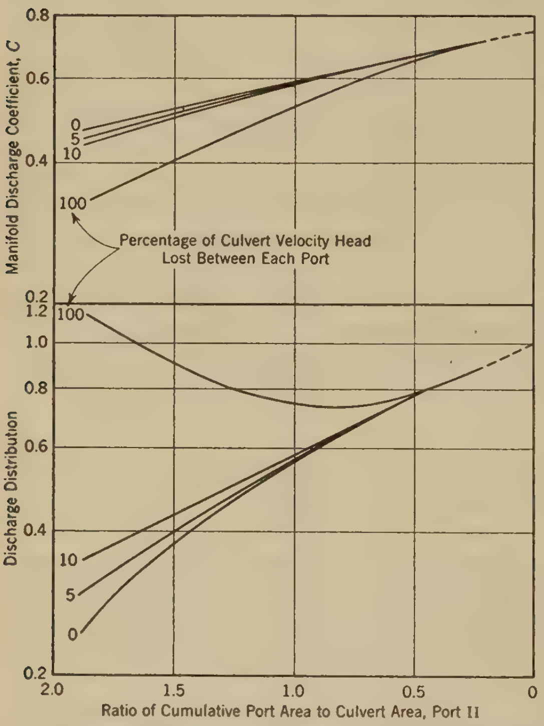

FIG. 20.—EFFECT OF FRICTIONAL LOSSES IN A CULVERT ON THE HYDRAULIC CHARACTERISTICS IN A MANIFOLD

An exponential equation was reported in 1937[20] for filling and emptying operations in lock models having port designs similar to Port X. In more recent (1944) tests on another lock model, the exponential equation was found applicable to steady flow calibrations and to varied flow operations as the number of lateral culverts (containing ports and connected to a main conduit) was varied. These results provide additional evidence of the relation that exists between single port calibrations and discharging manifolds.

The conduit friction has an appreciable effect on the characteristics of a discharge manifold when ports are widely spaced. An example for Port II is shown in Fig. 20. The conduit friction between each port is assumed as 0, 0.05, 0.10, and 1.00 of the velocity head immediately upstream of each port. (In Manning's equation these frictional values represent port spacings of roughly 0 ft, 1 ft, 2 ft, and 20 ft in a 6-in. square conduit with $n = 0.10$.)

The methods of testing and analysis presented by the authors are suitable for such designs as lock manifolds, storm sewers, sprinkler systems, and penstocks from a feeder line. With the graphs in Fig. 19 the design of manifolds, when ports are closely spaced, is further simplified and clarified.

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[19] "Manifold Studies, Part III," by Sydney Steinborn, The Panama Canal, Special Eng. Div., Diablo Heights, Canal Zone, December, 1941.

[20] "Laboratory Tests on Hydraulic Model of Filling and Emptying System for Proposed Watts Bar Project Lock," U. S. Engr. Sub-Office, Iowa City, Iowa, December, 1937.

Ralph G. Berk.[21]—A valuable method of predicting the hydraulic performance of lock manifolds is presented in this paper. The method can be applied to other types of manifolds, such as filter, bottom-lateral systems, sprinkling filter underdrain systems, irrigation piping distribution systems, and orange orchard, smudge-pot fuel-supply piping.

The authors' procedure is similar to the method employed for the calculation of backwater curves in open channels. Orifice coefficients are used for calculating the flow through each port in the manifold. These orifice coefficients are derived from data obtained in testing single port models. The test results are expressed by curves showing the variation of the orifice coefficient C with changes in the value of P (ratio of flow through the port to flow in the conduit).

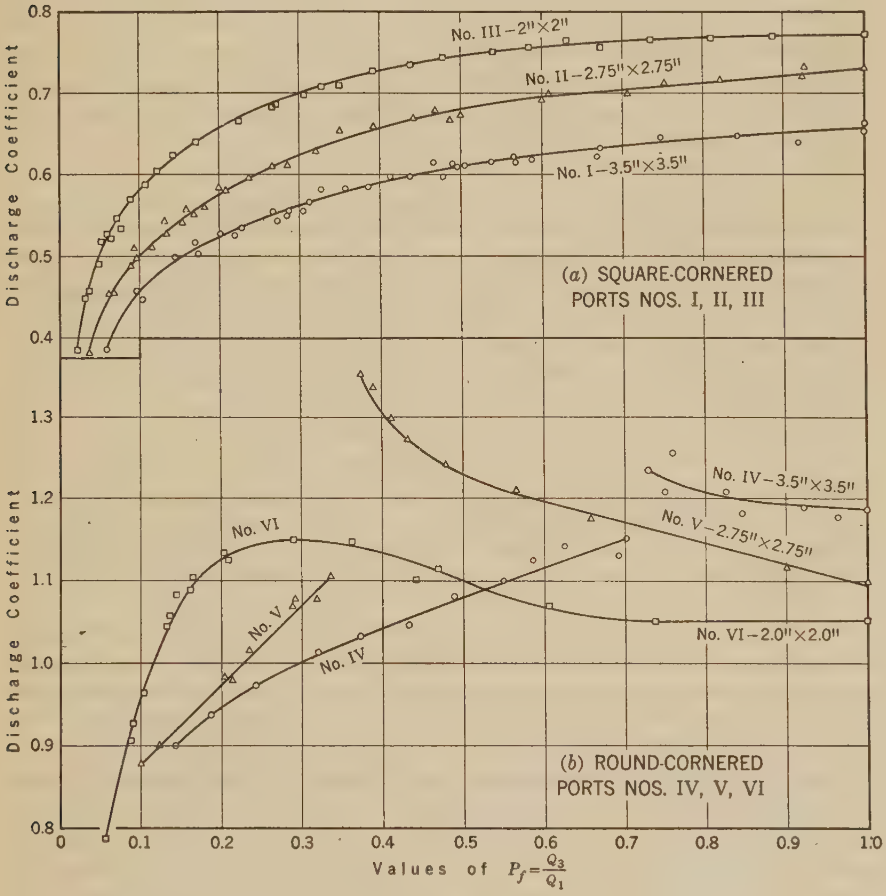

Fig. 21.—Curves of Port Characteristics, Ports I to VI, Inclusive

The orifice coefficients derived by the authors differ from the orifice coefficients ordinarily given in standard textbooks on hydraulics. The former reflect

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[1] Chf. Design Engr., Water Section, with Cons. Engrs., C. A., Caracas, Venezuela.

the inequality in flow through the ports of a manifold caused by abrupt changes in pressure and velocity of flow along the manifold tube, and by angularity of port discharge. None of these effects can be directly evaluated. Under the heading, "Single Port Tests: Analysis," the authors state that their derived orifice coefficients show no significant dependence upon variables other than P.

Despite the fact that the (C-P)-curves in Figs. 6 and 7 are reduced to a very small scale, it is still possible to observe that these curves assume different relative positions for varying sizes of ports. In Fig. 21 the same data are presented in a form designed for easier comparison. The (C-P)-curves for the square-cornered ports (Fig. 21(a)) indicate a consistent variation of C with the different sizes tested. This variation most likely conforms to the model laws expressed by Froude's criterion of dynamic similitude. However, the (C-P)-curves for round-cornered ports (Fig. 21(b)) show no consistent variation of C with the sizes tested. The shapes of these curves and the juxtaposition probably indicate no conformity with Froude's criterion.

In view of these differences, can one conclude that the geometrical shape of the port, together with variation in size, exercises some influence on the port orifice coefficient? Can tests with model ports of smaller scale provide enough information to permit the formulation of a model law defining the performance of a port of given shape in terms of its size? Discussing a specific example, what values of C for round-cornered ports (Ports IV, V, and VI) would the authors use in calculating the performance of a prototype lock manifold, with ports which should necessarily be much larger than the test ports? (The example shown in the paper is a calculation of the performance of a manifold having ports of the same size as those of the test model.)

To summarize, the writer believes that the authors do not present definite information for solving the practical problem of designing a prototype manifold larger in size than the test models and with port shapes other than square cornered. He must also conclude that, for purposes of practical design, it would be best to conduct tests on ports of prototype dimensions and shapes.

Ralph W. Powell,[22] M. Am. Soc. C. E.—As the authors state, the theoretical treatment of the pressures in manifolds has been very unsatisfactory. As one who, in the past, perhaps has helped to produce this situation, the writer feels moved to make another attempt to clarify the underlying hydraulics of the problem. Discussion will be limited to the case of a single port.

Fig. 22(a) is a diagram of a single manifold port with liquid entering through pipe 1, and leaving through pipe 2 and branch 3. The total pressure on section CD must exceed the total pressure on section AB by the loss of momentum per second, in the axial direction, of the liquid in the space ABCD. Since the liquid flowing into branch 3 will not cross section BC at right angles, it will carry away some of the momentum. The neglect of this condition vitiated the writer's former treatment.[23] Taking this fact into account,

$$\gamma A_e (Z_2 - Z_1) = \frac{\beta_1 Q_1 \gamma V_1}{g} - \frac{\beta_2 Q_2 \gamma V_2}{g} - \frac{\beta_3 Q_3 \gamma V_3 \tan \theta_3}{g} \quad \ldots (21)$$

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[22] Associate Prof. of Mechanics, Ohio State Univ., Columbus, Ohio.

[23] Discussion by R. W. Powell of "Hydraulics of Sprinkling Systems for Irrigation," by J. E. Christiansen, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), pp. 245–247.

in which θ_3 is the mean angle that the liquid crossing section BC makes with the axis of pipe 3, and β_1, β_2, and β_3 are the momentum factors in the respective pipes for correcting for the nonuniform velocity distribution. The other symbols are defined in the paper.

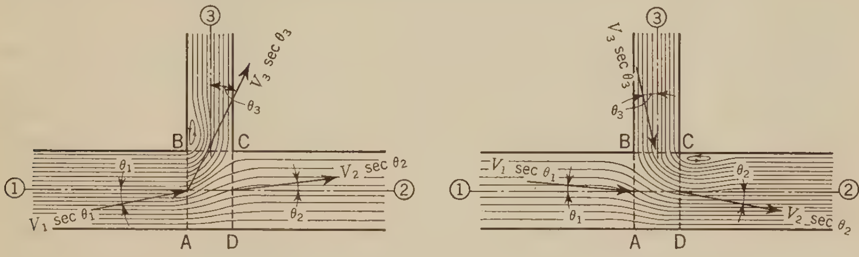
Eq. 21 reduces to

$$\frac{\Delta Z}{(V_1)^2/2\,g} = 2\left[\beta_1 - \beta_2\,(1 - P_d)^2 - (P_d)^2\,\frac{\beta_3\,A_c}{A_p}\tan\theta_3\right] \ldots\ldots (22)$$

Assuming that β_1 and β_2 are both unity, letting $\beta_3\,\frac{A_c}{A_p}\tan\theta_3 = \psi$ and equating to the value given by Eq. 2:

$$\psi = \frac{\beta_3\,A_c}{A_p}\tan\theta_3 = \frac{1 - k_d}{P_d} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

When P_d is small, the foregoing assumptions are probably nearly correct; but, for larger values of P_d, the factor β_2 will be much larger than unity.



(*a*) DISCHARGE PORT (*b*) INTAKE PORT

Fig. 22.—Diagrams of Idealized Flow

Using Eq. 23 and the data given or cited in the paper, the writer attempted to find the value of θ_3. About his only conclusion was that θ_3 approached 90° as P_d approached zero and that it also depended very much on the value of A_c/A_p. In fact, for any given value of P_d, when the side branch was made smaller with respect to the main pipe, θ_3 was increased enough that ψ remained more nearly the same for different values of A_c/A_p than did $\beta_3 \tan\theta_3$. In any given case, assuming a reasonable value of β_3, such as 1.05, an entirely reasonable value of θ_3 could be computed, but no law for forecasting the value could be deduced.

The case of inflow is shown in Fig. 22(*b*). The total pressure on section AB exceeds the total pressure on section CD by the gain in momentum per second, in the axial direction, of the liquid in the space ABCD. Thus:

$$\gamma\,A_c\,(Z_1 - Z_2) = \frac{\beta_2\,Q_2\,\gamma\,V_2}{g} - \frac{\beta_1\,Q_1\,\gamma\,V_1}{g} - \frac{\beta_3\,Q_3\,\gamma\,V_3\tan\theta_3}{g} \ldots (24)$$

which, assuming that β_1 is unity, reduces to

$$\frac{\Delta Z}{(V_2)^2/2\,g} = 2\left[\beta_2 - (1 - P_i)^2 - \beta_3\,(P_i)^2\,\frac{A_c}{A_p}\tan\theta_3\right] \ldots\ldots\ldots (25)$$

or letting $\frac{\Delta Z}{(V_2)^2/2\,g} = J$:

$$\frac{J}{2} = \beta_2 - (1 - P_i)^2 - (P_i)^2\,\psi \ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

In Eqs. 24, 25, and 26, β_2, β_3, and θ_3 are all unknown, and there seems little chance of determining them short of laborious experiments. By plotting J against P_i as in Fig. 7(*d*) and extending the curve back to $P_i = 0$, the value of β_2 can be estimated for small values of P_i. For Port VII the value of β_2 seems to be 1.12 and for the other ports it varies from 1.03 to 1.18. The data of Itaru Naramoto and Taijiro Kasai[24] gives, as nearly as can be estimated: $\beta_2 = 1.06$. The foregoing are for low values of P_i; when P_i becomes large, Ports I, II, III, VI, and VII show that the value of β_2 must increase, because otherwise the values of tan θ_3 become negative.

TABLE 5.—COMPUTATIONS FOR INTAKE PORT I

P_i (1)	J (2)	β_2 (3)	ψ (4)	θ_3 (5)
0.047	0.21	1.040	12.3	76°.6
0.088	0.34	1.040	4.9	59°.0
0.140	0.50	1.040	2.55	40°.9
0.152	0.53	1.040	2.42	39°.5
0.190	0.64	1.040	1.77	31°.0
0.240	0.80	1.040	1.07	20°.0
0.266	0.89	1.040	0.79	15°.1
0.320	1.03	1.040	0.62	11°.9
0.388	1.17	1.041	0.54	10°.4
0.429	1.32	1.043	0.31	6°.0
0.489	1.44	1.050	0.29	5°.6
0.567	1.69	1.087	0.171	3°.3
0.627	1.94	1.130	0.053	1°.0
0.668	2.06	1.170	0.067	1°.3
0.739	2.26	1.245	0.086	1°.7
0.768	2.45	1.290	0.019	0°.4

Table 5 gives a possible set of values for Port I. The values of P_i and J are scaled from the plotted values given in the authors' original paper. Only sixteen of the thirty points have been included, but they are distributed throughout the range of the experiment. Values of β_2 have been assigned arbitrarily, but in such a way as to give a smooth curve when plotted against P_i. The value of β_3 was taken as 1.00, and $A_c/A_p = 36/12.25 = 2.94$. The resulting values of θ_3 seem to be reasonable. Unfortunately, the writer has not been able to devise any equation that will forecast the values of β_2 and ψ or θ_3. For ports of the same proportions as those used by the authors, designs may be based on experimental results. For other proportions, new model tests will be needed.

EDWARD SOUCEK,[25] ASSOC. M. AM. SOC. C. E., AND E. W. ZELNICK,[26] JUN. AM. SOC. C. E.—As the principal conclusions of the paper appear to have been accepted by all who discussed it, this closure will be confined to a brief comment on each discussion and to clearing up a few minor points concerning which some difference of opinion appears to exist.

Mr. Pietrkowski enlarges upon the suggestion that the theoretical treatment adopted in the paper is related to the analysis of flow in side channel spillways. In his problem, the lateral discharge is out of the channel rather than into it. Although the basic principles are the same in the two cases, the direction of flow requires consideration as it appears that the conditions necessary for $k_d = 1$ cannot be obtained whereas $k_i = 1$ is quite possible. This point will be considered further in connection with Professor McNown's discussion. Mr. Pietrkowski's references concerning the conditions under which Bernoulli's theorem may be applied will be valuable to those who wish to explore the limitations of this important principle.

Professor McNown's study of the manifold problem is lucid and comprehensive; for research purposes, it may well supplant the brief theoretical study which appeared in the paper. For use in design, either treatment can be considered merely to support the observation that the characteristics of any port and culvert combination are expressible in terms of P_d and P_i. Professor McNown suggests clarification of the statement that "* * * Bernoulli's theorem is not applicable to the phenomenon under consideration, although a modified energy equation might be used." Bernoulli's theorem with a head-loss term is probably satisfactory for the analysis of discharge manifolds—tests of Boulder Dam penstocks were analyzed in this manner by the U. S. Bureau of Reclamation.[7] In intake manifolds or side channel spillways where flows from sources of differing energy content converge and mix in the culvert or channel as the case may be, Bernoulli's theorem would have to be modified by weighting the head from the two sources in proportion to the respective discharges, even if energy losses did not occur. In the side channel spillway, the energy of the flow received laterally is properly considered to be lost; with vanes to direct the inflow, a large part of the energy could be retained. Closely analogous conditions occur in an intake manifold. In connection with use of the energy principle for discharge manifolds, the Oakey-A curve indicates an energy gain in the direction of flow. The consistency of the A, B, C, and D curves makes experimental error an extremely unlikely explanation of the phenomenon. Professor McNown's other suggestion—that the lateral bleeds away enough low velocity water to increase the energy content of the remainder—seems more logical. However, there is another possibility—that in some manner mechanically similar to that in which water does work on a turbine blade, the lateral discharge actually yields a portion of its energy to the conduit flow. If future research should indicate that this is the correct explanation, it would be preferable to use the momentum principle for both intake and discharge manifolds.

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[24] "On the Loss of Energy at Impact of Two Confined Streams of Water," by Itaru Naramoto and Taijiro Kasai, *Memoirs of the College of Engineering*, Kyushu Imperial Univ., Fukuoka, Japan, Vol. VI, 1931–1932, p. 189.

[25] With Office, Chf. of Engrs., Washington, D. C.

[26] With The Panama Canal, Balboa Heights, Canal Zone.

Professor McNown stated that the mention of $k_d = 1$ has no practical significance. The remark in the paper was applied to insertion of values in Eq. 2 and Eq. 3, and consequently no practical significance was indicated. It is clear from the text that values of $k_d = 1$ were not anticipated. Moreover, the statement pertained equally to k_i and k_d; $k_i = 1$ is decidedly significant. It is well, however, that Professor McNown has clarified the matter. Mr. Pietrkowski has used $k_d = 1$ for analysis of a side spillway but, if the weir is constructed with side-walls so as to deflect the flow at right angles to the channel axis as assumed in the derivation, k_d will have a value less than unity. Conversely, if the weir is sharp-edged, the flow will not be diverted at right angles to the channel axis and a basic condition of the derivation—loss of momentum by the lateral flow—will not result. In either case, a rise in water levels substantially less than twice the difference in velocity heads may be anticipated.

It appears that Professor McNown may overestimate the significance of the correspondence of his c (= constant)-curves with the Enger-Levy curve. The theoretical bases for this curve involves flow through a longitudinal slit in a pipe. Although the substantial correctness of the curve is indicated by tests,[10] the agreement is not so close as to warrant adoption of the c (= constant)-curves at this time. The Oakey curves show a more rapid drop in the relative pressure rise for high values of P_d than do the most nearly comparable

c (= constant)-curves. The Bureau of Reclamation curve corresponds quite closely to the curve for $k_d = 0.3$ over a rather wide range. Of course, it is not necessary to determine the mathematical nature of the pressure-change curves, as they may be determined experimentally. Discharge from a longitudinal slit in a pipe—the basis for the Enger-Levy curve—is a phenomenon closely related to flow from a discharge manifold, but there are significant differences. Some of these were revealed in 1931 in discussions of a paper by Jakob Kunz.[27] The jet discharged from a continuous longitudinal slit suffers no loss of momentum in the direction parallel to the pipe axis but each element retains a longitudinal velocity component equal to the pipe velocity at the point of issue. The normal component of velocity appears to take place at the expense of pressure head only (it is assumed that the slit is at an extremity of a horizontal diameter). Shock loss is conspicuously absent. Under these conditions, there is every reason to expect that the pressure changes would occur in accordance with Bernoulli's theorem, although the momentum principle was used in deriving the curve. In a discharging manifold, shock loss (or unbalanced port pressure if the momentum principle is used) is the essence of the problem. Despite these minor differences of opinion, Professor McNown's basic ideas are accepted and endorsed—especially appreciated is his thorough treatment of the energy concept which was merely suggested in the paper. Although it is quite generally realized that the momentum and energy principles are but different statements of Newton's Second Law of Motion, statements that one or the other is exclusively applicable are fairly common. No statement in the paper was intended to suggest exclusion of the energy principle but the clarification suggested and largely supplied by Professor McNown was highly desirable.

Professor Edwards' discussion is most welcome; it was he who initiated the test program in the face of considerable evidence that the theory could not be developed in time for use on the Third Locks Project. As he states, the studies and tests did much to expedite the design and testing of complete lock models. The characteristic curves for Ports I to VI, inclusive, were much better defined than those for Port VII as shown in Fig. 21 by Mr. Berk but the verification tests were made for Port VII only. It is for this reason that Port VII is presented in detail. The port described by Professor Edwards has several desirable characteristics, the foremost of which are its high capacity, a relatively constant discharge coefficient over a considerable range of P_d, and elimination of an undesirable velocity component—lack of the latter characteristic to some extent defeats the purpose of the bottom-filling system of the original Panama locks. The similarity of the ports used in the MacArthur Lock and the one described by Professor Edwards has been noted. There is little doubt that this port shape is largely responsible for the quiet filling conditions in the MacArthur Lock—in addition to its other advantages, the flattened jet has a large surface area which is conducive to a high rate of energy dissipation and tends to minimize surface disturbances.

The curves introduced by Professor Lansford provide some basis for the analysis of flow from a perforated pipe but should eventually be supplemented by tests on a larger scale which will enable calculation of the pressure gradients.

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[27] "Jets from Manifold Tubes," by Jakob Kunz, *Transactions*, A.S.M.E., 1931, APM-53-14, p. 181.

Although Fig. 18 shows the "great effect the ratio of the kinetic energy to the potential energy has upon the flow through a manifold orifice in a pipe," it is not evident upon inspection that the data corroborate a functional relationship between the discharge coefficient and the ratio P_d. Starting with the basic orifice equation, successive transformations show that such is the case (Z_1 and H_d are equivalent).

$$C_d = \frac{Q_3}{A_p \sqrt{2\,g\,Z_1}} = \frac{P_d\,Q_1}{A_p \sqrt{2\,g\,Z_1}} = \frac{P_d\,A_c\,V_1}{A_p \sqrt{2\,g\,Z_1}} = \sqrt{\left(\frac{A_c}{A_p}\right)^2 (P_d)^2 \frac{(V_1)^2}{2\,g\,Z_1}} \quad \ldots (27)$$

Since A_c/A_p is a constant for any orifice and pipe combination, and as Fig. 18 shows that $(V_1)^2/(2\,g\,Z_1)$ is a function of P_d for each of the orifices tested, it follows that C_d for each orifice is expressible as a function of P_d alone. This experimental support of the theory presented in the paper is greatly appreciated.

The tests reported in the paper would be of little value if they did not lead to a method of designing manifolds. Mr. Kreiss has contributed a very practical means of applying the theory. In Eq. 18, the quantity $N\,A_3$ is the gross area of the ports downstream from the section at which the energy head is determined. In a simple manifold, for example, a lock culvert with wall ports, the energy head producing discharge is simply the gross head minus entrance losses. A few preliminary calculations will indicate whether or not friction losses are significant in any particular case—Fig. 20 serves as a guide for estimating the effect of friction losses. It should not be overlooked that calculations similar to those in Table 2 must be made in the preparation of Table 4. Thanks are due to Mr. Kreiss for undertaking this task and for presenting data on several more ports. It is gratifying to learn that he has made additional tests which tend to confirm the value of single port data in the design of manifolds.

Mr. Berk's comments are particularly interesting because they illustrate an error that is very likely to occur if it is forgotten that the characteristic curves are for a particular port and culvert combination—not for the port alone. Under the heading, "Single Port Tests: Limitations," it was stated that "The data are based on tests of ports of particular sizes, shapes, and lengths—attached to a conduit of a particular size and shape." Elsewhere in the paper, the effects of conduit area and conduit velocity were mentioned. Eq. 27, applicable to Professor Lansford's data, further shows that the culvert size has an important effect upon discharge coefficients.

The concept of a "port and culvert combination" clarifies the questions raised by Mr. Berk. The verification tests were based upon ports of the same size as the one used in single-port tests of Port VII; therefore, as Mr. Berk indicates, scale effects were not investigated. However, the phenomena considered are essentially dynamic in character and there is little question that Froude's law is applicable. The inconsistencies which Mr. Berk believes to exist are entirely due to incomplete geometric similarity in his comparisons. All data are based on a 6-in. square conduit. The values of C which would be used "in calculating the performance of a prototype lock manifold, with ports which should necessarily be much larger than the test ports" are the same as those obtained in small-scale model tests, but the characteristic curves would have to be based on complete geometric similarity. The culvert size and shape must not be ignored. In some cases, allowance for departures from geometric similarity could be determined by interpolation based on diagrams similar to Fig. 21.

Although Mr. Berk's conclusion regarding the value of small-scale tests must be rejected, his sincere discussion goes far toward clarifying the concept of the "port and culvert combination." Table 1 shows that Ports I to VI, inclusive, are all geometrically dissimilar. Even if the lengths of Ports I to III, inclusive, had been proportional to the heights and widths, the fact that all ports were attached to a 6-in. square conduit would prevent correspondence of the characteristic curves. Similar reasoning is applicable to Ports IV to VI, inclusive. Although interpolation in Fig. 21 between the curves for the round-cornered ports would be more difficult and less reliable than for the square-cornered ports, it would suffice for some purposes. For precise results, additional single port tests might be necessary.

The conclusions were limited specifically to the applications of single port test data. Although "definite information for solving the practical problem of designing a prototype manifold larger in size than the test models and with port shapes other than square cornered" is much to be desired, it is far beyond the scope of the paper. Mr. Berk raised the interesting question of scale effects, but this too is beyond the scope of the paper. However, there is much reason to believe, from the ever-increasing number of model-prototype comparisons, that scale effects would be very small in the present case. As explained in the paper, the allowances for conduit friction are independent of scale effect, so one important source of error in model indications has been eliminated.

Professor Powell has extended the analysis to cover momentum coefficients and angularity of velocities. It is unquestionable that a complete understanding of the mechanics of manifold hydraulics must be developed along these lines. Fortunately, design of manifolds need not await the isolation and analysis of these factors as their effects may be grouped together and determined experimentally. Table 5, prepared by Professor Powell, appears to indicate reasonable variation in angles and momentum coefficients for Port I although, as he states, determination of exact values would require "laborious experiments."

In closing, it should be emphasized that the use of characteristic curves for study of any port-and-culvert combination is simply a convenient method of reducing the number of variables. The fact that the characteristic curves may be plotted in terms of P_d and P_i is not a fortunate accident but has the same physical basis as the dimensionless representation of pressure variations in a bend. The ratios P_i and P_d establish the arrangement of stream paths and these patterns remain essentially unchanged by variations in the absolute values of the several variables if proportionality is maintained. One additional point which may merit mention is the fact that an entire lateral in a bottom-filling lock can be treated merely as a port on the longitudinal culvert. The interest shown in the paper is sincerely appreciated.

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AMPLIFIED SLOPE DEFLECTION

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WITH DISCUSSION BY MESSRS. LEON BESKIN, JOHN E. GOLDBERG, CAMILLO WEISS, WILLIAM C. SPIKER, ARTHUR B. MCGEE, JAROSLAV POLIVKA, A. A. EREMIN, AND RALPH W. STEWART.

SYNOPSIS

Slope deflection, as presented in current textbooks, is encumbered by the necessity of: (*a*) Reversing the signs of certain moments in writing equations; (*b*) Studying the significance of double signs (namely, $\mp$ and $\pm$) which are set up in the standard equations; and (*c*) a set of four standard equations when two are ample.

The purpose of this paper is to eliminate these difficulties, to present a complete physical interpretation of the components of slope-deflection equations, and to show how to use these components in such a manner that, for many problems, the algebraic work required will be only one half that required by the procedure taught in current texts.

DEFINITION OF SYMBOLS AND OTHER DATA

The abbreviation FEM, as used in this paper, denotes "fixed-end moment"—the moment that would exist at an end of a beam if both ends were fixed against rotation. The location of the FEM and the identification of the span to which it pertains are indicated by a subscript. Thus, FEM_{AB} means the FEM at end A of span AB; FEM_{DC}, the FEM at end D in span CD, etc.

The abbreviation FHEM means the fixed-end moment at one end of a beam when the opposite end is hinged, subscripts being used as for FEM.

The symbol Δ is used herein to represent the angle between the end tangents of a beam due to the application of a moment at one end when the other end is hinged. The reason for the selection of this symbol is that it is in common use in treatises on railway curves (notably transition spirals)[2] to indicate the angle between the end tangents of a curve. It measures the amount of curva-

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NOTE.—Published in September, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Bridge Engr., City of Los Angeles, Los Angeles, Calif.

[2] "The Railway Transition Spiral," by A. N. Talbot, McGraw-Hill Book Co., Inc., New York, N. Y. (6th Ed., 1927, out of print).

ture in the elastic curve of a flexed beam due solely to the end moment to which it is related.

The symbol θ is used to represent the angular change in direction, due to flexure, of the neutral axis of a beam at a support. Angles Δ and θ are identified as to location by subscripts.

Symbol A_o is the area of the $\frac{M}{E\,I}$-diagram $\left(\text{or } \frac{M}{I}\text{-diagram}\right)$ of a span when the span is considered as simply supported—that is, free from end restraints except simple supports acting at right angles to the beam.

A traverse of the elastic curves of a structure is a series of straight lines which delineate all the θ-angles and Δ-angles, and also, when necessary or desirable, the A_o-angles.[3] The lines of a traverse which pass through supports are tangent to the elastic curve at the supports.

Four important properties of an elastic curve traverse are:

(1) Each Δ-angle and each A_o-angle is located where the center of gravity of the $\frac{M}{E\,I}$-diagram appurtenant to such angle is projected upon the neutral axis of the member.

(2) Each Δ-angle or A_o-angle is numerically equal to the area of its $\frac{M}{E\,I}$-diagram. This follows because the Δ-angle or A_o-angle and the area of the appurtenant $\frac{M}{E\,I}$-diagram each measure the curvature in the beam due to the same moment.

(3) In any triangle formed by the lines of a traverse and the unsprung axis of a beam the angles are proportional to the opposite sides. The altitude of a traverse triangle for any ordinary beam is so small that the angles may be taken as equal to either their sines or their tangents. It is usually convenient in making computations to consider the tangent of an angle as representing its magnitude.

The basic stiffness of a beam is the magnitude of the end moment necessary to produce a unit Δ-angle, or the ratio of an end moment to the Δ-angle it produces. If a Δ-angle can be evaluated from the geometry of an elastic curve traverse it is converted into an end moment by multiplying it by the basic stiffness of the member. A tapered member will have a different basic stiffness for each end.

(4) Property (3) leads to the theorem that any Δ-angle multiplied by the appurtenant beam stiffness equals the adjacent end moment, and any end moment divided by the stiffness equals the appurtenant Δ-angle.

It is probably unnecessary to state that an elastic curve traverse, if correctly used, will close geometrically the same as a land survey. It should be mentioned that with amplified slope deflection, which gives geometrical values to the components of slope-deflection equations, the deflections of a structure are easily computed as a by-product of the stress analysis.

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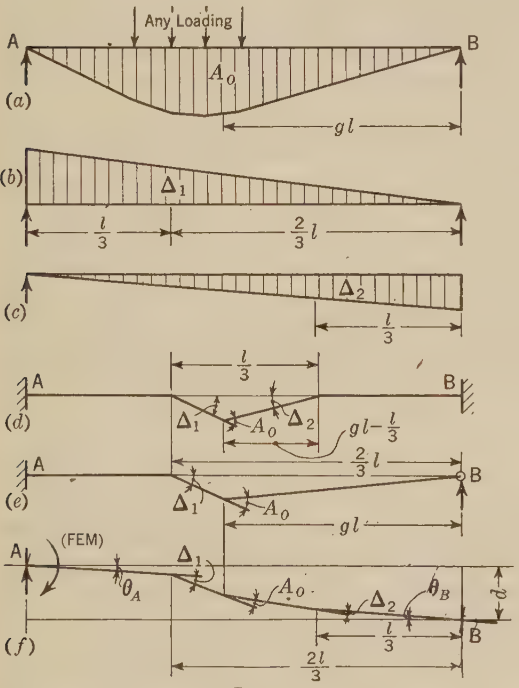
[3] "Relative Flexure Factors for Analyzing Continuous Structures," by Ralph W. Stewart, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 521 (see also references therein for demonstrations of the use of the elastic curve traverse).

Signs.—In slope-deflection equations those moments and joint movements which produce or tend to produce clockwise rotation at the joint for which an equation is written are taken as positive; the reverse, negative. In a traverse computation which progresses along a series of angles, the angles which turn the alinement of the traverse toward the right may be taken as positive; and those turning it toward the left, negative.

DEMONSTRATION PROBLEMS

Let member AB in Fig. 1 be a beam of constant section. Its basic stiffness will be $\frac{2\,E\,I}{l}$ since this is the ratio of an end moment to the area of the $\frac{M}{E\,I}$-diagram which it produces. Figs. 1(*a*), (1*b*), and (1*c*) show, respectively, the $\frac{M}{E\,I}$-diagrams for the simply supported condition, the condition of restraint at the left end only, and the condition of restraint at the right end only, each moment acting along the full length of the beam.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B
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A
B
(d)
$\frac{l}{3}$
Δ_1
A_0
Δ_2
$gl-\frac{l}{3}$
A
$\frac{2}{3}l$
B
(e)
Δ_1
A_0
gl
A
(FEM)
Δ_1
θ_A
A_0
Δ_2
θ_B
d
B
(f)
$\frac{l}{3}$
$\frac{2l}{3}$

FIG. 1

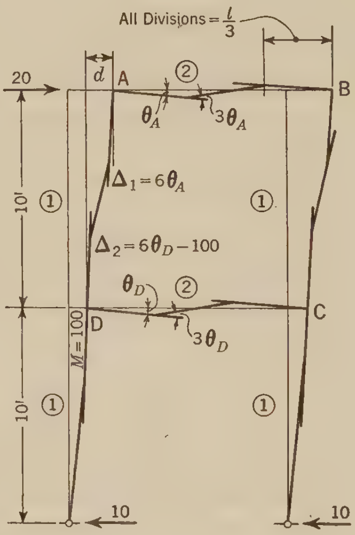
All Divisions = $\frac{l}{3}$
d
A
②
B
20
θ_A
$3\theta_A$
$\Delta_1 = 6\theta_A$
10'
①
①
$\Delta_2 = 6\theta_D - 100$
θ_D
②
C
D
M = 100
$3\theta_D$
10'
①
①
10
10

FIG. 2

Fig. 1(*d*) represents a traverse of the elastic curves of this beam with both ends fixed so that the θ-angles are zero. From traverse property (3) the angle Δ_1 will be equal to

$$\Delta_1 = \left[\left(g\,l - \frac{l}{3}\right) \div \frac{l}{3}\right] A_o = (3\,g - 1)\,A_o \ldots\ldots\ldots\ldots (1)$$

From traverse property (4) multiply Eq. 1 by the stiffness of the beam $\frac{2\,E\,I}{l}$ and obtain

$$\text{FEM}_\text{A} = \frac{2\,E\,I\,A_o}{l}\,(3\,g - 1)\ldots\ldots(2)$$

Fig. 1(*e*) is the traverse with the left end of the beam fixed and the right end hinged. From traverse property (3),

$$\Delta_1 = (g\,l \div \tfrac{2}{3}\,l)\,A_o = \tfrac{3}{2}\,g\,A_o\ldots\ldots(3)$$

and from property (4) of the traverse

$$\text{FHEM}_\text{A} = \frac{3\,E\,I\,A_o\,g}{l}\ldots\ldots(4)$$

Now consider the beam as one of a series of continuous spans so that the ends are subject to restraint less than fixation and some joint rotation can occur. Also, let the right end deflect a distance *d*. The traverse will then take the form of Fig. 1(*f*), which is drawn for loading and restraint conditions such that both Δ-angles, under slope-deflection sign rules, relate to positive end moments.

From the geometry of Fig. 1(*f*) the following equations may be written:

$$\theta_A + \Delta_1 - A_o - \Delta_2 - \theta_B = 0\ldots\ldots(5a)$$

and

$$\theta_A\,l + \tfrac{2}{3}\,l\,\Delta_1 - g\,l\,A_o - \tfrac{1}{3}\,l\,\Delta_2 = d\ldots\ldots(5b)$$

Solving,

$$\Delta_1 = A_o\,(3\,g - 1) - 2\,\theta_A - \theta_B + 3\,\frac{d}{l}\ldots\ldots(6)$$

Multiplying the Δ-value in Eq. 6 by the beam stiffness and letting $K = \frac{1}{l}$ it is found that

$$M_{AB} = \text{FEM}_\text{AB} - 2\,E\,K\left(2\,\theta_A + \theta_B - 3\,\frac{d}{l}\right)\ldots\ldots(7a)$$

If the beam is hinged at end B the angle Δ_2 will become zero, the traverse showing as a straight line from A_o to the right end. Solving for this condition by a procedure similar to that used to derive Eq. 7*a* it is found that:

$$M_{AB} = \text{FHEM}_\text{AB} - 2\,E\,K\left(1.5\,\theta_A - 1.5\,\frac{d}{l}\right)\ldots\ldots(7b)$$

Note that Eqs. 7 are the conventional slope-deflection equations found in the various textbooks except that all but two[4] of the textbooks, which the writer has consulted, reverse the sign of the fixed-end moments and enter the second term as positive. This leads to considerable confusion in the practical use of the equations. It is well known to all users of the slope-deflection method that the final moment at the end of a beam for which the moments are not dominated by joint movements is less than the fixed-end moment. The form in which Eqs. 7 are written may be expressed by the following rule:

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[4] "Statically Indeterminate Structures," by John I. Parcel and G. A. Maney, John Wiley & Sons, Inc., New York, N. Y., 2d Ed. 1936; also "Analysis of Statically Indeterminate Structures," by Clifford D. Williams, International Textbook Co., Scranton, Pa., 1943.

> "The moment at A in a beam AB is equal to the moment which would occur at A if the beam were fixed, less the relaxation in the moment due to the joint rotations, plus the moment due to the deflection of a support."

Under this rule the fixed-end moment is entered with its true sign. Note also that if the $\frac{d}{l}$-term is removed from the parentheses it will also have its true sign. This is logical as both the FEM and the deflection d create clockwise moment at end A in Fig. 1(f). The joint rotations θ reduce this moment and are therefore negative. However, amplified slope deflection will operate with either the direct or the reversed use of signs so any analyst can use his own system.

The important feature of amplified slope deflection developed by this paper is that, in any slope-deflection equation written in the form of either Eq. 7*a* or Eq. 7*b*, the expression inside the parentheses is the value of a Δ-angle in an elastic curve traverse. This is true for all special conditions such as tapering members and "semi-rigid" riveted or welded joints at intermediate points along members. The use of this property of the parenthetical expression in a slope-deflection equation offers short-cut solutions for many problems. For convenience in wording this parenthetical term will be called the "Δ-term."

A demonstration of a solution using the geometrical values of Δ-terms is presented in Fig. 2, which shows a symmetrical two-story frame subjected to a lateral force. The numerals in circles near the centers of the members show their relative stiffness values. From property (3) of the traverse the Δ-angles in the beams will be $3\,\theta_A$ and $3\,\theta_D$, as shown. (Note: In the conventional slope-deflection equation for beam AB the parenthetical term is $(2\,\theta_A + \theta_B)$. Since, due to symmetry, θ_A and θ_B are equal, this equals $3\,\theta_A$.) If a conventional slope-deflection formula like Eq. 7*a* were written for member AD, the Δ_1-term would be $2\,\theta_A + \theta_D - 3\,\frac{d}{l}$. This is inconvenient as it contains three unknown quantities. Amplified slope deflection offers a simpler value for Δ_1 containing only one unknown quantity derived as follows:

From traverse property (4)—

$$M_{AB} = 2\,(3\,\theta_A) = 6\,\theta_A \quad \text{(8a)}$$

Balancing moments about A—

$$M_{AD} = 6\,\theta_A \quad \text{(8b)}$$

and from traverse property (4)—

$$\Delta_1 = 6\,\theta_A \quad \text{(8c)}$$

In actual practice this procedure is shortened to

$$\frac{2}{1}\,(3\,\theta_A) = 6\,\theta_A \quad \text{(9)}$$

For joint D the sum of the column moments must balance the beam moment.

The bottom column moment is one half the lateral force times the column height, which is equal to 100. The beam moment is 6 θ_D. Subtracting the lower column moment from M_{DC} and observing that the relative stiffness of the upper column is 1:

$$M_{DA} = \Delta_2 = 6\,\theta_D - 100 \qquad (10)$$

Column AD carries one half the lateral force and the sum of its end moments will be equal to half the lateral force in the structure multiplied by the column height, from which

$$6\,\theta_A + 6\,\theta_D - 100 = 100 \qquad (11)$$

Next, write the equation for traverse angle closure in member AD as follows:

$$\theta_A + 6\,\theta_A - (6\,\theta_D - 100) - \theta_D = 0 \qquad (12)$$

Eqs. 11 and 12 can be solved simultaneously with a little less than half the algebraic work necessary if the conventional slope-deflection equations involving three unknown quantities were used. Solving, $\theta_A = 9.524$; $\theta_D = 23.81$; $M_A = 57.14$; and $M_{DA} = 42.86$; $M_{DC} = 142.86$.

If a pin connection is introduced at one end of a beam DC the unsymmetrical frame so formed will require six simultaneous equations for its solution by conventional slope deflection. Amplified slope deflection will reduce the number of equations to three.

If tapering members are involved the Δ-angle points will not be at the one-third length points. Their locations, and also the basic stiffnesses of the members, must be determined by a method described by the writer elsewhere[5] unless tables, which give coefficients for these beam constants, are available.

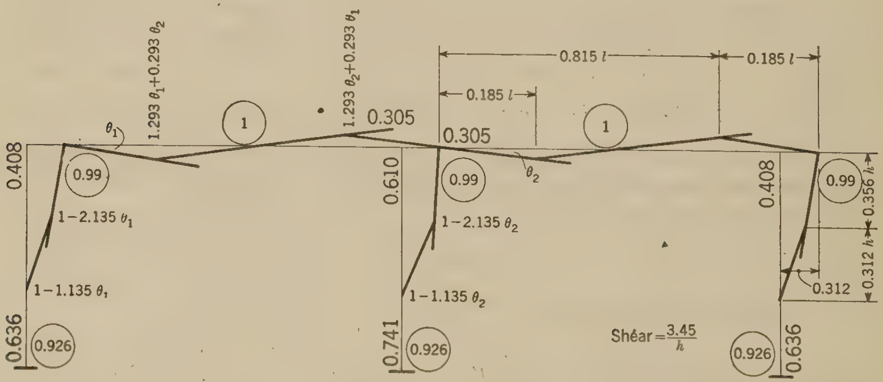
1.293 θ_1+0.293 θ_2
1.293 θ_2+0.293 θ_1
0.815 l
0.185 l
0.185 l
0.305
0.305
1
1
θ_1
θ_2
0.408
0.99
0.610
0.99
0.408
0.99
0.356 h
1−2.135 θ_1
1−2.135 θ_2
0.312 h
0.312
1−1.135 θ_1
1−1.135 θ_2
0.636
0.926
0.741
0.926
Shear = $\frac{3.45}{h}$
0.926
0.636

FIG. 3

Fig. 3 shows the solution of a symmetrical two-span frame[6] in which the beams are connected to the columns with semi-rigid joints; also the columns have semi-rigid connections to the foundations. This moves the Δ-points away from the one-third length points and alters the basic stiffness of each member. The beam and column properties of Fig. 3 were computed by a method demonstrated elsewhere.[7] Next, the lateral deflection of the tops of the columns was, for convenience, taken as 0.312 which is the coefficient for the distance between the column Δ-angles.

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[5] *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 1200.

[6] *Ibid.*, Vol. 107 (1942), p. 1029, Fig. 19(c).

All beams and column Δ-angles were then computed by the procedure used in discussing Fig. 1. Two slope-deflection equations were then written—one to express the condition of moment balance at the top of an end column and one to balance moments at the top of the center column.

The solution of these two simultaneous equations for the two unknown θ-angles enabled the moments to be evaluated and written on the frame as shown by large numerals.[7]

In conventional slope deflection the number of simultaneous equations required to solve a given problem is definite. In amplified slope deflection the number may be varied some in accordance with the analyst's judgment. In fact, all of the problems presented in this paper can be solved by using the elastic curve traverse in a manner which entirely avoids the use of simultaneous equations and does not use successive approximations.

One caution should be observed in the use of relative stiffness values. If all members have the same E-value and are of constant section the value of its $\frac{I}{l}$-ratio may be used to express the relative stiffness of each member. However, if some of the members in a structure have tapered sections and some have constant sections, it is necessary to use $\frac{2\,I}{l}$ for the stiffness factor of the constant section members. This is why Eq. 7*b* is written in the form shown with the factor 2 outside the parentheses.

CONCLUSIONS

If the slope-deflection equations are considered to be functions of only end slopes and deflections, their usefulness is substantially less than if the fact is recognized that they also are functions of Δ-angles which measure the changes in direction of the elastic curve within the beam. By recognizing and using all the angle components in slope-deflection equations, the computation labor required to use them is reduced by half for certain problems. Simplification of sign rules and a change in form and reduction in the number of standard equations also facilitate the work.

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[7] *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 1028.

DISCUSSION

Leon Beskin,[8] Assoc. M. Am. Soc. C. E.—In so far as this paper lacks a definite sign convention, it tends to obscure, rather than to clarify, the analysis of rigid frames. For example, the writer has been unable to interpret the sign convention the author has chosen for moments.

In Fig. 2, it seems that at node A, the condition of equilibrium, by Eqs. 8*a* and 8*b*, is

$$M_{AB} = M_{AD} \qquad (13a)$$

whereas, at node D, by Eq. 10,

$$M_{DO} + M_{DA} = M_{DC} \qquad (13b)$$

In Eq. 13*b*, the subscript DO refers to the lower left-hand column of Fig. 2.

A widely adopted sign convention requires that positive moments rotate all nodes in the same direction, with the consequence that the sum of all moments at a node is zero. This is not satisfied by Eqs. 13. In the theory of continuous beams, moments at the two sides of a node are equal. In that case, Eqs. 13 should be written

$$M_{AD} = \pm M_{AB} \qquad (14a)$$

and

$$M_{DO} - M_{DA} = \pm M_{DC} \qquad (14b)$$

the sign in the right-hand quantity being plus or minus depending on whether the member is vertical or horizontal, respectively. Eqs. 13 do not satisfy this second sign convention.

The foregoing remarks apply also to Fig. 3. One may assume that the author's sign convention is: Along columns, positive moments tend to rotate nodes clockwise; and, along beams, positive moments tend to rotate nodes counterclockwise. This convention is consistent with the conditions shown in Figs. 2 and 3, but inconsistent with Fig. 1, in which FEM_A is shown clockwise. Also, on this basis, Eq. 7*a* is inconsistent with Eq. 8*a*.

The comments in the text following Eq. 7*b* are also erroneous:

> "It is well known to all users of the slope-deflection method that the final moment at the end of a beam for which the moments are not dominated by joint movements is less than the fixed-end moment."

The final moment can be greater or smaller than the fixed-end moment, as demonstrated by a continuous beam on three supports for which the moment at the center support has an intermediate value between the fixed-end moments calculated for each span independently—it is greater than one moment, smaller than the other.

The rule following the foregoing quotation is not clear, as its wording seems to apply to absolute values, although the succeeding sentence expresses a contrary opinion. Moreover, the moment due to the deflection may have any sign, independent of the fixed-end moments.

The end-moment formula (Eq. 7*a*), when applied to the end B, can be written either

$$M_{BA} = \mathrm{FEM}_{BA} + 2\,E\,K\left(2\,\theta_B + \theta_A - 3\,\frac{d}{l}\right) \qquad (15a)$$

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[8] Structures Engr., Consolidated Vultee Aircraft Corp., San Diego, Calif.

or

$$M_{BA} = \mathrm{FEM}_{\mathrm{BA}} - 2\,E\,K\left(2\,\theta_B + \theta_A - 3\,\frac{d}{l}\right) \dots\dots\dots (15b)$$

It has been demonstrated that a minus sign should appear in the expression of M_{BA} which means that Eq. 15*b* must be used. Eqs. 8*b* and 10 tend to confirm this interpretation, although their signs are generally inconsistent with those in Eqs. 7*a* and 15*b*.

The claim for simplicity resulting from the use of the amplified slope deflections is not established, as shown by the following comparison, referring to Fig. 2: Applying Eq. 7*a* to ends A and D of member AD,

$$M_{AD} - M_{DA} = -(\theta_A - \theta_D) \dots\dots\dots (16)$$

Similarly, applying Eq. 7*a* to members AB and CD, and expressing the node equilibriums,

$$-M_{AB} = 2\,(3\,\theta_A) = 6\,\theta_A = M_{AD} \dots\dots\dots (17a)$$

and

$$-M_{DC} - 100 = 6\,\theta_D - 100 = M_{DA} \dots\dots\dots (17b)$$

Adding Eqs. 17*a* and 17*b*, and noting from equilibrium considerations that the total moment in member AD is equal to 100, the resulting equation is identical with Eq. 11. By equating Eqs. 17*a* and 17*b* and substituting the difference in Eq. 16, Eq. 11 becomes identical with Eq. 12. The only difference as compared with the author's calculation is that the foregoing has taken care of sign conventions—namely, all moments that tend to rotate nodes counterclockwise are positive.

Because he eliminates moments, the author asserts that he uses two unknowns. Actually, there are five unknowns, the three additional unknown moments being eliminated rapidly. It is possible to present the calculation with one apparent unknown M_{DA}. Thus, by equilibrium conditions and by deformation considerations:

$$-\frac{\theta_A}{6} = M_{AB} = -M_{AD} = -100 + M_{DA} \dots\dots\dots (18a)$$

and

$$-\frac{\theta_D}{6} = M_{DC} = -M_{DA} - 100 \dots\dots\dots (18b)$$

Substituting in Eq. 16 all quantities expressed as functions of M_{DA}, $100 - M_{DA} - M_{DA} + \frac{100 - M_{DA}}{6} - \frac{M_{DA} + 100}{6} = 0$; and $\frac{14}{6}\,M_{DA} = 100$. This development is even more rapid than that proposed by the author. The author's claim that fewer equations are used and half of the number of operations are necessary, as compared with conventional methods, is doubtful. In the case of the unsymmetrical framework, with a hinge at point D, a straightforward method applying strain energy requires the use of only three unknowns—the shear in the vertical member OD, the shear in member CD, and the axial load in member CD.

Of minor importance are the unjustifiable introduction of the symbol Δ from a technique that bears no remote relation to stress analysis,[2] and the author's adoption of the inaccurate term "tapered beams" to describe "unsymmetrical beams with variable moments of inertia."

In conclusion, it seems to the writer that slope-deflection methods do not require clarification. The author's effort to clarify them is imperfect to the extent that his lack of a precise sign convention has introduced unnecessary confusion.

JOHN E. GOLDBERG,[9] ASSOC. M. AM. SOC. C. E.—An amplification of the slope-deflection method is presented in this concise and excellent paper. The fundamental geometry of slope deflection is combined with the author's concept of the traverse angle as applied to members under flexure. The writer is gratified to perceive that, as this and other interesting papers indicate, the value of slope deflection as a powerful method of analysis is steadily becoming more appreciated.

Several of Mr. Stewart's statements deserve critical consideration. One of these is the sentence (see heading, "Demonstration Problems"): "In conventional slope deflection the number of simultaneous equations required to solve a given problem is definite." This standardization is a distinct advantage of the slope-deflection method from which both the experienced and the inexperienced engineer will benefit since any engineer can see at once the proper direct slope-deflection approach to a given problem. On the other hand, the slope-deflection method is so extremely flexible that it allows much leeway, when desired, in the solution of special problems.

Consider, for example, the problem of Fig. 1. Some years ago the writer developed an extremely simple analysis for problems of this type.[10,11] This method has the advantages of being readily applicable to any number of stories and of being extremely rapid since the basic equations converge very quickly even in extreme cases. Applying the method to Mr. Stewart's first problem, two equations would be set up immediately:

$$\theta_A\,(6 \times 2 + 1) = 0.5\,(20 \times 10) + \theta_D \ldots\ldots\ldots\ldots (19a)$$

and

$$\theta_D\,(6 \times 2 + 1) = 0.5\,(20 \times 10) + 0.5\,(20 \times 20) + \theta_A \ldots\ldots (19b)$$

The solution of Eqs. 19 ($\theta_A = 9.524$ and $\theta_D = 23.81$) agrees exactly with the author's solution. Thus, the method is shown to be simple and direct. Of course, this is merely proof of the fact that the most useful method generally is the one with which the engineer is most familiar.

Mr. Stewart goes on to mention the same frame with the lower girder pinned at one end and states that six simultaneous equations are required by the conventional slope-deflection method. This is absolutely true; but, when these equations are set up (and setting them up is in itself very simple), three of the equations may be eliminated in a very direct manner. Note also that a conventional strain-energy analysis can be made in terms of three redundants.

Mr. Stewart's point concerning sign conventions (see heading, "Definitions of Symbols and Other Data: Signs," and text following Eqs. 7) is well taken. In his original presentation[12] of slope deflection, G. A. Maney, M. Am. Soc. C. E., advanced the sign convention which the author advocates. In subsequent work,[13,14,15,16] both Professor Maney and the writer continued to use that convention, finding it simple and convenient.

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[9] Engr., Consolidated Vultee Aircraft Corp., San Diego, Calif.

[10] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 922.

[11] *Ibid.*, Vol. 110 (1945), p. 1512.

[12] *Engineering Studies No. 1*, by G. A. Maney, Univ. of Minnesota, Minneapolis, 1915.

The author's statement, discussing Eqs. 7, that "* * * the final moment at the end of a beam for which the moments are not dominated by joint movements is less than the fixed-end moment" is likely to be misinterpreted by the reader. The statement would be clarified greatly if the author had written that "the moment is less, algebraically, than the fixed-end moment."

Camillo Weiss,[17] M. Am. Soc. C. E.—The four "properties of an elastic curve traverse" presented in this paper are principles derived in part from "moment areas" and are important enough to be compared with the well-known principles of the moment-area method.

The concept of the Δ-angle is helpful in visualizing the effect of variations in cross sections or end restraints. The author is justified in stating that in many cases the number of simultaneous equations may be reduced by recognizing the significance of the Δ-angles.

The method would seem to deserve a more descriptive designation than "amplified slope deflection." Although related to slope deflection, the unknowns of the author's method consist exclusively of slopes and the method is based on its own distinctive principles.

The author should be commended for his clear and able presentation of these principles and for his skilful derivation and interesting interpretation of the slope-deflection equations.

William C. Spiker,[18] M. Am. Soc. C. E.—The "traverse" method described in this paper is creative and clever. However, because, as acknowledged, it is limited in application and because, in the writer's opinion, it requires more than the average amount of practice and facility for successful use, it should be simplified and amplified, if possible. Then, too, the writer believes that the questions of signs and "unknowns" in the basic slope-deflection method are not as serious as Mr. Stewart states. It is to be remembered, furthermore, that there are other, more simple methods.

As an illustration, the slope-deflection method can be applied to the case in Fig. 2, with perhaps a slight variation from standard routine. The writer prefers the following sign convention: In the setup equations all θ's have plus signs. The signs for the fixed-end moments refer to the moments which resist the fixed-end moments. They are plus when the resisting moments tend to turn the end of the member to the right and minus when the resisting moments tend to turn the end of the member to the left.

The detailed solution is as follows:

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[13] "Vertical-Load Analysis of Rigid Building Frames Made Practicable," by John E. Goldberg, *Engineering News-Record*, November 12, 1931, pp. 770–772.

[14] Simplified Methods for the Analysis of Multiple Joint Rigid Frames," by George A. Maney and John E. Goldberg, *Northwestern University Bulletin*, Vol. XXXIII, No. 7, 1932.

[15] "Wind Stresses by Slope Deflection and Converging Approximations," by John E. Goldberg, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), pp. 962–985.

[16] "Natural Period of Vibration of Building Frames," by John E. Goldberg, *Journal of the American Concrete Institute*, September, 1939, p. 81.

[17] Designer, Bethlehem Steel Co., Fabricated Steel Constr., Eng. Dept., Bethlehem, Pa.

[18] Cons. Engr., Mountain View, Ga.

Moment	Computation	Computed moment
M_{AB}	$12\,\theta\,A$	$+\ 57.18$
M_{AD}	$4\,\theta\,A + 2\,\theta\,D - 6\,R$	$-\ 57.15$
....	$16\,\theta\,A = +\,6\,R - 2\,\theta\,D$	....

Adding and transposing,

....	$\theta\,A = +\,0.375\,R - 0.125\,\theta\,D$	....
M_{DA}	$4\,\theta\,D + 2\,\theta\,A\ (= 0.75\,R - 0.25\,\theta\,D) - 6\,R$	$-\ 42.86$
M_{DC}	$12\,\theta\,D$	$+\ 142.8 \pm$
M_{DE}	$3\,\theta\,D - 3\,x\,R$	$-\ 100.00$
....	$18.75\,\theta\,D = +\,5.25\,R + 3\,x\,R$	....
....	$\theta\,D = +\,0.28\,R + 0.16\,x\,R$	$-\ 11.91$
....	$\theta\,A = +\,0.375\,R - (0.035\,R + 0.02\,x\,R)$	
	$= +\,0.34\,R - 0.02\,x\,R$	$-\ 4.765$

Therefore:

M_{AD}	$-\ 4.08\,R + 0.24\,x\,R$	....
M_{DA}	$-\ 4.20\,R + 0.60\,x\,R$	....
....	$-\ 8.28\,R + 0.84\,x\,R$	$-\ 100.00$
M_{DE}	$+\ 0.84\,R - 2.52\,x\,R$	$-\ 100.00$
....	$-\ 9.12\,R + 3.36\,x\,R$	0.00

Therefore, $x = 2.715$; $R = +\ 16.67$; and $x\,R = 45.23$. Then substitute into the basic equations the values of R, $x\,R$, $\theta\,A$, and $\theta\,D$ and add the moments opposite the equations as shown.

Note that, if the sway in both tiers is the same, the problem is as simple as that for two spans on three supports. The x disappears.

ARTHUR B. MCGEE,[19] ESQ.—Amplified slope deflection is in reality the traverse method[3] extended to include the displacement of joints, or sidesway. In a normal traverse solution, joint movement is restricted to rotation only. In general, the traverse of a member consists of three sides of known lengths and four angles—namely, a θ-angle at each end and two intermediate Δ-angles. If there is no joint displacement, any two angles are sufficient to determine the remaining two angles; but, if there is displacement of a joint, three angles are required to determine the fourth one. In the latter case, simultaneous equations are usually required.

Mr. Stewart has shown the relationship between the traverse method and slope deflection. The number of essential unknowns in a given rigid frame is the same regardless of the method used for analysis. However, if a method disposes of any of the unknowns in an easy manner and reduces the number of simultaneous equations necessary for the solution of a problem, it merits consideration.

Fig. 4(a) shows an unsymmetrical frame with members of uniform moments of inertia. The writer's method of procedure assumes a traverse of the deflected frame (Fig. 4(d)). Errors in the direction of deflection angles (Δ-angles) or in the direction of rotation of joints (θ-angles) will not affect the solution. The traverse must be drawn, however, to provide opposing moments at each joint. The assumed traverse is solved and values are found for the Δ-angles. If any angle proves to be negative, the angle was merely assumed in the wrong direction and the numerical value is still correct.

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[19] Structural Engr., Bridge and Structural Design Div., Bureau of Eng., City of Los Angeles, Los Angeles, Calif.

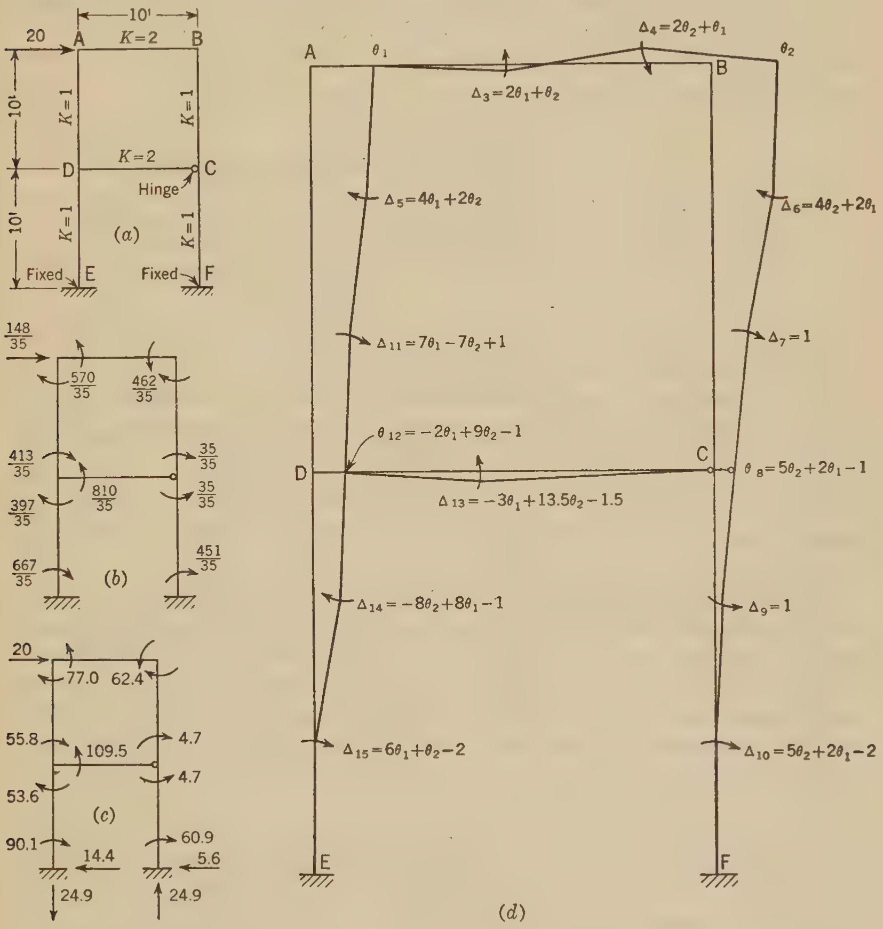
10'
20
A
K=2
B
10'
K=1
K=1
K=2
D
C
Hinge
10'
K=1
K=1
(a)
Fixed
E
Fixed
F
148/35
570/35
462/35
413/35
35/35
810/35
35/35
397/35
451/35
667/35
(b)
20
77.0
62.4
55.8
4.7
109.5
4.7
53.6
(c)
90.1
60.9
14.4
5.6
24.9
24.9
A
θ1
Δ4=2θ2+θ1
θ2
B
Δ3=2θ1+θ2
Δ5=4θ1+2θ2
Δ6=4θ2+2θ1
Δ11=7θ1−7θ2+1
Δ7=1
θ12=−2θ1+9θ2−1
C
D
θ8=5θ2+2θ1−1
Δ13=−3θ1+13.5θ2−1.5
Δ14=−8θ2+8θ1−1
Δ9=1
Δ15=6θ1+θ2−2
Δ10=5θ2+2θ1−2
E
F
(d)

FIG. 4

In Fig. 4(*d*), subscripts have been placed on the θ-angles and Δ-angles in the order of evaluating the angles for ease in following the solution. The Δ-angles are at the third points of the members. The angles of rotation of joint A and joint B are assumed to be θ_1 and θ_2, respectively. By geometry, $\Delta_3 = 2\,\theta_1 + \theta_2$ and $\Delta_4 = 2\,\theta_2 + \theta_1$. Since member AD is half as stiff as member AB, $\Delta_5 = 2\,\Delta_3 = 4\,\theta_1 + 2\,\theta_2$. Likewise, $\Delta_6 = 2\,\Delta_4 = 4\,\theta_2 + 2\,\theta_1$. By assumption, $\Delta_7 = 1$. Then, from joint B to joint C, $\theta_2 + \Delta_6 - \Delta_7 - \theta_8 = 0$; hence, $\theta_8 = 5\,\theta_2 + 2\,\theta_1 - 1$. Since $M_{CB} = M_{CF}$, $\Delta_9 = \Delta_7 = 1$. From joint C to support F, $\theta_8 - \Delta_9 - \Delta_{10} = 0$; therefore, $\Delta_{10} = 5\,\theta_2 + 2\,\theta_1 - 2$. The deflection in member AD must equal the deflection in member BC; so $3\,\theta_1 + 2\,\Delta_5 - \Delta_{11} = 3\,\theta_2 + 2\,\Delta_6 - \Delta_7$, and solving, $\Delta_{11} = 7\,\theta_1 - 7\,\theta_2 + 1$. From joint A to joint D, $\theta_1 + \Delta_5 - \Delta_{11} - \theta_{12} = 0$; and, therefore, $\theta_{12} = -2\,\theta_1 + 9\,\theta_2 - 1$. In the member DC, $\Delta_{13} = \frac{3}{2}\,\theta_{12}$; hence, $\Delta_{13} = -3\,\theta_1 + 13.5\,\theta_2 - 1.5$. The deflection in member DE must equal the deflection in member CF; so $2\,\theta_{12} + \Delta_{14} = 2\,\theta_8 - \Delta_9$ and $\Delta_{14} = -8\,\theta_2 + 8\,\theta_1 - 1$. From joint D to support E, $\theta_{12} + \Delta_{14} - \Delta_{15} = 0$ or $\Delta_{15} = 6\,\theta_1 + \theta_2 - 2$. About joint D, $\Sigma M = 0$; therefore, $\Delta_{14} + \Delta_{11} = 2\Delta_{13}$; and, substituting the values of the Δ-angles in terms of θ_1 and θ_2:

$$7\,\theta_1 - 14\,\theta_2 + 1 = 0 \ldots\ldots\ldots\ldots\ldots\ldots (20a)$$

The shear in the second story equals the shear in the first story; therefore,

$$\frac{\Delta_5 + \Delta_{11} + \Delta_6 + \Delta_7}{10} = \frac{\Delta_{14} + \Delta_{15} - \Delta_9 + \Delta_{10}}{10}$$ or, in terms of θ_1 and θ_2:

$$-3\,\theta_1 + \theta_2 + 8 = 0 \ldots\ldots\ldots\ldots\ldots\ldots (20b)$$

Eqs. 20 are solved simultaneously, and they yield $\theta_1 = \frac{113}{35}$ and $\theta_2 = \frac{59}{35}$.

Substituting these values in the Δ-angles of Fig. 4(*d*) and changing the angles to moments by the relationship $M = K\,\Delta$ give the results shown in Fig. 4(*b*). The shear in each story is $\frac{148}{35}$ which is the applied load producing the moments. Multiplying all the quantities shown in Fig. 4(*b*) by $20 \times \frac{35}{148}$ gives Fig. 4(*c*), which is the solution of the problem. Thus, this problem required two simultaneous equations.

It should be mentioned that, after drawing a logical traverse of the deflected structure, the analyst has numerous choices in starting the evaluation of angles. Some methods are easier than others but no set procedure has to be followed, and, as long as the assumed traverse is solved, the signs will take care of themselves automatically. If the members had had variable moments of inertia, both the stiffnesses and locations of the Δ-angles would have been different, but the procedure would have been the same.

JAROSLAV POLIVKA,[20] M. AM. SOC. C. E.—There are many interesting features in the author's method of simplifying slope deflection by using characteristic properties of a traverse of the elastic curve. Elastic deformation can be visualized more clearly, and automatic procedures can be reduced to a minimum. However, the author's claim that the method reduces by one half the work of computation required by the slope-deflection method, if applied generally, cannot be upheld. The author's procedure is limited to a few problems, such as the one demonstrated in Fig. 2, which was selected to show the usefulness of the method in a case where a symmetrical two-story frame, subjected to a lateral force, has pin connections with the bases.

It is true that, in the usual slope-deflection method, the joint expressions include terms of the unknown deflections; yet it is not difficult to reduce the simultaneous equations to a general form containing only deflection angles and load constants, as has been shown by A. Amirikian,[21] M. Am. Soc. C. E. This reduction is accomplished by substituting deflection values in terms of the deflection angles and the load constants (obtained from the shear equation of each story) for deflections in the joint expressions.

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[20] Lecturer E. D., Univ. of California, Cons. Engr., Berkeley, Calif.

Applying this principle to the author's example in Fig. 2 and referring to Mr. Amirikian's work, the following equations can be written: For joint A:

$$\left(2 + 1 - \frac{3}{4} \times \frac{1}{2} + 1 - \frac{3}{4 \times 2}\right)\frac{A}{2} + \left(\frac{1}{2} - \frac{2 \times 3}{4 \times 2}\right)\frac{D}{2} = \frac{20 \times 10}{2 \times 2};$$

or

$$13\,A - D = 200 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (21a)$$

Similarly, for joint D, the following equation results:

$$13\,D - A = 600 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (21b)$$

Since, in accordance with Mr. Amirikian's procedure, $A = 4\,\theta_A$ and $D = 4\,\theta_D$, Eqs. 21 become, respectively,

$$13\,\theta'_A - \theta'_D = 50 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (22a)$$

and

$$13\,\theta'_D - \theta'_A = 150 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (22b)$$

Eqs. 22 contain two unknowns and the solution is just as simple as that obtained by the author's method using the properties of the traverse of the elastic curve. The solution is $\theta'_A = \frac{100}{21}$ and $\theta'_D = \frac{250}{21}$, assuming the relative stiffness $K' = \frac{E\,I}{l}$, or, introducing the basic stiffness used in the paper, $K = \frac{2\,E\,I}{l}$ $\left(\text{ratio of an end moment to the area of the } \frac{M}{E\,I}\text{-diagram which it produces}\right)$, $\theta_A = \frac{200}{21}$, and $\theta_D = \frac{500}{21}$. The moments in the horizontal members are computed from the basic equations; thus: $M_{AB} = 2\left(\frac{3}{2}A\right) = 12\,\theta'_A = \frac{400}{7}$; and $M_{DC} = 2\left(\frac{3}{2}D\right) = 12\,\theta'_D = \frac{1{,}000}{7}$. From the shear equation for the symmetrical bent: $M_{AD} + M_{DA} = 100$; and $M_{DA} = \frac{300}{7}$. Finally, from the joint equation at point D: $M_{DA} = M_{DC} - 100 = \frac{300}{7}$.

A. A. Eremin,[22] Assoc. M. Am. Soc. C. E.—An important advantage of the amplified slope-deflection method is that the moments in the members are computed in conjunction with the sketching of the axes of the sloped members, and the moments can be checked simply by inspection. In his illustrative example, Mr. Stewart treats members with constant sections. By application of the simple formulas, the method may be extended to the members with variable sections. In a member AB with variable section, the distances p and q

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[21] "Analysis of Rigid Frames (An Application of Slope Deflection)," by A. Amirikian, Government Printing Office, Washington, D. C., 1942, p. 88.

[22] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

to the centroids of $\frac{M_{AB}}{E\,I}$ and $\frac{M_{BA}}{E\,I}$ from ends A and B, respectively, are

$$p = c_1\,l \qquad (23a)$$

and

$$q = c_2\,l \qquad (23b)$$

The stiffness factors of beam AB are

$$K_{AB} = \frac{K}{1.5\,e_1} \qquad (24a)$$

and

$$K_{BA} = \frac{K}{1.5\,e_2} \qquad (24b)$$

in which $K = \frac{I}{l}$. The values of the coefficients c and e in Eqs. 23 and 24 may be computed by the moment-area method or may be obtained from diagrams such as those presented elsewhere by the writer.[23]

The sign convention for the algebraic distribution of moments in continuous frames is often a serious source of error. As stated by the author, the interpretation of signs is simplified considerably by sketching the slopes of the axes. However, considerable skill is required. The interpretation of signs can be simplified further by distributing moments graphically. The moment closing line, constructed graphically by "crosslines,"[24] is shown in Fig. 5. Results are automatic and the signs cannot be misinterpreted. The resulting bending-moment diagram may be used directly for the design of the members.

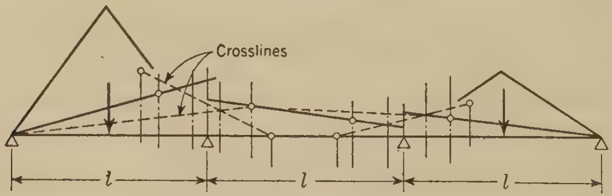

FIG. 5.—GRAPHICAL DISTRIBUTION OF MOMENTS

In the realm of fine arts, the artist's message is broadcast through various mediums—oil, water color, stone, or marble. The artist selects the medium that best agrees with his temperament. Likewise, in the applied art of adapting the science of mechanics to the determination of stresses, various algebraic and graphic procedures of distributing moments are available. All methods based on the elastic theory are equally exact for practical purposes. Mr. Stewart is to be commended highly for his valuable contribution to moment-distribution methods. With the addition of the new concept of the railroad traverse line, the stress analyst has a wider range of choice in selecting a method to satisfy his personal requirements.

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[23] "Analysis of Continuous Frames by Graphical Distribution of Moments," by A. A. Eremin, Sacramento, Calif., 1943.

[24] *Transactions*, Am. Soc. C. E., Vol. 110 (1945), p. 1543.

Ralph W. Stewart,[25] M. Am. Soc. C. E.—A number of questions about the paper have been raised by Mr. Beskin. His statement concerning the text following Eq. 7*b* has merit. The sentence he quotes should have an additional phrase as is indicated within the brackets:

> "It is well known to all users of the slope-deflection method that the final moment at the end of a beam for which the moments are not dominated by joint movements [or by the flexure of an adjoining span] is less than the fixed-end moment."

The other discussers apparently did not have Mr. Beskin's difficulties in applying signs. The paper states that any analyst can use his own system of signs and still take advantage of the interpretation of angle values—the basic idea of amplified slope deflection. Also, the statements regarding signs in the third and the last paragraphs of Mr. McGee's discussion should be noted.

The method followed by Mr. Beskin in Eqs. 16 and 17*a* involves the algebraic procedure of referring to Eq. 7*a* and transforming it by suitable substitutions. The writer's aim is to avoid such algebraic procedures. The solution of Fig. 2 with one apparent unknown, as developed by Eqs. 18*a*, 18*b*, and a subsequent unnumbered equation, also involves what the writer considers algebraic work of an undesirable type—namely, substituting in Eq. 16 which, in turn, is obtained from Eq. 7*a*. Direct use of the elastic curve traverse offers a solution with one apparent unknown which avoids reference to previously established equations. The elastic curve traverse also reduces the problem to a simple reading of visible geometrical alinement and the direct application of basic mechanical laws. This fact is demonstrated by Fig. 6, in which the values of the Δ-angles are written on each side of joint A in terms of M_A. Each value will be M_A divided by the member stiffness. By geometry, θ_A will be one third of the beam angle Δ. Since the sum of the column moments must be 100 for shear equilibrium, the value of Δ_2 is $100 - M_A$. By simple angle summation from A downward: $\theta_D = -100 + \frac{13\,M_A}{6}$. By geometry the lower beam angle Δ is 3 θ_b as shown in Fig. 6. The lower column moment is known to be 100 since the horizontal footing reaction is 10 and the column height is 10. The equation for equilibrium of moments about joint D is written:

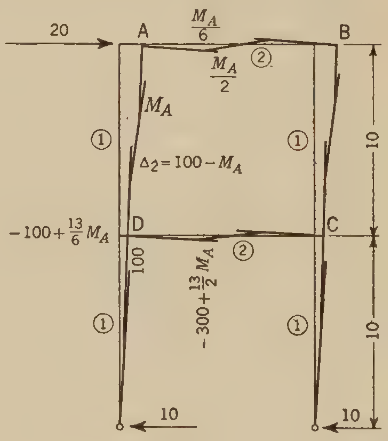

Fig. 6

$$100 + 100 - M_A - 2\left(-300 + \frac{13\,M_A}{2}\right) = 0 \quad \ldots\ldots\ldots\ldots (25)$$

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[25] Bridge Engr., City of Los Angeles, Los Angeles, Calif.

from which $M_A = 57.14$. Eq. 25 is shorter than the one following Eq. 18*b*; and, as the computation progresses, the physical significance of each term is shown by recording it in Fig. 6. Mr. Spiker has shown another, but longer, solution for Fig. 6.

As minor points, Mr. Beskin criticizes the use of the symbol Δ for angular change due to bending, as well as the use of the term "tapered beams." The term "tapered beams" has an excellent precedent.[26] The symbol Δ was selected because, in a number of treatises, it represents the angle between the tangents to a curve, and it seemed appropriate to extend its use to traverse diagrams which are solved largely as problems in geometrical alinement.

The writer is grateful to Messrs. Goldberg and Weiss for their comments. As Mr. Weiss suggests, the moment-area constants are the basis of the elastic curve traverse. If they are delineated as a traverse, the flexure of the structure can be visualized and the deformations due to flexure become easier to use than when pictured as areas. Such delineation also gives an opportunity to diversify and extend the use of moment-area constants. The object of the paper was to take advantage of this opportunity.

Mr. Polivka commented that "* * * the author's claim that the method reduces by one half the work of computation * * * if applied generally, cannot be upheld." The writer's claim was worded as follows: "* * * for many problems, the algebraic work required will be only one half that required by the procedure taught in current texts." It may be added that slope deflection, as amplified by the elastic curve traverse, will never, for any problem, involve more algebraic work than slope deflection as taught in current college textbooks.

The detailed equation given by Mr. Polivka, which he consolidates into Eq. 21*a*, represents either derivation of a formula by an algebraic procedure involving simultaneous equations as described by A. Amirikian,[21] M. Am. Soc. C. E., or handbook work which the writer wishes to avoid.

An important point which should not be overlooked is that the formulas derived by Mr. Amirikian are for frames composed only of members with constant cross sections. The introduction of tapering members would make no important changes in solutions which use the elastic curve traverse but it would have a serious effect on Mr. Amirikian's algebraic use of slope deflection.

The excellent and constructive work of Mr. McGee in presenting the complete detail of the solution for an unsymmetrical two-story frame with fixed-base columns is helpful. Following this step-by-step solution should clarify most of the situations which can arise in the general use of the method. As shown by Fig. 4(*c*), this problem involves eight unknown moments. The number of geometrical unknowns is six, comprising four joint rotations and two-story deflections. Only two unknowns (θ_1 and θ_2) appear in Fig. 4(*d*), the others having been eliminated in an easy manner so that only two simultaneous equations are needed for the solution.

Mr. Eremin offers suggestions regarding the extension of the method to structures having members of variable section. It is for such structures and also for such complicated conditions as "semirigid" joints that the use of an elastic curve traverse best shows its advantage.[7]

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[26] "Tapered Structural Members: An Analytical Treatment," by Walter H. Weiskopf and John W. Pickworth, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 1.

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STORAGE AND THE UNIT HYDROGRAPH

By C. O. Clark,[1] Jun. Am. Soc. C. E.

With Discussion by Messrs. James S. Sweet, Otto H. Meyer, L. K. Sherman, Gordon R. Williams, Ray K. Linsley, Jr., E. F. Brater, L. C. Crawford, Robert E. Kennedy, Victor H. Cochrane, Franklin F. Snyder, Howard M. Turner, Don Johnstone, and C. O. Clark.

Synopsis

During the period since 1930, study of the flow records of many and various types of streams by the engineers of the United States has resulted in an improved concept and understanding of the physical factors which influence runoff and the flood-producing capacity of streams. Two fundamental tools have emerged from these studies, the unit hydrograph and methods of flood routing (means of modifying a hydrograph by the effects of valley storage), both of which have been the subject of many papers.

The purpose of this paper is to clarify the inherent relationship between these fundamental tools and to show how this relationship may be used to derive accurate unit hydrographs for very short periods of initial runoff which accurately reflect the influence of shape of drainage area upon the shape of the hydrograph, allow the segregation of elements of the hydrograph attributable to particular components of the drainage area, and permit the definition of the calculation procedure to a degree which reduces the dependence of accurate stream flow calculations on intangible factors of personal judgment.

After showing that the constant time units characteristic of unit hydrographs are the characteristics induced by storage capacity of the streams and that storage capacity and discharge capacity are each a limiting factor on the other, the paper illustrates the incorporation of these characteristics in hydrograph calculations.

Definitions

Throughout this paper, the terms "unit hydrograph" and "unitgraph" are used interchangeably to mean a regimen of stream flow for a unit quantity of uniformly distributed runoff originating at a uniform rate during a specified

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Note.—Published in November, 1943, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Hydr. Engr., U. S. Engr. Office, Winchester, Va.

unit period of time. The time elements of the regimen are independent of the quantity of runoff and the discharge quantities are directly proportional to the quantity of runoff. Included are both the unit hydrograph of L. K. Sherman,[2] M. Am. Soc. C. E., and the distribution graph of Merrill Bernard,[3] M. Am. Soc. C. E. The terms "storage" and "valley storage" are applied to the entire volume of water stored between any two designated points along a river and include both the volume in the channel and on the flood plain. The same terms, expressed in time units, are used to designate the average ratio of storage volume to discharge capacity of the river reach.

The following definition of accuracy will apply to the writer's use of the term in subsequent paragraphs, and at the same time will illustrate some of the problems and aims of incorporating accuracy in stream flow calculations:

> An engineer should learn early to distinguish between the forms of accuracy which sometimes are called absolute, real, and relative. Absolute accuracy is the mathematical precision of correctly manipulated numbers, as used by engineers in computing payments to the cent from earthwork surveys by cross sections or soundings. Real accuracy is an evaluation of the closeness with which a measurement or computation approximates the truth, a quality inherent in the meaning of the engineering term "precision." Relative accuracy is an evaluation of the closeness with which a measurement or computation approximates another measurement or computation, preferably where both determinations are comparable, and frequently is expressed as a ratio or the difference between them, without necessarily giving an absolute value of either. The elevation of one bench mark with respect to another is a value that may have great relative accuracy without any absolute accuracy. The terms with which others designate these qualities of accuracy may differ, but the essential difference between them is acknowledged widely.

In the fields of evaluation and planning of river developments, relative accuracy is attainable at low cost, usually by thoroughly planned systematic analysis; and it is sufficient in the selection of the better of alternative plans. Real accuracy, as in the evaluation of the selected plan of construction, high-order surveys, or field observation and measurement of natural phenomena, such as river stage and discharge, is an elusive value pursued at great cost, with each additional fraction of attainment sometimes costing more than all prior to it. Absolute accuracy in most engineering offices is the product of a calculating machine, useful as a tool in attainment of relative accuracy regarding influences of small variables, but a source of colossal waste of time if confused with real or relative accuracy. One cannot start with an hypothesis having a real accuracy of 80% and develop therefrom conclusions of 95% real accuracy.

By any unmodified references to accuracy which may be made in the body of the paper which follows, the writer means only relative accuracy and the degree to which a computational determination by simplified procedure approximates the result of more laborious, accepted means of computation. No claims of unusual real accuracy are intended.

Investigators entering the field of flood flow analysis will do well to maintain an open mind on the subject of real accuracy, since the difficulties of field observation and measurement under severe flood conditions make it necessary to concede the possibility of errors as large as 20% (or more) in some items of published data. Such possibilities are indicated in many published records by the discussion preceding the tabulations, and in others by subsequent revisions when more reliable data are obtained.

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[2] "Stream Flow from Rainfall by the Unit Hydrograph Method," by Leroy K. Sherman, *Engineering News-Record*, Vol. 108, 1932, pp. 501–505.

[3] "An Approach to Determinate Stream Flow," by Merrill Bernard, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 347.

The attainment of a high order of relative accuracy in computation is expedited by concise, precise definition of procedure and the desired result. As an aid to the more rapid attainment of relative accuracy in the computation of flood hydrographs, the paper seeks to define the determination of the unit hydrograph of surface and subsurface runoff in terms of two, large, basic factors —the shape of the watershed from which runoff must come and the storage through which it must come.

The writer does not claim that the determinations will exceed in real accuracy the many excellent ones which have been obtained by trial and error, tried, tested, modified, and retried until they reproduce, well, the hydrographs of record. He does believe that the first determination of the unit hydrographs will prove satisfactory for use without modification in more cases than those determined by any of several other methods with which he is familiar.

INTRODUCTION

Rapid advances in the understanding of floodtime stream flow date from the Report of the Committee on Floods of the Boston Society of Civil Engineers in which it was recognized that, for storms of equal duration, the period of flood runoff was a constant not dependent upon the total volume of runoff and that the maximum rate of discharge was, therefore, a direct function of the total runoff. "For an instantaneous storm, the peak flow will vary directly with the maximum width of the drainage area, all other conditions remaining the same * * * unit peak flows will vary directly with the velocity of flow, all other conditions remaining the same" and "pondage tends to reduce the peak flow in the direct ratio that the volume of pondage bears to the total flood run-off." In these conclusions[4] lie the bases for many current concepts of the unit hydrograph, time-area concentration curves, and valley storage modification of flood flow.

A refinement of theory by Mr. Sherman[2] confirms the conclusion that the flood period is a constant and that the peak flow is proportional to the total volume of runoff, and extends the theory to show that, for a given unit quantity of runoff, uniform over the drainage area, originating in a small unit interval of time, the length of the hydrograph is a constant; that all ordinates will be proportional to the total runoff; and that, by the application of this so-called "unit hydrograph" to successive short periods of uniform runoff throughout the flood-producing storm period, the resulting hydrograph of stream flow could be reproduced with reasonable accuracy. This theory combines sufficient verification in many watersheds with convenience of use to a degree which has made it acceptable to many in the engineering profession and deserving of the status of an original contribution to fundamental hydrologic theory.

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[4] Report of the Committee on Floods of the Boston Society of Civil Engineers, *Journal*, Boston Soc. of Civ. Engrs., September, 1930.

By the limitation of its definition, as pointed out by Mr. Sherman, the unit hydrograph is correct and applicable only to areas from which the runoff for a given period of time is uniform and therefore to drainage areas for which the rainfall is essentially uniform over the entire basin. The extension of the theory to the areas of such size that this condition is not strictly true is a matter of practical expediency, but one which introduces fundamental discrepancies between theory and fact which are not a fault of the theory but of its application. In its original conception and application, the unit period was taken as one day, and the agreement of computed and observed discharges within that period of time was considered to be confirmation of the theory. The accuracy of the determination of the unit hydrograph was limited by the practical requirement of finding an isolated flood hydrograph resulting from a uniform unit rainfall. This latter requirement, while not a serious limitation on the use of the method for periods of one day, provides an almost insurmountable obstacle to the derivation of the unitgraph for very small drainage areas and very short unit periods of time.

A number of writers[5,6] have shown that the theory is equally applicable to very small drainage areas when the unit period used is sufficiently short to be in proper proportion to the total hydrograph.

In determinations of unit hydrographs for short periods of time, practical difficulties of finding uniform, short-period rainfall, which produced large enough stream flow to be analyzed accurately, make it necessary to determine unitgraphs from multiple periods of runoff. However, such a procedure is indeterminate, as there are always more equations than unknowns and a multiple of possible solutions.[7] For example, in determining a seven-period unitgraph from a flood resulting from three periods of runoff, it is possible to determine thirty-six unit hydrographs which, applied to the three items of runoff, will reproduce the composite hydrograph exactly. The approximate solutions are even more numerous. Consequently, the range of possible solutions by independent investigators is too large—far greater than the range of inaccuracy of stream discharge records. Most methods, graphical and mathematical, of selection of the most probable of these unitgraphs underestimate the capacity of a stream to reach a high instantaneous peak from short, intense rainfall and ignore tendencies toward double peaks which some streams exhibit following intense, short-period rainfall.

The range of possible unitgraph determinations can be reduced by correlating recognized discharge concepts with the physical limitations of valley storage and with the time elements which necessarily result from storage and discharge relations. The correlation can be utilized to develop, from time-area concentration curves of specific drainage areas, unit hydrographs with a small

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[5] "Relation between Rainfall and Run-off from Small Urban Areas," by W. W. Horner and F. L. Flynt, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 140.

[6] "The Unit Hydrograph Principle Applied to Small Water-Sheds," by E. F. Brater, *ibid.*, Vol. 105 (1940), p. 1154.

[7] "Engineering Procedure as Applied to Flood Control Reservoirs with Reference to the Muskingum Flood Control Project, Appendix IV," The Engineer School, Fort Belvoir, Va., 1936.

range of determination variability, independent of assumptions regarding run-off distribution in the flood-producing storm and reflecting influences of drainage area shape and stream pattern. The streams of Virginia and North Carolina, with a wide range of basin shapes, stream slopes, and patterns, present admirable opportunities to prove or disprove some of the theories advanced.

In some respects, the degree of proof or disproof is limited by the quality of records and the means of their interpretation. Although recording stream gages have been installed on many of the streams, few of them are rated to a discharge more than one half of the maximum stage of automatic record. For the very flashy streams it is almost impossible for measuring parties to reach the streams before they have fallen well below peak stage. The greatest limitations on the check of accuracy of the theories, however, are the lack of recording rainfall gages and contour maps. Because of rainfall data limitations, determinations of the time of precipitation without a possible error of 3 to 6 hours were seldom possible. If the reproduced hydrographs agreed with the observed within these limits of time, the theories were regarded as confirmed. In all but a few cases, the agreement was much closer than the allowable error.

The constant time elements which have been indicated as essential to the theory of the unit hydrograph are dictated by valley storage relationships. For that reason, some of the vital elements of valley storage concepts are reviewed.

VALLEY STORAGE

One of the principal characteristics of the passage of a flood wave down an open channel is the time elapsing between passage at one point and at some downstream point—frequently referred to as the time of travel. The concept of this time of travel as a function of the velocity of the water and therefore influenced by the slope of the river, channel friction, and other items which influence the velocity of flow in the river, is deeply rooted in the hydrology of streams. However, time of travel is more closely related to the variable storage capacity of the open channel; dimensionally, time is the ratio of storage to discharge. Flow in pipes is influenced by all the elements of slope, friction factor, etc., which influence velocity of water in an open channel; but there is no time of travel in a closed pipe, which is full at the beginning of the passage of the wave, where an increase in the velocity and discharge at the upstream end is accompanied almost simultaneously by an increase in velocity and a duplication of the waves at the exit end of the pipe. Time of travel is almost nonexistent in rigid pipes, such as cast iron and concrete, and becomes increasingly apparent in pipes which are elastic enough to have some variable capacity for storage, such as rubber hose and the arteries of the human body.

Most methods of taking account of the storage in a river valley involve the solution, either graphically or mathematically, of the storage equation:

$$\frac{I_1 + I_2}{2} t - \frac{O_1 + O_2}{2} t = S_2 - S_1 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which I is the total inflow rate and O the outflow rate, both in cubic feet per second; S is the total volume in storage in the reach in (cubic feet per second)-hours (12 (cu ft per sec)-hr = 1 acre-ft); and the subscripts 1 and 2 refer to the beginning and the end of the period respectively.[8] This is the arithmetic expression of the storage equation. The calculus equation for the same condition is:

$$dS = (I - O)\,dt \qquad (2a)$$

or

$$\frac{dS}{dt} = I - O \qquad (2b)$$

which is merely a mathematical way of saying that the rate of change of storage is equal to the difference between the inflow and the outflow rates. In Eqs. 2, t is the length of a period of time, in hours.

In general, the quantity of water stored in a short open channel increases with an increase in the flow in the channel, or:

$$S = C\,Q^n \qquad (3)$$

in which Q is the flow in the channel, n is a positive exponent, and C is a constant for the particular channel. Thus the storage in a river is a function of the flow and varies directly with some power of it.

In a short reach of river it may make little difference whether storage is considered as a function of inflow discharge or outflow discharge since there is little difference between them. In long reaches of the length considered practical for flood routing studies or of the length over which appreciable backwater effects may extend in floodtime, there is a great difference.

Methods of accounting for the influence of storage on discharge originally were used in analyzing flood reduction effects of reservoirs, where storage was a function of outflow. The accounting of storage between successive level pool surfaces whose elevation was controlled at the dam was a satisfactory approximation of the total storage. The amount of storage above the slack-water pool level and under the backwater curve was ignored as truly of small consequence.

The backwater storage, however small, existed because of the concurrent existence of inflow to the reservoir. Reduction of the inflow rate necessarily would be accompanied by a decrease in storage and an increase of outflow rate. Rapid inflow shutdown necessarily would be accompanied by a release of the backwater storage and a consequent increase in the slack-water storage or in the outflow discharge or in both. The released volume of water would be free to roll, as a pressure wave, to the lower end of the reservoir. Others have compared the movement to the rotation of saturated soil in a landslide. In either case, the lower end of the reservoir would have to rise slightly and the outflow discharge would have to increase.

The extension of concepts of storage modification to sloping channels and natural streams extended the practise of correlating storage with outflow. In some river reaches, the correlation with inflow is as close as the correlation with outflow. Some current concepts of the action of valley storage in reducing flood peak discharge are faulty because they are in terms of the original, limited theory. The storage which may be treated as a function of outflow reduces flood peak outflow discharge, but the portion which is a function of inflow increases outflow, so that the net decrease or increase of peak discharge depends on which storage portion is the greater.

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[8] "Flood Routing," by Edward J. Rutter, Quinton B. Graves, and Franklin F. Snyder, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 278.

The idea that there is some valley storage which does not decrease flood peaks is rather new. The nature of the actions is illustrated in Fig. 1, which shows water-surface profiles for a uniform, rectangular channel 500 ft wide, 20 ft deep, and with a fall of 10 ft in its length of 10,000 ft. Line b shows the profile for steady flow of 116,000 cu ft per sec throughout the channel length. Assume that the steady flow into the channel at the upper end is reduced quickly as by the closure of gates on a reservoir or the crevassing of a levee. With discharge reduced at the upper end of the reach and maintained at the lower end by storage release, the distribution of discharge in the reach would be similar to line c to which profile d would apply. The change from profile b to profile d involves the release from storage of 260 acre-ft during the period of inflow reduction and requires the outflow discharge to increase to accommodate such a release. However, as the outflow increases, the discharge distribution in the channel approaches profile e and the water-surface profile approaches that indicated by line f. Storage under profile f is the same as under profile b.

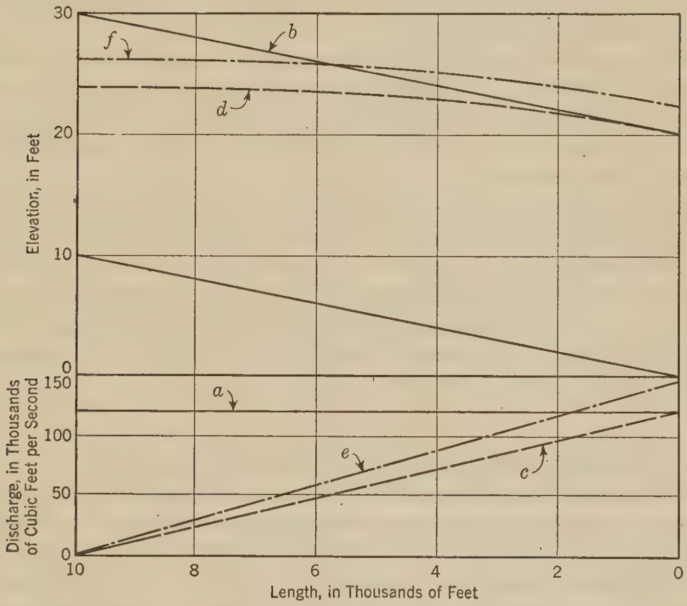

Fig. 1.—Effect of Decreasing Inflow on Flood Profile

A shift of water-surface profile from lines b to f takes place in a natural channel by mass rotation transmitted by means of pressure waves of small amplitude which surge back and forth through the reach, ceasing when the stable profile for the new discharge distribution has been attained. In natural channels under normal flow conditions, the reductions of inflow rates are gradual with the greater changes of discharge distribution occurring near peak stage. The surge of small amplitude transition waves is clearly apparent as a broad trace in many autographic records of stage. Following discharge change, the profile transition takes place in a very short time. The writer has a record of a natural flow reduction, caused by the crevassing of a large levee, in which the effect of the break in transforming the water-surface profile was transmitted 17 miles in as many minutes, whereas a normal flood wave transit of the same distance requires about 15 hours.

In the example of Fig. 1, if the water-surface elevation at the lower end is held constant by opening the gates of a dam, a material increase of outflow rate would be required. To prevent any increase in outflow discharge, a rise of about 3 ft would be required to store the initial volume as it shifted to the lower end of the reservoir.

In an unregulated reservoir of large capacity compared with the inflow to it, the storage may be solely a function of the outflow. In a long and uniform channel, the storage may be almost equally a function of the inflow and the outflow. All during the early stage of rapid increase in discharge in dry channels, such as occurs naturally in the arroyos of the Southwest, the storage is solely a function of the inflow. Thus, Q in Eq. 3 is to be considered as a weighted average value of the inflow and the outflow:

$$Q = x\,I + (1 - x)\,O \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which x is a coefficient with a value between 0.0 and 1.0.

According to Eq. 3, storage tends to increase with some power of the stream flow. There seems to be no theoretical reason why the power could not be either greater or less than unity. If it were less than unity, the peaks of large floods would occur earlier than those of small floods; if the power were greater than unity, the reverse would be true. If the power is unity, storage and discharge are in constant direct proportion, and time elements are constant regardless of flood magnitude.

Storage in a river channel does not necessarily increase directly with the discharge, but studies of the variation in a large number of streams indicate that this is the case for most smaller streams and is a condition closely approximated over large ranges of discharge in many large streams. That is, the exponent is 1.0 for small streams, and for many rivers of as much as 10,000 sq miles of drainage area it is less than 1.3. Even for the latter streams, over a large range the true storage may be closely approximated by storage directly proportional to discharge.

Storage which is directly proportional to discharge does not interfere with the accuracy of the unit hydrograph theory, as its characteristically constant time elements are compatible with, and necessary to, the similarly constant time elements of the unit hydrograph. Wherever the unitgraph theory is applicable, or wherever the time elements of flood passage are essentially constant, the storage is proportional to discharge.

There are a great many mathematical and graphical solutions for the storage equations, both for conditions where the storage is considered to vary directly with the discharge and where it is considered to vary directly with some exponential power of the discharge. All but a few of these solutions are for the special case in which the storage is a function of the outflow only and, therefore, are applicable only to routing floods through a very large reservoir. Use of these special solutions for natural channels, or for small reservoirs in which backwater storage is appreciable, leads to undesirable optimism regarding reductions of discharge by storage use.

The scope of this paper is limited to the general conditions in which the storage is considered to vary directly as the discharge, but where the discharge is a weighted value of the inflow and the outflow. Mathematically, these statements resolve to the three equations previously given:

$$dS = (I - O)\,dt \quad \text{(2a)}$$

$$Q = x\,I + (1 - x)\,O \quad \text{(4)}$$

$$S = K\,Q \quad \text{(5)}$$

from which it is evident that:

$$K = \frac{dS}{dQ} = \frac{(I - O)\,dt}{x\,dI + (1 - x)\,dO} \quad \text{(6a)}$$

and

$$O = I - K\,x\,\frac{dI}{dt} - K\,(1 - x)\,\frac{dO}{dt} \quad \text{(6b)}$$

Eq. 6*b* can be solved arithmetically for successive short periods of time, T in length, by designating the flows at the beginning of the short period as O_1 and I_1 and those at the end by O_2 and I_2, thus:

$$\frac{O_1 + O_2}{2} = \frac{I_1 + I_2}{2} - K\,x\,\frac{I_2 - I_1}{T} - K\,(1 - x)\,\frac{O_2 - O_1}{T} \quad \text{(6c)}$$

from which the following simple equation evolves:

$$O_2 = C_0\,I_2 + C_1\,I_1 + C_2\,O_1 \quad \text{(6d)}$$

In Eqs. 6:

$$C_0 = -\frac{K\,x - 0.5\,T}{K - K\,x + 0.5\,T} \quad \text{(7a)}$$

$$C_1 = +\frac{K\,x + 0.5\,T}{K - K\,x + 0.5\,T} \quad \text{(7b)}$$

$$C_2 = \frac{K - K\,x - 0.5\,T}{K - K\,x + 0.5\,T} \quad \text{(7c)}$$

and

$$C_0 + C_1 + C_2 = 1.00 \quad \text{(7d)}$$

This is the Muskingum method of flood routing developed by T. S. Burns and F. B. Harkness, Members, Am. Soc. C. E., and G. T. McCarthy,[7] Assoc. M. Am. Soc. C. E.

To illustrate the action of linear storage upon wave forms, two mathematical curves have been routed through various quantities of storage by the Muskingum method. Fig. 2 illustrates the modification of a cosine-type curve by storage represented by a value of K of four time units, with various values of x from 0.0 to 1.0. For this well-rounded initial wave form, the fundamental effect of the storage modification is to translate the entire wave approximately four time units, and to change the magnitude of the peak. The change of magnitude seems to be more a function of the value of x than of K, because when $x = 0.5$, the modified wave appears to be a close duplication of the initial wave, except for the parts of the curve before initial outflow and after final inflow.

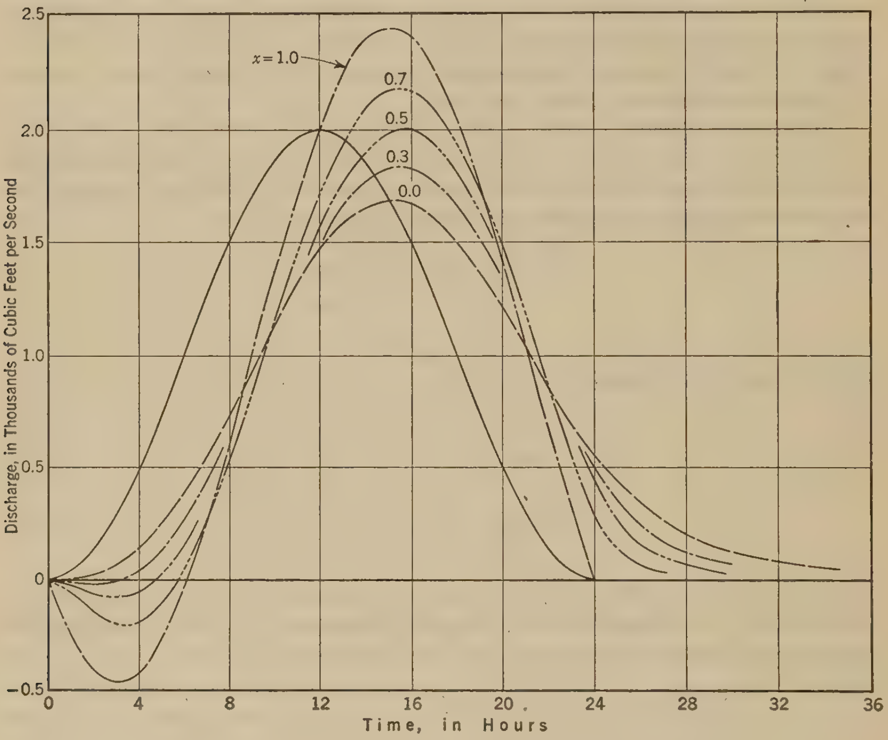

FIG. 2.—MODIFICATION OF COSINE TYPE OF WAVE BY STORAGE

It may be desirable to explain the unorthodox appearance of the modified wave forms (Fig. 2) between the initial inflow and initial outflow. The indicated negative values are partly erroneous and partly correct. A slight error arises in the theory assumption that the average discharge in the reach of river is indicated by a weighted value of the discharge at the ends and, therefore, that the storage prism between successive positions of the rising water surface may be approximated by two wedges of storage extending the length of the reach, one a function of inflow and the other a function of outflow. Considering the real bulbous shape of an advancing wave front in a near-dry channel, the true storage in the reach is less than the full-length wedge during the first part of the initial inflow. During this period, the actual outflow discharge is more than computed. At the instant the wave front reaches the lower end, the actual storage in the reach is somewhat more than that of a full-length wedge. At this time and for a short time after, the actual discharge at the lower end of the reach is less than indicated by the theory and the actual rise is later than that calculated. The inequalities between the theory and fact of wedge storage approximation are ironed out by the time the rates of flow increase at both ends are equal. Since the discharge at the lower end is closely indicated at this time, the true rise at the downstream end is more rapid than computed. The maximum total storage volume in the reach, represented by the area between the inflow and outflow hydrographs, is the same at this time for actual and computed modifications.

In the case of a wave passing into a partly full channel, into which flow is also coming from local streams, much less of the indicated negative discharge is to be regarded as error. The negative values indicate the capacity of the advancing wave not only to store its own waters, but to force the storage of the incoming local flows through the induced backwater effect. Only when the algebraic sum of all components of flow (including the negative values of the routed inflow wave) is less than the flow prior to the flood can the sum be discounted as error. Even then, the appearance of large negative values is an indication of a period of hydraulic instability which in extreme cases results in the formation of a hydraulic bore.

The negative values play an important part in the accuracy of the general storage theory and are to be discounted judiciously. The writer follows the practise of retaining the computed negative values, discounting them only in the drawing of the final hydrograph.

Fig. 3(a) shows the effect of storage that is equally a function of inflow and outflow; that is, $x = 0.5$ on a wave form with an initially very sharp peak. The first wave has been routed through 2 hours of storage in each of five successive reaches. In this case as in the case of the cosine curve, the translation effected by storage is approximately equal to the value of K; but some reduction of peak discharge is effected. However, the reductive influence of the storage decreases as the initial wave approximates the stable, well-rounded form.

The actual progressive modification by storage which is almost equally a function of inflow and outflow rates, illustrated in Fig. 3(a), has about the same effect as the bodily transposition of the initial wave form combined with a final modification of the transposed wave by a small amount of reservoir storage. The difference between the progressive calculation of storage effect, which is probably closer to truth, and the more easily computed transposition and single storage modification calculation is illustrated in Fig. 3(b), in which the wave form (a) has been progressively modified by passage through five reaches, each having 2 hours of storage (2 (cu ft per sec)-hr per cu ft per sec of discharge) to become wave form (b). The approximation wave form (c) was derived by bodily transposition of wave form (a) by 9 hours and routing it through reser-

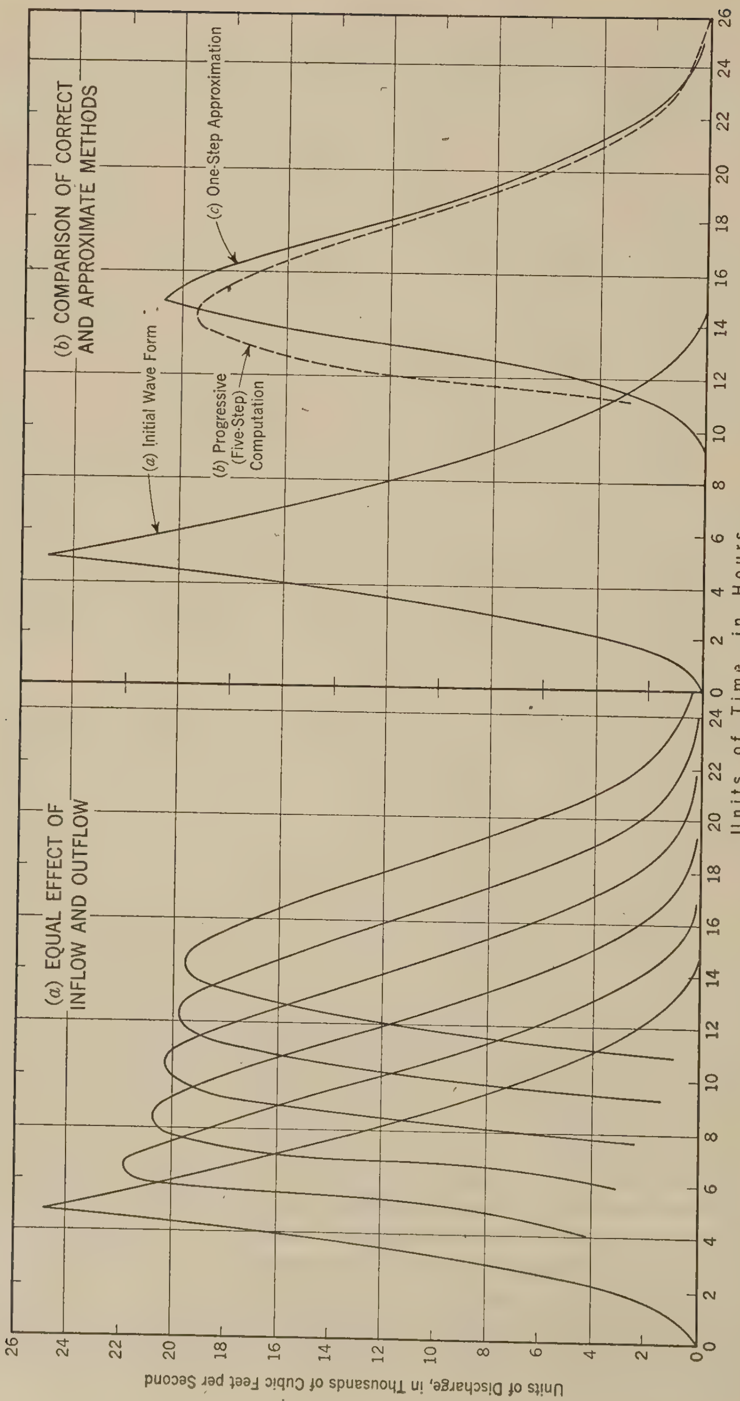

FIG. 3.—MODIFICATION OF A SHARP-CRESTED WAVE BY STORAGE

voir-type storage ($x = 0.0$) of 1 hour. The agreement between curves (b) and (c), far from perfect, is within the degree of accuracy usually attainable in runoff calculations.

This approximate form of storage calculation is used subsequently in the derivation of unit hydrographs; a time-area concentration curve of the drainage area (a composite of all of the transposed, unmodified wave elements of the watershed) is modified by a small amount of storage to form the basic unit-graph. The procedure will be explained in detail.

For many natural streams, the value of x, in Eq. 4, varies from 0.4 to 0.5, but the presence of a large flood plain carrying little flow near the lower end of a reach will cause the storage to be more of a function of outflow than inflow, and will reduce the relative influence of inflow, x, to 0.3 or lower. The back-water effect of a large incoming tributary may act in a similar manner. Conversely, if the large storage is near the upper end of the reach, the value of x may exceed 0.5 and result in an increase of crest discharge rather than the usually assumed decrease (the hydrographs in Fig. 12, introduced subsequently, indicate such an influence).

An understanding of the mechanical process of flow of water through a reach of river or a reservoir in light of the Muskingum routing theory will provide a clearer concept of what can and cannot be accomplished by valley storage. It also will show that, contrary to popular concept:

(*a*) Valley storage does not always decrease flood peaks; and

(*b*) The reservoir pool above a dam may not reduce flood discharges. In fact, if the pool level at the dam is held constant by gate control, the storage becomes the function of inflow solely, and the peak outflow may be larger than the peak inflow. Furthermore, such control, by reducing the effective storage, may decrease the time of travel through the reservoir to almost nothing, with a beneficial or detrimental effect as the case may be.

SIGNIFICANT STORAGE RELATIONSHIPS

The fundamental storage equation for storage which varies directly with a weighted average of the inflow and the outflow is (see Eq. 5*b*):

$$O + (1 - x)\,K\,\frac{dO}{dt} = I - K\,x\,\frac{dI}{dt} \qquad (8)$$

Although the Muskingum equations are easier to use in routing procedure, a knowledge of the following relationships which exist at certain points in the hydrographs of inflow and outflow is of great value in determining the quality and quantity of the storage effect in the reach. The symbols, $\frac{dI}{dt}$ and $\frac{dO}{dt}$, are used to represent the slope of the inflow and outflow hydrographs. When the slope of inflow and outflow hydrographs is equal:

$$\frac{dI}{dt} = \frac{dO}{dt} \qquad (9a)$$

and

$$I - O = K\,\frac{dO}{dt} \qquad (9b)$$

When inflow and outflow are equal:

$$I = O \quad \text{(10}a\text{)}$$

and

$$\frac{dO}{dt} = -K \frac{x}{1-x}\frac{dI}{dt} \quad \text{(10}b\text{)}$$

At peak inflow:

$$O = I - K \frac{dO}{dt}(1-x) \quad \text{(11}a\text{)}$$

At peak outflow:

$$O = I - K x \frac{dI}{dt} \quad \text{(11}b\text{)}$$

When inflow has ceased:

$$O = -\frac{K\,dO}{dt}(1-x) \quad \text{(12}a\text{)}$$

and

$$K = -\frac{O}{\frac{dO}{dt}(1-x)} \quad \text{(12}b\text{)}$$

Volume of runoff remaining in storage is

$$K\,O = -\frac{O^2}{\frac{dO}{dt}(1-x)} \quad \text{(13)}$$

Valley Storage and the Unit Hydrograph

A few years ago, the unit hydrograph was thought to be inapplicable to streams having a great amount of valley storage. However, when valley storage is defined as "all of the volume displaced by water during the passage of a flood wave, including the volume in the river channel," it is evident that no flood wave is unaffected by valley storage.

From a different point of view, the nature of the effect is clear. The essential difference between nonsteady flow in open channels and in pipes is in the time that elapses in the former only between the change of inflow rate and the change of outflow rate. The essential difference between an open channel and a pipe is the variable capacity of the former for storage as compared with the constant capacity of the latter. The time element and the storage capacity are related, for the ratio of storage capacity to discharge capacity is time; that is, $\frac{S}{Q} = T$.

The unit hydrograph theory assumes that the ordinates of any flood hydrograph originating from uniform runoff in the unit period of time will be directly proportional to the quantity of runoff, but that the abscissas, or time elements, will be constant and, therefore, unaffected by magnitude of discharge. Since the time elements are a function of the ratio of storage to discharge but not of discharge alone, it follows that the ratio of storage to discharge must be a constant throughout the entire range of discharge to which the unit hydrograph is to be applied. In other words, the storage-discharge curve for all reaches of river above the point to which the unit hydrograph is to be used must be straight lines when plotted on rectangular coordinate graph paper. This is the simplest of the storage discharge relationships for which methods of solution have been devised—the basis of the Muskingum method outlined by Mr. McCarthy.[9] An understanding of the manner in which it is possible for this type of storage to modify a flood wave will provide an understanding of several significant relationships which must exist in any actual unit hydrograph if the theory is applicable to the drainage area being studied.

THE HYDROGRAPH AS AN INDEX OF STORAGE

Any flood hydrograph is the resultant of all of the factors that have created it. Although no exact determination of the effect of any factor is possible without the segregation of the results of other influences, it is possible to determine the effect between certain limits. The accuracy of the determination will depend on the degree to which other influences can be segregated or discounted as insignificant.

The analysis of the hydrograph, as an index of previous storage effect, is based on the hypothesis that, after inflow to the channel has ceased, all water which will eventually become runoff, but which has not yet passed the gage, is in storage (axiomatic), and that such storage varies directly with the discharge (approximate, approaching truth in very small impervious watersheds). After such a time, the ratio of rate of change of discharge to total discharge reaches its greatest negative value, and remains constant thereafter:

$$\frac{\frac{dO}{dt}}{O} = -\frac{1}{K(1-x)} \qquad (14)$$

If all storage in a stream above a gage has been equally a function of inflow and outflow (that is, if $x = 0.5$), the greatest ratio of decrease of discharge to total discharge will correspond closely to a K-value of the time elapsing since the cessation of inflow. For the same amount of total storage, entirely a function of outflow, the ratio would have only half as large a magnitude, indicating a lesser slope of the depletion curve.

The falling legs of a large number of hydrographs of flood discharge for streams in major drainage basins from New Hampshire to North Carolina exhibit a ratio of decrease of discharge to total discharge which, for a value $x = 0.5$, indicates a value of K closely approximating the time elapsing between termination of runoff producing rainfall and the point of most rapid relative decrease in discharge. The fact that the hydrographs fall so rapidly is highly indicative that the storage affecting the hydrographs is of the balanced variety, and, therefore, chiefly effective in the translation of waves.

The writer has indicated that the effect of such storage may be approximated by (*a*) the bodily transposition of the runoff wave element from its source to the gage, and (*b*) the routing of the element through some amount of storage less than the total with a value of $x = 0$. The summation of all of the transposed elements for an entire drainage area is the same as a time-area concentration curve whose base length is the time between cessation of runoff and the time of most rapid relative decrease in discharge. The value of K selected for the routing of the time-area concentration curve is one corresponding to the ratio of discharge decrease to total discharge for an observed hydrograph (Eq. 12*b* with $x = 0.0$).

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[9] "The Unit Hydrograph and Flood Routing," by Gerald T. McCarthy, unpublished manuscript presented at the Conference of the North Atlantic Div., Corps of Engrs., U. S. Army, New London, Conn., June 24, 1938; printed by the U. S. Engr. Office, Providence, R. I.

The application of this method to a theoretical drainage area of rectangular shape with uniform instantaneous rainfall over the entire drainage basin is shown in Fig. 4(*a*). If the rainfall were to remain constant for a unit period of time, as is predicated in the theory of unit hydrographs, the flow concentration curve which was the time-area diagram (curve (a), Fig. 4(*a*)) is modified to become the time-flow diagram (curve (a), Fig. 4(*b*)) which, routed through the same storage, becomes curve (b), Fig. 4(*b*). The advantage of an instantaneous hydrograph is that it can be derived from the fundamental characteristics of the basin and then used with any length of unit period to determine the unit hydrograph, in which any ordinate of discharge is the average ordinate of the instantaneous hydrograph for the unit time period previous to time desired. Where the curvature of the instantaneous hydrograph is not excessive, the ordinate can be determined by averaging ordinates of that graph separated by the unit time period desired.

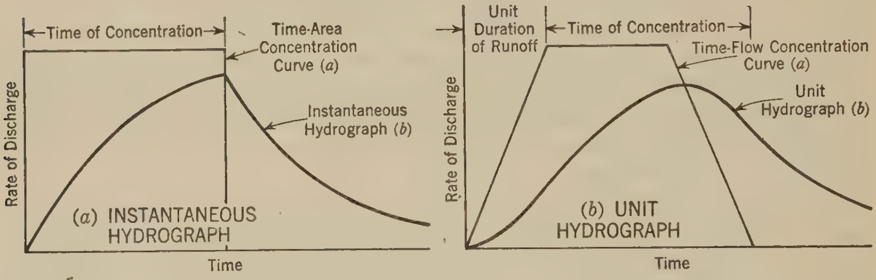

Fig. 4.—Hydrographs for a Rectangular Drainage Area

Determination of the Instantaneous Hydrograph from Stream Flow Records

Where adequate planimetric maps of a stream are provided in which recorder rainfall and discharge records are available, the elements to be determined are the base length of the time-area concentration curve, or the time of concentration, the time-area curve, and the magnitude of the unbalanced storage effect or the amount of valley storage (solely a function of discharge) by which the time-area curve is to be modified.

Referring to Eq. 12*b*, it will be seen that, when the inflow to the storage reach has ceased, the ratio of decrease of discharge to total discharge is a constant; or, as the Muskingum flood routing equation expresses it, the outflow at the end of any period of time bears a constant ratio, C_2, to the outflow at the beginning of the period of time.

Plotting the value of K which is indicated by the rate of decrease of discharge throughout the falling hydrograph, the indicated value should decrease to a minimum at the time that inflow ceased and remain constant thereafter. The time between cessation of runoff-producing rainfall and the minimum indicated value of K is the writer's concept of time of concentration and is so used in the remainder of this paper.

Taking the time of concentration derived in this manner as the time base length, a time-area concentration curve for the drainage area above the gage is prepared. This curve is used as the inflow to the river channel and is routed through an amount of storage represented by the indicated minimum K-value and an x-value of zero.

EXTENSION OF STORAGE THEORY TO GROUND-WATER FLOW

In practise, the indicated value of K does not remain constant after reaching its minimum value. The inflow from ground water continues and does not decrease as rapidly as the storage-controlled surface runoff. Therefore, it becomes an increasingly larger percentage of the total discharge and increases the indicated value of K.

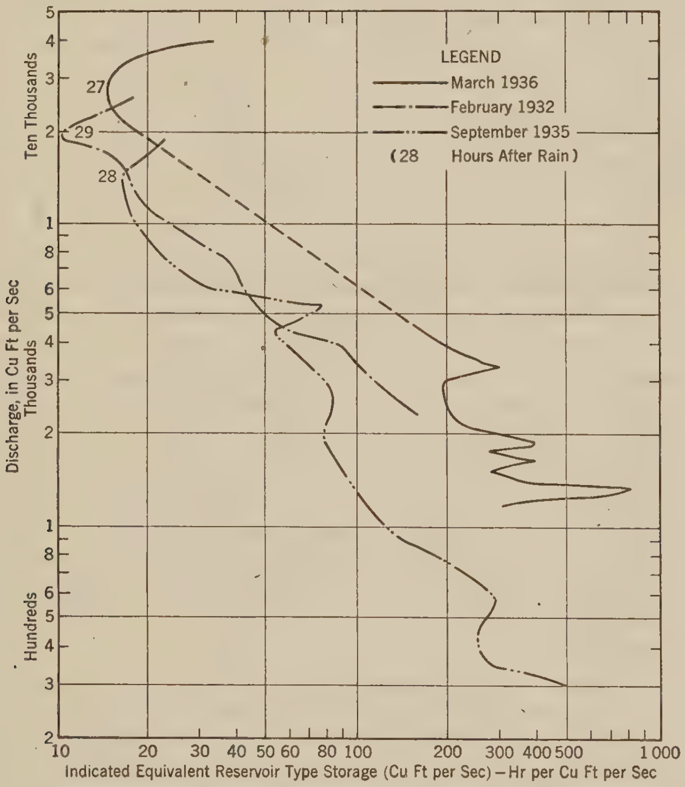

FIG. 5.—DETERMINATION OF EFFECTIVE STORAGE FROM RECESSIONAL HYDROGRAPH; JAMES RIVER AT LICK RUN

Actual curves of the indicated value of K are plotted in Fig. 5 against the discharge rate for the major part of the falling leg of several hydrographs at a stream gaging station at Lick Run, Va., on the James River (drainage area = 1,369 sq miles). The indicated value of K decreases to a certain minimum value which is essentially constant for all hydrographs and then increases from that point as the discharge decreases, becoming very large as the discharge becomes very small.

Application of this theory of runoff would imply a definition of surface runoff as being that quantity of total runoff which, after the cessation of inflow, is discharged in direct proportion to the amount of flow remaining to run off. Ground-water flow would include all other flow.

Since ground water is detained by storage also, it would be convenient for hydrologists if that storage were found to be in direct proportion to the rate of discharge. In analyzing the discharge hydrograph of any stream, it is rare to find one which, after reaching an abnormally high peak, continues to fall without further rainfall until it has reached an abnormally low flow. However, this was the case following the flood of September, 1935, on the James River. In that hydrograph, the discharge at Lick Run increased from 271 cu ft per sec to 19,400 cu ft per sec and then fell steadily to 258 cu ft per sec, the lowest discharge of the year. During the falling period, the rainfall was so small and scattered that it is doubtful if it contributed in any way to the stream flow. The indicated K-curve for this hydrograph is among those shown in Fig. 5.

After studying a large number of hydrographs extending into periods of low flow, both for this station and for several other stations in the James River basin, an approximate value of K of 200 hours was selected as an average representation of the ground-water storage effect in that watershed. For this station and for three others of good record, five hydrographs for each station were separated into the two volumes which could be represented by the channel storage modification outlined and the subsurface (ground-water) flow released from storage of 200 hours. No hydrographs were found in which the subsurface part was less than 30% of the total runoff, and there were no major flood hydrographs in which it exceeded 42%. Using a value of K of 200 hours and of $x = 0.0$, another unitgraph for subsurface flow was derived. Combination of 70% of the surface unitgraph and 30% of the subsurface unitgraph provided an excellent practical unitgraph for this drainage area, which included both surface and subsurface flow.

Thus in the derivation of a unitgraph for instantaneous origin of runoff, by the method proposed by the writer, a unitgraph of surface runoff, derived by routing the time-area concentration curve through an amount of unbalanced storage as outlined herein, is combined with a unitgraph of subsurface runoff determined by routing the same histogram through an appropriate amount of subsurface storage. What the ratio of these components will be in the final hydrograph will depend on the pertinent fact of how much of the water comes from surface runoff and how much from subsurface runoff in the particular watershed. For the streams analyzed along the eastern Appalachian slope, a 70 : 30 ratio seemed quite satisfactory. (If the additional work is justified, separate unitgraphs of surface and subsurface runoff may be utilized. With such use, no assumption of a fixed ratio is necessary.)

Although such a division is arbitrary, it is not as unsatisfactory as the current practise of attributing to ground-water discharge all flow beneath a straight line drawn from a point before the rise to an arbitrary and undefinable point on the descending hydrograph. The volume of flow so neglected frequently contains more than 20% of the total runoff. Any determination of runoff from subsequent storms, to be used in a unitgraph so derived, must be made in the light of such a derivation, with the result that, in times of great floods, the runoff percentage used for the flood-producing precipitation may be less than the average percentage runoff for the month corrected for prior and subsequent ground water.

Derivation of the Instantaneous and Unit Hydrographs

For the purpose of illustrating the method of derivation of unitgraphs, the Appomattox River at Petersburg, Va., was selected. There are several reasons why the drainage basin is not a good location for the application of the unitgraph principle; but, because of its peculiar shape, it is particularly adapted to the illustration of the benefits inherent in the method of unitgraph derivation proposed. Because it was not used in any way in the derivation of any of the data or principles on which this method is predicated and because the writer

TABLE 1.—Derivation of Instantaneous Hydrograph and 12-Hr Unitgraph on Appomattox River Near Petersburg

Time, in days	Time-area curve (%)	Routing Calculation[a]			Surface flow = 0.70 × Col. 5	Subsurface flow 0.30[b]	Instantaneous Hydrograph		12-hr unit hydrograph (cu ft per sec[c])
		$C_0 I_2 + C_1 I_1$ = 0.56 × Col. 2	$C_2 O_1$ = 0.44 × Col. 5	Routed time-area curve = Col. 3 + Col. 4			% per 12 hr[c]	Cu ft per sec[c]	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	....	....
0.5	1.8	1.01	0.0	1.01	0.70	0.03	0.73	500	250
1.0	3.8	2.13	0.44	2.57	1.80	0.10	1.90	1,300	900
1.5	6.9	3.86	1.13	4.99	3.50	0.22	3.72	2,550	1,920
2.0	10.8	6.05	2.19	8.24	5.75	0.40	6.15	4,220	3,380
2.5	19.1	10.70	3.63	14.33	10.00	0.72	10.72	7,360	5,790
3.0	7.6	4.25	6.32	10.57	7.40	0.81	8.21	5,630	6,490
3.5	6.5	3.64	4.64	8.28	5.80	0.88	6.68	4,580	5,100
4.0	5.5	3.08	3.64	6.72	4.70	0.93	5.63	3,870	4,230
4.5	9.0	5.04	2.96	8.00	5.60	1.03	6.63	4,550	4,210
5.0	14.0	7.83	3.52	11.35	7.80	1.22	9.02	6,180	5,360
5.5	9.5	5.32	4.98	10.30	7.20	1.32	8.52	5,850	6,010
6.0	5.5	3.08	4.53	7.61	5.33	1.34	6.67	4,580	5,210
6.5	....	....	3.35	3.35	2.35	1.25	3.60	2,470	3,520
7.0	....	....	1.47	1.47	1.03	1.18	2.21	1,510	1,990
7.5	....	....	0.65	0.65	0.46	1.11	1.57	1,110	1,310
8.0	....	....	0.29	0.29	0.20	1.04	1.24	850	980
8.5	....	....	0.13	0.13	0.09	0.98	1.07	735	800
9.0	....	....	0.06	0.06	0.04	0.92	0.96	660	700
9.5	....	....	0.03	0.03	0.02	0.86	0.88	600	630
10.00	....	....	....	....	....	0.81	0.81	560	580[d]

[a] Cols. 3, 4, and 5 are routing of Col. 2 with $K = 15$ hr, $x = 0.0$. [b] Col. 7 equals 0.30 × Col. 2 routed with $K = 200$ hr, $x = 0.0$. [c] Instantaneous rate of discharge. [d] 116,000 (cu ft per sec)-hr, or 13%, remaining to run off. Successive values of unitgraph hereafter are 0.942 × preceding value.

had done no work on this stream previous to the present calculation, this basin was used. The calculations in Table 1 show the first computations made and neither the original data, the first interpretations of these data, the calculations, nor the plotting, have been changed or altered in any way to make the computed hydrograph agree more closely with the observed hydrograph. (Only a slight change in estimated time distribution of runoff is necessary to improve the agreement.) This stream has a flatter slope and a longer time of concentration than any other stream of comparable drainage area in its immediate vicinity and therefore is adapted to illustrating some of the faults as well as the good points of the application. Fig. 6 shows the drainage area of the Appomattox River above Petersburg and the manner in which it was divided to provide the data for the time-area concentration curve; Fig. 7 is the profile of the river above the gaging station as taken from the best available U. S. Geological Survey topographic sheets; and Fig. 8, curve (a), shows, in bar diagram form, the time-area concentration curve converted to flow for 1 in. of runoff; and curves (b) and (c) show the time-area curve after routing through 9 hours and 200 hours, respectively, of storage. A combination of 70% of curve (b) and 30% of curve (c) is the instantaneous hydrograph, curve (d), for the Appomattox River at Petersburg. Fig. 9 shows the application of the 12-hr unitgraph derived from the instantaneous hydrograph to the flood of April, 1937. This is the second largest flood for which there is any available record at Petersburg and one of the largest floods that has occurred in the past 100 years. (The largest flood, August, 1940, caused by protracted rainfall, did not exhibit a double peak but is equally well reproduced with the same unitgraph.)

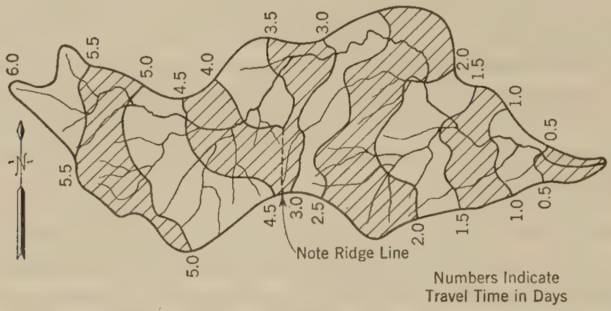

Fig. 6.—Drainage Area of the Appomattox River above Petersburg

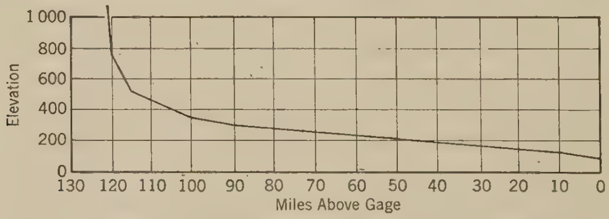

Fig. 7.—Profile of the Appomattox River above Petersburg

The following statements of fact must be taken into consideration in judging the degree of accuracy with which the computed record follows the observed record:

1. The area is too large for the proper application of a single unitgraph (drainage area, 1,335 sq miles). Because of the direction of the stream in comparison with the general east and west travel of storm fronts in this vicinity and because of the long time of concentration, a difference in rainfall both in the quantity and in time on the east part and west part of the drainage area may be considerable as it was in this case. Much better results could be obtained by using separate unitgraphs at the same gaging point for the upper and the lower drainage areas.

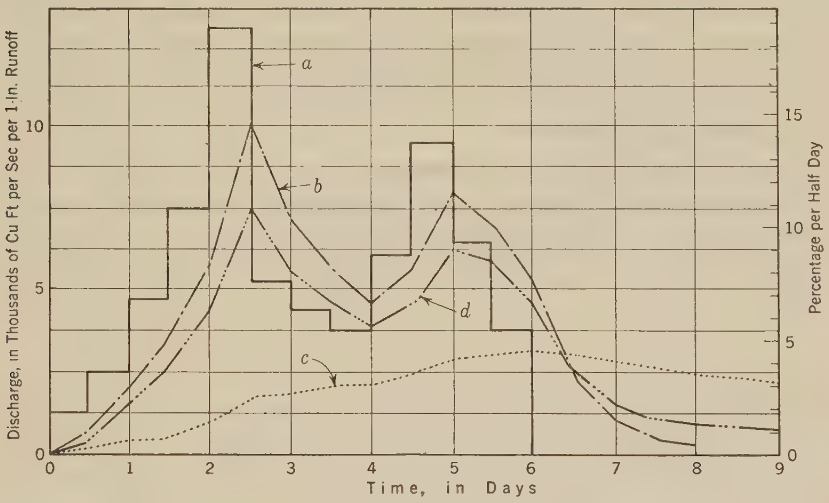

FIG. 8.—TIME-AREA CONCENTRATION CURVE, APPOMATTOX RIVER

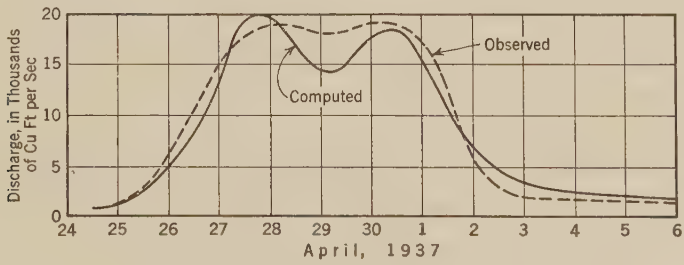

FIG. 9.—APPLICATION OF THE 12-HR UNITGRAPH, DERIVED FROM THE INSTANTANEOUS HYDROGRAPH, TO THE FLOOD OF APRIL, 1937

2. A more than usual degree of doubt as to the actual quantity of total precipitation is present in view of the small number of gaging stations (only one in the drainage basin), the nearest recording rainfall stations being some miles outside. The station in the center of the upper drainage area recorded 5.18 in.

of rain as compared with 2.58 in. recorded at Hopewell, Va., a station several miles east of Petersburg.

3. Observed discharges are from a single-stage discharge curve without slope correction. The gage is on a very flat stream and only 2.5 miles above a power dam. Although the gage is reported to be out of the backwater area in times of low water, the control for high water is thought to be the dam, in which case there is probably a large slope correction to discharges at this gage. If so, the observed stage hydrograph could not reflect the marked discharge decrease on April 29. At peak stage, 1 ft on the stage discharge curve is 3,000 cu ft per sec. This, and other features of the reported record, are sufficiently in doubt to cover the variation between observed and computed hydrographs.

4. On the other hand, the method may be no more accurate than is indicated by the agreement of the curves.

A comparison between the instantaneous unitgraph for the Appomattox River near Petersburg (drainage area, 1,335 sq miles; average slope of the river profile, 2.9 ft per mile; length above gage, 122 miles; and concentration time, 144 hours) and for the James River at Lick Run (drainage area, 1,369 sq miles; average slope 17.3 ft per mile; length, 78 miles; and concentration time, 28 hours) is shown in Fig. 10. Fig. 11(a) shows a similar comparison between the instantaneous hydrographs of the Smith River at Bassetts, Va. (drainage area, 265 sq miles; average slope, 16.6 ft per mile; length, 37 miles; and concentration

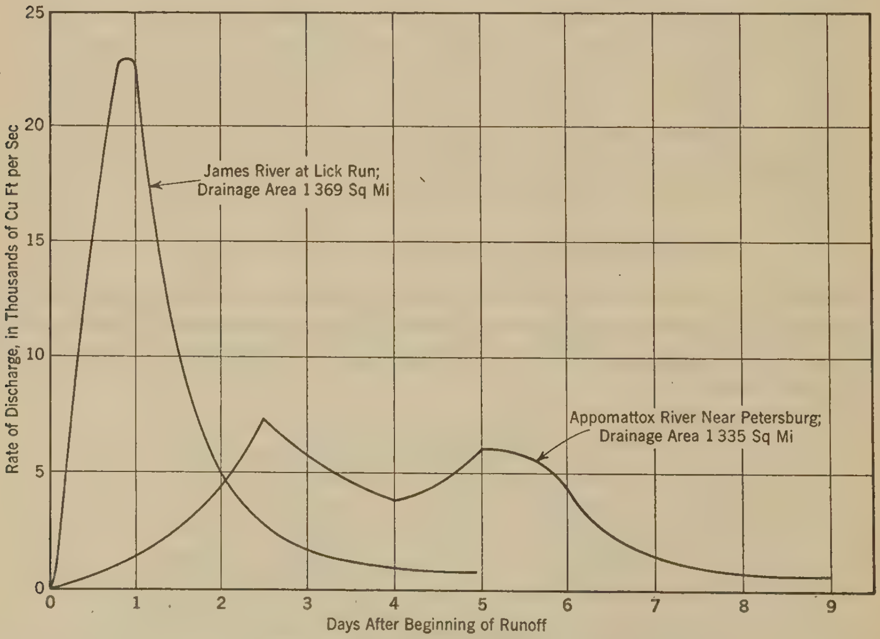

FIG. 10.—INSTANTANEOUS HYDROGRAPHS, APPOMATTOX RIVER NEAR PETERSBURG AND JAMES RIVER AT LICK RUN

time, 7 hours) and for the Meherrin River at Emporia, Va. (drainage area, 750 sq miles; average slope, 2.4 ft per mile; length, 86 miles; and concentration time, 78 hours). Fig. 11(*b*) shows a comparison of observed and computed flood hydrographs at Bassetts for the flood of August 18, 1939, and a similar comparison at Emporia for the flood of March 6, 1932.

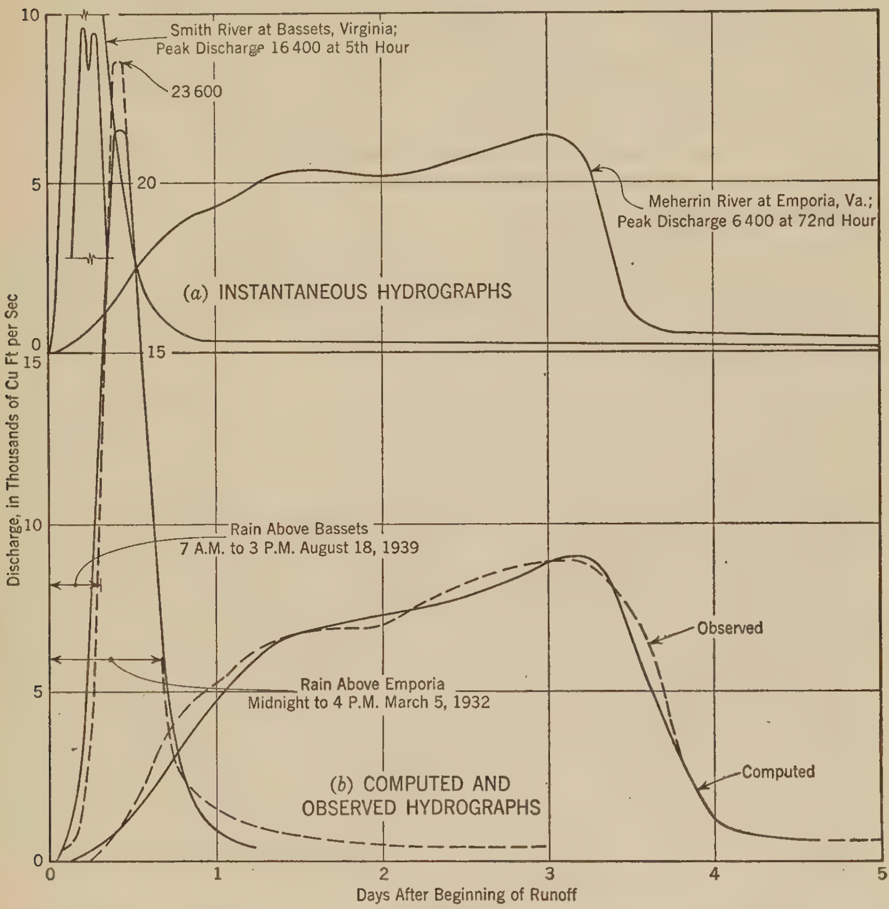

Fig. 11.—Hydrographs for the Smith River at Bassetts and the Meherrin River at Emporia

Determination of the Instantaneous Hydrograph for Streams Without Flow Records

The determination of any kind of unit hydrograph for a stream without records is a hazardous task. In most cases, it will be possible to install river stage recorders temporarily, obtain a record of rise and fall, compute a rating curve, and determine the elements of time of concentration, K and x, with a fair degree of accuracy. Since the record is utilized chiefly to determine the time of concentration, the proposed derivation of a unit hydrograph from a record of a small flood is not subject to the errors incurred by use of similar records in unit hydrograph derivations as originally proposed by Mr. Sherman. However, it is necessary occasionally to approximate the flow characteristics of a stream without records. For such attempts, a correlation of time of concentration or storage capacity with river slope would be adequate.

Attempting to correlate the storage per mile of river expressed as hours per mile (the reciprocal of velocity) with average slope of the stream above the gage, the writer found, as would be expected, that this item increased with the decrease of slope. The average storage in hours per mile was found to be numerically equal to the reciprocal of the square root of the slope in feet per mile from source to gage, when the slope is taken as that of a straight-line profile with the same area below it as the actual profile. For some streams the variation from the average value was large—sometimes as much as 100%. The storage was larger for streams in alluvial beds than for those in rock gorges, although the range in slope for which studies were made overlapped. The average storage above two or more gages on the same stream, although varying with the slope, was more nearly constant than indicated by the aforementioned average criterion.

How much of the variation in storage is to be attributed to the natural causes and how much to limits of data cannot be ascertained. Most of the stream profiles were determined from U. S. Geological Survey sheets of 100-ft contour interval, although some of the flatter streams crossed only two such contours in their entire length. For steeper streams, crossing enough contours to reduce the possible error in slope determination, the time of concentration became so small that the possible error of 6 hours in time of precipitation was a large percentage of the total time. Perhaps someone working in an area for which good maps were available and which contain recording rainfall gages may be able to find a more precise relationship. Also, the writer was unable to find a usable definition of average river slope that could evaluate the influence of local concentration of extreme slope as a truly small influence on flow-time of the entire stream, compared with well distributed fall of the same amount.

The amount of the storage by which the time-area concentration curve should be modified will approximate 10% to 25% of the time of concentration, with the smaller values being used with the longer times of concentration and extremely narrow drainage areas.

UNIT HYDROGRAPHS FOR LARGE DRAINAGE AREAS

By definition the application of the unit hydrograph is limited to areas over which the amount of runoff originating in any unit period may be considered as uniform within the required accuracy of the solution. For the flashy streams of the South Atlantic Coast, this is a drastic limitation, because they drain areas whose principal dimension is west to east, whereas the usual rainfall isohyets run north to south, and complete coverage of a major drainage basin by any storm is the exception rather than the rule. Because the streams are flashy, 6-hr unit periods are too long for component drainage areas, but the shortest practical period for entire principal river systems. Under such conditions, the difference between the theory and fact of uniform origin of runoff may be large even for drainage areas of 500 sq miles or less.

For these streams, the storage-discharge relationship in all reaches of many of the rivers may be approximated closely by a straight line throughout the range of flood discharge. This being the case, the Muskingum method of flood routing is applicable to the translation and storage modification of flood waves. Where the Muskingum method is applicable, the unit hydrograph theory may be used, by deriving the unit hydrograph for the component drainage areas of such size as to approximate the uniformity criterion and by routing the unit hydrographs themselves to the point for which a composite unit hydrograph is desired. The use of the proper amount and distribution of runoff with each routed component unitgraph will reproduce the component of runoff from the drainage area in question, and the sum of all components will be the desired composite flood wave. The degree of accuracy will be the same as though flood hydrographs had been computed for all tributary gages and then routed to the point for which the hydrograph is desired. In flood control work, the advantage of being able to identify individual components of a hydrograph is obvious.

TABLE 2.—UNIT HYDROGRAPH OF JAMES RIVER AT RICHMOND IN 1,000 CU FT PER SEC PER 1-IN. UNIFORM RUNOFF IN 6 HOURS

Time, in days[a]	James River at Lick Run, Va.	Craig Creek at Parr, Va.	Catawba Creek at Fincastle, Va.	Local at Buchanan, Va.	North River at Buena Vista, Va.	Local at Holcombs Rock, Va.	Local at Bent Creek, Va.	Buffalo River at mouth	Rockfish River at mouth	Local at Scottsville, Va.	Hardware River at mouth	Slate River near Arvonia, Va.	Rivanna River at Palmyra, Va.	Willis River at Flanagan Mills, Va.	Local at Cartersville, Va.	Local at Richmond, Va.	James River at Richmond, Va.	Time, in days[a]
Area[b]	1,369	331	104	280	660	506	421	417	245	238	135	235	675	247	379	515	6,757	Area[b]
0.25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.82	0.82	0.25
0.50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5.94	5.94	0.50
0.75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3.65	13.12	16.77	0.75
1.00	0	0	0	0	0	0	0	0	0	2.24	0	0	0	0	8.92	11.06	22.22	1.00
1.25	0	0	0	0	0	0	0	0	0	4.42	0	0	0	0	8.12	5.43	17.97	1.25
1.50	0	0	0	0	0	0	4.73	3.30	0	4.57	0.39	0.44	0	0.34	5.62	2.50	21.89	1.50
1.75	0	0	0	0	0	0.89	7.16	6.54	2.01	3.88	1.59	1.78	2.58	0.73	3.53	1.23	31.92	1.75
2.00	0	0	0	1.75	0	4.56	7.30	7.24	3.58	2.94	2.34	3.34	8.26	1.17	2.18	0.72	45.38	2.00
2.25	0	0	0.37	3.47	3.55	6.96	6.22	6.53	3.91	2.07	2.21	3.89	10.38	1.55	1.36	0.51	52.98	2.25
2.50	1.58	0	1.18	4.14	7.19	7.48	4.78	5.18	3.46	1.41	1.70	3.36	9.74	1.61	0.90	0.44	54.15	2.50
2.75	6.89	1.97	1.63	3.95	9.03	6.75	3.45	3.82	2.72	0.96	1.21	2.51	8.00	1.63	0.61	0.38	55.51	2.75
3.00	11.83	3.56	1.68	3.30	8.87	5.44	2.40	2.70	1.98	0.68	0.81	1.74	6.07	1.88	0.46	0.37	53.77	3.00
3.25	14.64	4.37	1.49	2.56	7.86	4.12	1.64	1.88	1.41	0.49	0.53	1.15	4.40	2.14	0.38	0.35	49.41	3.25
3.50	15.29	4.37	1.19	1.89	6.44	3.02	1.13	1.30	0.98	0.37	0.36	0.77	3.11	2.22	0.33	0.35	43.12	3.50
3.75	14.25	3.87	0.90	1.37	4.97	2.18	0.81	0.93	0.67	0.29	0.25	0.43	2.21	2.06	0.30	0.33	35.82	3.75
4.00	12.39	3.13	0.64	0.97	3.71	1.55	0.60	0.69	0.47	0.25	0.18	0.32	1.60	1.69	0.28	0.32	28.79	4.00
4.25	10.24	2.39	0.56	0.69	2.69	1.12	0.44	0.53	0.35	0.21	0.14	0.26	1.18	1.25	0.27	0.31	22.63	4.25
4.50	8.19	1.78	0.38	0.51	1.98	0.84	0.37	0.44	0.28	0.19	0.12	0.21	0.90	0.88	0.26	0.30	17.63	4.50
4.75	6.30	1.32	0.26	0.38	1.45	0.65	0.33	0.37	0.24	0.17	0.10	0.19	0.71	0.62	0.24	0.29	13.62	4.75
5.00	5.00	0.99	0.20	0.31	1.12	0.52	0.30	0.33	0.22	0.16	0.09	0.17	0.59	0.45	0.24	0.29	10.98	5.00
5.25	3.96	0.75	0.16	0.27	0.90	0.45	0.27	0.31	0.20	0.15	0.09	0.16	0.52	0.34	0.23	0.28	9.04	5.25
5.50	3.09	0.57	0.12	0.25	0.75	0.40	0.27	0.29	0.18	0.14	0.09	0.16	0.47	0.26	0.22	0.27	7.53	5.50
5.75	2.40	0.48	0.08	0.22	0.66	0.38	0.26	0.28	0.17	0.14	0.09	0.16	0.44	0.22	0.21	0.26	6.45	5.75
6.00[c]	1.88	0.41	0.06	0.21	0.59	0.35	0.25	0.26	0.15	0.14	0.09	0.16	0.42	0.19	0.21	0.25	5.62	6.00[c]

[a] After beginning of runoff. [b] Drainage area above station in square miles. [c] Values beyond 6 days omitted.

For forecast work, it is desirable not only to have a complete breakdown of the drainage area into small parts but also to have a table of summations of these component unit hydrographs in various applicable combinations. If, for any particular storm pattern, practical uniformity of runoff origin over several component drainage areas exists, considerable time may be saved by using the combination graphs rather than the separate components.

Table 2 is a typical arrangement of component unit hydrographs prepared for the James River at Richmond, Va. Illustrating the nonuniformity of occurrence of runoff typical of South Atlantic rivers, the largest known flood (May, 1771) at this city is reported to have occurred without sufficient rain in the city to settle the dust.

Determination of Approximate Routing Constants

There are several ways of determining the storage constants for use in the routing of the unitgraph components. If the inflow between the tributary gage and the outflow is small or if it can be determined with enough accuracy to give proper weight to its influence, the amount of storage effective between the tributary gage and the desired outflow point can be determined from the relationships of Eqs. 9 to 12; or, where the hydrographs are of similar shape, as is frequently the case, by measuring the displacement between the falling leg of the inflow and outflow hydrographs. Thus, in Fig. 12, the storage effect between Danville, Va., and South Boston, Va., on the Dan River, is 32 hours. Although there are 680 sq miles of intervening drainage area, the discharge from that area had decreased to insignificant proportions prior to the time which is used as a criterion.

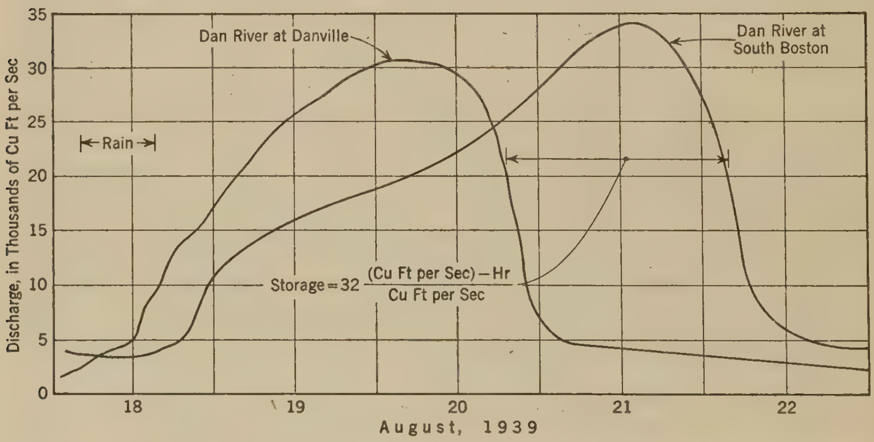

Fig. 12.—Approximate Determination of Storage from Hydrographs

The most accurate method of determining the storage-discharge relation is still from the accumulated differential between inflow and outflow, as outlined by Mr. McCarthy.[9]

SUMMARY

The paper has attempted to show that:

1. The constant time elements essential to the theory of the unit hydrograph, and common in many streams, are attributable to fundamental characteristics of the watersheds which maintain the storage capacities of the stream channels in constant direct proportion to the flows in them.

2. Such a relationship between storage capacity and discharge capacity produces certain characteristics in the rates of rise and fall of the hydrograph which are calculable by a simple established method.

3. These characteristics can be incorporated in a unit hydrograph, without the necessity of making or utilizing any quantitative determination of runoff distribution, in a few steps defined as follows:

(*a*) From a record of flood discharge, determine the time at which the rate of discharge decrease is greatest in relation to the prevailing discharge;
(*b*) The time elapsed between the end of runoff-producing rainfall and the time so determined is the base length (time) of a time-area concentration curve derived by dividing the watershed a long its water courses;
(*c*) The time-area concentration curve is routed as if through a reservoir with storage equal to K times the outflow when K is the time indicated by Eq. 12*b* at the time determined as in step (*a*) when $x = 0.0$;
(*d*) The result expressed in cubic feet per second per inch of runoff depth on the watershed at unit intervals of time is the instantaneous hydrograph; and
(*e*) The instantaneous hydrograph is converted to a unit hydrograph for any desired period of time by subdividing the instantaneous graph into periods of the desired unit length and averaging the ordinates over the preceding periods of time. Thus, in a 6-hr unit hydrograph, the rates of discharge at the end of 6, 12, and 18 hours, respectively, are the average ordinates to the instantaneous hydrograph in the 6-hr periods preceding the sixth, twelfth, and eighteenth hours.

4. A similar procedure may be applied to the determination of a unit hydrograph of subsurface flow, and the two unitgraphs may be combined in the average proportion applicable to the watershed, or used separately if desired.

5. The derivation of the unit hydrograph in this manner has some advantages:

(*a*) The procedure is definable so that all hydrographs derived by different individuals will be identical;
(*b*) The determination is independent of any knowledge of runoff distribution, except the time of ending, and therefore not subject to the errors due to indeterminability of runoff distribution which are factors in other methods known to the author;
(*c*) The influences of shape of drainage area and the capacity to produce large peak flows from concentrated runoff—both factors which are obscured by protracted runoff in other methods of derivation—are well indicated. The method also can be used to indicate the change in hydrograph form which is likely if a natural reach of river is replaced by a reservoir pool; and
(*d*) The unit hydrograph quantities are instantaneous rates of discharge at the time specified, not average values during the preceding unit period as in Mr. Sherman's original method, and the resulting hydro-

graph is properly drawn through all computed points, rather than as a bar diagram, and peaking of the hydrograph is no longer a matter of personal judgment. Hydrographs computed from the same distribution of runoff with 6-hr unitgraphs will agree with those by 1-hr graphs.

6. The derivation of the unit hydrograph in this manner has some disadvantages:

(*a*) Hydrographs are likely to fall a little too low on the receding leg between the point of maximum recession rate and the point where subsurface flow becomes predominant;
(*b*) When applied to too large drainage areas, the hydrograph may have too slow a rise and too rapid a fall, as a result of using the same storage factor for near and remote elements. This fault is corrected easily by subdividing the drainage area and computing separate unitgraphs for the components and combining them in the manner indicated in Table 2;
(*c*) Although this method reflects influence of shape of drainage area and capacity to produce high peaks well—better than any other one known to the author—it is possible that such influences are exaggerated slightly.

DISCUSSION

James S. Sweet,[10] Assoc. M. Am. Soc. C. E.—The relationship between flow and storage in open channels has been demonstrated in this paper. The analysis and the suggested procedures of the flow routing are a definite contribution to flood control design estimates. However, in forecasting work, where time of preparation and issuance of a forecast must necessarily be reduced to a minimum, the procedure must be simplified to effect a quick answer, without sacrificing the limits of relative accuracy. Mr. Clark's method involves time-consuming, laborious computations, and therefore is not adaptable for this type of work. On the James River where stage relationships between the gages along the entire length of the river are not always well defined, due to the nonuniformity of the distribution of the storms over the basin, it is best to use the method of the summation and routing of the discharges. Discharges in the upper reaches of the main stem and the tributaries should be ascertained either through the direct reports from the stations or through the simple rainfall-stage relations and should be routed to a downstream point on the main stem. Adjustments should be made to synchronize the arrival of the discharges at the desired point simultaneously. The discharges from the intervening ungaged areas should be adjusted, taking into consideration the ratio of index areas and rainfall over them and the area coefficient that reflects run-off characteristics and channel storage of the area. Determined empirically, these coefficients for various parts of the basin range from 0.6 to 0.7. Thus, discharge of the James River at Buchanan may be expressed as follows:

$$Q_B = Q_L + Q_P + Q_F + Q_U \qquad (15)$$

in which Q_B = discharge of the James River at Buchanan; Q_L = discharge of the James River at Lick Run, 9 hours previously; Q_P = discharge of Craig Creek at Parr, 7 hours previously; Q_F = discharge of Catawba Creek at Fincastle, 5 hours previously; and Q_U = discharge from the ungaged area (280 sq miles) above Buchanan.

For uniform storms, using the discharge at Lick Run as an index for the ungaged area discharge, Eq. 15 is further simplified:

$$Q_B = 1.123\, Q_L + Q_P + Q_F \qquad (16)$$

Discharges, including peak discharges, can be determined in a similar manner at all desired points along the main stem and converted into stages. With the equations prepared in a simple form such as Eqs. 15 and 16, actual computations take very little time after the headwater discharges become available to the forecaster.

It is regretted that Mr. Clark is introducing a new concept of time of concentration, thus adding another to the multitude of existing definitions. Not in the hope of relieving the confusion, but rather to add another choice for the future selection of an appropriate definition, the writer suggests that consideration be given to the definition of concentration time as time elapsed between the beginning of the effective rainfall (rainfall after the initial loss is satisfied) and the time of the peak rise of the stream.

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[10] Regional Hydrologic Engr., U. S. Weather Bureau, Flushing, N. Y.

Otto H. Meyer,[11] Assoc. M. Am. Soc. C. E.—A new tool for synthesizing runoff from rainfall is presented in this paper. It is a good and useful tool, as is shown by the excellent agreement of reconstituted hydrographs with natural hydrographs. Although this method will not replace other accepted techniques of synthesis entirely, it deserves to be used in a large proportion of cases because of both its simplicity and accuracy.

The author does well to emphasize relative accuracy. Estimates have often been condemned for lack of accuracy when they were in fact the best available.

Some conclusions, in the paper, such as increase in outflow following decrease in inflow, mass rotation of water-surface profiles, and negative outflows, are apparently deduced from empirical and approximate formulas. These conclusions should be examined critically. Also, the statement that K is constant (and the derivation of formulas based on that condition) involves rationalizing from an empirical assumption. The value of K can often be assumed constant, with good "relative accuracy" and in many cases with entirely satisfactory results; yet this value usually varies somewhat with S and Q; in fact, it is often so variable as to be unusable. The writer has developed numerous storage-outflow curves for streams in the Sacramento and San Joaquin valleys of California, and for good routing results these curved considerably; in several cases, there were sharp reversals of curvature. The principal effect of using a constant K, instead of a variable K, will probably be the shifting of high peaks in time relative to lower flows.

Mr. Clark mentions storage as "balanced" or "unbalanced." These two types of storage appear to refer to "reach" and "reservoir" storage, respectively.

The division of runoff into surface and ground-water flow, by the criterion that the runoff in direct proportion to storage is surface runoff, is arbitrary. This theory is permissible only for convenience or when no other means of division is practicable. An important part of the runoff other than surface runoff never reaches the true ground-water table, but flows quite near the surface, and responds promptly to rainfall. The writer has found an apparent correlation between this volume in subsurface storage and the infiltration from rainfall; that is, the subsurface storage determines not only the outflow but also the surface runoff factor. Thus, a part of the rainfall may be routed "through the ground"; but as it travels underground a short distance only (at most several hundred yards) the outflow should be added to the "net" pluviograph and synthesized into the hydrograph along with the direct surface runoff. Of course, the inflow into subsurface storage is not total rainfall less direct surface runoff, as evaporation must also be considered; likewise, some of the subsurface storage is lost to the true ground water, and some is lost in transpiration.

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[11] Maj., Field Artillery, U. S. Army, Corvallis, Ore.

The author constructs a time-area concentration curve, routes it through storage, and averages the outflow over a unit time to form a unitgraph. If the time-area concentration curve were averaged over a unit period, routing of the resulting graph might produce the same unitgraph. To extend this idea further, a summation of the products of a time-area concentration curve with successive parts of a net pluviograph could be routed through the storage with about the same labor as by the author's procedure. Although the suggested method would have the advantage of permitting use of a storage-outflow relation involving a variable K, it would involve more routing. However, there are less laborious procedures than the Muskingum method, such as "lag routing,"[12] or the use of a flood routing machine.

The application of Mr. Clark's technique is not necessarily limited to the Muskingum method of routing, but can be used with any of about a half-dozen methods in current use.

L. K. SHERMAN,[13] M. AM. SOC. C. E.—The definite, but complex, relations that exist between the hydrograph of surface runoff, the procedure of flood routing, and the law of proportionality of certain hydrographs, are presented in this paper.

All the objectives noted in the "Synopsis" have been fulfilled, with one very important exception. That exception is the author's statement, "to derive accurate unit hydrographs." No unit hydrographs or percentage distribution graphs have been derived in this paper.

Col. 10, Table 1, is labeled "12-hr unit hydrograph (cu ft per sec)." This caption is very misleading to a reader. It is not a unit hydrograph. The definition of a unit hydrograph, accepted and used since 1932, is: The hydrograph of surface runoff from a given basin due to a 1-in. depth of net rainfall, applied at a uniform rate in a unit of time. The ordinates of flow of this unit hydrograph (expressed in cubic feet per second) and the ordinates of flow of an observed hydrograph, due to a net rain of X inches in the same unit time, are proportional to their respective volumes of total surface runoff.

Net rain means rainfall minus infiltration and other losses.

The distribution graph is the unit hydrograph with ordinates expressed in percentage of the total volume of the observed hydrograph. The percentage distribution graph is commonly used in practice.

The proportionality of ordinates holds true for any two hydrographs of surface runoff on the same basin, provided the net rains in each case hold a direct relation of intensities, and follows identical sequence of downpours.

Let it be required to apply the foregoing criteria to this Appomattox River basin with a drainage area of 1,335 sq miles. If 1 in. of net rain fell on this basin, and if the rain followed identical times and similar intensities as that of April, 1937 (see Fig. 9), it would produce a hydrograph with the same time base as that observed in Fig. 9. The ordinates of runoff would be proportionate to the ordinates in Fig. 9, and the volume of surface runoff would be 1,335 in.-miles or 36,000 (cu ft per sec)-days or 72,000 (cu ft per sec)-half-days. The total surface runoff (by scaling ordinates in Fig. 9) in the flood of April, 1937, is

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[12] "Simplified Flood Routing," by Otto H. Meyer, *Civil Engineering*, May, 1941, p. 306.

[13] Cons. Engr., Chicago, Ill.

215,000 (cu ft per sec)-half-days. The base flow (also estimated from Fig. 9) was, approximately, $\frac{1,000 + 2,000}{2} \times 19$ half-days or 29,000 (cu ft per sec)-half-days. Then 216,000 − 29,000 = 187,000 (cu ft per sec)-half-days as the total volume of surface runoff due to the April, 1937, storm.

A similar storm producing 1 in. of runoff from this basin was 72,000 (cu ft per sec)-half-days.

The average depth of net rain, measured from the surface runoff hydrograph, is 187/72 = 2.6 in. of average depth over the Appomattox basin in the flood of April, 1937.

The comparable figures given in Table 1 are: Runoff for 1 in. of net rain = 60,000 (cu ft per sec)-half-days. This is the total of Col. 10. This amount is 87% of the total runoff. Therefore, 100% of 1 in. of net rain is 69,000 (cu ft per sec)-half-days. Accordingly, the depth of net rain utilizing the author's figure is 187/69 = 2.7 in.

This agreement by the author's elaborate routing analysis is an ingenious and possibly very useful accomplishment. It has no value as a derivative of the ratio of 1 in. of net rain to the total runoff from a particular storm. That is found readily in a few minutes from an observed hydrograph and a known size of drainage area. Col. 10, Table 1, is the runoff of 1 in. of net rain over the basin; but it certainly is not 1 in. applied over the basin in one unit of time. Therefore, it is not a unit hydrograph or a distribution graph. It certainly cannot be used to derive the hydrograph of runoff from the Appomattox basin under a storm of entirely different pattern. That is what the unit hydrograph is for. The same criticism applies to all the data in Table 2.

The author states (see heading, "Derivation of the Instantaneous and Unit Hydrographs") that another storm "is equally well reproduced with the same [so-called] unitgraph." He has not demonstrated the truth of this statement.

However, it would seem possible to utilize the methodology of Table 1. In this case the data furnished no record of the amounts of unit-time rainfalls on the basin. The total net rainfall averaged over the basin is simply and readily found as 2.6 in. or 2.7 in. How much of this fell in each of how many half-day periods is unknown. If this were furnished or found, a unit hydrograph could be derived. The unit hydrograph could be derived just as well if the 12-hr rainfalls for the period, April 25–29, were known—by the methods described by Mr. McCarthy[9] or the writer.[14] In deriving a unit hydrograph, the first step is to find the volume or volumes of net rainfall that fell in unit periods of time. These are the volumes that produce the hydrograph of surface runoff.

In the author's example of April, 1937, in Fig. 9, the total runoff was found to be 2.7 in., which is the sum of several unit-time downpours; but, in this case, the number or relative amounts of the unit volumes which total 2.7 in. is not known.

The bar pattern, Fig. 8, curve (a), indicates that the author has discovered that the 2.7-in. runoff was due to 12 unit periods of rain with ordinates proportional, or equal, to this bar pattern. Mr. Clark does not describe how he found this bar pattern. The number of blocks agrees with the number of time units in Table 1, Col. 2. The ordinates or heights of these blocks agree fairly well with Col. 2 averages.

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[14] "Physics of the Earth," National Research Council, Div. of Physical Sciences, Pt. 9—"Hydrology," by O. E. Meinzer, 1942, p. 521.

The total percentage of Col. 2, however, is not 100%, or even 87%, to correspond with the total of Col. 8, Table 1.

It is hoped that, in his closure, the author will consider this feature of Table 1 and the block pattern (a) of Fig. 8. He may have found a way of determining the unit volumes of net rainfall which created the observed hydrograph, without prior data on rainfall. Astounding as this statement may seem, there is nothing fundamentally impossible about it. The hydrograph of surface runoff reflects, not only the physical characteristics of the basin, but also the pattern or sequence of the creating net rainfall. In other words, with an observed hydrograph of surface runoff, it is possible to derive the unknown unit rains and thus the unit hydrograph. With the unit hydrograph and the observed rainfall, the corresponding hydrograph of runoff is found readily.

If the author presents further consideration of this feature, it is also hoped that he will follow the orthodox procedure of eliminating base flow. The problem is sufficiently complex without including it. In this connection, attention is called to the work of Robert E. Horton, M. Am. Soc. C. E., which includes routing and the derivation of unit hydrographs.[15]

The writer does not agree with the author's statement that 1,000 sq miles is a large basin upon which to apply the unit hydrograph or distribution graph. The distribution graph has been frequently and satisfactorily used for basins of 5,000, 10,000, and even 30,000 sq miles. The theory of the unit hydrograph assumes a uniform rate of rain for a relatively small fraction of the time of runoff. The volume in this unit time is the same volume as the nonuniform rain in this unit time. Each unit volume changes with the storm intensity. If the unit times are taken small enough, the runoff hydrograph will be exactly reproduced.

In practise, whether to use small areas and apply routing, or use large areas without routing, depends altogether upon the available data including the number of stream gages in the basin.

Gordon R. Williams,[16] Assoc. M. Am. Soc. C. E.—Numerous positive statements are made by the author which controvert accepted concepts and experience in the field of river hydraulics. These statements are not substantiated in the paper by experimental evidence or observations of actual conditions.

The most radical theory deals with "pressure" or momentum waves moving downstream through reservoirs and natural river reaches. The author claims that, if the inflow to a reservoir or river reach is decreased abruptly, the slope storage in the upper end of the reservoir or reach moves downstream as a pressure wave and increases the stage and discharge at the lower end of the pool relationship is not disturbed by any manipulation which follows. The relationship is not peculiar to instantaneous hydrographs, however. If two 6-hr unitgraphs are added with a 6-hr lag and the sum is divided by two, a 12-hr unitgraph results. The ordinate of the 12-hr graph at hour 12:00 is the sum of the 6-hr and 12-hr ordinates of the 6-hr graph divided by two, or the average ordinate of the 6-hr graph during the first twelve hours.

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[15] "Virtual Channel-Inflow Graphs," by Robert E. Horton, *Transactions*, Section of Hydrology, Am. Geophysical Union, Pt. III, 1941, p. 818 (4).

[16] Senior Engr., Office of the Chf. of Engrs., U. S. Army, Washington, D. C.

The author indicates an attempt to correlate storage with slope in order to devise a method for determining K for basins without adequate data. He found that the relation could be expressed approximately as: $\frac{K}{L} = \frac{1}{\sqrt{s}}$; or,

$$C_1 = \frac{K\sqrt{s}}{L} \qquad (18a)$$

in which L is the length of the stream in miles. Eq. 18*a* considers only the distance along the main stream and the slope. It would seem that other factors of importance would be the storage in tributaries and the comparative magnitude of flood flows to which the streams are subjected. If drainage area were introduced into Eq. 18*a*, it should indicate the effect of these two factors. Then: $\frac{K}{L} = \frac{\sqrt{A}}{\sqrt{s}}$; or,

$$C_2 = \frac{K\sqrt{s}}{L\sqrt{A}} \qquad (18b)$$

The writer has used the square root of A because peak flows tend to vary as the square root of the drainage area, and, in all probability, tributary storage also varies as some power of drainage area less than one.

TABLE 3.—COMPARISON BETWEEN EQS. 18*a* AND 18*b*

Stream	s (ft per mile)	L (miles)	K (hours)	A (sq miles)	C_1 (Eq. 18*a*)	C_2 (Eq. 18*b*)
Appomattox River near Petersburg, Va.	2.9	122	144	1,335	2.01	0.055
James River at Lick Run, Va.	17.3	78	28	1,369	1.49	0.040
Smith River at Bassetts, Va.	16.6	37	7	265	0.77	0.047
Meherrin River at Emporia, Va.	2.4	86	78	750	1.40	0.051
Middle Fork, Feather River at Bidwell Bar, Calif.	59	98	16	1,353	1.26	0.046
American River:						
North Fork, at Colfax, Calif.	80	50	8	308	1.43	0.082
North Fork, at Auburn, Calif.	80	56	10	619	1.60	0.064
South Fork, at Coloma, Calif.	91	60	12	635	1.90	0.075
Kings River:						
At Piedra, Calif.	72	86	9	1,694	0.89	0.036
Above North Fork, Calif.	132	60	7	956	1.34	0.043
San Joaquin River above Big Creek, Calif.	192	40	7	1,042	2.22	0.075

Table 3 affords a comparison between Eqs. 18*a* and 18*b*. The first four items are from data given by the author, and the last seven are based on data available to the writer. Unfortunately, determinations of K for streams of the Sierra Nevada drainage are complicated by the presence of a snow pack which effectively reduces the drainage area, length of channel, and time of concentration, in addition to the other problems indicated by the author. The criterion of Eq. 18*b* seems to be more nearly a constant than does the value of

$K \sqrt{s}/L$ from Eq. 18*a*. Values of $K \sqrt{s}/L \sqrt{A}$ have somewhat more of tendency to be higher for the California streams than do the values of Eq. 18*a*. This may indicate that a variation must be expected from region to region, but more likely it indicates that some power other than the $\sqrt{s}$ should be used.

Inspection of Fig. 6 indicates that the author divided the drainage area on the basis of equal units of stream mileage. Either Eq. 18*a* or Eq. 18*b* could be used to determine the subareas and should give a time-area concentration graph which more closely approximates the true graph. Eq. 18*a* is the simplest to apply, and, since the problem is merely to divide a basin for which K is already known, should be nearly as satisfactory as Eq. 18*b*. The number of reaches will equal K/T, and the values of $d/\sqrt{s}$ (d is the length of the reach) for all reaches should be equal. The solution is reached easily by approximations if a family of curves showing the variation of $d/\sqrt{s}$ with d, and with s, is plotted. A first approximation to the proper value of $d/\sqrt{s}$ will be the value of $T\,L/K\sqrt{s}$, which, for the Appomattox River, is 6.0. A final value of 5.75 was determined by the writer, which gives ten reaches of 8.6 miles, one reach of 12 miles, and one of 24 miles.

E. F. BRATER,[19] JUN. AM. SOC. C. E.—The unit hydrograph is the most useful tool available for the purpose of converting a quantity of surface water, resulting from precipitation, to a river discharge hydrograph. Since the unit hydrograph concept was introduced, a number of different interpretations and methods of application have been evolved. A description of these various concepts and techniques should be included in technical literature. Such a study and discussions of it should help to unify and improve engineers' understanding of the unit hydrograph. This paper is a valuable step in that direction. However, the engineer who may be desirous of making use of this technique will hope that a more detailed description of the methodology, with examples of the numerical computations, will be included in the author's closing discussion. For instance, the "time-area concentration curve" is an important feature of the method, but the author has not shown how one may be obtained for a given watershed.

In his method of flood routing, Mr. Clark sets up the relationship between storage and discharge in Eqs. 3 and 4. In Eq. 4, x is a term that is allowed to vary from 0 to 1. When $x = 0$, the storage becomes proportional to the outflow from the channel reach; when $x = 1$, the storage must be proportional to the inflow; and, when $x = 0.5$, the storage becomes proportional to both inflow and outflow in equal weight. The third condition assumes trapezoidal or double-wedge storage. Mr. Clark indicates that for most streams x is nearly 0.5. However, he chooses to discuss at considerable length the importance of the case in which x is greater than 0.5. In this connection, the writer does not agree with the author.

In the first place, the case of $x = 1$ is a physical impossibility in a real reach of channel, since, if storage is proportional to inflow, storage must become zero at the same instant that inflow becomes zero. For this to be true, the final portions of flood water would have to travel the length of the reach in-relationship is not disturbed by any manipulation which follows. The relationship is not peculiar to instantaneous hydrographs, however. If two 6-hr unitgraphs are added with a 6-hr lag and the sum is divided by two, a 12-hr unitgraph results. The ordinate of the 12-hr graph at hour 12:00 is the sum of the 6-hr and 12-hr ordinates of the 6-hr graph divided by two, or the average ordinate of the 6-hr graph during the first twelve hours.

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[19] Asst. Prof., Civ. Eng., Univ. of Michigan, Ann Arbor, Mich.

The author indicates an attempt to correlate storage with slope in order to devise a method for determining K for basins without adequate data. He found that the relation could be expressed approximately as: $\frac{K}{L} = \frac{1}{\sqrt{s}}$; or,

$$C_1 = \frac{K\sqrt{s}}{L} \qquad \text{(18a)}$$

in which L is the length of the stream in miles. Eq. 18*a* considers only the distance along the main stream and the slope. It would seem that other factors of importance would be the storage in tributaries and the comparative magnitude of flood flows to which the streams are subjected. If drainage area were introduced into Eq. 18*a*, it should indicate the effect of these two factors. Then: $\frac{K}{L} = \frac{\sqrt{A}}{\sqrt{s}}$; or,

$$C_2 = \frac{K\sqrt{s}}{L\sqrt{A}} \qquad \text{(18b)}$$

The writer has used the square root of A because peak flows tend to vary as the square root of the drainage area, and, in all probability, tributary storage also varies as some power of drainage area less than one.

TABLE 3.—COMPARISON BETWEEN EQS. 18*a* AND 18*b*

Stream	s (ft per mile)	L (miles)	K (hours)	A (sq miles)	C_1 (Eq. 18*a*)	C_2 (Eq. 18*b*)
Appomattox River near Petersburg, Va.	2.9	122	144	1,335	2.01	0.055
James River at Lick Run, Va.	17.3	78	28	1,369	1.49	0.040
Smith River at Bassetts, Va.	16.6	37	7	265	0.77	0.047
Meherrin River at Emporia, Va.	2.4	86	78	750	1.40	0.051
Middle Fork, Feather River at Bidwell Bar, Calif.	59	98	16	1,353	1.26	0.046
American River:						
North Fork, at Colfax, Calif.	80	50	8	308	1.43	0.082
North Fork, at Auburn, Calif.	80	56	10	619	1.60	0.064
South Fork, at Coloma, Calif.	91	60	12	635	1.90	0.075
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In the first place, the case of $x = 1$ is a physical impossibility in a real reach of channel, since, if storage is proportional to inflow, storage must become zero at the same instant that inflow becomes zero. For this to be true, the final portions of flood water would have to travel the length of the reach instantaneously. This paradox is illustrated by Fig. 2, in which, for $x = 1$, the inflow hydrograph and the routed or outflow hydrograph end at the same time. Thus, if such a graph were routed through successive reaches, the recession side of the graph could not advance in time. The result of several routings of the author's graph is a number of large plus and minus ordinates occurring at the end of the original hydrograph.

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[19] Asst. Prof., Civ. Eng., Univ. of Michigan, Ann Arbor, Mich.

Mr. Clark is of the opinion that storage of this type does occur and that it may cause outflow rates from a given reach to be greater than the inflow rates even without the addition of inflow from the local drainage area. In the third paragraph following Eq. 3 he states:

> "Reduction of the inflow rate [where backwater exists] necessarily would be accompanied by a decrease in storage and an increase of outflow rate. Rapid inflow shutdown necessarily would be accompanied by a release of the backwater storage and a consequent increase in the slack-water storage or in the outflow discharge or in both."

As an illustration of the nature of this action, the author takes the case of uniform discharge in the rectangular channel shown in Fig. 1. He assumes that the uniform flow of 116,000 cu ft per sec is suddenly stopped at the upper end of the reach and concludes that this will cause an increase in the depth at the lower end because 260 acre-ft of water are released from storage at the upper end. The writer wishes to point out that, since there is no change in the conditions at the lower end of the reach, the normal discharge of 116,000 cu ft per sec will continue there until the effect of this sudden stoppage can be transmitted through the length of the channel. A recession type of wave will be started at the instant of shutoff which, neglecting energy losses, might have a velocity as great as $V_w = \sqrt{g\,d} + V = \sqrt{32.2 \times 20} + 11.6 = 37$ ft per sec. Therefore, the earliest possible time that the effect will reach the lower end is $\frac{10,000}{37} \doteq 270$ sec after stopping inflow. During this time a volume of $\frac{270 \times 116,000}{43,560} = 720$ acre-ft will have passed out of the lower end of the reach. This is more than sufficient to make room for the 260 acre-ft which the author wishes to accommodate. There is in no sense a "release" of stored water, due to a decrease in the discharge at the upper end of the reach, either in a channel such as this or in the backwater above an impounding reservoir. Kinetic storage is held in place solely by the resistance to flow from point to point in the channel. It is already in complete equilibrium—that is, flowing as fast as possible with the energy available. A decrease in depth at the upper end of the channel can only decrease the energy and thus retard, rather than accelerate, flow.

L. C. CRAWFORD,[20] ASSOC. M. AM. SOC. C. E.—One phase of this interesting paper pertains to natural channel or valley storage and the relationship of this volume to discharge or stage. The channel-storage rating has become a valuable tool in flood routing, hydrograph analysis and synthesis, and infiltration studies. The author's comments on real and relative accuracy are very pertinent to unit hydrograph and channel-storage ratings which may be developed by indirect methods through the analysis of records of discharge during past major rises or capital floods.

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[20] Consultant, Iowa Inst. of Hydr. Research; Dist. Engr., U. S. Geological Survey, Iowa City, Iowa.

The basic stage and discharge records, which are cooperatively collected by the United States Geological Survey and other agencies, give an opportunity for potential channel storage and related studies that probably has not been sufficiently emphasized. Indirect methods of storage determination have been developed and are being further extended through new concepts, such as Mr. Clark presents, and through the use of basic records of stream flow. Thus, it is possible to ascertain with considerable relative accuracy various relations pertaining to natural channel or valley storage that otherwise would be too costly and wholly impracticable without extensive cross-sectional surveys.

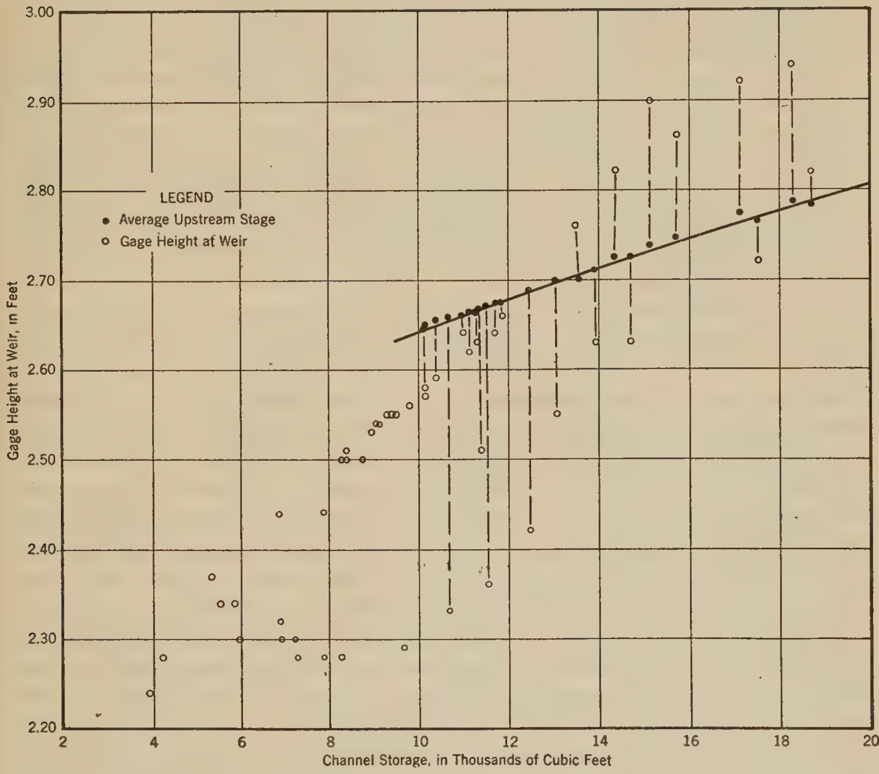

FIG. 13.—CHANNEL-STORAGE RATING CURVE FOR DIFFICULT RUN NEAR FAIRFAX, VA.

Practical experience with recession curves and storage relations have shown, however, that additional refinement in technique may be necessary and desirable. Such refinement is possible if sufficient hydrologic data are collected and correlated with stream flow records.

Fig. 13 shows determinations of gage height and storage in a small headwater area of about 1 sq mile on Difficult Run near Fairfax, Va. The method of channel-storage determinations was presented by O. E. Meinzer and other members of the United States Geological Survey,[21] in the following descriptive discussion:

> "The gaging station is equipped with a V-notch weir and a water-stage recorder. Measurements of channel storage were made by means of 134 secondary gages. The trunk-stream and each of the branches were divided into 100-foot segments, and at the middle of each segment a secondary gage was installed and a profile of the cross section of the channel was made. Each gage consists simply of a 2- by 2-inch stake driven into the bed of the stream so that the depth of water above the top of the stake can be measured with a scale. Secondary gages were installed and cross sections of the channel were made as far upstream on each of the four branches as any stream existed. The volume of water in each segment at any time was computed in cubic feet by multiplying the area of the cross section, in square feet, of the stream at the gaging station for that segment, by 100, and the channel storage in the entire stream system was computed as the sum of the volumes of water in all of the 134 segments. No account was taken, in the computations, of the dead storage below the level of the tops of the 2- by 2-inch stakes."

The observations and calculations of storage were made by V. C. Fishel of the United States Geological Survey in connection with an effluent seepage study.

Despite the probable precision in the determinations of storage volume, the relationship of the gage height at the weir to a storage volume is shown to be somewhat complicated and uncertain. However, if the computed storage is related to an average stage prevailing upstream as calculated from depth measurements from each of the 134 stakes, about two dozen of the storage determinations during appreciable outflow and channel storage may be alined as indicated in Fig. 13. This relationship is interesting in connection with Mr. Clark's discussion of the correlation of storage with discharge.

The measurements on Difficult Run indicate that the stage or discharge at the weir is a very rough index of stages and storage that may prevail in even such an extremely small basin. In addition, it is often presumed that the same relation exists between outflow and storage during rising stages as during the recession. Some methods recognize the existence of this hysteresis effect in connection with channel-storage studies but continue to assume that the net result is small or may be neglected. Some investigators in the field, nevertheless, recognize the inaccuracies in such an approach.

In any event, the study on Difficult Run demonstrates that more exact analysis is possible and probably desirable when the base data are suitable and sufficient. By relating mean upstream stages with channel-storage determinations, evaluations can be made of certain factors whose effect heretofore has been generally given only casual consideration. Moreover, computations involving the volume of channel storage during the rising side of a hydrograph can be made with some increased accuracy from such a storage curve as determined from the recession limb or by other methods.

The use of multiple gages for determining the channel-storage rating also affords a relatively simple method for larger basins. For example, several basins with areas greater than 3,000 sq miles have been examined to explore and demonstrate the procedure presented in this discussion. Fig. 14 shows the orientation of the channel-storage determinations as computed from recession curves for several major rises at the gaging station on the Wabash River at Logansport, Ind. The basin above Logansport comprises 3,760 sq miles and storage takes place in several tributaries with complicated inflow conditions.

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[21] "The Channel-Storage Method of Determining Effluent Seepage," by O. E. Meinzer, R. C. Cady, R. M. Leggette, and V. C. Fishel, *Transactions*, Am. Geophysical Union, 1936, Pt. 2, pp. 415–416.

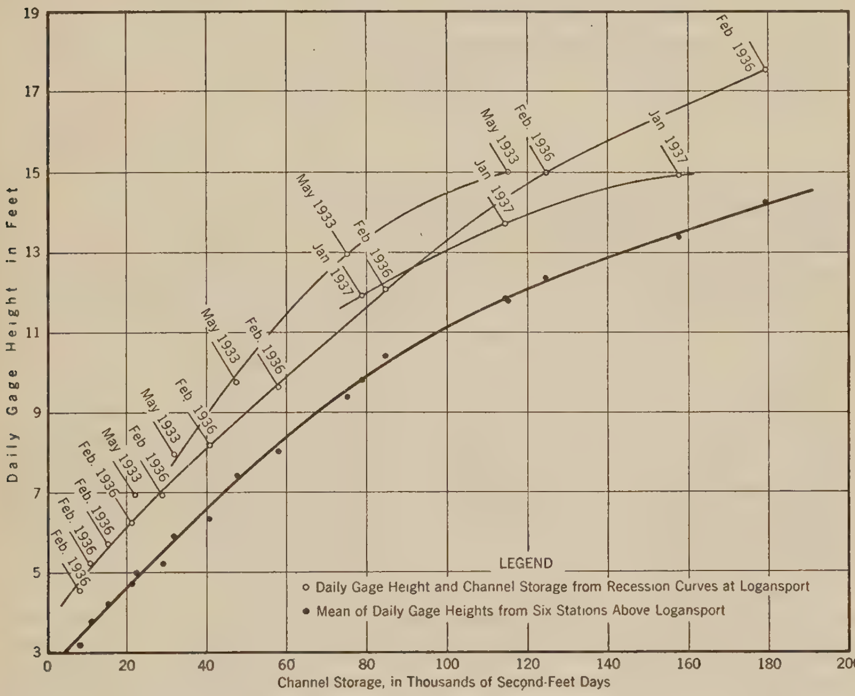

Fig. 14.—Channel-Storage Rating Curve for Wabash River at Logansport, Ind.

Figs. 13 and 14 illustrate important limitations in the assumption of Mr. Clark and others (see heading, "Valley Storage") that "over a large range the true storage may be closely approximated by storage directly proportional to discharge." The method, utilizing an average height or weighted upstream prevailing stage as an index for storage, appears also to provide a somewhat convenient approach for channel-storage determinations during the rising, as well as the falling, stream stages.

The opinions expressed herein are those of the writer and not necessarily those of the organizations with which the writer is identified.

Robert E. Kennedy,[22] M. Am. Soc. C. E.—At the risk of being more or less academic, the writer would like to comment on the old controversy over the basic assumption of the unit hydrograph.

In Fig. 15 the unit hydrograph, as labeled, is the runoff of a net rain of 1 in. over the watershed in a certain time unit—1 hr, 6 hr, or 12 hr—as may be chosen by the investigator. Then the basic assumption of the unit hydrograph idea is that, when a 2-in. net rain, for instance, falls on the watershed in the same length of time as the 1-in. net rain, the ordinates of the new hydrograph will be twice as high as those of the unit hydrograph, but the time of runoff will be the same for both storms. In Fig. 15 the hydrograph of a 2-in. net rain is labeled "Synthetic Hydrograph Computed by Proportional Ordinates of Unitgraph." The time length of both hydrographs, according to the basic definition of the unit hydrograph, is the line AE, for practical purposes. It is not claimed that the base lengths are exactly identical, which opens the question as to how much difference might occur under extreme conditions.

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[22] Asst. Engr., U. S. Bureau of Reclamation, Denver, Colo.

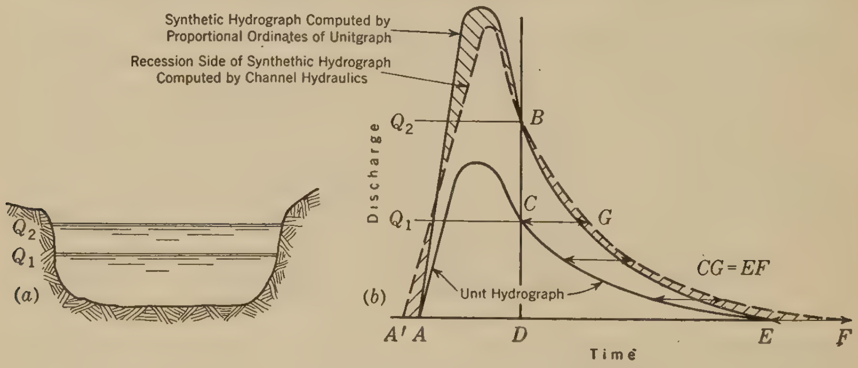
Synthetic Hydrograph Computed by Proportional Ordinates of Unitgraph
Recession Side of Synthethic Hydrograph Computed by Channel Hydraulics
Q_2
Q_1
(a)
Discharge
Q_2
Q_1
B
C
G
CG=EF
Unit Hydrograph
(b)
A′ A
D
E
F
Time

FIG. 15.—COMPARISON OF HYDROGRAPH COMPUTED BY UNITGRAPH METHOD WITH ONE COMPUTED BY CHANNEL HYDRAULICS ON RECESSION SIDE

A glance at the cross section of a stream channel in Fig. 15(*a*) shows that the channel storage of the two storms cannot be emptied in the same time. That would mean that the water stored in the channel at Q_2 and represented by point B on the hydrograph in Fig. 15(*b*) must run out in the same time as that stored at Q_1 in the channel and shown as point C on the smaller hydrograph, Fig. 15(*b*). This cannot occur because the water stored in the channel between Q_2 and Q_1 must go first, and that takes time. No matter how fast it may flow, the water cannot escape in "nothing flat"!

Just how much longer time the larger storage requires to empty is susceptible of mathematical treatment. The writer developed this relation and then found that his labor had been largely anticipated in mathematical analyses published by Robert E. Horton, M. Am. Soc. C. E., in 1936[23] and 1937.[24]

Any two floods can be so compared on the recession side of their respective hydrographs that the discharge at one point, for example, is twice that of the other. Let these two points be points of contraflexure. Two such stages are shown in Fig. 15(*b*) at point B on one hydrograph (dashed line) and point C on the smaller hydrograph below, ignoring for the present that the latter was used to illustrate the unit hydrograph

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[23] "Natural Stream Channel-Storage," by Robert E. Horton. *Transactions*, Am. Geophysical Union, 1936, Pt. II. pp. 406–416.

[24] "Natural Stream Channel-Storage" (Second Paper), by Robert E. Horton. *ibid.* 1937 Pt. II, pp. 440–456.

The difference in time required by the channel to empty itself of these two floods from this point of contraflexure is computed as $n^{0.20} - 1$ when n is the ratio of the ordinates of the two stages at this point. In Fig. 15, $n = 2$; and $2^{0.20} - 1 = 0.14$. Lines EF and CG, Fig. 15(*b*), are each 14% of line DE.

When the discharge of the larger flood is three times the smaller one at the point of contraflexure the larger flood requires 25% longer time to empty from that point on the hydrograph. If the larger flood discharge is ten times the smaller flood discharge, the larger flood would take 58% longer to empty. However, in practise the last featheredge of the storm water flow is so diluted with ground water or base flow that the end of the hydrograph is entirely indeterminate.

To complete the dashed-line hydrograph, resort was made to the non-mathematical assumption that the larger storm would take as much longer to fill the channel system as it does to empty it when compared to the time required for the smaller storm, so A′A was made 14% of AD. The rising leg of the dashed-line hydrograph was made parallel to that of the smaller flood.

The two areas shown crosshatched in one direction were made equal to the one area crosshatched in the other direction because the two hydrographs bounding these crosshatched areas represent the runoff from the same storm.

In conclusion it appears from the completed figure that the multiple ordinate concept of the unitgraph is not strictly correct because, when two such hydrographs as shown are superimposed, at only one point is the ordinate of the larger one exactly twice that of the smaller one. No matter how much shifting was done, all the ordinates of the larger or dashed-line hydrograph could not be twice those of the smaller one.

Nevertheless the unit hydrograph is a most useful tool in hydrology.

Victor H. Cochrane,[25] M. Am. Soc. C. E.—The relationship between the unit hydrograph and the storage capacity of the drainage system, as well as a simple and practical procedure for calculating hydrographs, is presented in this excellent paper. The author rightly states that the determination of a hydrograph depends upon two, large, basic factors—the shape of the watershed and the storage through which the runoff must come. The influence of these two factors is largely accounted for by the use of what the author calls the time-area concentration curve, and both the accuracy and simplicity of the method are due to this device.

Storage is related to time, and may be expressed in time units. The paper contains a discussion of storage concepts, and some rather surprising conclusions are drawn, but it does not appear that the author's procedure is dependent upon the validity of these conclusions. If the processes by which a hydrograph is built up are viewed from the standpoint of time instead of storage, a different, and perhaps more easily understood, explanation of the basis of the author's method is derived.

If a drainage basin is divided into a number of areas, or zones, such that their downstream boundaries are at equal time intervals of flow from the point of measurement, a bar diagram of these areas would be the time-area diagram for the watershed, the base of each bar being the constant time interval for flow through the zone. This diagram, when converted to flow in cubic feet per second for 1 in. of runoff in one period, becomes the time-area concentration curve.

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[25] Cons. Engr., Tulsa, Okla.

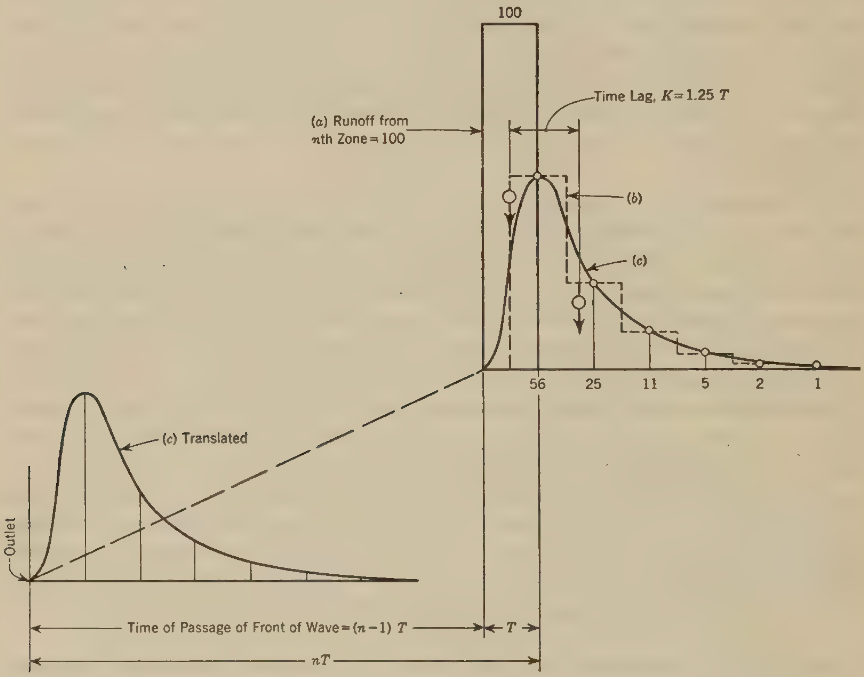


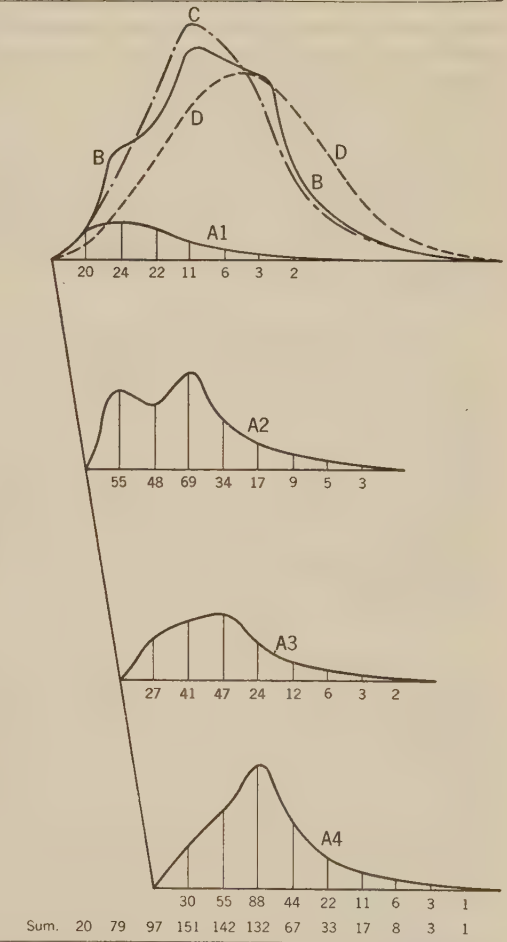
FIG. 16.—TYPICAL ZONE HYDROGRAPH

If all the runoff-producing rainfall in a unit period of time could be delivered instantaneously at the outlet (zero storage), the hydrograph would be a single bar with one time unit as the base and a height equal to the combined height of all the bars in the time-area concentration curve. If the net rainfall in each zone could be concentrated instantaneously at the zone outlet, and if the zone concentrations should then flow to the point of measurement at channel velocity and without further modification, the time-area curve would be the hydrograph of flow at the outlet. This latter process accounts for the greater part of the effect of time (or storage), and the time-area diagram resembles the actual hydrograph.

However, the zone contributions do undergo modification within the limits of the zone and during the passage downstream. In Fig. 16 the bar diagram represents the runoff originating in the nth zone above the outlet during the unit time T. By the time it reaches the outlet it is modified to a typical outflow hydrograph, such as curve (c), which is shown in relation to the precipitation and in the translated position. In this connection the author makes the valuable suggestion that the modification (which is in addition to that implicit in the time-area concentration curve) be determined by means of the Muskingum equations with x equal to zero and with K, the time lag in hours between center of mass of net zone rainfall and center of mass of zone runoff, equal to some small amount, such as 1.25 T. The modified hydrograph becomes the bar diagram (b), and if a smooth curve is drawn through the ordinates of (b) it will be the zone hydrograph (c).

TABLE 4.—COMPUTATION OF HYDROGRAPH; $K = 1.5\,T$

Zone Number	Area of Zone, in Square Miles	Net Rainfall: Depth, in Inches	Net Rainfall: Miles² Inches	Distribution to Periods: 1	2	3
1	40	2.2	88	40	28	20
2	100	2.4	240	110	40	90
3	60	2.7	162	54	54	54
4	100	2.6	260	60	80	120
Totals	300		750			



The author's use of a constant value for K seems to be based on the assumption that most of the modification occurs within the limits of the zone. This is a reasonable basis for practical use, for it will be found that the precise shape of the zone hydrograph is of little consequence. However, it is not difficult to make use of variable values of K. The summation of the zone hydrographs for a one-period rain may be used as a unit hydrograph, but it is better to complete each zone hydrograph for its entire runoff and then combine the flows.

The hydrograph is sensitive to the shape of the time-area diagram and to the number of periods of rainfall. Variations in rainfall intensity and distribution, infiltration losses, and other factors may be taken into account zone by zone. The method is simple and flexible. It is based on the same conception of constant time elements that underlies the unit hydrograph theory, but it is more adaptable to a variety of conditions. The hydrograph may be computed for an approximate time interval and then corrected to fit the proper time base. When the time-area diagram and other constants are properly determined, the hydrograph due to any type of storm can be calculated with satisfactory limits of approximation. Conversely, any kind of storm can be employed in deriving the constants.

Table 4 shows the suggested arrangement of data. The calculations are for a four-zone watershed having an area of 300 sq miles. The zones vary considerably in size, and a three-period rainfall is irregularly distributed both with respect to zones and time periods. Constant K equals 1.5 T, so that $C_0 + C_1 = C_2 = 0.50$. The modification of runoff for zone 1 is computed as shown in Table 5.

TABLE 5.—COMPUTATION FOR THE MODIFICATION OF RUNOFF, ZONE 1, TABLE 4

Line No.	Description	Runoff in Mile²-Inches							
		40	28	20	..	..	..	..	Remainder
1	Runoff, in mile²-inches × 0.50	20	14	10	..	..	..	..	..
2	Line 3 (advanced one period) × 0.50	..	10	12	..	..	..	..	..
3	Time-period ordinates	20	24	22	11	5.5	2.75	1.38	1.37
4	Recorded in Table 4, and shown graphically	20	24	22	11	6	3	2	..

The zone hydrographs, A1, A2, A3, and A4, are combined by adding the computed ordinates, resulting in the irregular hydrograph B (see Table 4). The relatively small amount of precipitation in zone 2 during the middle time period is the cause of the dip in A2 and the corresponding flattening of B. The sharp peak in A4 results in a bulge in B. If the same total net rainfall were uniform over the entire drainage area over the three time periods, the resulting hydrograph, C, is much less irregular, but the peak is changed only slightly. The same total rainfall uniformly distributed over five periods would produce hydrograph D, having a considerably lower peak.

If the surface and subsurface flows are separated, the foregoing method may be applied in the case of the Appomattox River (see heading, "Derivation of the Instantaneous and Unit Hydrographs"). Assuming a uniformly distributed net rainfall of 2.35 in. (3,140 mile²-inches), occurring in the three periods ending at noon of April 26, and a time lag (storage) of 15 hours, then adding subsurface flows amounting to 0.57 in. in 10 days in accordance with curve (c), Fig. 8, the results agree substantially with the computed hydrograph shown in Fig. 9.

In the "statement of fact" No. 1, Mr. Clark states that there may have been a considerable variation in rainfall from east to west both in quantity and time. The writer assumed a variation of from 1.7 in. to 3.1 in., with a weighted average of 2.35 in. over the entire basin, and found that, on the whole, the computed curve did not fit the observed hydrograph any better than did the author's hydrograph, Fig. 8. The effect of varying K from a minimum of 9 to a maximum of 20 was found to be negligibly small. It would seem that the time-area diagram is the most important factor in the computation. In all cases there was too much difference between the peaks and the valley between, as compared with the recorded flows. It is likely that the record is at fault, as suggested in the "statement of fact" No. 3, or that the largest zone areas have been somewhat overestimated.

FRANKLIN F. SNYDER,[26] ASSOC. M. AM. SOC. C. E.—Papers dealing with unit hydrograph and flood routing procedures have been quite numerous in recent years and the author is to be complimented for his efforts to clarify the inherent relationship between these hydrologic tools. However, the extent to which that objective has been accomplished is modified by the introduction of a number of confusing and unsupported statements about certain hydrologic phenomena.

In the "Introduction," the author, referring to an article,[7] states that "For example, in determining a seven-period unitgraph from a flood resulting from three periods of runoff, it is possible to determine thirty-six unit hydrographs which, applied to the three items of runoff, will reproduce the composite hydrograph exactly." Theoretically, if the unit hydrograph procedure were exact, there would be one unit hydrograph that would reproduce the composite hydrograph exactly. For illustration, one can assume a unit hydrograph and compute a composite hydrograph. There exists, therefore, a unit hydrograph which will reproduce the composite hydrograph exactly. In actual practice there is no unit hydrograph that will reproduce observed composite hydrographs exactly. However, if the composite hydrograph is not the result of a too complex storm, numerous methods are available for determining a unit hydrograph that will reproduce the observed composite hydrograph with a satisfactory degree of accuracy. The example[7] cited by the author illustrates this theory, as the most probable distribution-graph values obtained by "least squares" for a composite storm do not reproduce exactly the observed hydrograph.

Fig. 3 shows the results of a study of the effect of storage that is equally a function of inflow and outflow on a wave form with an initially very sharp peak. The author has made a common mistake[27] in obtaining an outflow hydrograph different from the inflow hydrograph after assuming that the reach storage is equally a function of inflow and outflow ($x = 0.5$, or that storage varies as the sum of inflow plus outflow).

The fact that the outflow hydrograph should be identical with the inflow hydrograph except for a time shift is readily proved from the general storage equation and is evident from a consideration of the Muskingum formulas,

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[26] Hydr. Engr., War Dept., Office of Chf. of Engrs., Washington, D. C.

[27] "A Direct Method of Flood Routing," by C. O. Wisler and E. F. Brater, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 1519.

Eqs. 6*d*, 7*a*, 7*b*, and 7*c*. Another result is obtained when certain basic requirements fundamental to most routing procedures are violated:[8] (1) The routing interval, T, must be small enough to define, satisfactorily, variations in rate of flow; and (2) the reach should be of such length that the time of travel is approximately equal to T. In the example under discussion a 2-hr value of time of travel, K, was assumed by the author for an imaginary reach of a river. The value of T that was used to derive the hydrographs in Fig. 3 is not given, but, according to requirement (2), T should be 2 hours. Then, with x equal to 0.5, $C_0 = 0$, $C_1 = 1.0$, $C_2 = 0$, and $O_2 = I_1$—that is, outflow at the end of the period equals inflow at the beginning. Requirement (1) was violated since a value of T equal to 2 hours is too large to define, adequately, the hydrograph in Fig. 3. However, O_2 will always equal I_1 from Eq. 6*d* as long as T is made equal to K.

On the same basis the outflow curve labeled $x = 0.5$ in Fig. 2 is in error and should be identical to the original curve except for a time shift of 4 hours. The negative values for the curve, $x = 0.5$, and for the other outflow curves, also, are definitely due to the selection of units and the limitations of the routing procedure, and to couple these values with physical phenomena either real or imaginary seems unwise.

Another erroneous result is often obtained when routing a hydrograph such as that for Danville in Fig. 12, through a reach with a value of K considerably greater than a reasonable value of T. The slow prolonged rise and rapid fall coupled with the assumptions of the Muskingum routing procedure result in a peak outflow rate higher than the inflow rate even though a value of x considerably less than 0.5 is assumed. Walter B. Langbein,[28] Assoc. M. Am. Soc. C. E., has suggested an adjustment of the Muskingum procedure to compensate for the inability of the weighted flow to represent storage in the reach adequately when either inflow or outflow is changing rapidly.

To justify the basic assumptions followed in developing the Muskingum routing procedure, the value of T should be made approximately equal to the value of K. If this is not done (and it is seldom possible in practical applications), the value of x is a function of the length of reach, or K, in addition to indicating the portion of the weighted flow that is derived from the inflow.

The attempt to prove that shifting a hydrograph in time and routing it through a small value of storage gives a hydrograph quite similar to that obtained with the Muskingum routing procedure using a value of $x = 0.5$ has resulted in a failure to make clear an important objective of the basic idea. Most flood routing procedures actually provide no time advantage for forecasting purposes—that is, the inflow data must be extended into the future just as far as it is desired to obtain values of outflow. The procedure proposed by the author—namely, shifting the inflow hydrograph in time and routing it through a predetermined amount of reservoir-type storage—does give a time advantage equal to the amount of the time shift.

The writer has been accumulating data in connection with such a routing assumption and, as would be expected from the ease with which routing procedures can be verified, has had little difficulty in obtaining satisfactory results when routing observed hydrographs for various streams in the eastern United States.

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[28] "Channel Storage and Unit Hydrograph Studies," by Walter B. Langbein, *Transactions*, Am. Geophysical Union, Pt. II, 1940, pp. 620–627.

In analyzing these records a procedure convenient for routing the transposed hydrograph through a certain amount of reservoir-type storage was developed. As stated by the author, where storage, S, and outflow, O, are uniquely defined by the elevation at the outlet such as in a reservoir ($x = 0.0$ in the Muskingum procedure), the ratio of successive increments of storage to increments of discharge, $\frac{\Delta S}{\Delta O}$, has the dimension of time and is represented in this discussion by the symbol T_s. If storage is in acre-feet and O in cubic feet per second, T_s is in units of days and equals (very nearly) $0.5 \frac{\Delta S}{\Delta O}$. Starting with a volumetric expression of the storage equation for some satisfactory routing period, the mean inflow, $\bar{I}$, minus the mean ouftlow, $\bar{O}$, equals the change in storage. For units of cubic feet per second for discharge, acre-feet for storage, and days for time:

$$\bar{I} - \bar{O} = \frac{\Delta S}{2\,t} \quad \text{(19)}$$

With the usual assumption of straight-line variation of inflow and outflow during the routing period, t, and, since $\bar{O}$ then equals $\frac{\Delta O}{2} + O_1$, with the subscript 1 indicating the value of the variable at the beginning of a routing period:

$$\bar{I} - O_1 = \frac{\Delta O}{2} + \frac{\Delta S}{2\,t} = \Delta O \left(0.5 + 0.5 \frac{\Delta S}{t\,\Delta O}\right) \quad \text{(20)}$$

Substituting T_s for $0.5 \frac{\Delta S}{\Delta O}$ and solving for ΔO gives

$$\Delta O = C\,(\bar{I} - O_1) \quad \text{(21)}$$

in which

$$C = \frac{1}{0.5 + \frac{T_s}{t}} \quad \text{(22)}$$

This solution of the storage equation lends itself readily to tabular computation and to the use of variable values of T_s by plotting the variation of C against O. When inflow is zero, ΔO equals $- C\,O_1$ and C equals $1 - C_r$ in which C_r is a recession coefficient of the outflow hydrograph equal to $\frac{O_2}{O_1}$.

Fig. 17 shows graphically some of the time relationships utilized in analyzing observed records. Local inflow has been separated from the outflow hydrograph so that the latter represents the total effect of the transposition, T_t, and the reservoir-type storage, T_s, on the hydrograph of inflow at the upper end of the reach. The total time shift between center of mass of inflow and outflow, T_T, is equal to the sum of T_t and T_s; that is,

$$T_T = T_t + T_s \quad \text{(23)}$$

The value of T_T is also equal to the value of K in the Muskingum routing procedure provided the fundamental routing requirements previously discussed are not violated. However, if the inflow hydrograph, I, of Fig. 17 is routed using the Muskingum coefficients with $K = 24$ hours, $x = 0.40$, and $t = 6$ hours (too long a reach), the time lag between center of mass of inflow and outflow, T_m, will be 21 hours instead of 24 hours. A quite similar outflow hydrograph is obtained by this procedure, using $T_t = 10$, $T_s = 11$, and $T_T = 21$ ($T_m = 21$).

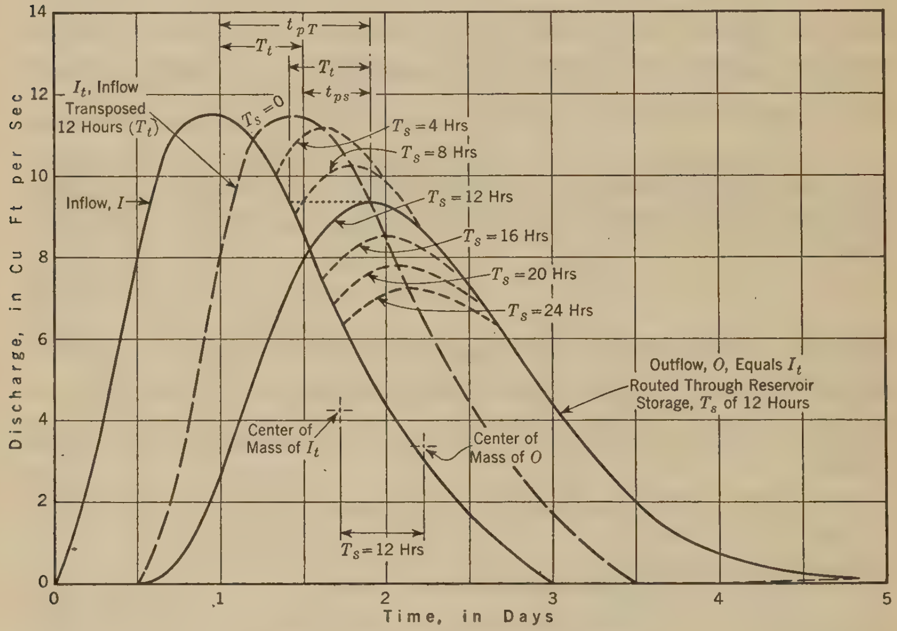

FIG. 17.—TIME AND STORAGE RELATIONSHIPS

The value of T_t in any particular case is available from inspection of the hydrograph and the following relationship is useful in estimating values of T_s: From Fig. 17, t_{pT} equals T_t plus t_{ps}, in which t_{pT} is the total time lag of the crest discharge, T_t is the crest lag due to the transposition, and t_{ps} is the lag of the crest discharge due to reservoir-type storage effect; therefore:

$$t_{pT} = T_t + t_{ps} \tag{24}$$

and, since T_s equals $T_T - T_t$,

$$T_s = T_T - t_{pT} + t_{ps} \tag{25}$$

The time between center of mass of the inflow and outflow hydrographs, T_T or T_m, is readily obtained by transferring the hydrograph shapes with carbon paper to a sheet of stiff paper, cutting out the hydrographs, and locating the centers of area.

In the beginning the writer had hoped that values of T_t and T_s could be assumed constant throughout a wide range of flow conditions, an assumption that is theoretically incorrect but one that is made in most routing procedures.

A variation of T_s can be handled without much difficulty but when both T_t and T_s vary so that their sum also varies, the problem becomes more difficult. As might be expected, observed hydrographs for many streams could be duplicated by assuming constant values of the time and storage factors. However, when a stream with a wide flood plain such as the Missouri River goes overbank, there is a variation of several hundred per cent in the factors.

Under the heading, "Valley Storage," the author states that "The reservoir pool above a dam may not reduce flood discharges." Most engineers will subscribe readily to this generality, and its acceptance is indicated by the fact that considerable attention is given to this problem in the design and operation of reservoirs for hydroelectric projects. The author also states that, " * * * if the pool level at the dam is held constant by gate control, * * * the peak outflow may be larger than the peak inflow." The writer could also agree with this statement if the words "natural peak" discharge at the dam site were substituted for the words "peak inflow"; but it will take considerable more evidence than the Muskingum routing hypothesis to convince him that the peak outflow can be greater than the peak inflow. As a matter of fact, because of backwater storage above a level reservoir pool, the level of the pool at the dam would have to be lowered somewhat to make the peak outflow of a flood as large as the peak inflow. The peak inflow in some cases is larger than the natural peak at the dam site would have been prior to construction of the project because of the elimination of some valley storage but principally because of the change in timing of the concentration of the runoff from the areas draining directly into the reservoir.

The author's extension of storage theory to ground-water flow is quite interesting. The use of a unit hydrograph for distributing ground-water runoff may be quite practicable, but the use of storage values determined from recession curves for deriving the rising part of the runoff hydrograph is subject to more criticism in the case of ground-water runoff than in the case of surface runoff, and the procedure is open to question even for surface runoff.

Other than the use of a constant factor for dividing total runoff into surface and ground-water runoff, the author's procedure probably would be satisfactory for many purposes. However, the writer takes exception to the inference that, in comparison, current practise in treating ground-water discharge is unsatisfactory. Reference is made to a paper[29] discussing this and other phases of runoff phenomena in which both direct runoff and ground-water runoff for the Schuylkill River above Pottstown, Pa. (drainage area, 1,147 sq miles) were computed storm by storm for a year of record using only observed values of precipitation and temperature. This article presents a graphical comparison of the computed ground-water discharge and the observed hydrograph.[30] The accuracy of the ground-water phase of the computations proved more than sufficient for any need of the writer in flood forecasting or hydrologic studies.

The author's use of the quantity, K, and the time of concentration in the development of unit hydrographs is confusing. Under the heading, "The

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[29] "A Conception of Runoff-Phenomena," by Franklin F. Snyder, *Transactions*, Am. Geophysical Union, Pt. IV, 1939, pp. 725–738.

[30] *Ibid.*, Fig. 4, p. 728.

Hydrograph as an Index of Storage," it is stated that, at the time of most rapid relative rate of decrease of discharge, "The falling legs of a large number of hydrographs of flood discharge for streams in major drainage basins from New Hampshire to North Carolina exhibit a ratio of decrease of discharge to total discharge which, for a value of $x = 0.5$, indicates a value of K closely approximating the time elapsing between termination of runoff producing rainfall and the point of most rapid relative decrease in discharge." With $x = 0.0$ the values of K for these same recessions determined by Eq. 12*b* would be equal to one half the value of time of concentration ($0.5\ T_c$). In the "Summary" it is stated that the time-area concentration curve should be routed through reservoir storage "equal to K times the outflow when K is the time indicated by Eq. 12*b*" at the time of most rapid relative rate of decrease of discharge when $x = 0.0$; that is, $K = 0.5\ T_c$. However, the author states (see heading, "Determination of the Instantaneous Hydrograph for Streams Without Flow Records") "the amount of storage by which the time-area concentration curve should be modified will approximate 10% to 25% of the time of concentration * * *." In the example for the Appomattox River above Petersburg, a value of $K = 0.06\ T_c$ was used ($K = 9$ hours, $T_c = 144$ hours) whereas the average value of K for the James River at Lick Run as determined from the data in Fig. 5 was $0.5\ T_c$ ($K = 14$ hours, $T_c = 28$ hours). With a somewhat similar procedure, which is about the most common and oldest known rational method of determining synthetic hydrographs, the writer has obtained satisfactory unit hydrographs from time-area curves routed through storage values equal to the unit hydrograph "lag" defined as time from center of mass of effective rainfall to peak of the unit hydrograph. Values of "lag" are almost always greater than one half the time of concentration as defined by the author.

Howard M. Turner, M. Am. Soc. C. E., and Allen J. Burdoin,[31] using a similar procedure, obtained fairly good reproduction of observed hydrographs for the Connecticut River at Sunderland, Mass., by routing a time-area or inflow hydrograph through a reservoir storage for which K equals 59 hours with a value of time of concentration equal to 50 hours ($K = 1.2\ T_c$).

The principal purpose of the preceding discussion is to emphasize the fact that there are still two variables in the development of a synthetic unit hydrograph: A time element (time of concentration, T_c, in this paper), and a storage factor (expressed as a percentage of T_c by the author). The use of a slope factor has been introduced by the author as an aid in determining the time factor, but the storage factor has not been correlated directly with any physical features of the drainage basin.

From the writer's experience in attempting to correlate the two variable factors with physical features,[32] he is not too optimistic about the author's use of a slope factor to determine the time variable except in localized areas. The river slope adopted by the author is believed to be a more representative slope factor than the average over-all slope usually obtained by dividing the total fall by the length. However, the data presented in the paper indicate a range of values of C, in which $T_c = C \frac{L}{\sqrt{s}}$, from 0.6 for the Smith River at Bassetts to 2.0 for the Appomattox River near Petersburg. (A value of average slope, as defined by the author, of 11.6 ft per mile for a length of 37 miles was determined from available reports and used for the Smith River at Bassetts instead of the value of 16.6 given in the paper.) Although the writer believes the exponent of the slope factor should be smaller than 0.5, variation of the exponent of s in the expression $T_c = C \frac{L}{\sqrt{s}}$ would have little effect on the relative range of values of C which vary from 0.6 to 2.0 for the Virginia drainage basins discussed in the paper.

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[31] "The Flood Hydrograph," by Howard M. Turner and Allen J. Burdoin, *Journal*, Boston Soc. of Civ. Engrs., Vol. XXVIII, No. 3, July, 1941, p. 232.

[32] "Synthetic Unit-Graphs," by Franklin F. Snyder, *Transactions*, Am. Geophysical Union, Pt. I, 1938, pp. 447–454.

The suggestion of the author that the determination of the time and storage factors for streams without records should be based on actual measurements should be emphasized. A time factor such as "lag" [32] can be obtained for any river station where discharge is principally a function of stage, without installing any equipment, by observing the time of crest stage and comparing it with field observations or official records of the rainfall characteristics of the storm. A storage factor is not so easily determined but it is not subject to as great a range in value as the time factor and the time factor provides a partial index of the actual value of the storage factor.

Howard M. Turner,[33] M. Am. Soc. C. E.—The first part of this interesting paper is a discussion of the channel storage in a reach of a river. The author compares the storage in this case with that in a reservoir, indicating the difference in the behavior, due to the fact that the relation between outflow and storage is not clear on account of the slope. The author cites an assumed case showing how a sharp diminution in inflow "requires" an increase in outflow at the outlet end of the reach. The writer has never seen any such effect. There is a sudden decrease in inflow at the outlet of many water power plants at 5 o'clock or 6 o'clock every afternoon for months during the year when there is no waste. The writer has examined many hydrographs taken at gaging stations some distance downstream from such stations and has never seen any evidence of any rise attributable to the cause given by the author. He admits the possibility, but believes that the author's statement is much too categorical in stating that it is "required" under such circumstances. There may be a slightly rising outflow because the high discharge existing at the upper end has not yet become stabilized at the lower end—that is, water is still going into storage and this rise will continue for a period of time after the power station shuts down due to the time lag between the two points. The outflow then drops off in the regular recession curve as the storage drains off. The difficulty in the author's contention is his statement (see heading, "Valley Storage") that the additional storage must be released "during the period of inflow reduction." This increased storage that must flow out may be released over a longer period of time.

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[33] Cons. Engr., Boston, Mass.; Lecturer on Water Power Eng., Harvard Univ., Cambridge, Mass.

The writer has analyzed several of these conditions and, by using the storage curve determined from the recession side of the hydrograph (that is, the storage as a function of the outflow), he has obtained very satisfactory agreements with an actual hydrograph by routing such power station discharges through this storage with time allowance for travel.[34] In two of the three cases considered, the hydrograph downstream did show higher flow than that upstream, but this was due to the flow from intervening drainage and the inaccuracy of computing the power station discharges from the kilowatt-hour output and the difference disappeared when the inflow hydrograph was adjusted to give the same total runoff. There was no evidence in these cases of any rise which the author states is required by sudden reduction of inflow to the reach. If the author has any actual examples showing this effect, it is hoped that he will present them as it is an extremely interesting and, as he states, a somewhat new conception.

The author then describes his distinction between storage as a function of the outflow and storage as a function of the inflow. The writer believes Mr. Clark has oversimplified this theory. Actually, storage is a function of neither inflow nor outflow alone, depending as it does on topography and the level of the water surface, which in turn are dependent upon slope. Therefore, any difference between the author's two concepts of storage is dependent on a difference in changing slope which with the topography of the banks affects the volume of storage. In this case a more exact statement would be that, in the case of the unsteady flow of a rising or falling river stage, the changing slope affects the storage.

The writer finds it difficult to accept some of the statements made in regard to the storage affected by the inflow. In Fig. 2 the author shows curves for routing a simple sine curve with different storages based entirely on inflow ($x = 1.0$) through to a curve based entirely on outflow ($x = 0.0$). In the case of the former the flow with storage is greater than without storage. Where $x = 0.5$, the hydrograph is very little changed. It is not until x equals less than 0.5 that the storage reduces the inflow, and not until $x = 0$ does the outflow hydrograph intersect the inflow hydrograph at the peak of the former, which is generally held as a basic requirement between inflow and outflow hydrographs.

As would naturally occur if these curves are correct, the curve of outflow with $x = 1.0$ intersects the curve of inflow at the peak of the former and thus the storage increases the peak discharge from the outlet of the reach by a very great amount. The writer gives $x = 0.3$ to 0.5 as the usual range of values. If this condition is valid, there will be many cases in which there will be little if any diminution of peak due to storage. The writer does not question the conclusions reached with the author's equations, but he has never seen any indication of such effects and therefore distrusts the original equation. He believes that it is based on too simple a function of the storage compared to that actually existing in any normal river channel.

The second part of the paper develops a method of constructing a unit hydrograph from an instantaneous hydrograph. In this case the author uses storage as a function of outflow alone ($x = 0$). The writer is not certain whether the author considers the condition to be somewhat different for a given reach than for the storage of the entire river system, since the storage used in this section of the paper does not seem to correspond with his previous statement regarding what he has found to be that on most rivers.

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[34] "The Flood-Hydrograph and Valley-Storage," by Howard M. Turner, *Transactions*, Am. Geophysical Union, 1943, Pt. II, p. 613.

The author's process consists of using storage based on the actual flood hydrograph at the beginning of the recession curve and routing through it a time-area concentration curve which is in effect an inflow hydrograph built from an analysis of the time required for the water from the different parts of the drainage area to reach the outlet. In doing this the same storage relation is used for the rising stage of the hydrograph as for the falling stage.

The writer was much interested in this procedure as it is based on the same principles that were used by him and Mr. Burdoin.[31] Using their method, a rational inflow hydrograph was constructed from the duration of the rainfall, the concentration time was obtained from the deflection point on the recession curve of the hydrograph, and the total flood volume was routed through storage computed from the recession curve to obtain the outflow or flood hydrograph. The author extends these principles somewhat further adding the feature of the instantaneous hydrograph.

The author's method differs with regard to the storage which he determines from only one point on the hydrograph—the inflection point on the descending limb. This assumes a straight line from this point down to zero, an assumption which may not always be correct. (On the Appomattox River at Petersburg, one of the examples cited, the curve is not a straight line all the way down.) Mr. Clark draws an instantaneous inflow hydrograph for a given river by actually figuring the time-area relation instead of using, as the writer did, a general one based on shape. This makes a much more flexible arrangement than the writer's and gives a more accurate inflow hydrograph. Such accuracy may not always be necessary, however, when the rainfalls over the area for different storms are considered. (For example, the Appomattox River at Petersburg has had other floods which present different characteristics. The one of March, 1932, with a peak of 8,000 cu ft per sec and about the same length of base shows no depression at the top of the hydrograph.) The writer found that an assumption of a rectangular shape gave satisfactory results in many cases that varied considerably from a rectangle.

The author's method, however, would permit extending the time-area concentration inflow not only to cover the shape of a given area with assumed uniform rainfall but also to include varying rates of rainfall on different parts of this area.

The author presents a new treatment of the base flow which, in the particular case of the Appomattox River, gives much better results than would the customary method—that is, a straight line at the base of the hydrograph, which, if used, gives a computed hydrograph less close to the actual one than the author's. This hardly can be taken as a proof that the author's method is any better since the entire matter of the rainfall distribution is unknown, as the author states. (Rainfall at Hopewell amounting to 23% of the total occurred during the two days, April 28 and 29, in the middle of the flood.)

There are many unknowns in the behavior of the ground storage during a flood. Unanswered are such questions as how the water gets into the ground with a rapidly rising river, how far back the rise in ground-water level continues before the river begins to drop again and the storage flows back into the river, and how much of this drainage from storage in the ground may be due to normal ground-water flow backed up by the rising level and thus not originating from the flood rainfall. The outflow hydrograph will also be much affected by the assumption as to the amount of flood appearing as ground-water flow assumed by the author as 30% of the total. The author suggests that this percentage, taken from records of another river, must vary with floods of different magnitude. To use 30% for an instantaneous hydrograph for all floods may lead to error in the case of much greater or much smaller floods. The author should give some examples of the proportion of ground-water runoff which his method discloses in small floods compared to very large floods. The writer has found that with all these possible differences, the simple method of drawing a straight line across the base of the hydrograph is sufficiently accurate for general use in spite of its theoretical limitations.

The author is to be congratulated for devising an additional tool to use in the attempt to analyze the steps between the rainfall, inflow hydrograph, and the flood hydrograph.

DON JOHNSTONE,[35] ASSOC. M. AM. SOC. C. E.—The difficulties of winnowing out the grains of wheat in technical literature are certainly not decreased by inventing new meanings for old phrases. Engineering terms should mean the same thing today as yesterday. If new concepts are introduced, new terms should be invented. The author, in applying the term "unit hydrograph" or "unitgraph" to a hydrograph which includes ground-water flow, is misusing a basic definition. Similarly, in applying the term "concentration time" to the time interval "between cessation of runoff-producing rainfall and the minimum indicated value of *K*" he is tying another time-honored phrase to a new concept. Mr. Bernard's definition of concentration time ("a time interval at the end of which all parts of the watershed are contributing to the flow at the point of observation") gives that particular combination of words a job to do and should pre-empt it from other uses.[36] Other examples of criticizable terminology include the use of "runoff" for "rainfall" in the third and fifth paragraphs of the "Introduction" and in the third paragraph of the section headed "Valley Storage and the Unit Hydrograph," followed by its use in an entirely different sense in the second paragraph of the next section; and the use of "the channel" in that same section without indicating whether reference is to the main stream alone or to any "channel" in the watershed.

Passing to other matters, attention is invited to the statement in the "Introduction" that the determination of unitgraphs from "multiple periods of runoff [sic] * * * is indeterminate, as there are always more equations than unknowns and a multiple of possible solutions." Presumably the author meant "more unknowns than equations," but, waiving that point, the statement is still incorrect. By unitgraph theory, surface runoff in the xth period resulting from a storm of n periods duration is $c_1 f(x) + c_2 f(x-1) + \cdots + c_n f(x-n+1)$, in which $c_1, c_2, \cdots, c_n$ are the net rainfalls for the corresponding periods and $f(x), f(x-1), \cdots, f(x-n+1)$ are the ordinates of the unitgraph for the xth, $(x-1)$th, $\cdots$, $(x-n+1)$th periods. If flow measurements and rainfall measurements were precise, if the base flow (ground-water flow) were known, if the net rainfalls for all periods were determinable, and, finally, if the stream behaved exactly in accordance with unitgraph theory, it would be possible to derive the unitgraph from the hydrograph of a multiple-period storm by successive subtractions. Actually, successive subtractions yield nothing of value, as none of the four "if's" is exactly in accordance with fact and as the process involves the accumulation of error from step to step. The concept is set forth here, however, to demonstrate that there are actually as many equations available as there are unknowns (so far as unitgraph theory is concerned), and that there should therefore be no possibility of multiple solutions from a given set of data if a proper technique of derivation is applied. One such technique has been outlined by William T. Collins,[37] Jun. Am. Soc. C. E.

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[35] Asst. Prof., Civ. Eng., Ohio State Univ., Columbus, Ohio (on military leave).

[36] "An Approach to Determinate Steam Flow," by Merrill Bernard, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 350.

It follows (see "Introduction") that "the practical requirement of finding an isolated flood hydrograph resulting from a uniform unit rainfall," which the author states "provides an almost insurmountable obstacle to the derivation of the unitgraph for very small [?] drainage areas and very short unit periods [intervals] of time," is no longer a requirement at all. As a matter of fact, a protracted period of heavy rainfall is the ideal period from which to derive the unitgraph, because (*a*) rainfall is more likely to be nearly uniform over the watershed, (*b*) infiltration conditions are more nearly uniform throughout the period of rainfall, and (*c*) the fact that rainfall actually extended over the selected unit time interval is assured. To carry this argument further would be beyond the proper scope of discussion; the writer acknowledges that despite the incorrectness of the quoted statements there is still need for a method that will permit deriving unitgraphs without reference to rainfall records.

The argument under "Valley Storage," down to the introduction of Eq. 4, is confusing. Particularly welcome would be a justification of the statement that "reduction of the inflow rate necessarily would be accompanied by * * * an increase of outflow rate." Fig. 1 and its explanation are unconvincing, and the writer would like to see (here as elsewhere) proof in the form of actual recorded data rather than in the form of a rather casual theoretical analysis.

The argument on the extension of storage theory to ground-water flow appears to "boil down" to this: It would be convenient if unitgraph principles were applicable to ground-water flow; therefore they are applicable. Without further proof, the author proceeds to apply them. There is something about neatly arranged tables of computations that seems to consecrate the result, and it is likely that computing a number of ground-water flow hydrographs by this method would confirm many a budding hydrologist in the belief that the implied relationships had been proved. Actually, the "refinement" attained by treating ground-water flow in this manner (if it be, in fact, a refinement at all) seems scarcely worth the effort; 30% of curve (c) in Fig. 8 could be replaced by the customary sloping straight line and the change in curve (d) would scarcely be noticeable.

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[37] "Runoff Distribution Graphs from Precipitation Occurring in More than One Time Unit," by William T. Collins, *Civil Engineering*, September, 1939, p. 559.

An even more basic criticism, however, is the inclusion of ground-water flow, by whatever means computed, in the unitgraph. By including it, the range of usefulness of the unitgraph is seriously impaired. Its major function is not—as emphasis in many papers would lead one to believe—to confirm itself repeatedly by reproducing second-rate floods with more or less accuracy. Rather, it is a tool to indicate what will probably happen when a really first-rate rainfall comes along; and it should be left to do that job unhampered by the introduction of ground flow factors that will likely be inapplicable at the big moment. Suppose the maximum storm hits the watershed when the ground is saturated—or frozen. A true unitgraph can handle that situation (compare the "pluviagraph" of Mr. Bernard[38]); a composite graph of surface and subsurface runoff cannot handle it—and it errs on the side of danger.

Passing to the author's example, there is much difficulty in following the detail. Table 1 shows $K = 15$, whereas the text states that $K = 9$. Also, the relationship between the plotted curves of Fig. 8 and the data of Table 1 is obscure. Should not curve (d) be the plotting of Col. 9?

However, the major shortcoming of the example is the failure to explain the construction of Fig. 6. This is the heart of the entire matter—the drawing of lines on a map in such a manner as to define areas that make their contributions to "the channel" in successive intervals of time. How is it done? What factors are taken into account? Why do the area boundaries cross the streams without bending stream-upward, when logic suggests that they should take the form of exaggerated contour lines? (See, for example, Mr. Bernard's statement:[36] "* * * the efficiency of the main channel, through higher velocities, places headwater areas much closer to the outlet in terms of time than extensive areas nearer but less advantageously located.") With a knowledge of how to chart these areas, the operator finds the remainder of the technique to be a matter of measuring areas by planimeter and of running a calculating machine; without it, he is at a loss where to start.

The justification for applying the same value of K to the contributions of all areas is not apparent. It would appear to be reasonable only on the assumption that storage conditions are the same in all areas and that no storage exists in the main channel; but obviously the author does not make any such assumption. Rather, he takes for the value of K the value existing at the moment when all surface runoff is presumed to have reached "the channel." It follows that K for the downstream areas should be smaller, and for the upstream areas larger, than the average.

Since delineating the areas on the map is subject to a wide range of personal choice, and since the use of a single value of K is open to question, the writer suggests that a somewhat different approach might yield equally satisfactory results with less effort. Using "concentration time" as defined by Mr. Bernard, the base length of any hydrograph of surface flow (including the unitgraph as defined by Mr. Sherman) is equal to the duration of runoff-producing rainfall plus the concentration time. This statement includes the instantaneous unitgraph, for which the base length is simply the concentration time itself. Concentration time, by the Bernard definition, can be determined from an inspection of hydrographs, with reasonable accuracy, as readily as concentration time by the Clark definition. Having determined it, divide the watershed map into the corresponding number of time-interval areas and determine the percentage area of each. The tabulation of these percentages, without any further operations, is in itself the instantaneous distribution graph of surface flow for the watershed.

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[38] "An Approach to Determinate Stream Flow," by Merrill Bernard, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 358.

It cannot be said that the effect of storage is overlooked in this proposed method, for it is reflected in the fact that concentration time (the Bernard definition) takes in the entire period of time required for a particle of water to travel from the hydraulically-most-remote point to the point of observation. Basically, the analysis seems as sound as that of the author's more complex procedure. It would be interesting to determine a unitgraph for the Appomattox River by this proposed method and test it, also, for the flood of April, 1937. Unfortunately, the necessary basic data are not available to the writer. Also, an adequate technique for dividing the watershed map presumably remains to be developed. This should be a fruitful field for further inquiry.

In making such a test, it would be necessary to assume a "reasonable" ground-water flow, and the writer would be content with a straight line joining the point on the observed hydrograph at which rainfall begins, with the point at which surface runoff ceases. Fig. 9 indicates the latter to be at the line marked "April 3."

C. O. CLARK,[39] JUN. AM. SOC. C. E.—The discussions represent much thought and labor at a time when most of the writers are actively engaged in entirely different matters. The expressions which corroborate the paper are gratifying and greatly appreciated. The divergent views outline some fertile fields for further investigation of problems which are still largely dependent on judgment and should stir stimulating and constructive thinking. Some of the queries and counterproposals presented lie beyond the ability of the writer to answer; some have been answered by other discussers; and a few are reviewed in the closing discussion. All are sincerely appreciated.

Mr. Sweet pertinently emphasizes that the labor involved in unit hydrograph and flood-routing techniques is excessive for routine river forecasting. Reduction of this labor by the development of simplified empirical coefficients is necessary and highly desirable. However, the simplification might be based upon a large number of hypothetical floods computed by appropriate methods as well as upon records of past floods. As Mr. Sweet states regarding the James River, nonuniformity of storm distribution may produce variable stage relationships between the gages along the river. If records of only a few large floods are available, it is easy to assume that relationships are fixed, only to be embarrassed when nature proves otherwise. However, unit hydrograph study of hypothetical adverse distributions of storm rainfall will reveal many of the variable possibilities which may serve as guides for good forecasts rather than as alibis for poor ones. The James River, with its two halves on either side of the Blue Ridge Mountains in Virginia, offers opportunities for illuminating study to any one who believes that nonuniformity of runoff distribution on an area is not significant, or that gage-to-gage stage relationships are simple or constant.

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[39] Hydr. Engr., U. S. Engr. Office, Winchester, Va.

Major Meyer clarifies several points in the paper, and appropriately warns of situations in which K is not a constant. Unit hydrograph theories and constant time elements of flow are convenient oversimplifications of a most complex form of open channel flow. They are by no means universally applicable. The paper treats only the elements common to unit hydrograph theory and to the theories of flood routing based upon constant values of K. "Lag routing," to which both Major Meyer and Mr. Snyder refer, is a very practical and expeditious method of flood routing. Fig. 3 and its supporting discussions demonstrate the essential agreement between this method and the much more laborious application of the Muskingum technique. The construction of the time-area concentration curve of Fig. 8 and its routing modification conform essentially to "lag routing" procedure.

Lag routing is an empirical approximation of some solutions of the storage equation; it is not itself a solution of the storage equation for any fixed relationship between storage capacity and discharge. It is an expedient method for use in a natural state of river development, as in flood forecasting or in computation of past floods. However, the writer would be unable to determine, with it, the effect of improvements which change the storage capacity along a river, such as extensive levee systems—or particularly those improvements which change the relative influence of inflow upon storage, such as a series of dams.

Thus, for solving routing problems which involve a change in storage-discharge relationships, the writer believes that the storage equation will be more reliable than empirical "lag routing." As Major Meyer writes, there are several methods other than the Muskingum method for the solution of Eqs. 2, 4, and 5.

Mr. Sherman—the creator of the unit hydrograph—reviews the underlying theory of this excellent hydrologic tool with masterliness. Possibly through misinterpretation of the bar pattern of Fig. 8 and through the assumption that curve a is a net rainfall pattern of twelve unit periods rather than a representation of shape of drainage area, Mr. Sherman concludes that the resulting derived hydrograph was not a unit hydrograph. In Fig. 8, a bar pattern representing net rainfall would be of infinitesimal duration at zero time. In Fig. 9, rainfall was principally confined to that on April 25 and that early on April 26.

An apology is made to Mr. Sherman and to others who found difficulty in checking the computations in Table 1, particularly the conversion of Col. 8 to Col. 9. The conversion at a rate of 69,000 (cu ft per sec)-half-days per in. is a rounded value appropriate to a drainage area of 1,300 sq miles, rather than to one of 1,335 sq miles. The former area is that for which the first derivation on this stream, determined for a dam site above the gage, was prepared. Intent upon presenting an unretouched, original, routine solution, the writer

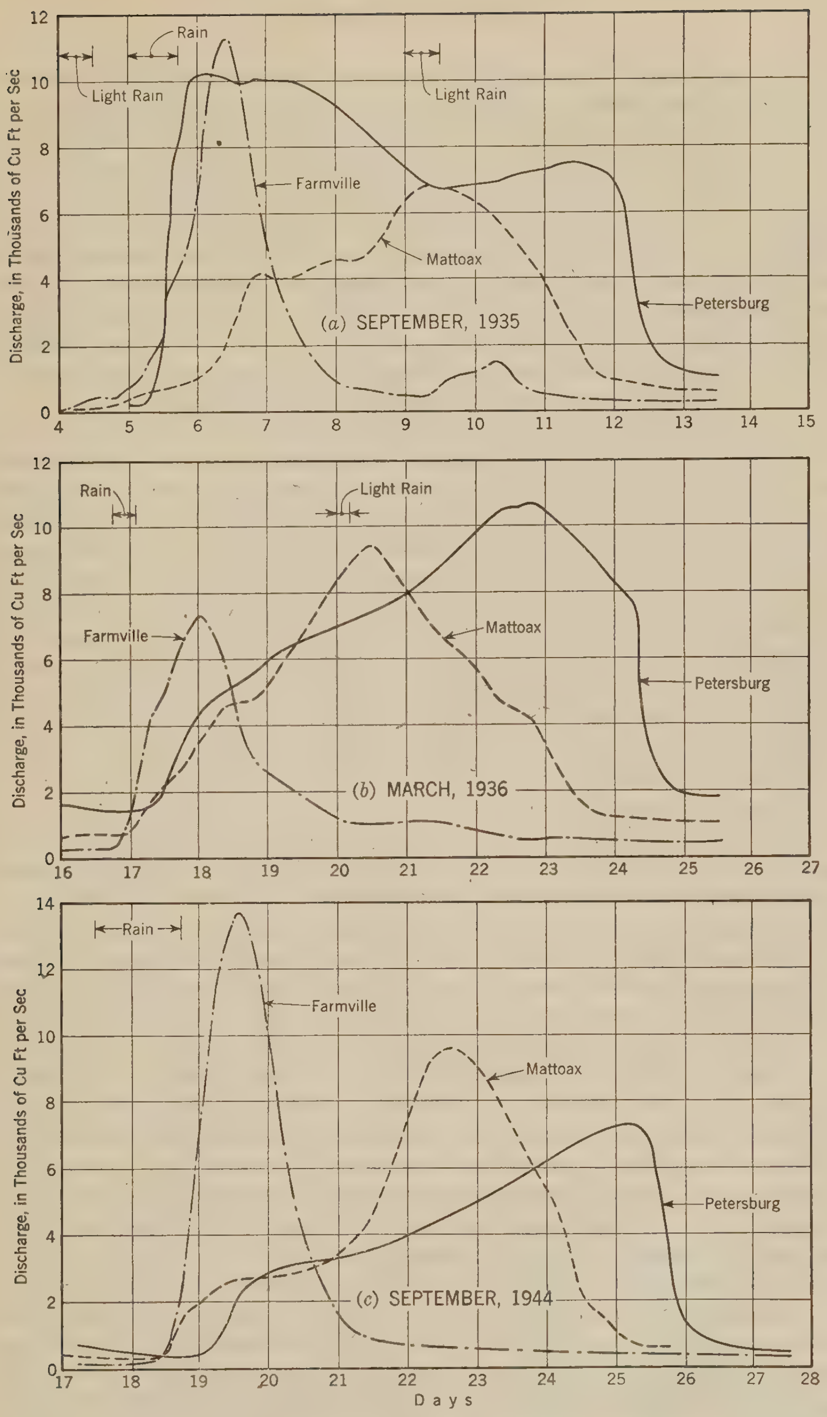

FIG. 18.—HYDROGRAPHS OF THE APPOMATTOX RIVER

failed to notice and properly explain the small difference in drainage area. The routing calculation illustrated is for 15 hours, as stated in Table 1, rather than for 9 hours, as stated in the text. Subsequent investigations indicated the smaller value to be more suitable. There are also some slight errors in Table 1, all well within the range of acceptable routine office practise, and there is much room for improvement in the analysis. The entire original analysis was prepared in 4 hours and was not then intended as a part of a published paper. The presentation of these data was intentional. The real value of a hydrologic tool is much better judged by the rough-hewn original product with its faults, than by the prettier refinements which most investigators possess and ultimately adopt for use.

Mr. Sherman comments on the size of watersheds to which unitgraphs may be applied. He correctly states that size is not a barrier to the adequate treatment of nonuniformity of rainfall with respect to time. On the other hand, nonuniformity with respect to areal distribution of the rainfall creates variations such as those shown in Fig. 18. Each of these hydrographs resulted from a rainfall brief enough to be considered a unit rain. The relative size of hydrographs at the three stations (gaging, respectively, 23%, 55%, and 100% of the Appomattox River above Petersburg) and rainfall records indicate heavier rainfall on one end or the other in each case. Of the hydrographs at Petersburg, none would be suitable for derivation of a unitgraph by any method which assumed that the hydrograph resulted from a uniform distribution of runoff. In the proposed method, which makes no such assumption, an equally suitable unitgraph could be derived from each of the records. However, neither of the hydrographs at Petersburg could be reproduced well by use of a single unit hydrograph, because, although a reproduction would represent flow from uniformly distributed runoff, the record does not. Subdivision of the area and the use of three component unitgraphs like those in Table 2 produce better results. The methodology as extended by Mr. Turner, incorporating both area and runoff depth in the time-area curve, gives much better results.

The possibilities of storage acting to increase outflow drew varied comment from Messrs. Meyer, Williams, Linsley, Brater, Snyder, Turner, and Johnstone, ranging from "controvert[ing] accepted concepts and experience" and "erroneous" to "familiar to all persons who have worked with reservoirs storing appreciable volumes of water under the backwater profile." The writer agrees heartily with those who suggest "These conclusions should be examined critically." The possibility should not be passed over lightly, however, as failure to recognize the possibility leads directly to overoptimism about the effectiveness of certain types of flood control dams, and to underestimation of the importance of avoiding fixed-pool operations at power and navigation dams and unnecessarily rapid gate closures.

The writer's comments about the inflow-controlled storage were intended to be qualitatively explanatory (rather than quantitatively assertive) of the statement (see heading, "Valley Storage"), "Valley storage does not always decrease flood peaks * * *." The excellent discussions reviewing the case against such a viewpoint appear to require some presentation of the data supporting an affirmative case.

The writer's use of a uniform channel in Fig. 1 for the qualitative explanation was ill chosen from a quantitative viewpoint. The uniform channel was selected because of the simple, familiar form of the two backwater profiles. Obviously, however, a uniform rectangular channel would not have an x-value in excess of 0.5. Therefore, Professor Brater's quantitative criticism is well founded. A quantitative demonstration might consider that the rectangular channel possesses a non-water-carrying flood plain several times as wide as the channel at the upper end, converging to the channel limits at the lower end. By appropriate selection of the flood plain width at the upper end of such a channel (which would be quite similar to the channels of many natural streams carrying relatively small flows in their extensive overbank areas), the volume of stored water under the backwater curve could easily be made to exceed 720 acre-ft, which is the volume Professor Brater's analysis indicates as the limiting volume that could be passed without surge.

Channels—the variable cross sections of which effectively converge in the direction of flow—might be expected to possess storage capacities influenced more by inflow than by outflow. Such channels might include:

(*a*) Channels converging in variable depth (such as low-slope channels approaching a steeper channel and thus flowing from a zone of great range in stage to one of small range in stage and at the same time possessing flood plains of about equal width, or such as reservoirs maintaining fixed levels at the dam with pronounced fluctuation at their upper ends, or such as reaches ending in tidal estuaries); and

(*b*) Channels converging in width but of relatively constant range in flood rise.

In the first class may be found certain South Atlantic streams which cross the Piedmont Plateau flowing toward the Fall Line. The Fall Line is a cascade zone which lies between the Piedmont Plateau and the Atlantic Coastal Plain. For example, the Appomattox River, whose drainage area above Petersburg was shown in Fig. 6, lies on the Piedmont Plateau. Two gaging stations are located above the Petersburg gage: Mattoax, Va., with a drainage area of 729 sq miles, is on the 2.0-day isochrone of Fig. 6, and Farmville, Va., with a drainage area of 306 sq miles, is on the 4.3-day isochrone of Fig. 6.

The typical flood rise at Mattoax in feet is much larger than that at either Farmville or Petersburg although flood plain widths are comparable for the three stations. The variable cross section of flood flow can be considered diverging in the direction of flow from Farmville to Mattoax and converging from Mattoax to Petersburg. Hydrographs of flow at these three stations resulting from rainfall of short duration are shown in Fig. 18. (Note that the later portions of the hydrographs at Mattoax and Petersburg must be essentially the flow component from above Farmville.) To occupy such a position in the hydrographs shown, it appears that the flood wave might have undergone flattening from Farmville to Mattoax and then have been built up from Mattoax to Petersburg. A more provable indication of the type of storage influence represented by x-values greater than 0.5 is the increase in recessional slope of the Petersburg hydrograph as compared with the Mattoax hydrograph.

In the second class might be short reaches of many rivers, as most flood plains alternately widen and narrow. Many rivers have long reaches through which flood waves flow without appreciable reduction.[40] It seems a little more rational to interpret such unreduced passage as the result of alternate slight increases and decreases in peak rather than as a passage of an unvaried peak. A record of a wave passage having slightly more increase than decrease was shown in Fig. 12, in which the hydrograph of flow at Danville appears to have built up slightly and shortened to become the later portion of the hydrograph at South Boston. The discharge from the intervening drainage area would have been insignificant on the twenty-first and twenty-second days.

Most rationalizations about the possibility of storage operating to build up peak flow turn upon the question: "Granting that a volumetric differential exists between the equilibrium profiles of flow applicable to flow distributions before and after a change in inflow, can the change from one profile to another take place rapidly enough to cause a surge at the downstream end?" Fig. 19 shows the experimental hydrographs which C. F. Izzard, Assoc. M. Am. Soc. C. E., and M. T. Augustine obtained by sprinkling a pavement.[41] In this case, the change from one flow distribution profile to another did take place rapidly enough to cause a surge. How fast does the change take place in rivers? How fast does storage volume react to a change in flow?

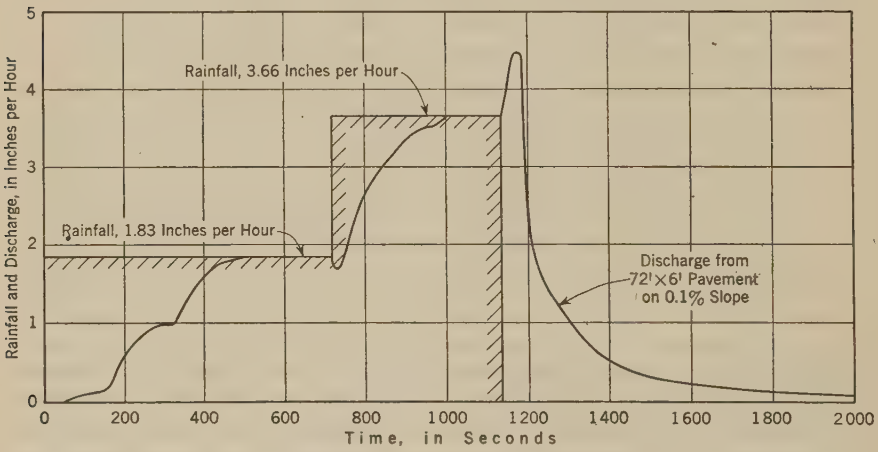

FIG. 19.—HYDROGRAPHS FROM EXPERIMENTS BY C. F. IZZARD AND M. T. AUGUSTINE

Several discussers mention the assumed instantaneous reaction of storage to a change in flow (which is characteristic of most flood-routing techniques and favorable to reduction of peak flow in the method discussed when x is less than 0.5) as the error which leads to the author's conclusion. The formula for reaction time presented by Messrs. Williams and Brater is quite favorably regarded by many, whereas Mr. Snyder appears to regard any value less than K as inadequate. Several intermediate viewpoints have some adherents. The time elements indicated by Eq. 17, first discussed by Mr. Williams, vary from very long values for a shallow depth at the beginning of a flood to crest-depth values which are very short in relation to the time elements of mean flood-wave travel. Thus, in the reach of river to which Mr. Williams applied the formula, the channel depth was more than 50 ft at the time of crevassing. For this depth, the formula discussed would indicate less than 40 min. (Neither the methods of observing stage, nor those of defining the time of crevassing of a levee are sufficiently reliable to distinguish between the time of transit as originally stated and that given by the formula.) This value would be insignificant in flood routing over a reach involving flood-wave travel time of 15 hours.

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[40] "The Floods of March 1936," *Water-Supply Paper No. 800*, U. S. Geological Survey, pp. 89 and 90 (last two hydrographs on each page).

[41] "Preliminary Report on Analysis of Runoff Resulting from Simulated Rainfall on a Paved Plot," by C. F. Izzard and M. T. Augustine, *Transactions*, Am. Geophysical Union, Vol. 24, 1943, Pt. I, p. 500.

The writer does not claim that the upgrading of flood crests by storage action has been proved, but merely offers the explanation and the examples of the paper in support of the statement, "There is some valley storage which does not decrease flood peaks." The upgrading of hydrographs does appear possible, however, and should be investigated. A closely analogous situation exists in electrical circuits, and the upgrading (in height, not discharge) of hydraulic waves in converging channels is a recognized phenomenon in tidal hydraulics.

Mr. Linsley clarifies several ideas in the paper by his discussion. The formula he presents for determining time of concentration in watersheds without records shows better correlation with data in the paper than the author's formula. To aid in further study by others, additional data are presented in the same manner in Table 6. The majority of values were developed from records available prior to 1938. The variation in the constants suggests judicious local application. Attention is again directed to the first two sentences under the heading, "Determination of the Instantaneous Hydrograph for Streams Without Flow Records," and to Mr. Snyder's closing two paragraphs.

Messrs. Linsley and Johnstone properly question the writer's use of uniform velocity in establishing time contours along the watercourses and suggest variations based on slope and efficiency of watercourses. These refinements were considered, but investigation showed their use to be unaccompanied by superior results. This may be a case of two wrongs making a right, as the likewise neglected principle of using greater storage values, K, on flows from more remote areas instead of a single average value is a logical refinement of opposite effect. In general, the use of a varied velocity of channel flow and a single value of K tends to increase the fault originally recognized by the writer in item 6(*b*) of the "Summary."

Messrs. Brater and Johnstone voice an interest in a more detailed description of the time-area concentration curve. As Mr. Snyder states, the curve is an old device[42,43] for calculating flow. It has fallen into disuse because its theory assumes the reductive influence of storage to be negligible. A storage correction factor was included in the paper to revitalize this useful tool. The time-area concentration curve is derived by marking time contours along the

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[42] "Sewerage and Sewage Disposal," by L. Metcalf and H. P. Eddy, McGraw-Hill Book Co., Inc. New York, N. Y., 1922, p. 94.

[43] "Calculation of Flood Discharge by the Use of a Time-Contour Plan," by Cecil N. Ross, *Transactions*, Institution of Engrs. (Australia), Vol. II, 1921, p. 85.

watercourses, the time to the most remote point being equal to the "time of concentration." For the latter term, a standard definition such as that quoted by Professor Johnstone is intended. Fig. 4 illustrates why the terminus of this time is not at the end of the hydrograph (theoretically infinite in length) but well up on the recessional leg.

TABLE 6.—ADDITIONAL COMPARISON BETWEEN EQS. 18*a* AND 18*b*

Stream	s (ft per mile)	L (miles)	K (hours)[a]	A (sq miles)	C_1 (Eq. 18*a*)	C_2 (Eq. 18*b*)
North Branch, Potomac River, at Bloomington, Md.	45	49	10	287	1.4	0.081
Savage River at Bloomington, Md.	65	26	11	115	3.4	0.32
Georges Creek at Franklin, Md.	78	18	5	72	2.4	0.29
Wills Creek near Cumberland, Md.	40	35	8	247	1.4	0.092
Patterson Creek near Headsville, W. Va.	19	40	12	216	1.3	0.089
South Branch, Potomac River, near Petersburg, W. Va.	24	60	15	642	1.2	0.048
South Fork, South Branch, Potomac River, near Moorefield, W. Va.	25	54	13	283	1.2	0.071
Cacapon River at Yellow Spring, W. Va.	16	51	10	306	0.78	0.044
Back Creek near Jones Springs, W. Va.	11	40	10	243	0.81	0.052
Conococheague Creek at Fairview, Md.	9.8	62	30	494	1.5	0.068
North River near Burketown, Va.	37	18	9	375	3.0	0.16
Middle River near Grottoes, Va.	14	54	12	360	0.83	0.044
South River at Waynesboro, Va.	17	24	12	144	2.1	0.17
North Fork, Shenandoah River, near Strasburg, Va.	12	96	20	772	0.72	0.026
Passage Creek at Buckton, Va.	31	32	6	87	1.0	0.11
Cedar Creek near Winchester, Va.	30	26	6	101	1.3	0.12
Antietam Creek near Sharpsburg, Md.	11	49	24	281	1.6	0.097
Monocacy River near Frederick, Md.	5.9	62	28	817	1.1	0.038
Goose Creek near Leesburg, Va.	11	33	17	338	1.7	0.093
Seneca Creek at Dawsonville, Md.	17	17	6	101	1.5	0.15
Jackson River at Falling Spring, Va.	22	47	12	409	1.2	0.059
James River at Lick Run, Va.	17	78	28	1,369	1.5	0.040
Cowpasture River near Clifton Forge, Va.	13	69	24	456	1.3	0.059
Calfpasture River at Goshen, Va.	23	36	12	147	1.6	0.13
North River at Rockbridge Baths, Va.	27	45	18	329	2.1	0.12
North River near Lexington, Va.	21	58	20	487	1.6	0.071
North River near Buena Vista, Va.	16	70	24	649	1.3	0.053
Hardware River near Scottsville, Va.	17	22	18	116	3.4	0.32
Slate River near Arvonia, Va.	9.0	34	30	235	2.6	0.17
Rivanna River at Palmyra, Va.	8.0	50	32	675	1.8	0.070
Willis River at Flanagan Mills, Va.	4.2	55	72	247	2.7	0.17
Appomattox River at Farmville, Va.	3.7	34	32	306	2.8	0.16
Appomattox River at Mattoax, Va.	3.0	87	114	729	2.3	0.084
Appomattox River near Petersburg, Va.	2.9	122	168	1,335	2.3	0.064
Meherrin River near Lawrenceville, Va.	3.2	67	60	553	1.6	0.068
Meherrin River at Emporia, Va.	2.4	86	78	750	1.4	0.051
Roanoke River at Roanoke, Va.	17	48	12	388	1.0	0.052
Blackwater River near Union Hall, Va.	14	46	18	208	1.4	0.10
Pig River near Toshes, Va.	8.7	72	24	394	0.98	0.049
Snow Creek at Sago, Va.	18	18	14	60	3.2	0.41
Goose Creek near Huddleston, Va.	14	33	12	187	1.4	0.099
Otter River near Evington, Va.	18	34	22	325	2.7	0.15
Falling River near Brookneal, Va.	11	23	22	228	3.2	0.22
Dan River near Francisco, N. C.	35	43	4	124	0.55	0.049
Mayo River near Price, N. C.	20	38	14	260	1.6	0.10
Smith River at Bassetts, Va.	17	37	7	265	0.77	0.047
Smith River at Martinsville, Va.	13	48	13	374	0.98	0.051
Smith River at Spray, N. C.	9.4	71	23	538	0.99	0.043
Sandy River near Danville, Va.	21	21	5	113	1.1	0.10
Banister River at Halifax, Va.	7.2	46	60	552	3.5	0.15

[a] Time of concentration, not reductive storage factor.

Mr. Crawford presents interesting analyses of channel-storage correlation with stages at the discharge point and upstream from it, and shows the superiority of correlations which give weight to upstream values of inflow and stage. He calls attention to the potential possibilities of determining from discharge records other storage relations which would be too costly by cross-sectional surveys. The writer has seen area-capacity curves for small dam sites so developed which would require very detailed field surveys for equal accuracy.

Mr. Kennedy emphasizes the fallacy of constant base length in a unit hydrograph. Much of this fallacy may originate in the assumption of a finite base length. If storage theories and unit hydrograph theories are to be correlated, the concept of finite base length in a unit hydrograph must give way to a concept of an infinite length, a characteristic possessed by the writer's instantaneous hydrographs and their derivatives. Nevertheless, the practical differences, as shown by Fig. 15, are within the range of usually acceptable hydrograph computations. This is probably the reason that the practical simplicity of finite constant base length has so long outweighed the theoretical superiority of infinite base length.

Mr. Cochrane has very thoroughly examined the procedure for deriving unitgraphs utilizing storage concepts, and presents some explanations which should be helpful to those who were mystified by the writer's presentation. In a simple manner, he clarifies the twofold effect of storage: First, the greater effect of storage in creating time lags between flows from different zones, or the time-area diagram; and, second, the further modification of hydrograph shape accomplished by storage.

The order in which these steps have been utilized is the reverse of that in Fig. 8, but the result is the same regardless of order. The order in Fig. 16 gives a clear picture of the mechanical effects, and would be necessary if one wished to use variable values of K for different zones. At the sacrifice of this possibility, the order of Fig. 8 involves less work. However, a unit hydrograph for an entire area may produce very disappointing results if applied to a storm characterized by great nonuniformity of runoff distribution. Mr. Cochrane presents a very practical application of storage principles which considers both nonuniformity of runoff distribution and some variation in storage effect.

Another method of accounting for nonuniformity, suggested by Mr. Turner, is to extend the time-area concentration curve to express not only shape but also different runoff distribution—that is, multiply the zone areas by the depth of runoff in each zone before embarking on the routing procedure of Table 1. This procedure takes care of distribution with respect to area and is a fine tool for studying the magnitude of the effect of areal nonuniformity of runoff distribution.

Both procedures abandon the unit hydrograph and require complete recalculation of each flood with attendant increased labor. Other less logical, but extremely useful techniques to account rapidly for nonuniformity include adjustments of chronological sequences of runoff used with the unit hydrograph. Thus, concentration of volume on more remote parts of a simple watershed can be simulated in some degree by using an effective runoff volume for a later time than actually was the case. Extreme intensity of rainfall within a unit period can be simulated by increasing the computational amount in that period and decreasing the values on either side of the period. The most flexible technique yet utilized by the writer is the development of unit hydrographs for very small zones, which are routed downstream to gaging stations and tabulated in the manner of Table 2. This procedure is followed for as small areas as desired to express nonuniformity to the necessary degree. These subunitgraphs can always be added together for use on areas as large as justified by the uniformity of any storm and can be used separately if necessary. Although this procedure was discussed under the heading, "Unit Hydrographs for Large Drainage Areas," any watershed is large from the viewpoint of detailed, exacting hydrograph computation.

Refined techniques are necessary and desirable to establish more suitably correct procedures and to confirm academic viewpoints as to the effect of certain qualitative variables. However, a computation is only a guide to judgment. After a hydrograph is computed with a unitgraph or other appropriate tool, judgment demands some modification of the answer if the premises of the methods and the facts of the occurrence are not in accord. At times a wide line, a small scale, or a little freehand "artistic license" may have as much merit as hours of computing. In their proper place, these too are proper tools.

Mr. Snyder presents several of the practical aspects of hydrograph calculation and the mathematical basis for a very simple and useful method of routing over long reaches of river where storage-discharge relationships are such as to produce essentially constant time elements. He calls attention to elements of logical and mathematical fallacy which may appear small to some investigators, but which must be faced squarely and solved before the hydrograph computations can be considered accurate enough to prove or disprove some of the questions raised in this paper.

As Mr. Snyder states, the Muskingum method of flood routing loses its usefulness when applied to too long reaches. However, this may also occur because the storage-discharge relationship for a long reach of river is much less closely approximated by a straight line than is the relationship for short reaches. The storage item in flood-routing formulas such as Eq. 5 deals with the volume stored in a reach at any instant of time, whereas the storage capacity of channels having nearly constant elements of flow time ($n = 1.0$ in Eq. 3) refers more closely to the volume under the high-water profile. Although this latter volume is the total storage volume utilized in the reach of river during flood passage, it is more than is used at any instant during that time. The difference becomes a material one in long reaches. Nevertheless, if storage solutions are confined to the length of reach for which Eq. 5 would be applicable, Eq. 6*b* is a correct solution. The Muskingum method is a correct solution of Eq. 6*b* only to the extent that a finite value of T is a satisfactory approximation of dt. Mr. Snyder expresses the opinion that values of T used in the preparation of Figs. 2 and 3 were too short.

The inadequacies of methods of mathematical solution receive too little recognition. Most engineers find it necessary to adopt arithmetic approximations, such as Eqs. 1 and 6*c*, for the solution of calculus equations, such as Eqs. 2*a* and 6*b*.[44] The practises are so common that they escape much comment. The simplicity of the Muskingum method encourages these procedures, since it depends first on the substitution of a finite value of time T for the infinitesimal time dt. Then, too, many convenient, quick solutions evolve from various lengths of time T which make one or more of the constants either 1.0 or 0. Values of T as large as K or larger conveniently obscure some of the effects of the inadequacy of the theory recognized by the writer in his discussion of Fig. 2, and have been accepted by some as a basic requirement. The differences are as given by Mr. Snyder. In an effort to prove whether such a basic requirement added anything to the over-all accuracy of hydrograph solutions, the writer has for several years assumed that the shortest practical time element, T, was the most appropriate. In general, practical applications have been no more faulty, and in some cases apparently better, whereas the practical benefits of greater mathematical consistency and rapid checking procedures accrue to (or result from) closer adherence to the mathematical premises.

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[44] "Graphical Integration of the Flood-Wave Equations," by Harold A. Thomas, *Transactions*, Am. Geophysical Union, Vol. 21, 1940, Pt. I, p. 597.

Mr. Snyder's presentation of the mathematics of a simple method of lag routing should increase the practical appeal and use of this simple procedure. To be completely general, the solution may need some provision (1) to cover the flow conditions in which the lag of the crest discharge is greater than the lag between the centers of mass of the inflow and outflow, and (2) to cover the conditions in which the recession rate of outflow exceeds the recession rate of inflow. Flow conditions requiring this modification are evident in Fig. 18, in which the hydrographs of the Appomattox River at Mattoax are shown to have a materially smaller rate of recession than does that portion of the hydrograph at Petersburg which encompasses the flow volume from above Mattoax.

Inspection of Fig. 17 and Eq. 25 suggests the applicability of a negative value of T_s. The writer finds the mathematics of a solution evasive, however, and doubts that this is a practical solution. Nevertheless, it is explanatory of a relationship between K and time of concentration (T_c), which still baffles him. However, it is assumed for the purpose of the next paragraph that negative values of T_s are applicable.

The failure of the writer to solve this possible adaptation of lag routing is the source of his confusion in statements under the heading, "The Hydrograph as an Index of Storage," about which Mr. Snyder rightfully comments. The values of K applicable to smaller watersheds appear to be as large as 0.5 of the time of concentration (T_c); yet those for larger watersheds have a much smaller ratio of T_c and occasionally a smaller dimensional quantity. Obviously this is possible if T_s could have a negative value. For the three gages on the Appomattox River—Farmville, Mattoax, and Petersburg, each site being downstream from the preceding site—the values of K which the writer regards as most appropriate are, respectively, 9 hours, 12 hours, and 9 hours, whereas the most acceptable times of concentration, T_c, are 32 hours, 114 hours, and 168 hours. (These values for Petersburg differ from values in Table 1, which were the very first computations made for that stream. The difference between the values of T_c equal to 144 hours and 168 hours is dimensionally large, but the same percentage difference in the more usual range of the writer's application, 24 hours or less, would represent but 4 hours, which is close to the limiting error fixed by rainfall data. Furthermore, that percentage difference is quite comparable with the expected difference due to seasonal variation in flow conditions, a factor usually ignored, but by no means absent.)

The Appomattox River presents the most perplexing flow conditions to which the writer has yet applied the unit hydrograph theory and the longest time of concentration for comparable drainage area that he has ever encountered, as well as a unique shape. Although the illustrated hydrographs for the Smith River at Bassetts and the James River at Lick Run are certainly the more common type, exceptions, like the illustrated hydrographs of Appomattox River and Meherrin River, are necessary to prove the rule. Judging from records of rounded and tree-shaped watersheds typical of the glaciated and alluvial streams of the Midwest, it would appear illogical that a stream could rise for a longer period than it fell, or that streams could have unit hydrographs like those presented for the Appomattox and Meherrin rivers. Mr. Sherman's doubt that any unit hydrographs were presented in the paper is therefore very understandable.

Mr. Turner provides a well-balanced discussion of the paper, confirming statements and expressing appropriate reserve and doubt where his experience and understanding of conventional concepts and procedures do not justify presented conclusions. His comparison of results obtained from the outlined unitgraph procedure and from his similar procedure[31] encourages confidence in the principles applied. His extension of the method to include varying rates of rainfall on different parts of the area is quite practical.

Professor Johnstone outlines some logical elements of greater and lesser refinement whose merit would depend on the extent of the use for them. He expresses several views in sharp conflict with those of the paper. The propriety of defining a unitgraph to include nonsurface runoff is basicly criticized Granting that some investigators have presented hydrographs attributed to surface runoff, the writer questions whether such a claim can be substantiated or disproved, since there is no feasible method of distinction, no acceptable specific definition of either kind of runoff, or any impervious boundary between the flow channels utilized by each. Practise in the application of the terms seems to justify distinctions satisfactory to the user and determinable only to the extent of their compatibility with a preferred theory. The writer indicated that separate unitgraphs could be used for surface and subsurface flow, if warranted, and that there was nothing fixed about the 70:30 ratio. However, in computations of subsurface flow there seems little reason to reject that definable quality which the use of the unit hydrograph theory has provided in surface flow analysis.

In his demonstration that there "* * * should * * * be no possibility of multiple solutions" of conventional unit hydrograph derivations from rainfall and stream flow data only, Professor Johnstone adequately presents the reasons why an investigator who uses all the data available can secure: (*a*) A distressingly large number of different solutions if he assumes he is able to determine runoff from rainfall; or (*b*) no solution if he admits inability to determine runoff from rainfall. In the storage concepts presented in the paper, there are additional considerations which eliminate the necessity for dependence on rainfall and runoff determinations. Until runoff determinations become more reliable, however, the real accuracy of the method must remain partly shrouded in unproved hope.

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ANALYSIS OF RIGID FRAMES BY SUPERPOSITION

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With Discussion by Messrs. F. S. Merritt, Ralph W. Stewart, John E. Goldberg, A. W. Fischer, Leon Blog, Arthur B. McGee, Jaroslav Polivka, Odd Albert, W. C. Spiker, David B. Hall, A. A. Eremin, Victor R. Bergman, and David M. Wilson.

Synopsis

The purpose of this paper is to demonstrate the usefulness of the principle of superposition in the analysis of rigid frames by the slope-deflection and moment distribution methods. A number of examples are presented to show how the principle may be used to advantage in attacking a variety of problems. The principle of superposition is usually stated in textbooks on structural analysis; but, in the writer's opinion, its application is not emphasized to the extent that its importance warrants. In the study of continuous structures, in particular, a thorough working knowledge of the principle is indispensable.

It will be noted that no new method of analysis is proposed. Instead, a method of procedure is outlined which coordinates the principle of superposition with existing methods and, at the same time, places emphasis where it properly belongs.

Preliminary Considerations

The scope of the paper is limited to the analysis of frames in which applied loads cause linear displacements of the joints, in addition to angular displacements (rotations), and consequent angular displacements of the members. It is assumed that each member of any frame considered has constant section throughout its length. However, the method of procedure which is presented is general, since it is based upon fundamental principles, and can be applied with little additional labor to the analysis of frames having members of varying section after fixed-end moments, carry-over factors, and stiffness values have been evaluated.

Notation.—The following letter symbols, used in this paper, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials,[2] prepared by a Committee of the American

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Note.—Published in February, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Prof., Civ. Eng., Univ. of Southern California, Los Angeles, Calif.

[2] ASA—Z10a—1932.

Standards Association, with Society representation, and approved by the Association in 1932:

C = a moment symbol commonly used in slope-deflection analyses; for example, C_{AB} = a fixed-end moment at end A of any member AB, caused by transverse loads on the member (see also M);

E = modulus of elasticity;

K = a ratio of moment of inertia of a section to the length of the member $= \frac{I}{L}$;

L = length of any member;

M = bending moment; for example, M_{AB} = resisting moment of end A of any member AB; M'_{AB} = fixed-end moment, caused by the angular displacement of the member (see also C);

R = angular displacement, in radians, of the straight line connecting the ends of the member $= \frac{\Delta}{L}$ (referred to as the angular displacement of the member);

Δ = relative linear displacement of one end of any member with respect to its other end, measured perpendicularly to the initial position of the member;

θ = angular distance or rotation; for example, θ_A = the rotation angle at joint A of any frame, or of end A of any member framing into joint A.

Convention of Signs.—The signs of the quantities used in the problems of this paper are determined by the following conventional rules:

The rotation angle, θ, is positive when it occurs in a clockwise direction and negative when it occurs in a counterclockwise direction.

The linear displacement, Δ, and the angular displacement, R, are both positive when the straight line connecting the ends of the member under consideration rotates in a clockwise direction and negative when this line rotates in a counterclockwise direction.

The resisting moment, M_{AB}, and the fixed-end moments, M'_{AB} and C_{AB}, are positive when they act in a clockwise direction and negative when they act in a counterclockwise direction.

Lengths of Members.—In the solutions of subsequent problems, the lengths of members are taken between axial intersections of members. Therefore, the moments obtained represent the moments at these intersections, not the moments at the faces of supporting members. The latter may be computed without difficulty, however, after the moments and shears at intersections have been determined.

Fixed-End Moments Induced by Angular Displacement, R.—The resisting moments at ends A and B, respectively, of any member AB are defined by the slope-deflection equations:

$$M_{AB} = 2\,E\,K\,(2\,\theta_A + \theta_B - 3\,R) + C_{AB} \qquad \text{(1a)}$$

and

$$M_{BA} = 2\,E\,K\,(2\,\theta_B + \theta_A - 3\,R) + C_{BA} \qquad \text{(1b)}$$

These equations may be expressed in the form:

$$M_{AB} = 2\,E\,K\,(2\,\theta_A + \theta_B) + (C_{AB} - 6\,E\,K\,R) \text{..........} (2a)$$

and

$$M_{BA} = 2\,E\,K\,(2\,\theta_B + \theta_A) + (C_{BA} - 6\,E\,K\,R) \text{..........} (2b)$$

Now, if both ends of the member are fixed against rotation so that $\theta_A = \theta_B = 0$, the fixed-end moments at A and B are obviously equal to $(C_{AB} - 6\,E\,K\,R)$ and $(C_{BA} - 6\,E\,K\,R)$, respectively. Furthermore, if the member carries no transverse loads so that $C_{AB} = C_{BA} = 0$, the fixed-end moments at A and B are each equal to $(-\,6\,E\,K\,R)$. Therefore,

$$M'_{AB} = M'_{BA} = -\,6\,E\,K\,R = -\,6\,E\,K\,\frac{\Delta}{L} \text{............} (3)$$

If one end (say, B) of the member is free to rotate and A is fixed against rotation so that $M_{BA} = 0$ and $\theta_A = 0$, it may be readily shown that the fixed-end moment at A, if the member carries no transverse loads, is equal to $(-\,3\,E\,K\,R)$. Under this condition, the rotational stiffness at A of the member is equal to three fourths the rotational stiffness at A when B is fixed against rotation.

Outline of Method of Procedure.—In the problems herein considered, the key to analysis in each case consists in the consideration of all possible conditions of deformation of the given frame. In accordance with the principle of superposition the resultant effects on the frame will be equal to the sum of the component effects produced by these separate deformations. The following outline of method of procedure presents, in logical order, the steps required for the solution of any rigid frame problem:

Step 1. Sketches showing all possible conditions of deformation of the frame are made. If transverse loads act on any member of the frame, these sketches will include one of the undeflected frame with any transverse loads that may be acting.

Step 2. Fixed-end moments set up by the transverse loads and unknown angular displacements of the members, before rotations of the joints are permitted, are determined and recorded on the appropriate sketches of step 1. These moments will be numerical values for the condition of transverse loads on members of the frame, and functions of the unknown angular displacements (or unknown joint displacements) for all other conditions of deformation (see Eq. 3).

Step 3. The joints are balanced for each condition of deformation in step 1 by either the slope-deflection or moment distribution method, and the final moments are shown on the deformation sketches.

Step 4. Equilibrium equations are written for the frame with the given loads; and, after proper substitutions, the unknown displacements are evaluated. These substitutions are determined from a consideration of appropriate free bodies on which the moments found in step 3 and all forces are shown acting. The moment at the end of any member in a free body must equal the sum of the moments obtained at the end of that member for the several deformation conditions. It will include a numerical value, if transverse loads act on any member, and additional values that are functions of the unknown displacements.

Step 5. With all displacements known, the moments in step 3 which are functions of these displacements are evaluated, and the final numerical values of the moments at the ends of all members are determined by summation.

It is important to observe that simultaneous equations obtained in the foregoing procedure will usually be the type which may conveniently be solved by successive approximations (method of iteration). This is of particular significance in problems involving more than three unknowns.

Rectangular Frames

Most frames in practice are rectangular in shape. In general, they must be designed to resist both vertical and horizontal loads. The following examples will illustrate the analysis of rectangular frames with different types of loading.

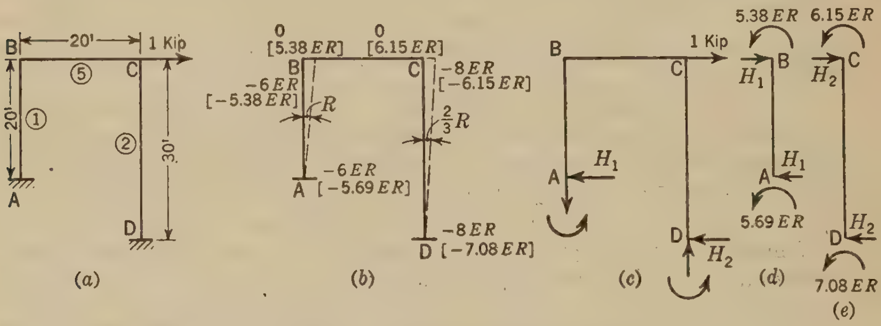


Fig. 1.—Bent with Legs of Unequal Length

Example 1. Bent with Legs of Unequal Length.—Fig. 1(*a*) represents a bent with legs of unequal length, subjected to a horizontal force of 1,000 lb at C. The relative *K*-values of all members are shown in circles. It is desired to compute the values of the moments at the ends of all members of the bent.

Step 1. The only possible condition of deformation of the bent is shown in Fig. 1(*b*). It is obvious that $R_{CD} = \frac{2}{3} R_{AB}$.

Step 2. The fixed-end moments at the ends of all members are determined by use of Eq. 3 as functions of $E\,R$ and are recorded in Fig. 1(*b*).

Step 3. In this problem, the joints in Fig. 1(*b*) are balanced by the slope-deflection method.

The moments at the ends of all members are defined by the slope-deflection equations as follows:

$$\left.\begin{aligned} M_{AB} &= 2\,E\,\theta_B - 6\,E\,R \\ M_{BA} &= 4\,E\,\theta_B - 6\,E\,R \\ M_{BC} &= 20\,E\,\theta_B + 10\,E\,\theta_C \\ M_{CB} &= 20\,E\,\theta_C + 10\,E\,\theta_B \\ M_{CD} &= 8\,E\,\theta_C - 8\,E\,R \\ M_{DC} &= 4\,E\,\theta_C - 8\,E\,R \end{aligned}\right\} \cdots\cdots\cdots\cdots (4)$$

At joint B: $M_{BA} + M_{BC} = 0$. Therefore,

$$24\,E\,\theta_B + 10\,E\,\theta_C = 6\,E\,R \qquad (5a)$$

Also, at joint C: $M_{CB} + M_{CD} = 0$; and

$$10\,E\,\theta_B + 28\,E\,\theta_C = 8\,E\,R \qquad (5b)$$

Solving Eqs. 5 simultaneously,

$$E\,\theta_B = 0.154\,E\,R \qquad (6a)$$

and

$$E\,\theta_C = 0.231\,E\,R \qquad (6b)$$

Substituting Eqs. 6 in Eqs. 4, the following values of the moments are obtained in terms of $E\,R$: $M_{AB} = -5.69\,E\,R$; $M_{BA} = -5.38\,E\,R$; $M_{BC} = 5.38\,E\,R$; $M_{CB} = 6.15\,E\,R$; $M_{CD} = -6.15\,E\,R$; and $M_{DC} = -7.08\,E\,R$. These moments are shown in brackets in Fig. 1(*b*).

Step 4. To determine the value of $E\,R$, an equilibrium equation for the bent must be written.

In Fig. 1(*c*), from $\Sigma F_x = 0$,

$$H_1 + H_2 = 1{,}000 \qquad (7)$$

In Fig. 1(*d*), from $\Sigma M = 0$,

$$H_1 = \frac{11.07\,E\,R}{20} \qquad (8a)$$

Similarly, in Fig. 1(*e*),

$$H_2 = \frac{13.23\,E\,R}{30} \qquad (8b)$$

Substituting Eqs. 8 in Eq. 7, the value of $E\,R$ may be found: $\frac{11.07\,E\,R}{20} + \frac{13.23\,E\,R}{30} = 1{,}000$. Solving, $E\,R = 1{,}005$.

Step 5. The final numerical values of the moments in foot-pounds are: $M_{AB} = -(5.69)(1{,}005) = -5{,}720$; $M_{BA} = -(5.38)(1{,}005) = -5{,}410$; $M_{BC} = (5.38)(1{,}005) = 5{,}410$; $M_{CB} = (6.15)(1{,}005) = 6{,}180$; $M_{CD} = -(6.15) \times (1{,}005) = -6{,}180$; and $M_{DC} = -(7.08)(1{,}005) = -7{,}110$.

Example 2. Unsymmetrical Three-Column Bent.—Fig. 2(*a*) represents an unsymmetrical three-column bent with a vertical load of 1,000 lb acting at the center of BC.

Step 1. Two possible conditions of deformation exist—one for the transverse load on BC, and the other for lateral deflection (sidesway) of the frame. These are shown in Figs. 2(*b*) and 2(*c*).

Step 2. Fixed-end moments are computed for the deformation conditions of Figs. 2(*b*) and 2(*c*) and recorded as shown.

Step 3. In this problem, and in subsequent problems, the joints are balanced by the moment distribution method. Fig. 3 shows the distributions of the fixed-end moments shown in Figs. 2(*b*) and 2(*c*). The final moments obtained from these distributions are shown in brackets in Figs. 2(*b*) and 2(*c*).

Step 4. To determine the value of $E\,R$, an equilibrium equation for the bent must be written.

In Fig. 2(d), from $\Sigma F_x = 0$,

$$H_1 + H_2 + H_3 = 0 \qquad (9)$$

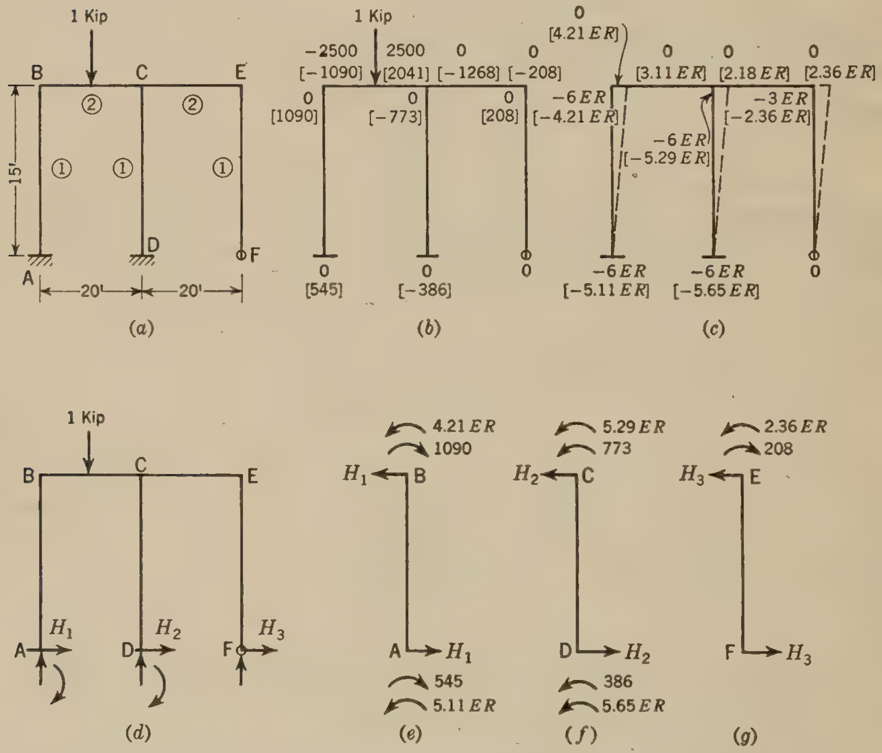

1 Kip
B
C
E
②
②
①
①
①
15'
A
D
F
20'
20'
(a)
1 Kip
−2500
[−1090]
2500
[2041]
0
[−1268]
0
[−208]
0
[1090]
0
[−773]
0
[208]
0
[545]
0
[−386]
0
(b)
0
[4.21 ER]
0
[3.11 ER]
0
[2.18 ER]
0
[2.36 ER]
−6 ER
[−4.21 ER]
−6 ER
[−5.29 ER]
−3 ER
[−2.36 ER]
−6 ER
[−5.11 ER]
−6 ER
[−5.65 ER]
0
(c)
1 Kip
B
C
E
H1
H2
H3
A
D
F
(d)
4.21 ER
1090
H1
B
A
H1
545
5.11 ER
(e)
5.29 ER
773
H2
C
D
H2
386
5.65 ER
(f)
2.36 ER
208
H3
E
F
H3
(g)

Fig. 2.—Unsymmetrical Three-Column Bent

In Fig. 2(e), from $\Sigma M = 0$, $15\,H_1 + 4.21\,E\,R + 5.11\,E\,R - 1{,}090 - 545 = 0$; and

$$H_1 = \frac{1{,}635 - 9.32\,E\,R}{15} \qquad (10a)$$

Similarly, in Fig. 2(f),

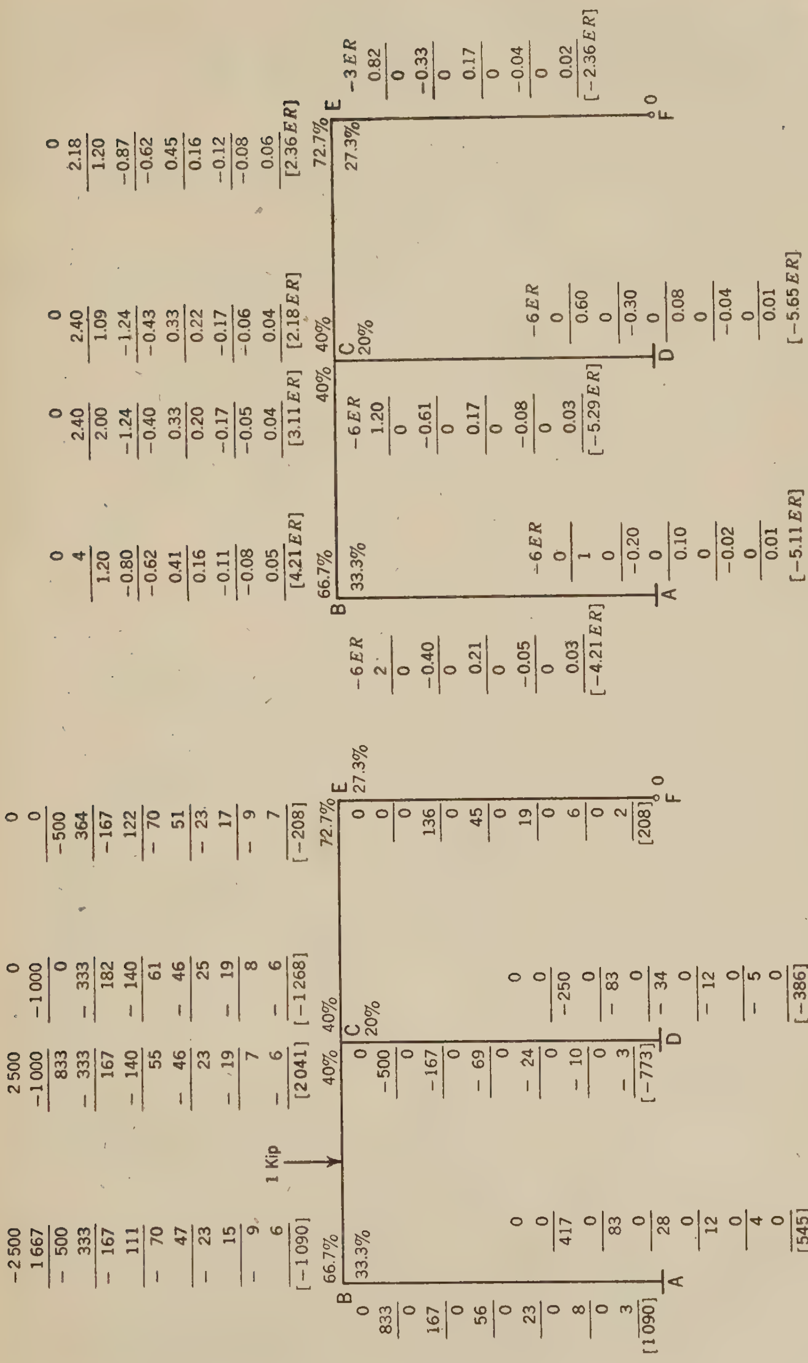
$$H_2 = \frac{-\,1{,}159 - 10.94\,E\,R}{15} \qquad (10b)$$

and, in Fig. 2(g),

$$H_3 = \frac{208 - 2.36\,E\,R}{15} \qquad (10c)$$

Substituting Eqs. 10 in Eq. 9, the value of $E\,R$ may be found; thus, $684 - 22.62\,E\,R = 0$ and $E\,R = 30.2$.

Step 5. The numerical values of the moments caused by lateral deflection are determined by substituting the value of $E\,R$ in the moment terms shown


-2 500
1 667
- 500
333
- 167
111
- 70
47
- 23
15
- 9
6
[-1 090]
2 500
-1 000
833
- 333
167
- 140
55
- 46
23
- 19
7
- 6
[2 041]
0
-1 000
0
- 333
182
- 140
61
- 46
25
- 19
8
- 6
[-1 268]
0
0
-500
364
-167
122
- 70
51
- 23
17
- 9
7
[-208]
1 Kip
66.7%
33.3%
40%
40%
20%
72.7%
27.3%
B
A
C
D
E
F
0
833
0
167
0
56
0
23
0
8
0
3
[1 090]
0
0
417
0
83
0
28
0
12
0
4
0
[545]
0
-500
0
-167
0
- 69
0
- 24
0
- 10
0
- 3
[-773]
0
0
-250
0
- 83
0
- 34
0
- 12
0
- 5
0
[-386]
0
0
0
136
0
45
0
19
0
6
0
2
[208]
0
0
4
1.20
-0.80
-0.62
0.41
0.16
-0.11
-0.08
0.05
[4.21 ER]
0
2.40
2.00
-1.24
-0.40
0.33
0.20
-0.17
-0.05
0.04
[3.11 ER]
0
2.40
1.09
-1.24
-0.43
0.33
0.22
-0.17
-0.06
0.04
[2.18 ER]
0
2.18
1.20
-0.87
-0.62
0.45
0.16
-0.12
-0.08
0.06
[2.36 ER]
-6 ER
2
0
-0.40
0
0.21
0
-0.05
0
0.03
[-4.21 ER]
-6 ER
0
1
0
-0.20
0
0.10
0
-0.02
0
0.01
[-5.11 ER]
-6 ER
1.20
0
-0.61
0
0.17
0
-0.08
0
0.03
[-5.29 ER]
-6 ER
0
0.60
0
-0.30
0
0.08
0
-0.04
0
0.01
[-5.65 ER]
-3 ER
0.82
0
-0.33
0
0.17
0
-0.04
0
0.02
[-2.36 ER]

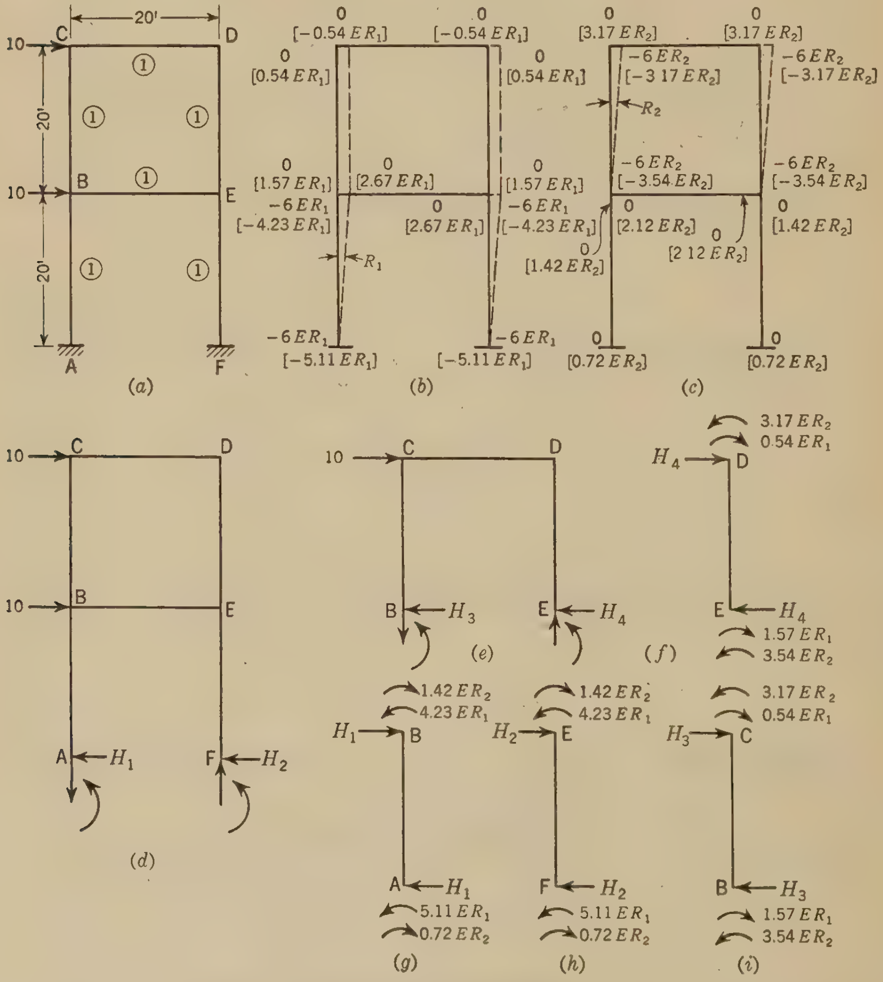
(a) SEE FIG. 2 (b)

(b) SEE FIG. 2 (c)

FIG. 3.—MOMENT DISTRIBUTION FOR EXAMPLE 2

in brackets in Fig. 2(*c*). By summation of the moments thus obtained and those shown in brackets in Fig. 2(*b*), the following final moments in foot-pounds are computed: $M_{AB} = 390$; $M_{BA} = 963$; $M_{BC} = -963$; $M_{CB} = 2{,}135$; $M_{CD} = -933$; $M_{DC} = -557$; $M_{CE} = -1{,}202$; $M_{EC} = -137$; and $M_{EF} = 137$.

Example 3. Two-Story Bent with Horizontal Loads.—Fig. 4(*a*) represents a two-story bent with horizontal loads applied at joints B and C.



20'
10
C
D
①
20'
10
B
E
20'
A
F
(a)
0
[−0.54 ER₁]
[0.54 ER₁]
0
[1.57 ER₁]
[2.67 ER₁]
−6 ER₁
[−4.23 ER₁]
R₁
−6 ER₁
[−5.11 ER₁]
(b)
0
[3.17 ER₂]
−6 ER₂
[−3.17 ER₂]
R₂
−6 ER₂
[−3.54 ER₂]
[2.12 ER₂]
[1.42 ER₂]
[0.72 ER₂]
(c)
10
C
D
10
B
E
A
H₁
F
H₂
(d)
10
C
D
B
H₃
E
H₄
(e)
3.17 ER₂
0.54 ER₁
H₄
D
E
H₄
1.57 ER₁
3.54 ER₂
(f)
1.42 ER₂
4.23 ER₁
H₁
B
H₂
E
3.17 ER₂
0.54 ER₁
H₃
C
A
H₁
5.11 ER₁
0.72 ER₂
(g)
F
H₂
5.11 ER₁
0.72 ER₂
(h)
B
H₃
1.57 ER₁
3.54 ER₂
(i)

FIG. 4.—TWO-STORY BENT WITH HORIZONTAL LOADS

Step 1. Two possible conditions exist—one for deformation of the first story, and the other for deformation of the second story of the frame. These are shown in Figs. 4(*b*) and 4(*c*).

Step 2. Fixed-end moments are computed as functions of $E\,R_1$ and $E\,R_2$ for the deformation conditions of Figs. 4(*b*) and 4(*c*) and recorded as shown.

Step 3. The joints in Fig. 4(*b*) and 4(*c*) are balanced by the moment distribution method, and the final moments, in terms of $E\,R_1$ and $E\,R_2$, respectively, are shown in brackets. The actual work of distribution is not shown in this or in subsequent examples, inasmuch as routine computations only are involved.

Step 4. To determine the values of $E\,R_1$ and $E\,R_2$, two equilibrium equations for the bent must be written. In Fig. 4(*d*), from $\Sigma F_x = 0$,

$$H_1 + H_2 = 20 \qquad (11a)$$

and, in Fig. 4(*e*), from $\Sigma F_x = 0$,

$$H_3 + H_4 = 10 \qquad (11b)$$

In Figs. 4(*g*) and 4(*h*), from $\Sigma M = 0$,

$$H_1 = H_2 = \frac{9.34\,E\,R_1 - 2.14\,E\,R_2}{20} \qquad (12a)$$

Similarly, in Figs. 4(*i*) and 4(*f*),

$$H_3 = H_4 = \frac{-\,2.11\,E\,R_1 + 6.71\,E\,R_2}{20} \qquad (12b)$$

Substituting Eqs. 12 in Eqs. 11.

$$18.68\,E\,R_1 - 4.28\,E\,R_2 = 400 \qquad (13a)$$

and

$$-\,4.22\,E\,R_1 + 13.42\,E\,R_2 = 200 \qquad (13b)$$

Solving Eqs. 13 simultaneously, $E\,R_1 = 26.7$; and $E\,R_2 = 23.4$.

Step 5. The numerical values of the moments caused by each deformation are determined by substituting the values of $E\,R_1$ and $E\,R_2$ in the moment terms shown in brackets in Figs. 4(*b*) and 4(*c*), and the final moments at the ends of all members in foot-pounds are computed by summation of these values: $M_{AB} = -\,120$; $M_{BA} = -\,80$; $M_{BE} = 120$; $M_{BC} = -\,40$; and $M_{CB} = -\,60$.

Example 4. Complex Three-Column Bent.—Fig. 5(*a*) represents a complex three-column bent (5*a*)[3] with a horizontal load of 500 lb applied at joint B.

Step 1. Two possible conditions of deformation exist—one for the part ABCD of the bent, and the other for primary deformation of part EDFG which in turn causes secondary deformation of part ABCD. These are shown in Figs. 5(*b*) and 5(*c*), respectively.

Step 2. Fixed-end moments are computed for the deformation conditions of Figs. 5(*b*) and 5(*c*) and recorded as shown. In this problem, the fixed-end moments are expressed in terms of the unknown linear displacements Δ instead of the angular displacements R as in previous problems. The method of solution, however, is not changed in any way.

Step 3. The joints in Figs. 5(*b*) and 5(*c*) are balanced by the moment distribution method, and the final moments, in terms of $E\,\Delta_1$ and $E\,\Delta_2$, respectively, are shown in brackets.

Step 4. To determine the values of $E\,\Delta_1$ and $E\,\Delta_2$, two equilibrium equations must be written for the bent. In Fig. 5(*d*), from $\Sigma F_x = 0$,

$$H_1 + H_2 = 500 \qquad (14a)$$

---

[3] Numerals in parentheses, thus: (5*a*), refer to corresponding items in the Bibliography (see Appendix).

and in Fig. 5(*e*), from $\Sigma F_x = 0$,

$$H_1 + H_3 + H_4 = 500 \qquad (14b)$$

In Fig. 5(*f*), from $\Sigma M = 0$,

$$H_1 = 0.359\ E\,\Delta_1 + 0.269\ E\,\Delta_2 \qquad (15a)$$

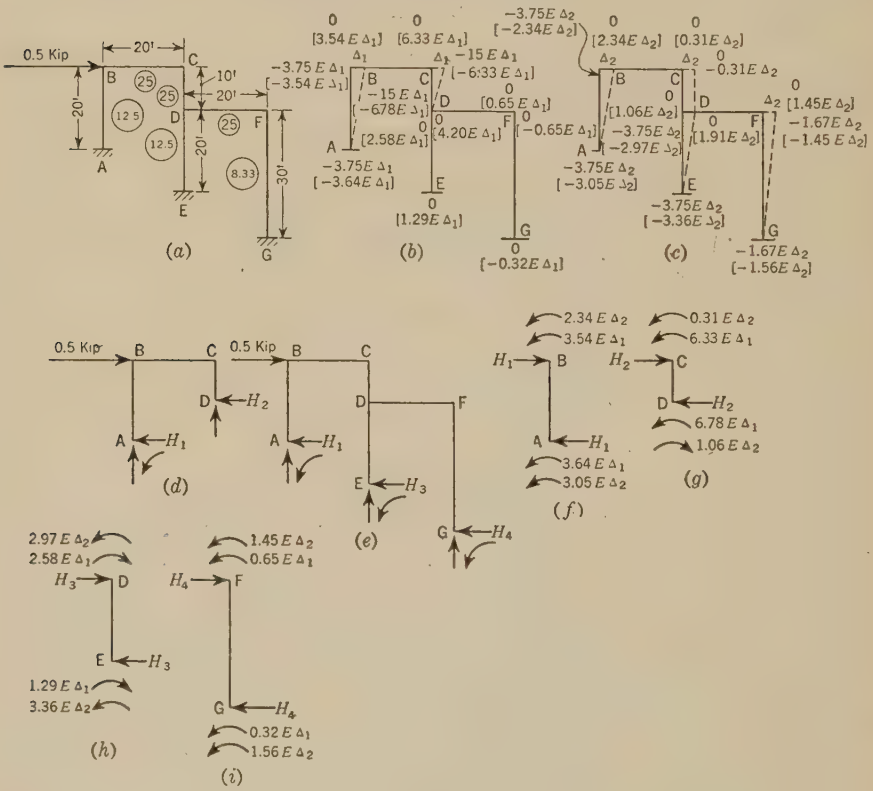

0.5 Kip
20'
B
C
10'
20'
D
F
A
E
G
20'
20'
30'
25
25
25
12.5
12.5
8.33
(a)
0
[3.54 E Δ1]
0
[6.33 E Δ1]
Δ1
Δ1
−15 E Δ1
[−6.33 E Δ1]
−3.75 E Δ1
[−3.54 E Δ1]
−15 E Δ1
[−6.78 E Δ1]
0
[0.65 E Δ1]
0
[2.58 E Δ1]
[4.20 E Δ1]
0
[−0.65 E Δ1]
−3.75 E Δ1
[−3.64 E Δ1]
0
[1.29 E Δ1]
0
[−0.32 E Δ1]
(b)
−3.75 E Δ2
[−2.34 E Δ2]
0
[2.34 E Δ2]
0
[0.31 E Δ2]
Δ2
Δ2
0
−0.31 E Δ2
0
Δ2 [1.45 E Δ2]
[1.06 E Δ2]
−3.75 E Δ2
[−2.97 E Δ2]
0
[1.91 E Δ2]
−1.67 E Δ2
[−1.45 E Δ2]
−3.75 E Δ2
[−3.05 E Δ2]
−3.75 E Δ2
[−3.36 E Δ2]
−1.67 E Δ2
[−1.56 E Δ2]
(c)
0.5 Kip
H1
H2
(d)
0.5 Kip
H1
H3
H4
(e)
2.34 E Δ2
3.54 E Δ1
H1
3.64 E Δ1
3.05 E Δ2
(f)
0.31 E Δ2
6.33 E Δ1
H2
6.78 E Δ1
1.06 E Δ2
(g)
2.97 E Δ2
2.58 E Δ1
H3
1.29 E Δ1
3.36 E Δ2
(h)
1.45 E Δ2
0.65 E Δ1
H4
0.32 E Δ1
1.56 E Δ2
(i)

FIG. 5.—COMPLEX THREE-COLUMN BENT

Similarly, in Fig. 5(*g*),

$$H_2 = 1.311\ E\,\Delta_1 - 0.075\ E\,\Delta_2 \qquad (15b)$$

In Fig. 5(*h*),

$$H_3 = -\ 0.193\ E\,\Delta_1 + 0.316\ E\,\Delta_2 \qquad (15c)$$

and, in Fig. 5(*i*),

$$H_4 = 0.032\ E\,\Delta_1 + 0.100\ E\,\Delta_2 \qquad (15d)$$

Substituting Eqs. 15 in Eqs. 14,

$$1.670\ E\,\Delta_1 + 0.194\ E\,\Delta_2 = 500 \qquad (16a)$$

and

$$0.198\ E\,\Delta_1 + 0.685\ E\,\Delta_2 = 500 \qquad (16b)$$

Solving Eqs. 16 simultaneously, $E\,\Delta_1 = 222$, and $E\,\Delta_2 = 666$.

Step 5. The numerical values of the moments caused by each deformation are determined by substituting the values of $E\,\Delta_1$ and $E\,\Delta_2$ in the moment terms shown in brackets in Figs. 5(*b*) and 5(*c*), and the following final moments in foot-pounds at the ends of all members are determined by summation: $M_{AB} = -2{,}838$; $M_{BA} = -2{,}345$; $M_{CD} = -1{,}612$; $M_{DC} = -799$; $M_{DE} = -1{,}407$; $M_{ED} = -1{,}954$; $M_{DF} = 2{,}203$; $M_{FG} = -1{,}109$; and $M_{GF} = -1{,}111$.

Example 5. Vierendeel Truss.—Fig. 6(*a*) represents a simple Vierendeel truss that supports a vertical load of 10 kips at the center of the chord BC. Values of E and I are assumed constant throughout.

Step 1. Three possible conditions of deformation exist—one for the transverse load on BC, and one for deformation of each panel of the structure. These are shown in Figs. 6(*b*), 6(*c*), and 6(*d*).

Step 2. Fixed-end moments are computed for the deformation conditions of Figs. 6(*b*), 6(*c*), and 6(*d*), and recorded as shown.

Step 3. The joints in Figs. 6(*b*), 6(*c*), and 6(*d*) are balanced and the final moments for each case are shown in brackets.

Step 4. To determine the values of $E\,R_1$ and $E\,R_2$, two equilibrium equations for the truss must be written. In Fig. 6(*e*), from $\Sigma F_y = 0$,

$$V_1 + V_2 = 7.5 \qquad (17a)$$

and, in Fig. 6(*f*), from $\Sigma F_y = 0$,

$$V_3 + V_4 = 2.5 \qquad (17b)$$

In Fig. 6(*g*), from $\Sigma M_c = 0$,

$$V_1 = \frac{55.38 + 6.23\,E\,R_1 - 1.82\,E\,R_2}{12} \qquad (18a)$$

In Fig. 6(*h*), from $\Sigma M = 0$,

$$V_2 = \frac{1.70 + 6.23\,E\,R_1 - 1.82\,E\,R_2}{12} \qquad (18b)$$

In Fig. 6(*i*), from $\Sigma M = 0$,

$$V_3 = \frac{-8.30 + 1.82\,E\,R_1 - 6.23\,E\,R_2}{12} \qquad (18c)$$

In Fig. 6(*j*), from $\Sigma M = 0$,

$$V_4 = \frac{1.24 + 1.82\,E\,R_1 - 6.23\,E\,R_2}{12} \qquad (18d)$$

Substituting Eqs. 18 in Eqs. 17,

$$12.46\,E\,R_1 - 3.65\,E\,R_2 = 32.92 \qquad (19a)$$

and

$$3.64\,E\,R_1 - 12.46\,E\,R_2 = 37.06 \qquad (19b)$$

Solving Eqs. 19 simultaneously, $E\,R_1 = 1.94$; and $E\,R_2 = -2.43$. (Note that these are relative R-values corresponding to $R = 0$ for the vertical members of the frame. The moments obtained using these R-values will be correct, however, as can readily be seen by reference to the general slope-deflection equation.)

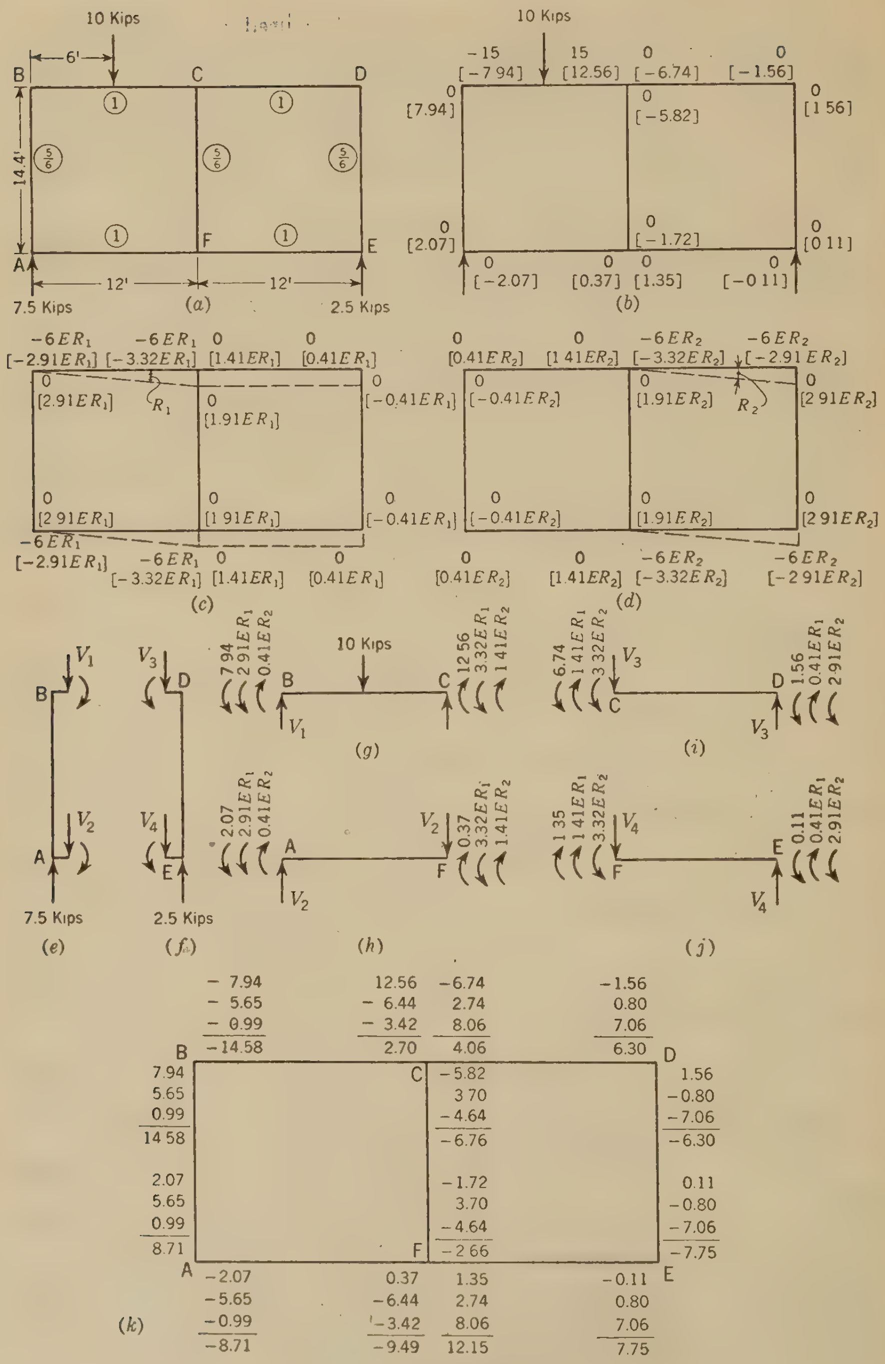

10 Kips
6'
B
C
D
①
⁵⁄₆
14.4'
A
F
E
12'
12'
7.5 Kips
(a)
2.5 Kips
10 Kips
-15 [-7.94]
15 [12.56]
0 [-6.74]
0 [-1.56]
0 [7.94]
0 [-5.82]
0 [1.56]
0 [2.07]
0 [-1.72]
0 [0.11]
0 [-2.07]
0 [0.37]
0 [1.35]
0 [-0.11]
(b)
-6ER₁ [-2.91ER₁]
-6ER₁ [-3.32ER₁]
0 [1.41ER₁]
0 [0.41ER₁]
0 [2.91ER₁]
R₁
0 [1.91ER₁]
0 [-0.41ER₁]
0 [2.91ER₁]
0 [1.91ER₁]
0 [-0.41ER₁]
-6ER₁ [-2.91ER₁]
-6ER₁ [-3.32ER₁]
0 [1.41ER₁]
0 [0.41ER₁]
(c)
0 [0.41ER₂]
0 [1.41ER₂]
-6ER₂ [-3.32ER₂]
-6ER₂ [-2.91ER₂]
0 [-0.41ER₂]
0 [1.91ER₂]
R₂
0 [2.91ER₂]
0 [-0.41ER₂]
0 [1.91ER₂]
0 [2.91ER₂]
0 [0.41ER₂]
0 [1.41ER₂]
-6ER₂ [-3.32ER₂]
-6ER₂ [-2.91ER₂]
(d)
V₁
B
V₂
A
7.5 Kips
(e)
V₃
D
V₄
E
2.5 Kips
(f)
7.94
2.91ER₁
0.41ER₂
B
10 Kips
C
12.56
3.32ER₁
1.41ER₂
V₁
(g)
6.74
1.41ER₁
3.32ER₂
V₃
C
D
1.56
0.41ER₁
2.91ER₂
V₃
(i)
2.07
2.91ER₁
0.41ER₂
A
V₂
F
0.37
3.32ER₁
1.41ER₂
V₂
(h)
1.35
1.41ER₁
3.32ER₂
V₄
F
E
0.11
0.41ER₁
2.91ER₂
V₄
(j)
-7.94
-5.65
-0.99
-14.58
12.56
-6.44
-3.42
2.70
-6.74
2.74
8.06
4.06
-1.56
0.80
7.06
6.30
B
C
D
7.94
5.65
0.99
14.58
-5.82
3.70
-4.64
-6.76
1.56
-0.80
-7.06
-6.30
2.07
5.65
0.99
8.71
-1.72
3.70
-4.64
-2.66
0.11
-0.80
-7.06
-7.75
A
F
E
-2.07
-5.65
-0.99
-8.71
0.37
-6.44
-3.42
-9.49
1.35
2.74
8.06
12.15
-0.11
0.80
7.06
7.75
(k)

FIG. 6.—VIERENDEEL TRUSS

Step 5. The summation of the moments in Figs. 6(*b*), 6(*c*), and 6(*d*) and the final values of the moments in foot-kips at the ends of all members are shown in Fig. 6(*k*).

NONRECTANGULAR FRAMES

Nonrectangular or quadrangular frames, such as trapezoidal bents, gable bents, and Vierendeel trusses with inclined upper chords, are frequently used in practice. Their behavior under the action of applied forces which cause linear displacements of the joints (deformations of the quadrangles) is radically different from that of rectangular frames under similar conditions, as will be noted in the problems of this section.

Relation Between Values of R for Members of a Quadrangle.—The quadrangle represented in Fig. 7 is deformed by external forces (not shown) so that joints B and C move to positions B′ and C′, respectively. The member AD is assumed to remain fixed in direction during the displacements of B and C. It is desired to find the relations between the values of R for AB, BC, and CD.

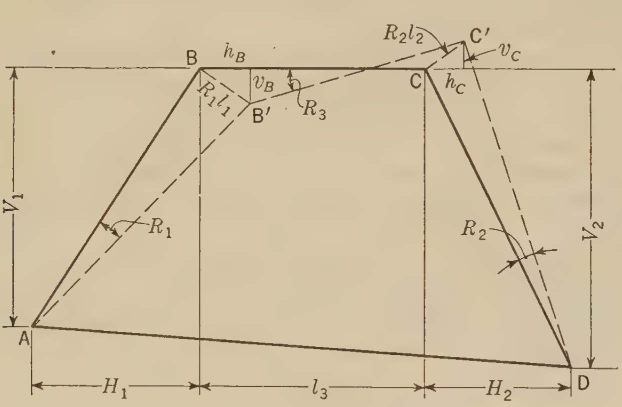


FIG. 7.—DEFORMATION OF A QUADRANGLE

Let l_1, l_2, and l_3 represent the lengths of members AB, CD, and BC, respectively, and R_1, R_2, and R_3 the corresponding values of R. Let H_1 and V_1 represent the horizontal and vertical projections of AB, and H_2 and V_2 the horizontal and vertical projections of CD. Let h_B and v_B represent the horizontal and vertical components of the displacement of B, and h_C and v_C the horizontal and vertical components of the displacement of C.

From similar triangles, the following relations may be written: $\frac{h_B}{R_1 l_1} = \frac{V_1}{l_1}$; and $\frac{v_B}{R_1 l_1} = \frac{H_1}{l_1}$. Therefore, $h_B = R_1 V_1$, and $v_B = R_1 H_1$. Similarly, it may be shown that $h_C = R_2 V_2$, and $v_C = R_2 H_2$. Since $h_B = h_C$, $R_1 V_1 = R_2 V_2$; and

$$R_2 = \frac{R_1 V_1}{V_2} \quad \text{(20a)}$$

Also, since the vertical displacement of C relative to B equals $-(v_B + v_C) \doteq -(R_1 H_1 + R_2 H_2)$,

$$R_3 = \frac{-(R_1 H_1 + R_2 H_2)}{l_3} \quad \text{(20b)}$$

Eqs. 20 are the desired relations which enable one to express R_2 and R_3 for members CD and BC, respectively, in terms of R_1 for AB.

In the quadrangles of many frames, the projections V_1 and V_2 of AB and CD, Fig. 7, will be equal. Eqs. 20 may then be written:

$$R_2 = R_1 \quad \text{(21a)}$$

and

$$R_3 = \frac{-R_1 (H_1 + H_2)}{l_3} \quad \text{(21b)}$$

Example 6. Trapezoidal Bent.—Fig. 8(*a*) represents a trapezoidal bent (9*a*) with a horizontal load of 200 lb applied at B and an unsymmetrical vertical load of 1,000 lb on BC.

Step 1. Two possible conditions of deformation exist—one for the transverse load on BC, and the other for lateral deflection (sidesway) of the frame. These are shown in Figs. 8(*b*) and 8(*c*), respectively.

Step 2. From Eqs. 21 in Fig. 8(*c*), $R_2 = R_1$ and $R_3 = -R_1$. Fixed-end moments are computed for this deformation condition in terms of $E R_1$, and for the condition of Fig. 8(*b*) in numerical values, and recorded on the appropriate sketches.

Step 3. The joints in Figs. 8(*b*) and 8(*c*) are balanced and the final moments for each case are shown in brackets.

Step 4. To determine the value of $E R_1$, an equilibrium equation for the bent must be written. In Fig. 8(*d*), from $\Sigma F_x = 0$,

$$H_1 + H_2 = 200 \quad \text{(22)}$$

In Fig. 8(*e*), from $\Sigma M_C = 0$,

$$V_1 = 221.4 - 1.384 E R_1 \quad \text{(23a)}$$

and, from $\Sigma M_B = 0$,

$$V_2 = 778.6 + 1.384 E R_1 \quad \text{(23b)}$$

The values of V_1 and V_2 in Eqs. 23 are shown acting in Figs. 8(*f*) and (8*g*), respectively. In Fig. 8(*f*), from $\Sigma M = 0$,

$$H_1 = \frac{20.30 E R_1 - 1{,}692}{15} \quad \text{(24a)}$$

and, in Fig. 8(*g*), from $\Sigma M = 0$,

$$H_2 = \frac{20.30 E R_1 + 4{,}908}{15} \quad \text{(24b)}$$

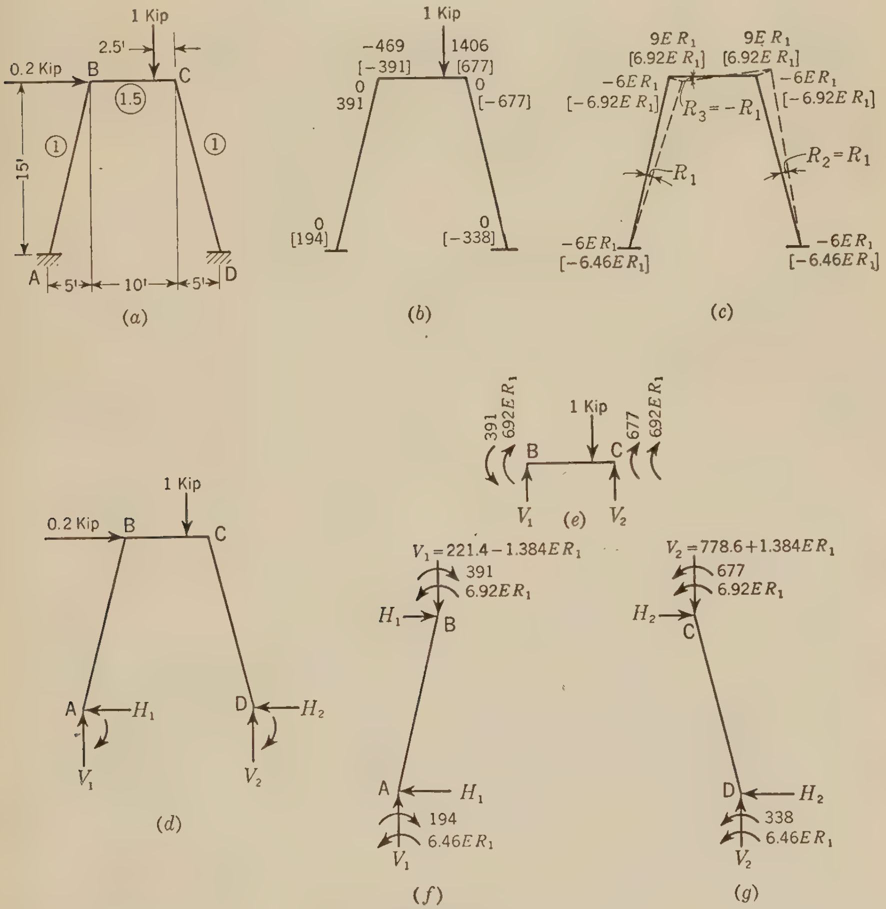
Substituting Eqs. 24 in Eq. 22,

$$\frac{20.30 E R_1 - 1{,}692}{15} + \frac{20.30 E R_1 + 4{,}908}{15} = 200 \quad \text{(25)}$$

and solving Eq. 25, $E R_1 = -5.32$.

Step 5. Summation of the moments in Figs. 8(*b*) and 8(*c*) gives the following final moments: $M_{AB} = 228$; $M_{BA} = 428$; $M_{BC} = -428$; $M_{CB} = 640$; $M_{CD} = -640$; and $M_{DC} = -304$.

Example 7. Two-Story Bent with Horizontal Loads.—Fig. 9(*a*) represents a two-story bent with inclined legs which carries horizontal loads applied at joints B and E.



1 Kip
2.5'
0.2 Kip
B
C
(1.5)
①
①
15'
A
D
5'
10'
5'
(*a*)
1 Kip
-469
[-391]
1406
[677]
0
391
0
[-677]
0
[194]
0
[-338]
(*b*)
9*E R*₁
[6.92*E R*₁]
9*E R*₁
[6.92*E R*₁]
-6*E R*₁
[-6.92*E R*₁]
-6*E R*₁
[-6.92*E R*₁]
*R*₃ = -*R*₁
*R*₂ = *R*₁
*R*₁
-6*E R*₁
[-6.46*E R*₁]
-6*E R*₁
[-6.46*E R*₁]
(*c*)
1 Kip
0.2 Kip
B
C
A
*H*₁
D
*H*₂
*V*₁
*V*₂
(*d*)
391
692*E R*₁
1 Kip
677
692*E R*₁
B
C
*V*₁
(*e*)
*V*₂
*V*₁ = 221.4 - 1.384*E R*₁
391
6.92*E R*₁
*H*₁
B
A
*H*₁
194
6.46*E R*₁
*V*₁
(*f*)
*V*₂ = 778.6 + 1.384*E R*₁
677
6.92*E R*₁
*H*₂
C
D
*H*₂
338
6.46*E R*₁
*V*₂
(*g*)

FIG. 8.—TRAPEZOIDAL BENT

Step 1. Two possible conditions of deformation exist—one for primary deformation of the first story which in turn causes secondary deformation of the second story, and the other for the deformation of the second story of the frame. These are shown in Figs. 9(*b*) and 9(*c*).

Step 2. By use of Eqs. 21, the *R*-values for all members in Fig. 9(*b*) have been expressed in terms of R_1 for AB, and for all members in Fig. 9(*c*) in terms of R_2 for BE. The corresponding fixed-end moments, in terms of $E\,R_1$ and $E\,R_2$, are recorded on these sketches.

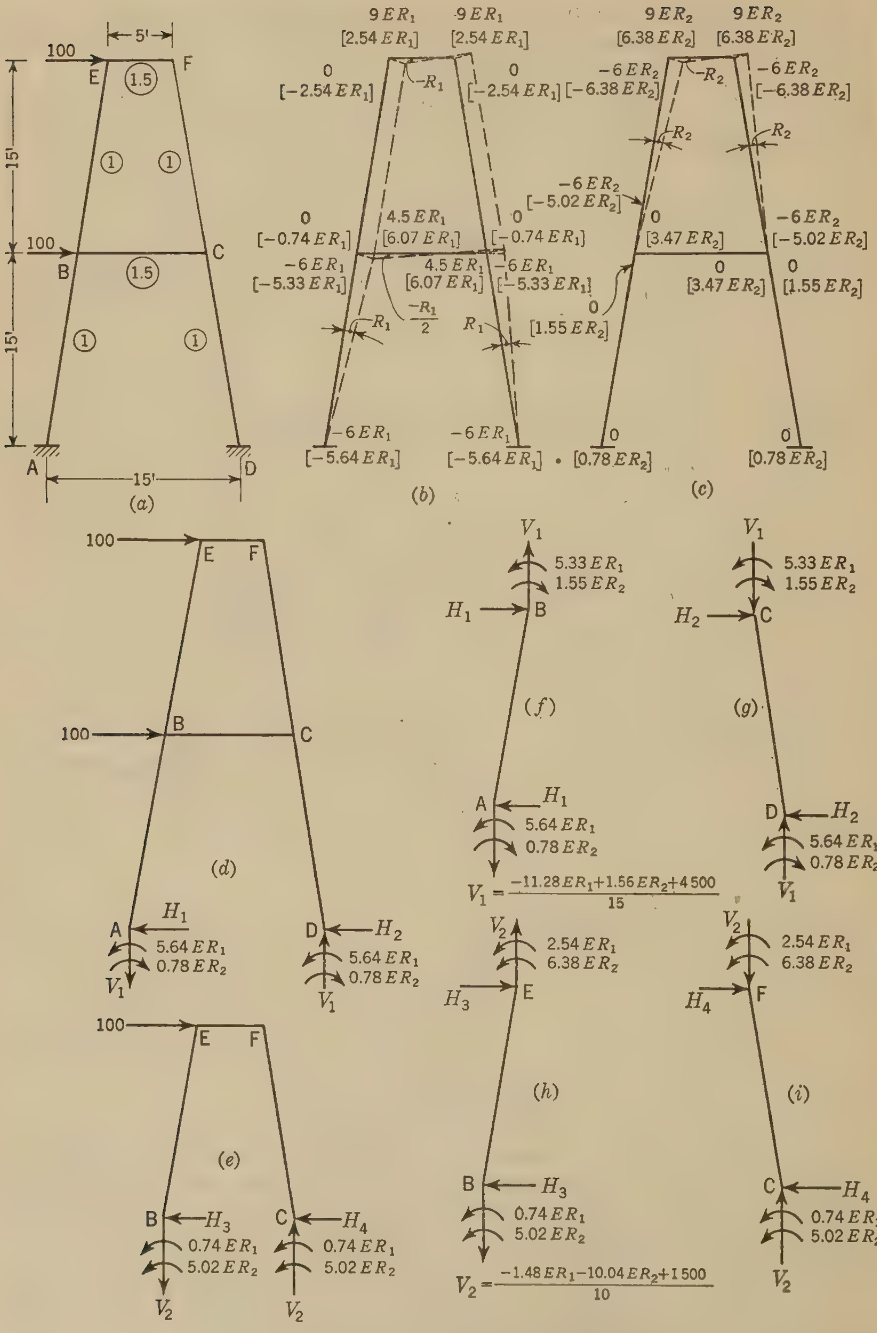

100
5'
E
F
1.5
15'
①
①
100
B
1.5
C
15'
①
①
A
D
15'
(a)
9ER₁
[2.54ER₁]
9ER₁
[2.54ER₁]
0
[−2.54ER₁]
−R₁
0
[−2.54ER₁]
0
[−0.74ER₁]
4.5ER₁
[6.07ER₁]
0
[−0.74ER₁]
−6ER₁
[−5.33ER₁]
4.5ER₁
[6.07ER₁]
−6ER₁
[−5.33ER₁]
−R₁/2
R₁
R₁
−6ER₁
[−5.64ER₁]
−6ER₁
[−5.64ER₁]
(b)
9ER₂
[6.38ER₂]
9ER₂
[6.38ER₂]
−6ER₂
[−6.38ER₂]
−R₂
−6ER₂
[−6.38ER₂]
R₂
R₂
−6ER₂
[−5.02ER₂]
0
[3.47ER₂]
−6ER₂
[−5.02ER₂]
0
[3.47ER₂]
0
[1.55ER₂]
0
[1.55ER₂]
0
[0.78ER₂]
0
[0.78ER₂]
(c)
100
E
F
100
B
C
(d)
A
H₁
5.64ER₁
0.78ER₂
V₁
D
H₂
5.64ER₁
0.78ER₂
V₁
100
E
F
(e)
B
H₃
0.74ER₁
5.02ER₂
V₂
C
H₄
0.74ER₁
5.02ER₂
V₂
V₁
5.33ER₁
1.55ER₂
H₁
B
(f)
A
H₁
5.64ER₁
0.78ER₂
V₁ = (−11.28ER₁+1.56ER₂+4500)/15
V₁
5.33ER₁
1.55ER₂
H₂
C
(g)
D
H₂
5.64ER₁
0.78ER₂
V₁
V₂
2.54ER₁
6.38ER₂
H₃
E
(h)
B
H₃
0.74ER₁
5.02ER₂
V₂ = (−1.48ER₁−10.04ER₂+1500)/10
V₂
2.54ER₁
6.38ER₂
H₄
F
(i)
C
H₄
0.74ER₁
5.02ER₂
V₂

Fig. 9.—Two-Story Bent with Horizontal Loads

Step 3. The joints in Figs. 9(*b*) and 9(*c*) are balanced and the final moments for each case are shown in brackets.

Step 4. To determine the values of $E\,R_1$ and $E\,R_2$, two equilibrium equations for the bent must be written. In Fig. 9(*d*), from $\Sigma F_x = 0$,

$$H_1 + H_2 = 200 \qquad (26a)$$

and, in Fig. 9(*e*), from $\Sigma F_x = 0$,

$$H_3 + H_4 = 100 \qquad (26b)$$

In Fig. 9(*d*), from $\Sigma M_D = 0$,

$$V_1 = \frac{-11.28\,E\,R_1 + 1.56\,E\,R_2 + 4{,}500}{15} \qquad (27a)$$

and, in Fig. 9(*e*), from $\Sigma M_C = 0$,

$$V_2 = \frac{-1.48\,E\,R_1 - 10.04\,E\,R_2 + 1{,}500}{10} \qquad (27b)$$

The values of V_1 and V_2 in Eqs. 27 are shown acting in Figs. 9(*f*), 9(*g*), 9(*h*), and 9(*i*). In Figs. 9(*f*) and 9(*g*), respectively, from $\Sigma M = 0$,

$$H_1 = H_2 = \frac{54.54\,E\,R_1 - 12.42\,E\,R_2 + 4{,}500}{90} \qquad (28a)$$

In Figs. 9(*h*) and 9(*i*), respectively, from $\Sigma M = 0$,

$$H_3 = H_4 = \frac{11.64\,E\,R_1 + 35.56\,E\,R_2 + 1{,}500}{60} \qquad (28b)$$

Substituting Eqs. 28 in Eqs. 26,

$$\left.\begin{aligned} 54.54\,E\,R_1 - 12.42\,E\,R_2 &= 4{,}500 \\ 11.64\,E\,R_1 + 35.56\,E\,R_2 &= 1{,}500 \end{aligned}\right\} \qquad (29)$$

Solving Eqs. 29 simultaneously, $E\,R_1 = 85.7$ and $E\,R_2 = 14.1$.

Step 5. The numerical values of the moments in Figs. 9(*b*) and 9(*c*) may now be calculated, and the following moments at the ends of all members are determined by summation: $M_{AB} = M_{DC} = -472$; $M_{BA} = M_{CD} = -434$; $M_{BE} = M_{CF} = -134$; $M_{EB} = M_{FC} = -308$; $M_{BC} = M_{CB} = 569$; and $M_{EF} = M_{FE} = 308$.

Example 8. Gable Bent.—Fig. 10(*a*) represents a gable bent with a horizontal load of 1,000 lb applied at joint B.

Step 1. Two possible conditions of deformation exist—one for deformation of EDCB, and the other for linear displacement of B which in turn causes an equal displacement of D (an alternate deformation of ABCD might be used instead). These are shown in Figs. 10(*b*) and 10(*c*), respectively.

Step 2. By use of Eqs. 20, the R-values for all members in Fig. 10(*b*) have been expressed in terms of R_1 for ED, and the R-value of ED in Fig. 10(*c*) has been expressed in terms of R_2 for AB. The corresponding fixed-end moments, in terms of $E\,R_1$ and $E\,R_2$, are recorded on these sketches.

Step 3. The joints in Figs. 10(*b*) and 10(*c*) are balanced and the final moments for each case are shown in brackets.

Step 4. To determine the values of $E\,R_1$ and $E\,R_2$, two equilibrium equations for the bent must be written.

In Fig. 10(d), from $\Sigma F_x = 0$,

$$H_1 + H_3 = 1{,}000 \qquad (30a)$$

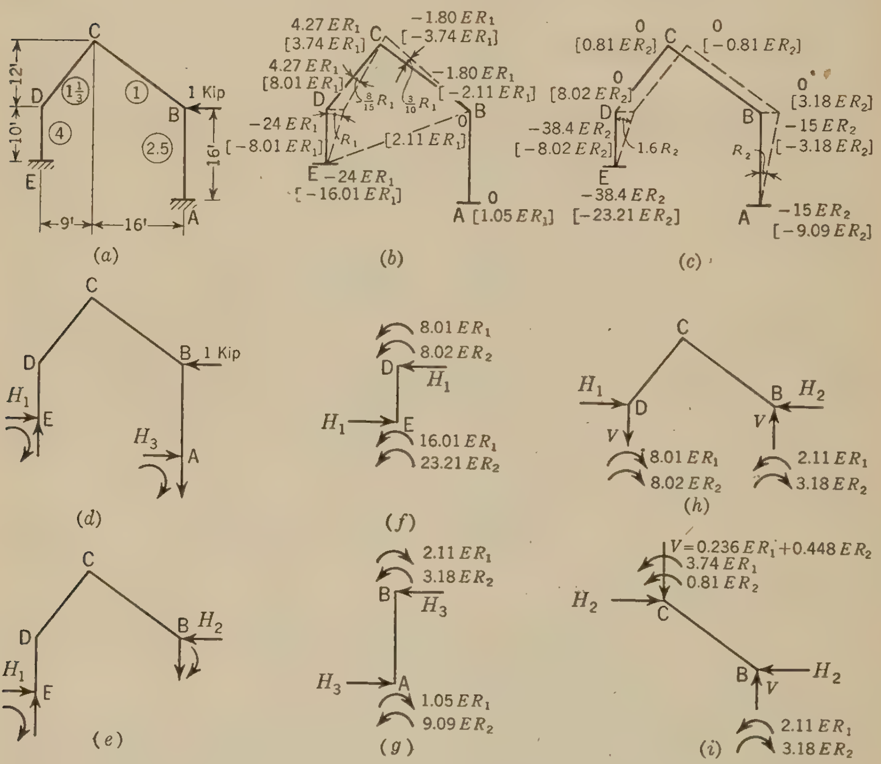

C
12'
10'
D
E
$1\frac{1}{3}$
1
4
2.5
1 Kip
B
16'
A
9'
16'
(a)
$4.27\,ER_1$
$[3.74\,ER_1]$
$-1.80\,ER_1$
$[-3.74\,ER_1]$
$4.27\,ER_1$
$[8.01\,ER_1]$
$-1.80\,ER_1$
$[-2.11\,ER_1]$
$\frac{8}{15}R_1$
$\frac{3}{10}R_1$
$-24\,ER_1$
$[-8.01\,ER_1]$
R_1
$[2.11\,ER_1]$
0
E $-24\,ER_1$
$[-16.01\,ER_1]$
0
A $[1.05\,ER_1]$
(b)
0
$[0.81\,ER_2]$
0
$[-0.81\,ER_2]$
0
$[8.02\,ER_2]$
0
$[3.18\,ER_2]$
$-38.4\,ER_2$
$[-8.02\,ER_2]$
$1.6\,R_2$
$-15\,ER_2$
$[-3.18\,ER_2]$
R_2
E
$-38.4\,ER_2$
$[-23.21\,ER_2]$
A
$-15\,ER_2$
$[-9.09\,ER_2]$
(c)
C
D
B 1 Kip
H_1
E
H_3
A
(d)
$8.01\,ER_1$
$8.02\,ER_2$
D
H_1
H_1
E
$16.01\,ER_1$
$23.21\,ER_2$
(f)
C
H_1
D
B H_2
V
V
$8.01\,ER_1$
$8.02\,ER_2$
$2.11\,ER_1$
$3.18\,ER_2$
(h)
C
D
B H_2
H_1
E
(e)
$2.11\,ER_1$
$3.18\,ER_2$
B
H_3
H_3
A
$1.05\,ER_1$
$9.09\,ER_2$
(g)
$V=0.236\,ER_1+0.448\,ER_2$
$3.74\,ER_1$
$0.81\,ER_2$
H_2
C
B
H_2
V
$2.11\,ER_1$
$3.18\,ER_2$
(i)

FIG. 10.—GABLE BENT

and, in Fig. 10(e), from $\Sigma F_x = 0$,

$$H_1 - H_2 = 0 \qquad (30b)$$

In Fig. 10(f), from $\Sigma M = 0$,

$$H_1 = -\,2.402\,E\,R_1 - 3.122\,E\,R_2 \qquad (31a)$$

in Fig. 10(g), from $\Sigma M = 0$,

$$H_3 = 0.198\,E\,R_1 - 0.767\,E\,R_2 \qquad (31b)$$

and, in Fig. 10(h), from $\Sigma M_B = 0$,

$$V = 0.236\,E\,R_1 + 0.448\,E\,R_2 \qquad (31c)$$

The value of V in Eq. 31c is shown acting in Fig. 10(i) in which, from $\Sigma M_B = 0$,

$$H_2 = 0.802\,E\,R_1 + 0.400\,E\,R_2 \qquad (31d)$$

Substituting Eqs. 31 in Eqs. 30

$$\left.\begin{array}{l} -2.204\,E\,R_1 - 3.889\,E\,R_2 = 1{,}000 \\ -3.204\,E\,R_1 - 3.522\,E\,R_2 = 0 \end{array}\right\} \qquad (32)$$

Solving Eqs. 32 simultaneously, $E\,R_1 = 748$; and $E\,R_2 = -680$.

Step 5. The numerical values of the moments in Figs. 10(*b*) and 10(*c*) may now be computed, and the final moments at the ends of all members are determined by summation: $M_{AB} = 6{,}970$; $M_{BA} = 3{,}740$; $M_{BC} = -3{,}740$; $M_{CB} = -2{,}250$; $M_{CD} = 2{,}250$; $M_{DC} = 550$; $M_{DE} = -550$; and $M_{ED} = 3{,}800$.

CONCLUSIONS

In this paper a method of procedure has been developed whereby the principle of superposition is utilized in conjunction with the slope-deflection or moment distribution method in the analysis of rigid frames.

The following conclusions are based on the foregoing analyses:

1. The method of procedure is general, and the results obtained are exact for the assumptions commonly made in rigid frame analysis.

2. Although many computations are required in the analysis of a frame having a large number of deformation conditions, the calculations are simple and all operations of multiplication and division may usually be performed with sufficient accuracy on a 10-in. slide rule.

3. At all times special care must be given to the correct formulation of equilibrium equations and, in the case of nonrectangular frames, to the determination of the relative R-values for the members of the quadrangles of the frame being considered.

4. For frames of unusual proportions where convergence by the moment distribution method is slow, it is recommended that the slope-deflection method be used to balance the moments at the joints in step 3 of the outline of method of procedure.

5. The principle of superposition is an indispensable aid in the analysis of rigid frames of all types in which applied loads cause linear displacements of the joints.

APPENDIX

BIBLIOGRAPHY

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(17) "Relative Flexure Factors for Analyzing Continuous Structures," by Ralph W. Stewart, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 521. (*a*) p. 542.

(18) "Simplified Methods for the Analysis of Multiple Joint Rigid Frames," by George A. Maney and John E. Goldberg, *Bulletin*, Northwestern Univ., Evanston, Ill., Vol. 33, No. 7, October 17, 1932.

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(20) "Simplified Determination of Moments in Multi-Story Structures," by A. W. Fischer, *Concrete*, November, 1938, p. 5.

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DISCUSSION

F. S. MERRITT,[4] JUN. AM. SOC. C. E.—Although no new method of analysis is proposed (see "Synopsis"), this paper is of value in illustrating the application of the principle of superposition to different types of frames. As Professor Wilson acknowledges, the principle of superposition is usually stated in textbooks on structural analysis; but, contrary to his opinion, it is also strongly emphasized in texts discussing moment distribution, since it is the most convenient tool for analyzing rigid frames by this method. However, nowhere in this paper is the principle of superposition, the subject of the paper, clearly stated or defined; nor are the limits of its validity described.

In the outline of the method of procedure, five steps are given for the solution of any rigid frame problem. Hardy Cross and N. D. Morgan, Members, Am. Soc. C. E., describe this same procedure simply and succintly as follows (3*a*):

> "In analyzing a bent by moment distribution we first assume no movement of the joints and analyze for this condition, find by statics the force necessary to prevent such movements and find the moments which it would produce, and then take the difference of the two results."

The writer has developed the following procedure (16):

> "1. Apply forces to the structure to prevent sidesway while the fixed-end moments due to loads are distributed.
>
> "2. Compute the moments due to these forces.
>
> "3. Combine the moments obtained in Steps 1 and 2 to eliminate the effect of the forces."

When moment distribution by the method of moment ratios (16) is used, there is never any need to revert to the slope-deflection method, because the question of convergence does not arise. The author's recommendation in conclusion 4, therefore, is not generally valid.

In applying either of these last two procedures, sketches may be desirable, but they are not necessary. From a practical viewpoint, the preparation of sketches requires time, and, where the calculations can follow a rigid procedure in which mistakes can be avoided without the use of sketches, such a procedure is recommended. In the classroom, sketches are useful in the introduction of an unfamiliar subject, but, in a drafting room, where time is money and a designer is assumed to be familiar with structural analysis, time-saving procedures should be adopted. Consequently, step 1 of the paper should be optional with the designer and should be included as a suggestion to facilitate determination of signs rather than as a step in the procedure.

In analyzing rigid frames, a designer generally wishes to obtain separately the effects of dead load, live load, wind, etc. The determination of live load moments might require the plotting of influence lines. In reality, this is the rigid frame problem, as contrasted to the classroom problem of a given structure with a fixed loading condition. Professor Wilson does not state how his procedure could be used under changing load conditions without the necessity of going through all five steps for each change in position or magnitude of loads. As a matter of fact, the writer has used the three steps outlined herein as the base of a simple procedure in which the principle of superposition is utilized to obtain the separate effects of varied loading conditions without the necessity of distributing fixed-end moments for every condition (16). The practice of computing and placing final moments on sketches, as recommended in steps 2 and 3 by the author, is cumbersome in view of the great number of sketches required and the difficulty in making summations and combinations from them.

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[4] Field Service Engr., D. W. Haering & Co., New York, N. Y.

The writer agrees with all the conclusions of the paper except conclusion 4.

Ralph W. Stewart,[5] M. Am. Soc. C. E.—In his opening sentence Professor Wilson states that the purpose of his paper "is to demonstrate the usefulness of the principle of superposition in the analysis of rigid frames by the slope-deflection and moment distribution methods." From an examination of the demonstrations the writer concludes that, for multistory frames with sloping columns and for the Vierendeel truss, the author has accomplished his purpose.

However, in his treatment of one-story and two-story frames with vertical columns, Professor Wilson introduces undesirable complexities and also so much unnecessary arithmetic that the writer does not consider this part of the paper useful. To uphold this conclusion the writer presents Fig. 11, which is the bent shown in Fig. 1, using the moment values in Eqs. 4. By equating the sums of the moments about joints B and C, respectively, to zero and then consolidating terms the author obtains Eqs. 5, containing three unknown quantities. To solve them he introduces Eqs. 7 and 8 from which $E R$ may be evaluated; and then Eqs. 5 can be solved for θ_B and θ_C.

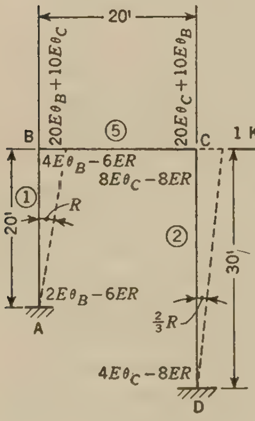
20'
$20E\theta_B+10E\theta_C$
$20E\theta_C+10E\theta_B$
B
⑤
C
1 Kip
$4E\theta_B-6ER$
$8E\theta_C-8ER$
①
R
②
20'
30'
$2E\theta_B-6ER$
A
$\frac{2}{3}R$
$4E\theta_C-8ER$
D

Fig. 11

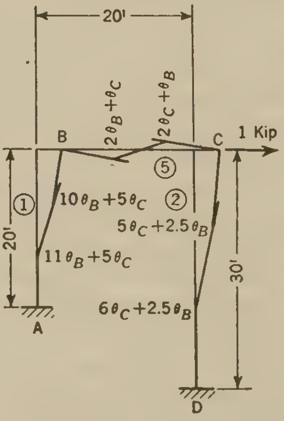
20'
$2\theta_B+\theta_C$
$2\theta_C+\theta_B$
B
C
1 Kip
⑤
①
$10\theta_B+5\theta_C$
②
$5\theta_C+2.5\theta_B$
20'
$11\theta_B+5\theta_C$
30'
$6\theta_C+2.5\theta_B$
A
D

Fig. 12

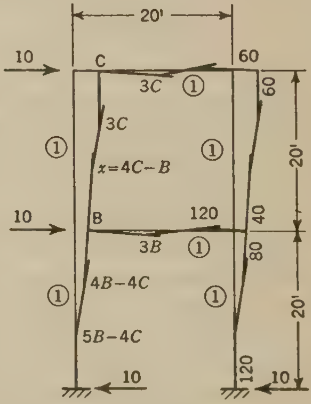
20'
10
C
60
3C
①
60
3C
①
①
x=4C−B
20'
10
B
120
40
3B
①
80
4B−4C
①
①
20'
5B−4C
10
120
10

Fig. 13

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[5] Bridge Engr., City of Los Angeles, Los Angeles, Calif.

Fig. 12 is a graph of the traverse of the elastic curves of Fig. 11. In Fig. 12 the geometrical values of the angles of flexure in terms of θ_B and θ_C are shown. For the beam each angle value is the same as the parenthetical term in a slope-deflection equation for this member. The properties of these angles have been published (17). They are located at the one-third points of the members.

Since the left column is only one fifth as stiff as the beam, its angle of flexure for the moment at B will be five times as great as the flexure angle in the beam, or $10\,\theta_B + 5\,\theta_C$. To obtain the flexure angle for M_A it is only necessary (from the geometry of the figure) to add θ_B to the flexure angle for the moment at B, giving $11\,\theta_B + 5\,\theta_C$. A similar procedure gives the flexure angles for the right column. The unknown R does not appear in these angle values. A property of an elastic curve traverse is that a flexure angle multiplied by the member stiffness equals the moment that the angle measures. By multiplying the flexure angles by the K-values a set of relative moment values for Fig. 12 is obtained. The geometry of the flexure of the columns gives the deflections at points B and C. Writing the equilibrium equation, in which the sum of the column shears equals 1 kip, and the geometrical equation, which expresses the fact that the deflections at B and C are equal, and consolidating terms:

$$1.383\,\theta_B + 1.233\,\theta_C = 1 \qquad \text{(33a)}$$

and

$$5.5\,\theta_B = 3.667\,\theta_C \qquad \text{(33b)}$$

These equations, with only two unknown quantities, replace Eqs. 5 with three unknown quantities.

Solving, $\theta_B = 0.3093$ and $\theta_C = 0.464$. The flexure angles can now be evaluated. Multiplying these angles by the K-values of the members gives a set of moments which check Professor Wilson's moments to slide-rule accuracy.

To solve the two-story symmetrical frame illustrated by Fig. 4, the author's procedure involved the following work: Steps 1 and 2, two end-moment distribution procedures for the frame; and, step 3, the development and solution of two simultaneous formulas (Eqs. 13). The two end-moment distribution operations were unnecessary as all of the work accomplished by these three stages of the author's solution can be accomplished by two simultaneous equations taken from the traverse diagram in Fig. 13.

To reduce the labor of typing and engraving, B and C have been substituted for θ_B and θ_C in Fig. 13 and the equations relating to this frame. The beam flexure angles $3\,B$ and $3\,C$ are shown in Fig. 13. Since the members are of equal stiffness, the uppermost column flexure angle is also $3\,C$. By angle summation for the upper column: $C + 3\,C - x - B = 0$, or $x = 4\,C - B$, as shown. Balancing moments about point B gives $4\,B - 4\,C$ as the upper flexure angle for the lower column. By angle summation $B + 4\,B - 4\,C$, or $5\,B - 4\,C$, is the bottom flexure angle as shown. Top-story shear balance gives: $7\,C - B = 100$; bottom-story shear balance gives: $9\,B - 8\,C = 200$; and solving: $B = 40$ and $C = 20$. The moments (which are the same as those in the paper) appear in the right half of Fig. 13. The labor involved in this solution is much less than that required by the author's multistage demonstration.

Even if the principle of superposition were mandatory for all the demonstration problems, Professor Wilson has, in general, performed more than double the necessary preliminary work by restricting his paper to slope deflection and end-moment distribution. For example, the work in Fig. 5(*a*) involved one hundred and twenty-eight arithmetical operations. With the traverse method, the results can be duplicated with thirty-seven arithmetical operations and two slide-rule settings from one of which seven readings are taken; and from the other setting, four readings.

The traverse solution also has some automatic checks which end-moment distribution does not. For example, in Fig. 5(*a*), M_{CE} should be exactly six times M_{EC}. The author's solution yields $6 \times 208 = 1{,}268$. The correct value of M_{EC} is 211.3. This difference in moments is not important but the fact that the traverse method offers checks not offered by end-moment distribution is important. Also the author's solution is applicable to only one position of the live load whereas the traverse solution can be used for any position of the live load.

For Fig. 4(*c*) all the arithmetical work needed to compute the moments by the traverse method is as follows: $2 + 3 = 5$; $4\,x = 6$; $x = 1\frac{1}{2}$; $3\,x = 4.5$; $3 + 10 - 4.5 = 8.5$; and $6 \times 1 = 6$. Finally:

$$\begin{array}{lll} & \text{Traverse results} & \text{Wilson results} \\ \dfrac{6\,E\,R}{8.5} \times & \left\{ \begin{array}{l} 4.5 = 3.176\,E\,R \\ 5.0 = 3.529\,E\,R \\ 3.0 = 2.121\,E\,R \\ 2 \quad = 1.412\,E\,R \\ 1 \quad = 0.706\,E\,R \end{array} \right. & \begin{array}{l} 3.17\,E\,R \\ 3.54\,E\,R \\ 2.12\,E\,R \\ 1.42\,E\,R \\ 0.72\,E\,R \end{array} \end{array}$$

The detail of Professor Wilson's end-moment distribution is not given, but it must certainly have involved several times as much work as the foregoing. The traverse solution of Fig. 4(*b*) is equally simple.

To summarize: The paper is of value to engineers who design framed structures. Its usefulness is considerably impaired, however, by unnecessarily complex solutions for some of the problems due to the exclusion of the elastic curve traverse from the scope of the work.

JOHN E. GOLDBERG,[6] ASSOC. M. AM. SOC. C. E.—A commendable résumé of the application of the principle of superposition to the analysis of certain types of rigid frame structures is presented by Professor Wilson. The paper is an excellent coordination of this principle with the particular types of structures described.

The method of superposition will give a reasonable analysis in many cases where the solution by more direct methods may be comparatively tedious. This is true because in direct methods of analysis a large number of simultaneous equations must be solved, whereas in the method of superposition a limited number of modes of deformation are first analyzed and subsequently used as parameters in the solution of the complete framework. Obviously, using the latter procedure there are a smaller number of equations to solve simultaneously. The method of superposition is particularly applicable to frames of what may be termed intermediate complexity.

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[6] Structures Engr., Consolidated Vultee Aircraft Corp., San Diego, Calif.

The writer has used similar methods for a number of years in conjunction with slope deflection and moment distribution for the analysis of several classes of structures—particularly Vierendeel trusses with cambered chords, A-frames, and simple or gabled bents—and also for the analysis of sidesway effects induced by unsymmetrical vertical loading or by transverse loading of building frames not more than a few stories in height. In such problems the method of superposition provides a simple and practicable method of analysis, particularly when used in conjunction with moment distribution or slope deflection (18)(19) and successive approximations or successive corrections. Certain of these problems have been analyzed by various writers in a number of texts and papers. The gabled bent, for example, has been "popular" in technical publications. The analysis of simple bents and bents with inclined legs likewise has been presented in several texts.

Some problems discussed by Professor Wilson may be solved at least as easily by direct methods without recourse to the method of superposition. Certain simple bents are generally in this class. A structure that may be analyzed easily by more direct methods is the Vierendeel truss with equal parallel chords or the structurally similar two-column bent with parallel equal columns. The attention of the writer was directed to this problem a number of years ago by the comparative impracticability of applying the usual popular methods to such structures when the outside members (that is, chords in the Vierendeel trusses or columns in the bents) are relatively stiff, say, five or more times as stiff as the interior members. In such cases moment or shear distribution is extremely slow in converging to the exact solution.

For a simple and practicable solution of this problem the writer derived the following exact formula from the basic slope-deflection equations:

$$\theta_n = \frac{1}{6\,K_{Gn} + K_{Cn} + K_{Co}} \left(\frac{M_n + M_o}{2} + K_{Cn}\,\theta_m + K_{Co}\,\theta_o \right) \ldots . (34)$$

in which K_{Gn} and K_{Cn} are the relative stiffnesses of the web members and of the chord members, respectively; and M_n is the product of the transverse external shear acting on the nth panel times the longitudinal dimension of the panel—that is, the distance between interior members. When the known K and M values are substituted, Eq. 34 reduces to an expression of the simple form,

$$\theta_n = A\,\theta_m + B\,\theta_o + C \ldots\ldots\ldots\ldots\ldots\ldots . (35)$$

A similar equation is set up for each panel point along either chord. The series of equations thus obtained converge very rapidly. For example, in cases where the chords are from eight to ten times as stiff as the interior members, three or four cycles of these extremely simple calculations will give an accuracy better than twenty cycles (approximately) by the usual moment distribution or shear distribution procedure. The web-member end moments are given by the formula,

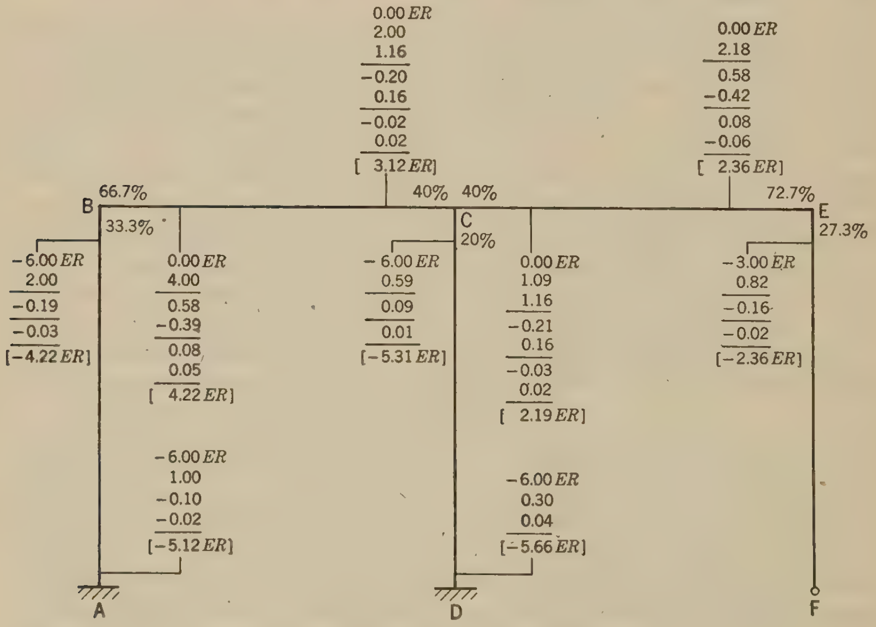
$$M_{Gn} = -3\,(K_{Gn}\,\theta_n) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (36a)$$

and the chord-member end moments by the formula,

$$M_{Cnm} = \frac{M_n}{4} + \left(\frac{\theta_m - \theta_n}{2}\right) K_{Cn} \ldots\ldots\ldots\ldots\ldots (36b)$$

Obviously, a number of problems not treated by Professor Wilson may also be analyzed by the method of superposition. The method of superposition is an extremely general and practicable method, and Professor Wilson is to be commended for his clear and concise outline.

A. W. Fischer,[7] Esq.—The analysis of rigid frames by superposition as presented by the author is certainly another valuable method for the analysis of rigid frames. The illustrative examples presented in the paper show that the method is general and can be used for complex frames. In 1938 (20) the writer found the method of superposition very practicable for calculating moments in structures several bays wide and several stories high.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40%
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C
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E
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−6.00 ER
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0.05
[4.22 ER]
−6.00 ER
0.59
0.09
0.01
[−5.31 ER]
0.00 ER
1.09
1.16
−0.21
0.16
−0.03
0.02
[2.19 ER]
−3.00 ER
0.82
−0.16
−0.02
[−2.36 ER]
−6.00 ER
1.00
−0.10
−0.02
[−5.12 ER]
−6.00 ER
0.30
0.04
[−5.66 ER]
A
D
F

Fig. 14.—Unsymmetrical Three-Column Bent

Example 2. Unsymmetrical Three-Column Bent.—The method of moment distribution as presented by Professor Wilson is general. In some cases considerable time can be saved by taking advantage of permissible short cuts. For example, the moment distribution in Fig. 3(*b*) can be shortened considerably by first balancing a joint and then carrying over one half of the balancing moment to the adjacent joints, before the adjacent joints are balanced. This method is correct theoretically, and the writer uses it in all his moment distribution analyses. Fig. 14 shows this method of balancing moments beginning at joint B. After carrying over one half of the balancing moment to the adjacent joints, the next step is to balance joint E. Finally, carry over one half of the balancing moment to joints adjacent to joint E and then balance joint C.

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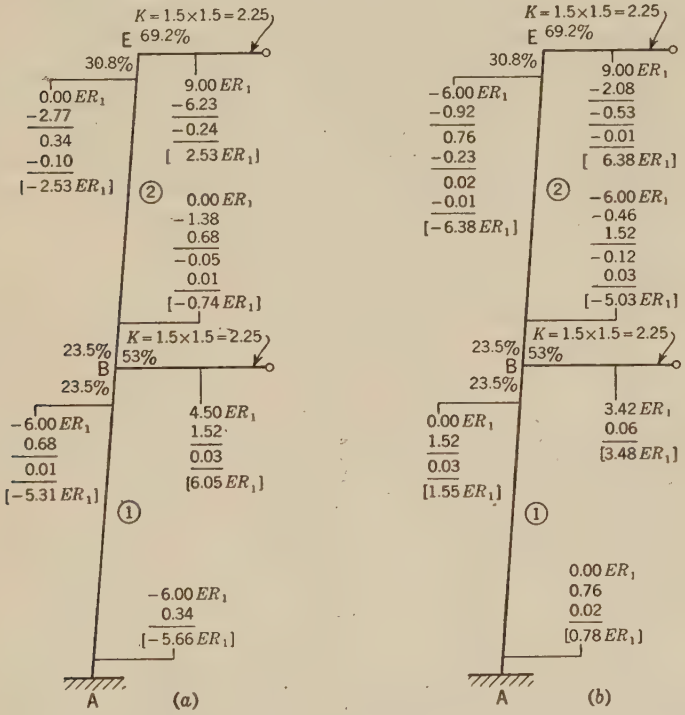
[7] Engr. (Naval Architect), Navy Dept., Washington, D. C.



K=1.5×1.5=2.25
E 69.2%
30.8%
0.00 ER₁
-2.77
0.34
-0.10
[-2.53 ER₁]
9.00 ER₁
-6.23
-0.24
[2.53 ER₁]
②
0.00 ER₁
-1.38
0.68
-0.05
0.01
[-0.74 ER₁]
K=1.5×1.5=2.25
23.5%
53%
B
23.5%
-6.00 ER₁
0.68
0.01
[-5.31 ER₁]
4.50 ER₁
1.52
0.03
[6.05 ER₁]
①
-6.00 ER₁
0.34
[-5.66 ER₁]
A
(a)
K=1.5×1.5=2.25
E 69.2%
30.8%
-6.00 ER₁
-0.92
0.76
-0.23
0.02
-0.01
[-6.38 ER₁]
9.00 ER₁
-2.08
-0.53
-0.01
[6.38 ER₁]
②
-6.00 ER₁
-0.46
1.52
-0.12
0.03
[-5.03 ER₁]
K=1.5×1.5=2.25
23.5%
53%
B
23.5%
0.00 ER₁
1.52
0.03
[1.55 ER₁]
3.42 ER₁
0.06
[3.48 ER₁]
①
0.00 ER₁
0.76
0.02
[0.78 ER₁]
A
(b)

FIG. 15.—TWO-STORY BENT WITH HORIZONTAL LOADS

On comparison with the formal procedure of the paper the amount of labor is found to be reduced considerably.

Example 7. Two-Story Bent with Horizontal Loads.—For a symmetrical structure, such as Fig. 9, the K-factors for the horizontal members can be modified slightly by assuming that there is a point of inflection, or hinge, at the center, which makes the K-factor equal to $\frac{I}{0.5\,L} \times \frac{3}{4} = 1.5\,\frac{I}{L}$. Thus, the K-factors for members BC and EF are equal to $1.5 \times 1.5 = 2.25$, and no balancing moment need be carried over to the adjacent joints for these members. Fig. 15(*a*) gives the moment distribution for the final moments shown in

Fig. 9(*b*), and Fig. 15(*b*) shows the moment distribution for the final moments shown in Fig. 9(*c*). With this slight modification, considerable labor can be saved and the method is theoretically correct.

Example 6. Trapezoidal Bent.—Fig. 16(*a*) represents a single-story trapezoidal bent similar to Fig. 8, but with a bottom member and a horizontal load of 1,000 lb applied at joint A (compare analysis of this bent by A. Amirikian, M. Am. Soc. C. E. (1*a*)). For this example, $E = 1$, $R_2 = R_1$, and $R_3 = -R_1$. Since K for member AD is equal to 2 (Fig. 16), the fixed-end moment $M_{AD} = M_{DA} = -2 \times 6\,R_1 = -12\,R_1$. The fixed-end moment $M_{AB} = M_{BA} = 2 \times 6\,R_1 = 12\,R_1$. The fixed-end moment $M_{BC} = M_{CB} = -3 \times 6\,R_1 = -18\,R_1$ (Fig. 16(*b*)). The first joint balanced is joint C and one half of the balanced moments are carried to the adjacent joints. Next balance joint D and carry one half of the balanced moments to its adjacent joints, balance joint B and carry one half of the balanced moments to its adjacent joints, and balance joint A, carrying one half of the balanced moments to its adjacent joints. Repeat the process in the order of joints C, D, B, and A. The final moments are shown in brackets in Fig. 16(*b*), thus accounting for steps 1, 2, and 3.

By step 4 in the author's procedure (see Fig. 16(*c*)):

$$H_1 + H_2 = 1{,}000 \quad \dots\dots\dots\dots (37)$$

also, in Fig. 16(*d*),

$$V_1 = 4.085\,R_1 \quad \dots\dots\dots\dots (38a)$$

and

$$V_2 = 4.085\,R_1 \quad \dots\dots\dots\dots (38b)$$

The values of V_1 and V_2 in Eqs. 38 are shown in Figs. 16(*e*) and 16(*f*), respectively. In Fig. 16(*e*),

$$H_1 = \frac{29.51\,R_1}{12} \quad \dots\dots\dots\dots (39a)$$

and, in Fig. 16(*f*),

$$H_2 = \frac{40.11\,R_1}{12} \quad \dots\dots\dots\dots (39b)$$

Substituting Eqs. 39 in Eq. 37 and solving, $R_1 = \dfrac{12{,}000}{69.62}$.

In step 5, substituting the value of R_1 from step 4 in Fig. 16(*b*) gives the following final moments in foot-pounds: $M_{AB} = 2{,}006 = -M_{AD}$; $M_{BA} = 2{,}218 = -M_{BC}$; $M_{DA} = -1{,}672 = -M_{DC}$; and $M_{CB} = -1{,}879 = -M_{CD}$.

Example 5A. Vierendeel Truss with Inclined Chords.—This problem, of course, differs somewhat from Example 5 of the paper (compare analysis by Mr. Amirikian (1*b*)). Fig. 17(*a*) represents a five-bay Vierendeel truss with a 200-lb load at each lower panel point. The value of E is considered as unity throughout, and the values of $I/L = K$ are as shown. Figs. 17(*b*) and 17(*c*) show the left portion of the truss in the distorted condition with the R-values expressed in terms of R_1 and R_2 using Eq. 21. For this particular symmetrical truss assume a hinge in the center of the members AA′, BB′, and CC′ and multiply each K-value by 1.5 so that, with this slight modification, no balancing moment is required to be carried over to the adjacent joint for these members.

The fixed-end moments 6 K R_1 for the various members are shown in Fig. 18(*a*). For the moment distribution method the sequence is: Joints A, C,

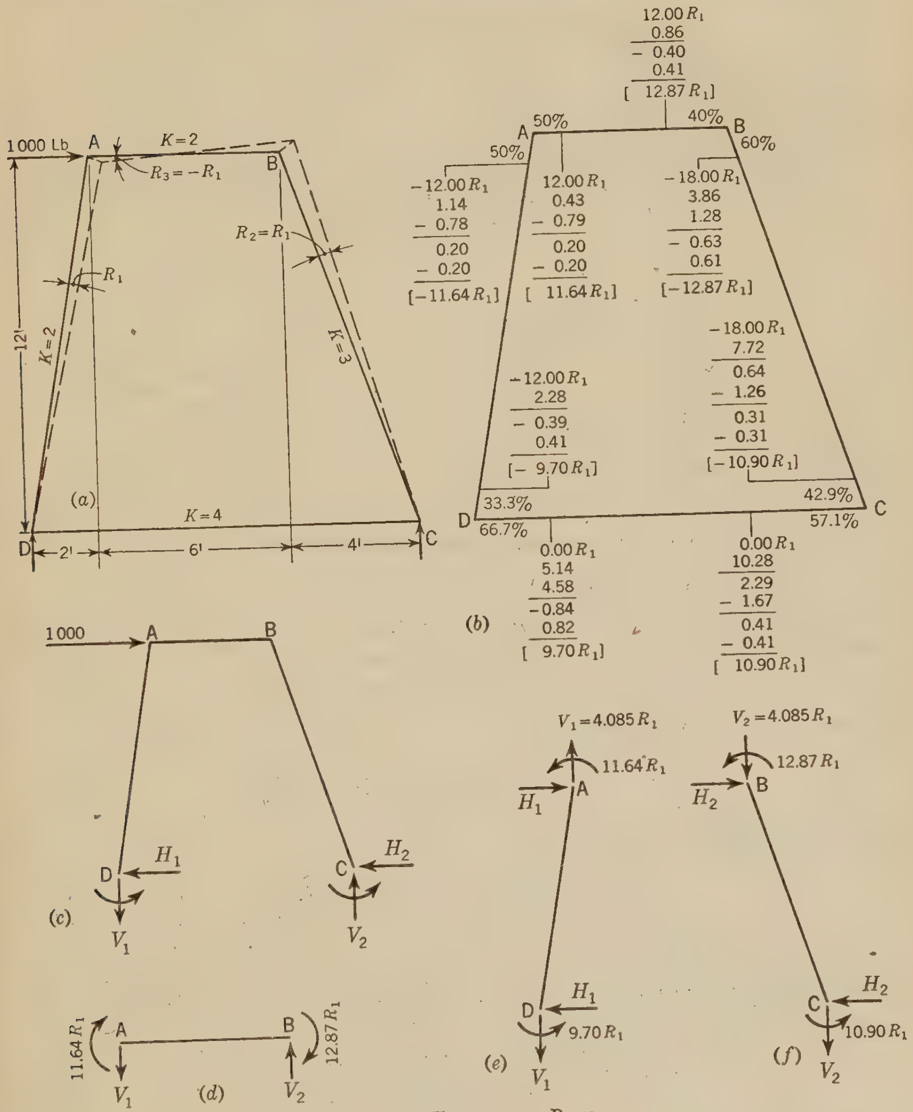

1000 Lb
A
K=2
B
R3=−R1
R2=R1
R1
12'
K=2
K=3
(a)
K=4
D
C
2'
6'
4'
12.00R1
0.86
− 0.40
0.41
[12.87R1]
A
50%
50%
40%
B
60%
−12.00R1
1.14
− 0.78
0.20
− 0.20
[−11.64R1]
12.00R1
0.43
− 0.79
0.20
− 0.20
[11.64R1]
−18.00R1
3.86
1.28
− 0.63
0.61
[−12.87R1]
−18.00R1
7.72
0.64
− 1.26
0.31
− 0.31
[−10.90R1]
−12.00R1
2.28
− 0.39
0.41
[− 9.70R1]
33.3%
66.7%
D
42.9%
57.1%
C
0.00R1
5.14
4.58
−0.84
0.82
[9.70R1]
0.00R1
10.28
2.29
− 1.67
0.41
− 0.41
[10.90R1]
(b)
1000
A
B
H1
H2
D
C
(c)
V1
V2
V1=4.085R1
V2=4.085R1
11.64R1
12.87R1
H1
A
H2
B
H1
D
9.70R1
(e)
V1
H2
C
10.90R1
(f)
V2
11.64R1
A
B
12.87R1
V1
(d)
V2

Fig. 16.—Trapezoidal Bent

and B. For member CD the carry-over balanced moment from joint D to joint C is one half of the balance moment at joint C with the sign changed, because then it is necessary only to apply the moment distribution method to the left part of the truss when the load and truss are symmetrical. The final moments in terms of R_1 are shown in brackets.

Fig. 18(*b*) gives the fixed-end moments in terms of R_2 and the sequence of balancing is: Joints B, A, and C, thus accounting for steps 1, 2, and 3.

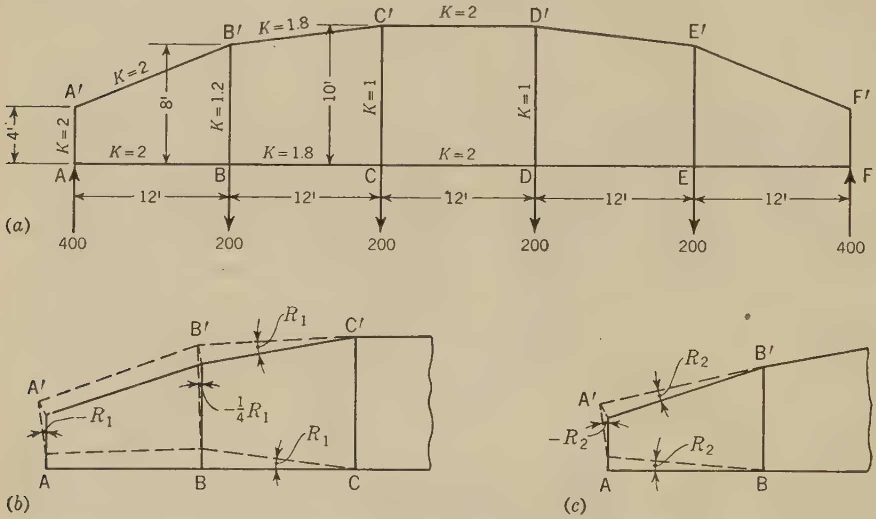

A′
B′
C′
D′
E′
F′
K=2
K=1.8
K=2
K=2
K=1.2
K=1
K=1
K=2
K=1.8
K=2
4′
8′
10′
A
B
C
D
E
F
12′
12′
12′
12′
12′
(a)
400
200
200
200
200
400
B′
C′
A′
R_1
$-R_1$
$-\frac{1}{4}R_1$
R_1
A
B
C
(b)
R_2
B′
A′
$-R_2$
R_2
A
B
(c)

FIG. 17.—VIERENDEEL TRUSS WITH INCLINED CHORDS

In Fig. 18(*c*), for step 4:

$$H_1 + H_2 = 200 \qquad (40a)$$

and, in Fig. 18(*d*),

$$H_3 + H_4 = 400 \qquad (40b)$$

In Fig. 18(*c*), with $\Sigma M = 0$,

$$V_1 = \frac{-10.72\,R_1 + 2.40\,R_2 + 7{,}200}{10} \qquad (41a)$$

and, in Fig. 18(*d*), from summation of moments = 0,

$$V_2 = \frac{1.54\,R_1 - 15.72\,R_2 + 4{,}800}{8} \qquad (41b)$$

The values of V_1 and V_2 in Eqs. 41 are shown acting in Figs. 18(*e*), 18(*f*), 18(*g*), and 18(*h*). In Figs. 18(*e*) and 18(*f*), respectively, with $\Sigma M = 0$, and Figs. 18(*g*) and 18(*h*),

$$H_2 = \frac{44.68\,R_1 - 22.25\,R_2 + 7{,}200}{60} \qquad (42a)$$

$$H_1 = \frac{11.08\,R_1 - 4.93\,R_2}{12} \qquad (42b)$$

$$H_4 = \frac{7.50\,R_1 + 21.26\,R_2 + 4{,}800}{24} \qquad (42c)$$

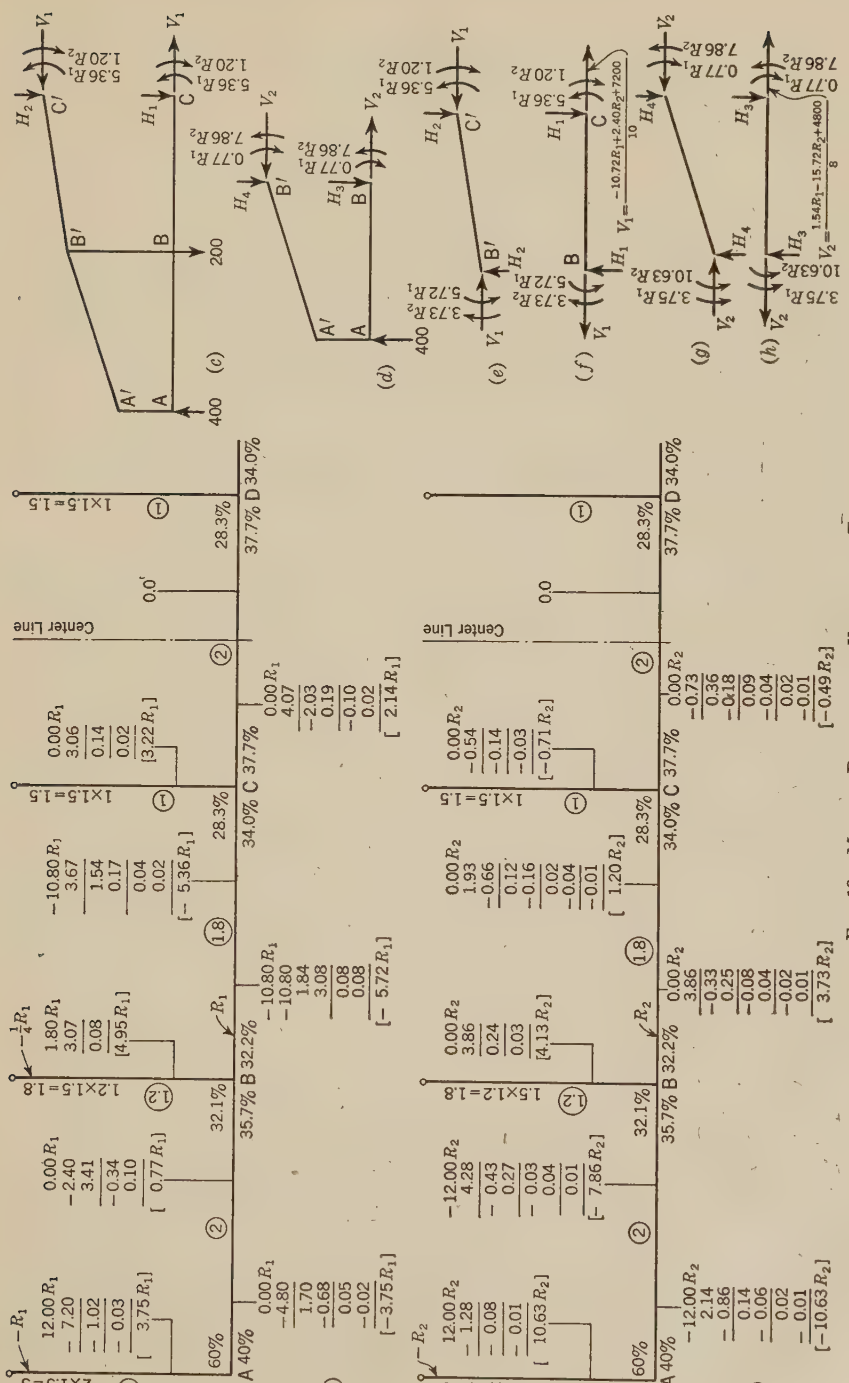

(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
2×1.5=3
1.2×1.5=1.8
1.5×1.2=1.8
1×1.5=1.5
Center Line
0.0
60%
A 40%
32.1%
35.7% B 32.2%
28.3%
34.0% C 37.7%
37.7% D 34.0%
12.00 R₁
−7.20
−1.02
−0.03
[3.75 R₁]
0.00 R₁
−2.40
3.41
−0.34
0.10
[0.77 R₁]
1.80 R₁
3.07
0.08
[4.95 R₁]
−10.80 R₁
3.67
1.54
0.17
0.04
0.02
[−5.36 R₁]
0.00 R₁
3.06
0.14
0.02
[3.22 R₁]
0.00 R₁
−4.80
1.70
−0.68
0.05
−0.02
[−3.75 R₁]
−10.80 R₁
−10.80
1.84
3.08
0.08
0.08
[−5.72 R₁]
0.00 R₁
4.07
−2.03
0.19
−0.10
0.02
[2.14 R₁]
12.00 R₂
−1.28
−0.08
−0.01
[10.63 R₂]
−12.00 R₂
4.28
−0.43
0.27
−0.03
0.04
0.01
[−7.86 R₂]
0.00 R₂
3.86
0.24
0.03
[4.13 R₂]
0.00 R₂
1.93
−0.66
0.12
−0.16
0.02
−0.04
−0.01
[1.20 R₂]
0.00 R₂
−0.54
−0.14
−0.03
[−0.71 R₂]
−12.00 R₂
2.14
−0.86
0.14
−0.06
0.02
−0.01
[−10.63 R₂]
0.00 R₂
3.86
−0.33
0.25
−0.08
0.04
−0.02
0.01
[3.73 R₂]
0.00 R₂
−0.73
0.36
−0.18
0.09
−0.04
0.02
−0.01
[−0.49 R₂]
R₁
−¼R₁
R₂
400
200
A A′ B B′ C C′
H₁ H₂ H₃ H₄
V₁ V₂
5.36 R₁
1.20 R₂
0.77 R₁
7.86 R₂
5.72 R₁
3.73 R₂
3.75 R₁
10.63 R₂
$V_1 = \frac{-10.72R_1 + 2.40R_2 + 7200}{10}$
$V_2 = \frac{1.54R_1 - 15.72R_2 + 4800}{8}$

FIG. 18.—MOMENT DISTRIBUTION, VIERENDEEL TRUSS

and

$$H_3 = \frac{2.98\,R_1 + 18.49\,R_2}{12} \qquad \text{(42d)}$$

Substituting Eqs. 42 in Eqs. 40,

$$100.08\,R_1 - 46.90\,R_2 = 4{,}800 \qquad \text{(43a)}$$

and

$$13.46\,R_1 + 58.24\,R_2 = 4{,}800 \qquad \text{(43b)}$$

Solving Eqs. 43 by determinants, $R_1 = 78.12$ and $R_2 = 64.36$.

The numerical values of the moments in Figs. 18(*a*) and 18(*b*) may now be calculated (step 5), and the following moments in foot-pounds at the ends of all members are determined by summation: $M_{AB} = M_{A'B'} = -977$; $M_{BA} = M_{B'A'} = -446$; $M_{BC} = M_{B'C'} = -207$; $M_{CB} = M_{C'B'} = -342$; $M_{CD} = M_{C'D'} = 136$.

The foregoing is intended to lend emphasis to the writer's sincere opinion that the method of superposition as presented by Professor Wilson is an unusually valuable tool for the solution of rigid frames.

LEON BLOG,[8] ESQ.—The importance of using the method of superposition when analyzing rigid frames is emphasized in this paper; but the paper has additional value. In the several examples, the author has explained various simplifications and clarifications previously unknown to the writer. Although superposition was not required for solving Example 1, there is considerable merit in the technique of its solution. By separating the R-term in Eqs. 1 and by writing it separately as in Eqs. 2, and then by expressing the R-term of other members in terms of that of one member, a considerable simplification in the statement of the values for the end moments occurs. Through stating the values of the angles of twist, θ, in terms of $E\,R$ as in Eqs. 6, it becomes unnecessary to evaluate θ and R quantitatively. Values of θ and R in radians are generally of a low order and lead to errors in the final moment values unless carried throughout the solution with many significant figures. Another simplification is that the value E occurs in both sides of Eqs. 6, so that, by the author's method, it need not be considered separately. Only the product $E\,R$ must be evaluated and substituted in the values of the end moments which are expressed solely in terms of $E\,R$. Use of the value of $E\,R$ leads to a minimum of error in the moment values sought. The technique just described is better than any known to the writer for the solution of rigid frames by either the slope deflection or the moment distribution methods.

Superposition is a method of solving, by steps, certain problems to which it is applicable. When solving problems dealing with rigid frames involving asymmetry of any kind, one-step solutions by moment distribution are not possible. Two steps are required: (*a*) Moment distribution to evaluate moments due to pure rotation of the joints; and (*b*) moment distribution to obtain moments due to movements occurring transversely to the axis of each member that can be deflected. The moments evaluated in each step are added algebraically at each joint to derive the required moment. Step 1 may be solved by any good system of moment distribution. Step 2 is often solved by the unbalanced shear method (2). Hardy Cross, M. Am. Soc. C. E., does not call step 2 anything, but unbalanced shears are actually the tools required when correcting for sidesway by his method.

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[8] Structural Engr., City of Los Angeles, Los Angeles, Calif.

Professor Wilson does not use the unbalanced shear method in step 2. In solving Example 2, Fig. 3(*b*), he distributes fixed-end moments which are expressed in terms of $E\,R$. Then he evaluates $E\,R$ and uses it to evaluate the distributed moments. The moments thus found are added algebraically to those in Fig. 3(*a*) to get the working moments. This consummates a very direct solution of Example 2.

Example 3 is an extension of the technique exhibited by Example 2. In analyzing two stories, it is necessary to use a set of R-values: R_1 appertains to the lower story and R_2 to the upper tier. After that variation from Example 2, the technique of solution parallels that for Example 2.

For the solution of Example 4, the author evaluates moments in terms of $E\,\Delta$. Here, inspection will not yield R-values for the vertical members in terms of the R-value of a selected vertical member as illustrated in Example 1. In this example it was evident that the R-values for vertical members joined by the same horizontal member are inversely proportional to the length of the vertical members. In Fig. 5(*a*), the joint at D between points C and E precludes evaluation of deflection angles by inspection. Otherwise the method of solution closely follows that for Examples 2 and 3.

For solving the Vierendeel truss of Example 5, Professor Wilson is able to profit by expressing moments in terms of $E\,R$. Otherwise, the solution is self-explanatory.

Equally valuable is the exposition of the relation between the values of R for the members of a quadrangle. The knowledge of this relation permits a very direct attack upon the type of problem considered in the ensuing comment. The meaning of R_3 in Fig. 7 may be made somewhat clearer by drawing a horizontal line through B' toward the right to meet a perpendicular drawn downward from C'. Then

$$R_3 = \frac{v_C + v_B}{l_3} \quad \text{.........................(44)}$$

For the solution of the frame of Example 6, the R-relation in Eqs. 20 and 21 was used. Fig. 8 is not a quadrangle, but the fixity of the legs AB and CD is analogous to having a member such as AD of Fig. 7. Therefore, Eqs. 20 and 21 are applicable to the solution of Example 6. The distributed moments are determined by using $E\,R_1$ in a manner closely resembling the solution of Example 2.

Example 7, Fig. 9, is solved easily by using the quadrangle concept. The lower story has legs which are fixed, and so a fictitious member AD may be assumed. The second story is a full quadrangle. Otherwise the solution closely follows that of Example 3.

In Fig. 10(*b*) of Example 8, the author uses the quadrangle. The dashed line EB corresponds to AD of the basic quadrangle. Moments are all expressed in terms of $E R_1$ and $E R_2$, and thereafter the equations of statics dominate the solution.

Finally, the writer believes that Professor Wilson has made a distinct contribution to the analysis of rigid frames of homogeneous material. Should any of the members be variable in shape, the resultant variation of end moment of inertias would preclude the simplification which results in Eqs. 4 by using the concept of relative *K*-values. However, each Eq. 4 would merely contain the *K*-value for that member and would lead to some additional numerical work. If the frame is not homogeneous as to material, modular ratios such as those for reinforced concrete may be used to establish all the equations in terms of the *E* of a selected member. This also would involve some extra numerical work, but the extra work would be common to other known methods.

ARTHUR B. MCGEE,[9] ESQ.—The principle of superposition is of great value for the analysis of rigid frames. The examples in the paper illustrate the method of procedure in a workable manner; but, since Examples 1, 2, 3, 4, 6, and 8 can be solved by shorter and easier methods, they do not demonstrate the value of the method. To do the method justice and to show its great value in solving difficult problems the writer wishes to show its application to a two-bay, saw-toothed structure with tapering members.

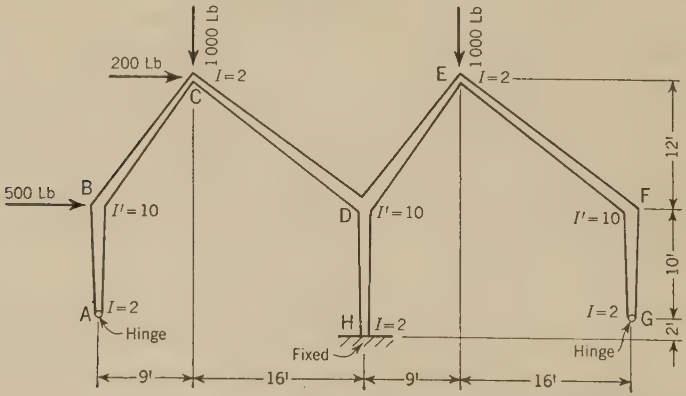

FIG. 19.—TWO-BAY, SAW-TOOTHED STRUCTURE WITH TAPERING MEMBERS

Fig. 19 shows such a frame loaded unsymmetrically with vertical and horizontal loads. The tapering members have been given the same moments of inertia at the ends so that one set of beam constants will suffice for all members. These constants are taken from the tables compiled by Walter Ruppel, Assoc. M. Am. Soc. C. E. (21). The application of these constants to the several members is given in Fig. 20 (17).

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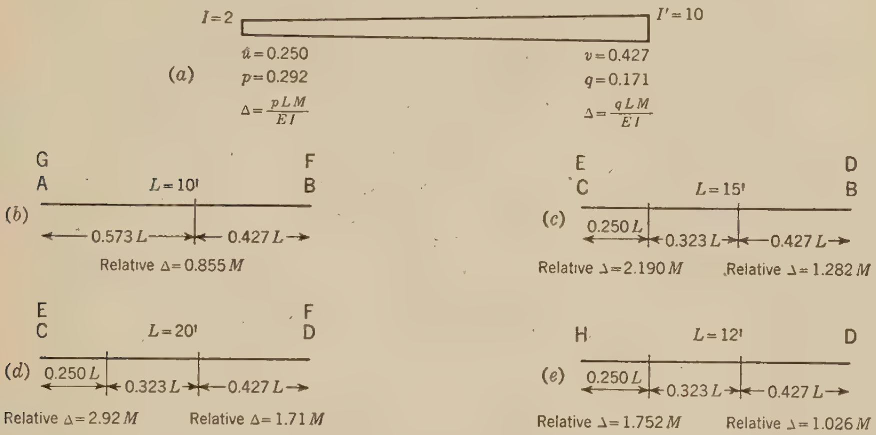
[9] Structural Design Engr., Bureau of Eng., City of Los Angeles, Los Angeles, Calif.

The notation used in this discussion varies slightly from the author's but should cause no confusion. The symbol d is used herein for the linear displacement of one end of a member, and Δ represents an angle equal to an $\frac{M}{EI}$-area. Also the writer considers an end moment as the moment in a member rather than a resisting moment acting on the member. This, of course, reverses the signs of the end moments from those used by Professor Wilson.



I=2
I'=10
(a)
u=0.250
p=0.292
Δ=pLM/EI
v=0.427
q=0.171
Δ=qLM/EI
G
A
F
B
L=10'
(b)
0.573 L
0.427 L
Relative Δ=0.855 M
E
C
D
B
L=15'
(c)
0.250 L
0.323 L
0.427 L
Relative Δ=2.190 M
Relative Δ=1.282 M
E
C
F
D
L=20'
(d)
0.250 L
0.323 L
0.427 L
Relative Δ=2.92 M
Relative Δ=1.71 M
H
D
L=12'
(e)
0.250 L
0.323 L
0.427 L
Relative Δ=1.752 M
Relative Δ=1.026 M

FIG. 20.—BEAM COEFFICIENTS

Table 1 gives the effect of applying a positive (clockwise) moment to each joint of the frame, without sidesway. The table may be computed by moment distribution. The writer, however, prefers the traverse method (22). For a moment at joint B the traverse is shown in Fig. 21. The moments on the diagram are multiplied by $\frac{1,000}{903.97}$ to obtain the results for joint B in Table 1. A corresponding traverse must be run for joint F. The intermediate joints, C, D, and E are easily calculated by using data taken from the two traverses already run. Table 1 will apply to any combination of fixed-end moments that may act on the frame.

TABLE 1.—EFFECTS OF MOMENTS APPLIED TO JOINTS
(Not Corrected for Sidesway)

+1,000 ft-lb moment applied at joint:	MEMBER											
	BA	BC	CB	CD	DC	DH	DE	ED	EF	FE	FG	HD
B	−569	−431	− 69	+ 69	+ 41	− 24	− 17	− 3	+ 3	+ 1	− 1	− 8
C	+249	−249	−558	−442	−260	+155	+105	+ 17	− 17	− 8	+ 8	+ 52
D	− 22	+ 22	+ 49	− 49	−240	−452	−308	− 49	+ 49	+ 24	− 24	−151
E	+ 10	− 10	− 22	+ 22	+107	+201	−308	−575	−425	−211	+211	+ 67
F	− 1	+ 1	+ 3	− 3	− 14	− 27	+ 41	+ 77	− 77	−371	−629	− 9

When the frame is permitted to deflect as indicated in Fig. 22(*a*), the relative displacements of the members AB, BC, and CD are shown in Table 2. In the triangle abc, side ab is drawn normal to member AB, side bc normal to member BC, and, likewise, side ca normal to member CD. These displacements are d, $-0.8\,d$, and $0.6\,d$, respectively. The equations for fixed-end moments, corresponding to $M'_{AB} = M'_{BA} = 6\,E\,K\,\frac{d}{L}$ for uniform moment of inertia members, are as follows:

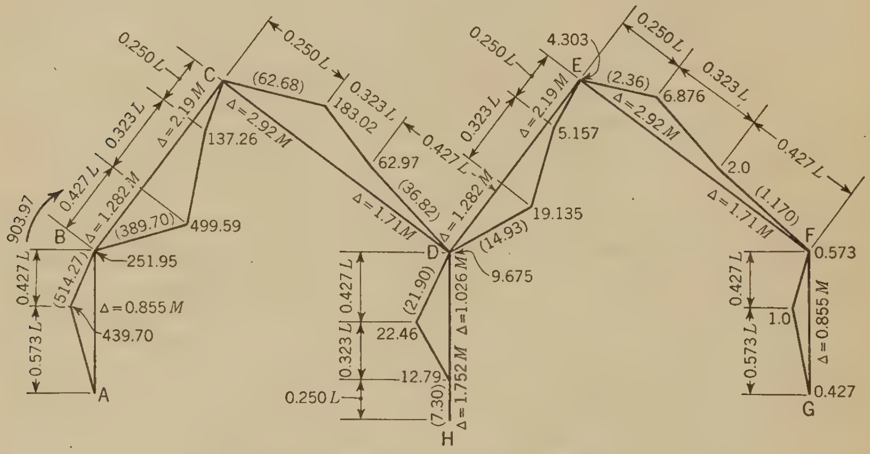

0.250 L
C
0.323 L
Δ=2.19 M
0.427 L
Δ=1.282 M
903.97
B
(62.68)
0.250 L
0.323 L
183.02
Δ=2.92 M
137.26
(389.70)
499.59
251.95
(514.27)
Δ=0.855 M
439.70
0.427 L
0.573 L
A
62.97
0.427 L
(36.82)
Δ=1.71 M
0.250 L
0.323 L
Δ=2.19 M
4.303
E
5.157
Δ=1.282 M
D
19.135
(14.93)
9.675
0.427 L
0.323 L
0.250 L
(21.90)
22.46
12.79
(7.30)
Δ=1.026 M
Δ=1.752 M
H
0.250 L
(2.36)
6.876
0.323 L
Δ=2.92 M
2.0
0.427 L
(1.170)
Δ=1.71 M
F
0.573
0.427 L
1.0
0.573 L
Δ=0.855 M
0.427
G

FIG. 21.—SOLUTION BY TRAVERSE METHOD

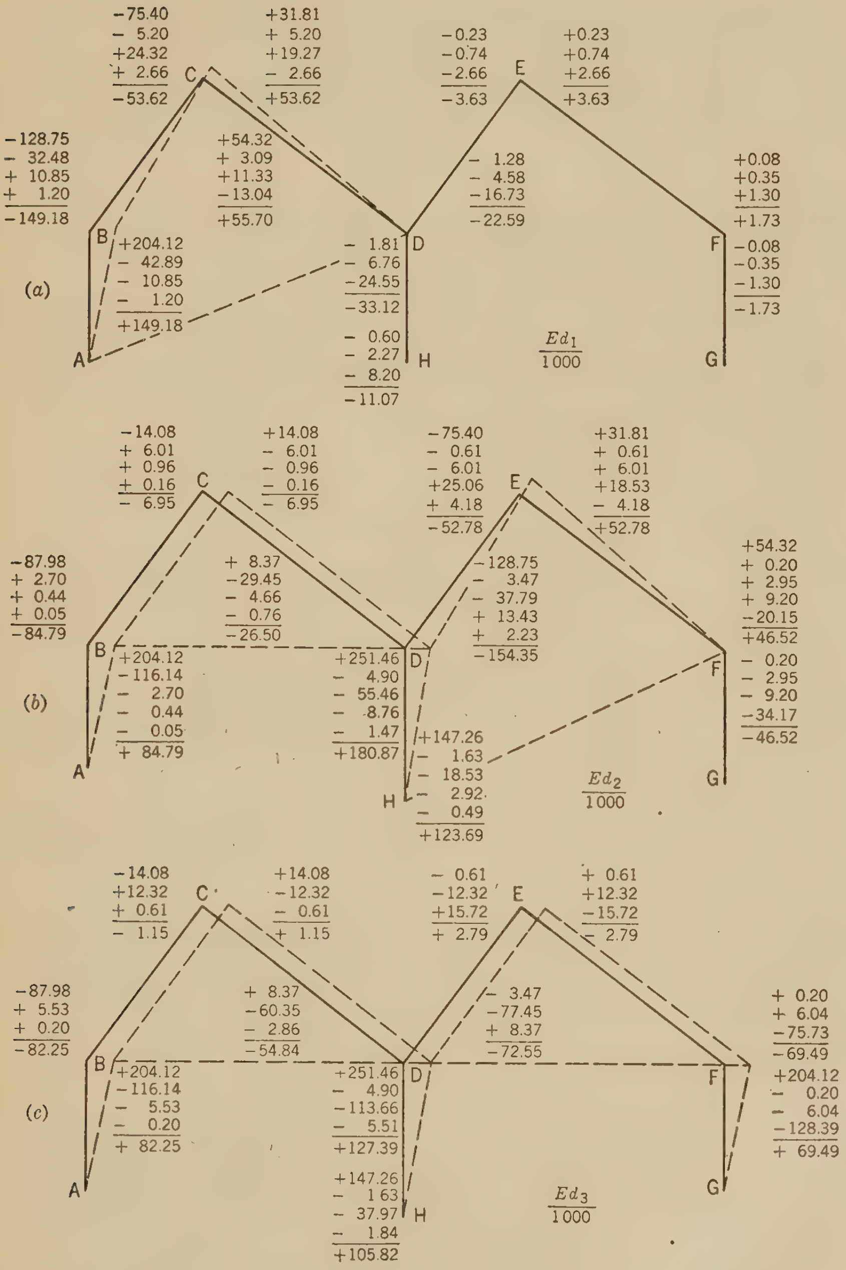
$$M_L = \frac{E\,I\,d}{(1 - u - v)\,p\,L^2} \quad \text{(45a)}$$

and

$$M_R = \frac{E\,I\,d}{(1 - u - v)\,q\,L^2} \quad \text{(45b)}$$

Table 2 shows the fixed-end moments for members AB, BC, CD, and DH, Fig. 19. Member DE is like member BC, member EF is like member CD, and member FG is like member BA.

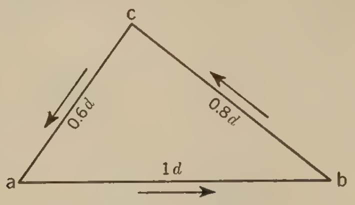
There are three conditions of deformation for this frame. The first condition is shown in Fig. 22(*a*). The coefficients of $E\,d$, as calculated in Table 2, have been multiplied by 1,000 arbitrarily to place the decimal point in a more convenient location. The fixed-end moment coefficients of $\frac{E\,d_1}{1{,}000}$ for members AB, BC, and CD are placed on the diagram in their respective locations and the moments are balanced by the use of Table 1. Thus, at joint B, $204.12 - 128.75 = 75.37$, a clockwise moment which must be balanced throughout the frame. Therefore, $\frac{75.37}{1{,}000}$ times the values in Table 1 for joint B will be added algebraically to the respective end moments of the members. Likewise, at joint C, $-75.40 + 31.81 = -43.59$, the unbalanced moment, and

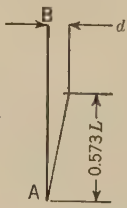
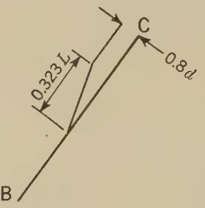
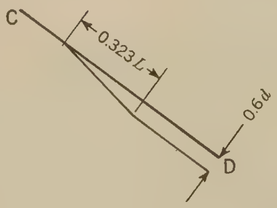
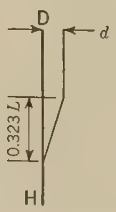


-75.40
- 5.20
+24.32
+ 2.66
-53.62
C
+31.81
+ 5.20
+19.27
- 2.66
+53.62
-0.23
-0.74
-2.66
-3.63
E
+0.23
+0.74
+2.66
+3.63
-128.75
- 32.48
+ 10.85
+ 1.20
-149.18
+54.32
+ 3.09
+11.33
-13.04
+55.70
- 1.28
- 4.58
-16.73
-22.59
+0.08
+0.35
+1.30
+1.73
B
+204.12
- 42.89
- 10.85
- 1.20
+149.18
- 1.81
- 6.76
-24.55
-33.12
D
F
-0.08
-0.35
-1.30
-1.73
(a)
- 0.60
- 2.27
- 8.20
-11.07
H
$\frac{Ed_1}{1000}$
A
G
-14.08
+ 6.01
+ 0.96
+ 0.16
- 6.95
C
+14.08
- 6.01
- 0.96
- 0.16
- 6.95
-75.40
- 0.61
- 6.01
+25.06
+ 4.18
-52.78
E
+31.81
+ 0.61
+ 6.01
+18.53
- 4.18
+52.78
+54.32
+ 0.20
+ 2.95
+ 9.20
-20.15
+46.52
-87.98
+ 2.70
+ 0.44
+ 0.05
-84.79
+ 8.37
-29.45
- 4.66
- 0.76
-26.50
-128.75
- 3.47
- 37.79
+ 13.43
+ 2.23
-154.35
B
+204.12
-116.14
- 2.70
- 0.44
- 0.05
+ 84.79
+251.46
- 4.90
- 55.46
- 8.76
- 1.47
+180.87
D
F
- 0.20
- 2.95
- 9.20
-34.17
-46.52
(b)
+147.26
- 1.63
- 18.53
- 2.92
- 0.49
+123.69
H
$\frac{Ed_2}{1000}$
A
G
-14.08
+12.32
+ 0.61
- 1.15
C
+14.08
-12.32
- 0.61
+ 1.15
- 0.61
-12.32
+15.72
+ 2.79
E
+ 0.61
+12.32
-15.72
- 2.79
-87.98
+ 5.53
+ 0.20
-82.25
+ 8.37
-60.35
- 2.86
-54.84
- 3.47
-77.45
+ 8.37
-72.55
+ 0.20
+ 6.04
-75.73
-69.49
B
+204.12
-116.14
- 5.53
- 0.20
+ 82.25
+251.46
- 4.90
-113.66
- 5.51
+127.39
D
F
+204.12
- 0.20
- 6.04
-128.39
+ 69.49
(c)
+147.26
- 1.63
- 37.97
- 1.84
+105.82
H
$\frac{Ed_3}{1000}$
A
G

FIG. 22

TABLE 2.—RELATIVE DISPLACEMENTS AND MOMENTS, MEMBERS AB, BC, CD, AND DH, FIG. 19



Member	Displacements	Moments
AB		$M_{BA} = \frac{d \times E \times 2}{0.573 \times 0.171 \times 100} = 0.20412\, E\, d$
BC		$M_{CB} = \frac{-0.8\, d \times E \times 2}{0.323 \times 0.292 \times 225} = -0.07540\, E\, d$ $M_{BC} = \frac{-0.8\, d \times E \times 2}{0.323 \times 0.171 \times 225} = -0.12875\, E\, d$
CD		$M_{CD} = \frac{0.6\, d \times E \times 2}{0.323 \times 0.292 \times 400} = 0.03181\, E\, d$ $M_{DC} = \frac{0.6\, d \times E \times 2}{0.323 \times 0.171 \times 400} = 0.05432\, E\, d$
DH		$M_{DH} = \frac{d \times E \times 2}{0.323 \times 0.171 \times 144} = 0.25146\, E\, d$ $M_{HD} = \frac{d \times E \times 2}{0.323 \times 0.292 \times 144} = 0.14726\, E\, d$

$\frac{-43.59}{1,000}$ times the moments shown for a +1,000 ft-lb moment at joint C in Table 1 will spread this unbalanced moment throughout the frame. The same procedure is followed for the unbalanced +54.32 moment at joint D. The end moments for each member are now added and the first condition of deformation is finished.

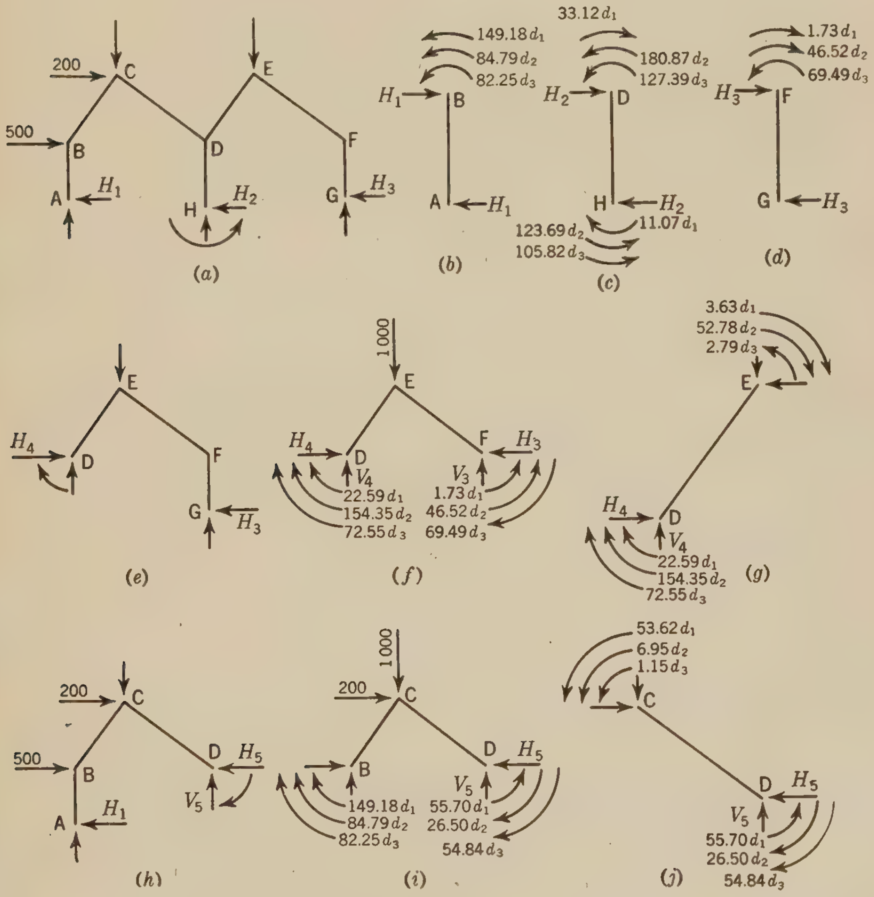

200
500
C
E
B
D
F
A
H_1
H
H_2
G
H_3
(a)
149.18d_1
84.79d_2
82.25d_3
H_1
B
A
H_1
(b)
33.12d_1
180.87d_2
127.39d_3
H_2
D
H
H_2
11.07d_1
123.69d_2
105.82d_3
(c)
1.73d_1
46.52d_2
69.49d_3
H_3
F
G
H_3
(d)
E
H_4
D
F
G
H_3
(e)
1000
E
H_4
D
V_4
22.59d_1
154.35d_2
72.55d_3
F
H_3
V_3
1.73d_1
46.52d_2
69.49d_3
(f)
3.63d_1
52.78d_2
2.79d_3
E
H_4
D
V_4
22.59d_1
154.35d_2
72.55d_3
(g)
200
C
500
B
D
H_5
V_5
A
H_1
(h)
1000
200
C
B
D
H_5
V_5
149.18d_1
84.79d_2
82.25d_3
55.70d_1
26.50d_2
54.84d_3
(i)
53.62d_1
6.95d_2
1.15d_3
C
D
H_5
V_5
55.70d_1
26.50d_2
54.84d_3
(j)

FIG. 23

Fig. 22(*b*) shows the second condition of deformation, and the coefficients of $\frac{E\,d_2}{1,000}$ are computed as before. The third condition is solved in Fig. 22(*c*).

In Fig. 23 and the remainder of this discussion, the expressions $\frac{E\,d_1}{1,000}$, $\frac{E\,d_2}{1,000}$, and $\frac{E\,d_3}{1,000}$ have been contracted to d_1, d_2, and d_3, respectively.

From Fig. 23(*a*) it is evident that

$$H_1 + H_2 + H_3 = 700 \tag{46a}$$

In Fig. 23(*b*), from $\Sigma M = 0$,

$$H_1 = 14.918\, d_1 + 8.479\, d_2 + 8.225\, d_3 \tag{46b}$$

Similarly, in Fig. 23(*c*),

$$H_2 = -3.682\, d_1 + 25.38\, d_2 + 19.434\, d_3 \tag{46c}$$

and, in Fig. 23(*d*),

$$H_3 = -0.173\, d_1 - 4.652\, d_2 + 6.949\, d_3 \tag{46d}$$

Substituting Eqs. 46*b*, 46*c*, and 46*d* in Eq. 46*a* gives

$$11.063\, d_1 + 29.207\, d_2 + 34.608\, d_3 = 700 \tag{46e}$$

From Fig. 23(*e*),

$$H_4 - H_3 = 0 \tag{47a}$$

In Fig. 23(*f*), taking moments about point F,

$$V_4 = 600 - 0.834\, d_1 - 4.313\, d_2 - 5.682\, d_3 \tag{47b}$$

In Fig. 23(*g*), taking moments about point E and substituting the value of V_4 from Eq. 47*b*,

$$H_4 = 480 + 1.560\, d_1 + 14.026\, d_2 + 1.552\, d_3 \tag{47c}$$

Substituting Eqs. 46*d* and 47*c* in Eq. 47*a* gives

$$480 + 1.733\, d_1 + 18.678\, d_2 - 5.397\, d_3 = 0 \tag{47d}$$

From Fig. 23(*h*),

$$H_1 + H_5 = 700 \tag{48a}$$

In Fig. 23(*i*), taking moments about point B,

$$V_5 = 456 + 3.739\, d_1 + 4.452\, d_2 + 5.484\, d_3 \tag{48b}$$

In Fig. 23(*j*), taking moments about point C and substituting the value of V_5 from Eq. 48*b* gives the equation,

$$H_5 = 608 + 14.095\, d_1 + 4.307\, d_2 + 2.838\, d_3 \tag{48c}$$

Substituting Eqs. 46*b* and 48*c* in Eq, 48*a* gives

$$29.013\, d_1 + 12.786\, d_2 + 11.063\, d_3 = 92 \tag{48d}$$

Solving Eqs. 46*e*, 47*d*, and 48*d* simultaneously gives $d_1 = -3.062$, $d_2 = -15.512$, and $d_3 = 34.280$.

The moments in Figs. 22(*a*), 22(*b*), and 22(*c*) multiplied by their respective *d*-values are shown in Table 3. The final values of the moments at the ends of all members are given in the bottom line of Table 3.

Additional bays can be added to the frame in Fig. 19 without increasing the problem unduly. Each additional bay would add one more condition of deformation. In other words, such a three-bay structure would require four conditions of deformation; a four-bay structure would call for five; etc.

TABLE 3.—END MOMENTS IN FOOT-POUNDS

Factors	Member											
	BA	BC	CB	CD	DC	DH	DE	ED	EF	FE	FG	HD
$d_1 = -3.062$	− 457	+ 457	+164	−164	− 170	+ 102	+ 69	+ 11	− 11	− 5	+ 5	+ 34
$d_2 = -15.512$	−1,315	+1,315	+108	−108	+ 411	−2,806	+2,394	+819	−819	− 722	+ 722	−1,918
$d_3 = 34.280$	+2,819	−2,819	− 39	+ 39	−1,880	+4,367	−2,487	+ 96	− 96	−2,382	+2,382	+3,627
Total....	+1,047	−1,047	+233	−233	−1,639	+1,663	− 24	+926	−926	−3,109	+3,109	+1,743

The solution of this type of problem by superposition may be rather long, but the individual steps are logical and easily followed and in the end a difficult problem has been solved.

JAROSLAV POLIVKA,[10] M. AM. SOC. C. E.—The principle of superposition is generally recommended and used in cases in which the direct analysis either involves the solution of a greater number of simultaneous equations or requires a relatively tedious determination of certain constants of the structure. For example, using the slope-deflection method, the examples presented in the paper require the solution of the following number of simultaneous equations:

Example No.	Number of equations	Example No.	Number of equations
1	3	5	8
2	4	6	3
3	4	7	4
4	6	8	5

In Examples 3 and 7, there are only four equations, of course, since the frames are symmetrical.

In most cases several load systems must be investigated, and for this reason preference should be given to a method of analysis by which certain constant values or characteristics of the given structure, common and applicable to any type of loading, can be determined. The method of analysis presented by the author possesses this advantage, since the computation of the internal forces (bending moments, shears, and thrusts) produced by lateral displacements of the joints is treated independently of transverse load.

Since an accuracy greater than 4% is usually not required for design purposes (except for checking the more complicated statically indeterminate structures), graphical or combined methods are useful for types of framed structures to which the method of superposition is applicable. One of the most promising methods is that of fixed (characteristic) points, which has not yet been fully utilized in the United States. The solution is general and can be applied to hinged and elastically yielding supports. The latter condition (yielding supports) is often more serious in engineering practice than either of the extreme conditions of fully restrained or hinged supports.

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[10] Research Associate, Univ. of California, Cons. Engr., Kaiser Co., Berkeley, Calif.

The designer can adapt graphical, semigraphical, or algebraic processes to his purpose. Graphical devices are quicker; they are easier to check; and they are as accurate as the precision of the draftsman who prepares them.

A graphical solution by fixed points, applied to Example 2, is demonstrated by reference to Fig. 24. The determination of the fixed (characteristic) points, U and V, of each structural member is shown in Fig. 24(*a*) using the values of the elastic weights, $G = \frac{L}{E I} = \frac{1}{E K}$, and the angular displacements, δ of the joints. The distances of the characteristic points, u and v, can be checked or calculated by the following relationships: $u_1 = \frac{L_1}{3}$ (full restraint at point A); $u_2 = \frac{L_2}{3} \frac{G_2}{G_2 + 2\,\delta_{BA}}$; and $u_4 = \frac{L_4}{3} \frac{G_4}{G_4 + 2\,\delta_{CAD}}$, in which $\delta_{CAD} = \frac{\delta_{CA}\,\delta_{CD}}{\delta_{CA} + \delta_{CD}}$, etc. The angular displacement δ_{BA} has the significance of the elastic weight of support of the member BC at point B and equals $\frac{G_1}{4}$, because of restraint at point A. The successive angular displacements can be expressed in terms of distances between the opposite fixed points and the supports; for example, $\delta_{CA} = \frac{G_2}{6}\left(3 - \frac{L_2}{L_2 - u_2}\right)$, etc.

Fig. 24(*b*) shows the bending moments of the bent, subjected to a central load on member BC, without considering the sidesway, found by the fixed points and the crosslines for the loaded span. The crosslines are determined by values K_B and K_C, derived from fixed-end moments or from formulas for each type of transverse loading.

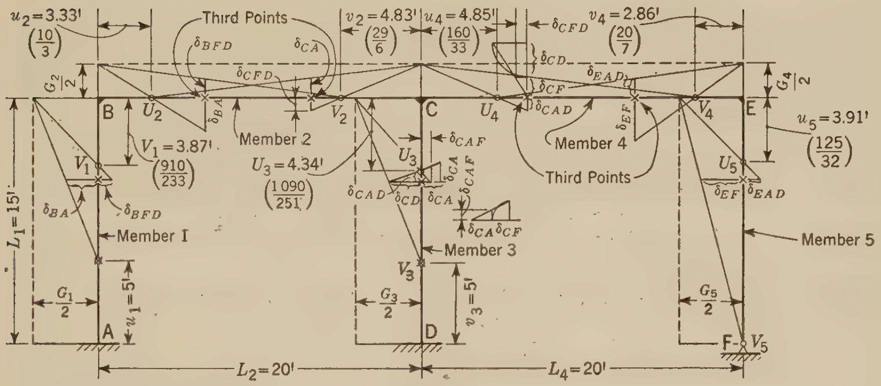
To take the lateral displacement of the column into account, the shears H_1, H_2, and H_3 are calculated from the moment diagram, and the sum, ΣH, of opposite sign, is applied at the temporary support, point E. The effect of this temporary reaction is obtained by allowing arbitrary translations $\Delta_B = \Delta_C = \Delta_E$ of the columns and calculating the shear in each column. The shear $H_{1\Delta}$ produced by translation Δ_B is calculated from $\Delta_B = H_{1\Delta}\, G'_1\, r^2_1$, in which G'_1 is the total elastic weight of column AB, restrained at point A and elastically supported at point B, and r_1 is the semiaxis of the ellipse of elasticity of this column. The elastic centroid and r_1 are found graphically as shown in Fig. 24(*c*). Simple relationships permit the computation of moments at points B and A, produced by $H_{1\Delta}$, as follows:

$$\bar{M}_{BA} = \frac{6\,\Delta_B\,v_1}{L_1\,G_1\,(L_1 - u_1 - v_1)} \dots\dots\dots\dots (49)$$

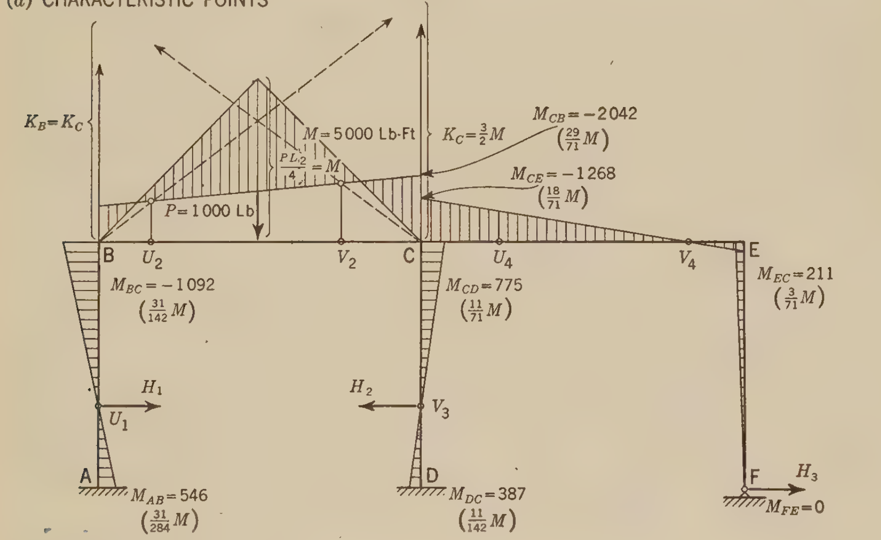
and

$$\bar{M}_{AB} = -\frac{6\,\Delta_B\,u_1}{L_1\,G_1\,(L_1 - u_1 - v_1)} = -M_{BA}\frac{u_1}{v_1} \dots\dots\dots\dots (50)$$

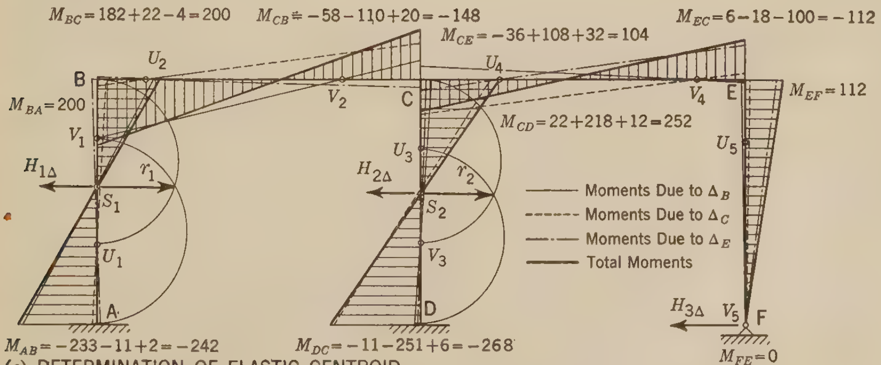
The moment $\bar{M}_{BA}$ is then distributed over all remaining structural members. Similarly, the moments due to $H_{2\Delta}$ and $H_{3\Delta}$, which are produced by $\Delta_C = \Delta_E$


$u_2 = 3.33'$
$\left(\frac{10}{3}\right)$
Third Points
δ_{BFD}
δ_{CA}
δ_{CFD}
$v_2 = 4.83'$
$\left(\frac{29}{6}\right)$
$u_4 = 4.85'$
$\left(\frac{160}{33}\right)$
δ_{CFD}
$v_4 = 2.86'$
$\left(\frac{20}{7}\right)$
δ_{CD}
δ_{EAD}
δ_{CF}
δ_{CAD}
$\frac{G_2}{2}$
$\frac{G_4}{2}$
B
U_2
δ_{BA}
V_2
C
U_4
δ_{EF}
V_4
E
$u_5 = 3.91'$
$\left(\frac{125}{32}\right)$
$V_1 = 3.87'$
$\left(\frac{910}{233}\right)$
Member 2
$U_3 = 4.34'$
$\left(\frac{1090}{251}\right)$
U_3
δ_{CAF}
δ_{CA}
δ_{CAF}
Member 4
Third Points
U_5
δ_{EF}
δ_{EAD}
$L_1 = 15'$
V_1
δ_{BA}
δ_{BFD}
Member I
δ_{CAD}
δ_{CD}
δ_{CA}
$\delta_{CA}\delta_{CF}$
Member 3
Member 5
V_3
$u_1 = 5'$
$v_3 = 5'$
$\frac{G_1}{2}$
$\frac{G_3}{2}$
$\frac{G_5}{2}$
A
D
F
V_5
$L_2 = 20'$
$L_4 = 20'$

(a) CHARACTERISTIC POINTS


$K_B = K_C$
$M = 5000$ Lb-Ft
$\frac{PL_2}{4} = M$
$K_C = \frac{3}{2}M$
$M_{CB} = -2042$
$\left(\frac{29}{71}M\right)$
$M_{CE} = -1268$
$\left(\frac{18}{71}M\right)$
$P = 1000$ Lb
B
U_2
V_2
C
U_4
V_4
E
$M_{BC} = -1092$
$\left(\frac{31}{142}M\right)$
$M_{CD} = 775$
$\left(\frac{11}{71}M\right)$
$M_{EC} = 211$
$\left(\frac{3}{71}M\right)$
H_1
U_1
H_2
V_3
A
D
F
H_3
$M_{AB} = 546$
$\left(\frac{31}{284}M\right)$
$M_{DC} = 387$
$\left(\frac{11}{142}M\right)$
$M_{FE} = 0$

(b) MOMENT DIAGRAM


$M_{BC} = 182 + 22 - 4 = 200$
$M_{CB} = -58 - 110 + 20 = -148$
$M_{EC} = 6 - 18 - 100 = -112$
$M_{CE} = -36 + 108 + 32 = 104$
U_2
U_4
B
$M_{BA} = 200$
V_2
C
V_4
E
$M_{EF} = 112$
V_1
$M_{CD} = 22 + 218 + 12 = 252$
U_3
U_5
$H_{1\Delta}$
r_1
$H_{2\Delta}$
r_2
S_1
S_2
Moments Due to Δ_B
Moments Due to Δ_C
Moments Due to Δ_E
Total Moments
U_1
V_3
A
D
$H_{3\Delta}$
V_5
F
$M_{AB} = -233 - 11 + 2 = -242$
$M_{DC} = -11 - 251 + 6 = -268$
$M_{FE} = 0$

(c) DETERMINATION OF ELASTIC CENTROID

FIG. 24.—GRAPHICAL SOLUTION OF EXAMPLE 2

$= \Delta_B$, are calculated. Finally, all moments produced by equal lateral displacement of the columns are summed up and the corresponding shear along member BCE is calculated.

Substituting the values in Eqs. 49 and 50: $\bar{M}_{BA} = \frac{182}{710}$ and $\bar{M}_{AB} = -\frac{233}{710}$. The moment $\bar{M}_{BA}$ is distributed to the other joints using the final carry-over factors which are determined by fixed points and angular displacements (at the joint C). The carry-over factor from point B to point C is $C_{BC} = \frac{v_2}{L_2 - v_2} = \frac{29}{91}$; the carry-over factor $\frac{M_{CD}}{M_{CB}} = \frac{11}{29}$; the carry-over factor $\frac{M_{CE}}{M_{CB}} = \frac{18}{29}$, etc. Similarly, the values $\bar{M}_{CD} = \frac{218}{710}$ and $\bar{M}_{DC} = -\frac{251}{710}$ (for column DC) and $\bar{M}_{EF} = \frac{112}{710}$ (for column FE) are obtained and carried over to the other joints (Fig. 24(*c*)). For the purpose of superposition, only the ratio between the temporary reaction at point E (Fig. 24(*b*)) and the total shear along member ECB (or the total thrust along this member) are considered. Therefore, the common denominator 710 can be eliminated (Fig. 24(*c*)). The temporary reaction at point E is $\frac{39}{284 \times 15} M$ (Fig. 24(*b*)) and the total shear produced by sidesway is $\frac{1{,}074}{15}$; hence, the ratio sought is $\frac{39 \times 5{,}000}{284 \times 1{,}074} = 0.63934$.

The additional moments produced by sidesway are calculated by multiplying the values in Fig. 24(*c*) by the ratio 0.63934.

Professor Wilson uses the moment distribution method (except in Example 1, in which the slope-deflection method is used) for the determination of the superimposing effects. The relative accuracy depends on the number of cycles and is usually completely sufficient for a rational design as shown by the values obtained by the author as compared with more accurate results of the writer's computations by the method of fixed (characteristic) points:

Example Moments

2 $M_{AB} = 390$ (391.05); $M_{BA} = 963$ (963.68); $M_{CB} = 2{,}135$ (2,136.88);
$M_{CD} = -933$ (−935.76); $M_{DC} = -557$ (−558.67);
$M_{CE} = -1{,}202$ (1,201.11); and $M_{EC} = -137$ (−139.86).

4 $M_{AB} = -2{,}838$ (−2,831.37); $M_{BA} = -2{,}345$ (−2,325.14);
$M_{CD} = -1{,}612$ (−1,612.93); $M_{DC} = -799$ (−808.81);
$M_{DE} = -1{,}407$ (−1,403.79); $M_{ED} = -1{,}954$ (−1,955.97);
$M_{DF} = 2{,}203$ (2,212.61); $M_{FG} = -1{,}109$ (−1,112.61); and
$M_{GF} = -1{,}111$ (−1,112.14).

5 $M_{AB} = 8.71$ (8.79); $M_{BA} = 14.58$ (14.65); $M_{CB} = 2.70$ (2.82);
$M_{CD} = 4.06$ (3.92); $M_{DC} = 6.30$ (6.31); $M_{ED} = -7.75$ (−7.75);
$M_{FE} = 12.15$ (12.02); $M_{CF} = -6.76$ (−6.74);
$M_{FC} = -2.66$ (−2.64); $M_{FA} = -9.49$ (−9.38); and
$M_{AF} = -8.71$ (−8.79).

Example	Moments
6	$M_{AB} = 228$ (230.93); $M_{BA} = 428$ (429.18); $M_{CB} = 640$ (642.25); and $M_{DC} = -304$ (-304.79).
7	$M_{AB} = -472$ (-474.36); $M_{BA} = -433$ (-433.76); $M_{BE} = -134$ (-135.19); $M_{EB} = -308$ (-307.40); and $M_{BC} = 569$ (568.95).
8	$M_{AB} = 6{,}970$ (6,986.53); $M_{BA} = 3{,}740$ (3,746.31); $M_{CD} = 2{,}250$ (2,256.11); $M_{DC} = 550$ (539.99); and $M_{ED} = 3{,}800$ (3,831.97).

In the foregoing, the writer's values are in parentheses. The results are accurate, the differences being less than 4%. In Example 3, no differences were found.

Odd Albert,[11] Assoc. M. Am. Soc. C. E.—Structural analysis by superposition, as proposed by Professor Wilson, is an excellent method. An improvement in procedure, however, is suggested by Example 2, an unsymmetrical three-column bent subjected to an off-center vertical load. The author solves this frame by determining the moment caused directly by the vertical load and then adjusts for the effect of sidesway in a separate operation. Final moments are obtained by adding the two sets of moments. A suggestion that results in more simple mathematics and one that is easier to visualize is presented herein, adapting the principle of slope deflection, as applied to a cantilever beam. For this purpose the deflections of the free end of a cantilever beam under various loadings are given in Fig. 25.

The sign conventions preferred by the writer are illustrated in Fig. 26—that is, when the end moment is negative, the arrow is drawn clockwise.

Vertical Load on Span BC.—If span BC, Fig. 27, is loaded as shown with the frame fixed to prevent sidesway, the moment transmitted to joint B travels to point A, and the moment transmitted to joint C separates and travels through point E to point F.

Referring to column EF, Fig. 27, the angle θ_E causes a deflection of $\theta_E L_{EF}$ at F_1, and the horizontal force H_F causes a deflection in the opposite direction (see Fig. 25):

$$\theta_E L_{EF} - \frac{H_F L^2_{EF}}{3E} = 0 \ldots\ldots\ldots\ldots (51)$$

Since $H_F = \frac{M_E}{L_{EF}}$,

$$\theta_E = \frac{M_{EF}}{3E} = -\frac{M_{EC}}{3E} \ldots\ldots\ldots\ldots (52)$$

Referring next to beam CE, Fig. 27, if end C were free to deflect:

$$\theta_E L_{CE} + \frac{V_C L^2_{CE}}{3E \times 2} + \frac{M_{CE} L_{CE}}{2E \times 2} = 0 \ldots\ldots\ldots\ldots (53)$$

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[11] Structural Engr., Belmar, N. J.

Substituting $V_C = -\frac{M_{CE}}{L_{CE}} - \frac{M_{EC}}{L_{CE}}$,

$$M_{EC} = \frac{M_{CE}}{6} \qquad (54)$$

In the same beam, if end E is free to deflect:

$$\theta_C L_{CE} + \frac{V_E L^2_{CE}}{3E \times 2} + \frac{M_{EC} L_{CE}}{2E \times 2} = 0 \qquad (55)$$

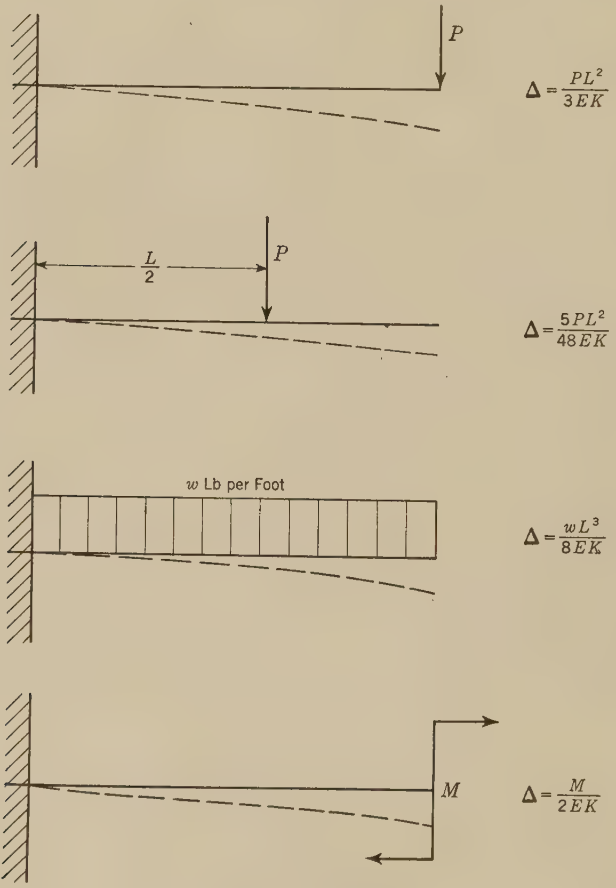

P
$\Delta = \frac{PL^2}{3EK}$
$\frac{L}{2}$
P
$\Delta = \frac{5PL^2}{48EK}$
w Lb per Foot
$\Delta = \frac{wL^3}{8EK}$
M
$\Delta = \frac{M}{2EK}$

Fig. 25

In this case,

$$V_E = -\frac{M_{CE}}{L_{CE}} - \frac{M_{EC}}{L_{CE}} = -\frac{7\,M_{CE}}{6\,L_{CE}} \qquad (56)$$

Substituting Eq. 56 in Eq. 55 and solving for θ_C:

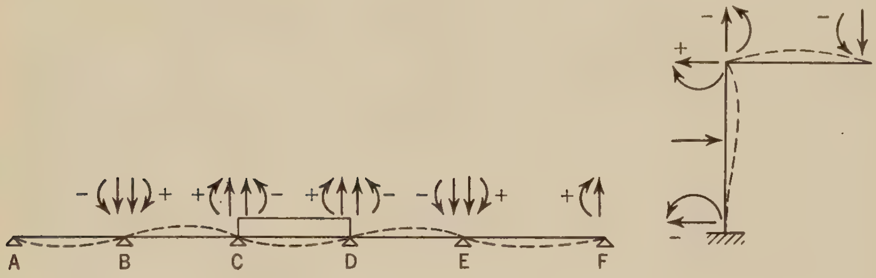
$$\theta_C = \frac{11\,M_{CE}}{72\,E} \qquad (57)$$

Column DC is treated similarly. Assuming point C free to deflect:

$$\frac{H_C\, L^2_{CD}}{3\, E} + \frac{M_{ED}\, L_{CD}}{2\, E} = 0 \quad \text{.................... (58)}$$

Substituting $H_C = -\frac{M_{CD}}{L_{CD}} - \frac{M_{DC}}{L_{CD}}$,

$$M_{DC} = \frac{M_{CD}}{2} \quad \text{........................... (59)}$$



(a) HORIZONTAL BEAM (b) VERTICAL BEAM

Fig. 26.—Sign Conventions

Assuming point D free to deflect, similarly:

$$\theta_C\, L_{CD} + \frac{H_D\, L^2_{CD}}{3\, E} + \frac{M_{DC}\, L_{CD}}{2\, E} = 0 \quad \text{.............. (60)}$$

With $H_D = -\frac{M_{CD}}{L_{CD}} - \frac{M_{DC}}{L_{CD}}$:

$$\theta_C = \frac{M_{CD}}{4\, E} \quad \text{.......................... (61)}$$

Since each member meeting at joint C rotates through an angle θ_C, Eqs. 57 and 61 must be equal; that is, $\frac{11\, M_{CE}}{72\, E} = \frac{M_{CD}}{4\, E}$ and

$$M_{CD} = \frac{11\, M_{CE}}{18} \quad \text{......................... (62)}$$

Distribute the moment M_{CB} to beam CE and column CD as M_{CE} and M_{CD}, respectively. In other words, since $M_{CB} = M_{CE} - M_{CD} = -M_{CE} - \frac{11\, M_{CE}}{18} = -\frac{29}{18} M_{CE}$,

$$M_{CE} = -\frac{18}{29} M_{CB} \quad \text{..................... (63a)}$$

and

$$M_{CD} = -\frac{11\,M_{CE}}{18} = -\frac{11}{18} \times \frac{18\,M_{CB}}{29} = -\frac{11\,M_{CB}}{29} \quad \text{......(63b)}$$

Finally (see Eqs. 57 and 61),

$$\theta_C = -\frac{11\,M_{CB}}{116\,E} = \frac{M_{CD}}{4\,E} = \frac{11\,M_{CE}}{72\,E} \quad \text{..............(64)}$$

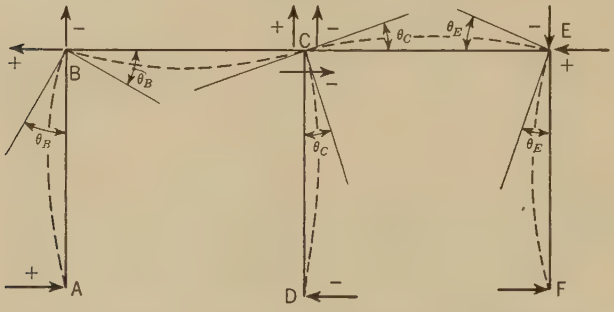


Fig. 27.—Vertical Load on a Frame Fixed Against Sidesway

Because column AB is identical to column CD, $M_{AB} = \frac{M_{BA}}{2}$, and:

$$\theta_B = \frac{M_{AB}}{4\,E} = -\frac{M_{BC}}{4\,E} \quad \text{.....................(65)}$$

If beam BC is free to deflect at end C (compare Eq. 53):

$$\theta_B\,L_{BC} + \frac{5\,P\,L^2_{BC}}{48\,E \times 2} + \frac{M_{CB}\,L_{BC}}{2\,E \times 2} - \frac{V_C\,L^2_{BC}}{3\,E \times 2} = 0 \quad \text{.........(66)}$$

Substituting $V_C = \frac{P}{2} + \frac{M_{BC}}{L_{BC}} + \frac{M_{CB}}{L_{BC}}$:

$$-5\,M_{BC} + M_{CB} = \frac{3\,P\,L_{BC}}{8} = 7{,}500 \quad \text{.............(67)}$$

If beam BC is free to deflect at end B, an equation similar to Eq. 66 $\left(\text{substituting } V_B = \frac{P}{2} - \frac{M_{BC}}{L_{BC}} - \frac{M_{CB}}{L_{BC}}\right)$ yields:

$$-91\,M_{CB} + 29\,M_{BC} = -\frac{3\,P\,L_{BC}}{8} \times 29 = -7{,}500 \times 29 \quad \text{.....(68)}$$

Solving Eqs. 67 and 68 simultaneously: $M_{BC} = -\frac{77{,}500}{71}$; $M_{CB} = +\frac{145{,}000}{71}$;

$M_{BA} = +\frac{77{,}500}{71}$; $M_{AB} = \frac{M_{BA}}{2} = \frac{1}{2} \times \frac{77{,}500}{71} = +\frac{38{,}750}{71}$; M_{CD} (Eq. 63*b*) $= -\frac{55{,}000}{71}$; M_{DC} (Eq. 59) $= -\frac{27{,}500}{71}$; M_{CE} (Eq. 63*a*) $= -\frac{90{,}000}{71}$; M_{EC} (Eq. 54) $= -\frac{15{,}000}{71}$; and $M_{EF} = +\frac{15{,}000}{71}$.

The horizontal force H_B is equal to $\frac{38{,}750 + 77{,}500}{71 \times 15} = \frac{7{,}750}{71}$. Similarly, $H_C = -\frac{5{,}500}{71}$ and $H_E = +\frac{1{,}000}{71}$. The horizontal force at the top required to prevent horizontal movement, then, is the sum of the three, or $\frac{3{,}250}{71}$.

Sidesway.—When columns are of equal height, the sidesway, or lateral deflection of the horizontal members, is equivalent to the corresponding lateral displacement of the column bases, assuming the girders to be fixed. The problem is approached by analyzing the effect of moving each column base separately.

For example (see Fig. 28), if support A is moved to the left a distance Δ, the moment generated in joint B will be distributed along the two paths

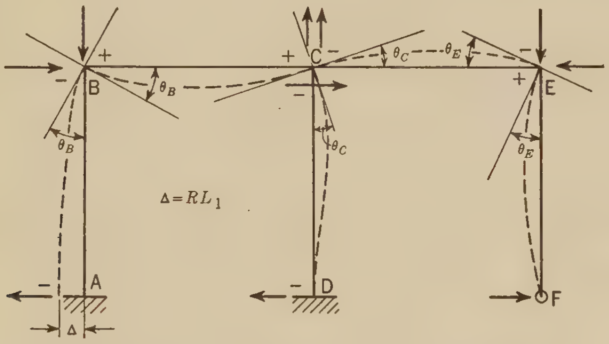


FIG. 28.—SUPPORT A MOVED TO THE LEFT A DISTANCE Δ

C-E-F and C-D. By Eq. 64, $\theta_C = -\frac{11\ M_{CB}}{116\ E}$ in terms of beam BC. If this beam is cut at end B,

$$\theta_C\, L_{BC} - \frac{V_B\, L^2_{BC}}{3\, E \times 2} + \frac{M_{BC}\, L_{BC}}{2\, E \times 2} = 0 \ldots\ldots\ldots\ldots\ldots (69)$$

As before, substituting $V_B = \frac{M_{BC}}{L_{BC}} + \frac{M_{CB}}{L_{BC}}$:

$$M_{CB} = \frac{29\ M_{BC}}{91} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (70)$$

Conversely, if joint B rotates with end C cut:

$$\theta_B L_{BC} - \frac{V_C L^2_{BC}}{3\,E \times 2} + \frac{M_{CB} L_{BC}}{2\,E \times 2} = 0 \ldots\ldots\ldots\ldots (71)$$

Substituting $V_C = V_B$, Eq. 71 becomes:

$$\theta_B = \frac{51\,M_{BC}}{4.91\,E} = -\frac{51\,M_{BA}}{4.91\,E} \ldots\ldots\ldots\ldots (72)$$

Isolate column AB, cut end B, and rotate support A, then:

$$\frac{H_B L^2_{AB}}{3\,E} + \frac{M_{BA} L_{AB}}{2\,E} = R\,L_{AB} \ldots\ldots\ldots\ldots (73)$$

Substituting $H_B = -\frac{M_{AB}}{L_{AB}} - \frac{M_{BA}}{L_{AB}}$:

$$M_{AB} = \frac{M_{BA}}{2} - 3\,E\,R \ldots\ldots\ldots\ldots (74)$$

Conversely, with end A cut:

$$\theta_B L_{AB} + \frac{H_A L^2_{AB}}{3\,E} + \frac{M_{AB} L_{AB}}{2\,E} = R\,L_{AB} \ldots\ldots\ldots\ldots (75)$$

Substituting $H_A = H_B$:

$$M_{BA} = -\frac{273\,E\,R}{71} \ldots\ldots\ldots\ldots (76)$$

Therefore: $M_{AB} = -\frac{273\,E\,R}{2 \times 71} - 3\,E\,R = -\frac{699\,E\,R}{2 \times 71}$; and, similarly: $M_{BC} = +\frac{273\,E\,R}{71}$; M_{CB} (Eq. 70) $= \frac{87\,E\,R}{71}$; M_{CD} (Eq. 63*b*) $= -\frac{33\,E\,R}{71}$; M_{DC} (Eq. 59) $= -\frac{33\,E\,R}{2 \times 71}$; M_{CE} (Eq. 63*a*) $= -\frac{54\,E\,R}{71}$; M_{EC} (Eq. 54) $= -\frac{9\,E\,R}{71}$; and $M_{EF} = +\frac{9\,E\,R}{71}$.

Deflecting support D to the left a distance Δ yields, by the same general procedure: $M_{CD} = -\frac{327\,E\,R}{71}$; $M_{DC} = -\frac{327\,E\,R}{2 \times 71} - 3\,E\,R = -\frac{753\,E\,R}{2 \times 71}$; $M_{CB} = +\frac{55 \times 327\,E\,R}{109 \times 71} = +\frac{165\,E\,R}{71}$; $M_{BC} = \frac{1 \times 165\,E\,R}{5 \times 71} = +\frac{33\,E\,R}{71}$; $M_{BA} = -\frac{33\,E\,R}{71}$; $M_{AB} = -\frac{33\,E\,R}{2 \times 71}$; $M_{CE} = \frac{54}{109} \times \frac{327\,E\,R}{71} = \frac{162\,E\,R}{71}$; M_{EC} (Eq. 54) $= \frac{1}{6} \times \frac{162\,E\,R}{71} = \frac{27\,E\,R}{71}$; and $M_{EF} = -\frac{27\,E\,R}{71}$.

Finally, deflecting support F a distance Δ to the left, and solving as before:

$$M_{EF} = -\frac{150\,E\,R}{71};\quad M_{EC} = +\frac{150\,E\,R}{71};\quad M_{CE} = \frac{8}{25} \times \frac{150\,E\,R}{71} = \frac{48\,E\,R}{71};$$

$$M_{CD} = -\frac{3}{8} \times \frac{48\,E\,R}{71} = -\frac{18\,E\,R}{71};\quad M_{DC}\ \text{(Eq. 59)} = -\frac{1}{2} \times \frac{18\,E\,R}{71}$$

$$= -\frac{9\,E\,R}{71};\quad M_{CB} = -\frac{5}{8} \times \frac{48\,E\,R}{71} = -\frac{30\,E\,R}{71};\quad M_{BC} = -\frac{1}{5} \times \frac{30\,E\,R}{71}$$

$$= -\frac{6\,E\,R}{71};\ M_{BA} = +\frac{6\,E\,R}{71};\ \text{and}\ M_{AB} = \frac{1}{2} \times \frac{6\,E\,R}{71} = +\frac{3\,E\,R}{71}.$$

The moments caused by a sidesway of Δ will then be equal to the sum of the moments generated by the separate movements of the supports; that is,

$$\left.\begin{array}{c} M_{AB} = -\dfrac{363\,E\,R}{71};\ M_{BA} = -\dfrac{300\,E\,R}{71};\ M_{DC} = -\dfrac{402\,E\,R}{71}; \\ M_{CD} = -\dfrac{378\,E\,R}{71};\ \text{and}\ M_{EF} = -\dfrac{168\,E\,R}{71} \end{array}\right\} \ldots (77)$$

By adding up the end column moments $\left(\frac{1{,}611\,E\,R}{71}\right)$, the horizontal force at the top of the frame required to move it sidewise a distance Δ will be $\frac{1{,}611\,E\,R}{71 \times 15}$ lb; but this force has already been found equal to $\frac{3{,}250}{71}$ lb (see text following Eq. 68). Equating the two quantities: $E\,R = \frac{16{,}250}{537}$ lb, a value that can be substituted in Eqs. 77.

The final moments are obtained by adding the moments caused by the vertical load (following Eq. 68) and those caused by sidesway; thus: $M_{AB} = +\frac{38{,}750}{71} - \frac{363 \times 16{,}250}{71 \times 537} = \frac{2{,}500 \times 28}{179}$. Similarly, $M_{BA} = +\frac{2{,}500 \times 69}{179}$; $M_{DC} = -\frac{2{,}500 \times 40}{179}$; $M_{CD} = -\frac{2{,}500 \times 67}{179}$; and $M_{EF} = +\frac{2{,}500 \times 10}{179}$.

Summary.—The writer can offer no criticism of the basic thesis presented by Professor Wilson. The foregoing comments are offered merely in the hope of suggesting an improvement in the complete procedure.

W. C. Spiker,[12] M. Am. Soc. C. E.—As the author states (see heading, "Synopsis"), in textbooks on structural analysis the application of the principle of superposition is not emphasized as much as its importance warrants. The writer has studied only one of the examples in the paper—Example 1. To that demonstration he wishes to suggest the following simplification: Omit

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[12] Engr. Adviser, Public Bldgs. Administration, Federal Works Agency, Washington, D. C.

Eqs. 5*a* and 5*b* of the simultaneous solution and proceed thus (omitting E for the sake of brevity):

At joint B—

$$4\,\theta_B - 6\,R + 20\,\theta_B + 10\,\theta_C = 24\,\theta_B - 6\,R + 10\,\theta_C = 0 \ldots. \quad (78)$$

Therefore,

$$\theta_B = 0.25\,R - 0.417\,\theta_C \ldots\ldots\ldots\ldots\ldots\ldots (79)$$

At joint C

$$20\,\theta_C + 10\,\theta_B + 8\,\theta_C - 8\,R = 0 \ldots\ldots\ldots\ldots\ldots\ldots (80)$$

In Eq. 81, substituting the value of θ_B from Eq. 80, and simplifying,

$$23.83\,\theta_C - 5.5\,R = 0 \ldots\ldots\ldots\ldots\ldots\ldots (81)$$

From the foregoing it appears to the writer that when practicable it is much simpler to substitute the value of θ from each preceding joint in the next succeeding equation than to carry "unknowns" through the fundamental equations and then write new equations and solve for the "unknowns."

David B. Hall,[13] Assoc. M. Am. Soc. C. E.—A simple orderly procedure for analyzing rigid frames with sidesway is presented in this paper. It is instructive to observe that relations exist between step 1, in which the geometrical properties of the cells are established, and step 4, in which conditions of equilibrium are established. It is the purpose of this discussion to point out these relations and to show that in the case of a single cell nonrectangular quadrangular frame they may be of considerable use.

In any structure the geometrical properties and static properties are closely interrelated through the law of conservation of energy. Thus, in most cases geometric methods and static methods are very similar, but because the relationship is an inverse one there will be occasions when the two are not equally simple. For example, the static analysis of a truss by vector diagram is simpler than the geometric analysis by Williot diagram because in the former all lines are parallel to members, whereas in the latter there are lines both parallel and perpendicular to members.

In the case of the quadrangular frame, an examination of the paper discloses that the statical analysis (Eqs. 22 to 25) requires a piecemeal method of approach, as contrasted with the simple geometric relation (Eq. 20). This difference appears to be characteristic of quadrangular cells. It seems logical, therefore, to apply the law of conservation of energy to geometric relations for the static analysis of the problem, just as it has been found advantageous to apply this law (as the method of virtual work) to find deflections of trusses from statical relations.

Before proceeding to do this, it will be advantageous to reestablish the geometrical relations in a manner slightly different from Eq. 20. When one side of a quadrangular cell remains fixed (AD, Fig. 29), the ends of the opposite member (BC) are constrained to move at right angles to their respective connecting members (AB and CD), and the member rotates about an instantaneous center located at the intersection of the connecting members (point O). In this connection it may be convenient to visualize triangle BCO as an imaginary rigid body supported on the two legs. If this imaginary body is given a rotation R_0, then:

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[13] Stress Analyst, Otis Elevator Co., New York, N. Y.

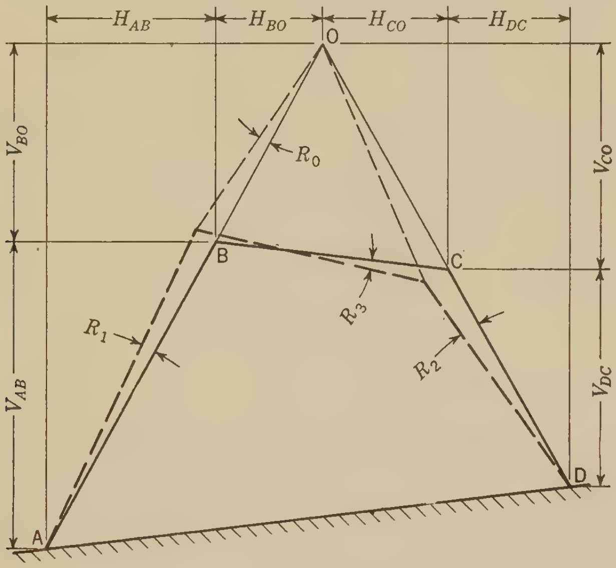

FIG. 29.—DEFORMATION OF A QUADRANGLE REFERRED TO INSTANTANEOUS CENTER

$$R_3 = R_0 \ldots\ldots\ldots\ldots (82a)$$

$$R_1 = -R_0 \frac{\overline{\mathrm{BO}}}{\overline{\mathrm{AB}}} = -R_0 \frac{V_{BO}}{V_{AB}} = -R_0 \frac{H_{BO}}{H_{AB}} \ldots\ldots\ldots\ldots (82b)$$

and

$$R_2 = -R_0 \frac{\overline{\mathrm{CO}}}{\overline{\mathrm{DC}}} = -R_0 \frac{V_{CO}}{V_{DC}}$$

$$= -R_0 \frac{H_{CO}}{H_{DC}} \ldots\ldots\ldots\ldots (82c)$$

Thus, the rotations of the various members are quite simply expressed in terms of a single parameter R_0.

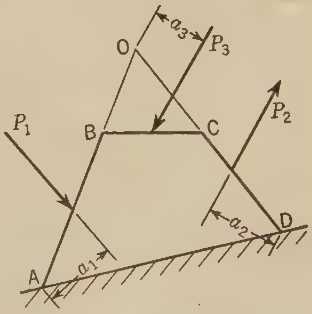

FIG. 30.—NOTATION FOR LOADS ON A QUADRANGULAR FRAME

Energy equations can now be written. For notation relating to loads refer to Fig. 30. Apply a rotation R_0 at the instantaneous center. The work done by loads on member BC equals $P_3 a_3 R_0$. The work done by loads on the legs equals $P_1 a_1 R_1$ and $P_2 a_2 R_2$. The work done by moments at A and D equals $M_{AB} R_1$ and $M_{DC} R_2$. The work done by moments at B and C equals

M_{BA} $(R_1 - R_3)$ and M_{CD} $(R_2 - R_3)$. Summing up the work done by all items and dividing by R_0,

$$P_3\,a_3 + P_1\,a_1\frac{R_1}{R_0} + P_2\,a_2\frac{R_2}{R_0} + M_{AB}\frac{R_1}{R_0} + M_{DC}\frac{R_2}{R_0} + M_{BA}\left(\frac{R_1}{R_0} - 1\right) + M_{CD}\left(\frac{R_2}{R_0} - 1\right) = 0 \ldots\ldots\ldots\ldots(83)$$

As an example this method will be applied to the trapezoidal frame shown in Fig. 8(*a*). The legs intersect on the center line 15 ft above BC—$-R_1 = -R_2 = R_3 = R_0$. There are two loads of the type P_3 and no others; thus—

$$\begin{aligned} -\,200 \text{ lb} \times 15 &= -\,3{,}000 \\ 1{,}000 \text{ lb} \times 2.5 &= 2{,}500 \\ \hline \sum P_3\,a_3 &= -\quad 500 \end{aligned}$$

and

$$-\,500 - M_{AB} - M_{DC} - 2\,M_{BA} - 2\,M_{CD} = 0 \ldots\ldots\ldots\ldots(84)$$

Values of moments in terms of loads and unknown displacements are given in Fig. 8(*b*) and Fig. 8(*c*). In Fig. 8(*c*) it would be more orderly, when using the proposed method, to change all signs and refer the moments to R_0, but this is not necessary. Substituting the values of moments from Fig. 8(*b*) and Fig. 8(*c*) in Eq. 84, the following equation results:

$$-\,500 - (194 - 6.46\,E\,R_1) - (-\,338 - 6.46\,E\,R_1) - 2\,(391 - 6.92\,E\,R_1) - 2\,(-\,677 - 6.92\,E\,R_1) = 0 \ldots\ldots\ldots(85)$$

Solving Eq. 85, $E\,R_1 = -\,5.32$, which agrees with the author's value.

After Eq. 83 has been derived by energy methods, it is not very difficult to retrace the solution and discover the significance of Eq. 83 as an equilibrium equation: Each term of the equation can be shown to represent a moment about point O, and the equation therefore is a statement that the net moment about point O is equal to zero. Thus, the equation could have been obtained without introducing energy, but its discovery would not have been so obvious and its derivation would not have been so simple.

A. A. EREMIN,[14] ASSOC. M. AM. SOC. C. E.—An interesting method of applying the principle of superposition to the algebraic distribution of moments in rigid frames is presented in this paper. The same results can be obtained, with less effort, by applying superposition principles to the graphical distribution of moments. The moment diagram derived by graphical distribution offers a means of comparing the directions of moments and forces visually, and obviates the need for approximate sketching, such as that suggested by the author. Furthermore, graphic methods can be checked readily, simply by inspection.

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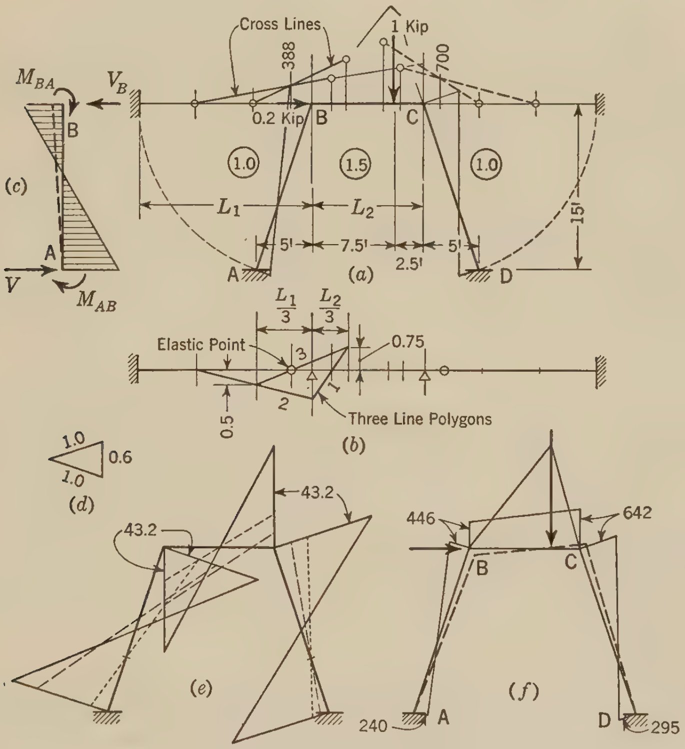
[14] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

For example, consider the frame in Fig. 31(*a*) in which the cross sections are constant. Basic stiffnesses, $\frac{I}{L}$, are shown in circles. The moments are determined graphically by the use of "three-line polygons" and "cross lines" (22) constructed at the "central points," points of inflection, and elastic points (see Fig. 31(*b*)) of each member.



Cross Lines
1 Kip
388
700
M_{BA}
V_B
0.2 Kip
B
C
(c)
1.0
1.5
1.0
L_1
L_2
15'
5'
7.5'
2.5'
5'
A
D
(a)
V
M_{AB}
$\frac{L_1}{3}$
$\frac{L_2}{3}$
Elastic Point
3
0.75
1
2
0.5
Three Line Polygons
(b)
1.0
0.6
1.0
(d)
43.2
43.2
446
642
B
C
(e)
(f)
240
A
D
295

FIG. 31.—GRAPHICAL SOLUTION BY SUPERPOSITION

When all members of the frame have uniform cross sections the "central points" are at the third points of the span, and the point of inflection is located by drawing line 1, Fig. 31(*b*), through the stiffness ordinate at the support and through the ordinate at the third point that marks one half the basic stiffness. The stiffness ordinate for each member is determined by drawing line 2, Fig. 31(*b*), through the point of inflection at the far end and through the ordinate at the third point that marks one half the basic stiffness of the span being analyzed. The point of inflection in the member with a fixed end is at the third point of the span. The elastic point for the intermediate joint B, Fig. 31(*a*), is determined by constructing a three-line polygon. Line 3, Fig. 31(*b*), is drawn through the intersection of line 2 with the vertical at the third point of the span and through the intersection of line 1 with the vertical at the third point in the adjacent span. The place where line 3, Fig. 31(*b*), intersects the axis of the member is the elastic point.

Moments due to the vertical force are determined graphically by "cross lines" (see Fig. 31(*a*)). When joint B is displaced a unit distance the resulting distortion diagram is as shown in Fig. 31(*d*); and the resulting moments in member AB, by the formula—

$$M_{AB} = \frac{600\, I\, a}{L^2_1\,(L_1 - a - b)} \qquad (86)$$

—are $M_{AB} = 29.6$ and $M_{BA} = 21.3$. In Eq. 86, a and b are the distances of the points of inflection from ends A and B, respectively. The numerical factor 600 is introduced for convenience of computation.

The manner of distributing the moments M_{AB} and M_{BA}, as well as the distortion moments for the other members of the frame, is indicated by the dotted lines in Fig. 31(*e*). The final moments are indicated by full lines. The moment diagram for member AB, showing also the direction of the moments and forces, is presented in Fig. 31(*c*) in a convenient form for memorizing.

By statics:

$$-\,2{,}812\, k = 200 \qquad (87)$$

in which $k = -\,0.071$. Eq. 87 is similar in form to Eq. 25. The final moments are equal to the algebraic sum of the moments in Fig. 31(*a*) and Fig. 31(*e*), multiplied by factor $k = -\,0.071$. As shown graphically in Fig. 31(*f*), these final moments differ slightly from those computed by the author.

The drawing can be included on a sheet the size of an ordinary business letter, although greater accuracy, where required, may be obtained by using a larger scale. The sidesway of members with haunches may be determined similarly, necessary constants being computed independently, or scaled (22) from diagrams.

To summarize: The author declared the purpose of his paper (see heading, "Synopsis") to be "to demonstrate the usefulness of the principle of superposition in the analysis of rigid frames by the slope-deflection and moment distribution methods." Granting the usefulness of the principle of superposition, it is the latter part of the foregoing statement which the writer questions. The writer's purpose has been to plead the virtue of more simple design procedures in demonstrating the acknowledged worth of the principle of superposition.

VICTOR R. BERGMAN,[15] ASSOC. M. AM. SOC. C. E.—For the analysis of those rigid frames in which the effect of joint translation as well as of joint rotation must be considered, Professor Wilson has presented a useful, interesting method of procedure involving superposition. Although some of the problems in this paper can be solved more readily by other approaches, all the examples have a definite value in developing and illustrating the method.

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One of the frames analyzed by Professor Wilson, the gable bent in Fig. 10, has enjoyed quite a vogue in technical literature. The author's treatment of this problem is instructive, and indicates the ease of application of the method to this type of structure, when the component members are of constant cross section. However, the growing demand for artistically pleasing structures leads to the construction of many of these frames with members of varying cross section. In this case, a gable bent often can be analyzed more advantageously by some other method such as column analogy.

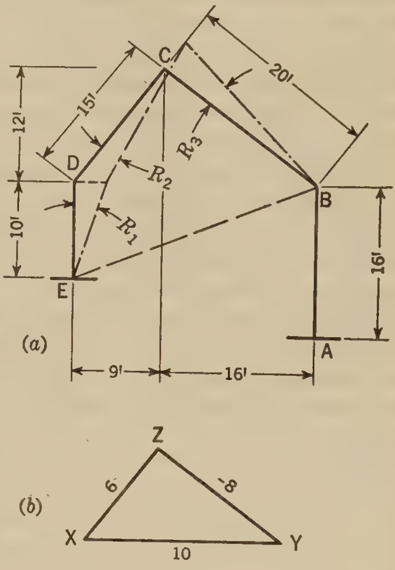

FIG. 32

It may be of value to point out that the "Relation Between Values of *R* for Members of a Quadrangle" (see subheading under "Nonrectangular Frames") can be determined readily by a semigraphical method. The gable bent in Fig. 10, redrawn in Fig. 32(*a*), will be used for purposes of illustration.

In Fig. 32(*b*), using any convenient scale, lay off a length, XY, of 10 units at right angles to leg DE of the gable bent. Through points X and Y draw lines perpendicular, respectively, to members CB and CD, intersecting at point Z. The values of XZ and YZ are found, by scaling, to be 6 and 8 units, respectively. Then: $R_1 = \frac{10}{10} R_1 = R_1$; $R_2 = \frac{8}{15} R_1$; and $R_3 = \frac{6}{20} R_1 = \frac{3}{10} R_1$.

DAVID M. WILSON,[16] M. AM. SOC. C. E.—Several of the discussions have concerned methods of balancing moments at the joints in step 3 of the paper. Messrs. Stewart and McGee prefer the traverse method (17) for this purpose, whereas Messrs. Polivka and Eremin use the graphical method of fixed points. Although these methods have definite limitations (17*a*) not possessed by the general slope-deflection and moment distribution methods, they will be found useful in the analysis of continuous beams and one-story frames. In this connection, a special technique in the use of slope deflection will be presented subsequently which accomplishes the same purpose as the traverse method.

Mr. Fischer presents some short cuts in the application of moment distribution to several of the simple examples of the paper. These are useful to the experienced designer in simplifying distribution calculations for many problems where they apply. In the paper, short cuts were intentionally avoided in order that the usefulness of superposition might be demonstrated without giving undue attention to distribution procedures. Furthermore, the slope-deflection and moment distribution methods, as used in the paper, are well known and perfectly general in their application to the analyses of all types of rigid frames.

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Mr. Merritt questions the validity of conclusion 4. The writer agrees with him that nearly all rigid-frame problems can be solved most expeditiously by moment distribution. However, in certain cases, slope-deflection procedures will furnish a more rapid convergence than moment distribution, as Mr. Goldberg states in his discussion. A comprehensive treatment of the limits of validity of the principle of superposition has been presented by J. A. Van den Broek, M. Am. Soc. C. E. (11*a*). The treatment of moving loads, suggested also by Mr. Merritt, will be given attention subsequently.

Mr. Albert suggests that the method of procedure is improved by determining the magnitude of the horizontal force required to prevent sidesway in Fig. 27. This method is presented in textbooks (3*b*) and elsewhere but, in the writer's opinion, is indirect and requires more computations than the method of the paper. Of course, certain forces must be assumed to act during the several deformation conditions of any frame; but it is not necessary to determine the magnitudes of these forces since they cancel each other when the resultant deformation effects are considered.

TABLE 4.—DISTRIBUTION OF APPLIED MOMENT OF 1,000 FT-LB AT JOINT B, FIG. 2 (CLOCKWISE)

Description	JOINT AND MEMBER (SEE FIG. 2(*a*))									
	A	B		C			D	E		F
	AB	BA	BC	CB	CD	CE	DC	EC	EF	FE
Distribution Factor		0.333	0.667	0.40	0.20	0.40		0.727	0.273	
Moment distributed		333	667							
Moment carried over				333						
Moment distributed				−133	−67	−133				
Moment carried over			−67					−67		
Moment distributed		22	45					49	18	
Moment carried over				22		24				
Moment distributed				−18	−10	−18				
Moment carried over			−9					−9		
Moment distributed		3	6					7	2	
Final moment	179	358	642	204	−77	−127	−38.5	−20	20	0

Mr. Hall presents an interesting and instructive discussion of the determination of the relative *R*-values for members of a quadrangle by reference to the instantaneous center of rotation. For the same purpose, Messrs. McGee and Bergman use a convenient semi-graphical method which is familiar to most structural analysts (3*c*). The question by Mr. Bergman concerning the analysis of frames having members of varying cross section is answered by Messrs. McGee and Blog and is given further consideration in this closure.

To clarify the general questions raised and problems suggested in the discussions, the following comments will deal with (*a*) the treatment of moving loads, and (*b*) the presentation of the general slope-deflection equation for members having varying moments of inertia in a form convenient for use in either the slope-deflection or moment distribution method of analysis. The part concerning moving loads will include the general solution by moment distribution and a special solution by slope deflection (applicable to continuous beams and one-story frames) which accomplishes the same purpose as the traverse method (17). The part dealing with varying moment of inertia will include a special solution by slope deflection, similar to that used for frames having members of constant section, of the frame analyzed by Mr. McGee.

Moving Loads.—In the solution of problems involving moving loads, it is recommended that the effect of an applied moment at each joint on all members of the frame be determined at the outset, either by moment distribution or by slope deflection. The resultant effects of different combinations of loads, both with and without sidesway, can then be determined readily by proportion and superposition without additional calculations involving the balancing of moments at the joints.

General Solution by Moment Distribution.—Example 2 of the paper has been selected to illustrate the foregoing procedure, using moment distribution. Table 4(a) shows the distribution of an applied moment of 1,000 ft-lb (assumed) at joint B (Fig. 2). Similarly, effects of applied moments of 1,000 ft-lb each at joints C and E are determined, and a summary of the results obtained is given in Table 5. From the values in Table 5 (called flexure factors by Mr. Stewart (17)), the distribution of a required balancing moment at any joint can be determined by simple proportion. The balancing moment may be either a numerical value, or a function of unknown displacements, as explained in the paper. After all joints are balanced, steps 4 and 5 give a solution for each critical position of the loads. If influence lines are desired, several solutions for varying positions of a unit load must be made.

TABLE 5.—EFFECTS OF A MOMENT OF 1,000 FT-LB APPLIED AT JOINTS B, C, AND E, FIG. 2. (CLOCKWISE)

Joint	MEMBER (SEE FIG. 2(a))									
	AB	BA	BC	CB	CD	CE	DC	EC	EF	FE
B	179	358	642	204	−77	−127	−38.5	−20	20	0
C	−38	−76	76	388	231	381	115	63	−63	0
E	13.5	27	−27	−141	−83	224	−41.5	704	296	0

Special Solution by Slope Deflection.—A special solution by slope deflection of continuous beams and one-story frames, which accomplishes the same purpose as the traverse method (17), is very useful. Example 2 of the paper also has been chosen to demonstrate this application of slope deflection.

For the purpose of determining the effects of an applied moment at any joint, the slope-deflection equation may be written conveniently in the following form, dropping the constant factor 2 E:

$$M\ (\text{near}) = K\ [2\ \theta\ (\text{near}) + \theta\ (\text{far})] \ldots\ldots\ldots\ldots\ldots (88)$$

In Fig. 33(*a*), it is first assumed that the rotation angle at joint F is equal to unity. By applying Eq. 88 successively to the several members of the frame, the following simple calculations are made. First assume $\theta_F = 1$; then:

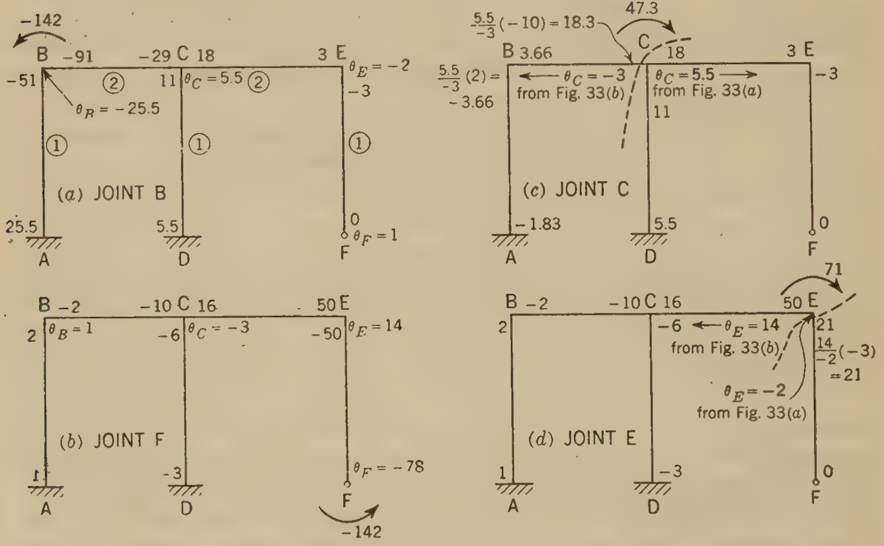
$$M_{FE} = 1\,(2\,\theta_F + \theta_E) = 0; \quad \theta_E = -2$$
$$M_{EF} = 1\,(2\,\theta_E + \theta_F) = -4 + 1 = -3$$

$$M_{EC} = 2\,(2\,\theta_E + \theta_C) = 3; \quad \theta_C = 1.5 + 4 = 5.5$$
$$M_{CD} = 1\,(2\,\theta_C + 0) = 11$$
$$M_{CE} = 2\,(2\,\theta_C + \theta_E) = 2\,(11 - 2) = 18$$

$$M_{CB} = 2\,(2\,\theta_C + \theta_B) = 2\,(11 + \theta_B) = -29; \quad \theta_B = -14.5 - 11 = -25.5$$
$$M_{BC} = 2\,(2\,\theta_B + \theta_C) = 2\,(-51 + 5.5) = -91$$

$$M_{BA} = 1\,(2\,\theta_B + 0) = -51$$
$$M_{AB} = 1\,(0 + \theta_B) = -25.5$$

The moment values thus obtained are shown at the ends of the members in Fig. 33(*a*). It is obvious that an applied moment of −142 ft-lb at joint B will induce these moments throughout the frame.



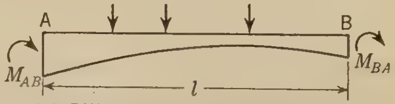
−142
B −91
−29 C 18
3 E
−51
②
11
$\theta_C = 5.5$
②
$\theta_E = -2$
−3
$\theta_B = -25.5$
①
①
①
(*a*) JOINT B
25.5
5.5
0
$\theta_F = 1$
A
D
F
$\frac{5.5}{-3}(-10) = 18.3$
47.3
B 3.66
C
18
3 E
$\frac{5.5}{-3}(2) =$
−3.66
$\theta_C = -3$ from Fig. 33(*b*)
$\theta_C = 5.5$ from Fig. 33(*a*)
−3
11
(*c*) JOINT C
−1.83
5.5
0
A
D
F
B −2
−10 C 16
50 E
2
$\theta_B = 1$
−6
$\theta_C = -3$
−50
$\theta_E = 14$
(*b*) JOINT F
1
−3
$\theta_F = -78$
A
D
F
−142
71
B −2
−10 C 16
50 E
2
−6
$\theta_E = 14$ from Fig. 33(*b*)
21
$\frac{14}{-2}(-3)$
= 21
$\theta_E = -2$ from Fig. 33(*a*)
(*d*) JOINT E
1
−3
0
A
D
F

FIG. 33

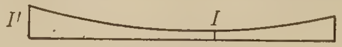
Similarly, if it is assumed that the rotation angle at joint B is equal to unity, the moment values shown in Fig. 33(*b*) are obtained. It will be noted that a moment of −142 ft-lb at joint F is required to produce unit rotation at joint B, whereas (see Fig. 33(*a*)) a moment of −142 ft-lb at joint B produces unit rotation at F. This is in agreement with Maxwell's law of reciprocity of displacement and constitutes a valuable check on the calculations.

From the moment values shown in Figs. 33(*a*) and 33(*b*), the effects of an applied moment at either joint C or joint E can be determined readily as shown in Figs. 33(*c*) and 33(*d*).

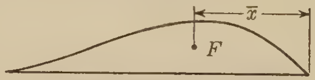
Varying Moment of Inertia.—In the paper, it was stated that the method of procedure therein described is general and can be applied with little additional labor to the analysis of frames having members of varying section after fixed-end moments, carry-over factors, and stiffness values have been evaluated. Mr. McGee has shown in an excellent manner the application of the method of procedure to a two-bay, saw-toothed structure with tapering members. Inasmuch as he used the traverse method in balancing moments at the joints, it is considered desirable to present the general slope-deflection equation for members having varying moments of inertia in a form convenient for use in either the slope-deflection or moment distribution method of analysis. Reference to Fig. 34 will assist in making clear the meaning of the symbols.



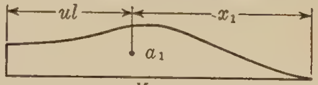
(*a*) BEAM AB OF VARYING SECTION



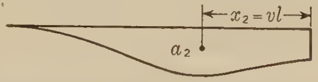
(*b*) *I*-DIAGRAM



(*c*) SIMPLE BEAM $\frac{M}{I}$-DIAGRAM FOR TRANSVERSE LOADS



(*d*) SIMPLE BEAM $\frac{M}{I}$-DIAGRAM FOR $M_{AB} = 1$



(*e*) SIMPLE BEAM $\frac{M}{I}$-DIAGRAM FOR $M_{BA} = 1$

FIG. 34.—TYPICAL $\frac{M}{I}$-DIAGRAMS

General Slope-Deflection Equations.—The general slope-deflection equations for members having varying moments of inertia may be written in the following form:

$$M_{AB} = S_A (\theta_A + c_A \theta_B) - S_A (1 + c_A) R + C_{AB} \quad (89a)$$

$$M_{BA} = S_B (\theta_B + c_B \theta_A) - S_B (1 + c_B) R + C_{BA} \quad (89b)$$

in which (see Fig. 34): S_A is the rotational stiffness at joint A with joint B fixed against rotation; S_B is the rotational stiffness at joint B with joint A fixed against rotation; c_A is the carry-over factor from joint A to joint B; and c_B is the carry-over factor from joint B to joint A. For members of constant section, $S_A = S_B = 4\,E\,K$ and $c_A = c_B = \frac{1}{2}$, which values, when substituted in Eqs. 89, yield the familiar form of Eqs. 2.

Stiffnesses—Far End Fixed.—The rotational stiffnesses are defined as follows:

$$S_A = \frac{E\,(l - x_2)}{a_1\,(x_1 - x_2)} = \frac{E\,I}{l}\,\frac{1 - v}{p\,(1 - u - v)} = \frac{E\,I}{l}\,\alpha \quad (90a)$$

$$S_B = \frac{E\,x_1}{a_2\,(x_1 - x_2)} = \frac{E\,I}{l}\,\frac{1 - u}{q\,(1 - u - v)} = \frac{E\,I}{l}\,\beta \quad (90b)$$

in which (21): $a_1 = \frac{p\,l}{I}$ = area of a simple-beam *M*-diagram for $M_{AB} = 1$ (assumed a positive quantity); $a_2 = \frac{q\,l}{I}$ = the area of a simple-beam *M*-diagram for $M_{BA} = 1$ (assumed a positive quantity); $x_1 = l - u\,l$ = the distance from the centroid of the $\frac{M}{I}$-diagram for M_{AB} to end B of the member; $x_2 = v\,l$ = the distance from the centroid of the $\frac{M}{I}$-diagram for M_{BA} to end B of the member; u and v are ratios introduced by Walter Ruppel, Assoc. M. Am. Soc. C. E. (21); and α and β are stiffness coefficients defined as follows:

$$\alpha = \frac{1 - v}{p\,(1 - u - v)} \qquad \text{(91}a\text{)}$$

and

$$\beta = \frac{1 - u}{q\,(1 - u - v)} \qquad \text{(91}b\text{)}$$

For members having constant section, $\alpha = \beta = 4$.

Carry-Over Factors.—The carry-over factors in Eqs. 89 are defined as follows:

$$c_A = \frac{x_2}{l - x_2} = \frac{v}{1 - v} \qquad \text{(92}a\text{)}$$

$$c_B = \frac{l - x_1}{x_1} = \frac{u}{1 - u} \qquad \text{(92}b\text{)}$$

Fixed-End Moments.—The fixed-end moments caused by transverse loads on the member AB are defined as follows:

$$C_{AB} = -\frac{F\,(\bar{x} - x_2)}{a_1\,(x_1 - x_2)} = -\frac{F\,(\bar{x} - v\,l)\,I}{p\,l^2\,(1 - u - v)} \qquad \text{(93}a\text{)}$$

$$C_{BA} = +\frac{F\,(x_1 - \bar{x})}{a_2\,(x_1 - x_2)} = +\frac{F\,(l - u\,l - \bar{x})\,I}{q\,l^2\,(1 - u - v)} \qquad \text{(93}b\text{)}$$

The fixed-end moments induced by angular displacement, R, are

$$M'_{AB} = -S_A\,(1 + c_A)\,R = -S_A\,(1 + c_A)\,\frac{\Delta}{l} \qquad \text{(94}a\text{)}$$

$$M'_{BA} = -S_B\,(1 + c_B)\,R = -S_B\,(1 + c_B)\,\frac{\Delta}{l} \qquad \text{(94}b\text{)}$$

It should be noted that I in the foregoing formulas for stiffnesses and fixed-end moments is the minimum moment of inertia of the member. A valuable check of the calculated values of stiffnesses and carry-over factors is given by the following relation:

$$S_A\,c_A = S_B\,c_B \qquad \text{(95)}$$

Special Slope-Deflection Equation.—If one end (say, end B) of the member is free to rotate so that $M_{BA} = 0$, the following special slope-deflection equation can be derived from Eqs. 89:

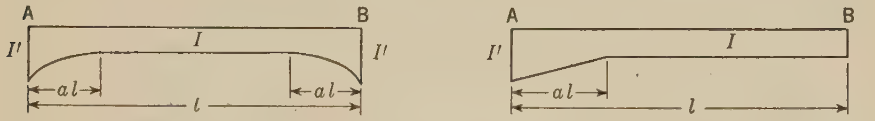
$$M_{AB} = S_A\,(1 - c_A\,c_B)\,\theta_A - S_A\,(1 - c_A\,c_B)\,R + H_{AB} \qquad \text{(96)}$$

in which H_{AB} = the fixed-end moment at end A caused by transverse loads on the member, with end B hinged—expressed as,

$$H_{AB} = C_{AB} - c_B\,C_{BA} \qquad \text{(97)}$$

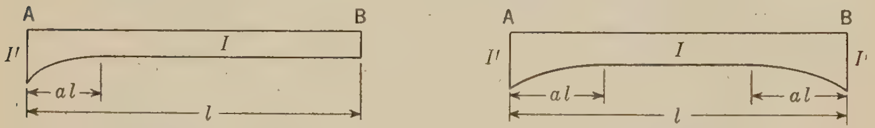
From Eq. 96, it is obvious that the rotational stiffness at end A of the member is $(1 - c_A\,c_B)$ times the rotational stiffness at end A when end B is

TABLE 6.—Beam Coefficients for Members of Varying Moment of Inertia



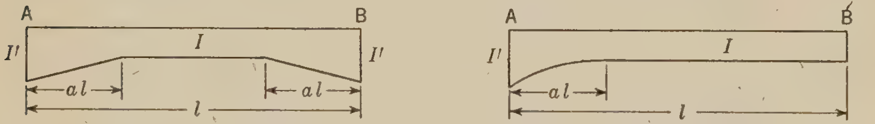
(*a*) SYMMETRICAL, SHARPLY CURVED HAUNCHES

(*d*) UNSYMMETRICAL, STRAIGHT HAUNCH



(*b*) UNSYMMETRICAL, SHARPLY CURVED HAUNCH

(*e*) SYMMETRICAL, PARABOLIC HAUNCHES



(*c*) SYMMETRICAL, STRAIGHT HAUNCHES

(*f*) UNSYMMETRICAL, PARABOLIC HAUNCH

$\left(S_A = \alpha \frac{EI}{l};\ \text{and}\ S_b = \beta \frac{EI}{l}\right)$

Quantity	Values of $b = I/I'$												
	1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02
(*a*) Symmetrical, Sharply-Curved Haunches													
$a = 0.50$—													
$c_A = c_B$	0.500	0.548	0.607	0.636	0.653	0.664	0.673	0.676	0.686	0.692	0.694	0.701	0.703
$\alpha = \beta$	4.00	5.54	7.85	9.17	10.00	10.63	11.10	11.50	12.01	12.33	12.60	13.00	13.23
$1 - c_A c_B$	0.75	0.700	0.631	0.595	0.574	0.560	0.547	0.542	0.529	0.521	0.518	0.509	0.506
$a = 0.40$—													
$c_A = c_B$	0.500	0.553	0.612	0.639	0.653	0.661	0.669	0.674	0.684	0.687	0.688	0.695	0.698
$\alpha = \beta$	4.00	5.32	7.18	8.16	8.73	9.12	9.45	9.75	10.12	10.28	10.43	10.70	10.88
$1 - c_A c_B$	0.75	0.694	0.625	0.592	0.574	0.563	0.553	0.545	0.533	0.528	0.526	0.517	0.513
$a = 0.35$—													
$c_A = c_B$	0.500	0.553	0.610	0.650	0.647	0.656	0.661	0.667	0.675	0.676	0.680	0.684	0.686
$\alpha = \beta$	4.00	5.19	6.79	7.59	8.08	8.38	8.63	8.85	9.19	9.30	9.43	9.55	9.70
$1 - c_A c_B$	0.75	0.694	0.628	0.578	0.581	0.569	0.563	0.556	0.545	0.542	0.537	0.533	0.528
$a = 0.30$—													
$c_A = c_B$	0.500	0.553	0.605	0.626	0.636	0.644	0.650	0.655	0.658	0.663	0.667	0.667	0.669
$\alpha = \beta$	4.00	5.07	6.41	7.03	7.38	7.65	7.84	8.02	8.15	8.30	8.43	8.45	8.57
$1 - c_A c_B$	0.75	0.694	0.634	0.608	0.595	0.585	0.578	0.570	0.567	0.560	0.556	0.556	0.553
$a = 0.25$—													
$c_A = c_B$	0.500	0.548	0.595	0.612	0.623	0.629	0.634	0.636	0.642	0.644	0.644	0.647	0.650
$\alpha = \beta$	4.00	5.00	5.98	6.46	6.74	6.90	7.04	7.15	7.31	7.38	7.40	7.48	7.56
$1 - c_A c_B$	0.75	0.700	0.646	0.625	0.612	0.604	0.599	0.595	0.588	0.585	0.585	0.581	0.578
$a = 0.20$—													
$c_A = c_B$	0.500	0.543	0.582	0.597	0.605	0.610	0.612	0.618	0.620	0.623	0.623	0.626	0.626
$\alpha = \beta$	4.00	4.76	5.57	5.92	6.10	6.23	6.30	6.41	6.49	6.55	6.57	6.64	6.65
$1 - c_A c_B$	0.75	0.705	0.661	0.643	0.634	0.628	0.625	0.618	0.615	0.612	0.612	0.608	0.608
$a = 0.15$—													
$c_A = c_B$	0.500	0.536	0.552	0.577	0.584	0.587	0.590	0.592	0.594	0.594	0.597	0.597	0.600
$\alpha = \beta$	4.00	4.39	5.16	5.37	5.53	5.59	5.64	5.69	5.74	5.75	5.81	5.82	5.86
$1 - c_A c_B$	0.75	0.712	0.695	0.667	0.658	0.655	0.652	0.649	0.647	0.647	0.644	0.643	0.640

TABLE 6.—(*Continued*)

QUANTITY	VALUES OF $b = I/I'$												
	1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02
(*b*) UNSYMMETRICAL, SHARPLY-CURVED HAUNCH													
$a = 1.00$—													
c_A	0.500	0.445	0.394	0.375	0.364	0.358	0.355	0.351	0.346	0.344	0.342	0.340	0.337
c_B	0.500	0.572	0.684	0.750	0.792	0.825	0.842	0.874	0.895	0.914	0.929	0.940	0.960
α	4.00	5.74	8.64	10.47	11.60	12.53	13.17	13.78	14.70	15.13	15.67	16.15	16.75
β	4.00	4.48	4.98	5.22	5.35	5.46	5.53	5.53	5.70	5.72	5.77	5.84	5.87
$1 - c_A c_B$	0.75	0.746	0.730	0.718	0.712	0.704	0.701	0.693	0.690	0.686	0.682	0.680	0.676
$a = 0.50$—													
c_A	0.500	0.468	0.445	0.436	0.432	0.429	0.428	0.426	0.424	0.424	0.422	0.422	0.420
c_B	0.500	0.587	0.692	0.739	0.765	0.785	0.794	0.810	0.823	0.830	0.840	0.844	0.851
α	4.00	5.26	6.91	7.72	8.19	8.51	8.74	9.00	9.20	9.38	9.49	9.58	9.74
β	4.00	4.19	4.43	4.54	4.63	4.65	4.70	4.72	4.74	4.79	4.78	4.80	4.82
$1 - c_A c_B$	0.75	0.725	0.692	0.678	0.669	0.663	0.660	0.654	0.651	0.649	0.645	0.644	0.642
$a = 0.40$—													
c_A	0.500	0.477	0.460	0.455	0.451	0.450	0.449	0.446	0.446	0.445	0.445	0.445	0.445
c_B	0.500	0.580	0.670	0.705	0.728	0.739	0.750	0.761	0.770	0.775	0.778	0.785	0.792
α	4.00	5.07	6.36	6.96	7.30	7.49	7.69	7.85	8.01	8.08	8.17	8.27	8.40
β	4.00	4.16	4.38	4.49	4.52	4.55	4.60	4.61	4.65	4.63	4.67	4.70	4.71
$1 - c_A c_B$	0.75	0.723	0.692	0.679	0.672	0.668	0.664	0.660	0.656	0.655	0.654	0.650	0.647
$a = 0.35$—													
c_A	0.500	0.481	0.468	0.464	0.462	0.460	0.460	0.457	0.457	0.455	0.455	0.455	0.455
c_B	0.500	0.575	0.654	0.686	0.702	0.716	0.724	0.730	0.740	0.746	0.748	0.754	0.759
α	4.00	4.97	6.08	6.57	6.83	7.02	7.15	7.26	7.42	7.49	7.54	7.65	7.72
β	4.00	4.16	4.35	4.44	4.49	4.50	4.54	4.55	4.58	4.58	4.60	4.62	4.62
$1 - c_A c_B$	0.75	0.723	0.694	0.682	0.676	0.671	0.667	0.666	0.662	0.660	0.660	0.657	0.655
$a = 0.30$—													
c_A	0.500	0.486	0.475	0.473	0.470	0.468	0.468	0.468	0.466	0.466	0.466	0.466	0.466
c_B	0.500	0.567	0.636	0.664	0.680	0.689	0.695	0.701	0.710	0.714	0.716	0.717	0.720
α	4.00	4.85	5.79	6.17	6.43	6.55	6.65	6.75	6.85	6.88	6.95	7.00	7.05
β	4.00	4.15	4.32	4.40	4.44	4.45	4.47	4.49	4.50	4.50	4.52	4.55	4.55
$1 - c_A c_B$	0.75	0.724	0.698	0.694	0.680	0.678	0.675	0.672	0.669	0.667	0.666	0.665	0.664
$a = 0.25$—													
c_A	0.500	0.490	0.481	0.478	0.479	0.476	0.476	0.476	0.476	0.475	0.475	0.475	0.475
c_B	0.500	0.560	0.618	0.639	0.652	0.659	0.663	0.670	0.674	0.676	0.680	0.685	0.687
α	4.00	4.74	5.49	5.80	5.97	6.06	6.14	6.20	6.30	6.33	6.36	6.40	6.45
β	4.00	4.14	4.28	4.33	4.38	4.38	4.41	4.42	4.45	4.44	4.44	4.45	4.46
$1 - c_A c_B$	0.75	0.726	0.702	0.694	0.688	0.686	0.684	0.680	0.679	0.679	0.677	0.675	0.674
$a = 0.20$—													
c_A	0.500	0.492	0.488	0.486	0.486	0.486	0.486	0.484	0.484	0.485	0.484	0.485	0.485
c_B	0.500	0.550	0.597	0.615	0.622	0.629	0.634	0.635	0.640	0.642	0.645	0.646	0.650
α	4.00	4.60	5.19	5.42	5.54	5.60	5.66	5.71	5.75	5.78	5.83	5.85	5.88
β	4.00	4.11	4.24	4.29	4.32	4.33	4.35	4.34	4.35	4.35	4.37	4.39	4.39
$1 - c_A c_B$	0.75	0.729	0.708	0.700	0.698	0.694	0.692	0.692	0.691	0.688	0.688	0.686	0.685
$a = 0.15$—													
c_A	0.500	0.497	0.492	0.492	0.493	0.492	0.490	0.490	0.490	0.490	0.490	0.490	0.490
c_B	0.500	0.541	0.575	0.587	0.594	0.596	0.600	0.603	0.605	0.607	0.607	0.610	0.610
α	4.00	4.46	4.88	5.04	5.13	5.17	5.20	5.25	5.28	5.30	5.30	5.34	5.35
β	4.00	4.10	4.19	4.22	4.25	4.25	4.25	4.25	4.26	4.28	4.28	4.29	4.29
$1 - c_A c_B$	0.75	0.731	0.717	0.711	0.707	0.706	0.706	0.704	0.703	0.702	0.702	0.702	0.701
(*c*) SYMMETRICAL STRAIGHT HAUNCHES													
$a = 0.50$—													
$c_A = c_B$	0.500	0.547	0.613	0.650	0.675	0.695	0.713	0.730	0.754	0.766	0.782	0.805	0.835
$\alpha = \beta$	4.00	5.69	9.25	12.32	15.15	17.80	20.40	24.05	29.10	34.00	40.00	50.25	70.40
$1 - c_A c_B$	0.75	0.700	0.624	0.578	0.545	0.517	0.492	0.467	0.432	0.413	0.388	0.352	0.304
$a = 0.40$—													
$c_A = c_B$	0.500	0.557	0.631	0.673	0.683	0.724	0.739	0.759	0.785	0.797	0.815	0.835	0.859
$\alpha = \beta$	4.00	5.50	8.36	10.68	12.70	14.60	16.25	18.50	22.30	24.70	28.10	33.30	41.80
$1 - c_A c_B$	0.75	0.690	0.602	0.547	0.534	0.476	0.455	0.425	0.384	0.364	0.336	0.303	0.263
$a = 0.35$—													
$c_A = c_B$	0.500	0.557	0.631	0.673	0.683	0.721	0.735	0.754	0.776	0.789	0.805	0.818	0.843
$\alpha = \beta$	4.00	5.35	7.84	9.79	11.43	12.87	14.10	15.79	18.23	19.90	22.30	24.90	30.10
$1 - c_A c_B$	0.75	0.690	0.602	0.547	0.534	0.480	0.460	0.432	0.397	0.377	0.351	0.331	0.290
$a = 0.30$—													
$c_A = c_B$	0.500	0.555	0.626	0.667	0.689	0.710	0.720	0.739	0.757	0.766	0.779	0.795	0.811
$\alpha = \beta$	4.00	5.19	7.26	8.85	10.00	11.10	11.87	13.13	14.61	15.52	16.88	18.70	21.05
$1 - c_A c_B$	0.75	0.692	0.608	0.556	0.525	0.496	0.481	0.455	0.426	0.413	0.394	0.368	0.342

TABLE 6.—(*Continued*)

QUANTITY	VALUES OF $b = I/I'$												
	1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02
(*c*) (*Continued*)													
$a = 0.25$—													
$c_A = c_B$	0.500	0.553	0.618	0.650	0.672	0.687	0.700	0.712	0.730	0.739	0.748	0.760	0.774
$\alpha = \beta$	4.00	5.03	6.71	7.81	8.67	9.32	9.96	10.62	11.65	12.25	12.94	13.88	15.02
$1 - c_A c_B$	0.75	0.694	0.618	0.578	0.548	0.528	0.510	0.493	0.466	0.454	0.441	0.422	0.401
$a = 0.20$—													
$c_A = c_B$	0.500	0.547	0.603	0.631	0.647	0.660	0.670	0.680	0.692	0.701	0.710	0.717	0.730
$\alpha = \beta$	4.00	4.86	6.10	6.92	7.45	7.90	8.22	8.65	9.17	9.55	9.94	10.40	11.10
$1 - c_A c_B$	0.75	0.700	0.636	0.602	0.581	0.564	0.551	0.537	0.521	0.508	0.496	0.485	0.466
$a = 0.15$—													
$c_A = c_B$	0.500	0.538	0.581	0.605	0.618	0.626	0.634	0.642	0.650	0.655	0.660	0.669	0.677
$\alpha = \beta$	4.00	4.65	5.51	6.02	6.37	6.60	6.81	7.05	7.31	7.48	7.68	7.95	8.26
$1 - c_A c_B$	0.75	0.710	0.662	0.634	0.618	0.608	0.599	0.588	0.578	0.570	0.564	0.552	0.541
(*d*) UNSYMMETRICAL, STRAIGHT HAUNCH													
$a = 1.00$—													
c_A	0.500	0.441	0.367	0.333	0.309	0.292	0.279	0.264	0.244	0.232	0.218	0.202	0.181
c_B	0.500	0.567	0.674	0.746	0.794	0.844	0.880	0.930	1.00	1.04	1.090	1.180	1.307
α	4.00	5.86	9.95	13.56	16.85	20.05	23.00	27.40	34.30	39.60	46.70	59.30	81.50
β	4.00	4.55	5.44	6.07	6.52	6.94	7.30	7.74	8.38	8.80	9.37	10.10	11.35
$1 - c_A c_B$	0.75	0.749	0.752	0.752	0.755	0.754	0.755	0.754	0.756	0.758	0.762	0.763	0.764
$a = 0.50$—													
c_A	0.500	0.464	0.422	0.404	0.393	0.383	0.377	0.370	0.363	0.357	0.351	0.345	0.337
c_B	0.500	0.595	0.739	0.835	0.905	0.960	1.005	1.060	1.133	1.175	1.230	1.290	1.380
α	4.00	5.40	7.88	9.75	11.27	12.50	13.63	15.09	17.02	18.25	19.85	22.00	25.10
β	4.00	4.21	4.50	4.74	4.89	5.00	5.12	5.26	5.45	5.54	5.69	5.86	6.11
$1 - c_A c_B$	0.75	0.724	0.688	0.663	0.644	0.632	0.621	0.607	0.589	0.580	0.569	0.555	0.535
$a = 0.40$—													
c_A	0.500	0.473	0.445	0.431	0.423	0.416	0.412	0.406	0.401	0.396	0.393	0.389	0.384
c_B	0.500	0.587	0.715	0.790	0.845	0.888	0.918	0.955	1.010	1.035	0.071	1.115	1.162
α	4.00	5.19	7.19	8.48	9.55	10.40	11.07	11.90	13.03	13.80	14.62	15.73	17.20
β	4.00	4.18	4.47	4.63	4.79	4.89	4.97	5.08	5.20	5.29	5.36	5.50	5.65
$1 - c_A c_B$	0.75	0.722	0.681	0.659	0.643	0.630	0.622	0.612	0.596	0.590	0.579	0.565	0.554
$a = 0.35$—													
c_A	0.500	0.479	0.455	0.444	0.439	0.433	0.429	0.424	0.420	0.419	0.414	0.410	0.405
c_B	0.500	0.581	0.695	0.762	0.810	0.840	0.869	0.900	0.940	0.965	0.990	1.020	1.062
α	4.00	5.08	6.77	7.90	8.73	9.30	9.86	10.50	11.33	11.90	12.50	13.15	14.20
β	4.00	4.19	4.44	4.63	4.73	4.79	4.86	4.96	5.08	5.15	5.21	5.30	5.40
$1 - c_A c_B$	0.75	0.722	0.684	0.662	0.645	0.636	0.628	0.619	0.604	0.595	0.590	0.582	0.570
$a = 0.30$—													
c_A	0.500	0.484	0.466	0.457	0.454	0.449	0.445	0.444	0.439	0.436	0.434	0.430	0.426
c_B	0.500	0.575	0.675	0.730	0.770	0.794	0.819	0.844	0.875	0.892	0.913	0.933	0.970
α	4.00	4.95	6.40	7.26	7.94	8.39	8.78	9.23	9.81	10.17	10.60	11.08	11.80
β	4.00	4.17	4.41	4.55	4.66	4.72	4.85	4.85	4.92	4.97	5.05	5.11	5.20
$1 - c_A c_B$	0.75	0.722	0.686	0.666	0.651	0.644	0.636	0.626	0.616	0.610	0.604	0.599	0.586
$a = 0.25$—													
c_A	0.500	0.489	0.475	0.468	0.466	0.462	0.460	0.457	0.454	0.454	0.452	0.449	0.446
c_B	0.500	0.566	0.649	0.695	0.724	0.746	0.762	0.780	0.807	0.818	0.834	0.852	0.875
α	4.00	4.83	5.96	6.64	7.11	7.42	7.72	8.05	8.44	8.67	8.96	9.30	9.74
β	4.00	4.15	4.36	4.46	4.56	4.61	4.66	4.71	4.75	4.81	4.85	4.91	4.96
$1 - c_A c_B$	0.75	0.723	0.692	0.675	0.662	0.655	0.650	0.643	0.634	0.629	0.622	0.618	0.609
$a = 0.20$—													
c_A	0.500	0.492	0.485	0.479	0.476	0.475	0.473	0.473	0.468	0.468	0.466	0.465	0.466
c_B	0.500	0.555	0.622	0.659	0.681	0.700	0.710	0.724	0.740	0.748	0.761	0.775	0.790
α	4.00	4.67	5.55	6.05	6.36	6.60	6.78	7.02	7.27	7.42	7.60	7.84	8.09
β	4.00	4.12	4.31	4.39	4.45	4.50	4.53	4.58	4.60	4.64	4.65	4.70	4.74
$1 - c_A c_B$	0.75	0.726	0.698	0.685	0.675	0.668	0.664	0.658	0.654	0.650	0.645	0.640	0.631
$a = 0.15$—													
c_A	0.500	0.495	0.490	0.488	0.486	0.484	0.485	0.484	0.481	0.481	0.481	0.479	0.479
c_B	0.500	0.544	0.595	0.620	0.636	0.646	0.655	0.663	0.674	0.685	0.689	0.699	0.705
α	4.00	4.50	5.15	5.49	5.69	5.84	5.94	6.08	6.23	6.35	6.45	6.58	6.73
β	4.00	4.10	4.24	4.31	4.35	4.38	4.40	4.43	4.45	4.47	4.50	4.51	4.55
$1 - c_A c_B$	0.75	0.731	0.709	0.698	0.690	0.687	0.682	0.680	0.675	0.670	0.668	0.666	0.662
(*e*) SYMMETRICAL, PARABOLIC HAUNCHES													
$a = 0.50$—													
$c_A = c_B$	0.500	0.550	0.618	0.652	0.680	0.698	0.713	0.730	0.750	0.762	0.779	0.798	0.821
$\alpha = \beta$	4.00	5.25	7.59	9.35	10.94	12.25	13.50	15.24	17.70	19.40	22.00	25.60	32.30
$1 - c_A c_B$	0.75	0.697	0.618	0.575	0.537	0.512	0.492	0.467	0.437	0.419	0.394	0.364	0.325

TABLE 6.—(*Continued*)

QUANTITY	VALUES OF $b = I/I'$												
	1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02
(*e*) (*Continued*)													
$a = 0.40$—													
$c_A = c_B$	0.500	0.550	0.612	0.647	0.673	0.689	0.703	0.718	0.739	0.748	0.762	0.779	0.798
$\alpha = \beta$	4.00	5.06	6.86	8.18	9.30	10.20	11.03	12.03	13.60	14.50	15.90	17.75	20.50
$1 - c_A c_B$	0.75	0.697	0.625	0.582	0.548	0.526	0.505	0.485	0.455	0.460	0.419	0.394	0.364
$a = 0.35$—													
$c_A = c_B$	0.500	0.548	0.607	0.639	0.664	0.676	0.689	0.703	0.720	0.732	0.743	0.758	0.779
$\alpha = \beta$	4.00	4.95	6.50	7.58	8.50	9.20	9.75	10.53	11.60	12.40	13.13	14.40	16.54
$1 - c_A c_B$	0.75	0.700	0.631	0.592	0.560	0.542	0.525	0.506	0.482	0.464	0.448	0.425	0.394
$a = 0.30$—													
$c_A = c_B$	0.500	0.545	0.600	0.629	0.646	0.660	0.675	0.687	0.701	0.710	0.720	0.732	0.747
$\alpha = \beta$	4.00	4.84	6.14	6.99	7.65	8.15	8.70	9.22	9.94	10.40	11.07	11.86	12.98
$1 - c_A c_B$	0.75	0.703	0.640	0.604	0.582	0.564	0.545	0.528	0.508	0.496	0.482	0.465	0.441
$a = 0.25$—													
$c_A = c_B$	0.500	0.541	0.590	0.615	0.631	0.644	0.652	0.660	0.674	0.684	0.691	0.703	0.715
$\alpha = \beta$	4.00	4.71	5.77	6.43	6.60	7.31	7.61	7.94	8.45	8.80	9.20	9.73	10.35
$1 - c_A c_B$	0.75	0.708	0.652	0.622	0.602	0.585	0.575	0.564	0.546	0.533	0.522	0.506	0.488
$a = 0.20$—													
$c_A = c_B$	0.500	0.534	0.577	0.597	0.610	0.620	0.625	0.636	0.646	0.652	0.658	0.667	0.676
$\alpha = \beta$	4.00	4.56	5.39	5.87	6.20	6.46	6.64	6.92	7.26	7.44	7.64	7.94	8.38
$1 - c_A c_B$	0.75	0.715	0.667	0.643	0.628	0.616	0.609	0.595	0.582	0.574	0.567	0.556	0.542
$a = 0.15$—													
$c_A = c_B$	0.500	0.529	0.560	0.577	0.587	0.595	0.600	0.605	0.613	0.618	0.623	0.629	0.639
$\alpha = \beta$	4.00	4.44	5.01	5.34	5.55	5.73	5.85	5.99	6.17	6.31	6.45	6.60	6.88
$1 - c_A c_B$	0.75	0.720	0.686	0.667	0.656	0.646	0.640	0.634	0.624	0.618	0.612	0.604	0.592
(*f*) UNSYMMETRICAL, PARABOLIC HAUNCH													
$a = 1.00$—													
c_A	0.500	0.451	0.394	0.364	0.346	0.331	0.320	0.307	0.290	0.280	0.269	0.256	0.238
c_B	0.500	0.581	0.714	0.796	0.864	0.918	0.970	1.030	1.112	1.162	1.233	1.333	1.485
α	4.00	5.51	8.55	10.92	13.10	15.00	16.75	19.25	22.85	25.50	29.00	34.70	44.60
β	4.00	4.30	4.71	5.00	5.24	5.40	5.55	5.74	5.96	6.15	6.36	6.66	7.15
$1 - c_A c_B$	0.75	0.738	0.718	0.710	0.701	0.696	0.690	0.682	0.678	0.675	0.668	0.659	0.647
$a = 0.50$—													
c_A	0.500	0.479	0.454	0.441	0.432	0.426	0.422	0.416	0.405	0.406	0.400	0.394	0.386
c_B	0.500	0.578	0.685	0.747	0.794	0.830	0.860	0.895	0.935	0.970	1.00	1.047	1.097
α	4.00	5.03	6.64	7.74	8.60	9.29	9.85	10.60	11.20	12.20	12.97	14.10	15.60
β	4.00	4.16	4.40	4.56	4.67	4.78	4.83	4.92	4.85	5.11	5.19	5.30	5.46
$1 - c_A c_B$	0.75	0.724	0.690	0.670	0.656	0.646	0.637	0.627	0.621	0.606	0.600	0.588	0.576
$a = 0.40$—													
c_A	0.500	0.486	0.468	0.460	0.454	0.450	0.445	0.441	0.436	0.435	0.430	0.426	0.421
c_B	0.500	0.567	0.659	0.710	0.746	0.775	0.794	0.822	0.856	0.874	0.895	0.929	0.970
α	4.00	4.84	6.15	6.95	7.52	8.02	8.40	8.86	9.54	9.90	10.40	11.02	11.90
β	4.00	4.15	4.36	4.49	4.58	4.65	4.71	4.78	4.85	4.92	4.99	5.05	5.17
$1 - c_A c_B$	0.75	0.724	0.692	0.674	0.658	0.651	0.647	0.637	0.626	0.620	0.615	0.604	0.591
$a = 0.35$—													
c_A	0.500	0.488	0.475	0.468	0.462	0.460	0.455	0.454	0.450	0.446	0.445	0.441	0.436
c_B	0.500	0.562	0.642	0.689	0.717	0.744	0.761	0.780	0.810	0.825	0.844	0.870	0.899
α	4.00	4.76	5.87	6.57	7.02	7.42	7.72	8.09	8.55	8.97	9.27	9.70	10.35
β	4.00	4.14	4.34	4.46	4.52	4.59	4.64	4.70	4.75	4.80	4.87	4.94	5.02
$1 - c_A c_B$	0.75	0.726	0.695	0.678	0.668	0.658	0.655	0.646	0.636	0.632	0.624	0.616	0.608
$a = 0.30$—													
c_A	0.500	0.490	0.481	0.475	0.470	0.468	0.466	0.465	0.462	0.460	0.457	0.455	0.451
c_B	0.500	0.555	0.625	0.663	0.689	0.710	0.725	0.744	0.762	0.775	0.793	0.810	0.835
α	4.00	4.66	5.60	6.15	6.53	6.83	7.08	7.35	7.71	7.92	8.23	8.56	9.00
β	4.00	4.12	4.30	4.40	4.46	4.51	4.55	4.60	4.67	4.70	4.74	4.81	4.86
$1 - c_A c_B$	0.75	0.728	0.698	0.685	0.676	0.668	0.662	0.654	0.648	0.643	0.637	0.631	0.623
$a = 0.25$—													
c_A	0.500	0.492	0.485	0.481	0.479	0.476	0.475	0.475	0.474	0.470	0.468	0.465	0.465
c_B	0.500	0.548	0.607	0.639	0.659	0.674	0.688	0.701	0.717	0.728	0.740	0.754	0.775
α	4.00	4.57	5.33	5.75	6.04	6.28	6.45	6.67	6.94	7.06	7.28	7.52	7.82
β	4.00	4.11	4.27	4.35	4.39	4.45	4.46	4.51	4.56	4.58	4.60	4.64	4.69
$1 - c_A c_B$	0.75	0.730	0.706	0.692	0.685	0.679	0.673	0.667	0.660	0.658	0.654	0.650	0.640
$a = 0.20$—													
c_A	0.500	0.495	0.490	0.488	0.486	0.486	0.484	0.484	0.481	0.481	0.479	0.479	0.476
c_B	0.500	0.540	0.587	0.613	0.629	0.639	0.650	0.660	0.674	0.680	0.688	0.701	0.716
α	4.00	4.45	5.06	5.38	5.57	5.75	5.88	6.05	6.21	6.32	6.45	6.64	6.85
β	4.00	4.10	4.21	4.27	4.31	4.36	4.39	4.43	4.45	4.47	4.49	4.53	4.56
$1 - c_A c_B$	0.75	0.732	0.712	0.701	0.694	0.689	0.686	0.680	0.676	0.672	0.670	0.664	0.658
$a = 0.15$—													
c_A	0.500	0.496	0.495	0.492	0.492	0.492	0.490	0.490	0.488	0.488	0.487	0.486	0.485
c_B	0.500	0.531	0.567	0.585	0.596	0.605	0.610	0.618	0.630	0.634	0.639	0.646	0.655
α	4.00	4.35	4.76	5.00	5.16	5.28	5.34	5.44	5.56	5.64	5.71	5.81	5.95
β	4.00	4.07	4.17	4.21	4.25	4.27	4.29	4.30	4.31	4.34	4.35	4.37	4.40
$1 - c_A c_B$	0.75	0.736	0.719	0.712	0.706	0.702	0.701	0.697	0.693	0.691	0.688	0.686	0.682

fixed against rotation. Also, the fixed-end moment at end A, which is induced by angular displacement R, is equal to $-S_A\,(1 - c_A\,c_B)\,R$.

Six tables giving carry-over factors and stiffness coefficients, and also values of the factor $(1 - c_A\,c_B)$, for members having varying moment of inertia have been prepared from the tables compiled by Mr. Ruppel (21) and are given in Tables 6(*a*) to 6(*f*), inclusive.

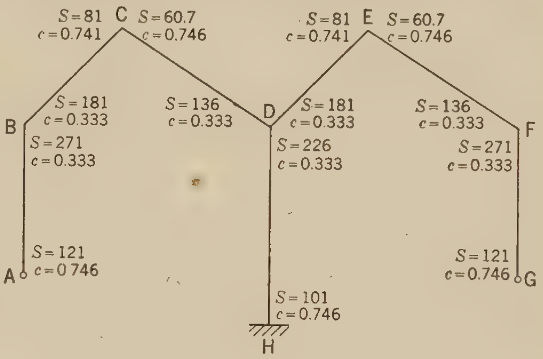


FIG. 35.—TWO-BAY, SAW-TOOTHED FRAME

Special Solution by Slope Deflection.—A special technique in the use of slope deflection, similar to that discussed previously herein for frames having members of constant section, will be applied to the frame analyzed by Mr. McGee. For the purpose of determining the effects of an applied moment at any joint of a one-story frame or continuous beam, the general slope-deflection equation for members having varying moments of inertia may be written conveniently in the following form:

$$M\,(\text{near}) = S\,(\text{near})\,[\theta\,(\text{near}) + c\,(\text{near})\,\theta\,(\text{far})] \quad \ldots\ldots\ldots (98)$$

Fig. 35 represents the frame analyzed by Mr. McGee with relative values of the stiffnesses and values of the carry-over factors (taken from Table 6(*d*)) recorded at the ends of the appropriate members. It was first assumed that $\theta_G = 0.427$, in order to show the relation between the slope-deflection and traverse procedures (ordinarily θ_G would be assumed equal to unity).

By applying Eq. 98 successively to the several members of the frame, the following simple calculations are made. First assume $\theta_G = 0.427$; then:

$$M_{GF} = 121\,(0.427 + 0.746\,\theta_F) = 0; \quad \theta_F = -\,0.573$$
$$M_{FG} = 271\,[-\,0.573 + (0.333)\,(0.427)] = -\,117$$

$$M_{FE} = 136\,[-\,0.573 + 0.333\,\theta_E] = 117; \quad \theta_E = 4.30$$
$$M_{EF} = 60.7\,[4.30 + (0.746)\,(-\,0.573)] = 236$$

$$M_{ED} = 81\,[4.30 + 0.746\,\theta_D] = -\,236; \quad \theta_D = -\,9.66$$
$$M_{DE} = 181\,[-\,9.66 + (0.333)\,(4.30)] = -\,1{,}490$$

$$M_{DH} = 226\,[-9.66 - 0] = -\,2{,}180$$
$$M_{HD} = 101\,[0 + (0.746)\,(-\,9.66)] = -\,730$$

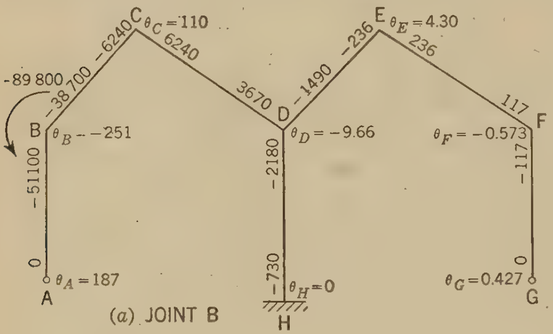
$$M_{DC} = 136\,[-9.66 + 0.333\,\theta_C] = 3{,}670; \quad \theta_C = 110$$
$$M_{CD} = 60.7\,[110 + (0.746)\,(-9.66)] = 6{,}240$$

$$M_{CB} = 81\,[110 + 0.746\,\theta_B] = -\,6{,}240; \quad \theta_B = -\,251$$
$$M_{BC} = 181\,[-\,251 + (0.333)\,(110)] = -\,38{,}700$$

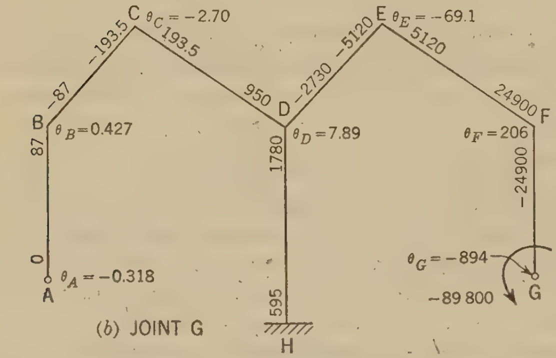
$$M_{AB} = 121\,[\theta_A + (0.746)\,(-\,251)] = 0; \quad \theta_A = 187$$
$$M_{BA} = 271\,[-\,251 + (0.333)\,(187)] = -\,51{,}100$$

The foregoing moment values are shown at the ends of the members in Fig. 36(*a*). It is obvious that an applied moment of −89,800 ft-lb at joint B will induce these moments throughout the frame, and an angular rotation at joint G equal to 0.427 (which was assumed). It will be observed that the values of θ obtained in the foregoing solution check those in Fig. 21, and that the calculated moments are proportional to the values shown in the same figure.

Similarly, if one starts at joint B and assumes $\theta_B = 0.427$, the moment values shown in Fig. 36(*b*) are obtained. It will be observed that the moment at joint G required to produce this angular rotation is also equal to −89,800 ft-lb. This constitutes a valuable check on the computations, in agreement with Maxwell's law of reciprocity of displacement. From the moment values in Figs. 36(*a*) and 36(*b*), the effects of applied moments at joints C, D, E, and F can be determined easily in a manner exactly similar to that shown for frames having members of constant section (see Fig. 33).


C
$\theta_C = 110$
−6240
6240
E
$\theta_E = 4.30$
−236
236
-89 800
−38 700
3670
D
−1490
117
F
B
$\theta_B = -251$
−51100
$\theta_D = -9.66$
−2180
$\theta_F = -0.573$
−117
0
$\theta_A = 187$
A
−730
$\theta_H = 0$
H
0
$\theta_G = 0.427$
G

(*a*) JOINT B


C
$\theta_C = -2.70$
−193.5
193.5
E
$\theta_E = -69.1$
−5120
5120
−87
950
D
−2730
24900
F
B
$\theta_B = 0.427$
87
$\theta_D = 7.89$
1780
$\theta_F = 206$
−24900
0
$\theta_A = -0.318$
A
$\theta_G = -894$
−89 800
G
595
H

(*b*) JOINT G

FIG. 36.—EFFECTS OF APPLIED MOMENTS AT DIFFERENT JOINTS

Summary.—It has been demonstrated in the paper, and also in the discussions, that the principle of superposition is indispensable in the analysis of rigid frames by the slope-deflection and moment distribution methods where applied loads cause linear displacements of the joints.

As stated in the paper, the key to analysis in any case consists in the consideration of all possible conditions of deformation of the given frame. This initial step of the method of procedure, which is based upon the validity of superposition, is very important and cannot be overemphasized. The balancing of fixed-end moments at the joints by the general methods of slope deflection or moment distribution (or by special methods where they apply) is but one step in the solution of a rigid frame problem involving unknown linear displacements of the joints, as the outline of the method of procedure clearly shows. The principle of superposition, on the other hand, is applied constantly throughout the analysis and, therefore, is deserving of special emphasis.

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Paper No. 2263

CONCENTRATION OF SEWAGE SLUDGE
A SYMPOSIUM

PAGE

Experience in Los Angeles, Calif.
By R. F. Goudey, M. Am. Soc. C. E. 1558

Back River Sewage Works, Baltimore, Md.
By C. E. Keefer, M. Am. Soc. C. E. 1562

Some Results in the State of New Jersey
By Willem Rudolfs, M. Am. Soc. C. E. 1565

Concentration of Sewage Sludges, New York, N. Y.
By Wellington Donaldson, M. Am. Soc. C. E. 1572

With Discussion by Messrs. C. R. Velzy, and R. F. Goudey.

Note.—Published in October, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

EXPERIENCE IN LOS ANGELES, CALIF.

BY R. F. GOUDEY,[1] M. AM. SOC. C. E.

SYNOPSIS

Interesting research on the thickening of various types of sludge was conducted on a practical scale by the Bureau of Water Works and Supply of the City of Los Angeles at a 200,000 gal per day demonstration plant which operated continuously from May 12, 1930, to January 6, 1934.[2] The plant, treating a raw sewage averaging 388 ppm total suspended solids and 394 ppm biochemical oxygen demand, included units of preliminary sedimentation, activated sludge, final sedimentation, coagulation and filtration of various types of sludges, sludge digestion, coagulation and filtration of activated sludge effluent, sludge concentration, sludge digestion, and vacuum filtration of sludge. The plant was radically designed in that the aeration period was 3 hr and was radically operated in four respects: Dissolved oxygen was maintained in the settled activated sludge; aeration tank solids were maintained at 1,000 to 1,600 ppm; returned sludge was carried at 45% or less; and returned sludge was added at different stages in the aeration process.

INTRODUCTION

Several axioms regarding sludge volumes must be kept in mind: First, as percentage removals of sludge from plain settling tanks increase, so the percentage moisture in the sludge increases. Second, sewage solids passing preliminary clarifiers into aeration or chemical processes become quite liquid even up to 99.5%. This means that in a complete treatment plant the greater the percentage removal of sludge in the preliminary clarifier, the lower will be the total sludge volume. Third, sludge-bulking characteristics apply not only to activated sludge, but to fresh and digested sludge as well. Fourth, washing sludge in clarifiers from storm flows or clear night flow or diluting sludges with fresh water (which practices have been recognized and used even prior to 1930) tend to produce denser sludges.

SLUDGE BULKING

Bulking of sludge is a phenomenon not restricted solely to activated sludge. For a given sewage, the fresh sludge moisture content increases with the percentage removal of total solids. Fresh sludge itself may be bulked from pH-variations above and below neutral, entrained carbon dioxide, carbonates, pectin, comminuted organic wastes of light specific gravity, and filamentous organic growths. Automatic pH-control, with lime, aids in producing fresh sludge with a low moisture content. Washing with water, whether from dilute sewage at low flow, storm water, or water added for elutriation, likewise aids.

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[1] San. Engr., Bureau of Water Works and Supply, Los Angeles, Calif.

[2] "Sewage Reclamation Plant for Los Angeles," by R. F. Goudey, *Western Construction News*, October 25, 1930, p. 519.

Mechanical mixing in the bottom of clarifiers or by mixing paddles for mechanical flocculation may assist in producing superior fresh sludges.

Bulking of activated sludge may be caused by many physical conditions similar to those for the bulking of fresh sludge. The process of "bulking" is not fundamentally biological because activated sludge can be bulked instantaneously by physical means long before bacteria or fungi could possibly multiply to account for it biologically.[3] Many organic and inorganic compounds that are laxative to man appear to be capable of bulking activated sludge. Pulp of fresh or dried apricots, figs, and tomatoes aerated in water become suspended in a relatively large volume. Onion and similar pulps, first alkalized and then made neutral, will also cause distended and bulky suspensions. When small amounts of activated sludge are added to pure pulp suspensions, the sludge becomes distributed throughout the pulp suspension and appears "bulked." A little activated sludge in a suspension of pure magnesium hydroxide appears enormous. In all these cases the activated sludge is distended and dispersed. Granite dust, sand, lime, or dilution with water, in all these cases, can reduce the "bulk" from 95% to 30% almost instantaneously, just as an increase in the pH-value in the magnesium hydroxide floc causes its immediate concentration. Lime is probably effective also in destroying enzymes, inverting colloidal phases, and in releasing ammonia.

Any sequence of treatment that controls bulking in all types of sludges and removes as large a part, as is practical, of the sewage solids as fresh sludge constitutes the first fundamental step in sludge concentration. The solids in passing preliminary clarifiers into aeration or chemical treatment units are converted into extremely liquid sludges, 97% to 99.5%, which constitute the main problem so far as concentration is concerned.

Excess Activated Sludge.—Three methods of excess activated sludge concentration were investigated by many months of practical and continuous operation. The first procedure was concentration by filling a tank with excess activated sludge, allowing it to settle by batch treatment for 24 hr, withdrawing water from clear zones between sludge and scum, and finally pumping the remaining sludge, which averaged 97.76% water. The second method was the return of excess activated sludge continuously to the preliminary clarifier which raised the moisture in the fresh sludge from 91.96% to 94.42%.

The third method consisted of continuous removal of excess sludge as equivalent aeration tank liquor, settling it at a rate of 600 gal per sq ft per day in a separate clarifier and treating the supernatant liquor to maintain a chlorine dose of 1 ppm over the settled sludge. The chlorine dose was equivalent to 20 lb per million gallons of sewage treated. The chlorine in the waste overflow was used to disinfect the final effluent from the entire plant.

No scum appeared on the thickening tank. Sludge was pumped once daily. This procedure produced an average moisture content of 94%. Washing with water produced sludges as low as 92%. The addition of lime averaged a sludge moisture of 90%. The addition of such inert materials as ground oyster shell, decomposed granite, and magnesite produced concentrated activated sludges as low as 76%.

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[3] "Demonstration on Bulking and Debulking of Activated Sludge at Escondido Plant," by R. F. Goudey, *California Sewage Works Journal*, Vol. V, No. 1, 1932–33, p. 61.

Excess activated sludge in Southern California can be reduced, readily and economically, to a moisture content of 92%, although a limit of 94% is sufficient where the sludge is pumped to digesters. The main secrets are: First, the settling of thin sludge from aeration tanks carrying 1,000 to 1,500 ppm of solids instead of settled sludge from the activated sludge clarifiers; second, settling at the rate of 600 gal per sq ft per day; third, the addition of chlorine of 1 ppm to prevent scum formation and to cause thickening; and, fourth, use of a continuous operating basis. If re-aeration tank liquor or settled activated sludge from the secondary clarifier is used, a smaller concentrating tank is required but the moisture contents will range from 96% to 97%.

In order to visualize the relative value of the foregoing four methods, a tabulation of the resultant combined volumes of fresh and activated sludges for a plant producing one ton of dried fresh solids at a moisture content of 91.96%, and two tons of dried excess activated sludge at 99.8% moisture, is as shown in Table 1.

TABLE 1.—COMBINED VOLUME OF WET, FRESH AND THICKENED ACTIVATED SLUDGE FOR VARIOUS METHODS OF CONCENTRATION

No.	Process	Moisture content (%)	Combined volume (gal)
1	Settled activated sludge mixed with fresh sludge	97.76	24,100
2	Excess sludge pumped to clarifier	94.42	12,900
3	Aeration liquor settled under chlorine and mixed with fresh sludge	92.00	9,000
4	Aeration liquor settled under chlorine and mixed with fresh and activated sludge	90.00	7,200

The City of Phoenix, Ariz., installed a sludge thickening tank[4] for both fresh and excess activated sludge. Dario Travaini, Assoc. M. Am. Soc. C. E., reports that fresh sludge has been reduced to 85% moisture but not without troublesome odors. The fresh sludge has an average moisture content of 95% and the excess activated sludge averages 97.4% moisture. These high values are due to chemical treatment prior to preliminary clarification and also to the fact that the thickening plant is too small to use aeration tank liquor rather than activated sludge itself. Mr. Travaini also reports that the Phoenix plant has "never successfully concentrated activated sludge in the primary clarifier" and that the digesters would "fail completely" if it were not for the sludge concentration tank employing chlorine.

It is apparent that the chlorine settling method is superior to that of pumping excess activated sludge to the preliminary clarifier where good removals are required and where digestion is involved. In case the final effluent needs to be chlorinated, the chlorine serves both for sludge concentration and mixture of the sludge liquor carrying sufficient chlorine to disinfect the final effluent. For moderate removals and where chlorination of the final effluent is concerned, concentration of activated sludge need not be carried below 94%.

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[4] "Sludge Thickening at Phoenix, Arizona," by Dario Travaini, *Water Works and Sewerage*, April 1934, p. 107.

Digested Sludge.—As early as 1930 successful experimental runs were conducted on digested sludge treated with various quantities of ferric chloride and lime followed by vacuum filtration. The average dose was 4% ferric chloride and 10% lime. Digested sludge produced a cake of 65% moisture content at a chemical and filtration cost of $3.50 per million gallons.[5]

Raw Sludge.—It was found that chemical doses calculated on the basis of dry solids used to concentrate digested sludge likewise concentrated fresh sludge; and this procedure permitted equally satisfactory dewatering of fresh sludge for either vacuum filtration or rapid sand bed drying. This indicates that digestion is an unnecessary step where chemical treatment of sludge is justified for other reasons.

Conclusions

All practices that will produce a concentrated sludge—whether fresh, activated, or digested—should be given due consideration. The cheapest way to remove solids of lowest moisture content is in the first clarifier where attention is paid to pH-control of the incoming sewage, mechanical flocculation, timing of sludge removal, and washing and dilution of sludge.

Excess activated sludge can be concentrated best by removing its equivalent as aeration tank liquor, diluted with water to 1,200 ppm and treated with lime occasionally, followed by sedimentation under a 1 ppm chlorine blanket. The withdrawal of aeration tank liquor should be continuous, the settling tank need never be emptied, and the thickened sludge should be pumped not more frequently than once a day. By this procedure, moisture contents of 94% can be attained.

When it comes to producing sludges of maximum concentration, digestion is unnecessary and wasteful. The economy of sludge concentration is based fundamentally on the degree of solids to be removed by a given plant, whether the final effluent needs chlorination, and whether vacuum filtration of sludge is required.

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[5] "Digestion and Disposal of Sewage Sludge," by R. F. Goudey, *Western Construction News and Highways Builder*, July 10, 1932, p. 381.

BACK RIVER SEWAGE WORKS, BALTIMORE, MD.

By C. E. Keefer,[6] M. Am. Soc. C. E.

Synopsis

When the preliminary plans for the additions to the Back River sewage works in Baltimore, Md., consisting of activated-sludge units, were made in 1937, it was decided to provide tanks for the thickening of the excess activated sludge. These tanks have been in almost continuous service since they were put in operation on November 27, 1939, and have given satisfactory service.

Description of Thickening Tanks

Two thickening tanks were constructed, each 26 ft in diameter with a working depth of 16 ft. The excess sludge enters the tanks through a vertical central column, and the supernatant flows radially outward and discharges over a V-notched weir at the tank periphery. A mechanism is supported by and rotates about the aforementioned column at a speed of one revolution in 14.75 min. The bottom of the mechanism is provided with squeegees for moving the settled sludge along the tank floor to the sludge discharge pipe near the center of the tank. Vertical steel angles, for thickening the sludge, are also attached to the mechanism. Projecting vertically downward 12 in. below the surface of the supernatant in each tank are 8-in. nozzles through which chlorine water can be discharged.

The two tanks were designed to thicken the excess sludge when the sewage flow treated by the activated-sludge units would amount to 40 mgd. It was estimated that 160,000 gal of excess sludge, containing 1.5% of dry solids, would result from treating the aforementioned quantity of sewage. With this flow of sludge the surface loading, with one thickening tank in service, would be 300 gal per sq ft daily. A total of 13 ft was allowed for deposited sludge and 3 ft for supernatant in the tanks. The supernatant from the tanks is pumped into the settled sewage being treated by the activated-sludge units, and the thickened sludge flows to two plunger pumps. These pumps discharge the material through a 6-in. cast-iron force main into a well, which receives the raw sludge from the preliminary sedimentation tanks. The mixture of raw primary and activated sludges is then pumped into heated digestion tanks.

This treatment of the activated sludge indicates the reasons for thickening it, namely:

1. To reduce pumping costs;
2. To save fuel for heating the sludge; and
3. To reduce the volume of space in digestion tanks occupied by water.

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[6] Associate Engr., Bureau of Sewers, Baltimore, Md.

OPERATING RESULTS

When the thickening tanks were designed, it was expected that each tank would treat the excess sludge resulting from the treatment of 20 mgd of sewage. Operating experience has indicated the desirability of frequently using two tanks even when the sewage flow is less than 20 mgd. Item 5, Table 2, indicates that during the 3-yr operating period from 1940 to 1942, inclusive, two tanks were in service from 59% to 64% of the time. When the sludge index is low one tank is sufficient to thicken the sludge, but when the index begins to increase materially two tanks are necessary.

TABLE 2.—OPERATING RESULTS AND DATA, ACTIVATED-SLUDGE THICKENING TANKS, BALTIMORE, MD.

No.	Description	1940	1941	1942
1	Quantity of sewage treated (mgd)	15.61	13.28	11.94
	Depth of Sludge in Tanks (Ft):			
2	From	7.7	4.9	9.1
3	To	14.0	11.7	
	Percentage of Time in Service:			
4	One tank	36	39	41
5	Two tanks	64	61	59
	Sludge to Thickener:			
6	Quantity (gal per million gal of sewage)	46,000	22,520	18,000
7	Solids (%)	0.9	1.1	0.9
8	Volatile matter (percentage dry weight)	77.1	79.5	80.1
	Sludge from Thickener:			
9	Quantity (gal per million gal of sewage)	5,370	4,231	4,500
10	Solids (%)	1.9	2.1	1.4
	Thickener Effluent (PPM):			
11	Five-day biochemical oxygen demand	88	....	....
12	Suspended solids	96	81	197
13	Chlorine applied to unthickened sludge (ppm)	16.5	29.4	10.0
14	Reduction in volume of sludge (%)	88.3	81.3	75.0

The performance of the thickening tanks from 1940 to 1942, inclusive, is indicated by Items 6 to 14 in Table 2. The reduction in the volume of the sludge (Item 14) has varied from 75% to 88.3%, with an increase in solids of from 0.9% or 1.1% (Item 7, Table 2) to 1.4% or 2.1% (Item 10, Table 2). The effluent from the thickeners (Item 11) had an average 5-day B.O.D. of 88 ppm in 1940 and an average yearly suspended solid content (Item 12) varying from 81 to 197 ppm. The chlorine applied to the unthickened sludge varied from 10.0 to 29.4 ppm (Item 13, Table 2).

A marked correlation was observed between the sludge index and the solids, in the unthickened and the thickened sludge (Fig. 1(*a*)). When the sludge index was low, the solid content in the unthickened and the thickened sludge was high; and, when the sludge index was high, the reverse condition was observed. The ratio of the solids in the thickened and the unthickened sludge (R) was also found to vary inversely with the sludge index (Fig. 1(*b*)). This relationship can be expressed by the formula

$$R = 7.94\ s^{-0.3125} \qquad \text{.........................(1)}$$

in which s is the sludge index. Eq. 1 is based on three years of operating results, and may require modification when more operating data are available. It is not intended to apply to other than Baltimore conditions. No correlation was found between the amount of inorganic matter in the sludge and the solid content and the sludge index, perhaps because there was insufficient variation in the amount of inorganic matter in the sludge.

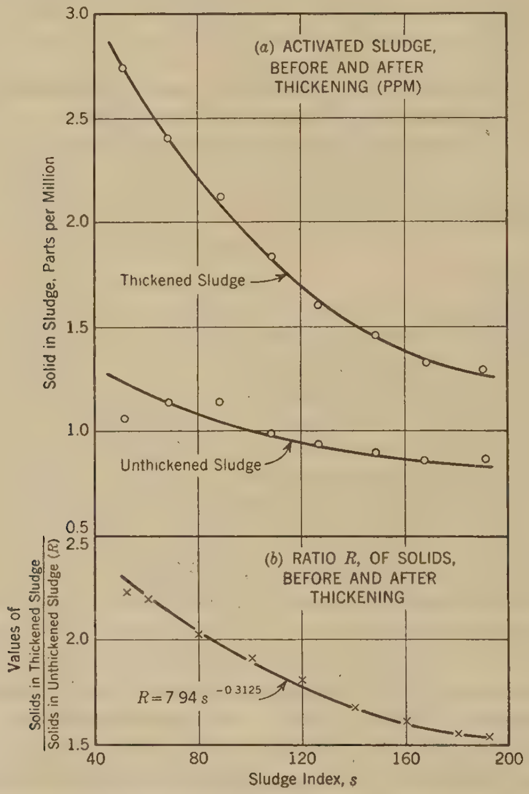

FIG. 1.—RELATIONSHIP BETWEEN THE SLUDGE INDEX AND THE SOLIDS IN THE SLUDGE

CONCLUSIONS

The thickening tanks have amply justified their installation, as they have materially simplified the problem of sludge digestion. The operating results indicate that the degree to which the sludge can be thickened depends upon two things: (1) The solid content of the sludge to be thickened; and (2) the sludge index of the unthickened sludge.

ACKNOWLEDGMENTS

The activated-sludge units were designed by the firm of Whitman, Requardt and Smith, consulting engineers. The Back River sewage works is under the general supervision of Nathan L. Smith, M. Am. Soc. C. E., chief engineer, Department of Public Works, and George E. Finck, M. Am. Soc. C. E., sewerage engineer.

SOME RESULTS IN THE STATE OF NEW JERSEY

BY WILLEM RUDOLFS,[7] M. AM. SOC. C. E.

SYNOPSIS

The purpose of sludge concentration or sludge compaction is primarily to reduce the bulk of the sludge and subsequently to effect economy in handling. The method of sludge concentration may be physical, chemical, or both. The ultimate sludge concentration obtained is affected by various factors, chiefly the character of the sludge, its initial concentration, time of compaction, temperature, and pressure. If chemicals are used, the type of chemical is important.

This paper is in the nature of a résumé and deals with several of these factors. The results reported are based upon laboratory and plant scale studies.

The character of the sludge to be concentrated is of greatest general importance. The effects of initial concentration, time of compaction, and temperature during concentration, as well as the effect of the chemicals used, vary for different types of sludges.

The studies on which this paper is based have been made on the three major types of sludges (fresh solids, digested sludges, and waste activated sludges). Because of the mass of data, only some of the results are discussed under separate headings.

FRESH SOLIDS

Initial Concentration.—The reduction in volume of sludge when it is quiescent is naturally greatest with the lowest initial concentration. As an illustration, fresh solids, allowed to compact for 120 hr at a winter temperature of 50° F, with initial concentrations of from 0.5% to 6.2%, show decreasing rates of compaction with increasing initial concentration (Fig. 2). The compaction is greatest during the first 24 hr.

Effect of Time.—Table 3 shows the effect of time on the concentration of fresh solids under actual plant conditions obtained at the Rahway Valley (New Jersey) Sewage Treatment Plant over a period of 3 yr. The average monthly results, irrespective of temperature effects, are shown in Fig. 3. The results confirm closely the laboratory results obtained with fresh solids from the same source as that shown in Fig. 2. Concentration is very rapid during the first 12 hr, followed by a gradual reduction in the rate of compaction.

TABLE 3.—MONTHLY AVERAGE, FRESH SOLIDS CONCENTRATION

Hours of concentration	PERCENTAGE OF TOTAL SOLIDS		
	Minimum	Maximum	Average
12 to 18	3.80	6.90	5.57
24	5.30	8.30	6.42
36 to 48	6.4	8.30	7.19

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[7] Chf., Dept. of Water and Sewage Research, New Jersey Agri. Experiment Station, New Brunswick, N. J.

Effect of Temperature.—Experiments and observations made at the Elizabeth (N. J.) Joint Meeting Sewage Treatment Plant, where the fresh solids are concentrated in tanks prior to barging to sea, show that concentration of solids is materially greater in summer than in winter. As an illustration, Fig. 4 compares the concentration of fresh solids kept at winter temperatures (47° F to 50° F), with similar solids concentration at summer temperatures (75° F to 80° F). The initial total solids concentration of 5.2% increased during 96 hr to 11.5% in summer as compared with 8.7% in winter. Practice for several years has shown that the sludge volume is materially less in summer than in winter, but handling the less concentrated sludge in winter is more difficult than handling the denser sludge produced in summer. The reason is primarily a question of viscosity of the sludge, which is appreciably greater in winter.[8]

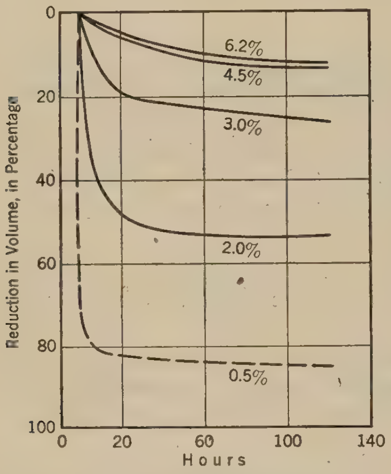

Fig. 2.—Relation Between Time of Compaction and Reduction in Sludge Volume

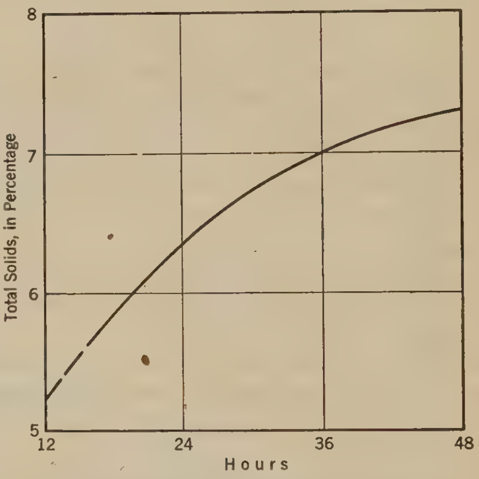

Fig. 3.—Average Results Obtained on Plant Scale During Three Years

The effect of sludge concentration on cost of operation may be illustrated by the volumes of sludge collected and the percentage reduction in volume obtained during 1940, 1941, and 1942. The sludge pumped in 1940 was 70,700 gal per day, which increased to 74,378 gal per day in 1941. On account of increased solids concentration, however, the reduction in volume amounted to 42.4% in 1941 as compared with 36.1% in 1940, so that less sludge had to be barged in 1941 than in 1940. During 1942 the sewage flow increased, and subsequently the sludge quantity increased to an average daily amount of 91,673 gal, or an increase in volume of 17%. Improved procedures in decanting led to increased solids concentration, reaching an average for the year of 8.14% total solids, so that only 83,000 tons, or about half of the increased quantity of sludge, needed barging.

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[8] "Properties of Sludge Which Affect Its Discharge Through 24-In. Pipe," by Willem Rudolfs and Leslie E. West, *Sewage Works Journal*, Vol. XII, 1940, p. 60.

Effect of Temperature on Pumping Time.—The concentrated sludge is barged to sea at approximately two-week intervals. The sludge is pumped through a 24-in. line about 4,000 ft long. The time required for pumping is an expensive item in the disposal of the sludge. The temperature of the sludge has a substantial effect on this time (see Table 4). During 1942 the pumping time amounted to 70 min per 1,000 tons of sludge with sludge temperatures of from 68° F to 73° F, in contrast to 120 min per 1,000 tons of sludge at temperatures of from 51° F to 57° F. Thus, about 80% more time was required during the winter period.

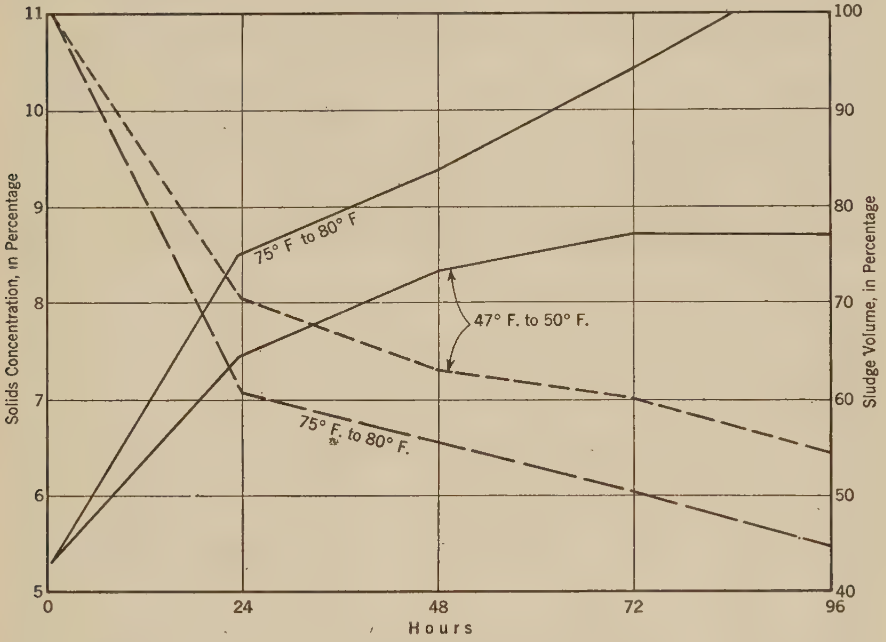

FIG. 4.—SUMMER AND WINTER TEMPERATURES AFFECTING CONCENTRATION OF FRESH SOLIDS

TABLE 4.—RELATION BETWEEN TEMPERATURE OF SLUDGE AND PUMPING TIME, 1942

(Elizabeth Joint Meeting Sewage Treatment Plant)

Number of bargings	Temperature range (degrees F)	Total solids (%)	Ash (%)	Tons barged	Average pumping time per 1,000 tons (min)	Average temperature (degrees F)
24	51–73	8.14	25.3	76,070	92	63.5
8	51–57	7.91	22.4	23,870	120	53.5
6	62–66	8.38	28.2	19,145	90	64.5
10	68–73	8.28	29.1	33,055	70	70.7

Plotting the 3-yr averages of individual temperatures prevalent during bargings clearly shows the relationship between temperature and time (see

Fig. 5). These results were actually obtained, and the curve cannot be drawn further without data to cover the extension. With the decreasing sludge temperatures the viscosity of the sludge increases, resulting in increased pumping time.[8]

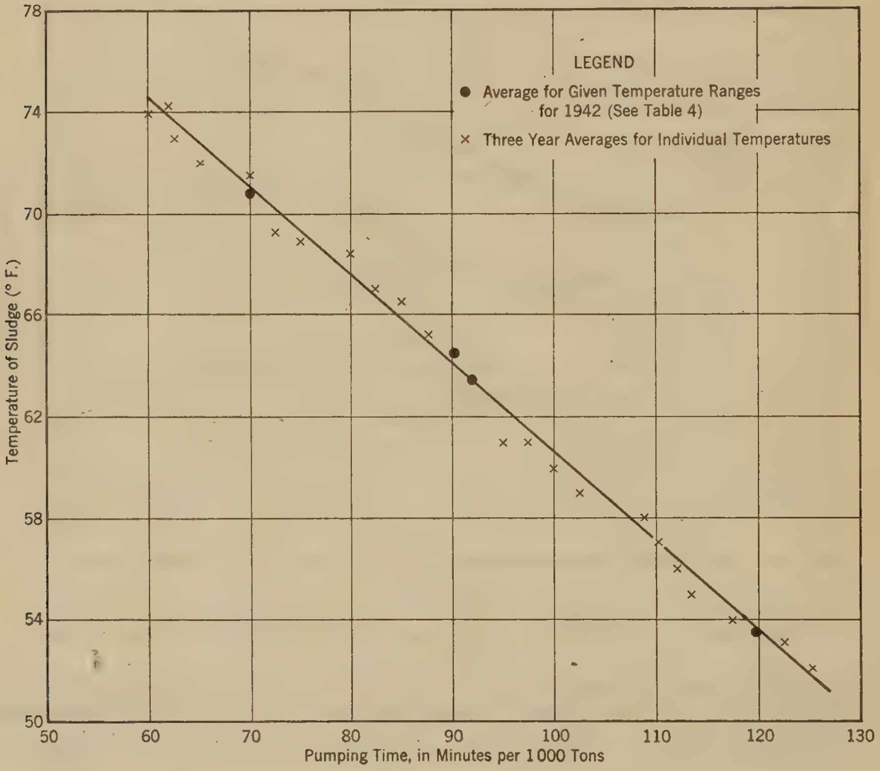

FIG. 5.—RELATION BETWEEN TEMPERATURE OF SLUDGE AND PUMPING TIME OBSERVED UNDER ACTUAL OPERATION

DIGESTED SLUDGE

The primary factors affecting fresh solids concentration—namely, initial concentration, time, and temperature—also affect concentration of digested sludge.

Initial Concentration.—The percentage reduction in volume decreases with increasing initial concentration. Compaction of ripe sludge proceeds in a manner similar to that of fresh solids, but the final solids concentration that can be obtained is greater. However, the fresh solids are more gelatinous in nature than the ripe sludge and they hold water more tenaciously, resulting in less dense sludge.[9]

Effect of Time.—The effect of time on ripe sludge compaction is similar to that on fresh solids. However, with initial higher solids contents, the concentration of ripe sludge is more rapid than similar concentration of fresh solids with equal elapsed time. For instance, with an initial concentration of 5%, fresh solids may be expected to concentrate 15% in 48 hr on an average, whereas ripe sludge increases under the same conditions an average of 30%. The percentage concentration varies with the length of time allowed for concentration.[10]

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[9] "Sludge Compacting," by Willem Rudolfs and Robert P. Logan, *Sewage Works Engineering and Municipal Sanitation*, Vol. 14, 1943, p. 10.

Effect of Temperature.—The effect of temperature is less on ripe sludge than on fresh solids. As an illustration, results obtained with ripe sludges of different initial concentrations allowed to compact at summer and winter temperatures are shown in Fig. 6(*a*). The curves indicate that, with an initial concentration of 5% solids, a decrease in volume of about 50% can be expected after 72 hr of compaction. The difference in compaction during summer and winter is very small as compared with differences in fresh solids concentration (see Fig. 6(*b*)). The greater difference in fresh solids compaction is due mainly to the slimy character of the material and the gases produced during the initial stages of decomposition. The acidic compounds formed during decomposition, together with the gas produced, cause the fresh solids to float, whereas the ripe sludge compacts at the bottom of the container or tank used.

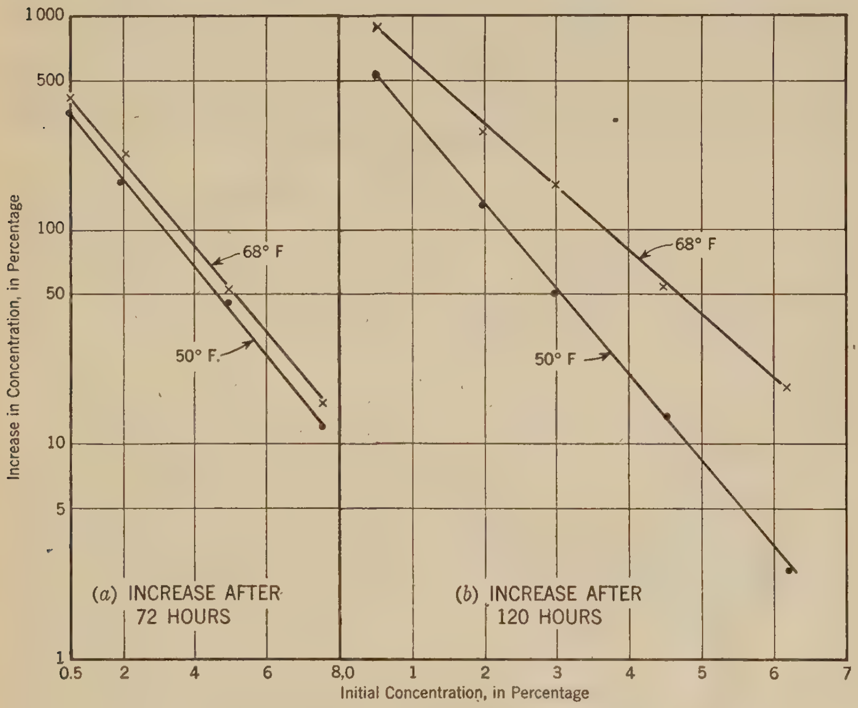

FIG. 6.—EFFECT OF WINTER AND SUMMER TEMPERATURES ON COMPACTION OF DIGESTED SLUDGE

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[10] "Effect of Temperature on Sludge Concentration," by Willem Rudolfs and R. P. Logan, *Sewage Works Journal*, Vol. XV, 1943, p. 894.

ACTIVATED SLUDGE

Activated-sludge compaction is subject to the same factors that influence fresh solids and ripe sludge. Material along these lines has been published previously.[11] For that reason, some results obtained with certain chemicals to induce concentration of activated sludge are presented.

Experiments conducted over several years on the compaction of activated sludge produced: Large flocs, small flocs, flotation, dehydration, weighting, retardation of decomposition, and the combination of these methods with the aid of chemicals, inert materials, poisons, and gases.[12]

From the mass of results obtained, some dealing with induced flotation are selected. A flotation process with the aid of chemicals for dewatering of sludge has been used for a number of years. The acid chemicals react with the alkaline substances present in the sludge or liquor, producing numerous small gas bubbles, which in turn cause the sludge particles to rise and aid in the separation of water from the sludge. If, at the same time, such a chemical is found to be a coagulant, which causes the flocs to shrink, prevents septicity, and does not result in materially increased oxygen demand of the liquor, the sanitary engineer is approaching something worth while.

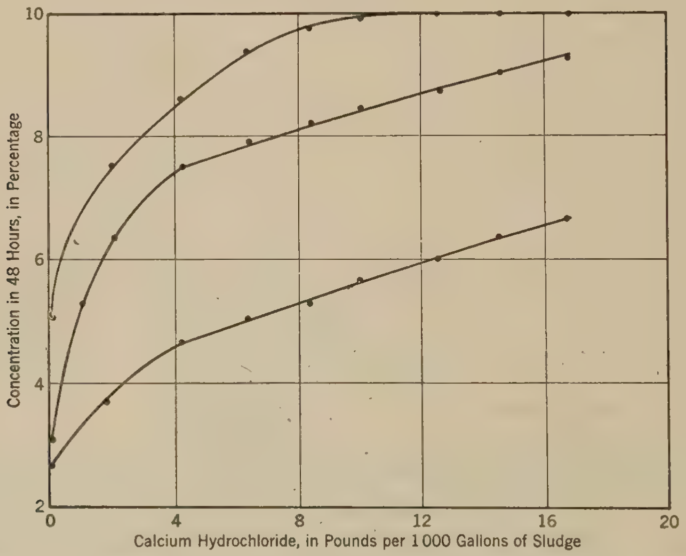

FIG. 7.—CHARACTER OF ACTIVATED SLUDGE IN RESPECT TO THE DEGREE OF OXIDATION

Results obtained with calcium hypochloride indicate that a dense, activated sludge may be obtained by flotation (Fig. 7). The concentration of the original waste, activated sludge (0.30% to 0.60%) does not appear to be a factor, but the character of activated sludge in respect to the degree of oxidation and the size of the floc seems to play a part. The 9% to 10% sludges produced by flotation are leathery in appearance, forming compact, cakelike sludges. The gases produced, which cause flotation, evolve freely when the sludge "cakes" are forming, but the sludge continues to compact materially after the first 24 hr. The chemical need not be added immediately upon discharge of the sludge. Letting the sludge stand for 10 hr at summer temperature and then adding the chemical produced the same sludge density after a total of 48 hr as that produced when the chemical was added at the beginning of the period. Allowing the sludge to become septic likewise had no effect. Neither age nor preliminary compaction, therefore, is detrimental or beneficial.

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[11] "Concentration and Distribution of Sewage Solids During Digestion," by Willem Rudolfs and I. O. Lacy, *Sewage Works Journal*, Vol. III, 1931, p. 64.

[12] "Concentration of Activated Sludge by Compacting and Flotation," by Willem Rudolfs, *ibid.*, Vol. XV, 1943, p. 642.

It is well known that excessive quantities of chlorine added to activated sludge cause a dispersion or have a peptizing action on the sludge. The same reaction occurs with the addition of calcium hypochloride, but to a lesser extent. The biochemical oxygen demand (B.O.D.) of the liquor increases, but the highest value recorded, with quantities of calcium hypochloride three to four times greater than required, was 600 ppm. The liquor can be readily clarified by additions of small quantities of ferric chloride (0.4 lb per 1,000 gal), ferric sulfate, or chlorinated copperas, resulting in a clear liquor with a B.O.D. of about 100 ppm. The iron salt can be added at the same time as the calcium hypochloride. The addition of about 2 lb of sulfuric acid with the calcium hypochloride produces the same result.

The drainability of compacted sludge is enhanced by the calcium hypochloride treatment. The sludge drains readily, forming a thin, dry sheet, without the production of odors on standing. Sludge treated by calcium hypochloride is not improved by vacuum filtration.

SUMMARY

Sludge compaction is affected by the type and character of the sludge, initial concentration, time, temperature, and pressure. If chemicals are used, the type of chemical is important.

Sludges may be concentrated by settling and compaction or by flotation and compaction. Temperature is an important influence on the thickening rate of sludge. The method of compaction to be used depends upon subsequent disposal of the sludge. For barging and incineration, flotation processes appear feasible and may be more economical than compaction by settling.

CONCENTRATION OF SEWAGE SLUDGES, NEW YORK, N. Y.

By Wellington Donaldson,[13] M. Am. Soc. C. E.

Synopsis

Operating experiences and operating data for the sewage treatment plants of New York City are presented in this paper.

Some of the reasons for wishing to concentrate sewage sludges are quite obvious. For instance, in the case of heated digester tanks, it is desirable to introduce the sludge into the digester in well-concentrated condition, to avoid chilling the digesters by unnecessary quantities of cool liquor and at the same time to avoid the necessity of decanting from the digesters an equal quantity of heated supernatant liquor. Likewise, in connection with artificial dewatering and drying processes, it is advantageous to secure initial concentration of sludge. Where the liquid sludge is transported some distance from the plant for disposal, as is the situation locally, concentration is of considerable economic importance.

The density of sludge from primary tanks up to some 10% solids is largely controllable by the operator, depending upon the sludge pumping schedule and method of operating sludge collecting mechanisms. As a matter of fact, difficulties with collector mechanism, pump, or sludge piping from too dense a primary sludge may discourage maximum concentration. Few operators of activated sludge plants will agree that a comparable consistent and dependable control of the density of activated sludge exists. In fact, the difficulty of consistently concentrating activated sludge is the principal demerit to an otherwise very fine process.

In reviewing briefly the experiences of the more important sewage treatment plants of New York City, attention is directed to Table 5, showing the quality of the various sewage sludges.

Coney Island

The Coney Island plant, which, in May, 1942, was increased in capacity from 35 mgd to 70 mgd, employs plain sedimentation for eight months and chemical precipitation and chlorination during the four summer recreational months. All sludge is digested, and the liquid digested sludge is ordinarily transported by scow to natural sand beds or lagoons remote from the plant. There is little significance in the sludge data shown in lines 1 and 2, Table 5, since no particular effort was made to concentrate the raw or digested sludge beyond following a convenient operating schedule.

During the summer months when a ferric salt was used to aid sedimentation, the primary sludge ran about 8.5% solids as compared with the average value of 7.80% shown in Table 5. Because the digested sludge from this plant is transported by scow to beds or lagoons on Marine Park, it is advantageous to keep digested sludge reasonably dense. However, the 7.71% solids shown for the digested sludge is governed more by the running schedule of the sludge scows than by any manipulation within the plant.

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[13] Chf., Bureau of Sewage Disposal Design, Dept. of Public Works, New York, N. Y.

TABLE 5.—AVERAGE QUALITY OF SEWAGE SLUDGES, NEW YORK CITY

No.	Description	Solids (%)	Volatile (%)
	(*a*) CONEY ISLAND PLANT		
1	Primary; 1941	7.80	68.1
2	Digested	7.71	49.0
	(*b*) WARDS ISLAND PLANT		
3	Primary; 1941	6.33	74.0
4	Aeration tank effluent	0.176	....
5	Returned sludge	0.48	....
6	Thickened activated sludge to storage	2.28	76.2
7	Weighted average, sludge to storage, primary to excess activated	4.30	....
8	Combined raw sludges to sea	4.45	74.6
	(*c*) BOWERY BAY PLANT		
9	Primary; 1941	7.10	68.2
10	Primary; 1942 (eight months)	5.38	62.9
11	Aeration effluent; 1942 (eight months)	0.16	....
12	Returned sludge; 1942 (eight months)	0.40	....
13	Thickened excess activated sludge; 1942 (eight months)	2.40	69.5
14	Combined raw sludges to digesters	4.03	....
	(*d*) TALLMANS ISLAND PLANT		
15	Primary; 1941 (January to May)	5.41	64.8
16	Primary; 1941 (June to December)	7.52	56.6
17	Aeration tank effluent (seven months)	0.152	....
18	Returned sludge (seven months)	0.51	....
19	Thickened activated sludge (five months)	1.35	56.5
20	Thickened activated sludge (seven months)	3.48	61.0
21	Combined raw sludges to digesters	6.41	....
22	Digested sludge to storage tank (twelve months)	5.94	....
23	Digested sludge to disposal (twelve months)	7.23	39.4

The wooden scows used for transporting the digested sludge from the plant to Marine Park are owned by the city. Towing is by contract, but the city personnel operate the gasoline driven pump on the scow for discharging the sludge through pipe and hose to the beds. Therefore, unnecessary scow trips would involve additional charges for towing, gasoline, and maintenance. During 1941 the disposal of digested sludge in this manner cost $4.31 per ton of dry solids delivered, capital charges included.

WARDS ISLAND

The Wards Island activated-sludge plant treats about 200 mgd of sewage. Both primary and excess activated raw sludge are finally disposed of by barging to sea in special Diesel-propelled sludge vessels, a distance of 34 miles from the plant. Therefore, it is of considerable importance that the sludge barged to sea be concentrated as much as possible. The primary settling tanks have retention periods of less than 1 hr. This probably limits the concentration of solids pumped from these tanks to the average of 6.33% solids shown in line 3, Table 5. Little can be done to change the density of the primary sludge, but the excess activated sludge requires continued watchfulness to avoid barging unnecessary water to sea. At this plant certain of the final settling tanks are set aside as sludge thickening tanks and operated at a slower rate to give a greater density of sludge than results in the regular final tanks. It will be noted from Table 5 that the return sludge averaged 0.48% solids (suspended solids, 4,800 ppm), whereas the thickened excess sludge from the special tanks averaged 2.28% solids. The computed weighted average of the combined primary and excess activated sludge was 4.30% solids. The difference between this and the succeeding value (4.45) represents a slight concentration secured in the sludge storage tanks by decantation before loading the sludge vessels. Decantation in this manner is practiced sparingly because of the adverse effect of the decanted liquor on the aeration tanks when the liquor is returned to the head of the plant.

Experience in New York City indicates that the rather common practice in smaller plants of concentrating excess activated sludge by wasting return sludge to the head of the primary tanks is not to be recommended unless the primary retention period is 2 hr or more. Shorter retention periods are apt to result in higher suspended solids in the effluent than in the raw sewage influent. Moreover, the septic character of the settled sewage is likely to be disturbing to the subsequent aeration process.

The motor vessels used in transporting sludge to sea are owned and operated by the city with a fixed personnel determined by U. S. maritime regulations. During 1941, each trip (1,500 wet tons) of sludge to sea cost about $202, for operating and maintenance charges. The cost of disposal of sludge to sea by the motor sludge vessels is estimated to be $3.13 per dry ton of solids transported, capital charges included. For New York City, sea disposal is the cheapest of methods available.

Bowery Bay

The 40-mgd activated-sludge plant at Bowery Bay was started on primary treatment in November, 1939, but secondary activated-sludge treatment was not begun until March, 1942. During the early operations the raw primary sludge was barged to sea. Later, as the digesters were put in service, the combined primary and excess activated sludges were digested and the digested sludge in most part was barged to sea by Diesel sludge vessels as at Wards Island. However, substantial amounts of liquid digested sludge have been carted by specially designed motor tank trucks to the unimproved areas of Flushing Meadow Park (formerly the site of the New York World's Fair), there to be used for soil improvement after natural drying.

During the period of primary treatment only, the raw sludge transported to sea averaged 7.10% solids, but the density decreased in the subsequent year to an average of 5.38%. The Bowery Bay plant has primary tanks of 1-hr retention. Wasting excess activated sludge ahead of the primary tank is not practiced, although a rather heavy decanted liquor from digesters is so wasted, and has proved disturbing to the aeration process. The decreased density in 1942 was probably due to the effect of decanting and to a higher rate of sludge pumping.

For concentration of the excess activated sludge, the plant has two thickening tanks. Provision is made for thickening either the return sludge or the aeration tank effluent—the latter is practiced. The aeration effluent (line 11, Table 5) averaging 0.16% solids (suspended solids, 1,600 ppm) was concentrated in the thickening tanks to 2.40% solids, whereas the return sludge averaged 0.40% (suspended solids, 3,980 ppm). The weighted average of combined primary and excess activated sludge pumped to the digesters was about 4%.

Since the same sludge vessels serve Wards Island, Bowery Bay, and Tallmans Island, the cost of $3.13 per dry ton given for Wards Island therefore applies also to the Bowery Bay plant, although the distance is longer and the loading more time consuming.

The other method of sludge disposal, mentioned previously, may need more explanation. The need for organic soil building material by the extensive park system of New York City led to the purchase of three special motor tank trucks to serve future park areas with liquid digested sludge. The cost of disposal by these trucks is estimated to be $3.50 per ton of dry solids removed, capital charges included, on the assumption that the trucks are kept rolling, which is not the case under wartime conditions.

TALLMANS ISLAND

The 40-mgd activated-sludge plant on Tallmans Island was placed in service in 1939. It treats an average flow of 15 mgd. At this plant, as at Bowery Bay, the excess activated sludge is concentrated in two special thickening tanks before pumping to digesters. The thickeners can be fed only with return sludge. During the first five months of 1941, when sludge thickening was under careful study, the thickened excess activated sludge was wasted to the head of the primary tanks having a retention of about 2 hr, and was recovered with the primary sludge. In the second half of 1941, the excess activated sludge, after thickening, was pumped separately to the digesters. When pumping to digesters, an effort was made to obtain the secure maximum concentration.

Table 5(*d*) shows several interesting facts concerning these operations. During the period when excess activated sludge was wasted to the head of the plant, the primary solids averaged 5.41%, compared with 7.52% solids when excess activated sludge was excluded from the primary tanks. With a feed of return sludge averaging 0.51% solids (suspended solids, 5,100 ppm), the thickeners during the first period concentrated the sludge to 1.35% solids, as compared with 3.48% solids during the more exacting second period. Combined sludges to digesters averaged 6.41%.

The difference between the values shown in lines 22 and 23, Table 5, of "digested sludge to storage tanks" and "digested sludge to disposal" is due to further concentration in the storage tanks by decantation. The digested sludge at Tallmans Island has been partly barged to sea and partly transported by the tank trucks to Flushing Meadow Park.

Special Thickening Tanks

The thickening tanks used at the Tallmans Island and Bowery Bay plants are of the conventional "picket fence" type. The Tallmans Island tank is illustrated in Fig. 8, and the Bowery Bay tank in Figs. 9 and 10. The main difference is that the Bowery Bay thickeners, later constructed, are shallower and of greater relative area. The Tallmans Island thickeners were each provided originally with a ring for introducing chlorine solution to the supernatant near the top of the tank, following the practice developed at certain western plants, but it was found that better results were obtained by discontinuing this arrangement and applying the chlorine solution direct to the

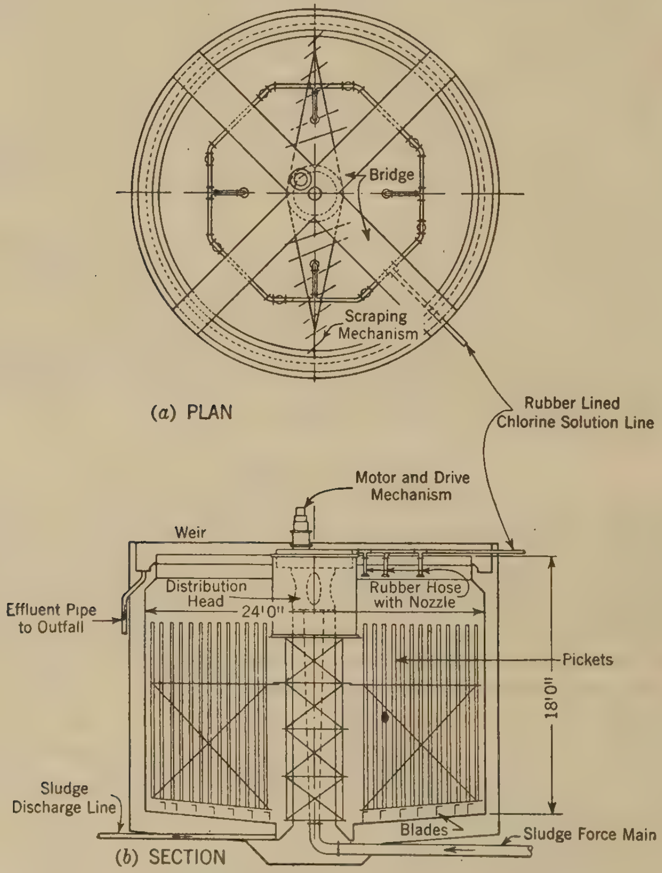
Bridge
Scraping Mechanism
(a) PLAN
Rubber Lined Chlorine Solution Line
Motor and Drive Mechanism
Weir
Distribution Head
24'0"
Rubber Hose with Nozzle
Effluent Pipe to Outfall
Pickets
18'0"
Sludge Discharge Line
Blades
Sludge Force Main
(b) SECTION

Fig. 8.—Sludge Thickening Tank, Tallmans Island

incoming sludge liquor. The distributing ring was omitted from the Bowery Bay and later designs.

The use of chlorine in connection with thickening is considered a helpful step during the summer months or on occasions when the settling characteristics of the sludge are not good. In general, chlorine helps when the dissolved oxygen in the entering liquor is low, the sludge index is poor, and the volatiles are high. Experience in New York City shows that better results can be obtained by thickening the aeration tank effluent in a single step than by rethickening the return sludge. This conclusion seems in agreement with the western experience, although apparently inconsistent with experience at Tallmans Island. It is recognized that the volatile content of activated sludge determines the degree of concentration possible.

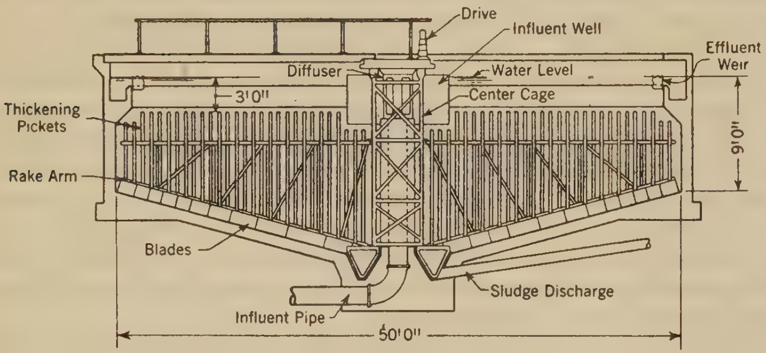
Drive
Influent Well
Effluent Weir
Diffuser
Water Level
3'0"
Center Cage
Thickening Pickets
9'0"
Rake Arm
Blades
Sludge Discharge
Influent Pipe
50'0"

FIG. 9.—SLUDGE THICKENING TANK, BOWERY BAY

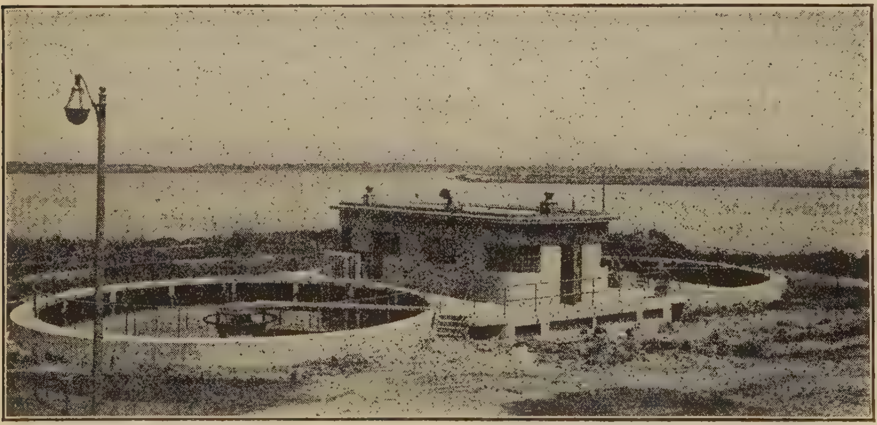

FIG. 10.—BOWERY BAY SLUDGE THICKENING INSTALLATION

At Tallmans Island, an effort to maintain a residual of 1.0 ppm in the overflow resulted in an average dose of 75 ppm of chlorine. At Bowery Bay, under wartime conditions, an average dose of 18 ppm of chlorine was used.

No residual chlorine was maintained in the overflow; hence the treatment could be considered only a palliative for a poor sludge condition and no answer to proper requirements.

CENTRIFUGE EXPERIMENTS

Aside from the usual operating methods described herein, it may be of interest to note some special experiments with concentrating sludges.

At the Jamaica (N. Y.) plant, a special centrifuge has been installed for the purpose of dewatering material caught by fine screens. During 1938 some thirty-five runs were made with this centrifuge on digested primary sludge from the Coney Island plant. The feed of the centrifuge averaged 11.1% solids; the cake, 50.2% solids; and the effluent, 3.4% solids. It was intriguing to find that within a few minutes digested sludge could be concentrated or partly dewatered to a friable inoffensive material ready to be hauled away and distributed on park areas, but the effluent from the centrifuge was still a sludge containing a high proportion of the grease, and no satisfactory way of disposing of this liquor was discovered.

At Wards Island in 1939, a continuous centrifuge was set up and experimented with to determine its possible utility for concentrating activated sludge. The following values were obtained on return sludge as feed:

Feed	Cake	Effluent
0.398%	4.72%	0.0932%

With excess sludge instead of return sludge as feed, the corresponding figures were:

Feed	Cake	Effluent
1.60%	2.59%	0.95%

Since the effluent of the centrifuge, as in the case of the Coney Island digested sludge, still presented a problem for final disposal, the experiments did not lead to any development along these lines.

Considerable study has been made on activated sludge in the laboratory in an effort to break down its gelatinous character, thus enabling the sludge to be separated from its voluminous water content. It is well known, of course, that ferric chloride of about 3% to 5% of the solids content will aid in this respect, but the cost of such treatment makes it adaptable only where sludge is to be dewatered on vacuum filters.

Some studies have been made of freezing methods suggested by John R. Downes of Plainfield, N. J., for dewatering primary digested sludge.[14] It was found that activated sludge could be concentrated to about 20% solids by freezing, quick thawing, and centrifuging. The liberated water, however, had to be separated immediately after each thawing, or the sludge became septic and less responsive to dewatering. The freezing method did not appear attractive for raw activated sludge.

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[14] *Proceedings*, New Jersey Sewage Works Assn.. 1939.

DISCUSSION

C. R. Velzy,[15] M. Am. Soc. C. E.—The subject of this Symposium is important to the designer who is faced with the problem of providing economical sludge disposal. The authors have treated the subject from the viewpoint of operation and research, and make valuable contributions in stressing some factors that aid in concentration, and also in indicating other factors which appear to be of no assistance.

Mr. Goudey indicates that raw sludge is concentrated with the same percentages of chemicals as are needed for digested sludge and therefore concludes (see heading, "Conclusions") that, for sludge concentration, "* * * digestion is unnecessary and wasteful." Digestion is a step in sludge disposal involving a number of factors rather than merely one method of concentrating sludge. Among these factors are an appreciable reduction in the quantity of solids, elimination of nuisance in sludge disposal, production of combustible gas, and other aids to sludge disposal. The reduction in solids by digestion results in the use of smaller quantities of chemicals for dewatering solids from a given quantity of raw sludge. Although digestion may be considered a means of thickening sludge, the reverse is often true—the concentration of sludge, in many cases, is an aid to digestion.

In Table 2 Mr. Keefer shows operating results at Baltimore, which appear to involve some discrepancies. For example, in the 1941 data, a quantity of 22,520 gal of sludge, 1.1% solids, is shown going to the thickener. The solids in the thickener effluent are negligible. Whereas the concentration of sludge is somewhat less than two to one, the reduction in volume is more than five to one. Stated in another way, the solids withdrawn from the thickener do not balance the solids discharged into the thickener. It is well known that in sewage treatment it is difficult to obtain representative samples and data truly characteristic of the actual conditions. Is it possible that more concentration was accomplished than is indicated by the data? Some comments by Mr. Keefer might be helpful.

Although consideration in the Symposium, and in some earlier work, has been given to concentration of primary and digested sludges, there is relatively not much to be gained in these applications. Digested sludge can be sufficiently concentrated in the bottom of digestion tanks. As Mr. Donaldson states, the density of primary sludge can be controlled adequately by the pumping schedule, and it is even possible to obtain primary sludge too dense for pumping.

With activated sludge in which concentration is likely to be from 1% to 2% solids, there is room for considerable gain by concentration. Even moderate results may reduce the volume to a half, or a third, of the original volume, which is a real gain for digestion, barging to sea, or any other process of sludge disposal.

Of the several factors affecting concentration of activated sludge, only a few can be applied practically by the designer. Mr. Rudolfs has shown that compaction may be affected by the character of sludge, initial concentration, time, temperature, pressure, and the use of chemicals. Of these elements, time and the use of chemicals have a direct bearing on design. Initial concentration is a factor in design because provision may be made to concentrate waste sludge from final settling tanks or aeration tank effluent, or to use either method at the option of the operator.

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Data in the several papers of the Symposium show that, at Los Angeles, Baltimore, and New York, satisfactory results in thickening activated sludge have been obtained. It is of interest to note that experience at Los Angeles and New York indicates that better concentration can be obtained by thickening aeration tank effluent than by thickening waste sludge. This fact permits simpler design and operation since the thickening tank can be built as a small unit of a group of final settling tanks, and an additional pumping operation on waste sludge is not required.

The designer at present finds a scarcity of conclusive data on sludge thickening. It is reasonably well established that time is a pertinent factor, but, over too long a period, sludge tends to become septic. This tendency can be controlled to some extent with chlorination. Other questions on which there is little information include the type of mechanism best adapted to concentrating and handling sludge, shape and size of sludge hoppers, and the degree of concentration to be expected. It is hoped that studies will be continued by operators having the facilities, and that particular emphasis will be placed on factors that may be incorporated in design and may be useful in operation.

R. F. GOUDEY,[16] M. AM. SOC. C. E.—Experience at the Los Angeles experimental sewage treatment plant, as previously discussed, indicates that, for a given volume of treated sewage, the chemical requirements for vacuum filtration of the fresh sludge and the equivalent requirements for the same fresh sludge digested are the same. It is realized that, volume for volume, digested sludge requires less chemical dosing than fresh sludge; but, considering the sewage producing fresh sludge and the corresponding reduced volume of digested sludge, the chemical doses are identical. It is quite logical, therefore, to question what advantage is actually gained from digesting sludge.

There are two purposes for digesting sludge: First, to obtain a sludge that can be dewatered on sand beds without creating a fly nuisance; and second, to obtain power. From the standpoint of fertilizing value, digestion reduces the nitrogen content from four to one per cent. On the other hand, chemically treated fresh sludge dewaters more rapidly than does digested sludge when placed on sand drying beds and does not create a fly nuisance. The chemically treated fresh sludge still retains a high nitrogen content but must be subjected to mold action for several months before it can be used satisfactorily as a fertilizer. Therefore, the broad question of whether or not sludge should be digested prior to chemical treatment seems far more involved than has yet been realized. It is hoped that further experiments in other parts of the United States will be made concentrating excess activated sludge under a chlorine blanket so that working knowledge of this superior method can be obtained firsthand.

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AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2264

WEDGE-SHAPED STRUCTURAL MEMBERS UNDER DIRECT STRESS AND BENDING

By Robert B. B. Moorman,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Phil M. Ferguson, Stanley U. Benscoter, George Winter, Walter H. Wheeler, Leon Beskin, and Robert B. B. Moorman.

Synopsis

Presented in this paper are expressions for computing compressive and shearing unit stresses in the concrete, tensile unit stresses in the reinforcement, and bond unit stresses between the reinforcement and concrete for wedge-shaped reinforced concrete members subjected to direct stress and bending. Curves to expedite the use of the expressions are also presented. Only approximate solutions are offered since the materials do not follow strictly the assumptions made in the derivations.

Introduction

The subject of stresses in wedge-shaped reinforced concrete members subjected to direct stress and bending has been treated in relatively few papers. E. Mörsch[2] presented a derivation pertaining to the shearing unit stress for the case in which the compressive surface is normal to the section on which the stress is considered acting, and the reinforcement inclined to that section. Mörsch's treatment serves as a basis for the more extensive derivations of expressions for shearing unit stress and bond unit stress found in this paper. V. Dašek[3] presented expressions with which to compute the shearing unit stress at any point along a section of the member, especially on the compressive side of the neutral axis. However, the shearing unit stress as used in design is a measure of diagonal tension, and for this reason only the stress on the tensile side of the neutral axis is of interest. On the compressive side of the neutral axis the actual shearing unit stress (maximum) would be found to be equal to

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Note.—Published in June, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Structures Consultant, Goodyear Aircraft Corp., Akron, Ohio.

[2] "Der Eisenbetonbau," by E. Mörsch, Konrad Wittwer, 1929, Vol. 1, Pt. 2, p. 290.

[3] "Die Schubspannungen bei aussermittiger Beanspruchung gerader Stäbe mit veränderlichem Querschnitt," by V. Dašek, *Beton und Eisen*, Vol. 37, 1938, p. 137.

one half the maximum compressive unit stress and would be acting on planes inclined at 45° to the compressive surface.

Notation.—The symbols used in this paper are defined where they first appear, either by text or diagram. They conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials[4] and American Standard Letter Symbols for Mechanics,[5] prepared by committees of the American Standards Association, with Society representation, and approved by the Association in 1932 and 1942, respectively.

Assumptions.—The usual assumptions that are made in deriving the ordinary expressions in reinforced concrete design are:

(1) A section plane before bending remains plane after bending;

(2) The reinforcement resists all the tension;

(3) The unit stress in the concrete is proportional to the distance from the neutral axis and in the reinforcement it is n times the proportionality for the concrete, in which n is the ratio between modulus of elasticity of steel and modulus of elasticity of concrete; and

(4) Perfect adhesion exists between the concrete and the reinforcement.

Bending Stresses

Fig. 1 shows part of a wedge-shaped reinforced concrete member acted upon by an external force, R. This force can be resolved into components N (normal) and V (shear) as shown. The total compression on the section is indicated by C, and the component, normal to the section A–A, of the total tension in the reinforcement by T. The compressive unit stress normal to and at the edge of the section is shown as f_0.

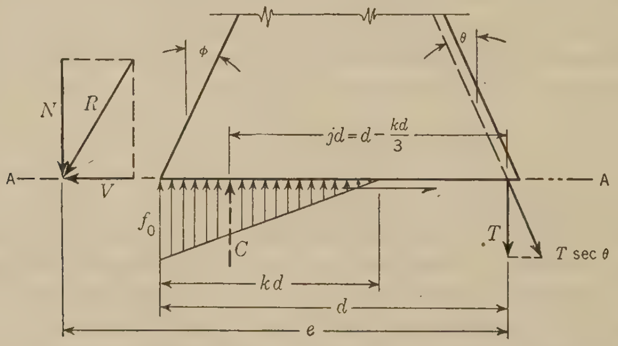

FIG. 1.—FORCES AND STRESSES ACTING ON PART OF A WEDGE-SHAPED MEMBER

In Fig. 2 assume that plane A–A′ and B–B′ bound a part of a wedge-shaped reinforced concrete member. The dashed line on the side of A′B′ represents the reinforcement. Assume that plane B–B′ is held in position while plane A–A′ is rotated through a very small angle α. The deformation, or change in length, of the reinforcement for length $\overline{rt}$ is $\overline{tu}$. Using the law of sines one may write

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[4] ASA—Z10a—1932.

[5] ASA—Z10.3—1942.

$$\frac{\overline{tu}}{\sin(90-\alpha)} = \frac{d(1-k)\tan\alpha}{\sin(90+\alpha-\theta)} \tag{1}$$

or

$$\overline{tu} = \frac{d(1-k)\tan\alpha\cos\alpha}{\cos(\theta-\alpha)} \approx \frac{d(1-k)\,\alpha}{\cos\theta} \tag{2}$$

since α is so small that $\tan\alpha = \sin\alpha = \alpha$ and $\cos\alpha = 1$, approximately. In Eqs. 1 and 2: d = the distance from the compression surface of the member to the center of the tensile reinforcement measured in the plane of section A–A; and k = the ratio of the distance from the neutral axis to the outer compressive fiber of a reinforced concrete member to the distance, d.

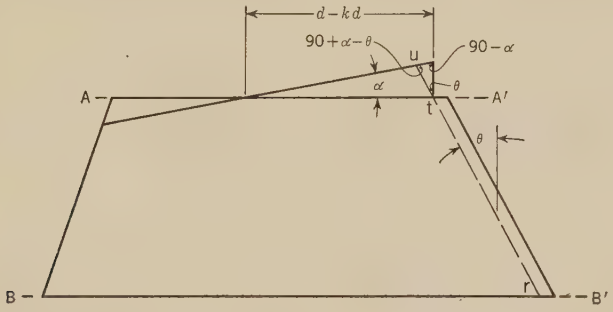

FIG. 2.—RELATION BETWEEN ANGLE CHANGE DUE TO BENDING AND DEFORMATION IN STEEL

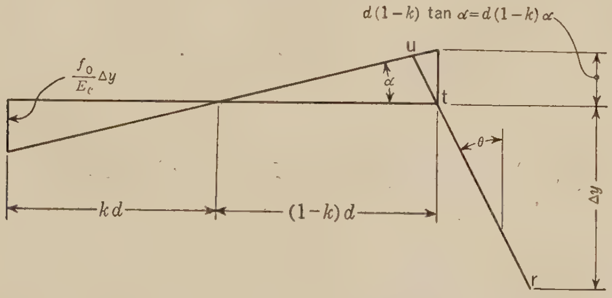

FIG. 3.—RELATION BETWEEN DEFORMATION OF CONCRETE AND DEFORMATION OF STEEL

Let Fig. 3 represent the deformations in length Δy. The angle α may be expressed in terms of the compressive unit stress in the concrete, f_0, normal to section A–A, as

$$\alpha = \frac{f_0\,\Delta y}{k\,E_c\,d} \tag{3}$$

and, multiplying both sides of Eq. 3 by $d(1-k)$, Eq. 3 becomes

$$d(1-k)\,\alpha = \frac{f_0}{E_c}\,\frac{1-k}{k}\,\Delta y \tag{4}$$

Now

$$\overline{\mathrm{tu}} \approx d\,(1-k)\,\alpha \sec\theta = \frac{f_0}{E_c}\,\frac{1-k}{k}\sec\theta\,\Delta y \qquad (5)$$

and

$$\overline{\mathrm{rt}} = \sec\theta\,\Delta y \qquad (6)$$

Therefore, the unit strain in the reinforcement can be expressed as

$$\frac{\overline{\mathrm{tu}}}{\overline{\mathrm{rt}}} = \frac{f_0}{E_c}\,\frac{1-k}{k} \qquad (7)$$

and the unit stress in the reinforcement as

$$f_s = E_s\frac{f_0}{E_c}\,\frac{1-k}{k} = n\,f_0\,\frac{1-k}{k} \qquad (8)$$

From a consideration of equilibrium (namely, the summation of vertical forces is equal to zero), the following expression may be written:

$$\tfrac{1}{2} f_0\, b\, k\, d = f_s\, p\, b\, d\cos\theta + N \qquad (9)$$

in which b = the width of the member; and p = the ratio of the area of the tensile reinforcement to the effective area of the concrete, $b\,d$. The area of the reinforcement is taken on a plane normal to the axis of the reinforcement.

Upon taking moments, about the point of application of the normal component of the resultant tensile stress in the reinforcement, of the forces acting on the section it is found that

$$\tfrac{1}{2} f_0\, k\, j\, b\, d^2 = N\,e \qquad (10)$$

Solving for N in Eq. 10 and substituting in Eq. 9,

$$\frac{1}{2} f_0\, k\, b\, d\left(1 - j\frac{d}{e}\right) = f_s\, p\, b\, d\cos\theta \qquad (11)$$

or

$$\frac{f_0}{f_s} = \frac{2\,p\cos\theta}{k\left[1-\left(1-\frac{k}{3}\right)\frac{d}{e}\right]} = \frac{2\,p\,\frac{e}{d}\cos\theta}{k\left(\frac{e}{d}-1+\frac{k}{3}\right)} \qquad (12)$$

Using Eq. 8 and Eq. 12 and eliminating $\frac{f_0}{f_s}$, one can write

$$k^2\left(\frac{e}{d}-1+\frac{k}{3}\right) = \frac{e}{d}(1-k)\,2\,p\,n\cos\theta \qquad (13)$$

or

$$\frac{k^3}{3} + k^2\left(\frac{e}{d}-1\right) + k\left(2\,p\,n\,\frac{e}{d}\cos\theta\right) = 2\,p\,n\,\frac{e}{d}\cos\theta \qquad (14)$$

Eq. 14 can be rearranged and a solution for $\frac{d}{e}$ computed as follows:

$$\frac{d}{e} = \frac{k^2 + k\,2\,p\,n\cos\theta - 2\,p\,n\cos\theta}{k^2\left(1-\frac{k}{3}\right)} = \frac{3}{3-k} + \frac{6\,(k-1)}{k^2\,(3-k)}\,p\,n\cos\theta \qquad (15)$$

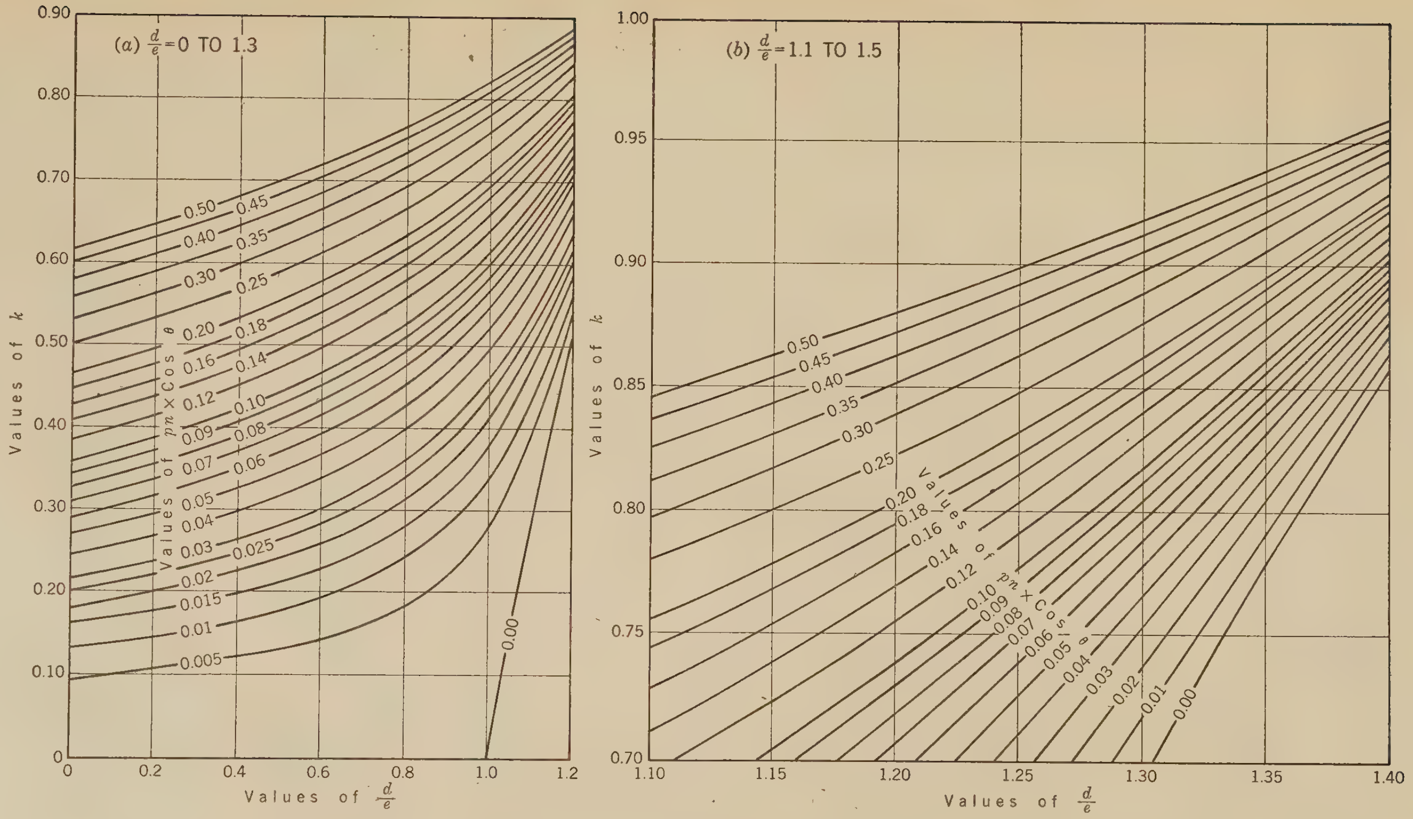

FIG. 4.—CURVES OF $\frac{d}{e} = \frac{3}{3-k} + \frac{6(k-1)}{k^2(3-k)} p\, n \cos\theta$

Eq. 15 has been used to plot the curves for various ($p\ n \cos \theta$)-values shown in Fig. 4. The $\frac{d}{e}$-values are the abscissas and the k-values the ordinates. The value of j may be computed by using the expression $j = 1 - \frac{k}{3}$.

The following expressions are presented for use in computing the maximum normal stresses:

$$f_c = f_0 \sec^2 \phi = \frac{2\ N\ e \sec^2 \phi}{k\ j\ b\ d^2} \qquad (16a)$$

and

$$f_s = n\ f_0 \frac{1-k}{k} = \frac{2\ n\ N\ e\ (1-k)}{k^2\ j\ b\ d^2} \qquad (16b)$$

In Eq. 16*a* it will be noted that the maximum compressive unit stress in the concrete is given as $f_c = f_0 \sec^2 \phi$. This expression[6] is for the unit stress on a plane normal to the compression face of the member.

In the case of a tee-shaped section in which the flange is in compression, $j\ d$ may be taken equal to $d - \frac{t}{2}$, approximately, in which t is the thickness of the flange. One may then use the following approximate expressions when computing the unit stresses due to direct stress and bending:

$$f_c = f_0 \sec^2 \phi = \frac{2\ N\ e \sec^2 \phi}{t\left(d - \frac{t}{2}\right) b_1} \qquad (17a)$$

and

$$f_s = \frac{N\left(e - d + \frac{t}{2}\right) \sec \theta}{A_s\left(d - \frac{t}{2}\right)} \qquad (17b)$$

in which b_1 = width of the compressive flange; and A_s = area of effective tensile reinforcement, for this case in width b_1.

Shear and Bond Stresses

Imagine Fig. 5 to be a part of a reinforced concrete member with a variable cross-section area. Assume that two sections are passed parallel to the horizontal plane a distance dy from one another. Taking moments about the intersection of the resultant of the compressive stresses with the section, $R\ r = T\ z'$; or

$$T = \frac{R\ r}{z'} \qquad (18)$$

in which: R = resultant of the external forces; r = perpendicular distance from line of action of the resultant, R, to the moment center; T = total tensile stress in the reinforcement; and z' = perpendicular distance from the line of action of the tensile force in the reinforcement to the moment center.

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[6] "Earth Pressure, Walls and Bins," by William Cain, John Wiley & Sons, Inc., New York, N. Y., 1st Ed., 1916, p. 240.

Considering changes to take place only in T, r, and z', the change in tensile stress in the reinforcement in distance dy may be expressed as

$$dT = R\,\frac{z'\,dr - r\,dz'}{(z')^2} = \frac{R\,dr}{z'} - \frac{R\,r\,dz'}{(z')^2} \qquad (19)$$

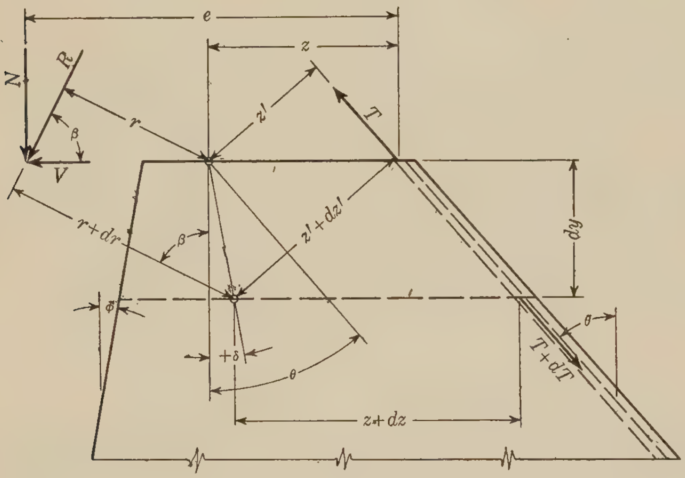

FIG. 5.—PART OF WEDGE-SHAPED MEMBER SHOWING MOMENT CENTERS

The following expressions for dr, dz', and z' may be derived from the geometrical relations (see Fig. 5):

$$dr = dy\cos\beta + dy\tan\delta\sin\beta \qquad (20a)$$

$$dz' = dy\sin\theta - dy\tan\delta\cos\theta \qquad (20b)$$

and

$$z' = z\cos\theta \qquad (20c)$$

Substituting Eqs. 20 in Eq. 19, the expression for dT is

$$dT = \frac{R\,dy}{z\cos\theta}(\cos\beta + \tan\delta\sin\beta) - \frac{R\,r\,dy}{z^2\cos^2\theta}(\sin\theta - \tan\delta\cos\theta) \qquad (21)$$

The differential tension, dT, in the reinforcement produces a unit shearing stress, v, in the adjacent concrete, which is parallel to the reinforcement, and an equal unit shearing stress v at right angles to the reinforcement.

If b is the width of the member, one may write

$$b\,v\,\frac{dy}{\cos\theta} = dT \qquad (22)$$

Substituting Eq. 21 in Eq. 22 and simplifying:

$$b\,v = \frac{R}{z}(\cos\beta + \sin\beta\tan\delta) - \frac{R\,r}{z^2}(\tan\theta - \tan\delta) \qquad (23)$$

From Fig. 4(b), also noting Eq. 18, it may be seen that

$$R \cos \beta = V \qquad (24a)$$

$$R \sin \beta = N \qquad (24b)$$

and

$$\frac{R\,r}{z \cos \theta} = T \qquad (24c)$$

Upon substitution of Eqs. 24 in Eq. 23, the expression for $b\,v$ becomes

$$b\,v = \frac{V + N \tan \delta}{z} - \frac{T\,(\tan \theta - \tan \delta)}{z} \cos \theta \qquad (25)$$

or, if the moment $R\,r$ of the external forces with respect to the selected moment center be indicated by M_0 (that is, $M_0 = z\,T \cos \theta$), Eq. 25 may be written as

$$b\,v = \frac{V + N \tan \delta}{z} - \frac{M_0\,(\tan \theta - \tan \delta)}{z^2} \qquad (26)$$

or

$$v = \frac{V + N \tan \delta}{b\,j\,d} - \frac{M_0\,(\tan \theta - \tan \delta)}{b\,(j\,d)^2} \qquad (27)$$

in which $j\,d$ = the distance, in the plane of the section, between the resultant of the compressive stresses and the resultant of the tensile stresses.

If it is desired to use the moment about the reinforcement as was done previously, the expression for M_0 becomes

$$M_0 = z\,T \cos \theta = R\,r = N\,(e - z) = N\,(e - j\,d) \qquad (28)$$

and substitution in Eq. 27 results in

$$v = \frac{V}{b\,j\,d} + \frac{N \tan \theta}{b\,j\,d} - \frac{N\,e}{b\,(j\,d)^2} (\tan \theta - \tan \delta) \qquad (29a)$$

In the case of a tee-shaped cross section, if the tee is on the compressive side, the position of the resultant of the compressive stresses changes so slightly with respect to the tee that $\tan \delta$ may be taken equal to $\tan(-\phi)$ without appreciable error. Eq. 29a can then be written as

$$v = \frac{V}{b'\,j\,d} + \frac{N \tan \theta}{b'\,j\,d} - \frac{N\,e}{b'\,(j\,d)^2} (\tan \theta + \tan \phi) \qquad (29b)$$

in which b' = the width of the stem of the tee-shaped member.

The expression for the unit bond stress, u, may be written

$$u = \frac{dT}{\dfrac{dy}{\cos \theta} \Sigma_o} = \frac{b\,v}{\Sigma_o} \qquad (30)$$

since, from Eq. 22, one has $\dfrac{dT}{\dfrac{dy}{\cos \theta}} = b\,v$. For rectangular sections, using

Eq. 29*a*, the expression for u becomes

$$u = \frac{V}{j\,\Sigma_o\,d} + \frac{N\tan\theta}{j\,\Sigma_o\,d} - \frac{N\,e}{(j\,d)^2\,\Sigma_o}(\tan\theta - \tan\delta)\ldots\ldots\ldots(31)$$

Eq. 31 may be used for tee-sections by substituting $-\phi$ for δ in the last quantity.

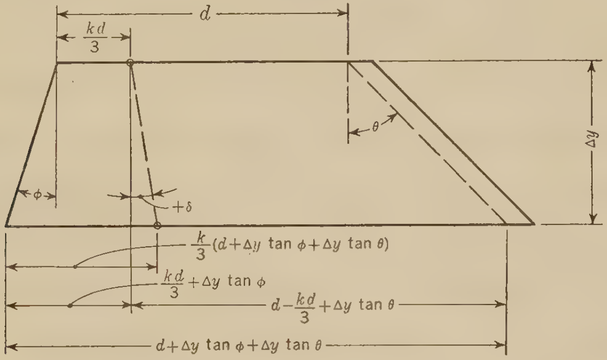
d
$\frac{kd}{3}$
θ
Δy
ϕ
$+\delta$
$\frac{k}{3}(d+\Delta y\tan\phi+\Delta y\tan\theta)$
$\frac{kd}{3}+\Delta y\tan\phi$
$d-\frac{kd}{3}+\Delta y\tan\theta$
$d+\Delta y\tan\phi+\Delta y\tan\theta$

FIG. 6.—CHANGE IN POSITION OF RESULTANT COMPRESSIVE STRESSES

An approximate expression for $\tan\delta$ may be derived in the following manner: Since k varies so slightly with y, especially over a distance Δy, one may write (noting Fig. 6),

$$\Delta y\tan\delta = \frac{k}{3}[d + \Delta y\,(\tan\theta + \tan\phi)] - \frac{k\,d}{3} - \Delta y\tan\phi\ldots\ldots(32)$$

Dividing by Δy the expression for $\tan\delta$ becomes

$$\tan\delta = \frac{k}{3}(\tan\theta + \tan\phi) - \tan\phi = k\left(\frac{\tan\theta}{3}\right) - j\tan\phi\ldots\ldots(33)$$

ILLUSTRATIVE EXAMPLE

Assume that it is desired to determine the unit stresses at section A–A in the part of the structure shown in Fig. 7. The force, R, per foot of member

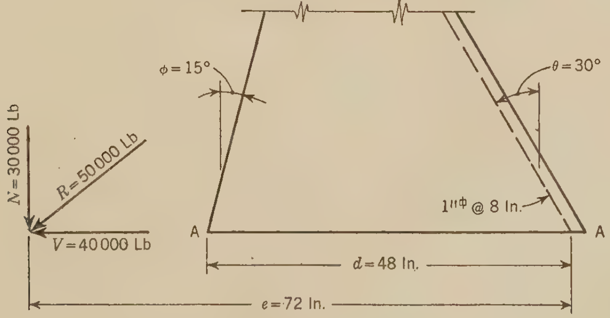
$\phi=15°$
$\theta=30°$
N=30000 Lb
R=50000 Lb
V=40000 Lb
A
A
1"φ @ 8 In.
d=48 In.
e=72 In.

FIG. 7.—ILLUSTRATIVE EXAMPLE

normal to the plane of the paper, is resolved into vertical (N) and horizontal (V) components. Other data are given in Fig. 7. The computation procedure is as follows: A_s per foot $= 1.5$ sq in.; Σ_o per foot $= 6$ in.; $p = \frac{1.5}{48 \times 12} = 0.0026$; $n = 10$ (given); and $p\,n = 0.026$. Enter Fig. 4(*a*) with $p\,n \cos\theta = 0.0225$ and $\frac{d}{e} = 0.667$; read $k = 0.29$; and compute $j = 1 - \frac{k}{3} = 0.903$. Use Eq. 10 to compute $f_0 = \frac{2 \times 30{,}000 \times 72}{0.29 \times 0.903 \times 12 \times 48^2} = 596$ lb per sq in.

From Eq. 16*a*, $f_c = 596 \times 1.072 = 639$ lb per sq in.; and, from Eq. 8, $f_s = 5{,}960\,\frac{0.71}{0.29} = 14{,}590$ lb per sq in. A value for $\tan\delta$ can be determined by use of Eq. 33 as equal to $0.29\,\frac{0.577}{3} - 0.903 \times 0.268 = -\,0.186$.

The shearing unit stress (as a measure of the diagonal tension) can be computed using Eq. 29*a*; thus, $v = \frac{40{,}000 + 30{,}000 \times 0.577}{12 \times 0.903 \times 48} - \frac{30{,}000 \times 72 \times (0.577 + 0.186)}{12\,(0.903 \times 48)^2} = 37$ lb per sq in. The bond unit stress can be computed by use of Eq. 30 as $u = \frac{12 \times 37}{6} = 74$ lb per sq in.

CONCLUSIONS

Formulas have been developed for compressive, bond and shearing stresses in the concrete and for tensile stress in the reinforcement for wedge-shaped reinforced concrete members. These expressions are applicable to members subject to bending and direct stress. The formulas are correct in so far as the assumptions used in the ordinary flexural theory are applicable to reinforced concrete.

DISCUSSION

Phil M. Ferguson,[7] M. Am. Soc. C. E.—Sometimes the reasonableness of a conclusion is as reliable a measure of an equation's worth as the reasonableness of the assumptions underlying this equation. As one who accepts conclusions very reluctantly when they do not appear to lead to reasonable results, the writer confesses he had difficulty with this paper. The following comments are largely critical of the results obtained by the proposed method.

The author has attacked a complex problem in an interesting manner. However, when the reader reaches Eq. 14, defining the location of the neutral axis, and discovers that this equation involves the slope of the reinforcing steel but not the slope of the compression face of the concrete, questions are raised. According to this equation, no matter what the concrete face slope (angle ϕ in Fig. 1) may be, the neutral axis will be the same relative distance $k\,d$ from the compression face; and this equation then leads to the further conclusion that f_0 and f_s are also independent of ϕ. The writer does not believe that this is true. On the contrary, he believes that all three of these terms are influenced by ϕ, f_0 and k being influenced to a very significant degree. It is probable that the influence of either ϕ or θ, which defines the slope of the steel, will be small when these angles are small; but it also seems probable that the influence is considerable when either angle is large. It is chiefly for these larger angles that concrete designers need specialized formulas.

It is probable that the basic assumption that plane sections before bending remain plane after bending is not acceptable in this problem, as satisfactory as it may have proved in other cases involving members of constant depth. It may be that this is the source of the following serious discrepancies.

Another contributing factor may be an inconsistency in the treatment of deformations at the beginning of the paper. The author is careful to relate the deformation of the steel to a certain vertical deformation and to his angle α. He makes no corresponding attempt to relate the outside concrete fiber deformation to the angle α. Instead, Eq. 3 states a supposed relation between f_0 and α which assumes that the unit concrete deformation vertically will be given by f_0/E_c. This relation ignores the fact that, with the principal stress parallel to the sloping concrete face, there exists a compressive stress perpendicular to f_0; and this stress will be large for large values of ϕ. It would seem that Poisson's ratio can scarcely be excluded from this equation. Even if this modification were made, it would seem safer to establish α or $k\,d$ from the inclined edge deformation which is more surely known in terms of f_c. A different location for the neutral axis might result from this treatment. However, it is doubtful whether any treatment based on straight-line deformations can be used in the general fashion indicated by this paper when face angles are of interesting magnitude.

The author implies that no limitations on the relative or absolute values of ϕ and θ are necessary. In the author's illustrative problem shown in Fig. 7, horizontal section A–A might have been taken at some other slope provided proper adjustment in e, N, V, and p accompanied this different choice of section. If the theory presented in this paper is correct, the concrete stress f_c at the left face at point A should be equally well determined by the author's section or by other inclined sections through the same point. Since the author's conclusions appeared open to considerable doubt, the writer calculated f_c at point A by the methods of this paper using sections A–B and A–C (Fig. 8(*a*)). The author reported f_c to be 639 lb per sq in. when based on section A–A. The calculated value of f_c at the same point on section A–B is 810 lb per sq in., and on section A–C it is 595 lb per sq in. If this theory were correct, there would not be this spread of more than one third in the resulting value of f_c when only the slope of the section analyzed is altered. One must conclude that the formulas are not correct for all sections—possibly not correct for any section.

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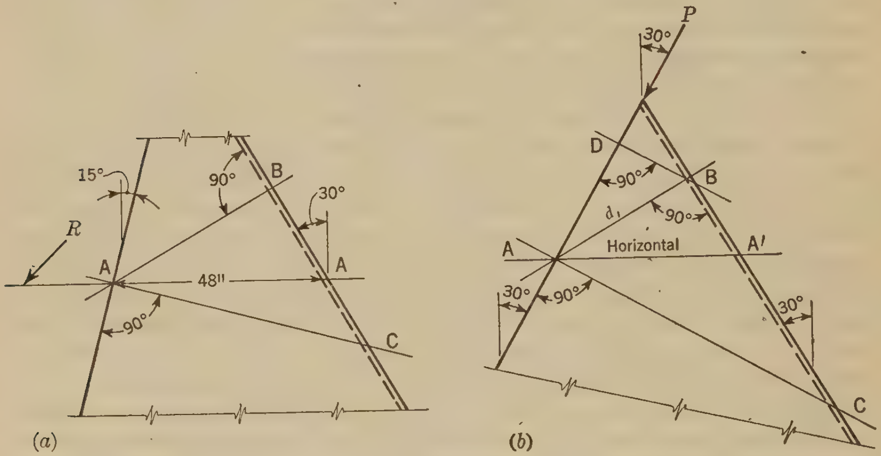

Fig. 8

To investigate this matter further, another wedge-shaped member was loaded in simple fashion, as shown in Fig. 8(*b*). With the load acting along one face, comparative analyses were simplified. The maximum concrete stress f_c at point A was calculated by the author's method applied to sections A–A′, A–B, and A–C. The steel stress f_s at point B was also calculated with sections A–B and D–B. The depth of section A–B was designated as d_1; the area of steel A_s was taken as $0.01\,b\,d_1$. The results show even more spread than in the foregoing case. A correct theory should give identical values of f_c and identical values of f_s for the points shown in Table 1.

Such divergent results lead to the conclusion that, until this method can be modified to give more consistent results on different sections, or until those sections are designated on which it may properly be used, the method must be rejected as one producing inconsistent results.

No lengthy comment will be made on the bond and shear formulas since these formulas must be subject to the errors of the bending formula. However, attention should be directed briefly to the theory behind the shear formula, because it might lead to unsafe design even when based on a correct tension formula. The author uses the shear adjacent to the steel as a measure of diagonal tension. This shear is not the largest shear in the case of a beam subjected to direct stress and bending. A larger shear exists within the compressive area, even in the case of a parallel sided member.[8] Moreover (although this is contrary to the conclusion of Stanley U. Benscoter and Samuel T. Logan, Juniors, Am. Soc. C. E.[8]), this larger shear will produce a considerably larger diagonal tension in the compression zone near the neutral axis than exists between the neutral axis and the steel. The small compression stress acting in this zone does make the diagonal tension smaller than the shear, but still leaves it greater than the lower shear value found near the steel. The shear adjacent to the steel is thus not a proper measure of the maximum diagonal tension and the author should not have used it as such.

TABLE 1.—DISCREPANCIES IN CALCULATED STRESSES

BASED ON SECTION:		Computed f_0	Computed f_c at point A	Computed f_s at point B
Fig. 8(*b*)	Description			
A–B	Perpendicular to steel	2.05 $P/b\,d_1$	8.20 $P/b\,d_1$	12.86 $P/b\,d_1$
A–A′	Horizontal	3.22 $P/b\,d_1$	4.32 $P/b\,d_1$	....
A–C	Perpendicular to compressive face	2.67 $P/b\,d_1$	2.67 $P/b\,d_1$	....
D–B	Perpendicular to compressive face	....	....	51.6 $P/b\,d_1$

S. Timoshenko[9] shows that large shears will accompany bending of a wedge-shaped beam of homogeneous material. In the case of a 15° slope of each face he shows that the maximum unit shear on a section perpendicular to the axis of symmetry is some 90% greater than on a beam of the same depth with parallel faces. This result does not necessarily apply to a reinforced concrete beam; but it should make one very conservative in the calculation of unit shears to be used as a measure of diagonal tension in the case of any member with sloping sides.

STANLEY U. BENSCOTER,[10] JUN. AM. SOC. C. E.—The concrete designer will find in this paper a group of formulas, for wedge-shaped members, developed on the basis of reasonable assumptions. Due to the lack of such formulas, internal stresses have been commonly computed for wedge-shaped members by using formulas and methods which should be applied only to prismatic members. The writer has been guilty of such erroneous stress analysis and has witnessed the same error by many designers. The Joint Committee Code renders little assistance in the analysis of combined stresses in wedge-shaped members, and it is of real value to the profession that formulas should become available.

It is possible to study the individual terms of some of the formulas and equations developed in this paper and thereby to establish some rules for design which tie in closely with past experience in the design of prismatic members. The cubic equation which governs the location of the neutral axis for a prismatic beam is well known. An examination of Eq. 14 indicates that one may replace the tensile steel area A_s, or the tensile steel area ratio p, by $A_s \cos \theta$ or $p \cos \theta$, respectively, and the cubic equation for prismatic beams will become the cubic equation for a tapered beam. This gives a simple rule for design. Project the normal cross section of the steel rods into the plane of the beam cross section to obtain a reduced steel area. Then calculate the neutral axis location just as for a prismatic beam. This rule is as applicable to compressive steel as to tensile steel. In the case of compressive steel, $(A_s)'$ and p' are replaced by $(A_s)' \cos \phi$ and $p' \cos \phi$, in which $(A_s)'$ and p' refer to the compressive steel area.

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[8] "Shear and Bond Stresses in Reinforced Concrete," by Stanley U. Benscoter and Samuel T. Logan, *Transactions*, Am. Soc. C. E., Vol. 110 (1945), p. 599.

[9] "Strength of Materials," by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N. Y., 2d Ed., Pt. II, 1941, p. 337.

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In solving the cubic equation for k a designer usually estimates the first significant figure correctly but seldom guesses the second significant figure correctly in an initial estimate. The graphs of Fig. 4 will give the second significant figure for members without compressive steel and will probably give a good estimate in most cases for members with compressive steel. They are equally useful for prismatic or tapered members. The third significant figure may be obtained from a few cycles of trial and error or by iteration computations. In research studies of shear and bond stresses the normal stresses must be computed accurately at sections 1 in. apart and a fourth significant figure for k is generally desirable.

Before considering the individual terms of the formulas for shear and bond stresses it is profitable to recall the formulas recommended by the Joint Committee Code for wedge-shaped members without thrust. The usual formulas for shear and bond stress for a prismatic beam may be used if the shear V is replaced by the modified shear V_1 given by,[11]

$$V_1 = V \pm \frac{M}{d} (\tan \theta + \tan \phi) \ldots\ldots\ldots\ldots\ldots\ldots (34)$$

in which the negative sign is to be used when the depth of the beam increases in the direction of increasing bending moment; and θ and ϕ are the angles, respectively, that beam faces make with a direction normal to the direction of the external shear. It is not difficult to see that this definition of θ and ϕ is sensible only in the case of a beam without thrust and cannot be extended to a beam-column such as a member of a rigid frame. In the case of a haunched beam-column, loaded normal to its straight surface, it is common practice to calculate stresses on a cross section normal to the loaded surface, thus assuming θ to be zero in regions of negative moment. Perhaps it would be more accurate to calculate the stresses on a plane that is normal to the bisector of the wedge angle. No matter what direction of cross section is considered by the designer as the one on which flexural theory will give the most accurate stress determination, the resultant force may be resolved into a normal and tangential component. The expression "external shear" creeps into the Joint Committee Code at several points, thus causing considerable confusion.

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[11] "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," Rept. of the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete, *Proceedings*, Am. Soc. C. E., June, 1940, Pt. 2, p. 56.

If the coefficient $\frac{M}{d}$ in Eq. 34 is replaced by $\frac{M}{j\,d}$, the formula has a simple rational explanation. This explanation has been given by the Joint Committee Code by stating that[12] "the resultant internal shear at any section is increased or decreased by the vertical components of the inclined stresses (tension or compression) * * *." Since $\frac{M}{j\,d}$ equals the normal tension T' or compression C' on a section without thrust, Eq. 34 may be modified to read (using the negative sign):

$$V_1 = V - (T' \tan \theta + C' \tan \phi) \qquad (35)$$

The normal forces T' and C' are components of the inclined forces T and C. Thus the terms $T' \tan \theta$ and $C' \tan \phi$ are components of T and C in the plane of the cross section. The force T acts along the center line of the tensile bar (or bars) whereas the force C acts parallel to the external surface in the compressive region. It would appear more reasonable if the angle ϕ were replaced by the angle $-\delta$ as introduced by Mr. Moorman, so that C acts along the line of centroids of the compressive stresses. The formula for δ as given by Eq. 33 may also be used for members without thrust.

It may be shown that Eq. 29*a* for v can be expressed in terms of the modified shear V_1 as given by Eq. 35. Regrouping the terms of Eq. 29*a* gives,

$$v = \frac{V}{b\,j\,d} - \left[\frac{N\,(e - j\,d)}{b\,(j\,d)^2} \tan \theta + \frac{N\,e}{b\,(j\,d)^2} \tan(-\delta)\right] = \frac{V_1}{b\,j\,d} \qquad (36)$$

in which

$$V_1 = V - [T' \tan \theta + C' \tan(-\delta)] \qquad (37)$$

When a section carries thrust as well as bending, the forces T' and C' are not equal but must be computed separately. Eq. 37 gives the designer a convenient rule for determining the shear and bond stresses in agreement with the assumptions of Mr. Moorman's analysis.

There are upper limits on the angles θ and ϕ beyond which the assumption of linear strain provides too much error. It is also known from the theory of elasticity that, in wedge-shaped beams, the shearing stresses have distributions which are greatly different from those in prismatic beams. The error involved in neglecting the rate of change of k along the member in evaluating δ from Eq. 32 is not known. These matters remain for future research study. It may be noted that, according to Eq. 33, the angle δ becomes zero when θ and ϕ are zero. Actually the angle δ is not zero for a prismatic beam that carries thrust. This error enters in due to the neglect of the rate of change of k along the member in writing Eq. 32. Neglecting the rate of change of j along the member leads to the Joint Committee Code formula, Eq. 34, and the formula is thus slightly in error.

GEORGE WINTER,[13] M. AM. SOC. C. E.—The ingenuous analysis of an important problem, as presented in this paper, is based on the assumption that plane sections remain plane in the deformed or stressed state of the members. This assumption is rigorously true for beams in pure bending. For transversely loaded beams it can be shown that the error introduced by such an assumption is insignificant provided the span : depth ratio is not too small. For beams of span : depth ratio smaller than about 3, such as occur occasionally in bunkers when the vertical walls are designed as beams spanning between supporting columns, the assumption of a straight-line strain distribution leads to very substantial errors.[14] This is true despite the fact that, in such beams, edges are parallel.

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[12] "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," Rept of the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete, *Proceedings* Am. Soc. C. E., June, 1940, Pt. 2, p. 55.

[13] Asst. Prof., Civ. Eng., Cornell Univ., Ithaca, N. Y.

In the case of a wedge-shaped member it appears pertinent to question whether the assumption of a straight-line strain distribution can be expected to give reasonably accurate results, for the following two reasons: (*a*) In such members, except for very small wedge angles, the ratio of length of element to average depth of cross section is rather small—by analogy with the aforementioned prismatic beams, one may then expect a nonlinear strain distribution for essentially identical reasons; and (*b*) in addition, the sloped shape of the member is likely to introduce further deviations from the familiar situation in long, shallow prismatic beams.

It is customary to assume that the strain distribution in a reinforced concrete member is the same as that in an analogous member made of homogeneous material. This enables one to apply the results of the theory of elasticity of such homogeneous members without substantial change to the design of reinforced concrete structures. Although it is likely that this assumption is not strictly correct, it has proved its worth in practical application. Proceeding on this assumption, it is necessary, in the present case, to investigate the strain distribution in a homogeneous, wedge-shaped member under bending plus thrust. The fundamental theory pertinent to this problem has been given by

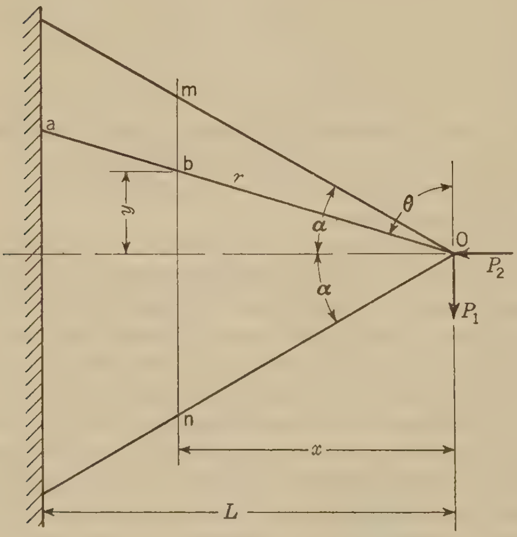

FIG. 9

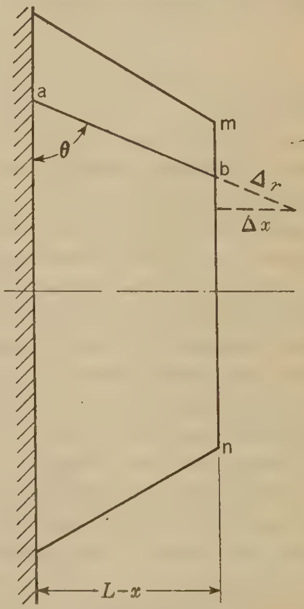

FIG. 10

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[14] "Über den Spannungszustand in hohen Trägern und die Bewehrung von Eisenbetontragwänden," by Hermann Bay, Berlin, 1931.

S. Timoshenko,[15] who shows that, in a wedge loaded as in Fig. 9, the principal tension and compression stresses are directed radially. The author's case of bending plus thrust can be resolved into the two individual cases of transverse bending by a force P_1 in Fig. 9 and of longitudinal thrust by a force P_2. The stress-strain distribution due to simultaneous action of P_1 and P_2 is then obtained by simple superposition.

The strain distribution in bending due to P_1 will first be investigated. The magnitude of the radial stress at point b (Fig. 9), distance r from point O on a radius subtending the angle θ with the direction of the force P_1,[16] is

$$f_r = \gamma_1 \frac{\cos\theta}{r} \qquad (38)$$

in which

$$\gamma_1 = -\frac{P_1}{\alpha - 0.5 \sin(2\alpha)} \qquad (39)$$

is a constant depending on the wedge angle and the magnitude of the load P_1. Consequently, the radial strain is

$$\epsilon_r = \frac{\gamma_1}{E}\frac{\cos\theta}{r} \qquad (40)$$

To obtain the shape of an originally plane cross section mn (Fig. 9), it is necessary to calculate the change of length Δ_{ab} of a fiber of undistorted length ab (Fig. 10). This change of length is obtained by integrating the radial strain (Eq. 40) over the length of fiber ab. Thus,

$$\Delta_{ab} = \frac{\gamma_1 \cos\theta}{E}\int_{r_b}^{r_a}\frac{dr}{r} = \frac{\gamma_1\cos\theta}{E}\log_e\frac{r_a}{r_b} \qquad (41)$$

in which r_a and r_b are, respectively, the radial lengths Oa and Ob (Fig. 9). From the angular relations of Fig. 9,

$$r_a = \frac{L}{\sin\theta} \qquad (42a)$$

and

$$r_b = \frac{x}{\sin\theta} \qquad (42b)$$

and, by substitution of these values in Eq. 41,

$$\Delta_{ab} = \frac{\gamma_1\cos\theta}{E}\log_e\frac{L}{x} \qquad (43)$$

The horizontal component of the displacement Δ_{ab} of point b is

$$\Delta x = \Delta_{ab}\sin\theta = \frac{\gamma_1}{2E}\left[\log_e\frac{L}{x}\right]\sin(2\theta) \qquad (44)$$

On the other hand, the magnitude of the distance from the axis, y (Fig. 9), is

$$y = x\cot\theta = x\frac{\sin(2\theta)}{1-\cos(2\theta)} \qquad (45)$$

A comparison of Eqs. 44 and 45 shows that, whereas y (the distance from the axis) is proportional to $\frac{\sin (2\,\theta)}{1 - \cos (2\,\theta)}$, the magnitude of the horizontal displacement Δx is proportional to $\sin (2\,\theta)$. It is thus established that, for the case of transverse bending, strains are not proportional to y in wedge-shaped members and that, consequently, originally plane cross sections become curved under such loading.

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[15] "Theory of Elasticity," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1934, pp. 93–95.

[16] *Ibid.*, Eq. 70, p. 94.

The shape of a cross section of a member subject to axial thrust is easily investigated in the same manner. With the angle θ as defined in Fig. 9, the radial stress in this case[17] is

$$f_r = \gamma_2 \frac{\sin \theta}{r} \quad \text{(46)}$$

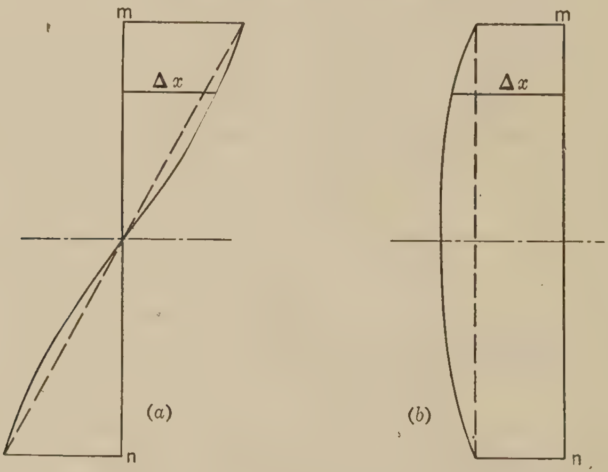
in which

$$\gamma_2 = - \frac{P_2}{\alpha + 0.5 \sin (2\,\alpha)} \quad \text{(47)}$$

The horizontal component of the displacement, obtained by the same reasoning as before, is, in this case,

$$\Delta x = \frac{\gamma_1}{2\,E} \left[\log_e \frac{L}{x} \right] \sin^2 \theta \quad \text{(48)}$$

This displacement is not constant over the cross section, but is proportional to $\sin^2 \theta$. Thus, as in the case of transverse bending, plane cross sections become curved in a wedge-shaped member subject to radial thrust.



(a) (b)

FIG. 11

To facilitate visualizing the magnitude of this effect, displacements were computed from Eqs. 43 and 48 for the case of a wedge with $\alpha = 30°$. The shapes of the distorted cross sections are plotted to scale—in Fig. 11(*a*) for transverse bending and in Fig. 11(*b*) for radial thrust. Even for this relatively small angle α, the deviations from planeness are substantial. For example, in the case of axial thrust, with $\alpha = 30°$, the horizontal displacement along the axis is one third larger than that at the outer fibers, whereas, if plane cross sections were maintained, these two displacements would have to be the same. Figs. 11(*a*) and 11(*b*) further show that in the case of simultaneous bending plus thrust, by superposition, the curvatures of the section subtract on the tension side, but add on the compression side. This complicates further the final shape of the cross section, which will also depend on the ratio $P_1 : P_2$.

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[17] "Theory of Elasticity," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1934, Eq. 69, p. 94.

An exact evaluation of the numerical influence of this distortion on the design values computed on the assumption of plane deformation is far beyond the scope of these sketchy comments. The numerical magnitude of the aforementioned discrepancies suggests, however, that practical application of the author's findings should probably be delayed until a more rigorous investigation of this problem becomes available.

WALTER H. WHEELER,[18] M. AM. SOC. C. E.—Lucidly and commendably, the author has developed for the use of the profession a comparatively simple set of formulas, for which he does not claim 100% accuracy. However, the formulas are accurate enough for practical purposes, and they are simple.

Although wedge-shaped members are not common in the average type of structure they do, or can, occur frequently in retaining walls (especially high ones), bridge piers, and abutments.

The writer suggests that, in his closure, the author indicate how the analysis could be modified to apply to pyramidal structures.

LEON BESKIN,[19] ASSOC. M. AM. SOC. C. E.—The equilibrium of a wedge is usually considered only in the case of forces applied at the apex. The general loading must include the effect of a pure couple. Even more general loading conditions could be examined. This problem has been analyzed by J. H. Michell[20] for the case of a plane element limited to two radii and to two concentric circles, and Mr. Michell's analysis has been completed by the writer.[21]

The problem is too general in scope to be examined herein. It involves an analysis of the influence of the loading distribution on the stress distribution. This influence is disregarded in the conventional analysis of straight beams. For such an analysis, the stress distribution is assumed to be a function only of the resultant

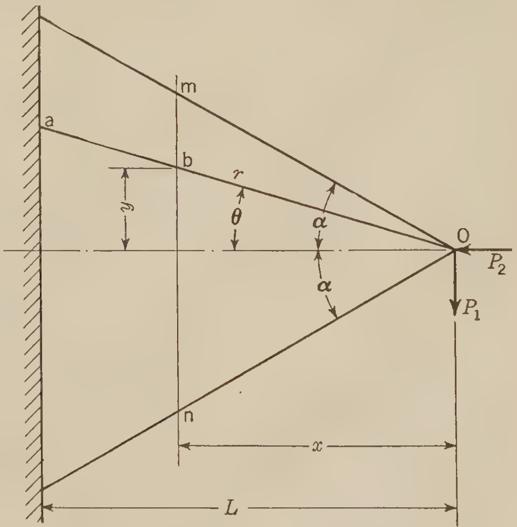

FIG. 12

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[20] *Proceedings*, London Mathematical Soc., Vol. 31, 1899, p. 100.

[21] *Comptes Rendus*, Académie des Sciences, Institut de France, Vol. 204, 1937, p. 1161.

of the loads and the moment about the centroid of the section at which the stresses are computed. For the same reasons as those given for straight beams, these effects will be neglected herein, and the problem will be limited to the case of a couple applied to the apex. The form of the corresponding stress function has been established by Mr. Michell[20] and reported by S. Timoshenko.[22]

For the case of a pure couple, the necessary terms of the stress function are:

$$\phi = a'_o\,\theta + d'_2 \sin 2\,\theta \qquad (49)$$

The corresponding stress components (see Fig. 12) are:[23]

$$\sigma_r = \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = -\frac{4\,d'_2}{r^2}\sin 2\,\theta \qquad (50a)$$

$$\sigma_\theta = \frac{\partial^2 \phi}{\partial r^2} = 0 \qquad (50b)$$

and

$$\tau_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \phi}{\partial \theta}\right) = \frac{1}{r^2}(a'_o + 2\,d'_2 \cos 2\,\theta) \qquad (50c)$$

Assuming that the shear is equal to zero along the two faces ($\theta = \pm\,\alpha$), the constant a'_o is defined as a function of d'_2 and the expression for the shear becomes:

$$\tau_{r\theta} = \frac{2\,d'_2\,(\cos 2\,\theta - \cos 2\,\alpha)}{r^2} \qquad (51)$$

The moment of the shear about the apex is equal to the applied couple M, per unit thickness of plate:

$$M = \int_{-\alpha}^{\alpha} r\,\tau_{r\theta}\,(r\,d\theta) = 2\,d'_2\,(\sin 2\,\alpha - 2\,\alpha \cos 2\,\alpha) \qquad (52)$$

Thus, $2\,d'_2 = M/K$; $K = \sin 2\,\alpha - 2\,\alpha \cos 2\,\alpha$; and Eqs. 50 become:

$$\sigma_r = -\frac{2\,M \sin 2\,\theta}{r^2\,K} \qquad (53a)$$

$$\sigma_\theta = 0 \qquad (53b)$$

and

$$\tau_{r\theta} = \frac{M\,(\cos 2\,\theta - \cos 2\,\alpha)}{r^2\,K} \qquad (53c)$$

In polar coordinates the strains and deformations are:[24]

$$\epsilon_r = \frac{\sigma_r - \nu\,\sigma_\theta}{E} = \frac{\partial u}{\partial r} \qquad (54a)$$

$$\epsilon_\theta = \frac{\sigma_\theta - \nu\,\sigma_r}{E} = \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta} \qquad (54b)$$

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[22] "Theory of Elasticity," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1934, Eq. 77, p. 114.

[23] *Ibid.*, Eq. 34, p. 53.

[24] *Ibid.*, Eqs. 45, 46, 47, and 48, pp. 62–63.

and

$$\gamma_{r\theta} = \frac{\tau_{r\theta}}{G} = \frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r} \quad \ldots\ldots (54c)$$

Substitution of Eqs. 53 in Eqs. 54 leads to the following expressions for the displacements:

$$u = \frac{2\,M \sin 2\,\theta}{E\,r\,K} \quad \ldots\ldots (55a)$$

and

$$v = \frac{M\,(1 - \nu)}{E\,r\,K}\,(\cos 2\,\theta - \cos 2\,\alpha) \quad \ldots\ldots (55b)$$

The component of the displacement parallel to the center line is

$$u_o = u \cos\theta - v \sin\theta = \frac{M \sin 2\,\theta}{2\,E\,x\,K}\,[(1+\nu)\cos 2\,\theta + 2 + (1-\nu)\cos 2\,\alpha] \,..(56)$$

in which the radius r has been replaced by the coordinate x along the center line.

Since there is a shearing deformation that adds its effect to that of the deformation due to normal stresses, it is useful to examine each deformation separately. This can be done by choosing $\nu = -1$, which defines a fictitious material that is absolutely rigid in shear. The corresponding deformation is

$$u'_o = \frac{M}{E\,x\,K} \sin 2\,\theta\,(1 + \cos 2\,\alpha) \quad \ldots\ldots (57)$$

Numerical values of u_o, u'_o, and $u_o - u'_o$ are given in Table 2 for a wedge with an angle $\alpha = 30°$. Three points dividing the half depth in equal elements are selected, thus permitting a direct comparison with a linear deformation. These results show definitely the limitations of the method suggested by Mr. Moorman.

TABLE 2.—COMPARISON OF LINEAR DEFORMATION WITH DEFORMATION CAUSED BY NORMAL STRESS AND SHEAR

Angle (θ)	Normal stress—$\frac{E\,x}{M}\,u'_o$	Shear—$\frac{E\,x}{M}\,(u_o - u'_o)$	Total—$\frac{E\,x}{M}\,u_o$	Linear deformation
30° 0′	3.80	0	3.80	3.80
21° 3′	2.94	0.30	3.24	2.53
10° 54′	1.62	0.31	1.93	1.27
0° 0′	0	0	0	0

For many practical purposes, however, it is probable that the errors introduced by the hypothesis of plane sections are not excessive, and are well within the range of the other countless approximations of conventional stress analysis. In the case of the illustrative example presented by the author, the linear distribution would lead to fiber stresses at the edge that would be greater than the actual fiber stresses, the difference being approximately 10%.

The writer feels that errors of 10% are acceptable and would not impair the safety of a structure, even if they were not on the conservative side. In addition, it must be noted that no better method is available, which makes the hypothesis of linear distribution imperative.

Robert B. B. Moorman,[25] Assoc. M. Am. Soc. C. E.—In general, the discussion of this paper is constructive. Technical advancement can be accelerated by such criticism.

Professor Ferguson believes that the angle ϕ should be included in Eq. 14, and the writer agrees. Professor Ferguson also demonstrates the multiplicity of results that can be obtained by selecting sections through a point on the compressive surface and at various angles thereto. Both of these faults can be corrected by following Mr. Benscoter's suggestion—namely, placing $\phi = \theta$. Then, throughout the derivation, ϕ and θ would be interchangeable. Eq. 29*b* would become

$$v = \frac{V}{b' j d} + \frac{N \tan \theta}{b' j d}\left(1 - \frac{2e}{j d}\right) \ldots\ldots\ldots\ldots (58)$$

and Eq. 33 could be written

$$\tan \delta = \left(\frac{2k}{3} - 1\right) \tan \theta \ldots\ldots\ldots\ldots (59)$$

On this basis, the illustrative example of Fig. 7 can be recomputed (see Fig. 13) as follows: $p = \frac{1.5}{45 \times 12} = 0.00278$; $p n = 0.0278$; $p n \cos \theta = 0.0257$; and $\frac{d}{e} = \frac{45}{74.35} = 0.606$. In Fig. 4(*a*) read $k = 0.29$ and compute $j = 1 - \frac{k}{3} = 0.903$. Eq. 10 gives $f_0 = 572$ lb per sq in. From Eq. 16*a*, $f_c = 582$ lb per sq in.; from Eq. 8, $f_s = 14{,}000$ lb per sq in.; from Eq. 59, $\tan \delta = -0.334$; from Eq. 29*a*, $v = 41$ lb per sq in.; and, from Eq. 30, $u = 82$ lb per sq in.

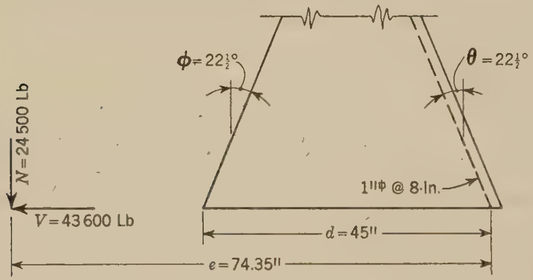

Fig. 13

Professor Ferguson states that "A larger shear exists within the compressive area * * *." He also states:

> "S. Timoshenko[9] shows that large shears will accompany bending of a wedge-shaped beam of homogeneous material. In the case of a 15° slope of each face he shows that the maximum unit shear on a section perpendicular to the axis of symmetry is some 90% greater than on a beam of the same depth with parallel faces."

Fig. 14(*a*) shows the distribution of such a unit shear, and it will be noted that Professor Ferguson's implication is quite misleading. The distribution of the unit shearing stress for a compressive load on the apex of a homogeneous wedge-shaped beam is shown in Fig. 14(*b*).

Mr. Benscoter gives some rules for design which should prove quite helpful. As he states: "There are upper limits on the angles θ and ϕ [now to be made equal] beyond which the assumption of linear strain provides too much error." The writer quite agrees and for this reason presents the data in Table 3. Although these data are for homogeneous beams, the practicing engineer can use them to determine the practical upper limit of θ. Referring to Fig. 12, the unit compressive stresses on section m–n due to the load P_2 may be expressed as

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[25] Engr., Johns Hopkins Laboratory of Applied Physics, Silver Spring, Md.

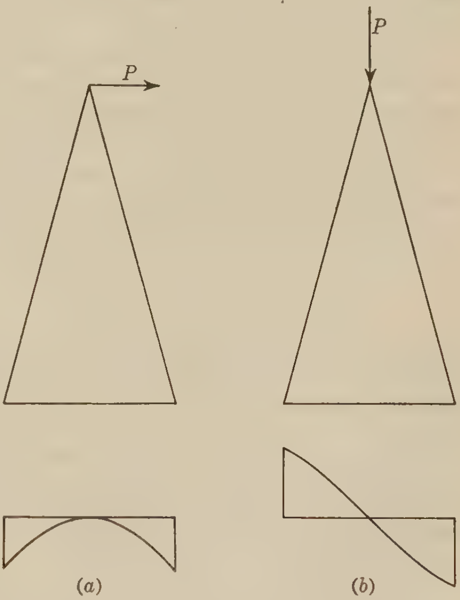
$$\sigma_x = -\frac{k_n P_2}{b\,d} \quad \ldots\ldots (60)$$

in which b is the width of section; d is the depth at section m–n; and k_n is a constant in Table 3—k_1 for point 0.1 d from the axis, k_2 for point 0.2 d from the axis, etc.

The unit stress on section m–n due to bending caused by the load P_1, Fig. 12, may be expressed as

$$\sigma_x = \pm \frac{c_n P_1 x\, d}{I} = \pm \frac{M c_n d}{I} \quad . (61)$$

in which I is the moment of inertia of section m–n $= \frac{1}{12} b\, d^3$; and c_n is a constant in Table 3—c_1 for point 0.1 d from the axis, c_2 for point 0.2 d from the axis, etc. The unit stress σ_x (for $n = 5$) should be multiplied by $\sec^2 \alpha$ to determine the maximum unit stress parallel to the surface.



(a) (b)

FIG. 14

TABLE 3.—COEFFICIENTS FOR USE IN EQS. 60, 61, AND 62

α	EQ. 60					EQ. 61					EQ. 62
	k_1	k_2	k_3	k_4	k_5	c_1	c_2	c_3	c_4	c_5	Q
5°	1.004	1.003	1.000	0.995	0.990	0.101	0.202	0.302	0.401	0.498	0.987
10°	1.018	1.011	0.998	0.981	0.960	0.103	0.205	0.304	0.399	0.488	0.952
15°	1.041	1.023	0.995	0.957	0.912	0.108	0.212	0.310	0.397	0.473	0.895
20°	1.074	1.041	0.989	0.923	0.847	0.115	0.223	0.317	0.395	0.453	0.820
25°	1.119	1.063	0.979	0.877	0.768	0.125	0.237	0.327	0.391	0.428	0.731
30°	1.176	1.088	0.962	0.820	0.679	0.138	0.255	0.339	0.385	0.398	0.631

Mr. Benscoter mentions problems that require future research. The writer would include the possibility of using the theory of plasticity as a basis for deriving equations comparable to those presented in the paper.

Professor Winter questions whether the assumption of a straight-line strain distribution can be expected to give reasonably accurate results. Essentially, the writer assumed straight-line stress distribution. His assumption was a practical expedient, and its accuracy can be gaged somewhat by use of the coefficients in Table 3. Specifications give allowable stresses with which to work, although it might have been more fitting to have developed specifications on the basis of allowable strains. Fig. 11 is interesting since it shows that the maximum strain for compression occurs at the position of minimum strain for bending.

Mr. Wheeler suggests that the writer indicate how the analysis could be modified to apply to pyramidal structures. Until some better method is developed, the writer believes that, if a section is taken perpendicular to the axis of the pyramid, the formulas as developed in the paper could be applied.

Mr. Beskin presents expressions for stresses and strains in a wedge for the case of a moment applied at the apex. By use of Eq. 53*a*, an expression for the maximum unit stress on the section m–n, Fig. 12, can be obtained as follows:

$$\sigma_r \text{ (max) } \cos^2 \alpha = \sigma_x \text{ (max) } = -\frac{M\,c}{I}\,\frac{\sin^3 2\,\alpha}{3\,K} = -\frac{M\,c}{I}\,Q \ldots\ldots (62)$$

in which c is the distance from the neutral axis to the extreme fiber; I is the moment of inertia of the section; and $Q = \frac{\sin^3 2\,\alpha}{3\,K}$. Values of Q are given in Table 3.

Mr. Beskin states that, in the illustrative example in the paper, the actual fiber stresses would be approximately 10% less than those computed by the method presented. The writer feels that, in general, conservative values would result from the use of the method.

In conclusion the writer would like to emphasize that (1) the angles ϕ and θ should be made equal, and (2) the coefficients in Table 3 can be used to gage the accuracy of the expressions presented.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2265

PRIVATE ENTERPRISE

ANNUAL PRESIDENTIAL ADDRESS

BY J. C. STEVENS,[1] PRESIDENT, AM. SOC. C. E.

Some groups, both inside and outside of the government, seem to believe that the natural laws of economics can be repealed like man-made laws and one would almost think that they are trying to repeal them. Acting under those natural laws this country has made the nearest approach to Utopia ever recorded in history. In the face of this historic fact it appears somewhat absurd and futile to attempt to substitute an artificial economy controlled by administrative edict and bureaucratic regulations.

THE CAPITALISTIC SYSTEM

This system, founded on the profit motive, has made the United States of America the outstanding nation of the world. We should not pussyfoot that statement but proclaim it boldly as a thing of which we have a right to be proud. That system produced the ships, the tanks, the planes, the munitions, the clothing, the foodstuffs that made it possible to outfit eleven million men and women and completely equip them for combat duty within a period of four years—an accomplishment unparalleled in the annals of the world.

Some may contend that the war capital was supplied by the government. Yes, but from whence came the government funds? The answer is obvious. They came from my earnings and yours and from those of every other taxpayer.

Let us face the facts. Private enterprise and the capitalistic system have, during the past 150 years, built the railroads; cleared the forests; built irrigation systems; cropped the lands; mined the coal, iron, and copper; launched ships; produced petroleum; erected the factories and power plants; designed and built automobiles, airplanes, caterpillar machinery; constructed highways; developed explosives, guns, electronics, communication systems; but, above all, they have produced officers of industry who, with the aid of free labor, knew how to turn their inventive and management genius to the business of winning a war. Patriotism supplanted profits and induced industry to carry on and produce in spite of many grueling regulations promulgated by well-intentioned but inexperienced and overlapping authoritative federal bureaus. Business has accepted this regimentation as a thing necessary to the war effort but is now demanding a respite which must be recognized.

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[1] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

The capitalistic system and free competitive enterprise have made the winning of this war possible. That same capitalistic system, with the stimulus of free competition in cooperation with free labor operating under the natural economic laws of supply and demand, will make it possible to liquidate the unprecedented debt that now hangs, like the sword of Damocles, over this nation.

Any attempt to continue the artificial manipulation of prices and the harrowing control of management after the war necessity therefor has passed can only result in inflation with its consequent repudiation of values and its unthinkable social effects. Taxes must be reduced. The present hazards of conducting business efficiently must be eliminated. Surplus earnings must be allowed in some reasonable degree to provide for expansion of industry. Risk capital must foresee reasonable profits or it will not be forthcoming but remain nestled among government securities. If private enterprise cannot take the helm, the national income will sink to sickening levels and we will again find ourselves in a Slough of Despond.

Regulation

Wholesome and intelligent regulation, by law, of business activities to insure that they are carried on in a truly competitive field will have the continued support of the nation. Such affairs as the Insull and Foshay debacles must not again be allowed to besmirch the good name of well-conducted enterprise and give ambitious politicians the ammunition for broadsides against corporations in general, and honestly acquired wealth and competence in particular.

Private enterprise is the only system that can solve unemployment. To do so, however, it must be permitted a fair field, unshackled by excessive taxes and meddlesome regulations, and stimulated by a stable policy of fair dealing with only sufficient and intelligent control to insure that its operations are truly competitive. Under such conditions small businesses will spring up all over the country; a portion of profits will go back into the businesses; financing by private sources will be the order of the day; expansions will follow; new plants will be built; and unemployment will be reduced to a minimum.

In brief, unhampered private enterprise operating under a constructive labor policy, encouraged by a reasonable scale of rewards to both labor and management, will keep this nation operating with a high degree of business stability, balance the national budget, and pay off an unprecedented national debt.

The Engineer Holds the Stage

The engineer will play a most important rôle in the revitalization of the world for peacetime pursuits. His is the task of converting industry to produce consumer goods in lieu of war materiel and munitions. He must design the highways and airways, modern streamliners of rail and stratosphere, improved radio, television, automobiles and trucks, household appliances, office equipment, air-conditioned housing, modern buildings, industrial instruments, communication systems, bridges, tunnels, and subways. He must not only design, but he must prepare specifications, execute contracts, and supervise construction. He just about occupies the center of the stage all the time.

All these—and yet he is virtually without honor save among his own colleagues; as witness the recent National Council for International Organization at San Francisco. The list of organizations asked to send representatives as consultants occupied several typewritten pages and included labor unions, service clubs, Jewish, Catholic, and Protestant groups, lawyers, guilds, many social and professional organizations, but not one representative of either a national or local engineering society. And why? Because the engineer has not organized his profession into pressure groups. He devotes time to civic service only on special request. He can solve technical problems but is rarely consulted on human problems. Public relations for him are merely word sounds without an urge to action. Politics is almost anathema in his thinking. He appears to prefer being left alone in his own technologic world without fanfare or encomiums.

The engineer does not yet fully realize that his usefulness to humanity would be enormously increased if he would take public relations to himself as a problem to be solved in the same earnestness with which he approaches the solution of a technical problem.

The engineer is no dullard. In cultural matters and accomplishments he will easily march abreast of those in other vocations. He can and must broaden his social horizon if he hopes to play effectively his part in getting this turbulent world back on an even keel.

He should take an interested part in civic affairs; join service and social clubs; chambers of commerce; serve on committees; learn to speak effectively in public; become interested in people and tolerant of their opinions. He will soon find himself on important committees, on boards of direction in industry and finance; and his advice and counsel will be solicited on all important matters.

Private enterprise must be manned and managed by men of integrity, of analytical mind, of technical ability, of sound judgment; men who are cooperative with labor and who so conduct their businesses as to hold the support and confidence of the public. To such positions the engineer should aspire.

Such enterprises, so managed, can accomplish the objectives herein set forth of balancing the budget, of maintaining the national income at a high level, and of paying off the most staggering debt that has ever been faced by any country in the world. It is vital that the public and Congress recognize the full import of that potentiality and give private enterprise the opportunity to justify our faith in it.

MEMOIRS OF DECEASED MEMBERS

JOHN FRANCIS COLEMAN, Past-President and Hon. M. Am. Soc. C. E.[1]

DIED JUNE 3, 1944

John Francis Coleman was born on November 23, 1866, in the small community known as Church Hill, Jefferson County, Miss., approximately twenty miles northeast of Natchez, Miss., on his grandfather's plantation known as "Wyolah." He was the eldest boy in a family of three boys and two girls born to James Wood Coleman and Elizabeth Treeby (Chaffe) Coleman, who shortly after their marriage had migrated to Mississippi from Minden, La., the home of Mr. Coleman's mother.

Mr. Coleman's brothers, Charles Coleman and Wood Coleman, died in their early youth, leaving a family of three children—Mr. Coleman and his sisters, Annie and Grace.

Mr. Coleman's paternal grandfather was Dr. Francis B. Coleman, a physician, who was beloved throughout the community in which he practiced.

Mr. Coleman's great-grandfather arrived in the United States from Ireland in the latter part of the eighteenth century, a refugee from the tyranny of the British Government, carrying with him documents relating to the age-old struggle of Ireland for independence led and sponsored by his associate and intimate friend, the Irish patriot, Robert Emmett. Mr. Coleman's mother, Elizabeth Treeby Chaffe Coleman, was the daughter of John Chaffe and Penelope Annie (Hamilton) Chaffe. Mr. Coleman derived his name from his maternal grandfather, John Chaffe, and his paternal grandfather, Dr. Francis B. Coleman.

When Mr. Coleman was twenty, the death of his father on March 21, 1886, necessitated his assumption of responsibilities not usually borne by a youth of his age.

From boyhood Mr. Coleman had been impressed with the skill of his grandfather as a family physician, and it was his greatest ambition to become a surgeon. However, this ambition was thwarted at an early age because he had both to earn his own livelihood and to aid in the support of his family.

After graduation from what was then Boy's High School, a New Orleans (La.) public school in June, 1884, since financial circumstances precluded a college education, Mr. Coleman, at the age of seventeen, accepted his first professional appointment as rodman on the location and construction of the Guatemala Northern Railroad. He remained with this organization until July, 1885, when he returned to New Orleans.

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[1] Memoir prepared by Robert J. Cummins and H. M. Gallagher, Members, Am. Soc. C. E., and Alonzo J. Hammond, Past-President and Hon. M. Am. Soc. C. E. (Mr. Hammond died on December 2, 1944.)

In September, 1885, Mr. Coleman became levelman on the location and construction of the Kansas City, Memphis and Birmingham Railroad (later part of the Frisco Lines), between Memphis, Tenn., and Birmingham, Ala., and was stationed at Aberdeen, Miss.

By diligent application and constant study, Mr. Coleman was soon advanced to positions of increased responsibility, finally being made resident engineer in charge of construction of approximately ten miles of grading and track laying.

At this period of his career Mr. Coleman became conscious of the embarrassing fact that he had been promoted over fellow members of his party, who were his seniors by a number of years, and he accordingly resolved to assume an older appearance by growing a beard. This Van Dyke beard, originally acquired to offset his youthful appearance, became a familiar mark of identification for Mr. Coleman and added immeasurably to his distinguished appearance.

Possessed of broad knowledge, a keen and photographic mind, and a magnetic personality, Mr. Coleman made friends wherever he went and to a host of his friends he typified a cultured Southern gentleman.

Upon completion of the Kansas City, Memphis and Birmingham Railroad between Memphis and Birmingham, Mr. Coleman accepted an appointment as assistant engineer on the construction of a smelting plant for the Philadelphia Smelting and Refining Company at Pueblo, Colo. He remained on this assignment until August, 1889.

In September, 1889, Mr. Coleman began his apprenticeship in hydraulic engineering when he became assistant engineer for the United States Engineers, making hydraulic observations on the Mississippi River and its tributaries. He remained with the United States Engineers until July, 1890. This assignment laid the foundation for his subsequent extensive experience on, and knowledge of, the Mississippi River and its control.

From July, 1890, to August, 1893, Mr. Coleman was assistant engineer for the Texas and Pacific Railway Company in responsible charge of both construction and maintenance of way, between Marshall, Tex., and New Orleans.

Contact with contracting operations often inspires young engineers to enlarge their experience by engaging in contracting themselves. Mr. Coleman was no exception, and in August, 1893, he became junior member of the contracting firm of Mott, Hanlon and Company, engaged in constructing earthwork levees and drainage facilities. This partnership was not successful, and in September, 1894, it was dissolved and Mr. Coleman returned to the practice of his profession.

The New Orleans and Western Railroad (later the New Orleans Terminal Company), a subsidiary terminal line of the Southern Railway Company, was being constructed at this time, and Mr. Coleman was appointed division engineer in charge of design and construction of interchange facilities, connecting the major trunk lines serving New Orleans, together with the construction of Chalmette Slip, a marine terminal on the Mississippi River built on the site of the Battle of New Orleans, fought in 1815 after peace between Great Britain and the United States had been declared.

In May, 1896, Mr. Coleman accepted his first and only political appointment as principal assistant city engineer for the City of New Orleans. During his four years in this assignment, the City of New Orleans undertook an ambitious program of drainage work, street paving, and other municipal improvements, the design and execution of which were under Mr. Coleman's immediate supervision and direction.

In January, 1900, Mr. Coleman began his career as a consultant by organizing the firm of Coleman, Malochee and Villere, specializing in civil, mechanical, and electrical engineering. This partnership was dissolved in the latter part of 1903. At this period of Mr. Coleman's career he was employed as engineer by the Board of Commissioners of the Port of New Orleans, an agency of the State of Louisiana, empowered by law to provide and administer port facilities within the Port of New Orleans.

The turn of the century witnessed the beginning of a change in transportation facilities serving the Port of New Orleans—the Mississippi River steamboat traffic was being gradually supplanted by a more modern means of river transportation, and New Orleans was becoming a seaport. Such a change demanded adequate shedded wharfage space to accommodate the foreign and domestic commerce which had replaced the local river traffic.

As engineer for the Board of Commissioners of the Port of New Orleans, Mr. Coleman was responsible for the design and construction of timber wharfs and steel frame transit sheds along the extensive New Orleans waterfront. These structures subsequently were standardized for the Port of New Orleans and served as models for many other port development projects throughout the United States.

In addition to serving as engineer for the Board of Commissioners of the Port of New Orleans, Mr. Coleman conducted a consulting practice, and was engaged on the following projects: Park improvements for Audubon Park in New Orleans; a water works and electric light plant for Hazlehurst, Miss.; a water works plant for Napoleonville, La.; irrigation and reclamation work for the Union Irrigation Company of southwest Louisiana; the reclamation of tidal overflow lands and subdivision for the New Orleans Land Company; and the construction of a race track for the New Orleans Jockey Club.

During 1907 and 1908, Mr. Coleman was chief engineer of the New Orleans Great Northern Railroad Company (later the Gulf, Mobile and Ohio Railroad Company) in charge of preliminary and location surveys, as well as of the construction of a single-track railroad between Jackson, Miss., and Slidell, La.

After he left the New Orleans Great Northern Railroad Company, the following projects represent some of his most important work as consulting engineer up to the declaration of war upon Germany in 1917: Construction of sawmill, narrow gage railroad, and industrial village for the Great Southern Lumber Company; design and construction of wharf and mechanical cargo handling equipment for the New Orleans Refinery of the American Sugar Refining Company; layout and plot plan of the City of Bogalusa, La.; construction of the Lafayette Branch of the Southern Pacific Lines between Lafayette and Baton Rouge, La.; opening and development of a large stone quarry in

Winnfield, La., for the Southern Mineral Company; and design and construction of storm sewer system and storm protection facilities for Port Arthur, Tex.

Ever mindful of his obligations in wartime and eager to contribute his share toward the winning of World War I, Mr. Coleman closed his office in November, 1917, following the declaration of war against Germany, and became general superintendent of the Chickasaw Shipbuilding and Car Company (a subsidiary of the U. S. Steel Corporation) near Mobile, Ala. This engagement involved the selection of the site for the large shipyard and supervision of the construction of shipbuilding facilities and ships, together with the construction of a village for employees and families with a population of five thousand, complete with sewerage, drainage, public utilities, and street railway accommodations to and from Mobile.

Upon the signing of the Armistice, Mr. Coleman returned to New Orleans in November, 1919, and with his son, Eugene Hunter Coleman,[2] M. Am. Soc. C. E., opened the office of the J. F. Coleman Engineering Company, conducting a general consulting practice. The firm was retained to supervise the liquidation of the International Shipbuilding Company of Pascagoula, Miss., which had become surplus property at the end of World War I in 1918. At this time, because of the demands for exports to the devastated countries of Europe, impetus was given to port development throughout the United States, and Mr. Coleman's previous experience with the Port of New Orleans was most valuable.

He served as a consultant for the following ports on the Atlantic Ocean and the Gulf of Mexico: Jacksonville, Fla.; Charleston, S. C.; Mobile, Ala.; Freeport, Tex.; Richmond, Va.; Houston, Tex.; Corpus Christi, Tex.; Beaumont, Tex.; and Lake Charles, La. In addition he was again retained by the Board of Commissioners of the Port of New Orleans as consulting engineer. In this capacity, Mr. Coleman was sent to Europe where he visited and inspected all the major ports of Northern Europe for the purpose of formulating and recommending a policy for the administration of the Inner Harbor Navigation Canal, built at New Orleans, connecting the Mississippi River with Lake Pontchartrain at a cost of $20,000,000.

Mr. Coleman's services as consultant on river hydraulics were retained in connection with: The diversion of the Brazos River in Texas, near its mouth; the control of the Mississippi River at Memphis to prevent damage to bridges; the control of Pearl River, near Columbia, Miss., to prevent the changing of its course; and the control of the Mobile and Tensaw rivers near Mobile to prevent damage to bridges.

In January, 1925, one of the writers (H. M. Gallagher) became a member of the firm of J. F. Coleman Engineering Company, and continued as a partner and intimate associate of Mr. Coleman until the latter's death.

Mr. Coleman, while attending an annual meeting of the Mid-South Section of the Society in Memphis in 1925, happened to be aboard the ill-fated steamer, *Norman,* inspecting improvements on the Mississippi River below Memphis, when the vessel capsized and eighteen lives were lost, including twelve distinguished members of the Society. The following account of the catastrophe[3] illustrates the fortitude and resourcefulness of the man:

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[2] For memoir, see *Transactions,* Am. Soc. C. E., Vol. 100 (1935), p. 1635.

> "On May 8, 1925, a party of distinguished engineers set out from Memphis, Tenn., to inspect improvements on the Mississippi River. During the course of the trip, their vessel, the ill-fated *Norman,* capsized and many were lost including a dozen members of the Society. A number, however, struggled to a sandbar in the river. Some of the later arrivals on this island were amazed to find a group that had reached there earlier huddled around a driftwood fire. In that group was a man of moderate height, friendly eyes, with stubby beard and mustache, calmly smoking a cigar. This striking picture is a typical characterization of John F. Coleman—resourceful in emergencies, uncomplaining of his lot, calm in the face of difficulties, and rising superior to every handicap."

Following the record high water with the attendant disastrous flood conditions which prevailed throughout the Mississippi Valley in 1927, the United States Engineers formulated and had enacted into law what was known as the Jadwin Plan of Flood Control for the Mississippi River and some of its tributaries. One of the most important features of this flood control plan was the protection of the City of New Orleans from further flood hazard by the diversion of a considerable proportion of the floodwaters from the Mississippi River, at a point twenty-six miles above the city, so the floodwaters could be discharged into Lake Pontchartrain at gulf level. The construction of the Bonnet Carre spillway was proposed for this diversion, and Mr. Coleman was retained by the United States Engineers to advise them with respect to the foundations for the concrete structure.

In 1930 Mr. Coleman was appointed by the U. S. District Court at New Orleans as the engineer member of a board of three commissioners to hear testimony and formulate recommendations as to the amount of damage sustained by the Louisiana State Highway Department, the Yazoo and Mississippi Valley Railroad Company, the Illinois Central Railroad Company, the Louisiana and Arkansas Railway Company, and other interested public utilities as a result of the construction and operation of the Bonnet Carre spillway and floodway. All recommendations submitted by this board were sustained by the court.

In 1932 Mr. Coleman was retained by the Freeport Sulphur Company to advise on the type and adequacy of foundations for plant structures at Grande Ecaille and Port Sulphur, La., where large new sulphur deposits had recently been discovered. The site proposed for the plant was sixty-five miles below New Orleans, on the west bank of the Mississippi River, in the midst of a flat alluvial salt water marsh, subject to overflow from the Gulf of Mexico at high tide—a soil aptly described by the early French settlers in Louisiana as *prairie tremblante* or "trembling prairie," because of its unstable condition. The construction of this plant was perhaps one of Mr. Coleman's greatest achievements in his long and successful career. The foundations supporting the heavy loads imposed by the machinery and equipment of the power plant, boiler room, compressor buildings, storage tanks, and other buildings have remained comparatively stable with very slight subsidence since erection.

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[3] *Civil Engineering,* January, 1936, p. 60.

Mr. Coleman continued in active practice as a consulting engineer until August, 1932, when he was appointed as one of five members of the Engineers Advisory Board of the Reconstruction Finance Corporation with offices in Washington, D. C. In this capacity, he served until the completion of the work for which he was engaged, returning to New Orleans to resume his practice in September, 1933. During this period funds for the construction of the Mississippi River Bridge at New Orleans were obtained from the Self-Liquidating Division of the Reconstruction Finance Corporation, and Mr. Coleman's knowledge of conditions at the site was invaluable in assisting the committee in making recommendations for the loan.

During 1934 and 1935, Mr. Coleman was executive vice-president and general manager of the Jefferson Lake Oil Company, in charge of all operations in connection with the production of sulphur and the exploration for new sulphur deposits.

In the fall of 1935, Mr. Coleman returned to his consulting practice. During the next succeeding three years, in association with Ralph Modjeski,[4] and Frank M. Masters, Members, Am. Soc. C. E., he was retained on projects by the New Orleans, Texas and Mexico Railway Company (Gulf Coast Lines) as well as by the Texas and New Orleans Railroad Company (Southern Pacific Lines). The former engagement concerned the design and construction of an extension to a railroad bridge over the Atchafalaya River at Krotz Springs, La. The latter project was for the design and construction of a double-track railroad swing bridge over Charenton Canal at Baldwin, La., a single-track through truss railroad bridge over Wax Lake Outlet near Franklin, La., and the raising of the single-track railroad bridge over the Atchafalaya River at Morgan City, La.

In August, 1938, Mr. Coleman returned to Washington, D. C., as engineer adviser to the Self-Liquidating Division of the Reconstruction Finance Corporation, and, with the approach of war in July, 1940, he was given the additional title of assistant chief engineer, Defense Plant Corporation, with its attendant responsibilities. He remained in Washington until his death.

Mr. Coleman's broad experience and sound judgment on all matters pertaining to both the vast engineering and the business problems confronting the Reconstruction Finance Corporation and Defense Plant Corporation were regarded with such esteem that officials of both corporations were prepared to accept his decisions and recommendations without question or further confirmation. The vastness of the responsibilities in connection with the defense and war programs, undertaken by the Defense Plant Corporation of which Mr. Coleman was an integral part, may be better visualized when it is revealed that the program included more than 2,200 projects, involving every conceivable type of production plant and facility, and cost more than ten billion dollars.

Although Mr. Coleman undertook certain full-time assignments, such as executive vice-president and general manager of the Jefferson Lake Oil Company and engineer adviser to the Reconstruction Finance Corporation, with the exception of his wartime service as general superintendent of the Chickasaw Shipbuilding and Car Company at Mobile, he maintained his office as consulting engineer in New Orleans continuously from the date of opening in January, 1900, until his death.

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[4] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 106 (1941), p. 1624.

On November 12, 1890, Mr. Coleman was married to Annie Hardee Hunter, daughter of Col. S. E. Hunter, an attorney of Clinton, La., and Stella (Taylor) Hunter. Mr. and Mrs. Coleman had one son, Eugene Hunter Coleman, who died on February 12, 1935. Mr. Coleman is survived by his sister, Miss Annie Hamilton Coleman; two nieces, Mrs. J. P. Shaw and Mrs. R. G. Deane; and his daughter-in-law, Mrs. Eugene Hunter Coleman.

No recital of Mr. Coleman's achievements, as an engineer and a loyal true friend, would be complete without a record of his undying devotion to his beloved wife, who always accompanied him to meetings of the Society. Although Mr. Coleman's devotion to Society affairs was outstanding to all who knew him intimately, his untiring love and devotion to his wife were his most noteworthy characteristics. Mrs. Coleman was nearly as well known to the profession as Mr. Coleman. It was this devotion and concern over the condition of Mrs. Coleman, who suffered a stroke of apoplexy in Washington, D. C., on January 15, 1944, and who died on January 24, 1944, which caused Mr. Coleman to suffer a severe heart attack on January 22, 1944. He never recovered and died in Baptist Hospital in New Orleans. Mr. Coleman could not have been a more ardent suitor during his courtship of his beloved wife than he was at all times, particularly during the closing days of Mrs. Coleman's life. His wife's death deprived Mr. Coleman of that zeal for living which had in the past always sustained him.

He practiced his profession with the zeal of a religion, and was noted for his meticulous regard for professional ethics, as promulgated by the various engineering organizations of which he was a member. His engineering contemporaries bear witness to his honorable practice and his unfailing consideration for the viewpoint of the other engineer.

Although having been denied the opportunity of an advanced formal schooling, he was a master of the technique of his profession and an old-school gentleman of broad reading and culture, well versed in the humanities. Mr. Coleman's leisure time for the most part was devoted to his wife, but he was fond of a game of golf for the recreation and the pleasant companionship which this sport afforded. If he had any hobby, it might be said to be an omnivorous desire to read. Reading, combined with a retentive mind, gave Mr. Coleman breadth of knowledge and a liberal education. He had the remarkable ability to grasp the intricate details of a situation quickly, and, discarding irrelevant data, to reach a prompt and logical decision.

Mr. Coleman was an engineer who possessed to an unusual degree the ability to apply common sense to all problems. He was outspoken in his views and tenacious of his opinions, broad-minded in his judgment, intensely loyal to his host of friends, and gifted with the faculty of endearing himself to all who were fortunate enough to know him.

Mr. Coleman was a member of the Episcopal Church, and at the time of his death was a member of Louisiana Lodge No. 102, F. and A. M., Orleans Delta Chapter, Royal Arch Masons. In 1906 he became a member of Jacques

De Molay Commandery No. 2, Knights Templar, from which he resigned in 1929. During World War I, while stationed in Mobile, Mr. Coleman organized and was the first master of the Pontalpha Lodge No. 804, F. and A. M., at Chickasaw, Ala. He was active in all the activities and ceremonies until his return to New Orleans in 1919.

At the time of his death, Mr. Coleman was a charter member, honorary member, and past-president of the Louisiana Engineering Society. In 1920 he was elected a member of the American Institute of Consulting Engineers and served as president in 1928. He was president of the American Engineering Council for one term in 1934 and 1935, and was always an ardent supporter of this organization. He belonged to the American Railway Engineering Association.

Mr. Coleman served the Society as Director from 1915 to 1917, as Vice-President in 1918 and 1919, and as President in 1930.

When Mr. Coleman was President of the Society, *Civil Engineering* was started, and his interest and enthusiastic support of this publication were manifested at all times.

In recognition of his attainments in his profession, Tulane University of New Orleans conferred upon Mr. Coleman the degree of Doctor of Engineering in 1935.

At the time of his death Mr. Coleman was vice-president of the Eye, Ear, Nose and Throat Hospital of New Orleans. He was most interested in this institution and gave unsparingly of his time in fostering its development.

Mr. Coleman was elected a Member of the American Society of Civil Engineers on October 2, 1901, and an Honorary Member on October 14, 1935.

ALONZO JOHN HAMMOND, Past-President and Hon. M. Am. Soc. C. E.[1]

Died December 2, 1944

Alonzo John Hammond, the son of John W. and Mary Ann (Padgett) Hammond, was born at Thorntown, Ind., on April 23, 1869. He came from a family of builders. His great-grandfather lived in Anne Arundel County, Maryland, and his grandfather began the westward trek into West Virginia. His father went to Thorntown and was married there. Alonzo Hammond received his early education in the public schools of Frankfort, Ind., a few miles from his birthplace.

Mr. Hammond was graduated from Rose Polytechnic Institute at Terre Haute, Ind., in 1889 with the degree of Bachelor of Science. He received the degrees of Master of Science in 1894, Civil Engineer in 1898, and Doctor of Engineering in 1933. During college, his ability as a student was recognized early. He was graduated with highest honors, being awarded the Hemingway Medal for the highest standing in the entire course, in a class in which one other besides himself became an Honorary Member of the Society—John D. Galloway.[2] During his college days, Mr. Hammond was gaining practical experience working on railway extensions, but he found time to be a leader in general social activities.

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[1] Memoir prepared by a Committee consisting of Charles B. Burdick, Chairman, Joshua D'Esposito, and Theodore L. Condron, Members, Am. Soc. C. E.

After graduation, he entered the employ of his father, who was an architect, serving for a short time as a draftsman and then resuming his studies at the Massachusetts Institute of Technology at Boston in 1890–1891.

Returning to Indiana, he at once became city engineer of Frankfort and, until 1898, was engaged in the design and construction of sewers, street paving, and city bridges. In 1898 he entered the office of the chief engineer, Terre Haute and Indianapolis Railroad Company, where he remained until 1901, as assistant engineer on masonry design for bridges and in charge of terminal facilities including freight and passenger depots at South Bend, Ind.

In 1901 he became city engineer of South Bend continuing until 1909, in charge of the design and construction of all municipal work under the control of the Board of Public Works, including sewers, street paving, a crematory, and a boulevard system. He also conducted a general consulting practice. In 1901 he became consulting engineer for the Indiana Railway Company in charge of surveys for the South Bend and Southern Michigan Railway from South Bend to St. Joseph, Mich.; design and construction of all bridges, track, and overhead construction on interurban lines from South Bend to Niles, Mich.; surveys for Indiana Western Railway from South Bend to Michigan City, Ind.; and construction from South Bend to New Carlisle, Ind. During his ten-year residence in South Bend, he designed miscellaneous projects including a sewer system, and hydroelectric power developments on the St. Joseph River at Elkhart, Ind., and at South Bend. He was bridge engineer for St. Joseph County and designed and constructed eight bridges across the St. Joseph River.

In 1911, since Mr. Hammond had had broad experience in various improvements pertaining to municipal public works, he was selected as chief engineer of the Bureau of Public Efficiency at Chicago, Ill.—a privately financed organization engaged in a study of public expenditures by the city. He then began his long residence in and near Chicago, which remained his home until he moved to Washington, D. C., in 1941.

In 1912 and 1913 Mr. Hammond served as engineer of bridges and harbors in the City of Chicago. During his tenure of office he directed extensive research which resulted in notable reductions in the cost of operating and maintaining the city's bridges and viaducts. The program of designing and constructing bridges which has been followed in the development of Chicago's celebrated bridge system was fostered under Mr. Hammond's supervision; and structural, mechanical, and artistic details of the Chicago type bridge were further perfected.

The original plans for the Michigan Avenue Bridge were drawn by Mr. Hammond's division in 1912, and the bridge as it stands at this time is substantially the same as that then conceived and planned. Many new projects were initiated in 1912—notably the 35th Street Trunnion Bascule Bridge and the 92d Street Trunnion Bascule Bridge. The Washington Street Bridge, begun in 1911, was practically completed during 1912. The final drawings of the Lake Street double-deck trunnion bascule bridge were started during 1912. This project, together with others, minor in character, required enlargement of the personnel and quarters of the designing section. The requisite arrangements were made without interfering with the routine maintenance and repair of existing structures.

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 109 (1944), p. 1451.

In 1914 he was engaged as consulting engineer by the Chicago Union Station Company and charged with the preparation of the design and construction of public facilities such as sewers, streets, and viaducts for this $75,000,000 project. From 1917 to 1919, during World War I, he acted as chief engineer in charge of the design of some of the most important functions and facilities of the terminal project. For a number of years, Mr. Hammond was engaged actively in construction; and, for three years (1922 to 1925), he was chief engineer of James O. Heyworth, Inc., contractors, largely on the construction of hydroelectric plants on the Mississippi, Chippewa, and Fox rivers in Wisconsin. From 1925 to 1927, he was chief engineer of the Mellon-Stuart Company in the Chicago office, representing that company on the formulation of a large internal improvement program in Cuba.

After 1927, and until his employment by the War Department in 1941, he conducted an office as consulting engineer in Chicago. During this time he was retained by the City of Chicago on various projects, such as river straightening, dock walls, bridges, and viaducts, and by numerous other cities, principally in connection with grade separations and passenger and river-rail terminals. During the 1930's, he frequently prepared confidential reports relating to loans and grants to municipalities and corporations for various federal agencies.

During World War II, and the earlier defense program, Mr. Hammond devoted his entire time to public service—from January, 1941, until his death. He was consulting engineer to the chief of the Construction Division, War Department, advising on camps and cantonments particularly during the organization and early construction periods of this important part of the war program. Thereafter, he was appointed a member of the Construction Advisory Committee (later merged into the Construction Contract Board of the Construction Division), War Department. This committee had the important task of advising on the employment of engineers, engineering organizations, and contracting organizations for the huge construction program necessary to mobilize resources and man power for war. The construction program being well under way in July, 1944, he was assigned to duty with the Price Adjustment Section, Construction Division, acting as deputy chief of the section. In this capacity he was engaged in the renegotiation of War Department contracts.

No one not closely associated with war activities can appreciate the long hours of hard work necessitated by the war program and the strenuous defense program which preceded it. Mr. Hammond's labors were continuous practically from the inception of these programs up to and including the day of his death. He was a man of robust health, but the last year was particularly trying because of the failing health of his wife who preceded him in death by only six weeks.

He took his job seriously. In one instance when an important decision on a 20-mile railroad line had to be made quickly, he examined the entire line on foot in less than two days, traversing difficult country that would have taxed the strength of a man half his age. Throughout his life, his work was characterized by ability, untiring industry, and the faculty of coordinating his work with that of others. He had the character and personality that made friends and kept them.

Mr. Hammond's career as an engineer is a striking example of the influences of both heredity and environment. His father and his grandfather were architects. Thus, he inherited the instincts of a professional man and an aptitude in designing and directing construction. His association with his father, before and after his technical education in Rose Polytechnic Institute, directly led him in the path of successful engineering practice with high ideals of integrity and professional conduct. By supplementing his four-year course in civil engineering at Rose Polytechnic Institute, with a special postgraduate engineering course at Massachusetts Institute of Technology, Mr. Hammond showed his own appreciation of the benefits of widening his training. His eminent success in the profession should be an inspiration to young men to acquire a broad education and to set high ideals of professional aims and conduct for themselves.

Mr. Hammond devoted all his spare time for many years to the service of the Society. He was Director (1926–1928), Vice-President (1929–1930), and President (1933), and remained for two years on the Board of Direction as a Past-President. In addition to serving in these offices, he was active in many other lines of Society endeavor. For five years he served on the Society's Committee on Professional Conduct, for the last two years of this period as chairman. He was chairman of the committee which made a study of American Engineering Council in 1928. He was always a strong advocate of co-operation among engineers and served as the president of American Engineering Council (1940–1941) and as president of the Construction League (1933–1934). He was chairman of the Functional Expansion Committee which sponsored several nontechnical Society activities. He was closely associated with the expansion of the Local Sections and the activities of Juniors and Student Chapters. He was instrumental in fathering such committees as those on engineering education, public education, legislation, and registration of engineers in public and quasi-public offices.

His work for engineers did not stop with the completion of his term on the Board of Direction of the Society. His service to the engineering profession is well stated in his citation for Honorary Membership which calls him "* * * Servant of the Society and other great professional bodies, increasingly honored by his fellow engineers for his personal qualities and his technical ability * * *." He also belonged to the American Institute of Consulting Engineers (vice-president in 1943), the Western Society of Engineers, the American Railway Engineering Association, the Indiana Society of Engineers, the Chicago Engineers Club, and the Shawnee Country Club.

Among other activities, Mr. Hammond found time to serve his alma mater, Rose Polytechnic Institute, as alumni member of the Board of Managers, from 1917 to 1921, and as a life member of the board until his death. During his residence in South Bend, in 1908, he helped to organize the Union Trust Company and was a director and vice-president. This organization was later merged with the First National Bank.

On May 23, 1893, Mr. Hammond was married to Flora Troll of Sullivan, Ind. Mrs. Hammond died on October 15, 1944. He is survived by two children, A. J. Hammond, Jr. (Lieutenant-Commander, U. S. Naval Reserve), and Mary (Mrs. George E. Davies). For the thirty years preceding his war work he resided at Evanston, Ill., a surburb of Chicago, where the family life centered around the First Presbyterian Church and the Shawnee Country Club. He is buried at South Bend—his home when the children were young.

Mr. Hammond was elected a Member of the American Society of Civil Engineers on June 1, 1904, and an Honorary Member on October 12, 1942.

MORTIMER ELWYN COOLEY, Hon. M. Am. Soc. C. E.[1]

Died August 25, 1944

Mortimer Elwyn Cooley, the son of Albert Blake Cooley and Achsah (Griswold) Cooley, was born at Canandaigua, N. Y., on March 28, 1855. He was a younger brother of the late Lyman E. Cooley, M. Am. Soc. C. E. The first nineteen years of his life were passed at Canandaigua in the agricultural district of the Finger Lakes region. He was a product of those great twin mentors, the American farm and the rural district school.

With this background, supplemented by his secondary school work at Canandaigua Academy, he took and passed the entrance examinations for the United States Naval Academy at Annapolis, Md., and entered as a Cadet Engineer in 1874. His roommate and classmate at Annapolis, who became his lifelong friend, was the late Ira N. Hollis, for many years a professor at Harvard University in Cambridge, Mass., and later president of Worcester Polytechnic Institute at Worcester, Mass. The lives of these two friends ran in parallel courses. Inseparable in school, assigned to the *U. S. S. Quinnebaug* on graduation in 1878, they were together on an extended cruise in the Mediterranean Sea in 1879 and eventually both of them were detailed by the Navy to teaching. Both resigned from the Navy and both became men of outstanding influence in engineering education. They were lifelong cronies.

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[1] Memoir prepared by a Committee consisting of Herbert S. Crocker and Henry E. Riggs, Past-Presidents and Hon. Members, Am. Soc. C. E.; Lewis M. Gram and Fred M. Zeder, Members, Am. Soc. C. E.; and Alfred H. White, Prof. Emeritus of Chemical Eng., Univ. of Michigan, Ann Arbor, Mich. (Past-President, Am. Inst. C. E.; Past-President, Society for the Promotion of Eng. Education.)

Following the year in Europe, Ensign Cooley spent a year aboard ship on the Atlantic Ocean and early in 1881, as a Lieutenant, was assigned to the Bureau of Steam Engineering. In the autumn of that year he was detailed by the Navy to inaugurate a course of instruction at the University of Michigan in Ann Arbor as "Professor of Steam Engineering and Iron Ship Building." He always was greatly pleased with this unique title given by the Navy to his teaching assignment although the university records indicate that officially he appeared on the payroll as professor of mechanical engineering from 1881 to 1922.

His four years in the Naval Academy together with cruises in European waters and with the Atlantic Squadron, plus his later experience with the Bureau of Steam Engineering, had changed the New York farmer's boy into a traveled, well-rounded man, who had made intimate contact with engineering problems and had secured not only a practical knowledge of steam engineering but confidence in himself. This is well illustrated by a story he has often told of his first faculty meeting at Ann Arbor. Some matter came up which required a vote of men of professorial rank only. When the ballot box came to Professor Cooley (he was then twenty-six years old), James B. Angell, then President of the University, raised the question as to whether "this young man" was entitled to vote. Before any one had a chance to discuss it, Professor Cooley rose, stated that he had been detailed by the Navy to the university as professor of steam engineering and that he considered that to be his rank, and proceeded to cast his ballot. Afterward no one ever questioned his title or rank.

An Engineering Department of very limited size and curriculum was established at the University of Michigan in 1857. In 1880, prior to the arrival of Professor Cooley, the faculty numbered three men and previous teaching had been limited to civil engineering and surveying. The total enrolment of engineering students in 1881 was 25. With the appointment of Professor Cooley the scope of the Engineering Department was extended to include courses and degrees in mechanical engineering. The first degree in mechanical engineering was given in 1883. The Department of Electrical Engineering was created in 1889 and the Department of Chemical Engineering in 1900. Thus, it is obvious that Professor Cooley can be rightfully classed as one of the pioneers in the founding of a great engineering college.

By 1895 the growth and development of curricula was such that the Regents established the College of Engineering and appointed Charles E. Greene,[2] M. Am. Soc. C. E., as the first dean. Professor Cooley became Dean of the College of Engineering in 1903 after the death of Dean Greene. In 1913 his title was changed to that of Dean of the Colleges of Engineering and Architecture. He retired in 1928 with the title of Dean Emeritus. During the twenty-four years of his administration, the departments of Architecture, Architectural Engineering, and Aeronautical Engineering were established. The total enrolment increased from 828 undergraduate and 7 graduate students in 1904 to 1,652 undergraduate and 81 graduate students in 1928.

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 849.

The significance of Dean Cooley's work at Michigan is not to be found in enrolment records, number of teachers, number of departments established, or courses offered. He clearly recognized the necessity for undergraduate education in the fundamental sciences with emphasis on a broad liberal training in the four-year course. In fact, he held that specialization in any field of engineering required a longer course or should be reserved for graduate work. Among other things he was insistent that every graduate have a reading and speaking knowledge of some foreign language. He believed that particular attention should be given to freshmen and was the founder of Michigan's mentor system. This plan has done much to help new students solve not only their problems of scholarship, but personal troubles as well.

Dean Cooley's conception of an engineering education was one of training for leadership, not only in design and construction of engineering projects, but in pure science, research, management of industrial properties, and good citizenship. It was with this vision of the engineer's function in society that he laid the foundations for the present very marked development of the College of Engineering as a center of graduate teaching and research attractive to men who have taken degrees in other engineering colleges.

His active service to the University of Michigan covered a period of forty-seven years. After retirement he resided in Ann Arbor, and maintained an office on the campus as adviser, counselor, and friend of both the faculty and the student body. Sixty-three years of his life were devoted to the service of the university as teacher, administrator, and adviser—this was his principal lifework. He left a record at the University of Michigan that will never be erased. The establishment by the Board of Regents of the Mortimer E. Cooley Foundation to

> "* * * insure the maintenance of a strong and adequate faculty for undergraduate teaching * * * to encourage and foster graduate work in Engineering * * * to encourage fundamental research in pure and applied science * * *"

will be a more lasting monument than marble or granite.

Professor Cooley resigned his commission in the Navy in 1885 but always maintained his keen interest in the service. On the outbreak of the Spanish American War in 1898 he re-entered the Navy with the rank of Chief Engineer and served first as Chief Engineer of *U. S. S. Yosemite,* and later at League Island Navy Yard, Philadelphia, Pa. He had long been interested in naval training on the Great Lakes, and from 1895 to 1911 he served as Chief Engineer Officer of the Michigan State Naval Brigade. Later, during World War I, he served as educational director of the seventy-two colleges and universities in Michigan, Wisconsin, and Illinois which comprised the Seventh District of the Students Army Training Corps.

His public service was of a widely diverse character. From 1888 to 1890 he was a member of the Board of Public Works of the City of Ann Arbor, and from 1890 to 1892 he was president of the Common Council of the city. From 1907 to 1912 he served as chairman of the Block Signal and Train Control Board of the Interstate Commerce Commission. In 1915 and 1916 he was chairman of the Board of Consulting Engineers which designed the present Belle Isle Bridge at Detroit, Mich. From 1920 to 1923 he was a member of the Advisory Council of the Joint Commission on Postal Service. In 1924 he was the Democratic nominee for United States Senator from Michigan and made a state-wide campaign for the office. He had no expectation of being elected but nevertheless received great pleasure from this campaign in which he argued that members of the engineering profession should be given greater prominence in public affairs. In 1933–1934 he served as state engineer for Michigan under the Federal Public Works Administration, becoming director in 1935. In 1934 he was appointed chairman of the Advisory Board to the Michigan State Highway Department.

Few if any citizens of Michigan have given the state more capable and varied service on commissions—with compensation consisting in large part of the satisfaction of promoting public welfare since only out-of-pocket expenses were paid. No other professional man ever won a place in the public affection and esteem of the whole state equal to that of Mortimer E. Cooley.

In the beautiful hill country of northern Michigan, State Highway No. 55 crosses the Manistee River in the Manistee National Forest over a long, high bridge which was selected as the most beautiful bridge of its type built in 1935. It was officially named the Mortimer E. Cooley Bridge and the bronze tablet at its east entrance records the high esteem in which the state held this citizen.

Throughout his life Dean Cooley was keenly interested in the advancement of the engineering profession and was active in state and national professional societies. He became a member of the Michigan Engineering Society in the early 1890's, served as its president in 1903, and was one of its first honorary members. He was vice-president of the American Society of Mechanical Engineers in 1902–1903 and president in 1919; he was made honorary member in 1928. He served as vice-president of Section D of the American Association for the Advancement of Science in 1898, as president of the Society for the Promotion of Engineering Education in 1920–1921, and as president of American Engineering Council from 1921 to 1923. He was an honorary member of the Engineering Society of Detroit, and a member of the National Association of Power Engineers and, until the time of his retirement, of the American Institute of Consulting Engineers.

During all the years that he was in active university work, Dean Cooley was called upon as a consultant in many matters. As a mechanical expert on patent litigation he was in frequent demand and testified in many cases between 1893 and 1925. He made a number of tests of water works pumping engines in Michigan and Ohio cities. He designed heating, lighting, and water plants for several of the large state hospitals and schools in Michigan.

His most outstanding work as a consultant was in the field of valuation. In 1900 the State of Michigan undertook the valuation of railroads, telephones, and other specific tax-paying properties in order to secure information to guide the legislature in the consideration of an ad valorem system of taxation. The Hon. Hazen S. Pingree, then Governor of Michigan, appointed Mortimer E Cooley appraiser and left the selection of staff wholly to him.

Prior to 1900 there had been a few water works valuations and perhaps half a dozen appraisals under the Texas stock and bond law. There was no literature on the subject of valuation. The only decision bearing on it was that in the famous case of Smyth versus Ames, which had been handed down two years earlier. There were no established principles or methods of valuation. Professor Cooley chose a staff of about one hundred engineers—the older men experienced and long in responsible charge of railroad construction and maintenance, the younger ones all graduate engineers with some experience. This was the first state-wide valuation in the United States. Definite rules of procedure were adopted. Forms were developed for the inventory of all kinds of property. Careful price studies were made. Cost of reproduction at then current price levels was adopted for the first time. Depreciation was determined by the inspection method. This valuation established precedents which were the models adopted in all subsequent state valuations and in the federal valuation of railroads. The Michigan valuation developed theories and methods which have been approved in a long line of federal and United States Supreme Court opinions in subsequent years.

The appraisal of 1900 was brought up to date in 1903 and 1905, and was followed from 1908 to 1920 by valuations of electric power and electric railway properties and gas plants for the State of Michigan—all under the direction of Mortimer E. Cooley and the staff selected by him. All this work, in this twenty-year period, was done as an engineering job, without any political interference and without the appointment of a single man on political recommendation.

During these twenty years Professor Cooley was called in as consultant by the State of Wisconsin, by Newfoundland, by Ontario (Canada), and by many cities and private corporations. One of his greatest contributions to the engineering profession was the establishment of a sound set of principles and methods upon which the valuation of utilities has since been developed.

During his busy years of university teaching and administration, public service, and private practice, many honors were conferred upon Mortimer E. Cooley. In 1885, at the time of his resignation from the Navy, the University of Michigan conferred on him the degree of Mechanical Engineer; in 1907 Michigan Agricultural College (Michigan State College of Agriculture and Applied Science at East Lansing) gave him the degree of Doctor of Laws; in 1911 the University of Nebraska at Lincoln granted him the degree of Doctor of Engineering; in 1923 Armour Institute of Technology at Chicago, Ill., conferred on him the degree of Doctor of Science; and in 1929 the University of Michigan gave him the honorary degree of Doctor of Engineering.

Sigma Xi, Tau Beta Pi, and Phi Kappa Phi honorary societies elected him to membership, and in addition he was elected to six of the chapters of the honor societies in the university by the student members. He was the recipient of the Navy Service Medal, the Detroit Naval Reserve Medal, and the Washington Award of the Western Society of Engineers. He served as Director of the American Society of Civil Engineers from 1914 to 1916.

On December 25, 1879, Mortimer Elwyn Cooley was married to Carolyn

Elizabeth Moseley. Their children were Lucy Alliance (Mrs. W. O. Houston), Hollis Moseley Cooley (Captain, U. S. Navy), Anne Elizabeth (Mrs E. C. Howe), and Margaret Achsah (Mrs. H. F. Cornwell).

The Cooley home in Ann Arbor abounded in hospitality and comfort. Scores if not hundreds of visitors to the city were cordially welcomed there. Mrs. Cooley was a lovely and highly cultured lady, greatly loved in the community, and active in church and charitable work. Her gracious welcome made any visit to her home a joy. She died on October 19, 1932, deeply mourned by a host of warm friends.

The foregoing paragraphs represent an attempt to summarize the principal facts regarding a long and busy life of service. They do not however picture the great, attractive personality that was Mortimer E. Cooley. To those who were privileged to work with him in the university for many decades his outstanding qualities were intense loyalty to the Engineering College, the desire to see it become a place where professional leadership could be developed, absolute freedom from prejudice in favor of any one department or field of professional work, the unerring ability to see the good points in young men and to sense the qualities that would make them good engineers, and complete and utter fairness to all. The task of the College of Engineering was to turn out good citizens and good engineers, leaders in their special field, and not to give a technical smattering which would qualify the graduate to hold an employee's job for life.

To those who have worked by his side in professional practice his abilities to analyze the work to be done, to plan the attack on it, to see both sides of every controversial question, and to insist upon careful analysis before reaching a conclusion were his outstanding qualities.

Mortimer E. Cooley sought the basic truth before attempting to solve any problem, engineering or educational. He always gave credit to members of his staff and never took full credit for the success of any undertaking. "There is always credit enough on a job that is well done to go around to all of those who did it" was a frequent saying of his, which always was evident in his reports.

He found time to write. His poems were real poetry. His papers on engineering education found a place in the great newspapers as well as in the technical press. His work on the Cooley genealogy of more than a thousand pages represents great research and must have taken a long time—yet he never talked much about it.

Dean Cooley's friends, neighbors, and colleagues will long remember the genial scholarly gentleman who could order the best of dinners and spin the best of yarns, whose smile was infectious, whose wit was always ready, and whose sarcasm, when occasion demanded, was biting. Those who have had the great privilege of being with him when he completely threw off the burden of work are the ones who have really seen the charming human side of this splendid gentleman.

He was a member of the Army and Navy Club of Washington, D. C., and of the Detroit and Yondotega clubs of Detroit. Those friends who have met him at any of these clubs saw him at his best with work and worry laid aside, free to discuss his many varied interests or his firm advocacy of the American way of life and of sound doctrines of politics and government.

His was a long life—full and useful. His influence will continue. His impress on the present generations of engineers has been deep and the influence of his teachings and the memory of his personality will not soon be lost. His principal monument is the great Engineering School, not alone the buildings (although he was in very large measure responsible for them), but the ideals of professional education which he did so much to establish.

Dean Cooley was elected a Member of the American Society of Civil Engineers on May 2, 1911, and an Honorary Member on October 14, 1935.

JOHN EDWIN GREINER, Hon. M. Am. Soc. C. E.[1]

Died November 15, 1942

John Edwin Greiner was born in Wilmington, Del., on February 24, 1859, the son of John and Annie (Steck) Greiner. At the age of nineteen his father moved from Ohio to Wilmington and during the Civil War served as a cavalryman in the Home Guards. Later he became a successful businessman. His mother, a native of Philadelphia, Pa., was well educated, fond of music, an accomplished pianist, and a good singer. From her Mr. Greiner inherited the fondness for music which was so evident throughout his life.

In his early childhood Mr. Greiner lived on a farm just outside of Wilmington. When young John was eight years old, his father sold the farm, and the formative years in the life of the youngster who was later to become such an eminent engineer were spent in the city of Wilmington.

His early education was obtained in the primary and secondary public schools of Wilmington, and at the age of fourteen he entered the Wilmington High School from which he was graduated in 1877. As a high school student he was interested in sports and in music, particularly in the violin which he played at both dances and concerts.

Mr. Greiner's record in high school was such that he was able to matriculate without examination at Delaware College (later the University of Delaware) at Newark, Del. At the conclusion of a three-year scientific course he was graduated in 1880 with the degree of Bachelor of Science. While in college Mr. Greiner engaged in extracurricular activities to a considerable extent —gymnastics, boxing, football, music, and dancing. He also organized the college orchestra.

Following graduation Mr. Greiner obtained a position as a draftsman in the Engineering Department of the Edge Moor Bridge Works, not far from his home in Wilmington. During the time he worked with this firm Mr. Greiner continued his studies and in 1883 he received the degree of Civil Engineer from Delaware College. In the same year he resigned from the employ of the Edge Moor Bridge Works and accepted a position as an assistant engineer with the Keystone Bridge Works in Pittsburgh, Pa. The notes in Mr. Greiner's uncompleted autobiography concerning his duties with the Keystone firm state, "The most interesting part of my work was when I assisted Mr. Thacher [the late Edwin Thacher,[2] M. Am. Soc. C. E.] (the chief engineer) in the design of the first bridge the Baltimore and Ohio Railroad Company built across the Susquehanna River at Havre de Grace [Md.]." In view of Mr. Greiner's later connection with this bridge—the present bridge of the Baltimore and Ohio Railroad Company at substantially the same location and the parallel highway bridge completed in 1940—one may comprehend why Mr. Greiner so vividly recalled his first association with a bridge spanning the Susquehanna River.

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[1] Memoir prepared by a Committee consisting of Ezra B. Whitman, Past-President, Am. Soc. C. E., and Gustav J. Requardt and Herschel H. Allen, Members, Am. Soc. C. E.

While in Pittsburgh Mr. Greiner received an offer from the late Gustav Lindenthal,[3] Hon. M. Am. Soc. C. E., to act as resident engineer for Mr. Lindenthal during the erection of the Seventh Street Bridge across the Allegheny River. Mr. Greiner accepted the position, and during this engagement he began to evidence those qualities of originality and genius which were to make him nationally and internationally known during later years.

Upon completion of the Seventh Street Bridge in 1885, Mr. Greiner returned to Wilmington for a well-earned vacation. The vacation however was of short duration for at the end of two weeks he was employed by the Baltimore and Ohio Railroad Company as a bridge draftsman. His advancement was rapid. The following year he was made bridge inspector and in 1887 chief draftsman. In 1889 he became assistant bridge engineer.

In 1891 feeling that his work with the Baltimore and Ohio Railroad Company was rather routine and desiring broader executive responsibilities, Mr. Greiner resigned and accepted a position as designing engineer with the Philadelphia Bridge Works at Pottstown, Pa. He remained at Pottstown only two years, however, as the Baltimore and Ohio recalled him in 1893. He was then appointed bridge engineer. In later years Mr. Greiner referred to this temporary severance of his connection with the railroad company as a "furlough." Mr. Greiner remained with the Baltimore and Ohio Railroad Company for the next fifteen years. In 1899 he was promoted to engineer of bridges and buildings and in 1905 to assistant chief engineer.

In 1908 Mr. Greiner, then forty nine, faced the problem of whether to remain with the Baltimore and Ohio Railroad Company and probably become, in subsequent years, its chief engineer or whether to resign and enter consulting practice. The notes for his autobiography concerning the reasons governing his decision to resign reveal both his modesty and the firm confidence which he had in his own ability. That this confidence was shared by others is evidenced by the fact that at the farewell reception given to Mr. Greiner the late D. D. Carothers,[4] M. Am. Soc. C. E., then chief engineer of the Baltimore and Ohio Railroad Company, announced the appointment of Mr.

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 918.

[3] For memoir, *ibid.*, Vol. 105 (1940), p. 1790.

[4] For memoir, *ibid.*, Vol. LXIV (1909), p. 581.

Greiner as consulting bridge engineer for the railroad company, a position which he held until his death.

Shortly afterward Mr. Greiner was appointed consulting engineer for the Erie Railroad Company, a relationship which continued until 1918. By 1911 the number of individual projects upon which Mr. Greiner's services were engaged had risen to more than thirty and by the end of 1916 to one hundred and eighteen. As his practice grew, Mr. Greiner added to his organization partners and associates and increased the number of the junior personnel. The work done by the firm included reports of various types in addition to complete design and supervision of construction. At the time of Mr. Greiner's death the value of the projects completed by the company was in excess of $300,000,000. Among the more important projects were eight large bridges across the Ohio River; the Soldiers and Sailors Memorial Bridge at Harrisburg, Pa.; the James River Bridge near Newport News, Va. (a structure approximately 4.5 miles long with a vertical lift span over the main channel); a concrete arch bridge over the James River at Richmond, Va.; a highway bridge approximately 2 miles in length over the Potomac River near Ludlow's Ferry, Maryland; the highway bridge across the Susquehanna River at Havre de Grace (awarded first prize by the American Institute of Steel Construction as being the most beautiful steel bridge constructed in America in 1940); the Howard Street Bridge in Baltimore, Md. (given honorable mention by the Institute); the Western Maryland Terminals in Baltimore; the Army Cantonment at Fort George G. Meade, Md.; the Marine Barracks at Jacksonville, N. C.; the Marine Air Base at Cherry Point, N. C.; and the Pennsylvania Turnpike, the four-lane toll highway extending from Carlisle, Pa., to Pittsburgh.

Mr. Greiner's great achievement was his contribution to the science of building bridges and other structures. In 1896 he ascertained that the same working stresses could be employed for both built-up tension members and annealed eyebars. Prior to Mr. Greiner's discovery, lower working stresses had been employed for built-up members. Before 1903, it was common practice to use two grades of bridge steel, mild and medium, with two sets of unit stresses. In 1901 Mr. Greiner had written, for the Baltimore and Ohio Railroad Company, specifications using a single grade of steel which he called structural steel with an average ultimate strength of 60,000 lb per sq in. These specifications were adopted as standard by the American Railway Engineering Association in 1903 and have remained standard, with modifications. In 1909 he formulated a method of classifying and rating bridges according to their load-carrying capacity. This classification was likewise adopted as standard by the American Railway Engineering Association and is standard today.

Mr. Greiner believed that an engineer should share his knowledge and discoveries with the other members of his profession. He practiced this belief by writing many articles for the publications of the various technical societies. The earliest of these was an article, published by the Engineering Society of Western Pennsylvania, describing a slide-moment diagram which Mr. Greiner had invented to facilitate the computation of train-load moments and shears.

Between 1891 and 1917 Mr. Greiner wrote nineteen articles which were published by the Society. For the article, "What is the Life of an Iron Railroad Bridge,"[5] he was awarded the Norman Medal in 1896. In 1915 the James Laurie Prize was presented to him for his paper on "Coal Piers on the Atlantic Seaboard."[6]

Mr. Greiner was a member of the following national technical societies, in addition to the Society: American Institute of Consulting Engineers, American Society for Testing Materials, American Railway Engineering Association, American Concrete Institute, and American Institute of Steel Construction. He participated actively in the affairs of these organizations, especially those of the Society, the American Railway Engineering Association, and the American Institute of Consulting Engineers. He was a Director of the Society from 1915 to 1917, and from time to time served as a member of various Society committees. For many years he was chairman of the Committee on Iron and Steel Structures of the American Railway Engineering Association. He was a charter member of the American Institute of Consulting Engineers and a member of the council from 1911 to 1914.

For years Mr. Greiner was an active member, and twice president, of the Engineers Club of Baltimore.

Mr. Greiner's services were not limited to those rendered to his employers and to the clients of his firm, nor to the professional knowledge he so willingly shared with his fellow engineers. As was to be expected of a man of his capabilities and character, he neither could nor did desire to confine his activities and energies to private engagements. At various times he served his city, his state, and his country—always with distinction. In 1904 he was appointed by the Hon. Robert M. McLane, then Mayor of Baltimore, as a member of the commission to inspect and report upon the damage done by the fire which ravaged the downtown section of Baltimore in February, 1904. Shortly thereafter Mayor McLane also appointed Mr. Greiner a member of the Commission for Revising the Building Code.

During World War I, Mr. Greiner was a member of the Council of National Defense. In 1917 he was also appointed a member of the American Advisory Commission of Railway Experts to Russia by the State Department. This commission passed on the strength and carrying capacity of the bridges on the Trans-Siberian and Russian railways. As a result of his endeavors the movement of traffic over the Russian lines was substantially increased. When he returned, Mr. Greiner received the official thanks of the State Department.

From 1920 to 1927 Mr. Greiner was chairman of the Port Development Commission of Baltimore. In 1932, after having previously served as the engineer member of the Maryland State Board of Health, he was appointed a member of the Maryland Waterfront Commission. Subsequently he became a member of the committee appointed by the Hon. William F. Broening, then Mayor of Baltimore, for the preparation of an ordinance providing for the electrification of the Pennsylvania Railroad in Baltimore.

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[5] *Transactions*, Am. Soc. C. E., Vol. XXXIV (1895), p. 294.

[6] *Ibid.*, Vol. LXXVII (1914), p. 454.

As was natural and appropriate, various honors were bestowed upon Mr. Greiner in recognition of his achievements. In 1917, Delaware College conferred upon him the honorary degree of Doctor of Engineering. In 1932, when Mr. Greiner was elected an Honorary Member of the Society, the late Francis Lee Stuart,[7] Past-President, Am. Soc. C. E., stated in part as follows:

> "His life work has run parallel with the advancement in modern bridge building and he has been such a part of that record that it is difficult to emphasize what are his important milestones in bridge design. Through his writings from time to time he has made valuable and permanent contributions to Engineering Literature."

After summarizing the more important of Mr. Greiner's achievements, Mr. Stuart said:

> "In my forty-eight years of professional and friendly contact with him I remember many other of his milestones which have been an example to the profession in ethics and citizenship, as well as ability."

In 1937 the Johns Hopkins University in Baltimore conferred on Mr. Greiner the honorary degree of Doctor of Engineering with the following citation:

> "Distinguished designer of bridges. Originator of methods, and contributor to the practice and standards of bridge building. His replacement of obsolete railroad bridges under traffic has required a judgment and an acceptance of responsibility before, during and after such replacement that are unique in the annals of his profession. His life work has run parallel with the advancement of modern bridge building and has been an essential factor therein."

In 1941 the Union Trust Company of Maryland at a ceremony which was broadcast presented Mr. Greiner with their "Plaque of Honor." This award which is in the form of a beautiful bronze plaque is given from time to time by the Trust Company to outstanding citizens. The plaque bears the following inscription:

> "For the great building feats which he has achieved throughout America; for unselfish service rendered to his country and the world; and for the renown in the field of scientific engineering which he has brought to his home city of Baltimore."

In all phases of his long and useful life Mr. Greiner was basically constructive. To both work and recreation he brought a physical and mental energy which was the envy of many of his associates. His sense of fairness and sportsmanship was never questioned and in all his relations with others he was essentially human.

In business Mr. Greiner was a firm believer in teamwork. Under his leadership and inspiration this idea permeated through and has remained in the organization he created. He insisted that employees were working with and not for the Greiner Company. Although varied degrees of responsibility were naturally borne by different members of the organization, there were "no bosses."

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[7] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1493.

Mr. Greiner had marked success in his selection of associates and key employees. In choosing them, he was guided by the following specifications, which he personally drafted. These are included not only because of their value, but also because of their applicability to Mr. Greiner.

Specifications for an Engineer

"An engineer, capable of taking responsible charge in the office of a consulting engineer, should have the following qualifications—

"He should be a graduate of a university or nationally recognized engineering school, a full member of a recognized national engineering society, interested and active in the technical papers and discussions of the Society and local clubs.

"He should be studious and in touch with important engineering construction, research and progress as well as with mathematics and the mechanics of engineering.

"He should be qualified to figure stresses, make designs and estimates for any kind of engineering structure with neatness, accuracy and dispatch.

"He should have experience and the ability to direct and supervise work in office or field, and to produce prompt results.

"He should be a thorough investigator and expert reviewer of drawings, be able to make clear, concise and adequate reports, specifications, contracts, etc., and to write business letters promptly, briefly and to the point, but never forgetting business courtesy.

"He should be broad-minded enough to handle efficiently a number of jobs concurrently without excitement or confusion, and have a strong enough mind to concentrate on his work and forget his personal troubles and worries while doing so.

"He should be physically strong and healthy, sure footed, able to climb, and walk on narrow supports at elevations when required in the course of field inspection.

"He should have good sense, tact, energy, decisiveness, loyalty, courtesy, and generally a character which will inspire confidence in his ability on the part of those with whom he deals.

"He should never confuse opinions and guesses with facts, make undigested statements which may possibly mislead, violate the recognized rules of engineering ethics, or do anything by act or work which may impair the confidence of those with whom he deals.

"He should not gloss over his errors or hunt excuses for them, or show resentment when they are called to his attention, but should always bear in mind that even a small error might result in considerable expense and lead to future trouble, and each discovered blunder, whether corrected in time or not, should be considered as a lesson to be remembered.

"Finally, he should have ambition, not merely for earning more pay, or for shining by reflected light, or for holding down his present job, but for earning a national reputation, for shining by the light of his own work, and for becoming a real business asset to the firm which employs him."

To his social contacts and hours of recreation, Mr. Greiner brought those same qualities which made him so successful in his profession. As previously mentioned, he was exceedingly fond of music, and was an excellent violinist. In his earlier years, he played at almost every opportunity, both with amateurs and professionals and with and without compensation. On one occasion, in company with the late Otis E. Hovey,[8] Hon. M. Am. Soc. C. E., and the late Ralph Modjeski,[9] M. Am. Soc. C. E., he played before a group of his fellow engineers in New York during a meeting of the Society. In later years his playing was largely confined to trios, quartets, and quintets composed of his close friends. He was devoted to classical music.

For outdoor recreation, Mr. Greiner liked swimming, golf, horseback riding, and hunting—particularly the latter. As a young man whenever occasion permitted, often on the coldest days, he would swim across the Delaware River at Wilmington. His hunting included shooting moose in Canada, caribou in Newfoundland, elk and goats in the Rocky Mountains, grouse and pheasants in England and Wales, ducks and geese on the Atlantic seaboard, and wild turkeys in Maryland and the South. Like all true sportsmen, he enjoyed his hunting trips because of the comradeship of his companions, the love of the chase, and the incidents which occurred, rather than because of the game he shot. However, he was quite successful, and in his office and home hang beautiful specimens of both large game and game birds.

Mr. Greiner was never idle, and his untiring energy as well as his many and varied achievements are an inspiration and a challenge to the members of the profession to which he contributed so greatly. Of him it may be truly stated that he filled "each unforgiving minute with sixty seconds worth of distance run."

In 1886 Mr. Greiner was married to Lily F. Burchell of Berryville, Va., who died in 1930. He is survived by two daughters, Lillian Burchell (Mrs. Alexander Duncan Richardson Collie) and Gladys Houston Greiner, and two grandsons, Duncan Eric Collie and Alastair Collie. Mr. Greiner's family life was a happy one and the previously mentioned notes for his autobiography contain references to the comradeship which existed between Mrs. Greiner and himself.

Mr. Greiner was elected a Member of the American Society of Civil Engineers on June 4, 1890, and an Honorary Member on October 3, 1932.

HENRY MATSON WAITE, Hon. M. Am. Soc. C. E.[1]

DIED SEPTEMBER 1, 1944

It has been said that the careers of some men are a commentary on the times in which they live. Henry Matson Waite had such a career.

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[8] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 1537.

[9] For memoir, *ibid.*, Vol. 106 (1941), p. 1624.

[1] Memoir prepared by a Committee consisting of Arthur S. Tuttle, Past-President and Hon. M. Am. Soc. C. E., and William B. Poland and E. W. Clark, Members, Am. Soc. C. E.

Henry Matson Waite began his career as an engineer with a railroad company when railroads were still in the expanding stage. Later, he was engaged in the development of coal mining in the early days of that industry. Then as a city engineer and following that as one of the first city managers in the country, he typified the civic aspirations of what was still a young America. At the start of World War I, he gave up civic work to go to France with the United States Army.

Returning from the war, Henry Waite planned port developments and constructed one of the finest railroad passenger terminals in the United States. In 1933 he became Deputy Administrator of the Public Works Administration, one of the best managed government agencies set up to combat the economic depression. Later, he was identified with the experimental work of the Nation Youth Administration, with the former National Resources Planning Board in its conservation and development work, and finally with the U. S. Bureau of the Budget, as the head of the War Projects Unit.

His family background was typically American. He was born on May 15, 1869, at Toledo, Ohio, the son of Henry Selden and Ione (Brown) Waite. His maternal grandfather was Gen. Joseph W. Brown, an active military figure in early days of the State of Michigan and in the War of 1812. His great-uncle, in the same line, was Gen. Jacob Brown, one of the commanders in the Mexican War. His paternal grandfather, Morison R. Waite, was Chief Justice of the Supreme Court of the United States, and Morison Waite's father was Chief Justice of the Supreme Court of Connecticut. It was in the spirit of these great forebears that Henry Waite carried on the traditional American life.

He received his early education at the Toledo High School, and his technical training at the Massachusetts Institute of Technology (M.I.T.) in Boston. One of the school friends of Henry Waite finds it interesting to recall that, in the struggles and successes of the undergraduate (particularly in the battles on the football field where Henry was a pillar of strength who would never case trying to make the always insufficiently trained M.I.T. team able to meet its rivals), the fine traits shown in Henry Waite's later career were all evident—capacity for hard work and never giving up, gaiety, ready sympathy, the ability to make and hold friends, and the power of accomplishment.

After leaving college in 1890, Henry Waite's first engineering work was with the Cleveland, Cincinnati, Chicago and St. Louis Railway Company (later a part of the New York Central Railroad Company) as a transitman and then as engineer of maintenance of way. In 1893 he became division engineer with the Cincinnati, New Orleans and Texas Pacific Railway Company (later a part of the Southern Railway Company), and during the following fourteen years he was promoted to bridge engineer, to roadmaster, and finally to division superintendent. In 1907 he was appointed superintendent of the Seaboard Air Line Railway Company, leaving in 1909 to become vice-president and chief engineer of the Clinchfield Coal Corporation at Dante, Va., where he remained until 1912.

Until this time, Henry Waite had conformed to the usual pattern followed by a successful engineer. A major change occurred, however, in his career when, in 1912, he accepted the appointment as city engineer of Cincinnati, Ohio, under what has been termed the first "reform" administration for that city. Thus, he began, in connection with his engineering, the civic side of his life; and his interest in municipal government never lessened. This experience in public administration led to his appointment as city manager of Dayton, Ohio, in 1914, at a time when the city manager form of government was in its trial period. Henry Waite made good as city manager of Dayton and, in doing so, attained national prominence. The success of the municipal administration in Dayton promoted the city manager system in the United States. Throughout his life, Henry Waite actively advocated this type of municipal government.

With the beginning of World War I, Henry Waite resigned as city manager of Dayton and volunteered his services to the United States Army. He was commissioned a Lieutenant Colonel in the Corps of Engineers and was sent to France. There he served with the Transportation Corps, American Expeditionary Forces, under Brig.-Gen. W. W. Atterbury,[2] M. Am. Soc. C. E.—first as Deputy Director General of Transportation, then as Engineer of Construction. Later, he was promoted to the rank of Colonel and was made Assistant Deputy General of Transportation, Second American Army, Advance Zone. Finally, he was transferred to the Third Army and was a member of the American Bridgehead Commission at Koblenz, Germany. For his meritorious services, he was awarded the Distinguished Service Medal of the United States and was made an officer of the Légion d'Honneur of France.

In the spring of 1919 Colonel Waite returned to the United States and, upon his discharge from the Army, became vice-president and chief engineer of the Lord Construction Company of New York, N. Y. In 1920 he was made president of the Lord Drydock Company which formulated plans for the development of the Port of New York. After winding up the affairs of the drydock company, he entered private practice as consulting engineer in New York City.

In 1927 Colonel Waite was called to Cincinnati to become chief engineer of the Cincinnati Union Terminal Company. In that position he was responsible for organizing, planning, and building a forty-million-dollar passenger terminal—one of the outstanding undertakings of its kind in the United States. He completed the project six months ahead of the original schedule and within the estimated cost. This terminal stands as one of the greatest monuments to his vision and ability.

Following the completion of the Cincinnati terminal in 1933, Colonel Waite went to Washington, D. C., to work with a small group in devising the policies and plans for what was later to become the Public Works Administration. Subsequently, after the enactment of the National Industrial Recovery Act and the establishment of the Public Works Administration, Colonel Waite was appointed Deputy Administrator, under the Hon. Harold L. Ickes, Secretary of the Interior, and was placed in charge of the three-billion-dollar public works program which took the form of grants or loans and grants to public agencies, thus enabling the agencies to proceed with worth-while improvements. The major share of the responsibility for organizing this public works program—one of the largest ever undertaken in the United States—rested upon Colonel Waite. At this particular time, the construction industry was dormant and unemployment was prevalent among engineers. Because of Colonel Waite's energy and administrative ability, coupled with the aid of a carefully selected staff, the program was advanced rapidly and engineers quickly began to enjoy comparatively prosperous days. The outstanding service rendered by Colonel Waite in this field placed the profession under a debt of gratitude which should never be forgotten. With the program well under way, he resigned in the autumn of 1934 to direct an unemployment survey in Cincinnati for the Regional Department of Economic Security, a nongovernmental organization. He inaugurated and conducted this work and completed it in 1937.

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1518.

In the latter part of 1937, Colonel Waite resumed his activities with the federal government as technical adviser to the National Youth Administration and as a consultant to the National Resources Committee (a predecessor of the National Resources Planning Board). For the National Youth Administration, he organized and operated the experimental project at Quoddy (near Eastport), Me., for training boys in various phases of mechanical work, and much of his time in 1937 and 1938 was spent at Quoddy. Subsequently, this project was used as a pattern for other similar schools.

After 1938 he served actively as chairman of the Chicago (Ill.) Subway Committee, which was responsible for approving the plans made by the Transportation Committee of the City of Chicago for the subway. This $56,000,000 project, for which the Public Works Administration had made a grant of $23,130,000, occupied much of Colonel Waite's time until his death.

For many years Colonel Waite had been an advocate of a national planning agency that would be primarily concerned with the physical development of the United States. While with the National Resources Committee and later while with the National Resources Planning Board, he served as chairman of the public works committees— a task which was undertaken at the request of the Hon. Franklin D. Roosevelt, President of the United States. These committees, under Colonel Waite's chairmanship, were responsible for developing procedures under which the planning, programming, and budgeting of federal public works projects could be coordinated into a unified operation, and for experimenting with methods by which state and local governments could prepare and keep up to date six-year programs of their public works activities. These latter experiments received wide publicity and were highly successful in various states and localities. Colonel Waite continued with the board until the autumn of 1940.

During 1939 and 1940, Colonel Waite also found time to serve as chairman of the Dayton and Montgomery County (Ohio) Survey for the improvement of public administration in the city and county governments.

Late in November, 1940, the Hon. Harold Smith, director of the U. S. Bureau of the Budget, requested Henry Waite to form and direct the War Projects Unit of the Bureau. Colonel Waite organized this work under the title of consultant to the Budget and head of the War Projects Unit. In this capacity he directed a field staff who surveyed and reported on the progress of government-financed war construction and, later, on the progress of the major war production programs. He was still actively engaged in this work at the time of his death.

In addition to his military honors, Colonel Waite received the honorary degree of Doctor of Laws from Miami University in Oxford, Ohio, and that of Doctor of Engineering from the University of Cincinnati. He served as vice-president of the Society in 1931–1932. Colonel Waite was a member of the Society of American Military Engineers (at one time vice-president), the International Association of City Engineers (past-president), the International Association of City Managers (one of the founders), the American Society of Municipal Engineers, the International Association of Public Works Officials, and the American Public Works Association.

On April 15, 1914, he was married to Mary Mason Brown of Lexington, Ky. Colonel Waite is survived by his widow. Any one who has ever enjoyed the hospitality of the Waite home would know the happiness which followed this union, and the inspiration Mrs. Waite always gave to Colonel Waite.

Throughout his varied and colorful career, Henry Waite brought a clear, logical mind to the solution of not only engineering but all other problems that he encountered in his lifetime. At the same time he never lost the human touch. His personality was such that, while leading men, he made them love him, as well as admire him. During his later years, he maintained a lively interest in the arts, being a lover of music, painting, and the theater. He also read widely. His life was an inspiration to all who knew him.

Colonel Waite was elected a Member of the American Society of Civil Engineers on April 1, 1914, and an Honorary Member on October 13, 1941.

EDWARD EVERETT WALL, Hon. M. Am. Soc. C. E.[1]

DIED JUNE 26, 1944

Edward Everett Wall was born at Cambridge, Saline County, Mo., on August 15, 1860. He was the son of John and Mary (Gault) Wall.

In 1833 John Wall, as a boy, was brought to Saline County. He fought in the Mexican War in Doniphan's noted regiment and then became one of the "forty niners" who went to California in a train of ox-drawn covered wagons. However, he soon returned to Cambridge and became sheriff as well as a merchant and a farmer. A little later, he became a soldier in the War Between the States. The fact that he fought on the Union side, although from a part of Missouri overwhelmingly Confederate in sympathy, is an example of the Wall family trait of independence. His son, Edward Everett, had the same characteristic of always acting as he thought right regardless of the politics of the occasion.

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[1] Memoir prepared by a Committee consisting of Baxter L. Brown, W. J. Burton, and W. W. Horner, Members, Am. Soc. C. E.

Edward Wall attended the public schools at Cambridge and then the University of Missouri at Columbia, from which he was graduated with the degree of Civil Engineer in the class of 1884. In September of the same year he began work with the Mississippi River Commission as recorder for a gaging party at Arkansas City, Ark., remaining until April, 1885. From September, 1885, to May, 1886, he served as leveler on a preliminary survey for a "short line" railroad (never built) between St. Louis and Kansas City, Mo. During the summer of 1886, he was transitman on preliminary surveys for another railroad project which did not reach the construction stage—the Kansas, Colorado and Texas. Then, from September, 1886, to January, 1887, he was employed as assistant division engineer on the construction of the Burlington and Missouri River Railroad (later part of the Burlington Lines) in Nebraska. His next engagement took him to Wichita, Kans., where, between April and October, 1887, he served as construction engineer on the Riverside and Suburban Street Railway (6 miles) and on the Wichita and Valley Center Railroad (12 miles). In August, 1888, he first entered the service of the St. Louis Water Department as an inspector, remaining until April, 1889. From this time until February, 1890, Mr. Wall was assistant engineer for the Missouri River Commission engaged on secondary triangulation of the upper Missouri River between Fort Benton, Mont., and Bismarck, N. Dak. Returning to St. Louis in February, 1890, he became deputy city surveyor for the W. W. Penney Surveying Company, continuing until May, 1892. This ended Mr. Wall's preparation for his lifetime service to the City of St. Louis.

Beginning as second assistant engineer in the Distribution System of the St. Louis Water Department in August, 1892, Mr. Wall rose to water commissioner and finally to director of public utilities in charge of not only the water supply but the city's power plants, street and public building lighting, city markets, and the railroad deck of the Municipal Bridge over the Mississippi River.

When he was assistant water commissioner, about the time of the Louisiana Purchase Exposition of 1904, Mr. Wall introduced the first of the processes by which the muddy water of the Mississippi and Missouri rivers not only is clarified and made safe but is transformed into one of the best large city supplies in the world. Much original research was involved in this work. The first efforts were directed toward settlement of the mud, hastened by chemical treatment to promote coagulation. Although the results were almost startling when compared with the brown liquid previously furnished to St. Louisans for water, this water was still not as clear or free from bacteria as was desirable. Accordingly, after extensive studies under Mr. Wall's direction, a very large sand filtration plant was decided upon. Such a filtration plant could not have been operated successfully without preliminary settlement and chemical treatment to remove the coarser silt particles; nor was it possible to produce satisfactory water without filtration. Accordingly, the process finally adopted included preliminary settlement of the raw water, chemical treatment followed by further settlement, then sand filtration, and finally chlorination. When completed in 1915, the Chain of Rocks filtration plant was the largest in the world and the product unsurpassed. When Mr. Wall began the improvement of the St. Louis water, the St. Louis typhoid death rate was 47 per 100,000 and the general death rate was 1,711 per 100,000. The purified water supply caused these rates to drop to 15 and 1,409 per 100,000, respectively. The Chain of Rocks water plant has a daily capacity of 160 million gal.

Shortly after the successful opening and operation of the Chain of Rocks plant, studies initiated by Mr. Wall indicated that the expected normal growth of St. Louis would require more water by 1935 than could be purified by the existing facilities. Investigations were made to determine whether it would be best to plan an enlargement of the Chain of Rocks plant or to build an entirely independent plant. These studies were by no means simple, as adequate supplies of raw water were possible from any one of three rivers in the area—the Mississippi, the Missouri, or the Meramec. Based on factors which included the center of expected population and water use, the changes necessary in the primary distribution system, and the effects of a major fire or an earthquake or other occurrence which might put the Chain of Rocks plant out of service, it was decided to build an entirely new plant which would duplicate the existing capacity but would be located at Howard's Bend on the Missouri River some thirty miles above the mouth. The studies showed that the cost advantages of the location, plus the certainty that the entire water supply could scarcely be interrupted by any even remotely possible major catastrophe, outweighed the additional expense of treating the muddier Missouri River water. The Howard's Bend plant was accordingly built in 1929. This construction involved some difficult foundation work in the alluvial river bottoms, but it was completed successfully.

In 1925 while the Howard's Bend plant was under construction, Mr. Wall was promoted to director of public utilities. As director he initiated the change from gas to electricity for city lighting—a long-time contract having prevented this change earlier. In connection with this program and very characteristically, he refused the request of the mayor to make certain personnel changes in his organization for purely political reasons and resigned instead. His adherence to principle was applauded and eventually led to the downfall of the mayor. After some five years in private practice, Mr. Wall was reappointed director of public utilities when a new administration was elected. He served until 1941 when advancing years caused him to retire.

Mr. Wall was quiet, unassuming, and modest. Those privileged to know him well remember him as kindly, generous, and considerate; an ever loyal friend. He greatly enjoyed his contacts with engineers. His particular hobby was the expression of his thoughts in verse and for a considerable period of years he exchanged his "poems" with J. D. von Maur, M. Am. Soc. C. E., a former St. Louis engineer living in Canada. These verses were collected and published for private distribution locally and at the time of Mr. Wall's death had filled two volumes[2] with a third in prospect. He wrote a somewhat similar but more pretentious prose effort, "Engineers, Engineering, and Some Vagaries."[3] The scope and charm of Mr. Wall's reasoning and attitude toward life are glimpsed by some of the chapter headings: "Advantages of Not Being Too Clever," "Engineering Ethics," "General Education for the Engineer," "Helping Each Other to Help the Profession," "Human Side of Engineering," "Luck," "Musical Tramp," "One Honest Man," and "Trees."

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[2] "Literary Productions of Two Civil Engineers," by J. D. von Maur and E. E. Wall, Locust Printing Co., St. Louis, Mo., 1940–1943.

[3] Engineers, Engineering, and Some Vagaries," by Edward E. Wall, Carter & Wilson, St. Louis, Mo., 1931.

Mr. Wall became a member of the St. Louis Engineers Club in 1889 (president in 1908). He belonged to the American Water Works Association (trustee from 1921 to 1923) and was on the standardization council in 1924–1925. He was a member of the American Public Health Association from 1915 to 1928, the American Society of Municipal Engineers from 1920 to 1927, the St. Louis Institute of Consulting Engineers (president in 1928), the St. Louis Academy of Science, the Circle Club, the Noonday Club, Tau Beta Pi fraternity, and the Masonic fraternity. In 1935 the University of Missouri conferred on him the honorary degree of Doctor of Laws.

In 1901, Mr. Wall was married to Jessie Towne of St. Louis. Mrs. Wall died in 1929 and was buried in Valhalla Cemetery at St. Louis. When Mr. Wall died, a Masonic service was held at the home and he was interred beside his wife. Many of his old-time friends served as honorary pallbearers in token of their high regard for him.

Always interested in Society affairs, Mr. Wall served as Director from 1918 to 1920 and as Vice-President in 1921 and 1922. In 1908 he was awarded the Thomas Fitch Rowland Prize for his paper on "Water Purification at St. Louis, Mo."[4]

Mr. Wall was elected a Member of the American Society of Civil Engineers on February 1, 1905, and an Honorary Member on October 10, 1938.

JEREMIAH AHERN, M. Am. Soc. C. E.[1]

Died April 19, 1944

Jeremiah Ahern was born on July 5, 1860, in Alameda County, California, near what was later called Grove Street and University Avenue in Berkeley.

Although it is not known in what year his parents, Jeremiah and Bridid (Marnen) Ahern, came to California, it can be truthfully said that he was the son of pioneers and that he grew up under pioneering conditions. Not until he was eight years old did the first continental railroad train arrive in California. In the same year, 1868, the University of California was incorporated, its site at Berkeley being within a quarter of a mile of the Ahern home.

Young Ahern was inspired by the great work of the late Theodore D. Judah,[2] M. Am. Soc. C. E., in locating and constructing the Central Pacific Railroad across the Sierra Nevada, and we know he was also familiar with the marvelous work other engineers were doing in the same mountains in storing and transporting water for use in gold mining. It was the last of the "Golden Era" in California and it was easy to foresee the unlimited opportunities open to the civil engineer in the development of the Pacific Coast.

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[4] Transactions, Am. Soc. C. E., Vol. LX (June, 1908), p. 170.

[1] Memoir prepared by Norwood Silsbee, Assoc. M. Am. Soc. C. E., and Joseph W. Gross, M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. XXXVIII (December, 1897), p. 448.

Therefore, when he was nineteen, young Ahern entered the University of California, in the Class of 1883.

Pioneering conditions still prevailed in the West. Travel was mainly by horses; Indians were not always friendly; outlaws were common; and sleeping and eating conditions outside the cities were primitive—but so strong was the appeal of the outdoor life to Jeremiah Ahern that for the twenty-five years of his active professional life he deliberately adopted it and enjoyed it.

For the first nine years after leaving college he served with the United States Geological Survey: From 1883 to 1888 as assistant topographer in California, Virginia, Alabama, and Georgia; in 1888 as topographer in charge of parties in Pennsylvania; and from 1889 to 1891 as topographer of the Powell Irrigation Survey in Montana. It was during these early years of the Geological Survey, organized in 1879 under Maj. John Wesley Powell, of Colorado River fame, that Mr. Ahern won the lasting friendship of so many of the engineers who afterward became noted in government work.

In 1892 Mr. Ahern left government employ and became associated with McCray Brothers, civil and hydraulic engineers of San Francisco, Calif. During the three years with this firm, he acted as assistant engineer on the South Gila Canal Project, in Arizona, making surveys and plans for a reservoir, canals, wagon roads, and irrigation system; as assistant engineer on location of a proposed railroad from Monterey Bay to Fresno, Calif., over the Coast Ranges; and in preparation of plans and estimates for irrigation storage dams on the San Luis Rey River, in California.

Returning to the United States Geological Survey in 1895, for the next three years Mr. Ahern was in charge of surveys in Indian Territory, which had been opened to settlement by the white man a few years earlier. From 1899 to 1901, he was in charge of topographic survey parties in Utah and Arizona; and in 1901 and 1902, as hydrographer, he was in charge of hydraulic investigations on the Gila and Salt rivers, in Arizona, and on the Colorado River between Needles, Calif., and Yuma, Ariz. The party on the Colorado River survey included the late J. B. Lippincott,[3] Hon. M. Am. Soc. C. E., and W. B. Clapp,[4] M. Am. Soc. C. E. Twenty-seven miles per day for eleven days were covered, using a specially designed flat-bottomed boat equipped with a sail. Mr. Ahern's story of the early history of the Colorado River was published in the "First Report of the U. S. Reclamation Service."[5]

One of the six large proposed irrigation projects selected for study when the Bureau of Reclamation of the U. S. Department of the Interior was organized in 1902 was the Uncompahgre Valley, in Colorado. Water from the Uncompahgre River was insufficient to irrigate the 175,000 acres of irrigable land in the valley and it was proposed to supplement the supply by diversion of 1,200 cu ft per sec from the Gunnison River (Colorado), running parallel in a deep gorge twenty miles distant. Under Mr. Ahern, the necessary topographic surveys for the project were completed in 1901, including a survey of the "Grand Canyon of the Gunnison," as it was named by the Hayden Survey. That portion of the Gunnison from which the seven-mile tunnel was to start had never been explored, and the gorge was so spectacular that the survey, and the ultimate rescue of some of the party, attracted nation-wide attention.

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[3] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 1543.

[4] For memoir, *ibid.*, Vol. LXXV (December, 1912), p. 1148.

[5] "First Annual Report of the Reclamation Service, from June 17 to December 1, 1902," pp. 112, 121.

The new Reclamation Service, under its chief engineer, the late F. H. Newell,[6] M. Am. Soc. C. E., attracted many engineers from the Geological Survey, among them being Jeremiah Ahern. His first job, that of supervising investigations for the DeSmet Project, in Wyoming, was so successfully executed that in 1903 he was placed in charge of investigations for the Shoshone Project, previously promoted by Col. W. F. Cody ("Buffalo Bill").

This project, also in Wyoming, originally provided for the construction of a storage reservoir on the Shoshone River and the irrigation of 125,000 acres of land. In 1904 Mr. Ahern's plans were approved, $2,250,000 was appropriated by the government for construction, and Mr. Ahern was appointed project engineer.

The Shoshone Dam was the first high concrete dam constructed by the Reclamation Service. The late H. N. Savage,[7] M. Am. Soc. C. E., was in charge of design. The dam is of arch design, 328 ft maximum height, 200 ft crest length, 10 ft wide on top, and 108 ft thick at the base. It contains only 78,600 cu yd of concrete, has spillway capacity for 30,000 cu ft per sec, and stores 456,600 acre-ft of water.

Mr. Ahern continued on this project until its successful completion in 1908, when he resigned from government service and returned to California. There he purchased a ranch near Dixon and engaged in the private practice of his profession for the next thirty-two years.

While building the Shoshone Project, Mr. Ahern became acquainted with his future wife, Julia Bessie Lane, whose sister, Dr. Frances Lane, had a hospital in Cody, Wyo. They were married in Denver, Colo., in 1910.

Both Mr. and Mrs. Ahern were seriously injured in an automobile accident in 1941. Mrs. Ahern passed away in the Woodland Clinic (Woodland, Calif.) six weeks later. Mr. Ahern eventually recovered but age was against him and on April 19, 1944, he died of heart trouble in the Dixon Hospital. He was buried from St. Michael's Catholic Church.

Members of Sacramento Section first became acquainted with Mr. Ahern some five years after the formation of the Section, when he called the Section's attention to the entire lack of public recognition given to Mr. Judah,[2] original promoter, locator, and first chief engineer of the first transcontinental railroad. Much of the publicity since given to Mr. Judah, and the erection in 1930 of a memorial to him on the grounds of the Sacramento Depot of the Southern Pacific Railroad by employees of that company, can be traced directly to Mr. Ahern's interest in the great engineer.

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[6] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1597.

[7] For memoir, *ibid.*, Vol. 103 (1938), p. 1873.

For many years Mr. Ahern did practically all the land surveying in northern Solano and southern Yolo counties and acted as consulting engineer on reclamation and irrigation problems. His intimate knowledge of the entire West and his broad business and professional experience caused him to be looked upon as an outstanding leader in the community and his opinions on public matters to be highly regarded.

As a local business friend expressed it: "He supported every activity which would benefit the community and gave generously to his church and to all worth-while charities. His neighbors were all fond of him, and his passing was a distinct loss to the community."

Mr. Ahern became a Life Member of the Society and of the Sacramento Section in 1938, being the Section's senior member in age and in length of Society membership. He was also a member of the Archæological Institute of America.

It was while he was in charge of the Shoshone Project that Mr. Ahern applied for membership in the Society, and the list of his references shows not only his standing among engineers at the time but the quality of his close friends. They were: Arthur P. Davis,[8] Past-President, Am. Soc. C. E., C. H. Fitch, J. B. Lippincott,[3] Frederick H. Newell,[6] William D. Pickett,[9] Hon. M. Am. Soc. C. E., J. H. Quinton, M. Am. Soc. C. E., and Herbert M. Wilson.

Mr. Ahern was elected a Member of the American Society of Civil Engineers on May 4, 1904.

JAMES PENNELL ANDERSON, M. Am. Soc. C. E.[1]

Died April 15, 1944

James Pennell Anderson, the son of William Edward and Mary (Wilson) Anderson, was born in Franklin, Williamson County, Tenn., on December 22, 1888. He was educated at Battle Ground Academy in Franklin, and at Branham and Hughes Junior College in Spring Hill, Maury County, Tenn.

His first engineering work was for the Louisville and Nashville Railroad Company in Nashville, Tenn., as a rodman in 1909 and later as instrumentman in Nashville and also in Montgomery, Ala. He entered the service of the Nashville, Chattanooga and St. Louis Railway in 1915 as assistant engineer. Later he was transferred to the Atlanta Division, as assistant division engineer, and was promoted to division engineer on January 1, 1926, which position he held until April 1, 1944, when he was appointed to the newly created position of engineer, maintenance of way and structures, for the entire system. He had assumed his new duties just prior to his death. He was in full charge of maintenance of way for the entire railroad with particular emphasis on roadway and track—a recognition of his outstanding maintenance work on the Atlanta Division.

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[8] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1582.

[9] For memoir, *ibid.*, Vol. LXXXI (December, 1917), p. 1671.

[1] Memoir prepared by a Committee of the Georgia Section consisting of C. A. Smith, J. H. Johnston, Members, Am. Soc. C. E., and J. A. Higgs, Jr., Assoc. M. Am. Soc. C. E.

The Atlanta Division is that part of the Nashville, Chattanooga and St. Louis Railway between Atlanta, Ga., and Chattanooga, Tenn., known as the Western and Atlantic Railroad of the State of Georgia, operated under a long-term lease from the state. It is probable that the Western and Atlantic is one of the crookedest trunk-line railroads in the United States, having a curvature that averages a little more than 90° to the mile. The terrain is mountainous and the line crosses many streams, necessitating bridge structures of varied designs and lengths. Almost immediately upon taking charge, Mr. Anderson made a thorough study of the conditions on this division, particularly alinement details. The data, from which the tabular functions were calculated, were worked out by Mr. Anderson for each curve as the center line was rerun, permanent monuments were established to mark ends of spirals, points of full elevation were fixed, rate of runoff was determined, and superelevation was computed. Since there are 316 curves in 137 miles, the magnitude of this job is better appreciated. So efficiently was this work done, and line and surface constantly maintained, that the smooth movement of trains passing around the curves has been a matter of favorable comment from railway officials of other lines after riding over the Western and Atlantic Railroad.

It is a generally recognized fact that more trains are handled over the line between Atlanta and Chattanooga than on any other single-track trunk-line railroad in the United States. In addition to its own business, the Nashville, Chattanooga and St. Louis Railway handles the trains of the Atlanta-Cincinnati Line of the Louisville and Nashville Railroad Company, over the 47 miles between Atlanta and Cartersville, Ga. Because of World War II the business of both railroads has been extremely heavy, severely taxing the operating officials to handle it promptly and efficiently. In 1943 and 1944 the Nashville, Chattanooga and St. Louis Railway installed centralized traffic control over this section which has greatly facilitated operations. The installation of this centralized traffic control was under the general supervision of Mr. Anderson.

It is the well-considered judgment of engineers of long experience in track maintenance who, with Mr. Anderson, made many inspections of the Western and Atlantic Railroad, that it has been maintained in a condition that compares favorably with that of the best railroads in the United States.

On June 23, 1915, Mr. Anderson was married to Roberta Reeves, of Montgomery, who, with a son, William Reeves, survives him. Also surviving are three sisters, Alma (Mrs. J. G. Lindeman), Mary (Mrs. A. H. Thomas), and Helen (Mrs. Clark A. Bean); and two brothers, William O. Anderson and R. Gordon Anderson.

Mr. Anderson was a charter member of Cascade Methodist Church of Atlanta, served as chairman of the Board of Trustees and the Board of Stewards, and was an active member of this church until his death. He was a Christian in his daily conduct. In dealing with the men in his department he was a strict disciplinarian, but was absolutely fair and just. He had the respect, confidence, and affection of all of them down to the last section hand.

He was highly regarded not only by his own men, but by men in other departments of the Atlanta Division. He was particularly interested in accident prevention work, and his successful handling of the Safety Department on his division repeatedly won the system banner for low accident record. He was a member of the American Railway Engineering Association.

Mr. Anderson was elected a Member of the American Society of Civil Engineers on December 28, 1931.

EDGAR ALCANDER BAYLEY, M. Am. Soc. C. E.[1]

Died November 8, 1943

Edgar Alcander Bayley was born at Petaluma, Calif., on September 19, 1877. He was the son of Alcander Alonzo and Bertha (Cooper) Bayley. He received his early education in the public school system of Los Angeles (Calif.), after which he attended Throop Polytechnic Institute (California Institute of Technology) at Pasadena for two years. He then entered the law school of the University of Southern California in Los Angeles, receiving the degree of Bachelor of Laws.

From 1901 until 1905 he worked in various engineering capacities for local electric and steam railway systems. From 1905 to 1910 he was locating and division engineer for the Los Angeles Aqueduct, making the preliminary and final locations of more than half the total length of 235 miles. Much of this work was done in difficult mountainous and desert country.

During 1910 he became assistant engineer of the San Pedro, Los Angeles and Salt Lake Railroad Company (later the Union Pacific Railroad Company), on location surveys in the State of Nevada. From 1911 to 1923, Mr. Bayley was assistant engineer for the Bureau of Water Works and Supply of the City of Los Angeles, in charge of field surveys, location, hydraulic design, and topographic mapping in connection with building the distribution system and terminal storage facilities for the Los Angeles Aqueduct. This work involved the construction of many reservoirs and large pipe lines.

From 1923 to 1930 Mr. Bayley was in charge of preliminary location, mapping, water right studies, and hydrographic studies for the new aqueduct from the Colorado River to Los Angeles. (This aqueduct has been finished and is operated by the Metropolitan Water District of Southern California.) On this project, it was necessary to map 25,000 sq miles of rough desert country and to prepare locations and estimates of all possible routes for the aqueduct. During the progress of the work as many as forty plane table and triangulation survey parties were in the field, and approximately sixty draftsmen and computers were in the Los Angeles and Beaumont (Calif.) offices. The territory covered extended from Los Angeles to the Colorado River on the east, to Las Vegas, Nev., on the north, and the Mexican border on the south. The cost of the completed aqueduct was about $225,000,000.

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[1] Memoir prepared by Ralph R. Proctor, M. Am. Soc. C. E.

During 1930 Mr. Bayley was assigned similar duties in connection with an extension of the Los Angeles Aqueduct northward into the Mono Basin. He prepared maps, estimates, etc., for two large reservoirs, with a tunnel, 11.3 miles long, and several miles of concrete conduit. The construction, land, and water right costs of this project were $20,000,000. This work was completed in 1937 and Mr. Bayley returned to the Los Angeles office to prepare studies on additional construction required for the joint use of the Los Angeles Aqueduct and the Colorado River Aqueduct in connection with the extended demand for water by the rapidly growing City of Los Angeles.

Mr. Bayley was widely known as an authority on methods of extending triangulation controls and precise levels into almost unexplored desert regions. He was particularly well versed in methods for locating conduits, tunnels, and steel pipe lines in combination to secure the most economical cost. Since he was deeply interested in financial studies, he assisted in selecting investments for the pension plan for the employees of the Department of Water and Power that was put into effect a few years prior to his death. An expert photographer, during the course of his work in remote regions he secured many outstanding photographs.

On October 27, 1903, Mr. Bayley was married to Grace Winifred Davis. He is survived by his widow and one daughter, Edith Winifred Bayley.

For many years Mr. Bayley was a member of the University Club of Los Angeles, and also was one of the founders of the Engineers Club of Los Angeles.

Mr. Bayley was elected a Member of the American Society of Civil Engineers on December 31, 1913.

PAUL LEONARD BEAN, M. Am. Soc. C. E.[1]

DIED JANUARY 29, 1944

Paul Leonard Bean of Auburn, Me., agent and engineer of the Union Water Power Company, the Androscoggin Reservoir Company, and the Union Electric Company of Lewiston, Me., from January, 1924, until his sudden death, was born at Saco, Me., on August 28, 1881. His parents were Joel and Ida Ellen (Hanson) Bean. He was educated at Thornton Academy, in Saco, and at the University of Maine, in Orono. In 1904 he was awarded the degree of Bachelor of Science in Civil Engineering and, in 1910, the degree of Civil Engineer, by the college of engineering of the university. Except for the first two years after graduation, when he was employed as a computer on special street railway work for William Wharton, Jr., and Company of Philadelphia, Pa., and as a draftsman for the Phoenix Bridge Company of Phoenixville, Pa., his entire life was spent in Maine.

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[1] Memoir prepared by Stanley B. Attwood, City Editor, Lewiston Daily Sun, Lewiston, Me.

He returned to the University of Maine in 1907 as an instructor in civil engineering, and was promoted to assistant professor in 1909 and associate professor in 1912. During the latter year he became associated with Harold S. Boardman, M. Am. Soc. C. E., later president of the university, in the engineering firm of Boardman and Bean.

In 1915 he was appointed chief engineer of the Maine Public Utilities Commission. He organized its engineering department, had responsible charge of stream gagings and of valuation of utilities, and compiled service standards for gas and electric utilities. Other work under his direction included grade crossing elimination; bridge investigation, including condemnation, design, and repair of structures; and inspection of dams.

Mr. Bean was in sole charge of a complete water power and storage survey, covering legal; engineering, and administrative phases, presented to the Maine Legislature in 1918.

He became associated in 1919 with the late Walter H. Sawyer,[2] M. Am. Soc. C. E., of Lewiston in the construction and hydraulic engineering firm of Sawyer and Bean, the firm continuing until Mr. Sawyer's death in 1924. He then succeeded Mr. Sawyer as agent of the three companies—Union Water Power Company, Androscoggin Reservoir Company, and Union Electric Power Company—owning and operating power on the Androscoggin River and the storage reservoirs at its headwaters (the Rangeley Lakes and Aziscohos or Sawyer Lake which was created by a dam on the Magalloway River, a tributary of the Androscoggin River). In this position he was in charge of the operation of the three companies and of the design, construction, and maintenance of their facilities. The Androscoggin River has a total length of 173 miles with a drainage area above Lewiston of 2,890 sq miles and a power development of about 25,000 hp. Its storage reservoirs have a capacity of 30,000,000,000 cu ft.

It has been claimed no river in the United States of comparable size is more completely developed for storage and power. The several dams—at Upper Dam, Middle Dam, Pond-in-the-River, Rumford, Livermore, and Lewiston in Maine and Pontoocook, Errol, Berlin, and Gorham in New Hampshire —have served not only their primary purposes but have proved entirely effective in flood control except in March, 1936, when the valley was swept by the highest water on record. Until that year a flow of 60,000 to 75,000 cu ft per sec was considered exceptional. The 1936 flood is estimated to have exceeded 200,000 cu ft per sec, although means of measurement were lacking. This flood resulted from a snow cover that was heavy but not of unusual depth, plus the unprecedented combination of heavy rain and temperatures around 55° at many upriver points in mid-March. During the flood, Mr. Bean refused to leave his office since an uninterrupted flow of reports from control stations gave him a good perspective of the entire area which he felt he could better retain without a personal inspection. A hospital bed was moved to his office and, until conditions approached normal, he was never more than a few feet from a telephone.

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 1437.

During the flood period his office was besieged by property owners; by town officers from Berlin, N. H., to Lewiston and from the downriver towns of Lisbon, Lisbon Falls, Pejepscot, and Brunswick, Me.; by the Portland office of the U. S. Weather Bureau in frequent consultations; by the newspapers; and by the general public. Mr. Bean insisted all information be given out as promptly as possible, and his unfailing courtesy in this emergency was acknowledged by public officials concerned with safety measures in the flood area.

Mr. Bean's principal attributes were integrity, frankness, determination, and unselfishness. His avocations were fishing, home gardening, and football —first as a player of all-Maine caliber at the university and later as a spectator at state series games.

One of the pioneers in snow measurement as a factor in control of river flow, he applied himself in later years to the study of meteorology, forestry, soils, and kindred subjects bearing on water storage. Perhaps no one had a more thorough knowledge of Maine topography. Mr. Bean's work in design and construction of bridges and dams, location and design of water utilities, and appraisals of various other engineering works took him to all parts of the state and eastern New Hampshire. He was intensely interested in having the United States Geological Survey complete the quadrangle mapping of Maine, still about 25% unfinished, contending that accurate maps were the first essential in any plan to develop the great wild land area of the state.

When stricken fatally ill, he had begun plans to draw a relief map of the Androscoggin watershed and to write a history of the development of the Androscoggin River, particularly as it pertained to the growth of industry through generation and distribution of hydroelectric power. In addition to a large technical library, he had acquired several hundred volumes dealing with Maine.

Mr. Bean was a consultant for the Lacroix Industries of Canada. He served frequently as an adviser to various city departments of Lewiston and Auburn and to the Auburn water and sewer districts. Although a resident of Auburn, he was named to the committee in charge of design and construction of Lewiston High School and of the west wing of the Central Maine General Hospital of Lewiston, of which he had been trustee for many years. He was a director of the Androscoggin County Savings Bank of Lewiston, a former trustee of the Auburn Public Library and the Lewiston Building and Loan Association, and for thirteen years a trustee of the Elm Street Universalist Church of Auburn. He was a member of Alpha Tau Omega fraternity, of the Masons and Knights Templar, and of Kora Temple of the Mystic Shrine.

Mr. Bean was a member of the Boston Society of Engineers, a former president of the Maine Association of Engineers, and a member of the Maine State Board of Registration for Professional Engineers and that organization's examining board.

On December 24, 1907, he was married to Delia Webster of Orono. He is survived by his widow; two sons, Paul Webster, Major, Infantry, U. S. Army, and Ralph Leonard, Private First Class, Infantry, U. S. Army; and three brothers, Roger J., Ralph N., and Philip H.

Mr. Bean was elected an Associate Member of the American Society of Civil Engineers on September 2, 1914, and a Member on December 15, 1924.

JAMES WALLACE BEARDSLEY, M. Am. Soc. C. E.[1]

DIED MAY 15, 1944

James Wallace Beardsley, the son of William Hurd and Catherine Tremper (Phillips) Beardsley, was born on September 11, 1860, in Coventry, Chenango County, N. Y. After completing the classical course at the State Normal School at Cortland, N. Y., in 1884, he entered Cornell University, in Ithaca, N. Y., receiving the degree of Civil Engineer in 1891.

During the summer of 1890, Mr. Beardsley served as assistant city engineer of Binghamton, N. Y. His first engagement after graduation was with the City of Newton, Mass., as assistant engineer (September, 1891, to September, 1892) on design and inspection of sewer construction. For the following six years (September, 1892, to August, 1898), he was employed by The Sanitary District of Chicago (Ill.) on the drainage canal—an outstanding engineering project of that period. After promotions from time to time, Mr. Beardsley was given charge of the Lemont-Lockport Division of the canal. His work included heavy earth and rock excavation and the construction of massive retaining walls, foundations for the beartrap dam, bridge foundations, and superstructures at Lockport, Ill.

When this project was completed in 1898, Mr. Beardsley was engaged until October, 1899, by the United States Board of Engineers on Deep Waterways, as assistant engineer. He was placed in charge of the surveys, designs, and estimates of cost for a ship canal (St. Lawrence River route) to connect the Great Lakes with the Atlantic Ocean—including water-power development at locks and dams. From 1900 to 1902 he was surveyor with the United States Corps of Engineers in Milwaukee, Wis., and Detroit, Mich. (Fox River surveys and Sand Beach harbor of refuge designs).

In July, 1902, Mr. Beardsley was commissioned as consulting engineer to the Philippine Commission, by the late Hon. William Howard Taft, then Governor General of the Philippine Islands. When the Bureau of Engineering was organized, he became its chief. In 1905 he was made Director of Public Works of the Philippines and he inaugurated extensive programs for good roads, public water supplies, sewerage, and general sanitation. In 1908 he was commissioned to investigate irrigation in Java, Burma, India, and Egypt, reporting his findings to the government of the Philippines.

On his return to the United States in 1909, he made investigations and reports on several irrigation projects in Oregon, Idaho, Washington, Montana, and Colorado for J. G. White and Company. However, he could not long resist the urge of foreign soil, and in July, 1910, Mr. Beardsley accepted the position of chief engineer of the Puerto Rico Irrigation Service. The extensive irrigation systems designed and constructed under his direction during the succeeding six years included: Earth dams of semihydraulic fill and rolled earth embankment with crest heights of 110 and 135 ft; concrete dams of the Ambursen type with crest heights of 65 and 115 ft above the river bed; aqueducts and tunnels; highway bridges; and an hydroelectric plant with a head of 760 ft. Because of the failure and inability of local contractors to progress satisfactorily, most of these projects were constructed by the Puerto Rico Irrigation Service under Mr. Beardsley's personal direction.

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[1] Memoir prepared by Glenn D. Holmes, M. Am. Soc. C. E.

After completion of the Puerto Rico work in August, 1916, he made a number of investigations and reports on irrigation and power projects in the western states. In March, 1918, he accepted employment as consulting engineer in the Inspection Division, U. S. Ordnance, at Washington, D. C.

From 1918 to 1920, Mr. Beardsley was assistant chief engineer of the Grand Canal Improvement Board of the Chinese Government, on surveys and designs for improvement of the Grand Canal, especially at the crossing of the Yellow River. Problems in connection with silting of streams and reservoirs were of unusual significance and required special investigation and study. When he had completed a report on this work, Mr. Beardsley was appointed chief engineer and engineer member of the Central Highway Commission of the Republic of Panama. During 1920 and 1921, under his supervision, a general highway policy was adopted, plans were prepared, and a contract was executed for the construction of 270 km (168 miles) of highways—an integral part of the intercontinental highway of the provinces. In addition, concrete and bituminous concrete roads were built near the City of Panama.

In October, 1921, Mr. Beardsley returned to the United States and was engaged in private consulting work until August, 1926. During the latter part of this period, he very graciously consented to assist the writer in the review and the preparation of a report on Onondaga Creek flood prevention, for the City of Syracuse, N. Y. This association was mutually pleasing and agreeable since it meant the revival of an old friendship between the "Chief" and one of his "boys," which began on the Deep Waterways Project in 1898.

This pleasant relationship was interrupted by an urgent request for Mr. Beardsley to become consulting engineer of the Public Works Bureau for the Dominican Republic. He accepted the position and directed the investigations, surveys, and designs for the improvement of the harbors of Santo Domingo, Puerta Plata, and Macoris. After three years of strenuous endeavor he returned to Syracuse in August, 1929, to engage again in private practice. During his many years of foreign service, Mr. Beardsley had an opportunity to explore remote and interesting places. Curios, works of art, tapestries, and jade and ivory carvings were included in his collection of mementoes. One hobby, photography, filled many albums with a record of his engineering accomplishments and observations. The albums are arranged according to subject matter—bridges, dams, monuments, irrigation, methods of transportation, and habits and costumes of various races—particularly throughout the Orient. Mr. Beardsley also collected postage stamps. Only by inquiry or after long acquaintance would one hear stories of his experiences or be shown some of these interesting collections, for Mr. Beardsley was more than unassuming and reticent regarding personal accomplishments.

Mr. Beardsley was a man of extremely high ideals. He possessed those qualities of sound reasoning, good judgment, and executive ability which particularly qualified him to be engineer, commissioner, and administrator of many large and important projects.

Mr. Beardsley was a member of the Western Society of Engineers, The Chinese and American Engineering Society, and the Technology Club of Syracuse (honorary member and past-president). For many years he belonged to the Cosmos Club of Washington, D. C.

On September 7, 1893, in Xenia, Ohio, he was married to Ellen Jane Pearne. He is survived by his widow; by a son, Wallace P. Beardsley; and by three grandsons.

Mr. Beardsley was elected a Member of the American Society of Civil Engineers on December 4, 1901.

RALPH NORMAN BEGIEN, M. Am. Soc. C. E.[1]

Died February 27, 1944

Ralph Norman Begien was born on March 15, 1875, in Somerville, Mass., the son of Henry Martin and Louise Florence (Thayer) Begien, of French Huguenot and old New England stock. While he was still a child, his family moved to Medford, Mass., where he received his elementary and high school education. He attended Harvard University at Cambridge, Mass., for two years, devoting most of his time and interest to mathematics and languages, and excelling in the former.

On December 11, 1900, he was married to Ida Davenport Rozzell, of Washington, D. C. He is survived by his widow; two sons, Ralph and John; and a daughter, Jeanne.

In 1897 Mr. Begien began his professional career by joining a group of young engineers under Francis Lee Stuart,[2] Past-President, Am. Soc. C. E., on surveys for the United States Nicaraguan Canal Commission and the Isthmian Canal Commission. He worked as instrumentman, draftsman, and junior assistant engineer. In 1900 he was assistant engineer in charge of corps on preliminary surveys and location for the Guayaquil and Quito Railroad in Ecuador, South America.

He was employed in December, 1900, by the United States Government, as assistant engineer in the Engineering Department of the District of

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[1] Memoir prepared by G. D. Brooke, E. M. Hastings, and J. E. Teal, Members, Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1493.

Columbia, under the Surveyor General's Office. In 1902 he resigned and returned to railroad work, this time with the Baltimore and Ohio Railroad Company, as assistant engineer (again under Mr. Stuart, who was at that time engineer of surveys for the Baltimore and Ohio). He was assigned to the study and location of various new lines, including a low-grade line from the Fairmont coal fields of West Virginia to tidewater, and had an important part in the development of a comprehensive plan of trunk-line improvements.

During the following years Mr. Begien held various positions in the Engineering, Maintenance of Way, and Operating departments of the Baltimore and Ohio Railroad Company:

Engineering Department—Draftsman, 1903–1904; assistant engineer in charge of corps on surveys for trunk-line improvements and assistant engineer at headquarters, Baltimore, Md., on special engineering investigations, 1904–1909.

Maintenance of Way Department.—Division engineer, Philadelphia Division, and acting engineer, maintenance of way, main line system, 1909–1910; assistant to chief engineer, system, 1910.

Operating Department.—Assistant to general manager, eastern lines, and assistant to third vice-president, system, in charge of operating efficiency work, 1910–1912; assistant general superintendent, main line system, 1912–1913; general superintendent, Baltimore and Ohio Southwestern Railroad Company (Baltimore and Ohio Railroad Company, successor) at Cincinnati, Ohio, in general charge of operating, engineering, maintenance of way, and equipment, 1913–1916.

Engineering Department.—Chief engineer, Baltimore and Ohio System, and Cincinnati, Hamilton and Dayton Railway Company under receivers, 1916–1917.

Operating Department.—General manager, eastern lines, Baltimore and Ohio, in general charge of operation, engineering, maintenance of way, and equipment, 1917–1918; assistant to federal manager, United States Railroad Administration, in charge of operating and engineering of the Baltimore and Ohio (eastern lines), Western Maryland Railway Company, Cumberland Valley Railroad Company, Cumberland and Pennsylvania Railroad Company, Coal and Coke Railway Company, and Wheeling Terminal Railway Company, 1918–1919; federal manager, United States Railroad Administration, in general charge of operation, traffic, engineering, maintenance of way and equipment of Baltimore and Ohio (western lines), Dayton and Union Railroad Company, and Dayton Union Railway Company, 1919–1920; general manager, Baltimore and Ohio (western lines), Dayton and Union Railroad Company, Dayton Union Railway Company, Sandy Valley and Elkhorn Railway Company, and Long Fork Railway Company, 1919–1923.

From 1903 to 1906, while assistant engineer for the Baltimore and Ohio, Mr. Begien, because of his seemingly unlimited capacity for work, established enviable records for rapid surveys in running location lines through mountainous territory in West Virginia. This he was able to do by utilizing available maps of the United States Geological Survey and because of his practical skill in reconnaissance methods. Each night he would spend some time in recording and plotting the notes of the day's work, preparing paper locations, and planning the work for the following day. His high quality of leadership was further evidenced in his personal direction of field parties, and he was the equal of any man on the party in the performance of physical tasks, often using this ability to relieve men, temporarily, who were fatigued or needed elsewhere.

While he was assistant to the general manager of the Baltimore and Ohio, Mr. Begien made an extensive study of railway operating efficiency. He established an Operating Statistical Bureau and organized methods of competition between operating divisions, comparing their operating performance on properly equated efficiency units. A number of young engineers were under his direction, who, after being trained in the use of practical statistics, put into effect the train load formula and adjusted tonnage factor for loading freight trains, which Mr. Begien had developed as chairman of the Committee on Economics of Railway Location of the American Railway Engineering Association. The operating results obtained by the use of these methods were notable and contributed to Mr. Begien's prominence as an operating officer.

Mr. Begien's extensive experience in railway engineering and operation acquired during his more than twenty years with the Baltimore and Ohio Railroad Company and his association with prominently capable railway engineering and operating men rounded out the development of an unusually active, aggressive, and prolific mind. When federal control of railroads ended in 1920, Mr. Begien was well known as a railroad man of ability and vision, with a progressive attitude toward the relations of labor and management.

On June 1, 1923, Mr. Begien became vice-president of the Chesapeake and Ohio Railway Company and Hocking Valley Railway Company and was placed in general charge of maintenance and operation. In May, 1930, when the Hocking Valley was consolidated with the Chesapeake and Ohio, he continued as vice-president in charge of engineering and related activities. On July 1, 1932, due to declining health, he was appointed vice-president—advisory.

During his association with the Chesapeake and Ohio and under his general direction, a broad program of maintenance rehabilitation and additions and betterments, including the development of the Presque Isle Coal and Ore Docks, was completed. In the decade ending in 1932, the Chesapeake and Ohio expanded its properties by acquisition and construction—595 miles of road, 287 miles of second, third, and fourth main tracks, and 789 miles of yard and industrial tracks. The work included a 65-mile, double-track, low-grade railway, connecting the trackage of the Chesapeake and Ohio and the Hocking Valley, constructed in the remarkably short time of ten months. This was indeed a fine engineering achievement. Mr. Begien was retired from active service with the Chesapeake and Ohio Railway Company on May 1, 1938.

In his younger days, Mr. Begien was quite athletic, indulging in tennis and baseball, and later in golf. He was a lifelong baseball fan. He enjoyed a vacation spent at a mountain resort, and, being a good sailor, was particularly fond of a long sea voyage. His abilities were diversified. He could run a lathe in a machine shop and write an excellent engineering report. He was a proficient and rapid draftsman, a capable locating engineer, and a mathematical expert.

Of a sociable nature, kindly and considerate, with a cheerful and optimistic disposition, he was well regarded by all who knew him, and his associates soon became his staunch friends. The possessor of an analytical and ingenious mind, with unusual clarity of vision, Mr. Begien had original and broad ideas on many phases of location and operation. He was zealous in his devotion to duty, possessed intense energy and a high degree of personal efficiency. His absorbing interest—a love for his work—made him an effective executive and an acknowledged leader in the transportation field.

He applied his thorough knowledge of railway engineering and operation and his broad experience acquired in the course of his railroad career effectively to devise improved methods and to promote efficiency in railway practices. He advocated a definite line of authority which permitted the individual a maximum scope of responsibility and the greatest opportunity for exercising initiative. His ideas on organization have been the source of a number of articles relating to transportation matters.

Mr. Begien believed that a good railroad officer should know the problems of other departments as well as of his own. Accordingly, he would call departmental staff meetings to bring about better understanding of related problems, to foster unified and cooperative action, and so to improve the results of operation. He considered loyalty a most important attribute, and once was heard to say that he would rather have one man who understood his plans and was 100% loyal than a dozen genii.

In his relations with his fellow officers and employees, Mr. Begien encouraged their progress and development as future railway men, and he had the faculty of imparting to them his enthusiasm and knowledge. Several of these men have become heads of important transportation systems; so, if it is true, as has been said, that the Baltimore and Ohio is the mother of railroad presidents, then certainly Mr. Begien's influence on his friends and associates made him the big brother of railroad presidents.

Mr. Begien belonged to a number of clubs, including the Commonwealth Club, the Country Club of Virginia (Richmond), and the University Club (Cincinnati). He was an Episcopalian, and in things political adhered to the Republican Party.

He was a devoted husband, and as a father took a keen interest in the education and welfare of his children. After he retired he lived on Douglasdale Road in Richmond.

A member of the American Railway Engineering and Maintenance of Way Association (later American Railway Engineering Association) in 1907 (life member in 1940), he served on Committee 16—Economics of Railway Location (1908–1923, inclusive) as vice-chairman from 1911–1913 and as chairman in 1914 and in 1919–1920. He also was on the Committee on Wood Preservation (1909–1910) and the Special Committee on Cooperative Relations with Universities (1924–1927). While a member of Committee 16, he directed many dynamometer tests on Baltimore and Ohio freight trains, the results of which were basic freight train tonnage rating formulas.[3] Mr. Begien was the author of a number of papers concerning railway engineering and operation, and served from time to time as a member of the board of directors of a number of corporations allied with the transportation industry.

He was interested constructively in the activities of the Society, especially in the field of railway engineering. In 1923–1924, he was a Director from Zone III.

Mr. Begien was elected an Associate Member of the American Society of Civil Engineers on June 7, 1905, and a Member on March 13, 1917. He became a Life Member in 1940.

WILLIAM ETHELBERT BELKNAP, M. Am. Soc. C. E.[1]

DIED OCTOBER 1, 1942

William Ethelbert Belknap was born in Yonkers, N. Y., on July 20, 1867, one of a large and very distinguished family. His father was Ethelbert Belknap, a highly esteemed and successful hat manufacturer in Yonkers, and his mother was Lydia Elizabeth (Frazee) Belknap. His ancestor, Abraham Belknap of Warwickshire, England, settled in Lynn, Mass., in 1637. Another ancestor, Joseph Belknap, was the founder of the Old South Church, Boston, Mass. In the same line, Samuel Belknap fought in the French and Indian War. John Frazee (1790–1852), Mr. Belknap's grandfather on his mother's side, was an outstanding American architect. He designed the U. S. Sub-Treasury Building on Wall Street, New York, N. Y. (1834–1841). As a great sculptor, Mr. Frazee was a connecting link between American plastic art and the modern period. In 1824–1825, Mr. Frazee made the first marble bust in America—that of John Wells, a New York City attorney. He also did seven busts for the Boston Athenaeum.

William Ethelbert Belknap was prepared for the Yonkers High School in the Rev. Montgomery R. Hooper's Academy for Boys, and was graduated from New York University in New York in 1888 with the degrees of Bachelor of Science and Civil Engineer. Naturally, with his ability and genial disposition, he was elected to Phi Beta Kappa and Psi Upsilon fraternities.

During the summer of 1888 he acted as chainman for the New York State Adirondack Survey.

From February, 1889, to November, 1892, he was with the Pennsylvania Railroad Company in Jersey City, N. J., first as rodman for the engineer of maintenance of way with duties of general assistant on grades, lines, inspection, and surveys. He was promoted to field assistant in responsible charge of location and construction of track and grading for a large roundhouse, storehouses, and boilerhouses. During this same period, he worked on a water supply system in connection with the establishment of the Waldo Avenue Yard in Jersey City, and made miscellaneous surveys for the location of rights of way, sidings, etc.

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[3] "Basic Freight Train Tonnage Rating Formulas," Committee 16, *Proceedings*, A.R.E.A., Vol. 12, Pt. 1 (1911), p. 330.

[1] Memoir prepared by T. Kennard Thomson, M. Am. Soc. C. E.

He was assistant engineer in responsible charge of construction of the Bay Street Ferry Station in Jersey City and of construction of undergrade crossings for the Port Reading Railroad Company (heavy masonry bridge abutments, elevation of tracks, trestle work, etc.). Mr. Belknap then was engaged on the design and construction of the Jersey City powerhouse of the Consolidated Traction (Railway) Company, dealing especially with trestles, sidings, coal pockets, etc. In addition, he was employed on the plans and construction of the Jersey City electric light and powerhouse for the Pennsylvania Railroad Company, miscellaneous reports, estimates, surveys, etc.

Continuing his service with the Pennsylvania Railroad Company, in October, 1892, Mr. Belknap was transferred to the office of the engineer of maintenance of way, at Altoona, Pa., as assistant engineer in responsible charge of construction of the Altoona and Logan Valley Electric Railway (20 miles of city and suburban track, powerhouse, etc.). This was followed by the construction of Lakemont Park, near Altoona (earth dam, park buildings, etc.), for branch lines, changes of line, etc.

From May to September, 1894, Mr. Belknap was associated with S. L. Cooper,[2] M. Am. Soc. C. E., in making plans for the sewer system at Hastings-on-Hudson, N. Y. He acted as assistant engineer in charge of construction of the extension of Warburton Avenue, near Yonkers; and as consulting engineer for the Lancaster Sea Beach Improvement Company in Far Rockaway, N. Y., on plans for ocean shore protection, roads, and other improvements. Also, Mr. Belknap was consulting engineer for the Brooklyn (N. Y.) Sanitary Company, designing the foundations and planning the wharf work for the garbage utilization plant at Barren Island, New York.

From February, 1895, to January, 1897, he was engineer for the Wallabout Improvement, Department of City Works, Brooklyn, in charge of the design and construction of a large shipping basin (bulkhead, walls, piers, and other wharfage facilities).

From January, 1897, to December, 1902, Mr. Belknap was assistant engineer, Department of Docks and Ferries, New York City, in charge of waterfront operation in the Borough of Brooklyn, dredging, construction of various piers, pier sheds, yacht landing basin, bulkhead walls, crib bulkheads, pile tests, etc.

Mr. Belknap was engaged in the coal business from December, 1902, until 1914—as superintendent on a coaling pier in Jersey City for the Communipaw Coal Company, and as secretary and treasurer of this company; and as secretary and treasurer of Burns Brothers (coal company) whose work was centered in New York and New Jersey. In addition, he had charge of the design and reconstruction of coaling piers, coal pockets, etc.

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXVI (December, 1913), p. 2212.

As Mr. Belknap happened to be in England in 1914 when World War I started, he assisted in the evacuation of Americans then in England. He returned to America during the summer of 1914, and, near the end of that year, took his family to Hollywood, Calif.

Early in 1916 he bought the Orchard Gables estate in Hollywood, and that fall moved his family there permanently. He worked for the Red Cross in Washington, D. C., in 1917–1918.

He lived in California until he died, practicing architecture and engineering. Among other buildings, he designed his daughter's house. He was also a water color artist. His favorite sports were boating, fishing, tennis, and golf. He was a good Episcopalian, and a Republican until 1932 when he voted Democratic.

He was married to Helen Brett Runyon on April 22, 1897, who survives him. They were blessed with three children, Carman Runyon, who died in 1937; Ethelbert, who died in infancy; and Isabelle Fitz Randolf (Mrs. Paul Kunasz). He is also survived by three sisters, Ada Blanche Belknap, Edith Beatrice Belknap, Athenia Agnes (Mrs. Thomas Wood Hastings); one brother, Christopher Irving Belknap; and three grandchildren, Paul Belknap Kunasz, Peter Brett Kunasz, and William Runyon Belknap.

William Ethelbert Belknap lived up to the great standard of his outstanding family as a man, engineer, business man, artist, and sportsman. He was always genial, reliable, and true to his family, friends, and associates, whose admiration and affection he constantly held. It was a privilege to have known him. Whatever Mr. Belknap did was modestly and unassumingly well done.

Among his writings are many discussions published in the Society's *Transactions*. He served the Society well, especially when he was a Director for three years from 1910 to 1912.

Mr. Belknap was elected a Junior of the American Society of Civil Engineers on March 31, 1891; an Associate Member on March 6, 1895; and a Member on December 1, 1897.

EDWARD FRANKLIN BERRY, M. Am. Soc. C. E.[1]

DIED AUGUST 28, 1944

Edward Franklin Berry was born on October 1, 1889, in Fredericton, New Brunswick, Canada, the son of Edward Albert and Margaret Jane (Todd) Berry. He received his early education in the public schools of Fredericton, and later attended the University of New Brunswick at Fredericton from which he was graduated in May, 1913, with the degree of Bachelor of Science in Civil Engineering.

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[1] Memoir prepared by Louis Mitchell, M. Am. Soc. C. E., and J. O. F. Eichler, Assoc. M. Am. Soc. C. E.

While an undergraduate, Mr. Berry held several minor positions during the summer months on surveying parties, and in 1912 was assistant resident engineer on the construction of the St. John and Quebec Central Railroad, in charge of a field party.

After graduation, he became instrumentman for the Quebec Central Railway Company on preliminary and location surveys. From 1914 to 1916, Mr. Berry was chief draftsman on the federal valuation of the Maine Central Railroad Company.

In September, 1916, Mr. Berry was appointed instructor of civil engineering at Lehigh University in Bethlehem, Pa., where he served for three years. During this time he acted as consultant on several structural engineering projects. From 1919 to 1920, he was employed by the Texas Company as editor of its technical periodical, *Lubrication*. He also wrote two textbooks dealing with the subject of lubrication, for schools conducted by the Texas Company.

In September, 1920, Mr. Berry was appointed to the civil engineering staff of the Lyman Cornelius Smith College of Applied Science, Syracuse University, in Syracuse, N. Y., where he served as assistant professor, associate professor, and from 1939 until the time of his death as professor of civil engineering and head of the department of civil engineering, specializing in structural engineering. From 1923 to 1939, he had charge of the Materials Testing Laboratory which was rebuilt under his direction. On numerous occasions he was engaged as consulting engineer.

In addition to the Society, Professor Berry belonged to the following technical and professional organizations: Society for the Promotion of Engineering Education, American Association of University Professors, the American Society for Testing Materials, and the Technology Club of Syracuse. He was one of the founders and the first president of the Upper New York State Section of the Society for the Promotion of Engineering Education. He was also president of the Syracuse Section of the American Society of Civil Engineers in 1935, and a member of the Board of Directors of the Technology Club of Syracuse. In addition to his professional affiliations, Professor Berry belonged to Sigma Xi, Phi Kappa Phi, Theta Tau, and Tau Beta Pi fraternities. He was a Thirty-Second Degree Mason and a member of the University Methodist Church of Syracuse.

Professor Berry was married to Norma J. Vader of Syracuse in 1936. He is survived by his widow and a daughter, Winona N. (Mrs. James Dee).

As a citizen, as an engineer, and as a teacher, Professor Berry had the highest ideals, character, and integrity.

Professor Berry was elected a Member of the American Society of Civil Engineers on November 11, 1929.

PAUL ALEXANDER BLACKWELL, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 9, 1943

Paul Alexander Blackwell was born in Ghent, Carroll County, Ky., on February 13, 1879, the son of James Shannon Blackwell, noted linguist, and Mary Evelyn (Smith) Blackwell.

He attended the University of Missouri at Columbia and received the degree of Bachelor of Science in Civil Engineering in June, 1900. He was a member of Phi Gamma Delta fraternity. Several years after graduation his education was supplemented by study of statically indeterminate structures under J. W. Turm, a graduate student of Prof. J. Melan, of Prague.

In 1900 he was commissioned a Second Lieutenant, Battalion of Cadets, National Guard of Missouri. In 1917 he enlisted as a private in the Virginia Volunteer Infantry, serving, successively, as First Sergeant, Second Lieutenant, First Lieutenant, and Captain.

His engineering career began in 1900 as a structural draftsman with the Koken Iron Works, of St. Louis, Mo. Following this engagement, he served the Fort Pitt Bridge Works, in Pittsburgh, Pa., first as a structural draftsman from 1901 to 1903 and then as designer and estimator from 1903 to 1906. From August, 1906, to April, 1907, he worked for the Riverside Bridge Company, Martins Ferry, Ohio, as designer, estimator, and contracting agent.

In April, 1907, Mr. Blackwell was employed by the Virginia Bridge and Iron Company of Roanoke, Va., as an estimator and designer. A month and a half later, he was made assistant chief engineer. In 1922 he was promoted to chief engineer. He continued in this position until 1936 when the Virginia Bridge and Iron Company became a subsidiary of the U. S. Steel Corporation. The company's name was then changed to Virginia Bridge Company and Mr. Blackwell remained as chief engineer until his retirement on January 31, 1940.

During the years when Mr. Blackwell was assistant chief engineer and chief engineer for the Virginia Bridge and Iron Company, the company experienced its greatest expansion, both as to extent of territory and capacity of plants As chief engineer, he was in charge of designs, estimates, and details of all classes of steel structures furnished by his company. The many and varied types of bridges, buildings, etc., which were designed under his supervision and which are giving excellent service today, testify to his ability as an engineer.

Mr. Blackwell was endowed with an active and receptive mind. Quick to grasp an engineering problem, he always made decisions promptly. He was very systematic in his personal affairs and instilled this attribute into his organization.

In addition to his engineering ability, Mr. Blackwell was a student of several languages. An inquiring mind led him to study and investigate many subjects unrelated to his chosen profession. His quick wit and ready repartee, together with an exceptional fund of information, made him an interesting conversationalist and a person who will not be quickly forgotten by his friends and associates.

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[1] Memoir prepared by C. W. Ogden, M. Am. Soc. C. E., and George P. Jones and C. R. Wentworth, Assoc. Members, Am. Soc. C. E.

He was married on November 19, 1902, to Emily Frances Little, who survives him.

Mr. Blackwell was elected a Junior of the American Society of Civil Engineers on December 4, 1900; an Associate Member on January 3, 1906; and a Member on January 20, 1920. He became a Life Member in January, 1941.

CLARENCE MOORE BLAIR, M. Am. Soc. C. E.[1]

DIED MAY 21, 1944

Clarence Moore Blair, the son of William Henry and Mary (Moore) Blair, was born on August 21, 1885, in New Haven, Conn., in which city he lived throughout his entire life.

Following his early education in the public schools of New Haven, including Hillhouse High, he entered the Sheffield Scientific School of Yale University, specializing in civil engineering. Receiving honorable mention for excellence in mathematics in his freshman year, honors for excellence in all studies in his junior year (Sheffield at that time had no sophomore year) and general two-year honors in his senior year, he was graduated in 1904 with the degree then given graduates in science: Bachelor of Philosophy.

He at once entered the office of the late Albert B. Hill, M. Am. Soc. C. E., one of the outstanding consulting engineers of Connecticut, and, starting as rodman, by 1913 he had become first assistant in charge of all work. Mr. Hill died in 1930, and Mr. Blair and the late Orson H. Marchant, M. Am. Soc. C. E.—a classmate of Mr. Blair's who also had entered the Hill office on graduation—took over the practice under the name of Blair and Marchant, Inc., Mr Blair being president and treasurer. On the death of Mr. Marchant in 1938, the business was continued as Clarence M. Blair, Inc., Mr. Blair remaining as president and treasurer, and taking in his former assistants in the company as partners.

From its foundation by Mr. Hill the office had been noted for the thoroughness and accuracy of its work. Mr. Hill, a gentleman of the old school, inculcated in his men the same high standard of living and work by which he himself was guided, and Mr. Blair maintained Mr. Hill's principles in full. The reputation of the office brought to it much important work, giving the younger men unusual experience and opportunity. On his first assignment, while nominally rodman on the construction of an interurban electric railway from Cheshire to Waterbury, Conn., across heavy country, Mr. Blair showed such ability that he was put in charge, successively, of the sewage disposal plant for the Taft School at Watertown, Conn.; sewers for Litchfield, Conn.; the Wepawaug tunnel, Watrous dam, and the Gaillard dam and tunnels for the New Haven Water Company; and a number of smaller water and sewerage projects.

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[1] Memoir prepared by Clarence Blakeslee and Charles Rufus Harte, Members, Am. Soc. C. E.

Upon the death of Mr. Hill, Mr. Blair's work became more of an executive character, but until his final illness he kept actively and closely in touch with the construction as well as the designing and general engineering for a number of Connecticut utilities, towns, and the state itself; and personally represented many of these clients in valuation, rate, and other court cases, the thoroughness of his preparation and his poise making him a particularly valuable witness.

A firm believer in the advantages of organized association, Mr. Blair became a member of Sigma Xi and of The Yale Engineering Association before his graduation. Immediately thereafter, in August, 1904, he was elected a member of The Connecticut Society of Civil Engineers, Inc., and, serving efficiently on many of its committees, he also had the honor, shared by no other member of that society, of having held, successively, every one of the offices from director to president. As secretary-treasurer in the years from 1923 to 1931, he brought the affairs of the society from the very serious condition, in which they were turned over to him, to a high degree of prosperity. In 1941 he was elected an honorary member.

Always interested in the Society, in 1940 Mr. Blair was elected Director for District No. 2, and he served on several important committees. He was active in organizing the Connecticut Section of the Society, of which he was a charter member, and he served as its secretary-treasurer from 1921 to 1931, inclusive. He joined the New England Water Works Association on January 13, 1915, and the American Water Works Association in January, 1938.

In 1930, to fill the vacancy caused by Mr. Hill's death, the Hon. John H. Trumbull, Governor of Connecticut, appointed Mr. Blair a member of the Connecticut Board of Civil Engineers, which at that time had supervision over dams and reservoirs throughout the state. He served on that board and its successor, the State Board for the Supervision of Dams, Dikes, Reservoirs and Other Similar Structures, until his death. He was very active in securing the passage of the state act for the registration of engineers, and, in recognition of his efforts, when he became a "Registered Professional Engineer and Land Surveyor," he was given the first certificate, No. 6, issued to any other than members of the board—the act assigning Nos. 1 to 5, inclusive, to the board. In 1925 he was appointed a member of the Board of Control of the Safety Council of New Haven.

Of a deeply religious nature, Mr. Blair became a member of the Dwight Place Congregational Church when but eleven years old. In 1910 he was elected a member of the financial board of the church, becoming a trustee when the church was incorporated in 1920, and continuing to serve in that capacity until 1924 when he was made a deacon. He held this latter office until his death. From 1914 to 1918 he was associate superintendent of the Sunday school, and was superintendent from 1918 to 1919.

Carrying his belief in the wisdom of organized association into his non-professional life, Mr. Blair was a member and a past-president of the New Haven Rotary Club, of which he had also been a director; and he was a member of the New Haven Chamber of Commerce, of which he was a director from 1937 to 1939, inclusive. University conscious, he was chairman of the "'04 S" (Class of 1904, Sheffield Scientific School) Reunion Committee for both 1934 and 1939, and he was vice-president of his class. On the social side he was a member of the Graduates' Club of New Haven, and of the New Haven Country Club.

To the problems laid before him Mr. Blair brought a well-trained, analytical mind; his decisions and reports, made only after thoughtful and thorough study of all sides of the question, could be depended upon absolutely, and any construction entrusted to him was supervised with the same meticulous care. Unlike some other men of marked professional ability, not only was he widely recognized as an outstanding engineer, but he also held the high personal regard of all who were privileged to know and to work with him.

Returning from a brief trip into New Hampshire, on September 8, 1942, at Northfield, Mass., as he was about to enter his car, Mr. Blair slipped and fell, fracturing his hip. The break did not heal properly; several operations were performed; and following the fifth one he died on May 21, 1944. His death removed a fine Christian gentleman, a valuable citizen, and a most capable engineer, of whose life no better epitaph can be written than the two words: "Well done!"

On April 15, 1909, at New Haven, Mr. Blair was married to Fanny Belle Rand, the daughter of Stillman Nehemiah and Belle (Morrison) Rand, who survives him.

Mr Blair became an Associate Member of the American Society of Civil Engineers on December 5, 1911, and a Member on May 15, 1917.

ERIC EDWIN BLOMGREN, M. Am. Soc. C. E.[1]

Died October 11, 1944

Eric Edwin Blomgren, the son of John E. and Maria (Erickson) Blomgren, was born on August 21, 1889, in Norway, Mich. He received his early education in Norway and was graduated from the University of Michigan at Ann Arbor in June, 1916, with the degree of Bachelor of Civil Engineering. For two years, he studied at the graduate school of the University of Michigan, from which he received the degree of Master of Science in Civil Engineering in June, 1918.

Immediately after graduation, Mr. Blomgren became maintenance engineer for the Washtenaw County Good Roads District at Ann Arbor, and in January, 1919, he was appointed assistant engineer for the Wayne County

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[1] Memoir prepared by H. C. Coons, M. Am. Soc. C. E.

Road Commission in Detroit, Mich. In 1920 because of the illness of his father, a road contractor with headquarters in Norway, he resigned this position.

After his father's death, he was planning engineer for the Ford Motor Company in the "Upper Peninsula" district of Michigan from 1923 to 1926. In 1926 he became plant engineer for the Nash Motor Company in Milwaukee, Wis., and in 1930 he was employed as resident engineer for the Illinois Division of Highways on the Outer Drive in Chicago, Ill.

On July 1, 1933, Mr. Blomgren was appointed maintenance engineer for the Michigan State Highway Department and continued in that capacity until October, 1938, when he became district engineer of the southwest district of Michigan, comprising ten counties. He continued in this position until his death.

Mr. Blomgren was a man of high ideals and unquestionable integrity. His professional ethics were especially high, with the result that he was admired and respected by those with whom he came in contact.

Charles M. Ziegler, then state highway commissioner of Michigan, considered Mr. Blomgren one of his best engineers and an excellent representative in the field who had done outstanding work for the department as district engineer in the Kalamazoo (Mich.) office.

Mr. Blomgren was an active member of the First Presbyterian Church and the Lions Club of Kalamazoo. He was a past-president of the Kalamazoo Engineering Society and a director at the time of his death. He also belonged to the Michigan Engineering Society.

On June 23, 1917, he was married to Ruth Nord of Iron Mountain, Mich. He is survived by his widow; one daughter, Ruth (Mrs. Richard Publow); and two grandsons, Richard Henry Publow, Jr., and Peter Eric Publow.

Mr. Blomgren was elected a Member of the American Society of Civil Engineers on March 10, 1941.

SHERMAN WORCESTER BOWEN, M. Am. Soc. C. E.[1]

DIED AUGUST 19, 1944

Sherman Worcester Bowen, the son of Sherman Worcester and Julia Antoinette (Howard) Bowen, was born in Kimmswick, Jefferson County, Mo., on July 24, 1877. He received his elementary education at Bowen School in Jefferson County, which was named in memory of his father. He was graduated from the St. Louis (Mo.) Manual Training School in 1895 and from Washington University at St. Louis in 1901, with the degree of Bachelor of Science in Civil Engineering. He received the degree of Civil Engineer from Washington University in 1932.

During summers, while attending Washington University, Mr. Bowen worked as draftsman and inspector for the Koken Iron Works in St. Louis and for Brennecke and Fay, consulting engineers. He served with the latter firm from 1901 to 1914 as draftsman, designer, and engineer. During this period he was resident engineer on the St. Louis Municipal Bridge across the Mississippi River. He designed and supervised construction of four reinforced concrete viaducts in Fort Worth, Tex., investigated and reported on river terminals for the City of St. Louis, designed and reconstructed Compton Hill Reservoir in St. Louis, and designed the Robert Roy Bridge for the Cotton Belt Railroad Company. He also constructed various buildings and bridges as a contractor's engineer.

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[1] Memoir prepared by A. P. Greensfelder, M. Am. Soc. C. E.

From 1914 to 1916 he was office engineer and field engineer for the Department of Public Utilities of the City of St. Louis, supervising construction of an intake tower for the Water Department in the Mississippi River and studying a municipal dock at St. Louis. From 1916 to 1927 Mr. Bowen was engineer on construction for Fruin-Colnon Contracting Company, general constructors of St. Louis, on numerous industrial and commercial structures in the St. Louis metropolitan area. He specialized in foundations and structural work. In 1927 he was elected vice-president of Fruin-Colnon Contracting Company, which position he held until retirement a few weeks before his death.

On October 16, 1901, he was married to Edith Louise Holbert of Bloomington, Ill. He is survived by his widow and four children, Laura Antoinette (Mrs. Charles W. Crews); Sherman Worcester Bowen, III; Charles Holbert Bowen, Lieutenant (j.g.), U. S. Navy; and Howard Basford Bowen, Technical Sergeant, U. S. Army.

Mr. Bowen was a trustee of The First Unitarian Church of St. Louis, a Third Degree Mason of Rose Hill Lodge in St. Louis, and a member of the St. Louis Academy of Science, St. Louis Engineers' Club, and Royal Society of Arts, London (fellow). He was the author of various articles in the technical press—in the Society's *Transactions,*[2] *Engineering News,*[3] *Engineering Record,*[4] and *Civil Engineering*[5]—on work with which he was connected.

Mr. Bowen was elected an Associate Member of the American Society of Civil Engineers on September 7, 1904, and a Member on July 9, 1912.

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[2] "The Design and Construction of Four Reinforced Concrete Viaducts at Fort Worth, Texas," by S. W. Bowen, *Transactions,* Am. Soc. C. E., Vol. LXXVIII (1915), p. 1206; *ibid.,* Vol. LXXV (December, 1912), p. 353; and *ibid.,* Vol. LXXVII (December, 1914), p. 709.

[3] The Substructure of the Municipal Bridge over the Mississippi River at St. Louis, Mo.," by S. W. Bowen, *Engineering News,* Vol. 65, No. 11, March 16, 1911, p. 320; and "The St. Louis Municipal Bridge," by S. W. Bowen, *ibid.,* Vol. 67, No. 6, February, 8, 1912, p. 231.

[4] "Proposed Section for Heavy Compression Members" (an illustrated letter to the editor), by S. W. Bowen, *Engineering Record,* Vol. 58, No. 25, December 19, 1908, p. 707; and "Heavy Compression Members" (a letter to the editor), by S. W. Bowen, *ibid.,* Vol. 62, No. 27, December 31, 1910, p. 783.

[5] "Use of Caissons in St. Louis Sub-Soil," by S. W. Bowen, *Civil Engineering,* January, 1931, pp. 309–311.

SAM WIGFALL BRADSHAW, M. Am. Soc. C. E.[1]

Died January 20, 1945

Sam Wigfall Bradshaw was born on April 13, 1874, in Steelton, Pa. His parents, James Gibson Bradshaw and Sarah (Wigfall) Bradshaw, came from England.

After being educated in the Steelton public schools, he left high school one year before graduation to take a position as draftsman with the Bridge and Construction Department of the Pennsylvania Steel Company. From April, 1896, to July, 1897, he was employed by Milliken Brothers in Staten Island, N. Y., as a checker on structural steel drawings. He then returned to the Pennsylvania Steel Company as a drafting room squad leader, advancing to assistant engineer in the Estimating and Designing Department. In 1916 the Pennsylvania Steel Company was purchased by the Bethlehem Steel Company, and Mr. Bradshaw moved to Bethlehem, Pa. He was employed by the Bethlehem Steel Company as assistant engineer.

His was a strong character; and, despite the early termination of his formal schooling, Mr. Bradshaw was, for many years, a devoted student of engineering, particularly in the field of structures. He was closely associated with the designing and detailing of many important bridges, especially with the mechanical features of movable bridges and turntables. His accumulated notes and sketches on these topics were particularly valuable.

A Mason of long standing, he was a life member of the Blue Lodge, a member of the Knights Templar and the Mystic Shrine, and a Thirty-Second Degree Mason of the Scottish Rite. For years he took a very active part in freemasonry, especially in the dramatic consistorial degrees. He also was well versed in Masonic history.

Mr. Bradshaw was a man of broad and distinctly esthetic interests, a lover of flowers, a student of astronomy, and a violinist of real ability. In his younger days he was, for a number of years, an organist in the Protestant Episcopal Church, and participated in community musical ensembles. He was a vestryman of the church, and was versed in church history to an unusual degree for a layman. He was beloved and respected by his many associates in industry and in his community.

On June 22, 1899, in York, Pa., he was married to Etta A. Smith, of Steelton, who survives him.

Mr. Bradshaw was elected a Junior of the American Society of Civil Engineers on October 2, 1900; an Associate Member on February 4, 1903; and a Member on July 6, 1920. He became a Life Member in January, 1938.

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[1] Memoir prepared by Charles H. Mercer, M. Am. Soc. C. E.

BENJAMIN EDWARD BRIGGS, M. Am. Soc. C. E.[1]

DIED OCTOBER 1, 1944

Benjamin Edward Briggs was born at Sullivan, Ind., on July 26, 1860, the son of Murray and Margaret (Briggs) Briggs.

He received his education in the Indiana Public Schools and studied engineering with the International Correspondence Schools. His first engineering work was on a survey of the Wabash River bottom lands in Sullivan County, Indiana, in 1883. Mr. Briggs served as county surveyor for Sullivan County, and later as engineer for the Indiana and Illinois Southern Railway. His next engagement was with the Coronado Beach Company of San Diego, Calif.

In 1890 Mr. Briggs came to Erie, Pa., and served in the employ of the Pennsylvania Railroad Company. On November 1, 1893, he was appointed assistant city engineer of Erie, and three years later he was promoted to the position of city engineer. He served in that capacity until 1916, and he is credited with many municipal improvements, including the Buffalo Road Viaduct. After 1916 Mr. Briggs was engaged in private consulting practice and was active until a short time before his death.

Mr. Briggs was a highly respected and faithful public servant, who enjoyed the high regard and esteem of all who knew him. He maintained throughout his life the highest standards of conduct, understanding, tolerance, fellowship, human relations, and professional integrity, which have accrued, in his passing, as a legacy of credit to his profession.

On September 19, 1883, Mr. Briggs was married to Cora Saxton. Mrs. Briggs died on June 1, 1937, and in 1939 Mr. Briggs was married to Mrs. Vera Steinrock. He is survived by his widow and a daughter by his first marriage, Margaret (Mrs. Oliver Horton).

Mr. Briggs belonged to the American Society of Municipal Improvements, a national organization of city engineers, and was president in 1912. A founder of the Engineering Society of Northwestern Pennsylvania in 1898, he served as president for two years. He was a member of the Erie Chapter of the Pennsylvania Society of Professional Engineers and also of the Rotary Club of Erie.

Mr. Briggs was elected a Member of the American Society of Civil Engineers on November 4, 1908. He became a Life Member in January, 1933.

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[1] Memoir prepared by Clarence Pulling, Asst. City Engr., Erie, Pa.

ORRIN LAWRENCE BRODIE, M. Am. Soc. C E.[1]

DIED OCTOBER 5, 1943

Of high intellectual attainments, Orrin Lawrence Brodie combined those qualities of mind, heart, and hand that endeared him, beyond expression, to all with whom he came in contact.

Orrin Lawrence Brodie was born on July 5, 1879, in Pittsburgh, Pa., the son of Henry T. Brodie and Agnes (Bartberger) Brodie. He was educated in the public and high schools of that city. After preliminary surveying work around Pittsburgh, he attended Columbia University in New York, N. Y., from which he was graduated with the degree of Civil Engineer in 1901. He was known to his classmates as "Steve." Following his graduation, he taught surveying and geodesy at Columbia. He then worked for the New York Central and Hudson River Railroad as inspector of retaining walls and bridge abutments; and, later, on the construction of the Pennsylvania Railroad Tunnels under the Hudson River leading into New York. From 1903 to 1906, he was engineer in immediate charge of repaving streets in the Borough of Richmond.

With the New York City Board of Water Supply from 1906 to 1914, Mr. Brodie directed investigations and designs, prepared contract and working drawings and specifications, and conducted special structural and hydraulic studies, all connected with the Catskill Water Supply System.

Mr. Brodie served three years as associate efficiency engineer with the Board of Estimate and Apportionment and Commissioner of Accounts of the City of New York. For one year he was assistant to the Consulting Engineer of the Borough of Manhattan. In World War I, he was engineer for the government nitrate plant at Muscle Shoals, Ala.

From 1919 to 1927, as assistant designing engineer with the Holland Tunnel organization, Mr. Brodie originated highly important methods of tunnel ring stress determination[2] and silt pressure investigations. He returned to the Board of Water Supply in November, 1927, as designing engineer, which position he held at the time of his death. In addition to his routine designing supervision, Mr. Brodie completed the Grant City model studies concerned with the diversion tunnel of the Rondout River at the Merriman Dam, investigations on caisson sinking, and many other studies. He assisted Charles M. Allen, M. Am. Soc. C. E., with the tests at Worcester Polytechnic Institute in Worcester, Mass., for the Rondout stilling pool.

In 1910 Mr. Brodie was the co-author of a well-known technical book, "High Masonry Dam Design."[3] The second edition of this book appeared in 1916 under the title "Masonry Dam Design Including High Masonry Dams." At times, Mr. Brodie was lecturer and instructor in "extramural courses" of the College of the City of New York on "Masonry Construction and Design" and was, also, an instructor at New York University. At other times, he designed dams and reservoirs for private consultants.

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[1] Memoir prepared by Thomas K. A. Hendrick, Assoc. M. Am. Soc. C. E.

[2] *Transactions*, Am. Soc. C. E., Vol. 109 (1944), p. 737.

[3] "High Masonry Dam Design," by Charles E. Morrison and Orrin L. Brodie, John Wiley & Sons, Inc., 1st Ed., New York, N. Y., 1910.

In the military sphere, Mr. Brodie was a captain, commanding the Third Company, Ninth C. A. C., New York Guard, until placed on the officers retired list. His company of the State Guard protected the Catskill Aqueduct during World War I.

Among Mr. Brodie's hobbies were: An unusually complete workshop in his home, archery, automobiles, and people—his interest in people knew no bounds. Each associate was to him a living entity with aspirations for mutual cooperation. He served as assistant minister at the Protestant Episcopal Church of the Ascension, Staten Island, New York. Also, Mr. Brodie was Past Master of the Richmond Lodge of Free and Accepted Masons. Many people from near and far sought his advice and counsel. He lived the maxim of never causing pain to any one. A guiding quotation, he often mentioned, was: "Before a conscience can be quickened or a will inspired, the mind must know." He placed a high value on his membership in the Society. He also belonged to the Association of Engineers of New York, Inc., and The Society of American Military Engineers; and he was an original subscriber and charter member of the New York Society of Military and Naval Officers, World War I. He was a pre-eminent exemplar of technical and professional efficiency in the civil service, as attested by the following editorial published on October 8, 1943, in the *Staten Island Advance:*

EMINENT PUBLIC SERVANT

> "Mr. Orrin L. Brodie, who died recently at his home in Westerleigh, was an outstanding illustration of the civil service employe who performs a role of profound public importance behind the cloak of comparative obscurity which hides the operations of the entire civil service force.
>
> "His name might never have become widely known even on the Island beyond the confines of his own field of professional activity if a story in this newspaper several years ago had not described a series of tests which he conducted in the Department of Water Supply, Gas and Electricity station at Grant City.
>
> "Mr. Brodie, a designing engineer with the city's Board of Water Supply, built an exact scale model of a multi-million-dollar diversion tunnel and reservoir spillway which his board was planning to construct upstate in connection with the vast municipal water supply expansion program. The model was correct to the fraction of an inch in every particular; it was the prototype of the projected tunnel and spillway in every detail, and was used in experiments to determine how various features of the actual project would function under all manner of conditions. By experimenting with the model, Mr. Brodie and his staff were able to decide the best method of designing the ultimate improvement. Incidentally, they saved the taxpayers a considerable sum of money through what they learned in the tests; they produced the most satisfactory final blueprints that could be devised; and they gained some new scientific data which had not been known before.
>
> "Within his own professional circle, Mr. Brodie was highly respected for his accomplishments. These accomplishments—all of them in the public welfare—were many and diverse.

> "He assisted in planning the Ashokan dam in upper New York State and the Muscle Shoals dam in Alabama, where he was stationed during World War I. He worked with the Port of New York Authority on the Holland Tunnel plans, and between times held key assignments on other major civic works.
>
> "Through the years he did all this without glorification or acclaim.
>
> "He was an eminent public servant, desiring neither glory nor acclaim for his unremitting efforts."

His sparkling brown eyes bespoke a mental activity of a steel trap variety that was equaled only by his genuine interest and enthusiasm in his fellow Americans without regard to race, color, or creed.

In addition to the inspiration which he gave so generously, Mr. Brodie was most interested in and cordial to his friends and associates. He must know, now, in spirit, that he is enshrined in their hearts.

On October 3, 1905, he was married to Eleanor Van Cott of an historic New York City family. He is survived by his widow; four daughters, Janet (Mrs. Charles F. Flint), Agnes (Mrs. Eduard F. Von Wettberg), Margaret (Mrs. James J. Broderick), and Eleanor (Mrs. Charles McKirdy); seven grandchildren, Eleanor Thayer Flint, Janet Crawford Von Wettberg, Eduard F. Von Wettberg III, Lawrence Orrin Broderick, James Michael Broderick, Agnes Brodie McKirdy, and Charles Robert McKirdy; and one sister, Althaea (Mrs. Martin G. Kindlund).

Mr. Brodie was elected an Associate Member of the American Society of Civil Engineers on July 10, 1907, and a Member on September 3, 1913.

ALBERT MARSHALL BROSIUS, M. Am. Soc. C. E.[1]

DIED OCTOBER 9, 1944

Albert Marshall Brosius, the son of Samuel Hartmann Brosius and Laura (Stephens) Brosius, was born at Baltimore, Md., on July 4, 1869. He was a descendant of John Abraham Brosius of Zweibrücken, Germany, who, on September 24, 1742, arrived in Philadelphia, Pa., on the ship *Robert and Alice.*

From 1885 to 1887 he studied mechanical engineering at Purdue University in Lafayette, Ind.

Until 1905 Mr. Brosius was engaged as draftsman and assistant engineer, on surveys and construction for railroads, irrigation projects, cable and electric street railways, sewers, real estate developments, and subways in California, Georgia, Maryland, Washington, D. C., and New York. He was in charge of sewer design and construction for the City of Baltimore from August, 1898, to March, 1900, and in charge of design and construction of sewers and subsurface structures for Jacobs and Davies on the Hudson River tunnels (New York, N. Y.) from April to December, 1905.

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[1] Memoir prepared by John W. Van Denburg and Alfred M. Wyman, Members, Am. Soc. C. E.

In January, 1906, Mr. Brosius joined the engineering force of the Sewerage Commission of Baltimore as first assistant engineer. At times he served as acting chief engineer. In 1912 he was appointed chief engineer of the Havana Sewerage and Paving Contract in Havana, Cuba. He remained in Havana until 1914.

For two years he practiced privately in Los Angeles, Calif., during which time he conducted some of the early experiments on mechanical aeration of activated sludge. In 1918 he became superintendent of transportation and materials for the Degnon Contracting Company on the construction of the large depot and ordnance warehouse at Charleston, S. C., for the U. S. Quartermaster Department. From 1919 to 1921, he was senior engineer for the United States Shipping Board, Emergency Fleet Corporation, Washington, D. C. In 1921 he resumed sanitary engineering, in which he was deeply interested, and spent several years with Fuller and McClintock in New York City. He then went to Guatemala for the J. G. White Engineering Corporation to aid in the preparation of a comprehensive report on water supply, sewerage and irrigation, hydroelectric development, and electric railways.

In 1928 Mr. Brosius was employed by the Westchester County Sanitary Commission in Westchester County, New York, where he was engaged in the selection of the general layout of the extensive trunk sewer system for the southern part of that county and on various phases of its design. Subsequently, from 1932 to 1939, as designing engineer with the Department of Public Works of New York City, he took part in the development of a comprehensive plan of sewerage collection and treatment—one of the major projects of its kind in the United States. From 1940 until his death he was in private practice as a consulting engineer on civil and sanitary projects in New York City.

Mr. Brosius commanded the respect of his associates by his wide and detailed knowledge of his profession. He will always be remembered by his friends for his unfailing energy and courtesy and his ready cheerfulness under even the most trying circumstances. He was a regular attendant of St. Bartholomew's Episcopal Church in White Plains, N. Y.

On April 28, 1903, he was married to Edith Helen Coakley, who, with a son, Albert Marshall Brosius, Jr., survives him.

Mr. Brosius was elected a Member of the American Society of Civil Engineers on March 1, 1910.

FREDERICK CHARLES CARSTARPHEN, M. Am. Soc. C. E.[1]

DIED JANUARY 8, 1944

In the death of Mr. Carstarphen the State of Colorado and the City of Denver lost a wise and cooperative leader and counselor; the engineering profession, a valuable member; and society, a respected and loved friend.

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[1] Memoir prepared by John E. Field and M. C. Hinderlider, Members, Am. Soc. C. E.

Mr. Carstarphen, a native son of Colorado, was born in Denver on April 1, 1881. He attended grade school, and the old East Denver High School at 19th Street and California Street from which he was graduated in 1900. He was graduated from the Colorado School of Mines at Golden in 1905. He received the degree of Doctor of Engineering from the School of Mines in 1932 for his outstanding contributions and achievements in the mining field, and for promoting technical and scientific treatment of engineering problems.

Dr. Carstarphen was of pioneer stock. Among his ancestors was Charles Carroll, of Carrollton, Md., a signer of the Declaration of Independence. His mother, Annie Josephine Carstarphen, was a Carroll and came as a little girl to Central City, Colo. His father, H. W. Carstarphen, was originally from Hannibal, Mo., the home of Mark Twain. His father first went to Leadville, Colo., in 1879, working on numerous mining town newspapers before he moved to Denver, where he has been for many years on the staff of the *Rocky Mountain News,* the oldest newspaper in Colorado.

Throughout Dr. Carstarphen's professional career new and difficult engineering problems interested him, and he contributed widely to their discussion and mathematical solution not only in mining but also in general engineering. A keen analyst of engineering problems, he was the author of many brochures and articles on technical subjects. His ability with higher mathematics was such that fellow engineers sometimes brought him their problems on which he worked for the sheer pleasure of conquering them. He took a leading part in local activities of engineering groups. At the outbreak of World War II, Dr. Carstarphen was appointed local chairman of the Society's National Committee on Civilian Protection in War Time.

As early as 1900 young Carstarphen worked and experimented on aeronautical problems and phenomena, becoming a kite expert—especially on the lifting power and construction of the box kite to obtain high efficiency. Although only nineteen years old, he achieved such a local reputation that he was employed by one of the Denver newspapers to travel around Colorado and show youngsters how to build and fly a kite.

He was an expert in the designing of reinforced concrete. Among the structures constructed by Dr. Carstarphen is the Ideal Building in Denver (1908)—the first reinforced concrete building in the city. He also designed the outlet and spillways of the Kenwood Dam on Cherry Creek (1934), accepted as one of the best designed and safest dams in Colorado. This dam was constructed to protect Denver against floods.

If Dr. Carstarphen had a specialty, it was the design of aerial tramways. From 1913 to 1923 he was chief aerial tramway engineer for the American Steel and Wire Company of Trenton, N. J. He was engineer for the Colorado Portland Cement Company for some years prior to going to Trenton. From 1923 to 1926 he was vice-president and chief engineer of the Manufacturers Selling Company, at Trenton. He was, at times, consulting engineer to the Bureau of Reclamation of the State of Colorado and to the City of Denver.

A life member of the Colorado Section of the Society, he was also president of the Colorado Society of Engineers (member of the board for eight years). In addition to these engineering societies, he belonged to the American Association for the Advancement of Science, and the American Institute of Mining Engineers (life member). Dr. Carstarphen was a Thirty-Second Degree Mason and a Shriner. Active in civic affairs, he was a recognized leader in promoting the development of the resources of Colorado, not only in the mining field, but also in irrigation farming and related industries.

A detailed list of Dr. Carstarphen's work may be found in *Who's Who in Engineering* for 1921.

He was married to Kate Fullerton in Denver, Colo., on June 23, 1908. Mrs. Carstarphen died in April, 1943. Surviving him are his father; an uncle, W. P. Carstarphen; a brother, Carroll W. Carstarphen; and his four children: Catherine (Mrs. Wells), Charles (Lieutenant, U. S. Navy), Florence, and Marion.

Dr. Carstarphen's outstanding characteristic was neither his technical knowledge nor his ability to meet new problems in a practical way, but his teaching ability—his desire to pass along to others his knowledge and his experience. He was essentially a teacher who delighted in his work and in helping others. The young engineer found in him a wise counselor, and one who was able to impart his knowledge in terms within the understanding of younger and less experienced men. He not only taught and helped young men but also was always available to discuss and to analyze the problems of engineers who had grown rusty, or who had not kept up with engineering progress. This was especially true in the case of higher mathematics when the latest developments were pertinent. He was a man who never lost his sense of humor, and never violated the admonition of "Don't take yourself too seriously." In his contacts he met every one with dignity, respect, and consideration, he never criticized the man, nor questioned a man's motives. Although he criticized and discussed projects, theories, and conclusions, personalities never entered the discussion. His criticisms were constructive and analytical.

Dr. Carstarphen lived comfortably and well and was content and appreciative of his opportunities and rewards, with the result that he did not acquire the wealth which he might have if he had devoted more thought to his own financial advantage. He raised and educated four children, a boy and three girls, in the best American traditions—responsibility, self-respect, and duty. His children and the many young men whom he influenced are the greatest and most enduring monuments to a life well spent.

Dr. Carstarphen was elected an Associate Member of the American Society of Civil Engineers on April 6, 1909, and a Member on May 15, 1917.

CHARLES ROBERT FORAN COUTLEE, M. Am. Soc. C. E.[1]

DIED JANUARY 1, 1944

Charles Robert Foran Coutlee was born at "Oakland Lodge," near Aylmer, Que., Canada, on January 29, 1867, the son of Maj. Louis Maurille Coutlee, sheriff of the District of Ottawa, Ont., Canada, and Jane Maria (Clegg) Coutlee. He was a descendant of Louis Coutlée, or Coutelet, who, as an ex-soldier of France, emigrated to Canada in the sixteenth century. He received his primary education at Aylmer, and, after being graduated from the Ottawa Collegiate Institute, he entered the Royal Military College of Canada, at Kingston, Ont., Canada, receiving a diploma from that institution in 1886.

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[1] Memoir prepared by K. M. Cameron, Chf. Engr., Dept. of Public Works of Canada, Ottawa, Ont., Canada.

From 1886 to 1889 he worked for the Canadian Pacific Railway Company as rodman and as assistant engineer on the company's "Short Line," through Maine and on the Crow's Nest Pass tunnel near Fernie, B. C., Canada. He entered the engineering service of the Canadian Department of Railways and Canals in 1889, as leveler on the Soulanges Canal, and from 1892 to 1901 was assistant engineer in charge of the construction of the Cascades Division which included four lift locks with weirs, bridges, and heavy rock and earth excavation. During this period he examined and reported on the proposed canalization of the French River in Ontario, Canada.

When the Soulanges Canal was completed, Mr. Coutlee became a consulting engineer in Vancouver, B. C., Canada—on coal mine surveys, irrigation projects, and railway location near Frank, Alta., Canada (the scene of the disastrous slide which practically obliterated that community). He was also an expert adviser on several court actions. In 1904 the Province of Nova Scotia, Canada, embarking on a program of highway improvment and development, engaged Mr. Coutlee as technical adviser. His report formed the basis of the subsequent provincial highway system.

For many years, the project of a ship canal to connect Montreal, Que., Canada, on the St. Lawrence River with the Great Lakes system, via the Ottawa and French rivers, following the historic route of the early voyagers (of whom Samuel de Champlain is the most notable) had been discussed. The Government of Canada decided on a complete investigation of the possibilities of this scheme, and Mr. Coutlee was assigned to the eastern section (some 140 miles long) from Montreal to Ottawa. After the survey of the Ottawa-Des Joachims section, under E. J. Rainboth, was completed and Mr. Rainboth had resigned, that section (140 miles long) was combined with the Montreal-Ottawa section for final planning under Mr. Coutlee's direction. The result of the investigation is contained in the "Georgian Bay Ship Canal Report, 1908." Although the canal project was not completed, the report, data, and plans have been consulted continually with reference to power developments and related matters. The Georgian Bay Survey was conducted by the Department of Public Works, with which department Mr. Coutlee remained until his retirement in 1932. After the completion of the Georgian Bay Ship Canal Survey, Mr. Coutlee continued to work on surveys and compilation of data for the water resources of the Ottawa River watershed. In this connection, he also studied the possible economic utilization of the watershed, and had responsible charge of the construction and operation of the regulating dams built on that river by the Department of Public Works.

Because of his wide experience and sound judgment, he became counselor to the successive heads of the Engineering Service in the Department of Public Works. Mr. Coutlee cultivated a natural aptitude for observation and deduction, as well as the ability to express himself clearly and concisely in both conversation and writing. An outstanding raconteur, his friendship and companionship were greatly valued. His many acts of charity and kindness sprang from deep-seated personal feeling.

In September, 1895, Mr. Coutlee was married to Daisy Pringle, who predeceased him. Later he was married to Beatrice Maude Fleming, who survives him.

He joined the Canadian Society of Civil Engineers as a student member in 1888, the year after the formation of that society, and was the first chairman of its Ottawa Branch. He was a member of the Astronomical Society of Canada and a past-president of the Royal Military College Club.

Mr. Coutlee was elected a Member of the American Society of Civil Engineers on March 1, 1905.

JAMES COWIN, M. Am. Soc. C. E.[1]

DIED DECEMBER 17, 1943

James Cowin was born in Adrian, Minn., on June 13, 1884, the son of James and Rhoda C. (Moberly) Cowin. He attended the School of Mines of the University of Minnesota at Minneapolis, and was graduated with the degree of Mining Engineer in 1907. He continued his studies at Stanford University in Stanford University, Calif., for a short time. In 1916, when he returned to Stanford for special work, he was given the degree of Civil Engineer. He was a member of the Chi Psi fraternity.

In the summer of 1907, Mr. Cowin was in charge of a party on geological survey for coal outcrops in Utah. In the fall of 1908 he went to Mexico and was cyanide foreman for the Real Del Monte y Pachuca Mining Company at Pachuca, Mexico. For a short time he served as engineer for Spurr and Cox, Consulting Engineers of Mexico City, Mexico.

In 1909 Mr. Cowin entered the office of C. A. P. Turner, M. Am. Soc. C. E., reinforced concrete engineer of Minneapolis, as draftsman and designing engineer. In 1910 he opened an office in Winnipeg, Manitoba, Canada, for Mr. Turner, of which he was local manager until 1913. In that year he was transferred to Houston, Tex., as local manager. In 1914 he organized and was president of the company of Cowin and Fee, Ltd. (later Cowin and Company, Ltd.) in Winnipeg. The company specialized in reinforced concrete engineering. In 1915 Mr. Cowin returned to Minneapolis and founded the engineering firm of Cowin and Company, Inc., of Minneapolis.

After the United States entered World War I, he went to St. Louis, Mo., as chief engineer at the St. Louis Clothing Depot for the U. S. Army. In 1918 he returned home and formed the Indian Bolt and Nut Company as an affiliate of Cowin and Company, Inc. This company was later sold. He also incorporated the Steel Service, Inc., of San Francisco, Calif., for warehousing reinforcing steel. He sold his interest in this company in 1929. His engineering companies both in Canada and the United States prepared plans for many notable structures in all of which Mr. Cowin took a personal interest.

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[1] Memoir prepared by H. B. Henderson, M. Am. Soc. C. E.

Mr. Cowin had vision and an inventive mind. He built up Cowin and Company, Inc., from a small company devoted exclusively to the engineering and fabrication of concrete reinforcement into a successful manufacturing firm with many diverse products. The company's output included unique, modern types of steel storage buildings, corn cribs, and other farm equipment designed in steel by Mr. Cowin.

After the entry of the United States into World War II, Cowin and Company, Inc., turned to the production of steel products for war use. Difficult forgings, a special flexible bending die for shaping steel angle frames for oil tankers, and many other ingenious and difficult operations were developed by Mr. Cowin to aid the war effort. Most of the company's facilities were engaged in war work at the time of his death.

He was an associate editor of "Reinforced Concrete and Masonry Structures,"[2] writing the chapter on "Finishing Concrete Surfaces."

Mr. Cowin was a Christian gentleman, quiet and unassuming. His many kindly acts were done unobtrusively. He was a good friend and intensely loyal. Although he was modest, he was determined and persistent in accomplishing anything he had made up his mind to do.

Mr. Cowin was a member of the Minneapolis Club, the University Club of St. Paul, Minn., the Engineers Club of San Francisco, and Minnesota Engineers Club. He belonged to the American Institute of Mining and Metallurgical Engineers. He was a member of Westminster Presbyterian Church of Minneapolis.

Mr. Cowin was married on February 28, 1912, to Lorilla Martha Rice of Minneapolis, who died in 1928. Two children, Lorilla Mary (Mrs. Elwood Molander) and James, Jr., Lieutenant in the U. S. Army Air Force, survive him. On September 24, 1936, Mr. Cowin was married to Marion Parsons of Fergus Falls, Minn., who also survives him. Other survivors are a sister, Ethelwyn (Mrs. H. Morton); and three brothers, Alton, Percy, and Clifford.

Mr. Cowin was elected an Associate Member of the American Society of Civil Engineers on March 14, 1916, and a Member on June 2, 1920.

LEONARD MARTIN COX, M. Am. Soc. C. E.[1]

DIED DECEMBER 12, 1943

One of the Civil Engineers whose service in the Navy was intimately associated with the early development of the steel floating dry dock died at the age of seventy three, at Saint Helena, Calif. His active professional career as an officer in the Civil Engineer Corps of the Navy covered the period from 1899 to 1923.

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[2] "Reinforced Concrete and Masonry Structures," G. H. Hool and W. S. Kinne, Editors-in-Chief, 1st Ed., McGraw-Hill Book Co., Inc., New York, N. Y., 1924, p. 132.

[1] Memoir prepared by C. A. Carlson, M. Am. Soc. C. E.

Leonard Martin Cox was born in New Liberty, Ky., on March 21, 1870, the son of Attilla and Kate (Martin) Cox, members of a pioneer Kentucky bluegrass family.

After attending the public schools of Louisville, Ky., and being graduated from the Louisville High School, Mr. Cox entered Rensselaer Polytechnic Institute in Troy, N. Y., from which he was graduated in 1892 with the degree of Civil Engineer. While at college he was elected to membership in the Theta Delta Chi fraternity.

From 1892 to 1895 Mr. Cox was employed as an assistant engineer on railroad work, bridge erection, and maintenance of way by the Louisville, Henderson and St. Louis Railway Company. From 1895 to 1899 he was with a power and manufacturing plant in Louisville; served one year as professor of chemistry at the Louisville Male High School; and, from June, 1898, to January, 1899, was assistant engineer for the Louisville and Nashville Railroad Company on the Nashville (Tenn.) terminals.

His naval career began as the result of an open competition examination, whereby he won a commission as Assistant Civil Engineer, U. S. Navy, with the rank of Lieutenant (j.g.), in January, 1899. He was advanced through the succeeding grades up to that of Captain prior to his retirement in 1923.

His first duty was at the New York (N. Y.) Navy Yard under one of the senior Civil Engineers of the Navy, Peter C. Asserson,[2] M. Am. Soc. C. E., then Captain (C.E.C.), U. S. Navy, and Head of the Yards and Docks Department of the yard. At that time, the reconstruction of Dry Dock No. 2 was the major project.

From 1901 to 1903 Lieutenant Cox served as Head of the Department of Yards and Docks under the Governor of Guam. Probably one of the earliest public works undertaken by the United States in its new capacity in colonial government was the dam and storage reservoir built by the young engineer to furnish a water supply to the native town of Agaña, also the seat of the naval government. His report on the Island of Guam was later published as a public document.

From July, 1903, to February, 1906, Lieutenant Cox served as Supervising Civil Engineer in charge of construction of the floating dry dock *Dewey,* then building at Sparrows Point, Md. This duty, combined with his earlier experience at the New York Navy Yard on the construction of masonry graving docks, led him to a study and investigation of the comparative merits of graving docks versus floating docks for naval purposes. Upon completion, the *Dewey* was towed to Solomons Island, Maryland, where sufficient depth of water permitted complete tests to be conducted, including the docking of the battleships *Colorado* and *Iowa,* and the complete cycle of self-docking operations during the month of July, 1905. The *Dewey,* having completed satisfactory contract requirements, was accepted by the Navy Department.

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[2] For memoir, see *Transactions,* Am. Soc. C. E., Vol. 96 (1932), p. 1397.

Civil Engineer Cox resigned from the Navy in February, 1906, in order to assist his father, who was ill, by serving as chief engineer of the Louisville, Henderson and St. Louis Railway Company.

At the December, 1906, meeting of the Society, Mr. Cox presented a paper on "The Naval Floating Dock—Its Advantages, Design and Construction."[3] This paper was judged worthy of special commendation for its merit as a contribution to engineering science, and the author was awarded the Norman Medal by the Society in 1907.

In April, 1907, by special act of Congress, Mr. Cox was reinstated in the Navy as a Civil Engineer, with the rank of Lieutenant. In the period between 1907 and 1909, he served at the Norfolk (Va.) and the New York navy yards, and at the latter yard was placed in immediate charge of the construction of Dry Dock No. 4 by direction of the Chief of the Bureau of Yards and Docks.

From 1909 to 1911, Lieutenant Cox was assistant to the Chief of the Bureau of Yards and Docks, Richard C. Hollyday,[4] M. Am. Soc. C. E., then Rear Admiral (C.E.C.), U. S. Navy, serving as Executive Officer, and also in immediate charge of design of maritime works at a time when planning for future dry dock construction was being given much consideration. He was one of the senior Civil Engineers who was called into consultation by the Chief of the Bureau of Yards and Docks on the solution of the foundation problems encountered in the building of the Pearl Harbor (Hawaii) Dry Dock, and is credited with contributing materially to the final plans under which the dry dock was successfully built.[5]

From September, 1911, to January, 1915, Lieutenant-Commander Cox was Public Works Officer at Norfolk Navy Yard, in complete charge of public works, including the extension of Dry Dock No. 3. During this tour of duty, he was appointed by the Hon. William Howard Taft, then President of the United States, as a member of the First Alaska Railroad Commission, of which J. J. Morrow,[6] M. Am. Soc. C. E., then Colonel, Corps of Engineers, U. S. Army, was president. The Commission traveled some 13,000 miles, investigating various railroad sites in Alaska, and submitted a report which was published as a public document and formed the basis for subsequent preparation for that work. In June, 1912, Lieutenant-Commander Cox was appointed Navy delegate to the International Navigation Congress, held at Philadelphia, Pa.

From January, 1915, to October, 1916, he served as Public Works Officer, New York Navy Yard, in general charge of public works construction, including construction of piers and a new layout of yard for battleship construction, as well as of constructing the first modern shore establishment supply buildings.

From October, 1916, to February, 1917, under special orders of the Navy Department, Commander Cox investigated all available sites for naval bases in the West Indies, making examinations of the various available harbors of the islands of Cuba, Haiti, Santo Domingo, and the St. Thomas group; also, he investigated and reported on possible sources for a water supply to the naval base at Guantánamo, Cuba.

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[3] "The Naval Floating Dock—Its Advantages, Design and Construction," by Leonard M. Cox, *Transactions*, Am. Soc. C. E., Vol. LVIII (June, 1907), p. 97.

[4] For memoir, *ibid.*, Vol. 102 (1937), p. 1564.

[5] "Pearl Harbor Dock," by H. R. Stanford, *ibid.*, Vol. LXXX (December, 1916), p. 223.

[6] For memoir, *ibid.*, Vol. 103 (1938), p. 1846.

From September, 1917, to November, 1918, he was Public Works Officer of what was later the Eleventh Naval District, where he initiated layouts, design, and construction of naval improvements at San Diego and San Pedro, Calif., including the Naval Air Station on North Island; the Marine Expeditionary Base, Training Camp, and preliminary studies for the new Naval Hospital in San Diego. From December, 1918, to June, 1919, he was assistant manager of Division Shipyard Plants, Emergency Fleet Corporation in Philadelphia.

From June, 1919, to May, 1923, Captain Cox served as Public Works Officer at the Mare Island (Calif.) Navy Yard charged with all public works and investigation of naval base sites on San Francisco (Calif.) Bay, as well as the project for the conservation of the Mare Island Navy Yard as a docking and repair base for capital ships.

He was a member of the Engineering Institute of Canada; the Army and Navy Club in Washington, D. C.; and the Engineers Club, the Commercial Club, and the Commonwealth Club of San Francisco.

Captain Cox retired from naval service in May, 1923, due to physical disability. After his retirement, he lived in Vallejo, Calif., where he became interested in civic affairs, and was one of the founders and directors of the Mechanics and Merchants' National Bank. In June, 1927, he became co-publisher with Luther E. Gibson of the *Vallejo Evening Chronicle*. In 1932 he gave up active interest in the newspaper and built a home called "Malvern Woods," just north of Saint Helena to secure some relief from acute attacks of asthma.

Throughout his career, Captain Cox was most keenly interested in plans to augment major shore facilities of the Navy such as dry docks, building ways, quay walls, piers, power plants, and transportation systems, so that the navy yards would be able to meet the needs of the fleet in time of war. His early studies and experience with floating docks brought him into contact with some of the leading civil engineers of England, such as L. E. Clark and Charles Standfield, under whose design the New Orleans (La.) or Algiers (La.) floating dock had previously been built. His paper on floating docks entered fully into the design and construction of such docks, and presented a detailed analysis for the dry dock *Dewey* with respect to loadings, stability, strength, and deflection calculations. This paper received serious consideration from other floating dock designers and led to interesting discussions. The merit of floating docks as compared with graving docks from a naval standpoint, on account of their mobility, was advanced. Civil Engineer Cox agreed with the idea that a floating dock could be used as a part of a repair station that could be taken to locations most accessible to the fleet in time of war. Such a project had previously been suggested by A. C. Cunningham,[7] M. Am. Soc. C. E., then Civil Engineer, U. S. Navy, in an article entitled "The Movable Base."[8] These two officers, having been closely associated in the design and construction of the floating dry docks, were no doubt in a large measure responsible for bringing to the general attention of naval officers, and the service at large, the military value of the floating dock as a naval auxiliary for service with the fleet.

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[7] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXI (December, 1917), p. 1691.

[8] *U. S. Naval Institute*, March, 1904, p. 181.

Captain Cox, through his duty at various stations and his contact with officers of ability in the line and staff corps, was recognized as being a man who was broadly interested in the Navy as a whole and a man whose professional ability was outstanding. His duties at times necessarily required him to present comments and recommendations not only on strictly professional problems, but on matters most pertinent to the administration of the public works activities, both at naval stations and in the Bureau of Yards and Docks. Even though the question might be highly controversial, his personality and integrity were such that his views were given the fullest and most serious consideration. Undoubtedly, the soundness of his forward looking plans as to measures for the advancement of the Navy in respect to its shore bases has been reflected in developments later applied to the logistic phases of World War II.

Captain Cox was held in high esteem by his superiors and his immediate staff, and inspired in his associates a zeal to carry out the task at hand. He left many friends among the workmen of Mare Island and citizens of Vallejo, and, although his death occurred twenty years after his retirement from the Navy, many were the sincere expressions of sympathy and appreciations of his high character and accomplishments.

On October 16, 1895, Leonard M. Cox was married to Jane Castleman, of Louisville. He is survived by his widow and a daughter, Katherine Castleman Cox.

Captain Cox was elected an Associate Member of the American Society of Civil Engineers on October 4, 1899, and a Member on January 5, 1904.

CLARE HARMON CURRIE, M. Am. Soc. C. E.[1]

Died September 17, 1942

Clare Harmon Currie was born in Webster City, Iowa, on May 15, 1882, the son of Frank S. and Sarah (Risdon) Currie. He was educated in the Webster City public schools and was graduated from the local high school in 1900. He entered Iowa State College at Ames and was graduated in 1905 with the degree of Bachelor of Science in Civil Engineering. While in college, he excelled in track and maintained a keen interest in Iowa State College athletics throughout his lifetime.

In 1906 and 1907 Mr. Currie was with the Oregon Short Line Railroad Company on railroad location, where he received his pioneer training in the wild west country of Oregon, Utah, and Wyoming. Immediately after graduation he had worked in Salt Lake City, Utah, for a paving and water works contractor and later for the city.

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[1] Memoir prepared by Anson Marston, Past-President and Hon. M. Am. Soc. C. E., and Fred H. Austin and J. S. Dodds, Members, Am. Soc. C. E.

The county drainage program was beginning in Iowa about that time, and in the fall of 1907 Mr. Currie left railroading and returned to Hamilton County, Iowa, where he was county drainage engineer for nearly two years. Then, in the summer of 1909, he became city engineer of Webster City in charge of a new paving program. He also continued as consulting engineer on land drainage matters. Between 1911 and 1919 he was a consultant on municipal engineering projects.

In 1919 Mr. Currie organized the Currie Engineering Company, which was active, until his death, in the municipal engineering field in Iowa, Minnesota, Illinois, Arizona, and California. Although Mr. Currie specialized in sanitary engineering and designed many very successful sewage disposal plants, he also was interested in paving, water works, land drainage, and valuation projects in the area served by his firm. He was a pioneer in the beautification of municipal sewage disposal plants.

One of his last important assignments in Iowa was on the Okoboji-Spirit Lake area sewage disposal improvement for the state. Earlier Iowa sewage disposal installations by Mr. Currie are at Boone, Clear Lake, Clarion, Creston, Decorah, Iowa City, Osage, Storm Lake, and Webster City. During this time he was also consulting engineer on water plants at Eagle Grove, Hamburg, and Avoca, Iowa. Mr. Currie applied many newly discovered principles to his sanitary engineering designs. At Mason City, Iowa, Mr. Currie, working with the Iowa Engineering Experiment Station at Ames, developed a very successful double filtration sewage disposal plant for the effluent from the Decker Packing Plant, thus solving a most unsatisfactory stream pollution problem.

The Webster City sewage plant, however, was his pride and joy. The last installation which he had designed and directed personally, he considered it to be of the finest and most modern construction and regarded it as a monument, both to himself and the city.

The Currie firm had an office at San Bernardino, Calif., directed by Mr. Currie's brother, Frank S. Currie, M. Am. Soc. C. E. In California some beautiful sewage disposal plants were designed for various towns, including Colton, El Monte, Huntington Beach, Laguna Beach, Newport Beach, Oxnard, Pasadena, Redlands, San Bernardino, San Clemente, Seal Beach, and Visalia. These sewage plants serving beach cities involved interesting features such as underwater outlets into the ocean, tank skimming by compressed air, and controlled ventilation. Every effort was made to create attractive plants.

An active and valuable member of the Iowa State Board of Engineering Examiners for twelve years, Mr. Currie belonged to the Iowa Engineering Society and was president of the Iowa Section of the Society. He helped to organize the Iowa Sewage Works Conference and was a member of the American Water Works Association.

Mr. Currie had a lovable nature and was devoted to his family and his engineering affiliations. His acquaintances included prominent men in all walks of life and in all parts of the United States. A genial companion, he was ever ready with a good story to illustrate his conversation.

In 1910, at Fairmont, Minn., he was married to Jessie Wohlhuter. Besides his widow, he is survived by two sons, David, Assoc. M. Am. Soc. C. E., First Lieutenant, U. S. Army, and Frank; and by a daughter, Jane (Mrs. William McGahey).

Mr. Currie was elected an Associate Member of the American Society of Civil Engineers on April 1, 1914, and a Member on October 8, 1918.

EDOUARD JEAN BERNARD DE MEY, M. Am. Soc. C. E.[1]

DIED NOVEMBER 20, 1943

Edouard Jean Bernard de Mey was born at Detroit, Mich., on October 25, 1889, the son of Emile J. de Mey and Marie R. (Bagard) de Mey. Until about 1914, his father was American representative of Count Alfred de Monti Bello, and then he became Belgian Consul in Philadelphia, Pa.

Mr. de Mey's early training was obtained in private schools in Boston, Mass., and Philadelphia, and his technical education at the University of Pennsylvania at Philadelphia, from which he was graduated with the degree of Bachelor of Science in Civil Engineering in 1910.

His first engineering experience was with the S. Morgan Smith Company, of York, Pa., where, as construction engineer, he had charge of the erection of pattern and mill buildings. Following this, for two-and-one-half years, he served the Baltimore and Ohio Railroad Company as chainman, leveler, and finally inspector in charge, representing the company on the Ludlow Avenue Viaduct in Cincinnati, Ohio. He next was employed by the City of Cincinnati, as assistant engineer, Division of Structures. During this time he had immediate charge of extensive reinforced concrete viaduct construction.

In 1915 Mr. de Mey became associated with Toupet, Bell and Conley, Inc., Contracting Engineers of Pittsburgh, Pa., as construction engineer in charge of supervision, design, and office work. From 1917 to about 1922, he was chief engineer in general charge of the company's affairs. In 1923, with Mr. Conley, he formed the partnership of Conley and de Mey. This firm specialized in building construction. On the death of Mr. Conley in 1928, Mr. de Mey practiced general engineering until 1932 when he joined the firm of Edward Crump, Jr., Inc., Contracting Engineers of Pittsburgh, as chief engineer and vice-president, remaining until his death.

Mr. de Mey was also interested in technical writing and translated various German and French technical publications into English, among them *Treatise on French Locomotives* by Pechot.

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[1] Memoir prepared by Eber J. Ellsworth and Bennett Taylor Gordon, Assoc. Members, Am. Soc. C. E.

A life member of the Congress of Navigation, Brussels, Belgium, he also belonged to the University Club and the Pittsburgh Athletic Association. He was a member of Calvary Protestant Episcopal Church. A loyal alumnus, he was very active in the work of the alumni association of the University of Pennsylvania.

Mr. de Mey was elected a Junior of the American Society of Civil Engineers on January 2, 1912; an Associate Member on May 15, 1917; and a Member on January 18, 1921.

HERMAN FRANCOIS DOELEMAN, M. Am. Soc. C. E.[1]

DIED MAY 30, 1944

Herman Francois Doeleman, the son of Adriaan and Wilhelmina (Was) Doeleman, was born on August 25, 1874, in Hooge Zwaluwe, Netherlands, with which locality his family had been identified for several hundred years. He attended the Zierikzee High School and upon graduation went to Amsterdam, Netherlands, and studied engineering at the Royal Institute of Engineers.

In 1893 Mr. Doeleman came to the United States. It was natural that the Chicago (Ill.) World's Fair, which was then opening, attracted him. In Chicago he found employment, but several years passed before he was able to obtain a position in which he could apply his engineering knowledge.

During the period from 1896 to 1904, he was with various iron concerns in Chicago and New York, N. Y. In Chicago Mr. Doeleman was engaged as a draftsman by the Chicago Architectural Iron Works and for a short time by Winslow Brothers. In May, 1900, he went to New York and was employed as a draftsman by Levering and Garrigues and later as a designing engineer in the office of George K. Hooper, consulting engineer. In October, 1900, he was a draftsman for Milliken Brothers, and within the next four years became a checker and finally a squad man.

In March, 1904, Mr. Doeleman started with the American Bridge Company as estimator and designer in the New York office. Four months later he was transferred to Baltimore, Md., as the office engineer and assistant to H. Ashton Ramsay, the contracting manager. After serving for eight years in this capacity, he became chief engineer at the Baltimore plant of the Carnegie Steel Company. During this time Mr. Doeleman made many friends in Baltimore, where he was to reside for the remainder of his life. These friends were his greatest asset in the following years.

In January, 1913, he left the Carnegie Steel Company to go into private practice. He established his office in Baltimore as a consulting engineer, specializing in the design and supervision of the construction of steel and reinforced concrete structures, mainly buildings.

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[1] Memoir prepared by Howard L. Crandall, M. Am. Soc. C. E.

Shortly after the United States entered World War I, Mr. Doeleman was selected for and accepted a commission as Major under Lincoln Bush,[2] Past-President and Hon. M. Am. Soc. C. E., Colonel in the Construction Division of the United States Army. Mr. Doeleman and some of his former employees were in Washington, D. C. After the war, he was an officer in the Corps of Engineers reserve and received active training at the Reserve Officers' Training Camp. In 1930 he was commissioned a Lieutenant Colonel.

After World War I, in 1919, Major Doeleman reopened his office in Baltimore and remained in private engineering practice until his retirement. In private practice he was the consulting engineer on the construction of a number of industrial plants and buildings, a complete list of which would be too lengthy to include herein. A few of the structures are the Baltimore Trust Building (O'Sullivan Building), Standard Oil Building, the Baltimore War Memorial, a number of buildings in the Johns Hopkins Hospital and University groups, the Western and Eastern schools in Baltimore, and the Gymnasium Building at the United States Military Academy in West Point, N. Y.

In 1940 Millard R. Tydings, U. S. Senator from Maryland, then chairman of the Committee on Appropriations of the U. S. Senate, having considered several reports on the condition of the roof framing over the House and Senate wings of the Capitol, suggested that a report be obtained from still another engineer, this report to be final. Colonel Doeleman was designated to make the necessary survey and render a report. His investigation sustained the previous reports and Congress authorized the reconstruction. Colonel Doeleman was consultant to the Office of the Architect of the Capitol under whose supervision this work was done. His paper on the "Investigations for Roofs of U. S. Capitol"[3] was delivered before the Structural Division of the Society at the 1941 Spring Meeting in Baltimore.

Colonel Doeleman was liked and respected by his associates and subordinates. One of his characteristic traits was helpfulness to his employees. To work under him was an excellent education. He required each man to understand every step of any work, instead of merely following orders.

Although engineering was his lifework, Colonel Doeleman was noted, too, for business and financial judgment and for knowledge of men and events, contemporary and historic. In his writing, in speeches, and in conversation, his erudition and delightful wit were equally apparent. His searching intellect found expression and satisfaction in world travel, omnivorous reading, and in the study of languages—Dutch, English, French, German, Spanish, Italian, and, finally, Russian and Sanskrit—as well as in the more prosaic recreations of bridge playing and gardening.

He was rejected for active military duty in World War II, but gave valuable civilian service as chairman of the Structural Division of the Maryland Section of the Committee on Civilian Protection in War Time and as Protective Construction Specialist of the Third Civilian Defense Region.

During his professional career, Colonel Doeleman was quite active in various technical societies including—besides the Society—the American Concrete

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 1695.

[3] *Civil Engineering*, June, 1941, p. 327.

Institute and the American Institute of Consulting Engineers (elected member of council in 1943 for a three-year term). He was President of the Maryland Section of the Society in 1926. In addition, he was a member of the Engineers Club and the Merchants' Club of Baltimore and trustee of the Baltimore Efficiency and Economy Commission. In his later years he was, also, president of Belt's Wharf Warehouses, Inc.

Colonel Doeleman was married twice. His first wife, the former Emma Blankfard of Baltimore, to whom he was married in 1903, died in 1930. He is survived by his second wife, Natalie Brainerd Doeleman, of Westfield, N. J. (sister of Harold A. Brainerd, Assoc. M. Am. Soc. C. E.), to whom he was married in 1931. There were no children.

Colonel Doeleman was elected an Associate Member of the American Society of Civil Engineers on June 24, 1914, and a Member on September 11, 1917.

THOMAS LINUS KEITH DONNELLY, M. Am. Soc. C. E.[1]

Died December 20, 1943

Thomas Linus Keith Donnelly was born at Bay City, Mich., on May 16, 1882, the son of Thomas Brady and Lillie (Keith) Donnelly. After graduation from Saint James High School at Bay City in 1900, he studied civil engineering at the University of Notre Dame du Lac at Notre Dame, Ind., and was awarded the degree of Civil Engineer.

After spending two years in the field for the Michigan Central Railroad Company and a short period with the Chicago and North Western Railway Company, he went with the Chicago, Rock Island and Pacific Railway Company on construction and location. He was assistant engineer in the Gary (Ind.) city engineer's office, assistant engineer, Constructing Quartermaster's Division, U. S. Army, and then superintendent for the American Asphalt and Rubber Company, before rejoining the staff of the Michigan Central Railroad Company as assistant engineer of terminal construction. He remained with this company from 1909 until 1914.

When the Pere Marquette Railway Company was reorganized in 1914, Mr. Donnelly was employed on the valuation of that railroad for the federal court by Mortimer E. Cooley, Hon. M. Am. Soc. C. E., and Henry E. Riggs, Past-President and Hon. M. Am. Soc. C. E. He continued this work on the valuation of the New Jersey Public Service Electric Railway Company and on the valuation of the Detroit Edison Company. As copartner of the firm of Wenzell, Donnelly and Company, Mr. Donnelly engaged in private practice in Detroit, Mich., and concurrently was assistant professor of civil engineering at the University of Detroit.

Commissioned a Captain in the Engineer Reserve Corps, he entered active duty in World War I as Commander of Company A, Forty-Eighth Engineers, and served as division engineer, First Division Railways, Advance Section, American Expeditionary Forces. Mr. Donnelly for many years participated actively in the Reserve Corps of the Army, attaining the rank of Lieutenant Colonel, Corps of Engineers, Army of the United States. He was a member of the Reserve Officers Association, the Veterans of Foreign Wars of the United States, and the American Legion.

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[1] Memoir prepared by Eugene I. Van Antwerp, Assoc. M. Am. Soc. C. E.

After the war Colonel Donnelly returned to the firm of Cooley, Riggs and Anderson to work on public utility appraisals. The next six years he spent on docking, dredging, and highway construction, as engineer and as principal engineer. He assisted Professor Riggs on depreciation studies and valuations for many railroads and other utilities. For two years he served as engineer for the Construction Division of the S. S. Kresge Company, for four years as district maintenance engineer for the Michigan State Highway Department, and for two years he practiced privately. Since 1938 Colonel Donnelly had been engaged in private and consulting practice in Bay City.

Professor Riggs has said of Colonel Donnelly:

> "He was a thoroughly competent railway maintenance and construction Engineer. He was a quiet, unassuming, and dependable worker, a good man to work with and be with, and a general favorite in any organization on account of these qualities. I rated him as a high class dependable member of the staff, a competent and experienced engineer and a good citizen. It was with very sincere regret that I read in *Civil Engineering* of his passing."

One of his former associates, Frank M. Bowen, M. Am. Soc. C. E., stated:

> "Tom Donnelly was an engineer of proven ability and possessed a never failing sense of humor. His presence in any organization served as a welcome tonic to everyone. I do not believe he ever caused injury to anyone by word or deed, intentionally. He was an excellent husband and father, a real friend and citizen, and a man whose passing we could ill afford."

In 1907 Colonel Donnelly was married to Magdalen Whetstone, daughter of Mr. and Mrs. Charles Whetstone of Niles, Mich. He is survived by his widow; three children, Thomas Keith, Magdalen, and Polly A.; and five grandchildren, Thomas L. K., Dennis Louis, Brian Kevin, Beirbre, and Michael Donnelly.

Colonel Donnelly was elected a Member of the American Society of Civil Engineers on August 29, 1927.

EDWARD BROWN DONOHUE, M. Am. Soc. C. E.[1]

DIED FEBRUARY 9, 1944

Edward Brown Donohue was born in the Village of Cherokee, Iowa, on March 28, 1896. His parents were Florence and Matilda (Brown) Donohue.

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[1] Memoir prepared by Fred G. Birch, Assoc. M. Am. Soc. C. E.

His father died when Brown Donohue was eight years old; soon thereafter the family moved to Glendive, Mont., where they made their home until they went to Thermopolis, Wyo. There young Brown took an active part in school affairs and developed into a baseball and basketball player of note; later, while at the University of Notre Dame in Notre Dame, Ind., he became a high jumper of exceptional skill.

His schedule of educational and technical activities, submitted with his application for membership in the Society, is a very modest résumé of more than ordinary attainment. This record reveals his graduation from Thermopolis high school in 1914; his study of civil engineering at the University of Notre Dame from 1915 to 1917; and a year (1917) as chief of party at Thermopolis doing mapping, subdivision, and location of oil placer claims. Then, in 1918, heeding the national call for competent men, he enlisted with the Fourteenth Transportation Corps of the United States Army, becoming a Sergeant First Class and later acting as chief clerk and assistant engineer, Maintenance of Way, Touraine Division, in France.

After the Armistice and until 1921, he was employed, successively, as an engineer by the Wyoming Highway Commission, the City of Helena (Mont.), and the Montana State Highway Commission in many capacities. He then became attached to the highway engineering staff and in the years until 1937 was regularly advanced from one responsibility to another through many departments, being a resident engineer, a division engineer, state maintenance engineer, and finally assistant chief engineer handling a road program worth approximately $15,000,000. As an extra activity, he organized and developed the Montana Highway Patrol to a position of national prominence.

A change in state administration retiring him from the highway staff, Mr. Donohue became chief engineer of the Montana State Water Conservation Board and in November, 1938, assumed the additional duties of state engineer; his responsibility in these two positions entailed the construction of approximately $8,000,000 worth of public projects. It is notable that during his employment by the State Highway Commission he was able to complete his college work and in June, 1924, was graduated from the University of Wisconsin, at Madison, with the degree of Bachelor of Science in Civil Engineering.

In August, 1941, the Hon. Sam C. Ford, Governor of Montana, granted him a leave of absence to act as a project manager responsible for the design of a 35,000-man United States Army cantonment—a $30,000,000 project to be erected at Henrys Lake at West Yellowstone, Mont. The magnitude and difficulties of this undertaking began to reveal in Edward Brown Donohue an unusual capacity as a skilful organizer and a most competent executive—qualities which were later to be employed usefully in even greater efforts.

Immediately on the completion of the West Yellowstone duties, the engineering construction firms of Foley Brothers, Inc., and Spencer, White and Prentis, Inc., employed Mr. Donohue as highway engineer to take charge of the design and construction of highways in Iran near the Persian Gulf, which these firms (as joint venturers) were building for the United States Army

Engineers. By reason of the precarious state of his own health and the terrific climatic conditions in Persia, the Army physicians—and his own—refused to permit Mr. Donohue to leave for this overseas assignment.

In an advisory capacity, on special engineering investigation, he was employed by Foley Brothers, Inc., until April 24, 1942, at which time The Pleasantville Constructors (Foley Brothers, Inc., Spencer, White and Prentis, Inc., Winston Brothers Company, and W. E. Callahan Construction Company with their associates) engaged Mr. Donohue to accompany its party, with the Army engineers, on a preliminary visit to the proposed site of two large airfields which, together with buildings, water and sewer supply, and power installations, were to be built in Nassau, Bahamas, British West Indies, for the United States Government. Mr. Donohue remained in Nassau to establish local connections and to build a field office. Two weeks later he was appointed assistant general manager of the project. On January 7, 1943, he was promoted to general manager and remained in that capacity until the completion of the work late in December, 1943. His management of this project was outstanding and was obviously the high point of his career. The United States and British officials and the construction firms he represented outspokenly commended his work; the Duke of Windsor, Governor General of the Bahama Islands, who had formed a warm attachment for him, commended him personally for the diplomatic manner in which he handled the many perplexing, difficult problems affecting labor affairs and relations between American and British interests.

In Mr. Donohue, personal charm, sound engineering judgment, and business acumen were combined; wherever he went and whatever his position, he had a host of good and lasting friends; whatever his problem, he solved it with judgment. Standing six feet four inches—gaunt, lean, powerful looking—his was a commanding figure, yet one of modest carriage. As a public official his reputation for integrity was unblemished; in his career as a construction superintendent he demonstrated not only exceptional business judgment and technical knowledge but also the rare talent of diplomacy. Without exception his associates—in low places and in high position—commented on his kindness, complete sense of justice, and his courtly, gentlemanly character.

Strain, overwork, and impaired health caused his death on February 9, 1944, and terminated a brilliant engineering and construction career.

Edward Brown Donohue was forty-seven years old at the time of his death. He was married in 1926 to Gretchen Coates, who died in 1933, leaving two sons, Allan and George. In 1938 he was married to Kathleen McGrath of Butte, Mont., who survives him.

Mr. Donohue was elected an Associate Member of the American Society of Civil Engineers on April 20, 1925, and a Member on September 9, 1940.

MAX HARRY DOYNE, M. Am. Soc. C. E.[1]

DIED AUGUST 1, 1943

Max Harry Doyne was born in New York, N. Y., on April 6, 1891, the eldest of the four children of Charles Doyne and Ida (Singer) Doyne. He received his early education in the public schools of New York City. From 1909 to 1911 he attended Cornell University at Ithaca, N. Y. He completed his engineering training by taking a night course at Cooper Union Institute in New York, from which he was graduated in 1915 with the degree of Bachelor of Science in Civil Engineering. In 1918 he received the degree of Civil Engineer from Cooper Union.

In August, 1916, after several years of junior engineering experience with New York engineers and architects, Mr. Doyne went to St. Louis Mo., as assistant engineer with C. E. Smith and Company, consulting engineers, becoming chief engineer and manager in 1928. In 1941 he was appointed Director of Public Utilities of the City of St. Louis by the Hon. William Dee Becker, then Mayor of St. Louis. On August 1, 1943, together with Mr. Becker and eight others, Max Harry Doyne met with an untimely death in a glider exhibition accident at Lambert Field in St. Louis.

Mr. Doyne made engineering studies and reports on passenger terminals for Montreal, Canada, and New Orleans, La., and on a Fruit and Produce Terminal for New Orleans. He prepared appraisals and valuations for several stockyard companies, for presentation before the U. S. Department of Agriculture. He was consultant to the City of St. Louis on matters before the Missouri Public Service Commission, involving rate of return of the various large utilities operating in St. Louis. He represented numerous short line railroads in federal valuation cases, appearing before the Interstate Commerce Commission. He was a formidable witness and always well prepared. In 1920–1922 he did the field work and assisted C. E. Smith, M. Am. Soc. C. E., in preparing a comprehensive report on the St. Louis-East St. Louis railroad terminals.

While Mr. Doyne was manager of C. E. Smith and Company, the firm prepared the designs and supervised the construction of all the recent railroad approaches to the St. Louis Municipal (Douglas MacArthur) Bridge. The firm also drew up all the construction plans for the new approaches to the McKinley Bridge at St. Louis. In addition, numerous new and reconstructed industrial plants were planned and constructed.

In 1936 and 1937, as consulting engineer for the Kaw Valley Drainage District, Kansas City, Kans., Mr. Doyne prepared the plans and supervised the raising of levees and the construction of concrete levee walls and slope revetments on the right bank of the Kaw River, thus providing flood protection for the industrial district at Kansas City.

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[1] Memoir prepared by Samuel B. May, M. Am. Soc. C. E.

Mr. Doyne's professional experience was quite varied, covering surveys, designs, construction, special investigations, appraisals, reports, and city and regional planning problems. His construction experience covered many types of work, including municipal improvements, such as sewerage, water systems, streets, and flood protection; bridges; industrial buildings; railroads; and terminal facilities.

As Director of Public Utilities of the City of St. Louis, Mr. Doyne was very active in all matters under his jurisdiction. He arrived at his desk about 6 a.m. each day and stayed long after his staff had left. As chairman of the Airport Commission, he gave vigorous attention to the expansion of Lambert Airport and to the studies involving the acquisition by the city of an additional airport site. He likewise eagerly pursued the development of the "Jefferson National Expansion Memorial" on the St. Louis river front. He was chairman of the St. Louis Municipal Bridge Commission, a member of the St. Louis Defense Council, and Function Chief of the Utilities Division in charge of blackout preparation. All of these special assignments he handled capably, in addition to his numerous regular routine duties.

On June 21, 1915, Mr. Doyne was married to Rose Schneiderman of New York City. He is survived by his widow and a son, Morgan I. Doyne.

Max Harry Doyne was very active in various technical and fraternal groups, including—in addition to the Society—the American Railway Engineering Association, the American Water Works Association, St. Louis Institute of Consulting Engineers (past-president), the Engineers' Club of St. Louis, Jerome L. Goldman Post of the American Legion (past commander), the Society of Forty and Eight (past Chef de Gare), St. Louis Lodge No. 20, A. F. and A. M. (past master), Missouri Consistory No. 1, Ancient and Accepted Scottish Rite, and the St. Louis Chamber of Commerce.

Mr. Doyne was elected a Junior of the American Society of Civil Engineers on January 19, 1920; an Associate Member on June 6, 1921; and a Member on July 16, 1928.

SAMUEL MORRISON ELLSWORTH, M. Am. Soc. C. E.[1]

DIED AUGUST 13, 1944

Samuel Morrison Ellsworth, the son of George Gower and Helen (Morrison) Ellsworth, was born at Braintree, Mass., on January 4, 1895. After attending the local schools, he studied sanitary engineering at the Massachusetts Institute of Technology in Cambridge. He was graduated with the degree of Bachelor of Science in Sanitary Engineering in June, 1916.

Mr. Ellsworth's career in civil and sanitary engineering was broad in compass and steady in growth. His search for experience was purposefully directed toward the ultimate goal of establishing himself as a consultant in sanitary engineering. Only for a short time was his choice of work determined by lack of opportunity for suitable employment. Mr. Ellsworth preferred to overcome the obstacles that often beset the path of young engineers rather than to exchange his ambitions for the tenuous satisfaction of economic security in an available post. His years of apprenticeship included service with the Massachusetts Department of Public Health; the municipal sewerage authority of Rye, N. Y.; the North Jersey District Water Supply Commission; and three firms of consulting engineers—Weston and Sampson, Morris Knowles, Inc., and Metcalf and Eddy. Included in his experience were surveying and field studies, supervision of construction, preparation of preliminary as well as final designs for sanitary works, and responsibility for engineering investigations and reports of varied character. This portion of his career occupied twelve years during which he became familiar with the thinking and methods of some of the leaders of the profession, including Harrison P. Eddy,[2] Past-President, Am. Soc. C. E., and X. H. Goodnough,[3] Robert Spurr Weston,[4] and Morris Knowles,[5] Members, Am. Soc. C. E. These men were good masters under whom to learn and splendid exemplars of the service to society which Mr. Ellsworth wanted eventually to perform in his own right.

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[1] Memoir prepared by Gordon M. Fair, M. Am. Soc. C. E.

When the United States entered World War I in 1917, Mr. Ellsworth first served the American Red Cross and U. S. Public Health Service on extra-cantonment sanitation. Later he joined the American Expeditionary Force, United States Army, being promoted from a private in the infantry to Second Lieutenant in water sanitation with the First Army and ultimately to area water inspector at the American Embarkation Center at Le Mans, France.

Characteristic of Mr. Ellsworth's steadfastness of purpose was his seizing upon the depression years of the early nineteen thirties to become a consulting engineer. Some months after an extended trip to Europe to study the latest developments in sanitary works abroad—while on leave of absence from the firm of Metcalf and Eddy—he started his consulting practice in Boston, Mass. This practice grew slowly but steadily and was devoted to investigations, reports, design, and supervision of construction of water supply and sewerage works for municipalities and industries in New England. Mr. Ellsworth's solution of the engineering problems involved was characterized by a ready grasp of the essentials, willingness to give adequate analysis, and directness of execution.

As the clouds of world-wide war gathered for a second time during his life, Mr. Ellsworth directed all his energies toward the design and construction of essential military installations. Either in his own name or in collaboration with others he became associated with engineering work for Camp Edwards, Mass.; Weymouth (Mass.) Lighter-than-Air Base; Naval Advance Base Depot at Davisville, R. I.; Camp McCain at Grenada, Miss.; the Holston Ordnance Works at Kingsport, Tenn.; and the Pratt and Whitney Works at East Hartford, Conn. Without giving concern to his own welfare, he devoted long hours to the completion of the tasks that were to become so important in the winning of the war. His untimely dealth may reasonably be traced to his failure properly to husband his strength during three years of continuous application to these problems.

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 1867.

[3] For memoir, *ibid.*, Vol. 101 (1936), p. 1565.

[4] For memoir, *ibid.*, Vol. 109 (1944), p. 1542.

[5] For memoir, *ibid.*, Vol. 98 (1933), p. 1567.

From his Pilgrim ancestors, Mr. Ellsworth inherited a rugged idealism which was linked to reality by a gentle sense of humor. This gave comfort to him as well as to those who sought his judgment on some of the perplexing social questions of the times. His interest in public affairs made him conscious, in particular, of the public responsibilities of the engineer. His own responsibilities he met with sincerity and integrity. He served the Town of Braintree as chairman of the Sewer Commission; the City of Boston as a member of the Works Projects Board; Harvard University, at Cambridge, as a lecturer on sanitary engineering design; and the federal government as a water consultant. In addition, Mr. Ellsworth held office in many of the engineering societies of which he was a member. He served as president of the Northeastern Section of the Society and of the New England Sewage Works Association, and was secretary of the New England Section of the American Water Works Association. When he died, he was president of the Boston Society of Civil Engineers and director-elect of the New England Water Works Association.

Mr. Ellsworth wrote a number of papers describing the engineering works that he constructed. Among his most thought-provoking writings, however, were those relating to the welfare of the engineering profession and a paper in which, well in advance of others, he raised the question of whether infantile paralysis could be spread by bathing in sewage-polluted waters.

Samuel Ellsworth was a true son of New England. As such he possessed rectitude of thought in full measure. However, he did not permit his thinking to be hampered by traditionalism. Instead he was willing to accept changing social practices as evolutionary in the American way of life. His friends will remember him for the stimulating arguments which he presented for keeping an open mind on important questions and for advancing the welfare of the profession of engineers as well as that of all men. Often he argued to clarify his own thinking on timely subjects. This fact he would wistfully acknowledge if forced to do so—a trait that endeared him to his listeners.

Mr. Ellsworth was a lover of good music and an accomplished violinist. In professional and social contacts he possessed the rare ability to find something in the character or behavior of most persons which he could admire and through which he could reach a common meeting ground. In consequence he was respected by simple folk as well as by more broadly educated people. He had quarrels as do all men who feel deeply and are honestly outspoken; but his disputes were directed toward the exposure of shallow and dishonest thinking or thoughtless acts. In this he manifested an urge for reforming people and circumstances—a vestige probably of his descent from a long line of teachers and preachers.

His death, on the threshold of the years during which he might have made his engineering ability and personal force felt more widely, has deprived the profession of a man who represented the finest in engineers and his friends of a wise counselor and congenial companion.

On November 6, 1926, Samuel Ellsworth contracted a happy marriage with Marjorie Ann Thomas of Summit, N. J. He is survived by his widow, together with two sons, Samuel Morrison, Jr., and Thomas Allen.

Mr. Ellsworth was elected an Associate Member of the American Society of Civil Engineers on September 10, 1923, and a Member on April 15, 1929.

FRANK HARVEY ENO, M. Am. Soc. C. E.[1]

Died August 7, 1943

Frank Harvey Eno was born in Elmwood, Ill., on July 27, 1865, the eldest of the three sons of Harvey Eno and Cecilia (Kellog) Eno. His father was a farmer and schoolteacher, and Frank spent his first ten years on a farm. In 1875, because of poor health, his father moved to Pomona, Calif.

After graduation from high school, he entered the University of Southern California at Los Angeles where he studied for two years. In the meantime he had become interested in civil engineering by watching the construction of a railroad through his home town of Pomona. Since, at that time, there were no engineering courses at the University of Southern California, he returned to his native state, entered the University of Illinois at Urbana in 1887, and was graduated with the degree of Bachelor of Science in 1891.

During the summer vacations between his sophomore and junior and his junior and senior years he worked as a rodman and instrumentman for Cole, Alvord and Shields of Chicago, Ill., on street, subdivision, and sewer work in the Burchwood subdivision, later a part of Chicago.

Immediately after graduation he went to work for the World's Columbian Exposition Company at Chicago, as first assistant engineer in the water supply and sewerage department. He later became superintendent of the Department of Water Supply, Sewerage and Fire Protection at the Exposition. Based on this experience, he prepared a thesis in partial fulfilment of the requirements for the professional degree of Civil Engineer, which he received from the University of Illinois in 1894. When the Exposition was closed, Mr. Eno became assistant engineer for John A. Cole, consulting engineer of Chicago. From 1895 to 1898 he was Superintendent of Public Works and Village Engineer for Winnetka, Ill. In 1898 he was employed by the City Engineer's office in Columbus, Ohio, as assistant engineer and chief draftsman. In this capacity he supervised the preparation of maps and plans for the new city water supply and storage dam on the Scioto River and for the street and drainage problems of the city.

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[1] Memoir prepared by a Committee of the Central Ohio Section consisting of Clyde T. Morris, M. Am. Soc. C. E., Chairman, and C. W. Allen, Assoc. M. Am. Soc. C. E., and C. B. Hoover, M. Am. Soc. C. E.

After the death in 1902 of C. N. Brown, who had been head of the Civil Engineering Department at Ohio State University in Columbus, Mr. Eno was appointed Associate Professor of Civil Engineering. In 1905 he was promoted to Professor of Municipal and Sanitary Engineering, and in 1926 he was made Research Professor of Highway Engineering, continuing in this position until his retirement in 1935.

In addition to his professorial duties, Professor Eno was active in civic affairs and local and national engineering organizations. He was a charter member of, and served for ten years on, the City Planning Commission of Columbus. For two years he was chairman of the Highway Research Board of the National Research Council. He also served as governor of the Ohio Good Roads Federation and as director of the Ohio Highway Research Board. He belonged to the Society for the Promotion of Engineering Education, Sigma Xi fraternity, the Engineers' Club of Columbus, and the Faculty Club.

In 1890, Professor Eno was married to Nellie C. Kellogg, who continued as his constant companion and loyal helpmate until his death. He is survived by his widow, and one brother, Imle L. Eno.

Professor Eno was active in the organization of the Central Ohio Section of the Society and served as its first president in 1921–1922. The Section suffers a deep sense of loss in the death of Frank Harvey Eno. We knew him as an excellent teacher, an engineer of wide experience, an enthusiastic investigator, a loyal friend, and a true Christian gentleman. He possessed the unqualified confidence of his colleagues. He never hesitated where right and justice were involved, possessing that rare quality of saying the gracious thing and always thinking kindly of those with whom he might disagree.

Professor Eno was elected a Member of the American Society of Civil Engineers on March 4, 1903. He became a Life Member in 1936.

FREDERIC HAROLD FAY, M. Am. Soc. C. E.[1]

DIED JUNE 5, 1944

Frederic Harold Fay was born in Marlboro, Mass., on July 5, 1872, the only child of John Sawyer Fay and Elizabeth (Ingalls) Fay. He was a direct descendant (eighth generation) of John Fay who was born in England in 1648, sailed from England on the *Speedwell,* landed in the Massachusetts Bay Colony on June 27, 1656, and settled in Marlboro. John Fay participated in King Philip's War. Elizabeth Ingalls was a descendant (ninth generation) of Edmund Ingalls who came from England in 1629 and settled in Lynn, Mass.

John Sawyer Fay was a veteran of the Civil War who lost a hand and a leg in battle, and, while in a field hospital, was captured by the Confederates and confined in Libby Prison in Richmond, Va. He once made an interesting Memorial Day address at Marlboro describing his experiences after receiving his wounds and while in Libby Prison. He had the distinction of having been appointed postmaster of Marlboro by every president from Andrew Johnson, in 1865, to Theodore Roosevelt with the exception of James A. Garfield who died before Mr. Fay's term of office expired. He occupied this position for forty-nine years until his death in 1914, during which time the post office was raised from a fourth-class post office to a second-class post office. He was also a vice-commander of the Massachusetts Department of the Grand Army of the Republic.

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[1] Memoir prepared by C. M. Spofford, M. Am. Soc. C. E.

Frederic Fay, after his education by private instructors and in the public schools of Marlboro, entered the Massachusetts Institute of Technology at Boston in 1889 and received the degrees of Bachelor of Science in Civil Engineering in 1893 and Master of Science in Civil Engineering in 1894, being the first man to receive the degree of Master of Science in Civil Engineering from that institution.

Prior to his graduation, he served as a transitman for the city engineer of Marlboro and, after graduation, was employed for a brief time by the Boston Bridge Works. In February, 1895, he entered the Engineering Department of the City of Boston as draftsman and later became assistant engineer in charge of design and construction of the city's bridges and of waterfront developments. After the death of William Jackson,[2] M. Am. Soc. C. E., then city engineer, Mr. Fay assisted in the development of the plan for the consolidation of the engineering, street, and water departments of Boston into a Department of Public Works, and in July, 1911, he was appointed division engineer in charge of the Bridge and Ferry Division of that department. Under this division were placed the design, construction, maintenance, and operation of the municipal bridges and ferries and also the abolition of grade crossings and the general engineering work formerly done by the city engineer. In addition, Mr. Fay was made the commissioner representing Boston on the Boston and Cambridge Bridge Commission. During this period as an associate of John E. Cheney,[3] M. Am. Soc. C. E., and also acting independently, he served as consultant on sundry bridge and structural engineering projects and acted as expert in legal cases.

In 1914 he resigned his position with the city and, in association with the writer and Sturgis H. Thorndike,[4] M. Am. Soc. C. E., organized the engineering firm of Fay, Spofford and Thorndike. This firm (in whose operations he played a leading part until a few months before his death) made numerous engineering and economic reports on various undertakings, and it also designed and gave engineering supervision during construction to many government, state, municipal, and private projects, including army bases, bridges, airfields, harbor works, industrial buildings, sewerage, and water works systems. Among the more important of these projects were the Boston Army Supply Base, built during World War I; the Bourne and Sagamore bridges across the Cape Cod (Massachusetts) Canal, for the former of which the firm received the award of the American Institute of Steel Construction for the most beautiful bridge built at a cost of more than $1,000,000 in 1934; a bridge across Lake Champlain, which received a Phebe Hobson Fowler Architectural-Engineering Award in 1930; the Hampden County Memorial Bridge at Springfield, Mass.; the State Pier at Portland, Me.; the water works system of Warwick, R. I.; and the sewer system and sewage treatment works of Cranston, R. I. During 1941–1944, the firm was the engineer member of the architect-engineer organization which designed fifteen northern North American and off-continent bases for the U. S. War Department, and during this same period was associated with other engineering firms under the title of Dry Dock Engineers, which organization designed numerous dry docks and other harbor works for the U. S. Navy Department.

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXIV (December, 1911), p. 504.
[3] For memoir, *ibid.*, Vol. LIX (December, 1907), p. 537.
[4] For memoir, *ibid.*, Vol. 93 (1929), p. 1910.

Following Mr. Fay's resignation from the Boston Bridge and Ferry Division, he continued actively in civic affairs and served for many years as member and chairman of the Boston City Planning Board, chairman of the Boston Board of Zoning Adjustment from its establishment, member of the Special Committee to Investigate Boston Intermediate Thoroughfares, member of the Boston Advisory Commission on Building Heights, and member of the Special Commission on Fort Point Channel. He also served as vice-president and director of the Boston Chamber of Commerce and for many years was chairman of the Committee on Municipal and Metropolitan Affairs. Mr. Fay always retained much interest in the activities of the Massachusetts Institute of Technology, serving at one time as president of its alumni association and later as alumni member of its Corporation. The class of 1893, of which he was a member, participated ardently for many years in Technology affairs—for which Mr. Fay, as secretary of the class from its graduation, was largely responsible. The class history, which he prepared on the occasion of the class reunion in 1923, was a notable example of class literature. Mr. Fay's death occurred exactly one year after the class had celebrated its fiftieth anniversary.

To the engineering profession at large Mr. Fay gave valuable service as a director and a member of numerous committees of the Society, as president of the American Institute of Consulting Engineers and member of its council, as president of the Boston Society of Civil Engineers, and as director and vice-president of the Society of Terminal Engineers. He was also a member of the New England Water Works Association, the Permanent International Association of Navigation Congresses, the Engineering Society of Western Massachusetts, the Engineering Institute of Canada, the American Concrete Institute, and the Massachusetts Highway Association. His scientific societies included the American Academy of Arts and Sciences and the American Association for the Advancement of Science.

Mr. Fay was the author of numerous papers and articles appearing in the publications of the Society,[5] the Boston Society of Civil Engineers,[6] and *The*

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[5] "Excess Condemnation in Massachusetts," *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 816; "The Development of Mariemont, Ohio," *ibid.*, Vol. 91 (1927), p. 737; "Advantages of Orderly Planning," *ibid.*, Vol. 104 (1939), p. 636; and *ibid.*, Vol. 92 (1928), p. 1162.

[6] "Boston Army Supply Base—General Features of Project," *Journal*, Boston Soc of Civ. Engrs., Vol. VI, No. 3, 1919, p. 67; "The Port of Hamburg," *ibid.*, Vol. XVII, No. 8, 1930, p. 481; and "Planning Boston's Thoroughfares," *ibid.*, Vol. XVIII, No. 5, 1931, p. 153.

Technology Review (Massachusetts Institute of Technology),[7] and elsewhere.[8] He presented many addresses to various societies dealing with engineering, transportation, and city planning matters. The memorial volume "Fifty Years of Boston" contained an article by him entitled "The Planning of a City." He was also the author of "The Population and Finances of Boston."[9]

During much of his life Mr. Fay gave considerable time to the work of the American Unitarian Association, and served as vice-president of the Unitarian Laymen's League and as chairman of the board of trustees of the First Parish Church in Dorchester, Mass.

He was a member for many years of Northeastern University (Boston) Corporation, and served on its board of trustees and on its executive committee. He was also chairman of its committee on buildings.

His clubs at the time of his death were The Union Club of Boston, The Engineers Club of Boston, The Engineers Club of New York, and The Technology Club of New York.

Mr. Fay had many qualities to insure success. His physical strength during his prime enabled him to work longer and more intensely than most men. He took nothing for granted, investigated thoroughly everything he undertook, and had the vision to foresee the important factors that would affect any problem under his consideration. His public spiritedness is shown by his numerous civic activities, and in his private life he was ever mindful of the welfare of his family, associates, and employees and gave generously of his time and energy to all who came to him for advice.

On April 21, 1897, at Quincy, Ill., Mr. Fay was married to Clara May Potter, a descendant of a Massachusetts family then living in Illinois. Mrs. Fay at one time was a prominent member of the Daughters of the American Revolution. Mr. Fay is survived by his widow; five daughters, Beatrice (Mrs. Frederic B. West), Mildred Elizabeth (Mrs. Robert S. Boland), Dorothy Clara (Mrs. Joseph Finestein), Eleanor Potter (Mrs. Wentworth Kennard), and Elizabeth Fay; and five grandchildren, Peter, Richard, and Fay Boland, and Joan and Dwight Kennard. A son, Allen Potter, died at the age of eight.

Mr. Fay was elected a Junior of the American Society of Civil Engineers on October 2, 1894; an Associate Member on April 2, 1902; and a Member on January 5, 1904.

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[7] "Building a Great City—The Engineer's Contribution to Metropolitan Boston," *The Technology Review,* Vol. 32, February, 1930, p. 189.

[8] "Protection of Metal Structures," *Proceedings,* Engrs.' Soc. of Western Pennsylvania, Vol. 31, 1915, p. 115; "The Quequechan River Improvement, Fall River," *American Journal of Public Health,* Vol. IV, 1914, p. 1187; "Boston as a World Port," *The Street,* January 21, 1920; "Modern Traffic Highways Planned for the Port of Boston," *World Ports,* 1931; "Construction Industry," *Boston Business,* June, 1939; and, in collaboration with Charles M. Spofford and John C. Moses, "Boylston Street Bridge, Boston, from 1888 to the Present Time," *Journal,* Assn. of Eng. Societies, Vol. XLIII, No. 6, 1909, p. 234.

[9] "The Population and Finances of Boston," by Frederic H. Fay, Municipal Printing Office, Boston, Mass., 1901.

EDWIN STANTON FICKES, M. Am. Soc. C. E.[1]

Died December 19, 1943

The industrial age has brought into prominence a new type of executive —one possessing not only the essential qualifications for leadership that have always brought men to the top throughout the world's history but also a keen sense of the great importance of sound economics, engineering, and thorough research. Mr. Fickes was such a man. His whole mature life was closely associated with the remarkable growth of the aluminum industry, and he helped to establish many of the markers reached and passed by that industry in its unbroken march of progress.

Edwin Stanton Fickes, the son of George Jackson and Mary (Akin) Fickes, was born at Steubenville, Ohio, on November 3, 1872. His early education was obtained in the public schools of that city. In 1894 he was graduated with the degree of Civil Engineer from Rensselaer Polytechnic Institute at Troy, N. Y. During the summer vacation of 1892, he began his engineering career as a masonry inspector for the Pittsburgh, Cincinnati, Chicago and St. Louis Railroad Company. After graduation, he was employed from July, 1894, to October, 1895, by the late William Glyde Wilkins,[2] M. Am. Soc. C. E., and the late George S. Davison, Past-President and Hon. M. Am. Soc. C. E., consulting engineers of Pittsburgh, Pa., on the design and construction of water works, coal handling works, and bridges. From October, 1895, to January, 1896, he was engaged on surveys for Gustave Kaufman,[3] M. Am. Soc. C. E., of Pittsburgh. From January, 1896, to February, 1897, he was an assistant to Hermann Laub,[4] Assoc. M. Am. Soc. C. E., consulting engineer of Pittsburgh, on construction of highway bridges and manufacturing plants. From February, 1897, to January, 1899, he was again employed by Wilkins and Davison on the design of water works. From January to October, 1899, he worked for the Metropolitan Street Railway of New York, N. Y. During these years he was also identified with estimates and design of power stations, ore docks, and street railways for São Paulo, Brazil; Sidney, Nova Scotia, Canada; Georgetown, British Guiana; Havana, Cuba; and Paris, France.

In October, 1899, Mr. Fickes began his association with The Pittsburgh Reduction Company, shortly thereafter becoming its chief engineer. His initial work with that company was on the design of new plant buildings being constructed at Niagara Falls, N. Y. About 1900 a subsidiary company was organized for the manufacture of aluminum in Canada at Shawinigan Falls, Quebec. Mr. Fickes planned and superintended the construction of the plant. During the next two years a number of plant expansions at New Kensington,

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[1] Memoir prepared by a Committee of the Pittsburgh Section consisting of Robert F. Ewald, F. W. Ely, and Myron W. Hanson, Members, Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1742

[3] For memoir, *ibid.*, Vol. LXXXVII (1924), p. 1367.

[4] For memoir, *ibid.*, Vol. LXXXIII (1919–1920), p. 2263.

Pa., Niagara Falls, and Shawinigan Falls occupied much of his time. He also gathered data which led in 1901 to the beginning of bauxite mining operations in Arkansas and to the construction of a plant for the manufacture of alumina at East St. Louis, Ill. In 1902 under Mr. Fickes' guidance the first aluminum producing furnaces were built at the great Massena (N. Y.) works.

The period from 1905 to 1907 was one of exceptional activity and expansion. During that time The Pittsburgh Reduction Company adopted the policy of developing its own hydroelectric power and began by taking over the plant of the St. Lawrence Power Company at Massena, which was in financial difficulties. As chief engineer of The Pittsburgh Reduction Company, Mr. Fickes was prominent in all these activities. In addition, he took charge of the purchasing department in 1901, the operations of the American Bauxite Company and direction of all rail and ocean traffic in 1907, and the operation of the alumina plant at East St. Louis in 1909.

The officials of The Pittsburgh Reduction Company early realized the possibilities in developing the Long Sault Rapids of the St. Lawrence River as a source of energy for the manufacture of aluminum. In 1906 the company started an active campaign to purchase property, to secure engineering data, and to prepare the necessary plans. All this work was under the jurisdiction of Mr. Fickes. The campaign continued for years until it became evident that the project could not be completed because of conflicting political interests.

In 1907 The Pittsburgh Reduction Company became the Aluminum Company of America, and in the same year Mr. Fickes made the first of many trips to Europe. During the period from 1907 to 1910, he was very busy on a wide expansion program to better existing facilities, personally suggesting and directing a number of marked improvements in the Bayer process of alumina manufacture. In addition, he was active in the search for new sources of bauxite in various parts of the world. It became evident about 1910 that the company would be unable to bring into harmony the many conflicting interests involved in the development of the Long Sault Rapids. A search was begun for other hydro power, and as an initial step in the development of such sources a number of smaller developments were constructed in northern New York, the power being transmitted to Massena. About the same time the Aluminum Company of America became interested in a number of hydro sites on the Little Tennessee River and in 1913 erected its first southern aluminum plant at Alcoa, Tenn., near Knoxville, the first block of power being taken from the Tennessee Electric Company, which was then building a series of plants on the headwaters of the Tennessee River. In addition to these activities, Mr. Fickes also devoted much personal effort to locating new supplies of bauxite in the United States, Europe, and South America. His investigations finally took him to South America in the latter part of 1913, when he realized that a very large proportion of the company's future supplies would come from the Guianas. Steps were immediately taken to acquire and develop necessary properties. The first South American ore was shipped to the United States in 1918.

In 1915 the Aluminum Company of America purchased the holdings of the Southern Aluminium Company, a French company, which had been building a hydroelectric development and aluminum plant on the Yadkin River at Badin, N. C., and had been compelled to drop the project at the beginning of World War I. Mr. Fickes was instrumental in the development of new plans for the aluminum plant. Much work done by the French Company was scrapped, and the plant was constructed at a different site.

In the latter part of 1916 Mr. Fickes took a long trip through Russia, Siberia, and Japan, studying the possibilities for the development of aluminum industries in those countries. In 1918 he brought into reality a project that had long been in the back of his mind—the organization and development of a capable research department. This department soon became a leader in the development of strong aluminum alloys and methods in the manufacture and application of aluminum in all its forms, as well as in research in scores of allied fields. In 1919 Mr. Fickes became a vice-president of the Aluminum Company of America.

From the close of World War I until the latter part of 1925, Mr. Fickes' activities were primarily connected with improvements at the company's existing plants and the development of the bauxite mines in the Guianas. He made a number of trips to Europe in his ceaseless search for new and improved methods. The principal new development of this period was the construction and placing in operation, in 1920, of the first units of the great aluminum rolling mills at Alcoa. In 1924 Mr. Fickes was placed in full charge of all the company's research activities.

A new upsurgence in the growth of the aluminum industry in America took place in 1924 when the Aluminum Company of America began the construction of its Santeetlah hydroelectric development in the Little Tennessee watershed in North Carolina. In 1925 the company became interested in the hydroelectric development of the Saguenay River in Quebec, Canada, and built the first units of the aluminum plant at Arvida, Quebec, Canada—later the greatest in the world. In 1927 construction was started on the third hydroelectric development on the Yadkin River in North Carolina (High Rock). In the same year the company began to build Chute à Caron Dam on the Saguenay River—the first step in what has become the world's greatest single hydroelectric power station—namely, Shipshaw, with 1,200,000 hp under one roof. In 1928 construction began on Calderwood, the third of five hydroelectric developments built by the company on the Little Tennessee River. As head of the Aluminum Company's engineering division, Mr. Fickes had a very active part in all these works. In 1929 he became a director of the company and in 1931 senior vice-president. The economic depression, which began early in 1930, ended all major expansion and not until 1937 was any new work, beyond minor additions to existing facilities, attempted. In that year a new fabricating plant was built at Lafayette, Ind., a new foundry and forge shop at Vernon, a suburb of Los Angeles, Calif., and a new alumina plant at Mobile, Ala.

In recognition of his great services in promoting high standards of engineering and his interest in research on problems connected with the manufacture and use of aluminum and the development of important improvements in those processes, Rensselaer Polytechnic Institute conferred the honorary degree of Doctor of Engineering on Mr. Fickes in 1936. This event was the culmination of a long unbroken career in engineering achievements.

In 1937 his health began to fail. Mr. Fickes then resigned from all his offices and retired to a life of relative quiet in Washington, Conn., and Annapolis Royal, Nova Scotia, Canada, with winter sojourns in California. After his retirement from active duties he served the Office of Production Management as a special adviser.

On April 10, 1909, Mr. Fickes was married to Marguerite Webb, of Warwick, N. Y. Their children are: Harriet Webb (Mrs. Donald W. Chadwick), George J., and Charles W.

His principal hobbies were the collection of etchings and of books and manuscripts relating to American history previous to 1860. He was also greatly interested in art, architecture, gardening, and music, and was a director on the board of the Pittsburgh Symphony Orchestra until his retirement. His beautiful home on Woodlawn Road in Pittsburgh, the scene of his gardening activities, was given to the Pennsylvania College for Women at Pittsburgh after his retirement.

Mr. Fickes possessed a quiet but very forceful personality and an exceptionally well-developed sense of the practicability of any project, large or small. He was especially appreciative of the importance of research and the application of sound engineering principles. Much of the success of the Aluminum Company can be attributed to his strict adherence to the principles of thorough research and sound engineering and construction. His extensive travels placed him in close touch with the leaders in the aluminum industry throughout the world and his keen insight into the details of the almost innumerable ramifications of that business was the marvel of all those who knew him well. No one took a greater and livelier interest in anything that affected the usefulness of aluminum or in the scores of materials and endless variety of equipment required in its production.

He was a member of the American Institute of Mining and Metallurgical Engineers, Rensselaer Society of Engineers, American Association for the Advancement of Science, Historical Society of Western Pennsylvania, American Federation of Arts, University Club of New York, Engineers Society of Western Pennsylvania, Caxton Club of Chicago, Quivera Society of Albuquerque, N. Mex., Sigma Xi, and Engineers Club of New York. Until his retirement in 1937, he had been on the Metallurgical Advisory Board of Carnegie Institute of Technology at Pittsburgh. He also belonged to Duquesne Club, Junta Club, University Club, and Fox Chapel Country Club—all of Pittsburgh.

Mr. Fickes was elected a Junior of the American Society of Civil Engineers on January 4, 1898; an Associate Member on February 6, 1901; and a Member on December 6, 1904.

HAROLD CHAMBERLAYNE FISKE, M. Am. Soc. C. E.[1]

DIED JANUARY 8, 1942

Only too often does genius, because of innate modesty, fail to receive due recognition until after death. This is true in the case of Harold Chamberlayne Fiske, Colonel, Corps of Engineers, U. S. Army, whose vision and undaunted spirit, in the face of what at the time seemed insuperable obstacles, led the way to the comprehensive development of the larger river systems of the United States for power production, navigability, and abatement of flood damage. What had long been regarded by engineers and legislators as mere wishful thinking was shown by him to be practicable. Quoting the *Chattanooga Times* of January 9, 1942: "* * * in a large sense the Tennessee Valley program and public power in this region are monuments to his [Colonel Fiske's] foresight." It remained for others to mature the plan, to provide the necessary legislation, and to build. It seems appropriate to recount briefly, with due regard to historic accuracy, Colonel Fiske's contribution to engineering progress and his military achievements, for Colonel Fiske was a resourceful and gallant soldier with an excellent war record known to but few.

Harold Chamberlayne Fiske was born on March 1, 1880, at Jamestown, N. Y. He attended Union College in Schenectady, N. Y., from September, 1897, to June, 1899. He then went to the United States Military Academy at West Point, N. Y., being graduated as a Second Lieutenant, Corps of Engineers, U. S. Army, on June 11, 1903, ranking third top man in his class. (Among his classmates were such officers as Gen. Douglas MacArthur; Maj. Gen. Julian L. Schley (retired), M. Am. Soc. C. E.; Maj. Gen. Ulysses S. Grant, III, Assoc. M. Am. Soc. C. E.; Maj. Gen. R. C. Moore; Brig. Gen. Max C. Tyler, M. Am. Soc. C. E.; the late Brig. Gen. Hugh L. Johnson; and Maj. Charles T. Leeds (retired), M. Am. Soc. C. E.)

On October 8, 1910, he was married to Mary Mitchell, the sister of the late Col. Americus Mitchell and of the late Gen. William A. Mitchell. First Lieutenant Fiske and Miss Mitchell were the first couple to wed in a formal ceremony in the then newly finished West Point Chapel. Colonel Fiske is survived by his widow; by a daughter, Rebecca A. (Mrs. Saint Julien Fiske Marshall); and by two granddaughters, Louise and Virginia Marshall.

At the age of twenty three, immediately following graduation, Second Lieutenant Fiske was sent to the Philippines in charge of land surveys, bridge construction, and the extension of the Jolo harbor jetty. He was promoted to First Lieutenant on April 23, 1904. After a year of postgraduate work (1905–1906) at the Engineer School, Washington Barracks, Washington, D. C., he spent a year in Cuba, making extended reconnaissances for railroad bridges, and in immediate charge of building construction of public works (Obras Publicas) for Cuba. This was followed by additional postgraduate work and by military surveys of Staten Island, New York, in command of a company of engineer troops on both garrison and field duty. From August, 1908, to August, 1912, he taught at West Point Military Academy, first as instructor and later as assistant professor of civil and military engineering. He was promoted to Captain on February 27, 1911.

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[1] Memoir prepared by Gerard H. Matthes, Hon M. Am. Soc. C. E.

From August 12, 1912, to October 19, 1915, Captain Fiske was assigned to river and harbor work in the United States, first as assistant to the district engineer (and for six months with the title of acting district engineer) at the U. S. Engineer Office at Pittsburgh, Pa., on construction and repairs of locks and dams of the Ohio, Monongahela, and Allegheny rivers. This was followed by a tour of duty as district engineer at the U. S. Engineer Office in Vicksburg, Miss., from October, 1915, to June, 1917. He was promoted on January 12, 1916, to the rank of Major. At Vicksburg, Major Fiske's principal work was on the construction and maintenance of revetments and levees and on channel dredging operations on the Red and Yazoo rivers, tributaries of the Mississippi River. In the five years preceding World War I, he developed a profound interest in the potentialities of commercial river navigation and the potentialities, also, of flood control. The latter field, then in its infancy, had been brought into focus during this period by the passage of the Conservancy Act of Ohio in 1914 and by the first Flood Control Act passed by Congress in 1917, covering the Sacramento and Mississippi rivers. Major Fiske's interest and studies in these two directions were interrupted in 1917 by the entrance of the United States into World War I.

In June, 1917, he was placed in command of the First Battalion, First Engineer Regiment. His first task was the training of engineer troops at Washington Barracks for war service. Promoted to Lieutenant Colonel (temporary) on August 5, 1917, he arrived in France with the first engineer regiment on August 22, 1917, and immediately began organizing the Army Corps engineer training school which he conducted as its director until February, 1918. On December 20, 1917, he was promoted to the rank of Colonel (temporary). Colonel Fiske saw active service as Commanding Officer of the One Hundred and Seventh Engineer Regiment of the Thirty-Second Division overseas, in the Advance Section from February, 1918, to January, 1919. During the period of the regiment's most active operations, in the course of which it took part in four offensives, Colonel Fiske's work as division engineer elicited the highest commendation from his superior officers. His accomplishments are summarized in a letter from Maj. Gen. W. G. Haan, U. S. Army, commanding the Seventh Army Corps, dated December 10, 1918, addressed to the Commander in Chief of the American Expeditionary Forces, in which he recommended Colonel Fiske for award of the Distinguished Service Medal (which was never made), stating that he had earned it for:

> "* * * engineering work of the highest character accomplished during the active operations of the Division and particularly in the following actions:
>
> "(*a*) Second Battle of the Marne, July 29–Aug. 7, while the Division was serving with the 6th French Army.
>
> "(*b*) Near Soissons, Aug. 30–Sept. 5, while the Division was serving with the 10th French Army.

> "(*c*) In the vicinity of Romagne, Northwest of Verdun, Sept. 29–Oct. 20, where the Division was serving with the First American Army.
>
> "2. During these operations and during the periods of preparation for battle, Colonel Fiske showed an unusual knowledge and insight into the Engineering requirements for the battles, and his plans for accomplishing results were always of the most satisfactory kind, and in executing these plans during the actions the work of himself and his Regiment was tireless, and totally disregarded danger in working near the front lines. Colonel Fiske himself was constantly directing these operations and saw that the plans of the Division Commander for keeping roads in order and other Engineering operations up to date were fully and completely carried out.
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> "3. In actually accomplishing useful work during battle and in preparation before battle, I consider Colonel Fiske has shown himself to be one of the most useful and accomplished Engineer officers we have in France, and never lost an opportunity in having his work in full preparation, whether that required night work or not.
>
> "4. I consider that Colonel Fiske has fully earned the award of the Distinguished Service Medal, and more, if more could be given him."

For service in World War I, he was made an Officer of the Legion of Honor of France, received the Croix de Guerre and the Victory Medal, and was awarded the Order of the Purple Heart. Colonel Fiske, on April 19, 1919, was awarded a citation by Gen. John J. Pershing, Commander in Chief of the American Expeditionary Forces, as an expression of appreciation of his services as division engineer, Thirty-Second Division. After November 11, 1918, Colonel Fiske served as Corps Engineer of the Third Corps, Third Army, in its march to and across the Rhine River and in its operations in the occupied sector, east of the Rhine until July, 1919, when the Corps was disbanded. The crossing of the Rhine at Hönningen, Germany, on May 9, 1919, was a remarkable feat in ponton-bridge construction, performed in 2 hr and 25 min across this deep and swift river at a point where it was 1,440 ft wide. Infantry and artillery passed safely over the bridge immediately upon its completion.

Upon his return from France, and having reverted to his permanent rank of Major, Corps of Engineers, U. S. Army, on August 19, 1919, he was reassigned in September, 1919, to river and harbor work as district engineer at Chattanooga, Tenn. For about two years Major Fiske served also as district engineer for the Nashville (Tenn.) District, until relieved by Maj. Julian L. Schley. In this dual capacity, he had charge of the operation and maintenance of locks and dams and of channel improvements on the Tennessee and Cumberland rivers. In 1920 he began the "Survey of the Tennessee River and Tributaries," ordered by Congress — the first study of a river system to be undertaken by the Corps of Engineers, in which the development of hydroelectric power and the control of floods were made integral parts of the plan for the development of navigation. Because of the remarkable potentialities of Major Fiske's plan, the rather strictly observed rule, which limits the tour of duty of officers of the Corps of Engineers at any one place not to exceed four years, was extended, and he continued to serve as district engineer at Chattanooga until November, 1926, his total incumbency at that post covering a little more than seven years. In permitting the extension, the War Department acceded to insistent requests from local authorities to keep Major Fiske at Chattanooga.

Before leaving Chattanooga, Major Fiske was promoted, on September 22, 1926, to the permanent rank of Lieutenant Colonel, Corps of Engineers, U. S. Army. He was placed in command of the Sixth Regiment of Engineers at Fort Lewis, Wash., in November, 1926, continuing in command of this regiment after it was moved to Fort Lawton, Wash. On June 15, 1930, he was retired upon his own application, having served continuously for thirty years in the Corps of Engineers, and, on June 21, 1930, he was designated Colonel, retired.

However, until January, 1933, Colonel Fiske was active as division engineer in charge of the Personnel Division of the Board of Transportation of New York, N. Y., assisting the late Col. J. R. Slattery,[2] M. Am. Soc. C. E., on the construction of the Eighth Avenue Subway, a difficult position for which, by reason of his extensive experience in handling men, he proved eminently fitted.

Retiring from active life, Colonel Fiske returned to Chattanooga to make his home in pleasant surroundings on Missionary Ridge. He continued to take a keen interest in the development of the Tennessee River Valley, serving as chairman of the Electric Power Board of Chattanooga until June, 1938, when he retired from active work on account of heart trouble and moved to Long Island, New York, in 1940, where he bought a home at Manhasset. He died there on January 8, 1942, from a heart attack induced by intense worry over his friend and classmate, Gen. Douglas MacArthur, at that time in great peril at Corregidor, in the Philippine Islands.

Colonel Fiske was active in engineering circles. He was a past-president (1929) of the Western Washington Section of the Society, and of the Seattle Chapter, Society of American Military Engineers; an honorary member and past-president of the Chattanooga Engineers Club; an honorary member of the Kiwanis Club of Chattanooga; and a member of the International City Managers' Association, the Army and Navy Club of Washington, D. C., and the Engineers Club of New York City.

Colonel Fiske was known to his associates as a modest man who was scrupulously honest with all with whom he came in contact, as well as with himself. His clear-thinking mind, genial personality, and resourcefulness won him the admiration of all his personnel.

Major Fiske's plan for the Tennessee River which contemplated coordinating the improvement of navigation with hydroelectric power development, flood peak reduction, and wider utilization of Tennessee's mineral resources met at first with both criticism and hearty approval. Criticism mostly came from officialdom which held that the Corps of Engineers under then existing statutes was concerned neither with power production nor with the abatement of flood damage. The power interests were skeptical—some of their engineers freely declared that it was inconceivable to them how navigation dams could be of value for power production. Convinced that his plan was technically as well as economically sound, Major Fiske continued his surveys and studies and eventually overcame many of the doubts which had been expressed as to the practicability of his plan. He urged substituting for the proposed project (specifying thirty-two low dams with small locks designed to make the Tennessee River navigable as far upstream as Knoxville for barges drawing only 6 ft) six power dams in addition to the then existing power dam at Hales Bar and the Muscle Shoals dam which was under construction. Thus, a navigable depth could be provided for the 9-ft draft which at that time had become standard for the Ohio and Mississippi rivers. Major Fiske further proposed that the cost of constructing the dams and the power plants be borne by power companies although the federal government was to assume the cost of constructing the locks and other navigation improvements. His plan also embraced the tributaries. Among the more important reservoir sites covered in his surveys were the Cove Creek site (later Norris Dam) on the Clinch River, where storage capacity sufficient to double the low-water flow and hence the primary power at the main river dams could be provided. He stressed the importance of water power development as necessary to the economic and industrial welfare of the Tennessee River Valley, and the material flood-stage reductions which would result from the operation of the storage dams were considered by him as an incidental although a profitable by-product. His early reports dated January 15, 1921, and March 15, 1922,[3] give evidence of Major Fiske's remarkably broad conception and sound vision. The power interests found his plan practical and far from visionary. In 1923, when his surveys had reached an advanced stage, the power companies had filed with the Federal Power Commission under the then new Federal Power Act, applications for permits to build dams and power plants at all the sites recommended by Major Fiske, totaling 1,500,000 hp of installed capacity. In an occasional instance more than one power company made application for the same site.

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 1511.

Another valuable contribution to engineering progress was made by Major Fiske in the course of the Tennessee River survey by the introduction of aerial photographic mapping. His survey was the first instance of an extensive river or harbor project in the United States being mapped by this method. This, together with his undaunted request for more than half a million dollars for making his studies and surveys—a sum unheard of in those days for a river survey by the Corps of Engineers—was typical of his courage in breaking away from established precedents. Although the aerial survey, followed by sketching contours on photographs by the plane-table method, was executed by personnel entirely new at these arts, acceptable results were produced, and time and money were saved. Congress was not accustomed to appropriating a half million dollars for river studies but became convinced of the merit of the project and made the funds available.

During a two-day hearing on the findings of the Tennessee River Survey before the House Committee on Rivers and Harbors (Sixty-Eighth Congress, First Session, March 31 and April 1, 1924), after a thorough discussion, one committee member referred to Major Fiske's plan as "astounding and amazing," and at this session the suggestion was first made that similar surveys should be instituted on other rivers of the United States. As a result, the River and Harbor Act of March 3, 1925, in Section 3, authorized and directed the Secretary of War, through the Corps of Engineers, United States Army, and the Federal Power Commission jointly:

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[3] *House Doc. No. 319*, 67th Cong., 2d Session.

> "* * * to prepare and submit to Congress an estimate of the cost of making such examinations, surveys, or other investigations, as in their opinion, may be required of those navigable streams of the United States, and their tributaries, whereon power development appears feasible and practicable, with a view to the formulation of general plans for the most effective improvement of such streams for the purposes of navigation and the prosecution of such improvement in combination with the most efficient development of the potential water power, the control of floods, and the needs of irrigation."

There followed that historic instrument known as "House Document 308, 69th Congress, 1st Session" signed by Maj. Gen. H. Taylor, Chief of Engineers, and O. C. Merrill, M. Am. Soc. C. E., Executive Secretary, Federal Power Commission, dated April 7, 1926, in which they recommended that surveys be made for the comprehensive development of 183 specified rivers of the United States together with some of their tributaries. It would be out of place to recall the sequence of events which grew out of these recommendations, except to state that, directly in line with these recommendations, Congress, in the River and Harbor Act of January 21, 1927, authorized examinations and surveys of type similar to that made by Major Fiske on the Tennessee River to be made of all the rivers specified at expenditures estimated to total $7,322,400. These surveys marked the beginning of the most ambitious program of river development that has been witnessed in the annals of engineering in the United States—a program which has made available millions of hydroelectric horsepower and caused thousands of miles of navigable river channels to be improved, in addition to providing extensive flood protection benefits through the construction of high dams for flood-water storage. For the completion of the Tennessee River survey, additional funds were made available, followed by an act of Congress approved on May 18, 1933, which created a special corporate federal agency known as the Tennessee Valley Authority, charged with undertaking actual development work primarily for the promotion of navigation and the control of floods and clothed with authority to develop and sell power. The project as a whole was to be operated by the federal government for the benefit of the people. Colonel Fiske lived to see the day when a major part of his cherished plan, as modified by this act, had become a reality.

Colonel Fiske was elected a Member of the American Society of Civil Engineers on August 4, 1924.

CHARLES BROE GALVIN, M. Am. Soc. C. E.[1]

Died November 18, 1943

Charles Broe Galvin was born at Jersey City, N. J., on March 9, 1873, the son of Charles Dowling and Ester (Broe) Galvin. He received his education from the International Correspondence School in Scranton, Pa.

In 1890 Mr. Galvin began his career as an apprentice and assistant in the machine shops and engine and dynamo rooms of the Pennsylvania Steel Works at Steelton, Pa. From 1894 to 1896, he was assistant in the street railway electric power plant of the Union Railroad Company in New York, N. Y. In 1896 he served as machinist for the De La Verne Machine Company of New York. From 1897 to 1904, he was employed by the Knickerbocker Athletic Club in New York as a licensed steam engineer.

Because of his health, Mr. Galvin decided to work outdoors and, from 1904 to 1906 he was, respectively, axman, chainman, rodman, and instrumentman for the Highway Engineer Corps of the Borough of the Bronx in New York City. In 1906 he entered the employ of the New York City Board of Water Supply as rodman acting as instrumentman. He then was promoted, successively, to leveler acting as instrumentman; engineer in charge of field party; transitman and computer in charge of field party on preliminary surveys and construction, Hudson River siphon; and finally to assistant engineer.

Leaving the practice of engineering temporarily during 1910 and 1911, Mr. Galvin worked on an invention for surveying deep diamond drill holes, which he later patented. From 1911 to 1913 he traveled extensively in Europe, Asia, and Africa, studying important engineering works; and, from 1913 to 1920, he managed and directed the financial affairs of an estate.

In 1920 Mr. Galvin returned to engineering as draftsman for the New York City Department of Docks (later the Department of Marine and Aviation). He was promoted in 1927 to assistant engineer in charge of drafting, contracts, specifications, and plans for miscellaneous pier and bulkhead structures—a reinforced concrete pier, a three-story hospital superstructure, and floating property and yards. He retired on July 30, 1941.

On October 18, 1910, Mr. Galvin was married at Cornwall-on-Hudson, N. Y., to Antoinette Elizabeth Gazzam. Surviving are three sons, Rodrigo Sheil, Ralph Broe, and Ormond Reading, all in the armed services of the United States. Mrs. Galvin died in September, 1940.

Mr. Galvin was notably of a quiet and serene disposition, and was never known to speak ill of any one. From his father, a journalist, he inherited an acutely analytical mind. In addition to his engineering knowledge, he was widely read in history and astronomy. His interpretation of current events and personages, weighed and appraised in the light of long years of experience, was interesting and instructive. He was musical and was especially fond of early twentieth century classics. His rendition of the old favorites on the violin was as pure and correct as it was modest and unassuming. During the years he traveled extensively in Europe and the Near East, he wrote delightful letters on surveying methods in Cairo, the aqueducts of Rome and Nîmes, the sewers of Paris, and the docks of Liverpool.

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[1] Memoir prepared by J. P. Hurley and W. H. Yates, Members, Am. Soc. C. E.

Of steadfast and reliable character, at an early age, Mr. Galvin was able to take care of himself under any circumstances. He leaves behind him a host of friends who feel that life has been the better because they knew him.

Mr. Galvin was a member of the National Society of Professional Engineers; Municipal Engineers, City of New York; and Society of American Military Engineers.

Mr. Galvin was elected an Associate Member of the American Society of Civil Engineers on March 15, 1926, and a Member on October 14, 1935.

JAMES MADISON GARRETT, M. Am. Soc. C. E.[1]

Died December 20, 1943

James Madison Garrett was born at Mount Willing, Lowndes County, Ala., on September 24, 1864, the eldest of eight children born to James Madison and Belle (Mooney) Garrett. His ancestors were among the early settlers of the State of Alabama.

After attending schools in Lowndes County, in 1878 Mr. Garrett entered Strata Academy, which became Highland Home College (Highland Home, Ala.) in 1880. After his graduation in 1882, he served for four years as an instructor in English, Latin, and Mathematics at Highland Home College, resigning in 1886 to enter the employ of the Atlantic Coast Line Railroad Company as transitman.

During the succeeding four years Mr. Garrett was engaged in railroad location and construction work for several railway systems which were expanding rapidly in Alabama at that time. His assignments included those of chief of party on location and resident engineer on construction. During this period he made a survey for a complete street railway system in Montgomery, Ala. In 1890 Mr. Garrett left the field of railroad location and construction to become chief engineer for the Standard Coal Company, of Brookwood, Ala. In this capacity, he resurveyed two old mines, supervised the construction of some two hundred coke ovens, and handled other engineering duties.

In the latter part of 1890, Mr. Garrett returned to railroad work, as division engineer for the Louisville and Nashville Railroad Company (Sylacauga-Anniston, Alabama Division) from August, 1890, until January, 1891. In January, 1891, he became resident engineer for the Montgomery-Northwestern Railway Company. In July, 1891, he was appointed county engineer of Montgomery County, Alabama. For the succeeding twelve years, in this capacity,

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[1] Memoir prepared by a Committee of the Alabama Section consisting of S. W. Harbin, Assoc. M. Am. Soc. C. E., Chairman, J. L. Land, M. Am. Soc. C. E., and A. Reese Harvey, Jr., Assoc. M. Am. Soc. C. E.

Mr. Garrett built a splendid system of county roads and pioneered in the use of local materials for surfacing. During this same period he was county surveyor for Montgomery County, and conducted a general engineering practice in Alabama, Florida, and Georgia. Resigning as county engineer in 1903, Mr. Garrett became chief engineer of the Birmingham, Columbus and St. Andrews Railroad Company, and during the next three years supervised the location survey work on two hundred miles of line and the construction of forty miles of railroad.

In 1906, Mr. Garrett entered the employ of the City of Mobile, Ala., and made a complete city survey in 1907. During this period he also served as county engineer for Mobile County, Alabama. Railway location again appealed to Mr. Garrett, and so in 1908 he became chief engineer on the location of a projected railway from Andalusia, Ala., to Pensacola, Fla.—a distance of about ninety-five miles.

Much work done by Mr. Garrett up to this time had been on a contract basis. Beginning in 1908 and continuing through 1917, he conducted a general engineering practice, principally in Alabama. He was especially interested in highways, street systems, and railroads. In 1918 he organized the engineering firm of Garrett and Slack, specializing in the preparation of surveys and plans on a contract basis for state highway and bridge construction in Alabama and Georgia. During the period from 1916 to 1920 some forty-two state projects covering several hundred miles of roads were located and designed under the supervision of Mr. Garrett. Later the firm of Garrett and Slack did considerable work for various counties and municipalities.

Mr. Garrett was appointed city engineer of Montgomery in 1924, which position he held continuously until 1941. While he was city engineer, he developed a splendid system of paved streets and other municipal improvements, including substantial additions to the water supply and distribution system. A fine municipal airport was designed and constructed under his supervision.

From 1941 until his death, Mr. Garrett was a consulting engineer in partnership with his son, Roy S. Garrett, M. Am. Soc. C. E., in general engineering practice in Montgomery and its vicinity. During the last year of his life his advanced age and failing health kept him from active practice.

Mr. Garrett was a man of splendid character and of fine personality. He was straightforward in his actions. In his profession he employed a code of ethics which created the greatest respect for his decisions. His judgment was respected by all engineers.

He was married in 1889 to Lucy Tankersley of Hope Hull, Montgomery County, Ala., who died in 1910. In 1911 he was married to Mary Barnes, the eldest daughter of his former teacher, J. M. Barnes, one of the founders of Highland Home College. She died in 1930. Three sons by his first marriage—James M., Colonel in the U. S. Army, Roy S., and George T., Captain in the U. S. Army—and one son by his second marriage—Frank B., Major in the U. S. Army—survive him.

Mr. Garrett was a member of the Kiwanis Club, the Society of American Military Engineers, the American Association of Engineers, and a charter member of the Engineer's Club of Montgomery.

Mr. Garrett was elected a Member of the American Society of Civil Engineers on April 17, 1917. He became a Life Member in January, 1942.

WILLIAM GERIG, M. Am. Soc. C. E.[1]

Died April 3, 1944

William Gerig was born on March 25, 1866, at Ashland, Mo., the son of Joseph and Caroline (Degen) Gerig. He was the eldest of six children, four boys and two girls. Five of the children were graduated from the University of Missouri at Columbia, and four were winners of Phi Beta Kappa keys. William Gerig was graduated in 1885 and received the degree of Civil Engineer in 1886. Fifty-two years later the University of Missouri honored their distinguished graduate by conferring on him the degree of Doctor of Laws.

While stationed in Arkansas as a young engineer, working on railroad construction, he met and, in 1890, was married to Frances Crow of Arkadelphia, Ark., who survives him. In 1940 they celebrated their golden wedding anniversary in the same home in which they were married. He is also survived by two children, Frank and Mildred (Mrs. J. L. Newberry).

Beginning his career as a rodman on railroad construction in Arkansas, Mr. Gerig progressed through three years of railroad work, fifteen years with the Mississippi River Commission, three years as division engineer on the Panama Canal, six years as chief engineer and general manager of the Pacific and Eastern Railway in Oregon, six years as consulting engineer and assistant chief engineer of the Alaskan Railroad, and fifteen years as consulting engineer in the Office of the Chief of Engineers, U. S. Army, retiring with the title of head engineer.

In 1889 Mr. Gerig was employed by the Mississippi River Commission, performing such distinguished service that his work was recognized as outstanding. He became an authority on hydraulics, revetments, and dredging. He designed a number of dredging plants and dams, and was given credit for the use of many new theories of hydraulics in dredging. He assisted in the development of Fascine River mats for flood control. He remained with the Mississippi River Commission until 1905. During this period he served also as assistant engineer for the Chicago Drainage Canal. His varied experiences with the River Commission and his successful designs made possible his future eminence as an authority on lock and dam construction, earthworks, dredging, and waterways, and also led, in 1905, to his appointment as division engineer on the Panama Canal.

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[1] Memoir prepared by a Committee of the Mid-South Section consisting of J. M. Page, Brig. Gen. Max C. Tyler, and Harry N. Pharr, Members, Am. Soc. C. E.

With his family, Mr. Gerig lived in Panama for three years, and contributed much to the construction of that celebrated waterway. His particular problem was "to keep the Pacific out." It is a matter of interest that three dams had been recommended on the Pacific side, but, when the chief engineer, the late John F. Stevens, Past-President and Hon. M. Am. Soc. C. E., asked for Mr. Gerig's opinion, Mr. Gerig advised two dams instead of three. This recommendation was accepted, thus saving the government millions of dollars and rescuing much needed land space for terminal facilities. Mr. Gerig was later transferred to the Atlantic side of the Panama Canal where he was division engineer on the Gatun Locks and Dam. Mr. Gerig served under the three chief engineers in charge of the huge project—Mr. Stevens, John F. Wallace,[2] Past-President, Am. Soc. C. E., and George W. Goethals,[3] M. Am. Soc. C. E. He formed lifelong friendships on this project.

In 1909 Mr. Gerig left Panama to return to railroad construction as vice-president, directing chief engineer, and general manager of the Pacific and Eastern Railway with headquarters in Portland, Ore. During this time he also was consulting engineer for the Spokane, Portland and Seattle Railway Company, and in 1916 consulting engineer for the St. Paul Union Depot Company.

Ever sought as a builder, when but for his modesty and love of his chosen profession he might have become an executive, in 1917 he began, as assistant to the chief engineer, construction of the railroad in Alaska for the federal government. His experiences in Alaska gleaned from letters and newspaper clippings read like a saga. He was not only the master builder, but the master planner—the one person who envisaged the development of transportation as the means of evolving a great new world in the north country. The Anchorage *Daily Times* gives William Gerig the credit for the Anchorage Ocean Dock: "* * * to him alone is due the distinction of working out in all the multiplicity of detail, the finished work, the completed structure, the new dock at Anchorage." This port connected the interior of Alaska—the coal mines, the fisheries, the agricultural industries—with the outside world.

So well, so quietly, and with such a zealous devotion to duty and to his profession did Mr. Gerig work that, in 1923, he was called to Washington as head consulting engineer in the Office of the Chief of Engineers, U. S. Army. He remained with that office until he was retired in 1938.

As consultant on dredging and river and harbor work, Mr. Gerig was a member of the board that assisted in preparing the Jadwin Flood Control Plan, adopted by Congress in 1928, for the Mississippi River. He was consulting engineer on the Florida Ship Canal. (Previously he had been an expert consultant on the New York Barge Canal in 1916, Lake Okeechobee, and various other excavation projects.) As an authority on dams and earthworks, his was the authoritative word on Fort Peck Dam in Montana, Wappapello Dam in Missouri, Sardis Dam in Mississippi, Conchos Dam in New Mexico, Denison Dam in Texas, and work in the Oklahoma-Denver Division.

An unusual distinction was given him in that his retirement age limit was twice extended by Presidential Executive Order, "because the public interest so requires." Even after his retirement, Mr. Gerig remained for several years as consulting engineer on a number of projects, such as the Lake Okeechobee Flood Control Project and the Fort Peck Dam (a $100,000,000 project), and was frequently called to the board meetings concerning them.

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1635.

[3] For memoir, *ibid.*, Vol. 93 (1929), p. 1813.

He contributed many articles to professional magazines, such as the *Military Engineer* and *Dredge Design and Operation.*

William Gerig was one of America's most eminent engineers. He was a master builder, a man of vision with the courage and force to carry out his convictions. His devotion to duty was his most remarkable characteristic; his devotion to his profession was inherited. It has been stated, "* * * if there is a continent to cut in two there will be a Gerig there." Mr. Gerig's father was an engineer on the Suez Canal, Mr. Gerig was at Panama, Frank Gerig is an engineer in Arkansas, and Capt. Frank Gerig, Jr., who was graduated from the U. S. Military Academy in West Point, N. Y., in 1941, is in the service of the U. S. Engineer Corps.

The fact that William Gerig was congenial with his fellow workers is well illustrated by his years of service with government officials. He served under four Chiefs of Engineers, U. S. Army. To the men who worked with him, sometimes high ranking in Army and Navy, his word was the final authority. Many testimonials of his careful concern and influence in their behalf have come to his family.

In Arkadelphia where he chose to make his home, he was loved for his friendliness and sincerity. His quiet dignity concealed the fact that he had achieved eminence. His modest retiring but courtly bearing covered the greatness of his accomplishments and made him revered as a friend—a citizen of a small town. It is fitting that we quote from the *Arkadelphia Standard:*

> "William Gerig left a great deal more than useful monuments of his engineering skill in this world as he went along. * * * His quiet dignity and respectability, his belief in the integrity of men, brought out these qualities in others and gave encouragement and inspiration to those with whom he progressed along the pathway of life.
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> "Such a man creates no grief in the world but gives off happiness and reassurance. In the warm glow of such virtues we find it hard to mourn, even though we do miss his lovable personality."

He belonged to the Society of American Military Engineers, the Masonic Order, and the Presbyterian Church. He was a member of Tau Beta Pi (national honorary engineering fraternity), Phi Beta Kappa, and held the degree of Doctor of Laws from the University of Missouri.

Mr. Gerig was elected a Member of the American Society of Civil Engineers on March 5, 1902. He became a Life Member in 1937.

HERBERT THURSTON GERRISH, M. Am. Soc. C. E.[1]

Died September 6, 1943

The statement that Herbert Thurston Gerrish spent most of his active life near the sea would certainly not be an exaggeration. Mr. Gerrish loved the ocean, whether he was sailing boats with his children off Falmouth on Cape Cod, Massachusetts, or supervising dredging projects in the waters of the Atlantic. However, Mr. Gerrish's major love was his profession. He was a very hard worker and his only real extended vacation from his home in Melrose, Mass., was his honeymoon.

Herbert Thurston Gerrish, the son of John and Ida (Thurston) Gerrish was born in Portland, Me., on July 6, 1886. After being graduated from Massachusetts Institute of Technology in Boston in 1908, he immediately became connected in business with his father who was the president of the Eastern Dredging Company. At this period the Eastern Dredging Company was the largest dredging organization in the northeastern United States, owning, approximately, 34 dredges, 60 to 70 towboats, and more than 200 dump scows.

Mr. Gerrish surpervised the dredging of rivers and harbors in New England, New York, and New Jersey. In Massachusetts he directed work on Cape Cod, the Charles River Dam, West Boston Bridge, Charles River conduits, and filling for the Boston Fish Pier. At this time, Mr. Gerrish became quite interested in Florida, then being developed into a major resort. In fact, the Eastern Dredging Company was one of the first companies to operate a dredge at Miami, Fla. During this period Mr. Gerrish supervised dredging of Matanzas Inlet from St. Augustine, Fla., to Lake Okeechobee, Florida, and dredging at Miami and Miami Beach. From 1916 to 1919 Mr. Gerrish was with the Army engineers. At the close of World War I, he was Captain, commanding Company F, 218th Engineers, Camp Travis, Texas.

In 1917, after his release from the United States Army, he became treasurer and general manager of the Gerrish Dredging Company and the Trimount Dredging Company. In this capacity, he directed such projects as large playground improvements at Rye, N. Y., dredging and filling on Cape Cod, and dredging in the rivers and harbors of New England, New York, and New Jersey.

As an alumnus of Massachusetts Institute of Technology, he was very much interested in working with professors on problems of soil erosion and soil mechanics. Mr. Gerrish was a Past Master of Wyoming Lodge (Masonic Order) and a member of Waverly Royal Arch Chapter, Hugh de Payons Commandery, and The Aleppo Temple. He belonged to the Boston Society of Civil Engineers and the Boston Engineers Club.

At Somerville, Mass., in September, 1913, Mr. Gerrish was married to Edna Whitney. He is survived by his widow; a daughter, Nancy; a son, Whitney,

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[1] Memoir prepared by William H. Ellis, Affiliate Am. Soc. C. E.

Master Sergeant in the U. S. Army; five sisters, Louise Gilbert, Alice Hanson, Lelia Phillips, Alma Hight, and Ruth M. Gerrish; and two brothers, Maurice S. and John Jordan.

Mr. Gerrish was elected an Associate Member of the American Society of Civil Engineers on October 14, 1919, and a Member on July 16, 1928.

LESLIE WARREN GODDARD, M. Am. Soc. C. E.[1]

Died April 6, 1944

Leslie Warren Goddard was born on October 30, 1860, at Lena, Ill., the son of Warren Caleb Goddard and Perly (Skeel) Goddard.

He was educated in rural schools and taught in rural schools prior to entering high school in Ann Arbor, Mich., where he prepared for the University of Michigan, also in Ann Arbor. He was graduated in the class of 1886, with the degree of Bachelor of Science in Civil Engineering.

From July, 1886, to June, 1890, Mr. Goddard worked on railroads and on city surveying in Chicago, Ill., and in Los Angeles, Calif. In June, 1890, he came to Grand Rapids, Mich., as assistant engineer in the U. S. Engineer Department, and continued to work on the harbors of Lake Michigan, ultimately having local charge of the Grand Rapids office under officers of the Corps of Engineers, U. S. Army. The district included all rivers and harbors along the east shore of Lake Michigan. He retired from active duty after forty years of service in November, 1930, continuing to reside in Grand Rapids and wintering in Saint Petersburg, Fla.

Mr. Goddard's work concerned design, construction, and maintenance of harbor structures and navigable channels. In 1905 he was in charge of the improvement of the Grand River from Grand Rapids to Grand Haven, Mich. In this connection, he wrote occasional articles on the effects of storms.[2] A trip to the Panama Canal in 1914 furnished him with pictures and engineering data for lectures on that interesting project. He served as a member of the International Congress of Navigation and was greatly interested in the economics of navigation on the Great Lakes. However, because of political aspects, he never felt free to publish his views on controversial matters. As an employee of the U. S. Engineer Corps, he fulfilled his duty by making the best possible public use of the funds granted by Congress; responsibility for prescribing the expenditures rested on the elected officials.

In spite of his exceeding modesty, his exemplary character was admired by all who knew him; and many a young engineer was inspired by his life and counsel. Mr. Goddard was especially interested in work for boys and young men, and gave freely of his time in teaching at church and the Young Men's Christian Association. He had been a member of the official board of the

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[1] Memoir prepared by Robert H. Merrill, M. Am. Soc. C. E.

[2] *Engineering News*, 1908, I, p. 594; 1912, II, pp. 91 and 171.

First Methodist Church since 1894 and became secretary of the board of trustees in 1914. For more than forty years he served the church school. In 1934 he was appointed a member of the board of trustees of the Clark Memorial Home in Grand Rapids. He was church historian since the building of a new edifice in 1936.

Mr. Goddard was a member of York Lodge, F. and A. M.; the Sons of the American Revolution; and Round Table International. He was a life member of the Western Society of Engineers, and a charter member of the Grand Rapids Engineers' Club. He was a loyal supporter of the University of Michigan alumni association.

On September 26, 1914, Mr. Goddard was married to Mina Bodine of Saline, Mich., who died in 1932. On August 23, 1938, he was married to Mrs. Nellie McNaughton Kendrick of Flint, Mich., who survives him.

Mr. Goddard was elected an Associate Member of the American Society of Civil Engineers on March 2, 1892, and a Member on October 6, 1908.

PAUL HANSEN, M. Am. Soc. C. E.[1]

DIED FEBRUARY 6, 1944

Paul Hansen was born at Arlington, Va., on August 9, 1879, the only son of John and Pauline (Meyenberg) Hansen. His father was a prominent jeweler in Washington, D. C.

Mr. Hansen received his early education in the public schools of Washington, D. C. Greatly interested in the High School Cadet Corps, he became a captain in his senior year. In 1903 he was graduated from the Massachusetts Institute of Technology at Boston, with the degree of Bachelor of Science in Sanitary Engineering.

While an undergraduate, he worked, in the summer of 1901, with a survey party of the U. S. Geological Survey, and, in the summer of 1902, with the engineering department of the Massachusetts State Board of Health.

After graduation, from June, 1902, to May, 1904, Mr. Hansen served in the engineering department of the Massachusetts State Board of Health. He then joined the staff of the late John H. Gregory,[2] M. Am. Soc. C. E., as an assistant engineer on the Improved Water and Sewerage Works at Columbus, Ohio. From January to October, 1905, he was chief engineer of the Pittsburgh Filter Manufacturing Company in Pittsburgh, Pa., installing water purification plants. From 1906 to 1910, he served as first assistant and later as chief engineer of the Ohio State Board of Health. From August, 1910, to October, 1911, he was state sanitary engineer of Kentucky. In 1911 Mr. Hansen became the engineer of the Illinois State Water Survey, and lectured at the University of Illinois in Urbana on Sanitary Engineering. In 1915 he was appointed chief engineer of the Illinois State Board of Health, remaining in this position until 1917, when he volunteered for service in the U. S. Army. His knowledge of military science and sanitary engineering attracted attention. He served for twenty-one months during World War I (1917–1919) as Captain with the American Expeditionary Force in France on the staff of Gen. John J. Pershing, specializing in water supply problems.

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[1] Memoir prepared by W. W. De Berard, Samuel A. Greeley, and Langdon Pearse, Members, Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 1552.

After the Armistice, Mr. Hansen returned to the Illinois State Board of Health. Desiring to enter private practice, in May, 1920, he joined in forming the firm of Pearse, Greeley and Hansen with headquarters in Chicago, Ill. Later, in 1931, upon the retirement of Mr. Pearse, the firm became Greeley and Hansen. The firm specialized in water supply, water purification, sewerage and sewage, and refuse disposal. Mr. Hansen was keenly interested in water works. Among the water works projects with which he was particularly identified were the earth dams at Bloomington, Ind., and Litchfield, Ill.; and water supply works at Centralia, Evanston, Highland Park, Kenilworth, Lake Forest, Moline, Wilmette, and Winnetka, Ill.; Benton Harbor, Mich.; Danville and Harrodsburg, Ky.; East Chicago and Hammond, Ind.; Eau Claire and Portage, Wis.; and Sandusky and Toledo, Ohio.

At the time of his death Mr. Hansen was engaged, for the firm, as consultant on the South District filtration works for the City of Chicago and for the Washington (D. C.) Aqueduct Commission on additions to the water supply works of the District of Columbia. In the water works field, he served as an expert witness in various rate cases, including that of the Indianapolis (Ind.) Water Company, usually on behalf of the municipality or state. In the sewerage field, he was active on projects for Buffalo, N. Y.; Peoria, Ill.; Philadelphia (Pa.) Authority; Hampton Roads (Va.) Sewerage Commission; Austin, Minn.; Eau Claire and Portage, Wis.; Hinsdale, Ill.; Evansville, Ind.; Sioux Falls, S. Dak.; and many other municipalities.

During World War II he was actively engaged in advising the War Department on various projects, as well as in planning and constructing water and sewerage facilities for Camp Forrest, Tenn., and for several ordnance works.

Among the earlier achievements of his career, Mr. Hansen esteemed highly the organization of the public health engineering work for the state boards of health in Kentucky and Illinois. In the forty years of his professional life he accomplished his ambition—becoming one of the prominent sanitary engineers in the United States. He was a skilful and resourceful engineer, and withal a student of human relations. Traveling widely in his professional work, he possessed a large circle of friends. Although a busy man, he found time to serve on various committees of the Society and of the American Water Works Association. He also prepared a number of papers on topics mostly concerned with water purification or public health. His kindly humor enlivened many a meeting. He was active in the technical societies. His works will live after him as a worthy monument. Those who knew him well mourn the loss of a firm friend.

He enjoyed occasional relaxation in an auto or fishing trip. Although he had no special hobbies, he liked fine china, and good music, plays, and reading.

Paul Hansen belonged to the Western Society of Engineers, American Public Works Association, American Association of Engineers, American Water Works Association, New England Water Works Association, Central States Sewage Works Association, Society of American Military Engineers, and Sigma Xi fraternity. He was a fellow of the American Public Health Association. In addition to his technical activities, he was a member of the University Club of Chicago, Rotary Club of Chicago, Executives Club of Chicago, American Legion, and Theta Xi.

On October 8, 1905, he was married to Alison May Scott. He is survived by his widow; two children, Elizabeth Scott (Mrs. Henry Pope, Jr.) and Dr. Paul Scott Hansen; and five grandchildren, Alison Scott Pope, Nancy Pope, Katherine Pope, Judith Coit Hansen, and Celia Coit Hansen.

On February 6, 1944, after a short illness, Mr. Hansen died of a coronary thrombosis at his residence in Chicago. For a number of years he had suffered from high blood pressure and overweight, and had been warned to be careful, but the responsibilities and worries of war time caught him unprepared.

Mr. Hansen was elected a Junior of the American Society of Civil Engineers on January 3, 1905; an Associate Member on April 1, 1908; and a Member on December 3, 1913.

CLINTON LEE HARRIS, M. Am. Soc. C. E.[1]

DIED NOVEMBER 15, 1944

Clinton Lee Harris, the son of Thomas Reyford Harris and Margaret (Lee) Harris, was born on January 28, 1886, at Fair Play, S. C.

His paternal ancestors came to the United States from Wales. The earliest ancestor that has been identified in America was the Rev. John Harris—a minister, educator, and Revolutionary War soldier, who came from the eastern shore of Maryland and who by 1778 resided in Boonesborough, Long Cane Section, South Carolina. Mr. Harris' earliest maternal ancestor in America was the Rev. David Humphreys, a minister who, with other refugees, came from England to Philadelphia, Pa., in 1682, seeking religious freedom.

After attending the public schools at Fair Play, Mr. Harris entered The Citadel, a military college in Charleston, S. C., receiving the degree of Bachelor of Science in Civil Engineering in 1909. He received the degree of Civil Engineer in 1916 from The Pennsylvania State College at State College. Immediately after graduation from The Citadel, Professor Harris served for two years as principal and teacher of mathematics in the high school at Latta, S. C. During the academic year of 1911–1912, he was the commandant of cadets at the Nebraska Military Academy in Lincoln, Nebr. During the academic year of 1912–1913, he served as professor of mathematics at the University School in Montgomery, Ala.

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[1] Memoir prepared by R. M. Gerhardt, Asst. Dean, School of Eng., The Pennsylvania State College, State College, Pa.

In 1913 Professor Harris joined the faculty of the School of Engineering of The Pennsylvania State College, as an instructor in drawing and descriptive geometry. He became an instructor in architectural engineering in 1916, assistant professor in 1917, associate professor in 1919, and professor of architectural engineering in 1926. He was professor of architectural engineering until his death.

During the summers, Professor Harris practiced engineering. In 1917 he was with the Bethlehem Steel Corporation Plant at Steelton, Pa., designing and detailing falsework for the steel railroad bridge across the Ohio River at Louisville, Ky. In 1918 he was engaged by The Austin Company of Cleveland, Ohio, on the erection of factory buildings at Columbus, Ohio, and on the erection of the roundhouse at Nelsonville, Ohio. In 1919 he worked on the design, detailing, and erection of two engineering buildings at The Pennsylvania State College.

As a registered professional engineer and architect in Pennsylvania, Professor Harris had an active practice. In State College, he worked on the structural design and erection of the addition to the State College High School, the erection of the addition to the Methodist Episcopal Church, construction of the Baptist Church, the erection of the Episcopal Church, and the building of the Faith Reformed Church. Professor Harris also acted as engineer for The Pennsylvania State College on such projects as the installation of the Imhoff tank and of sludge and drying beds; the preliminary studies for the Recreational Building; and the steel design of the Horticultural Building. The last professional work upon which Professor Harris was engaged was the design of a large concrete tank for use in the manufacture of shells for a war plant at Williamsport, Pa.

On sabbatical leave from the college in 1932 and 1933, Professor Harris was a member of the Research Association of the National Bureau of Standards, at Washington, D. C. The results of his research during these years were published in "Influence of Neighboring Structures on the Wind Pressure on Tall Buildings"[2] and in technical periodicals both in the United States and abroad. He conducted such research projects at The Pennsylvania State College as "Determination of the Stress of Cables Under Varying Conditions of Loading" and "The Structural Design and Support of Camouflage Nets." Professor Harris was also an associate author of "Building or Buying a House,"[3] which he completed shortly before his death.

A recognized leader in community affairs at State College, Professor Harris served as vice-president of the board of trustees and chairman of the building committee of St. Paul's Methodist Church, as president of the Wesley Foundation, and as a director of the First National Bank. He was a member of the borough council of State College from 1927 to 1930 and president during 1930. He was a member of the following professional and honorary organizations, in addition to the Society: American Association for the Advancement of Science, Society for the Promotion of Engineering Education, American Association of University Professors, Sigma Xi, Sigma Tau, Scarab, and Pi Gamma Alpha. He was a Mason, and belonged to the college "X-Club" and the Centre Hills Country Club.

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[2] "Influence of Neighboring Structures on the Wind Pressure on Tall Buildings," by Clinton Lee Harris, *Bulletin No. 43,* The Pennsylvania State College, State College, 1934.

[3] "Building or Buying a House," by Johnstone and Associates, Whittlesey House of McGraw-Hill Pub. Co., Inc., New York, N. Y.

Professor Harris was a modest, conscientious man who was always praiseworthy and well qualified. His unselfishness, sincerity, insight, and astuteness, coupled with the objective viewpoint that he applied to every problem, made his counsel and advice sought and welcomed by men in the many stations of life. In all things he was thorough and untiring, with infinite patience, always giving unsparingly of himself to his students, to his professional duties, and to his many other interests.

For more than thirty years, his life was devoted to the instruction of the many students in engineering who were enriched by his teaching. His remarkable memory, especially for former students, identified him as one of the most unusual of men. Although he never temporized when a question of principle or discipline was involved, Professor Harris was revered and known affectionately among the students as "Poppy" or "Coach." He was their devoted friend, wise in counsel, tolerant in judgment, and beloved by all.

On July 23, 1913, in Due West, S. C., Professor Harris was married to Lula A. McGee. He is survived by his widow and by a son, Rhett Gibbs Harris (Major, Army of the United States).

Professor Harris was elected an Associate Member of the American Society of Civil Engineers on August 28, 1922, and a Member on November 9, 1936.

ELMO GOLIGHTLY HARRIS, M. Am. Soc. C. E.[1]

DIED DECEMBER 21, 1944

Elmo Golightly Harris was born on June 27, 1861, near Spartanburg, S. C. He was the son of David Golightly and Emily (Liles) Harris. His paternal forebears were of early American stock and came to Boston, Mass., from London, England, finally settling in North Carolina during the early 1700's. His father, David Golightly Harris, owned and operated a large cotton plantation with all the necessary buildings and appurtenances, including slaves. Since Mr. Harris was only four years old when the Civil War ended, his boyhood was spent during the most strenuous and trying years of the reconstruction era. These early pioneer conditions profoundly affected his life, fixed his pleasures, decided the field of endeavor in which he later reached eminent success, and molded the outstanding qualities of his sterling character.

His early education was acquired in a one-room log cabin. He went to school when plantation work was slack. He later attended a private school or

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[1] Memoir prepared by E. W. Carlton and Joe B. Butler, Members, Am. Soc. C. E.

"Academy" and a high school in Spartanburg. After completing high school he entered the University of Virginia at Charlottesville in 1880 to study civil engineering. The course of study was undeveloped and satisfactory textbooks were nonexistent. Teaching was chiefly by lectures—hence, little ground could be covered. After two years of work at the university, he received the degree of Civil Engineer on his twenty-first birthday (June 27, 1882). Among his classmates at the University of Virginia was William H. Echols, a very talented mathematician, who proved a most loyal friend and benefactor during the years to come.

At the time of Mr. Harris' graduation, many railroads were being constructed in the United States, and he played a prominent part in surveying routes for many of these lines in the South and Southwest. He was in charge of surveying for the location of the Missouri Pacific Railroad Company from Van Buren, Ark., to Kansas, planning lines that have been in operation for more than half a century. Mr. Harris was later employed by the Cotton Belt Railroad Company in charge of rebuilding the bridge over the Arkansas River at Rob Roy near Pine Bluff, Ark. This task involved great responsibilities and experiences. New piers had to be built under the old bridge while traffic was maintained. These piers were placed by the pneumatic process; and, in watching and studying the process of blowing out the sand with compressed air, Mr. Harris discovered, independently, the principles involved in the "Air-Lift Pump." Although these principles had been discovered previously, nothing had been published and the theory had not been evolved. Through his study and development of the theory of the air-lift pump, he became interested in compressed air. He later published a textbook on compressed air,[2] which enjoyed wide use in colleges.

In July, 1895, Mr. Harris wrote an article on the "Theory of the Air-Lift Pump."[3] (He had applied for a patent on the air-lift pump which had been twice rejected by the patent office because "it was contrary to the laws of physics and would not work." Nevertheless, a patent was granted a few months later to another man on claims that were absurd and contrary to the laws of physics. Mr. Harris noted all these facts in his article.[3]) As a result, the air-lift pump was freed of patent restrictions. Mr. Harris designed and laid out the first water works and sewerage system for the City of Rolla (Mo.), and the air-lift pump was used on Rolla's deep well water supply installation. Later, he developed a return air pump that was widely used until the development of the electrically driven centrifugal pump. The return air pump is still valuable, however, in pumping gritty material such as sand, gravel, and so forth. An early outstanding installation of this type of pump was on the New York City (N. Y.) water supply.

For several years Mr. Harris practiced civil engineering in Little Rock, Ark. During this time he explored and surveyed much of the great timber area in the St. Francis swamp region, together with some of the earlier highways in Arkansas.

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[2] "Compressed Air," by Elmo G. Harris, McGraw-Hill Book Co., Inc., New York, N. Y., 1910.

[3] "The Theory of the Air-Lift Pump," by Elmo G. Harris, *The Journal of the Franklin Institute,* July, 1895, p. 32.

Through the influence of his good friend, Mr. Echols, Mr. Harris joined the faculty of the Missouri School of Mines and Metallurgy at Rolla in the fall of 1891 as director and professor of civil engineering. Two years later, he gave up his directorship and devoted all his time to teaching. He specialized in a wide range of subjects—mining, engineering drawing, surveying, hydraulics, highway engineering, concrete, structures, railroad engineering, water supply, sewage, contracts, and specifications. Many of his students have attained world-wide distinction in various fields of engineering. One of the more famous was D. C. Jackling, who revolutionized copper mining methods and whose discoveries have doubled the world's supply of copper.

Professor Harris continued his research and developed many revolutionary ideas which attracted international attention. He published the first treatise on centrifugal pumps (which lift a greater volume of water than all other pumps combined). In connection with the computation of the flow of water in uniform channels, he developed a chart for the Manning formula covering one page of a textbook—replacing some two hundred pages of tables formerly used for the same purpose.

Professor Harris' written contributions to the engineering profession are along numerous lines, and many of them appear as bulletins of the Missouri School of Mines and Metallurgy. His early studies were mainly on compressed air and hydraulics, whereas his later studies were largely on concrete arch dams, retaining walls, and highway bridges. All in all, Professor Harris wrote some thirty-four published technical articles and bulletins and also numerous unpublished studies in the field of structures and economics. Professor Harris first called attention to the possibility of a hydroelectric power plant on the Ouachita River in Arkansas (at the site of the Carpenter Dam which furnishes power for Hot Springs, Little Rock, and elsewhere). He was a strong advocate of good roads and in 1919 wrote a treatise on "Road Problems in the Ozarks" which was widely used during the highway development of southern Missouri.

In the fall of 1931 Professor Harris was appointed professor emeritus of civil engineering, and in 1938 a new building on the campus to house civil engineering was named Harris Hall in his honor.

Throughout his life Professor Harris had a diversity of interests. He especially loved the Ozark hills, woods, and streams. He was a noted hunter and woodsman and an active gardener, cultivating flowers, fruits, and vegetables. He gave much attention to building and maintaining residences. Professor Harris was a Southern gentleman in its truest and highest sense, a devoted husband and father, a good neighbor, and a loyal supporter of religious and civic activities.

The death of Professor Harris breaks the direct contact of the faculty of the School of Mines with the nineteenth century and the early days of the school. His service to the School of Mines covers fifty-three of the seventy-four years of its existence. More than 98% of the 3,100 alumni were graduated while Professor Harris was with the school.

A few years before he died Professor Harris stated in his memoirs:

> "The Period of 'rugged individuality,' wholesome achievement, peace and security in which I have lived is past. We are at a turning point in human affairs."

Professor Harris' philosophy of life is best expressed in a prayer written by himself and often repeated by his family:

> "God Thou Great Mystery. Thou hast made me I know not why. Pity my ignorance, forgive my weakness. Thou hast given a blessed life. I am grateful and trust Thee to direct me through to some good purpose according to your all wise plan."

On April 24, 1890, Professor Harris was married to Zoe Barrow, who preceded him in death. He is survived by four daughters, Jean (Mrs. George C. Willson), Zoe Harris, Dixie (Mrs. Clarence H. Burrage), and Emily (Mrs. Glenn A. Dooley); seven grandchildren, Mrs. Charles A. Bemis, George C. Willson, III (Ensign, U. S. Navy), Jean Harris Dooley, Zoe Harris Dooley, Jean Burrage, Emily Barrineau Burrage, and Mary Ella Burrage; and one great-grandchild, Victoria Ann Bemis.

Professor Harris was an honorary member of Phi Kappa Phi and Sigma Xi. He also belonged to various Masonic orders and was active in local civic organizations.

Professor Harris was elected a Member of the American Society of Civil Engineers on October 2, 1901. He became a Life Member in January, 1932.

BERTHOLD FRANCIS HASTINGS, M. Am. Soc. C. E.[1]

Died August 11, 1944

Berthold Francis Hastings, the son of Joel W. and Esther (Lowenthal) Hastings, was born in Brooklyn, N. Y., on October 20, 1888.

After graduation from high school, he entered the Sheffield Scientific School, Yale University, in New Haven, Conn., from which he was graduated with the degree of Bachelor of Philosophy in 1910. He received the degree of Mining Engineer in 1912. During summer vacations while at Yale, he was engaged in general engineering work in various mining camps in different sections of the United States.

After graduation, during 1912 and 1913, he worked on erection and design, gaining both field and office experience, for the United States Reclamation Service on the Elephant Butte Dam in New Mexico. Immediately following this engagement, he became instructor in various civil engineering subjects, including the theory of reinforced concrete, at the University of Pennsylvania, in Philadelphia. From 1914 until 1916, he taught mathematics at the Bridgeport (Conn.) High School, and conducted a private practice in general engineering. He then became structural draftsman on hydroelectric and transmission line design for the New England Power Company, at Worcester, Mass.

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[1] Memoir prepared by F. H. Frankland, M. Am. Soc. C. E.

During 1917 Mr. Hastings was designer and estimator for the Porcupine Steel Company of Bridgeport. In this capacity, he worked on the design of various types of steel structures, the most important of which was the Bullard Gun Plant for the manufacture of ordnance. Following this engagement, he became assistant to J. E. Kirkham, M. Am. Soc. C. E., professor of structural engineering at Iowa State College in Ames, on consulting work for the Iowa State Highway Commission, and at the same time was given charge of instructing upper classmen and graduate students in the design of steel and reinforced concrete. At the end of the school year he undertook a special assignment in the bridge department of the New York, New Haven and Hartford Railroad Company at New Haven, analyzing the capacity of existing bridges on the railroad and directing the design work involved in the necessary strengthening for increased load capacity. This work required a high degree of judgment and design skill.

From the early part of 1919 to the latter part of 1921, Mr. Hastings served as designing structural engineer for the late H. G. Balcom, M. Am. Soc. C. E., of New York, N. Y. While with the Balcom organization, Mr. Hastings had complete charge of the structural design of many important buildings, such as the Ritz-Carlton Hotel and the Ambassador Hotel, both at Atlantic City, N. J. The Ambassador Hotel was the first large structure in southeastern New Jersey to be built on spread footings, instead of piles. Another important structure for which he had direct charge of the determination of stresses, deflections, design of members, and details was the Sixteenth Street Bridge in Pittsburgh, Pa. Following the completion of the design for the Sixteenth Street Bridge, he became representative engineer for Warren and Wetmore, the architects on the project, and was in general charge of the erection and inspection in the mill, shop, and field.

In the autumn of 1922, Mr. Hastings was appointed associate professor of structural engineering at Carnegie Institute of Technology, in Pittsburgh, serving in that position until June, 1928. While with Carnegie Institute of Technology, he was also engaged in private practice as a consulting structural engineer on such buildings as the William Penn Hotel Annex, a 35-story hotel, and the Keystone Athletic Club, a 27-story residential club—both in Pittsburgh.

From June, 1928, until his death, Mr. Hastings served as district engineer for the American Institute of Steel Construction, with headquarters in Philadelphia. His work during this period consisted mainly of engineering and economic consultations with architects, engineers, and financial interests, relating to the proper and economic use of structural steel in buildings and bridges.

Mr. Hastings' training, experience, and inclination led him to intensive studies on the economics of structural design. He was keenly interested in the effect of sociological changes on political trends. He was active in local and national engineering organizations, and was President of the Philadelphia Section of the Society in 1937. As chairman of Committee A–57 on Building

Code Regulations for Steel and Iron of the American Standards Association, he steered the development of these regulations with great success until they became an American standard in 1943. While he was with the American Institute of Steel Construction, he was frequently called upon by various municipalities for advice on building code work and on various bridge design problems. He was a registered professional engineer in New York and Pennsylvania.

Mr. Hastings was qualified, as few are, as a teacher and was remarkably successful in this field. His integrity never permitted him to deviate from the search for truth in the many engineering investigations with which he was connected. This quality and that of loyalty to his friends endeared him to all those who were privileged to know him well.

In 1913 Mr. Hastings was married to Loretta M. Buckley of Bridgeport, who died in 1940. He is survived by a son, Frank Hastings (Sergeant, U. S. Army Air Forces), and a daughter, Virginia Hastings. Another daughter, Joan Margaret, died in 1939.

Mr. Hastings was elected an Associate Member of the American Society of Civil Engineers on July 6, 1920, and a Member on December 4, 1922.

EMILE HENRY HEILBRON, M. Am. Soc. C. E.[1]

Died February 10, 1945

Emile Henry Heilbron was born at Rochester, N. Y., on November 14, 1872. He was the son of Christian and Mary (Snyder) Heilbron. He was reared at Sea Breeze, near Rochester, and was educated in the public schools of Rochester.

In 1891 Mr. Heilbron moved to Chicago, Ill.; and, in 1892, he entered the service of The Sanitary District of Chicago. He remained in the employ of The Sanitary District of Chicago until his death—a period of fifty-two years. Beginning as a flagman on preliminary surveys for the Summit and Willow Springs divisions of the Chicago Drainage Canal, he gradually rose through the grades of computer, subinstrumentman, and subassistant engineer on canal construction, to assistant engineer in 1902. From 1902 to 1905 he directed topographic surveys in the Illinois River Valley and acted as a hydrographer for the U. S. Geological Survey, on discharge measurements of the Des Plaines, Illinois, Fox, and Sangamon rivers in Illinois.

From 1906 to 1913, as division engineer, he made a detailed topographic survey of the Illinois River Valley and compiled data relative to overflow lands, involving damage suits totaling some $12,000,000 against The Sanitary District of Chicago resulting from the diversion of water from Lake Michigan through the Chicago Drainage Canal into the Illinois River. From 1913 until his death, Mr. Heilbron was Illinois Valley engineer for the sanitary district, reporting to the attorney of the district. During this period, surveys were made and data were compiled for the Illinois River and its bottom lands. Topographic surveys and maps of more than 200,000 acres were completed, as were resurveys to determine crop areas on nearly 600,000 acres. Location and profiles were obtained for the 390 miles of levees that protect nearly half the bottom lands along the lower 200 miles of the Illinois River. Many reports were prepared relating to overflow lands, flood heights, and damages. In all this work Mr. Heilbron was closely connected with the defense of the sanitary district against overflow claims and suits in the Illinois Valley, and with the settlement of claims. Until 1925 some 237 suits, of the 458 filed, were pending. In 1925 the board of trustees of the Sanitary District created an Illinois Valley Claims Commission. New suits and claims brought the total filed to 1,827. When Mr. Heilbron died, 575 suits and claims, involving alleged damages of about $8,000,000 to nearly 100,000 acres, were still pending. As a result of his years of contact with the problems of the Illinois River Valley, Mr. Heilbron was probably the best informed engineer in Illinois on the status of overflow lands, crops, and reclamation, covering the 285 miles of the Illinois waterway from Joliet to Grafton, Ill., with an overflow area of about 400,000 acres.

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[1] Memoir prepared by Langdon Pearse, H. P. Ramey, and H. S. Ripley, Members, Am. Soc. C. E.

In 1910–1911 Mr. Heilbron also served as consulting engineer for the Submerged and Shore Lands Legislative Investigating Committee of the Forty-Ninth General Assembly, State of Illinois. This committee presented a comprehensive report covering the entire state. At other times he advised on the reclamation of swamp areas in Illinois, Wisconsin, and Missouri, and testifed as an expert in the case of the United States versus the Economy Light and Power Company, which involved the question of navigability of the Des Plaines River.

At the time of his death he was a life member of the Western Society of Engineers which he joined on March 7, 1900. He also belonged to the Illinois Society of Engineers from 1913 to 1932. He was the third oldest member of the Masonic Blue Lodge at La Grange, Ill. (which he joined in 1895), and also a Knight Templar and a Shriner (Medinah Temple, Chicago, since 1903).

After 1909 Mr. Heilbron resided at Chillicothe, Ill. He was physically strong and a born outdoor man, very fond of athletics, boating, riding, hunting, and fishing. Duck shooting was his favorite recreation. For years he owned and supervised a duck club on the Illinois River. From 1921 to 1931 he also maintained a hunting base in Wyoming. His home was on the bluff above the Illinois River, surrounded by a show yard, including a rock garden, sunken garden, and pools. On February 10, 1945, the automobile which Mr. Heilbron was driving was struck by a train in Chillicothe. He was killed instantly, together with his daughter-in-law, Mrs. Harriet Heilbron, and a youth, Patrick Mooney.

Mr. Heilbron was married three times: In 1895 to Mona Abbitt, in 1907 to Ella Walther, and in 1943 to Mrs. Berdie (Gegenbach) Booker. He is survived by his widow; three daughters, Ruth Heilbron, Marie (Mrs. Howard Hart), and Ella Henrietta Heilbron; two sons, Lee Abbitt Heilbron and

Edward Henry Heilbron (Lieutenant, U. S. Army); and a sister, Cordelia (Mrs. Francis J. Nugent).

Mr. Heilbron was elected a Member of the American Society of Civil Engineers on September 10, 1923.

GEORGE HEWITT, M. Am. Soc. C. E.[1]

Died June 24, 1944

George Hewitt was born in Paterson, N. J., on January 30, 1876, the son of William and Mary (Rainey) Hewitt.

He attended the local grade and high schools and the Rogers and Magee Preparatory School. In 1892 he entered Stevens Institute of Technology at Hoboken, N. J., being graduated in June, 1896, with the degree of Mechanical Engineer.

From 1896 to 1906 Mr. Hewitt was employed by the Passaic Rolling Mill Company, of Paterson, of which George H. Blakeley,[2] M. Am. Soc. C. E., was chief engineer.

On July 1, 1906, he joined the writer in forming the partnership of Lee and Hewitt, Engineers and Architects, with offices in New York, N. Y., and Paterson. The firm did general engineering work, specializing in manufacturing, industrial and power plants, and school buildings.

From January 1, 1934, until December 20, 1938, Mr. Hewitt served as county engineer and road supervisor for Passaic County, New Jersey. During this time he was also a member of the Passaic County Planning Board and the Passaic Valley Flood Control Commission (treasurer).

During World War I, Mr. Hewitt was engineering manager of the Boston (Mass.) Ordnance Office, and, during World War II, he served on the construction of the new naval base at Bayonne, N. J., on the new dry dock at this base, and with the Wright Aeronautical Corporation in the Engineering Department.

Mr. Hewitt was married twice. His first wife was Nellie Latham to whom he was married on April 14, 1903. She died on April 3, 1923. Two children of this marriage, Frances Elizabeth and William Latham, survive him. On November 9, 1925, he was married to Irma Marselus Post, who, with a son, Edward Spear, also survives him.

Music was Mr. Hewitt's principal pastime and enjoyment. An accomplished pianist and organist, he was organist and choir director for many churches.

Mr. Hewitt was a trustee of the Paterson Orphan Asylum, a member of the Friday Afternoon Music Club of Paterson, the Reformed Church in America, the Scottish Rite, and Past Master of Ivanhoe Lodge No. 88, F. and

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[1] Memoir prepared by E. M. Lee, M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 1560.

A. M. He belonged to the American Institute of Architects, Passaic County Engineering Society, and State Association of County Engineers (New Jersey).

Mr. Hewitt was elected an Associate Member of the American Society of Civil Engineers on October 3, 1906, and a Member on March 5, 1912.

GEORGE HIGGINS, M. Am. Soc. C. E.[1]

Died August 21, 1943

George Higgins was born in Borrisokane, Tipperary County, Ireland, on March 31, 1856, the son of the Rev. John Higgins, a Wesleyan Methodist minister, and Anne (Bournes) Higgins. His school days were spent in Abbeyleix, Newry, Dublin, Clonmel, and Wexford—all in Ireland.

In November, 1869, he left Ireland and arrived in Melbourne, Australia, in February, 1870. For a few years he was engaged in commercial, shipping, and banking work.

Mr. Higgins entered the University of Melbourne on March 31, 1875, and received the certificate of Civil Engineer in November, 1877. During 1878, he studied mining, metallurgy, and higher mathematics, while earning some fees for private teaching and for drafting work in the Crown Lands Department. He was awarded the Argus Scholarship in Engineering in February, 1879. Early in 1879, he entered the Victorian (Australia) Railways Department, doing office work and field work on railway surveys and later on railway construction. He also acted as engineer for contractors on the railways to Mudgee, Jerilderie, and Hawkesbury, in New South Wales, Australia.

In 1885 he directed operations for the erection and working of a large stone-crushing plant at Wandong to supply ballast for the North Western Railway Line in Victoria. From November, 1885, to June, 1888, as contractor's engineer, Mr. Higgins supervised the building of Prince's Bridge in Melbourne, and worked on the reclamation of the Elwood Swamp at St. Kilda, Australia.

In July, 1888, he visited California and arranged with the patentee to use a patented system of suction dredging for the construction and operation of a dredge in Australia. In August, 1888, he returned to Melbourne and constructed the dredge *Elwood.* Besides reclaiming the Elwood Swamp with that dredge, he supervised excavation of a canal and dock for the Railway Department of Victoria on the West Melbourne Swamp, the dredged material being used to reclaim adjacent land, later occupied by railway yards and sheds. The land on which the Yacht Club's building was erected at St. Kilda was also reclaimed from the sea by the *Elwood.*

During these operations with the *Elwood* and afterward, Mr. Higgins designed and superintended the construction of the suction dredge with which the Albert Park Lake, near Melbourne, was deepened and adjacent low lying land was reclaimed. He also designed and constructed in Sydney, Australia, machinery for the suction dredge, *Groper*, which has been used by the New South Wales government at both Sydney and Newcastle, Australia. Later, he planned cutting machinery for the government suction dredge, *Dictys*, which operated in soft sandstone on the Richmond River in New South Wales. Suction-dredging machinery was also designed by Mr. Higgins for the Queensland (Australia) government, for dredging contractors in Queensland, and for the contractors on the Adelaide (Australia) outer harbor. He was employed by the firm of William Simons and Company, of Renfrew, Scotland, to design cutting machinery for dredges operating in Durban, Union of South Africa, and Rangoon, Burma.

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[1] Memoir prepared by G. O. Higgins, on service with United States Foreign Economic Administration in South Pacific.

In New Zealand, Mr. Higgins designed dredging machinery for the Otago Harbour Board and for a west coast gold dredging company. From 1905 until 1922, he lectured on civil engineering and (for about five years) on surveying in the University of Melbourne, being appointed dean of the faculty of engineering and acting professor for one year. His private practice during these seventeen years consisted mainly in advising the government of Victoria in connection with accidents on railways at Richmond, Australia, and near Castlemaine, Australia. He was a member of a board of inquiry appointed to examine a proposal for using minting machinery to manufacture cartridges and also of a board which reported to the Victorian government on silt disposal.

Mr. Higgins contributed to various technical journals, and was the author of two books, "Centrifugal Pumps"[2] and "Water Channels."[3] He translated literature on the water hammer from Italian and "Chambres d'Équilibre"[4] from French. Much of the years 1927 and 1928 was devoted to the preparation of a third book on "The Flow of Liquids in Pipes." However, this work was suspended when he was appointed engineer to the Warrnambool Harbour Board in Victoria, early in 1929. He was in England when this appointment was made, and came to Australia by the first steamer. He prepared a scheme for a harbor at Warrnambool, based on surveys and borings which he supervised. The plan was approved by the Harbour Board, by the Victorian Department of Public Works (to whom it had to be submitted), and by the State Executive Council. At this stage, the Victorian government declared its inability to provide funds for the work. His engagement with the Harbour Board terminated in the middle of 1930.

After 1930, instead of completing his third engineering book, Mr. Higgins devoted most of his time to lecturing and writing on economic subjects, being impressed with the urgency of the financial situation—the virtual financial collapse not only of Australia but of many other countries. His conviction that the American economist, Henry George, had discovered the root cause of financial collapse, poverty, unemployment, and war led to Mr. Higgins participation in the work of organizations which proposed to apply practically the methods recommended by Mr. George. Mr. Higgins also began to write a book on "The Natives' Land in Africa."

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[2] "Centrifugal Pumps," by George Higgins, Lockwood, London, 1926.

[3] "Water Channels," by George Higgins, Lockwood, London, 1927.

[4] "Théorie des Chambres d'Équilibre," by J. Calame and D. Gaden, Gauthier-Villars, Paris, 1926.

In addition to his trip to California, Mr. Higgins spent about two years in Great Britain, which included several visits. While correcting proofs for his published books and preparing material for the third, he lived for nearly twelve months in Rome, Italy. He visited Holland, Belgium, Germany, Russia, France, Cape Town (Union of South Africa), Durban, Colombo (Ceylon), and all the Australian states—his chief interest when traveling being harbor works.

Always interested in professional affairs, Mr. Higgins was a member of the following engineering associations: The Victorian Institute of Engineers (former president), the Melbourne University Engineering Society (former president), the Institution of Civil Engineers of England, the Verein Deutscher Ingenieure of Germany from 1902 to 1914 (attached to the Wurtemburghischer Bezirksverein), the International Association for Testing Materials, the Symposium on Water Hammer of New York, N. Y. (honorary member), and the International Congress of Navigation. In addition, Mr. Higgins belonged to the following social and civic organizations: The Town Planning Association of Victoria, the Australian Forest League (committee of the Melbourne Branch), the League of Nations Union of Melbourne (member of the council), the Australian Student Christian Movement (vice-president of the senior branch), the Melbourne Branch Peace Society, the Melbourne University Rowing Club (life member), the Yorick Club of Melbourne, the Malvern Chess Club of Melbourne, the Royal Melbourne Golf Club, the Metropolitan Golf Club of Melbourne, various golf clubs in Sydney and New Zealand, The Henry George Club and The Henry George League (both of Melbourne), and the English League for the Taxation of Land Values. As he grew older, Mr. Higgins became increasingly deaf, and he was unable to participate actively in meetings of the various groups.

Concerning Mr. Higgins' personality and character, a former student, C. Carre-Riddell, wrote:

> "There is another most important part of his life, of which there is no record, except in the hearts and memories of those he met, particularly among his old students. Most of us met him for the first time as leader of the annual survey camps which were our introduction to the life of engineering on which we had then embarked.
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> "After the field work of the day, during and for half an hour after the evening meal, before the night's recording and checking of results commenced, he would sit and talk with us. During these talks he taught us much without appearing to do so. Incidents in his own experience, difficulties overcome, many a tale, sometimes against himself, but so often containing a moral partly hidden in dry humour, taught us much in those happy evenings.
>
> "We learnt then that engineering means much more than the mere calculation of strength and stress and quantity of earth or rock to be excavated, or of cost per cubic yard or linear mile; he taught us how important were those things which cannot be weighed or measured, how success and

> failure so often depend on human nature. He showed how we would be dealing quite as much with men as with steel and stone. Men of all types, some employees under our control, some such as town councillors who would be in a sense our employers, and the engineer stood between, responsible to both: To one for carrying out the work cheaply and efficiently, and to the other for a fair deal with strict, but not stern, discipline.
>
> "His teaching all led up to this that the engineer must show both sides that he knew his job and intended to do it to the best of his ability, that all men could approach him and would get a courteous hearing either to complaint or suggestion, and above all that he did what he considered just, honest and right in all circumstances.
>
> "So often I have heard the remark from one of his hearers after such a talk 'George is a real white man.' There could be no higher compliment and none deserved it more.
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> "A great English poet [Tennyson in 'Ulysses'] has said that 'I am a part of all that I have met' and throughout this land and scattered in many other places there are men who, having met and absorbed a part of him, are better engineers but, above all, better men, for having met George Higgins.
>
> "As long as I live, I will be grateful that I am one of them."

On August 29, 1892, in St. George's Church, Malvern, Melbourne, Mr. Higgins was married to Beatrice Elizabeth Shuter. He is survived by his widow and two sons, the writer and C. G. Higgins, who is serving with the Royal Air Force in the European theater.

Mr. Higgins was elected a Member of the American Society of Civil Engineers on October 2, 1907.

FRED KEATING HILT, M. Am. Soc. C. E.[1]

DIED JULY 11, 1944

Fred Keating Hilt was born at Perryville, Md., on April 30, 1872, the son of Oscar and Zoe (Owens) Hilt. He was born into construction work, as his father, who came originally from Maine, was a contractor and bridge builder and was engaged in the construction of the old double-deck iron railroad bridge over the Susquehanna River from Perryville to Havre de Grace, Md., when he met Zoe Owens, a resident of Perryville.

Young Hilt's boyhood home was in Rensselaer, N. Y., where he received his preliminary education and then attended and was graduated from Rensselaer Polytechnic Institute at Troy, N. Y., with the degree of Civil Engineer in 1894. After graduation, for about one year he was with his father on general heavy construction, including some cofferdam work. He then was employed by the Hilton Bridge Construction Company of Albany, N. Y., as a template maker and draftsman.

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[1] Memoir prepared by B. S. Murphy, Mech. Engr., Jersey City, N. J.

In August, 1896, he began the type of engineering work that he was to do so successfully for many years, when he went with the U. S. Corps of Engineers, first as a civilian instrumentman and later as chief of hydrographic party on the upper Hudson River improvements. Since this was seasonal employment, Mr. Hilt spent his winters as inspector of masonry on the Erie Canal improvements for the New York State Superintendent of Public Works.

In February, 1901, Mr. Hilt began to specialize in tunnel work, continuing in this field until his death. His first engagement was with the Pennsylvania Railroad Company on the preliminary surveys and borings and then on the precise triangulations and topographical surveys for the New York (N. Y.) tunnels. In 1903 he was with the firm of Jacobs and Davies (Charles M. Jacobs [2] and John Vipond Davies,[3] Members, Am. Soc. C. E.), as an assistant engineer on the same type of precise work for the southern tunnels of the Hudson Companies (later Hudson and Manhattan Railroad Company) under the Hudson River from Jersey City, N. J., and Hoboken, N. J., to downtown Manhattan. Also, during this time he made the preliminary surveys for the proposed gas tunnel under the East River from Long Island to 111th Street in Manhattan.

With the beginning of construction on the Hudson and Manhattan tunnels, Mr. Hilt, as division engineer, continued in responsible charge of the alinement and field engineering during the building of the river tunnels, stations, and approaches of the northern division from Hoboken to 12th Street and Sixth Avenue in Manhattan. This project included 7.5 miles of underground and subaqueous railroad tunnels, the terminal at Hoboken, caissons at junction points, and the building of the stations at Christopher Street and 9th Street in Manhattan.

On the completion of this work, Mr. Hilt was transferred to the section extending from Exchange Place to the surface in Jersey City. This was extremely interesting construction as it was under what was then the main passenger line of the Pennsylvania Railroad into New York City and the tunnels extended through the fill of this four-track steam railroad. All this work was done by compressed-air tunnel construction methods until the roof of the tunnel was at street grade, with no interruption to the heavy steam railway traffic.

After the Hudson and Manhattan tubes were in full operation, Mr. Hilt was made works manager and division engineer, in charge of all heavy construction and tunnel structure repairs. During this period he designed and constructed an additional platform, about 350 ft long, with the necessary passenger passageways at the Erie Station in Jersey City. All this work was below water level in soft muck and sand, and construction was completed without closing off the station to the public or without interrupting train movement.

Mr. Hilt resided in Ridgewood, N. J., for a number of years and gave much time to voluntary work with the school board on construction matters.

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–1920), p. 2236.

[3] For memoir, *ibid.*, Vol. 105 (1940), p. 1826.

On February 8, 1935, in Ridgewood, he was married to Mrs. May Lee. He is survived by his widow; a daughter, Janet (Mrs. Winthrop Foster), by a former marriage; and one grandson, Winthrop Foster, Jr.

Mr. Hilt was elected a Member of the American Society of Civil Engineers on November 28, 1916.

JOHN BENJAMIN HITTELL, M. Am. Soc. C. E.[1]

Died July 4, 1944

John Benjamin Hittell was born in Wilton, Iowa, the son of Lydia and Thomas Hittell. When he was very young, his parents moved to Allentown, Pa. He was graduated from Lehigh University at Bethlehem, Pa., with the degree of Bachelor of Science in Civil Engineering in 1887.

His first employment was with the Atchison, Topeka and Santa Fe Railway Company. He then went to Chicago, Ill., as an assistant engineer for the Town of Lake View which was annexed to the City of Chicago in 1889. He worked for the City of Chicago for fifty-one years. In 1941 he retired as chief engineer of streets in the Board of Local Improvements—a title that he received in 1901. Upon retirement from the service of the City of Chicago, Mr. Hittell formed the partnership of Grimes-Hittell, consulting engineers, with offices in Chicago.

During the forty years that Mr. Hittell was chief engineer of streets for the City of Chicago, he planned and supervised the construction of about 4,000 miles of city pavement which cost approximately $275,000,000. Some of the most important street improvements accomplished under Mr. Hittell's direction were the widening and paving (100 ft wide) of Ashland Avenue and Western Avenue. This work extended from the northerly to the southerly city limits. These two street improvements, 20½ miles long and 24 miles long, cost $26,000,000 and $22,000,000, respectively.

During the economic depression, Mr. Hittell prepared a large program of street improvements, eligible for the use of Federal Public Works Administration funds. He obtained these funds together with funds from the motor fuel tax to construct simultaneously twenty-six street improvements in various sections of the city, during 1937 and 1938. These were the first new pavements in the City of Chicago to be financed in this manner.

During the period that he served the City of Chicago, Mr. Hittell witnessed probably the most sensational rehabilitation of a system of streets ever to occur in any large city in the world. A sprawling city with millions of people, Chicago in 1900 had narrow, broken, cobblestone, wood-block, and macadam pavements, which were unsuited and inadequate for modern traffic. When he retired, the city had an almost complete renewal of pavements, with many important streets widened, extended, and unobstructed.

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[1] Memoir prepared by Arthur Engh, Assoc. M. Am. Soc. C. E., and James E. Grimes, Chicago, Ill.

He was a member of Tau Beta Pi, past-president of the Illinois Society of Engineers, Inc., and one of the founders of the American Road Builders' Association.

On September 29, 1919, in Buena Presbyterian Church at Chicago, Mr. Hittell was married to Corinna Beatrice Williams. He is survived by his widow and one brother, R. James Hittell.

Mr. Hittell was elected a Member of the American Society of Civil Engineers on July 1, 1909.

ELMER THOMAS HOWSON, M. Am. Soc. C. E.[1]

Died September 1, 1944

American railway transportation systems long felt the impact of the mind and pen of Elmer Thomas Howson, western editor of *Railway Age,* editor of *Railway Engineering and Maintenance,* and editor of the "Railway Engineering and Maintenance Cyclopedia."[2] His keen analytical appraisal of men and methods, his dynamic presentation of fundamentals, and his ability to organize and inspire others resulted in his leaving accomplishments far exceeding those usually found in a busy life terminating at sixty years.

Elmer Thomas Howson was born near Clinton, Iowa, on May 23, 1884. He was the son of Thomas Henry and Anna Cora (Wessels) Howson. He died at Chicago, Ill., on September 1, 1944.

His early education was acquired in rural schools, which he attended until reaching high school age. The family moved to Clinton when he was thirteen years old, and Mr. Howson attended the Clinton High School, being graduated in 1902.

In the fall of that year he entered the College of Engineering of the University of Wisconsin at Madison, from which he was graduated with the degree of Bachelor of Science in Civil Engineering in 1906. The degree of Civil Engineer was conferred upon him by the University of Wisconsin in 1914. While in the university, he was an honor student, a member of Tau Beta Pi, and president of the civil engineers' society.

Early in life, Mr. Howson chose railway transportation as his specialty. Like many others with a similar leaning, he started at the bottom, pursuing the course that had been charted by his engineering training, and was willing to live and learn. However, since he was endowed with an unusual capacity for both living and learning, his advance was destined to be rapid—beyond what were his most ambitious hopes.

Probably he would have risen to a commanding post in the engineering councils of some great railway. However, fate had decreed a broader field for him, one that would embrace all railroads and all phases of railroading where his influence for higher standards and constant betterment would reach all ranks of railway men in all parts of the United States—the field of technical railway journalism in which he was destined to attain a position of pre-eminence.

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[1] Memoir prepared by Louis R. Howson, M. Am. Soc. C. E.; Samuel O. Dunn, chairman of board of Simmons-Boardman Pub. Corp. and editor of *Railway Age;* and Neal D. Howard, editor of *Railway Engineering and Maintenance.*

[2] "Railway Engineering and Maintenance Cyclopedia," edited by E. T. Howson, Simmons-Boardman Pub. Corp., Chicago, Ill., 1st Ed., 1921.

While still at the University of Wisconsin, during his summer vacation in 1903, Mr. Howson was first employed on railway work by the Iowa and Illinois Railway (later the Clinton, Davenport and Muscatine). Returning to the same railroad during the summer of 1904, he was appointed instrumentman. After his graduation in 1906, he immediately became resident engineeer for the Chicago, Burlington and Quincy Railroad Company, soon advancing to assistant engineer on that railroad's heavy reconstruction work in Wisconsin and Illinois. In 1911 Mr. Howson was promoted to division engineer of the railroad's La Crosse division, with headquarters at La Crosse, Wis., in charge of maintenance and extensive second-track work. Having attracted the favorable attention of higher officers of the railroad, he was definitely on his way to still greater responsibilities when, early in 1911, he was asked to join the staff of the *Railway Age* (then the *Railway Age Gazette*). This publication needed a man of broad capabilities to give it further engineering prestige.

At first, Mr. Howson was reluctant to leave active railway service, since, by that time, he had acquired an absorbing love for railroading that was never to leave him. However, convinced that the opportunity opened broader horizons for that love, with a larger field for service, he became engineering editor of the *Railway Age* on February 1, 1911. In taking this step he was also influenced, no doubt, by an early interest in the journalistic field and an innate ability with the pen. In his school days, he had inaugurated a monthly high school magazine, of which he served as editor. At the University of Wisconsin he was editor of the *Wisconsin Engineer,* the publication of the Engineering College.

As the first engineering editor of the *Railway Age,* Mr. Howson soon became important, not alone through the pages of that publication (which immediately began to assume leadership in the coverage of current railway construction and maintenance practices), but also in the railway field generally. His influence was the result of his extensive traveling and insatiable desire to know railway men and to study their problems. Not satisfied, however, with the outlet which the *Railway Age* alone offered for disseminating engineering and maintenance information (the space available for covering the activities of each railway department was necessarily limited) and convinced that there was a field for an associated publication dealing primarily with matters relating to track, bridges, buildings, and water service facilities, he was soon promoting the creation or acquisition of such a publication. As a result, in June, 1916, the publisher of the *Railway Age,* the Simmons-Boardman Publishing Corporation, purchased the then outstanding publication in this field—*Railway Engineering and Maintenance of Way*—and incorporated it into the corporation's growing group of transportation publications under the name of *Railway*

Maintenance Engineer. Continuing as engineering editor of the *Railway Age,* Mr. Howson also became editor of this new publication, the name of which was subsequently changed, in 1923, to *Railway Engineering and Maintenance.*

In 1919, in recognition of the fact that his interests and activities transcended the field of railway engineering and maintenance, Mr. Howson was appointed to the newly created position of western editor of the *Railway Age;* and, subsequently, in 1936, he was elected vice-president, assistant secretary, and assistant treasurer of the Simmons-Boardman Publishing Corporation. In 1937 he was elected a director of the corporation.

As western editor of the *Railway Age* and directing head of *Railway Engineering and Maintenance,* Mr. Howson's principal and constant endeavor was to render maximum help to railway officers through the pages of these publications. He accomplished this purpose not only by reviewing current activities in the field, but also through his editorial writings, which were designed to stimulate constantly improved construction, maintenance, and operating practices.

In 1919, convinced that beyond the publications of his company there was still further opportunity to serve those concerned directly or indirectly with railway engineering and maintenance matters, Mr. Howson sponsored and was appointed editor of the first edition of the "Railway Engineering and Maintenance Cyclopedia," a volume treating all branches of railway construction and maintenance authoritatively. This was being revised for a sixth edition when he died.

Mr. Howson believed that there was no substitute for personal observation in securing information in the railroad engineering field. Accordingly, he traveled extensively on the railroads of the United States, Canada, and Mexico. His writings were authoritative discussions of the problems and accomplishments of these railroads. He watched with great interest the nationalization of railroads in Mexico and his last contribution to the *Railway Age,* which dealt with the problems of the Mexican railroads, was published after his death.

Through his multitude of connections, his extensive travels, and widespread observations of railroad practices, Mr. Howson obtained an unparalleled fund of practical information. To him, this information was not just something to acquire and retain—but to make available to others. He shared his knowledge freely with the men with whom he came in contact, either personally or through his writings; and, in return, he received and cherished the respect and intimate friendship of large numbers of railway engineering maintenance, operating, and executive officers.

In the interest of his profession as an engineer and railway editor, Mr. Howson had long been a member of many technical associations, societies, and clubs, always taking an active part in their affairs. Among these groups were the Illinois Section of the Society (President in 1927); the American Railway Engineering Association (a director, chairman of the arrangements committee, and chairman of the committee on cooperative relations with universities when he died); the American Association of Railroad Superintendents (chairman of the program and arrangements committee); the Western Society of Engineers (president in 1924–1925); the American Wood-Preservers' Association (president in 1932); the Roadmasters' and Maintenance of Way Association (president in 1934); the American Railway Bridge and Building Association (president in 1927); the Western Railway Club (for a number of years chairman of the program and arrangements committee); the National Conference of Business Paper Editors (president in 1925–1926); the Associated Business Papers, Inc., a national organization of the publishers of business papers (president in 1934–1935); and the Maintenance of Way Club of Chicago (charter member in 1921 and chairman of the program committee). In addition to his membership and activities in these technical associations and societies, Mr. Howson belonged to the following clubs in Chicago: The Union League Club, the South Shore Country Club, and the Chicago Engineers' Club.

Elmer Howson had limitless energy supported by a robust physique. His capacity for work was enormous. To be associated with him was a challenge. His habit of driving himself inspired others to do their best. He did not spare himself and expected others to follow his pace—a feat that was not always easily accomplished. However, he never expected of others more than he was ready to do himself.

Perfection in whatever he undertook was almost an obsession with Elmer Howson. He strove for it himself and inspired others to work for the same goal. He was impatient with careless or indifferent work, but was ever ready, through direction and encouragement, to assist others to exert their best efforts. He was particularly delighted with, and interested in, young men, and inspired them constantly to higher achievements and confidence in their ability. Although critical of inferior work, he was able to appraise and bring out the best in others and to organize the efforts of his staff so as to secure the most effective utilization of the potentialities of his associates.

From youth, Mr. Howson was an organizer and he never allowed an organization, once started, to lapse through inattention on his part. The numerous technical associations that he headed; the Howson Fellowship Bible Class, which he organized; the Howson annual family reunion, which he fostered; and even a long-standing association of his high school class members, which he helped to sustain—all testify to the fact that he felt a continuing responsibility for every activity in which he participated.

This cumulative sponsorship of many activities required time and energy in ever-increasing amounts, which in some way Mr. Howson managed to summon to his command. His counsel was sought and freely given to the many associations and societies in which he played so prominent a part. His administration of their affairs was uniformly aggressive and rich in accomplishments. Several associations largely owe to him their survival during the depression and their existence today as virile influences for the betterment of railway transportation services.

A vital force in the life of Elmer Howson was his deep and firm religious convictions. He was a lifelong member of the Methodist Church and was ever active in its service. Since 1911 he resided in Chicago and served the Woodlawn Methodist Church as a member of the board of stewards, chairman of the finance committee, chairman of the board of trustees, and lay delegate to the Rock River Conference. In many other ways, he devoted his time and energy to religious work.

In 1911 he organized and assumed leadership of a Young Men's Bible Class, which became one of the major activities in his busy life. Under his inspirational leadership, the class developed into one of the largest and most active in Chicago. In the thirty-three years of its existence prior to Mr. Howson's death, more than 10,000 men had been members of the class, and its influence is felt by men scattered throughout the world. In 1931 the class name was changed to the Howson Fellowship Bible Class. He continued as the class leader through all the years and gave much of his thought and energy to the class. Of all the many and varied activities in which he participated, the class which bore his name was perhaps the dearest to him,

On June 5, 1907, he was married to Mae McCulloh, who survives him.

Mr. Howson was elected an Associate Member of the American Society of Civil Engineers on May 28, 1912, and a Member on March 9, 1920.

HAROLD WALTON HUDSON, M. Am. Soc. C. E.[1]

DIED JANUARY 15, 1943

Harold Walton Hudson devoted the greater part of his active professional career to the direction of large engineering operations rather than to their design. He was more at home on the job in high boots and an old hat than at a desk in the office. His ability to deal with men and to supervise their activities won him a high place in the civil engineering and general construction field. He spoke and understood the language of the construction man. Despite his blunt and often gruff manner of speech, he was both liked and respected by all the men with whom he came in intimate contact on the projects he directed.

Mr Hudson was born in Pittsburgh, Pa., on September 29, 1874, the son of Thomas James and Elizabeth (Reilly) Hudson. He studied civil engineering at the University of Minnesota at Minneapolis for three years, and then left college to take up the railroad and general engineering work in the West, Midwest, and Southwest that doubtless gave him his love of the outdoors and construction camps. He was instrumentman on the Florence and Cripple Creek Railroad in Colorado and held various jobs with the engineering firm of Tuttle and Pike. In May, 1900, he joined the engineering force of the Missouri Valley Bridge and Iron Works and was sent to Tampico, Mexico, on wharf construction. A year later he returned to the United States to resume railroad work in the Southwest.

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[1] Memoir prepared by V. T. Boughton, M. Am. Soc. C. E.

For a decade Mr. Hudson devoted himself largely to railroad work. In 1903 he inaugurated his long connection with the Pennsylvania Railroad Company, and its subsidiaries, being employed in various capacities for twenty-five years except for short intervals. After service as assistant engineer on the Philadelphia Division and a brief period in private practice, Mr. Hudson came to New York, N. Y., in November, 1906, as assistant engineer for the Pennsylvania Tunnel and Terminal Railroad Company on the Sunnyside Yards in Long Island, part of the great new terminal development of the Pennsylvania Railroad System. On the conclusion of that work, Mr. Hudson spent about a year on a power project in Tennessee, but in 1912 he returned to New York to become engineer of construction on the Hell Gate Bridge and its approaches. That position he held until May, 1917, when, as a Captain in the Engineer Reserve Corps, he was called into active service to command Company A of the 1st Reserve Engineers, later the 11th Engineers (Railway), U. S. Army.

Captain Hudson sailed with the 11th Engineers in July, 1917, but soon after the regiment reached France he was drafted by Brig.-Gen. W. W. Atterbury,[2] M. Am. Soc. C. E., then head of the Transportation Corps of the American Expeditionary Force (for whom Captain Hudson had worked while with the Pennsylvania Railroad Company), as one of the officers of his staff on the Transportation Corps. Captain Hudson served in this capacity until after the armistice, being promoted to the rank of Lieutenant Colonel. After the armistice he rejoined the 11th Engineers, returning as commanding officer of one of the two units into which the regiment was divided for transportation back to the United States. For his war service he was awarded the Purple Heart by the United States Government and was made a Chevalier of the Legion of Honor of France.

After World War I, Colonel Hudson was chief engineer of the Interstate Railroad Company in Virginia on location, surveys, and construction, from 1919 to 1924. Subsequently, he became superintendent of construction for Gibbs and Hill, engineers, of New York, on electrification of the Virginian Railway. On completion of that work he held the same position with Gibbs and Hill on the electrification of the Bay Ridge Division of the Long Island Railroad Company.

Leaving railroad work in 1928, Colonel Hudson went to the New Jersey State Highway Department to participate in the building of new highway approaches to New York City from the southwest. He took an active part in the selection of a high-level viaduct (later the Pulaski Skyway) for carrying the new highway in New Jersey between Newark and Jersey City, and was engineer in charge of construction. After this project was completed, he was appointed Assistant State Highway Engineer, holding this position until March, 1934, when he resigned to become chief construction engineer of the Triborough Bridge Authority on the Triborough Bridge and the Bronx-Whitestone Bridge. He later joined the engineering firm of Madigan-Hyland, of New York, remaining until his death on January 15, 1943.

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[2] For memoir, see *Transacations*, Am. Soc. C. E., Vol. 101 (1936), p. 1518.

From 1939 to 1941, Colonel Hudson was a Director of the Society. In 1937–1938, he served as president of the Metropolitan Section.

Colonel Hudson was elected an Associate Member of the American Society of Civil Engineers on May 3, 1905, and a Member on September 3, 1912.

MARTIN JOACHIMSON, M. Am. Soc. C. E.[1]

DIED FEBRUARY 7, 1944

Martin Joachimson, the son of Fritz and Bertha (Riegner) Joachimson, was born in Breslau, Germany, on June 27, 1867. He was educated at the Breslau Gymnasium, and received the degree of Mechanical Engineer from the College of Mechanics in Berlin, Germany, in 1892.

After his arrival in New York, N. Y., he worked for the Electric Construction and Supply Company from 1893 to 1895. In 1895 and 1896 he was assistant engineer for the American Sugar Refining Company at Baltimore, Md. He served as chief engineer of Koster and Beal's Theatrical Enterprises of New York in 1896 and 1897. From 1897 to 1904 he was employed as mechanical engineer by R. Hoe and Company, manufacturers of printing presses, in New York City.

Mr. Joachimson entered the service of the City of New York in 1904, under the civil service title of draftsman, in the Department of Bridges (which in 1916 became the Department of Plant and Structures and finally was merged into the present Department of Public Works on January 1, 1938). Within a few years he was promoted to the grade of assistant engineer.

The department had under its jurisdiction many movable bridges of the swing-draw and bascule types and the mechanical and electrical equipment for these structures soon came within the scope of Mr. Joachimson's work. Many movable bridges were built by the department, for which Mr. Joachimson designed all machinery and electrical equipment. Among these were the University Heights, the Eastchester, the Greenpoint Avenue, the Metropolitan Avenue, the Roosevelt Avenue, the Eastern Boulevard, and the Westchester Avenue bridges.

The Department of Plant and Structures also constructed several garbage incinerators and a number of other city plants. Mr. Joachimson had charge of the design of the service installations. In 1924 Mr. Joachimson was promoted to the newly created office of engineer of equipment which he held with distinction until his retirement at the age of seventy one, on July 1, 1938.

Mr. Joachimson was both an expert in the field of mechanical engineering and a competent structural engineer. In his earlier years in the Department of Bridges, he worked on general bridge design. Mr. Joachimson also was a patent attorney, practicing as often as his regular duties permitted. After his retirement from the department he maintained an office, for several years, as consulting engineer and patent attorney.

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[1] Memoir prepared by Isidore Delson, M. Am. Soc. C. E.

He found time for the hobby of photography and also for extensive travel in the United States, Canada, South America, and Europe. After his retirement he spent his winters in Florida. Mr. Joachimson was an active member of Beethoven Lodge, Free and Accepted Masons, of New York. He belonged to the Society of Professional Engineers.

On November 4, 1900, he was married to Lena Kallman of New York City. He is survived by his widow; by his daughter, Beatrice (Mrs. Fred Behr); by two grandchildren, Donald and Cecily Behr; and by a sister, Clara (the widow of Dr. J. Leon Lascoff).

Mr. Joachimson was elected a Member of the American Society of Civil Engineers on March 4, 1914. He became a Life Member in January, 1939.

FRANK MINITREE JOHNSON, M. Am. Soc. C. E.[1]

Died March 27, 1944

Frank Minitree Johnson was born at his paternal grandfather's home near Berryville, Va., on November 4, 1872. His parents were Robert Marshall and Katherine (Minitree) Johnson, both descendants of well-known Virginia families. His family home was in Washington, D. C., where his father was president of the Metropolitan Iron Works.

F. V. Hayden, then head of the U. S. Geological and Geographical Survey of the Territories, in the eighteen seventies, was a frequent visitor at the Johnson home. Young Frank as a small boy was fascinated by Dr. Hayden's experiences in the West, and he would ask numerous questions about the survey work. From these contacts Mr. Johnson acquired his interest in the outdoors and in survey work.

Mr. Johnson was educated in private schools in Washington, D. C., and was graduated from Central High School. He then attended the Corcoran Scientific School of the Columbian University (later George Washington University) for a year. For the next five years he was employed on various engineering jobs, including two years as a draftsman at the Cramp shipyards in Philadelphia, Pa.

In 1895 the U. S. Geological Survey began its subdivision of the lands of the Five Civilized Tribes in the Indian Territory, and, welcoming an opportunity to do survey work in the West, Mr. Johnson, in September, 1895, joined that organization as an axman on one of the field parties. A few months later he was advanced to surveyor in charge of a field party and continued in charge of various field parties until the completion of the field work in June, 1898. For the next five months he was engaged on office computations, and resigned on November 30, 1898.

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[1] Memoir prepared by Robert Follansbee, M. Am. Soc. C. E.

As a result of his experience in making public land surveys, he was appointed an examiner of surveys in the General Land Office in March, 1899, and during 1899 and 1900 he spent the greater part of the time in New Mexico in connection with the survey of the Las Vegas Grant. Because of his friendliness and genuine interest in people, Mr. Johnson acquired a wide acquaintance with many old timers who were instrumental in shaping New Mexico's history. Mr. Johnson continued as an examiner of surveys engaged on examinations, engineering investigations, and special reports, chiefly in the Rocky Mountain area, until April, 1905, during which period he spent a six-month leave of absence abroad. While in Cairo, Egypt, he met Major Edwards, a British engineer in charge of making surveys for the enlargement of the Aswan Dam, and joined the Edwards party for a few months.

In 1905 the Shoshone Indian lands in Wyoming were to be subdivided preparatory to being opened for settlement, and, under the system then in effect, the work was to be done by private surveyors under contract. This was a project of considerable magnitude, and, in April, 1905, Mr. Johnson resigned from the General Land Office and opened an engineering office in Cheyenne, Wyo., chiefly for the purpose of making those surveys. Upon the completion of that work, which was done in record time, due to the early date at which the lands were to be allotted, he resurveyed lands in western Wyoming within the Union Pacific Land Grant, and also made subdivisional resurveys and determination of the boundaries of private lands, title to which had passed from the government. During these years in Wyoming, which continued until 1910, Mr. Johnson was able to settle many long-standing disputes between stockmen and settlers over the limiting boundaries of their holdings. This was accomplished not only because of his familiarity with the engineering problems involved, but also because of his complete understanding of human nature and its many frailties. He had the rare gift of being able to instil, in those affected, confidence in the fairness of his decisions.

In 1910 Congress, to meet more adequately the increasing need for public land surveys to provide for the rapid settlement then taking place and because of the increasing difficulty in obtaining contracts within authorized rates of compensation, abolished the old contract system, and directed the General Land Office to make future land surveys with its own personnel. From his experience both as an examiner of surveys for the General Land Office and later as a contract surveyor, Mr. Johnson was eminently fitted to head the new organization. Accordingly, in July, 1910, he was appointed U. S. Supervisor of Surveys to organize the Cadastral Engineering Service, the personnel of which consisted of former examiners of surveys and contract surveyors who had been previously associated with him. Headquarters was established in Denver, Colo., and within a short time thirteen regional offices were established in the public land states and Alaska.

Beginning with a dozen chiefs of field parties, the number was increased to one hundred and sixty chiefs of field parties by 1915. At first the Cadastral Engineering Service confined its activities to making the field surveys, and the office work connected with preparing the land plats was under the Surveyor General in each public land state. This division was not wholly satisfactory, and in 1925 Mr. Johnson was largely instrumental in having legislation enacted which abolished the position of Surveyor General and transferred the organizations to the Cadastral Engineering Service, thus bringing into one organization both the field and the office work. Under Mr. Johnson's wise leadership the land surveys were raised to a high standard of accuracy without materially increasing the cost of the work. During those years many seemingly unsurmountable obstacles were encountered, but through his efforts these crises were overcome. As a result of his genuine interest in people, his office door was always open to the lowest field assistant, who came out feeling that his chief was very much interested in his welfare. Mr. Johnson had the priceless attribute of inspiring loyalty in his associates who have been unanimous in giving him a major part of the credit for the successful organization and operation of the Cadastral Engineering Service.

Paying tribute to Mr. Johnson, the Secretary of the Interior, Harold L. Ickes, stated:

> "The death of Mr. Johnson is a distinct loss to the Government which he served so long and so well. Not only was he recognized as an outstanding authority on public land matters, but the skill and painstaking care with which he installed in 1911 (1910) and supervised until the time of his death the Government's Cadastral Engineering Service, will stand as a lasting monument to his memory."

On October 17, 1917, Mr. Johnson was married to Lucy E. Barham, of Texas, who survives him. This was a happy marriage, and, in the spirit of southern hospitality, the Johnsons kept a delightful open house for the officers of the Cadastral Engineering Service and for their many friends among the engineers.

One of Mr. Johnson's diversions was the study of history, particularly that of the West, and because of that study, as well as because of his many years of experience, he was an authority on the settlement of the West. During his earlier years, particularly, he knew some of its most colorful as well as notorious characters, and toward the latter he showed the tolerance, forbearance, and kindness of heart which he always felt toward his fellow men.

Mr. Johnson was deeply interested in boys approaching manhood and was ever ready to counsel and advise them against the pitfalls of life. He derived great satisfaction from giving six boys a college education.

Mr. Johnson was elected a Member of the American Society of Civil Engineers on August 29, 1927.

FRANK RHYMAL JUDD, M. Am. Soc. C. E.[1]

Died March 19, 1944

Frank Rhymal Judd was born in Hamilton, Ont., Canada, on May 28, 1882, of English parentage, the son of Albert Judd and Eliza Jane (Dillon) Judd. Through naturalization he became an American citizen on October 14, 1904. His father was a veteran railroad man in England, British India, and, subsequently, in Canada and the United States.

When Mr. Judd was young, his parents moved from Canada to Chicago, Ill. His boyhood days were largely spent in that city, and he received his education in Chicago. Upon being graduated from the Forrestville Grade School, he enrolled in the Chicago Manual Training Institute where he took a preparatory engineering course, including shop practice, being graduated in June, 1899. He later supplemented this training with courses in civil engineering, bridge engineering, and mechanical engineering.

After his graduation from the Chicago Manual Training Institute, he obtained employment in the Pullman Company Shops at Chicago. In November, 1899, he entered the service of the Illinois Central Railroad Company as a draftsman. During his employment with that company he held positions at various times as a chainman, rodman, instrumentman, structural designer, chief draftsman, assistant engineer, and Engineer of Buildings. With the exception of about ten months, when he worked as an instrumentman on a survey in Texas for the Chicago, Rock Island and Gulf Railway Company and as a draftsman with the Crane Company in Chicago, his entire professional career was with the Illinois Central Railroad Company.

Mr. Judd was appointed Engineer of Buildings for the Illinois Central Railroad Company in November, 1915, and held this position until ill health caused his retirement on August 31, 1943. While holding this position, he was in charge of the design and construction of all buildings on the railroad system and responsible, among many other projects, for the design and construction of the important railroad shops at Paducah, Ky., and Markham Yards, Chicago; water storage reservoirs; the passenger and freight stations at Baton Rouge, La.; the passenger stations at Champaign, Ill., and Jackson, Miss.; the railroad company's nine-story office building at 63d Street in Chicago; and levee protection for the railroad company's properties at New Orleans, La.

In addition to his membership in the Society, Mr. Judd belonged to the American Railway Engineering Association and the American Society for Testing Materials. While a member of the American Railway Engineering Association, he served on various committees, was chairman of Committee No. 6 for buildings, and was responsible for the preparation of specifications for railway buildings, which are used extensively on railroads in the United

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[1] Memoir prepared by W. Raynes, Chf. Clerk, Bldg. Dept., Ill. Cent. R. R. Co., Chicago, Ill.

States. He was licensed to practice as a structural engineer in the State of Illinois, as a civil and structural engineer in the State of Indiana, and as a civil engineer in the State of Louisiana.

Mr. Judd enjoyed playing tennis and lawn bowling and took a keen interest in following the tennis championship matches. Interested in singing, at various times he sang in church choirs. He was an Episcopalian. He was a member of Garden City Lodge, Ancient Free and Accepted Masons No. 141, Chicago; Member of Ancient Scottish Rite of Free Masonry of Valley of Chicago, Thirty-Second Degree; member of Medinah Temple, Ancient Arabic order, Nobles of the Mystic Shrine, Chicago.

On September 27, 1910, in Chicago, he was married to Janet Gilmour MacLeish. He is survived by his widow and two daughters, Elizabeth Janet (Mrs. Holbrook P. Frazier) and Marjorie Jane (Mrs. Walter S. Smith, Jr.).

Mr. Judd was elected an Associate Member of the American Society of Civil Engineers on March 2, 1909, and a Member on September 8, 1941.

JULIUS KAHN, M. Am. Soc. C. E.[1]

DIED NOVEMBER 4, 1942

Julius Kahn was born at Münstereifel, Germany, on March 8, 1874. His parents, Joseph and Rosalie (Kohn) Kahn, brought their gifted children to America in 1881, settling in Detroit, Mich. This immigrant family of six sons was destined to influence greatly the architecture and building construction of many lands.

Julius Kahn was educated in the public schools of Detroit and at the University of Michigan, in Ann Arbor, from which he was graduated in June, 1896, with the degree of Bachelor of Science in Civil Engineering. Three years later the University of Michigan awarded him the degree of Civil Engineer.

During the three years after his graduation (June, 1896, to October, 1899), Mr. Kahn supplemented his theoretical engineering education by drafting and field work with (1) Union Bridge Company of Athens, Pa., (2) Bureau of Yards and Docks, U. S. Navy, and (3) C. W. Hunt Company, New York, N. Y.

Julius Kahn had a keen interest in Japan throughout most of his adult life. His first trip to that country was made in 1899, when he went to Tokyo to join the engineering staff of Frazar and Company. While employed by C. W. Hunt Company in New York, he received an offer through E. W. Frazar to become engineer on the development of a sulfur mine on the Island of Etorofu-jima in the Kurile group. Mr. Kahn accepted this position with his usual enthusiasm. He spent most of 1900 on that assignment, which he completed successfully. The experience had a pronounced bearing on his later life. He studiously acquired a knowledge of the Japanese language. In addition, he noted the differences between Japanese and American ideals and morals.

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[1] Memoir prepared by O. W. Irwin, Assoc. M. Am. Soc. C. E.

Mr. Frazar was so impressed with Mr. Kahn's zeal, enthusiasm, and ability, especially in mathematical analysis, that he offered him another engagement, but at the same time recommended that he return to the United States where his unusual ability should assure him a successful career.

On returning to Detroit in January, 1901, Julius Kahn became associate partner to his older brother, Albert Kahn. This was the beginning of a lifetime team in America's construction era: Julius Kahn, the inventor and manufacturer, and Albert Kahn, the artist and architect, both specializing in industrial construction.

The young engineer and the young architect pioneered in concrete and reinforced concrete. Julius Kahn soon became aware of the diagonal tension problem in this new building material when subjected to bending. His solution was to attach shear members rigidly on the main reinforcing bar ("Kahn Trussed Bar"). The success of the idea was first demonstrated on the Agricultural Building of the University of Michigan in Ann Arbor.

To promote the "Kahn System" of construction, the Trussed Concrete Steel Company was incorporated in October, 1903. The brothers and a few friends furnished the modest capital required (about $55,000), and Julius Kahn was elected president and general manager. When it became evident that the average contractor would not risk his reputation by building in "mud reinforced with whiskered rods," Mr. Kahn organized the Concrete Steel and Tile Construction Company to do the field work, often giving bond to keep the structure standing for an initial period of five or more years. This was the modest beginning of a steel fabricating business which reached a volume of about $50,000,000 per yr under Mr. Kahn's guidance.

In spite of his financial success, Mr. Kahn desired recognition as an engineer rather than as a business executive. There was always a drawing board in his home and he was constantly refreshing his memory on engineering matters. Always active himself, he questioned the ability of any man who did not display energy and enthusiasm. He had confidence only in a draftsman who worked in his shirt sleeves.

As president of the Trussed Concrete Steel Company, later the Truscon Steel Company, Mr. Kahn was a frequent visitor at the application desk of the U. S. Patent Office. From August 18, 1903, to April 9, 1940, seventy-four patents were issued in his name. Scores of others, issued in the names of his employees, bear the imprint of his versatile mind.

Among the innovations evolved by him may be mentioned:

1. The use of clay fillers between reinforced concrete floor joists.

2. The substitution of permanent sheet steel fillers for clay tile to reduce the dead weight of the floor and decrease the number of joists.

3. The use of expanded sheet steel (known as metal lath) as a base for plastering and as concrete floor slab centering.

4. The use of united steel windows, the beginning of the present large steel window and steel partition industry.

5. The development of light steel joists for floors, ranging from the original double-channel "metal lumber," cold-formed from steel strip, through the spot-welded plate girder from strip, to the so-called "bar joist" and the very light "nailer joist."

6. The planning of many load transfer and anti-crack devices for use in concrete pavements.

7. Composite steel and concrete beams, the structural shapes to carry the construction loads and then to serve as reinforcing for the concrete after the latter had set—a system later used extensively on subway and building construction.

8. The use of reinforced concrete for railway cross-ties.

In the miscellaneous fields covered by Kahn patents are improvements in: Steel hangers for wood joists, expanded metal for slab reinforcement, concrete piles, stair treads, shaft hanger supports in sash hanging and operating mechanisms, screeds for anchoring wood floors to concrete, electric welding machines, expanded steel poles and towers for transmission lines, pressed-steel tunnel-liner plate, sound-excluding windows, highway traffic markers, sheet steel culverts, and even thread protectors. Thirty seven of the seventy-four patents, most of them basic, were issued in eleven years from 1904 to 1914. The exploitation of these patented ideas contributed much to the early growth and financial success of the company.

Mr. Kahn's first love was devising new forms of reinforced concrete. The structural frameworks of many of Detroit's auto manufacturing plants are monuments to his ingenuity and to his trust in this new material. He created his own design formulas for many a solid flat slab, with or without dropped panels, even raising the panel around the column head. He originated the so-called "cantilever" floor slab, very arbitrarily selecting points of inflection which more mathematical studies later confirmed. He was never more thoroughly "at home" than when, waiving aside a draftsman who was floundering in his own mathematics, he would take the stool and sketch in how the problem should be solved.

Mr. Kahn again became interested in Japan in 1910, when the Trussed Concrete Steel Company, after becoming successful in the United States, organized an export department and began to ship its products to foreign markets. In that year the company sent its first representative to Japan. Soon after the end of World War I, the Truscon Steel Company decided to establish a factory in Japan for the manufacture of steel building materials. The plant was built at Kawasaki between Tokyo and Yokohama and the enterprise was successful from the start. At times more than a thousand people were employed and the capital increased from 500,000 yen in 1910 to 5,000,000 yen in 1941. The factory was wrecked in the great earthquake of 1923, but was soon rebuilt on a larger scale and became Japan's largest enterprise of its kind.

In 1934 Mr. Kahn visited Japan again for the purpose of negotiating a merger between the Truscon Steel Company of Japan and a competing company which was owned by the great Mitsui Company. This transaction was concluded satisfactorily and resulted in the formation of the Oriental Steel Products Company which continued to operate successfully until the start of World War II.

Mr. Kahn made lasting friendships with a number of Japan's business leaders whom he liked to entertain in his spacious home, whenever they visited the United States. He often expressed sympathy with what he considered their legitimate aspiration to control and operate their own industries, but was deeply disappointed at the growing military control and nationalism in Japan.

In 1914 the headquarters of Truscon Steel Company were moved from Detroit to Youngstown, Ohio, in order to be near the sources of steel supply. Local fabricating shops were gradually established in eighteen other cities, from Boston, Mass., to Los Angeles, Calif., and from Minneapolis, Minn., to Jacksonville, Fla. Under Mr. Kahn's guidance the company became a vast training school for reinforced concrete engineers. Many a structural engineer was given a real postgraduate course in building design and construction, not to mention salesmanship, in the Youngstown Home Office, before being transferred to one of the more than forty sales offices. Many later "graduated" into responsible positions, with consulting engineers and with contractors.

As an industrialist, Mr. Kahn recognized his social responsibility—beginning with financial assistance to deserving and promising students and often to messengers and office boys. The company early adopted incentive wage plans. Departmental heads were rewarded with liberal bonuses, based on over-all company profits. Machine operators received extra payments for output in excess of 80% of the rated capacity of the machine. Keymen named their own beneficiaries in large life insurance policies, the premiums being paid by the company. District sales managers earned handsome bonuses as their district profits exceeded their district sales expenses. Attractive prizes (usually cash) were offered in monthly sales contests where the accredited "points" were computed on each sales contract in proportion to the estimated profit in the contract. The rivalry to excel was intense, but no employee disregarded the president's motto "all for one, one for all."

Among other social ideals, Mr. Kahn early instituted "Safety" as the factory slogan. He felt that, to avoid accidents in the plant, home conditions must be favorable and encouraged his supervisors to interest themselves in the family problems of their subordinates. The company was among the first to adopt group insurance, both accident and life, to alleviate financial worry in the homes of its employees. A full-time attorney was available gratis to employees to help them with any real or imaginary legal problems. Many a mortgage foreclosure and some incipient divorces were prevented. Employees were protected in auto and streetcar accidents.

The Truscon Laboratories in Detroit, created in 1906 during the earlier pioneering days of reinforced concrete construction, also enjoyed Mr. Kahn's active leadership. The laboratories, however, were quite independent of the Truscon Steel Company. Mr. Kahn, with his scientific approach, quickly recognized the need for a laboratory not only to test Portland cement but also to provide technical research and leadership in the development of basic concrete practice. The laboratories made outstanding contributions in improving the methods of testing and inspection of Portland cement and in standardizing the technique of proportioning, placing, and handling concrete. Later the need of various dampproofings, waterproofings, and special technical coatings for concrete became apparent to Mr. Kahn, and the laboratories initiated and completed the research involved in the development of a complete line of specialized products for use with concrete construction. Although Mr. Kahn assigned the development work to his staff of chemists and laboratory technicians, he maintained a close interest and his great vision and tireless energy were constant stimulants to the laboratory organization.

In this field Mr. Kahn's genius for appropriate and effective advertising flowered. He was a firm believer in the efficacy of printer's ink and the drawing power of beautiful pictures. He made it easy for the prospective buyer to ask questions by tearing the corner off the advertisement. He preached that no question should go unanswered, no matter how illiterate the inquirer, and he boasted that some of his best sales resulted from following up postal card inquiries.

In 1935 Mr. Kahn and the directors arranged for the stockholders of the Truscon Steel Company to exchange their stock for that of the Republic Steel Corporation on a basis which Mr. Kahn felt was very favorable to the stockholders. At that time the Truscon Laboratories separated entirely from the parent company, and Mr. Kahn became chairman of the board of the laboratories, continuing until his death.

When the exchange of Truscon stock was completed in October, 1935, Mr. Kahn resigned the presidency of Truscon Steel Company to become vice-president of Republic Steel Corporation, in charge of product development, and moved to Cleveland, Ohio. To assist in this work he requisitioned several of his former associates in the Truscon Steel Company, and during the next four years he made many trips to the U. S. Patent Office.

By 1939 Mr. Kahn was homesick for the active commercial life, and, consequently, aided (with a substantial loan) four of his "boys" to incorporate the United Steel Products of Wooster, Ohio. He was a regular visitor at the Wooster plant and was very free with advice and assistance. It is related that he arrived one morning, with tickets in his pocket for a Mediterranean cruise for Mrs. Kahn and himself. When he learned that his "boys" were striving to design and sell a large order of pressed steel plates in Honolulu, Hawaii, he telephoned to Cleveland to cancel the cruise and asked Mrs. Kahn to be ready to start that evening for San Francisco, Calif., and Honolulu. When the design was completed and the order secured, the Kahns vacationed in Hollywood, Calif., for a few weeks.

After his resignation from Republic Steel Corporation in 1939, Mr. Kahn continued to live in Cleveland, looking after his many investments. He retained his connection with the Truscon Laboratories and became president of the Kahn Realty Company, both in Detroit. His office in the Terminal Building in Cleveland was the focal point for scores of ex-Truscon men, until his too early death. His visitors were never disappointed in his friendly interest, and always were surprised by his marvelous memory for mutual interests of bygone years.

Probably his most outstanding characteristic, that impresses one who has been associated with Julius Kahn for many years, was his truly democratic spirit. Notwithstanding his success in his chosen field, both from the professional and materialistic viewpoints, he remained, in the truest sense of the word, a real democrat. Any one in his organization—no matter how lowly his position—could approach and see Mr. Kahn at any time. In fact, as success crowned his efforts, the democratic spirit inherent in Mr. Kahn seemed to expand, and his greatest joy and relaxation seemed to come from an association with the men who had spent practically their entire business careers with him.

His keenness of mind and readiness in making decisions marked him as a true leader of men. In addition, he had a thirst for knowledge in every field and a mental capacity able to absorb an unlimited amount of information, which truly made him one of the intellectual giants in the steel industry of his generation.

His inventive genius, combined with his determination to succeed, gave him the incentive to contribute enormously to the introduction and progress of a new type of construction—reinforced concrete—in the building field. His daring in design and his willingness to experiment formed a combination, the results of which reached to all corners of the world. The Marlboro-Blenheim and the Traymore hotels at Atlantic City, N. J., the trestle of the Chesapeake and Ohio Railway Company at Richmond, Va., and reinforced concrete structures throughout the United States and other parts of the globe stand as lasting monuments to his genius—far more enduring than any tribute of words.

Julius Kahn was a member of the American Society of Mechanical Engineers at the time of his death.

On December 24, 1903, Mr. Kahn was married to Margaret Kohut of New York. As usual, he was most indifferent to matters of personal comfort and welfare. For his approaching wedding he neglected the buying of appropriate clothes and a ring until the day he was leaving Detroit for New York to be married. The charm, tact, and beautiful cooperation of his wife were decided factors in his success. It was not unsual for Mrs. Kahn to play the part of hostess at dinner while her husband was trying to solve an engineering problem with a pencil on the white tablecloth. Into this perfect home were born two daughters, Gisela (Mrs. William Gresser) and Katherine Kahn; and a son, Julius Kahn, Jr.

Mr. Kahn was elected a Junior of the American Society of Civil Engineers on May 3, 1898; an Associate Member on October 3, 1900; and a Member on January 16, 1917.

WINFIELD SCOTT KING, M. Am. Soc. C. E.[1]

Died November 15, 1944

Winfield Scott King, the son of Jacob King and Christine (Christiansen) King, was born in Platte County, Nebraska, on January 19, 1862.

He attended high school in Omaha, Nebr. In 1883, at the age of twenty one, he was employed by the Oregon Short Line Railroad Company, directing surveys from Wyoming and Montana to Oregon. In 1889 he became chief engineer on the construction of the Omaha stockyards. Later he did similar work in St. Paul, Minn., Lincoln, Nebr., and Fort Worth, Tex. In 1910 he entered the paving construction business with headquarters in Omaha, and in 1915 he opened an office in Omaha as a consulting engineer.

Mr. King entered the United States Army in 1917, retiring with the rank of Captain in 1920. He then moved to Tucson, Ariz., where he continued to practice his profession. In 1921 he supervised a location survey for a railroad, connecting with the Southern Pacific Company's line in Arizona, which was seeking a port on the Gulf of California. Later he was associated with the city engineering office in Tucson. He retired from active practice in 1930.

On January 4, 1888, Mr. King was married to Margaret Catherine Hall. He is survived by two sons, Ralph H. King and Fred King, and two daughters, Margaret (Mrs. Oney Anderson) and Vera (Mrs. J. H. Thomas).

Mr. King was a man of determined character with a high regard for, and understanding of, his fellow men.

Mr. King was elected a Member of the American Society of Civil Engineers on December 6, 1899. He became a Life Member in January, 1932.

WALTER HAROLD KIRKBRIDE, M. Am. Soc. C. E.[1]

Died December 20, 1944

Walter Harold Kirkbride, the son of Charles Henry and Louise (Holloway) Kirkbride, was born in Pueblo, Colo., on January 22, 1874. He went to California at an early age. After attending grammar school and the College of the Pacific at Stockton, Calif., he entered Stanford University in Palo Alto, Calif., with the pioneer class of 1891. He was graduated with the degree of Bachelor of Science in Civil Engineering in 1895.

Mr. Kirkbride began his engineering career in 1895 as a United States deputy mineral surveyor, but in 1898 identified himself with railroad construction in the capacity of assistant chief engineer on location and construction for the Sierra Railway Company of California from Oakdale to Jamestown, Sonora, Tuolumne, and Angels Camp, Calif. From February, 1902, to August, 1902, he was chief of field party on a survey for the Shasta Mineral Belt Railway Company from Anderson to Alturas, Calif.

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[1] Memoir prepared by John H. Gardiner, M. Am. Soc. C. E.

[1] Memoir prepared by C. R. Harding, M. Am. Soc. C. E.

He became associated with the Southern Pacific Company on August 13, 1902, as assistant engineer on the Sacramento Division in California, and later held the following engineering positions: From 1904 to 1906, roadmaster at Red Bluff, Calif.; from 1906 to 1909, assistant resident engineer, Sacramento Division; and, from 1909 to 1917, division engineer on Sacramento and Coast divisions and also assistant superintendent of the Sacramento Division with headquarters at Sacramento, Calif. In 1918 he was appointed chief engineer of the Pacific Lines, Southern Pacific Company, for the United States Railroad Administration, serving during World War I until March 1, 1920, when federal control of railroads ceased. After the railroads were returned to private ownership, Mr. Kirkbride was engineer, maintenance of way and structures, for the Southern Pacific Company. He held this position until 1932, when he was appointed chief engineer. Mr. Kirkbride was also chief engineer of the Northwestern Pacific Railroad Company and the San Diego and Arizona Eastern Railway Company, subsidiaries of the Southern Pacific Company.

As chief engineer of the Southern Pacific Company, Mr. Kirkbride supervised about 9,000 road miles of railroad in seven western states and participated in much of the railroad development that occurred in these states during the last forty years. Between 1918 and 1944 he supervised the expenditure of approximately $1,000,000,000 for maintenance of track, roadway, and structures, and for additions and betterments to railroad facilities.

Among the projects with which he was associated were the Redding to Delta (Calif.), 30.1-mile-long change of tracks, occasioned by the construction of Shasta Dam and reservoir for the Central Valley Project of the federal government; the Martinez-Benicia double-track railroad bridge across the Straits of Carquinez in California; the Los Angeles (Calif.) Union Passenger Terminal; the extensive railroad line change through San Jose, Calif.; railroad line relocations and reconstructions in Southern California after the 1938 floods; construction of the 400-ft span crossing the Colorado River at Yuma, Ariz., including the terminal yard tracks at Yuma and the relocation of the main line eastward of Yuma; and bridges over the Sacramento, American, San Joaquin, Los Angeles, Santa Ana, and other rivers.

After nearly forty-two years of service, Mr. Kirkbride, at the age of seventy, retired as chief engineer of the Southern Pacific Company on January 31, 1944. Because of his interest in, and knowledge of, civic affairs he was appointed by the Hon. Roger Dearborn Lapham, Mayor of San Francisco, as a member of the Board of Permit Appeals of that city. He held this office until his death at Dunsmuir, Calif., on December 20, 1944, after a brief illness.

Mr. Kirkbride was highly successful in stabilizing sinking roadbeds by applying the principles of soil mechanics. Notable among his accomplishments were the roadbeds on the Suisun marshes and Elkhorn slough in California, and on the Lucin cutoff across Great Salt Lake in Utah. He also introduced the principle of drain tunnels to correct sliding fills. In connection with the adaptation of hydraulics to river control, he personally developed certain types of jetties and other protective devices which resulted in the permanent protection of the railroad from flood erosion by rivers and by streams flowing in alluvial soil on steep gradients.

Mr. Kirkbride was consulted by the first group of engineers engaged to study the problem of the San Francisco (Calif.) Bay Bridge crossing; and, since the final construction plans for the San Francisco-Oakland Bay Bridge provided for the operation of the Southern Pacific electric lines, he took a prominent part in the studies relating to the terminals of the bridge and in the construction problems affecting the substructure. He was also on the committee appointed by the Hon. James Rolph, Jr., then Governor of California, to make a preliminary investigation of the Central Valley Project. This study culminated in the adoption of the site of the Shasta Dam and its ultimate construction.

During his years of railroading, Mr. Kirkbride was responsible for the development of many railroad devices, including the rail gage plate, of which he was sole inventor, adopted in 1930 and used as a standard on the Southern Pacific Lines to protect railroad ties from mechanical wear and to prevent spread track. He prepared numerous papers on engineering and construction subjects. He received the Thomas Fitch Rowland Prize from the Society in 1935 for his paper on "The Martinez-Benicia Bridge."[2] Mr. Kirkbride participated in the editing of "Marine Borers and Their Relation to Marine Construction on the Pacific Coast," in 1927[3] and "Termites and Termite Control," in 1934.[4] He was chairman of the standardization committee of the Southern Pacific Lines.

Mr. Kirkbride conceived the idea for the monument erected at the Sacramento railroad passenger station, in memory of Theodore Dehone Judah,[5] M. Am. Soc. C. E., the first chief engineer of the pioneer Central Pacific Railroad (later the Southern Pacific Lines). This memorial was dedicated on February 25, 1931, and a mural was placed in the Sacramento station showing the "Driving of the Last Spike." These tributes to Mr. Judah were financed by voluntary subscriptions from the personnel of the Southern Pacific Company.

A member of the American Railway Engineering Association in 1915, Mr. Kirkbride was director from 1925 to 1928, and served on the rail committee from 1933 to 1944. He also belonged to the American Wood-Preservers' Association (life member in December, 1944); the Engineers', Commercial, and Commonwealth clubs, all of San Francisco; the Pacific Railway Club (past-president); the California Historical Society; and the Seismological

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[2] "The Martinez-Benicia Bridge," by W. H. Kirkbride, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 154.

[3] "Marine Borers and Their Relation to Marine Construction on the Pacific Coast," Final Report of the San Francisco Bay Marine Piling Committee, C. L. Hill and C. A. Kofoid, Editors-in-Chief, published by the Committee, San Francisco, Calif., 1927.

[4] "Termites and Termite Control," Charles A. Kofoid, Editor-in-Chief., Univ. of California Press, Berkeley, 1934.

[5] For memoir, see *Transactions*. Am. Soc. C. E., Vol. XXXVIII (1897), p. 448.

Society of America (director from 1933 to 1944 and president when he died). He was a member of California Lodge No. I, Free and Accepted Masons, in San Francisco, and of Phi Kappa Psi fraternity. He was a Past-President and former Director of the San Francisco Section of the Society.

Recognized as one of the nation's foremost railroad engineers, Mr. Kirkbride was noted for his thorough knowledge of the engineering details encountered in daily activities, and for his earnestness and integrity of purpose and judgment in the solution of the many intricate problems which were his responsibility. Possessing a vigorous mentality and high moral courage, he could present his conclusions with forceful ability and usually convinced those who sought his counsel.

In 1902 he was married to Martha Sims of Nevada City, Calif. He is survived by his widow and a daughter, Audrey (Mrs. Harold Miles Tucker). Mr. Kirkbride had a pleasing personality and was modest and unassuming; he was a devoted husband and father—in short, he was a good American and no higher tribute can be paid to any man.

Mr. Kirkbride was elected a Member of the American Society of Civil Engineers on November 3, 1915. He would have become a Life Member in January, 1945.

FREDERIC ARNOLD KUMMER, M. Am. Soc. C. E.[1]

Died November 22, 1943

Through the death of Frederic Arnold Kummer the American Society of Civil Engineers lost one of its most individual and internationally known members. He was born in Catonsville (Baltimore), Md., on August 5, 1873. After receiving his preliminary education in the public schools and high schools of Baltimore, he was graduated from Rensselaer Polytechnic Institute at Troy, N. Y., in 1894.

Mr. Kummer was first employed, for more than a year, on the editorial staff of the *Railroad Gazette*. In 1896 he resigned to become salesman and engineer for the American Wood Preserving Company of Philadelphia, Pa. He remained with this company until 1898 designing and erecting small plants for the treatment of railroad ties and bridge timbers for various rail and railroad companies. He resigned in 1898 to accept employment as New York representative and later as general manager at Catskill, N. Y., for the Eastern Paving Brick Company. In this capacity, Mr. Kummer had direct charge of all the paving work and contracts for paving brick amounting to more than twelve million bricks per year, and also directed the extensive plant manufacturing of the company. Two improved permanent down-draft brick kilns of the largest size (five 100-ft brick chimneys) were designed by him and erected under his supervision. He also directed track construction in and around shale clay banks.

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[1] Memoir prepared by Frederick Philip Stieff, Vice-Pres., Chas. M. Stieff, Inc., Baltimore, Md.

On June 6, 1900, Mr. Kummer presented a paper on "A Proposed Method for the Preservation of Timber,"[2] for which he won the Society's Collingwood Prize for Juniors in 1902.

In 1901 he was appointed general manager of the United States Wood Preserving Company of New York, N. Y. While with this company, he was in sole charge of constructing modern wood block pavements in most of the principal cities of the United States for streets, bridge floors, etc. This work included most of the main streets in lower New York, including Broadway, and many other paving contracts presenting unusual and novel features. He also directed the extensive plant producing the paving material. The projects under Mr. Kummer's supervision exceeded a quarter of a million square yards of pavement a year—a substantial amount for those days.

In spite of the meticulous exactitude required of an engineer, beneath the surface of Frederic Arnold Kummer there lay a definite tendency toward the artistic, the creative. He had casually toyed with painting in oils and was finally importuned by his artistic friends to show his work at one of the better annual exhibitions. In 1903 he sent a canvas, "A Summer Sea," to the Seventy-Second Annual Exhibition of the Pennsylvania Academy of Fine Arts. It was accepted and hung with canvases of Whistler, Sargent, Homer, Oakley, Innes, Borglum, Sloan, Henri, and other first ranking contemporary artists.

Mr. Kummer also continued his literary career. On November 21, 1906, he read a treatise on "The Development of Wood Block Pavements in the United States" before the Boston Society of Civil Engineers.[3]

In 1907 Mr. Kummer and his second wife traveled extensively in Europe, presumably in the interests of his profession. However, he saw and absorbed far more than engineering data. Later this was to serve him well. At this juncture in his life a holocaust occurred which caused a complete metamorphosis of his entire career, for, on his return to the United States, he found that the firm by which he had been retained had ceased to exist. As a result, Mr. Kummer had to pause to consider his next step carefully—as to what business or other career offered an advantageous future. His pen had upheld his engineering development well from the start, and his imagination had fed both his pen and his profession. Thus, it was that Frederic Arnold Kummer decided to write. He contributed short stories to the old *Smart Set, Blue Book, Red Book, Ainslee's,* and many other of the most popular current periodicals.

His was a unique case. Many authors, or followers of the arts, have begun their careers in distasteful employment, either through misguidance or duress, and only later have they been freed from uncongenial oppression to follow their literary bents. For example, Sir Arthur Conan Doyle had been a doctor; Rimski-Korsakov was trained as an officer in the Russian Navy. Unlike these men, Mr. Kummer had shown definite qualifications as an established engineer, pointing to future and greater successes in that profession.

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[2] *Transactions,* Am. Soc. C. E., Vol. XLIV (December, 1900), p. 181.

[3] "The Development of Wood Block Pavements in the United States," by Frederic A. Kummer, *Journal,* Assn. of Eng. Societies, November, 1906.

The accuracy of his intuition was evidenced by the immediate and continued success of his short stories. This was responsible in 1910 for his first play, "Mr. Buttles," which was developed from a novelette and starred Henry Dixey, then a well-known actor. In Mr. Kummer's first year as a playwright, he wrote his second play, "The Other Woman," starring Blanche Walsh. Then there fairly burst forth from his versatile pen, books of verse (blank and otherwise), novels, detective stories, histories, sophisticated satire, juvenile books for musical comedies, a libretto for an opera, plays, movies, etc. He was the most prolific and versatile literary personality Baltimore has produced with the exception of Edgar Allan Poe. Among his works are:

1911: "The Green God," a novel.

1912: "The Brute" and "The Diamond Necklace," both novels also produced as plays; and, under the nom de plume of Arnold Fredericks, "One Million Francs," a novel, and "The Ivory Snuff Box" (first a serial, then a novel, later a motion picture, and finally a book translated into Braille).

1913: "The Second Sixpence," a novel, and "The Painted Woman," a drama (produced in Baltimore and New York, starring Florence Reed).

1914: "A Lost Paradise," a novel, and "Many Happy Returns" (a production of the Paint and Powder Club of Baltimore).

1915: "The Blue Lights" and "The Little Fortune," both novels under the name of Arnold Fredericks; and "The Second Coming," a novel written in collaboration with Harry Pratt Janes.

1916: "The Slave Market" (a new version of "The Painted Woman" produced as a motion picture by Jesse Lasky with Pauline Fredericks in the leading rôle; in 1917 this play was also published as a novel).

1917: "The Film of Fear," a novel under the name of Arnold Fredericks.

1919: "The Battle of Nations," a children's history of World War I (published in both England and Canada); two novels, "The Web" and "Peggy-Elsie" (in collaboration with Mary Christian); and two musical comedies, "The Magic Melody," with music by Sigmund Romberg, and "My Golden Girl," with music by Victor Herbert.

1920: "The Bonehead," a play.

1921: "The Pipes of Yesterday," in collaboration with Mary Christian, and "Plaster Saints," both novels.

1922: "The First Days of Man," the first volume of "The Earth's Story." (This series was later translated into German, Norwegian, Swedish, and Danish.)

1923: "The First Days of Knowledge," the second volume of "The Earth's Story," and "The Voice," a play.

1924: "The First Days of History," the third volume of "The Earth's Story," and "Phryne," a drama in blank verse.

1925: "The Road to Fortune," a novel.

1927: "Love's Greatest Mistake," a novel.

1928: "Ladies in Hades," a sophisticated satire.

1929: "Maypoles and Morals," a satire; and, as Arnold Fredericks, "The Mark of the Rat."

1930: "Gentlemen in Hades," a satire.

1931: "Forbidden Wine," a novel.

1932: "The Golden Piper," a novel.

1933: "Red Clay," a novel; and, under the pseudonym of Arnold Fredericks, "The Spanish Lady," also a novel.

1934: "Manhattan Masquerade," a detective story.

1935: "The Captive," a grand opera ("The Painted Woman") with music by Gustav Strube, first conductor of the Baltimore Symphony Orchestra, successfully produced in Baltimore.

1936: "Design for Murder," a detective story.

1937: "Death at Eight Balls," a detective story.

1938: "The Scarecrow Murders" and "The Twisted Face," both detective stories; and "The Great Road," a child's book.

1939: "Leif Erikson the Lucky," an historical story for adolescents.

1940: "Courage Over the Andes," another juvenile novel.

1941: "Torch of Liberty," his last book for children.

1943: "Perilous Island," a novel.

In 1941 Mr. Kummer's health began to fail. Although for long periods he was confined to his bed, his keen and alert mentality and indefatigable courage never failed him. He never "merely wrote." His works were sponsored by the leading publishers of his day, and during his final ailing years "The Great Road," "Leif Erikson the Lucky," and "Courage Over the Andes" were chosen by the Juvenile Book of the Month Club. The meticulousness his engineering training had bred in Mr. Kummer never permitted him to compromise with true merit. His last books maintained the standard from which he had never deviated during his entire literary career.

"Perilous Island," staged in the Aleutians, written just before his death was his last published novel and an extremely successful one. Posthumously, his serial "Michael Blake and the Minotaur" appeared in *Boys' Life*, the monthly magazine of the Boy Scouts of America.

During his lifetime, Mr. Kummer also continued his art career and painted numerous pictures, among them "The Yellow Pawn," "Motherhood," and "The Belgian."

"Fritz" Kummer, as he was affectionately known to his intimates, was a definite personality. He had a most alert and agreeable mentality. His letters to the discussion columns of the daily press covered fields ranging from the Spanish Revolution to the pay of local school teachers. Vitally interested in all paramount problems of the day, he followed the progress of the world and of the war via radio with utmost fidelity.

In 1894 Mr. Kummer was married to Clare Redmond Beecher, later to become known as the playwright Clare Kummer. A daughter, Marjorie Beecher Kummer, was married to Roland Young, the stage and screen actor. A second daughter, Frederica, died in infancy. His first marriage ended in divorce, and he was married for the second time in 1907 to Marion J. McLean. By his second marriage he had three children, Marion McLean (Mrs. Ernest E. Wachsmuth), Frederic A., Jr., and Joseph T. T. Mr. Kummer was essentially a man who lived with, and for, his family. Always interested in aspirants for literary achievement, he was ever willing to help with advice and counsel and proved a valuable literary "father confessor" to many. His counsel was, however, honest and sincere and not for those who preferred praise to improvement. Consequently his advice was desired and esteemed.

Frederic Arnold Kummer was a member of the Society of American Dramatists and Composers, The Authors League of America, and Chi Phi fraternity. He belonged to the Paint and Powder Club of Baltimore. This unique organization has been in existence for more than a half century. An amateur theatrical club, until recently of stag membership, during the years of its existence, it has operated only in behalf of charity and contributed to charitable organizations more than a hundred thousand dollars. Among its members are such nationally known personalities as Harry Lehr and Baldwin Sloan, the latter being the only Baltimorean except Mr. Kummer who had three plays playing on Broadway simultaneously. In 1922 Mr. Kummer was elected president of the club and that year sponsored a most successful revival of "The Runaway Girl."

Baltimore lost an outstanding personality on the death of engineer-author-playwright-artist Kummer; the Society, a most highly esteemed member; and the world at large, an able and respected historian, a successful playwright, and a teller of tales of deservedly international popularity.

Mr. Kummer was elected a Junior of the American Society of Civil Engineers on April 30, 1895; an Associate Member on March 4, 1903; and a Member on October 2, 1906.

EDWARD ROWLAND LEWIS, M. Am. Soc. C. E.[1]

Died August 13, 1944

Edward Rowland Lewis, the son of David Mynard and Mary (Rowland) Lewis, was born in Raritan, N. J., on November 20, 1869. His father was a civil engineer who practiced in New York, N. Y., and later moved to New Jersey, where he became interested in railroad contracting. This business took him to various states in the Middle West and his family followed him. Young Edward's primary education and high school education were received in the public schools of Champaign, Ill. In 1885 he entered the University of Illinois at Urbana, remaining for one year.

In the summer of 1886, the family removed to Atlantic, Iowa; and in September, 1886, Edward Lewis entered the University of Iowa, at Iowa City, from which he was graduated in 1890 with the degree of Bachelor of Science in Civil Engineering. In 1906 he received the degree of Civil Engineer from this same institution.

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[1] Memoir prepared by J. E. Bebb, Assoc. M. Am. Soc. C. E.

During his summer vacations while at the University, he worked for the Missouri Pacific Railroad Company as an axman, rodman, and clerk on location. After graduation, he served as assistant engineer of construction for the Omaha Southern Railway. In June, 1892, he went to Sheridan, Wyo., where he engaged in private practice, specializing in survey and irrigation work. He compiled a map of the City of Sheridan and laid out many of the streets, sewers, and other municipal works. In May, 1896, he accepted a position as division engineer, Mississippi River Improvements, for the U. S. Government, on levee location and construction, soundings, and mapping.

He did not, however, remain long in government employ. Early in 1899 Mr. Lewis went to England and then to South Africa where he worked for a short time for the Electric Tramways Company of Cape Town. Later he became associated with the British Cape Government Railways as assistant engineer on location and construction of government-owned railway lines, including the design and construction of bridges, buildings, water service, and other railroad facilities. This work extended from Cape Colony well into equatorial Africa.

Since the Boer War, which began in 1899, caused a cessation of railroad construction, Mr. Lewis returned to the United States in 1901. He accepted a position as engineer of construction for the White River Railroad (later a part of the Missouri Pacific Railroad Company) at Batesville, Ark., and later was appointed division engineer of the Fort Smith and Western Railroad. In the fall of 1902, he returned to South Africa and resumed his connection with the Cape Government Railways, as locating engineer and district engineer at Port Elizabeth and Mafeking.

In the early winter of 1905, he returned to the United States and became assistant engineer on the Keweenaw Central Railroad Company (later part of the Copper Range Railroad Company). In August, 1906, he was appointed division engineer, Michigan Central Railroad Company, with headquarters at Bay City, Mich. He held this position until June, 1912, when he became assistant to the general manager for the Duluth, South Shore and Atlantic Railway Company at Duluth, Minn., in charge of an extensive program of bridge and building renewals. In 1918, he was appointed chief engineer of the Duluth, South Shore and Atlantic Railway Company, with headquarters, as before, at Duluth.

Mr. Lewis was keenly interested in technical journalism and had contributed many articles to the technical press. He was the author of a volume entitled "Winter Track Work."[2] Thus, it was an entirely logical decision on his part when, in March, 1919, he accepted a position on the editorial staff of the Simmons-Boardman Publishing Corporation of Philadelphia, Pa., publishers of *Railway Age* and *Railway Engineering and Maintenance.* He became managing editor of the "Maintenance of Way Cyclopedia"[3]—a monumental work embodying all phases of railway design, construction, and maintenance. This volume was completed early in 1921 and in March, 1921, Mr. Lewis was appointed office engineer of the Michigan Central Railroad Company with offices at Detroit, Mich. In January, 1924, he was appointed principal assistant engineer and served in that capacity until June, 1933, when he resumed his former position as office engineer. He retired on December 1, 1939.

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[2] "Winter Track Work," by E. R. Lewis, Ry. Educational Press, Inc., Chicago, Ill., 1917.

[3] "Maintenance of Way Cyclopedia," edited by E. T. Howson and others in cooperation with the A.R.E.A., Simmons-Boardman Pub. Corp., New York, N. Y., 1921.

After his retirement, Mr. Lewis made his home at St. Petersburg, Fla., but spent the summer months each year in the north. He died at St. Petersburg on Sunday, August 13, 1944.

In June, 1901, Mr. Lewis was married to Madeline Young, of La Crosse, Wis., who survives him. They had no children. He was a member of the Presbyterian Church.

Among railroad engineers, Mr. Lewis was recognized as an authority on track construction and maintenance, and his associates knew him as a kindly gentleman and a loyal friend. His long experience and world-wide travel gave him a fund of knowledge from which, when the spirit moved him, he delighted to draw and regale his friends. He was a raconteur of no mean ability. His loss will be keenly felt.

In 1906 he became a member of the American Railway Engineering Association and served for many years on various important committees. He was chairman for three years of the committees on railway labor. He also belonged to the Detroit Engineering Society and the Detroit Economic Club.

Mr. Lewis was elected a Member of the American Society of Civil Engineers on May 17, 1913.

FREDERICK CHARLES LOHMANN, M. Am. Soc. C. E.[1]

DIED JANUARY 1, 1944

Frederick Charles Lohmann was born in Gibson City, Ill., on October 26, 1884, the son of Louis Henry and Etta (Harnack) Lohmann.

After completing his secondary and high school studies in Gibson City, Mr. Lohmann entered the University of Illinois, at Urbana, from which he was graduated in 1911 with the degree of Bachelor of Science in Civil Engineering.

Immediately after graduation, Mr. Lohmann became an inspector on the construction of the high school in Gibson City. In November, 1911, Mr. Lohmann was appointed assistant city engineer of Champaign, Ill., and, in May, 1915, he became city engineer of Champaign. From 1927 to 1931, Mr. Lohmann was superintendent of building construction for the University of Illinois.

In 1931 he again became city engineer of Champaign, continuing in this capacity until 1935. From 1935 to 1939 he served as director for the Works Progress Administration in Champaign County. From 1939 until his death, Mr. Lohmann was employed by the U. S. Army Engineers. His first work with the Army Engineers was the rebuilding and enlarging of Chanute Field,

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[1] Memoir prepared by Wallace M. Lansford, M. Am. Soc. C. E.

Ill. Later he was transferred to the Columbus (Ohio) Office of this department.

On June, 5, 1915, he was married to Ada L. Kennedy of Brookston, Ind. Mrs. Lohmann died in April, 1943, after an illness of more than eight years. Mr. Lohmann is survived by two sisters, Emma (Mrs. L. F. Whallon) and Laura; and a brother, Lawrence.

Mr. Lohmann belonged to the First Presbyterian Church of Champaign and the Masonic Fraternity, Western Star Lodge of Champaign. He was a member of the Western Society of Engineers, the Illinois Society of Engineers, Inc. (past-president), and the Central Illinois Section of the American Society of Civil Engineers (past-president).

He was honored and respected by his many friends for his sterling character and for his professional loyalty, honesty, and perseverance.

Mr. Lohmann was elected a Member of the American Society of Civil Engineers on January 15, 1923.

MOSES JEROME LOOK, M. Am. Soc. C. E.[1]

DIED JULY 11, 1944

Moses Jerome Look, the son of Warren Jerome and Delphina (Look) Look, was born at Addison, Me., on February 24, 1869. He attended the grammar and high schools of Boston, Mass., and studied civil engineering at Massachusetts Institute of Technology in Boston.

After graduation in 1892, Mr. Look was in charge of a field party on various engineering projects for E. A. W. Hammatt,[2] M. Am. Soc. C. E. In August, 1893, he entered the field of water supply engineering in which he was destined to continue for the remainder of his life. At that time the Massachusetts State Board of Health, of which Frederic P. Stearns,[3] Past-President, Am. Soc. C. E., was chief engineer, was just beginning surveys and investigations for a metropolitan water supply, and Mr. Look became transitman on one of the survey parties which operated in the vicinity of West Boylston and Clinton, Mass., in an area which was later to become the site of the Wachusett reservoir. He served in this position until February, 1895, and then was briefly engaged on the construction of Payson Park reservoir for the City of Cambridge (Mass.) and on surveys for improvement of the Neponset River.

When the Metropolitan Water Board was established in December, 1895, an engineering force was organized on which, until 1905, Mr. Look served as assistant engineer and division engineer for the Wachusett reservoir on surveys, studies, plans, and construction, including work on north and south dikes and the location and construction of the Central Massachusetts Railway and of several miles of highways made necessary by the building of the reservoir.

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[1] Memoir prepared by George W. Booth, M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 1698.

[3] For memoir, *ibid.*, Vol. LXXXIII (1919–1920), p. 2132.

MacArthur Brothers and Winston and Company was the contractor for the main Wachusett dam and certain related works. The energy and executive talents of the young engineer were so impressive that, in 1905 upon completion of the Wachusett work, this firm employed Mr. Look as engineer and general superintendent. From 1917 until 1930 he remained with MacArthur Brothers and Winston and Company in the same position and as vice-president and manager for Winston and Company.

During these twenty-five years Mr. Look was engaged on the following construction contracts, most of them of outstanding importance and many involving difficult problems: Cross River dam and reservoir at Katonah, N. Y., for the City of New York, N. Y.; Ashokan dam and dikes and the Catskill headworks; a dam and a reservoir for Pittsfield, Mass.; shipbuilding at Kingston, N. Y., and West Point, Va.; Camp Abraham Eustis and a balloon school in Virginia; Skiff Creek dam and reservoir for Newport News, Va.; Scituate dam, dikes, and purification works for Providence, R. I.; Cooper Lake dam and reservoir, Kingston, N. Y.; Cobble Mountain dam and reservoir, Springfield, Mass.; and Alcove dam, Albany, N. Y. During this period Mr. Look conducted numerous investigations and made reports and estimates for bids on various water supply and railroad projects.

In 1930, having already had experience as consultant and expert engineering witness, Mr. Look established his office as a consulting engineer, practicing for the remainder of his life in that field. He was engaged in numerous instances by New York City, Springfield and Fitchburg, Mass., and the Commonwealth of Massachusetts.

Mr. Look was married in 1893 to Cora Groves. Mrs. Look died in 1895. In 1900 he was married to Amy G. Wood. He is survived by his widow; a son by his first marriage, Frederick W. Look; and two daughters by his second marriage, Laura (Mrs. Galland Frank) and Miriam (Mrs. Donald B. MacMillan).

The fact that Mr. Look was so successful was evidence of his versatility, his fine character, and good judgment. He possessed tremendous energy and was never content to be idle. Insight into a man's real life can often be obtained by observation of his off-hour activities, his hobbies, and his so-called play. Born of a seafaring family on one of the finest seacoasts in the world—that of Maine—naturally Mr. Look was a lover of the sea, and all that pertains to it. From the door of his summer home in Casco Bay he could see Eagle Island and the cottage of Robert E. Peary, Admiral, U. S. Navy. Mr. Look's interest in northern adventure and arctic exploration was such that he was elected president of the MacMillan Arctic Association year after year.

When Donald B. MacMillan, Commander, U. S. Navy, and his seven men were marooned for four years far up on the northern shores of Greenland, Mr. Look was the first to contribute toward their relief which was accomplished successfully in 1917 after repeated failures.

Commander MacMillan, through his intimate associations with Mr. Look when "off the job," knew him as few men did, and paid him the following tribute:

> "We are all the better for his having lived and his influence will be with us always, as he will be. When a man lives on in the hearts of those who love and have loved him, then he has fulfilled his mission in life. Praise and honors and standing in the community are nothing in comparison to this. The rank of any man is his power to uplift others, contribute something in the altogether too short time allotted us. One is indeed fortunate if he happens to be blessed with that power, and Jerry had it."

Mr. Look was a member of the Society of American Military Engineers, the New England Water Works Association, the Museum of Natural History, the Boston City Club, the Alumni Association of Massachusetts Institute of Technology, and the Boylston Lodge, A. F. and A. M., of which he was Master in 1900. He was licensed as a professional engineer in the states of Massachusetts and New York.

Mr. Look was elected an Associate Member of the American Society of Civil Engineers on May 6, 1903, and a Member on April 4, 1905. He became a Life Member in 1938.

JOHN HERBERT McMANUS, M. Am. Soc. C. E.[1]

Died September 6, 1941

John Herbert McManus, former chief of the Bureau of Claims of the Board of Water Supply, of the City of New York, N. Y., died on September 6, 1941, at Orthmann Sanitarium, in Kingston, N. Y., following a long illness. He had retired on February 28, 1941, because of ill health, after almost thirty-five years of continuous service with the Board of Water Supply.

Mr. McManus was born in Boston, Mass., on July 9, 1882, the son of Charles Aloysius and Agnes (Kidney) McManus. He received his early education in that city, and studied sanitary and civil engineering at the Massachusetts Institute of Technology, in Boston, from which he was graduated in June, 1905, with the degree of Bachelor of Science. He then taught at the Institute for one year as assistant instructor in civil engineering.

On June 13, 1906, Mr. McManus reported to the Board of Water Supply of the City of New York and was assigned to the Reservoir Department at Brown Station, New York. For the next seven years he was engaged on construction work in the Ashokan Reservoir area, first in the West Hurley section, then at the Olive Bridge Dam during the most important stages of the work there, and later at the headworks of the Catskill aqueduct where he helped to build a section of the aqueduct, along which was located the then largest venturi meter in the world. He assisted in the construction of the Ashokan screen chamber and lower gate chamber, the waste channel and reinforced concrete bridge spanning it, and the aerator. In addition, he did considerable incidental work on superstructures, roads, landscaping, and several reinforced concrete bridges.

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[1] Memoir prepared by Edward P. Ward, Asst. Engr., Board of Water Supply, and James A. Guttridge, M. Am. Soc. C. E.

While on construction work, Mr. McManus aided considerably in the preparation for trial of various claims against the city, and as result was assigned to the Board's Bureau of Claims in 1913 to record, classify, investigate, and report upon claims of all kinds filed in connection with the construction of the water works and to assist the city's Law Department at the trial of such claims. Early in 1919 he was appointed chief of the Bureau and continued in this capacity until his retirement. While Mr. McManus was chief of the Bureau, his judgment was frequently utilized for important determinations of policy in the matter of the disposition of claims brought for all sorts of damages by persons whose properties were involved in the condemnations necsary for the great work with which Mr. McManus was identified.

Although he had no formal legal training, his familiarity with the many and complicated leading cases which serve as precedents in condemnation proceedings made Mr. McManus invaluable to the legal staff. This litigation frequently involved engineering problems and concepts with which even outstanding members of the legal profession could be expected to have little acquaintance. His early experience as instructor aided Mr. McManus in imparting to his legal associates sufficient technical data to enable them to explain many things in understandable terms to commissions and courts, not ordinarily familiar with engineering methods.

His wide acquaintance with leading members of his own profession contributed in no small measure to his fund of versatile information, so essential in his highly specialized activities. Naturally of a studious nature, Mr. McManus possessed a more comprehensive and detailed knowledge of the New York City water supply system than many engineers engaged in the construction of its component parts.

With his associates, including those under his personal direction, he always was considerate, making their problems his own and considering it his obligation not only to work zealously for the interests of the City of New York, but to assist in the advancement of other employees. His loyalty exemplified the universally recognized high morale of the Board of Water Supply.

Of modest and unobtrusive personality, Mr. McManus directed, without display or ostentation, but with efficiency, dispatch, and completeness to the last detail, the varied activities of the Bureau of Claims.

A great lover of the outdoors, his recreational activities included baseball, tennis, and hockey, and during his later years he was also interested in skating, skiing, hiking, and golfing. There were few of the highest peaks of the Catskills upon which he had not walked or camped. His favorite study was geology, with which he was unusually familiar, and he keenly enjoyed discovery and identification of specimens in the field.

A licensed professional engineer in the State of New York, Mr. McManus was also a member of the New England Water Works Association.

Surviving are his widow, the former Mary F. Cusack, to whom he was married on September 10, 1913; three sons, John H. McManus, Jr., Thomas C.

McManus, and Edwin C. McManus; a daughter, Mary C. McManus; and a brother, Charles A. McManus.

Mr. McManus was elected a Member of the American Society of Civil Engineers on October 1, 1928.

DAVID WHITE McNAUGHER, M. Am. Soc. C. E.[1]

Died November 24, 1943

David White McNaugher, the son of Joseph and Jessie (White) McNaugher, was born in Allegheny (later Pittsburgh), Pa., on November 23, 1859. He attended public schools and Newell Institute in Pittsburgh until 1878, when he entered Westminster College at New Wilmington, Pa., as a sophomore, being graduated in 1881 with the degree of Bachelor of Arts. The following fall he entered the freshman class at Rensselaer Polytechnic Institute in Troy, N. Y., from which he was graduated in 1885 with the degree of Civil Engineer.

During vacations from Rensselaer Polytechnic Institute, Mr. McNaugher worked in the engineering office of James P. Witherow and also took a special course in chemistry. During the summer after graduation from the Institute, he was again engaged by Mr. Witherow. In the fall of 1885 he entered the employ of the Troy Steel and Iron Company (of which the late Robert W. Hunt,[2] M. Am. Soc. C. E., was general superintendent) as assistant chemist and also as an assistant in the engineering corps in charge of the reconstruction of the steel plant.

When Captain Hunt left the Troy Steel and Iron Company and opened the Bureau of Inspection, Tests and Consultation at Chicago, Ill., Mr. McNaugher went with him and took charge of the laboratory and also the inspection of rails and other materials. Later he was sent to Pittsburgh to supervise inspection at the Duquesne Steel Works.

In 1889 Mr. McNaugher joined the engineering and inspection firm of G. W. G. Ferris and Company of Pittsburgh, the partners being G. W. G. Ferris, James C. Hallsted, M. Am. Soc. C. E., W. F. Gronau, and Mr. McNaugher. This company designed the Ferris Wheel, of Chicago Exposition fame, in 1893. In the design and fabrication of the wheel many problems had to be solved which were new at that time. In 1894 Mr. Gronau withdrew from the firm and two years later Mr. Ferris died. The title of the firm was then changed to that of Hallsted and McNaugher. The company continued until 1900, when it was merged with the Robert W. Hunt Company, of Chicago. Mr. McNaugher had charge of the Pittsburgh office and became vice-president and treasurer of the Robert W. Hunt Company.

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[1] Memoir prepared by C. G. Dunnells, M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 1360.

Mr. McNaugher belonged to the American Institute of Mining Engineers, Engineers' Society of Western Pennsylvania, American Railway Engineering Association, and American Society for Testing Materials.

He was a director of the Monongahela National Bank of Pittsburgh, a trustee of the Dollar Savings Bank of Pittsburgh, a trustee and the treasurer of Westminster College, a trustee of the Western Pennsylvania School for the Deaf, and president of the Board of Corporators of the Union Dale Cemetery of Pittsburgh. He was a member of the Duquesne Club of Pittsburgh and the Engineers Club of New York, N. Y. He belonged to the Fourth United Presbyterian Church of Allegheny, was a member of the Session, and was always active in its welfare and financial support.

On November 22, 1900, Mr. McNaugher was married to Josephine Scott, of Allegheny, who died on December 9, 1939. He is survived by two sons, David White McNaugher, Jr., and Robert Scott McNaugher, both graduates of Rensselaer Polytechnic Institute.

Mr. McNaugher was elected a Member of the American Society of Civil Engineers on June 3, 1908.

CHARLES STERLING MILLARD, M. Am. Soc. C. E.[1]

DIED JUNE 5, 1942

Charles Sterling Millard was born in Louisville, Ky., on May 3, 1874. He was descended from French and English stock. His father, formerly in the iron business in Utica, N. Y., moved to Indianapolis, Ind., and organized a railroad car building company. His maternal grandfather, Elisha Morgan Gilbert, was one of the organizers and first president of the Michigan Central Railroad Company.

As a boy Mr. Millard was educated in Clinton, Conn.; Lawrenceville Academy in Lawrenceville, N. J.; and the Holbrook School in Ossining, N. Y. After a year at Union College in Schenectady, N. Y., he entered the Sheffield Scientific School of Yale University in New Haven, Conn., where he taught for a short period after being graduated in 1896.

Returning to Indianapolis, he entered the maintenance of way department of the Pennsylvania Railroad Company. Next, he worked for the Peoria and Eastern Railway Company. In 1898, after brief service in the Spanish-American War, he returned to railroading as assistant engineer, maintenance of way, for the Peoria and Eastern Railway Company and soon was promoted to engineer, maintenance of way. In 1901 he became division engineer for the Delaware, Lackawanna and Western Railroad Company. Later he served for short periods as engineer, maintenance of way, for the Peoria and Pekin Union Railway Company, and as assistant engineer of construction for the Illinois Central Railroad Company.

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[1] Memoir prepared by C. O. Sherrill and F. F. McMinn, Members, Am. Soc. C. E.

In 1903 he returned to the Peoria and Eastern Railway Company as engineer, maintenance of way, and three years later was promoted to engineer, maintenance of way, for the Cleveland, Cincinnati, Chicago and St. Louis Railway Company (Big Four). In 1910 he was made engineer of track and roadway and two years later was promoted to division superintendent of the Big Four. In 1914 he was returned to the Peoria and Eastern Railway Company where he was division superintendent for two years. In 1916 he was transferred to the Big Four as division superintendent. In 1918 he was appointed assistant general superintendent; and, in 1924, general manager of the Big Four, with headquarters at Cincinnati, Ohio. In 1930 he was appointed vice-president of the New York Central Railroad Company.

Aside from routine railroad operation, Mr. Millard was, perhaps, best known for his advanced principles in negotiation of labor relations. In addition to writing many of the rules governing labor relations on the Big Four, he was chosen independent arbitrator in a number of labor disputes. He was an active representative of the New York Central Railroad Company on the construction of the Cincinnati Union Terminal and served two terms as president and four as chief operating officer.

Interested in golf, during his middle age Mr. Millard helped to organize the Indianapolis Country Club. He also was a cabinet maker. During two summer vacations, when a boy, he was apprenticed in the pattern making shop of his father's car building company. Throughout his life this hobby provided him with tremendous interest and enjoyment.

Mr. Millard was director of ten corporations, principally subsidiaries of the New York Central Railroad Company. When he was elected an Associate Member of the Society in 1901, he was the youngest man to achieve this honor in the Society's history. He also belonged to many railroad associations.

On June 5, 1942, he died of coronary occlusion in Holmes Hospital in Cincinnati. He is survived by his widow, Elizabeth Ekin Millard; two sons, Charles S. Millard, Jr., U. S. Navy, and William E. Millard; a daughter, Mrs. Henry Lyman Greer; and four grandchildren.

Mr. Millard was elected a Junior of the American Society of Civil Engineers on May 2, 1899; an Associate Member on June 5, 1901; and a Member on January 31, 1905. He became a Life Member in 1936.

EGBERT JESSUP MOORE, M. Am. Soc. C. E.[1]

Died February 1, 1944

Egbert Jessup Moore was born on July 10, 1877, at Cutchogue, L. I., N. Y., the son of Isaac T. Moore and Susan (Jessup) Moore. He was educated in the schools of Middletown, N. Y., being graduated from the local high school in

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[1] Memoir prepared by J. P. H. Perry, M. Am. Soc. C. E.

1895. He then entered Cornell University at Ithaca, N. Y., and was graduated with the degree of Civil Engineer in 1899.

After his graduation from Cornell Mr. Moore worked in the field and on steel erection for the Berlin Iron and Bridge Company until August, 1900. From September, 1900, to December, 1902, he served in the drafting rooms of the American Bridge Company at East Berlin, Conn., Buffalo, N. Y., and Philadelphia, Pa. He was a draftsman for the Phoenix Iron Company at Phoenixville, Pa., from December, 1902, to April, 1904. From April, 1904, until March, 1905, he was with S. Pearson Sons Company, contractors on the construction of the East River (New York, N. Y.) tunnels for the Pennsylvania Railroad Company. During these six years after his graduation from Cornell, Egbert Moore clearly showed the talents which later became outstanding in his life—that is, he demonstrated to his superiors his willingness to accept responsibility, his great capacity for hard work, and his dependability in all things. His unusually penetrating mind and his solid substantial character based on upright living which was to guide the remainder of his life also became evident.

In March, 1905, Mr. Moore entered the employ of the Turner Construction Company as engineer. The Turner Construction Company at that time was a small concern engaged in building construction, doing less than half a million dollars worth of work a year. This company specialized in reinforced concrete, then in its infancy as far as commercial usage went. Since there was little precedent for engineering design of buildings constructed of this material, originality, research, and experimentation were required.

From 1907 to 1918 Mr. Moore was chief engineer of the Turner Construction Company. In 1917 he became vice-president and a director. He retired from active duty on June 1, 1931, but continued as a director until his death, serving in that capacity for nearly twenty-seven years. He was a vice-president of the company through 1942.

During the thirty-nine years that Mr. Moore was connected with the Turner Construction Company he not only saw but played an important part in its expansion from a concern doing $442,000 worth of business yearly to one doing $93,000,000 worth of business yearly.

From the day Mr. Moore joined the Turner Company forces until the beginning of World War I he took a leading part in the development of sound design of reinforced concrete as a building structural material. He served as a member of the famous "Joint Committee" (representing various engineering organizations including, in addition to the Society, American Society for Testing Materials, American Concrete Institute, and Railway Engineering Maintenance of Way Association) which, after several years of arduous and sometimes controversial service, established principles for the design of reinforced concrete, particularly for spiral columns and flat slab floor construction, as well as for the more usual beam and girder floor system. Mr. Moore's contribution to the work of this committee was accepted throughout the engineering profession as outstanding.

While Mr. Moore was chief engineer of the Turner Company he directed its drafting rooms in preparing working details, shop drawings, and often fundamental designs for the 389 reinforced concrete buildings which the company was constructing.

During World War I, Mr. Moore was responsible for all problems of the company's personnel in regard to military service. He also selected men for the company's office and field forces, during the period when the company expanded from a $6,639,000 concern in 1916 to a $35,226,000 concern in 1918. After becoming vice-president, Mr. Moore for many years continued to function as the de facto chief engineer of the company. About 1923 he became an operating officer. Until his retirement in 1931 he fulfilled his duties most successfully.

After retiring from active participation in the management of the Turner Construction Company, Mr. Moore was promptly elected a member of the Board of Trustees of the Yonkers (N. Y.) Savings Bank and, when the United States entered World War II, he became chairman of Selective Service Draft Board No. 732 in Yonkers. Mr. Moore was a member of the Cornell Society of Civil Engineers, Cornell Club of New York, and the Hudson River Country Club.

On September 2, 1903, Mr. Moore was married to Georgia Holley. Their son, Clifford Moore, died in 1925 during his junior year at Cornell University. Mrs. Moore; a daughter, Sylvia (Mrs. James C. Freer); and a sister, Mrs. Herbert Woodhull, survive him.

A member of the Board of Trustees of the First Presbyterian Church of Yonkers for many years and chairman of that board for the past five years, Mr. Moore took an active part in the social and religious life of Yonkers. He was always a friend to young people and helped several girls and boys to obtain a higher education. His hobby was gardening. He was most interested in buying tiny evergreen trees by the hundred and watching them grow. When they became sizable trees, he gave them to friends and neighbors.

Mr. Moore was elected a Junior of the American Society of Civil Engineers on March 31, 1903; an Associate Member on February 7, 1906; and a Member on May 31, 1910.

JOHN PATRICK MYRON, M. Am. Soc. C. E.[1]

DIED MARCH 21, 1944

John Patrick Myron was born at Peninsula, Summit County, Ohio, on September 7, 1880, the son of Michael Myron and Margaret (Mourn) Myron.

Mr. Myron was educated in the grade and high schools of Peninsula. At the age of eighteen, being unable to continue his education, he moved to Pittsburgh, Pa., where there were greater opportunities for engineering experience.

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[1] Memoir prepared by F. O. Leopold, President, F. B. Leopold Co., Inc., Pittsburgh, Pa.

In November, 1898, shortly after his arrival in Pittsburgh, he secured a position as timekeeper in charge of labor on construction work with the Jones and Laughlin Steel Company where he remained until March, 1902. Being determined to satisfy his desire for further technical training, he enrolled with the International Correspondence Schools studying civil and bridge engineering at night. After arduous work and study, he received a certificate as bridge engineer from the International Correspondence Schools in 1907.

Not being satisfied as timekeeper with his technical knowledge, he left the Jones and Laughlin Steel Company, and in March, 1902, secured a position with the Fisher Foundry and Machine Company of Pittsburgh, as designing draftsman. From March, 1903, until March, 1904, he was employed by the Mesta Machine Company as designing draftsman. While he was with the Mesta Machine Company, Mr. Myron was always alert for further knowledge and advancement. In March, 1904, he became designing draftsman for the Pittsburgh Filter Company of Pittsburgh. He remained with this company for sixteen years, advancing from designing draftsman to chief draftsman, chief engineer, secretary, and general manager of the Oil City (Pa.) manufacturing plant of the enlarged Pittsburgh Filter and Engineering Company. During his many years with the Pittsburgh Filter and Engineering Company, Mr. Myron had charge of the design and construction of many outstanding municipal water filtration plants—in all, more than two hundred municipal plants. In addition to design and construction, he directed the design and manufacture of all types of water purification plant equipment.

In March, 1920, he left the Pittsburgh Filter and Engineering Company to enter into a partnership with Leo Hudson of Pittsburgh. The firm specialized in water works and sewerage projects. During the ten years, until 1930, he supervised the design and the construction of many water works projects. He was intensely concerned with the best interests of his clients, who remained his lifelong friends.

In January, 1930, he joined F. B. Leopold Company, Inc., of Pittsburgh. Many of his former associates in the Pittsburgh Filter and Engineering Company (which had dissolved in 1922) were also in the employ of this company. Mr. Myron was interested not only in the design of water filtration plants, but in their actual construction and in the design and constant improvement of water filtration plant equipment.

During the early years of the economic depression and until December 7, 1941, the water works field suffered possibly more than a great many other activities. Many adjustments and readjustments occurred during the emergency period. Since Mr. Myron was intensely interested in all phases of water works development, this situation without a doubt helped to cause his early death.

Like that of other companies, the business of the F. B. Leopold Company, Inc., increased greatly due to wartime demands, following the attack on Pearl Harbor, Hawaii, on December 7, 1941. This increased activity found a great response in the efforts of Mr. Myron, and, disregarding all warnings, he, like many others, tried to do more than usual, only aggravating his then unknown ill health. When he finally consulted a physician, it was too late to repair the damage done by his intense interest and relentless energy.

During his thirteen years with F. B. Leopold Company, Inc., Mr. Myron made outstanding contributions—innovations and improved methods and equipment—to the water treatment field. He supervised all engineering work and construction and designing, as well as serving as vice-president and secretary. At his death he was vice-president and treasurer in complete charge of the company's operations. During his many years in the water works field, he avoided publicity and personal promotion, being at all times more interested in his work and the over-all good that his efforts might produce.

Although Mr. Myron took an active part in professional affairs, he remained out of the limelight. His only hobby was his sincere interest in engineering and the welfare of his family. His vacations were few, and they were usually spent in behalf of others. Mr. Myron derived his greatest pleasure from helping other people.

Mr. Myron was one of the first residents of Brookline (later part of Pittsburgh). He was instrumental in the formation of the Brookline Savings and Trust Company and was secretary and director from its inception in 1926. He helped to found the Brookline Building and Loan Association of which he was a director from its beginning in 1922.

On June 28, 1905, Mr. Myron was married to Rose A. Kelly of Pittsburgh. Besides his widow, he is survived by three daughters, Grace, Helen, and Rita; two sons, John F., a cadet at the United States Military Academy in West Point, N. Y., and Thomas L., Lieutenant, U. S. Army; and a sister, Nellie.

Mr. Myron was elected a Member of the American Society of Civil Engineers on April 19, 1920.

DAVID LEONARD NEUMAN, M. Am. Soc. C. E.[1]

DIED NOVEMBER 24, 1944

David Leonard Neuman, the son of Samuel and Ernestine (Rosette) Neuman, was born in New York, N. Y., on August 27, 1891. After being educated in the local public schools, he entered Columbia University, also in New York City, from which he was graduated in 1913 with the degree of Civil Engineer. Following his graduation, he was engaged on various types of construction work, beginning as an instrumentman and advancing to construction engineer.

Mr. Neuman joined the New York National Guard on July 4, 1916, and served as a Private, First Class, with Company A of the Twenty-Second Engineers during the border dispute with Mexico. On March 6, 1917, he was commissioned a Second Lieutenant in the Officers Reserve Corps and subsequently was appointed to the Corps of Engineers of the United States Army. He was promoted to the rank of Captain on June 4, 1917, and was assigned to Camp Forrest, Georgia, as Executive Officer.

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[1] Memoir prepared by C. L. Hall, M. Am. Soc. C. E.

From 1919 to 1922, Captain Neuman was in charge of surveying military roads in Panama. In 1923 he was graduated from the course for company officers given by the Engineer School at Fort Belvoir, Va., and was assigned to Fort Sam Houston, Tex. In 1926 he became Assistant to the District Engineer of the U. S. Engineer Office in Milwaukee, Wis. In 1928 he was appointed Engineer Supply Officer of the Columbus General Depot at Columbus, Ohio. He successfully completed the course at the Command and General Staff School, Fort Leavenworth, Kans., in 1933. On March 1, 1934, he was promoted to Major and assumed command of the First Battalion, Thirteen Engineers, at Fort Belvoir.

In 1935 Major Neuman became Assistant to the Division Engineer, South Atlantic Division, in Atlanta, Ga. He was promoted to the permanent rank of Lieutenant Colonel on July 1, 1940, and returned to Fort Belvoir to assume command of the Second Training Group, Engineer Reserve Training Corps. He was promoted to the temporary rank of Colonel in the Army of the United States on February 1, 1942.

In March, 1942, Colonel Neuman was placed in command of the Ninety-Fifth Engineer Regiment in the South Sector of the Alcan Highway in Canada. Suffering from a knee injury five months later, he was sent to Walter Reed Hospital in Washington, D. C., for treatment. Upon recovery, Colonel Neuman returned to the Columbus Army Service Forces Depot as Engineer Supply Officer. In November, 1943, he was appointed Commanding Officer of the Marion Engineer Depot in Marion, Ohio, where his untimely death occurred while in the performance of duty on November 24, 1944. He was buried with full military honors in Arlington National Cemetery in Arlington, Va., on November 28, 1944.

Colonel Neuman served the United States faithfully as an officer in the Corps of Engineers, and his loss is keenly felt by his friends in and out of the service. He died, as he would have wished, serving his country in time of war.

In 1929 Colonel Neuman was married to Ruth Cott of Columbus, Ohio. He is survived by his widow; a son, Robert Neumann; and two daughters, Myrna and Marcia Neuman.

He was a charter member of the Society of American Military Engineers.

Colonel Neuman was elected a Junior of the American Society of Civil Engineers on February 4, 1914; an Associate Member on November 9, 1920; and a Member on April 13, 1936.

JOSEPH PETTIS NEWELL, M. Am. Soc. C. E.[1]

DIED DECEMBER 5, 1938

Joseph Pettis Newell was born on May 11, 1866, at Mt. Tabor (later within the corporate limits of Portland, Ore.), on a homestead which was the end of the covered wagon trail for his parents, John S. and Sarah E. (Kearns) Newell. They had arrived, strangers to each other, from Iowa and Indiana in 1852, and were principals in the first wedding at Mt. Tabor.

When Joe had finished his education in the log schoolhouse of the Mt. Tabor neighborhood, there was ne'er a bridge across the Willamette River. He rode a pony four miles and ferried to attend the Portland High School. In 1884 he matriculated at Massachusetts Institute of Technology (M. I. T.) in Boston, Mass., and is believed to have been the first student from the "Oregon Country" to slake his thirst for knowledge at that celebrated school of scientific learning. In that day "Boston Tech" was a far cry from Mt. Tabor in Oregon, notably so for young Joe Newell who had neither New England pedigree, nor engineering heritage, nor come-by patrimony to dissipate. Getting there however was a sign of a dominant engineering bent in a budding mind that visioned opportunity in engineering in a promising wilderness. Indeed he had decided on his vocation when he entered high school.

Definitely his thinking throughout a long career was alined to "tasks of applying means to definite ends in the material life of men under the limitations of economic laws."[2] Note especially the word "task." J. P. Newell could go into theories with you but he was essentially a doer, an evaluator, a man of direct methods and processes toward concrete ends. He was intrinsically genuine and substantial, gifted with a probity both moral and intellectual that formed the warp of his homespun character. The weft was never coarse nor common nor flashy.

"Native" describes the career of Joe Newell. He was born, grew seventy-two inches tall, lived all of his seventy-two years, and died in Portland, while the population of that city waxed from six thousand to three hundred and fifty thousand. To attain the eminence that he reached in his profession he naturally moved in a larger circle than the compass of the home town, but was never away from home for any permanency. For him ever there was no place like home. On the other hand, he gave no impression of provincialism.

He was at M. I. T. three years, and in the spring of 1887 had practically completed a four-year course, but was not granted a diploma. He might have graduated with his class cum laude with a bit of readying for commencement, but the grain of Boston had been garnered, he needed no hallmark, he had come of age and was busily engaged in his life's work. He had a job as draftsman with the Cincinnati Southern Railway, and held it a year or so; then in

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[1] Memoir prepared by C. P. Keyser, M. Am. Soc. C. E.

[2] From a definition by Nicholas Murray Butler, *Engineering News-Record*, January 24, 1924, p. 161.

1888 he returned to Oregon and grew up with the Oregon Railway and Navigation Company which he served, with some minor leaves, until 1907.

In the 80's and 90's, before the Harriman merger, these lines had seen an interesting two decades building and rebuilding under William H. Kennedy,[3] M. Am. Soc. C. E., as chief engineer. From him Newell got his training in practical problems of railroad location and construction, along with some dogma from A. M. Wellington's "Economic Theory." Writing of Kennedy in his memoir[4] the late Virgil G. Bogue, M. Am. Soc. C. E., states, "He may safely be called one of the great engineers of his time and country." Newell was an apt scholar, and became one of his chief's most able assistants, handling most important assignments, main line, branch, and terminals, location, construction, and reconstruction. During the last three years of his service with the Oregon Railway and Navigation Company he was division engineer of the Oregon Division, which comprised 600 miles and included the Portland rail and water terminals.

In his railroading experience he demonstrated initiative, ingenuity, and administrative ability in the full measure that it required. For instance, when in his early track work, he came across trackmen with lining bars "easing" the points of curve and tangency with more sense than science, he deduced an equation and tabulated a set of graduated shifts or radial offsets with corresponding superelevations. His method was simple enough for use by track foremen without the benefit of engineer surveyors.[5] It proved so satisfactory that he converted it to a set of spirals stated in a flexible system of deflections from tangent for transit setting, made the Newell spiral standard for the Oregon Railway and Navigation Company, and over a period of years got the entire system properly spiraled. The late Elliot Holbrook, M. Am. Soc. C. E., developed the same curve, an approximation to the cubic parabola, about the same time,[6] unbeknown to Newell, and it was later elaborated and a working manual[7] was prepared by Prof. Arthur Newell Talbot,[8] Past-President and Hon. M. Am. Soc. C. E.

He was smart, and ready and able in emergency. Once upon a time, a short time in spite of his vigilance, a main line tunnel collapse became suddenly imminent, and would have caused a serious interruption of traffic if Newell, the division engineer right on the job, had not thrown an 18° "shoofly" around it in time to save the day. It was merely an incident that the master mechanic protested that his new long-wheel-base locomotives would not take a curve of so short a radius; but Newell had reckoned the gage spread and it all worked out satisfactorily after he had seen to it that more slack was allowed in some peculiar stay chains on the engine trucks—something that mere civil engineers were not supposed to understand or to tamper with. Virtue was its own reward in this instance, as in many another, for the general superintendent took all of the credit and Joe remarked dryly, "Everybody was satisfied."

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[3] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 2176.

[4] *Ibid.*, p. 2183.

[5] *Engineering News*, Vol. XLI, June 22, 1899, p. 393.

[6] See mention of "Holbrook Spiral Curve," 1880, *Who's Who in Engineering*, 1931, p. 617.

[7] "The Railway Transition Spiral," by Arthur Newell Talbot, Univ. of Illinois, Champaign, 1899.

[8] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 1530.

A romantic story has been written, founded on the Indian legend of the Bridge of the Gods. The story prevails that there was a natural bridge across the Columbia River where it has cut through the backbone of the Cascade range down to sea level 100 miles inland from its mouth. According to J. P. Newell there might have been a natural bridge, and it might have afforded a crossing for the aborigines. He thought, however, that he was not bridge engineer enough to attempt to restore it even in his mind's eye, with the abutments now standing three thousand feet high and nearly five miles apart. But he was long and intimately connected with railways through the gorge on both banks of the river—railways that had not waited for the ground to come to rest following the failure of the bridge, mythical or actual.

He was well informed on the ostensive causes of the storied failure, or, more to the point, what to do to keep the tracks in alinement as they traversed the restless banks of the mighty stream. The Indians might hand down a story, but it was characteristic of Newell to garner and interpret the indications, and one could well believe what he reported even to his observations on the Indians themselves, to whom he was no stranger.

In 1907 he was 41 years old, ripe in experience, and exceptionally well informed on the economics of transportation in particular, and utilities in general. He quit the railroad, and hung out his shingle in Portland as a consulting engineer, an event in his life that marked decisively the turn from the formative to the definitive stage of his career.

Thus he was available as a qualified expert to enter a long tenure with the Oregon (State) Public Service Commission where he rendered yeoman service. Perhaps his most conspicuous achievement for his state and community was the bringing of victory in the celebrated Columbia Basin rate case in 1919, which was decided by the Interstate Commerce Commission mainly on Newell's analysis of the basic considerations in the points at issue. His studies of "equated mileage" disclosed unanswerably the difference in tonnage costs via inexpensive and safe transit through the gorge of the Columbia on water grades to and from maritime connection at Portland, as compared with the haul over the Cascade mountains to Puget Sound ports. Incidentally, in 1917 he wrote a Report to the Oregon Public Service Commission entitled "Study of Cost of Freight Service, O. W. R. & N. Ry.," which would have made Newell's reputation even if he had not brought victory at said hearing, or later figured so prominently in Canadian railway arbitration.

In 1918 the rate structure of the three main railway systems of Canada had become involved and out of harmony; at least two were fraught with crucial financial difficulties. So a thoroughgoing investigation and arbitration were brought about through legislative action of the National Parliament by appointment of a special Arbitration Board. To amass and evaluate the data pursuant to the findings it was necessary to engage an engineer competent in valuation and operation cost analysis; and particularly one who could report authoritatively the underlying basis of rate making. In this case rates were complicated not only within Canada but also by transhipment over rail connections in the United States, and the implications reflected on obligations to stockholders and also to wage interests of employees. It was a momentous job which began by sifting the Canadian Northern and later worked over the Grand Trunk. The officials of Ottawa, Ont., Canada, asked the United States Interstate Commerce Commission to recommend three engineers for them to choose from. Mr. Newell was the first and most highly recommended of the three names submitted and so he was engaged by the Canadian Government first from February to June, 1918, on the Canadian Northern Railway Arbitration at Toronto, Ont., Canada, and again from May, 1920, to July 1921, on the Grand Trunk Railway Arbitration at Montreal, Que., Canada. Mr. Newell was named the Canadian Government's official engineer in taking over the Grand Trunk.

Out of the mass of fact-finding in these cases he prepared a paper [9] which won for him the first award of the Society's A. M. Wellington prize instituted by *Engineering News-Record.*

He was intimately connected with the board and committee workings of the Portland City Club, an unchartered civic institution, notable for nonpartisan findings in matters of local governmental processes, city planning, etc., and he served one year as its president. He was a member of the National Conference on City Planning. He was the first chairman and thereafter the perennial consultant to the Portland City Planning Commission, and made cogent reports on transportation and traffic, particularly highway, rail, and water borne, in relation to major schemes of articulation and terminal factors as affecting land use of urban areas. He was one of the founders of the Portland (Ore.) Section of the Society (1913), and continued to be an active Past-President, doing his part of the cambium activities that keep these limbs of the parent trunk from dying of dry rot. He headed a committee of the Portland Section that made an exhaustive investigation and printed a report in 1915 on causes and remedies of public works unemployment, really an analysis of fluctuations in demand on the man-power supply, entitled: "Report on Winter Unemployment." He was an active member of the Engineering Institute of Canada.

Nominally a Republican, he "had no truck" with the slough of politics, although ever constant in duties of citizenship. He favored prohibition and was an orthodox Methodist, but his ways "were ways of pleasantness, and all his paths were peace." A man of high ideals and no illusions, he did not quarrel but lived in a human world content and self-sufficient. If he could be accused of a hobby it was critical and intensive study of the world's "best seller" which he engaged in to the extent of teaching Sunday School classes and even unto delving into Greek for a working knowledge in order to interpret the original text of the New Testament for his own edification. His recreation was in useful toil in his garden or in voracious reading.

He had in nice combination a faculty for rapid assimilation and an exceptionally accurate and retentive memory. Few people in everyday and upper walks of life had a tidier vocabulary, and, even fewer, a more facile flow of clear and simple thought-expressing language. He employed few expletives and conceits of expression, and was not much given to metaphor. Early in life he became proficient in shorthand, and habitually used it in composition, side notes, and memoranda. He kept a diary in shorthand consistently. His amanuensis could transcribe his notes as readily as her own, a real advantage in reaping a busy man's mental harvest. It was a pleasure to "listen in" on his dictation.

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[9] "Analysis of Cost of Freight Service, Grand Trunk Railway Company of Canada," by J. P. Newell, *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 351.

He enjoyed legitimate drama, but was not intrigued by movies. He abhorred idleness in any form like nature abhors a vacuum, so much so that he did not even want to think of taking a vacation; and he couldn't be bothered with insomnia. He did not fret himself with counting sheep or worry about wakefulness. As he put it, he rested in bed while he took advantage of the quiet to think constructively, glad to avail himself of the opportunity. But mind you, he had no addictions except honest hard work, not even to mild stimulants. "Hells' bells" never jangled him out of his mental equilibrium. Except for an injury to a knee late in life which laid him up for a time, he was always abundantly able to go about and get where he wanted to be.

He had little interest in gadgets for his personal use. A rapid sure-fire mental calculator, he found no need for a slide rule. He never owned or operated an automobile, although he did not underestimate or fail to appreciate its value as a modern convenience to a useful, as differentiated from a wastrel, purpose. To him an artificial thing of beauty might be a joy forever only if it had an excuse, or more properly a cause, in usefulness. But the pristine beauty of a young tamarack—that would be an unalloyed joy.

He was gifted with a keen sense of humor and a ready dry wit. He was the sort of solid individual that seemed to have no loopholes for an antagonist to work on—that is, he would be responsive, generous, and deferential, but like all men of caliber, was not easily influenced. Yet withal he was interesting, and agreeable in his contacts, the kind of personality that any one could enjoy conversing with.

The summit of his career was marked by the Canadian Railway Arbitration engagement referred to previously, but perhaps the most flattering offer he ever received came when he was nearing sixty. This was an overture from the Persian Government to reconstruct and modernize their national transportation system on his own terms. After considerable study he declined the offer. At the time he was engrossed in another large thoroughgoing utility appraisal, but it will not enhance the history to hazard that he might have delegated the drudgery of that engagement. Then why did desert Persia fail to lure him from verdant Oregon? Deponent sayeth not in very sooth; but to one who knew him well, a sufficient answer would be not difficult to ascribe. Civil engineers, notably those who become eminent in their profession, are the sort of stones that go rolling more or less. Newell was not given naturally to temporary sojourning. His locale remained centered in the home life, private and public, of his native heath all of his days. He went on living with the same bride, with one son to bless the union, in the home with spacious garden that he brought her to from southern Oregon in 1892, until deaths did part in 1938, hers in July, his in December, both suddenly.

All the while he kept up the place joyously by his own toil and care. He was a pillar of his church a few blocks away. He was public minded in his professional service; sound and clearheaded and essential in his numerous civic activities. Never self seeking nor a weather vane, his friendship and his friendships were valued; in fine, a generous and constant contributor to the good life and toward its betterment in his home town and state.

There was no outcrop from his make-up of a lust for power or glory or pelf; seemingly indifferent to fame, he was ever content with life's compensations where he chose to find them. So—lovely mosses had gathered on this stone and it had settled in situ in threescore years. Shouldn't it have taken some prizing to dislodge it and set it going toward remote Persia?

He lived in an abiding faith in the ultimate worth of mankind, and the fitness of mundane cause and effect, neither a borrower nor a lender of trouble. Here was one of those rare characters of ability and distinction, who move ever forward in full power, unaccused of envy, avarice, or a "yen" to glitter. An abstemious thoroughgoing person, nothing small, who wrought and dwelt constantly for the benefit of the people and the region to which he belonged.

Mr. Newell was elected a Member of the American Society of Civil Engineers on October 2, 1907.

WALTER HENRY NORRIS, M. Am. Soc. C. E.[1]

DIED JULY 11, 1942

Walter Henry Norris, the son of Edward John and Clarissa (Norton) Norris, was born on March 6, 1870, in Charlestown (Boston), Mass. He was graduated from the Massachusetts Institute of Technology at Boston in 1893 with the degree of Bachelor of Science in Civil Engineering. During college vacations he worked for the Canadian Bridge and Iron Company of Montreal, Que., Canada, and for the St. Johnsbury and Lake Champlain Railroad Company as rodman and instrumentman.

After graduation, Mr. Norris entered the Engineering Department of the Boston and Maine Railroad. He remained with this organization until 1909. From June, 1893, to January, 1899, he was draftsman in the Bridge Department, investigating the strength of structures and inspecting bridges in the field. From January, 1899, to August, 1909, he was assistant bridge engineer, designing and preparing working plans for grade-crossing eliminations, freight houses, engine houses, coaling plants, and docks.

In 1909 Mr. Norris joined the engineering department of the Maine Central Railroad Company and remained with that company until his death. From 1909 to May, 1933, he was bridge engineer, and, from 1933 to 1939, assistant engineer of structures. From 1939 until his death he was consulting structural engineer.

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[1] Memoir prepared by Edward M. Hunt, M. Am. Soc. C. E.

The largest of the many bridges Mr. Norris designed for the Maine Central Railroad Company is the five-span Sheepscot River Bridge at Wiscasset, Me. This bridge, completed in 1916, consists of two through-plate girder spans, one of which is a draw span, and three riveted truss spans, one of which is 400 ft long.

Mr. Norris designed and supervised the reinforcement of the Frankenstein Trestle near Crawford Notch in the White Mountains of New Hampshire. This work, completed in 1930, was done without interrupting train service, and showed superior ability in treating difficult design problems.

In 1920 the Hon. Percival P. Baxter, then Governor of Maine, appointed Mr. Norris on a three-man board of engineers for the construction of the highway memorial bridge across Portsmouth Harbor between Maine and New Hampshire. The board employed the firm of Waddell and Hardesty, of New York, N. Y., to design the bridge, which was built by the American Bridge Company and completed in 1923 at a cost of $1,500,000.

Mr. Norris was a conscientious worker and loyal to the teachings of his instructors and to the natural laws that govern good design. He deemed steel the best medium for strength, endurance, and economy in heavy bridges. To his eye, a well-designed bridge, although plain and severe in appearance, was beautiful because it served a useful purpose. Embellishment he regarded as extravagance.

His domestic and social principles were puritan like those of his ancestors. He sought to teach young people that a firm financial foundation is a prime requisite for life. Mr. Norris had deep sympathy and deemed a timely lift to an associate a pleasure to the giver. He had been a member of the Congregational Church since 1889, and expressed his religion in practical ways that endeared him to many. His courage, cheerful disposition, and spontaneous sense of humor were appreciated by his associates.

In 1897 Mr. Norris was married to Effie L. Shapleigh, who survives him, as do two daughters, Elizabeth (Mrs. D. Kilton Andrew) and Katherine Norris; and two sons, Edward J. Norris (Captain, United States Army Engineer Corps) and Emerson S. Norris. Mr. Norris' sons and his brothers (the late Webster Norris and the late George L. Norris) all were graduated from Massachusetts Institute of Technology.

Mr. Norris was elected an Associate Member of the American Society of Civil Engineers on September 6, 1905, and a Member on June 12, 1939.

CLIFFORD OLDER, M. Am. Soc. C. E.[1]

Died November 28, 1943

Clifford Older was born in Lincoln, Adams County, Wis., on November 17, 1876, the son of Milton DeWitt and Elizabeth Ann (Clark) Older. His parents soon moved to Portage, Wis., where he grew up and attended high school. He chose engineering as his life work and received his training at the University of Wisconsin in Madison, being graduated in 1900 with the degree of Bachelor of Science in Civil Engineering.

From 1900 to 1906 Mr. Older was employed on railroad work—for the Pennsylvania Railroad Company, the Wabash-Pittsburgh Terminal Railroad Company, and the Chicago and Alton Railroad Company. In 1906, five months after the organization of the Illinois Highway Commission, Mr. Older was appointed Bridge Engineer for the State of Illinois. At that time very few roads in Illinois were improved, and much of his time was devoted to emphasizing to township and county officials and to the general public the benefits of permanent road improvements. He initiated and developed the construction of reinforced concrete bridges in Illinois and did much to popularize such construction. In 1917 he was appointed by the Hon. Frank E. Lowden, then Governor of Illinois, as Chief Highway Engineer. In that capacity he continued to apply the initiative and originality that had characterized his work as Bridge Engineer.

Mr. Older participated in the formulation of the $60,000,000 Bond Issue Road Law—the first step in making a paved road system possible in Illinois. Looking forward to the thousands of miles of road to be built and to the constantly increasing traffic, he conceived an extensive research project to determine what design and type of construction would be best and most economical. Under his direction the famous Bates Road Tests were planned and executed. Small sections of various types of pavement were built where traffic could be completely controlled and were subjected to actual truck traffic. Numerous field observations were made and laboratory tests conducted to check the behavior of the sixty-seven sections. When the research was completed, the profession possessed a scientific basis for designing pavements based on tests made under actual traffic conditions. International recognition was given to the importance of this study. Designs of pavement slabs developed as the result of this research were adopted not only throughout Illinois but, to a large extent, throughout the entire world.

In 1924 Mr. Older resigned as Chief Highway Engineer of Illinois to enter private practice as senior partner of the firm of Consoer, Older and Quinlan, in Chicago, Ill. During ten years of active service with this firm, he directed a large variety of engineering design and construction and also conceived and developed several successful patented appliances in the highway construction field. After his retirement from this firm in 1934, he served as consultant on highways for several railroad associations and worked on inventions connected with rigid pavement design.

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[1] Memoir prepared by George O. Consoer, M. Am. Soc. C. E.

On May 28, 1903, in Lowville, Wis., he was married to Kitty May Drake. He is survived by his widow and their five children—Fern Elizabeth (Mrs. Harold Lundberg); Kitty Beatrice (Mrs. Lloyd Quayle); Clifford D., Lieutenant in the U. S. Navy (U.S.N.R.), serving in the South Pacific area; D. Drake; and Grenfell.

Always interested in photography, Mr. Older enjoyed the technical problems of color photography and secured patents on two of his ideas in this field. Although barred by increasing lameness from many activities, he took many automobile trips with Mrs. Older through the United States. Mr. Older's keen mind and love of nature made each excursion a delightful experience.

Mr. Older was a member of the Western Society of Engineers, the American Society of Municipal Engineers, the Mississippi Valley Conference of State Highway Departments, the American Society of State Highway Officials, and the American Society for Testing Materials. He served for many years on the Design Commission of the Highway Research Board of the National Research Council. He belonged to the Masonic Order and the Methodist Church.

Occasionally one meets a man of such marked ability, character, and affability that he commands immediate respect and also induces a warm personal liking. Mr. Older was a man of this type. As a public servant, he was a shining example of integrity, devotion to duty, and technical competence. On his associates he left an indelible impression of kindliness, unfailing courtesy, and adherence to high ideals.

Mr. Older was elected an Associate Member of the American Society of Civil Engineers on October 4, 1905, and a Member on July 7, 1915. He became a Life Member in 1940.

DAVID KIRK ORR, M. Am. Soc. C. E.[1]

Died November 9, 1943

David Kirk Orr was born on November 24, 1865, in Allegheny City (later the north side of Pittsburgh), Pa., the son of Samuel Mowry Orr and Jane (Kirk) Orr.

He was educated in the public schools of Allegheny City; at the Western University of Pennsylvania (later the University of Pittsburgh) in Pittsburgh; and at Geneva College in Beaver Falls, Pa., being graduated from the latter with the degree of Bachelor of Arts in 1889.

Mr. Orr's professional career was devoted to railroading. First, in 1889 he was employed by the Norfolk and Western Railway Company as topographer's assistant. In 1891 he obtained a position with the Pittsburgh and

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[1] Memoir prepared by G. H. Burnette, M. Am. Soc. C. E., and J. W. Boyd, Supt., Monongahela Railway Co., Brownsville, Pa.

Lake Erie Railroad Company as instrumentman. From 1893 to 1901 he was associated with David B. Kirk and Company, flour blenders and exporters of Kansas City, Mo. In 1901 he returned to the Pittsburgh and Lake Erie Railroad Company as assistant engineer, remaining until the Monongahela Railroad Company was organized in 1903.

Beginning as engineer for the Monongahela Railroad (later Monongahela Railway) Company, he advanced to assistant chief engineer in charge of construction and maintenance. Under his guidance, extensions were made to the main line and several branches during this period. In 1919 he was promoted to superintendent. Following a reorganization in 1933, he was made roadmaster. He retired on June 1, 1937, after nearly half a century of railroading.

Mr. Orr was known affectionately as "D.K." to old and young alike. He was identified with almost every community activity and received a citation from the local American Legion Post for outstanding community service.

The first president of the Brownsville (Pa.) Rotary Club, at the time of his death he had been serving as the club's executive secretary. Also he was secretary-treasurer of the Brownsville Board of Trade; a member of the Session of the First Presbyterian Church; past-president of the South Brownsville School Board, Brownsville Trust Company, South Brownsville Improvement Company, and Brownsville Athletic Association; and treasurer of the Ward Supply Company.

Mr. Orr belonged to a number of fraternal organizations and lodges including F. and A. M. (32d Degree), the Shrine, Benevolent and Protective Order of Elks, Nemacolin (Pa.) and Morgantown (W. Va.) country clubs, Engineering Society of Western Pennsylvania, American Railway Engineering Association, and Railway Club of Pittsburgh.

On September 14, 1897, he was married to Margaret McMaster at Broughton, Pa. He is survived by three sons—Gilbert, Samuel, and John—and two daughters—Jennie and Mary.

A man of simple tastes, high ideals, and integrity, his life was filled with active service to his fellow men. All who knew Mr. Orr mourn the loss of an able, efficient, and wise associate, counselor, and friend.

Mr. Orr was elected a Member of the American Society of Civil Engineers on April 4, 1911. He became a Life Member in 1936.

GEORGE ALEXANDER ORROK, M. Am. Soc. C. E.[1]

Died April 6, 1944

George Alexander Orrok ("Uncle George," as many of his intimates affectionately called him) was an essence of his ancestry—a finely distilled and stimulating essence, such as cognac from wine or Calvados from Norman cider.

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[1] Memoir prepared by Dugald C. Jackson, M. Am. Soc. C. E.

Few came to know his full and sparkling merit until after extended acquaintance.

He was born at Dorchester, Mass., on July 3, 1867, a descendant of old-time European and American families. Late in the eighteenth century a member of the Orrok family came to America from Great Britain, but presumably Mr. Orrok's paternal ancestors migrated to England from Denmark during the Danish invasion in about 900. A village east of Edinburgh still bears the name of Orrok. These people were seamen, merchants, business men, and mechanics. Alexander Orrok came to America; family tradition associates him with Paul Revere and with the Boston Tea Party. He, like others of the family, was a skilled mechanic. His grandson, James, married Laura Davenport, and "Uncle George" was their son. A Davenport relative is reputed to have been aide of General Lafayette during the Revolutionary War. These American ancestors were from Boston, Mass., and Dorchester, and generally seem also to have been skilled mechanics in ship chandlery or ship operation. George Orrok's father was a heavy, strong man, who was a member of one of the volunteer fire companies of the day and is said to have lifted the rear wheel of a three-horse steam fire engine for a stunt.

His mother died when George Orrok was four years old, and he and a younger brother were mostly cared for by their father. A younger sister was raised by relatives. With the interested attention of father and aunts, young George grew up through an active and happy but very busy life. By the time he was of school age he was largely earning his own living and also had become adept at amateur theatricals, a lover of instrumental music, singing, and dancing, and a follower of tennis. He undertook to play various musical instruments and at one time supported himself by playing the cornet in a brass band. In later years he played the flute and became a member of the New York (N. Y.) Flute Club.

Mr. Orrok prepared for college at the Dorchester Grammar School and the Mechanic Arts High School of Boston, with the summers affording him foundry, wagon shop, tack factory, and other varied employment. His plan for life was to become a steel chemist and he entered Massachusetts Institute of Technology at Boston in 1885, enrolling in the chemistry course; but providence thwarted the ambition. In 1888, on account of his eyes, the family physician directed him to work no more in laboratories and to secure employment that would keep him outdoors as much as possible.

Thereupon, Mr. Orrok started upon four years of a somewhat gypsy series of engineeering employment which took him from Alabama, to Wisconsin, to Utah, and back to New England. Then he began the fine, creative professional career in engineering that ultimately led to the great distinction of honorary membership in the American Society of Mechanical Engineers, a society of which he became a member in 1902 and for which he had great affection.

Having returned to Boston, he was employed in 1892 by F. S. Pearson,[2] M. Am. Soc. C. E., who was a distinguished consulting engineer in the electric and cable street railway fields, and whose particular clients were Henry M.

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 1402.

Whitney of Boston and William C. Whitney of New York. As the Messrs. Whitney had wide interests beyond their own localities and also beyond the street railway field, Mr. Orrok had a roving commission, ranging from examining coal mines in Rhode Island, extensive designing of coal-mine developments in Nova Scotia, designing and supervising the construction of a cable railway in New York City, designing and supervising underground trolley street railways in New York City, to designing exceptionally large coal piers in Nova Scotia, coal-carrying ships, and mine hoists for the coal mines. Associated with all these projects were a variety of others (mostly in Nova Scotia), which required a high degree of initiative as well as mechanical skill.

Returning to Mr. Pearson's main office in New York City, Mr. Orrok worked on electric power stations for street railways from New York City to Winnipeg, Man., Canada; sugar refineries at St. John, N. B., Canada, and Baltimore, Md.; an underground trolley system for Birmingham, England; and various projects for Brazil, Cuba, and the Island of Jamaica, B. W. I. When Mr. Pearson was taken ill and retired from his practice as a consulting engineer, Mr. Orrok, after some intervening employment, in the spring of 1898 was engaged by the New York Edison Company, which had great problems regarding its growing electric power plants and distribution system—all of them in the hands of the brilliant John Van Vleck, consulting engineer for the company. At this time the New York Edison Company was concentrating its generation of electric power in great water-side alternating-current generating plants and substituting rotary-converter substations for the old-time direct-current primary generating stations. In all this novel work, Mr. Orrok was in his preferred element—on work which required great concentration of effort but which returned great satisfaction from accomplishment. In 1928 he established his own office as consulting engineer, under the name of Orrok, Myers and Shoudy (later Orrok and Myers Associates). Clients came to this firm from many parts of the United States and various foreign countries.

Mr. Orrok always retained his intimate contact with the mechanical engineering affairs of the New York Edison Company. His was a long and contributive career in electric power plant design, with particular emphasis on the thermodynamic aspects. Thus, when the growing use of higher pressures in the steam boilers of power plants became important, Mr. Orrok sought adequate information regarding the qualities of steam at ultrahigh pressures. He therefore participated in the development of extended and improved steam tables—a piece of eminent research in which he gained the adherence and aid of important investigators in the United States and in England. Financial contributions for the study were obtained from educational institutions and from manufacturing companies interested in the improvement of thermodynamic equipment. The work was conducted under the patronage of a committee of the American Society of Mechanical Engineers, which society is proud of the achievement.

Becoming interested in the needs of World War II, Mr. Orrok joined the firm of Binger-O'Connor-Orrok to deal with important problems of design and construction facing the armed forces of the United States. Many of these problems called for unusual initiative.

As chairman of committees and member of other committees, especially of the American Society of Mechanical Engineers, he contributed much to the progress of engineering practice. He was the author of articles in the Proceedings of the Societies to which he belonged. Jointly with the late Dean Robert H. Fernald of the University of Pennsylvania at Philadelphia, he wrote the book entitled "Engineering of Power Plants."[3] His Newcomen Society essay entitled "Engineering Recollections, 1868–1898" is a delicious exposition of his engineering philosophy, which arose from his thrifty Scotch ancestry and his own notable original mindedness.

Mr. Orrok possessed an interesting versatility in general learning; and, with his intimates, he loved to illustrate his "shop talk" with quotations from a wide range of classical and modern literature, all apposite and impressive. As a fisherman, he was unusually familiar with the various fish and shell fish of American waters and also in the general aspects of foods. These qualities attracted to him a group of men who habitually took lunch at a round table in the Engineers' Club (New York), for whom he was the admired companion and wit.

In the days when he lived in an apartment near Grand Central Station in New York City, he invited his friends arriving from out of town to breakfast with him, and to them Mrs. Orrok hospitably served stacks of tempting hot cakes and maple syrup with delicious coffee. Together with his cherished wife and son, these breakfast-enjoying friends seemed to compose his family. He was not a physically robust man, but he was an ardent attendant at engineering meetings. In the evenings he loved best to rest at home where he read with pleasure a wide range of ancient and modern literature, or played lively tunes on his flute. His was a soul of restful sociability but of distinguished initiative in engineering professional affairs.

Mr. Orrok became a member of the American Society of Mechanical Engineers in 1902 and a fellow in 1936. In 1936 he was elected an honorary member of that society—a distinguished honor which he greatly appreciated. He also belonged to the American Society of Civil Engineers, the American Institute of Mining and Metallurgical Engineers, the American Institute of Consulting Engineers, the Institution of Mechanical Engineers (Great Britain), and other societies which cherished his membership. The Stevens Institute of Technology in Hoboken, N. J., conferred on him the honorary degree of Mechanical Engineer. He lectured on steam engineering and power plant engineering at Yale University in New Haven, Conn.; Harvard University in Cambridge, Mass.; the University of Pennsylvania in Philadelphia; and at other institutions. He was a Brackett Lecturer at Princeton University in Princeton, N. J.; and, from 1911 to 1914, he taught steam engineering in the Polytechnic Institute of Brooklyn at Brooklyn, N. Y.

On December 24, 1898, Mr. Orrok was married to Jessie Waldo, who died in February, 1921. George A. Orrok, Jr., their son, follows in his father's footsteps as engineer for a great electric power company. Mr. Orrok was married on October 20, 1923, to Elene E. Geer, who survives him. He owned a place in Connecticut where he enjoyed summer holidays, often with his son and family, but the abode which many of his friends knew was the apartment in New York City.

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[3] "Engineering of Power Plants," by Robert H. Fernald and George A. Orrok, McGraw-Hill Book Co., Inc., New York, N. Y., 3d Ed., 1927.

Mr. Orrok was elected a Member of the American Society of Civil Engineers on June 6, 1906.

ARTHUR EDMUND OWEN, M. Am. Soc. C. E.[1]

Died March 21, 1944

Arthur Edmund Owen was one of the intrepid group of railroad men who never considered personal comfort before duty. Thus, on March 21, 1944, at the end of a busy day of office routine and executive conferences, his strength was too depleted to do more than relax into the eternal sleep from which no man may be awakened by earthly commands. He died in a railway coach of the "Crusader" near Elizabeth, N. J., on the train that usually carried him to his home at Abington, Pa.

Arthur Edmund Owen was born at Montclair, N. J., on January 19, 1876. His parents, James Owen and Mary Caroline (Dunham) Owen, were certain one more civil engineer had been born into the family. His father, who had emigrated from London, England, where he had been graduated from London University, for more than forty years was county engineer of Essex County, New Jersey, building many famous roads and writing authoritative articles for magazines and technical journals on road building and allied topics. Mary Caroline Dunham Owen was descended from a long line of colonial ancestors, many of whom had contributed both governmental and engineering services to the United States.

After being educated in the grade and high schools of Montclair, Mr. Owen attended Rutgers College, at New Brunswick, N. J., being graduated in 1897. In June, 1932, at the one hundred and sixty-sixth commencement of Rutgers University he received the honorary degree of Master of Science.

In 1898 Mr. Owen was employed in the tax agent's office of the Central Railroad Company of New Jersey, and he remained with that railroad throughout his entire engineering career. In August, 1899, he was transferred from the Tax Department to the chief engineer's office as a rodman, and the following November he was appointed assistant engineer, in charge of work at Mauch Chunk, Pa. In 1901 he was again transferred, this time to Jersey City, N. J., in the capacity of assistant engineer, holding that title until January, 1907, when he was made principal assistant engineer. After the death of Joseph O. Osgood, M. Am. Soc. C. E., Mr. Owen was made chief engineer on July 1, 1916, and held that position continuously until 1944.

During the first World War he assumed the duties of chief engineer of the Reading System, in addition to those of the Central Railroad of New Jersey, and in 1938 he was again officially appointed chief engineer of the Reading System and also continued his duties as chief engineer of the Jersey Central until the fall of 1943 when he retired from service on the Reading System. Such a service record illustrates, but can in no way measure, the qualities of the man and his unique characteristics.

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[1] Memoir prepared by J. J. Yates, M. Am. Soc. C. E.

Mr. Owen was modest and unassuming and never advertised his versatile achievements. He recognized merit, and always credited a man for good qualities and effort, finding ways in which encouragement could be given. One man stated that "* * * he was the first and only man who ever took a rap for me," and others have asserted that "* * * he was a man above men."

Mr. Owen was a meticulous recorder of daily events. His diaries cover every important incident in the history of the Central Railroad of New Jersey and the Reading System, in such factual detail as to be reliable references on numerous important occasions.

Shelton Pitney and Walter P. Gardner, trustees of the Jersey Central, and William Wyer, chief executive officer, wrote of Mr. Owen in the railroad's annual report:

> "Mr. Owen was a very capable officer and a man of keen judgment and high integrity. Through his long connection with the road he knew Jersey Central history more intimately than any other officer, and his counsel and advice were of the greatest value. He served the company well and faithfully and his passing is a distinct loss to the company and to those associated with him."

Mr. Owen and his able associates completed novel construction enterprises such as harbor improvements, docks and ferries, engine terminals, railroad passenger stations, railroad and county bridges, the important Newark Bay (N. J.) Bridge with its lift and accompanying structures, the valuable reconstruction of the Jersey City train shed, and the elimination of dangerous grade crossings requiring the elevation of tracks and complete changes in the terrain of whole communities. Flood control was sponsored throughout the State of Pennsylvania, as was coastal protection in southern New Jersey.

In addition to his membership in railroad and engineering societies, Mr. Owen belonged to various philatelic and recreational groups. His opinions were frequently expressed in magazine articles, or quoted by authorities who sought his counsel. The golf course became an absorbing interest when Mr. Owen found ways to make such a desert produce first-class greens—the grower's instinct combined with engineering knowledge to produce a landscape of admirable quality. In the field of philately he achieved national prominence as an authority on the stamps of the United States. In a full-page article on Mr. Owen's death, the magazine *Stamps* stated that he "* * * was known wherever United States stamps are collected, and was probably one of the most liked and honored men in the entire fraternity," and that

> "* * * to many collectors he is probably best known for his work with the Bureau Issues Association * * *. He prepared one of the most complete records of the plate numbers used on United States stamps in existence [published by the Bureau Issues Association]. In fact, it was reported

that even the Bureau of Engraving and Printing called upon him for his record in order to correct their own."

Mr. Owen was married to Anne G. Surtees of Ottawa, Ont., Canada, in 1903. Mrs. Owen died in 1933. Mr. Owen is survived by three children, Margery S., James, and Arthur E., Jr., Sergeant, U. S. Army; and two grandchildren, James Owen, Jr., and Charles Arthur Owen. In January, 1936, Mr. Owen was married to Lillian A. Garing of New York, N. Y. Miss Garing was the former vice-chairman of the Republican organization in New York State and Queens County, New York.

Mr. Owen was elected an Associate Member of the American Society of Civil Engineers on May 3, 1905, and a Member on September 9, 1919. He became a Life Member in 1940.

ARCHIBALD E. PALEN, M. Am. Soc. C. E.[1]

Died May 1, 1944

Archibald E. Palen was born on June 1, 1882, in Niobrara, Nebr., the son of Henry Dean and Jennie (McDonald) Palen. He received his early education in Niobrara, and was graduated from the University of Nebraska at Lincoln on February 15, 1907, with the degree of Bachelor of Science in Civil Engineering.

Immediately after graduation, he became an assistant to the county engineer of Lancaster County at Lincoln. From July, 1907, to June, 1908, he worked for the Red Canyon Mining Company at Lander, Wyo., conducting preliminary canal surveys on flumes, tunnels, and siphons for a water power project. On this same project he made his initial venture into the location and construction of a mountain road—a class of work which played a prominent part in his future career.

In July, 1908, Mr. Palen sailed for the Philippine Islands and for nearly four years served as assistant engineer to the district engineer of the Highway Division of the Bureau of Public Works on the construction and maintenance of highways and other public improvements. Securing a leave of absence, from April to August, 1912, he visited Europe and studied highway location, construction, and maintenance, particularly of the mountain roads in the Swiss Alps. He then returned to the United States and was engaged on two short assignments with irrigation and mining companies in Wyoming and Colorado.

During July, 1913, he was with the U. S. Forest Service as a highway engineer in charge of federal aid construction in the national forests of Colorado, Wyoming, and South Dakota. In June, 1914, Mr. Palen was transferred to the Bureau of Public Roads, thereby becoming one of the pioneer engineers of this organization (later the U. S. Public Roads Administration). As assistant to the district engineer of the Public Roads Administration, he handled, very successfully, forest highway location, construction, and maintenance in the Rocky Mountain States of Colorado, Wyoming, and New Mexico and in the Black Hills region of South Dakota during the period when the early mountain trails were being developed and transformed into higher type mountain highways.

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[1] Memoir prepared by a Committee of the Colorado Section consisting of Jesse E. Williams, Assoc. M. Am. Soc. C. E., and Clyde E. Learned and Carl A. Gould, Members, Am. Soc. C. E.

On April 1, 1929, Mr. Palen was appointed acting district engineer of the Public Roads Administration at Denver, Colo., in charge of federal aid, forest highway, and national park major road construction. On February 1, 1933, he was transferred to the St. Paul, Minn., office as district engineer of the Public Roads Administration, handling all federal aid highway activities in the states of Minnesota, Wisconsin, North Dakota, and South Dakota, until his death.

Probably Mr. Palen's most outstanding characteristic that many of his associates will remember was his balanced and cheerful disposition, even under the most trying circumstances. In all his dealings Mr. Palen had a keen sense of responsibility, and he was a man of unimpeachable integrity. His professional ethics were especially high, with the result that he was admired and respected by those with whom he came in contact. He was very loyal to his organization, had the ability to select good men and train them, and was very deliberate in studying and arriving at fair and just decisions.

Mr. Palen was an active member of the Christian Science Church and was a member of the Masonic Fraternity.

On July 16, 1910, in Manila, Philippine Islands, he was married to Edna E. Chidester. He is survived by his widow and two daughters, Katherine (Mrs. B. K. Atkins) and Mary.

Mr. Palen was elected an Associate Member of the American Society of Civil Engineers on January 17, 1916, and a Member on April 26, 1921.

THEODORE BISSELL PARKER, M. Am. Soc. C. E.[1]

Died April 27, 1944

In the untimely death of Theodore Bissell Parker the engineering profession suffered the loss of an outstanding exponent of many of the highest traits of his profession. Forceful and energetic by nature, with a comprehensive grasp of technical matters, he possessed a combination of the qualities of leadership, executive ability, and technical skill which peculiarly fitted him for his career as an administrative engineer.

Descended from several generations of New England stock, Theodore Bissell Parker was born in Roxbury, Mass., on August 20, 1889, the son of Franklin Wells and Sarah (Bissell) Parker. Although his father died when

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[1] Memoir prepared by C. E. Blee and R. T. Colburn, Members, Am. Soc. C. E., and Sherman M. Woodward, Hon. M. Am. Soc. C. E.

Theodore was but ninteen, his mother was determined that he should complete his college course and prepare for a profession. After graduation from high school in Wellesley, Mass., in 1907, he entered the Massachusetts Institute of Technology (M. I. T.) at Boston, and was graduated in June, 1911, with the degree of Bachelor of Science. He was elected to membership in the historic Alpha Theta Chapter of Sigma Chi fraternity.

It was not by mere chance that Mr. Parker was elected first marshal of his class, for, throughout his school life, he was recognized as a leader whose integrity was unusual, whose judgment was sound, and whose friendship was available to all. While in school, in addition to his ability as an outstanding leader, he was noted for his all-around athletic prowess and his inherent sense of fairness. He was fond of sports—golf, tennis, squash, basketball, and baseball. Excelling in most games, he was above the average in all.

After graduation, Mr. Parker was an assistant in the Department of Civil Engineering at M. I. T. for a year and then joined the staff of the late H. C. Keith, M. Am. Soc. C. E., consulting engineer of New York, N. Y., as draftsman and computer on highway and railroad bridges. Mr. Parker began to specialize in hydroelectric development in 1912 when he was employed as assistant hydraulic engineer by the Utah Power and Light Company. Hydroelectric projects remained his principal professional interest thereafter. While with the Utah Company, Mr. Parker was connected with the construction of new power developments on the Bear River in Idaho and the reconstruction of the existing hydroelectric system in Utah.

Then, the United States entered World War I in 1917, and he joined the Army as First Lieutenant, later becoming Captain and commander of Company D of the Twenty-Sixth Engineers in service overseas. After returning from abroad, in April, 1919, Mr. Parker joined the Electric Bond and Share Company as assistant hydraulic engineer, engaged in investigations and reports regarding power sites on the Platte, Loup, and Niobrara rivers in Nebraska, and on the Susquehanna, Delaware, and Lehigh rivers in Pennsylvania. In October, 1920, Mr. Parker again entered the Army, serving as Captain in the Corps of Engineers until November, 1922. This tour of duty included a year of service with troops and a year at the Army Engineer School, in Fort Humphreys, Va., from which he was graduated in 1922.

In December, 1922, he resumed civilian life as an hydraulic engineer with the Stone and Webster Engineering Corporation. In this capacity he was active on several important hydroelectric developments, including the Bartlett's Ferry Project near Columbus, Ga., and the Rock Island development on the Columbia River in Washington, serving as project engineer in charge of design on both of these projects.

In the spring of 1933 Mr. Parker, as a reserve officer, attended the U. S. Army Command and General Staff School at Fort Leavenworth, Kans., where he received the commission of Lieutenant Colonel in the Corps of Engineers Reserve. In August, 1933, he became a member of the engineering staff of the Federal Emergency Administration of Public Works, serving, successively, as engineer examiner, state engineer, and acting state director for Massachusetts.

Colonel Parker joined the Tennessee Valley Authority (TVA) in November, 1935, as chief construction engineer, responsible for the general supervision of lock, dam, powerhouse, and appurtenant construction work. At this time the Norris and Wheeler projects were under construction; the Pickwick Landing project had just been started; and the Chickamauga, Guntersville, and Hiwassee projects had only been authorized. In May, 1938, Colonel Parker was named chief engineer, in charge of all engineering and construction work for the entire Tennessee River development program—an unprecedented system of projects for water control in the Tennessee Valley.

As chief engineer of TVA, he had general responsibility for the planning, design, and construction of the Kentucky, Watts Bar, Fort Loudoun, Cherokee, Douglas, Apalachia, Ocoee, Nottely, Chatuge, and Fontana dams and the Watts Bar Steam Plant. Substantially all the work on these projects was done by force account. The greatest number of engineers employed at any one time was about 2,500, and the maximum total number of employees in the Engineering Department and the Construction Department was about 29,000.

Colonel Parker was devoted to the promise and completion of his work in the Tennessee Valley, interested in, and proud of, the accomplishments of his men. Some two years before the completion of Douglas Dam, an urgently needed war project, his alma mater, M. I. T., called him. He wanted to go—to return to the scene of his youth and to teach—but he delayed his acceptance until Douglas Dam was finished. Then he felt free to go—and not before. Thus, in June, 1943, he left the Tennessee Valley and returned to Massachusetts to become head of the Department of Civil and Sanitary Engineering at M. I. T.

The high esteem the members of the TVA Engineering Department and the Construction Department held for Professor Parker is best indicated by the wording of a scroll presented to him at the time of his leaving:

> "We, members of the TVA engineering and construction staff, desire to express our sincere appreciation of the privilege of having been able to serve under your intelligent and able leadership. We bid you Godspeed in your new duties and want to assure you of our intention to carry on the high ideals of service and the standards of practice you have inculcated and left with us here in the Authority. We view with regret TVA's loss and look forward to sharing M. I. T.'s gain."

Professor Parker's professional career can be measured only in part by the many notable engineering projects with which he was associated. The full measure of his success must include the encouragement he so generously gave the younger members of the profession, the friendship he tendered his associates, and the honor and esteem in which his colleagues hold his memory.

The engineering problems involved in the development of the natural resources of foreign countries devastated by war were very interesting to him. Professor Parker believed that these problems could best be coped with, and ultimately solved, by competent, loyal engineers of the countries themselves. In his characteristic, straightforward manner he was taking steps to help train these men.

He was active in the affairs of local and national engineering organizations. At one time he was a member of the Boston Society of Civil Engineers, from which organization he, together with the late Harry A. Hageman, received the Desmond Fitzgerald Medal for a joint paper on the Bartlett's Ferry Project,[2] which was considered the most outstanding paper of 1926. Professor Parker also was the author of numerous other papers.[3,4,5,6,7] He was a member of the Water Resources Committee of the National Resources Planning Board and a director of the Society of American Military Engineers.

He participated in the activities of the Tennessee Valley Section of the Society as well as in the activities of the Socitey as a whole. His last trip, undertaken even while ill, was to attend the Society's Annual Meeting in January, 1944, at which time he presented a paper on "Major Hydraulic Objectives of the Tennessee Valley Authority."

To his associates Professor Parker was an intense, dynamic personality; an outstanding leader; an engineer of wide experience, prompt in making decisions and always willing to assume responsibility; and a loyal friend who at all times commanded confidence and respect. With a touch of shyness and an extreme modesty, in spite of pressing demands on his time, he remained ever sensitive to the trials and efforts of his associates; and in his own quiet, friendly manner, either by a visit or a personal note, he made other men aware of his interest in their welfare. He was never too preoccupied to greet and encourage the younger members of the profession. His strength of character, his sincerity of purpose, and his desire to serve his fellow men influenced the lives of innumerable associates. Through them his ideals live on.

As a tangible and lasting evidence of esteem, his friends and associates have created an education fund in his memory, known as The Theodore B. Parker Memorial Fund. On the occasion of initiating this endeavor, one of the speakers representing the management of the TVA aptly stated,

> "Colonel Parker's personality, his engineering competence, and his administrative sense are written forever into the TVA and the Tennessee Valley. The attribute he possessed of putting first things first made him a bold engineer and a sometimes impatient administrator. He tempered boldness and courage with a commendable quality of New England caution. * * * 'Make sure of what you're doing—then go ahead' was a characteristic epigram in his discussions of a program."

On May 10, 1913, Professor Parker was married to Estelle Peabody of Wellesley. He is survived by his widow; his mother; and two children,

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[2] "The Bartlett's Ferry Hydroelectric Development," by H. A. Hageman and T. B. Parker, *Journal*, Boston Soc. of Civ. Engrs., March, 1926, p. 93.

[3] "Stresses in Horizontal Pipes on Rigid Saddles," by Theodore B. Parker, *Engineering News-Record*, Vol. 87, 1921, p. 306.

[4] "Rock Island—A Power Development on the Columbia River," by T. B. Parker, *Electric Light and Power*, May, 1931, p. 22.

[5] "The TVA Construction Program," by Theodore B. Parker, *The Military Engineer*, January–February, 1939, p. 42.

[6] "The Construction Program of TVA," by Theodore B. Parker, *Civil Engineering*, June, 1939, p. 355.

[7] "Allocation of the Tennessee Valley Authority Projects," by Theodore B. Parker, *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 174.

Franklin P. Parker (Major, Corps of Engineers, U. S. Army) and Nancy Parker.

Professor Parker was elected an Associate Member of the American Society of Civil Engineers on August 9, 1920, and a Member on March 30, 1931.

GEORGE WELLINGTON PICKELS, M. Am. Soc. C. E.[1]

Died December 2, 1944

George Wellington Pickels, the son of George Wellington Pickels and Minnie (Parrish) Pickels, was born on September 5, 1883, in Richmond, Ky., where he grew to manhood. In 1904 he received the degree of Bachelor of Civil Engineering from the University of Kentucky at Lexington and in 1911 was awarded the professional degree of Civil Engineer by the University of Illinois at Urbana.

He began his professional career as resident engineer on the construction of the Murrayville (Ill.) cutoff of the Chicago and Alton Railroad Company. In the fall of 1907 he accepted a position as instructor in civil engineering at the University of Illinois; and, during his thirty-seven years of service at the University, distinguished himself as a teacher, author, and investigator in the fields of hydrology, land drainage, and flood control, and became a recognized authority on those subjects. When he died, he held the rank of professor of civil engineering in the fields of drainage and flood control engineering.

During World War I, Professor Pickels served as head of the Department of Aids to Flight in the United States School of Military Aeronautics at the University with the rank of First Lieutenant. Following the war, he took an active part in many investigations of the problems of land drainage and flood control in Illinois and for several summers he served as drainage engineer for the Illinois State Geological Survey.

The intimate knowledge of the hydrological and other characteristics of the state gained in this work made Professor Pickels unusually adept in selecting possible sites for reservoirs and lakes in various parts of the state. He lived to see the consummation of a major project of this kind which he outlined. This is the Crab Orchard Dam, built by the U. S. Resettlement Administration a few miles east of Carbondale, Ill., on a tributary of the Big Muddy River, creating the largest artificial body of water in the state with immediate and important economic results and great future potentialities.

Professor Pickels' report on this project formed the major part of a document entitled "A Reconnaissance Report on the Basin of the Big Muddy River" issued jointly by the University, the Illinois State Geological Survey, the Illinois State Natural History Survey, and the Illinois State Water Survey in 1935. He contributed a significant section to a similar but more elaborate document entitled "The Physical, Social, and Economic Aspects of the Valley of the Kaskaskia River" issued in 1937 by the same agencies. (These two documents were not printed but issued in rather large mimeographed editions. They were prepared for specific purposes and are mentioned because they have been of immense value in the State of Illinois and because of Professor Pickels' outstanding contributions to them.)

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[1] Memoir prepared by C. C. Wiley, M. Am. Soc. C. E.

His other writings consisted of technical articles, reports, bulletins, and textbooks. As coauthor with F. B. Leonard, he prepared a bulletin on the "Engineering and Legal Aspects of Land Drainage in Illinois,"[2] published by the State Geological Survey in 1921. He also wrote two bulletins for the University of Illinois Engineering Experiment Station on "Run-Off Investigations in Central Illinois"[3] and "Magnitude and Frequency of Floods on Illinois Streams."[4] "A Textbook on Railroad Surveying"[5] and a larger and more complete volume on "Route Surveying"[6] were prepared in collaboration with the writer. Perhaps Professor Pickels' outstanding contribution to engineering literature was his book entitled "Drainage and Flood-Control Engineering."[7]

In addition to serving on Society committees, he was a charter member of the Central Illinois Section and later served as its Secretary-Treasurer, Vice-President, and President. He was a member and a past-president of the Illinois Society of Engineers and belonged to the Society for the Promotion of Engineering Education, the American Geophysical Union, and the American Water Works Association. He was also founder and chairman of the drainage conferences held at the University of Illinois from 1916 to 1921, inclusive.

Professor Pickels possessed a most engaging personality, combining a quiet dignity with a ready wit and a warm and genial disposition which endeared him to everyone. He was keen, competent, and meticulous in his work and tolerant and cooperative with others. It was both pleasant and stimulating to associate and to work with him. He enjoyed being with students and possessed to a high degree the qualities of a good teacher. As a result he was held in high regard by his students as well as by his colleagues. He was interested in civic affairs and served a four-year term as a member of the Urbana City Council.

On May 16, 1907, in Winchester, Ky., he was married to Elizabeth S. Robinson. Professor Pickels is survived by his widow; a daughter, Elizabeth C. Pickels; and a son, George Wellington Pickels—fourth successive bearer of the name.

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[2] "Engineering and Legal Aspects of Land Drainage in Illinois," by G. W. Pickels and F. B. Leonard, *Bulletin No. 42,* Illinois State Geological Survey, Urbana, 1921; 1st Ed. Revised, 1929.

[3] "Run-Off Investigations in Central Illinois," by G. W. Pickels, *Bulletin No. 232,* Eng. Experiment Station, Univ. of Illinois, Urbana, 1931.

[4] "Magnitude and Frequency of Floods on Illinois Streams," by G. W. Pickels, *Bulletin No. 296,* Eng. Experiment Station, Univ. of Illinois, Urbana, 1937.

[5] "A Textbook on Railroad Surveying," by G. W. Pickels and C. C. Wiley, John Wiley & Sons, Inc., New York, N. Y., 1914.

[6] "Route Surveying," by G. W. Pickels and C. C. Wiley, 2d Ed., John Wiley & Sons, Inc., New York, N. Y., 1939.

[7] "Drainage and Flood-Control Engineering," by G. W. Pickels, 2d Ed., McGraw-Hill Book Co., Inc., New York, N. Y., 1941.

In the death of Professor Pickels the profession lost a valuable member; the University, one of its ablest men; the community, an outstanding citizen; and his associates, a beloved friend.

Professor Pickels was elected a Member of the American Society of Civil Engineers on May 19, 1924.

JOHN RICHARDS GRAY PILL, M. Am. Soc. C. E.[1]

DIED JANUARY 29, 1944

John Richards Gray Pill was born on July 6, 1866, in the village of Perranporth, Cornwall, England, the son of William and Mary (Gray) Pill. For generations, his paternal forebears had been associated with the tin and copper mining industry of England. His mother was Scotch and a daughter of an officer of the British Army.

He acquired his early education at Truro Academy in Truro, Cornwall, England, and at the National School, near Toulon, France. He won scholastic honors at both schools. His parents moved from France to St. Johns, Newfoundland, where he completed his engineering education at Methodist College (of St. Johns) receiving the degree of Civil Engineer.

From 1885 to 1887, Mr. Pill was in the employ of the Engineering Department of the Newfoundland government and served as assistant construction engineer on the Placentia Railway Project. Later, he headed an expedition sent by the Newfoundland government to prospect for mineral deposits in Labrador. The party spent more than a year in this search, and secured many valuable data on these resources. For this work he was highly commended by his superiors.

In 1888 he entered private practice with Payne and Howell, Civil Engineers of Knoxville, Tenn. In 1891 he was employed as chief assistant engineer on the Knoxville and West Knoxville sewer systems. In 1894 Mr. Pill became assistant engineer, Construction Division of the Southern Railway Company, and was assigned to the offices at Knoxville and Chattanooga, Tenn., and Birmingham, Ala.

In January, 1901, he resigned his position with the Southern Railway Company and was named chief engineer of the Galloway Coal Company of Carbon Hill, Ala. Later in 1901 he was promoted to general superintendent and was placed in charge of direction and development of extensive coal mining operations in Walker County, Alabama. In 1907, he also became vice-president and general manager of the Choctaw Coal and Mining Company and assumed the additional duty of directing its mining operations.

Early in 1909, Mr. Pill was appointed president and general manager of the Birmingham Fuel Company with the duty of opening and developing mining operations at Townley, Ala. By 1912 the output of these mines was 230,000 tons of coal annually. In March, 1912, the Birmingham Fuel Company acquired the holdings of the Corona Coal and Iron Company in Walker County. The properties were merged and operated as the Corona Coal and Iron Company. Mr. Pill was made general manager of the new company and moved with his family to Corona, Ala. He managed eight mines for the company and increased the output more than 1,000,000 tons annually. During his active mining career, he was in responsible charge of engineering and construction work from ground breaking to complete operation of ten or more mines.

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[1] Memoir prepared by a Committee of the Alabama Section consisting of W. G. McConnel and C. A. Wilmore, Members, Am. Soc. C. E.

Mr. Pill was most successful in directing and handling men. Always solicitous as to their safety, he did not ask employees to begin dangerous operations until he had led the way. He provided accident insurance, medical care, and early established a system to provide for the welfare of the men and their families.

During World War I, Mr. Pill served, at personal sacrifice, with the Engineers Committee of the United States Fuel Administration in Washington, D. C. He was cited for distinguished service by the administration.

In 1920 Mr. Pill opened consulting offices in Birmingham. He acquired an interest in and was vice-president of the Southern Mineral Land Company. His practice included extensive service to the State of Alabama and various mining companies, as consultant on mineral properties in the state. He also served as land trustee for Alabama College, Montevallo, Ala. —an association which he thoroughly enjoyed.

He entered the employ of the Woodward Iron Company of Woodward, Ala., in 1927, as manager of the Land Department and continued in that capacity until his death. To this work, he brought a wide knowledge of the coal and iron industry and of mineral property values. His services were of such value and quality as to merit the praise of fellow officials and the respect of employees. No other tour of duty afforded him more satisfaction than his service with the Woodward Iron Company.

Throughout his long residence in Alabama, Mr. Pill studied the state's mineral lands and deposits. By personal exploration (often on foot) and the study of books, technical articles, and papers, he gained a vast store of information. Mr. Pill loved the outdoors, was almost completely free of illness of any kind, and was very active until he was hospitalized before his death. His recreations were fishing, hunting, checkers, and chess at which he was very adept. He was an inveterate reader—at least four hours daily—and a constant student.

Although he was quiet, reserved, almost to the point of shyness but with a keen wit, his friends were limited only by his acquaintance. He was a man who was seen and known too little by too few. He loved his home, and the family was one of mutual devotion.

Mr. Pill was a fellow of the Royal Society of Arts (Great Britain). He belonged to the American Institute of Mining and Metallurgical Engineers, the Rocky Mountain Club (New York, N. Y.), and the Episcopal Church.

On October 31, 1894, he was married at Knoxville to Katherine Rant, who died in July, 1943. One son, Donald Rant Pill, died in 1914. Mr. Pill is survived by one daughter, Mary Louise (Mrs. W. S. Green), and one son, Howard Edgar Pill.

Mr. Pill was elected an Associate Member of the American Society of Civil Engineers on February 7, 1906, and a Member on March 4, 1913.

JAMES CHARLES PINNEY, M. Am. Soc. C. E.[1]

Died December 2, 1944

James Charles Pinney was born at Sturgeon Bay, Wis., on October 17, 1882. He was the son of James C. and Abigail (Hannan) Pinney. He was graduated from Fargo College at Fargo, N. Dak., in 1905, receiving the degree of Bachelor of Arts in Mathematics. In 1910 he received the degree of Civil Engineer in structures and hydraulics from the University of Wisconsin at Madison.

After graduation from Fargo College, Mr. Pinney spent five years in various field positions: Chief of party for the reconstruction of the main line of the Great Northern Railway Company; assistant engineer in charge of field party on maintenance and right of way for the Minneapolis and St. Louis Railroad Company; assistant city engineer of Fargo on office and field work for the city's sewer and water extension program; and finally county surveyor for Door County, Wisconsin.

In 1910 he accepted an assistant professorship at Marquette University in Milwaukee, Wis., where he taught courses in structural engineering and in material testing. During his stay at Marquette University he acted as a consultant for the City of Milwaukee; and in 1912 he became superintendent of bridges and public buildings, in charge of the design and construction and repairs and maintenance of all bridges, public buildings, and waterfront structures, costing more than one million dollars. Having completed most of the construction planned by the city, Mr. Pinney returned to Marquette University in 1917 as dean of the College of Applied Science and Engineering where he established a cooperative engineering course that permitted students to obtain practical experience while attending the university. He was retained as a private consultant by the City of Milwaukee on various engineering work—including the design of the North Avenue Viaduct, Scott Street Bridge at Fond du Lac, Wis., and the car ferry terminal for the Milwaukee Association of Commerce.

In 1925 Mr. Pinney entered private consulting practice and became a member of the firm of Lee County Engineering Company at Fort Myers, Fla. He specialized in municipal engineering and surveying, and worked on the subdivisions of Riverside, Twin Palms, York Manor, and Orangewood, all at Fort Myers. He owned a local orange grove which was his pride and joy. After ten years in private practice, he entered the service of the federal government as chief of party for the U. S. Biological Survey. He then was employed by the U. S. Coast and Geodetic Survey, and in 1939 he became engineer in the Dams Division for the Bureau of Reclamation, U. S. Department of the Interior, in Denver, Colo., remaining until his death.

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[1] Memoir prepared by Dirk A. Dedel, Assoc. M. Am. Soc. C. E.

Mr. Pinney was a quiet, hard-working man who won the esteem of his associates by his profound knowledge of engineering principles and his high regard for engineering ethics. He belonged to the Engineering Society of Wisconsin (past-president and honorary member), the Society for the Promotion of Engineering Education, the American Railway Engineering Association, the Florida Engineering Society, and the Engineers and Architects Club of Fort Myers.

In March, 1936, at Galveston, Tex., he was married to Mabel Stiff. He is survived by his widow.

Mr. Pinney was elected a Member of the American Society of Civil Engineers on April 16, 1918.

FRED ASHLEY PNEUMAN, M. Am. Soc. C. E.[1]

DIED NOVEMBER 30, 1942

Fred Ashley Pneuman was born in Cortland, N. Y., on November 12, 1899. At an early age he moved with his parents to Georgetown, Colo., where he received his primary education in the public schools. In 1912, when the family moved to a ranch near Denver, Colo., he continued his studies at the North Denver High School. After graduation, he entered the University of Colorado at Boulder, receiving the degree of Bachelor of Science in Civil Engineering in 1922.

Shortly after his graduation from college, Mr. Pneuman entered the employ of the American Bridge Company at Gary, Ind., as a structural draftsman and remained continuously with that company until his death. During this period he was employed in various engineering capacities. From 1922 to 1929, he worked at the company's Gary plant as a draftsman and checker on structural steel for office buildings, mill buildings, and bridges of both the fixed and movable types. During this period Mr. Pneuman developed a particular interest in the mechanical features of movable bridges, and, because of his aptitude for this type of work and his fertile imagination in developing new ideas, he was transferred to the designing office of the American Bridge Company at Chicago, Ill., in July, 1929. In this office, he was employed for four years as a designer on various types of steel structures, particularly movable bridges and operating machinery. He assisted in the design of the mechanical features for the vertical lift span in the bridge of the Missouri, Kansas and Texas Railway Company over the Missouri River at Boonville, Mo., and for the vertical lift bridge of the Elgin, Joliet and Eastern Railway Company over the Illinois River at Joliet, Ill. He also designed the machinery for three large bascule bridges.

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[1] Memoir prepared by T. A. Jordan, A. S. Marvin, and George W. Serat, Members, Am. Soc. C. E.

Late in 1932 the contract for the construction of the San Francisco-Oakland Bay (Calif.) Bridge was awarded to the American Bridge Company, and in January, 1933, Mr. Pneuman was transferred to the Erecting Department to assist in the development of construction plans and in the design of equipment required for the erection of this unusual structure. By January, 1935, the work on the bridge was progressing satisfactorily, and the erectors were ready to place the suspension spans. At this time Mr. Pneuman was transferred to San Francisco where, during the next year and a half, he was largely responsible for the continuous operation of the cable spinning equipment. During this period, he conceived and developed many improvements in the method of spinning cable for suspension bridges, and several of his innovations have since been adopted as standard practice.

After completion of this assignment in 1936, Mr. Pneuman was transferred to the New York (N. Y.) office of the American Bridge Company to take charge of the engineering work in connection with the erection procedure and equipment for the Marine-Parkway Bridge—a vertical lift structure, in New York. His next assignment was as resident engineer on the Bronx-Whitestone Bridge project, also in New York City. This bridge—a suspension bridge of unusual proportions and design—gave Mr. Pneuman an excellent opportunity to apply the knowledge and experience he had gained on the San Francisco-Oakland Bay Bridge. He planned the erection and also designed the erecting equipment.

Upon the completion of the Bronx-Whitestone Bridge in 1939, Mr. Pneuman returned to the designing office of the American Bridge Company at Chicago where he remained until his death. During this period he was given several important engineering assignments, including the design of a bascule bridge for the Southern Pacific Company at Beaumont, Tex., and the erection plans for the Kettle Falls Bridge in the State of Washington.

Mr. Pneuman was an active member of the Masonic Lodge and was also interested in all forms of athletics. His particular hobby, however, was amateur photography, and he acquired considerable skill in taking both still and motion pictures. His motion pictures of field work have been of considerable value and interest to his engineering associates. He possessed a magnetic personality which endeared him to all who knew him, particularly to the younger engineers who frequently received his patient assistance and guidance.

In 1923 he was married to Mildred Young, who, together with one son, Frederick, survives him. A daughter, Shirley, died in 1939.

Mr. Pneuman was elected a Member of the American Society of Civil Engineers on July 10, 1940.

CORYDON TYLER PURDY, M. Am. Soc. C. E.[1]

DIED DECEMBER 25, 1944

Corydon Tyler Purdy, the son of Samuel J. and Emma Jane (Tyler) Purdy, was born on May 17, 1859, in Wisconsin Rapids, Wis. He was of English ancestry, dating back to the fourteenth century.

When he was still a small boy, a new Howe-truss timber bridge was built across the Wisconsin River in the lower part of Wisconsin Rapids. Mr. Purdy in later years told of his absorbed interest in the building of this bridge. He spent as much time watching the workmen as his parents would permit. Apparently greatly impressed, he announced then and there that he would become an engineer when he grew to manhood. He never changed his mind, nor his determination to carry out his plans.

He received his elementary and high school education in Wisconsin Rapids and then attended the University of Wisconsin at Madison. Because of difficulties with his eyesight, he was forced to interrupt his studies at the university for several years. During this time, he was employed periodically on railroad survey work and also as a draftsman.

In 1882 he was appointed assistant engineer by the Chicago, Milwaukee, St. Paul and Pacific Railroad Company, in charge of field surveys for a new railroad entrance to Chicago, Ill., through Evanston, Ill.

In 1883 he returned to the University of Wisconsin to complete his course, being graduated in 1885 with the degree of Civil Engineer. Three months before his graduation, he was elected city engineer of Eau Claire, Wis. He resigned from this position in the fall of 1887 and for the next two years worked as a material inspector in Pittsburgh, Pa., and also in the Engineering Department of the Keystone Bridge Works.

In 1889 Mr. Purdy opened his first office in Chicago as a consulting engineer on the design of bridges. Chicago at this time was experiencing considerable expansion in building construction and the attention of real estate owners, architects, and contractors was concentrated on the problem of erecting tall structures on the valuable property in the business sections of the city. In a general way, the construction of higher buildings followed the lines of earlier construction of relatively low buildings in that the important vertical supports were still the masonry exterior walls and interior piers. Cast-iron columns and iron floor beams were beginning to be used extensively in the interior construction of buildings, but masonry was being strained to support taller buildings. As an example, the Monadnock Building, seventeen stories high and probably one of the highest masonry commercial structures ever built, had walls seven feet thick on the ground story.

Mr. Purdy observed and studied these conditions and began to visualize the enormous possibilities in the adaptation of steel supporting members for building construction. This theory, given more and more attention during the next few months, caused him to revise the plans he had prepared for his future as an engineer. He decided to turn from bridge construction and bend every effort to develop a field of structural engineering in building construction, an endeavor for which he was already equipped by education and training. He soon became one of a group of men intent on inaugurating a new form of building construction based on the use of a structural steel frame. In his own words, "We were not discovering, nor inventing; we were evolving." The evolution became very rapid. Before the Columbian Exposition opened in 1893, buildings of unprecedented height had been designed and erected and the tremendous advantages of steel frame construction were fully demonstrated.

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[1] Memoir prepared by P. J. Reidy, Assoc. M. Am. Soc. C. E.

During this period, with a member of his office staff, Lightner Henderson, Mr. Purdy formed the partnership of Purdy and Henderson. Many important features of steel construction in buildings, methods of calculations, and standard details were originated by this partnership, an association which endured until Mr. Henderson's death in 1916.

The Columbian Exposition had given considerable publicity to steel frames in high buildings and, as a result, Mr. Purdy was asked to lecture before engineers and engineering students at Boston, Mass., and at Cornell University in Ithaca, N. Y.

In 1894 Mr. Purdy decided that the field of structural design, which had been developed largely through his efforts, would be particularly adaptable to New York, N. Y., although conditions at that time, in the opinion of most of his Chicago associates, made it difficult to prove the value of such engineering service to the owner, architect, or builder. However, Mr. Purdy proceeded with his intention to open a New York office and, after almost a year of patient study and introductory effort, he obtained his first important commission—namely, the design of the foundations and superstructure of the Waldorf-Astoria Hotel. Thereafter, Mr. Purdy remained in the New York office which, with the increase in the volume and importance of the business, became the headquarters of Purdy and Henderson in 1901, when the partnership was incorporated. Branch offices were subsequently opened in Boston, Mass.; Montreal, Que., Canada; Seattle, Wash.; and Vancouver, B. C., Canada.

In 1896 George A. Fuller, a prominent contractor of Chicago, requested Mr. Purdy to establish a New York office for his firm. For the following year Mr. Purdy had charge of Mr. Fuller's business while his partner, Mr. Henderson, managed the New York office of Purdy and Henderson. In 1897 Mr. Purdy resigned from the Fuller organization and resumed the management of Purdy and Henderson.

When the United States Realty and Construction Company was organized in 1898, it was the largest combination of realty and construction interests at that time. Mr. Purdy was requested to join the organization as chief engineer; but, in accordance with his wish to remain effective head of his own business, an arrangement was made to retain Purdy and Henderson as structural consultants for all the Realty Company's activities.

In 1900 Mr. Purdy prepared the provisions under which the ownership of the site of the proposed New York Times Building was transferred to the New York Times Company. This building at Times Square, New York City, is directly over the subway, with four tracks running through the building above the pressroom in the subbasement. Some years later, Mr. Purdy prepared a paper for the Institute of Engineers of Great Britain containing an analysis of structural vibration in buildings, for which he was awarded the Telford Premium. At the request of the United States Government, Mr. Purdy organized and supervised the preparation of an exhibit for the Paris (France) World's Fair of 1900 to show the details and methods of modern steel frame construction of buildings, as well as all the other features of a multistory office building. A building then under construction in New York was used as a model.

In 1900 Mr. Purdy went to Cuba and, as a result of his survey of prospects, opened an office in Havana. The subsequent rapid development of Cuba brought a large volume of business to this office. Although originally the Havana office was to be an engineering office similar to those in the United States, it soon became evident that this idea was not adaptable to general business practice in Cuba. To continue the office, it was found necessary to enter the construction field. In turn, a subsidiary company had to be organized for the importation and distribution of building materials and equipment. The company formed for this purpose was the Purdy and Henderson Trading Company, which later specialized in general importing. One of the firm's Cuban activities and interests was the construction of branch banks for the Royal Bank of Canada in all parts of Cuba and on some of the other islands in the Caribbean area. This arrangement worked out so well that, at the request of the Royal Bank, the Purdy and Henderson Company, Inc., was asked to build branch banks throughout Canada. To accomplish this, the Purdy and Henderson Company, Ltd., was organized in 1909. Representative of the construction in Cuba were the Cuban Capital, Banco Nacional de Cuba, Centro Gallego, Centro Asturiano, and the Nacional Hotel. For a time, a branch of the company was operated in Puerto Rico.

In 1905, and again in 1908, Mr. Purdy visited England and made extensive surveys of building construction design and methods. While there, he delivered several lectures on the construction procedures being developed in the United States and had a long interview, during the 1908 trip, with Sir Guilford Molesworth, then president of the Institute of Engineers of Great Britain, who subsequently nominated Mr. Purdy for membership in the institute.

After the Baltimore (Md.) fire in 1904, Mr. Purdy was retained by the Hon. E. Clay Timanus, then Mayor of Baltimore, as consultant to the commission created to revise the building code.

Mr. Purdy resigned as president of the Purdy and Henderson Company, Inc., in 1917, but continued his active association with the firm as chairman of the board of directors, a position he filled until his death. During his years as active head of his company, he was responsible for the structural design of many prominent buildings. Illustrative of the range of the firm's accomplishments are the following: Waldorf-Astoria Hotel, New York Times Building, Flatiron Building, New York Municipal Building, R. H. Macy store, Pennsylvania Station concourse, Hudson Terminal, and New York Hippodrome in New York, N. Y.; Custom House Tower in Boston, Mass.; Cook County Court House and McCormick Building in Chicago, Ill.; Willard Hotel in Washington, D. C.; B. F. Goodrich Plant in Akron, Ohio; Capwell store in Oakland, Calif.; Royal Bank Building in Toronto, Ont., Canada; and Ocean Terminal Station in Halifax, N. S., Canada.

In later years Mr. Purdy devoted much time to the operation of his farm near Monroe, Orange County, N. Y. This property had been owned by his ancestors for about two hundred years. The original deed, still in existence, was written on two sheets of parchment in 1734. Another interest to which he devoted much time and effort, particularly after 1920, was the welfare of youth, first locally in Orange County and then in New York City. He later became a director of the National Child Welfare Association.

In his relationships with his associates and employees, Mr. Purdy always showed keen interest in their advancement, both technically and financially, if they were trustworthy and dependable. Most of the men who, in later years, directed the affairs of Purdy and Henderson Company, Inc., were encouraged in their efforts by Mr. Purdy's willing recognition of their individual merits. He was fond of quoting a part of an essay which he learned to speak as a schoolboy in Wisconsin Rapids:

> "Opportunity has hair in front, behind he is bald. If you seize him by the forelock you may hold him, but if suffered to escape, not Jupiter himself can catch him again."

His own career is an example of the road to a successful and full life: His early determination to be an engineer; his single-minded efforts to obtain the education and training necessary to equip him for his vocation; his untiring work to succeed as an engineer; his willingness to share success with his associates; and his ability to act promptly when opportunity presented itself.

Mr. Purdy was a member of the Western Society of Engineers, the Institute of Engineers of Great Britain, the Engineers' Club of New York, the New York State Engineering Society, the Arctic Club of Seattle, and the Civitan Club of New York.

In 1890 he was married to Eugenia Cushing of Turner, Me., who died within a year. In 1892, in Columbus, Ohio, Mr. Purdy was married to Rose E. Morse. He is survived by his widow; a son, Corydon P. Purdy; and a grandson, Corydon P. T. Purdy.

Mr. Purdy was elected a Junior of the American Society of Civil Engineers on February 2, 1887, and a Member on December 6, 1893.

IRA LAWRENCE PYLE, M. Am. Soc. C. E.[1]

DIED MAY 1, 1944

Ira Lawrence Pyle was born in Philadelphia, Pa., on March 27, 1877, and was educated in private schools in that city. The son of Abram Pyle and Ella (Sweeney) Pyle, he was of Irish and Dutch ancestry and combined the imagination and keenness of the former race with the thoroughness and patience of the latter race.

Early evincing a preference for the profession of engineering, Mr. Pyle began his career in 1901 in the drafting room of the Pennsylvania Railroad Company in Philadelphia. At the suggestion of the late Col. Joseph U. Crawford, engineer of branch lines of the Pennsylvania Railroad Company, that the coal and timber lands of southern West Virginia, then being developed, offered especially attractive opportunities for ambitious young engineers, Mr. Pyle secured a job with a field party making land surveys in West Virginia, doing both instrument and drafting work. In 1902 he transferred to the Chesapeake and Ohio Railway Company. At that time, this line was beginning to reach out into the rich coal and timber fields of southern West Virginia. During his subsequent career, Mr. Pyle was identified with the Chesapeake and Ohio Railway Company.

From 1902 to 1922, Mr. Pyle advanced from draftsman with a field party, to levelman and transitman, and finally to resident engineer and assistant engineer in charge of heavy construction. During these two decades his promotions were slow, but it was characteristic of him that he mastered all the details of his work and prepared himself for the more important duties he was later to assume. On January 1, 1923, he was appointed assistant chief engineer. Following the death of C. W. Johns,[2] M. Am. Soc. C. E., chief engineer of the company, on September 18, 1939, Mr. Pyle was appointed chief engineer, remaining in this capacity until his death.

During the 20-yr period from 1923 to 1943, when Mr. Pyle was assistant chief engineer and then chief engineer, the total investment of the Chesapeake and Ohio Railway Company in road and equipment more than doubled. All classes of physical property were included in this tremendous increase, and improvements were made and facilities added to every division of the system from Hampton Roads, Va., to the Great Lakes. Millions of dollars were spent annually, even during the economic depression, on the construction of new main lines and branches; on the construction and improvement of yards and terminal facilities; on the elimination, enlargement, and improvement of tunnels; on the strengthening and renewal of bridges; and on the construction of wharfs and docks. In addition, new engine houses and shops for heavier motive power and equipment were provided as were many other facilities and improvements that have played an important part in making the

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[1] Memoir prepared by Crosby Miller, Assoc. M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 105 (1940), p. 1866.

Chesapeake and Ohio Railway Company one of the great coal carrying railroads of the United States.

Mr. Pyle, with the assistance of capable associates, planned and executed the many important projects entrusted to him and retained his interest and enthusiasm for his work until his death.

Mr. Pyle was a man of wide interests and varied but quiet tastes. He was keenly interested in fine horses and dogs and delighted in good music and books and in a well-arranged garden. A man of broad sympathies, he had a deeply religious nature. A member of the Overbrook Presbyterian Church of Richmond, Va., for a quarter of a century, he was elected a representative from the presbytery to the general assembly and was regarded as a leader in the affairs of the Presbyterian Church in the South.

On July 1, 1903, he was married to Bessie Boozer of Philadelphia. Mr. Pyle is survived by his widow.

Mr. Pyle was elected a Member of the American Society of Civil Engineers on December 9, 1940.

MILTON ALLENDER PYLE, M. Am. Soc. C. E.[1]

Died April 20, 1944

Milton Allender Pyle was born in Baltimore, Md., on August 24, 1894, the son of Benjamin Franklin Pyle and Laura (Allender) Pyle.

He attended the Baltimore public schools including the Baltimore Polytechnic Institute, from which he entered the University of Maryland at College Park. He was graduated in 1918 with the degree of Bachelor of Science in Civil Engineering. In 1937 the university conferred on him the professional degree of Civil Engineer.

Because of his excellent undergraduate academic record, Mr. Pyle was appointed instructor in civil engineering at his alma mater following graduation and began teaching in September, 1918. In October, 1932, he was promoted to assistant professor and in October, 1942, to associate professor of civil engineering.

Professor Pyle's chief teaching interest was in the fields of surveying, drawing, and descriptive geometry. He spent several of his summer vacations with the Washington Suburban Sanitary Commission on surveys and construction to develop sanitary facilities in the metropolitan area surrounding Washington, D. C. Other summers he spent surveying and mapping the property of the University of Maryland and in laying out a large subdivision in the vicinity of College Park. In the summer of 1937 he served as instructor in surveying at the Yale Engineering Camp at East Lyme, Conn.

During the summer of 1938 he was engineer of surveys for the firm of the late J. E. Greiner, Hon. M. Am. Soc. C. E., of Baltimore, which was consultant to the Maryland State Roads Commission. Professor Pyle had charge of several field parties on the preliminary surveys for the primary bridge program of Maryland. The projects consisted of the bridge at Havre de Grace across the Susquehanna River; the bridge at Ludlow's Ferry across the Potomac River; the proposed bridge across the Patapsco River in Baltimore; and the proposed bridge across the Chesapeake Bay at two tentative locations, one in the vicinity of Baltimore, from Miller's Island to Tolchester, and the other in the vicinity of Annapolis from Sandy Point to Kent Island. Most recently he served as engineer for the Advisory Board to the County Commissioners of Prince Georges County, Maryland.

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[1] Memoir prepared by S. S. Steinberg, M. Am. Soc. C. E.

Professor Pyle was unusually successful as a teacher. He spent considerable time and effort revising and perfecting his courses to keep his students in touch with the newest developments in his special fields. He exemplified to a high degree the qualities of loyalty and devotion to duty, and his passing is a great loss to his colleagues on the faculty, as well as to the University of Maryland in whose service he spent more than a quarter of a century.

He was a member of Tau Beta Pi, the National Society of Professional Engineers, and of Sigma Phi Sigma social fraternity.

On June 5, 1919, Professor Pyle was married to Anna Elizabeth Hunter at Baltimore. He is survived by his widow and a daughter, Shirley Dell (Mrs. Paul T. Lanham).

Professor Pyle was elected a Member of the American Society of Civil Engineers on November 12, 1940.

ALFRED MERRITT QUICK, M. Am. Soc. C. E.[1]

Died March 31, 1944

Alfred Merritt Quick was born at Seymour, Conn., on February 10, 1867, the son of Abram J. and Frances (Merritt) Quick. He was graduated from Brown University in Providence, R. I., in 1887 with the degree of Bachelor of Philosophy. Immediately after graduation, he entered the city engineer's office in Providence, where he was engaged as a draftsman on a new sewerage system, designing buildings for the Public Works Department.

Early in Mr. Quick's career, he was undecided as to whether to practice as an engineer or an architect, but eventually he decided to become an engineer.

Until 1894, with the exception of six months as a draftsman in the office of the city engineer at Lockport, N. Y., Mr. Quick continued to work in the city engineer's office in Providence. Because of his interest in water works and sewer systems, Mr. Quick decided to further his knowledge in these fields and entered the employ of the late Samuel M. Gray,[2] M. Am. Soc. C. E., a consulting engineer of Providence. In this capacity, he designed and wrote reports on water works and sewer systems. From 1897 to 1898, he was assistant engineer in the Water Department of Baltimore, Md., in charge of $2,000,000 worth of new improvements. Through experience, he finally determined that the water systems were most appealing to him. Between 1898 and 1900 he engaged in private consulting practice in Baltimore with the late Nicholas S. Hill, Jr.,[3] and the late Kenneth Allen,[4] Members, Am. Soc. C. E., under the firm name of Hill, Quick and Allen. Although the firm specialized in designing and investigating water works, it was also engaged on sewer systems, street railways, general engineering, and work for electric light and gas companies.

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[1] Memoir prepared by George E. Finck, M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1679.

In 1900 Mr. Quick was offered a position as chief engineer of the Baltimore Water Department and also was made president of the Baltimore Water Board. In addition, he was a member of the Board of Public Improvement of Baltimore. He served in these positions during three civic administrations. One of the first engineers to utilize chlorine in the treatment of water, Mr. Quick had studied the further treatment of water by use of sand beds although he seemed to favor the slow sand filter method of treatment. During this period, he was in charge of all construction of reservoirs, standpipes, and buildings and also was consulting engineer to a number of water companies, making investigations and reporting on improvements.

Between 1911 and 1922, because of a change in city adminstration, Mr. Quick entered private practice as consulting engineer on water works, sewer systems, hydroelectric plants, and mining, principally in Maryland, Virginia, West Virginia, and Pennsylvania.

About this time the United States Fidelity and Guaranty Company contemplated the establishment of an Engineering Department—an unusual action for a bonding company. In 1922 Mr. Quick was selected as chief engineer of this department. He served in this capacity until his death, acting as an adviser on contract bond underwriting and settling claims, as well as giving any necessary engineering advice.

Mr. Quick was modest in his dress and taste. He was liked by his associates, and all those who came in contact with him respected not only his engineering ability, but also his integrity. Although he gave up architecture as a vocation, it was evident that his ability as an excellent freehand sketcher would have been of material help in this particular profession.

Mr. Quick's personality is well summarized in the following statement by the Rev. A. Brown Caldwell, pastor of the Walbrook (Baltimore) Presbyterian Church, which Mr. Quick attended:

> "Alfred M. Quick was one of those unusual characters of whom it is difficult to form an estimate, except that he was an unusual man, that he did not conform to pattern or standard and that you could expect from him a refreshing attitude and a startling response.
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> "He was a man of sharp, precise and unorthodox opinions. He was strong in both his attachments and his prejudices. Few men of my acquaintance have been more devoted to their family than was he, and equally few men as intolerant of the foibles and misrepresentations which creep into our lives. He would make any sacrifice, sometimes seeming to border on

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[3] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1808.

[4] For memoir, *ibid*, Vol. 95 (1931), p. 1437.

the unreasonable, for his family, yet he would refuse even a crumb of charity if he believed the seeker to be unworthy or the cause in any way unjust.

"He was consistent in this attitude to Spartan extent. His wife recounted how he was once offered a princely sum of money to betray some information which would enable others to make gain but his stern sense of honor made him scornfully refuse to enrich himself because he regarded his public trust so highly. Yet he would lavishly minister to the whims of his children and friends with no selfish thought for himself at all.

"He seconded, throughout his family life, the deep religious devotion of Mrs. Quick. He gave of himself unstintingly to the service of the Walbrook Presbyterian Church to which he belonged. His service as Chairman of the Board of Trustees of that Church for many years resulted in improvements in its physical property and advantage in its spiritual outreach. He liked his minister and was markedly devoted to the program. But he would return to his home without entering the Church on a given Sunday if he found a stranger in the pulpit whom he did not know on occasions when his own minister might be called from the Church for some other service. His expressions of appreciation of a sermon presentation, or of some other service by the minister or a church officer, were of the same emphatic nature as was most everything else Mr. Quick did. They were most decided, almost superlative, if given at all. And he was just as ready to condemn aggressively and emphatically that with which he found cause to disagree. His attitude in politics, in social matters and in nearly all that he did was one of extremes.

"Mr. Quick was a voracious reader and in this field he carried his attitude of extremes to an unusual end. Conceiving the idea that American writers produced inferior works, he consistently refused to read the works of American authors, especially those currently issuing books, and read instead the works of other writers.

"Perhaps his attitude may have been born of an innate shyness and a deep humility of nature. But this very fact enabled him to develop his own mind so that he was capable of nearly instant judgment both of human character and in other matters. His well-trained engineering mind was able very quickly to form a judgment about a problem presented, or a case submitted. He preferred to take the matter in hand to his own mind, form his own opinion and deliver his judgment. He was not given to laboring the point with others, did not customarily indulge in debate, but was firm in his convictions when once established and held tenaciously to his judgments when given."

Mr. Quick never joined any fraternal organizations as he often expressed the opinion that his Society badge was all the professional entrée that was needed. He was efficient in his work and strong in his likes and dislikes. When preparing any papers or articles, he was very thorough and spent much time obtaining accurate information. His papers included articles on "The Cost of Modern Fireproof Buildings," "The Analysis of the Mississippi Flood Control Project and Cost," and "Water Service of the Baltimore Fire in February 1904." He wrote a comprehensive small history of the Baltimore Engineer's Club. He was a member of Sigma Xi fraternity and the Casualty and Survey Club of Baltimore, and an honorary engineer associate of the Engineer's Club of Baltimore.

Mr. Quick was elected an Associate Member of the American Society of Civil Engineers on October 5, 1898, and a Member on May 3, 1910.

AARON ISRAEL RAISMAN, M. Am. Soc. C. E.[1]

Died July 11, 1943

Aaron Israel Raisman, a native New Yorker, was born on November 9, 1873, the son of Joseph and Minna (Rubin) Raisman. He received his education in New York City—first in the public schools and later at Columbia University, from which he was graduated with the degree of Civil Engineer in 1898.

His entire professional career extending over four decades (except for two years on the engineering staff of the Central Railroad Company of New Jersey after his graduation from Columbia) was devoted to the planning and design of rapid transit railroad structures for the City of New York. He was first employed as draftsman and assistant engineer under the late William Barclay Parsons,[2] Hon. M. Am. Soc. C. E., then chief engineer of the Rapid Transit Commission, on the first subway built in New York. Mr. Raisman's outstanding ability was soon recognized. He was promoted to positions of progressively increasing responsibility and closed his career as chief designing engineer in 1938—when the last construction contracts of the $670,000,000 Independent Division of the New York City Transit System, popularly known as the Eighth Avenue Subway, were nearing completion. He was one of the small pioneer group of engineers who developed the basic design features of New York City subways.

The engineering design of the Eighth Avenue Subway—consisting of a four-track trunk line, a distributing loop in the inner part of the city, and branches extending into four of the five boroughs of New York—was under Mr. Raisman's direction from the beginning of construction in 1924 until his retirement in 1938. This project comprises fifty-nine route miles of subways, land tunnels, and subaqueous river crossings, in addition to yards, shops, and auxiliary structures. This municipally planned and operated railroad includes two hundred miles of operating track and more than one hundred passenger stations of various types. As of 1944 it carried one-and-one-half-million passengers a day with an ultimate capacity of two million passengers a day. During the height of the work when construction proceeded at the rate of $250,000 per day (believed to be a prewar record for a single project) Mr. Raisman directed a designing force of seven hundred engineers and draftsmen.

His previous work—on the planning of subways, tunnels, and elevated railroads of the first rapid transit subway in the Borough of Manhattan and the Borough of the Bronx and extensions into the Borough of Brooklyn, the Center Street Loop in Manhattan, the Brooklyn-Fourth Avenue Subway, and the dual system subways—would in itself be considered a major achievement for an active lifetime. Many of the improvements in the basic plan and in the design, keeping pace with new and improved materials and changing methods of construction, originated with Mr. Raisman.

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[1] Memoir prepared by Alfred Brahdy, M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1485.

His services were not confined to rapid transit problems of the metropolis. His advice was often sought on other municipal engineering works, and he was also consulted on rapid transit projects of other cities both in the United States and abroad. To this work he applied a combination of enthusiasm, endurance, patience, and understanding that promoted teamwork and achieved results. His keen analytical mind was quick in sizing up the fundamentals of an engineering project, and he was able also to visualize the details as well as the collateral problems. He was generous to a fault in giving credit when suggestions were made by others, and, sometimes, when things did not turn out as had been expected, he assumed the responsibility on behalf of his subordinates. His primary consideration always was the good of the work rather than personal success.

Mr. Raisman's life was one continuous record of unselfish public service for the City of New York and of loyalty to his superiors and to those who worked with him or for him. His helpful understanding extended far beyond his professional duties and many a man is indebted to him for guidance and good advice. His retirement was keenly felt by all of his associates. After he retired as chief designing engineer for the Board of Transportation of the City of New York in April, 1938, he devoted his time to travel and to his family.

Mr. Raisman recorded some of his ideas and achievements in technical publications.[3,4,5] However, the enduring record of his achievements is incorporated in the steel and concrete of the municipal rapid transit structures of the City of New York and in the hearts of his associates.

On September 10, 1900, in New York City, Mr. Raisman was married to Emma Behrman. He is survived by his widow; two children, Lucy (Mrs. Leonard Passell) and Victor Raisman, M. D.; and two grandchildren, Margery Raisman and Nicholas Passell.

Mr. Raisman was elected a Member of the American Society of Civil Engineers on June 16, 1924.

FREDERICK HOSEA RICHARDSON, M. Am. Soc. C. E.[1]

DIED NOVEMBER 5, 1944

Frederick Hosea Richardson, the son of Samuel H. and Henrietta (Hutaff) Richardson, was born in Detroit, Mich., on December 10, 1882. He attended

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[3] "The Evolution of the Subway Station," by A. I. Raisman, *Civil Engineering*, October, 1932, p. 605.

[4] "Advances in Subway Design and Equipment," by A. I. Raisman, *ibid.*, January, 1933, p. 21.

[5] "Some Elements of Subway Design," by A. I. Raisman, *Municipal Engineers Journal*, First Quarterly of 1926, p. 15.

[1] Memoir prepared by Ralf R. Woolley and R. K. Brown, Members, Am. Soc. C. E.

the public schools of Seattle, Wash., and the civil and mechanical engineering schools of the University of Washington at Seattle. He was a member of the Sigma Nu fraternity.

Mr. Richardson served the Oregon Short Line Railroad Company, the Belcher Mountain Railway Company, and the Clear Lake Railway Company on location, construction, and maintenance for about two years. For the next three years he was chief engineer on reconnaissance, location, and construction for the San Francisco, Idaho and Montana Railway Company (250 miles) in Montana.

From 1909 to 1911 he was a member of the firm of Richardson and Williams, civil engineers of Caldwell, Idaho, specializing in highway, municipal, irrigation, and reclamation work. During this period he was also city engineer of Caldwell in charge of various improvements to the water supply and distribution system, sewers and sewage disposal, complete city resurvey, street paving, sidewalks, etc. At this time he was also water commissioner for the Third District of the State of Idaho.

From the middle of 1911 until 1913, he was a member of the firm of Richardson and Richardson, civil and mining engineers, at Republic, Wash., and was also deputy county engineer of Ferry County, Washington. For the next year and a half he was assistant engineer for the Washington State Highway Department in charge of location and construction of state highways in Ferry County.

From September, 1919, to December, 1920, Mr. Richardson served as examiner, senior examiner, senior negotiator, and assistant to the adjuster of the United States Shipping Board, Emergency Fleet Corporation, in charge of appraisals of all materials on hand in all shipyards in the Pacific Northwest.

Beginning in January, 1921, for the next fourteen months he was assistant engineer for the Washington State Highway Department on location and construction in Jefferson County. On March 2, 1923, Mr. Richardson became field engineer for the Portland Cement Association at Spokane, Wash. He served in this capacity for twenty-seven months and then for five years as district engineer. During the next ten and one-half years he was consulting engineer for the same association at Salt Lake City, Utah. He was employed by the Portland Cement Company of Utah until his death.

While working for the Portland Cement Association at Salt Lake City, he was chairman of the Salt Lake County Committee of the Civil Works Administration, member of the board of consulting engineers for projects of the Reconstruction Finance Corporation in Utah, member of the Utah State Board of Engineering Examiners, member of the Utah Water Storage Commission, and member of the Salt Lake County Relief Committee.

During World War I, from January, 1918, to August, 1919, Mr. Richardson was Captain, Corps of Engineers, U. S. Army, serving as Company Commander and later as Commanding Officer of the 526th Engineers. He was also Engineer Officer in charge of the construction of various projects for the Advanced Section, the Services of Supply, and the American Expeditionary Forces and was in France for one year.

While in Salt Lake City he became a Major in the Corps of Engineer Reserve, U. S. Army, serving under Gen. Walter C. Sweeney, Commanding Officer at Fort Douglas, Utah. Major Richardson made comprehensive studies and reports on the theory that, in case of war, Fort Douglas would become headquarters for the 9th Corps Area (later the Ninth Service Command) which it did in January, 1942. For that purpose he planned and had charge of extensive improvements at Fort Douglas.

On November 2, 1940, Major Richardson was ordered to active duty as a Lieutenant Colonel, Corps of Engineers, and was in turn Executive Officer at the Recruit Reception Center at Fort Douglas; Constructing Quartermaster, Area Engineer, at Fort Dix, N. J.; Assistant to District Engineer at Philadelphia, Pa., as Chief of Operations and Executive Officer; student, Engineer Supply School; Assistant to Engineer Supply Officer as Chief, Control Unit, at the Columbus (Ohio) Quartermaster Depot; and Engineer Supply Officer, Richmond, Va. (Quartermaster) Army Service Forces depot.

On September 30, 1943, Colonel Richardson was granted a leave of absence until December 30, 1943, at which time he reverted to inactive status. He rendered signal service to the United States—in fact, he so faithfully and fully devoted himself to his Army duties that his health was seriously impaired, and a month or so before his death he entered the Veterans Hospital for observation and treatment. However, this was to no avail as his heart finally failed and he died on November 5, 1944.

In November, 1943, he returned to Salt Lake City, of which he was very fond, with the expectation of making it his permanent residence.

On March 31, 1918, Colonel Richardson was married to Mary E. Lucas of Seattle, at Petersburg, Va. He is survived by his widow and two sons, William H. (Second Lieutenant, Corps of Engineers, U. S. Army) and Frederick L. (Second Lieutenant, U. S. Army Air Corps).

Colonel Richardson was a member of the American Concrete Institute, the Salt Lake City Rotary Club, Salt Lake City Chamber of Commerce (for which he served on many committees), and the Utah Society of Professional Engineers. He was licensed to practice engineering in Utah. He was also second vice-president of the Utah Chapter of the Society of American Military Engineers and was a Thirty-Second Degree Mason, a Noble of the Mystic Shrine, and a member of El Kalah Temple in Salt Lake City. Colonel Richardson belonged to the Advisory Board of the Utah Engineering Experiment Station. Previously he had served as a member of the Advisory Committee of the Civil Engineering Department of the University of Utah, at Salt Lake City. During the winter of 1943–1944 he was a part-time instructor in Civil Engineering at the university. For many years, until his death, he served ably as Secretary of the Utah Section (later the Intermountain Section) of the Society.

Much interested in good government, he was active in many phases of civic life and service and was particularly concerned about the welfare of the younger members of the engineering profession. He possessed a genial personality with a broad outlook on life and was highly respected for his fairness and constructive vision. His death was a distinct loss to the community.

Colonel Richardson was elected an Associate Member of the American Society of Civil Engineers on September 5, 1911, and a Member on July 8, 1941.

ALLAN TOWNSHEND RICKETTS, M. Am. Soc. C. E.[1]

Died October 31, 1944

Allan Townshend Ricketts, the son of Thomas Hugh Ricketts and Cora Hanson (Townshend) Ricketts, was born in Baltimore, Md., on June 23, 1883.

He proved an exception to the rule that men who are employed for several years immediately after graduation from high school seldom obtain a university education. Mr. Ricketts was graduated from high school in Summit, N. J., in 1903 and immediately started his career as a rodman, instrumentman, and assistant engineer on municipal and railroad construction and maintenance. He left the employ of the Cincinnati, Hamilton and Dayton Railway Company at Cincinnati, Ohio, in 1911, to enter the University of Michigan at Ann Arbor.

Allan Ricketts was graduated from the University of Michigan in 1915 with the degree of Bachelor of Science in Civil Engineering. His major subject was sanitary engineering. From 1915 to 1927 he was engaged by N. S. Hill, Jr.,[2] M. Am. Soc. C. E., as assistant engineer and later as associate engineer on design and construction of water and sewer systems of considerable magnitude, including treatment plants for both water and sewage. In 1927 he was appointed deputy chief engineer of the Public Works Engineering Corporation and later of its successor, the Trojan Engineering Corporation. Both these companies were engineers for the Federal Water Service Corporation. During this engagement he conducted investigations for new water supplies throughout the United States. He also supervised design and construction of many important municipal water supplies and distribution systems.

Mr. Ricketts entered government service in Washington, D. C., in 1932 and remained in federal employ until failing health compelled his retirement in 1943. He first joined the staff of the Reconstruction Finance Corporation. In July, 1933, he transferred to the Public Works Administration (PWA). In PWA he served, successively, as engineer examiner, assistant chief engineer, and chief engineer. After the Federal Works Agency (FWA) was established, he became consultant in the office of the chief engineer of that agency, remaining until his death.

Many million dollars worth of federally financed sanitary engineering construction felt Mr. Ricketts' sound and conservative influence when he was with PWA and FWA. His broad engineering experience, knowledge, and ability were recognized as of outstanding value in water works, sewerage, and municipal engineering. His booklet on "Diagram for the Solution of Problems by Williams and Hazen Formula for Discharge of Pipes," first distributed in 1934, has provided a most useful method for direct solution of hydraulic problems in water works engineering.

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[1] Memoir prepared by a Committee consisting of H. H. Jewell and W. N. Carey, Members, Am. Soc. C. E.; H. S. Devlin, Assoc. M. Am. Soc. C. E.; and G. L. Rounds, Engr., Federal Works Agency, Washington, D. C.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1808.

In addition to his membership in the Society, which he prized, Mr. Ricketts belonged to the American Society for Testing Materials, American Public Health Association, Federal Sewage Research Association, Washington Golf and Country Club, and Capital City Chess Club. His devotion was first to his family and then to his profession. His principal hobby was golf at which he was expert. He made and retained a host of friends who will miss him. The engineering profession has lost an able leader.

He is survived by his widow; a son, Allan T. Ricketts, Jr.; and a daughter, Jane (Mrs. James G. Thomas).

Mr. Ricketts was elected a Member of the American Society of Civil Engineers on September 9, 1919.

FRANKLIN HENRY ROBBINS, M. Am. Soc. C. E.[1]

Died March 12, 1944

Franklin Henry Robbins was born in Yonkers, N. Y., on August 31, 1872, the son of Charles T. and Mary L. (Dewers) Robbins.

During his boyhood the family moved to Corsicana, Tex.; then to Toledo, Ohio, and finally to Kingston, Mass., where Mr. Robbins was graduated from the Kingston High School in 1890. Mr. Robbins attended the Massachusetts Institute of Technology, then in Boston, being graduated in 1894 with the degree of Bachelor of Science in Mechanical Engineering.

Before graduation, during his summer vacation in 1893, he acquired practical engineering experience as transitman on street railway construction for Hodges and Harrington of Boston. Soon after graduation he worked for five months as draftsman for the Peckham Motor Trunk and Wheel Company of Kingston, N. Y. From December, 1894, to February, 1897, he was assistant in mechanical drawing and descriptive geometry at the Massachusetts Institute of Technology. During the first of his summer vacations he served as draftsman and instrumentman on maps, surveys, and estimates for the city engineer of Cambridge, Mass., and during the second as machinist for the E. H. Howard Watch and Clock Company of Boston.

From February, 1897, to March, 1903, Mr. Robbins was transitman with the Boston Water Works and designer for the Metropolitan Water Board of Massachusetts on surveys, experimental work, inspections, designs, and estimates. From March, 1903, to December, 1903, he was draftsman and computer on designs and estimates for the Commission on Additional Water Supply of New York, N. Y. He then worked three years and four months for the Bureau of Filtration of Pittsburgh, Pa.—for eleven months as assistant engineer on design, and in charge, under the principal division engineer, of the preparation of contract drawings and specifications for a pumping station; and then for two years and five months until September, 1907, as division engineer in charge of construction of the pumping station, river intake, river protection, and the examination of submitted drawings of machinery and boilers.

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[1] Memoir prepared by Halsey French and Wilbur T. Wilson, Assoc. Members, Am. Soc. C. E.

From Pittsburgh, Mr. Robbins went to New York and in August, 1907, began thirty-five years of service in the Headquarters Department of the Board of Water Supply of the City of New York. For twenty years he was assistant designing engineer, and for the remaining fifteen years until his retirement on August 31, 1942, he was a designing engineer. During this period his work on the Catskill Aqueduct and the Delaware Aqueduct projects of the Board included designs, basic hydraulic determinations, experimental work, specifications and investigations for cut-and-cover aqueducts, grade tunnels, shafts, pressure tunnels, dams, bridges, steel and cast-iron pipe lines, sewage disposal plants, sanitary works, aerators, coagulating plants, chlorinating installations, and flow control chambers, together with all incidental and appurtenant designs.

He was a life member of the Boston Society of Civil Engineers and the New England Water Works Association; a member of the Municipal Engineers of the City of New York; and, for some thirteen years, an active member of the American Water Works Association.

On September 5, 1899, he was married to Marie Joubert-LaLoge in Boston. He is survived by his widow; two sons, Charles LaLoge Robbins and Henry Fuller Robbins; a granddaughter, Jean, daughter of Charles L. Robbins; a brother, Leavitt T. Robbins; and two sisters, Emily C. Robbins and Helen D. Robbins.

From boyhood, Mr. Robbins was very fond of music and excelled as a cornet player. He frequently attended the opera and concerts of symphony orchestras. While a resident of Cambridge, he was a member of the Bohemian Orchestral Club, an amateur musical organization composed largely of Harvard University men.

For many years he spent his vacations in New Hampshire fishing or occupying himself with craftwork in copper or britannia metal. He was very skilful in this craftwork and made many objects—both useful and ornamental.

He was gifted as an engineer and possessed a keen perception, an analytical mind, a most retentive memory, and was painstaking, thorough, and indefatigably industrious. Mr. Robbins left behind him a record of marked excellence and accomplishment on major engineering work. An innate modesty and reserve never concealed, however, his outstandingly high qualities of heart and mind which won for him the highest respect and esteem of those who knew him.

Mr. Robbins was elected an Associate Member of the American Society of Civil Engineers on July 10, 1907, and a Member on October 7, 1929. He became a Life Member on January 1, 1942.

JOHN McCLURE SALMON, M. Am. Soc. C. E.[1]

DIED JANUARY 30, 1945

John McClure Salmon, the son of Anderson and Edna (McClure) Salmon, was born in Washburn, Mo., on June 3, 1878. He received his education in public schools and at the University of Missouri in Columbia, from which he was graduated in 1902 with honors in Civil Engineering.

From May, 1902, to June, 1902, Mr. Salmon was employed by the St. Louis, Memphis and Southeastern Railway. He was with the consulting engineering firm of Brenneke and Fay from June, 1902, to June, 1903, and with the Terminal Railroad Association of St. Louis (Mo.) from June, 1903, to June, 1906.

He began to work for the Louisville and Nashville Railroad Company in June, 1906. He remained with this company until April, 1914. During this period he was engaged in design work, as well as in field supervision of wrecks and storms.

In April, 1914, Mr. Salmon left the Louisville and Nashville Railroad Company to become chief engineer and general manager of the Louisville Bridge and Iron Company, continuing until September, 1917, at which time he returned to the Louisville and Nashville Railroad Company as bridge engineer. He served in this capacity until his death.

Some of the most important structures which Mr. Salmon designed and constructed for the Louisville and Nashville Railroad Company are the Rigolets and Chef Menteur bridges in Louisiana; the Tensas River Bridge near Mobile, Ala.; the Ohio River bridge at Henderson, Ky.; and the Tennessee River Bridge at Danville, Tenn.

Mr. Salmon had many friends. He was a fine, honest, upstanding man, and all those who knew and worked with him loved him. Honesty, loyalty, the spirit of self sacrifice, belief in fair play, and tolerance were his outstanding characteristics. His willingness to help a friend or associate and his kindliness toward others were among his finest traits. He took great pride in his profession and work. He was a member of Tau Beta Pi, an honorary engineering fraternity, and the American Railway Engineering Association. He was active on many committees.

On November 27, 1902, Mr. Salmon was married to Elizabeth Jane Wood, at Independence, Mo. He is survived by his widow; two sons, J. M. Salmon, Jr., and Gordon W. Salmon; one granddaughter, Barbara; and three grandsons, Gordon, Jr., J. McClure II, and Robert M.

Mr. Salmon was elected a Member of the American Society of Civil Engineers on June 4, 1913.

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[1] Memoir prepared by H. G. Livesay, Assoc. M. Am. Soc. C. E.

GEORGE ARTHUR SEAVER, M. Am. Soc. C. E.[1]

Died January 20, 1945

George Arthur Seaver was born in New Orleans, La., on November 23, 1886. He was the son of Marjorie (Ford) Seaver of Virginia and George Arthur Seaver of New York State.

He attended Ferrill's School and upon graduation went to Tulane University in New Orleans where he entered the School of Civil Engineering. Desiring to earn his own way through college he worked during his vacations, and remained out of college for about two years, gaining valuable experience.

In 1903 he worked in the Johnson Ship Yards in Algiers, New Orleans. From June, 1904, to October, 1905, he was with Sanderson and Porter, engineers of New York, N. Y., as rodman, draftsman, and instrumentman on underground utilities and cables for the New Orleans Public Service. He also worked on the construction of the Market Street powerhouse and Elysian Fields Avenue substation and tunnel. From June, 1907, to October, 1908, he was assistant general superintendent of construction for A. B. Stannard on the building of the New Orleans Court House. In 1910 he was graduated from Tulane University with the degree of Bachelor of Engineering in Civil Engineering.

After graduation, Arthur Seaver entered the employ of George J. Glover, a general contractor specializing in building construction. In 1924 when the organization was incorporated as George J. Glover Company, Inc., he was taken into the firm. Mr. Seaver became well known for handling subcontracts and directing construction. He was project manager on the construction of the LaGarde General Hospital for the U. S. Army, which was the first of the rush projects to be built in New Orleans, and then took over the direction of the construction of the Consolidated Vultee Aircraft Plant, a $7,000,000 project built by his company for the Defense Plant Corporation.

In 1916, during the Mexican border trouble, Arthur Seaver enlisted in Battery C of the Washington Artillery and served with this unit on the Mexican border as a chief mechanic.

Arthur Seaver delighted in doing work with tools, repairing and constructing around his home. Tennis was his lifelong hobby, and he participated actively in this sport throughout his lifetime. He was a member and past-president of the New Orleans Lawn Tennis Club, one of the oldest tennis clubs in the United States; and, while engaged in a match of doubles, he was fatally stricken on January 20, 1945. He studied the design and construction of tennis courts, and developed a design for a brick dust court which was extremely popular and successfully used at the New Orleans Lawn Tennis Club and in the parks of New Orleans. He was always experimenting with new ideas for the improvement of this type of court.

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[1] Memoir prepared by a Committee of the Louisiana Section consisting of Bernard H. Grehan, Chairman, and John Riess, Members, Am. Soc. C. E., and Charles M. Kerr, Assoc. M. Am. Soc. C. E.

In the professional field, by virtue of his large circle of acquaintances and his experience and ability, Mr. Seaver became an outstanding figure in the building construction field. His death was a great loss, not only to his associates, but to the entire building industry in Louisiana. He was a life member of the Louisiana Engineering Society.

In 1921 he was married to Nellie Fischer. Mr. Seaver is survived by his widow and by three children, Elizabeth (Mrs. Gelon Hann Doswell), George Arthur Seaver, III, and John Wyman Seaver.

Mr. Seaver was elected a Member of the American Society of Civil Engineers on May 18, 1943.

CHARLES FRANK SEIFRIED, M. Am. Soc. C. E.[1]

Died July 21, 1943

Charles Frank Seifried, the son of Frank and Josephine (McGowan) Seifried, was born at Chicago, Ill., on October 3, 1885. While he was still quite young, his parents moved to Denver, Colo., where he completed the regular public grade and high school courses. He studied general engineering subjects at Stanford University, Stanford University, Calif., from September, 1903, to January, 1905. In September, 1906, he entered Cornell University at Ithaca, N. Y., from which he was graduated with the degree of Civil Engineer in June, 1909. From January, 1905, to September, 1906, and during the summers of 1907 and 1908 while attending Cornell University, he worked for the Union Pacific Railroad Company in Colorado—first as rodman, later as instrumentman, and finally as foreman on miscellaneous railroad construction and maintenance jobs, including some highway and power-line construction.

Like many young engineers, his first few years after leaving school were spent in various places on different assignments. From September, 1909, to December, 1917, Mr. Seifried held the following positions:

September, 1909, to December, 1910—with the St. Louis, Iron Mountain and Southern Railway Company at Van Buren, Ark., first as assistant in the engineer's corps, and later as assistant engineer on general maintenance work.

December, 1910, to February, 1911—with contractors on highway improvement work in Colorado, in charge of concrete bridge and culvert construction.

May, 1911, to May, 1912—with Reitze, Story and Duffy, at Seattle, Wash., for six months as instrumentman and inspector on street improvement work and for six months as chief of survey party on railroad location.

May, 1912, to May, 1913—with the Clear Lake (Washington) Lumber Company, on location of logging roads, construction of trestle bridges, and topographic mapping.

June, 1913, to December, 1914—with Boynton and Church and Guthrie-

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[1] Memoir prepared by W. H. Fisher, Assoc. M. Am. Soc. C. E.

McDougal, contractors, in the State of Washington on heavy railroad construction, including five months in charge of a grading camp of fifty men and considerable time in charge of concrete and other construction.

May, 1915, to December, 1917—with the U. S. Interstate Commerce Commission, Division of Valuation, as junior railway civil engineer, in charge of cross-section work and track parties on inventory for various western railroads.

In May, 1917, he applied for a commission in the U. S. Army Engineer Corps for service in World War I. He was assigned to Officers' Training Camp at Petersburg, Va., in January, 1918, where he was made a Second Lieutenant. Assigned to the Twenty-Third Engineers at Camp Laurel, Maryland, he later sailed for France on March 28, 1918, and saw fourteen months of active service there in charge of highway and railroad construction and maintenance, during which time he became a First Lieutenant. He was honorably discharged from the Army in July, 1919.

In August, 1919, immediately after his discharge from the Army, Mr. Seifried was employed by the Wyoming State Highway Department, as resident field engineer on highway construction. Within one month, however, he was called into the headquarters office of the department at Cheyenne, Wyo., as chief draftsman and head designing engineer. He here had full charge of preparing the detail plans for all future highway projects, for eighteen years, or until September, 1937. During this period he was directly responsible for the complete plans and specifications used on the construction of some 3,300 miles of primary and secondary state highways.

In September, 1937, the State Highway Commission, having long since recognized his ability, appointed Mr. Seifried state highway superintendent, the highest office in the department. In March, 1939, because of a change in the state administration, he temporarily relinquished this position to become chief engineer of the department. In this latter capacity, he acted as principal assistant and consultant to the superintendent, but still retained full charge of all field and office engineering personnel and work, supervised the expenditure of all departmental funds, and handled a major part of the main office correspondence.

During May, 1942, while on a routine inspection trip in the northeastern part of Wyoming he was suddenly stricken by a severe heart attack. Brought to Cheyenne by ambulance, Mr. Seifried remained hospitalized until the following December before being able to resume work. In January, 1943, the State Highway Commission reinstated him as superintendent with the title of state highway superintendent and chief engineer. He retained this position until his death.

On July 21, 1943, while on an inspection trip with the state highway commissioners, at Cody, Wyo., he was suddenly stricken by a second heart attack, which at once proved fatal. His sudden death came as a great shock, not only to the entire personnel of the State Highway Department, among whom he was always familiarly known as "Si," but likewise to his many hosts of friends both in Cheyenne and over the entire state.

Despite his arduous duties in the State Highway Department, Mr. Seifried had always managed to find time through the years to engage in many worthwhile outside activities, in which he filled many positions of honor and responsibility. During the last three years of his life, he had served as "charge lay leader" in the Cheyenne Methodist Church, a position next in importance to that of pastor. He was vitally active in Blue Lodge and Scottish Rite Masonry, and had been Master of Blue Lodge, Acacia No. 11, at Cheyenne. He was actively interested in Boy Scout work, having served for several years as Cub Scout Leader. Always interested in professional affairs, he was president of the Western Association of State Highway Officials from June, 1941, to June, 1942; vice-president for Fourth District of the American Association of State Highway Officials from October, 1941, to December, 1942; president of the Cheyenne Engineers Club for 1925; president of the Wyoming State Board of Examining Engineers from February, 1935, until his death; president of the Wyoming Section of the Society for 1940; and vice-president of the Wyoming Engineering Society from January, 1943, until his death.

While in Seattle in 1911, Mr. Seifried first met his future bride, Lilian McGrew. They were married at Tacoma, Wash., in September, 1917. Two children graced their happy marriage—a son, Robert Dayton Seifried (in the U. S. Navy), and a daughter, Josephine Louise Seifried. By her whole-hearted devotion and high Christian ideals maintained through the years, Mrs. Seifried contributed much not only to the life and achievements of Mr. Seifried, but also to the welfare of the entire community. Mr. and Mrs. Seifried were at all times keenly active in the Cheyenne Methodist Church to which they belonged, and both always participated in any religious, educational, or civic betterment movement—either local, state, or national.

Any appraisal of Mr. Seifried's personality must of necessity list the following characteristics as being outstanding: His constant faith in his fellow men and his extreme optimism; his dynamic personality and the seemingly boundless energy and enthusiasm with which he would undertake a new job or grapple with an old one; his quickness to accept new situations as they arose and to act accordingly; his open frankness in all dealings; and his amazing capacity for work, matched only by his zeal for accomplishment. With such qualities it is only logical that it was with some difficulty that he could tolerate anything lackadaisical, either in word or in action. He was not a stickler for extreme nicety of form or for finespun theory beyond the lines of its practical application to some definite problem. These traits of character, too, were nicely balanced on occasion by his possession of a very keen sense of humor.

Remarkably noteworthy was Mr. Seifried's ability to remember details of any kind—whether these details were of some recent or of some long-past activity or conversation, or whether of some printed text or plan. Thus, his many inspection trips over Wyoming had made him thoroughly familiar with its every topographic detail, and during these trips he could commonly and offhand not only recall all essential construction details for any given highway section, but could also cite the date when built and the name of the constructing contractor. This happy faculty of recalling minute details, seemingly without effort, was naturally of great help to him throughout the years, particularly when state highway superintendent.

To summarize, Charles Frank Seifried was a man beloved by all who knew him. His sterling character and radiant good humor, his boundless optimism and enthusiasm in any undertaking, his stalwart Christian faith as exemplified in his daily life, and his tireless faithfulness to duty—these were qualities which drew to him the respect and admiration not only of his immediate associates in the State Highway Department, but of all others who knew him. In conclusion, as Edwin Markham has so aptly written of Abraham Lincoln, it may be truly said of Mr. Seifried:

"He held his place—
Held the long purpose like a growing tree—
Held on through blame and faltered not at praise.
And when he fell in whirlwind, he went down
As when a lordly cedar, green with boughs,
Goes down with a great shout upon the hills,
And leaves a lonesome place against the sky."

Mr. Seifried was elected a Member of the American Society of Civil Engineers on January 9, 1939.

GILMAN WALTER SMITH, M. Am. Soc. C. E.[1]

Died July 12, 1944

Gilman Walter Smith was born on December 1, 1859, in Red Wing, Minn., the son of Otis Smith and Margaret (Densmore) Smith. His forebears were all of good American stock. He was graduated from the University of Minnesota at Minneapolis in 1880, receiving the degree of Bachelor of Science.

Mr. Smith was first employed by the Chicago, Milwaukee, St. Paul and Pacific Railroad Company as a member of a field surveying party reporting to the division engineer's office in Minneapolis. In this capacity he was engaged on the location of a railway line from Norwood to Mankato, Minn. From 1881 to 1886 he worked for the same company in various positions beginning as rodman and advancing to assistant engineer, chiefly on maintenance of way. In 1886 he was transferred to the bridge and building department where he acted as chief carpenter, largely on inspection and construction of timber bridges for the South Minnesota Division. In 1888 he was promoted to assistant engineer on the inspection of bridges reporting to general headquarters at Milwaukee, Wis.

In 1890 the general offices were removed to Chicago, Ill., and he was transferred as assistant engineer to the staff of Onward Bates,[2] Past-President and Hon. M. Am. Soc. C. E., engineer and superintendent of bridges and buildings. In 1895 Mr. Smith became assistant superintendent of bridges and buildings, remaining with the railway company in this capacity until he resigned in 1901 to become erecting manager for the Western Division of the American Bridge Company of New York, N. Y., with headquarters in Chicago.

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[1] Memoir prepared by Herbert S. Crocker, Past-President and Hon. M. Am. Soc. C. E.
[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 1484.

There is no detailed record of his railway experience, but it is evident that Mr. Smith performed his duties most efficiently, or he would not have been consistently promoted to added responsibilities. He was painstaking and thorough in all the duties assigned to him.

After 1901 Mr. Smith enjoyed greater opportunities for responsibility and administration of the important work for which he had trained himself. As erecting manager for the Western Division of the American Bridge Company, he supervised the building of many large structures and bridges, including two railroad bridges across the Mississippi River—one at Thebes, Ill., and one at St. Louis, Mo.—and two bridges across the Missouri River—one at Omaha, Nebr., and the other at St. Joseph, Mo. His duties included the direction of arrangements, selection of methods and equipment, and responsibility for the safety, economy, and accounting of operations. The erection of bridges is not a simple procedure since varying sites and conditions demand methods entirely beyond routine and involve more careful planning, imagination, and courage than are generally realized either by the public or by the profession at large.

After leaving the bridge company in 1915, for two or three years Mr. Smith was engaged in personal affairs apart from engineering. From October, 1917, to March, 1918, he was employed by Metcalf and Eddy, consulting engineers of Boston, Mass., as resident engineer on the construction of a reinforced concrete reservoir for the city water supply of Dayton, Ohio.

Anxious to serve the United States in World War I, on March 15, 1918, Mr. Smith became a civilian employee of the Construction Division of the Army as personnel assistant to the executive officer of the building division. This work included interviewing, corresponding with, and judging the fitness of, applicants for commissions. He also supervised the records giving information regarding vacant positions and assignment of officers to duty.

On October 21, 1918, Mr. Smith was commissioned a Major, Quartermaster Corps, U. S. Army, and was assigned to duty with the administrative division. Later he was liaison officer to the commissioned personnel branch of the General Staff. He was honorably discharged on October 30, 1920. From November 1, 1920, to June 30, 1921, he was employed in the Real Estate Service of the War Department and in the Purchase, Storage, and Traffic Division of the General Staff.

From August 28, 1922, to August 31, 1924, Major Smith was senior structural engineer, with the Bureau of Valuation of the Interstate Commerce Commission. From October, 1924, until June 30, 1934, when he was retired by the U. S. Civil Service, he served as chief contract and record secretary, Construction Service, U. S. Veterans Administration.

After his retirement, he made a number of tours, including one to England, and finally settled in Los Angeles, Calif., where he supervised certain family estates and lived in comfort and good health.

A few months before his death, Major Smith entered the James Wadsworth Veteran's Hospital near Santa Monica, Calif., as the result of an accident. He was buried in Arlington Cemetery in Washington, D. C.

The writer, who has known Major Smith since 1888, can truly state that Gilman W. Smith was an able engineer and an excellent administrator—logical and thorough in all his methods, fond of analysis, honest and loyal to those with whom he served. He had many admirable traits of character, being unselfish, always thoughtful of others, and active in the promotion of the welfare of his friends. Major Smith was a man of good habits, unpretentious but outstanding in his profession. There should be more engineers like him.

He was married in Minneapolis, Minn., in 1883 to Addie Anna Tidd. He was a member of the Universalist Church and, with Mrs. Smith, was always interested in church matters. After nearly fifty years of happily married life Mrs. Smith died in May, 1932, in Washington, D. C. Major Smith is survived by one son, Donald T. Smith.

From September, 1901, to December 31, 1919, Major Smith was a member of the Western Society of Engineers.

Major Smith was elected a Member of the American Society of Civil Engineers on March 2, 1909.

HOWARD EVERETT SMITH, M. Am. Soc. C. E.[1]

DIED JUNE 27, 1944

Howard Everett Smith, the son of Henry and Mary (Gay) Smith, was born at Waltham, Mass., on February 19, 1870.

His interest in engineering began with his employment by a firm of sanitary engineers in Natick, Mass., in 1888. This led to his attending the Worcester Polytechnic Institute at Worcester, Mass., from 1892 to 1894, when he shifted over to the Massachusetts Institute of Technology at Boston for the junior and senior years, graduating in 1896, with the degree of Bachelor of Science in Sanitary Engineering.

The next two years were spent on surveys for the Boston (Mass.) subway system then under construction. After the subway was opened, he became assistant to the civil engineer in charge at the Portsmouth (N. H.) navy yard, on the layout of buildings, installation of water and sewerage systems, and construction of a dry dock. From 1900 to 1906 he was an assistant civil engineer for the War Department in Ohio, constructing a breakwater at Fairport Harbor and having charge of a residency which included Ashtabula and Conneaut harbors.

In 1906 he went to New York State as an assistant engineer in the Department of the State Engineer and Surveyor, on the Barge Canal and the state highways, later with departmental rank of resident engineer. In 1909 when the highways were transferred from the control of the state engineer to a separate commission, Mr. Smith, with many other engineers, went over to the new department. He functioned as a resident engineer in the Syracuse Division and later as division engineer at Binghamton. In 1923 he transferred to the Rochester division. Not long afterward, in consequence of the merger of the several New York State construction bureaus into the State Department of Public Works, Mr. Smith became district engineer charged with design and construction of state and county highways in Monroe, Ontario, Livingston, Wyoming, Genesee, and Orleans counties, with general supervision of town and county road work, operation and maintenance of the Barge Canal from Lyons to Lockport, N. Y., and miscellaneous engineering projects inherited from the former state engineer's department. He retired in 1940.

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[1] Memoir prepared by Percy Waller, M. Am. Soc. C. E.

Mr. Smith was of a rather reserved, retiring disposition, but behind this there was a great kindliness and richness of friendship. He was a man of high principles, of sound judgment, and of great executive ability—an excellent judge and manager of men. He was endowed with an exceptionally keen mind which evidenced itself in the clarity of his ideas and of his speech.

A prominent engineer, a friend of Mr. Smith for half a lifetime, writes: "Howard Smith was one of the finest men I ever knew insofar as his personal habits and traits were concerned and the attention which he devoted to his invalid wife and his family."

In 1901 Mr. Smith married Elizabeth Goddard. A son, Thorold Smith, and a daughter, Louise (Mrs. A. H. Knight), survive. Mrs. Smith died in 1938. It is almost a legend among the many who knew him how, when his wife became unable to walk, Howard Smith would carry her in his arms into church, concert, theater, or other places. In 1943 Mr. Smith married, this time Mabel C. Keehn of Rochester, who survives him.

He was a member of the national and New York State societies of professional engineers, a Thirty-Second Degree Mason, and a Rotarian.

Mr. Smith was elected a Member of the American Society of Civil Engineers on November 6, 1907.

GEORGE FRANCIS SPARHAWK, M. Am. Soc. C. E.[1]

DIED MAY 3, 1944

George Francis Sparhawk was born in Randolph, Vt., on September 15, 1863, the eldest son of Luther Tucker and Josephine (Bean) Sparhawk. His grandfather and great-grandfather were Congregational ministers in Vermont.

His preparatory education was obtained in Randolph, and he was a member of the first class to be graduated from the high school. He entered Dartmouth College, at Hanover, N. H., in September, 1885, and was graduated in 1889 with the degree of Bachelor of Literature. He continued his engineering education in the Thayer School of Civil Engineering from which he was graduated in 1891 with the degree of Civil Engineer.

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[1] Memoir prepared by C. F. Goodrich, M. Am. Soc. C. E.

After graduation, Mr. Sparhawk began at once to practice structural engineering—in which field he continued to specialize throughout his professional career. His first position was as assistant engineer with the R. F. Hawkins Iron Works in Springfield, Mass. He served in a similar capacity for George S. Morison,[2] Past-President, Am. Soc. C. E., consulting engineer; the Berlin Iron Bridge Company; and the Boston and Albany Railroad Company. In 1896 he became assistant engineer for the Union Bridge Company at Athens, Pa.

When the Union Bridge Company was incorporated in the American Bridge Company in 1901, Mr. Sparhawk became assistant engineer for the new firm. In 1903 he was transferred to the Ambridge (Pa.) Plant of the American Bridge Company, where he was promoted, successively, to squad foreman in the drawing room, to drawing room engineer, and, just before his retirement, to assistant to the plant engineer, as general consultant on special problems in the various drawing rooms. He retired in 1931 at the age of sixty eight from the American Bridge Company and from active engineering. All but five of the forty years of his professional career had been devoted to the American Bridge Company.

After his retirement from his chosen profession, Mr. Sparhawk sold insurance until he was forced to retire from all business activities because of failing health.

Those who knew George Sparhawk, who worked with him, or who were his neighbors in Beaver, Pa., will long remember his hearty manner, his ready smile, and his keen interest—not only in his work but in the humanistic side of life. He seldom wanted to talk about himself but always evinced much interest in other people. Always ardently enthusiastic about the Thayer School and Dartmouth College, he was able to attend the fortieth and fiftieth reunions of his class. He was a strong supporter of the American Society of Civil Engineers, attending the meetings of the Pittsburgh Section regularly and the Annual Meetings in New York, N. Y., whenever possible.

Mr. Sparhawk was an enthusiastic horseback rider and during the greater part of his active life kept one or two very fine saddle horses—some with such spirit that few besides himself dared ride them.

Mr. Sparhawk was married in 1896 to Jessie Burt of Orange, N. J. In addition to his widow, he is survived by two sons, John Burt and George Francis, Jr., and two grandchildren.

Mr. Sparhawk was elected a Member of the American Society of Civil Engineers on October 10, 1916.

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LIV (June, 1905), p. 513.

ALLEN WHITMORE STEPHENS, M. Am. Soc. C. E.[1]

Died March 31, 1944

Allen Whitmore Stephens was born in Patten, Penobscot County, Me., on July 19, 1879, the son of Charles Walter and Susan Adella (Chadbourne) Stephens.

Mr. Stephens was graduated from the University of Maine, at Orono, in 1899, with the degree of Bachelor of Civil Engineering. In 1923 his alma mater recognized his outstanding engineering ability by conferring on him the honorary degree of Master of Science. He was a member of the Phi Kappa Sigma, Phi Kappa Phi, and Tau Delta Pi fraternities.

In June, 1899, he entered the engineering field as a draftsman for the New England Structural Company, at Everett, Mass. In March, 1901, he was employed by the American Bridge Company, as a checker and squad boss. In December, 1903, Mr. Stephens moved to Phoenixville, Pa., as a squad boss for the Phoenix Iron Company.

These positions were all in the field of structural steel design. The reinforced concrete industry, then in its infancy, attracted Mr. Stephens' attention. In September, 1905, he entered the employ of the Turner Construction Company, pioneers in concrete construction, first as a draftsman, then as a checker, chief draftsman, designing engineer, and finally as engineer. In May, 1919, he became chief engineer, and, from December, 1925, to September, 1926, he acted as general manager of the Chicago (Ill.) branch office. During these twenty-one years, all men who came under his direction learned to appreciate his kindly ways and honest dealings.

Mr. Stephens became vice-president of the Dresser Company of Cleveland, Ohio, in September, 1926, and continued in the field of general contracting until November 28, 1928, when he became a consulting engineer specializing in structural steel and concrete work and in expert court testimony. From November, 1933, to June 30, 1934, he was acting professor of civil engineering at the University of Delaware, at Newark, teaching highway, municipal, and structural engineering. He also served from May 1, 1935, until his death as a member of the Advisory Committee on Engineering, Public Buildings Administration, Federal Works Agency, Washington, D. C. As a consultant, he specialized in prefabricated demountable house design from October, 1940, to March 1, 1942.

Although an extremely busy man, Mr. Stephens found time to be chairman of the Building Code Committee of the American Concrete Institute, director in the New York City (N. Y.) Sunday School Association, trustee of the Moravian College and Seminary, in Bethlehem, Pa., and director of a boys' club. He was a member of the First Moravian Church of New York City and president of the Board of Trustees for many years. He served as chairman of the Board of Trustees of the Larger Life Foundation of the Moravian Church in America for twenty-four years.

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[1] Memoir prepared by Alfred B. Heiser, M. Am. Soc. C. E.

Mr. Stephens belonged to the Engineers Club of New York, the Structural Engineers' Society of New York, the American Concrete Institute, the Western Society of Engineers, and the Newcomen Society.

He received the Maine Plaque with "M" for his services to the University of Maine through the alumni association, of which he was president for ten years.

On December 27, 1905, in Bethlehem, he was married to Marion Levering, who survives him.

Mr. Stephens was elected an Associate Member of the American Society of Civil Engineers on June 6, 1911, and a Member on May 8, 1922.

CHARLES HENRY STEVENS, M. Am. Soc. C. E.[1]

Died February 3, 1944

Attributes of character are formed during youth, and men are what their early experiences have made them. It is not accidental that many men who have risen to prominence in their chosen fields did so from a background of very limited opportunity, for responsibility in early life develops character. This is well illustrated in the career of Charles H. Stevens, whose father died when the boy was about eight years of age, leaving the widow and four children almost penniless.

Charles Henry Stevens was born in Harrisburg, Pa., on October 21, 1875, the son of William Franklin and Katherine (Marion) Stevens. Shortly after his father's death, Charles noticed one day that his mother appeared troubled. Upon inquiring, he learned that the mother's concern was due to lack of the necessary household money. The boy advised his mother not to worry, since he had sufficient money in his child's bank to take care of everything, and he thereupon turned over its contents for that purpose. The bank contained just two dollars! Here was a spirit unusual in a child, but one which was to be evidenced soon again.

Charles attended the public schools of Harrisburg, but left school, apparently at the age of eleven, and came to Philadelphia, Pa., with his mother in October, 1887. At the age of twelve, Charles entered into a nine-year apprenticeship agreement with a Philadelphia mechanical engineer, in order to provide for his own needs and for his mother's as far as he was able. The agreement is reproduced in full to illustrate the terms to which the boy voluntarily bound himself. While probably not unusual for the period, these terms speak for themselves.

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[1] Memoir prepared by Philip H. Carlin, Scott B. Lilly, and Henry J. Sherman, Members, Am. Soc. C. E.

"THIS INDENTURE,

"Witnesseth, that Charles H. Stevens being a minor, and of the age of 12⅓ years, by and with the consent of his mother hath put himself, and by these presents, doth voluntarily, and of his own free will and accord, put himself Apprentice to T. Shaw, M. E., to learn the art and trade of Machinist business, and after the manner of an Apprentice to serve the said T. Shaw, M. E., until he shall have attained the age of twenty-one years, to wit: the 21st day of October in the year of our Lord one thousand eight hundred and ninety-six. During all which time the said Apprentice doth covenant and promise, that he will serve his master faithfully, ten hours each day—Sundays, Christmas day, and Fourth of July excepted—keep his secrets, and obey his lawful commands; that he will do him no damage himself, nor see it done by others, without giving him notice thereof—that he will not waste his goods, nor lend them unlawfully—that he will not contract matrimony within the said term—that he will not play at cards, dice, or any other unlawful game whereby his master may be injured—that he will neither buy nor sell, with his own goods or the goods of others, without license from his master—and that he will not absent himself day nor night from his master's service, without his leave—nor haunt ale houses, taverns or play-houses, but in all things behave himself as a faithful apprentice ought to do during the said term, and use his utmost endeavors to instruct other apprentices, when required. And the said master on his part, doth covenant and promise, that he will use the utmost of his endeavors to teach, or cause to be taught or instructed, the said Apprentice, in the art and trade of Machinist business, and will make allowance to his mother during said term, in lieu of sufficient meat, drink, clothing, lodging, washing, and all other necessaries fitting for an Apprentice, as follows:

$2.50 per week, for the 1st year; and $3.00 per week for the 2nd year
$3.50 per week for the 3rd year; and $4.00 per week for the 4th year
$4.50 per week for the 5th year; and $5.00 per week for the 6th year
$5.50 per week for the 7th year; and $6.00 per week for the Balance of the Term.

for every week in which said Apprentice shall be at work. In case of sickness, provided a Doctor's certificate be furnished for the time he is absent, the allowance shall not be deducted except in case of sickness and injuries brought on by his own imprudence. If a Doctor's certificate be not furnished, all allowance shall cease till said Apprentice resumes work. Should the said Apprentice, except in case of sickness, absent himself from work, without his master's permission, he shall forfeit at the rate of double the above wages for every working day in which he shall so absent himself. Should the said Apprentice, through idleness, negligence, or carelessness, neglect to perform his allotment of work, or fulfil it of inferior quality, or shall break any rule of the factory, he shall forfeit a just proportion of above allowance for such negligence or carelessness. Any compensation for overwork shall be subject to forfeiture for any breach of the order or discipline of the factory.

"And for the true performance of all and singular the covenants and agreements aforesaid, the said parties bind themselves each unto the other, firmly by these presents. In witness whereof, the said parties have interchangeably set their hands and seals hereunto. Dated the 27th day of February in the year of our Lord one thousand eight hundred and eight-eight. Sealed and delivered before me, the Subscriber, Cora I. Shaw

Thomas Shaw, M. E.
Charlie H. Stevens
Mrs. K. M. Stevens"

After completing his apprenticeship, Mr. Stevens served as draftsman with the Reading Railway Company for one and one-half years and then, in 1900, began his long career with the City of Philadelphia, initially as bridge draftsman in the Bureau of Surveys. With the exception of four years, from 1903 to 1907, which he spent with the Philadelphia Rapid Transit Company (later the Philadelphia Transportation Company) on designs for the Market Street Subway-Elevated Railway, Mr. Stevens was to continue in the service of the city in various capacities until 1940—a period of more than thirty-seven years.

During the years of his apprenticeship and early engineering employment, Mr. Stevens made good use of every opportunity to obtain the education of which he had been deprived by the circumstances of early life. He attended evening classes at several technical schools during this period, and also pursued extensive studies at home throughout his early life. The result was that, as his responsibilities increased, he was always able to supplement his practical engineering knowledge with the theoretical background necessary for application to new problems. In this connection, although Mr. Stevens was preeminent in administrative work rather than in technical design, he nevertheless displayed unusual ability in analyzing design problems.

Mr. Stevens returned to the Bureau of Surveys in 1907 as assistant engineer in the Grade Crossing Division, in charge of design, preparation of plans and specifications, and estimates of cost of construction. The principal activity of the division at that time was elevation of the tracks of the Philadelphia and Reading Railway Company (later the Reading Company) to abolish grade crossings between Spring Garden Street and Broad Street, between 16th Street and Wayne Junction, and along the Richmond Branch from B Street to Richmond Street.

On completion of this work in 1913, Mr. Stevens was employed for four months in superintending the installation of a system of high-pressure fire mains in the northeast section of the city.

At about this time there was a growing realization of the advantages of high-speed transit facilities over the surface trolley system which prior to 1907 had provided all public transportation service in Philadelphia. Although at that time the passenger automobile was just coming into use, it had already created a certain amount of dissatisfaction with the comparatively slow surface system. Various areas of the city, particularly the northern and northeastern sections, were in need of faster service, and the phenomenal growth of the 69th Street section at the western terminus of the Market Street Subway-Elevated Railway was not without effect on the imagination of real-estate operators in other areas.

All these factors contributed to a growing demand for additional high-speed facilities. At the same time it became evident that the Philadelphia Rapid Transit Company was not in a position to finance further extensive construction work, and municipal financing would be required.

In view of this situation, the city council on May 13, 1912, appointed A. Merritt Taylor as transit commissioner to study the matter, and began the legal steps necessary to acquire authority to construct and own transit facilities. The engineering firm of Ford, Bacon and Davis was retained in a consulting capacity for the transit investigation. On July 14, 1913, Mr. Taylor submitted to Rudolph Blankenburg, then Mayor of Philadelphia, a comprehensive high-speed transit program calling for construction of the Frankford Elevated Railway and the Broad Street Subway, among other projects. These recommendations (although with later modifications) were destined to provide the foundation for Philadelphia's high-speed transit construction program.

Meanwhile the necessary enabling legislation had been passed at Harrisburg, and on June 26, 1913, the city council created a Department of City Transit, to have charge, control, and management of the transit facilities of the city. Organization of the department began immediately, and on July 1, 1913, Charles H. Stevens was transferred to the department as chief draftsman. From that time on, the life of Mr. Stevens is in a very real sense the history of high-speed transportation development in Philadelphia.

The first project undertaken by the new department was the design and construction of the Frankford Elevated Railway. This is a two-track structure extending from Front and Arch streets in Front Street, Kensington Avenue, and Frankford Avenue to Bridge Street in Frankford, a distance of 6.4 miles. In his capacity as chief draftsman, Mr. Stevens supervised preparation of plans and specifications for the entire structure and for its equipment.

During the period from 1913 to 1918, design work also went forward for various parts of the Broad Street Subway, including sections under, and adjacent to, City Hall, and several construction contracts were awarded. However, progress was slow. The entry of the United States into World War I prevented the award of further contracts after April, 1917, and finally, in July, 1918, forced termination of all remaining construction activities.

However, during the war and early postwar years, the department completed its designs for the Frankford Elevated Railway and Bustleton Surface Line. The Frankford elevated line, including terminal yard, three electric substations, and one hundred cars, cost about $15,000,000. Construction was delayed by World War I, but the line was finally completed by the department and was opened to service in November, 1922, by the Philadelphia Rapid Transit Company, lessee and operator of all city-owned lines. Unusual design features of the Frankford elevated line are the passenger station buildings, which house the stairs to relieve the sidewalks of obstruction, and the cruciform-bent steel structure of the elevated in Frankford, which frees the narrow sidewalks of column and pole obstructions. The Bustleton surface line, a much smaller project completed at about the same time, is a $700,000 single-track surface-car feeder extending from Frankford Avenue and Bridge Street via Bustleton Pike to Byberry.

Little new work could be undertaken during the period from 1918 to 1923 because basic questions had arisen as to the adequacy of several of the remaining projects as originally proposed, and in 1923 a transit advisory board appointed by J. Hampton Moore, then Mayor of Philadelphia, recommended extensive modifications to these projects. Additional changes were recommended by a councilmanic transit commission during the course of the year, and, at the November, 1923, general election, the voters approved the projects in their revised form and voted funds to reopen the work.

The department once again had a definite program and sufficient funds with which to proceed. However, its staff had shrunk to a very small size in the meantime, and in March, 1924, Mr. Stevens was appointed engineer of design with the task of assembling an adequate staff of engineers and draftsmen for the design of the projects as authorized.

The design staff was quickly expanded, and the department went ahead with plans and specifications for the Broad Street Subway north of Stiles Street. This work was completed and the entire section between Stiles Street and Courtland Street was placed under contract during 1924. In 1925, contracts were awarded for the remaining sections of the North Broad Street Subway.

The design work continued at a rapid rate under Mr. Stevens' direction, and, by the end of 1927, plans and specifications had been prepared and contracts let for the Fern Rock Terminal Yard, with its service buildings, and for the subway under South Broad Street north of Kater Street (near South Street).

Designs for the substation buildings necessary to provide direct current for subway operation were next completed. Construction work went on without the earlier delays, and in September, 1928, the North Broad Street Subway was opened for operation. Service was extended south to Kater Street in April, 1930.

The sections of the Broad Street Subway completed and opened for service during Mr. Stevens' incumbency as engineer of design consist of 6½ miles of four-track structure and ¼ mile of two-track structure at the south end. Only two tracks are operated. The total cost of this work, including terminal yard and service buildings, four electric substations, and one hundred and fifty cars, was about $110,000,000. The most advanced designs were utilized throughout the work. Station platforms are 550 ft long, permitting the operation of eight-car trains. Some interesting and difficult problems were involved in the work, particularly in the underpinning of City Hall, a massive masonry building with walls up to 12 ft thick and with foundations of uncertain character in some places.

In 1928 the department began work on plans and specifications for the relocation of the Market Street Subway tracks at City Hall in connection with the City Hall Concourse, and for the construction of the Ridge Avenue Branch of the Broad Street Subway.

When the Market Street Subway was constructed, both high-speed and subway-surface tracks had been detoured around City Hall to avoid the necessity for underpinning that structure, but this arrangement added appreciably to the time and cost of operating cars between the 13th Street and 15th Street stations. The new designs overcame the more serious of these conditions by providing a direct east-west route under City Hall for the high-speed trains. The subway-surface tracks were then relocated in the space previously occupied by the high-speed tracks, and the areas thus cleared were renovated to provide an underground concourse for pedestrians.

The subway in Ridge Avenue was planned to extend from a connection with the Broad Street Subway at Fairmount Avenue, in Ridge Avenue, Eighth Street, and Locust Street to 18th and Locust streets, and ultimately to West Philadelphia. Because of a shortage of funds, only that section of the line north of Ranstead Street (near Market Street) has actually been completed and put into operation (December, 1932), although subway construction was continued west to 18th Street. The cost of constructing the entire 2.28 miles of this two-track subway was $17,300,000.

In 1929 plans and specifications were begun under Mr. Stevens' direction for the first section of the two-track Market Street Subway Extension, from 24th Street to 32d Street. This section included a tunnel under the Schuylkill River designed as an integral part of the foundations for a new two-arch highway bridge to be constructed over the river on the line of Market Street. The subway was planned to extend ultimately from 22d Street to about 46th Street, permitting removal of the existing elevated structure between those points, but financial difficulties intervened. The work was closed up in 1932, after completion of the 24th Street-32d Street section at a cost of $6,900,000. The river crossing presented some difficulties because of the condition of the soil under the river, where both rock and a deep layer of mud were encountered.

In the midst of the City Hall Concourse and Ridge Avenue subway work, the department suffered the loss by death of its very able chief engineer, Frank R. Fisher. Mr. Stevens had been directing the design work with speed, effectiveness, and a high degree of technical competence, and in recognition of his ability he was appointed chief engineer, succeeding Mr. Fisher in August, 1930.

The economic depression of the 1930's curtailed further work on the Locust Street Subway and Market Street Subway Extension, but permitted work at a slow pace on the two-track South Broad Street Subway. The section of this line between Kater Street and Snyder Avenue was completed at a cost of about $8,000,000 and was put into operation in September, 1938, bringing to an end, for the time, construction of high-speed facilities.

Mr. Stevens retired as chief engineer of the Department of City Transit in January, 1940. During his twenty-seven years of service with the Department of City Transit, he had been in charge of designs for, and construction of, many kinds and types of work. As chief engineer, he directed not only the actual design and construction of subway structures with all the attendant problems of underpinning and protecting existing buildings along the streets and the underground service structures under these and intersecting streets, but also the installation of services and equipment to provide a completed system, ready for operation when leased by the operating company. His work included the installation of trackage, complete automatic signal systems, and automatic train control, as well as station finish and controls for the collection of fares. Further technical problems involved were design and construction of substation buildings, transformers, converters, rectifiers, and all appurtenant equipment required to transform and convert 13,200-v alternating current to 600-v direct current; installation of underground cables for distribution of power and for electric light and telephone service; lighting of subways and stations; and installation of complete subway drainage systems with sumps from which the accumulated water is pumped into the city sewerage system above the roof of the subway. Under his able leadership, the design and construction of passenger cars incorporated many new features which contributed both to ease and economy of operation, and to the safety of passengers and operating crews; and terminal yards and shop buildings were developed and constructed with facilities for the storage and maintenance of cars particularly adapted to the problems of operation of high-speed transit lines.

Preparations by the federal government for the approaching war emergency drew Charles H. Stevens back into public service in January, 1941, when he was appointed chief zone engineer, Construction Division, Quartermaster Corps, War Department, for Zone III, comprising Pennsylvania, Maryland, and Virginia, with headquarters in Baltimore, Md. In this capacity, he supervised the work of architect-engineers and contractors throughout the tristate area in connection with the construction of cantonments, quartermaster and ordnance depots, airfields, and lease-lend warehouses, among other structures. The cost of such work done under Mr. Stevens' supervision amounted to several hundred million dollars.

Upon completion of the duties assigned too him, Mr. Stevens resigned from this service on April 2, 1942. From then until the time of his death, Mr. Stevens was not very active professionally because of his health. He had expended a tremendous amount of mental and physical energy in planning construction for the Quartermaster Corps. However, he was called upon from time to time as a consultant on transportation and on subway and subsurface construction problems. Although he had enjoyed excellent health until about 1940, a heart condition was developing which on more than one occasion prostrated him for short periods. His heavy responsibilities as chief engineer of the Philadelphia Department of City Transit and as chief zone engineer for the Constructing Quartermaster undoubtedly contributed to the failure of his health. However, he continued to give his time without stint to these services and, with characteristic determination, was preparing to undertake new responsibilities shortly before his final illness. This began on February 2, 1944, and on the following day he died without regaining consciousness.

Mr. Stevens was active in the affairs of the Society for many years. He was particularly active in the Philadelphia Section of the Society, which he served with distinction as Secretary-Treasurer from 1922 to 1924, as President in 1924–1925, and as chairman of the Board of Trustees of the Richard L. Humphrey Memorial Fund (which he largely helped to create) from 1932 until his death. From 1930 to 1932, he served as Director of the Society for District 4, in which capacity he was a member of the Committee on Publications that established *Civil Engineering*. In 1941 and 1942 he was Vice-President of the Society for Zone II, and to this office also he devoted much time and effort.

As a result of Mr. Stevens' long experience in transportation and subway work, he became recognized as an expert in the design and construction of rapid-transit lines and equipment. His active life was rich in experiences covering a wide field of engineering, and was also rich in associations with civil engineers of such ability and prominence as George S. Webster,[2] Past-President, Am. Soc. C. E.; and C. E. Myers,[3] William S. Twining,[4] Samuel T. Wagner,[5] and Henry H. Quimby, Members, Am. Soc. C. E.

In addition to his activities in the Society, Mr. Stevens was a charter member of the Society of Municipal Engineers of Philadelphia, which he served at various times as secretary-treasurer and as president. In 1925 this society conferred upon him its medal for twenty years of meritorious service to the City of Philadelphia. He was also a past director and past vice-president of the Engineers Club of Philadelphia, a member of the American Society for Testing Materials, and a former member of the American Concrete Institute.

Throughout a long and unusually active business life, Mr. Stevens found time for many church activities. He was an active member of the Methodist Church throughout adult life, and at the time of his death was president of the Board of Trustees of the Mount Carmel Methodist Church.

Integrity and sincerity were outstanding characteristics of Charles H. Stevens. Another was the determination with which he pushed forward at all times toward his goal, whether this was a technical problem which required solution or a more personal matter. This determination sometimes expressed itself in a certain ruthlessness toward those who failed to agree with him and even toward those who expressed divergent opinions. However, none could doubt the sincerity of Mr. Stevens' belief in the rectitude of his course, or the integrity of his character.

Mr. Stevens was married in 1904 to Emily E. James of Philadelphia, and had two sons and a daughter. He is survived by his widow and sons (Charles J. Stevens, Assoc. M. Am. Soc. C. E., and Henry B. Stevens, an architect).

Mr. Stevens was elected an Associate Member of the American Society of Civil Engineers on May 6, 1914, and a Member on January 20, 1920.

WILLARD WILBERFORCE STONE, M. Am. Soc. C. E.[1]

Died February 11, 1944

Willard Wilberforce Stone was born in Brookline, Mass., on October 23, 1873. His father, Grafton Willard Stone, was born in Newton, Mass., on September 22, 1835, and died in Brookline on March 17, 1879. His mother, Betsey Carpenter (Babbitt) Stone, was born in Berkley, Mass., on August 31, 1843, and died in Berkley on August 4, 1903. Grafton Willard Stone and Betsey Carpenter Babbitt were married in Berkley on November 16, 1865.

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 1423.

[3] For memoir, *ibid.*, Vol. 108 (1943), p. 1612.

[4] For memoir, *ibid.*, Vol. 102 (1937), p. 1662.

[5] For memoir, *ibid.*, Vol. 96 (1932), p. 1574.

[1] Memoir prepared by Charles D. Thomas, M. Am. Soc. C. E., with the assistance of Florence H. Stone, Taunton, Mass.

Mr. Stone was descended from a long line of ancestors prominent in colonial times and in England before coming to the United States. His father's family were well known:[2]

> "The Stone family is among the forty-nine best families selected by the American Historical-Genealogical Society for whom the society has published family histories during the past few years. The Stone family has been prominent in the British Empire and in United States, its members have played important roles in war and in peace."

In "Simon Stone Genealogy"[3] there is a family tree of the Stone family from Walter Atte Stone "born probably as early as 1285 in the reign of Edward I," to Simon Stone with a note under the latter name "went to America." Other excerpts from this book may be of interest:

> "Deacon Simon Stone and his brother Deacon Gregory Stone pioneers in New England in 1635 * * * descended mainly from the old Anglo-Saxon and Danish stock conquered by the Normans * * * lived in Essex and adjoining Suffolk from a period in Saxon time long prior to the Norman Conquest. The earliest mention that has been found of the name there is in 1302 under the form of Atte Stone and this form continued in use until about 1450 when the 'Atte' disappears. It therefore seems obvious that this particular Stone family derived their name from the fact that a remote ancestor in the thirteenth century assumed or was termed 'Atte Stone' from residing at or near some prominent stone landmark in the region."
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> "On April 29, 1904 a beautiful stained glass window in memory of Simon and Gregory Stone was unveiled and dedicated at Great Bromley, Essex, England, funds having been raised among their descendants in America. Deacon Gregory and Deacon Simon Stone * * * were baptized and four generations of their ancestors were christened there and had sepulture in its church yard."

The records show that through several generations in the United States the Stone family has furnished officers in the militia, town officials, representatives to the General Court of Massachusetts, many deacons of the church, and a councilor of Massachusetts.

On his mother's side, Mr. Stone also had illustrious ancestors.[4] The Babbitt family was descended from Edward Bobit who came to the United States in 1638, settled in Berkley (then a part of Taunton, Mass.), purchased land there in 1652, became one of the proprietors in the "Bloomerie" as the ancient Iron Works of Taunton were named, was married to Sarah Tarne in 1654, and was killed by the Indians in June, 1675. The Babbitt family has been prominent in the localities where its members have lived, furnishing officers of the militia, town officials, members of committees for settling disputes with the Indians, and a representative to the General Court at Boston, Mass., in 1787.

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[2] "Stone Family Records," by J. Montgomery Seaver, Genealogical Research and Publishing Co., Philadelphia, Pa., 1929.

[3] "Simon Stone Genealogy," by J. Gardner Bartlett, published for the Stone Family Assn. of Boston, Mass., Pinkham Press, Boston, 1926.

[4] "The Babbitt Family History," by William Bradford Browne, published for the Babbitt Family Assn. of Taunton, Mass., C. A. Hack & Sons, Taunton, 1912.

Grafton Stone, Mr. Stone's father, was in business in Newton, until about the time of his marriage when he moved to Brookline, where all his four children—Carrie M., Grafton Edgar, Willard Wilberforce, and Florence Helen—were born. After her husband's death, Betsey Stone and her two surviving children, Willard and Florence, returned to Berkley, her native town. (Carrie Stone died on March 11, 1879, and Grafton Edgar Stone died on January 4, 1873.)

From the time he was about six years old, Mr. Stone was brought up and educated in a locality rich in historic events and folklore. The house in which he lived was the old Babbitt homestead built in Berkley on the bank of the Taunton River by his maternal grandfather, John Dean Babbitt, in 1836. A few miles north, up the river in Massachusetts, was the flourishing iron manufacturing city of Taunton and to the east were historic Plymouth and Cape Cod. Mr. Stone attended the local public schools and the high school at Taunton from which he was graduated in 1893.

After graduation from high school, Mr. Stone was employed in the Engineering Department of the City of Taunton until September, 1896, as rodman, instrumentman, inspector of construction on a $400,000 sewer system, and assistant engineer in charge of field party on surveys for sewer, street, and paving work. The City of Taunton at that time adopted and started construction on a comprehensive system of sewers which, with extensions, is still in use. He was also instrumentman on the layout of the Taunton-Fall River Street Railway.

Mr. Stone's associates in the office of the city engineer heard many stories of early historical events along the Taunton River and on Cape Cod. One day in the spring, a group of engineers visited East Taunton, Mass., and enjoyed the remarkable sight of herring running up the Taunton River and passing through the fishways at the dam. Below the dam the fish were so thick that it appeared as if one could walk across the river on a floor of fish.

From September, 1896, to June, 1900, Mr. Stone attended Massachusetts Institute of Technology in Boston, being graduated with the degree of Bachelor of Science in Civil Engineering. From August, 1900, until his death, he held the following interesting positions:

August, 1900, to July, 1902—Draftsman and designer on a sewerage disposal system in Washington, D. C., designing sewers up to 18 ft in diameter, cofferdams, tide gates, outfall sewers, suction and discharge conduits, foundations, and accessory structures for a $1,000,000 sewerage pumping station.

July, 1902, to February, 1906—Assistant engineer for the State of New York acting as resident engineer in charge of construction of roads and bridges (seven contracts) in Onondaga County, among which were the Fabius-Apulia Road and the Skaneateles East Lake Road.

February, 1906, to June, 1913—Assistant engineer, Board of Water Supply, City of New York, N. Y., in charge of topographical surveys and field location of aqueduct; senior assistant engineer in charge of two aqueduct contracts totaling $1,000,000 (one, a $250,000 contract for a steel pipe line 9½ ft in diameter, and the other, a $750,000 cut-and-cover contract); engineering expert, Newburgh Division, Board of Water Supply of New York City, on the appraisal of property taken by the city for construction of the aqueduct (about 20 miles long with property claims amounting to about $2,500,000); and consulting engineer on water supply for Walden, N. Y.

June, 1913, to April, 1914—Senior highway engineer for the United States Office of Public Roads at Washington, D. C., in charge of surveys, designs, estimates of cost, and construction of roads and bridges in Virginia and North Carolina.

April, 1914, to April, 1916—Assistant engineer, New York State Highway Commission, in charge of designs, estimates of cost, and construction of brick, bituminous, and concrete highways costing $800,000. (Mr. Stone returned to New York State because his health was poor in the hot climate of the South.)

April, 1916, to July, 1917—Road engineer for the Portland Cement Association, in charge of designs, estimates of cost, and construction of concrete highways costing $2,000,000.

July to December, 1917—Engineer for Frazier Brace and Company at the Federal Shipyard in Kearny, N. J., in charge of making designs for deepening and widening the Hackensack River at the plant and designing docks.

January, 1918, to November, 1919—Engineer with Snare and Triest Company, construction engineers on the construction of Raritan Arsenal at Metuchen, N. J., in charge of building storm and sanitary sewers, a sewage disposal plant, and a water-works system (a $900,000 project).

December, 1920, to June, 1921—Engineer assistant, Valuation Department, Baltimore and Ohio Railroad Company, in charge of field work in Pennsylvania and Illinois, inventorying special structures.

July, 1921, to September, 1922—Resident engineer for Fuller and McClintock, sanitation engineers, in charge of construction of sewer extensions and pumping stations at Glen Cove, N. Y. (a $250,000 project); and in charge of designing works for additional water supply at Newburgh, N. Y., including earth and masonry dams, gatehouses, pipe lines, and special structures (a $200,000 project).

September, 1922, to February 11, 1944—On the engineering force in the Office of Nassau County Engineer (which became, on January 1, 1938, the Department of Public Works), in charge of an engineering party in the Division of Highways. In this capacity, Mr. Stone supervised necessary investigations and highway surveys and also office work. He plotted surveys, prepared construction drawings and plans for acquisition of the property necessary to locate and widen arterial streets in Nassau County, arranged the field work, and supervised the construction of the highways. Among the numerous arterial highways in Nassau County under his direction were parts of Merrick Road, Hempstead Avenue from Lynbrook, N. Y., to Hempstead, N. Y., Clinton Road, and the Horse Brook improvement. He designed and had charge of construction of the bridge that carries Atlantic Avenue over Milburn Creek at Freeport, N. Y.

Mr. Stone was very accurate in designing and checking engineering work and his associates considered him the best mathematician in the office. Many men in executive positions have profited greatly from being with Mr. Stone. He played an important part in the engineering department while Nassau County was developing from a farming community to a metropolitan area. Because of an increase in population from approximately 150,000 in 1920 to about 450,000 in 1944, which caused greater motor vehicle traffic, 400 miles of highways had to be straightened, widened, and reconstructed. He was well liked by all his associates, many of whom sought his advice on engineering subjects. Henry A. Van Alstyne, M. Am. Soc. C. E., a former state engineer of New York, wrote on December 11, 1917:

> "I have known Mr. Stone for over fifteen years. He is a man of high character, good habits and excellent reputation. * * * He is a capable, reliable, energetic and trustworthy engineer."

Mr. Stone was 6 ft 6 in. tall and during his early manhood was thin but later he became heavier and at one time weighed about 225 lb. He was a very sociable man, and attended the meetings of his club and of engineering societies. He attended the Carpenter Memorial Methodist Church at Glen Cove, he participated in church activities, and purchased and presented to the church a war service flag. He made many friends and always remembered the smallest details about the people he knew. In spite of his sociable nature and his grace in talking with the ladies, he never married. During the last twenty-two years of his life, his tall stately form has been a familiar sight wherever Nassau County highway work was in progress.

For about three months before his death at the Nassau Hospital in Mineola, N. Y., Mr. Stone was in poor health. He was buried in Newton, Mass. A sister, Florence H. Stone, survives him.

Mr. Stone belonged to the Republican Club of Glen Cove, and the Society of American Military Engineers. He was a registered professional engineer and land surveyor in New York State.

Mr. Stone was elected an Associate Member of the American Society of Civil Engineers on February 7, 1906, and a Member on January 19, 1923.

MERTON ROGERS SUMNER, M. Am. Soc. C. E.[1]

DIED NOVEMBER 5, 1943

Merton Rogers Sumner was born at South Paris, Me., on June 11, 1886, the son of R. C. and Lillian Sumner.

After being graduated from the University of Maine, at Orono, with the degree of Bachelor of Science in Civil Engineering, he was employed by the Fort Worth-Cleborne Interurban Railroad Company on construction at Fort

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[1] Memoir prepared by A. C. Ackenheil, M. Am. Soc. C. E.

Worth, Tex. He then worked for Stone and Webster Engineering Corporation on hydroelectric development in California. Returning to the University of Maine for further study, he received the degree of Civil Engineer in 1918.

Resuming his career, Mr. Sumner then was in charge of design for Fred T. Ley and Company and the Fuller Industrial Engineering Company on the construction of the U. S. Bomb Loading Plant at Rockwell Park, Del. After this engagement, he was employed by the Byllesby Engineering Corporation, advancing to general superintendent of construction for the Pittsburgh (Pa.) district. While with this company he contributed numerous articles to the technical press. He also gave lectures on advanced economics at the University of Pittsburgh, in Pittsburgh.

When the United States entered World War II, Mr. Sumner became project engineer for the John F. Casey Company on the construction of the Neville Island Parkway for the Navy Department. When this work was completed, he became associated with the Defense Plant Corporation as office engineer in Pittsburgh.

Affable and of an engaging manner, Mr. Sumner had many friends who mourn his passing. His engineering ability was of a very high standard, and he had the respect and confidence of his associates.

He is survived by his widow and three children, Virginia, Hugh, and Paul. The boys are both serving in the armed forces of the United States.

Mr. Sumner was elected an Associate Member of the American Society of Civil Engineers on May 13, 1918, and a Member on March 8, 1921.

JOHN CALVIN SWEENEY, M. Am. Soc. C. E.[1]

DIED AUGUST 16, 1944

John Calvin Sweeney, the son of John and Mattie (Bomar) Sweeney, was born in Paris, Tenn., on August 9, 1885. His father was a prominent Paris lawyer and at one time a federal judge in the Philippine Islands. John received his early education in the Paris public schools, was graduated from the United States Naval Academy at Annapolis, Md., in 1905, and completed a graduate course at the Massachusetts Institute of Technology in Boston in 1910.

He then began a most unusual and varied career in engineering and construction of railroads, hydroelectric developments, buildings, dams, shipyards, and ports and terminals. His career culminated in his last position as resident manager in charge of naval base construction at Trinidad, B.W.I., for the James Stewart Associates. His professional work took him to most parts of the United States and Canada and to some foreign countries.

In 1910 he went to Canada, as superintendent and chief engineer for William Carson, of Winnipeg, Man., for whom he designed and constructed hotels, post offices, railroad roundhouses, bridges, stations, and one hydroelectric station for the Canadian Pacific Railway. In 1912 he became chief engineer and manager for Michael Connolly in Montreal, Que., in charge of harbor works and the breakwater at St. John, N.B.; piers at Three Rivers, Que., and at Goderich, Ont.; a paper mill and hydroelectric development at St. Raymond, Que.; and water works at Quebec.

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[1] Memoir prepared by Gardner S. Gould, M. Am. Soc. C. E.

In 1914 he became district and supervising engineer for the Department of Public Works of Canada, in charge of design and supervision of all public works in western Canada, including the lock and dam at St. Andrews; flood control works on the Red and Assiniboine rivers; protection work at the mouth of the Red River; land reclamation near Lake Manitoba in the Province of Manitoba; and shipways and yard at Selkirk; together with dredging and bridges over navigable rivers, buildings, etc., which came under the jurisdiction of the Department. Late in 1914 he was employed by the Toronto (Ont.) Harbor Commission on a harbor development which was under contract to the Canadian Stewart Company. Subsequently he resigned his position with the Toronto Harbor Commission and joined the Stewart organization. He later supervised war emergency work, enlarging harbor and pier facilities at St. John.

When the United States entered World War I in 1917, he returned to the United States and, as engineer for James Stewart and Company, worked on cantonment construction at Little Rock, Ark. (Camp Pike); Fayetteville, N. C. (Camp Bragg); Leon Springs, Tex.; and Fort Douglas, Utah. In this same assignment were included the Naval Operating Base at Sewall's Point, Va.; bulkheads at Hampton Roads, Va.; a seaplane base at Bayside, L. I., N. Y.; and a housing development at Erie, Pa.

In 1918 he returned to Canada for the Canadian Stewart Company to construct a plant and dam for the Abitibi Paper and Power Company and also various buildings for General Motors of Canada, Fisher Body Company, and Burroughs Adding Machine Company.

At this time, Mr. Sweeney suffered his first attack of pulmonary tuberculosis, which forced him to leave Canada and go to Saranac Lake, N. Y., for treatment. He continued, however, to act as advisory engineer for James Stewart and Company on New York State Barge Canal contracts and for the Standard Oil, Gulf Refining, and Humble oil companies on plants, buildings and piers.

In 1924 Mr. Sweeney opened his own office as a consulting engineer and, during the next ten years, was actively engaged on diversified projects for many clients, including Cuban highways for James Stewart and Company; a hydroelectric development on the Deschutes River in Oregon; harbor developments and a paper mill at Vancouver, B. C., Canada; various bond issues for E. H. Rollins and Company; harbor and dredging projects in Canada for the Canadian Dredging Company; dams, power developments, dredging, and other improvements for Paul Smith Electric Power and Railroad Company and the International Paper Company; and buildings and improvements at the Lake Placid (N. Y.) Club. He made an examination and report on the principal hydroelectric dams and developments in the United States for F. W. Stehr and W. D. Ross.

In 1928 he moved to California, continuing his consulting practice there. He acted for Merritt, Chapman and Scott on piers and harbor developments and on a proposed project in Mexico; for the Continental Oil Company and for Republic Steel on piers and other structures; and for the Rohl-Connolly Company on the El Capitan Dam, a rock-fill and hydroelectric-fill structure. After the 1933 earthquake, Mr. Sweeney went to Long Beach and volunteered his services as inspector. These were accepted and he took charge of inspection forces until September, when reconstruction was practically complete. For this work, he received the thanks of the city manager and council. Following this, in continuation of his work on earthquake damage, he made a general survey and damage appraisal for the J. V. McNeil Company and an investigation and report of the earthquake in collaboration with Prof. R. R. Martel, M. Am. Soc. C. E. He also served the United States Coast and Geodetic Survey in an advisory capacity. He was consultant on San Gabriel Dam No. 1 in California; but, as he did not concur in the design and methods adopted, he resigned in 1934. His opinions were later upheld and engineering forces of the Los Angeles Flood Control District were reorganized, construction halted, and the design changed to meet his recommendations.

In 1935 Mr. Sweeney returned to the Atlantic coast and was employed by the Corps of Engineers of the United States Army on the Passamaquoddy (Maine) tidal power project. He made weather and current studies and reports and drew up recommendations for proposed construction equipment and methods, power requirements, etc. In August, 1935, he was placed in charge of all foundation explorations and designed and built the special equipment necessitated by the extreme depths involved and by the severe tidal and weather conditions. All this was successfully accomplished at a cost much below the appropriations. He was then appointed senior superintendent of construction in charge of all permanent work; and, before the Passamaquoddy project was closed, he had opened several quarries, built railroads, and constructed three minor dams. He was transferred to Fitchburg, Mass., as engineer in charge—for the Corps of Engineers—of a $1,800,000 flood control project, handling all administrative, as well as engineering, features of the work.

In August, 1937, Mr. Sweeney returned to Tennessee, in the hope that he could remain there permanently. He entered very actively into water and power work in western Tennessee, particularly in Paris. At the request of the municipal government, he organized all communities served by the Kentucky-Tennessee Light and Power Company into a unit which he represented in negotiations resulting in the purchase of all the properties of the company. He was asked to assume the management of the Paris Electric and Water System and accepted the position. He put the system on an exceptionally profitable basis during its first year of operation, at the same time effecting complete rehabilitation with the aid of a grant from the Public Works Administration.

Soon after the beginning of World War II, Mr. Sweeney found that he would not be permitted to end his days peacefully in Tennessee. In late 1940, when the great need of war shipping became evident and the inadequacy of shipyards was realized, he was called to take charge of the construction of the new yard for the Todd-Bath Corporation at South Portland, Me. This was the first large-scale instance on the east coast of building ships in prepared basins, floating them out as soon as they would float, and completing them afloat, thus permitting a quicker turnover than is possible on the customary sliding ways. Mr. Sweeney began this work in the early part of a New England winter and had most of the area down to grade and much of the side-walls completed by April. Hulls were under construction in July. The Atlantic Ocean was on one long side of the project and a difficult ground-water condition existed ashore; yet he handled the water problem so skilfully that, when the writer visited the job in April, the bottom of the excavation was drier than the surrounding uplands.

Late in 1941, soon after he left Portland, Me., Mr. Sweeney was asked by James Stewart Associates, who had been awarded the contract for the construction of the Trinidad and British Guiana naval air stations, to go to Trinidad as resident manager. Although the project in Trinidad began as a naval air base, it was soon enlarged to an operating and repair base for both air and surface ships. Mr. Sweeney's health had been poor for several years; and, although he fully realized that his illness would probably be aggravated by living in the tropics, he unhesitatingly took the job. With the help of a nucleus of extremely able superintendents and foremen, he welded together a most efficient organization and succeeded in completing the work and its great additions so rapidly and in such a manner that, in March, 1943, the company was awarded the Army-Navy "E." As in the case of most bases built in foreign lands, the contractors had to depend on a nucleus of American labor in key and skilled positions, using native labor to the greatest extent possible elsewhere. Mr. Sweeney was notably successful in the adaptation, training, and handling of native labor on this job, and the efficiency of this labor far exceeded expectations. Mr. Sweeney displayed great ingenuity in the use of makeshifts—methods, equipment, and materials—necessitated by the isolation of the job from ordinary sources of supply and by the lack of shipping facilities.

Mr. Sweeney's exertions, combined with the effects of the tropical climate, proved too much for his impaired health. Early in 1943, he suffered his third attack of tuberculosis, aggravated by other ills, and had to be brought back to the United States. He immediately entered a hospital in New York, N. Y., later going to the Oakville Sanatorium at Memphis, Tenn. He never recovered from this attack of illness and died at his home in Paris on August 16, 1944.

An appraisal of the qualities of a man like John Sweeney, whose life and activities were so varied and numerous, can only be a consensus of the opinions of the many men who knew and lived and worked with him. No man saw all of Mr. Sweeney. He was primarily a construction man—energetic, keenly practical, ingenious, and a hater of waste motion. However, these qualities were controlled by a sound engineering education and instinct. He was a driver, but always drove himself harder than he did his men. Impatient with, and somewhat contemptuous of, well-meaning regulations which tended to hamper progress, he never hesitated to break rules if he thought that, by so doing, better or quicker results could be attained.

Caustically critical of inefficiency and carelessness, he was generous in praise of good work and glad to give full credit to others. He always dominated any project with which he was connected, but in a way that won for him the admiration and respect of his co-workers. The writer had occasion to follow Mr. Sweeney on two of his last jobs and was amazed to note the almost reverential manner in which the field organization spoke of, and felt toward, him.

Mr. Sweeney was a keen judge of men, an excellent organizer, and wise in his ability to delegate authority. A Sweeney job was always a safe job and always a colorful one. The engineering profession and the construction industry have lost one of their ablest men in the death of John C. Sweeney.

He is survived by two sons: John C. Sweeney, III (in the "Seabees," U. S. Navy), and Richard H. Sweeney.

Mr. Sweeney was elected a Member of the American Society of Civil Engineers on December 22, 1930.

CHARLES MASON TALBERT, M. Am. Soc. C. E.[1]

Died January 25, 1944

Charles Mason Talbert was born near Shelbyville, Shelby County, Ind. on September 27, 1870, the son of William and Amanda (Carter) Talbert. While still quite young he went with his parents to southwest Missouri where he grew up, developing a rugged pioneer constitution.

He received his education in the schools of southwestern Missouri and his training as a civil engineer at the University of Missouri in Columbia in the early nineties. Upon leaving the university, his first employment was with the Missouri River Commission for about three months. He then worked as leveler and recorder on levee work for the late Julius Pitzman,[2] M. Am. Soc. C. E., in St. Louis, Mo. In the spring of 1893 he went with the Mississippi River Commission as recorder on triangulation. He then acted, successively, as hydrographer, leveler, topographer, chief of party on surveys and discharge work, superintendent of a dredge and a dredging district, and general superintendent of dredging operations.

In 1902 he was employed by the Louisiana Purchase Exposition Company in charge of field work which included the construction of the Cascades on Art Hill in Forest Park. This was the most important landscape effect of the Exposition. During the year of the St. Louis Fair he was superintendent of building and installation of the exhibits in the Liberal Arts Building.

After the Fair he served as engineer for the president of the Board of

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[1] Memoir prepared by John T. Garrett, M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 1426.

Public Improvements of St. Louis. During this time Mr. Talbert helped in the engineering work leading to the adoption of a new charter for the city. After it was adopted he was appointed to one of the most important positions created by it—that of Director of Streets and Sewers. He served in this capacity for sixteen years. During these years, since traffic was changing from horse-drawn vehicles to motor transportation, innovations were required such as a new type of pavement for the streets, the elimination of sharp curves, the marking out of safety zones, the installation of the first stop-and-go signs, and the designation of some of the narrower streets for one-way traffic. Because the coming of the motor cars caused a sharp increase in the number of traffic accidents on the streets and highways, new safety measures on the streets were necessary. As a result, the St. Louis Safety Council was organized in 1916. In 1920 Mr. Talbert was made president of the council. He later became an officer in the National Safety Council.

In 1919, seeing an opportunity in the insurance field, Mr. Talbert gave up public life and entered the insurance business, making a speciality of industrial buildings and engineering structures. He built up a substantial business in this line and remained in insurance until his death. He served as president of the St. Louis Fire Underwriters Association, later the Insurance Board of St. Louis, and for many years was a member of the City Plan Commission.

His interest in civic and municipal affairs prompted Mr. Talbert to become an early advocate and supporter of the Municipal Theater Association. At the time of his death he was a member of the association's board of directors and of the executive committee. In addition to these duties he handled the traffic problem during the yearly twelve-week opera season, in such a way that, even though there were thousands of cars to park every night, there was little or no delay in arriving or leaving the theater.

A man of high principles, sound judgment, and fine character, Mr. Talbert had the invaluable faculty of endearing himself to all those with whom he came in contact, both in a personal and business way. He possessed a refreshing sense of humor and had the ability to see the human comedy in routine matters about him. He liked people and they liked him. He kept himself well informed on all current subjects so that he was ready at all times to take part in any discussion. He was a great lover of books and had many rare and valuable editions on the shelves of his extensive library.

While he was superintendent of the Liberal Arts Building at the St. Louis World's Fair which housed extensive exhibits of oriental art, he came to know and admire oriental ideals and artistic skill. When the Fair was over and the exhibits were being removed, those in charge of the Oriental Exhibition presented him with a few samples of the art. These he took home, later to find that they were several hundred years old and admirable specimens of the Chinese art of that period. This started him on a hobby of collecting specimens of eastern art. Everywhere he traveled he visited out-of-the-way places and shops where he found many curious and rare pieces of oriental art. In his ivory collection, he had many old and valuable netsukes, small button-like appendages, intricately carved and used by the orientals since the Middle

Ages to hang from their girdles, attached to pipe cases, or medicine boxes, much the same as a watch fob.

All who knew Mr. Talbert well valued his friendship highly and found his companionship a pleasure and delight. As Hamlet said,

> "He was a man, take him for all in all,
> I shall not look upon his like again."

During World War II Mr. Talbert served as coordinator in the Office of Civilian Defense. He was a 32d degree Scottish Rite Mason and a member of the Mystic Shrine. He belonged to the Missouri Athletic Club and the Normandy Golf Club.

Mr. Talbert was married on April 12, 1899, to Mary Morse Davitt. He is survived by his widow and one son, William R.

Mr. Talbert was elected a Member of the American Society of Civil Engineers on March 4, 1914.

PETER GANSEVOORT TEN EYCK, M. Am. Soc. C. E.[1]

Died September 2, 1944

Peter Gansevoort Ten Eyck, the son of Abraham Cuyler and Margaret Matilda (Haswell) Ten Eyck, was born on November 7, 1873, in Bethlehem, Albany County, N. Y.

Mr. Ten Eyck obtained his early education at the Albany (N. Y.) Boys Academy. In 1892 he entered Rensselaer Polytechnic Institute at Troy, N. Y., where he studied civil engineering for three years. His first employment was as engineer in charge of laying out and developing Beaver (later Lincoln) Park in Albany.

In 1896 Mr. Ten Eyck accepted a position with the Signal Department of the New York Central Railroad Company. After working as electrician, repair foreman, and supervisor of signals, he was promoted to signal engineer for the New York Central Railroad Company in 1902. In 1905 he resigned to become chief engineer of the Federal Railway Signal Company in North Albany. While he was with this company, Mr. Ten Eyck developed several signal devices, many of which are still in use. He became vice-president of the company in 1907. In 1908 he resigned to devote his time to real estate development.

For the next three years, in collaboration with his brother, he developed a tract of family acres into home sites. This property, a part of the old Gansevoort Farm in the southwestern quadrant of Albany, became a well-established residential section.

In 1912 Peter Ten Eyck began his long service to the public. In this year he was elected to the United States House of Representatives where he served until 1915. After a lapse of five years during which he was engaged in real estate development and as a consulting engineer, he was again elected to Congress, serving until 1923.

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[1] Memoir prepared by John A. O'Connor, M. Am. Soc. C. E.

From 1923 to 1934 he was in the real estate business, practiced as a consulting engineer, and served as chairman of the Deeper Hudson River Commission. His unselfish interest in improving inland waterways, particularly the Hudson River, was manifest at all times. From 1923 to 1925 he supervised the compilation of a brief for the Port of Albany. The soundness of his work is evidenced by the construction of this port, which was begun in 1928. Mr. Ten Eyck was chairman of the Albany Port District Commission from 1928 to 1934 and had responsible charge of the construction of the Port of Albany.

Peter Ten Eyck was interested in many other fields than those of engineering and real estate. He belonged to the following learned and public societies: The American Society of Electrical Engineers, American Association of Signal Engineers, Albany Society of Engineers, Albany Realty Board, New York State Dairymen's Association, New York State Waterways Association, and numerous other societies and clubs. However, his true love and devotion were for the soil. For many years he successfully operated his "Indian Ladder Farms"—a dairy and fruit farm with considerable acreage in choice apples—in the Town of New Scotland (N. Y.). Mr. Ten Eyck had a keen interest in increasing the milk production of the Albany area, and did much pioneering work. Recognizing his leadership in this field, the Hon. Herbert H. Lehman, then Governor of New York State, appointed Mr. Ten Eyck director of the New York State Farm Bureau. In connection with agriculture, he was a member of the Board of Visitors for New York State School of Agriculture and president of the Capitol District Guernsey Breeders Association. He belonged to various other farm societies, and was one of the organizers of the American Farm Bureau Federation.

Mr. Ten Eyck had a fine personality. He was a dignified and courteous person with a host of friends who called him "Pete"; he liked club life and the society of men; had a certain flair for politics and a fine reputation as a public speaker. He gave unsparingly of his time in behalf of the many civic organizations in which he was interested.

On April 15, 1903, at Albany, he was married to Bertha F. Dedrick. He is survived by his widow and a son, Peter.

Mr. Ten Eyck was elected a Member of the American Society of Civil Engineers on February 19, 1934.

FRANK ELLSWORTH TRASK, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 4, 1944

Frank Ellsworth Trask was born at Industry, Me., on July 26, 1863. He was the son of Nathaniel and Betsey Helen (Wills) Trask.

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[1] Memoir prepared by William C. Hogoboom, Assoc. M. Am. Soc. C. E.

Mr. Trask was educated at Gould's Academy in Bethel, Me., and at the University of Maine in Orono, from which he received the degree of Bachelor of Civil Engineering in 1887 and the degree of Civil Engineer in 1890. He was a member of Phi Gamma Delta fraternity.

In 1887 Mr. Trask went to Southern California and engaged in general engineering at Ontario. This practice consisted largely of design, construction, and reports upon irrigation districts and municipal water systems, with some incidental work on land subdivision. In 1888 he was engineer for the San Antonio Heights Railway Company on the construction of eight miles of street railway.

During the 1890's, Mr. Trask was chief engineer for the San Antonio Water Company in connection with the $400,000 reconstruction and extension of an irrigation system and water supply. At the same time he was engineer for the Escondido (Calif.) Irrigation District, he designed and constructed a water works system for the City of Ontario, he made a report to the City of Pomona (Calif.) on municipal water supply, and he was employed as engineer by the Sycamore Water Development Company, the Pomona Orange Belt Irrigation District, and the Ontario Electric Company. For the last-named company, he designed and built eight miles of electric railway and a hydroelectric power plant.

After 1900 Mr. Trask served as consulting engineer for many municipalities, irrigation districts, and private companies on various types of water problems. This work involved construction of tunnels, dams, and reservoirs; reports and appraisals of water systems; and presentation of hydrographic data in court cases. Among the cities, irrigation districts, and companies which employed him as consulting engineer were: Whittier Water Company, Ojai Mutual Water Company, Consolidated Water Company, Arrowhead Reservoir and Power Company, Fontana Development Company, Mill Creek and Crafton Water Company, West Los Angeles Water Company, Union Hollywood Water Company, Rindge Company, Traders' Oil Company, Azusa Foothills Citrus Association, Mojave River Irrigation District, Santa Ana River Association, San Gabriel River Association, and the cities of Pomona, Escondido, Ontario, Covina, Redlands, and Huntington Park, Calif.; and Yuma, Ariz.

In 1933–1934, Mr. Trask served as California state engineer for the Public Works Administration (PWA), and in 1934–1935 he was a member of the Technical Board of Review of PWA at Washington, D. C. In 1935 he was retained as a permanent consulting engineer by the Los Angeles (Calif.) County Flood Control District.

For many years Mr. Trask was a leader in the development of underground waters in the Southwest through pumping and tunnel beds. He was the author of numerous papers on irrigation, and a contributor to engineering journals and textbooks. Having practiced in Southern California for more than fifty years, Mr. Trask was very well known in the engineering profession. He was admired and loved by the young engineers who knew him, because of his untiring efforts to help younger men and to advance the engineering profession in general.

On July 1, 1909, in Los Angeles, Mr. Trask was married to Carlotta Thornton. He is survived by three sons, George Harlan Trask, Olin Wills Trask, and Elwood Ellsworth Trask; a sister, Blanche (Mrs. Erwin Phillips); and three brothers, Loren Trask, Richard Trask, and Elmer Trask.

Mr. Trask was elected an Associate Member of the American Society of Civil Engineers on June 5, 1895, and a Member on January 2, 1901.

HARRY EDWARD TYRRELL, M. Am. Soc. C. E.[1]

Died December 5, 1943

Harry Edward Tyrrell was born in Philadelphia, Pa., on July 31, 1882, the son of Richard D. and Mary Emma (Sykes) Tyrrell. He received his education in Philadelphia public schools and the Technical High School, from which he was graduated in 1901.

Mr. Tyrrell was a self-educated man in many respects. He often spoke of the difficult course in mathematics and mechanics at the Northeast Manual School, in which he learned the first rudiments of engineering. He served an apprenticeship in the Pencoyd (Pa.) Plant of the American Bridge Company in 1902. Later his background was broadened by experience in minor capacities with the West India Construction Company, Norfolk and Western Railway Company, Bell Telephone Company, Pennsylvania Railroad Company, and Philadelphia and Reading Railway Company. In 1906 he was employed by the engineering department of the Southern Railway Company. His promotion was rapid, and in July, 1915, he became engineer, maintenance of way, in charge of the Western District of the Southern Railway Company, with offices at St. Louis, Mo. He was appointed chief engineer, maintenance of way and structures, for the Central Lines of the Southern Railway System on August 1, 1934, continuing in this position until his death.

Mr. Tyrrell had an unusual interest in his work, and was very successful. He was a resident of St. Louis for nineteen years, was active in the Engineers' Club of that city, and had a wide acquaintance. When he was transferred to Knoxville, Tenn., he became associated with the Technical Society, which includes in its membership representatives of all branches of the engineering profession.

Although Mr. Tyrrell was eminent in his chosen field, he himself felt that his outstanding achievement was in directing men. He was justly proud that he was able to perform effectively the duties of management as differentiated from an impersonal practice of engineering. He handled the personnel under him successfully, and organized his men into an efficient working unit. He knew that knowledge and practice of civil engineering are prime requisites for an engineering career, but he felt that the basis of larger success was in personality and in qualities of leadership. He strove earnestly to attain this end.

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[1] Memoir prepared by J. B. Akers, M. Am. Soc. C. E.

Mr. Tyrrell took pride and found satisfaction in furthering the careers of young engineering school graduates who were employed by the Southern Railway Company. Many young engineers were trained and guided in their formative years by Mr. Tyrrell.

Mr. Tyrrell was a lovable man, and had a great many friends. He was kindly, unpretentious, and richly deserved the confidence placed in him by all who knew him.

Mr. Tyrrell did not find much time for sports, although he was very much interested in them. In his later years he was a member of the Holston Hills Golf and Country Club at Knoxville, where he tried earnestly to develop a good game of golf. Only a few days before his death he said that he had about "lost hope," and that he still could not shoot better than one hundred —he even admitted to occasionally missing the ball. For many years he and his family made an annual pilgrimage to the New Jersey seacoast to fish, and, at the same time, to spend a happy period with his parents. He was an active member of the American Railway Engineering Association.

On April 10, 1907, Mr. Tyrrell was married to Amanda M. Haines in Washington, D. C. He is survived by his widow; his two sons, Harry E., Jr., and Maxwell H. Tyrrell; his mother; his father; and a sister, Mrs. Elsie Tyrrell Hebrew.

Mr. Tyrrell was elected a Member of the American Society of Civil Engineers on May 28, 1923.

THOMAS UTEGAARD, M. Am. Soc. C. E.[1]

Died December 17, 1943

Thomas Utegaard was born in Eau Claire, Wis., on December 9, 1893, the son of Ole T. and Ragna (Brekken) Utegaard. After being graduated from high school, he entered the College of Engineering of the University of Wisconsin at Madison. In 1917 after his graduation from the University with the degree of Bachelor of Science in Civil Engineering, he worked for a while for the Dravo Construction Company, Pittsburgh, Pa. In January, 1918, he enlisted in the U. S. Army as a private at Columbus, Ohio. Later he entered officer's training school at Camp Grant, Ill., and was commissioned a Second Lieutenant. He served for nine months in France with the 56th Coast Artillery Regiment.

After being discharged from the Army early in 1919, Thomas Utegaard worked for a time for the St. Joseph Stockyards Company, South St. Joseph, Mo., later that year entering the employ of the City of Janesville, Wis., in the City Engineer's department. On September 1, 1920, he was employed by the Consolidated Water Power and Paper Company, and moved to Wisconsin Rapids, Wis. He remained with this company until early in 1943, when he was commissioned a Lieutenant Commander in the Navy.

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[1] Memoir prepared by Carl A. Cajanus, M. Am. Soc. C. E.

Between 1920 and 1926, Thomas Utegaard worked as draftsman, structural designer, instrumentman, and chief of party. The Consolidated Water Power and Paper Company was, at that time, laying the groundwork for a 20-yr program of expansion. Thomas Utegaard conducted the early surveys and investigations for the conservation of water resources and the development of power on the Wisconsin River and its tributaries, between Stevens Point and Mosinee and Petenwell and Germantown in Wisconsin. During this period he also drew the plans for a large warehouse at the Wisconsin Rapids mill of the Consolidated Water Power and Paper Company.

Between 1926 and 1943, Thomas Utegaard was assistant chief engineer of Consolidated Water Power and Paper Company, handling all the civil engineering work of the company. His first important assignment was the construction of a complete mill (a sulphite pulp mill, a two-machine paper mill, and a steam power plant) for the production of newsprint at Port Arthur, Ont., Canada.

In addition to the main plant at Wisconsin Rapids and the newly constructed mill at Port Arthur, the Consolidated Water Power and Paper Company operated plants in Wisconsin at Biron, Stevens Point, and Appleton. Because raw materials had become scarce in Wisconsin, the production of newsprint was gradually discontinued and this product was manufactured more and more in Canada, where the wood supply was still plentiful. This trend made it necessary for the Consolidated Water Power and Paper Company to discontinue the production of newsprint and to convert its facilities to the production of other kinds of paper. The transition from the manufacture of newsprint to better grades of paper required a great deal of equipment and additional buildings. During this period, Thomas Utegaard supervised the design and construction work at all the plants of the Consolidated Water Power and Paper Company.

At the Wisconsin Rapids plant, additions to the digester building of the sulphite pulp mill were built in 1926 and 1937. Because the company was making a high-grade white paper, a filter plant for the filtration of water was needed and was erected in 1927 with additions in 1929 and 1937. The steam power plant was somewhat antiquated and an entirely new boiler house and steam turbine plant were built in 1927 with additions in 1938 and 1941. The quality of the paper required not only clean water, but also the bleaching of pulp for which a plant was built in 1929 and an addition in 1937. The manufacture of paper required much more floor space than had been needed previously and additions for the handling, finishing, and loading of the paper were built in 1929, 1937, and 1941. In 1929, an eighteen-car garage was provided for the storing of cars and trucks. . In 1937 a very large expansion program was completed, consisting of a building for a new large paper machine of the latest design, which considerably increased the output of the mill. In the same year a large four-story addition was necessary to house the mill offices.

The factors which influenced the changes at the Wisconsin Rapids plant also affected the production at the other plants of the company. Between 1929 and 1936, Thomas Utegaard designed and supervised the construction of certain additions at the Biron mill of the company, consisting of a water filtration plant, a department for the cleaning and preparing of the pulpwood, and a building for the sorting, packing, and shipping of the finished paper. Between 1928 and 1931, the sulphite pulp mill at Appleton was entirely rebuilt under his direction. At the Stevens Point plant, a storage building was added in 1935.

Thomas Utegaard also designed and supervised construction of a paper box plant in 1930 and a paper carton plant in 1939 for the Ahdawagam Paper Products Company, a subsidiary of the Consolidated Water Power and Paper Company. A short time before he left to assume his duties with the U. S. Navy, he designed and began construction on a plastics press building.

Two of the power dams owned by the Consolidated Water Power and Paper Company, at Wisconsin Rapids and at Biron, were of wood-crib construction. In 1936 the Biron dam had to be replaced by a new concrete dam. The dam at Wisconsin Rapids was replaced with a concrete dam in 1941. Thomas Utegaard designed and supervised the construction of both of these structures.

A project, for which Thomas Utegaard made preliminary surveys in the early 1920's, was the Big Eau Plaine reservoir, completed in 1936. It consisted of damming up the Big Eau Plaine River (a tributary of the Wisconsin River) with an earth dam more than a mile long. Another project for which the preliminary work was done in the 1920's was the Du Bay power project, consisting of a power house with generating equipment of 9,000 kva, under a 25-ft head and an earth dam one-and-three-quarters miles long. This was completed under the direction of Thomas Utegaard in 1942 and early 1943. In 1942 he designed the structural framework and supervised the construction of a hospital in Wisconsin Rapids for Childs and Smith, architects of Chicago, Ill.

In 1933, at the request of state authorities, Thomas Utegaard was given a leave of absence from the Consolidated Water Power and Paper Company for about three months to enable him to assume the position of Regional Director of the Civil Works Administration.

Thomas Utegaard was respected in the community where he had lived for almost twenty-three years. His ability as an engineer was recognized by his employer, as is shown by the following statement of Stanton W. Mead, vice-president of the Consolidated Water Power and Paper Company:

> "Tom Utegaard's service of 23 years to Consolidated was of a kind that is impossible to replace. His interests and the responsibilities which he always took eagerly included engineering tasks of every size and scope. Every one of these tasks undertaken by him or his department was done thoroughly and well. In each instance he planned for the future stability and performance. Work and service were an enjoyment for Tom Utegaard. Those who worked with him soon sensed his enjoyment in work well done and readily followed his leadership.

> "Buildings which Tom Utegaard engineered and supervised while under construction are prominent in our mills and in the community and are monuments to his fine ability."

The following editorial was published in *Wisconsin Rapids Daily Tribune* on December 23, 1943:

> "On Wednesday the comrades and friends of Lieutenant Commander Tom Utegaard gathered for a final tribute to a man who twice offered his services to his country in world-wide conflicts. That distinction was not the only one Tom Utegaard possessed, although it is one he bore with real honor.
>
> "Tom Utegaard was a builder, a constructor of things which will endure because he planned them well and demanded the best of materials and workmanship.
>
> "He knew that no edifice would long stand that was not founded upon solid foundations, erected with strong and lasting materials and fabricated with careful craftsmanship. The many buildings he engineered for the Consolidated, the installations such as the Du Bay and Wisconsin Rapids dams, will outlive all of us now on this earth. They are truly fine monuments to a man with a capable mind and an honest spirit.
>
> "As we review his contribution in a material way, we have explored his character too. The solid physical edifices were only a symbol of his character. He knew his philosophy too, and it was based on the same forthright reasoning that the only enduring human relationship was built of the same qualities.
>
> "There was only one route so far as Tom Utegaard was concerned—the right way—and it was the straight line between two points both honestly conceived. Two and two in human relationships didn't add up to either three or five with him. There was only one answer, the same answer that he got on his drawing board and work sheets.
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> "On the drawing board of life, as well as in the blueprint laboratory, two and two made four, not as a matter of expediency but because there could be no compromise with its validity. That was the rule on which Tom Utegaard based his life."

Thomas Utegaard died on December 17, 1943, at the Navy Hospital in Brooklyn, N. Y., where he had been a patient since May, 1943. After his training at Norfolk, Va., he had been assigned as Public Works Officer of the U. S. Naval Supply Depot, at Bayonne, N. J.

He was a member of the American Concrete Institute, and of the American Legion, the Elks Lodge, and the Bull's Eye Country Club at Wisconsin Rapids.

On May 31, 1918, Thomas Utegaard was married to Isabel E. Fletcher at Columbia, S. C., just before sailing overseas. Besides his widow, he leaves three sons, Thomas F., John H., and Rolf; and four brothers, Olin, Arthur, Olaf, and Harold.

Thomas Utegaard was elected a Member of the American Society of Civil Engineers on July 13, 1942.

JEPTHA ALAN WADE, M. Am. Soc. C. E.[1]

DIED JULY 4, 1944

Jeptha Alan Wade, the son of George Henry Wade and Matilda S. (Clements) Wade, was born in Eaton County, Michigan, on February 8, 1887.

Jeptha Wade attended local schools in Eaton County and was graduated from high school at Charlotte, Mich. He then attended Olivet College at Olivet, Mich., and was graduated *summa cum laude* in 1908 with the degree of Bachelor of Arts. From 1908 until 1911, he was a high school teacher and principal of schools at St. Johns and Monroe, Mich., and at Reno, Nev. He entered the University of Michigan at Ann Arbor in 1911 and was graduated in 1913 with the degree of Bachelor of Civil Engineering.

For the first fourteen years of his professional practice he served in various positions which gave him a thorough background for his later work:

July, 1913, to July, 1916—draftsman for S. Morgan Smith Company, manufacturers of hydraulic turbines.

July, 1916, to December, 1916—draftsman with George F. Hardy, M. Am. Soc. C. E., on design of paper mills, etc.

January to July, 1917, and December, 1917, to February, 1918—engineer with British Chemical Company of Trenton, Ont., Canada, first on field and office work in charge of design and construction of water and sewer systems for a trinitrotoluene (TNT) plant; and during the last two months as assistant chief engineer in charge of new construction.

September to December, 1917—superintendent of construction on steam power plant for Esselstyn and Murphy of Detroit, Mich.

February to September, 1918—hydraulic designing engineer for Graham, Anderson, Probst and White of Chicago, Ill., in charge of design of a large pumping station at the United States Explosive Plant at Nitro, W. Va.

September to December, 1918—assistant to area supervisor in nitrating area, plant C at Nitro, in responsible charge of construction work.

January to July, 1919—engineer on valuation work for Henry Earle Riggs, Past-President and Hon. M. Am. Soc. C. E., in Ann Arbor, in charge of unit price investigations, editing reports, etc.

July to December, 1919—field engineer, Realty Construction Company of Flint, Mich., in charge of engineering on the construction of large hotel and bank buildings.

December, 1919, to February, 1920—special assistant, Technical Section, War Department Claims Board, reviewing and reporting on claims in connection with canceled war contracts.

February to October, 1920—assistant to engineer in charge of hydraulic structures, J. G. White Engineering Corporation, making and checking calculations for stability of dams and powerhouses; designing powerhouses, penstocks, head gates, hoists, canals, etc.; and making computations for available power, storage, required flywheel for speed regulation, etc.

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[1] Memoir prepared by Howard A. Harris, Jr., Assoc. M. Am. Soc. C. E.

October, 1920, to February, 1921—hydraulic engineering assistant to chief engineer, Guggenheim Brothers of New York, N. Y., in charge of design of hydraulic plants and of investigations and reports on hydraulic problems.

February, 1921, to July, 1923—engineer with James H. Fuertes,[2] M. Am. Soc. C. E., consulting engineer of New York City in charge of design and construction of municipal pumping station and of pipe lines at Harrisburg, Pa.

July, 1923, to October, 1924—engineer with Viele, Blackwell and Buck on power plant design.

February, 1925, to July, 1927—designer with Nicholas S. Hill, Jr.,[3] M. Am. Soc. C. E., on various municipal water and sewer projects.

From July, 1927, to March, 1933, Jeptha Wade was associated with the Federal Water Service Company as assistant chief engineer. During this period he supervised the design and construction of water softening plants at Massillon and Marysville, Ohio. In April, 1933, he went to California as chief engineer of the California Water Service Company. In this position he was responsible for the design and construction of all types of water works facilities on twenty operating properties—numerous wells and pumping stations, three water treatment plants, a 10-mgd water supply for industrial plants, and many miles of transmission and distribution mains. Mr. Wade was author of numerous unpublished technical reports on water resources, plans for developing and enlarging existing plants, and papers presented before the California section of the American Water Works Association.

On July 9, 1918, Jeptha Wade was married to Jessie Ada Baker in Flint. He is survived by his widow and two children, Jeptha A. Wade, Jr., and Margaret Ann Wade.

A man of high character, Mr. Wade was very thorough in his work. He was respected both by those who employed him and by his fellow workers. He was extremely sincere and honest in all his work and dealings. His professional ethics were the highest and he followed his beliefs in all his actions. He was a genial companion and a cooperative associate and was held in highest regard by all who knew him. Interested in his fellow men, particularly in those young engineers who worked under him, he gave much helpful assistance both professional and personal.

He was a member of Sinfonia, Tau Beta Pi, and Sigma Xi engineering societies; the Engineers Club of San Francisco, Calif.; the Peninsula Water Works Association; and the American Water Works Association (director of the California section for two years). He was a member of Maple Masonic Lodge, Maplewood, N. J., and a member of the Congregational Church.

Mr. Wade was elected an Associate Member of the American Society of Civil Engineers on November 27, 1917, and a Member on October 15, 1923.

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1482.

[3] For memoir, *ibid.*, Vol. 103 (1938), p. 1808.

JOSEPH PALMER WADHAMS, M. Am. Soc. C. E.[1]

Died January 17, 1944

Joseph Palmer Wadhams was born in Goshen, Conn., on April 22, 1877, the son of Francis M. and Frances (Palmer) Wadhams.

The family later moved to New Haven, Conn., and, after completing the college preparatory studies at Hillhouse High School, Joseph Wadhams entered the Sheffield Scientific School of Yale University in New Haven, from which he was graduated in 1899 with the degree of Bachelor of Philosophy.

In 1900 Mr. Wadhams established a private practice as consulting engineer in Torrington, Conn. At this time he was also engineer for the Torrington-Winchester Street Railway Company. In 1905 he entered the employ of the New York, New Haven and Hartford Railroad Company, and was engaged in the electrification of the New York Division from Woodlawn, N. Y., to New Haven, and of the Harlem River Branch from New Rochelle, N. Y., to the Harlem River. He later moved to Boston, Mass., where he was in charge of the office, preparing the electric traction plans for the electrification of the Boston Terminal and the Roxbury-Dedham branches of the Boston and Providence Railroad Corporation.

From 1912 to 1915 Mr. Wadhams was engineer in charge of the firm of McHenry and Murray in New Haven. During 1915–1916 he served as street commissioner and city engineer of Norwalk, Conn.

In 1919 Mr. Wadhams was appointed engineer for the Public Utilities Commission of the State of Connecticut, in which position he ably served until his death. He was the commission's chief expert on evaluation of water companies and an authority on rates. He represented the National Society of Public Utilities Engineers at various meetings in Washington, D. C.

Mr. Wadhams was long a member of Plymouth Congregational Church of New Haven, being on the board of deacons and president of the finance committee. He belonged to the Graduate Club of New Haven, University Club of Hartford, Conn., the Torrington Country Club, and the Masonic fraternity of Torrington. A member of the Connecticut Society of Civil Engineers, Mr. Wadhams was elected president in 1926, became a life member in 1938, and was chosen an honorary member in 1942. A former President of the Connecticut Section of the Society, for a period of years, he served as Secretary and Treasurer of the Section.

In 1902 he was married to Ethel Hubbell Clarke of New Haven. He is survived by his widow; a daughter, Dorothy Palmer Wadhams (Mrs. Stuart Cleaveland); and a granddaughter, Dorothy Wadhams Cleaveland.

During his entire career, Mr. Wadhams practiced engineering. A man of sterling character and high ideals, he was quiet and reserved. He was greatly admired by all with whom he came in contact, both socially and professionally, for his honesty, loyalty, perseverance, and for his charming and genial good nature.

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[1] Memoir prepared by Clarence M. Blair, M. Am. Soc. C. E. Mr. Blair died on May 21, 1944.

Mr. Wadhams was elected an Associate Member of the American Society of Civil Engineers on October 31, 1911, and a Member on June 24, 1914.

ELTON DAVID WALKER, M. Am. Soc. C. E.[1]

Died February 24, 1944

Elton David Walker was born on March 8, 1869, in Taunton, Mass., the son of Benjamin D. and Ruth (Cobb) Walker. He received his education in the public schools of Taunton and at the Massachusetts Institute of Technology in Boston from which he was graduated with the degree of Bachelor of Science in Civil Engineering in 1890. After one year as assistant in civil engineering at his alma mater, he gained practical experience in several types of engineering work before finally returning to the teaching profession. During the latter half of 1891 he was civil engineer in charge of construction with the Constructing Quartermaster at Fort Sheridan, Ill. For the next three years he was a member of the engineering firm of Walker and Gallagher of Chicago, Ill. From 1894 to 1900 he taught civil engineering at Union College in Schenectady, N. Y.—the first three years as instructor and the last three as assistant professor. During his last two years at Union he was also resident hydrographer for the U. S. Geological Survey.

In October, 1900, Professor Walker went to The Pennsylvania State College at State College where he taught until he retired in July, 1939. Successively, he held the rank of Assistant Professor of Civil Engineering, Professor of Hydraulic and Sanitary Engineering, and, during the last thirty-three years, Head of the Department of Civil Engineering. From 1913 to 1915 he was also acting Dean of the School of Engineering.

From 1917 to 1919 Professor Walker served in France as Captain with the 15th Engineers, U. S. Army, engaged on construction work—railroads, water supply and drainage systems, warehouses, hospitals, barracks, and preliminary studies of harbor projects. He was awarded the Order of the Purple Heart, and a citation by Gen. John J. Pershing "for exceptionally meritorious and conspicuous services with the 15th Railway Engineers at Jonchery and Liffol-le-Grand." After the war he continued in the Army Reserve Corps and became a Colonel of Engineers, participating actively in the Corps until 1942. He was president of the Centre County Chapter of the Reserve Officers Association of the United States, and later first vice-president of the Pennsylvania department of that organization.

Professor Walker had a consulting engineering practice from 1892 until about two years before his death. He designed and directed construction of sewage treatment plants and sewer and water supply systems, and did valuation work. He served as expert witness in water works cases; and, as special engineer for the Pennsylvania State Department of Health, investigated and reported on cases of alleged water pollution, on applications for permits to construct water works and sewage systems, and on requests for water company charters.

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[1] Memoir prepared by L. S. Rhodes, Associate Prof. of Civ. Eng., The Pennsylvania State College, State College, Pa.

While teaching at Penn State, Professor Walker supervised the operation of the college sewage treatment plant for a time, and later acted in an advisory capacity. At this treatment plant he conducted research on open versus closed trickling filters, liquid chlorine versus bleaching powder as a disinfectant for sewage plant effluents, and treatment of creamery wastes.

In addition to the Society, Professor Walker belonged to the following technical organizations: The New England Water Works Association, the Society for the Promotion of Engineering Education, the American Association of University Professors, and the Society of American Military Engineers. He was a fellow of the American Public Health Association. In 1926 he called a conference at State College from which developed the Pennsylvania Water Works Operators' Association and the Pennsylvania Sewage Works Association. These two organizations were originally one association of which Professor Walker was the first president. He was also a director of the Engineers' Society of Pennsylvania.

A director of the University Club of State College, Professor Walker was its president from 1912 to 1915, during which time the present clubhouse was built. He was active in St. Andrews Episcopal Church of State College, being for a long time a member of the vestry and taking an important part in the construction of the present church edifice. He was a member of Sigma Xi, Phi Kappa Phi, Tau Beta Pi, Chi Epsilon, and Scabbard and Blade honor societies; and of Delta Kappa Epsilon, a social fraternity.

While at Union College, in 1896 Professor Walker was married to M. Louise Brownell of Schenectady. Two children, Dudley Brownell and Robert Brownell, were born to them. Dudley died in February, 1907, and Robert in April, 1912. Both Mr. and Mrs. Walker came from early New England families. He is survived by his widow and by a brother, Chester E.

As a citizen, as an engineer, and as a teacher, Professor Walker had the highest ideals, character, and integrity. His life exemplified these qualities. The exactness of his memory was always a source of amazement to his friends.

Professor Walker was elected a Junior of the American Society of Civil Engineers on October 4, 1892; an Associate Member on June 1, 1898; and a Member on September 5, 1900.

JOSEPH WEIDEL, M. Am. Soc. C. E.[1]

DIED JANUARY 6, 1945

Joseph Weidel was born on February 22, 1873, on a farm near Lagrange, Fayette County, Tex. He was the son of Joseph Weidel and Elisabeth (Seibert) Weidel. His father and mother and older sister, Marie, came to Texas from Germany in 1871. His mother died when he was seven years old.

Mr. Weidel obtained his early primary education in the public schools at Kinkler, a small community about seven miles north of Hallettsville, Tex. At the age of thirteen he left the farm, learned city ways at Houston and Galveston, Tex., and within two years decided to become an engineer. He studied intensively at night school in San Antonio, Tex., and entered the Agricultural and Mechanical College of Texas at College Station, in the fall of 1889. He was graduated from the College of Civil Engineering at that institution in 1893, with the degree of Bachelor of Civil Engineering.

His engineering career began in 1895 as draftsman for M. Guggenheim's Sons at Monterrey, Mexico, assisting on the plans for a refinery being planned for construction at Perth Amboy, N. J. In a short time he was transferred to New Jersey and worked on the design and plans for a variety of furnaces; machinery, gas-producing, and distributing facilities; electrolytic plant; and structural steel buildings. The plant was placed in operation in 1896.

During the winter of 1896–1897, he taught mathematics and drafting in a private school in Chicago, Ill. He then returned to Texas because of poor health.

Late in 1899 he joined the Buck Exploration Company as a surveyor and draftsman on a prospecting expedition across Mexico from the Rio Grande to the State of Sinaloa. In October, 1900, he entered railroad engineering as chainman for The Atchison, Topeka and Santa Fe Railway Company at Pueblo, Colo. In 1901 he was promoted to transitman, in charge of second-track construction between Morley and Wootton in southern Colorado. In 1902 he acted as building inspector on the company's new shops and terminal being constructed at Topeka, Kans., where he conducted an investigation of systems of heating for large plants such as the railroad shops.

In September, 1902, Mr. Weidel returned to Mexico as division engineer on construction for the Mexican Central Railroad Company on the line from San Pedro to Paredon in the State of Coahuila. In 1903 he became field engineer in charge of a party on railroad location for the railroad company in the State of Veracruz. On March 1, 1904, he returned to The Atchison, Topeka and Santa Fe Railway Company as assistant engineer on the construction of new shops and terminals at La Junta, Colo. In January, 1905, he directed construction on the Shoemaker to Watrous change of line in northern New Mexico, and in May he was transferred to the Canadian to

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[1] Memoir prepared by G. W. Harris, M. Am. Soc. C. E., J. DeN. Macomb, Assoc. M. Am. Soc. C. E., and F. B. Baldwin, who succeeded Joseph Weidel as Valuation Engr. System, The Atchison, Topeka and Santa Fe Ry. Co., Chicago, Ill.

Glazier change of line in the Texas panhandle. In 1906 he was in charge of construction of the Holly and Swink Railway in eastern Colorado.

During 1907 and 1908 Mr. Weidel supervised the construction of a new tunnel for The Atchison, Topeka and Santa Fe Railway Company at Raton Pass, just south of the Colorado-New Mexico boundary. From 1908 to 1912 he was engaged in the standardization of plans for track, bridges, and buildings. From 1912 to 1914, he was in charge of constructing the line from Lubbock, Tex., to Texico, N. Mex. In 1915 he was appointed valuation engineer, System. He retired on January 15, 1943.

Eminently qualified by education and experience as an engineer to locate and build railroads, Mr. Weidel also had that tenacity of purpose and mental training which enabled him to analyze and unfold the great mass of details and facts involved in the stupendous task of inventorying, classifying, and evaluating the physical properties of a large railway system. He was well known by his associates for his ability on construction, but of greater importance was his contribution to the valuation field. As a valuation engineer, he was known and highly regarded throughout the United States.

A familiarity with the German language gained early in life and a knowledge of Spanish acquired later not only gave him an entree to literature, but also equipped him as a translator. He was a member of the American Railway Engineering Association and for many years a member of the Western Society of Engineers.

In 1909 Mr. Weidel was married to May Genevieve Baston of Hamilton, Ont., Canada. He is survived by his widow; a son, Joseph Charles; and two daughters, Wilhelmina Elisabeth (Mrs. Verne J. McCaul) and Marie Josephine (Mrs. Wesley A. Hodgetts).

Mr. Weidel was elected an Associate Member of the American Society of Civil Engineers on July 10, 1907, and a Member on November 26, 1918. He became a Life Member in January, 1942.

WILLIAM HENRY WETZLER, M. Am. Soc. C. E.[1]

Died December 23, 1944

William Henry Wetzler, the son of Joseph and Crescenz (Ruf) Wetzler, was born at Monroe, Wis., on March 13, 1880. Later, his parents moved to Beloit, Wis., where he spent his boyhood and youth in grade school and high school. He entered the University of Wisconsin at Madison in the fall of 1902 and was graduated in 1906. He was a self-supporting student—earning his way through the university and at the same time studying the very difficult civil engineering course.

Mr. Wetzler's early experience, like that of most young engineers, was diversified both as to type and location. However, most of his professional career was devoted to the design of hydraulic structures of all kinds, at which he was a recognized expert.

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[1] Memoir prepared by L. F. Harza, M. Am. Soc. C. E.

Immediately after graduation, he entered the employ of Daniel W. Mead, Hon. M. and Past-President, Am. Soc. C. E., where he remained for one year. Then, after taking a competitive examination, he was appointed an assistant engineer with the Board of Water Supply of New York City (N. Y.). He spent two and one-half years on design and construction in connection with the Olive Bridge Dam and Ashokan Reservoir and on the design for a sanitary trunk sewer in the Esopus watershed. He then served the Arnold Company of Chicago, Ill., on the design of irrigation and hydroelectric structures for two and one-half years. From 1912 to 1915 he was in direct charge of hydroelectric structures for the H. M. Byllesby Company of Chicago. For the next two years he supervised the design of a hydroelectric project for James O. Heyworth,[2] M. Am. Soc. C. E., in Canada.

Like most engineers, during World War I, Mr. Wetzler left his usual activities for war work and devoted his time to the design of ships for the Emergency Fleet Corporation at Washington, D. C. He was also a surveyor of ships for the American Bureau of Shipping in New York City.

After the war, he returned to his chosen field of hydraulic design, and was with the L. E. Myers Company of Chicago, on the design of hydroelectric plants for two years. He then served as chief draftsman for the Sinclair Refining Company of Chicago for two years on refinery design—another departure from his chosen work. Evidently he did not like oil as well as water and, in 1924, returned to the design of hydraulic structures—this time for the Harza Engineering Company of Chicago. From 1925 to 1929 he was in California, being employed first by the Department of Water and Power of Los Angeles on the Colorado River Aqueduct and then by the Los Angeles County Flood Control District on the design of gravity and multiple arch dams. From 1929 to 1942 he was employed by the Bureau of Water of the City of Chicago, in charge of a section of the designing division. On November 30, 1942, he moved to Southern California—the place he loved most.

Mr. Wetzler was always a thorough student of the problems before him; a tireless and efficient worker, who sought perfection as far as humanly possible in design and execution. He loved his work, took it seriously, and never neglected any detail which might help to improve the final product. To all executives, a most difficult problem is to find men who can be assigned a job without further worry, supervision, checking, conferences, and frequent revisions. From long experience as a colleague or employer, the writer can state that Mr. Wetzler was one of the few engineers to whom a difficult problem could be assigned with complete confidence. Mr. Wetzler was one of those men to whom engineering was not merely a means of livelihood. He put his very best energy, talent, and perseverance into the job—in fact, his work was his life and his one great supreme interest next to his wife. He belonged to the Society of Naval Architects and Marine Engineers.

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1826.

On June 1, 1907, in Buffalo, N. Y., he was married to Mary Crook. He is survived by his widow.

Mr. Wetzler was elected a Member of the American Society of Civil Engineers on April 14, 1919.

LINDSEY WOOD WHITEHEAD, M. Am. Soc. C. E.[1]

Died July 15, 1944

Lindsey Wood Whitehead, the son of Bower T. and Harriet (Wood) Whitehead, was born on June 16, 1888, in Brookings, S. Dak. He received his early education in the Brookings public schools. In 1908 he was graduated from South Dakota State College of Agriculture and Mechanical Arts at Brookings, with the degree of Bachelor of Science in Mathematics. For the next two years he served his alma mater as an assistant in mathematics.

From June, 1911, to August, 1912, he was rodman and instrumentman for the Grand Trunk Pacific Railroad Company, engaged on construction of bridges and trestles and on inspection of cement and concrete. In September, 1912, he entered the Massachusetts Institute of Technology at Boston, receiving the degree of Bachelor of Science in Civil Engineering in 1913. In 1914 he was assistant in civil engineering at the Massachusetts Institute of Technology. In June, 1914, Mr. Whitehead again left the teaching field to become instrumentman and inspector for E. Worthington, a consulting engineer of Dedham, Mass. In this capacity, he worked on general municipal engineering, surveying, and highway construction.

Mr. Whitehead returned to teaching in September, 1916, when he was appointed instructor in civil engineering at The Pennsylvania State College in State College. Later he became associate professor of civil engineering in charge of the division of surveying, which position he held until his death. Professor Whitehead was largely responsible for the design and construction of the college's surveying-instruction summer camp, which has permanent buildings with water supply, sewage disposal, and electric lighting. This camp was his pet hobby. During his vacations and his spare time he worked for the Department of Grounds and Buildings of the college on extensions to the sewer system, prepared plans for the growth of the campus, and made an accurate survey of campus buildings, streets, and of water, sewer, electric, and steam lines. He also served in 1931 as assistant borough engineer for the Borough of State College, in charge of plans, specifications, and construction of a storm sewer.

In addition to the Society, Professor Whitehead was a member of Chi Epsilon (honorary civil engineering fraternity), the Society for the Promotion of Engineering Education, and the American Association of University Professors.

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[1] Memoir prepared by L. S. Rhodes, Associate Prof. of Civ. Eng., The Pennsylvania State College, State College, Pa.

In 1913, while at the Massachusetts Institute of Technology, Professor Whitehead was married to Marie Laut of Bremen, Germany. He is survived by his mother; his widow; four daughters, Grace (Mrs. F. G. Koch), Lyndall (Mrs. J. C. Wert), Esther (Mrs. L. H. Casper), and Louise (Mrs. J. K. Poorman); and one son, Bower T. Whitehead (in the United States Army).

Larry, as Mr. Whitehead was known to all his associates, lived in Boalsburg, Pa. (near State College), where he was a member of the Independent Order of Odd Fellows. He was most active in the Pennsylvania State College Student Chapter of the Society, serving as faculty adviser and assisting at all meetings. His dealings with students and associates were always characterized by great fairness and justice, and he held the respect and affection of all who worked with him.

Professor Whitehead was elected a Member of the American Society of Civil Engineers on August 12, 1935.

JULIUS EDGAR WILLOUGHBY, M. Am. Soc. C. E.[1]

DIED MARCH 11, 1944

Julius Edgar Willoughby was born on October 12, 1871, in Arkadelphia, Ala. He was the son of John Paul and Mary Jane (Cosby) Willoughby and the grandson of John Hincky and Amanda (Tyler) Willoughby. The Willoughbys were descended from a long line of distinguished English ancestors and the Tylers belonged to a Virginia family well known in state and national affairs.

John Paul Willoughby was schoolmaster at Arkadelphia, and later a merchant at Blount Springs, Ala. However, business life was not congenial and he gave up that pursuit to enter the service of the Louisville and Nashville Railroad Company as land agent. After various promotions he was appointed general land agent and geologist for the State of Alabama, which position he held at the time of his death in 1910. Julius Willoughby's grandfather, John Hincky Willoughby, was a physician and Presbyterian minister—ministering to both the physical and spiritual welfare of man.

Descended on both sides from professional forebears, it was natural for Mr. Willoughby to seek a college education. He was graduated from the University of Alabama at University in 1892 with the degree of Bachelor of Civil Engineering. Later he received the degree of Civil Engineer from the same institution. In recognition of his scholarship, he was senior speaker at the commencement exercises—the highest honor conferred at that time. Since it was a state-supported institution, the University of Alabama included military training as part of the curriculum. When he was graduated, Mr. Willoughby was Captain of Company D, and was listed in the Army register as a desirable officer for the U. S. Army. He was a member of the Kappa Alpha fraternity.

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[1] Memoir prepared by George H. Maurice, M. Am. Soc. C. E.

Immediately after graduation Mr. Willoughby entered the employ of the Louisville and Nashville Railroad Company and thus began his career in railroad engineering. His first assignment was in the Land Office Department, followed in May, 1900, by his appointment as assistant chief engineer on the construction of new lines in Alabama. In April, 1901, he was promoted to engineer of construction for the Alabama, Florida and Gulf Railroad Company on about twenty-five miles of road, including a drawbridge. From March to September, 1902, he served as division engineer on location and construction; and from September, 1902, to January, 1905, as chief engineer for the Knoxville, La Follette and Jellico Railroad Company on about 105 miles of track with eight tunnels 300 ft to 3,600 ft long. He also supervised the building of terminals in Knoxville, Tenn. From January, 1905, to March, 1912, he was engineer of construction for the Louisville and Nashville Railroad Company on the reconstruction of 110 miles of track for the Atlanta, Knoxville and Northern Railway Company and other divisions. He was also engineer of construction on 88 miles of track for the Cambria Southern Railway Company and the Cartersville Branch of the Atlanta, Knoxville and Northern Railway Company (32 miles of double track from Corbin to Livingston, Ky., the West End Freight Yards in Atlanta, Ga., and many branch lines and spurs). Also, during this period, he was chief engineer of the Cairo and Thebes Railroad Company. In April, 1912, he was appointed chief engineer of the Caribbean Construction Company and of the National Railroad of Haiti.

Mr. Willoughby returned to the United States in June, 1913, as assistant engineer for the Atlantic Coast Line Railroad Company, being promoted to chief engineer in September, 1915. He retired on January 1, 1942, and became a consulting engineer. He was also chief engineer of certain subsidiaries and some affiliated lines. During this time, the double track of the Atlantic Coast Line Railroad Company was built, and the entire roadway and track were strengthened.

In 1895, at Blount Springs, he was married to Mary A. Byars, who, with two daughters, Mary and Ruth, survives him. An only son, William, died in 1936. From the time of his connection with the Atlantic Coast Line Railroad Company until his retirement, Mr. Willoughby resided in Wilmington, N. C. He then returned to his home town, Blount Springs. In 1942 he moved to Sarasota, Fla., where he died on March 11, 1944.

Mr. Willoughby took an active part in the North Carolina Section of the Society, serving on numerous committees and as President in 1928–1929. He was also a member of the North Carolina Society of Engineers, American Railway Engineering Association, Society of American Military Engineers, American Association for the Advancement of Science, and North Carolina Academy of Science. He was the author of a paper[2] and numerous discussions in the Society's publications.[3,4]

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[2] "Reconstructing the Sandbar Ferry Bridge," by J. E. Willoughby, *Civil Engineering,* October, 1932, p. 633.

[3] *Transactions,* Am. Soc. C. E., Vol. LXXII (June, 1911), p. 193; Vol. LXXXV (1922), p. 999; Vol. 91 (1927), p. 733; Vol. 96 (1932), p. 898; Vol. 99 (1934), p. 36 and p. 895; Vol. 102 (1937), p. 785; and Vol. 103 (1938), p. 1310 and p. 1561.

[4] *Civil Engineering,* February, 1932, p. 109; and March, 1936, p. 205.

A mere recital of a man's professional accomplishments, although interesting to his fellow engineers as a gage of ability, is not sufficient to evaluate his worth to his fellow men. From personal contacts and many letters received from his warmest friends and associates, the character of Mr. Willoughby shines out. The person who probably knew him best paid tribute to his quiet, unassuming, genial, and happy disposition. He was devoted to his family, loyal to his friends, and generous to a fault, giving freely to every worthy cause. Other friends, especially J. Houstoun Johnston, M. Am. Soc. C. E., praised Mr. Willoughby's zeal and energy, coupled with an unflagging loyalty to duty. He met all issues squarely and never shirked responsibility. As an example of this quality, when the Black Oak Ridge tunnel in eastern Tennessee on the Knoxville, La Follette and Jellico Railroad was constructed, at great personal risk he inspected every foot of the bore, through treacherous and extremely dangerous strata where slips and falls constantly occurred. There is a legend, probably founded on fact, that, after he was made assistant chief engineer of the Atlantic Coast Line Railroad Company, he walked over and inspected every mile of the system—about 4,500 miles. His work was his hobby, although he found recreation in reading and in national sports. He was a fine and genial host, with an interesting sense of humor. He was a staunch member of the Methodist Church for fifty years, and served for a great part of that time as steward. With his concise and penetrating mind, Mr. Willoughby saw issues clearly and made sound judgments. He was liberal in his donations to church buildings, college debts, college scholarships, orphanages, and the poor. In truth, it may well be stated, "The world is better for his having passed this way."

Mr. Willoughby was elected a Member of the American Society of Civil Engineers on June 1, 1909. He became a Life Member in January, 1942.

DONALD WITTEN, M. Am. Soc. C. E.[1]

Died October 4, 1944

Donald Witten, the son of Thomas A. and Carrie (Bailey) Witten, was born in Kansas City, Mo., on February 20, 1890. He was of English descent, both his maternal and paternal ancestors having settled in America many years prior to the American Revolution.

Mr. Witten decided on an engineering career at an early age and even acquired the special interest in bridges which he retained through life while still an undergraduate at the University of Missouri, in Columbia. To become a bridge designer in the shortest possible time, he left the university in his senior year to work for the consulting engineering firm of Waddell and Harrington. He remained with this firm and its successor, Harrington, Howard and Ash, as draftsman, designer, and resident engineer until April, 1918. His work included the designing and supervision of construction of heavy bridges of steel and reinforced concrete, including heavy movable spans. Some of the more interesting bridges which he helped to design were the Twelfth Street Trafficway Viaduct, Central Avenue Viaduct, and other important bridges at Kansas City; the Arroyo-Seco Viaduct, Pasadena, Calif.; the North Hill Viaduct, Akron, Ohio; the Willamette River Bridge, Portland, Ore.; the Arkansas River Bridge at Tulsa, Okla.; the Colorado River Bridge at Austin, Tex.; and bascule bridges in Miami, Fla.

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[1] Memoir prepared by Emmett H. Karrer, Assoc. M. Am. Soc. C. E.

Mr. Witten was resident engineer on the construction of the Eleventh Street Bridge across the Arkansas River in Tulsa. Impressed by the rapid growth of the city, he established the engineering firm of Wood, Elston and Witten in Tulsa in April, 1918. He remained a member of this firm and of its successor, Wood and Witten, specializing in location, design, and supervision of construction of highways, bridges, and municipal pavements, and in occasional special surveys and reports, until 1935. Among the more important projects on which the firm was engaged during this period were concrete highways and bridges for Washington County, Oklahoma, and Montgomery County, Kansas; the Twenty-Third Street Bridge over the Arkansas River in Tulsa; the Third Street Viaduct in Tulsa; and the widening of the Eleventh Street Bridge, which had been built originally under his supervision.

Mr. Witten joined the staff of the U. S. Public Roads Administration in 1935, and he was assigned to the District Six Office in Fort Worth, Tex. He acted as chief analyst for the Public Roads Administration on the design of the bridge across the Mississippi River at Baton Rouge, La. In 1938 he was transferred to District Ten in Washington, D. C., and for several years was in charge of the bridge division of that district, supervising the building of federal-aid bridges in Delaware, Maryland, Ohio, Pennsylvania, and the District of Columbia.

During the thirty-two years of his engineering career, Mr. Witten devoted practically all his time and energy to his specialty, bridge design. Entering that field when concrete design was in its infancy, he followed its development through the years, being a pioneer in many ideas which have become standard practice. He was an excellent structural analyst, and his decisions on engineering matters were characterized by sound common sense and good judgment. His joy in his work was one of his outstanding characteristics. His vocation was his avocation.

On June 11, 1914, he was married to Margaret Elston. Mr. Witten is survived by his widow; a daughter, Sarah Caroline Witten; a son, Thomas A. Witten (Captain, U. S. Army Medical Corps); a grandson, Thomas A. Witten, Jr.; and a sister, Hazel Witten.

Mr. Witten was a member of the Unitarian Church, the Rotary Club of Tulsa, and Phi Kappa Psi fraternity. His genial personality, backed by a keen sense of humor which he judiciously used, made him very popular with his fellow workers, professional associates, and many friends.

Mr. Witten was elected an Associate Member of the American Society of Civil Engineers on September 10, 1918, and a Member on October 22, 1924.

JOSEPH HENRY YOUNG, M. Am. Soc. C. E.[1]

DIED JANUARY 21, 1944

In the death of Joseph Henry Young, the State of Utah, the United States Public Roads Administration, and the field of highway engineering lost a prominent and active man who was endowed with a sixth sense—a requisite for deciding upon highway locations or revisions of existing highways. Mr. Young's work was outstanding and was greatly appreciated by the Utah State Highway Department.

Joseph Henry Young, a native son of Massachusetts, was born in Boston on February 12, 1874. He came from old American stock, his father having been a veteran of the Civil War. He attended grade school and the Dorchester High School in Boston. To further his education, he enrolled in college extension classes, including those of the University of Wisconsin at Madison. It was one of Mr. Young's great regrets that he did not receive a college degree—as a matter of fact, natural ability is of more importance than technical training when deciding highway locations.

In September, 1892, Mr. Young began his engineering career. Throughout his lifetime he continued to work on engineering projects, except during World War I, at which time he was an inspector of ordnance in a government arsenal.

His experience was rather diversified, including aqueduct and reservoir construction for the Commonwealth of Massachusetts and barge canal construction for the State of New York. He also was employed on highway work by the New York State Highway Department.

In April, 1918, Mr. Young joined the United States Bureau of Public Roads and from that time until his death served as a highway engineer in the government service. He received marked credit for his outstanding ability, which was recognized by his principals as well as by the several chief engineers serving the State of Utah during this period.

A Past-President of the Intermountain Section of the Society, he was also president of the Ogden (Utah) Engineers Club. He was a Mason and was active in civic affairs.

Mr. Young's outstanding characteristic was his generosity. Although never a man of considerable means, he contributed to many charities. Throughout the economic depression, he employed many workmen rather than have them jobless. He lost heavily by this philanthropy. To many others he gave food and shelter, and sent them on their way with increased hope and vigor. Mr. Young started many businesses, several of which benefited others rather than himself.

Always a good companion on the long inspection trips throughout Utah, he never lost his sense of humor and obtained joy from the little things of life.

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[1] Memoir prepared by a Committee of the Intermountain Section consisting of O. C. Lockhart, M. Am. Soc. C. E., R. R. Mitchell, Assoc. M. Am. Soc. C. E., and R. E. Simpson, Jun. Am. Soc. C. E.

In all his contacts with others he was courteous and dignified, and commanded the respect and esteem of brother engineers.

On September 17, in Marlboro, Mass., he was married to Minnie Adler. Mr. Young is survived by his widow and a son, Carleton Young (Captain, U. S. Army Air Forces).

Mr. Young was elected a Member of the American Society of Civil Engineers on November 15, 1926.

ELISHA BROWN BAKER, Assoc. M. Am. Soc. C. E.[1]

DIED JULY 12, 1942

Elisha Brown Baker was born in Kingsboro, N. Y., later a part of the City of Gloversville, N. Y., on March 6, 1870, the son of Manville Cook Baker and Harriet (Brown) Baker. He was graduated from Union College at Schenectady, N. Y., in 1890, with the degree of Civil Engineer. In college he was a member of Delta Phi fraternity and in his senior year was elected to the scientific society of the Sigma Xi.

After his graduation, Mr. Baker served for a time as clerk for the Board of Water Commissioners and later as city engineer of Gloversville. In 1897 Mr. Baker began his career as a contractor. In partnership with Albert M. Banker of Gloversville, he entered into a contract for the improvment of the Erie Canal at Middleport, N. Y. Upon completion of that work in 1898, Mr. Baker went to Pittsburgh, Pa., and, from 1898 to 1907, he built locks and dams on the Ohio and Monongahela rivers.

In 1907 he returned to Schenectady to begin construction on the first of a long series of contracts for parts of the New York State Barge Canal. For twenty-five years, Mr. Baker was actively engaged as a contractor, building many locks, dams, and other structures on the Canal, chiefly along the Mohawk River from Cohoes, N. Y., to and including the terminal at Utica, N. Y. He also was awarded many other contracts for state highways and other public projects.

In 1932 Mr. Baker retired from active work as a contractor, but after that date he served for a time as assistant city engineer of Utica and as a member of the Municipal Civil Service Commission of Utica.

On November 28, 1906, Mr. Baker was married to Sarah Pfeil, of Pittsburgh. Mrs. Baker and a daughter, Elizabeth Brown Baker, survive him.

Mr. Baker was a trustee of Westminster Church, in Utica. He was a Thirty-Second Degree Mason, and a member of the Fort Schuyler Club, the Torch Club, and the Yahnundasis Club of Utica; and the National Society of Professional Engineers.

Mr. Baker was elected an Associate Member of the American Society of Civil Engineers on April 1, 1896. He became a Life Member in 1931.

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[1] Memoir prepared by Frederick W. Newton, Assoc. M. Am. Soc. C. E.

ERNEST DANIEL BEAN, Assoc. M. Am. Soc. C. E.[1]

Died February 20, 1943

Ernest Daniel Bean was born at Haverhill, Mass., on July 13, 1884, the son of John Ames and Henrietta (Brown) Bean. He was educated in the public schools of Haverhill, and the University of Maine at Orono, from which he was graduated in 1907 with the degree of Bachelor of Science in Civil Engineering. He made an enviable record in athletic and social activities.

As with most engineers, his professional career started with minor positions during college vacations and continued thereafter in successive engagements of increasing responsibility. By 1911 he had progressed to general superintendent for Barrally and Ingersoll on a three-mile section of the New York State Barge Canal. Following the completion of this work, he became chief engineer of the Palisades Interstate Park Commission and supervised design and construction on this 35,000-acre playground until early in 1915. In 1917 Mr. Bean was superintendent for W. R. Phillips Company and directed construction on a plant for 6-in. guns at Fairfield, Conn.

After several years of diversified engagements on construction and appraisal work, he joined the J. G. White Management Corporation as hydraulic engineer in 1923 and for fifteen years continued in this field with that company and its successor, the Associated Gas and Electric Company. During this period he worked on many hydroelectric installations—sometimes as designer, sometimes as supervisor, sometimes as construction superintendent, and not infrequently as all three. The project which gave him the greatest satisfaction was the 16,000-kw development at Botocan near Manila, Philippine Islands. As chief engineer of construction, he spent two years during 1929–1931 building this $5,000,000 project which involved a concrete dam, 4,000 ft of rock tunnel, and a power house in which water under a 647-ft head motivated the equipment. The plant was flooded in 1934 when 27 in. of rain fell on the watershed within a few days. Mr. Bean spent most of 1935 at Botocan, devising, designing, and constructing the necessary diversion and protection works.

From July, 1938, to July, 1940, he served as engineer, senior engineer, and expert hydraulic engineer to the Power Division of the Public Works Administration and the Federal Power Commission, investigating the possibilities of many hydraulic development projects and several phases of the Shasta (California), Santee-Cooper (South Carolina), and Tennessee Valley Authority developments.

After a brief engagement on cantonment construction at Starke, Fla., he was employed by the Hercules Powder Company as supervising engineer in January, 1941, and directed construction of that company's $12,000,000 ordnance plant at Pulaski, Va. Late in 1941, when this plant was substantially completed, poor health forced him to resign.

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[1] Memoir prepared by Cuyler W. Lush, M. Am. Soc. C. E.

Twice in 1942—once with the Rust Engineering Company and once with the Tuller Construction Company—he resumed active work with his customary enthusiasm; but each time failing health forced his resignation in a few months.

Known to his friends as "Beano," Mr. Bean possessed the happy faculty of making whatever work was at hand a pleasure for his fellow workers. Many were the testimonials given to him by associates on various engagements and many are the pleasant memories of his genial humor and good cheer which linger with those who knew him. He was a Protestant, a Mason, and a member of the Phi Eta Kappa society.

He was married on July 17, 1909, to Beth E. Hoyt.

Mr. Bean was elected an Associate Member of the American Society of Civil Engineers on October 1, 1912.

FRANK JOSEPH BOYLE, Assoc. M. Am. Soc. C. E.[1]

Died January 20, 1944

Frank Joseph Boyle was born in Hazleton, Pa., on June 3, 1896, the son of Patrick M. and Bridget (Moore) Boyle.

His early education was received in the grade schools of Wilkes-Barre, Pa. He attended the Mining and Mechanical Institute in Freeland, Pa., and, in September, 1914, he entered Pennsylvania State College, in State College. At the end of his junior year, since the United States had entered World War I, Mr. Boyle decided to join the Army, and he enlisted on August 15, 1917, at Madison Barracks, in Watertown, N. Y. After a training period, he was commissioned a Second Lieutenant in the Signal Corps and was transferred to Fort Omaha, Nebraska, where he served as an instructor in aerial artillery observation and map reading. He received his discharge on January 16, 1919, at Camp Grant, Ill. In the fall of 1919, he reentered Pennsylvania State College and received the degree of Bachelor of Science in Sanitary Engineering in June, 1920.

In 1913 Mr. Boyle began his engineering career, working as a rodman and chainman on a surveying party for L. S. Howe, of Kingston, Pa. After graduation from college, he worked for two years as transitman and chief of party on general engineering surveys. From September, 1922, to April, 1931, he was associated with his brother, the late James A. Boyle, Assoc. M. Am. Soc. C. E., in Wilkes-Barre, on sewerage systems and highways for various municipalities and townships and on the construction of roads, sewers, and bridges for the Pennsylvania Highway Department.

From June, 1931, to January, 1936, Mr. Boyle was employed as assistant engineer by Luzerne County (Pennsylvania) on the design of steel and concrete bridges. From February, 1936, to October, 1938, he served as construction superintendent for the Works Progress Administration in the Wilkes-Barre District in charge of the bridge program. From November, 1938, to September, 1940, he was resident inspector for the Public Works Administration in the Pittsburgh (Pa.) District. In November, 1940, Mr. Boyle accepted a position as designer in the Bureau of Design of the Allegheny County (Pennsylvania) Department of Works, checking final estimates on bridge construction.

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[1] Memoir prepared by R. S. Quick, M. Am. Soc. C. E.

Mr. Boyle was well liked by his associates. At all times he showed a willingness to assist others with their problems. His experience in the Air Corps in World War I made him keenly interested in the progress of aviation during World War II. He was very proud that two of his sons were in this branch of the service—one as Second Lieutenant in the Army Air Corps and the other as Ensign in the Navy Air Corps.

On January 20, 1920, Mr. Boyle was married to Martha Seitzinger. His widow and three sons, Richard J. (Second Lieutenant, U. S. Army Air Corps), James (Ensign, U. S. Navy Air Corps), and Frank Joseph, Jr., survive him.

Mr. Boyle was elected an Associate Member of the American Society of Civil Engineers on November 9, 1942.

THOMAS BYRNES BROGAN, Assoc. M. Am. Soc. C. E.[1]

Died June 11, 1944

Thomas Byrnes Brogan was born in New York, N. Y., on July 4, 1880, the son of John J. Brogan and Maria (Fence) Brogan.

He received his early education in New York City at Public School No. 3 in Greenwich Village and the Dwight School. In 1903 he was graduated from New York University with the degree of Bachelor of Science in Civil Engineering.

During the summer of 1902 he worked as instrumentman for the Erie and Central New York Railroad Company. From July, 1903, to January, 1904, he was employed as instrumentman by C. Robinson Griggs in New York City. In 1905 he became map draftsman and field topographer for E. Belcher Hyde. He was in charge of shield-driven tunnels in compressed air from July, 1905, to September, 1909, and was responsible for line and grade as tunnel inspector at the Weehawken (N. J.) Shaft of the Pennsylvania Railroad Tunnels.

Entering the employ of the City of New York, from October, 1909, to May, 1913, he was employed as engineer inspector and assistant engineer on the construction of the Catskill Aqueduct for the Board of Water Supply. From May, 1913, until his death, Mr. Brogan was connected with the Bureau of Highways—later the Bureau of Highways and Sewers—in the office of the President of the Borough of Brooklyn, City of New York. Until 1922 he was engaged on surveys for all highway improvements in that borough, and, from 1938 to 1941, he was in charge of construction of all highway improvements in the southerly and easterly sections of Brooklyn. From 1941 until his death he was in charge of the preparation of final estimates for all highway work of the Bureau. While on active field duty, Mr. Brogan suffered a heart attack and died after three days of illness.

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[1] Memoir prepared by Joseph C. O'Dea, Assoc. M. Am. Soc. C. E.

Mr. Brogan was a quiet, unassuming gentleman. He was always kind and considerate and enjoyed the affection and respect of those who were associated with him. He was conscientious and painstaking in all his efforts.

In 1912, he was married to Bertha Melius who died in 1920. He is survived by a brother, John J. Brogan, and a sister, Mrs. Edward F. Faulkner.

Mr. Brogan was a member of the Delta Upsilon fraternity and the New York Lodge, F. and A. M.

Mr. Brogan was elected an Associate Member of the American Society of Civil Engineers on February 1, 1910.

WALTER JOHN BUCHANAN, Assoc. M. Am. Soc. C. E.[1]

Died August 14, 1944

Walter John Buchanan, the son of Peter and Margaret (Pender) Buchanan, was born at Hamilton, Scotland, on October 31, 1888. He was educated at Allan Glen's School, in Glasgow, Scotland, and at the Royal Technical College of the University of Glasgow, from which he obtained an Associateship in Civil Engineering and a Diploma in Mining Engineering.

After spending several years with the firm of Sir Robert McAlpine and Sons, in 1915 he joined the staff of the late Prof. T. Gray at the Royal Technical College. In this capacity, he assisted in the toluene control for Scotland during World War I.

In 1919 he was appointed to the teaching staff of the Royal Technical College and was largely responsible for the supervision of the Fuel Laboratories until his death. For many years he endeared himself to his colleagues by his genial disposition and to his students by the painstaking assistance he rendered them in their practical work.

Mr. Buchanan was well known in Glasgow and was a keen golfer. He was a bachelor and is survived by three brothers and a sister.

Mr. Buchanan was elected a Junior of the American Society of Civil Engineers on April 30, 1912, and an Associate Member on July 6, 1920.

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[1] Memoir prepared by W. M. Cumming, "Young" Prof. of Technical Chemistry, The Royal Technical College, Glasgow, Scotland.

EDWARD LARAMIE CARPENTER, JR., Assoc. M. Am. Soc. C. E.[1]

DIED JULY 18, 1943

Edward Laramie Carpenter, Jr., was born at Vancouver, Wash., on March 3, 1900, the son of E. L. Carpenter, and Harriet (Brown) Carpenter. He was educated in the public schools of Aberdeen, Wash., and was graduated from the University of Washington at Seattle in 1924 with the degree of Bachelor of Science in Civil Engineering.

Edward Carpenter's boyhood was spent near or on engineering projects in the Pacific Northwest which were under the direction of his father, a senior civil engineer with the U. S. Engineer Department. This early experience, coupled with his technical engineering training, equipped him for his chosen field of general contracting.

In 1924, immediately after graduation, Mr. Carpenter entered the employ of the Wallace Bridge and Structural Steel Company of Seattle, first as steel designer and later as squad leader in the drafting room. In 1927 he was promoted to resident engineer, and served in that capacity on several important steel erection jobs, including the Spokane Street Bridge, the Northern Life Tower, and the Washington Athletic Club, all in Seattle; the Longview Bridge at Longview, Wash., across the Columbia River; and the St. John's Bridge at Portland, Ore.

In 1930 Mr. Carpenter joined the contracting firm of Henry and McFee, and assisted in the construction of a large repair and outfitting pier at the Puget Sound Navy Yard, Bremerton, Wash. He then became assistant superintendent for the same organization on the construction of a large reinforced concrete pier for the American Can Company at Seattle.

In 1931 he returned to the employ of the Wallace Bridge and Structural Steel Company as resident engineer in charge of steel erection for the dirigible hangar at the Naval Air Station at Sunnyvale, Calif., one of the largest structures of its kind in the world.

In 1932 Mr. Carpenter became associated with the Puget Sound Bridge and Dredging Company of Seattle, a pioneer construction organization of the Northwest. His first duties were as superintendent of construction of a rock mound breakwater for Santa Monica, Calif., followed by a similar position on the construction of the first unit of the San Pedro (Calif.) detached breakwater. For the ensuing ten years, as a member of the Puget Sound Bridge and Dredging Company organization, Mr. Carpenter had responsible charge of many other large and important construction projects throughout the Pacific Coast States and Alaska, included in which were the improvement of Cordova Harbor and dredging in Wrangell Narrows, Alaska. He was assistant superintendent on the construction of the Lake Washington Floating Bridge at Seattle, the largest floating bridge in the world; and superintendent in charge of field construction of a large U. S. Naval Air Station at Kodiak, Alaska, a work of unusual responsibility and magnitude.

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[1] Memoir prepared by H. W. McCurdy, M. Am. Soc. C. E., and J. Adron Troxell, Jun. Am. Soc. C. E.

In January, 1942, Mr. Carpenter was called into active service in the U. S. Naval Reserve with the rank of Lieutenant Commander, and was assigned to the Admiral's staff in the Alaska Sector. The following July, he was killed in an airplane crash while on an inspection trip.

Commander Carpenter was a man of fine character, an engineer and contractor of extremely wide experience, and a thoroughly capable operator and organizer.

On August 7, 1924, in Seattle, he was married to Margaret Kamps. He is survived by his widow, Margaret; two children, Margaret Jean and Edward; his mother, Mrs. H. H. Kamps; and a sister, Frances (Mrs. Loren H. Holden).

Commander Carpenter was elected an Associate Member of the American Society of Civil Engineers on November 27, 1933.

ROBERT PICKET CARR, Assoc. M. Am. Soc. C. E.[1]

Died February 23, 1945

Robert Picket Carr, the son of Albert and Josie (Butterfield) Carr, was born at East Orange, N. J., on January 17, 1896. After attending the East Orange High School and the Peddie School in Hightstown, N. J., he spent two years at Lafayette College in Easton, Pa.

When the United States entered World War I, he tried to enlist, but an old injury to his back caused his rejection. For nearly a year he worked for International Coal Products Company in Newark, N. J. In June, 1918, he was drafted by the U. S. Army, and went to Edgewood, Md., in the Chemical Warfare Service. His final rating was Sergeant First Class. In December, 1918, he was honorably discharged from the Army.

After a few months in Cincinnati, Ohio, in 1919 Mr. Carr went to Michigan College of Mining and Technology at Houghton, being graduated in 1921 with two degrees—Bachelor of Science and Engineer of Mines. His first mining experience was with Chontales Mines, Ltd., at Santo Domingo, Nicaragua, as mine engineer and shift boss. In 1922 he was employed by the Braden Copper Company in Rancagua, Chile, and remained three years. During 1923 and 1924, he served as safety engineer for the entire property. In 1925 he worked on the exploration, tunneling, etc., of two proposed mines in Chile.

In 1926 he was employed by Cia Chilena de Electricidad of Santiago, Chile. He remained with this company for eight years, serving in several very important capacities. For six years he was foreman of the construction department, chief of transport, and superintendent of work on hydroelectric plants, transmission lines, and concrete buildings. He rose to the position of assistant chief engineer, and became acting chief engineer when the chief engineer, A. C. Sandstrom, M. Am. Soc. C. E., died very suddenly on August 4, 1929. During his last years with the company, he concentrated on general office work and special reports.

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[1] Memoir prepared by Mrs. Albert Carr, East Orange, N. J.

In 1935 Mr. Carr was called to Oruro, Bolivia, by the International Machinery Company, as manager for Bolivia, continuing in that position for six years. In addition to regular work on industrial and electrical equipment, he handled all government priorities. In 1941 he was employed as technical adviser by the large mining and industrial organization of M. Hochschild and Company. At the end of his three-year contract, in the autumn of 1944, he went to New York, N. Y., for a few months, but decided to accept another contract with the same company; and, in January, 1945, he left New York for Lima, Peru, where he died on February 23, 1945.

On January 16, 1926, Robert Carr was married to Tulia Ehijo, of San Bernardo, Chile. He is survived by his widow and three daughters, Jacqueline del Carmen Carr, Josie Maria de la Luz Carr, and Marie Angelica Carr.

He was a member of the American Institute of Mining Engineers and the Trinity Congregational Church of East Orange. In a letter, a fellow engineer in Chile testifies to Mr. Carr's fidelity, industry, indomitable perseverance, and conscientious effort to produce the best possible results.

Mr. Carr was elected an Associate Member of the American Society of Civil Engineers on November 11, 1929.

DAVID WILSON CROFOOT, Assoc. M. Am. Soc. C. E.[1]

Died September 20, 1944

David Wilson Crofoot, the son of George L. and Mary A. (Thomas) Crofoot, was born on January 3, 1902, in Vandling, Pa. He was graduated from Cornell University at Ithaca, N. Y., in 1923 with the degree of Bachelor of Arts.

His first engineering employment after graduation was with the Pennsylvania Department of Highways as draftsman and inspector. From October, 1925, to October, 1929, he served the Alabama Highway Department as transitman, party chief, and resident engineer. In October, 1929, he entered the employ of the Missouri Highway Department as designer and later was promoted to project engineer. In February, 1942, he was appointed construction office engineer in Joplin, Mo.

On November 22, 1942, he enlisted in the United States Army. Later he was commissioned a Second Lieutenant in the Corps of Engineers and was in action on Angaur Island in the central Pacific area when he received the wounds which caused his death on September 20, 1944.

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[1] Memoir prepared by a Committee of the Kansas City Section consisting of Chester A. Smith, M. Am. Soc. C. E., and E. A. Blanpied, Assoc. M. Am. Soc. C. E.

Lieutenant Crofoot was a cultured, refined Christian gentleman of pleasing personality, always quiet, reserved, and considerate of others. He was ambitious and his work was his first concern, but he spent his spare time reading and studying which made him exceedingly well versed in many subjects.

On June 29, 1929, in Clayton, Ala., he was married to Helen Clayton Quillin. He is survived by his widow.

Lieutenant Crofoot was elected a Junior of the American Society of Civil Engineers on November 11, 1929, and an Associate Member on April 25, 1932.

LUIS AGUSTIN DELIZ, Assoc. M. Am. Soc. C. E.[1]

Died February 17, 1944

Luis Agustin Deliz was born in the Town of Aguadilla, Puerto Rico, on May 5, 1890, the son of Jose M. Deliz and Ana P. (Mendez) Deliz.

After receiving his early schooling in his home town, he continued his education in the United States, entering Baltimore (Md.) Polytechnic Institute from which he was graduated in 1909. He later studied engineering at Lehigh University in Bethlehem, Pa. In 1923 he received the degree of Bachelor of Science in Civil Engineering from the University of Puerto Rico at Rio Piedras.

From 1911 to 1920 Mr. Deliz worked at intermittent periods for the insular government of Puerto Rico in various capacities—as transitman, inspector, and assistant chief engineer in charge of design and construction of various public works. In 1920 he accepted a position as assistant engineer with the American Railroad Company of Puerto Rico. From 1926 until 1929, he was superintendent of public works for the City of San Juan, Puerto Rico. In this capacity he supervised the construction and maintenance of all streets, municipal roads, water works, public buildings, etc.

From 1930 until his death, Mr. Deliz held various positions—assistant engineer with James H. Fuertes,[2] M. Am. Soc. C. E., of New York, N. Y.; assistant engineer, Board of Transportation, New York City; designing engineer, Department of the Interior of Puerto Rico; assistant chief, Bureau of Translation of the Government of Puerto Rico; and assistant engineer, U. S. Engineer Department, San Juan.

In addition to his engineering work, Mr. Deliz was engaged throughout his life in other activities and civic affairs. His unusual ability as a translator and his knowledge of Spanish and English made him valuable as assistant chief of the Bureau of Translations of the Government of Puerto Rico. He was an active and prominent Mason and was worshipful master of the Estrella de Luquillo Lodge No. 5. He was for some time president of the charitable society, Shoes for Poor Children, and was a prominent member of the political party, Union Republicana, advocating the granting of statehood to the Island of Puerto Rico by the Congress of the United States.

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[1] Memoir prepared by Etienne Totti y Torres, M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1482.

Mr. Deliz belonged to the Sociedad de Ingenieros de Puerto Rico and the Institute of Engineers of Puerto Rico. He was an active member of the Puerto Rico Section of the Society, serving as Secretary-Treasurer and as Vice-President. His activity and zeal contributed much to Society affairs.

Mr. Deliz died at a time when he could have been of most service to Puerto Rico. His sense of civic duty and responsibility added to his quiet and simple manners and his ability to make friends made those who knew him feel that a valuable citizen had been lost.

In 1920 in Yauco, Puerto Rico, Mr. Deliz was married to Mary Roig. He is survived by two sons, Luis R. Deliz and Norris E. Deliz.

Mr. Deliz was elected an Associate Member of the American Society of Civil Engineers on February 10, 1930.

CARL E. DOWNING, Assoc. M. Am. Soc. C. E.[1]

DIED FEBRUARY 27, 1942

Carl E. Downing was born at Perlee, Iowa, on November 1, 1884, the son of Philip Henry and Ermina (Snook) Downing. He was graduated from the State University of Iowa at Iowa City in 1911 with the degree of Bachelor of Engineering, and in 1915 he was awarded the professional degree of Civil Engineer by his alma mater.

In 1911 Mr. Downing entered the employ of the Morgan Engineering Company in Memphis, Tenn., and remained with that company as rodman and instrumentman for a year and a half, except for two months spent with the Chicago Rock Island and Pacific Railway Company as masonry inspector.

From December, 1912, until November, 1918, he was employed by the Belzoni Drainage District at Belzoni, Miss. After about a year as assistant engineer, he became chief engineer on the construction of a large flood control and drainage project for the reclamation of agricultural land.

For a short time in 1919, Mr. Downing was assistant engineer with the U. S. Reclamation Service in Wyoming. For nearly three years, during two periods—first in 1919 and again in 1921 and 1922—he conducted a private engineering practice at Belzoni, and also served as chief engineer for the City of Belzoni and as engineer for surrounding towns and communities on the construction of sewer, street paving, road, and drainage projects.

In May, 1919, he entered the employ of the Mississippi State Highway Department and, with the exception of about two years as city engineer of Belzoni and in private practice, remained with this department until January, 1940. For five years he served as project engineer on road construction, for one year as assistant district engineer, and for nearly thirteen years as district engineer in charge of road construction and maintenance, with headquarters first at New Albany, then at Holly Springs, and finally at Oxford, Miss. The most important work of his career was done for the Mississippi Highway Department. During this period the State of Mississipi constructed an extensive state-wide system of modern paved highways, and Mr. Downing's sound engineering judgment and integrity were largely responsible for the excellent system of roads in northern Mississippi.

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[1] Memoir prepared by Leroy L. Hidinger and Lee H. Johnson, Jr., Members, Am. Soc. C. E.

During 1940 and 1941 he was employed by the Work Projects Administration as project engineer and superintendent in charge of work on roads and city streets for Oxford and near-by communities.

Mr. Downing was a quiet, steady, efficient engineer and a man of sterling character, highly respected by his associates and in the communities where he lived. Although his personal life was quiet and unassuming, he left a favorable lasting impression on all those with whom he came in contact.

On July 14, 1920, Mr. Downing was married to Edna Armstead Southall of Belzoni. He is survived by his widow; a daughter, Patricia Armstead Downing; and a son, Carl Seldon Downing.

Mr. Downing was elected a Junior of the American Society of Civil Engineers on October 1, 1913, and an Associate Member on December 6, 1915.

CHARLES DORMAN EVANS, Assoc. M. Am. Soc. C. E.[1]

Died June 30, 1944

Charles Dorman Evans was born in Galveston, Tex., on June 19, 1879, the son of Charles and Mary Cecile (Chambers) Evans.

His high school education was completed at Bicker Academy, in Austin, Tex., and he was graduated from the Agricultural and Mechanical College of Texas at College Station in 1899 with the degree of Bachelor of Science in Civil Engineering.

Most of his early professional experience was in railroad work. After working a few months as architectural draftsman immediately after graduation from college, Mr. Evans started as chainman on railroad location and construction, progressing successively through various field and office engineering positions to chief engineer. His railroad experience was gained with various branches of the Southern Pacific Railroad Company, the Louisiana Railway and Navigation Company, and the International and Great Northern Railway Company. With the exception of intermittent periods while employed on other work, Mr. Evans continued on railroad location, design, construction, and maintenance through 1913. During this time he had the opportunity to design not only the necessary engineering structures but also the more complicated structures, such as depots, warehouses, coaling stations, freight terminals, and other similar units.

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[1] Memoir prepared by Elmer L. Ferguson, Clovis J. Harkrider, and R. L. Tatum, Assoc. Members, Am. Soc. C. E.

After a short period in private practice, Mr. Evans assumed the management of a 1,500-acre plantation. His engineering education, knowledge, and background were of material benefit in enabling him not only to design a complete water pumping plant and distribution system and other necessary improvements, but also to establish a type of management which resulted in a profitable self-supporting concern.

In 1919 he again entered private engineering practice as senior member and manager of the engineering firm of Evans and Buxton at Shreveport, La., and in 1921 he became sole owner of the firm. His practice included design and supervision of water supply and distribution systems, sanitary sewer systems and disposal plants, steel and reinforced concrete bridges, storm sewers, street paving, reclamation and drainage, land developments, athletic fields and campus improvements, landscaping, oil field surveys, tank-farm layouts, pipe-line locations, investigations and reports, court evidence, and similar work in north Louisiana, south Arkansas, and east Texas.

In 1930 Mr. Evans was elected city commissioner of Shreveport, and served in both this capacity and as superintendent of streets and parks for two successive terms until 1934. Besides the regular duties normally indicated by his title, he had charge of the planning and maintenance of a municipal airport and made an extensive study of the garbage collection and disposal problem of the city, resulting in the construction of a modern incinerator.

During the early part of 1941 he was chief engineer for the Louisiana State Conservation Department in charge of design and construction of park improvements in coordination with the National Park Service, including water systems, sewer systems, beach improvements, roads, and bridges. For a short time before beginning these duties, he was superintendent of construction on a four-story hospital and several other masonry buildings.

Mr. Evans started service with the federal government early in life as civil engineer with the Bureau of Yards and Docks, at the New Orleans (La.) Naval Station for a short period in 1904. He again entered government service in 1935 as resident engineer inspector with the Public Works Administration, and continued in that capacity through April, 1940. He represented the federal government on the supervision of several large sewage collection systems and treatment plants, drainage structures and canals, pumping and power plants, and numerous public buildings of various types.

When Congress passed the Community Facilities Act in furtherance of the defense and war efforts early in 1941, the services of Mr. Evans were requested to assist in the reorganization of the former Public Works Administration to administer the funds appropriated to carry out the purposes of the act. Although his permanent civil service status would have been placed in jeopardy if he did not return to government service before July 1, 1941, Mr. Evans insisted upon delaying acceptance of the appointment until his former employer could be given sufficient notice to permit his replacement. This action is typical of Mr. Evans' fairness in his relationships with other people.

From August, 1941, until his death he served as examining engineer and senior examining engineer for the Public Works Agency which was later designated as the Federal Works Agency. In this capacity, he made investigations of complicated prospective projects to determine need and scope. He also assisted in the construction from inception of the plans to completion of the work. His broad and versatile experience enabled him to handle a wide variety of projects.

Mr. Evans was a member of the First Presbyterian Church of Shreveport and had been a deacon for many years. He was also a Knight Templar, but liked his home too much to be a very regular attendant at lodge meetings. He belonged to the Louisiana Engineering Society.

A great lover of the outdoors, Mr. Evans fished and hunted all his life. He knew the names and habits of all the southern birds, and was an expert taxidermist in his youth. He loved all growing things—his garden was full of plants and vines from places he had visited. Art was another of his avocations, and it was his ambition to paint. In 1939 he joined an art club and painted several excellent landscapes in pastels. He also collected coins and had a very good collection of old firearms.

Mr. Evans was an excellent engineer and a man of fine personality and high ideals. His standing in Shreveport, where he had maintained residence since the beginning of the century, was the best. He was highly esteemed as an efficient and accommodating public servant, and his loss is deeply regretted by his many friends.

On November 7, 1912, in Shreveport, La., he was married to Nell Taylor. He is survived by his widow; one daughter, Genevieve (Mrs. A. F. Merewether); four grandchildren, Diana Genevieve, Catherine Evans, Frances Charles, and James William Merewether; a sister, Mrs. Cecile Oliphant; and three brothers, Stanley, W. C., and J. P.

Mr. Evans was elected an Associate Member of the American Society of Civil Engineers on March 7, 1921.

LOUIS FOCHT, Assoc. M. Am. Soc. C. E.[1]

DIED JUNE 4, 1944

Louis Focht, the son of Hiram and Georgette (Rex) Focht, was born in Elizabeth, N. J., on September 5, 1863. He attended Lehigh University at Bethlehem, Pa., where he studied civil engineering.

In 1883 he entered the engineering department of the Lehigh Valley Railroad Company, as rodman, when that company was emerging from the status of a local railroad to that of a trunk line, actively engaged in extensive construction. He remained with the Lehigh Valley Railroad Company until 1897, during the last ten years acting as division engineer in charge of all construction and maintenance on the New Jersey, Lehigh, and Pottsville divisions.

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[1] Memoir prepared by Edwin W. Denzler, Jr., Assoc. M. Am. Soc. C. E.

During this period Mr. Focht worked on the location and construction of the main line from South Plainfield, N. J., to Newark Bay, New Jersey, and the Raritan River, Flemington, Pittstown, and Bloodgood branches. Under his supervision, the Musconetcong Branch was extended by constructing a dam on the Musconetcong River, coal and freight docks, and terminal yard facilities at Perth Amboy, N. J., and relining the Musconetcong Tunnel. Also, he constructed and operated a creosoting plant at Perth Amboy and a coal storage plant at South Plainfield. He was engaged actively in the location and construction of a connection for the Easton and Northern Railroad Company at Easton, Pa., including the design and construction of a new bridge across the Lehigh River. He had charge of maintenance of way for the Lehigh Valley Railroad Company from Pottsville, Pa., and Pennhaven Junction, Pennsylvania, eastward to Newark and Perth Amboy, covering both main line and branches, including property of the Pennsylvania, Poughkeepsie and Boston Railroad (later a part of the Lehigh and New England Railroad Company). In addition, Mr. Focht acted under special assignment on the investigation of many new projects, one of which was a new independent railroad line from Buffalo, N. Y., to Toledo, Ohio.

In 1897 he resigned from the Lehigh Valley Railroad Company to become chief engineer of the Lehigh Construction Company, at that time constructing a large hydroelectric power development on the St. Lawrence River. He remained with this company until May, 1898. Mr. Focht then became engineer for the State Board of Assessors of New Jersey. In this connection, from that time until his death, he valued all railroad property in the state for assessment purposes, with different titles and in the employ of various boards, such as the State Board of Assessors, the State Board of Taxes and Assessment, and the State Tax Department. From 1910 to 1912 he acted as chief engineer in charge of revaluation of all railroad and canal property in New Jersey. From 1931, when the State Tax Department was created, he was director and chief engineer, in charge of the Division of Engineering and Railroad Taxes.

On special assignment, Mr. Focht spent some time in Venezuela and Colombia, South America, investigating the Táchira Railroad, for a syndicate of American bankers. This assignment included reconnaissance work from Lake Maracaibo to the valley of the Orinoco River and a report on the cost, probable traffic, income, and other problems pertaining to the economics of the project, during 1897. In 1901–1902 he acted as chief engineer and superintendent on construction and operation for the New Jersey Terminal Railroad Company. In 1907 he was chief engineer of the Mount Desert Railway Company, in Maine, making preliminary surveys, locations, estimates, and designs of railway structures. The panic of 1907 resulted in the suspension of this project. From 1912 to 1914 he was consulting engineer of the Virginia Products Corporation of Baltimore, Md. In this capacity he designed factory buildings and equipment and developed an economical process for concentrating glycerine.

From 1912 to 1913 Mr. Focht was also engineer to the Godfrey (New Jersey) Legislative Commission then investigating the status of the Morris

Canal, submitting several reports covering the valuation of the property and the economical features relating to the possible enlargement of the canal. In 1923, through legislative action, he was made a member of the Morris Canal Abandonment Commission. This commission was given plenary powers by the New Jersey Legislature to enter into a compact with the Lehigh Valley Railroad Company for the abandonment of the canal property.

From 1921 until his death Mr. Focht was a member, chief engineer, and general superintendent of what was later the Delaware River Joint Toll Bridge Commission, which purchased and operated sixteen highway bridges over the Delaware River between New Jersey and Pennsylvania, from Trenton, N. J., on the south, to Milford, Pike County, Pa., on the north. During this period and under his direction, four new highway bridges were designed and constructed. In 1938 the $2,500,000 highway bridge connecting Easton and Phillipsburg, N. J., was designed and constructed under his supervision, and is operated as a toll bridge.

Louis Focht was an able engineer. He exercised good judgment and common sense at all times. He took a keen personal interest in every phase of the work being done under his supervision and, because of his wide experience, was always able to offer helpful criticism. He made many contacts and associations which resulted in lasting friendships with many men in the engineering profession, and he will be missed. As an executive, he was strict but always just. He was very quiet and unassuming and never sought publicity for himself, nor limelight for his actions. He was meticulous in all undertakings and any one having dealings with him would soon discover that he was ruggedly honest.

On November 16, 1899, in Bethlehem, Pa., Louis Focht was married to Pauline Detwiller Doster, of Bethlehem, who survives him. He is also survived by a son, Doster, and three grandsons.

He was a licensed professional engineer and land surveyor in the State of New Jersey. For five years he served as chairman of the Local Membership Committee of the Philadelphia Section of the Society, for South Jersey. Mr. Focht belonged to the American Institute of Mining and Metallurgical Engineers, the American Chemical Society, the Society of Chemical Industry of England, and the Society of American Military Engineers (life member).

Mr. Focht was elected an Associate Member of the American Society of Civil Engineers on May 4, 1892. He became a Life Member in 1927.

HOWARD LEWIS FRANCIS, Assoc. M. Am. Soc. C. E.[1]

Died December 11, 1936

Howard Lewis Francis was born at Charlevoix, Mich., on November 27, 1884. His parents were William Helson Francis, of English ancestry, and

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[1] Memoir prepared by Verne L. Havens, M. Am. Soc. C. E. Mr. Havens died on August 12, 1944.

Christy Ann (McLean) Francis, of Scottish ancestry, both of Canadian birth. Mr. Francis was educated in the public schools of his native city and was graduated from Michigan State College in East Lansing with the degree of Bachelor of Engineering in 1906.

Two major interests in Mr. Francis' early life affected all his later activities. One was the proximity of Lake Michigan where he became an expert in the handling of catboats and other small craft. The other was an innate interest in people on both sides of the Canadian-American border and later in the peoples of other lands. Even before his graduation from college, his plans were made for a first-hand study of the history, resources, and language of the Caribbean area and Mexico, as well as of the peculiar problems of the people living there. The following record indicates Mr. Francis' professional activities:

June, 1906—Instrumentman on railroad construction with the Chicago, Rock Island and Pacific Railway Company at Moorland, La.

February 1, 1907—Draftsman, Trussed Steel Concrete Company, designing reinforced concrete structures in Chicago, Ill.

May 1, 1907— Assistant resident engineer on Long Key Viaduct of the Key West Extension of the Florida East Coast Railway Company, in charge of party responsible for measurements pertaining to construction operations.

September 1, 1907, to December 1, 1908—First assistant engineer, Department of Public Works of Cuba, during the armed intervention by United States forces; one of a commission of three to study the design of aqueducts and water distribution systems for cities in Santa Clara Province, in direct charge of surveys and assistant on design.

January, 1909—Bridge engineer, Southern Pacific Railroad Company of Mexico in charge of a 1,200-ft steel and concrete bridge over the Piaxtla River in Sinaloa; completed grading on a 12-mile residency including steam-shovel work.

January and February, 1910—Engineer in charge of subdivision of a 40,000-acre estate in Veracruz, Mexico.

March 10, 1910—Superintendent of bridges and buildings for the Mexico Tramways Company, the Mexican Light and Power Company, and related enterprises; assistant to the writer on design, inspection, and maintenance, supervising a labor force ranging from 800 to 1,200 men on various jobs worth from $10,000 to $50,000—receiving station, substations, carbarns, fire protection work, office buildings, foundations, and boiler maintenance projects. (Mr. Francis was at times required to bid against independent contractors and always to comply with building codes. The revolution led by Francisco Madero caused cessation of the work.)

July, 1912—Engineer on the writer's staff in Uruguay in connection with the Port Paloma to Cebollati Railroad of the Farquhar syndicate; in local charge of construction of the first section of the line. (Cessation of this work was caused by the Balkan War which interfered with the credit agreements between the syndicate and French bankers.)

January, 1914, to January, 1915—Independent contractor in Mexico City, Mexico.

1915 to December, 1920—Chief civil engineer of the Mexico Tramways Company under the government of Venustiano Carranza; after Señor Carranza's assassination, chief engineer of the Central Railroad of Santo Domingo and at times general manager.

October, 1921, to 1932—Independent operator engaged in various commercial enterprises; later counsel in clearing up pending matters between Mexican and American interests. (He was quite successful in that field because of his complete familiarity with the problems, languages, and psychology of both peoples.)

During his years in Mexico, among the many problems which faced Mr. Francis was that of labor supply for construction activities in and near Mexico City. This was a great difficulty since labor had not yet been organized in 1910, and Mexican labor instinctively distrusted foreigners. Mr. Francis had to adapt Anglo-Saxon psychology to the Mexican laborer whose life had been spent close to the soil. True to the instincts of a gentleman in a foreign land, he quickly acquired an understanding of the slang and idioms and the customs of the common people.

His time and sympathy were given freely to assist in "ironing out" the problems that appear when labor is in the throes of organization, and the laborers learned to trust him. In the course of time, the governor of the federal district began to inquire about "Howwow," which was as close to Howard as possible in Spanish. Mr. Francis became an authority on the folklore and the songs of the people. His geniality in Spanish and English simultaneously was a source of wonderment to the peon and to the governor alike. One of his pleasures was to write Mexican parodies of American popular songs and play the guitar as an accompaniment. His voice was that of an average engineer, making his parodies the more popular. On two different occasions Mr. Francis saved the lives of Americans who did not know that the Mexican pride is very easy to wound.

Having salvaged a modest competency from these activities, Mr. Francis retired from active affairs and established his home in San Antonio, Tex. During this period he cleared up all pending matters, and thus left no problems for his family to solve after his death.

On March 10, 1926, Mr. Francis was married to Emma Velasco, daughter of Panfilo José Velasco and his wife, Maria, of Mexico, City. The sole survivors of Mr. Francis and his parents are his widow; his daughter, Christina, born on October 3, 1930; and his sister, Mrs. John O. Plank.

Mr. Francis was long associated with the writer, both professionally and socially, and the writer reposed great confidence in him. His brightest accomplishment was in mapping his life and achieving all his major objectives. Few can hope for more or strive for less.

Mr. Francis was elected an Associate Member of the American Society of Civil Engineers on October 1, 1913.

LAWRENCE GRIFFITH, Assoc. M. Am. Soc. C. E.[1]

Died February 8, 1944

Lawrence Griffith was born in Baltimore, Md., on February 6, 1869, the son of Goldsborough and Eleanor (Michael) Griffith. He was educated in the public schools of Baltimore and was graduated from the Maryland Institute School of Art and Design in Baltimore.

His early engineering experience was on railroad work. From March, 1886, to June, 1890, he served as an apprentice in the machine shops of the Baltimore and Ohio Railroad Company in Baltimore. He then was employed by the Northern Central Railway Company in various capacities: From June, 1890, until December, 1891, he served as rodman on the Baltimore Division; from December, 1891, to January, 1894, he was assistant supervisor, maintenance of way and construction on the Baltimore Division; and, from January, 1894, to October, 1894, he held a similar position with the Canandaigua Division on maintenance of way and surveys for a viaduct over Rock Stream gorge. During the second period he was in charge of (1) the construction of a 104–lever mechanical interlocking machine at the junction of the Baltimore and Potomac Railroad and the Northern Central Railway; (2) changes to bridges at Charles Street, Maryland Avenue, and Lombard Street in Baltimore; and (3) surveys for changes of part of the line, reconstruction of No. 3 yard, and shops at Canton, N. Y.

In October, 1894, Mr. Griffith was placed in charge of surveys for change of line of the Sunbury Division, Philadelphia and Erie Railroad Company. In January, 1895, he was employed by the New York Division of the Pennsylvania Railroad Company on construction and maintenance of mechanical and electric pneumatic interlocking and block signals. From May, 1899, until 1900 he was supervisor of the Grand Central Station Yard in New York, N. Y., directing construction and maintenance of tracks, signals, and interlocking devices.

During World War I from 1915 to 1918, Mr. Griffith was greatly interested in the U. S. Army. He went to the Officers' Training School at Plattsburg Barracks, N. Y., and was a member of Company K, First Provisional Regiment. He was in Plattsburg from September 8 until October 6, 1915, and was interested in riflery.

Much of Mr. Griffith's engineering career centered around the City of Yonkers, N. Y. He was twice city engineer of Yonkers—from January 1, 1918, to December 31, 1921, and from January 1, 1924, to December 31, 1926. During his first period of service, World War I curtailed actual construction, but he supervised and prepared many studies and plans which were later used by the city. Among these were zoning maps (the first to be prepared for Yonkers and the second group to be made in New York State) and a new assessment map using the section block system. During his second term as city engineer, Mr. Griffith revamped much of the Yonkers water supply system and planned the Hillview Reservoir to connect with the Ashokan Water Supply of New York. In addition, he planned and built a model firehouse for suburban areas in the Bronxville section, which received much publicity. On February 9, 1934, Mr. Griffith became president of the Yonkers Mortgage Corporation.

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[1] Memoir prepared by George W. Kittredge, Hon. M. Am. Soc. C. E.

Always a diligent public servant, Mr. Griffith was a member of the Yonkers Waterfront Commission and engineer and consultant to the Westchester County Transit Commission.

Throughout his entire career Mr. Griffith performed his duties well and was held in high regard, respect, and esteem by all with whom he came in contact.

On June 21, 1899, in New York, he was married to Kate H. Belloni. He is survived by his widow; two daughters, Mrs. Louis G. Butts and Mrs. John S. Priest; a sister, Mrs. James Booker Clark; and a brother, Goldsborough S.

Mr. Griffith was elected an Associate Member of the American Society of Civil Engineers on May 2, 1900.

JAMES RAYMOND HEAD, Assoc. M. Am. Soc. C. E.[1]

Died July 19, 1943

James Raymond Head, the son of L. Walter and Edith Eleanor (Webster) Head, was born at Garden Grove, Orange County, Calif. In 1906 and 1907, he attended the University of California at Berkeley, majoring in civil engineering. He was a member of Phi Gamma Delta fraternity.

In 1907 his engineering career began when he was employed by the Union Lumber Company at Fort Bragg, Calif., on railroad surveys. From 1908 to 1911 he served as engineer on the construction of hydroelectric power plants for the Arizona Power Company at Prescott, Ariz. In 1911 and 1912 he was engaged in general engineering work at Los Angeles, Calif. From 1912 to 1918 he was assistant engineer for the Southern Pacific Railroad Company, Bakersfield Division. Deserting engineering temporarily in 1918 and 1919, he farmed near Tulare Lake, California, and at Chico, Calif.

In 1919 he resumed his professional career, being associated with the firm of Polk and Robinson, Civil Engineers, of Chico, as resident engineer on irrigation, reclamation, and general construction until 1928. During this time, in 1924, he was resident engineer on surveys and investigations for a report on the watershed and storage reservoirs of the Santa Maria Valley in California. In 1929 he was appointed city engineer of the City of Chico. In December, 1929, he became sales engineer with the California Corrugated Culvert Company of Berkeley, Calif., for the northern district of California, remaining in this position until his death. Since the territory of the district included all of northern California, he dealt with city, county, and state officials, the personnel of irrigation, drainage, and reclamation districts, and construction men of all types.

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[1] Memoir prepared by a Committee of the Sacramento Section consisting of Martin Collins Polk, M. Am. Soc. C. E., Chairman, and Harold Fred Percival and Frederick T. Robson, Assoc. Members, Am. Soc. C. E.

Mr. Head's exceptionally fine personality, honesty, and technical knowledge made him very valuable to his employers. He had a host of friends.

On November 29, 1912, in Bakersfield, Calif., he was married to Rhoda H. Nilsson. His widow and two daughters, Barbara Louise and Marie, survive him.

Mr. Head was a member of Chico Lodge No. 111, F. and A. M., and of the Scottish Rite bodies.

Mr. Head was elected an Associate Member of the American Society of Civil Engineers on February 23, 1932.

WILLIAM JOHNSTON HENDERSON, Assoc. M. Am. Soc. C. E.[1]

Died May 10, 1945

William Johnston Henderson, the son of Edgar Clark Henderson and Adelyia Young (Price) Henderson, was born at Glen Gardner, N. J., on October 31, 1899.

He received the degree of Bachelor of Science in Civil Engineering from Lafayette College at Easton, Pa., in 1922 and the degree of Civil Engineer in 1926. After his graduation, Mr. Henderson spent three years with the Bethlehem Steel Company in the design and fabrication departments. His work consisted of detailing structural steel, of directing the assembly and finishing of structural members and, finally, of estimating and designing. In September, 1925, he became an instructor in structural engineering at Purdue University in Lafayette, Ind. Later he was promoted, successively, to assistant professor and associate professor.

During his teaching days at Purdue University, Professor Henderson progressed in his profession by contacts with various structural firms and engineers. He checked the steel and reinforced concrete design of a large power plant in Cleveland, Ohio, designed several steel and reinforced concrete bridges for the Indiana State Highway Commission, and prepared both the steel and reinforced concrete designs for several of the buildings on the campus. Walter Scholer, the architect for Purdue University, stated that he depended a great deal upon Professor Henderson for the design of the structural parts of several of the university buildings.

Professor Henderson was an excellent teacher—his work was well organized and clearly presented. He believed that the student should be guided by the instructor but should do his own thinking. He maintained high standards and made the students work. They liked this attitude and no complaints were ever heard. With an even temper and without incurring resentment, he pointed out students' faults. The balanced combination of practicality and systematic presentation of theory which he brought into the classroom was valued highly by his students and colleagues. In undertaking any task, either special or routine, he could be depended upon to do a thorough, workmanlike job. Because of his accuracy and reliability, Professor Henderson was given many extra assignments.

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[1] Memoir prepared by C. A. Ellis and R. B. Wiley, Members, Am. Soc. C. E., and P. E. Soneson, Assoc. M. Am. Soc. C. E.

Outside of the classroom he was the friend of every one, faculty and students alike. As a member and past-president of the Faculty Bowling Club, his unruffled, genial manner and cordial companionship, win or lose, endeared him to all. He was a member of Alpha Chi Rho fraternity, Chi Epsilon, and the American Concrete Institute.

Professor Henderson was married to Ruth Elizabeth Banghart of Glen Gardner on October 4, 1924. He is survived by his widow, his mother, and a brother, Abram B. Henderson.

Professor Henderson was elected an Associate Member of the American Society of Civil Engineers on August 29, 1927.

MAXWELL HENRY, Assoc. M. Am. Soc. C. E.[1]

Died April 7, 1944

Maxwell Henry was born in New York, N. Y., on December 26, 1897, the son of Simon Henry and Maria Johanna (Meiers) Henry.

His early education was acquired in the public elementary schools and Townsend Harris High School of New York City.

Even as a young man Mr. Henry disclosed those traits and characteristics which ultimately gave him a brilliant career in engineering and teaching. Early evidence of a mechanical bent and desire for information and perfection and his aptitudes in engineering fields led him in his youth to become a skilled craftsman in the electrical field. In 1918, at the age of twenty one, he was one of the few who succeeded in obtaining a license as a master electrician in New York City. For a time, he was an illumination expert for many theatrical and operatic enterprises. During this same period, Mr. Henry was active in the New York City Federation Settlement, directing boys' activities in mechanical and workshop courses and in the technique of theatrical illumination.

In 1919 Mr. Henry interrupted his engineering studies at the College of the City of New York to become instructor, for the college, in electrical wiring and installation on the Rehabilitation Program for Disabled Veterans of World War I. He was in full charge of that department from 1920 until the end of the program in 1924. Mr. Henry received the degrees of Bachelor of

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[1] Memoir prepared by Sidney L. Cahn, New York, N. Y.

Science and Civil Engineer from the College of the City of New York at a late date—1924—because of the interruption occasioned by his work for the disabled war veterans. He was then invited to, and did join, the college's staff as instructor of engineering in the School of Technology. In 1937 he was promoted to associate professor of Electrical Engineering.

While with the School of Technology, Mr. Henry designed and developed the laboratories and courses in the civil, electrical, and mechanical engineering departments. Because of his varied and expert abilities and talents, he played a most important part in developing and supervising the power and communication laboratories of the electrical engineering department and of the materials testing laboratories of the civil engineering department. He was almost exclusively responsible for all the course work in electrical measurements and illumination. In the field of electrical engineering he became a recognized expert.

In addition to his regular college duties, Professor Henry was called upon, throughout the years, to serve on practically every committee concerned with the buildings and physical improvements of the colleges everywhere in New York City—particularly those of Queens and Hunter colleges. His work in this field, building projects and directing mechanical and electrical improvements for all the colleges of the City of New York, was perhaps one of his greatest contributions to the Board of Higher Education.

In addition to his membership in the Society, Professor Henry was active professionally as a member of the American Institute of Electrical Engineers. He was licensed as a professional engineer in New York State and was engineering consultant to several industrial firms and to the Civil Service Commission of the City of New York. For the last several years he acted as expert examiner for the Civil Service Commission.

As the School of Technology of the College of the City of New York expanded, Professor Henry limited his teaching to the electrical engineering department. For many years he was executive officer of this department. It is doubtful whether the people of the City and State of New York have ever seen a more energetic and capable public servant. No work was too small or menial for Professor Henry, none too large or exacting. His able hand was visible in the new projects for all the colleges everywhere in New York City.

In these colleges, as elsewhere, there were few people, if any, who had not at one time or another sought and obtained Professor Henry's opinion or advice or in some way leaned upon him. During his long connection with the college, he rejected many highly remunerative offers to enter the industrial world, for his heart was in teaching "his boys." His door was always open and all who have passed through his classrooms have known him as a brilliant and effective teacher and adviser and as a kindly and considerate guide. His ability to impart his vast fund of knowledge to others and his patience and tact with his students and the younger men in the departments endeared him in the hearts and minds of his friends and associates throughout the twenty-five years of his devoted service to the College of the City of New York.

Professor Henry's intense activities in college matters during all times of the day, night, summer, and winter were beyond human endurance. He died in the service of the College of the City of New York at the age of forty six, an irreplaceable loss to the people of the City of New York.

He is survived by his widow, Pauline Lichtenstein Henry, D.D.S., to whom he was married on December 24, 1924; a son, Paul Maxwell; and a daughter, Carol Zena.

Mr. Henry was elected an Associate Member of the American Society of Civil Engineers on June 27, 1932.

CURT JOSEPH HEYMANN, Assoc. M. Am. Soc. C. E.[1]

Died March 30, 1944

Curt Joseph Heymann, the son of Joseph and Odile (Cahen) Heymann, was born of Luxemburgian parents in Aachen, Germany, on October 10, 1910. He attended the Aachen Gymnasium School and the Technische Hochschule Zu Aachen, from which he was graduated with the degree of Bachelor of Science in 1934.

During the years he studied for his degree, Mr. Heymann, by coaching students in Greek, Latin, and mathematics, earned his university fees. After his graduation in June, 1934, and until his departure for South Africa in October, 1936, he was associated with an engineering firm in Paris, France. Arriving in South Africa on October 31, 1936, he proceeded to Johannesburg, Union of South Africa, where he was employed by Spotkin and Newfield, designing reinforced concrete structures.

In March, 1937, he joined the Reinforcing Steel Company, of Johannesburg; and, after a month with this firm, he was transferred to the Durban, Natal, branch, as a structural engineer—estimating, designing, detailing, and supervising the construction of reinforced concrete structures, having full charge and reporting to the manager. During the three and one-half years he was associated with this firm—the boom period of building in Durban—he was responsible for the structural design and the supervision of the erection of many of the fine modern buildings in Durban.

From July to September, 1940, Mr. Heymann was associated with Messrs. Stott, Sullivan and Barbour, surveyors and architects of Durban, as assistant land surveyor. In September, 1940, he joined the Durban Corporation as an engineering assistant, and in this capacity he was engaged on a water system planned by the Durban Corporation. This project includes a gravity dam (storage capacity, 5,000,000,000 gal) with flood diversion control works and an aqueduct 27.7 miles long, consisting of 48-in. reinforced concrete pipe and 42-in. and 36-in. steel pipes in twenty-one tunnels 7.30 miles long. The system, when completed, will provide Durban with an additional 50,000,000 gal of water per day.

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[1] Memoir prepared by Mrs. Curt Joseph Heymann and H. A. Smith, Assoc. M. Am. Soc. C. E.

Under the direction of the resident engineer, Mr. Heymann completed the design of the flood control gates to the diversion canal. (These gates will divert all silt-laden flood waters from the main storage dam.) In addition, he supervised construction work in the early stages. There are six gates, 18 ft wide by 30 ft long, which will be operated electrically or manually. He also supervised the design of the multiple arch dam, an alternative to the gravity dam, and the design of the 48-in. diameter concrete pipe, a part of the aqueduct.

He was engaged on this work when he was stricken with heart collapse, in December, 1943. Mr. Heymann was confined to his bed, until his death. Thus, the career of a brilliant young man, who showed great promise in the engineering profession of South Africa, was brought to an untimely conclusion. At the end of December, 1943, after residing for the requisite five years in South Africa, he applied for citizenship and was accepted as a naturalized British subject.

On December 26, 1939, Mr. Heymann was married in Dundee, Natal, to Elizabeth Williamson, of the well-known pioneer mining family in that district, who survives him.

Always interested in professional affairs, in June, 1938, he was elected an associate member of the South African Society of Civil Engineers; in February, 1939, an Associate Member of the Institute of Structural Engineers (British); and, in 1942, a member of the British Society of Civil Engineers.

Apart from his great devotion to his work, Mr. Heymann was a brilliant classical scholar and musician.

Mr. Heymann was elected an Associate Member of the American Society of Civil Engineers on October 19, 1942.

JOHN BREWSTER HODGDON, Assoc. M. Am. Soc. C. E.[1]

DIED MARCH 16, 1935

John Brewster Hodgdon, the son of George Washington and Mary Jane (Hobbs) Hodgdon, was born on October 22, 1861, in Rochester, N. H. He received his early education in the Rochester High School, and was graduated with the degree of Bachelor of Science from Dartmouth College at Hanover, N. H., in 1885. As a student at Dartmouth he was a member of Vitruvian (later Beta Theta Pi fraternity), and was active in the outside activities of his class.

A year after graduation he was employed by Knight and Bontecou, of Kansas City, Mo., where he remained only until July, 1886, when he became transitman on location for the Wyandotte and Northwestern Railroad Company in the same city. In 1888 he surveyed and mapped about ten thousand acres of mineral and timber land in Wise County, Virginia; and in 1889 he worked on location for the Kansas City Link Railroad (never built).

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[1] Memoir prepared from information on file at the Headquarters of the Society.

The next ten years were spent in varying activities. He went to Joplin, Mo., in 1890, supervising construction of the Joplin Electric Railway. The following year he became assistant city engineer in charge of sewer construction. From 1891 to 1892 he worked on the location and the construction of the Southwest Missouri Electric Railway between Webb City and Prosperity, Mo. In January, 1892, he went to Galena, Kans., where he was engaged as a civil engineer on mining and surveying for five years. In 1896 he built the Joplin and Galena Electric Railway, and was chief engineer for the Southwest Missouri Electric Railway Company (later the Southwest Missouri Railroad Company). He then became assistant engineer for the Kansas City, Fort Scott and Memphis Railroad Company, with headquarters in Springfield, Mo., in charge of general inspection and repair work on all bridges from Springfield to Memphis, Tenn., with the exception of the Memphis Bridge. He returned to Joplin in 1899 as chief engineer for the Southwest Missouri Railroad Company.

Mr. Hodgdon became resident engineer for the Kansas City, Memphis and Birmingham Railroad Company, in 1901, with headquarters at Amory, Miss. In 1902 he made land surveys in Mississippi County, Arkansas, and government surveys into the margin of Big Lake (the survey embraced more than 12,000 acres). From 1903 to 1904 he was chief engineer of the Mineral Cities Railway Company, from Joplin to Pittsburg, Kans. From 1904 to 1907 he served as local assistant engineer of the Missouri, Kansas and Texas Railway Company, and as chief engineer of the Southwest Missouri Railroad Company; he also rebuilt tracks at the Galena yards for the Missouri, Kansas and Texas Railway Company.

In 1907 Mr. Hodgdon was appointed city engineer of Joplin, under the councilmanic form of government, and served in that capacity until 1911. In 1914 he became the first commissioner of streets under the commission form of government, resigning in 1918 to superintend the cutting of stone for the Denver Municipal Park Grounds, at Stone City, Colo.

As superintendent of the Baxter Springs, Kans., water works and sewer system from 1920 to 1923, he was instrumental in the installation of the two systems there. Returning to railroad engineering, from 1924 to 1925 he served as engineer for the Kansas, Oklahoma and Gulf Railroad Company.

The City of Joplin reappointed Mr. Hodgdon city engineer in 1930, which position he held until his death. With his wide engineering experience in city, railroad, electric railway, and land surveying; mining engineering; and bridge construction as a background, Mr. Hodgdon specialized during the last several years of his life in municipal street paving. Through experimentation he is responsible for pavement made out of Joplin "chats," a material made from the waste of the lead and zinc mines. This material has proved so satisfactory that a great business has been opened to the mine operators, and these chats have been shipped all over the United States for use in making concrete material.

On August 24, 1887, he was married to Lula I. Akin, of Olathe, Kans., who survives him, together with two sons, Otto Fox Hodgdon and Frank Ralph Hodgdon; a daughter, Ruth May (Mrs. Guy Mills); a brother, Frank Hodgdon; and seven grandchildren.

Mr. Hodgdon was a Thirty-Second Degree Mason and a member of the Scottish Rite and of Joplin Lodge No. 335, A. F. and A. M. He belonged to the Byers Avenue Methodist Church.

Mr. Hodgdon was elected an Associate Member of the American Society of Civil Engineers on April 3, 1907.

JOHN WARDWELL HOWARD, Assoc. M. Am. Soc. C. E.[1]

DIED JANUARY 25, 1944

John Wardwell Howard, the son of Charles Henry Howard and Susan (Wilson) Howard, was born in Providence, R. I., on March 20, 1879. During his boyhood the family moved to Boston, Mass., and made their home in the Savin Hill section of Dorchester where they lived for a great many years. Mr. Howard attended the Mechanic Arts High School in Boston from which he was graduated in 1899. He then entered the Massachusetts Institute of Technology (M.I.T.), at Boston, being graduated with the degree of Bachelor of Science in Civil Engineering in 1903.

Following his graduation he joined the staff of the Department of Civil and Sanitary Engineering at M.I.T., remaining for forty years. He was an assistant from 1903 to 1905, an instructor from 1905 to 1913, and an assistant professor from 1913 to 1922. He held the rank of associate professor of topographical engineering from 1922 until his death.

Prior to the establishment of the M.I.T. Surveying Camp in Maine in 1912, Professor Howard devoted his summers to outside professional work. His first position was that of rodman with the Pennsylvania Railroad Company from June to September, 1902, between his junior and senior years. During the summer of 1903, he was a member of the staff at the Surveying School held that summer at East Machias, Me. During August and September he served as a field assistant with the Commission on Additional Water Supply for the City of New York (N. Y.).

He worked on the Town Boundary Survey of Massachusetts during the six summers from 1905 to 1910. This project was conducted by the Massachusetts Harbor and Land Commissioners under the supervision of Henry B. Wood,[2] M. Am. Soc. C. E. The work consisted of actually finding all the town boundary corners or angle points and then locating them with reference to the U. S. Coast and Geodetic Survey triangulation system in Massachusetts so that complete town boundary atlases could be prepared. During the summers Professor Howard spent on these surveys, the work principally involved reconnaissance and triangulation in the western part of the state where the terrain was rough and wooded. Considerable skill and ingenuity were required to make satisfactory progress under these conditions. This summer employment gave him an unusually broad and practical experience which was helpful in his teaching.

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[1] Memoir prepared by John B. Babcock, 3d, M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 1155.

From January to October, 1912, he was an assistant engineer for the Costa Rica-Panama Boundary Commission. These countries had referred their boundary dispute to the Chief Justice of the United States Supreme Court and a commission of engineers had been appointed to supervise a survey of the territory involved. Frank W. Hodgdon,[3] M. Am. Soc. C. E., one of the engineers of this commission, was chief engineer of the Massachusetts Harbor and Land Commissioners. Since Mr. Hodgdon was familiar with Professor Howard's capable work on the Town Boundary Survey of Massachusetts, he selected Professor Howard to head one of the field parties. There were four field parties, each consisting of an assistant engineer, a transitman, and three rodmen. The group left New York early in January, 1912, for Bocas del Toro, Panama, on the Atlantic Coast. During the actual work in the field, each party had a cook and fifteen to twenty machete men. The first work was a topographical survey of the northerly watershed of the Sixaola River, principally by stadia methods. By June the survey had proceeded so far that the problem of supply became difficult. The parties returned to the coast and were transported by rail across Costa Rica to Puntarenas on the Pacific Coast. With a native Indian village as a base, the survey of the headwaters in the mountains was then completed. This stage occurred during the rainy season when it was possible to work only a few hours each morning. For this reason, and because of unfavorable topographic features, triangulation methods were used for locating this part of the divide. When the survey was finished, the parties returned to New York, arriving in October, 1912. Many hardships were endured in Costa Rica—snakes and malaria causing the chief difficulties. Although a number of the men were stricken with fever, there was but one fatality—a laborer who was bitten by a poisonous snake.

During World War I, in addition to his regular duties, Professor Howard served as an instructor in the U. S. Army School of Military Aeronautics at M.I.T. He was also in charge of a School of Navigation at Portland, Me., for the United States Shipping Board. After the war, he served as coordinator for students attending M.I.T. under the auspices of the Federal Board for Vocational Education, later consolidated in the Veterans' Bureau. His sincere interest in these returning men and his understanding of their problems made his services very valuable.

For nearly thirty years Professor Howard was a member of the staff at Camp Technology, the M.I.T. Summer Surveying Camp on Gardner Lake near East Machias. Although he was on the Costa Rica-Panama Boundary Survey when the camp was opened in 1912, he attended all but two sessions from 1913 to 1942. He was in charge of both administration and instruction from 1935 to 1942, after which the camp was temporarily suspended because of M.I.T.'s adoption of a continuous academic program for the duration of World War II.

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[3] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 1355.

From 1916 to 1922, students in mining engineering attended the camp for a five-week course in plane and topographic surveying before beginning underground surveying. Professor Howard was in charge of the program for the mining engineering students both at Camp Technology and at mines in Nova Scotia, Vermont, and New Jersey.

His influence on the students who attended the surveying camp was marked. He was recognized by them not only as an able teacher of surveying practice but as one who was always ready to cooperate with them in their activities. He was a sincere friend with whom they could discuss their personal problems. Professor Howard once said,

> "Aside from the many advantages of teaching practical surveying at the Camp during the summer, I have always considered the intimate contact between the staff and students to be of inestimable value to both. We know our men."

How well this ideal was realized is indicated by a quotation from the *Benchmark,* the annual publication by the camp students, in which it was stated,

> "It is an excellent place for work and play, and it offers to us what other courses do not:—the breaking down of the barrier between students and instructor."

In addition to teaching astronomy, geodesy, and plane and topographic surveying, Professor Howard was interested in aerial surveying and photogrammetry and acted as consultant to Army officers interested in this field.

Besides the Society, he belonged to the Boston Society of Civil Engineers, Society for the Promotion of Engineering Education, American Society of Photogrammetry, American Geophysical Union, American Congress on Surveying and Mapping, and Lambda Chi Alpha fraternity. He was a member of the Society's Surveying and Mapping Division and served on its Committee on Definition of Surveying Terms. He was at one time president of the Mechanic Arts High School Alumni Association.

In addition to his professional accomplishments, he was a man who was loved by all who knew him. He was ever ready to sacrifice his personal interest to help a student or a fellow staff member. His office door stood open to all, and within awaited a cordial and warm greeting.

On August 29, 1934, he was married to Alice Elizabeth Welch of Dedham, Mass. He is survived by his widow and by two sisters, Katherine and Mabel Howard.

Professor Howard was elected an Associate Member of the American Society of Civil Engineers on March 4, 1913.

HOWARD CHAPIN IVES, Assoc. M. Am. Soc. C. E.[1]

Died October 6, 1944

Howard Chapin Ives, the son of Howard C. and Julia (Dunham) Ives, was born on March 20, 1878, at Cheshire, Conn. He was graduated from the Sheffield Scientific School, Yale University, at New Haven, Conn., in 1898. He received the degree of Civil Engineer in 1900.

After being employed by the Pennsylvania Railroad Company on its lines west of Pittsburgh, Pa., he joined the staff of the Worcester Polytechnic Institute (W.P.I.) in Worcester, Mass., in 1900 as an instructor in civil engineering. Three years later he went to the University of Pennsylvania in Philadelphia as assistant professor of civil engineering, and then returned to W. P. I. in 1906 in a similar capacity. He was promoted to professor of railroad engineering in 1912. He resigned from this position in 1924. In 1902 Professor Ives was an instructor in the first W. P. I. summer surveying camp at Chaffin, Mass., organized by Arthur W. French, M. Am. Soc. C. E., and after 1906 he was in charge of the annual session.

In collaboration with Professor French, he prepared a textbook on "Stereotomy."[2] Later, with H. E. Hilts, Assoc. M. Am. Soc. C. E., he wrote "Problems in Surveying, Railroad Surveying and Geodesy."[3] He also revised and edited "Field Engineering"[4] by William H. Searles,[5] M. Am. Soc. C. E., and compiled a surveying manual[6] and several books of mathematical and surveyors' tables. With H. R. Saunders he published a textbook on "Map Drafting and Lettering."[7]

In 1908 he was married to Mary Belle Young of Hartford, Conn. Professor Ives is survived by his widow and a brother, Harvey Ives.

After resigning from the faculty of Worcester Polytechnic Institute, Professor Ives moved to California where for a time he was employed in the office of the city engineer of Los Angeles, Calif. He subsequently resided in Laguna Beach, Calif., occupied with revisions of his books. He returned to Waterbury, Conn., in June, 1944.

Professor Ives was a member of the Society for the Promotion of Engineering Education, the National Institute of Social Sciences, and Sigma Xi. He was a Mason and a member of the Independent Order of Odd Fellows.

Professor Ives was elected a Junior of the American Society of Civil Engineers on December 3, 1901, and an Associate Member on April 5, 1910.

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[1] Memoir prepared by Jerome W. Howe, M. Am. Soc. C. E.

[2] "Stereotomy," by Arthur W. French and Howard C. Ives, 1st Ed., John Wiley & Sons, Inc., New York, N. Y., 1902.

[3] "Problems in Surveying, Railroad Surveying and Geodesy," by Howard C. Ives and Harold Ezra Hilts, 1st Ed., John Wiley & Sons, Inc., New York, N. Y., 1906.

[4] "Field Engineering," by W. H. Searles, 21st Ed., revised and enlarged by H. C. Ives, John Wiley & Sons, Inc., New York, N. Y., 1936.

[5] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1720.

[6] "Surveying Manual Designed for the Use of First-Year Students in Surveying, and Especially for the Use of Non-Civil Engineering Students," by Howard C. Ives, 2d Ed., John Wiley & Sons, Inc., New York, N. Y., 1931.

[7] "Map Drafting and Lettering," by Howard R. Saunders and Howard C. Ives, Benson Book Co., Inc., Los Angeles, Calif., 1931.

EDWARD RESOLVED MACK, Assoc. M. Am. Soc. C. E.[1]

Died March 21, 1944

Edward Resolved Mack was born in Hardwick, Vt., on November 19, 1876, the son of Henry Resolved and Lois (Williams) Mack. He received his preparatory schooling at Hardwick Academy, and was graduated in 1898 from the University of Vermont at Burlington, with the degree of Bachelor of Science in Mechanical Engineering.

Following his graduation, Mr. Mack was employed for a year as draftsman by the Cramp Shipbuilding Company of Philadelphia, Pa. In August, 1899, he entered the Transport Service of the U. S. Quartermaster Department where he remained until May, 1901. During this period he was a member of the engineering staff of the U. S. Army Transport *Thomas,* plying between San Francisco, Calif., and the Philippine Islands. In 1902 he was employed as draftsman by Moran Brothers Shipbuilding Company in Seattle, Wash., on the construction of the battleship, *U. S. S. Nebraska.*

Returning east in July, 1902, Mr. Mack had short engagements as draftsman with the Woodbury Granite Company of Hardwick, and with the New York Shipbuilding Company of Camden, N. J. On January 1, 1904, he entered the employ of the Water Department of the City of Wilmington, Del., in charge of surveys for a water filtration plant and a concrete conduit line. When this work was completed in September, 1906, he was appointed park engineer and secretary of the Board of Park Commissioners of the City of Wilmington. He continued in this position until his death.

Edward Mack brought to his work a great love of nature and its beauties, together with a consuming interest in the young and underprivileged. The coordination of the esthetic and the recreational phases of park and playground systems with the development and benefit of the youth of the city was his high ideal and one he persistently and steadfastly followed. During the thirty-eight years that Mr. Mack served as park engineer for the City of Wilmington, the park and playground systems were greatly enlarged and modernized by the addition of swimming pools, playgrounds, athletic fields, trails, and picnic sites.

He made several trips abroad and had visited and studied many of the park systems of Europe, Canada, and the United States.

Mr. Mack was the first scoutmaster in Delaware, organizing the initial Boy Scout troop in 1910. He retained his interest in scouting until his death. In 1940 he was awarded the Silver Beaver "for outstanding service to boyhood" by the Del-Mar-Va Council, Boy Scouts of America. He was a charter member of the Social Service Club and a member of the Rotary Club and Grace Methodist Church. He also belonged to the American Association of Park Executives and the Regional Planning Commission of New Castle County. From 1928 to 1932, he served on the Technical Advisory

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[1] Memoir prepared by W. W. Mack, Chf. Engr., State Highway Dept., Dover, Del.

Committee of the Regional Planning Federation of the Philadelphia Tri-State District.

On September 2, 1914, Mr. Mack was married to Ruth Kellogg of Columbus, Ohio, who survives him.

Mr. Mack was elected an Associate Member of the American Society of Civil Engineers on June 7, 1905.

JAMES ALEXANDER McFADDEN, Assoc. M. Am. Soc. C. E.[1]

Died October 22, 1944

James Alexander McFadden, the son of Dr. John James and Elizabeth (Williams) McFadden, was born in Brandon, Man., Canada, on November 5, 1900. He was descended from a family who for generations had lived in Canada. His grandfather, Moses McFadden, was one of the Dominion land surveyors and engineers who laid out the roads of Manitoba.

After a preparatory education in the schools of his home town, he entered the University of Manitoba at Winnipeg, Man., Canada, where he completed two years of liberal arts followed by a four-year course in civil engineering, being graduated in 1925 with the degree of Bachelor of Science in Civil Engineering. When the university reopened in the autumn of 1925, he became instructor in civil engineering, and, in addition to teaching, he did postgraduate work in hydraulics.

Eager to be "in the field," Mr. McFadden left the university at the end of the 1926 semester and obtained employment in Chicago, Ill., with Consoer, Older, and Quinlan, municipal engineering consultants. He was with this firm for nearly four years, engaged principally on paving, sewer, and watermain construction projects. Starting as inspector, he advanced from assistant resident engineer to resident engineer, and finally to designer.

In 1929 he went with the Commonwealth Edison Company, where, as structural engineer, he worked for more than two years on designs related to steam power stations. He then was appointed civil engineer for both the United States Forest Service and the National Park Service at Ames, Iowa, on a program of park improvement projects. During this time he designed small dams, bridges, sewers, and water-supply systems. In January, 1936, he transferred to the Public Works Administration in the Des Moines (Iowa) area as assistant resident engineer on sewer and water supply projects.

Mr. McFadden went to Knoxville, Tenn., in December, 1936, to work for the Tennessee Valley Authority (TVA), bringing with him a strong enthusiasm for the kind of engineering that was being undertaken by that organization. As an assistant structural engineer in the Construction Plant Division, he was engaged on many phases of design connected with the construction of TVA dams and power plants, including the Hiwassee Dam cableway and the suspension bridge on the Watts Bar project. In April, 1939, he was promoted to associate structural engineer and was placed in charge of a design squad. In July, 1941, Mr. McFadden was transferred to the Civil Design Division, which was expanding rapidly because of increasing war activities. Here he continued as a squad supervisor. When, in the summer of 1942, the position of administrative assistant to the head of the Civil Design Division became vacant, Mr. McFadden was the logical choice because he already demonstrated a marked ability to handle personnel and production work for a large group of engineers. In line with his new responsibilities, he was promoted to civil engineer, and, from the time he assumed his new duties until his forced retirement shortly before his death, he was deeply concerned with the TVA emergency program and its continuously changing problems involving procurement and high-pressure production schedules. All this he managed in his efficient manner, and his unfailing sense of humor helped enormously to boost the efforts and morale of his co-workers. It was a vexatious problem indeed that could take the ready smile from his face or eliminate the quizzical sparkle from his eye. He had a habit of standing at the door of his office in the morning or just after lunch, cigarette in the hand raised against the door frame. Just to see him standing there relaxed, with his pleasant greeting ready for everybody, was a good beginning for the day. Any one approaching him, whether for information or to pass a moment in discussion, was struck by his essential gentility and kindness. At the same time, his almost-red hair indicated determination and mental courage.

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[1] Memoir prepared by P. J. Gordon, Associate Structural Engr., TVA, Knoxville, Tenn., and B. H. Arneberg and Adolf A. Meyer, Members, Am. Soc. C. E.

Mr. McFadden was married on June 30, 1928, to Ingrid Haerem. He is survived by his widow; three children, John Alf, Barbara, and Elizabeth Ann McFadden; his mother, Mrs. J. J. McFadden; two sisters, Eleanor Gertrude McFadden and Margaret Elizabeth (Mrs. J. H. Grahame); and two brothers, Hamilton Saunders McFadden and Jack N. McFadden.

The last years of Mr. McFadden's life were spent in Norris, Tenn., where he took an active part in community affairs. He was a member of the board for the Norris Cooperative and a member and treasurer of the Norris Forum Committee, in which capacity he aided greatly in arousing public interest in postwar problems. For some time he also served on the executive committee of the Norris Council, Boy Scouts of America. He held the license of professional engineer in the State of Iowa.

Mr. McFadden was elected an Associate Member of the American Society of Civil Engineers on July 11, 1938.

JOHN FRANCIS MINIHAN, Assoc. M. Am. Soc. C. E.[1]

Died August 14, 1944

John Francis Minihan, the son of Daniel and Margaret (Connolly) Minihan, was born in San Francisco, Calif., on May 8, 1896. He attended Mission

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[1] Memoir prepared by M. B. McGowan, Jr., Assoc. M. Am. Soc. C. E.

High School in San Francisco and then studied civil engineering at the University of California at Berkeley and was graduated in May, 1918, with the degree of Bachelor of Science in Civil Engineering.

He began his professional career on the design and detail of steel structures for the Judson-Pacific Company, under E. O. Burgess, and then proceeded to other professional assignments as follows:

1920–1922—Engineer of design and detail of high-tension electric transmission towers for the Pacific Coast Steel Company of San Francisco.

1922–1927—Estimator and office engineer on building construction of all types for Cahill Brothers, Inc.

1927–1932—Estimator, office engineer, and assistant superintendent on building construction for J. A. Bryant, Inc.

1932–1937—Estimator and office engineer on bridges, pile foundations, wharves, and cofferdam construction for M. B. McGowan, Inc.

1937–1939—Chief estimator for the Golden Gate International Exposition.

1939–1940—Vice president of the Lane-Minihan Construction Company.

1941–1942—Office engineer in charge of all field operations for construction of piers and buildings in ammunition area at the Mare Island Navy Yard, California, for Cahill-Gerwick Company.

1942–1943—Engineer in charge of estimates, estimates for payment, and plant history of methods on 5,000 prefabricated housing units (Federal Housing Project) at Norfolk, Va., for Barrett and Hilp, Inc.

1943–1944—Chief inspector for the Federal Housing Administration at the $3,000,000 Ridge Point project (2,000 dwelling units including utilities) at Hunters Point, San Francisco.

At the time of his death Mr. Minihan was checking material lists prepared by J. H. Pomeroy for the U. S. Navy, and was considering opening an estimating and quantity survey office to serve the construction industry.

As a specialist in the most difficult quantity survey work and heavy construction pricing, Mr. Minihan had few equals; and his own exhaustive job cost records, from actual experience, could well have made him famous had he chosen to publish rather than to apply them. He held great hopes for the future of prefabrication. Complete works on prefabrication history and construction methods were in his extensive historical and engineering library. He was identified during his career with some of the great engineering achievements of all time, such as the San Francisco-Oakland Bay Bridge, the Golden Gate Bridge, and the Golden Gate International Exposition. He left a record that few men in his field have been able to attain.

John Francis Minihan was well known to, and beloved by, many members of engineering and construction firms in the San Francisco Bay area. His cheery disposition and his keen wit made him one of the most intelligent and lovable characters in the California construction industry.

On May 7, 1932, in San Francisco, he was married to Anne Cooney. Mr. Minihan is survived by his widow and two sisters, Agnes (Mrs. C. A. Young, Jr.) and Helen (Mrs. J. P. Cassidy).

His death was deeply felt by the many engineers and contractors with whom he had so long been friendly, and all of them will join in wishing him Godspeed to the Valhalla of good engineers. His wit and his charm will live long in the hearts of his many friends.

Mr. Minihan was elected an Associate Member of the American Society of Civil Engineers on August 29, 1927.

WILLIAM JOHNSTON MITCHELL, Assoc. M. Am. Soc. C. E.[1]

DIED MAY 19, 1943

William Johnston Mitchell, the son of David and Mary (Johnston) Mitchell, was born at New Bedford, Mass., on November 2, 1890. He received his preliminary education in Fairhaven High School, Fairhaven, Mass. In 1913 he was graduated from the University of Maine at Orono with the degree of Bachelor of Science in Civil Engineering. When at college, he was elected to the honorary engineering society, Tau Beta Pi.

After graduation, Mr. Mitchell obtained a position with the American Bridge Company as structural detailer at the Brooklyn (N. Y.) plant. In February, 1915, he was detailer and designer of reinforced concrete structures for the Trussed Concrete Steel Company of New York, N. Y. Six months later he was employed by Felnheimer and Long, Architects of New York City, to assist in the design of railroad structures.

In October, 1915, Mr. Mitchell became a checker of structural steel details with Gunvald Aus Company of New York, continuing until June, 1916. At this time he transferred to the Leetsdale (Pa.) plant of the Riter-Conley Manufacturing Company of Pittsburgh, Pa., where he remained until he enrolled for service in World War I.

Mr. Mitchell enlisted in the U. S. Army on May 17, 1918, at Washington, D. C., as a private in the 29th Engineers. After a few months of training, he sailed from Hoboken N. J., on July 8, 1918, on the steamer *Toba*. On July 19 he arrived in France, landing at Brest, then being sent to Fort de Saint Mange, where he served as instructor in flash ranging until December 26. He sailed from Saint-Nazaire, France, on February 28, 1919, and arrived at Newport News, Va., on March 11, 1919. He was discharged from Camp Dix (New Jersey) on March 24, 1919.

Mr. Mitchell resumed his civilian life as designer and estimator for the Berlin Construction Company of Berlin, Conn. In September, 1922, he accepted a position as structural designer with the Mining Department of the Bethlehem Steel Corporation in Bethlehem, Pa., specializing in the design of wood, steel, and concrete structures for the mines and quarries of the Bethlehem Steel Corporation. In February, 1932, drastic curtailment of work caused by the economic depression gave Mr. Mitchell an opportunity to retire and supervise the real estate properties his father had left when he died. At this time Mr. Mitchell (who never was married) lived with his mother. In 1934 they decided to spend a winter in Florida, liking it so well that from then on they spent their summers in Fairhaven and their winters in Saint Petersburg, Fla.

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[1] Memoir prepared by E. C. Waite, Assoc. M. Am. Soc. C. E.

The outbreak of World War II, with the resultant vast industrial program, led Mr. Mitchell to decide to help by taking a job at the Bushnell Lyons Iron Works, at Tampa, Fla. This organization did considerable work on the construction of phosphate plants but finally specialized in landing barges.

Because of the pressure of work Mr. Mitchell did not return to Massachusetts in 1942 but continued with the Iron Works until he became ill in February, 1943. He was admitted to the hospital in Tampa and was later transferred to the Bay Pines (Fla.) Hospital, where he died on May 19, 1943. A military funeral was held on May 20, and he was buried in the Bay Pines Cemetery.

An active member of the American Legion, Mr. Mitchell also belonged to La Société des 40 Hommes et 8 Chevaux.

Mr. Mitchell was elected an Associate Member of the American Society of Civil Engineers on May 25, 1931.

RUSSELL MELVIN OBERT, Assoc. M. Am. Soc. C. E.[1]

Died June 5, 1943

Russell Melvin Obert was born at Columbus, Ohio, on November 5, 1901, the son of William and Emma (Reichert) Obert. His maternal grandfather and paternal grandparents were natives of Germany. Educated in the public schools of Columbus, he entered Ohio State University, also in Columbus, in September, 1920, and was graduated with the degree of Bachelor of Civil Engineering in June, 1924.

From June, 1924, to August, 1925, he was engaged as transitman with the Bureau of Water Works Extension of the City of Columbus, in connection with the construction of the O'Shaughnessy Dam. From August, 1925, to October, 1926, he was with the Pure Oil Company, as draftsman and, for the latter part of the engagement, as designer in charge of concrete and structural work at Chicago, Ill. From November, 1926, to December, 1927, he was employed by the International Derrick and Equipment Company of Columbus as a structural draftsman, superintending construction of shop building extensions and serving as engineering representative on office building construction.

In January, 1928, he became an assistant engineer for The Columbia Engineering Corporation at Columbus and until his death served this organization in various capacities, being principally engaged in valuation and appraisal work for various companies held by the corporation in Ohio, Pennsylvania, West Virginia, and Kentucky. A fitting eulogy to Mr. Obert was expressed by an associate at the time of Mr. Obert's death in the *Broadcaster*, a company publication from which the following is taken:

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[1] Memoir prepared by John A. Rousculp, Assoc. M. Am. Soc. C. E.

> "Russell (Russ) Obert passed from our midst on June 5th, but is not forgotten. Russ was the victim of a heat stroke, while working in his victory garden, and never regained consciousness. With his untimely death there comes a distinct sense of loss to his family, community, and our company, because he was always eager to do more than his part, going out of his way, voluntarily, to do little favors for others. A hard-working individual, who was not satisfied with anything but his best performance on any assignment.
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> "Russ was employed by The Columbia Engineering Corporation on January 16, 1928, as an Assistant Engineer. Later he served as Senior Engineer in charge of material pricing, inventories and appraisals. Among other activities he was President of the Employees' Credit Union in 1942 and 1943, and also served as a Gasco Club Officer and Merchandise Sales Chairman. He was a member of the Quarterback Club, a Red Bird baseball team booster, and a flower and garden enthusiast."

In 1923 he was married to Lola Mildred O'Brien, of Columbus. He is survived by his widow, his parents, and a brother, Floyd H.

He was a lifelong member of St. Johns Evangelical Church of Columbus.

Mr. Obert was elected a Junior of the American Society of Civil Engineers on April 12, 1926, and an Associate Member on November 26, 1934.

WILLIAM SCHUYLER POST, Assoc. M. Am. Soc. C. E.[1]

Died January 24, 1945

William Schuyler Post was born on August 21, 1871, in Vienna, Austria. He was the son of Gen. Philip Sidney Post and Cornelia (Almira) Post. His father had served with distinction in the Civil War and was, at the time of William Post's birth, United States Consul General in Austria. General Post's tour of duty as Consul ended in 1876, at which time the family returned to the United States and settled in Galesburg, Ill., where young Post spent his boyhood. He attended the public schools and later entered Knox College in Galesburg, being graduated at the age of seventeen. He then spent two years at the Corcoran Scientific School in Washington, D. C., and one year at Johns Hopkins University in Baltimore, Md., studying electrical engineering.

After graduation—during 1889 and 1891 and 1893 and 1897—Mr. Post served as chief of party on topographic surveys for the U. S. Geological Survey. This work included primary triangulation in Wyoming and South Dakota. In 1898 he was topographer for the same department on the exploration of southwestern Alaska. In 1900, after a short period as engineer for the Port

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[1] Memoir prepared by Harry Day Chapman, Assoc. M. Am. Soc. C. E.

America Railway Company in Puerto Rico, Mr. Post moved to Los Angeles, Calif.

During World War I, Mr. Post served as Major in the 316th Engineers and later on the Regimental Staff.

Although Mr. Post's engineering activities covered a large and varied field, he specialized in hydraulics. He served as chief engineer for the Guyamaca Mutual Water Company on reconstruction, flumes, concrete siphons, and appraisements, and he appeared before the California Railroad Commission on company affairs. He was chief engineer of the Vulcan Light and Water Company and the San Dieguito Mutual Water Company (later combined in the Lake Henshaw System for the supply of water to San Diego, Calif.). He also served as engineer in charge of hydraulic investigations and water installations in Kings County and in Whittier, Calif., under J. B. Lippincott,[2] Hon. M. Am. Soc. C. E.

Mr. Post did much work for the State of California. From 1921 to 1923, as hydraulic engineer, he was one of the three principal assistants to the deputy state engineer on water resources investigation, in charge of reservoir studies and dam estimates. From 1927 to 1929, under the same title, he made and was in charge of "The Santa Ana River Investigation" and was the author of a report on the flood control and conservation of water of the Santa Ana River.[3]

During 1929 and 1930 Mr. Post was resident engineer in charge of the construction of the Hogan Dam, built on the Calaveras River for flood control purposes by Stockton, Calif. He made reconnaissance and hydrographic surveys for this project; and, in association with Frederick H. Tibbetts,[4] M. Am. Soc. C. E., consultant of San Francisco, Calif., was joint designer of the dam, constructed at a cost of $1,000,000.

In 1931 Mr. Post, having passed the civil service examination for senior civil engineer (hydraulics) as No. 2 eligible man, was appointed director of irrigation, U. S. Indian Service, in charge of twenty-six projects. Later he became supervising engineer at large. He left the U. S. Indian Service in 1934 and entered the U. S. Park Service, with headquarters at Death Valley, Calif.

In 1935 Mr. Post was employed by the Public Works Administration. He acted as resident engineer inspector during the raising of the Hetch Hetchy Dam for the City of San Francisco and later on the water supply project at Tilden Park, for the cities of Oakland and Berkeley, Calif. In 1940 he again entered federal service as hydraulic engineer, U. S. Engineer Department, in Los Angeles, remaining in that position until he was retired. After his retirement, Mr. Post was employed by the Pacific Naval Air Bases—a combination of contractors for constructing and handling air bases on the Pacific coast—at Ventura, Calif. When he died, he was in charge of the library for that outfit.

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 1543.

[3] *Bulletin No. 19*, Div. of Eng. and Irrig., State of California, Sacramento, 1929.

[4] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 105 (1940), p. 1924.

William Schuyler Post was a great reader and student. He knew his work from beginning to end. Since he had a wonderful memory and a great knowledge and understanding of general and current affairs, he was interested in public matters and did not hesitate to transmit his views and comments to his Senator or Representative or to county and town officials. His was the courtesy of the old school—with the "saving grace of humor." He was a fine "Chief"—capable, kind, and considerate of his subordinates; a fine husband and father; and a good citizen and a loyal friend.

In 1900, in Los Angeles, Mr. Post was married to Estelle C. Johnson, whom he had met when he lived at Galesburg. He is survived by his widow and two children, William Schuyler Post, Jr. (Commander, U. S. Navy Submarine Service), and Carolyn (Mrs. Evan E. Fickling).

He was a member of the Los Angeles Section of the Society for many years before his death.

Mr. Post was elected an Associate Member of the American Society of Civil Engineers on May 1, 1901.

MARION deKALB SMITH, JR., Assoc. M. Am. Soc. C. E.[1]

Died January 20, 1944

Marion deKalb Smith, Jr., was born in Chestertown, Kent County, Md., on July 30, 1879, the son of Marion deKalb Smith and Addie (Burchinal) Smith. His primary education was received in the public schools of Chestertown, and, after being graduated from the local public school, he entered Washington College in Chestertown. He was graduated from this college with the degree of Bachelor of Arts in 1898, after which he entered Cornell University at Ithaca, N. Y., being graduated with the degree of Civil Engineer in 1901.

Immediately after graduation from Cornell, he took a position with the Pennsylvania Railroad Company, and was with this railroad until 1924. Following the death of Mr. Smith's parents, he resigned from the Pennsylvania Railroad Company. Because there were only two surviving members of Mr. Smith's family—himself and his sister, neither of whom ever married—it seemed advisable for him to return to the old family homestead in Chestertown. Thus, in 1924, Mr. Smith began to participate actively in the affairs not only of the City of Chestertown, but also of Kent County. There was no necessary public or civic enterprise of any size or value to the community which was not willingly undertaken by him without compensation.

The Maryland State Roads Commission called upon Mr. Smith for expert advice on the future development of roads not only in his own county, but throughout the state, and several schemes for the development of through highways were presented by Mr. Smith during this period. The citizens of Chestertown sought his advice and counsel in organizing and building the Chester

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[1] Memoir prepared by Henry Powell Hopkins, Architect, Baltimore, Md.

River Yacht and Country Club. He was a charter member of this organization and remained, until his death, a close adviser and participator in its every activity.

Long a champion of school improvement in Kent County, Mr. Smith gave unstintingly of his personal knowledge and his time to improve the school system not only in the City of Chestertown, but also in Kent County, and it was unfortunate that he did not live sufficiently long to see his efforts bear fruit. As an active member of the Kent County Historical Society, he was instrumental in having a monument erected on the public square in Chestertown to commemorate the three hundredth anniversary of Kent County.

Mr. Smith was one of the founders of the Rotary Club of Chestertown and until his death held a 100% record for attendance as a member of this organization. He attended all the district conferences and was sent as a delegate from his club to the Rotary International in Europe in 1927.

Washington College, his early alma mater, Cornell University, his second alma mater, and last, his home community, the City of Chestertown in Kent County, lost, upon the death of Mr. Smith, a valuable public-spirited citizen.

Mr. Smith was elected a Junior of the American Society of Civil Engineers on October 1, 1901, and an Associate Member on September 5, 1911.

GEORGE PHILIP ALEXANDER STAPE, Assoc. M. Am. Soc. C. E.[1]

Died December 3, 1943

George Philip Alexander Stape was born in Rochester, N. Y., on August 2, 1887, the son of Amelia (Nagle) and Philip Stape.

Mr. Stape was graduated from East High School in Rochester and in 1909 from the Rochester Mechanics Institute. He then attended the University of Michigan in Ann Arbor and was graduated with the degree of Bachelor of Science in Civil Engineering in 1914. In preparatory school and in college, he was interested in all sports, but wrestling, football, and baseball were his favorites. At Michigan, he was class champion in wrestling in his weight class and played baseball and football on class teams. While attending school, he worked summers for the construction department of the Rochester Gas and Electric Company as a surveyor on the construction of the Central Avenue dam and the rock tunnel through the Genesee River gorge in Rochester.

After graduation, Mr. Stape's early professional work was with the automobile and aircraft industries. From 1914 to 1916, he worked as maintenance engineer on factory maintenance for the Cadillac Motor Car Company in Detroit, Mich. In 1916 and 1917, he was engineer in charge of new buildings, maintenance, and factory equipment for the Saxon Motor Car Corporation in Detroit. From 1917 to 1920, he served as planning engineer on equipment, tools, building construction, and plant maintenance for the Dayton Wright Airplane Company of Dayton, Ohio.

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[1] Memoir prepared by Harry W. Eustance, M. Am. Soc. C. E.

In 1920 he returned to Rochester, his home town, and entered the employ of the Eastman Kodak Company. He remained with this company for twenty-four years until his death. Most of his work was for the Engineering and Maintenance Department of Kodak Park, designing and supervising the construction of buildings and equipment specially devised for the manufacture of photographic film and paper.

Mr. Stape was an excellent associate and his unassuming manner matched the daily evidence of his quiet competence. He had few intimate friends, but those who had the opportunity to know him well valued his friendship and respected him most highly.

To dwell on Mr. Stape's professional work is to neglect what was, to him, the one important task he must do well. Only those who knew him best were aware of the depth of his devotion and the scope of his constant service to his wife, his mother, and his younger sister.

In 1914 Mr. Stape was married to Mary Bernadette Christner of Rochester. After only one year of normal happily married life, Mrs. Stape contracted tuberculosis and lived for ten years as an invalid. Mr. Stape willingly gave up most outside interests and activities to spend as much time as possible in an attempt to help his wife regain her health. Mr. and Mrs. Stape spent much time in the Adirondack Mountains and in the Finger Lake region of New York State. Mr. Stape willingly suspended his professional career to make all these trips possible.

After the death of his wife in 1924, Mr. Stape lived with his mother, remaining with her until her death in 1940. She also was an invalid for several years and was devotedly cared for by her son who was her constant attendant. After his mother's death, Mr. Stape made his home with his sister, Bertha L. Stape, in Rochester. He was devoted to his family and in turn dearly beloved by them. In addition to his sister, Bertha, Mr. Stape is survived by three other sisters, Helen (Mrs. E. B. Gottschalk), Amelia M. (Mrs. August Heim), and Charlotte (Mrs. E. A. Pappert); and two brothers, Harry J. and Howard A.

Mr. Stape was elected an Associate Member of the American Society of Civil Engineers on April 14, 1919.

CHARLES AUGUST THANHEISER, Assoc. M. Am. Soc. C. E.[1]

Died February 8, 1944

Charlie Thanheiser, as he was known to all his friends in this section of the United States, was born in Fayetteville, Tex., on November 3, 1881.

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[1] Memoir prepared by Charles R. Haile and W. W. Washburn, Members, Am. Soc. C. E.

He attended the public schools in Fayette County, and continued his education at the Agricultural and Mechanical College of Texas in College Station, where he was a distinguished student and completed the four-year course in three years. He was graduated in June, 1901, with the degree of Bachelor of Science in Civil Engineering.

Upon graduation from college, Charlie Thanheiser immediately went to work for the Southern Pacific Lines of Texas as rodman, and continued until November, 1912, advancing to instrumentman, locating engineer, roadmaster, resident engineer, and division superintendent. He severed his connection with the Southern Pacific Lines of Texas in November, 1912, to accept a position with the Missouri, Kansas and Texas Railway Company of Texas, and served this company until 1916, as chief engineer, maintenance of way, and division superintendent.

From July, 1916, to October, 1922, he was consulting engineer for the Youngstown Sheet and Tube Company and Brier Hill Steel Company of Youngstown, Ohio. He was consultant on the construction of the large expansion program during World War I for these companies and on the construction of the necessary railroad and dock terminals.

In December, 1922, he organized the Southwestern Construction Company of Houston, Tex., and remained as president and principal owner until his retirement in 1932. This company, under his ownership and management, constructed many of the large buildings in the Gulf Coast area of Texas, some of which are: The Plaza Hotel, the City of Houston Public Library, and St. Paul's Methodist Church, in Houston; and the La Salle Hotel, in Beaumont.

From 1932 until his death, Mr. Thanheiser owned considerable property in Houston and also a Brazos River bottom farm, to which he devoted his time.

In addition to serving his own company, he contributed much to the development of the construction industry in south Texas. For seven terms he was president of the Houston Builders Exchange. Throughout his entire career, Charlie Thanheiser took an active part in civic affairs, serving on many committees of the Houston Chamber of Commerce, and until shortly before his death he was chairman of the athletic committee of the Chamber of Commerce. He loved sports of all kinds and took an active part in the direction of sports programs in southern Texas.

Charlie Thanheiser was known and loved by every man who ever attended Texas Agricultural and Mechanical College. He gave generously and was liberal in any movement that was to the benefit and betterment of the college. He served the Ex-Students Association of the college as state president, and several times was president of the Houston Agricultural and Mechanical Club. For a number of years, Charlie Thanheiser was the representative of the Ex-Students Association on the athletic committee of the Agricultural and Mechanical College of Texas. It was through his advice and counsel that a program was arranged which cleared the athletic plant and stadium of the well-known Texas Aggies of all indebtedness.

He was a member of the Texas Section of the Society, the Houston Club, and the Trinity Episcopal Church. He was a Thirty-Second Degree Mason and a Shriner.

In the death of Charlie Thanheiser, Houston lost one of its most respected engineers and constructers and an active participant in civic affairs.

Mr. Thanheiser was married to Adele Lindemann on January 7, 1920. He is survived by his widow and two sons, Charles A. Thanheiser, Jr. (Second Lieutenant, Coast Artillery, U. S. Army), and Will Hugh Thanheiser (a midshipman at Northwestern University in Evanston, Ill.).

Mr. Thanheiser was elected an Associate Member of the American Society of Civil Engineers on July 10, 1907.

JOSEPH JEANES WALKER, Assoc. M. Am. Soc. C. E.[1]

DIED JULY 26, 1944

Joseph Jeanes Walker was born in New Centerville, near Philadelphia, Pa., on November 4, 1871. He was the son of Joseph Robinson and Ellen Wells Walker. His family was one of the old Quaker families of Pennsylvania, and he was the eighth Joseph Walker, in direct descent, to be born in the home built by his original ancestor, Lewis Walker, on land granted by William Penn.

Mr. Walker attended the Friends' Central School in Philadelphia and Swarthmore College in Swarthmore, Pa., from which he was graduated in 1892 with the degree of Civil Engineer. He belonged to Delta Upsilon fraternity and was elected a member of the honorary scientific society of Sigma Xi.

He began his active engineeering career as a rodman in the engineering corps of the Pennsylvania Railroad Company, under W. L. Ziegler, inspecting masonry and brickwork during the construction of the Broad Street Train Shed. In 1894 he entered the employ of the Shiffler Bridge Company of Pittsburgh, Pa., first as a detail draftsman and later as a designer and estimator. In 1896 he was transferred to the Philadelphia office of the company and placed in charge of designing, estimating, and contracting. In 1897 he was transferred to the New York (N. Y.) office as assistant engineer in charge of designing, estimating, and contracting.

In 1900 he joined the newly formed American Bridge Company (a subsidiary of the United States Steel Corporation) and was assigned to the Structural Engineering Department at the East Berlin (Conn.) office as assistant engineer on cost estimates for mill buildings and on miscellaneous structural work. In 1901 he was transferred to the New York office and placed in charge of cost estimates for all classes of bridge and structural steelwork. Among some of the steel structures for which Mr. Walker supervised the cost estimates were New York subways, New York Terminal of the Pennsylvania Railroad Company, Hudson Terminal Building, Rockefeller Center buildings, Equitable Life Insurance Building, New York Life Insurance Building, Empire State Building, the Trylon and Perisphere of the New York World's Fair, and the Bronx-Whitestone Bridge.

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[1] Memoir prepared by J. E. Wadsworth, M. Am. Soc. C. E.; Warren D. Benedict, Dobbs Ferry, N. Y.; and Charles W. Geoghegan, Asst. Engr., American Bridge Co., New York, N. Y.

In 1938, upon reaching the retirement age for employees of the United States Steel Corporation, Mr. Walker opened an office in New York City as consulting engineer. He was engaged as a consultant on the 60,000-ton steel contract for the Parkchester Apartments, built by the Metropolitan Life Insurance Company in the Borough of the Bronx in New York City.

Mr. Walker was a resident of Dobbs Ferry, N. Y., and always took an active interest in civic affairs. In 1905 he was elected president of the Dobbs Ferry Chamber of Commerce. From 1907 to 1933 he was a member of the Board of Trustees of the village. In 1933 he was elected Mayor of Dobbs Ferry. Every two years since 1933 he was reelected and, when he died, he was serving his sixth term. For the last two terms he had no opposition for election.

His engineering experience was a great asset in his work as Trustee and Mayor of the Village of Dobbs Ferry. In 1938 he designed and supervised the construction of a retaining wall, costing $24,000, along the tracks of the New York Central Railroad in Dobbs Ferry. He designed and secured the approval of two Works Project Administration (W.P.A.) projects for paving and street work—one costing $40,000 and the other, $10,000. In 1941 he designed and supervised the construction of a $20,000 sewer system for the village.

Mr. Walker was a member of the American Bridge Engineering Club of New York (one of the organizers), the American Iron and Steel Institute, the Hendrik Hudson Boy Scout Council (past-president), the New York State Conference of Mayors, and the Westchester County Village Officer's Association (former president). With the entrance of the United States into World War II, Mr. Walker was appointed deputy director of civilian protection in Westchester County. He immediately organized an active defense organization in his village and worked tirelessly.

Mr. Walker was well born; and, from his Quaker ancestors, he inherited honesty, integrity, and loyalty. He did not hide these qualities but developed them into a well-rounded Christian character. He had many friends to whom he was always loyal. To him a promise meant that it must be fulfilled.

On October 6, 1904, in Dobbs Ferry, he was married to Florence Amy Davidson. He is survived by his widow; a son, Joseph P. Walker (Lieutenant, U. S. Navy); a daughter, Elizabeth (Mrs. Alonzo Milton George); and a brother, Charles W. Walker.

Mr. Walker was elected an Associate Member of the American Society of Civil Engineers on November 5, 1902.

HERBERT KIRKMAN WARD, Assoc. M. Am. Soc. C. E.[1]

Died May 28, 1942

Herbert Kirkman Ward was born in San Diego, Calif., on September 22, 1882, the son of Sampson Lett and Ella (Kirkman) Ward. He attended the public schools of San Diego and completed an engineering course in the International Correspondence School.

Mr. Ward began his professional career in 1901 as rodman and chainman for the San Diego and Eastern Railway. From September, 1902, to November, 1903, he was employed as rodman on the subdivision of government lands. From March, 1904, to February, 1905, he was rodman and timekeeper for the California Mountain Water Company, which supplied San Diego and surrounding areas; and, from June, 1905, to June, 1906, he was resident engineer in charge of construction of five miles of wood-stave pipe and erection of a filter plant and reinforced concrete water tower at the Chollas Heights Reservoir and of a pipe line from the reservoir to the City of San Diego.

From July, 1906, to February, 1908, Mr. Ward was engaged in private practice with his father in San Diego County on land subdivision and resurveys. From March to September, 1908, he was levelman for the San Diego and Arizona Railway Company. He was employed by the San Diego County Highway Commission from October, 1908, to September, 1909, successively as transitman and assistant engineer on highway location.

In October, 1909, Mr. Ward returned to the San Diego and Arizona Railway Company, serving as resident engineer in charge of grading and pile bridge construction until April, 1911. From May, 1911, to June, 1914, he was assistant engineer and, from July, 1914, to April, 1917, division engineer, in charge of both location and construction through mountainous country. In April, 1917, he entered the service of the Alaska Engineering Commission at Anchorage, Alaska, as locating engineer. He was later transferred to Nenana, Alaska, as assistant to the engineer in charge of construction and operation and remained until February, 1920.

In May, 1920, Mr. Ward began a long period of service with the Division of Highways of California. He was employed in District II, with headquarters in Dunsmuir, Calif., and later in Redding, Calif., as locating engineer, superintendent of day labor construction, and resident engineer. At the time of his death Mr. Ward was resident engineer on a 33-mile mineral-access highway in the Klamath River area. He had many important assignments during his long service with the Division of Highways.

In 1922 Mr. Ward was married to Mrs. Sarah Hill of Redding. There were no children, and he was left a widower in 1939. He is survived by two brothers, R. Ernest Ward and Beverly T. Ward.

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[1] Memoir prepared by F. W. Haselwood, M. Am. Soc. C. E., and George A. Tilton, Jr., Assoc. M. Am. Soc. C. E.

Mr. Ward was a member of the Redding Chapter of the B.P.O.E. He became a subscribing member of the Sacramento Section of the Society in 1938.

Mr. Ward was elected an Associate Member of the American Society of Civil Engineers on May 15, 1917.

TIMOTHY SILAS WILLIAMS, Assoc. M. Am. Soc. C. E.[1]

Died May 12, 1944

Timothy Silas Williams was born at Brooklyn, N. Y., on August 28, 1883, the son of Sanford H. Williams and Eliza Rebecca (Wiley) Williams. He was graduated from Pratt Institute in Brooklyn with the degree of Mechanical Engineer in 1902, received the degrees of Naval Architect and Marine Engineer from Webb Academy in New York, N. Y., in 1906, and the degree of Civil Engineer from Brooklyn Polytechnic Institute in 1910.

From June, 1906, to October, 1907, Timothy Williams was with the engineer corps on the Brighton Beach Improvement and on surveys, track layouts, and construction for the Brooklyn Rapid Transit System, as rodman, transitman, and later in charge of party. This work gave him the practical experience of the surveyor, which, together with his technical engineering training, equipped him well for his chosen field of designing engineer.

From 1908 to 1910, Mr. Williams served as civil engineering instructor at Cooper Union in New York City.

In 1910 Mr. Williams entered the employ of Westinghouse, Church, Kerr and Company, of New York City, as a structural designer. From 1912 to 1914, he was a structural steel and concrete designer at the Montreal (Canada) office of the company. Upon his return to the New York office, he served again as a structural designer until May, 1916, when he became assistant to the concrete engineer in charge of concrete and foundation design. During this ten-year period, Mr. Williams was engaged in the design of various types of steel, timber, and concrete construction—power houses, bank buildings, steel bridges and viaducts, transmission lines, coal and ore bins, crane runways, and flat slab construction.

From 1920 to 1924, Mr. Williams was employed by several engineering organizations, including The Foundation Company, Walter Kidde Company, Meinike and Frank, and the Standard Oil Company—all of New York City. During this time he supervised the design and preparation of construction drawings for industrial buildings, apartment houses, foundations, bulkheads, and piers.

In 1924 Mr. Williams joined Thomas E. Murray and Company, of New York City, as concrete engineer in responsible charge of the design of heavy power plant concrete foundation and water front construction. This experience included the use of timber, concrete, and tubular steel piling; the design of open cofferdams sealed by tremie concrete; and the construction of timber decks with concrete retaining walls, for such outstanding projects as the East River Station of the New York Edison Company and the Hell Gate Station of the United Electric Light and Power Company. Large intake flume construction below the water level designed for the support of heavy power house building and equipment foundations and bulkheads of timber and concrete with provision for intakes and screens of considerable extent made this four-year period a valuable and interesting one. For the Thomas E. Murray Company Mr. Williams wrote specifications for concrete construction, inspected sites before designing, and had frequent conferences with contractors on construction methods.

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[1] Memoir prepared by Herbert J. Raff, Chemical Constr. Corp., New York, N. Y.

He worked for the United Engineers and Constructors of Philadelphia, Pa., from 1928 to 1931, on the preparation of estimates and plans for the West Philadelphia improvement work of the Pennsylvania Railroad Company. He was in charge of the estimating and of the complete working drawings for the $3,000,000 Railway Express Agency Building, with trackage and platform space of about 7½ acres. From 1933 to 1938, Mr. Williams had a number of professional engineering assignments with such companies as Phelps Dodge Copper Products Corporation, Wigton Abbott Company, The Texas Company, and Chemical Construction Corporation—all of New York City. He was employed as assistant structural engineer by The New York World's Fair Corporation, designing various structures and approving designs submitted by outside engineers from 1938 to 1939.

In October, 1939, Mr. Williams returned to the employ of Chemical Construction Corporation as concrete engineer and remained until his death. He was in responsible charge of design and the preparation of plans for all concrete construction and foundation work. Among the projects built from his designs were chemical plants, in the United States and abroad; the Jayhawk, Ozark, and Cactus ordnance plants for the War Department; and a number of projects sponsored by the Defense Plant Corporation.

Mr. Williams was a man of fine character, a capable structural engineer, and a good organizer and supervisor of the work of others.

On February 21, 1931, he was married to Dorothy E. Hall in Melrose, Mass. He is survived by his widow; one daughter, Elizabeth-Jeannette; and a brother, William Shaw.

Mr. Williams enjoyed music, tennis, and boating. He was an independent Republican and a Methodist. He was a licensed Professional Engineer in the states of New York and New Jersey.

Mr. Williams was elected an Associate Member of the American Society of Civil Engineers on June 16, 1919.

MERTON HORATIO WILLIS, Assoc. M. Am. Soc. C. E.[1]

DIED JANUARY 7, 1943

Merton Horatio Willis was born in Burr Oak, Iowa, on August 26, 1880, the eldest of seven children of Davis Burr and Ida (Starr) Willis. His forebears were some of the earliest settlers in New England. Several years after his birth the family moved to Colorado where he spent his boyhood and young manhood.

Other than grade school, Mr. Willis had no formal education. As a boy, he was employed by a surveying party, and the chief of party encouraged him to study engineering. He began studying at night after his work for the day was finished. In this fashion he kept up his studying until at last he finished the course, continuing his active work in engineering in the meantime.

Mr Willis obtained his early engineering training with the engineering departments of the Denver, Northwestern and Pacific Railway (Moffat Road), and the Cananea, Rio Yaqui and Pacific Railway (subsidiary of Southern Pacific Company), and with contractors on railway construction work in Colorado, California, and Arizona.

After serving a three-year apprenticeship on railroad work, he commenced his engineering career in Mexico where he was destined to live and practice his profession for the next twenty years.

From 1907 to 1912, Mr. Willis was engaged chiefly in pioneer railway location and construction work in the states of Sinaloa and Jalisco, in western Mexico. Since development and operation of the many mines in these states also attracted his interest, he gradually shifted his activities from railroad engineering to mining.

In 1912 Mr. Willis was employed by the Huasteca Petroleum Company of Tampico, Mexico. One of his first and most important single tasks was the construction and initial operation of the company's first oil refinery in Mexico, followed by construction of a topping plant for the company at Desterhan, La.

On his return to Mexico in 1917 Mr. Willis was made assistant chief engineer of the Huasteca Petroleum Company and in that capacity was in immediate charge of the extensive and varied construction work required in the development of a large oil field. In addition to his interest in refinery operation, he devoted much time to the pipe line transporting oil from the company's Ebano oil field to the loading docks at Tampico. The highly viscous Mexican crude oil required an elaborate series of pumping and heating plants to handle it.

In 1927 activity in the oil fields had been brought almost to a standstill by the increasingly restrictive policies of the Mexican government. The Standard Oil Company of Indiana, which had bought out Huasteca Petroleum several years previously, in late 1927 transferred Mr. Willis to their Lake Maracaibo field in Venezuela.

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[1] Memoir prepared by Merton Joseph Willis, Jun. Am. Soc. C. E.

After serving for a year as chief engineer, he resigned to enter contracting in the Lake Maracaibo area and on the island of Aruba, Dutch West Indies.

Mr. Willis suffered from chronic malaria for most of his twenty-five years in the tropics, and by 1932 he was in such bad health that he was told to remain for several years in a cold climate to bring the malaria under control. From 1932 to 1936 he lived first in Idaho and later in California in virtual retirement.

From 1936 until his death he was engaged in surveying and general engineering work in the vicinity of Walnut Creek, Calif.

Mr. Willis was well known in Mexican and Venezuelan oil circles as a man of integrity and outstanding practical engineering ability.

He was a member of the Knights Templar in the Masonic Order (Tampico).

On October 7, 1922, he was married in Tampico to Mary Clyde Melton. He is survived by his widow; a son, the writer; three brothers, Lawrence, Leslie I., and Burton D., Colonel in the U. S. Army; and two sisters, Una (Mrs. H. P. O'Reilly) and Clara (Mrs. E. E. Siegley).

Mr. Willis was elected an Associate Member of the American Society of Civil Engineers on January 14, 1918.

HAROLD AARON WOOD, Assoc. M. Am. Soc. C. E.[1]

Died July 17, 1944

Harold Aaron "Pete" Wood, the son of Samuel Emmor and Anne (Scott) Wood, was born in Gallup, N. Mex., on May 12, 1904.

After graduation from high school, he attended the University of New Mexico in Albuquerque for three years. He was employed by his father's engineering firm, the S. E. Wood Engineering Company of Gallup, from 1920 to 1925. In May, 1927, Mr. Wood accepted a position with the J. G. White Engineering Corporation in Panama, as instrumentman on the location and construction of railroads, and remained there until February, 1928. The following year he returned to the United States, and, after spending a year working with his father's engineering firm in Gallup, he returned to the University of New Mexico, being graduated in June, 1932, with the degree of Bachelor of Science in Civil Engineering.

For two years following his graduation, Mr. Wood was employed by the New Mexico State Highway Department, and, from February, 1934, until December, 1934, he served as surveyor for the United States Indian Irrigation Service, Department of the Interior, in Albuquerque. During 1935 Mr. Wood spent three months as project supervisor of a malaria-control project for the

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[1] Memoir prepared by a Committee of the New Mexico Section consisting of Burton L. Smith and John H. Bliss, Assoc. Members, Am. Soc. C. E., and Berkeley Johnson, M. Am. Soc. C. E.

New Mexico State Health Department; four months as draftsman and designer on irrigation structures for the United States Indian Irrigation Service; and three months as instrumentman for W. R. Holloway, consulting engineer of Tulsa, Okla., on the investigation of a two-million-dollar domestic water supply for the City of Albuquerque.

In October, 1935, Mr. Wood returned to the employ of the United States Indian Irrigation Service in Albuquerque as junior engineer, being promoted to assistant civil engineer in July, 1939.

When the United States entered World War II, he offered his services to his country and was commissioned a Lieutenant (jg) in the United States Navy in May, 1942. He spent a year on duty in the South Pacific, and in March, 1943, was promoted to Lieutenant, stationed at Port Chicago, Calif. On July 17, 1944, he was reported missing in the Port Chicago explosion. Funeral services were conducted in Albuquerque on August 5, 1944.

Pete, as he was known to the engineers of New Mexico, and to his many business and personal acquaintances, was Secretary-Treasurer of the New Mexico Section of the Society from December, 1935, to December, 1936, and Second Vice-President from January, 1942, until he entered the Navy. To these positions he gave his usual untiring and enthusiastic effort. Work and service were enjoyable to Pete, as all those who had the privilege to be associated with him can testify. He was a registered professional engineer in New Mexico, a member of the New Mexico Society of Engineers, a Mason, and a member of Kappa Alpha fraternity (southern section).

In November, 1934, he was married to Doris Whitney of Salisbury, Vt. In addition to his widow, he is survived by a son, Samuel W., and his mother.

Lieutenant Wood was elected a Junior of the American Society of Civil Engineers on November 28, 1932, and an Associate Member on May 17, 1937.

DEANE FRANCIS BIXBY, Jun. Am. Soc. C. E.[1]

Died December 4, 1944

Deane Francis Bixby, the son of Neil F. Bixby and Bertha (Hoffman) Bixby, was born in Portland, Ore., on January 29, 1921. His parents both were born in Blue Earth County, Minnesota. They were married in Portland, where Mr. Bixby has been continuously employed with the United States Army Engineers since March 30, 1919.

Mr. Bixby received his early schooling in Parkrose grade and high schools. He attended Multnomah College in Portland for two years and then went to Oregon State College at Corvallis from which he was graduated with honors and the degree of Bachelor of Science in Civil Engineering in 1942. During his senior year at Oregon State College he was honored by election to Tau Beta Pi fraternity.

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[1] Memoir prepared by R. E. Hickson, M. Am. Soc. C. E.

On June 4, 1942, three days after his graduation, Mr. Bixby enlisted in the United States Army and was assigned to the 826th Engineer Battalion (Aviation), at Fort George Wright in Spokane, Wash. In August, 1942, he was sent to England as a Corporal in this battalion and was engaged in the construction of airfields for the British. Almost a year later he was ordered to Officer Candidate School, at Fort Belvoir, Va., from which he was graduated as a Second Lieutenant, Corps of Engineers, on December 1, 1943. He remained at Fort Belvoir, receiving further training until he was again ordered to England in August, 1944. A month later he crossed the English Channel to France, assigned to Company C, Eighth Engineer Combat Battalion. Lieutenant Bixby fought and performed the arduous duties demanded of an officer in the combat engineers during the historic campaign which drove the Germans out of France. He was seriously wounded in action in Germany on November 27, and died on December 4, 1944. He is buried "somewhere in Belgium."

Lieutenant Bixby was an engineer of promise; he was true to his many friends who sincerely miss him. He had hopes and aspirations for success and service in his chosen profession, and above all he had a fervent desire to serve his country.

In addition to his parents, Lieutenant Bixby is survived by three sisters, Shirley, Patricia, and Evelyn (Mrs. H. R. Strong).

Lieutenant Bixby was elected a Junior of the American Society of Civil Engineers on October 19, 1942.

JOHN EDWARD BUSSELL, Jun. Am. Soc. C. E.[1]

Died June 6, 1944

John Edward Bussell, the son of Edward E. and Lora Mabel (Miles) Bussell, was born in Coffeyville, Kans., on November 17, 1919. He was graduated from St. Joseph Junior College at St. Joseph, Mo., in 1938 and from the University of California at Berkeley in 1941.

From May to November, 1941, Mr. Bussell was employed as junior engineer by East Bay Municipal Utility District of Oakland, Calif. He then spent several months as student engineer of the Special Engineer Division on preliminary work for the Third Locks of the Panama Canal.

Returning to the United States, he enlisted in the U. S. Naval Reserve (U.S.N.R.). Mr. Bussell was appointed Midshipman in March, 1943, and was sent to Naval Reserve School in New York, N. Y. In June, 1943, he was appointed Assistant Civil Engineer with the rank of Ensign, CEC–V(S), U. S. N. R., and, after several months of training with an Amphibious Demolition Unit, was sent overseas in January, 1944.

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[1] Memoir prepared by a Committee of the Kansas City Section consisting of Chester A. Smith, M. Am. Soc. C. E., and E. A. Blanpied, Assoc. M. Am. Soc. C. E.

On June 6, 1944, he was killed on Omaha Beach in Normandy, France, where his unit of Force O formed part of the first wave of the attack and met devastating artillery, machine gun, and sniper fire but blasted five gaps for the assault forces and removed German traps from 85% of the Omaha Beach area. The unit received a citation for the action but lost over 50% of its men in killed and wounded.

He was a member of the Chi Epsilon, civil engineering professional honor society.

Ensign Bussell was elected a Junior of the American Society of Civil Engineers on October 9, 1941.

HARRY HAMILTON HOLTON, Jun. Am. Soc. C. E.[1]

Died July 10, 1944

Harry Hamilton Holton was born in Tupelo, Miss., on September 12, 1920, the son of John Calvin and Ruth (Crook) Holton. After graduation from the Jackson (Miss.) High School in 1937, he entered Mississippi State College, at State College, from which he was graduated in June, 1941. He majored in civil engineering, and was a member of the Mississippi State College Student Chapter of the Society. He was very popular with students and faculty members, and was quite active in campus affairs, serving as an officer in his social fraternity, Kappa Sigma, for three years. He also made many friends when he traveled with his father on trips while the latter was Commissioner of Agriculture for the State of Mississippi.

As a result of his training in the Reserve Officers' Training Corps at Mississippi State College, Mr. Holton was eligible for a commission as a Second Lieutenant in the U. S. Army Reserve Corps upon graduation; but this was not granted until February, 1942, when he was twenty-one years old. From June, 1941, to January, 1942, he was with the Tennessee Valley Authority, in charge of a mapping and surveying crew. In the meantime he transferred from the Coast Artillery to the Air Corps, and was called to active duty as an Administrative Officer, reporting to Patterson Field, Ohio, in March, 1942. Imbued with the spirit of rendering the greatest possible service, Lieutenant Holton applied for pilot training, reported to Maxwell Field, Alabama, in June, 1942, and received his "wings" at Spence Field, Moultrie, Ga., in January, 1943.

Although he was trained for fighter plane duty, he was first assigned to the Ferrying Command, flying planes and supplies from the Northwest, particularly Billings, Mont., and Minneapolis, Minn., to Nome and Fairbanks, Alaska. He was sent overseas in June, 1943. In July, 1943, he was assigned to the Air Transport Command, flying over the "Hump" (Himalaya Mountains) from India to China—probably the most hazardous duty in the entire war theater. He had this assignment for a little more than a year, with the exception of about two months when, because of his civil engineering training, he was called on to serve as a technical adviser on the selection of airport sites and on the inspection of established fields in India. Lieutenant Holton received a Presidential Citation for "meritorious service" in the spring of 1944, and was promoted to First Lieutenant. It was rumored that a captaincy was to follow. He was killed in an airplane crash at Kunming, China.

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[1] Memoir prepared by Robert E. Bowlus. Arlington, Va.

On May 1, 1943, Lieutenant Holton was married to Jean Morris, daughter of Mr. and Mrs. Lloyd Morris of Yazoo City, Miss. Mrs. Holton was then a junior at Vanderbilt University in Nashville, Tenn., and, in the fall of 1943, following Lieutenant Holton's assignment overseas, she entered Millsaps College, in Jackson, where she completed her college work in February, 1944. Lieutenant Holton's death came as a very great shock to the Holton and Morris families, as he was expecting to return to the United States in the fall of 1944, having already served for nearly thirteen months overseas. He is survived by his parents; his widow; and a sister, Dorothy (Mrs. A. B. Kelly).

Lieutenant Holton was elected a Junior of the American Society of Civil Engineers on October 9, 1941.

EUGENE CLARE MILLER, Jun. Am. Soc. C. E.[1]

Died February 10, 1944

Eugene Clare Miller, the son of Ernest and Clara Blanche (Snyder) Miller, was born in Glendora, Calif., on July 6, 1913. Mr. Miller spent his early life in Glendora and Whittier, Calif. Upon completion of high school, he entered the California Institute of Technology at Pasadena. In June, 1935, he was graduated with the degree of Bachelor of Science in Civil Engineering and immediately started to work for the United States Forest Service in Glendora as a junior engineer. Shortly thereafter, he became a concrete inspector for A. C. Martin, an architect of Los Angeles, Calif.

During 1936, he was timekeeper, draftsman, and engineer for both the Jahn and Bressi Construction Company and H. C. Nelson, structural engineer, in Los Angeles. He later became a bridge steel inspector for the United States Army Engineers. In December, 1936, he started work as a draftsman for Consolidated Steel Corporation, Ltd., in Los Angeles. For the next few years he worked as a detailer (structural and reinforcement) and as a labor and material estimator. At the beginning of 1942 he was appointed chief estimator for Consolidated Steel Corporation, Ltd.

In this capacity, with his background of education and experience, Mr. Miller was an invaluable man to the company, as his duties required a combination of capability, personality, experience, and tact. Mr. Miller had all these qualities, as was evidenced by his many friendships—both business and social. His services were invaluable for he was careful and accurate in his work, and possessed driving energy and leadership. In spite of the tremendous effort and long hours required of him, he was still able to find time to enjoy sports of all kinds, being a particular devotee of bowling and fishing, especially deep-sea fishing.

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[1] Memoir prepared by Edward R. Bosley, Jr., Director of Public Relations, Consolidated Steel Corp., Ltd., Los Angeles, Calif.

During February, 1944, Mr. Miller was on his way to Washington, D. C., on important war business, when the ill-fated plane on which he was a passenger crashed into the Mississippi River, and all aboard were lost.

On September 24, 1936, in Glendora, he was married to Grace Wills. He is survived by his widow; his young son, Ernie, aged four; his parents, Mr. and Mrs. Ernest Miller; a brother, Victor A., Ensign, United States Navy; and a sister, Betty Louise (Mrs. Robert McPhetridge).

Mr. Miller was elected a Junior of the American Society of Civil Engineers on December 9, 1935.

HARRY ALFRED PARR, Jun. Am. Soc. C. E.[1]

Died November 27, 1944

Harry Alfred Parr, the son of Harry and Hetty (Feist) Parr, was born in Roslindale, Mass., on March 25, 1917. He was graduated from the University of New Hampshire, at Durham, in 1941 with the degree of Bachelor of Science in Civil Engineering.

After graduation from college, Mr. Parr entered government service in the United States Engineering Department, remaining until he was commissioned a Second Lieutenant in the Corps of Engineers, United States Army.

Lieutenant Parr received his basic training at Fort Belvoir, Va. He then went to Camp Gruber, Okla., where he was promoted to First Lieutenant. In January, 1944, he went overseas, and was killed in action on November 27, 1944.

His commanding officer, Lt.-Col. Philip C. Satterthwaite, wrote of Lieutenant Parr:

> "First Lt. Harry A. Parr * * * was a fine gentleman and an excellent officer. He was admired and respected by everyone for his ability and pleasant personality. He fully deserved a promotion to Captain which I had, as yet, been unable to give him * * *."

Harry A. Parr was a very modest young man, sensible, and courageous. He was persevering in anything he undertook to do. While a student at the university, he was a member of the Reserve Officers' Training Corps. Both in high school and at college, he was a member of the football teams.

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[1] Memoir prepared by Mrs. H. A. Parr, Hampton, N. H.

While at the university, Lieutenant Parr belonged to the local Student Chapter of the Society, and was president during his senior year. When he was graduated from Hampton High School, he received a gold honor medal.

In 1942 he was married to Evelyn Ruth Brown of Camden, Me. He is survived by his widow; a son, Randall Alfred Parr, whom he had never seen; his parents; two brothers, Richard A. and Robert G. Parr; and two sisters, Doris I. and Lorraine E. Parr.

Lieutenant Parr was elected a Junior of the American Society of Civil Engineers on October 9, 1941.

MILTON HARRY SCHOERPNER, Jun. Am. Soc. C. E.[1]

Died September 12, 1944

Milton Harry Schoerpner was born in the Borough of the Bronx, New York, N. Y., on July 24, 1917. Both his father, Rudolph Schoerpner, and his mother, Olga (Kraus) Schoerpner, were born in what was later Czechoslovakia; both came separately to the United States when quite young and were married in this country. In spite of the difficulties and handicaps which confront recent immigrants, Rudolph Schoerpner was able eventually to go into business for himself and to operate his own restaurant until his death in 1930.

Mr. Schoerpner obtained both his secondary and college education in New York City. After graduation from Stuyvesant High School in 1934, he entered New York University. For the following eight years, he attended evening classes, working during the day to earn a livelihood and at the same time to finance his education. He was employed as stock clerk in the firm of G. Schirmer in New York City, and held clerical positions in other New York City and Long Island firms, among them the A. C. Horn Company, where he became assistant office manager. Although the positions he held involved full-time employment with varying degrees of responsibility and although he was required to meet the same high scholastic standards in his evening classes as students in the regular sessions, Mr. Schoerpner, in his determination to enter the professional world, took full advantage of every opportunity to realize his ambition. By combining industry and native ability, he carried on his intensive double program of work and scholastic endeavor without interruption until July, 1942, when he was graduated from New York University with the degree of Bachelor of Civil Engineering.

The attainment of this engineering degree in spite of such great handicaps is ample evidence of Mr. Schoerpner's ambition and of his determination to achieve his goal. If success consists in the attainment of one's ideals, he was successful before his career as an engineer actually began.

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[1] Memoir prepared by Arthur W. Harrington, M. Am. Soc. C. E.

Immediately after graduation, Mr. Schoerpner received an appointment as junior hydraulic engineer and was assigned to the office of the United States Geological Survey, Water Resources Branch, in Albany, N. Y. He remained with this organization until his induction into the United States Army. His investigational work with the Geological Survey was of excellent quality, and he displayed more than sufficient ability to assure his engineering future.

Mr. Schoerpner was inducted into the Army on September 2, 1943, and reported for active duty at Camp Upton, Long Island, N. Y., on September 23, 1943. About two weeks later, he was sent to Camp Claiborne, La., for basic training, after which he was transferred to the Granite City Engineering Depot in Granite City, Ill., for special training. Early in May, 1944, he left with his outfit for England and from there he was shipped to France, landing about August 15, 1944. Comprehensive details with respect to his Army work, which concerned supplies to embattled areas, cannot be released, but it is known that his group was engaged in experimental work to facilitate the movement and repair of heavy equipment.

On September 12, 1944, Corporal Schoerpner was killed near Longny, France, as a result of a motor transport accident. He is buried in the American Military Cemetery at St. Corneille, France, where, on September 16, 1944, officers and men of his company paid him full military honors. In the Army, he was highly regarded by both superiors and subordinates, and, for efficiency and fidelity in service, Corporal Schoerpner was awarded the Good Conduct Medal posthumously.

Hampered both by lack of time and of resources, Mr. Schoerpner had made only a fair start on such hobbies as a collection of electric trains (which included, and was built up from, the first set which had been given to him as a boy) and a collection of coins. His particular enthusiasm was photography.

On November 15, 1942, in New York City, he was married to Madeline Beer. He is survived by his widow.

Mr. Schoerpner belonged to the Albany Society of Engineers and to the American Geophysical Union. While attending New York University, he was an active member of the Student Chapter (Evening Section) of the Society, and served as Secretary for one year.

Mr. Schoerpner was elected a Junior of the American Society of Civil Engineers on November 9, 1942.

HOWARD EARLE SHAW, JR., Jun. Am. Soc. C. E.[1]

DIED OCTOBER 2, 1943

Howard Earle Shaw, Jr., the son of Howard Earle and Kate (Worley) Shaw, was born on December 7, 1913, in Andover, Mass.

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[1] Memoir prepared by W. S. Evans, M. Am. Soc. C. E.

He received his preparatory education at Deering High School and attended the University of Maine, at Orono, being graduated with the degree of Bachelor of Science in Civil Engineering in June, 1937. The next year he attended the Graduate School of Sanitary Engineering at Harvard University, in Cambridge, Mass., and received the degree of Master of Science.

In September, 1938, Mr. Shaw was employed by the Portland (Me.) Water District as research engineer on the development of algae strainers. A year later he was employed as junior engineer inspector, Public Works Administration, on office and field work for the inspection of a submarine water main at Bath, Me. In the fall of 1939 he was construction engineer for Sanders Engineering Company on hydroelectric work at Hiram Falls, Me.

During college, Mr. Shaw was active in military affairs, being elected to Scabbard and Blade. He was commissioned a Second Lieutenant in the Reserve Officers Corps upon graduation. While employed as safety engineer by the Liberty Mutual Insurance Company of Philadelphia, Pa., he was called to active duty on May 5, 1941, as a First Lieutenant, assigned to 8th Infantry, Fort Benning, Ga., and later to 385th Infantry, 76th Division, and was promoted to rank of Captain on August 11, 1942. He attended the Infantry School in Fort Benning, and the Command and General Staff School, in Fort Leavenworth, Kans., then rejoined the 385th Infantry at Fort George G. Meade, Md., on April 10, 1943. He left the United States on July 15, 1943, for Casablanca, Morocco. He was in charge of water and sanitation at a large prisoner-of-war camp somewhere in the North Africa area. Following this he was transferred to Sicily, Italy, on September 15, 1943, reporting to 3d Division Headquarters and soon was assigned to command of Company M, 30th Infantry, operating somewhere in Italy. At about nine o'clock on the morning of October 2, 1943, while making a reconnaissance for suitable position to employ his heavy machine guns and mortars, he was instantly killed by a sudden burst of fire from a German machine pistol. Captain Shaw was buried in an American Military Cemetery in Italy on October 4, 1943.

Captain Shaw possessed a genial and winning personality. Cheerful and optimistic by nature, daily association with him in his work and elsewhere was most congenial and pleasant. Well trained, capable, loyal, and conscientious in his work, his brief career as a civil engineer will always reflect honor upon his name and upon the profession to which he belonged.

On September 1, 1940, in Andover, Captain Shaw was married to Eleanor Elizabeth Manning. Besides his parents, he is survived by his widow; a daughter, Susan Margaret, born on November 14, 1942; and a sister, Katharine (First Lieutenant in the Women's Army Corps).

Captain Shaw was elected a Junior of the American Society of Civil Engineers on April 10, 1940.

FREDERICK QUAAS TEICHERT, Jun. Am. Soc. C. E.[1]

DIED MAY 12, 1944

Frederick Quaas Teichert was born in Sacramento, Calif., on August 7, 1915, the son of Adolph Teichert, Jr., and Augusta (Quaas) Teichert. He attended the public schools of that city and was valedictorian of his class upon his graduation from the Sacramento High School.

After majoring in civil engineering for two years at the Sacramento Junior College, he continued his engineering education at the University of California in Berkeley, receiving the degree of Bachelor of Science in Civil Engineering in 1936. He was a member of Tau Beta Pi, engineering honor society, and served as president of the university chapter. In addition, he was affiliated with the Psi Upsilon fraternity, was president of the Student Chapter of the Society, and a member of the Student Engineering Council.

After graduation, Mr. Teichert joined his father as a member of the old and established contracting firm of A. Teichert and Son, Inc. He held various positions in the firm before advancing to that of vice-president. During the years he was in business with his father, Mr. Teichert worked on various airport, highway, and levee construction projects. Two of his outstanding accomplishments were coordinating the construction of the Sierra Ordnance Depot, a $12,000,000 project in Lassen County, California, which was completed in five months during 1942, and the construction of the Debris Dam on the North Fork of the American River (California) for the U. S. Army Engineers. He was a director of the Northern California Chapter of Associated General Contractors of America.

Mr. Teichert possessed initiative, leadership, and originality to an unusual degree, and displayed exceptional capacity for success in the engineering profession. His superior executive ability, which was far beyond his years, qualified him to execute important engineering projects.

Popular and likable, Mr. Teichert was a young man of high character, integrity, and ideals, and his passing is a great loss to the community, as well as to his many friends and relatives. He is survived by his widow, Norma Watrous Teichert, to whom he was married on January 20, 1940, in Sacramento; his father, Adolph Teichert, Jr.; a sister, Nancy Teichert; and two brothers, Adolph H. Teichert and Capt. Henry Teichert of the U. S. Army Air Corps.

Mr. Teichert was presented the Junior Prize for the year 1936 by the Sacramento Section of the Society. He was selected as recipient for this award on the basis of scholarship, his creditable record of activities in the Student Chapter of the Society, and his capacity for success in his chosen profession.

Mr. Teichert was elected a Junior of the American Society of Civil Engineers on December 14, 1936.

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[1] Memoir prepared by O. Haven Hart and Roy J. Sykes, Assoc. Members, Am. Soc. C. E.

ALFRED EPHRAIM KORNFELD, Affiliate Am. Soc. C. E.[1]

DIED APRIL 8, 1944

One of the small group of able men who developed American engineering journalism in its early days passed away with the death of Alfred Ephraim Kornfeld in New York, N. Y., on April 8, 1944. His share in the upbuilding of *Engineering News,* as its business manager, may justly be ranked as a contribution to the development of the civil engineering profession in the United States. The editorial leadership of Arthur M. Wellington,[2] M. Am. Soc. C. E., and his technical collaborators found a necessary supplement in the business genius of Mr. Kornfeld in association with that of George H. Frost,[3] Assoc. M. Am. Soc. C. E. They gave the enterprise vitality and stability.

Alfred Ephraim Kornfeld, the son of Karl and Julia (Grögor) Kornfeld, was a native New Yorker, born in the metropolis on October 24, 1869. He was educated in the public schools and the College of the City of New York. His business abilities revealed themselves at an early age and he was employed in the commercial and advertising departments of technical papers while still in his teens. This work inspired him with the initiative and confidence to propose to Mr. Frost, the founder of *Engineering News,* that he be entrusted with the advertising representation of the paper. Mr. Frost's insight into human nature led him to agree to the proposal, and Mr. Kornfeld's immediate success under discouraging circumstances (the dark days of the 1893 economic depression) justified the wisdom of the decision.

The quick understanding, clear business sense, integrity, and human qualities responsible for his success soon carried Mr. Kornfeld farther—to the business managership of the publication and participation in its ownership. He continued to be a leading spirit in the direction of *Engineering News* until 1912, when Mr. Frost's retirement led to its transfer to the Hill Publishing Company (later merged with the McGraw Publishing Company to form the McGraw-Hill Publishing Company). Mr. Kornfeld turned over his work to younger hands, although he maintained his active interest in *Engineering News* and its successor, *Engineering News-Record,* up to the time of his death.

In his later years, Mr. Kornfeld's interest was centered in philanthropic and civic work, and even during occasional reentry into business ventures these activities continued to hold first place. He gave help to both individuals and groups, and had the satisfaction of making many people happier.

On June 21, 1891, Mr. Kornfeld was married to Hennie Eisenberg, of New York City, who, with a daughter, Muriel (Mrs. Franklin Hollander), survives him.

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[1] Memoir prepared by F. E. Schmitt, M. Am. Soc. C. E.

[2] For memoir, see *Proceedings,* Am. Soc. C. E., Vol. XXI (1895), p. 199.

[3] For memoir, see *Transactions,* Am. Soc. C. E., Vol. LXXXI (December, 1917), p. 1827.

His technical and scientific memberships included the American Society of Mechanical Engineers, the American Association for the Advancement of Science, the Society of American Military Engineers, and the American Institute of the City of New York.

Mr. Kornfeld was elected an Affiliate of the American Society of Civil Engineers on October 7, 1908.

TRANSACTIONS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

INDEX

VOLUME 110

1945

Subject Index, Page 1924
Author Index, Page 1932

Titles of papers are in quotation marks when given with the author's name.

VOLUME 110

SUBJECT INDEX

ADDRESSES

"Private Enterprise." Annual Presidential Address. J. C. Stevens. 1605. (*Annual Address presented through the pages of "Civil Engineering," August 1945, because of wartime conditions prohibiting the holding of conventions.*)

AERODYNAMICS

"Rigidity and Aerodynamic Stability of Suspension Bridges." D. B. Steinman. (With Discussion.) 439.

AIRPORTS

"Military Airfields." A Symposium. 669. Discussion: James B. Newman, Jr., Thomas E. Stanton, W. E. Howland, David S. Jenkins, William E. Rudolph, Raymond L. Irwin, G. G. Greulich, Hibbert Hill, Jacob Feld Robert Horonjeff, Clarence S. Jarvis, D. D. McGuire, E. D. Parmer, A. Casagrande, Harry E. Cotton, D. O. Murray, Thomas B. Pringle, John R. Haswell, W. J. Turnbull and William H. Jervis, Ralph R. Proctor, D. M. Burmister, Ralph Hansen, H. V. Pittman, G. R. Schneider, H. M. Williams, Ira B. Mullis, C. F. Izzard, L. J. Mensch, Hibbert Hill, and Gail A. Hathaway. 734.

BEAMS

"Amplified Slope Deflection." Ralph W. Stewart. (With Discussion.) 1401.

"Design Constants for Beams with Nonsymmetrical Straight Haunches." August L. Ahlf. 1019. Discussion: Robert O. Jameson, John Sherman, Dale H. Rea, I. B. Hosig, and August L. Ahlf. 1037.

"Shear and Bond Stresses in Reinforced Concrete." Stanley U. Benscoter and Samuel T. Logan. (With Discussion.) 599.

BIBLIOGRAPHIES

Cofferdams. 1114.

Construction and design of airfields, and design of drainage facilities. 730.

Frames. 1507.

Highway engineering. 293.

Hydraulics. 1251.

Sewage disposal. 241.

BORINGS

"Unusual Cutoff Problems—Dams of the Tennessee Valley Authority." A Symposium. (With Discussion.) 947.

BRIDGES

"Rainbow Arch Bridge Over Niagara Gorge." A Symposium. 1. Discussion: Egidio O. Di Genova, Charles Mackintosh, C. M. Goodrich, T. Kennard Thomson, C. M. Spofford, L. J. Mensch, O. H. Ammann, Louis Balog, William G. Rapp, Leon Beskin, Neil Van Eenam, C. H. Gronquist, M. Hirschthal, George Stern, Shortridge Hardesty, J. M. Garrelts, and I. G. Hedrick, Jr., Elmer K. Timby, Lawrence M. Mead, Jr., and Robert McEldowney, Jr., and John I. Copp, Karl de Vries, William H. Jameson, and Jonathan Jones. 120.

"Rigidity and Aerodynamic Stability of Suspension Bridges." D. B. Steinman. 439. Discussion: Edward Adams Richardson, R. K. Bernhard, J. M. Robertson, C. H. Gronquist, F. B. Farquharson, H. Reissner, Blair Birdsall, Hardy Cross, Frederick C. Smith and Hermann Friedlaender and George S. Vincent, Louis Balog, and D. B. Steinman. 476.

CANALS

"Economical Canal Cross Sections." Victor L. Streeter. 421. Discussion: V. A. Endersby, Julian Hinds, D. C. McConaughy, C. J. Posey, and Victor L. Streeter. 431.

"Surges in Panama Canal Reproduced in Model." F. W. Edwards and Edward Soucek. (With Discussion.) 345.

CHANNELS

"Translatory Waves in Natural Channels." J. H. Wilkinson. (With Discussion.) 1203.

CLAY

"Pressure on the Lining of Circular Tunnels in Plastic Soils." D. P. Krynine. (With Discussion.) 1048.

"Simultaneous Consolidation of Contiguous Layers of Unlike Compressible Soils." Hamilton Gray. (With Discussion.) 1327.

COFFERDAMS

"Stability and Stiffness of Cellular Cofferdams." Karl Terzaghi. 1083. Discussion: G. G. Greulich, Raymond P. Pennoyer, Dean P. Tsagaris, R. T. Colburn, A. F. Hedman, Adolph J. Ackerman, Savile Packshaw, Martin W. Lautz, J. S. Miller, Robert F. Legget, D. P. Krynine, J. J. Polivka, and Karl Terzaghi. 1120.

COLUMNS

"Shear Effect on the Strength of Struts." S. Sergev. 391. Discussion: Leon Beskin, Marshall Holt, David B. Hall, and S. Sergev. 404.

COMPRESSED AIR

Action of. 1064.

CONCRETE

"Economical Canal Cross Sections." Victor L. Streeter. (With Discussion.) 421.

CONSTRUCTION PLANT

See **CONTRACTORS' PLANT**

CONTRACTORS' PLANT

"Erection of Hydraulic Turbines and Generators." Andrew M. Komora. (With Discussion.) 581.

"Rainbow Arch Bridge Over Niagara Gorge." A Symposium. (With Discussion.) 1.

CONTRACTS

"Original and Readjusted Repayment Contracts, Boulder Canyon Project." Clay C. Elder. 363. Discussion: W. L. Chadwick, A. R. Arledge, and Clay C. Elder. 384.

COSTS

Cost of canals with various cross sections. 436.

Cost of work, Rainbow Arch Bridge. 169.

"Dewatering, Incineration, and Use of Sewage Sludge." A Symposium. (With Discussion.) 179.

Operating cost per foot for drilling, TVA foundations. 987.

"Original and Readjusted Repayment Contracts, Boulder Canyon Project." Clay C. Elder. (With Discussion.) 363.

CUTOFFS

"Unusual Cutoff Problems—Dams of the Tennessee Valley Authority." A Symposium. (With Discussion.) 947.

DAMS

"Original and Readjusted Repayment Contracts, Boulder Canyon Project." Clay C. Elder. (With Discussion.) 363.

"Unusual Cutoff Problems—Dams of the Tennessee Valley Authority." A Symposium. 947. Discussion: Berlen C. Moneymaker, Portland P. Fox, J. K. Black, L. C. Glenn, E. H. Burroughs, James B. Hays, and L. A. Schmidt, Jr. 995.

DRAINAGE

"Flood Formulas Based on Drainage Basin Characteristics." H. B. Kinnison and B. R. Colby. (With Discussion.) 849.

"Military Airfields." A Symposium. (With Discussion.) 669.

EARTH PRESSURES

"Pressure on the Lining of Circular Tunnels in Plastic Soils." D. P. Krynine. (With Discussion.) 1048.

ECONOMICS

"Economical Canal Cross Sections." Victor L. Streeter. (With Discussion.) 421.

ELECTRIC GENERATORS

"Erection of Hydraulic Turbines and Generators." Andrew M. Komora. (With Discussion.) 581.

ENGINEERS AND ENGINEERING

"Geology in Highway Engineering." Marshall T. Huntting. (With Discussion.) 271.

"Private Enterprise." Annual Presidential Address. J. C. Stevens. 1605.

(Annual Address presented through the pages of "Civil Engineering," August 1945, because of wartime conditions prohibiting the holding of conventions.)

FABRICATION

"Rainbow Arch Bridge Over Niagara Gorge." A Symposium. (With Discussion.) 1.

FERTILIZERS

“Dewatering, Incineration, and Use of Sewage Sludge.” A Symposium. (With Discussion.) 179.

FILTERS

See FILTRATION

FILTRATION

“Dewatering, Incineration, and Use of Sewage Sludge.” A Symposium. (With Discussion.) 179.

FLOODS

“Flood Formulas Based on Drainage Basin Characteristics.” H. B. Kinnison and B. R. Colby. 849. Discussion: Clarence S. Jarvis, L. R. Beard, M. J. Ord, Howard M. Turner, Byron O. McCoy, C. O. Clark, and H. B. Kinnison and B. R. Colby. 877.

“Storage and the Unit Hydrograph.” C. O. Clark. (With Discussion.) 1419.

FOUNDATIONS

“Simultaneous Consolidation of Contiguous Layers of Unlike Compressible Soils.” Hamilton Gray. (With Discussion.) 1327.

“Stability and Stiffness of Cellular Cofferdams.” Karl Terzaghi. (With Discussion.) 1083.

“Unusual Cutoff Problems—Dams of the Tennessee Valley Authority.” A Symposium. (With Discussion.) 947.

FROST

Design of pavement to avoid action of. 684.

Problem in highway engineering. 278.

FRAMEWORKS

See STRUCTURES, THEORY OF

GENERATORS

See ELECTRIC GENERATORS

GEOLOGY

“Geology in Highway Engineering.” Marshall T. Huntting. (With Discussion.) 271.

“Unusual Cutoff Problems—Dams of the Tennessee Valley Authority.” A Symposium. (With Discussion.) 947.

GROUND WATER

“Unusual Cutoff Problems—Dams of the Tennessee Valley Authority.” A Symposium. (With Discussion.) 947.

GROUTING

“Unusual Cutoff Problems—Dams of the Tennessee Valley Authority.” A Symposium. (With Discussion.) 947.

HIGHWAYS AND ROADS

"Geology in Highway Engineering." Marshall T. Huntting. 271. Discussion: W. W. Crosby, Carl B. Brown, F. H. Kellogg, Berlen C. Moneymaker, E. F. Bean, A. T. Bleck, Lyman W. Wood, Philip Keene, Jacob Feld, H. E. Marshall, K. B. Woods, D. J. Belcher, R. Woodward Moore, George E. Ekblaw, Allen S. Cary and Ade E. Jaskar, Karl Terzaghi, D. P. Krynine, Robert F. Legget, Irving B. Crosby, Hyde Forbes, A. A. Eremin, and Marshall T. Huntting. 298.

HYDRAULICS

"Coefficients for Velocity Distribution in Open-Channel Flow." William S. Eisenlohr, Jr. (With Discussion.) 633.

"Construction of the Flow Net for Hydraulic Design." H. Alden Foster. 1237. Discussion: J. W. Howe, Clarence S. Jarvis, and H. Alden Foster. 1253.

"Economical Canal Cross Sections." Victor L. Streeter. (With Discussion.) 421.

"Hydraulic Model Investigation of Miter Gate Operation." Maurice N. Amster. (With Discussion.) 1297.

"Lock Manifold Experiments." Edward Soucek and E. W. Zelnick. (With Discussion.) 1357.

"Surges in Panama Canal Reproduced in Model." F. W. Edwards and Edward Soucek. (With Discussion.) 345.

HYDROGRAPHS

"Flood Formulas Based on Drainage Basin Characteristics." H. B. Kinnison and B. R. Colby. (With Discussion.) 849.

"Military Airfields." A Symposium. (With Discussion.) 669.

"Storage and the Unit Hydrograph." C. O. Clark. 1419. Discussion: James S. Sweet, Otto H. Meyer, L. K. Sherman, Gordon R. Williams, Ray K. Linsley, Jr., E. F. Brater, L. C. Crawford, Robert E. Kennedy, Victor H. Cochrane, Franklin F. Snyder, Howard M. Turner, Don Johnstone, and C. O. Clark. 1447.

INCINERATION

"Dewatering, Incineration, and Use of Sewage Sludge." A Symposium. (With Discussion.) 179.

LANDSLIDES

Prevention of, in highway engineering. 281.

LOCK GATES

"Hydraulic Model Investigation of Miter Gate Operation." Maurice N. Amster. (With Discussion.) 1297.

LOCKS

"Lock Manifold Experiments." Edward Soucek and E. W. Zelnick. 1357. Discussion: J. Pietrkowski, John S. McNown, F. W. Edwards, W. M. Lansford, Robert F. Kreiss, Ralph G. Berk, Ralph W. Powell, and Edward Soucek and E. W. Zelnick. 1377.

MEMOIRS OF DECEASED MEMBERS
See name of member in Author Index; also Contents, page x.

MILITARY ENGINEERING
"Military Airfields." A Symposium. (With Discussion.) 669.

MODELS
"Hydraulic Model Investigation of Miter Gate Operation." Maurice N. Amster. 1297. Discussion: Edward Soucek, Isaac DeYoung, F. W. Edwards, William A. Tripp, and Maurice N. Amster. 1314.
"Rigidity and Aerodynamic Stability of Suspension Bridges." D. B. Steinman. (With Discussion.) 439.
"Surges in Panama Canal Reproduced in Models." F. W. Edwards and Edward Soucek. 345. Discussion: Harold A. Weggel, Harry Leypoldt, C. J. Posey, and F. W. Edwards and Edward Soucek. 356.
Test procedures for Rainbow Arch Bridge. 35.

MOMENTS
"Amplified Slope Deflection." Ralph W. Stewart. (With Discussion.) 1401.
Design Constants for Beams with Nonsymmetrical Straight Haunches." August L. Ahlf. (With Discussion.) 1019.

PAVEMENT
"Military Airfields." A Symposium. (With Discussion.) 669.

POWER GENERATION
A by-product of sewage incineration. 238.

POWER PLANTS
"Original and Readjusted Repayment Contracts, Boulder Canyon Project." Clay C. Elder. (With Discussion.) 363.

PRESSURES
See EARTH PRESSURES

RAINFALL
"Flood Formulas Based on Drainage Basin Characteristics." H. B. Kinnison and B. R. Colby. (With Discussion.) 849.
"Storage and the Unit Hydrograph." C. O. Clark. (With Discussion.) 1419.

REINFORCED CONCRETE
"Shear and Bond Stresses in Reinforced Concrete." Stanley U. Benscoter and Samuel T. Logan. 599. Discussion: F. R. Shanley, B. J. Aleck, Dean Peabody, Jr., William E. Wilbur, L. E. Grinter, Anders Bull, Victor R. Bergman, Phil M. Ferguson, and Stanley U. Benscoter and Samuel T. Logan. 607.
"Wedge-Shaped Structural Members Under Direct Stress and Bending." Robert B. B. Moorman. 1581. Discussion: Phil M. Ferguson, Stanley U. Benscoter, George Winter, Walter H. Wheeler, Leon Beskin, and Robert B. B. Moorman. 1591.

RIVERS

"Translatory Waves in Natural Channels." J. H. Wilkinson. (With Discussion.) 1203.

RUNOFF

"Flood Formulas Based on Drainage Basin Characteristics." (With Discussion.) 849.

"Storage and the Unit Hydrograph." C. O. Clark. (With Discussion.) 1419.

SEWAGE DISPOSAL

"Concentration of Sewage Sludge." A Symposium. 1557. Discussion: C. R. Velzy, and R. F. Goudey. 1579.

"Dewatering, Incineration, and Use of Sewage Sludge." A Symposium. 179. Discussion: A. L. Genter, Willem Rudolfs, Lloyd M. Johnson, Isador W. Mendelsohn, Langdon Pearse, C. R. Velzy, and George J. Schroepfer. 243.

SEWAGE TREATMENT

See SEWAGE DISPOSAL

SEWAGE WORKS

See SEWAGE DISPOSAL

SIMILARITY, THEORY OF

See MODELS

SOILS

"Geology in Highway Engineering." Marshall T. Huntting. (With Discussion.) 271.

"Military Airfields." A Symposium. (With Discussion.) 669.

"Pressure on the Lining of Circular Tunnels in Plastic Soils." D. P. Krynine. (With Discussion.) 1048.

"Simultaneous Consolidation of Contiguous Layers of Unlike Compressible Soils." Hamilton Gray. 1327. Discussion: Edward S. Barber, Jacob Feld, George W. Glick, and Hamilton Gray. 1345.

SPILLWAYS

"Construction of the Flow Net for Hydraulic Design." H. Alden Foster. (With Discussion.) 1237.

STRAIN

See STRESS AND STRAIN

STREAM FLOW

See WATER, FLOW OF, IN OPEN CHANNELS

STRENGTH OF MATERIALS

"Shear Effect on the Strength of Struts." S. Sergev. (With Discussion.) 391.

STRESS AND STRAIN

"Analysis of the General Two-Dimensional Framework." Yves Nubar. (With Discussion.) 905.

"Characteristic Redundants Used for Analyzing Statically Indeterminate Structures." John B. Wilbur. (With Discussion.) 1257.

"Rainbow Arch Bridge Over Niagara Gorge." A Symposium. (With Discussion.) 1.

"Shear and Bond Stresses in Reinforced Concrete." Stanley U. Benscoter and Samuel T. Logan. (With Discussion.) 599.

"Shear Effect on the Strength of Struts." S. Sergev. (With Discussion.) 391.

"Wedge-Shaped Structural Members Under Direct Stress and Bending." Robert B. B. Moorman. (With Discussion.) 1581.

STRUCTURES, THEORY OF

"Amplified Slope Deflection." Ralph W. Stewart. 1401. Discussion: Leon Beskin, John E. Goldberg, Camillo Weiss, William C. Spiker, Arthur B. McGee, Jaroslav Polivka, A. A. Eremin, and Ralph W. Stewart. 1408.

"Analysis of Rigid Frames by Superposition." David M. Wilson. 1489. Discussion: F. S. Merritt, Ralph W. Stewart, John E. Goldberg, A. W. Fischer, Leon Blog, Arthur B. McGee, Jaroslav Polivka, Odd Albert, W. C. Spiker, David B. Hall, A. A. Eremin, Victor R. Bergman, and David M. Wilson. 1509.

"Analysis of the General Two-Dimensional Framework." Yves Nubar. 905. Discussion: Leon Beskin, Jaroslav Polivka, A. A. Eremin, and Yves Nubar. 933.

"Characteristic Redundants Used for Analyzing Statically Indeterminate Structures." John B. Wilbur. 1257. Discussion: Charles A. Ellis, Ralph W. Stewart, Myle J. Holley, Jr., M. W. Huggins, Leon Beskin, Frederick S. Merritt, and John B. Wilbur. 1280.

"Design Constants for Beams with Nonsymmetrical Straight Haunches." August L. Ahlf. (With Discussion.) 1019.

"Rainbow Arch Bridge Over Niagara Gorge." A Symposium. (With Discussion.) 1.

"Wedge-Shaped Structural Members Under Direct Stress and Bending." Robert B. B. Moorman. (With Discussion.) 1581.

STRUTS

See COLUMNS

TORSION

"Rigidity and Aerodynamic Stability of Suspension Bridges." D. B. Steinman. (With Discussion.) 439.

TUNNEL LINING

"Pressure on the Lining of Circular Tunnels in Plastic Soils." D. P. Krynine. 1048. Discussion: M. A. Drucker, John M. Bird, and D. P. Krynine. 1066.

TURBINES (WATER)

"Erection of Hydraulic Turbines and Generators." Andrew M. Komora. 581. Discussion: Paul E. Gisiger, Frank I. Morgan, J. F. Roberts, D. E. Brainard, and Andrew M. Komora. 593.

WATER, FLOW OF, IN OPEN CHANNELS

"Coefficients for Velocity Distribution in Open-Channel Flow." William S. Eisenlohr, Jr. 633. Discussion: A. A. Kalinske, Edward H. Taylor, E. R. Van Driest, Hunter Rouse and John S. McNown, and William S. Eisenlohr, Jr. 645.

"Flood Formulas Based on Drainage Basin Characteristics." H. B. Kinnison and B. R. Colby. (With Discussion.) 849.

"Hydraulic Model Investigation of Miter Gate Operation." Maurice N. Amster. (With Discussion.) 1297.

"Translatory Waves in Natural Channels." J. H. Wilkinson. (With Discussion.) 1203.

WATER STORAGE

"Storage and the Unit Hydrograph." C. O. Clark. (With Discussion.) 1419.

WAVES

"Surges in Panama Canal Reproduced in Model." F. W. Edwards and Edward Soucek. (With Discussion.) 345.

"Translatory Waves in Natural Channels." J. H. Wilkinson. 1203. Discussion: Harold A. Weggel, W. M. Lansford, M. A. Churchill, and J. H. Wilkinson. 1226.

WIND

Critical wind velocities in bridge design. 505.

AUTHOR INDEX

ACKERMAN, ADOLPH J.

Cellular cofferdams. 1155.

AHERN, JEREMIAH

Memoir of. 1638.

AHLF, AUGUST L.

"Design Constants for Beams with Nonsymmetrical Straight Haunches." 1019.

ALBERT, ODD

Analysis by superposition. 1533.

ALECK, B. J.

Shear and bond. 607.

AMSTER, MAURICE N.

"Hydraulic Model Investigation of Miter Gate Operation." 1297.

AMMANN, O. H.
Rainbow Arch Bridge. 135.

ANDERSON, JAMES PENNELL
Memoir of. 1641.

ARLEDGE, A. R.
Repayment contracts. 388.

BAKER, ELISHA BROWN
Memoir of. 1864.

BALOG, LOUIS
Aerodynamic stability. 522.
Rainbow Arch Bridge. 137.

BARBER, EDWARD S.
Consolidation of soils. 1345.

BAYLEY, EDGAR ALCANDER
Memoir of. 1643.

BEAN, E. F.
Highway engineering. 305.

BEAN, ERNEST DANIEL
Memoir of. 1865.

BEAN, PAUL LEONARD
Memoir of. 1644.

BEARD, L. R.
Flood formulas. 882.

BEARDSLEY, JAMES WALLACE
Memoir of. 1647.

BEGIEN, RALPH NORMAN
Memoir of. 1649.

BELCHER, D. J.
Highway engineering. 316.

BELKNAP, WILLIAM ETHELBERT
Memoir of. 1653.

BENSCOTER, STANLEY U.
"Shear and Bond Stresses in Reinforced Concrete." 599.
Wedge-shaped members. 1593.

BERGMAN, VICTOR R.
Analysis by superposition. 1544.
Shear and bond. 619.

BERK, RALPH G.
Lock manifold experiments. 1393.

BERNHARD, R. K.
Aerodynamic stability. 485.

BERRY, EDWARD FRANKLIN
Memoir of. 1655.

BESKIN, LEON
Amplified slope deflection. 1408.
Characteristic redundants. 1284.
Rainbow Arch Bridge. 139.
Strength of struts. 404.
Two-dimensional framework. 933.
Wedge-shaped members. 1599.

BIRD, JOHN M.
Pressure in plastic soils. 1068.

BIRDSALL, BLAIR
Aerodynamic stability. 509.

BIXBY, DEANE FRANCIS
Memoir of. 1912.

BLACK, J. K.
Cutoff problems. 1003.

BLACKWELL, PAUL ALEXANDER
Memoir of. 1657.

BLAIR, CLARENCE MOORE
Memoir of. 1658.

BLECK, A. T.
Highway engineering. 305.

BLOG, LEON
Analysis by superposition. 1520.

BLOMGREN, ERIC EDWIN
Memoir of. 1660.

BOWEN, SHERMAN WORCESTER
Memoir of. 1661.

BOYLE, FRANK JOSEPH
Memoir of. 1866.

BRADSHAW, SAM WIGFALL
Memoir of. 1663.

BRAINARD, D. E.
Erection of turbines. 596.

BRATER, E. F.
Storage. 1455.

BRIGGS, BENJAMIN EDWARD
Memoir of. 1664.

BRODIE, ORRIN LAWRENCE
Memoir of. 1665.

BROGAN, THOMAS BYRNES
Memoir of. 1867.

BROSIUS, ALBERT MARSHALL
Memoir of. 1667.

BROWN, CARL B.
Highway engineering. 299.

BUCHANAN, WALTER JOHN
Memoir of. 1868.

BULL, ANDERS
Shear and bond. 616.

BURMISTER, D. M.
Military airfields. 809.

BURROUGHS, E. H.
Cutoff problems. 1005.

BUSSELL, JOHN EDWARD
Memoir of. 1913.

CARPENTER, EDWARD LARAMIE, JR.
Memoir of. 1869.

CARR, ROBERT PICKET
Memoir of. 1870.

CARSTARPHEN, FREDERICK CHARLES
Memoir of. 1668.

CASAGRANDE, A.
Military airfields. 780.

CARY, ALLEN S.
Highway engineering. 323.

CHADWICK, W. L.
Repayment contracts. 384.

CHURCHILL, M. A.
Translatory waves. 1229.

CLARK, C. O.
Flood formulas. 892.
"Storage and the Unit Hydrograph." 1419.

COCHRANE, VICTOR H.
Storage. 1461.

COLBURN, R. T.
Cellular cofferdams. 1136.

COLBY, B. R.
"Flood Formulas Based on Drainage Basin Characteristics." 849.

COLEMAN, JOHN FRANCIS
Memoir of. 1608.

COOLEY, MORTIMER ELWYN
Memoir of. 1619.

COPP, JOHN I.
"Rainbow Arch Bridge Over Niagara Gorge (Symposium): Fabrication and Erection Controls." 50.

COTTON, HARRY E.
Military airfields. 784.

COUTLEE, CHARLES ROBERT FORAN
Memoir of. 1670.

COWIN, JAMES
Memoir of. 1672.

COX, LEONARD MARTIN
Memoir of. 1673.

CRAWFORD, L. C.
Storage. 1456.

CROFOOT, DAVID WILSON
Memoir of. 1871.

CROSBY, IRVING B.
Highway engineering. 334.

CROSBY, W. W.
Highway engineering. 298.

CROSS, HARDY
Aerodynamic stability. 511.

CURRIE, CLARE HARMON
Memoir of. 1677.

DELIZ, LUIS AGUSTIN
Memoir of. 1872.

de MEY, EDOUARD JEAN BERNARD
Memoir of. 1679.

de VRIES, KARL
"Rainbow Arch Bridge Over Niagara Gorge (Symposium): Fabrication and Erection Controls." 50.

DeYOUNG, ISAAC
Miter gate operation. 1316.

Di GENOVA, EGIDIO O.
Rainbow Arch Bridge. 120.

DOELEMAN, HERMAN FRANCOIS
Memoir of. 1680.

DONALDSON, WELLINGTON
"Concentration of Sewage Sludge (Symposium): Concentration of Sewage Sludges, New York, N. Y." 1572.

DONNELLY, THOMAS LINUS KEITH
Memoir of. 1682.

DONOHUE, EDWARD BROWN
Memoir of. 1683.

DOWNING, CARL E.
Memoir of. 1873.

DOYNE, MAX HARRY
Memoir of. 1686.

DRUCKER, M. A.
Pressure in plastic soils. 1066.

DUNDAS, WILLIAM A.
"Dewatering, Incineration, and Use of Sewage Sludge (Symposium): Experience of Chicago, Ill., in the Preparation of Fertilizer." 218.

DURKEE, E. L.
"Rainbow Arch Bridge Over Niagara Gorge (Symposium): Erection of Steel Superstructure." 82.

EDWARDS, F. W.
Lock manifold experiments. 1385.
Miter gate operation. 1319.
"Surges in Panama Canal Reproduced in Model." 345.

EISENLOHR, WILLIAM S., JR.
"Coefficients for Velocity Distribution in Open-Channel Flow." 633.

EKBLAW, GEORGE E.
Highway engineering. 322.

ELDER, CLAY C.
"Original and Readjusted Repayment Contracts, Boulder Canyon Project." 363.

ELLIS, CHARLES A.
Characteristic redundants. 1280.

ELLSWORTH, SAMUEL MORRISON
Memoir of. 1687.

ENDERSBY, V. A.
Canal sections. 431.

ENO, FRANK HARVEY
Memoir of. 1690.

EREMIN, A. A.
Amplified slope deflection. 1415.
Analysis by superposition. 1542.
Highway engineering. 341.
Two-dimensional framework. 943.

EVANS, CHARLES DORMAN
Memoir of. 1874.

FARQUHARSON, F. B.
Aerodynamic stability. 492.

FAY, FREDERIC HAROLD
Memoir of. 1691.

FELD, JACOB
Consolidation of soils. 1349.
Highway engineering. 307.
Military airfields. 763.

FERGUSON, PHIL M.
Shear and bond. 622.
Wedge-shaped members. 1591.

FICKES, EDWIN STANTON
Memoir of. 1695.

FISCHER, A. W.
Analysis by superposition. 1514.

FISKE, HAROLD CHAMBERLAYNE
Memoir of. 1699.

FOCHT, LOUIS
Memoir of. 1876.

FORBES, HYDE
Highway engineering. 337.

FOSTER, H. ALDEN
"Construction of the Flow Net for Hydraulic Design." 1237.

FOX, PORTLAND P.
Cutoff problems. 997.

FRANCIS, HOWARD LEWIS
Memoir of. 1878.

FRIEDLAENDER, HERMANN
Aerodynamic stability. 512.

FUHRMAN, RALPH E.
"Dewatering, Incineration, and Use of Sewage Sludge (Symposium): Vacuum Filters Used in Primary Sludge Dewatering." 188.

GALVIN, CHARLES BROE
Memoir of. 1705.

GARRELTS, J. M.
"Rainbow Arch Bridge Over Niagara Gorge (Symposium): Design." 6.

GARRETT, JAMES MADISON
Memoir of. 1706.

GENTER, A. L.
Use of sewage sludge. 243.

GERIG, WILLIAM
Memoir of. 1708.

GERRISH, HERBERT THURSTON
Memoir of. 1711.

GISIGER, PAUL E.
Erection of turbines. 593.

GLENN, L. C.
Cutoff problems. 1004.

GLICK, GEORGE W.
Consolidation of soils. 1351.

GODDARD, LESLIE WARREN
Memoir of. 1712.

GOLDBERG, JOHN E.
Amplified slope deflection. 1410.
Analysis by superposition. 1512.

GOODRICH, C. M.
Rainbow Arch Bridge. 123.

GOUDEY, R. F.
"Concentration of Sewage Sludge (Symposium): Experience in Los Angeles, Calif." 1558.

GRAY, HAMILTON
"Simultaneous Consolidation of Contiguous Layers of Unlike Compressible Soils." 1327.

GREINER, JOHN EDWIN
Memoir of. 1625.

GREULICH, G. G.
Cellular cofferdams. 1120.
Military airfields. 758.

GRIFFITH, LAWRENCE
Memoir of. 1881.

GRINTER, L. E.
Shear and bond. 614.

GRONQUIST, C. H.
Aerodynamic stability. 490.
Rainbow Arch Bridge. 150.

HALL, DAVID B.
Analysis by superposition. 1540.
Strength of struts. 410.

HAMMOND, ALONZO JOHN
Memoir of. 1615.

HANSEN, PAUL
Memoir of. 1713.

HANSEN, RALPH
Military airfields. 812.

HARDESTY, SHORTRIDGE
"Rainbow Arch Bridge Over Niagara Gorge (Symposium): Design." 6.

HARRIS, CLINTON LEE
Memoir of. 1715.

HARRIS, ELMO GOLIGHTLY
Memoir of. 1717.

HASTINGS, BERTHOLD FRANCIS
Memoir of. 1720.

HASWELL, JOHN R.
Military airfields. 796.

HATHAWAY, GAIL A.
"Military Airfields (Symposium): Design of Drainage Facilities." 697.

HAYS, JAMES B.
"Unusual Cutoff Problems—Dams of the Tennessee Valley Authority (Symposium): Deep Solution Channel, Kentucky Dam, Kentucky." 948.

HEAD, JAMES RAYMOND
Memoir of. 1882.

HEDMAN, A. F.
Cellular cofferdams. 1145.

HEDRICK, I. G., JR.
"Rainbow Arch Bridge Over Niagara Gorge (Symposium): Design." 6.

HEILBRON, EMILE HENRY
Memoir of. 1722.

HENDERSON, WILLIAM JOHNSTON
Memoir of. 1883.

HENRY, MAXWELL
Memoir of. 1884.

HEYMANN, CURT JOSEPH
Memoir of. 1886.

HEWITT, GEORGE
Memoir of. 1724.

HIGGINS, GEORGE
Memoir of. 1725.

HILL, HIBBERT.
Military airfields. 758; 836.

HILT, FRED KEATING
Memoir of. 1728.

HINDS, JULIAN
Canal sections. 433.

HIRSCHTHAL, M.
Rainbow Arch Bridge. 159.

HITTELL, JOHN BENJAMIN
Memoir of. 1730.

HODGDON, JOHN BREWSTER
Memoir of. 1887.

HOLLEY, MYLE J., JR.
Characteristic redundants. 1283.

HOLT, MARSHALL
Strength of struts. 408.

HOLTON, HARRY HAMILTON
Memoir of. 1914.

HORONJEFF, ROBERT
Military airfields. 766.

HOSIG, I. B.
Haunched beams. 1044.

HOWARD, JOHN WARDWELL
Memoir of. 1889.

HOWE, J. W.
Flow net. 1253.

HOWLAND, W. E.
Military airfields. 738.

HOWSON, ELMER THOMAS
Memoir of. 1731.

HUDSON, HAROLD WALTON
Memoir of. 1735.

HUGGINS, M. W.
Characteristic redundants. 1284.

HUNTTING, MARSHALL T.
"Geology in Highway Engineering." 271.

IRWIN, RAYMOND L.
Military airfields. 752.

IVES, HOWARD CHAPIN
Memoir of. 1892.

IZZARD, C. F.
Military airfields. 827.

JAMESON, ROBERT O.
Haunched beams. 1037.

JAMESON, WILLIAM H.
"Rainbow Arch Bridge Over Niagara Gorge (Symposium): Fabrication and Erection Controls." 50.

JARVIS, CLARENCE S.
Flood formulas. 877.
Flow net. 1254.
Military airfields. 768.

JASKAR, ADE E.
Highway engineering. 323.

JENKINS, DAVID S.
Military airfields. 743.

JERVIS, WILLIAM H.
Military airfields. 797.

JOACHIMSON, MARTIN
Memoir of. 1737.

JOHNSON, FRANK MINITREE
Memoir of. 1738.

JOHNSON, LLOYD M.
Use of sewage sludge. 254.

JOHNSTONE, DON
Storage. 1474.

JONES, JONATHAN
"Rainbow Arch Bridge Over Niagara Gorge (Symposium): Fabrication and Erection Controls." 50.

JUDD, FRANK RHYMAL
Memoir of. 1741.

KAHN, JULIUS
Memoir of. 1742.

KALINSKE, A. A.
Velocity distribution. 645.

KEEFER, C. E.
"Concentration of Sewage Sludge (Symposium): Back River Sewage Works, Baltimore, Md." 1562.

KEENE, PHILIP
Highway engineering. 306.

KELLOGG, F. H.
Highway engineering. 300.

KENNEDY, ROBERT E.
Storage. 1459.

KING, WINFIELD SCOTT
  Memoir of. 1748.
KINNISON, H. B.
  "Flood Formulas Based on Drainage Basin Characteristics." 849.
KIRKBRIDE, WALTER HAROLD
  Memoir of. 1748.
KOMORA, ANDREW M.
  "Erection of Hydraulic Turbines and Generators." 581.
KORNFELD, ALFRED EPHRAIM
  Memoir of. 1921.
KREISS, ROBERT F.
  Lock manifold experiments. 1389.
KRYNINE, D. P.
  Cellular cofferdams. 1175.
  Highway engineering. 328.
  "Pressure on the Lining of Circular Tunnels in Plastic Soils." 1048.
KUMMER, FREDERIC ARNOLD
  Memoir of. 1751.
LANSFORD, W. M.
  Lock manifold experiments. 1388.
  Translatory waves. 1227.
LAUTZ, MARTIN W.
  Cellular cofferdams. 1165.
LEGGET, ROBERT F.
  Circular cofferdams. 1173.
  Highway engineering. 332.
LEWIS, EDWARD ROWLAND
  Memoir of. 1755.
LEYPOLDT, HARRY
  Canal surges. 356.
LINSLEY, RAY K., JR.
  Storage. 1452.
LOGAN, SAMUEL T.
  "Shear and Bond Stresses in Reinforced Concrete." 599.
LOHMANN, FREDERICK CHARLES
  Memoir of. 1757.
LOOK, MOSES JEROME
  Memoir of. 1758.
MACK, EDWARD RESOLVED
  Memoir of. 1893.

MACKINTOSH, CHARLES
Rainbow Arch Bridge. 121.

McCONAUGHY, D. C.
Canal sections. 435.

McCOY, BYRON O.
Flood formulas. 891.

McELDOWNEY, ROBERT, JR.
"Rainbow Arch Bridge Over Niagara Gorge (Symposium): Moments in Flexible Two-Hinged Arches." 35.

McFADDEN, JAMES ALEXANDER
Memoir of. 1894.

McGEE, ARTHUR B.
Amplified slope deflection. 1412.
Analysis by superposition. 1522.

McLAUGHLIN, C. P.
"Dewatering, Incineration, and Use of Sewage Sludge (Symposium): Experience of Chicago, Ill., in the Preparation of Fertilizer." 218.

McMANUS, JOHN HERBERT
Memoir of. 1760.

McNAUGHER, DAVID WHITE
Memoir of. 1762.

McNOWN, JOHN S.
Lock manifold experiments. 1378.
Velocity distribution. 651.

McQUIRE, D. D.
Military airfields. 776.

MARSHALL, H. E.
Highway engineering. 309.

MEAD, LAWRENCE M., JR.
"Rainbow Arch Bridge Over Niagara Gorge (Symposium): Moments in Flexible Two-Hinged Arches." 35.

MENDELSOHN, ISADOR W.
Use of sewage sludge. 255.

MENSCH, L. J.
Military airfields. 831.
Rainbow Arch Bridge. 128.

MERRITT, FREDERICK S.
Analysis by superposition. 1509.
Characteristic redundants. 1291.

MEYER, OTTO H.
Storage. 1448.

MILLARD, CHARLES STERLING
Memoir of. 1763.

MILLER, EUGENE CLARE
Memoir of. 1915.

MILLER, J. S.
Cellular cofferdams. 1170.

MINIHAN, JOHN FRANCIS
Memoir of. 1895.

MITCHELL, WILLIAM JOHNSTON
Memoir of. 1897.

MONEYMAKER, BERLEN C.
Cutoff problems. 995.
Highway engineering. 302.

MOORE, EGBERT JESSUP
Memoir of. 1764.

MOORE, R. WOODWARD
Highway engineering. 320.

MOORMAN, ROBERT B. B.
"Wedge-Shaped Structural Members Under Direct Stress and Bending." 1581.

MORGAN, FRANK I.
Erection of turbines. 594.

MULLIS, IRA B.
Military airfields. 826.

MURRAY, D. O.
Military airfields. 790.

MYRON, JOHN PATRICK
Memoir of. 1766.

NEUMAN, DAVID LEONARD
Memoir of. 1768.

NEWELL, JOSEPH PETTIS
Memoir of. 1770.

NEWMAN, JAMES B., JR.
Military airfields. 734.

NORRIS, WALTER HENRY
Memoir of. 1775.

NUBAR, YVES
"Analysis of the General Two-Dimensional Framework." 905.

OBERT, RUSSELL MELVIN
Memoir of. 1898.

OLDER, CLIFFORD
Memoir of. 1777.

ORD, M. J.
Flood formulas. 886.

ORR, DAVID KIRK
Memoir of. 1778.

ORROK, GEORGE ALEXANDER
Memoir of. 1779.

OWEN, ARTHUR EDMUND
Memoir of. 1783.

PACKSHAW, SAVILE
Cellular cofferdams. 1161.

PALEN, ARCHIBALD E.
Memoir of. 1785.

PARKER, THEODORE BISSELL
Memoir of. 1786.

PARMER, E. D.
Military airfields. 776

PARR, HARRY ALFRED
Memoir of. 1916.

PEABODY, DEAN, JR.
Shear and bond. 610.

PEARSE, LANGDON
Use of sewage sludge. 261.

PENNOYER, RAYMOND P.
Cellular cofferdams. 1124.

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PIETRKOWSKI, J.
Lock manifold experiments. 1377.

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Memoir of. 1792.

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Memoir of. 1794.

PITTMAN, H. V.
Military airfields. 814.

PNEUMAN, FRED ASHLEY
Memoir of. 1795.

POLIVKA, JAROSLAV
Amplified slope deflection. 1414.
Analysis by superposition. 1529.
Cellular cofferdams. 1178.
Two-dimensional framework. 940.

POSEY, C. J.
Canal sections. 436.
Canal surges. 358.

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Memoir of. 1899.

POWELL, RALPH W.
Lock manifold experiments. 1394.

PRINGLE, THOMAS B.
Military airfields. 792.

PROCTOR, RALPH R.
Military airfields. 799.

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Memoir of. 1803.

RAISMAN, AARON ISRAEL
Memoir of. 1806.

RAPP, WILLIAM G.
Rainbow Arch Bridge. 139.

REA, DALE H.
Haunched beams. 1042.

REISSNER, H.
Aerodynamic stability. 501.

RICHARDSON, EDWARD ADAMS
Aerodynamic stability. 476.

RICHARDSON, FREDERICK HOSEA
Memoir of. 1807.

RICKETTS, ALLAN TOWNSHEND
Memoir of. 1810.

ROBBINS, FRANKLIN HENRY
Memoir of. 1811.

ROBERTS, J. F.
Erection of turbines. 595.

ROBERTSON, J. M.
Aerodynamic stability. 487.

ROUSE, HUNTER
Velocity distribution. 651.

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"Concentration of Sewage Sludge (Symposium): Some Results in the State of New Jersey." 1565.
Use of sewage sludge. 251

RUDOLPH, WILLIAM E.
Military airfields. 751.

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Memoir of. 1813.

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Military airfields. 818.

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Memoir of. 1815.

SERGEV, S.
"Shear Effect on the Strength of Struts." 391.

SHANLEY, F. R.
Shear and bond. 607.

SHAW, HOWARD EARLE, JR.
Memoir of. 1918.

SHERMAN, JOHN
Haunched beams. 1041.

SHERMAN, L. K.
Storage. 1449.

SMITH, FREDERICK C.
Aerodynamic stability. 512.
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SNYDER, FRANKLIN F.
Storage. 1465.
SOUCEK, EDWARD
"Lock Manifold Experiments." 1357.
Miter gate operation. 1314.
"Surges in Panama Canal Reproduced in Model." 345.
SPARHAWK, GEORGE FRANCIS
Memoir of. 1821.
SPIKER, WILLIAM C.
Amplified slope deflection. 1411.
Analysis by superposition. 1539.
SPOFFORD, C. M.
Rainbow Arch Bridge. 127.
STANTON, THOMAS E.
Military airfields. 737.
STAPE, GEORGE PHILIP ALEXANDER
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STEINMAN, D. B.
"Rigidity and Aerodynamic Stability of Suspension Bridges." 439.
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STERN, GEORGE
Rainbow Arch Bridge. 163.
STEVENS, CHARLES HENRY
Memoir of. 1824.
STEVENS, J. C.
"Private Enterprise." 1605.
STEWART, RALPH W.
"Amplified Slope Deflection." 1401.
Analysis by superposition. 1510.
Characteristic redundants. 1282.

STONE, WILLARD WILBERFORCE
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STRATTON, JAMES H.
"Military Airfields (Symposium): Construction and Design Problems." 670.

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Memoir of. 1835.

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Memoir of. 1836.

SWEET, JAMES S.
Storage. 1447.

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Velocity distribution. 646.

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Memoir of. 1920.

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TERZAGHI, KARL
Highway engineering. 324.
"Stability and Stiffness of Cellular Cofferdams." 1083.

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THOMSON, T. KENNARD
Rainbow Arch Bridge. 125.

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"Rainbow Arch Bridge Over Niagara Gorge (Symposium): Moments in Flexible Two-Hinged Arches." 35.

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Miter gate operation. 1320.

TSAGARIS, DEAN P.
Cellular cofferdams. 1133.

TURNBULL, W. J.
Military airfields. 797.

TURNER, HOWARD M.
Flood formulas. 888.
Storage. 1471.